

## Research Article

# An Integrated Eco-Epidemiological Plant Pest Natural Enemy Differential Equation Model with Various Impulsive Strategies

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We established a mathematical model based on the sense of biological survey in the field of agriculture and introduced various control methods on how to prevent the crops from destructive pests. Basically, there are two main stages in the life cycle of natural enemies like insects: mature and immature. Here, we construct a food chain model of plant pest natural enemy. In natural enemies, there are two stages of construction. Also, we consider three classes of diseases in the pest population, namely, susceptible, exposed, and infectious in this proposed work. In order to categorize the considered models into the class of Impulsive Differential Equations (IDEs), in our study, we specifically consider two ecosystems, which define the impact of control mechanisms on the impulsive releasing of virus particle natural enemies and infectious pests at particular time. Additionally, the importance of spraying virus particles in pest control is discussed; then, we obtain two types of periodic solutions for the system, namely, plant pest extinction and pest extinction. By utilizing the small amplitude perturbation techniques and Floquet theory of the impulsive equation, we obtain the local stability of both periodic solutions. Moreover, the comparison technique of IDE shows the sufficient conditions for the global attractivity of a pest extinction periodic solution. With the assistance of the comparison results, we draw a numerical calculation for the addressed models. Finally, we extend the study of the two models for pest management models: with and without the existence of virus particle.

## 1. Introduction

Of late, it has been of immense interest to inspect the dynamical properties of impulsive perturbations on population models. In the field of agriculture, the main problem faced by farmers is to find an effective pest control method. Numerous methods like physical, chemical control or biological methods can be used to control pests. It is widely acknowledged that pest is a destructive insect and its surge

affects economic as well as ecological problems critically [1, 2]. Evidence demonstrates that annually the pests induce 25% shrinkage in rice, 30% in pulses, 5 – 10% in wheat, 20% in sugar cane, 35% in oil seeds, and 50% in the case of cotton [3]. One of the most important population models is the predator-prey system, which has been discussed by various authors. Acknowledging the predatory-prey model (stocking or harvesting) is essential, as it comprises human actions. Human action invariably happens instantaneously or in a

short time. Then, we reintegrate an impulsive perturbation after detaching the action of humans and models. These human models are based on short-term perturbations which are generally in the structure of IDE in the modeling. Therefore, IDEs give instinctive descriptions of similar systems [4]. Various illustrations are provided by Bainov [5]: relating impulsive vaccination [6], impulsive birth [7, 8], population ecology [9, 10], and chemotherapeutic treatment of disease [11–13]. While considering plant preservation, we affix considerable emphasis to managing insect pest systems and diseases biologically, depending on the respective food chain, and systematically discharge the natural enemies of pests in order to accomplish or eliminate pests' purpose. As an illustration, to effectively control the ectoparasitoid, *Scleroderma guani* is discharged periodically to execute the dissemination medium of *Bursaphelenchus xylophilus* Nickel and *Monochamus alternatus* Hope. There are ample measures of literature which can be used in the control of microbial disease to conceal pests and making use of mathematical model to discuss the dynamics of it [14–20]. And for forest insect pests, only a few applications are there. To manage forest insect pests, discharging of the natural enemies can be considered an effective method.

In 1760, Daniel Bernoulli, a pioneer in the field, presented a solution for his mathematical model in small pox. Mathematical methods are widely used for finding the mechanisms behind the spreading of infectious diseases; in particular, the epidemic outbreak among animals has gained a lot of attention. Epidemic models found an important category of mathematical ecology. Anderson and May [21, 22] studied different types of SIR epidemic models. The disease's incubation time is negligible in SIR models, and as a result, each vulnerable individual becomes infected and then recovers with either temporary or permanent immunity. In many recent works, researchers divided the diseased population into 2 or 3 components as in SI and SEI models, using microbial diseases as a control input. Researchers like Jiao, Wu, Shi, and Song [23–26]

consider SEI models as they give a more realistic explanation of biological problems where the susceptible population moves to exposed pests. Xiang [27] made a study on a relevant pest management SEI model and hence proposed a model given as follows:

$$\left\{ \begin{array}{l} \frac{dS(t)}{dt} = rS(t) \left( 1 - \frac{S(t)}{K} \right) - \frac{\vartheta_1 S(t) I(t)}{1 + \widehat{\sigma} S(t)} \\ \frac{dE(t)}{dt} = \frac{\vartheta_1 S(t) I(t)}{1 + \widehat{\sigma} S(t)} - (\omega + \delta_1) E(t) \\ \frac{dI(t)}{dt} = \omega E(t) - \delta_1 I(t) \end{array} \right\} t \neq nT, \quad (1)$$

$$\left\{ \begin{array}{l} S(t^+) = S(t) \\ E(t^+) = E(t) \\ I(t^+) = I(t) + \widehat{\eta}_1 \end{array} \right\} t = nT.$$

