Hindawi
Mathematical Problems in Engineering
Volume 2021, Article ID 9082320, 15 pages
https://doi.org/10.1155/2021/9082320

Research Article

On Analysis of Topological Aspects for Subdivision of Kragujevac Tree Networks

Salma Kanwal,[1] Shanshan Shang,[2] Muhammad Kamran Siddiqui,[3] Tahira Sumbal Shaikh,[1] Ammara Afzal,[1] and Anton Asare-Tuah[4]

[1]*Department of Mathematics, Lahore College for Women University, Lahore, Pakistan*
[2]*Shanghai University of Engineering Science, Shanghai 200336, China*
[3]*Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Lahore, Pakistan*
[4]*Department of Mathematics, University of Ghana, Legon, Ghana*

Correspondence should be addressed to Anton Asare-Tuah; aasare-tuah@ug.edu.gh

Received 13 August 2021; Revised 24 September 2021; Accepted 6 October 2021; Published 3 November 2021

Academic Editor: Ali Ahmadian

In this paper, we have taken review of certain topological topological characteristics of subdivision and the line graph of subdivision of Kragujevac tree. A Kragujevac tree is denoted by K, $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$, with order $r(2t+1)+1$ and size $r(2t+1)$, respectively. We have computed the Zagreb polynomials, forgotten polynomial, and M-polynomial for Kragujevac tree. Moreover, we have computed topological indices like Zagreb-type indices, reduced reciprocal Randić indices, family of Gourava indices as well as forgotten index. Further, some topological indices that can be directly derived from M-polynomial, i.e., first and second Zagreb index, modified second Zagreb index, Randić and reciprocal Randić index, symmetric division and harmonic index, and inverse sum and augmented Zagreb index are also computed.

1. Introduction

A topological index is a mathematical formula which is applicable to any graph that represents a molecular structure. It is also known as molecular descriptor. Using these indices, we can analyze the numerical out comes, and various physio-chemical characteristics of a molecule can be investigated. Therefore, use of topological indices is efficient way to avoid highly expensive and time taking laboratory experiments [1, 2].

Recent innovation and advancement in mathematical chemistry and simulation of chemical graphs is due to the numerous research studies done in the field of chemical graph theory. Chemical graph theory enables the researchers to create a relationship between graph theory and chemical structures of compounds [3]. Due to the rapid increments in production of new drugs and chemical compounds by chemical industries, the workload in chemical graph theory has been increased. Molecular descriptors are playing significant role in chemistry, pharmacology, and biological and physical sciences [4]. Topological indexes via M-polynomials are the prominent component for studying the various properties of chemical graphs [5, 6].

Topological indices do not relay on labeling and graphical representation of some graphs. Degrees and distances between vertices give rise to some chemical invariants called topological indices. Gutman and Trinajstić [7] three decades before put forward the Zagreb indices. Later on, Balaban et al. [8, 9] termed the Zagreb indices as Zagreb group indices.

Then, the first Zagreb index is expressed as follows:

$$M_1(H) = \sum_{p\in V(H)} \left(d_p\right)^2 = \sum_{kp\in E(H)} d_k + d_p. \tag{1}$$

The second Zagreb index is expressed as follows:

$$M_2(H) = \sum_{kp\in E(H)} d_k d_p. \tag{2}$$

Fath-Tabar [10] presented the third Zagreb index in 2011 symbolised by $M_3(H)$ for some graph H and is expressed as follows:

$$M_3(H) = \sum_{kp\in E(H)} \left|d_k - d_p\right|. \quad (3)$$

Zagreb polynomials were brought into notice in the same year by the same author, named as 1st, 2nd, and 3rd Zagreb polynomials and are presented as follows:

$$M_1(H,w) = \sum_{kp\in E(H)} w^{d_k+d_p},$$
$$M_2(H,w) = \sum_{kp\in E(H)} w^{d_k d_p}, \quad (4)$$
$$M_3(H,w) = \sum_{kp\in E(H)} w^{\left|d_k-d_p\right|},$$

respectively. Another variant of Zagreb indices is modified Zagreb indices [11] which also deal with degrees of vertices. The first and second modified Zagreb indices are denoted by ${}^mM_1(H)$ and ${}^mM_2(H)$ individually and are as follows:

$${}^mM_1(H) = \sum_{kp\in E(H)} \frac{1}{d_k + d_p},$$
$${}^mM_2(H) = \sum_{kp\in E(H)} \frac{1}{d_k d_p}. \quad (5)$$

The hyper-Zagreb index was presented by Shirdel et al. [12] as follows:

$$HM(H) = \sum_{kp\in E(H)} \left(d_k + d_p\right)^2. \quad (6)$$

The redefined Zagreb indices were put forward in 2013 by RanHini et al. [13] and were defined as follows:

$$\text{redefined first Zagreb Index} = \mathrm{Re}M_1(H) = \sum_{kp\in E(H)} \frac{d_k + d_p}{d_k d_p}, \quad (7)$$

$$\text{redefined second Zagreb Index} = \mathrm{Re}M_2(H) = \sum_{kp\in E(H)} \frac{d_k d_p}{d_k + d_p}, \quad (8)$$

$$\text{redefined third Zagreb Index} = \mathrm{Re}M_3(H) = \sum_{kp\in E(H)} \left(d_k d_p\right)\left(d_k + d_p\right). \quad (9)$$

The forgotten index symbolised by $F(H)$ for a graph H was introduced in [14] by Furtula et al. and defined as follows:

$$F(H) = \sum_{p\in V(H)} \left(d_p\right)^3 = \sum_{kp\in E(H)} \left[\left(d_k\right)^2 + \left(d_p\right)^2\right]. \quad (10)$$

Parallel to the notion of the forgotten topological index is that of forgotten polynomial. Forgotten polynomial of a graph H is given by

$$F(H,w) = \sum_{kp\in E(H)} w^{\left[\left(d_k\right)^2 + \left(d_p\right)^2\right]}. \quad (11)$$

Reciprocal Randić, reduced second Zagreb, and reduced reciprocal Randić indices were proposed by I. Gutman et al. [15] in 2014 and are defined as follows:

$$\mathrm{RR}(H) = \sum_{kp\in E(H)} \sqrt{\left(d_k\right)\left(d_p\right)}, \quad (12)$$

$$RM_2(H) = \sum_{kp\in E(H)} \left(d_k - 1\right)\left(d_p - 1\right), \quad (13)$$

$$\mathrm{RRR}(H) = \sum_{kp\in E(H)} \sqrt{\left(d_k - 1\right)\left(d_p - 1\right)}. \quad (14)$$

Many new graph invariants [16–19] which are well-known family of Gourava indices were brought into existence by V. R. Kulli in 2017 and are described as follows:

$$\text{first Gourava index} = \mathrm{G_1O}(H) = \sum_{kp\in E(H)} \left[d_k + d_p + d_k d_p\right], \quad (15)$$

$$\text{second Gourava index} = \mathrm{G_2O}(H) = \sum_{kp\in E(H)} \left[\left(d_k + d_p\right)\left(d_k d_p\right)\right], \quad (16)$$

$$\text{product connectivity Gourava index} = \mathrm{PGO}(H) = \sum_{kp\in E(H)} \frac{1}{\sqrt{\left(d_k + d_p\right)\left(d_k d_p\right)}}, \quad (17)$$

$$\text{sum connectivity Gourava index} = \text{SGO}(H) = \sum_{kp\in E(H)} \frac{1}{\sqrt{(d_k + d_p) + (d_k d_p)}}, \tag{18}$$

$$\text{first hyper} - \text{Gourava index} = \text{HGO}_1(H) = \sum_{kp\in E(H)} \left[(d_k + d_p) + (d_k d_p)\right]^2, \tag{19}$$

$$\text{second hyper Gourava index} = \text{HGO}_2(H) = \sum_{kp\in E(H)} \left[(d_k + d_p)(d_k d_p)\right]^2. \tag{20}$$

M-polynomial [20] is given as

$$M(H; w, x) = \sum_{\delta\le r\le s\le\Delta} y_{rs}(Hw^r x^s). \tag{21}$$

With δ and Δ as, respectively, minimum and maximum of vertex degrees in graph, H and y_{rs} represent the number of edges $kp \in E(H)$ such that $\{d_k, d_p\} = \{r, s\}$ (see [21]). We can deduce various indices from M-polynomial such as follows:

$$1^{\text{st}}\text{Zagreb index} = M_1(H) = (D_w + D_x)(M(H; w, x))_{w=x=1}, \tag{22}$$

$$2^{\text{nd}}\text{Zagreb index} = M_2(H) = (D_w D_x)(M(H; w, x))_{w=x=1}, \tag{23}$$

$$\begin{aligned}\text{modified } 2^{\text{nd}}\text{Zagreb index} &= {}^m M_2(H)\\ &= (S_w S_x)(M(H; w, x))_{w=x=1},\end{aligned} \tag{24}$$

$$\begin{aligned}\text{Randić index} = R_\alpha(H) &= (S_w^\alpha S_x^\alpha)(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} (d_k d_p)^\alpha,\end{aligned} \tag{25}$$

$$\begin{aligned}\text{inverse Randić index} &= \text{RR}(H)\\ &= (D_w^\alpha D_x^\alpha)(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} \sqrt{d_k d_p}.\end{aligned} \tag{26}$$

