

## Research Article

# Subspace Based Adaptive Beamforming Algorithm with Interference Plus Noise Covariance Matrix Reconstruction

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As we all know, the model mismatch, primarily when the desired signal exists in the training data, or when the sample data is used for training, will seriously affect algorithm performance. This paper combines the subspace algorithm based on direction of arrival (DOA) estimation with the adaptive beamforming. It proposes a reconstruction algorithm based on the interference plus noise covariance matrix (INCM). Firstly, the eigenvector of the desired signal is obtained according to the eigenvalue decomposition of the subspace algorithm, and the eigenvector is used as the estimated value of the desired signal steering vector (SV). Then the INCM is reconstructed according to the estimated parameters to remove the adverse effect of the desired signal component on the beamformer. Finally, the estimated desired signal SV and the reconstructed INCM are used to calculate the weight. Compared with the previous work, the proposed algorithm not only improves the performance of the adaptive beamformer but also dramatically reduces the complexity. Simulation experiment results show the effectiveness and robustness of the proposed beamforming algorithm.

## 1. Introduction

Adaptive beamforming uses adaptive spatial filtering and interference suppression to enhance the desired signal and suppress the interference signal to make the desired signal output more powerful. At present, it has been widely used and rapidly developed in fields such as radar, sonar, exploration, seismology, radio astronomy, wireless communication, acoustics, medical imaging, biomedical engineering, and other fields [1–3]. Without the required directional knowledge, beamforming based on blind source separation uses the transverse mode of the signal to recover the desired signal, such as wireless communication. On the contrary, when the desired signal direction of arrival (DOA) is available, the Capon adaptive beamformer can output the most prominent signal to interference plus noise ratio (SINR) in the array, and it is the best spatial filter [4–6]. However, it is well known that when there is a model mismatch, especially when there is a desired signal component in the sampling covariance matrix, it has a severe impact on its performance. In this case, the performance of

Capon beamformer is seriously degraded. In addition, in practical applications, due to the limited number of training samples, it is very difficult to perfectly estimate the required interference plus noise covariance matrix (INCM). Therefore, the adaptive beamforming method must be robust to the uncertainty of covariance matrix [7–10].

The diagonal loading (DL) algorithm is a classic and easy to implement robust adaptive beamforming technology [11]. The main idea is to add a weighted identity matrix before the inversion of the received signal sample covariance matrix of the array and improve the robustness of the beamformer by reducing the dispersion of the small eigenvalues corresponding to the noise. Then, scholars proposed many improved algorithms to improve the performance and robustness of the algorithm [12–16]. However, there is no clear theoretical guide to choose the best loading factor in different situations. However, in the practical application environment, the mismatch degree of the steering vector cannot be accurately known in advance, so it is difficult to determine the best loading factor for the best beam performance, which can only be

determined by the designer subjectively. Therefore, this kind of algorithm is minimal to improve the performance of adaptive beamformer.

The main idea of beamforming technology based on feature subspace projection [17] is to project the steering vector (SV) of the desired signal on the signal-interference subspace and then use the projected SV to replace the original assumed SV of the desired signal for beamforming. However, the performance of the algorithm will be seriously degraded at low input signal to noise ratio (SNR), because when the input SNR is low, the signal-interference subspace and noise subspace will be aliased, resulting in serious subspace exchange.

Since 2003, researchers have proposed a series of robust adaptive beamforming (RAB) algorithms based on uncertain desired signal vector sets to further improve the robustness of RAB algorithms. They are beamformer based on worst-case performance optimization [18], beamformer based on covariance fitting [19], robust Capon beamformer (RCB) [20], and robust minimum variance beamformer (RMVB) [21]. This kind of algorithms is unified in the worst-case performance optimization (WCPO) criterion, and the worst-performance optimization can also be seen as a kind of DL technology. However, the worst case is difficult to occur in the actual environment, and we usually know the upper bound of the norm of the mismatch vector. Therefore, the worst-case optimization in this case is still suboptimal [22–24].

Unfortunately, the above methods improve performance by estimating the sampling covariance matrix of the signal received by the array, rather than removing the desired signal. Since the sampled data always has the desired signal component, self-destructive phenomena will occur when the input SNR is high, and the output performance of the above-mentioned beamforming algorithm will be reduced. In recent years, a robust adaptive beamforming algorithm based on the reconstruction INCM idea has been proposed in order to achieve the goal of removing the desired signal component in the sampling covariance matrix as much as possible [22]. In practical application scenarios, the number of sampling snapshots is generally very limited. In the case of limited snapshots, the sample covariance matrix will have a significant error compared with the ideal covariance matrix. At this time, the performance and robustness of beamformer will be seriously affected. Therefore, matrix reconstruction technology uses the reconstructed INCM to replace the estimated covariance matrix and can not only eliminate the desired signal component but also reduce the covariance matrix mismatch caused by small snapshots, thereby significantly improving the performance of the beamformer. In 2012, The RAB algorithm based on the INCM reconstruction idea was first proposed in [19]. The main idea of the algorithm is to reconstruct the INCM by integrating the reconstruction expression constructed by Capon power spectrum in the interference signal sector and then use the reconstructed INCM to construct a quadratic constrained quadratic programming (QCQP) to estimate the SV of the desired signal. As soon as the reconstruction method of INCM is proposed, it has attracted wide attention of scholars. Then researchers have proposed many improved algorithms [25–31], which reduce the complexity of the

algorithm, further improve the output performance of beamformer, and improve its robustness against array structure error [32]. In reference [25], using the sparsity of source DOA, INCM is improved to be reconstructed through a compressed sensing problem. According to the definition of INCM, the algorithm reconstructs the INCM directly from the covariance term of interference signal and noise, which avoids the process of integration and improves the accuracy of reconstruction. In reference [27], by extending the integral to high-dimensional annular interval, the ability of reconstruction algorithm to deal with array calibration error is improved.