Here, time  $t$ ,  $S(t)$ ,  $E(t)$ , and  $I(t)$  denote densities of susceptible, exposed, and infectious pests, respectively. In the lack of  $I(t)$ , the growth of  $S(t)$  exponentially with carrying capacity  $K$  and  $r$  is the intrinsic birth rate constant, the individual susceptible population  $S(t)$  and infectious population  $I(t)$  get in contact with them until the contact rate is given by  $\vartheta_1 S(t) I(t) / (1 + \widehat{\sigma} S(t))$ , and  $\omega$  represents the inverse of the latent period.  $K$ ,  $r$ ,  $\vartheta_1$ ,  $\widehat{\sigma}$ ,  $\delta_1$ , and  $\widehat{\eta}_1$  are positive constants, and the death rate of the infectious and exposed pests is denoted by the parameter  $\delta_1$ . If  $T$  is the period of the impulse effect, then the release amount of the infected pests is denoted by  $\widehat{\eta}_1$  at  $t = nT$ ,  $n \in \mathbb{N}$ .

The above model is modified by adding natural enemies, and the corresponding model is given in (2) and (3). Many authors [24, 28–31] studied the predator-prey system, which is a relevant population model. Also, we assume that the ability to attack prey is almost the same for each individual predator in the classical predator-prey model [32].

$$\left\{ \begin{array}{l} \frac{dS(t)}{dt} = rS(t) \left( 1 - \frac{S(t)}{K} \right) - \frac{\vartheta_1 S(t) I(t)}{1 + \widehat{\sigma} S(t)} - \vartheta_2 S(t) N(t) \\ \frac{dE(t)}{dt} = \frac{\vartheta_1 S(t) I(t)}{1 + \widehat{\sigma} S(t)} - (\omega + \delta_1) E(t) \\ \frac{dI(t)}{dt} = \omega E(t) - \delta_1 I(t) \\ \frac{dN(t)}{dt} = \frac{\delta \vartheta_2 S(t) N(t)}{1 + h \vartheta_2 S(t)} - \delta_2 N(t) \end{array} \right\} t \neq nT, \quad (2)$$

$$\left\{ \begin{array}{l} S(t^+) = S(t) \\ E(t^+) = E(t) \\ I(t^+) = I(t) + \hat{\eta}_1 \\ N(t^+) = N(t) + \hat{v}_2 \end{array} \right\} t = nT. \tag{3}$$

The individuals of the natural enemy are classified into either mature or immature in this paper; also, we assume that immature natural enemy does not attack the prey. This can be considered reasonable as in the case of many mammals in which the immature natural enemy is brought up by their parents. In these cases, the reproductive rate and the attacking rate are negligible. Inspired by these models, we analyze the stability of the eco-epidemiological plant pest natural enemy model with different impulsive strategies, but it lacks the ability to attack the infected pest. In our approach, pests and natural enemies are treated as prey and predators, respectively. Severe assumptions are made for the mathematical simplicity of this model. Some details are given in the coming section. The key results of this paper are summarized as follows:

- (i) Based on the survey, only a few studies have been done on plant pest natural enemy paradigm in eco-epidemiology. In particular, we are considering the different life stages of natural enemies and also diseases in pest populations with three classes.
- (ii) This manuscript deals with different impulsive strategies; particularly, models with viruses and without viruses are discussed. The numerical investigation concludes with a comparative study of these two models.
- (iii) In our model, we are using multiple impulsive strategies. So, in case of any shortage in any of the impulsive control inputs, we can stabilize our system by altering other control inputs. That's why this model is practically more useful compared to other models.
- (iv) This model is very effective and the period of releasing these impulsive controls can be lengthened compared to other integrated models.

And the remaining work is constructed as follows. This paper is structured into 5 sections. In S, we construct an eco-epidemiological model with stage structure and formulate two different models such as with virus particle and without virus particle. Section 3 deals with periodic solutions and main lemmas, followed by global attractivity and local stability of periodic solutions which are investigated in Section 4. Comparative study and discussions are given in Section 5. In the final section, future works and conclusion are given.

