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Research Article

New Weighted Lomax (NWL) Distribution with Applications to Real and Simulated Data

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The rationale of the paper is to present a new probability distribution that can model both the monotonic and nonmonotonic hazard rate shapes and to increase their flexibility among other probability distributions available in the literature. The proposed probability distribution is called the New Weighted Lomax (NWL) distribution. Various statistical properties have been studied including with the estimation of the unknown parameters. To achieve the basic objectives, applications of NWL are presented by means of two real-life data sets as well as a simulated data. It is verified that NWL performs well in both monotonic and nonmonotonic hazard rate function than the Lomax (L), Power Lomax (PL), Exponential Lomax (EL), and Weibull Lomax (WL) distribution.

1. Introduction

From the last few years, it is usual practice to make a contribution to the existing theory of probability due to its wide application in different fields of sciences, for example, in reliability analysis, signal processing, survival analysis, and so on. Due to the advanced computer technology and statistical software, many researchers have developed new probability distributions to improve the goodness of fit measures. For example, Lemonte et al. [1] introduced the additive Weibull distribution by adding the two Weibull distributions, Al-Aqtash et al. [2] presented the new family of distribution with a logit function, Aldeni et al. [3, 4] explored by employing the quantile function, Alzaatreh et al. investigated the gamma-normal distribution [5], and references [6–11] presented new probability distributions using transmutation technique. Alzaghal et al. [12] introduced an exponentiated T-X family of distribution. Extended Lomax distribution was introduced by Lemonte and Cordeiro [13].

The fundamental goal of this paper is to present a new life-time probability distribution that improves the flexibility of the model and also provides a better fit in monotonic and nonmonotonic hazard function than other existing probability models.

1.1. Lomax Distribution. Let a positive random variable be $Y \sim L(\alpha, \beta)$; the CDF is given by

$$F(y) = 1 - \left[1 + \left(\frac{y}{\beta}\right)\right]^{-\alpha}, \quad y > 0 \text{ and } \alpha, \beta > 0. \tag{1}$$

The PDF related to (1) is defined as

$$f(y)=\frac{\alpha}{\beta}\left[1+\left(\frac{y}{\beta}\right)\right]^{-(\alpha+1)}, \quad y>0. \tag{2}$$

Equation (2) is one of the right skewed distributions and has been applied by many researchers to real data sets found in business science, engineering, computer, survival analysis, and some others.

To increase the flexibility of the model, modification of this distribution has been done by many researchers; for example, Ashour and Eltehiwy [10] introduced transmuted Lomax distribution, Ashour and Eltehiwy [11] transmuted Exponentiated Lomax distribution, Lemonte and Cordeiro [13] explored the extended Lomax, Cordeiro et al. [14] defined gamma-Lomax, Ghitany et al. [15] presented Marshall–Olkin extended Lomax and discussed their applications to censored data, Al-Zahrani and Sagor [16] modified Poisson Lomax distribution. El-Bassiouny et al. [17] defined Exponential Lomax, and Shams [18] presented Kumaraswamy-generalized Lomax distribution. Ijaz et al. [19] worked on the Flexible Lomax distribution, ZeinEldin et al. [20] presented Alpha power transformed inverse Lomax distribution, Almetwally and Gamal [21] defined Alpha Power Inverse Lomax, ul Haq et al. [22] discussed Marshall–Olkin power Lomax distribution, Lemonte and Cordeiro [13] explored an Extended Lomax, and Cordeiro et al. [14] worked on gamma-Lomax distribution. Kilany [23] worked on the Weighted Lomax distribution, and Ahmad et al. [24] derived a Length-Biased Weighted Lomax distribution. For other modifications, refer [25–38].

1.2. A New Weighted Lomax (NWL) Distribution. In this paper, we developed a highly flexible Lomax distribution by replacing a Lomax random variable y by e^y and using the inner product of $(\beta/(\beta+1))$. The suggested distribution is called a New Weighted Lomax distribution or in short NWL distribution. The shape and scale parameter of this distribution are α and β, respectively. The proposed distribution in this paper provides more flexibility and provides the best fit than other existing distributions.

Definition 1. . Considering a continuous random variable Y, the CDF of a New Weighted Lomax distribution is defined by

$$F(y)=1-\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)\right]^{-\alpha}, \quad y>0 \text{ and } \alpha,\beta>0. \tag{3}$$

The corresponding PDF is given by

$$F(y)=\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha}e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha+1)}, \quad \text{where } \alpha,\beta>0. \tag{4}$$

Figure 1 shows the behavior of the PDF and CDF of the NWL(α,β) distribution.

Figure 1 shows the probability and distribution function of the New Weighted Lomax distribution with different parameter values.