Although the aforementioned matrix reconstruction algorithm greatly improves the performance of beamforming, it needs to be integrated in the calculation process, thereby increasing the complexity of beamforming. To solve this problem, this paper proposes a practical low complexity INCM reconstruction algorithm based on subspace. In this paper, since the eigenvector of the desired signal is in the same subspace as the SV, the eigenvector of the desired signal can be used to estimate the SV. The reconstructed INCM is composed of the SV and power of the signal. Therefore, the reconstructed covariance matrix removes the signal component, which reduces the problem of signal self-fading. Previous reconstruction methods are different. In this paper, we estimate the SV of the desired signal and reconstruct the INCM directly based on subspace. Because there is no complex operation such as integral, the complexity of the algorithm is low. Simulation experiments show that the performance of the algorithm is close to the ideal output SINR in a wide input SNR range. At the same time, the complexity of the algorithm is low.

The main work and contributions of this paper can be summarized as follows:

- (i) In this paper, a subspace based adaptive beamforming algorithm for reconstruction of INCM is proposed. The algorithm realizes the organic combination with the subspace algorithm and uses the idea of reconstructing INCM to calculate the weight vector. Therefore, compared with the general algorithm, the output SINR performance of the algorithm is greatly improved.
- (ii) Compared with other reconstruction algorithms, the complexity of the proposed algorithm is greatly reduced. Because the algorithm uses the parameters generated by the DOA estimation process to reconstruct the INCM, it does not need complex operations such as integration, so the algorithm has low computational complexity.
- (iii) We derive the corresponding relationship between eigenvalue and power. And the deduced relationship is verified by mathematical formula. Thus, the eigenvalues obtained by eigendecomposition of covariance matrix can be used as the power estimation of signal and noise.
- (iv) We obtained the one-to-one correspondence between the direction vector and the power by combining the formulas. According to the corresponding relationship between direction vector and

eigenvector, eigenvector and eigenvalue, and eigenvalue and power, the corresponding relationship between direction vector and power is obtained.

In this article, we use uppercase and lowercase bold letters to represent matrices and vectors, respectively. Given a matrix  $\mathbf{A}$ , we use  $\mathbf{A}^T$ ,  $\mathbf{A}^H$ , and  $\mathbf{A}^*$  to denote the transpose, the Hermitian transpose, and the conjugate of  $\mathbf{A}$ , respectively.  $\mathbf{E}(\cdot)$  is used to express statistical expectations.

We organize the rest of this article as follows: Section 2 describes the signal model. Section 3 estimates the steering vector and reconstructs the INCM. Section 4 gives the algorithm simulation experiments under different conditions. Section 5 summarizes the work of this paper.

## 2. Preliminaries

In this section, we will briefly review the signal model and minimum variance distortionless response (MVDR) adaptive beamforming, which are needed in this paper [21].

*2.1. The Signal Model.* A uniform linear array (ULA) composed of  $Q$  array elements is considered. At the same time,  $J$  far-field narrow-band signals of wavelength are incident on the array ( $Q > J$ ). It is assumed that each element in the array is an omnidirectional antenna and is isotropic, and there are no factors such as channel inconsistency and mutual coupling between elements. The array element spacing  $d$  is set to half of the wavelength of the incident signal ( $d = \lambda/2$ ). Assuming that the first element is the reference element, the output of the physical array at time  $t$  can be expressed as

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, 2, \dots, L, \quad (1)$$

where  $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_J(t)]^T \in \mathbb{C}^{J \times 1}$  is the incident signal vector and  $\mathbf{n}(t) \in \mathbb{C}^{Q \times 1}$  is a Gaussian white noise with mean value of 0 and variance of  $\sigma_n^2$ , which is independent of each other.  $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_J)] \in \mathbb{C}^{Q \times J}$  is the array manifold matrix. The direction vector of the  $k$ th signal is defined as

$$\mathbf{a}(\theta_k) = [e^{-j\pi d_1 \sin \theta_k}, e^{-j\pi d_2 \sin \theta_k}, \dots, e^{-j\pi d_Q \sin \theta_k}]^T, \quad (2)$$

where  $d_k$  is the distance between the  $i$ th element and the reference element and  $d_k = k - 1$ ,  $\theta_k$  is the azimuth of the  $k$ th signal ( $k = 1, 2, \dots, Q$ ).

Under ideal conditions, the received signal covariance matrix of the array can be expressed as

$$\begin{aligned} \mathbf{R}_{xx} &= \mathbf{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} \\ &= \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma_n^2\mathbf{I}_Q. \end{aligned} \quad (3)$$

The signal covariance matrix is defined as  $\mathbf{R}_{ss} = \mathbf{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_J^2\}$ , and  $\sigma_j^2$  ( $j = 1, 2, \dots, J$ ) represents the power of the  $j$ th signal.  $\sigma_n^2$  is the power of the noise, and  $\mathbf{I}_Q \in \mathbb{C}^{Q \times Q}$  is the identity matrix whose main diagonal is 1.