The symmetric division deg index is as follows:

$$\begin{aligned}\text{SDD}(H) &= (D_w S_x + D_x S_w)(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} \left\{\frac{\min(d_k, d_p)}{\max(d_k, d_p)} + \frac{\max(d_k, d_p)}{\min(d_k, d_p)}\right\},\end{aligned} \tag{27}$$

$$\begin{aligned}\text{harmonic index} = H(H) &= 2S_w H(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} \frac{2}{d_k + d_p},\end{aligned} \tag{28}$$

$$\begin{aligned}\text{inverse sum indeg index} &= \text{ISI}(H)\\ &= S_w H D_w D_x(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} \frac{d_k d_p}{d_k + d_p},\end{aligned} \tag{29}$$

and the augmented Zagreb index is as follows:

$$\begin{aligned}A(H) &= S_w^3 Q_{-2} D_w^3 D_x^3(M(H; w, x))_{w=x=1}\\ &= \sum_{kp\in E(H)} \left\{\frac{d_k d_p}{d_k + d_p - 2}\right\}^3,\end{aligned} \tag{30}$$

where

$$\begin{aligned}D_w M(H; w, x) &= w\frac{\partial(M(H; w, x))}{\partial w},\\ D_x M(H; w, x) &= x\frac{\partial(M(H; w, x))}{\partial x},\\ S_w M(H; w, x) &= \int_0^w \frac{M(H; t, x)}{t}\,\mathrm{d}t,\\ S_x M(H; w, x) &= \int_0^x \frac{M(H; w, t)}{t}\,\mathrm{d}t,\\ HM(H; w, x) &= M(H; w, w), Q_\alpha M(H; w, x) = w^\alpha M(H; w, x).\end{aligned} \tag{31}$$

2. The Structure of Kragujevac Tree

A Kragujevac tree [22–24] is a proper tree holding a central vertex of degree at least 3, in which branches of the form $Q_1, Q_2, Q_3, \ldots$ are joined. Let $Q_2, Q_3, Q_4, \ldots$ be branches whose form is given in Figure 1. $\text{Kg}_{q,r}$ denotes the group of proper Kragujevac trees K of order q and central vertex with degree r (see details in [25–28]). If an additional vertex of degree 2 is added on one pendant line of proper Kragujevac tree, then we get a new family of Kragujevac trees named as improper Kragujevac tree, denoted by $\text{Kg}_{q,r}^*$.

The branches attached to the central vertex are given by $Q_{a_1}, Q_{a_2}, \ldots, Q_{a_r}$ where $a_i \ge 2$ for all $\{i = 1, \ldots, r\}$. In this paper, we are concerned with special case of Kragujevac tree

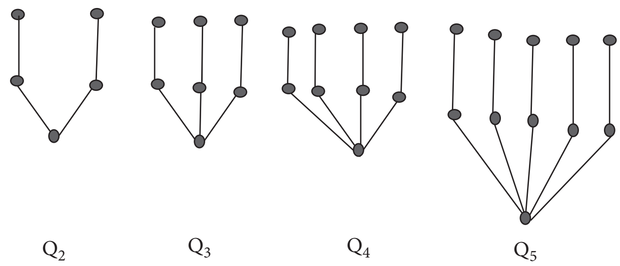

FIGURE 1: Branches (a) Q_2. (b) Q_3. (c) Q_4 (d) Q_5.

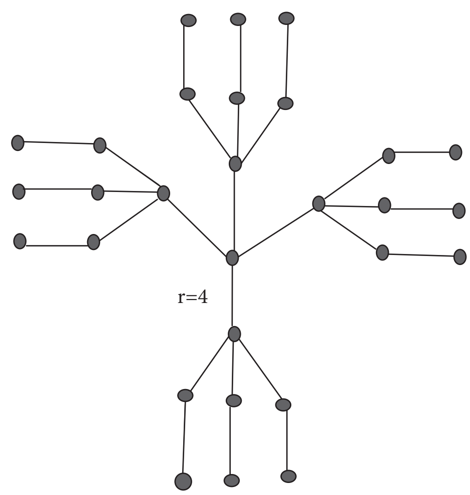

FIGURE 2: Kragujevac tree K from class $Kg_{29,4}$.

when $Q_{a_1}, Q_{a_2}, \ldots, Q_{a_r}$ are same (or isomorphic), i.e., $a_1 = a_2 = \cdots = a_r = t$. Hence, the order and size of Kragujevac tree $K \in Kg_{q,r}$ are $r(2t+1)+1$ and $r(2t+1)$, respectively. For example, see Figure 2.

Graphs under inspection in this paper are undirected and simple, without various edges and finite. Considering a graph H, the sets of nodes and lines are symbolised by $V(H)$ and $E(H)$, individually. Also, degree, i.e., number of lines attached to a node is denoted as $d_H(p)$ for node p.

A huge amount of research is made on graphs, several operations on graphs, and chemical invariants since last three decades. Eliasi et al. [29] proposed operations on graphs as S, R, Q, and T named like subdivision, semitotal point graph, semitotal line graph, and total graph. Subdivision of graph H denoted by $S(H)$ is derived by putting an extra vertex (mentioned as white vertex) into each edge of H. To make a contrast between already existing and newly inserted vertices, vertices of H are known as black vertices. Two white vertices will be related to each other if their correlating edges in H are adjacent and likewise two black vertices will be called related if they are adjacent in H.

Subdivision and line graph of subdivision of Kragujevac tree are presented in Figures 3 and 4, respectively.

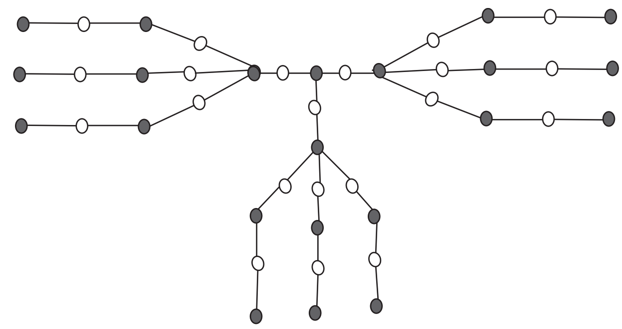

FIGURE 3: Subdivision graph of Kragujevac tree.

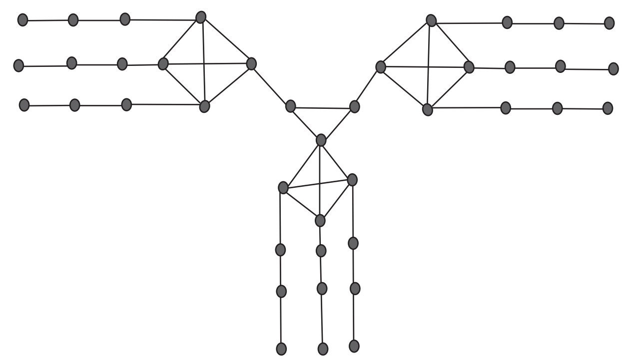

FIGURE 4: Line graph of subdivision of Kragujevac tree.

3. Certain Topological Indices and Polynomials of $S(K)$

In this section, we compute certain topological indices, polynomials, and several other chemical indices in terms of these polynomials as stated in the previous section for the subdivision of Kragujevac tree. The subdivision of Kragujevac tree is shown in Figure 3.

For $H = S(K)$, $K \in Kg_{q=r(2t+1)+1,r}$, forgotten and Zagreb polynomials are as follows:

$$\begin{aligned}
F(H,w) &= r\left[w^{r^2+4} + w^{t^2+2t+5}\right] + rt\left[w^{t^2+2t+5} + 2w^8 + w^5\right],\\
M_1(H,w) &= r\left[w^{r+2} + w^{t+3}\right] + rt\left[w^{t+3} + 2w^4 + w^3\right],\\
M_2(H,w) &= rt\left[w^{2t+2} + 2w^4 + w^2\right] + rw^{2t+2} + rw^{2r},\\
M_3(H,w) &= rt\left[w^{t-1} + w + 2\right] + rw^{t-1} + rw^{r-2}\right].
\end{aligned} \tag{32}$$

Consider the subdivision of Kragujevac tree, denoted by $S(K)$, where $K \in Kg_{q=r(2t+1)+1,r}$. We categorize the lines of $S(K)$ according to the lines of type $E_{(d_k,d_p)}$, where $kt \in E(S(K))$. The line categorization is presented in Table 1.

Table 1: Line categorization in $S(K)$, where $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$.