2. Survival and Hazard Function

The survival function of NWL(α,β) is defined by the expression as under

$$S(y)=P(Y>y), \quad y>0. \tag{5}$$

Using (3), we get

$$S(y)=1-\left\{1-\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)\right]^{-\alpha}\right\}=\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)\right]^{-\alpha}. \tag{6}$$

The hazard function or failure rate of a NWL distribution is defined by using the formula as under $h(y)=(f(y)/(1-F(y)))$; recalling (3) and (4), we have

$$h(y;\alpha,\beta)=\frac{\alpha}{\alpha\beta}e^y\left(1+\frac{e^y}{\beta}\right)^{-1}; \quad y>0,\ \alpha,\beta>0. \tag{7}$$

Figure 2 delineates the capability of the suggested distribution to model the nonmonotonically hazard function.

3. Mode

The mode or a point by which the probability density function of a NWL will reach to its maximum point is defined as

$$f'(y)=\frac{d}{dy}\left[\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha}e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha+1)}\right]. \tag{8}$$

In order to find the maximum point, we have to equate this expression equal to zero and then solve for Y, and we get

$$(\alpha e^y-\beta)=0. \tag{9}$$

The mode is obtained as follows:

$$y_m=\log\left(\frac{\beta}{\alpha}\right). \tag{10}$$

4. Quantile and Median Function

The QF is the real solution to the inverse cumulative distribution function of NWL distribution having two parameters. This function will help in providing the median but also in generating random data from NWL distribution. The QF is defined as $F(y)=u$ where $u\sim U(0,1)$.

By using equation (3), we have

$$1-\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)\right]^{-\alpha}=u. \tag{11}$$

When we solve the above function for a variable Y, we obtained

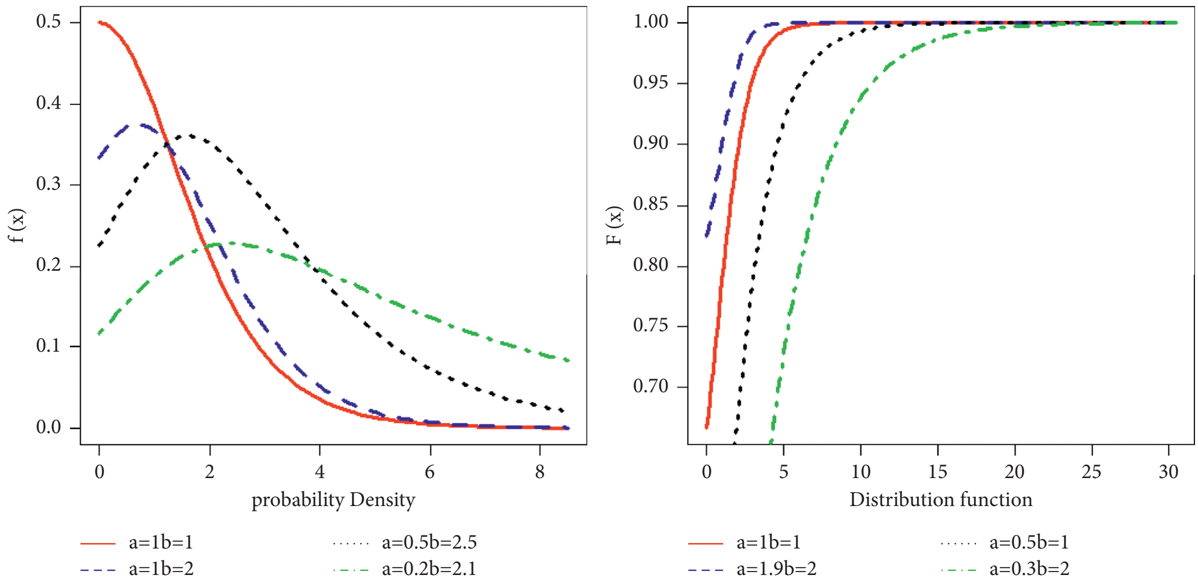

Figure 1: The PDF and CDF of the NWL(α, β) distribution.

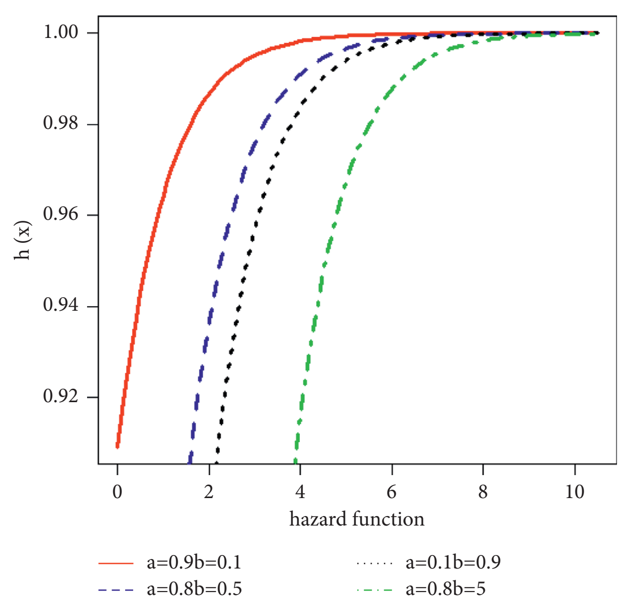

Figure 2: The hazard function of NWL(α, β) distribution.