*2.2. Robust Adaptive Beamforming.* In this part, the desired signal is defined as  $s_1(t)$ , and the remaining  $J - 1$  signals are defined as interference signals. The output formula of the array can be determined by the following formula:

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{d}(t) + \mathbf{i}(t) + \mathbf{n}(t) \\ &= \mathbf{a}(\theta_1)s_1(t) + \sum_{i=2}^J \mathbf{a}(\theta_i)s_i(t) + \mathbf{n}(t), \end{aligned} \quad (4)$$

where  $\mathbf{d}(t) = \mathbf{a}(\theta_1)s_1(t)$  is the desired signal vector and  $\mathbf{i}(t) = \sum_{i=2}^J \mathbf{a}(\theta_i)s_i(t)$  represents the vector of the interference signals. The output of the beamformer can be expressed as

$$\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t), \quad (5)$$

where  $\mathbf{w} \in \mathbb{C}^{Q \times 1}$  is the weight vector.

We define SNR as the ratio of desired signal to noise power:

$$\text{SNR} = \frac{\sigma_1^2}{\sigma_n^2}. \quad (6)$$

SINR is the power ratio of desired signal to interference signals plus noise. The output SINR of the array is used to evaluate the performance of the beamformer, which is defined as follows:

$$\text{SINR} = \frac{\sigma_1^2 |\mathbf{w}^H \mathbf{a}(\theta_1)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \quad (7)$$

where  $\sigma_1^2$  is the power of the desired signal and INCM is  $\mathbf{R}_{i+n}$ , which is given by the following formula:

$$\begin{aligned} \mathbf{R}_{i+n} &= \mathbf{E}\{[i(t) + n(t)][i(t) + n(t)]^H\} \\ &= \sum_{i=2}^J \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I}_Q. \end{aligned} \quad (8)$$

Here,  $\mathbf{E}(\cdot)$  is used to express statistical expectations.

Among the various beamforming criteria proposed, the output SINR maximization is the most popular one. The definition is as follows:

$$\max_{\mathbf{w}} \frac{\sigma_1^2 |\mathbf{w}^H \mathbf{a}(\theta_1)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}. \quad (9)$$

We can achieve the largest output SINR by solving the MVDR problem [33]. We can construct the MVDR beamformer by solving the following constraints:

$$\begin{aligned} \min_{\mathbf{w}} & \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \\ \text{s.t.} & \mathbf{w}^H \mathbf{a}(\theta_1) = 1, \end{aligned} \quad (10)$$

where the solution is given by

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_1)}{\mathbf{a}^H(\theta_1) \mathbf{R}_{i+n}^{-1} \mathbf{a}(\theta_1)}, \quad (11)$$

where  $\mathbf{w}_{\text{opt}}$  is also called Capon beamformer. It is difficult to get the covariance matrix of interference noise  $\mathbf{R}_{i+n}$  directly in practical application. Therefore, the sampling covariance matrix  $\mathbf{R}_x$  can be used instead:

$$\mathbf{R}_x = \frac{1}{L} \sum_{t=1}^L \mathbf{x}(t)\mathbf{x}^H(t), \quad (12)$$

where  $L$  denotes the number of snapshots.

It should be noted that when  $L$  is very small, the gap between  $\mathbf{R}_x$  and  $\mathbf{R}_{i+n}$  is relatively large, which will make the desired signal be suppressed as interference; with the improvement of SNR, the proportion of required signal components will become larger, which will lead to serious signal self-zeroing phenomenon, especially in the case of high SNR. Therefore, it is necessary to remove the desired part when estimating the covariance matrix. In addition, the performance of the adaptive beamforming algorithm is very sensitive to the direction vector, so we need to accurately estimate the direction of arrival of the signal to accurately reconstruct the direction vector.

### 3. The Proposed Algorithm

In this section, a new adaptive beamforming algorithm is proposed to estimate the direction vector and power of the signal to construct the weight vector. According to formula (11), the weight vector of beamformer based on MVDR criterion is determined by the INCM  $\mathbf{R}_{i+n}$  and the desired signal SV  $\mathbf{a}(\theta_1)$ . Therefore, the idea of adaptive beamforming algorithm is to make full use of parameters of subspace algorithm to accurately reconstruct the core parameters of these two beamforming devices, so that the designed beamformer performance approaches the theoretical value.

First, the subspace algorithm is used to estimate the DOA of the signals, and the maximum correlation coefficient is used to correct the estimated direction vector of the desired signal. Then the power of the signal and noise is estimated through the eigenvalues to reconstruct INCM. Finally, the weight vector is calculated according to the reconstructed INCM and estimated SV.

Before the research, we need to take the number of signals as the prior information of the algorithm. Here we use Minimum Description Length (MDL) criterion to get the number of signals [34]. Because it is not the focus of this paper, we will not repeat it here.

*3.1. Estimation of Direction Vector and SV.* The adaptive beamforming algorithm in this paper is based on subspace algorithm. Multiple signal classification (MUSIC) is one of the classical subspace algorithms, so MUSIC algorithm is introduced in this part, which provides the necessary basis for the estimation of SV and the reconstruction of INCM.