Lines of type	Number of lines
$E_{(r,2)}$	r
$E_{(t+1,2)}$	r
$E_{(t+1,2)}$	rt
$E_{(2,2)}$	$2rt$
$E_{(2,1)}$	rt

$$
\begin{aligned}
F(H,w) &= \sum_{kp\in E(H)} w^{(d_k)^2+(d_p)^2} \\
&= rw^{(r)^2+(2)^2} + rw^{(t+1)^2+(2)^2} + rtw^{(t+1)^2+(2)^2} \\
&\quad + 2rtw^{(2)^2+(2)^2} + rtw^{(2)^2+(1)^2} \\
&= r\left[w^{r^2+4} + w^{t^2+2t+5}\right] + rt\left[w^{t^2+2t+5} + 2w^8 + w^5\right], \\
M_1(H,w) &= \sum_{kp\in E(H)} w^{d_k+d_p} \\
&= \sum w^{r+2} + \sum w^{t+1+2} + \sum w^{t+1+2} + \sum w^{2+2} + \sum w^{2+1} \\
&= r\left[w^{r+2} + w^{t+3}\right] + rt\left[w^{t+3} + 2w^4 + w^3\right], \\
M_2(H,w) &= \sum_{kp\in E(H)} w^{d_k d_p} \\
&= rw^{r\times 2} + rw^{2\times(t+1)} + rtw^{2\times(t+1)} + 2rtw^{2\times 2} + rtw^{2\times 1} \\
&= rt\left[w^{2t+2} + 2w^4 + w^2\right] + rw^{2t+2} + rw^{2r}, \\
M_3(H,w) &= \sum_{kp\in E(H)} w^{\left|d_k - d_p\right|} \\
&= rw^{r-2} + rw^{t+1-2} + rtw^{t+1-2} + 2rtw^{2-2} + rtw^{2-1} \\
&= rt\left[w^{t-1} + w + 2\right] + rw^{t-1} + rw^{r-2}.
\end{aligned}
\tag{33}
$$

For $H = S(K)$, $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$, we have

$$
\begin{aligned}
HM(H) &= r^3 + 4r^2 + 7rt^2 + rt^3 + 13r + 56rt\mathrm{Re}, \\
\mathrm{Re}M_1(H) &= \frac{rt}{2}\left[\frac{4}{t+1} + 7\right] + \frac{rt^2}{2(t+1)} + \frac{r}{2} + 1 + \frac{3r}{2(t+1)}\mathrm{Re}, \\
\mathrm{Re}M_2(H) &= 2rt\left[\frac{2}{t+3} + \frac{4}{3}\right] + \frac{2r}{t+3}\left(t^2+1\right) + \frac{2r^2}{r+2}\mathrm{Re}, \\
\mathrm{Re}M_3(H) &= 2r\left(r^2 + t^3\right) + 4r^2 + 8rt^2 + 6r + 48rt, \\
{}^{m}M_1(H) &= rt\left[\frac{1}{t+3} + \frac{5}{6}\right] + \frac{r}{t+3} + r\frac{1}{r+2}R, \\
\mathrm{Re}M_2(H) &= r^2 + rt^2 + 3rt - r\mathrm{RRR}(H), \\
\mathrm{RRR}(H) &= r\left[t\left(\sqrt{t} + 2\right) + \sqrt{t}\,\right] + r\sqrt{r-1}, \\
\mathrm{GO}_1(H) &= rt\left[3(t) + 29\right] + 7r + 3r^2, \\
\mathrm{GO}_2(H) &= 2r\left[r^2 + t^3\right] + 52rt + 4r^2 + 10rt^2 + 6r, \\
\mathrm{HGO}_1(H) &= rt\left[9\left(t^2\right) + 208\right] + 29r + 12r^2 + 39rt^2, \\
\mathrm{HGO}_2(H) &= 4rt\left[t^4 + 30t^2 + 36t + 170\right] + 36r + 4r^5 \\
&\quad + 16r^3 + 16r^4, \\
\mathrm{PGO}(H) &= rt\left[\frac{1}{\sqrt{2\left(t^2+4t+3\right)}} + \frac{1}{2} + \frac{1}{\sqrt{6}}\right] \\
&\quad + \frac{r}{\sqrt{2\left(t^2+4t+3\right)}} + \frac{r}{\sqrt{2r^2+4r}}, \\
\mathrm{SGO}(H) &= rt\left[\frac{1}{\sqrt{3t+5}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}}\right] + \frac{r}{\sqrt{3t+5}} + r\frac{1}{\sqrt{3r+2}}, \\
F(H) &= rt\left[3t + 28\right] + 9r + rt^3 + r^3.
\end{aligned}
\tag{34}
$$

For line categorizations shown in Table 1, using formulas of (6)–(10), (13)–(20), and (24) will give desired above expressions.

For $H = S(K)$, $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$, M-polynomial and certain topological indices deduced from M-polynomial are

$$\begin{aligned}
M(H;w,x) &= rw^{r}x^{2} + (r+rt)w^{t+1}x^{2} + 2rtw^{2}x^{2} + rtw^{2}x,\\
M_1(H) &= r^2 + rt^2 + 5r + 15rtM_2(H)\\
{}^{m}M_2(H) &= \frac{r}{2(t+1)} + rt\left[\frac{1}{2(t+1)} + 1\right] + \frac{1}{2}R_{\alpha},\\
R_{\alpha}(H) &= r\left[\frac{1}{(2)^{\alpha}(r)^{\alpha}} + \frac{1}{(t+1)^{\alpha}(2)^{\alpha}}\right] + rt\left[\frac{1}{(t+1)^{\alpha}(2)^{\alpha}}\right.\\
&\quad \left. + \frac{2}{(2)^{\alpha}(2)^{\alpha}} + \frac{1}{(2)^{\alpha}(1)^{\alpha}}\right],\\
\mathrm{RR}_{\alpha}(H) &= r\left[(2)^{\alpha}(r)^{\alpha} + (2)^{\alpha}(t+1)^{\alpha}\right] + rt\left[(2)^{\alpha}(t+1)^{\alpha} + 2(2)^{\alpha}(2)^{\alpha} + (1)^{\alpha}(2)^{\alpha}\right],\\
\mathrm{SDD}(H) &= rt\left[\frac{3}{2} + 6 + \frac{2}{t+1}\right] + r\left[\frac{1}{2} + \frac{2}{t+1}\right] + rt^{2}\left[\frac{1}{2}\right] + \frac{r^{2}}{2},\\
H(H) &= r\left[\frac{2}{r+2} + \frac{2}{t+3}\right] + rt\left[\frac{2}{t+3} + \frac{5}{3}\right],\\
\mathrm{ISI}(H) &= \frac{2r^{2}}{r+2} + r\left[\frac{2}{t+3}\right] + rt^{2}\left[\frac{2}{t+3}\right] + rt\left[\frac{4}{t+3} + 2 + \frac{2}{3}\right],\\
A(H) &= r[16] + rt(2)^{3}\left[(t+1)^{3} + 3\right].
\end{aligned} \tag{35}$$

For line categorizations shown in Table 1, using formulas (21)–(30) will give above desired expressions.

4. Certain Topological Indices and Polynomials of $L(S(K))$

In this section, we calculate several topological indices, polynomials, and several other chemical indices in terms of these polynomials as mentioned in the first section for the line graph of subdivision of Kragujevac tree. The line graph of subdivision of Kragujevac tree is presented in Figure 4.

For $H = L(S(K))$, $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$, forgotten and Zagreb polynomials are as follows:

$$\begin{aligned}
F(H,w) &= \frac{r^{2}}{2}\left[w^{2r^{2}}\right] + r\left[u^{r^{2}+t^{2}+2t+1} - \frac{1}{2}w^{2r^{2}}\right] + \frac{rt^{2}}{2}\left[w^{2t^{2}+4t+2}\right]\\
&\quad + rt\left[\frac{w^{2t^{2}+4t+2}}{2} + w^{t^{2}+2t+5} + w^{5} + w^{8}\right],\\
M_1(H,w) &= r^{2}\left[\frac{w^{2r}}{2}\right] + r\left[w^{r+t+1} - \frac{w^{2r}}{2}\right] + \frac{rt^{2}}{2}w^{2t+2} + rt\left[\frac{w^{2t+2}}{2} + w^{t+3} + w^{3} + w^{4}\right],\\
M_2(H,w) &= r^{2}\left[\frac{w^{r^{2}}}{2}\right] + r\left[w^{rt+r} - \frac{w^{r^{2}}}{2}\right] + \frac{rt^{2}}{2}w^{t^{2}+2t+1} + rt\left[w^{t^{2}+2t+1} + w^{2t+2} + w^{2} + w^{4}\right],\\
M_3(H,w) &= r^{2}\left[\frac{1}{2}\right] + r\left[w^{r-t-1} - \frac{1}{2}\right] + \frac{rt^{2}}{2} + rt\left[\frac{1}{2} + w^{t-1} + w + 1\right].
\end{aligned} \tag{36}$$

Consider the line graph of subdivision of Kragujevac tree, symbolised by $L(S(K))$. We categorize the lines of $L(S(K))$ according to the lines of type $E_{(d_k,d_p)}$, where $kp \in E(L(S(K)))$. The line categorization is shown in Table 2.

Table 2: Line categorization of $L(S(K))$.