$$y_Q = \log\left(\frac{\beta}{\left((\beta/(\beta+1))^{\alpha}(1-u)\right)^{1/\alpha}} - \beta\right). \quad (12)$$

Now, if we are interested to find the median of the data understudy, we can easily measure the median value using the above equation by just placing $u = 0.5$. Hence, the median function is obtained as follows:

$$y_M = \log\left(\frac{\beta}{\left((\beta/(\beta+1))^{\alpha}(0.5)\right)^{1/\alpha}} - \beta\right). \quad (13)$$

5. Bowley Skewness (*S*) and Moors Kurtosis (*K*)

The mathematical equation of the Bowley Skewness and Moors Kurtosis [39, 40] is given by

$$S_K = \frac{Q(3/4) + Q(1/4) - 2Q(2/4)}{Q(3/4) - Q(1/4)},$$
$$K_M = \frac{Q(7/8) + Q(3/8) - Q(5/8) - Q(1/8)}{Q(3/4) - Q(1/4)}, \quad (14)$$

where Q represents different quartile values. The numerical values of Skewness and Kurtosis using different parameter values are given in Table 1.

6. Order Statistics

If $Y_1, Y_2, Y_3, \ldots, Y_n$ are ordered variables, then the minimum (1st) and maximum (nth) CDF of the order statistics of NWL(α, β) distribution are defined by

$$F_{Y(1)}(y) = 1 - [1 - F(y)]^n,$$
$$F_{Y(n)}(y) = [F(y)]^n. \quad (15)$$

The corresponding PDF is defined by

$$f_{Y(1)}(y) = n[1 - F(y)]^{n-1} f(y),$$
$$f_{Y(n)}(y) = n[F(y)]^{n-1} f(y). \quad (16)$$

Hence, the CDF and PDF of the 1st and nth order statistics of a new WL, respectively, take the following form:

$$F_{Y(1)}(y) = 1 - \left[\left(\frac{\beta}{\beta+1}\right)\left(1 + \frac{e^y}{\beta}\right)^{-\alpha}\right]^n,$$
$$F_{Y(n)}(y) = \left[\left(\frac{\beta}{\beta+1}\right)\left(1 + \frac{e^y}{\beta}\right)^{-\alpha}\right]^n,$$
$$f_{(1;n)}(y_i) = \frac{n!}{(n-1)!}\frac{\alpha}{\alpha\beta}\left(\frac{\beta}{\beta+1}\right)^{\alpha} e^y \left(1 + \frac{e^y}{\beta}\right)^{-(\alpha+1)} \left(\left[\left(\frac{\beta}{\beta+1}\right)\left(1 + \frac{e^y}{\beta}\right)\right]^{-\alpha}\right)^{n-1},$$
$$f_{(n;n)}(y_i) = \frac{n!}{(n-1)!}\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha} e^y \left(1 + \frac{e^y}{\beta}\right)^{-(\alpha+1)} \left(1 - \left[\left(\frac{\beta}{\beta+1}\right)\left(1 + \frac{e^y}{\beta}\right)\right]^{-\alpha}\right)^{n-1}. \quad (17)$$

7. Parameter Estimation

In statistical inference, the estimation of the unknown parameters of the model is an important phase. In general, the parameters are unknown constant; we obtain their representative value through sample data. Under this section, we have considered the following likelihood function to estimation the parameters of NWL distribution:

$$L = \prod_{i=1}^{n} \left(\frac{\alpha}{\alpha\beta}\left(\frac{\beta}{\beta+1}\right)^{\alpha} e^y \left(1 + \frac{e^y}{\beta}\right)^{-(\alpha+1)}\right). \quad (18)$$

After applying the log function, we obtain

$$l = n\log\left(\frac{\alpha}{\alpha\beta}\left(\frac{\beta}{\beta+1}\right)^{\alpha}\right) + \sum y_i - (\alpha+1)\log\left(1 + \frac{e^{\sum y_i}}{\beta}\right), \quad (19)$$

where α and β are estimated by partially differentiating (19) with respect to α and β and will give the following results:

$$\frac{dl}{d\alpha} = \frac{dl}{d\alpha}\left(n\log\left(\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha}\right) + \sum y_i - (\alpha+1)\log\left(1 + \frac{e^{\sum y_i}}{\beta}\right)\right), \quad (20)$$

which finally becomes

$$= n\left(\frac{1}{\alpha} + \log\left(\frac{\beta+1}{\beta}\right) - \log\beta\right) - \sum\left(1 + \frac{e^{y_i}}{\beta}\right). \quad (21)$$

Now, differentiating (19) with respect to β, we have

$$\frac{dl}{d\beta} = \frac{dl}{d\beta}\left(n\log\left(\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha}\right) + \sum y_i - (\alpha+1)\log\left(1 + \frac{e^{\sum y_i}}{\beta}\right)\right),$$
$$\frac{dl}{d\beta} = n\left(-\frac{\alpha(\beta+2)}{\beta(\beta+1)}\right) + \sum\left(\frac{(\alpha+1) + e^{y_i}}{\beta^2}\right). \quad (22)$$

Table 1: Skewness and Kurtosis of NWL(α, β).