According to  $J$  received signal vectors, the estimated value of sampling covariance matrix as shown in formula (12) is obtained. The covariance matrix obtained by formula (12) is decomposed into eigenvalues:

$$\mathbf{R}_x = \mathbf{U}_S \boldsymbol{\Sigma}_S \mathbf{U}_S^H + \mathbf{U}_N \boldsymbol{\Sigma}_N \mathbf{U}_N^H, \quad (13)$$

where  $\mathbf{U}_S$  is the signal subspace and  $\boldsymbol{\Sigma}_S$  is a matrix with diagonal elements corresponding to the eigenvalues of  $J$  signals and the remaining elements are 0.  $\mathbf{U}_N$  is the noise subspace, and  $\boldsymbol{\Sigma}_N$  is the diagonal matrix of  $Q - J$  small eigenvalues. From the definition of eigenvalue and eigenvector, we know

$$\mathbf{R}_x \mathbf{U}_N = \boldsymbol{\Sigma}_N \mathbf{U}_N = \sigma_n^2 \mathbf{U}_N. \quad (14)$$

And from formula (12) we can also get

$$\mathbf{R}_x \mathbf{U}_N = \mathbf{A}(\theta) \mathbf{R}_S \mathbf{A}^H(\theta) \mathbf{U}_N + \sigma_n^2 \mathbf{U}_N. \quad (15)$$

According to formulas (14) and (15), we can get

$$\mathbf{A}(\theta) \mathbf{R}_S \mathbf{A}^H(\theta) \mathbf{U}_N = 0. \quad (16)$$

Since the signal covariance matrix  $\mathbf{R}_S$  is full rank and nonsingular, its inverse exists. Equation (16) can be rewritten as

$$\mathbf{A}^H(\theta) \mathbf{U}_N = 0. \quad (17)$$

This means that the column vector in matrix  $\mathbf{A}$  and the noise corresponding eigenvector are orthogonal to each other, so there are some advantages:

$$\mathbf{U}_N^H \mathbf{a}(\theta_k) = 0, \quad k = 1, 2, \dots, J. \quad (18)$$

Since the eigenvectors corresponding to the noise and direction vector are orthogonal to each other, the spatial spectrum function of the array is obtained as follows:

$$\mathbf{P}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{U}_N \mathbf{U}_N^H \mathbf{a}(\theta)}. \quad (19)$$

DOA  $\theta_k$  ( $k = 1, 2, \dots, J$ ) of the desired signal and interference signal can be obtained by searching the peak [35, 36]. After obtaining the DOA of the signal, the manifold matrix  $\mathbf{A}_1$  is reconstructed by formula (2):

$$\mathbf{A}_1 = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_J]. \quad (20)$$

From formula (20), we can get the SV of desired signal  $\mathbf{a}_1$ . However, due to the correlation between the noise and the signals, an error will occur in the estimated SV of desired signal. Therefore, it is necessary to correct the desired signal SV. Formula (13) can be written as

$$\mathbf{R}_x = \sum_{i=1}^Q \lambda_i \mathbf{e}_i \mathbf{e}_i^H = \mathbf{U}_S \boldsymbol{\Sigma}_S \mathbf{U}_S^H + \mathbf{U}_N \boldsymbol{\Sigma}_N \mathbf{U}_N^H. \quad (21)$$

Among them,  $\lambda_i$  ( $i = 1, 2, \dots, Q$ ) is the feature value of  $\mathbf{R}_x$  and arranged from large to small, and  $\mathbf{e}_i$  is the eigenvector corresponding to the feature value.

$$\mathbf{U}_S = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \dots, \mathbf{e}_J], \quad (22)$$

where  $\mathbf{U}_S$  is the signal subspace. We know that the eigenvector and the desired signal SV are in the same space, so replacing the estimated desired signal SV  $\mathbf{a}_1$  with the

eigenvector will achieve better performance. Since the direction vectors of different signals are orthogonal to each other, we have

$$\mathbf{e}_j^H \mathbf{a}(\theta_i) = \begin{cases} \sqrt{Q}, & i = j, \\ 0, & i \neq j. \end{cases} \quad (23)$$

Only when the eigenvector and the direction vector correspond, the inner product between them can get the maximum value. Therefore, we define the correlation coefficient to find the eigenvector corresponding to the desired signal SV  $\mathbf{a}_1$  estimated by formula (20) [37]. Assuming that the corresponding eigenvector is  $\mathbf{e}_s$ , the correlation coefficient can be expressed as follows:

$$\begin{aligned} \text{cor}(\mathbf{e}_s, \mathbf{a}_1) &= \max_{\mathbf{e}_i} |\text{cor}(\mathbf{e}_s, \mathbf{a}_1)| \\ &= \max_{\mathbf{e}_i} \frac{|\mathbf{e}_i^H \mathbf{a}_1|}{\|\mathbf{e}_i\| \|\mathbf{a}_1\|}. \end{aligned} \quad (24)$$

By substituting the  $J$  eigenvectors obtained from formula (22) into (24), we can obtain  $J$  correlation coefficients. Due to the maximum correlation between the SV and the eigenvector of the desired signal, we can get the eigenvector  $\mathbf{e}_s$  of the desired signal. We assume that the SV of the desired signal is  $\mathbf{a}_s$ . Substituting  $\mathbf{a}_s$  and  $\mathbf{e}_s$  into formula (23) can be rewritten as

$$\mathbf{e}_s^H \mathbf{a}_s = \sqrt{Q}. \quad (25)$$

Then we can get

$$\mathbf{a}_s = \sqrt{Q} \mathbf{e}_s. \quad (26)$$

**3.2. Reconstruction of INCM.** We can write formula (21) as

$$\mathbf{R}_x = \sum_{j=1}^Q \lambda_j \mathbf{e}_j \mathbf{e}_j^H = \sum_{i=1}^J \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I}_Q. \quad (27)$$