Lines of type	Number of lines
$E_{(r,r)}$	$r(r-1)/2$
$E_{(t+1,r)}$	r
$E_{(t+1,t+1)}$	$rt(t+1)/2$
$E_{(t+1,2)}$	rt
$E_{(2,1)}$	rt
$E_{(2,2)}$	rt

$$
\begin{aligned}
F(H,w) &= \sum_{kp\in E(H)} w^{(d_k)^2+(d_p)^2} \\
&= \frac{r(r-1)}{2}w^{(r)^2+(r)^2} + rw^{(r)^2+(t+1)^2} + \frac{rt(t+1)}{2}w^{(t+1)^2+(t+1)^2} \\
&\quad + rtw^{(t+1)^2+(2)^2} + rtw^{(2)^2+(1)^2} + rtw^{(2)^2+(2)^2} = \frac{r^2}{2}\left[w^{2r^2}\right] + r\left[w^{r^2+t^2+2t+1}\right. \\
&\quad \left.-\frac{1}{2}w^{2r^2}\right] + \frac{rt^2}{2}\left[u^{2t^2+4t+2}\right] + rt\left[\frac{w^{2t^2+4t+2}}{2} + w^{t^2+2t+5} + w^5 + w^8\right], \\
M_1(H,w) &= \sum_{kp\in E(H)} w^{d_k+d_p} = \frac{r^2}{2}w^{r+r} - \frac{r}{2}w^{r+r} + rw^{r+t+1} + \frac{rt^2}{2}w^{t+1+t+1} \\
&\quad + \frac{rt}{2}w^{t+1+t+1} + rtw^{t+1+2} + rtw^{2+1} + rtw^{2+2} = r^2\left[\frac{w^{2r}}{2}\right] + r\left[w^{r+t+1} - \frac{w^{2r}}{2}\right] \\
&\quad + \frac{rt^2}{2}w^{2t+2} + rt\left[\frac{w^{2t+2}}{2} + w^{t+3} + w^3 + w^4\right], \\
M_2(H,w) &= \sum_{kp\in E(H)} w^{d_k d_p} \\
&= \frac{r^2}{2}u^{r\times(r)} - \frac{r}{2}w^{r\times r} + rw^{r\times(t+1)} + \frac{rt^2}{2}u^{t+1\times(t+1)} + \frac{rt}{2}w^{t+1\times(t+1)} \\
&\quad + rtw^{2\times(t+1)} + rtw^{2\times(1)} + rtw^{2\times(2)} \\
&= r^2\left[\frac{w^{r^2}}{2}\right] + r\left[w^{rt+r} - \frac{w^{r^2}}{2}\right] + \frac{rt^2}{2}w^{t^2+2t+1} + rt\left[w^{t^2+2t+1} + w^{2t+2} + w^2 + w^4\right], \\
M_3(H,w) &= \sum_{kp\in E(H)} w^{\left|d_k-d_p\right|} = \frac{r^2}{2}w^{r-r} - \frac{r}{2}w^{r-r} + rw^{r-t-1} + \frac{rt^2}{2}w^{t+1-t-1} + \frac{rt}{2}w^{t+1-t-1} \\
&\quad + rtw^{t+1-2} + rtw^{2-1} + rtw^{2-2} = r^2\left[\frac{1}{2}\right] + r\left[w^{r-t-1} - \frac{1}{2}\right] + \frac{rt^2}{2} + rt\left[\frac{1}{2} + w^{t-1} + w + 1\right].
\end{aligned}
\tag{37}
$$

For $H = L(S(K))$, $K \in \mathrm{Kg}_{q=r(2t+1)+1,r}$, we have

$$
\begin{aligned}
HM(H) &= 2r^4 - r^3 + r + 38rt + 2rt^4 + 7rt^3 + 13rt^2 + 2r^2t + 2r^2, \\
\mathrm{Re}M_1(H) &= r\left[1 + \frac{1}{t+1}\right] + rt^2\left[\frac{2}{t^2+2t+1} + \frac{1}{2t+2}\right] + rt^3\left[\frac{1}{t^2+2t+1}\right] + rt\left[\frac{1}{t^2+2t+1}\right] \\
&\quad + rt\left[\frac{3}{2t+2} + \frac{5}{2}\right] + \frac{t}{t+1} + \frac{1}{t+1} - 1, \\
\mathrm{Re}M_2(H) &= r^3\left[\frac{1}{4}\right] + r^2\left[\frac{1}{r+t+1} - \frac{1}{4}\right] + rt^4\left[\frac{1}{4t+4}\right] + rt^3\left[\frac{1}{2t+2} + \frac{1}{4t+4}\right] \\
&\quad + rt^2\left[\frac{1}{4t+4} + \frac{1}{2t+2} + \frac{2}{t+3}\right] + rt\left[\frac{1}{4t+4} + \frac{2}{t+3} + \frac{5}{3}\right] + \frac{r^2t}{r+t+1}, \\
\mathrm{Re}M_3(H) &= r^5 - r^4 + r^3 + r^2 + r^3t + 2r^2t + r^2t^2 + rt^5 + 4rt^4 + 8rt^3 + 12rt^2 + 29rt, \\
{}^{m}M_1(H) &= r\left[\frac{1}{4} + \frac{1}{r+t+1}\right] + rt\left[\frac{1}{4t+4} + \frac{1}{t+3} + \frac{7}{12}\right] + rt^2\left[\frac{1}{4t+4}\right] - \frac{1}{4}, \\
RM_2(H) &= r^4\left[\frac{1}{2}\right] - r^3\left[\frac{3}{2}\right] + r^2\left[\frac{3}{2}\right] - r\left[\frac{1}{2}\right] + r^2t + \frac{rt^4}{2} + \frac{rt^3}{2} + rt^2, \\
\mathrm{RRR}(H) &= \frac{r^2}{2}\sqrt{r^2 - 2r + 1} - \frac{r}{2}\sqrt{r^2 - 2r + 1} + \frac{rt^3}{2} + \frac{rt^2}{2} + rt\left[\sqrt{rt - t} + \sqrt{t} + 1\right], \\
\mathrm{GO}_1(H) &= r^4\left[\frac{1}{2}\right] + r^3\left[\frac{1}{2}\right] + r^2 + r + r^2t + \frac{5rt^3}{2} + \frac{rt^4}{2} + \frac{13rt^2}{2} + \frac{41rt}{2}, \\
\mathrm{GO}_2(H) &= r^5 - r^4 + r^3 + r^3t + 2r^2t + r^2t^2 + rt^5 + 4rt^4 + 8rt^3 + 12rt^2 + 29rt + r^2, \\
\mathrm{HGO}_1(H) &= r^5\left(\frac{3}{2}\right) + 2r^3 + r + \frac{95}{2}rt^2 + \frac{41rt^4}{2} \\
&\quad + \frac{67rt^3}{2} + \frac{9rt^5}{2} + 4r^3t + r^3t^2 + 2r^2t^2 + 6r^2t + \frac{241rt}{2} + \frac{r^6}{2} + 4r^2 + \frac{rt^6}{2}, \\
\mathrm{HGO}_2(H) &= 2r^8 - 2r^7 + r^5t^2 + 4r^3t^2 + 2r^4t^3 + r^5 \\
&\quad + 4r^3t^2 + 2r^4 + r^3 + 2rt^8 + 30rt^6 + 14rt^7 + 22rt^5 + 50rt^4 + 94rt^3 + 14rt^2 + 328rt, \\
\mathrm{PGO}(H) &= r^2\left[\frac{1}{2\sqrt{2r^3}}\right] + r\left[\frac{1}{\sqrt{r^2t + rt^2 + 2rt + r^2 + r}} - \frac{1}{2\sqrt{2r^3}}\right] + rt\left[\frac{1}{\sqrt{2t^2 + 8t + 6}}\right] \\
&\quad + rt^2\left[\frac{1}{2\sqrt{2t^3 + 6t^2 + 6t + 2}}\right] + rt\left[\frac{1}{2\sqrt{2t^3 + 6t^2 + 6t + 2}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{16}}\right], \\
\mathrm{SGO}(H) &= r^2\left[\frac{1}{2\sqrt{2r + r^2}}\right] + r\left[\frac{-1}{2\sqrt{2r + r^2}} + \frac{1}{\sqrt{2r + rt + t + 1}}\right] + rt\left[\frac{1}{2\sqrt{6t + 2t^3 + 2 + 6t^2}}\right] \\
&\quad + rt\left[\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{3t + 5}}\right] + rt^2\left[\frac{1}{2\sqrt{6t + 2t^3 + 2 + 6t^2}}\right], \\
F(H) &= r^4 + r + rt^4 + 4rt^3 + 6rt^2 + 21rt.
\end{aligned}
\tag{38}
$$

TABLE 3: Numerical results for derived topological indices for K, $K \in \mathrm{Kg}_{q=r(2s+1)+1,r}$.