## 2. Formation of Mathematical Modelling

The following assumptions are established in order to create a mathematical model that discusses the entire behavior of a plant, pest, virus, and natural enemy.

- (i)  $\mathcal{H}_1$  Logistically, the plant population  $P(t)$  is increasing. The density of pest population  $S(t)$  captures plant  $P(t)$  represented by  $P(t)S(t)$  with rate  $p_1$ : plant predation rate by susceptible pests. Thus, the evolution equation is

$$\frac{dP(t)}{dt} = P(t)(1 - P(t)) - p_1P(t)S(t). \tag{4}$$

$\mathcal{H}_2$  Diseases in pest populations can be divided into three categories: susceptible, exposed, and infectious. Varied contexts necessitate different functional reactions, according to Holling [33] in 1965. As a result, the typical Lotka-Volterra systems were more practical than they had ever been. Holling II response function means nonlinear saturated incidence rate,  $\vartheta_1 S(t)I(t)/1 + \bar{\omega}S(t)$ , where  $\vartheta_1$  is the contact number of susceptible pest and infected pest per unit time, so  $\vartheta_1 I(t)S(t)$  gives the force of infection and the effect of inhibition caused by behavioral changes in sensitive individuals owing to their increased numbers or crowding effect is determined by  $1/1 + \bar{\omega}S(t)$ .  $\omega$  denotes the inverse of the latent period. Natural enemies' mature and immature life phases are represented by  $N_1$  and  $N_2$ , respectively. As the density of pests increases, the natural enemies with a predation rate  $\vartheta_2$  can only consume a limited quantity of pests. Susceptible pests are consumed only by the mature natural enemy, while the exposed and infective pests are not affected by them.  $\delta$  is the death rate of susceptible pests. Also, the mortality rates of exposed and infected pests are represented by  $\delta_1$ . Thus, the equations are

$$\begin{aligned} \frac{dS(t)}{dt} &= p_1P(t)S(t) - \frac{\vartheta_1 S(t)I(t)}{1 + \bar{\omega}S(t)} - \vartheta_2 S(t)N_2(t) - \delta S(t), \\ \frac{dE(t)}{dt} &= \frac{\vartheta_1 S(t)I(t)}{1 + \bar{\omega}S(t)} - (\omega + \delta_1)E(t), \\ \frac{dI(t)}{dt} &= \omega E(t) - \delta_1 I(t). \end{aligned} \tag{5}$$

$\mathcal{H}_3$   $N_1(t)$  depends on  $N_2(t)$  and the natural enemy's death rate and maturity rate are  $\delta_3$  and  $\kappa$ , accordingly. Then, the corresponding equation is

$$\frac{dN_1(t)}{dt} = \vartheta_2 S(t)N_2(t) - (\delta_3 + \kappa)N_1(t). \tag{6}$$

$\mathcal{H}_4$  At a rate of  $\kappa$ , when the immature natural enemy population  $N_1(t)$  grows, the mature natural enemy

population  $N_2(t)$  grows as well.  $\delta_4$  is the mortality rate of a mature enemy population. Thus, evolution model is

$$\frac{dN_2(t)}{dt} = \kappa N_1(t) - \delta_4 N_2(t). \quad (7)$$

$\mathcal{H}_5$  Releasing the number of infected pests, the immature and mature natural enemies are  $\hat{\eta}_1, \hat{\nu}_1$ , and  $\hat{\nu}_2$ , respectively, which are released periodically at a

particular time  $t = nT$ , where  $T$  is the impulsive period and  $n \in \mathbb{Z}^+$ .

By using the above-stated hypotheses, we will propose 2 mathematical models: the first model: without a virus particle and the second model: with a virus particle.

### 2.1. Model without Virus Particle

$$\left. \begin{cases} \frac{dP(t)}{dt} = P(t)(1 - P(t)) - p_1 P(t)S(t) \\ \frac{dS(t)}{dt} = p_1 P(t)S(t) - \frac{\vartheta_1 S(t)I(t)}{1 + \widehat{\sigma}S(t)} - \vartheta_2 S(t)N_2(t) - \delta S(t) \\ \frac{dE(t)}{dt} = \frac{\vartheta_1 S(t)I(t)}{1 + \widehat{\sigma}S(t)} - (\omega + \delta_1)E(t) \\ \frac{dI(t)}{dt} = \omega E(t) - \delta_1 I(t) \\ \frac{dN_1(t)}{dt} = \vartheta_2 S(t)N_2(t) - (\delta_3 + \kappa)N_1(t) \\ \frac{dN_2(t)}{dt} = \kappa N_1(t) - \delta_4 N_2(t) \end{cases} \right\} t \neq nT, \quad (8)$$

$$\left. \begin{cases} P(t^+) = P(t) \\ S(t^+) = S(t) \\ E(t^+) = E(t) \\ I(t^+) = I(t) + \hat{\eta}_1 \\ N_1(t^+) = N_1(t) + \hat{\nu}_1 \\ N_2(t^+) = N_2(t) + \hat{\nu}_2 \end{cases} \right\} t = nT. \quad (9)$$