It can be seen from formula (27) that the eigenvalue of the covariance matrix eigenvalue decomposition has a certain corresponding relationship with the signal power. It is available to transform formula (27):

$$\begin{aligned} \lambda_j &= \mathbf{e}_j^H \mathbf{R}_x \mathbf{e}_j \\ &= \mathbf{e}_j^H \left( \sum_{i=1}^J \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I}_Q \right) \mathbf{e}_j \\ &= \sum_{i=1}^J \sigma_i^2 \mathbf{e}_j^H \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) \mathbf{e}_j + \sigma_n^2. \end{aligned} \quad (28)$$

By substituting formula (23) into formula (28), we can get

$$\lambda_j = \begin{cases} Q\sigma_j^2 + \sigma_n^2, & j \leq J, \\ \sigma_n^2, & J < j \leq Q. \end{cases} \quad (29)$$

Since the first  $J$  eigenvalues correspond to signal power, the following  $Q - J$  eigenvalues correspond to noise power. Therefore, the small eigenvalue of the matrix is the noise power. In practice, in order to obtain higher accuracy, we average the noise power:

$$\bar{\sigma}_n^2 = \frac{1}{Q-J} \sum_{j=J+1}^Q \lambda_j. \quad (30)$$

By substituting formula (30) into formula (29), we can get

$$\lambda_j = Q\sigma_j^2 + \bar{\sigma}_n^2, \quad j \leq J. \quad (31)$$

Therefore, we use the large eigenvalue to subtract the small eigenvalue corresponding to the noise power and then divide it by the number of array elements  $Q$  to estimate the power of the interference signal:

$$\tilde{\sigma}_i^2 = \frac{(\lambda_i - \bar{\sigma}_n^2)}{Q}, \quad i = 1, 2, \dots, J. \quad (32)$$

In order to prove that the power estimation result we derive is correct, we verify it through the following derivation. Simultaneous formulas (2) and (27) can be obtained:

$$\mathbf{R}_x = \begin{bmatrix} G & a_{12} & a_{13} & \cdots & a_{1Q} \\ a_{21} & G & a_{23} & \cdots & a_{2Q} \\ a_{31} & a_{32} & G & \cdots & a_{3Q} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{Q1} & a_{Q2} & a_{Q3} & \cdots & G \end{bmatrix}, \quad (33)$$

where  $G = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_J^2 + \sigma_n^2$  and  $a_{ij}$  ( $i = 1, 2, \dots, Q; j = 1, 2, \dots, Q$ ) is the array element. The trace of the matrix can be expressed as

$$\text{tr}(\mathbf{R}_x) = QG. \quad (34)$$

**Theorem 1.** *The trace of the matrix is equal to the sum of the eigenvalues of the matrix.*

In this paper, let  $\lambda = \sum_{j=1}^Q \lambda_j$ , and then we have  $\lambda = \text{tr}(\mathbf{R}_x)$ . Accumulate the eigenvalues of formula (29) to get

$$\lambda = Q \sum_{i=1}^J \sigma_i^2 + Q\sigma_n^2 = QG. \quad (35)$$

From formulas (34) and (35), we know that the estimated power is correct. However, there is a problem: we cannot determine the one-to-one correspondence between the direction of arrival and the power of the desired signal and interference signal. Therefore, we define the correlation coefficient:

$$f = \left| \mathbf{a}^H(\theta) \mathbf{U}_S \mathbf{U}_S^H \mathbf{a}(\theta) \right|, \quad (36)$$

where we substitute the eigenvector  $\mathbf{e}_i$  into  $\mathbf{U}_S$  and  $\mathbf{a}(\theta)$  is the direction vector composed of (2) and (19). When the eigenvectors  $\mathbf{e}_i$  correspond to the direction vectors  $\mathbf{a}(\theta)$ , their relations are not orthogonal to each other, so the coherence

coefficient is the largest. From formula (21) we get the corresponding relationship between the eigenvector and the eigenvalue, so we get the corresponding relationship between the eigenvalue and the direction of arrival. From formula (32), we get the corresponding relationship between eigenvalue and power, and finally we get the corresponding relationship between power and direction of arrival.

The estimated power and direction of arrival are substituted into (8) reconfigurable INCM as

$$\tilde{R}_{i+n} = \sum_{i=2}^J \tilde{\sigma}_i^2 \mathbf{a}_i \mathbf{a}_i^H + \tilde{\sigma}_n^2 \mathbf{I}_Q. \quad (37)$$

Finally, by substituting the desired signal SV  $\mathbf{a}_s$  of formula (26) and the reconstructed INCM  $\tilde{R}_{i+n}$  of formula (37) into (11), the weight vector of the proposed algorithm can be expressed as

$$\tilde{\mathbf{w}}_{\text{opt}} = \frac{\tilde{R}_{i+n}^{-1} \mathbf{a}_s}{\mathbf{a}_s^H \tilde{R}_{i+n}^{-1} \mathbf{a}_s}. \quad (38)$$

The implementation steps of the proposed algorithm are summarized in Algorithm 1. The complexity analysis is as follows. Since the performance of the proposed algorithm and reconstruction algorithm is much higher than other algorithms, we only discuss the complexity of the proposed algorithm and reconstruction algorithm. The complexity of estimating the steering vector of the desired signal is  $\mathcal{O}((J+2)Q)$ , the complexity of obtaining the corresponding relationship between direction vector and power is  $\mathcal{O}(2JQ)$ , the complexity of reconstructing INCM is  $\mathcal{O}((J-1)Q)$ , and the complexity of calculating weight vector is  $\mathcal{O}((Q^2+2Q+1)Q)$ . Therefore, the complexity of the proposed algorithm is  $\mathcal{O}((Q^2+2Q+4J+2)Q)$ . The computational load of this method is mainly caused by the calculation of weight vector process in step 4, and all MVDR beamforming algorithms need to calculate the weight vector. And the complexity of the reconstruction algorithm is  $\mathcal{O}(Q^2N)$  ( $N \gg Q$ ), where  $N$  is the number of samples in  $\tilde{\Theta}$ . From the above analysis, it can be seen that the complexity of the algorithm is much lower than that of the reconstruction algorithm. Table 1 shows the comparison of computational complexity of the SMI [7], DLSMI [11], WORST-CASE [18], RECONSTRUCTION [25], and the proposed algorithm.