α	B	S	K
0.9	0.6	0.21263546	1.262644
0.5	0.5	0.24844780	1.298883
0.2	0.4	0.26177711	1.306311
0.4	0.3	0.25919498	1.305287
0.6	0.2	0.25555346	1.301593
1.1	2.1	0.12763767	1.217210
1.2	2.2	0.11767883	1.210175
1.3	2.3	0.10862779	1.204149
1.4	2.4	0.10036278	1.198988
1.5	2.5	0.09278102	1.194565

Since the two expressions (21) and (22) are not in closed form, we can obtain the asymptotic confidence bounds for the population parameter of a new WL distribution. To achieve the asymptotic confidence bounds, we need the second time partial derivative of the parameters, and we have

$$\frac{\mathrm{d}l}{\mathrm{d}\alpha^2} = I_{11} = n\left(-\frac{1}{\alpha^2}\right) + 0,$$
$$\frac{\mathrm{d}l}{\mathrm{d}\beta^2} = I_{22} = n\left(-\frac{\beta+2}{\beta(\beta+1)}\right) + \sum\left(\frac{e^{y_i}}{\beta^2}\right), \tag{23}$$
$$\frac{\mathrm{d}l}{\mathrm{d}\alpha\beta} = I_{12} = n\left(-\frac{1}{\left(\beta+\beta^2\right)} - \frac{1}{\beta}\right) + \sum\left(0 - \frac{e^{y_i}}{\beta^2}\right).$$

The information matrix is then obtained as

$$I = -\begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}. \tag{24}$$

The approximated variance-covariance matrix is defined as

$$V = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}^{-1}. \tag{25}$$

The approximated ml estimates are given by

$$\hat{v} = \begin{bmatrix} \hat{I}_{11} & \hat{I}_{12} \\ \hat{I}_{21} & \hat{I}_{22} \end{bmatrix}^{-1}. \tag{26}$$

Using (26), we can easily obtain the $(1-\gamma)$ 100% confidence bounds for the unknown parameters α and β in the following forms:

$$\alpha \pm Z_{\gamma/2}\sqrt{\operatorname{var}(\alpha)},$$
$$\beta \pm Z_{\gamma/2}\sqrt{\operatorname{var}(\beta)}. \tag{27}$$

8. Mean Residual Life (MRL)

In reliability analysis or survival analysis, the mean residual life is also an important aspect of the probability model. The MRL is used to measure the remaining mean life of an object given that the object has survived until the time y. Let a random variable y represent the life expectancy of an object, then the MRL is define as follows.

$M_{\mathrm{NWL}}(y) = E(((Y-y)/Y) > y)$; it can be expressed as

$$= \frac{1}{S(y;\alpha,\beta)}\int_y^\infty t f(t;\alpha,\beta)\mathrm{d}t - y, \tag{28}$$

where

$$S(Y;\alpha,\beta) = \left(\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^{y_i}}{\beta}\right)\right)^{-\alpha}. \tag{29}$$

By employing these functions in (28), we get

$$\int_y^\infty t f(t;\alpha,\beta)\mathrm{d}t = \int_y^\infty t\frac{\alpha}{\alpha\beta}\left(\frac{\beta+1}{\beta}\right)^\alpha e^t\left(1+\frac{e^t}{\beta}\right)^{-(\alpha+1)}\mathrm{d}t$$
$$= \frac{-t\alpha\beta}{\alpha^2\beta}\left(\frac{\beta+1}{\beta}\right)^\alpha\left(1+\frac{e^y}{\beta}\right)^{-\alpha}. \tag{30}$$

Replacing (6) and (30) result in (28), the result of the MRL is obtained:

$$M_{\mathrm{NWL}}(y) = \frac{1}{((\beta/(\beta+1))(1+(e^{y_i}/\beta)))^{-\alpha}}\frac{-t\alpha\beta}{\alpha^2\beta}\left(\frac{\beta+1}{\beta}\right)^\alpha\left(1+\frac{e^y}{\beta}\right)^{-\alpha} - y = \frac{-t\alpha\beta}{\alpha^2\beta} - y. \tag{31}$$

9. Stress Strength Parameter

Let us consider Y_1 and Y_2 as the two IRV which follow a new WL distribution with parameters (α_1, β) and (α_2, β), then the stress strength of the New Weighted Lomax distribution is defined by the following expression:

$$
\begin{aligned}
S_{\text{stress,strength}}(y) &= \int_0^\infty f_1(y)F_2(y)\mathrm{d}y \\
&= \int_0^\infty \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1} e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\left(1-\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)^{-\alpha_2}\right]\right)\mathrm{d}y \\
&= \int_0^\infty \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1} e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\mathrm{d}y - \int_0^\infty \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1} e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\left(\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)^{-\alpha_2}\right]\right)\mathrm{d}y.
\end{aligned} \tag{32}
$$

The solution to the first integral function in the above equation is given by

$$
\begin{aligned}
&= \int_0^\infty \frac{1}{\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1} e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\mathrm{d}y \\
&= \left(\frac{\beta+1}{\beta}\right)^{\alpha_1}\int_0^\infty \frac{1}{\beta}e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\mathrm{d}y = \frac{1}{\alpha_1}.
\end{aligned} \tag{33}
$$