$[r,s]$	$\mathrm{HM}(H)$	$\mathrm{ReM}_1(H)$	$\mathrm{ReM}_2(H)$	$\mathrm{ReM}_3(H)$	${}^{m}\mathrm{M}_1(H)$	$\mathrm{RM}_2(H)$	$\mathrm{RRR}(H)$
[1, 1]	82	7	5.33	70	2.083	4	4
[2, 2]	346	21	19.8667	332	5.033	22	11.442
[3, 3]	876	115	48.6	918	10.1	60	43.0272
[4, 4]	1972	73	77.0476	2240	16.85	124	78.9282
[5, 5]	3190	111	118.809	3830	25.297	220	87.0820

For line categorizations shown in Table 2, use of formulas of (6)–(10), (13)–(20), and (24) will give desired above expressions. For $H = L(S(K))$, $K \in Kg_{q=r(2t+1)+1,r}$, M-polynomial and certain topological indices that can be deduced from M-polynomial are

$$\begin{aligned}
M(H;w,x) &= \frac{r^2}{2}w^r x^r - \frac{r}{2}w^r x^r + rw^{t+1}x^r + \frac{rt^2}{2}w^{t+1}x^{t+1} \\
&\quad + \frac{rt}{2}w^{t+1}x^{t+1} + rtw^{t+1}x^2 + rtw^2x + rtw^2x^2, \\
M_1(H) &= r^3 + rt^3 + 3rt^2 + 12rt + r, \\
M_2(H) &= r^4\left[\frac{1}{2}\right] - r^3\left[\frac{1}{2}\right] + r^2 + rt^4\left[\frac{1}{2}\right] + rt^3\left[\frac{3}{2}\right] + rt^2\left[\frac{7}{2}\right] + r^2t + rt\left[\frac{17}{2}\right], \\
{}^{m}M_2(H) &= rt^2\left[\frac{1}{2t^2+4t+2}\right] + \frac{1}{t+1} + rt\left[\frac{1}{2t^2+4t+2} + \frac{1}{2t+2} + \frac{3}{4}\right] + \frac{1}{2} - \frac{1}{2r}, \\
R_\alpha(H) &= \frac{r^2}{2(r)^\alpha(r)^\alpha} - \frac{r}{2(r)^\alpha(r)^\alpha} + \frac{r}{(t+1)^\alpha(r)^\alpha} + \frac{rt}{2(t+1)^\alpha(t+1)^\alpha} \\
&\quad + \frac{rt^2}{2(t+1)^\alpha(t+1)^\alpha} + \frac{rt}{(2)^\alpha(t+1)^\alpha} + \frac{rt}{(2)^\alpha} + \frac{rt}{(2)^\alpha(2)^\alpha}, \\
\mathrm{RR}_\alpha(H) &= \frac{r^2(r)^\alpha(r)^\alpha}{2} - \frac{r(r)^\alpha(r)^\alpha}{2} + r(t+1)^\alpha(r)^\alpha + \frac{rt^2(t+1)^\alpha(t+1)^\alpha}{2} \\
&\quad + \frac{rt(t+1)^\alpha(t+1)^\alpha}{2} + rt(t+1)^\alpha(2)^\alpha + rt(2)^\alpha + rt(2)^\alpha(2)^\alpha, \\
\mathrm{SDD}(H) &= r^2\left[1 + \frac{1}{t+1}\right] + rt^3\left[\frac{1}{t+1}\right] + rt^2\left[\frac{2}{t+1} + \frac{1}{2}\right] + rt\left[\frac{3}{t+1} + 5\right] + t + 1], \\
H(H) &= \frac{r}{2} + r\frac{2}{r+t+1} + rt^2\left[\frac{1}{2t+2}\right] + rt\frac{1}{2t+2} + rt\frac{1}{t+1} + rt\frac{7}{6} - \frac{1}{2}, \\
\mathrm{ISI}(H) &= \left[\frac{r^4}{4} - r^2\left[\frac{1}{4} + \frac{1}{t+r+1}\right] + rt^3\left[\frac{3}{4t+4}\right] + rt^2\left[\frac{3}{4t+4} + \frac{2}{t+3}\right]\right. \\
&\quad \left. + tr^2\left[\frac{1}{r+t+1}\right] + rt\left[\frac{1}{4t+4} + \frac{2}{t+3} + \frac{5}{3}\right]\right], \\
A(H) &= \left[\frac{r^2(r)^3(r)^3}{2(2r-2)^3} - \frac{r(r)^3(r)^3}{2(2r-2)^3} + \frac{r(t+1)^3(r)^3}{(r+t-1)^3} + \frac{rt^2(t+1)^3(t+1)^3}{2(2t)^3}\right. \\
&\quad \left. + \frac{rt(t+1)^3(t+1)^3}{2(2t)^3} + \frac{rt(t+1)^3(2)^3}{(2t+1)^3} + rt(2)^3 + \frac{rt(2)^3(2)^3}{(2)^3}\right].
\end{aligned} \tag{39}$$

Table 4: Numerical results for derived topological indices for $S(K)$, where $K \in Kg_{q=r(2s+1)+1,r}$.

$[r,s]$	$GO_1(H)$	$GO_2(H)$	$HGO_1(H)$	$HGO_2(H)$	$PGO(H)$	$SGO(H)$	$F(H)$
[1, 1]	42	76	297	1020	1.816	2.308	38
[2, 2]	166	364	1394	6632	5.1651	7.932	178
[3, 3]	403	1116	3849	25452	10.45141	14.497	468
[4, 4]	732	2200	8436	76560	17.9521	24.3831	996
[5, 5]	1210	4180	16145	447180	26.3589	35.7707	1870

Table 5: Numerical results for derived topological indices for $L(S(K))$, where $K \in Kg_{q=r(2s+1)+1,r}$.

$[r,s]$	$HM(H)$	$ReM_1(H)$	$ReM_2(H)$	$ReM_3(H)$	$^mM_2(H)$	$RM_2(H)$	$RRR(H)$
[1, 1]	66	75.75	4.33	60	1.4133	3	3
[2, 2]	482	19.556	23.867	608	8.117	41	28.3137
[3, 3]	1713	41.5625	72.6428	3348	9.7143	228	106.63305
[4, 4]	4868	71.36	170.4171	12960	16.8087	822	281.4256
[5, 5]	14630	109.3048	344.0553	39500	25.4079	2285	607.7042

Table 6: Numerical results for the derived topological indices for $L(S(K))$, where $K \in Kg_{q=r(2s+1)+1,r}$.

$[r,s]$	$GO_1(H)$	$GO_2(H)$	$HGO_1(H)$	$HGO_2(H)$	$PGO(H)$	$SGO(H)$	$F(H)$
[3, 3]	34	60	249	569	1.5665	1.9816	34
[4, 4]	216	668	2696	15352	4.7948	6.18966	310
[5, 5]	777	3348	16149	253647	9.5496	14.9748	1002
[6, 6]	2140	12960	66732	1868864	15.8902	18.9035	3028
[7, 7]	4980	39500	223805	144566875	24.0314	23.4994	20130

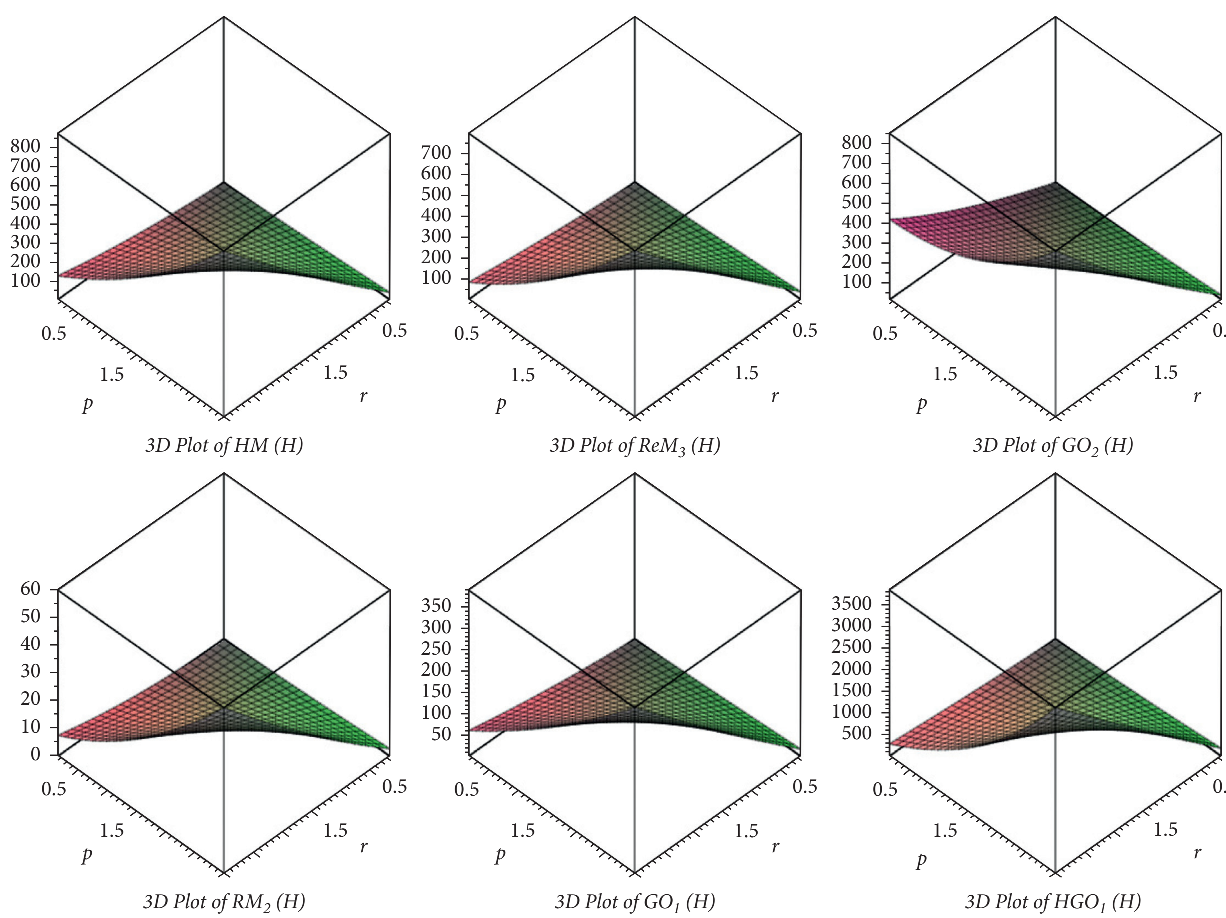

Figure 5: The graphical representation of (a) $HM(H)$, (b) $ReM_3(H)$, (c) $GO_2(H)$, (d) $RM_2(H)$, (e) $GO_1(H)$, and (f) $HGO_1(H)$.