#### 4. Simulation Results

Our simulation experiment considers an omnidirectional uniform linear array with  $Q = 10$  elements, and the spacing between the elements is half a wavelength. It is assumed that the array receives three signals, among which the desired signal comes from direction  $\theta_s = 5^\circ$ , the interference signal comes from directions  $\theta_1 = -10^\circ$ , and  $\theta_2 = 20^\circ$ , and the incident signals are independent of each other. The interference to noise ratio (INR) of each element is fixed to 30 dB. The additive noise is modeled as a complex Gaussian random process with a mean value of 0 and a variance of 1. When comparing the output SINR of the adaptive beamforming algorithm with the input SNR, the number of

snapshots is set to  $L = 30$ . When comparing the curve of the output SINR with the number of snapshots, the SNR of the array element is assumed to be 20 dB. For each scenario, we conduct 500 Monte-Carlo experiments.

The proposed algorithm will be compared with the traditional sampling covariance matrix inverse beamforming algorithm [7], diagonal loading algorithm [11], worst-case performance optimization [18], and covariance matrix reconstruction [25] based beamforming algorithm in the output SINR performance. In the figure, these algorithms use "PROPOSED", "SMI", "DLSMI", "WORST-CASE", and "RECONSTRUCTION" as the illustration.

Our simulation shows a diagonal loaded beamformer with a loading factor of 10 and a worst case performance optimized beamformer with a parameter of  $0.3Q$ . In the reconstruction based beamformer and the proposed beamformer, we assume that the angular region of the desired signal is  $\Theta = [0^\circ, 10^\circ]$ ; therefore,  $\tilde{\Theta} = [-90^\circ, 0^\circ] \cup (10^\circ, 90^\circ]$ .

*Simulation 1.* Exactly known signal SV.

In the first simulation example, the true direction of the desired signal is considered to be consistent with the assumed desired signal direction; that is, it is assumed that the SV of all signals are accurately known. Under this simulation condition, the performance of the beamformer will still be affected when the input SNR is high and the number of snapshots is limited. Figure 1 depicts the relationship between the output SINR and the input SNR of the beamformer under the fixed snapshot number  $L = 30$ . Obviously, the proposed beamformer always achieves near optimal performance in a wide input SNR range. This is because the proposed algorithm eliminates the desired signal by reconstructing INCM, thus avoiding the self-destructive phenomenon of the signal. However, the other beamformers do not remove the desired signal in the covariance matrix except reconstruction algorithms. Therefore, when the SNR is high, there will be different degrees of performance degradation. Although the performance of the proposed algorithm is only slightly higher than that of covariance matrix reconstruction algorithm, the complexity of the algorithm is greatly reduced. Figure 2 shows the relationship between the output SINR and the number of snapshots when the beamformer has a fixed SNR = 20 dB. The simulation results show that the proposed algorithm has excellent performance under ideal conditions, and the output SINR is higher than all other tested algorithms.

*Simulation 2.* Fixed look direction mismatch.

This simulation experiment studies the output SINR performance when there is a look direction mismatch in beamforming. The real direction of the desired signal is selected as  $\theta_s = 5^\circ$ , while the assumed direction of the desired signal is selected as  $8^\circ$ , and the corresponding observation direction of  $3^\circ$  does not match. Figure 3 shows the relationship between the output SINR and the input SNR of the beamformer, where the number of snapshots remains constant at  $L = 30$ . It can be observed that the SINR of the proposed beamformer is only slightly lower than that of

Step 1: The DOA results are obtained using the subspace algorithm in (19), and the direction vector of the signals is reconstructed by formula (20)

Step 2: The SV of the desired signal is corrected by formula (24), and the corrected desired signal SV is obtained using (26)

Step 3: The correspondence between eigenvalues and power is derived using (28) and (29), and (30) and (32) are used to calculate the power of noise and interference signals respectively. Then we proved that the corresponding relationship between the derived eigenvalues and power is correct

Step 4: The corresponding relationship between power and DOA is obtained by (21), (32) and (36)

Step 5: The interference plus noise covariance matrix  $\tilde{R}_{i+n}$  is reconstructed by the direction vector and power using (37)

Step 6: The estimated steering vector and the reconstructed INCM are used to calculate the weight vector  $\tilde{w}_{opt}$  in (38)

ALGORITHM 1: The proposed algorithm steps.

TABLE 1: Comparison of computational complexity.