Now, consider the second part of (32):

$$
\begin{aligned}
&= \int_0^\infty \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1} e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1+1)}\left(\left[\left(\frac{\beta}{\beta+1}\right)\left(1+\frac{e^y}{\beta}\right)^{-\alpha_2}\right]\right)\mathrm{d}y \\
&= \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1+\alpha_2}\int_0^\infty e^y\left(1+\frac{e^y}{\beta}\right)^{-(\alpha_1-\alpha_2+1)}\mathrm{d}y \\
&= \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1+\alpha_2}\left(\frac{1}{\beta}\right)^{(-\alpha_1-\alpha_2+1)}\sum_{n=0}^{\infty}\beta^n\binom{-\alpha_1-\alpha_2}{n}\left[\frac{1}{\alpha_1+\alpha_2}\right].
\end{aligned} \tag{34}
$$

Combining the result of (33) and (34) gives the stress strength parameter of a new WL:

$$
S_{\text{stress,strength}}(y) = \frac{1}{\alpha_1} - \frac{\alpha_1}{\alpha_1\beta}\left(\frac{\beta+1}{\beta}\right)^{\alpha_1-\alpha_2}\left(\frac{1}{\beta}\right)^{(-\alpha_1-\alpha_2+1)}\sum_{n=0}^{\infty}\beta^n\binom{-\alpha_1-\alpha_2}{n}\left[\frac{1}{\alpha_1+\alpha_2}\right]. \tag{35}
$$

10. Rank Regression on *Y*

CDF of a NWL distribution is defined as

$$
\begin{aligned}
1-F(t) &= \left[\left(\frac{\beta+1}{\beta}\right)\left(1+\frac{e^t}{\beta}\right)\right]^{-\alpha}, \\
\log(1-F(t)) &= -\alpha\log\left(\frac{\beta+1}{\beta}\right) = -\alpha\log\left(1+\frac{e^t}{\beta}\right).
\end{aligned} \tag{36}
$$

By comparing (36) with a simple linear regression model, we have

$$
\begin{aligned}
y &= \log(1-F(t)), \\
a &= -\log\left(\frac{\beta+1}{\beta}\right), \\
b &= \alpha.
\end{aligned} \tag{37}
$$

From the least square equation method, the parameters are estimated by using the following two equations:

$$\hat{a} = \frac{\sum_i y_i}{N} - \hat{b}\frac{\sum_i x_i}{N},$$
$$\hat{b} = \frac{\sum_i x_i y_i - \left(\left(\sum_i x_i \sum_i y_i\right)/N\right)}{\sum_i x_i^2 - \left(\sum_i x_i^2/N\right)}. \tag{38}$$

So, in the current case, we have to replace

$$y = \log(1 - F(t)),$$
$$x = \log\left(1 + \frac{e^t}{r}\right). \tag{39}$$

So, the regression equations related to a new WL distribution are described as

$$\hat{a} = \sum \ln(1 - F(t)) - \hat{b}\ln\left(1 + \frac{e^t}{r}\right),$$
$$\hat{b} = \frac{\sum_i \ln\left(1 + \left(e^t/r\right)\right)\ln(1 - F(t)) - \left(\left(\sum \ln\left(1 + \left(e^t/r\right)\right)\sum \ln(1 - F(t))\right)/N\right)}{\sum \ln\left(1 + \left(e^t/r\right)\right)^2 - \left(\left(\sum \ln(1 + (e^t/r))\right)^2/N\right)}. \tag{40}$$

Note. ln = log and $F(t)$ values are estimated from the median ranks.

11. Total Time on Test (TTT)

The TTT plot identifies various shapes of the hazard function. The TTT plot exhibits a straight line (diagonal) for a constant failure rate. For nonmonotonic failure rates, this plot would first decrease and then increase or vice versa. For monotonic failure rates, the TTT plot will be decreased if it is convex and increases if it is concave. The general formula of the TTT plot is given by

$$G\left(\frac{r}{n}\right) = \frac{\sum_{i=1}^{r} x_{i:n} + (n-r)x_{i:n}}{\sum_{i=1}^{n} x_{i:n}}, \quad r = x_{i:n} = 1, 2, 3, \ldots, n, \tag{41}$$

where $x_{i:n}$ are the order statistics.

12. Applications

Under this section, we provided applications to the proposed probability model using two real-lifetime data sets. To decide the best among other models, we considered goodness of fit statistics including AIC, CAIC, BIC, HQIC, W (Cramer-von Mises), and A (Anderson Darling). It is noted that a probability model with less value of AIC, CAIC, BIC, and HQIC and with a greater value of W and A will be considered the best one among others.

12.1. Wind Catastrophes Data. The data set represents the losses (in millions of dollars) due to wind catastrophes recorded by Boyd [41]. The data set consists of the following information:

2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 8, 8, 9, 15, 17, 22, 23, 24, 25, 27, 32, 43.