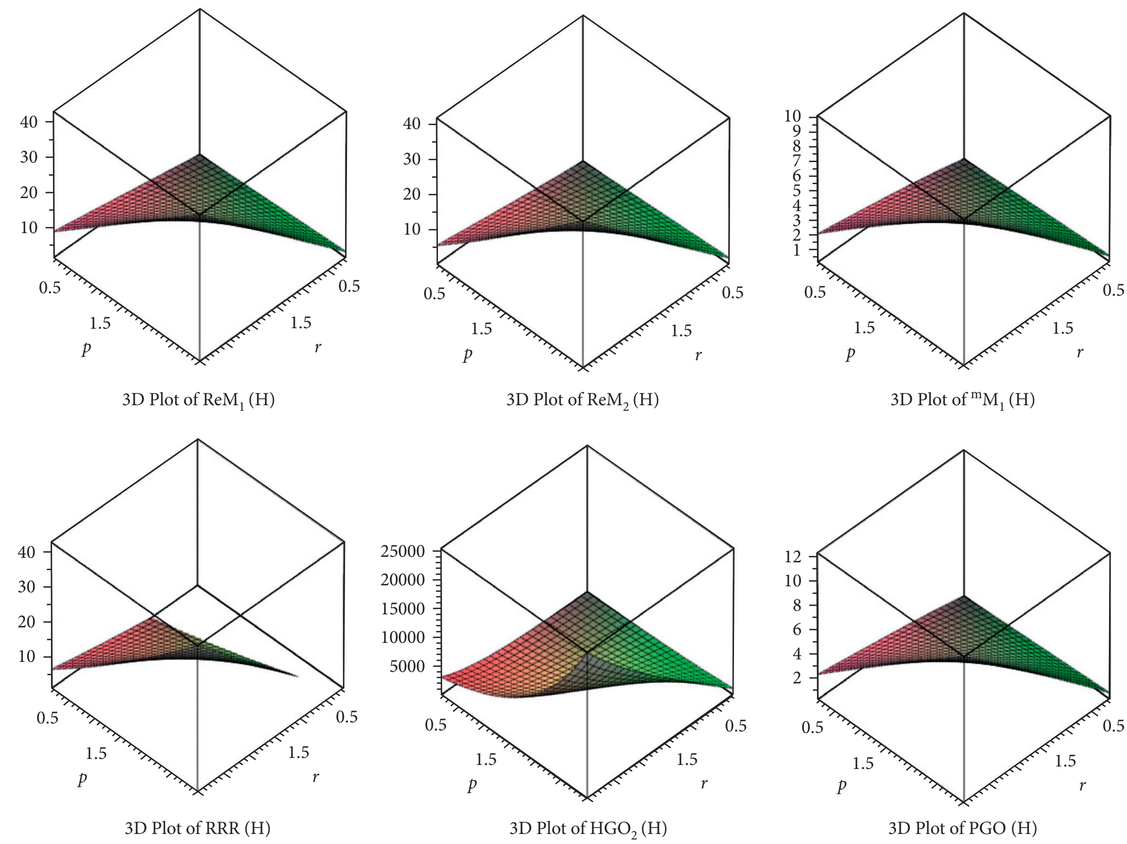

FIGURE 6: The graphical representation of (a) $\mathrm{Re}M_1(H)$, (b) $\mathrm{Re}M_2(H)$, (c) ${}^{m}M_1(H)$, (d) $\mathrm{RRR}(H)$, (e) $\mathrm{HGO}_2(H)$, and (f) $\mathrm{PGO}(H)$.

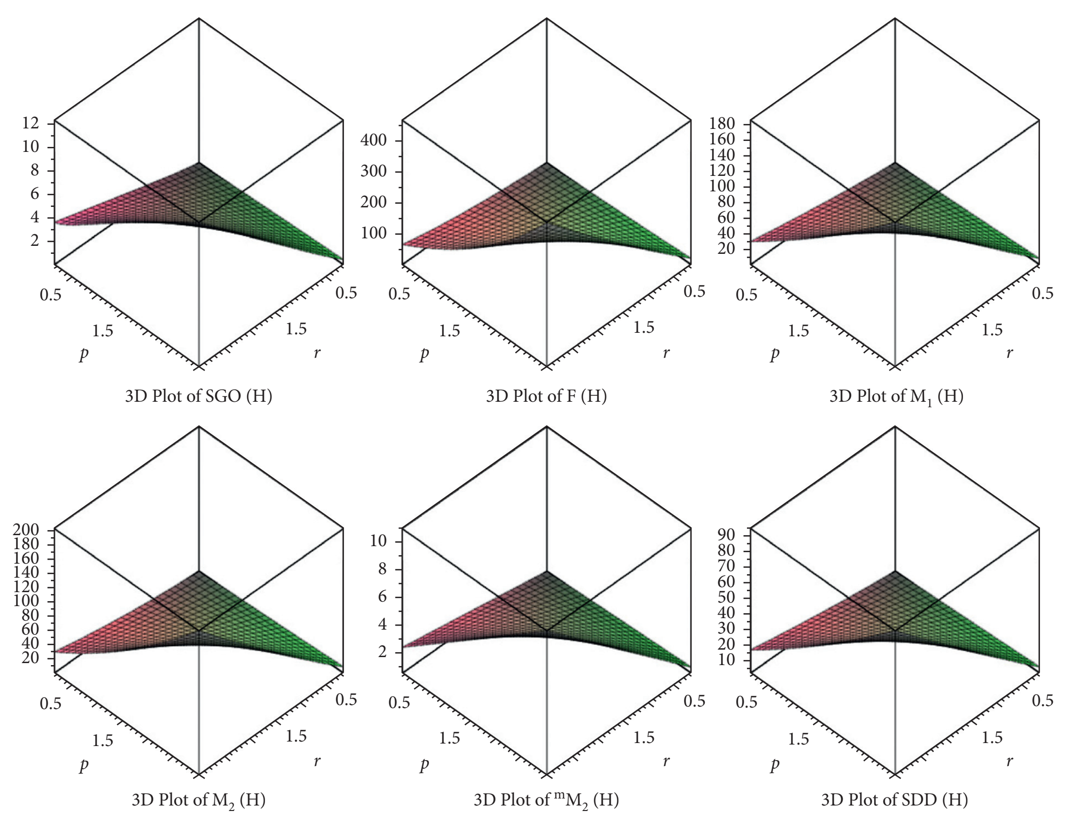

FIGURE 7: The graphical representation of (a) $\mathrm{SGO}(H)$, (b) $F(H)$, (c) $M_1(H)$, (d) $M_2(H)$, (e) ${}^{m}M_2(H)$, and (f) $\mathrm{SDD}(H)$.

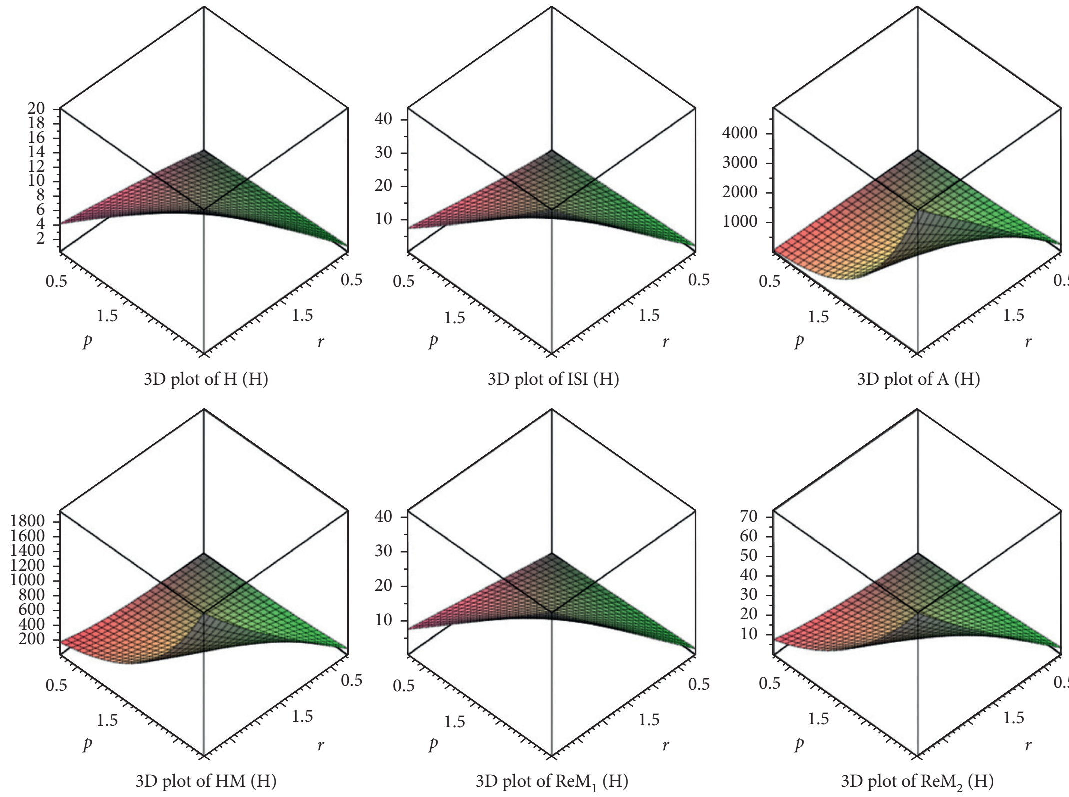

FIGURE 8: The graphical representation of (a) $H(H)$, (b) $\mathrm{ISI}(H)$, (c) $A(H)$, (d) $HM(H)$, (e) $\mathrm{Re}M_1(H)$, and (f) $\mathrm{Re}M_2(H)$.