Algorithms	Computational complexity
The SMI algorithm [7]	$\mathcal{O}((Q^2 + 2Q + 1)Q)$
The DLSMI algorithm [11]	$\mathcal{O}((Q^2 + 2Q + 1)Q)$
The WORST-CASE algorithm [18]	$\mathcal{O}((5Q^2 + 2Q + 1)Q)$
The algorithm [25]	$\mathcal{O}(Q^2N) (N \gg Q)$
The proposed algorithm	$\mathcal{O}((Q^2 + 2Q + 4J + 2)Q)$

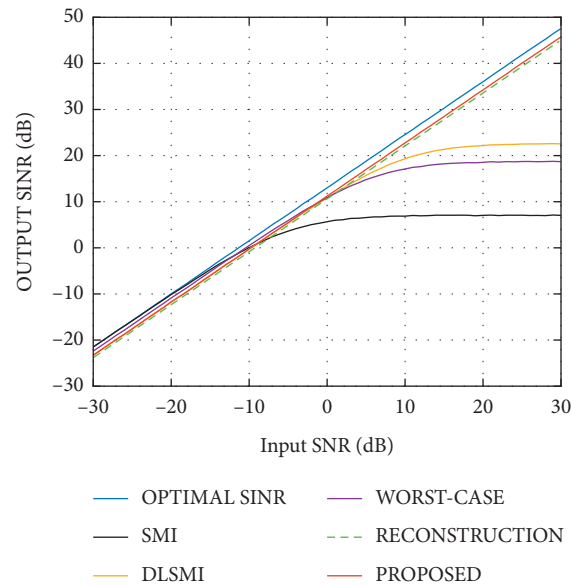


FIGURE 1: Exactly known signal SV: output SINR versus the input SNR.

covariance matrix reconstruction beamformer, but the complexity of the proposed algorithm is much lower than that of covariance matrix reconstruction beamformer. With the increase of the input SNR, the covariance matrix reconstruction and the proposed beamformer can realize that the output SINR changes linearly with the input SNR, because they remove the desired signal in the sampling covariance matrix. The performance of SMI, DLSMI, and worst case beamformers at high SNR degrades due to the mismatching of SV due to the mismatching of direction. Assuming  $\text{SNR} = 20$  dB, the relationship between the performance curve and the number of snapshots is shown in Figure 4. The results show that, under the condition of fixed look direction mismatch, the performance of the proposed

beamformer is slightly worse than that of covariance matrix reconstruction beamformer and is significantly better than other comparable beamformers. The simulation results prove the effectiveness of the proposed algorithm when there is a fixed look direction mismatch. Although the performance is slightly lower than covariance matrix reconstruction, the complexity of the algorithm is much lower than that of the algorithms in [24].

### Simulation 3. Random SV mismatch.

In this simulation experiment, we consider the impact of an uncertain interference in the desired signal on the performance of the beamformer, that is, the performance of the

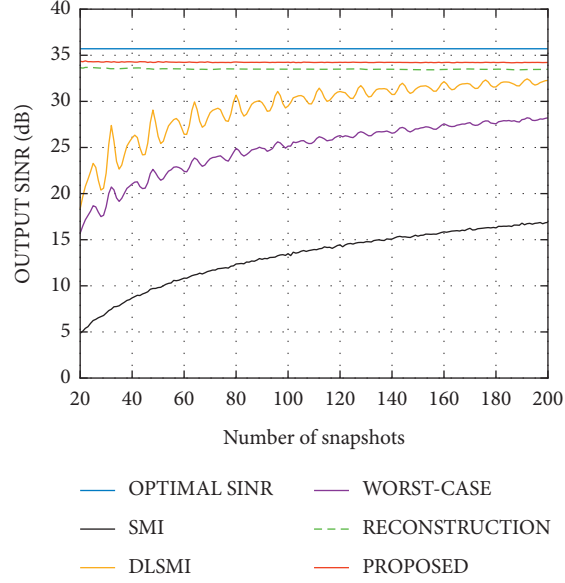


FIGURE 2: Exactly known signal SV: output SINR versus the number of snapshots.

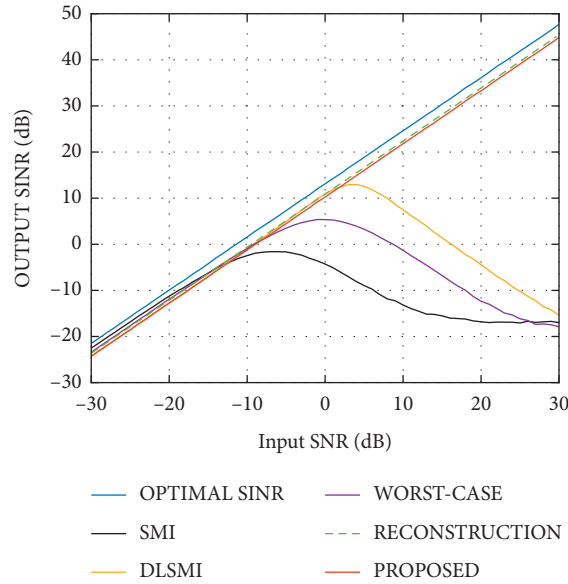


FIGURE 3: Fixed look direction mismatch: output SINR versus the input SNR.