The fitted line in Figure 3 shows that the data follow a constant failure function.

Table 2 reflects the values of ml estimates, and their corresponding standard error is attached in the parentheses. Table 3 defines the values of the goodness of fit measures, and it has been observed that the values of AIC, CAIC, BIC, and HQIC are less while W and A statistics are larger for the New Weighted Lomax distribution than other probability models. Hence, a new WL leads to a better fit than Lomax (L), Power Lomax (PL), Exponential Lomax (EL), and Weibull Lomax (WL).

Figure 4 shows the empirical and theoretical PDF and CDF of the proposed distribution $\mathrm{WL}(\alpha, \beta)$ and other existing distributions for the losses due to wind catastrophes.

12.2. Bladders Cancer Patients. The data set represents the remission times (in months) of 128 bladders cancer patients and is taken from Aldeni et al. [3]. The data set values are given as follows:

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69.

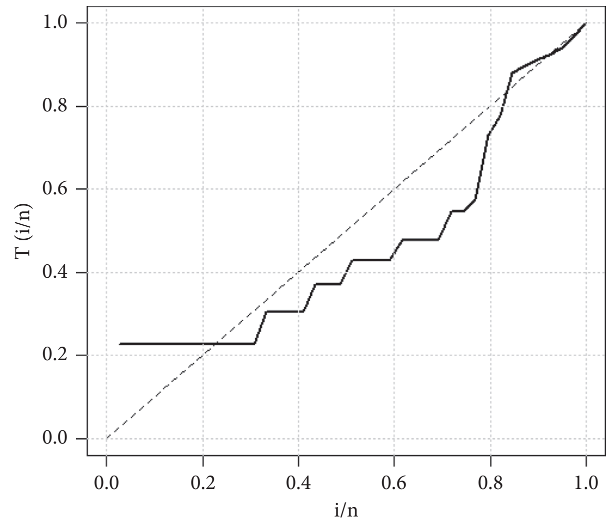

Figure 3: TTT plot of the losses due to wind catastrophes using WL distribution.

Table 2: ML estimates.

Model	Estimates			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\vartheta}$
NW-Lomax	0.1115898 (0.0178886)	24.7376349 (4.6011156)		
Lomax	2.259102 (0.9034286)	13.107217 (6.7006737)		
P-Lomax	0.1612794 (0.0677728)	5.4172791 (1.7104091)	20.3051382 (13.2356863)	
W-Lomax	2.8345778 (1.10337502)	1.9742578 NaN	1.0284592 NaN	0.2073842 (0.03295629)
E-Lomax	28.842426 (36.1915908)	1.481920 (0.2297605)	2.482791 (2.5815765)	

Table 3: Goodness of fit measures for losses due to wind catastrophes.

Model	AIC	CAIC	BIC	HQIC	−log	W	A
NW-Lomax	52.2338	52.56714	55.56093	53.42755	24.1169	0.5933578	3.367013
Lomax	252.6833	253.016	256.0104	253.877	124.3416	0.2949964	1.946171
P-Lomax	235.6173	236.303	240.608	237.4079	114.8086	0.1678204	1.406701
W-Lomax	249.5339	250.7104	256.1881	251.9214	120.7669	0.2982084	1.942425
E-Lomax	237.7877	238.4734	242.7784	239.5783	115.8939	0.1700244	1.383728

The fitted line in Figure 5 is concave-convex type; hence, we determined that the bladder cancer patient data follow a nonmonotonic hazard function.

The ml estimates and their standard error in braces are given in Table 4. Table 5 explains the goodness of fit measures, and it has been noted that the proposed model provides a better fit to these data as compared with other probability models including Lomax (L), Power Lomax (PL), Exponential Lomax (EL), and Weibull Lomax (WL).

Figure 6 shows the empirical and theoretical CDF and CDF of the proposed distribution WL(α, β) and other existing distributions for the Bladder cancer patients.

13. Simulations

The simulation study also plays an important role in making a decision that whether the given model provides a better fit or not. In order to get random data from the New Weighted Lomax distribution, equation (12) would be considered. The random experiment is replicated 100 times with different samples of sizes n with different values of parameters. The