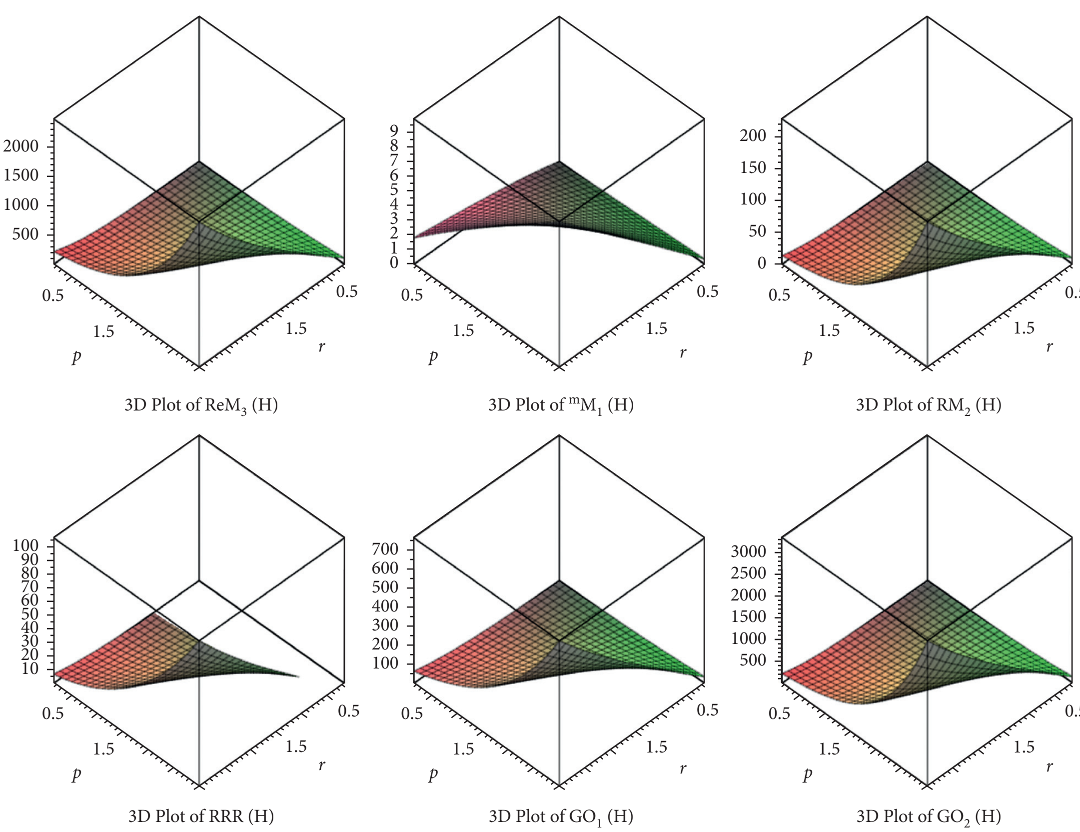

FIGURE 9: The graphical representation of (a) $\mathrm{Re}M_3(H)$, (b) ${}^{m}M_1(H)$, (c) $RM_2(H)$, (d) $\mathrm{RRR}(H)$, (e) $\mathrm{GO}_1(H)$, and (f) $\mathrm{GO}_2(H)$.

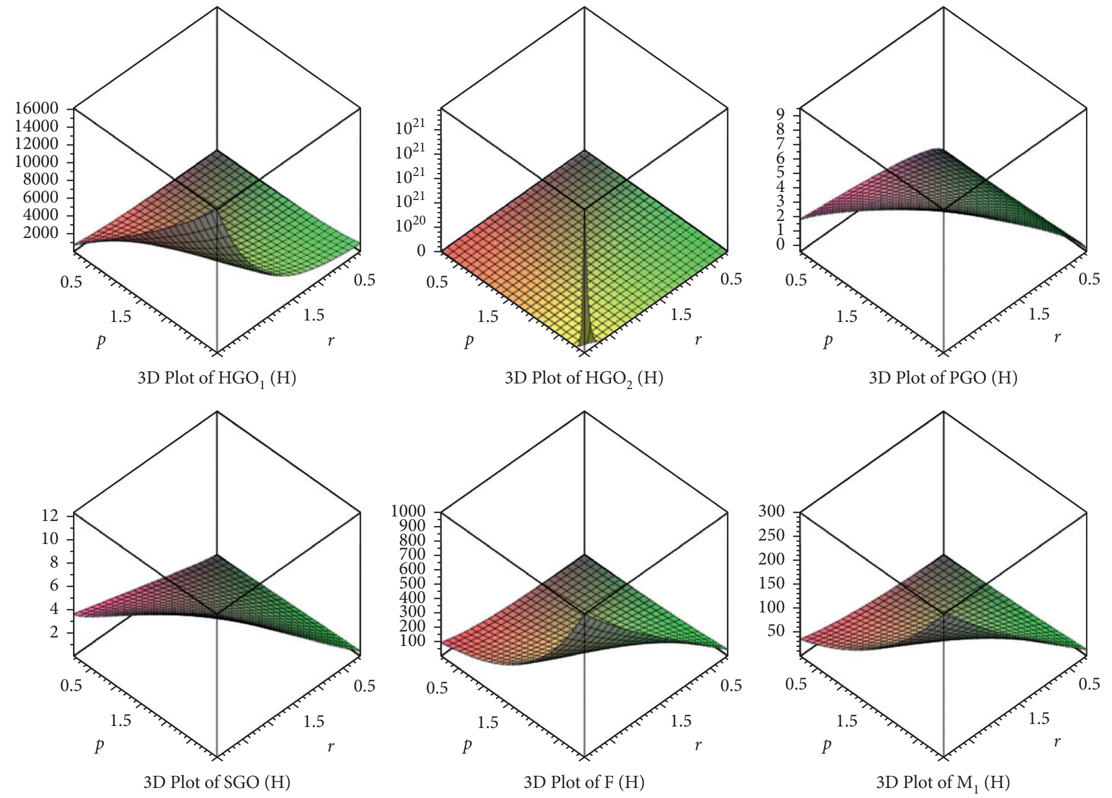

FIGURE 10: The graphical representation of (a) $HGO_1(H)$, (b) $HGO_2(H)$, (c) $PGO(H)$, (d) $SGO(H)$, (e) $F(H)$, and (f) $M_1(H)$.

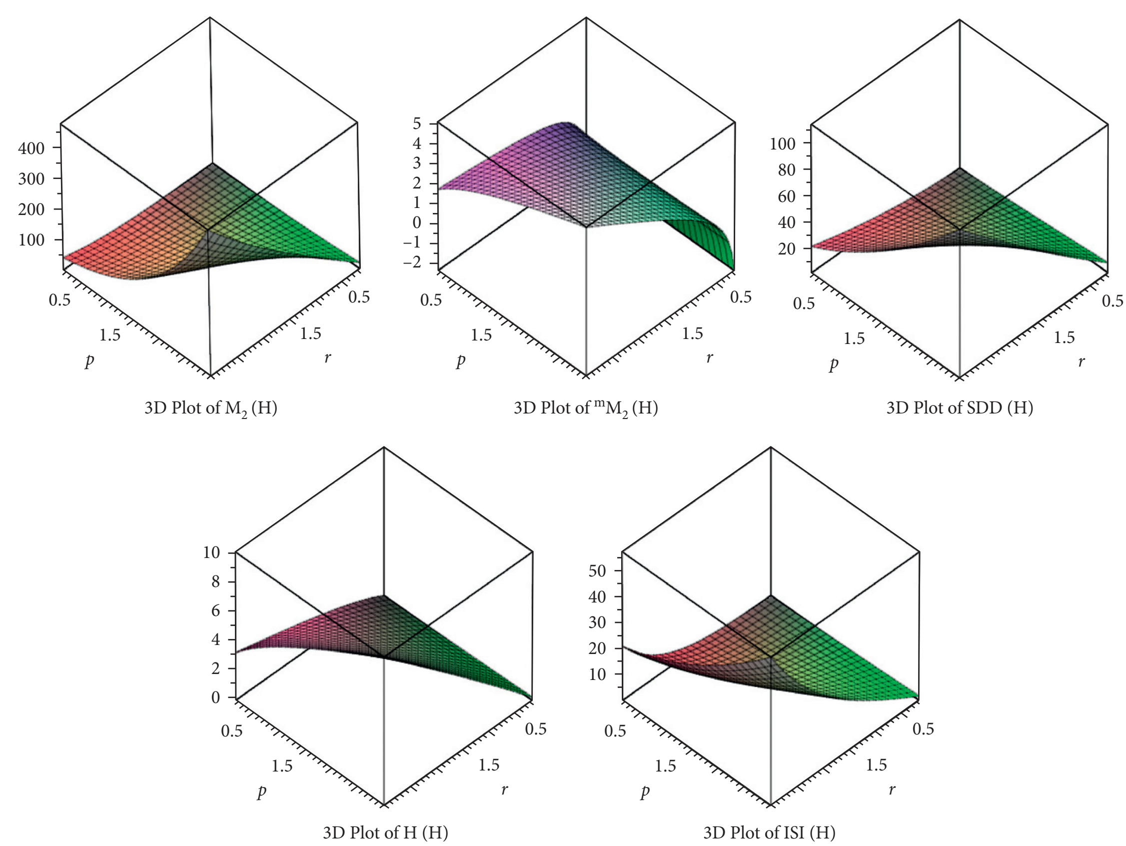

FIGURE 11: The graphical representation of (a) $M_2(H)$, (b) ${}^{m}M_2(H)$, (c) $SDD(H)$, (d) $H(H)$, and (e) $ISI(H)$.