algorithm when the random SV mismatch. In this case, the real SV can be expressed as

$$\mathbf{a} = \mathbf{a}(\theta_1) + \boldsymbol{\delta}, \quad (39)$$

where  $\mathbf{a}(\theta_1)$  is a hypothetical SV with direction of  $\theta_1$  and  $\boldsymbol{\delta}$  is a random SV mismatch generated by the uncertainty set, as follows:

$$\boldsymbol{\delta} = \frac{\alpha}{\sqrt{Q}} [e^{j\phi_1}, e^{j\phi_2}, \dots, e^{j\phi_M}]^T, \quad (40)$$

where  $\alpha$  is the norm of  $\boldsymbol{\delta}$ , which is generated randomly from one run to another in  $[0, \sqrt{0.5}]$ .  $\phi_u$  ( $u = 1, 2, \dots, Q$ ) represents the coordinate of  $\boldsymbol{\delta}$  independently generated from  $[0, 2\pi]$  in the  $u$ th run. Figures 5 and 6 show the relationship between the input SNR, the number of snapshots, and the output SINR. Obviously, as the input SNR increases, the output SINR of the proposed algorithm is always close to the ideal value, and convergence requires less snapshots. The simulation results prove that the algorithm is effective in the case of random SV mismatch. In addition, we can see that the performance of the



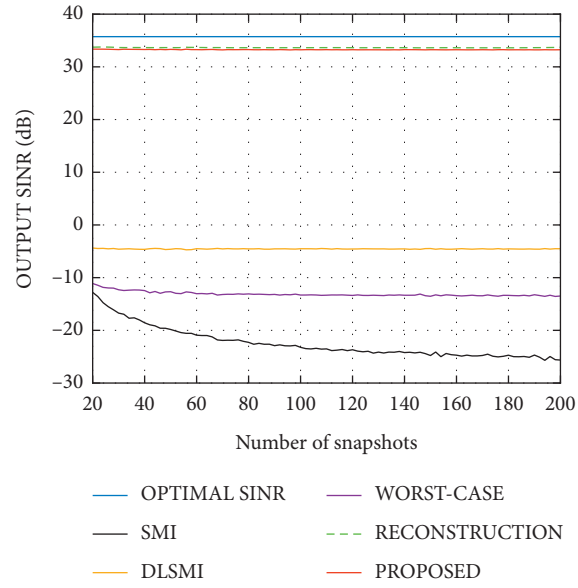


FIGURE 4: Fixed look direction mismatch: output SINR versus the number of snapshots.

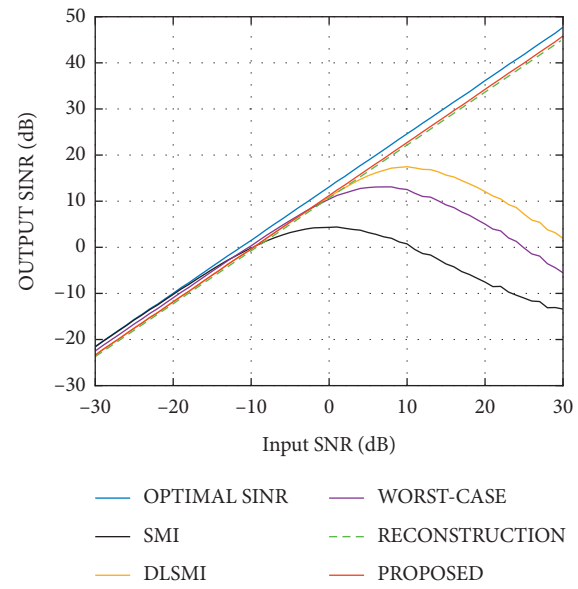


FIGURE 5: Random SV mismatch: output SINR versus the input SNR.

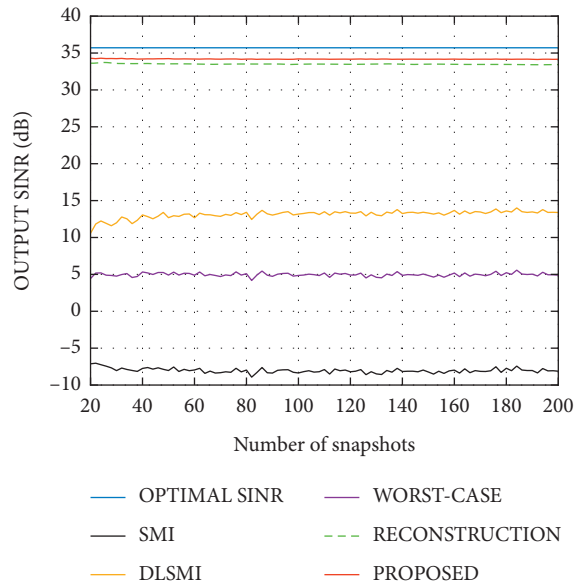


FIGURE 6: Random SV mismatch:output SINR versus the number of snapshots.

beamformer is significantly better than other test beamformers, and we conclude that the beamformer is more robust to random SV mismatches.

## 5. Conclusion

In this paper, an effective and low complexity adaptive beamforming algorithm based on subspace algorithm of DOA estimation was proposed. The eigenvalue and signal subspace were obtained by decomposing the eigenvalues of covariance matrix, and DOA estimation of desired signal and interference signal is obtained by spectral peak search. Since the desired signal eigenvector in the signal subspace has the greatest correlation with the desired signal SV, we can use the desired signal eigenvector to estimate its SV. Then, we used the small eigenvalue of eigenvalue decomposition to replace the noise power, the result of subtracting the small eigenvalue from the large eigenvalue to replace the power of the desired signal and the interference signal, and the spectral peak search result to replace the direction of the desired and interference signal, and reconstructed the INCM according to the corresponding relationship between the power and the direction of the interference signal determined by the correlation coefficient. Finally, we use the estimated required signal SV and the reconstructed INCM to calculate the weight vector of the proposed algorithm. We prove the effectiveness of the algorithm through simulation experiments. Compared with the existing algorithms, this method has lower computational complexity while maintaining close to the optimal performance.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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