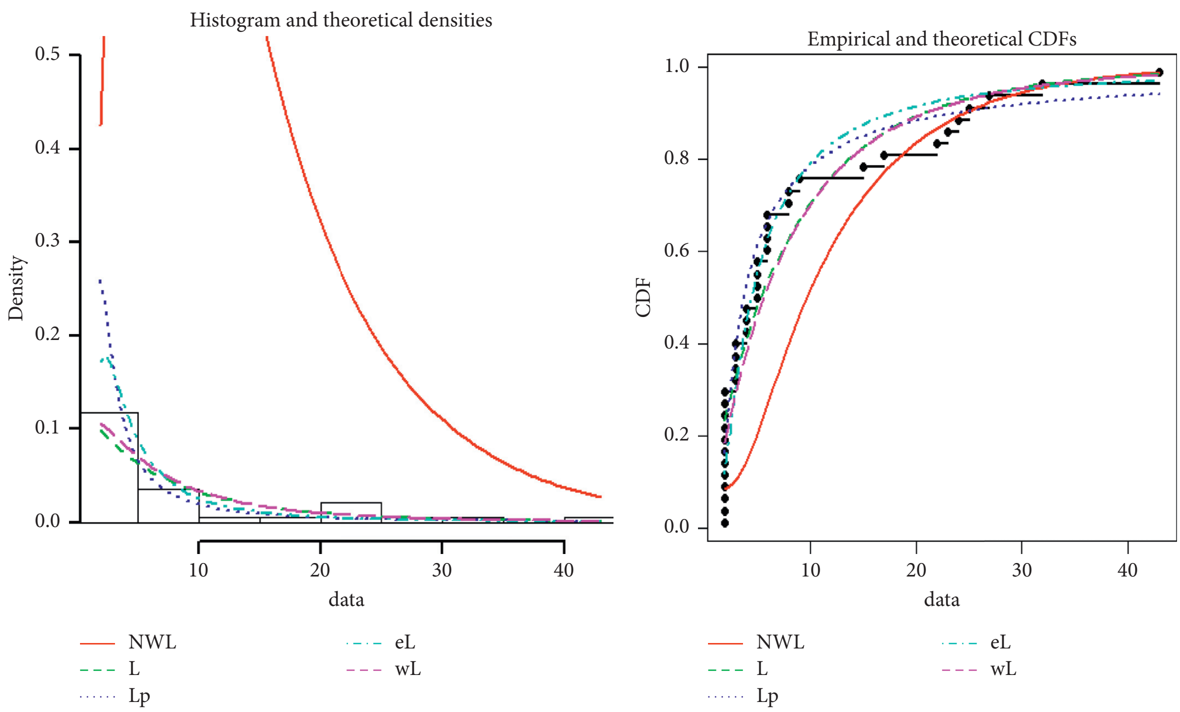

Figure 4: The empirical and theoretical PDF and CDF of the WL(α, β) distribution.

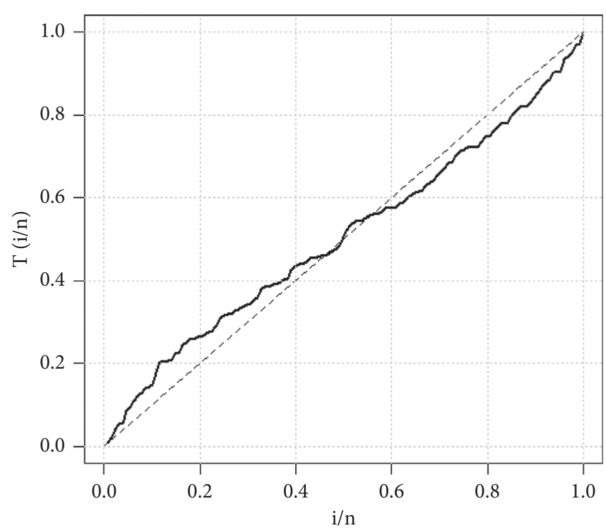

Figure 5: TTT plot of the bladders cancer patients using NWL distribution.

Table 4: ML estimates.

Model	Estimates			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\vartheta}$
NW-Lomax	0.10613 (0.009386)	31.98979 (3.188287)		
Lomax	3.8661 (1.1079)	28.4134 (9.4998)		
P-Lomax	1.404926 (0.4085148)	1.438151 (0.1522463)	19.819986 (5.4992568)	
W-Lomax	5.4732949 (0.0506327)	1.5096438 NaN	4.7310404 NaN	0.2643016 NaN
E-Lomax	10.0160681 (2.41150223)	9.7029282 (1.84701652)	0.1310464 (0.03225501)	

TABLE 5: Goodness of fit measures for bladder cancer patient.

Model	AIC	CAIC	BIC	HQIC	−log	W	A
NW-Lomax	100.39	100.49	106.10	102.71	48.199	0.56604	3.7004
Lomax	835.54	835.64	841.25	837.86	415.77	0.034874	0.224
P-Lomax	827.8986	828.0921	836.4547	831.375	410.9493	0.02481589	0.1860483
W-Lomax	828.6928	829.018	840.1009	833.328	410.3464	0.03741088	0.2432686
E-Lomax	305.2265	305.4765	313.0421	308.3896	149.6133	0.3014044	1.615242