For line categorizations shown in Table 2, using formulas given in (21)–(30) will give desired expressions.

5. Numerical and Graphical Representation of Results

In this segment, numerical results were computed for different Zagreb-type indices. Moreover, Tables 3–6 show numerical results of the aforementioned topological indices for various parameter values r and s for the subdivision graph and for the line of subdivision graph of K where $K \in \mathrm{Kg}_{q=r(2s+1)+1,r}$. We have used different values of r and s and compute the numerical results. Moreover, we have plotted the graphs for Kragujevac tree to study the behavior of above computed topological descriptors.

Figures 5–8 display different topological indices and polynomials for subdivision of Kragujevac tree.

Figures 9–11 display the pictorial representation of different topological indices and polynomials for line graph of subdivision of Kragujevac tree.

6. Conclusion

In chemical graph theory, the calculation of topological indices of any graph is significant. In this article, subdivision of graph and line graph of subdivision of Kragujevac tree is considered and we computed polynomials like Zagreb polynomials, forgotten polynomial, and M-polynomial, and also computation of chemical indices like hyper Zagreb, redefined Zagreb, modified first Zagreb, reduced second Zagreb, reduced reciprocal Randic index, family of Gourava Indices, and forgotten index is made. Further, some topological indices that can be directly derived from M-polynomial, i.e., 1st Zagreb, 2nd Zagreb, modified 2nd Zagreb and augmented Zagreb index, Randic and reciprocal Randic index, symmetric division deg index, harmonic index, and inverse sum indeg index are also computed. However, much work still needs to be done in this area.

Data Availability

The data used to support the findings of this study are cited at relevant places within the text as references.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

This work was equally contributed by all authors.

References

[1] Y. Ji, X. W. Jin, Z. S. Xu, and S. J. Qu, "A mixed 0 – 1programming approach for multiple attribute strategic weight manipulation based on uncertainty theory," *Journal of Intelligent and Fuzzy Systems*, 2021.

[2] Y. Ji, H. Li, and H. Zhang, "Risk-averse two-stage stochastic minimum cost consensus models with asymmetric adjustment cost," *Group Decision and Negotiation*, vol. 14, pp. 1–31, 2021.

[3] D. Afzal, F. Afzal, M. R. Farahani, and S. Ali, "On computation of recently defined degree-based topological indices of some families of convex polytopes via M-polynomial," *Complexity*, vol. 2021, Article ID 5881476, 11 pages, 2021.

[4] T. Gao and I. Ahmed, "Distance-Based polynomials and topological indices for hierarchical hypercube networks," *Journal of Mathematics*, vol. 2021, pp. 1–11, Article ID 5877593, 2021.

[5] Yu-M. Chu, M. Imran, A. Q. Baig, S. Akhter, and M. K. Siddiqui, "On M-polynomial-based topological descriptors of chemical crystal structures and their applications," *Eur. Phys. J. Plus*, vol. 135, pp. 1–19, 2020.

[6] X.-L. Wang, J.-B. Liu, M. Ahmad, M. Kamran Siddiqui, M. Hussain, and M. Saeed, "Molecular properties of symmetrical networks using topological polynomials," *Open Chemistry*, vol. 17, no. 1, pp. 849–864, 2019.

[7] M. K. Siddiqui, M. Imran, and M. Saeed, "Topological properties of face-centred cubic lattice," *Hacettepe Bulletin of Natural Sciences and Engineering*, vol. 49, no. 1, pp. 195–207, 2020.

[8] M. H. Khalifeh, H. Yousefi-Azari, and A. R. Ashrafi, "The first and second Zagreb indices of some graph operations," *Discrete Applied Mathematics*, vol. 157, no. 4, pp. 804–811, 2009.

[9] H. Deng, D. Sarala, S. K. Ayyaswamy, and S. Balachandran, "The Zagreb indices of four operations on graphs," *Applied Mathematics and Computation*, vol. 275, pp. 422–431, 2016.

[10] A. E. Nabeel, "Zagreb polynomials of certain families of bendrimer nanostars," *Tikrit Journal of Pure Science*, vol. 20, no. 4, pp. 25–35, 2015.

[11] J. Hao, "Theorems about zagreb indices and modified zagreb indices," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 65, pp. 659–670, 2011.

[12] G. H. Shirdel, H. Rezapour, and A. M. Syadi, "The hyper-Zagreb index of graph operations," *Iranian Journal of Mathematical Chemistry*, vol. 4, no. 2, pp. 213–220, 2013.

[13] P. S. Ranjini, V. Lokesha, and I. N. Cangül, "On the zagreb indices of the line graphs of the subdivision graphs," *Applied Mathematics and Computation*, vol. 218, no. 3, pp. 699–702, 2011.

[14] B. Furtula and I. Gutman, "A forgotten topological index," *Journal of Mathematical Chemistry*, vol. 53, no. 4, pp. 1184–1190, 2015.

[15] I. Gutman, B. Furtula, and C. Elphick, "Three new/old vertex-degree based topological indices," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 72, no. 24, pp. 617–632, 2014.

[16] V. R. Kulli, "The gourava indices and coindices of graphs," *Annals of Pure and Applied Mathematics*, vol. 14, no. 1, pp. 33–38, 2017.

[17] V. R. Kulli, "The product connectivity gourava index," *Journal of Mathematical and Computational Science*, vol. 8, no. 6, pp. 235–242, 2017.

[18] V. R. Kulli, "On the sum connectivity Gourava index," *International Journal of Mathematical Archive*, vol. 8, no. 6, pp. 211–217, 2017.

[19] V. R. Kulli, "On hyper-gourava indices and coindices," *International Journal of Mathematical Archive*, vol. 8, no. 12, pp. 116–120, 2017.

[20] M. S. Ahmad, W. Nazeer, S. M. Kang, and C. Y. Jung, "M-polynomials and degree based topological indices for the line graph of Firecracker graph," *Global Journal of Pure and Applied Mathematics*, vol. 13, no. 6, pp. 2749–2776, 2017.

[21] M. Irfan, H. U. Rehman, H. Almusawa, S. Rasheed, and I. A. Baloch, "M-polynomials and topological indices for line graphs of chain silicate network and H-naphtalenic nanotubes," *Journal of Mathematics*, vol. 2021, Article ID 5551825, 11 pages, 2021.

[22] S. A. Hosseini, M. B. Ahmed, and I. Gutman, "Kragujevac trees with minimal atom-bond connectivity index," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 71, pp. 5–20, 2014.

[23] B. Basavanagoud and S. Timmanaikar, "Computing first zagreb and forgotten indices of certain dominating transformation graphs of kragujevac trees," *Journal of Computer and Mathematical Sciences*, vol. 8, no. 3, pp. 50–61, 2017.

[24] M. Rezaei, M. S. Hosamani, M. R. Farahani, and M. K. Jamil, "On the terminal wiener index and Zagreb indices of Kragujevac tree," *International Journal of Pure and Applied Mathematics*, vol. 5, no. 113, pp. 617–625, 2017.

[25] B. Basavanagoud and S. Timmanaikar, "On the wiener and hyper wiener indices of certain Dominating Transformation Graphs of Kragujevac Trees," *International Journal of Mathematical*, vol. 8, no. 1, pp. 128–138, 2017.

[26] B. Basavanagoud, "Computing certain topological indices of some dominating graph operations of Kragujevac Trees," *Asian Journal of Mathematics and Computer Research*, vol. 15, no. 1, pp. 30–40, 2017.

[27] B. Basavanagoud and S. Timmanaikar, "Study on some topological indices of semientire and entire dominating graphs of kragujevac trees," *Journal of Computer and Mathematical Sciences*, vol. 9, no. 2, pp. 94–109, 2018.

[28] R. Cruz, J. Rada, and I. Gutman, "Topological indices of Kragujevac trees," *Proyecciones (Antofagasta)*, vol. 33, no. 4, pp. 471–482, 2014.

[29] M. Eliasi and B. Taeri, "Four new sums of graphs and their wiener indices," *Discrete Applied Mathematics*, vol. 157, no. 4, pp. 794–803, 2009.