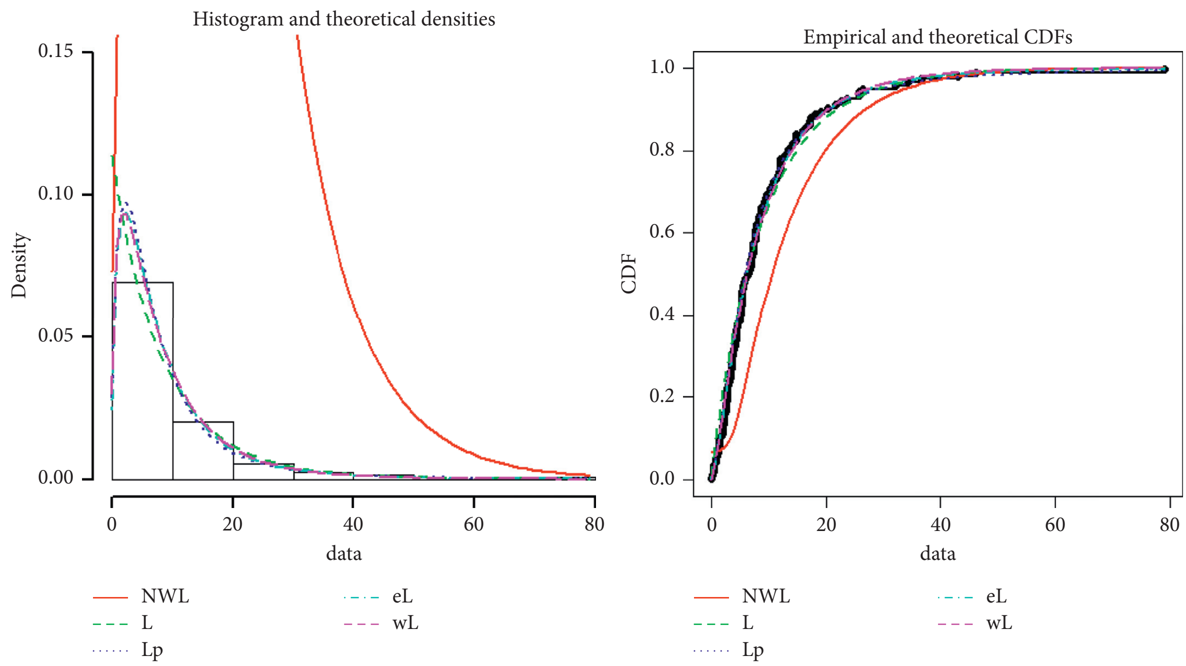

FIGURE 6: The empirical and theoretical PDF and CDF of the WL(α, β) distribution.

TABLE 6: Bias and MSE of NWL(α, β) distribution.

α	B	N	MSE (α)	MSE (β)	Bias (α)	Bias (β)
		30	$3.996406e-05$	15.18753	0.00570363	3.460371
	16.87	60	$3.967258e-05$	2.213798	0.005476957	0.9058611
		80	$6.484166e-06$	0.183196	0.002486657	0.3842137
		30	$3.170174e-05$	14.62279	0.00233932	3.752701
0.033	17.1	60	$1.079283e-05$	4.312617	0.002267292	1.901363
		80	$4.80265e-06$	2.457985	0.0009147354	0.8291876
		30	$2.453543e-05$	16.9866	0.003788296	3.993388
	17.5	60	$1.591147e-05$	8.288684	0.003596319	2.036911
		80	$3.825583e-05$	3.029128	0.0007795378	0.8864978
		30	$4.484847e-05$	54.86977	0.005403825	7.251341
	21.29	60	$2.466488e-05$	28.85079	0.004804561	5.304583
		90	$6.124085e-06$	10.57544	0.002321392	3.138554
		30	$4.484847e-05$	55.74353	0.005403825	7.311341
0.036	21.35	60	$2.466488e-05$	30.46467	0.004804561	5.454583
		90	$6.085762e-06$	10.95351	0.002315325	3.198279
		30	$5.25542e-05$	57.16769	0.005964773	7.383071
	22.45	60	$1.331295e-05$	36.26379	0.003408052	6.00667
		90	$5.233742e-06$	7.576437	0.002065704	2.621302

Table 6: Continued.

α	B	N	MSE (α)	MSE (β)	Bias (α)	Bias (β)
		90	$3.126413e-05$	32.53668	0.005583154	5.696644
	23.2	120	$1.527273e-05$	7.007146	0.003620997	2.272011
		150	$1.384378e-05$	2.284185	0.003689726	1.413837
		90	$1.818826e-05$	36.94084	0.004142978	5.972791
0.04	24.2	120	$1.527273e-05$	12.55117	0.003620997	3.272011
		150	$1.426254e-05$	6.008216	0.003729036	2.364439
		90	$3.126413e-05$	32.53668	0.005583154	5.696644
	23.4	120	$1.527273e-05$	7.007146	0.003620997	2.272011
		150	$1.384378e-05$	2.88972	0.003689726	1.613837

result given in Table 6 declares that both the Bias and MSE are continuously decreased as the sample size increases.

14. Conclusion

The basic aim of this paper is to make a further contribution to the existing theory of the probability models. The paper presents a New Weighted Lomax (NWL) distribution model with two parameters, which is very versatile than others. Various statistical properties are discussed like hazard function, mean residual life function, and stress strength function. To make a comparison with other existing distributions, we have considered two real data sets. The first data set follows a monotonic hazard shape while the second data set (bladder cancer patients) has a nonmonotonic (bathtub) hazard shape. The results demonstrated in both data sets that a new WL model is too much better and provides an adequate fit than the Lomax, P-Lomax, W-Lomax, and E-Lomax distribution.

Data Availability

The data sets used to support the finding of this study are taken from the literature.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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