The use of pesticides can even affect nontarget species badly. To prevent this, we can use some biological control like viruses. Take, for example, the case of North America where certain forest areas to a large extent were affected by defoliation due to the presence of larvae of gypsy moths. Here, *Lymantria dispar* multicapsid nuclear virus was sprayed extensively to control the larvae. Those larvae that consumed the virus perished and the carcasses that remained on foliage further facilitated the presence of the virus to infect the rest of the larvae still present on the foliage as well. Another prime example for the use of viruses as a biological control measure was seen in Australia where the mammalian virus and rabbit hemorrhagic disease virus were used to control the invasive European rabbit population. However, the control measure turned counterproductive since an extensive amount of rabbit population in the country got eliminated when some rabbits which were under quarantine managed to get away [34]. To

control specific insect pests, baculoviruses are well-known substitutes for chemical pesticides. These viruses cause infection only if they get exposed to the host [35]. But certain viruses can live on nonliving organisms for a time duration, and when the host gets in control of these viruses, it will get infected. As we know, even COVID-19 can live for about 3 hr in air, 4 hr in copper, 24 hr in cardboard, 2–3 days in stainless steel, and 3 days in polypropylene plastic. The most commonly used virus or baculovirus will hereafter refer to nucleopolyhedroviruses. And these viruses are characterized for their species-specific, narrow spectrum insecticidal applications. Also, they are characterized for not having any negative impacts on any other living species like plants, birds, mammals, etc. From these, we can see the importance of using virus particles in pest control. Now, we are going to modify the above mathematical model by introducing the virus particle as one more control input.

2.2. *Model with Virus Particle.* The virus is thought to spread mainly from pests to pests. This can happen between pests that are in close contact with one another. But the virus can also spread from contact with infected surfaces or objects. For example, if we spread a virus particle, a pest can be infected by touching a surface or object that has the virus on it. By using this concept, we modify the above model.

Further assumptions can be added to the above assumptions  $[\mathcal{H}_1] - [\mathcal{H}_5]$  for modification of system (8) and

(9). Let  $V(t)$  be the virus particle; they attack susceptible pests and make them infected. Infected pests when die release the virus.  $\vartheta_3$  is the production rate of virus from infected pests and  $\delta_2$  represents the death rates of virus particles.

If the virus is released periodically, with releasing amount  $\widehat{\eta}_2$ , when  $t = nT, n \in \mathbb{Z}^+$ . Then, (8) and (9) becomes

$$\left\{ \begin{array}{l} \frac{dP(t)}{dt} = P(t)(1 - P(t)) - p_1P(t)S(t) \\ \frac{dS(t)}{dt} = p_1P(t)S(t) - \frac{\vartheta_1S(t)I(t)}{1 + \widehat{\sigma}S(t)} - \vartheta_2S(t)N_2(t) - \vartheta_3S(t)V(t) - \delta S(t) \\ \frac{dE(t)}{dt} = \frac{\vartheta_1S(t)I(t)}{1 + \widehat{\sigma}S(t)} - (\omega + \delta_1)E(t) + \vartheta_3S(t)V(t) \\ \frac{dI(t)}{dt} = \omega E(t) - \delta_1I(t) \\ \frac{dV(t)}{dt} = \vartheta_3\delta_1I(t) - \delta_2V(t) \\ \frac{dN_1(t)}{dt} = \vartheta_2S(t)N_2(t) - (\delta_3 + \kappa)N_1(t) \\ \frac{dN_2(t)}{dt} = \kappa N_1(t) - \delta_4N_2(t) \end{array} \right\} t \neq nT, \quad (10)$$

$$\left\{ \begin{array}{l} P(t^+) = P(t) \\ S(t^+) = S(t) \\ E(t^+) = E(t) \\ I(t^+) = I(t) + \widehat{\eta}_1 \\ V(t^+) = V(t) + \widehat{\eta}_2 \\ N_1(t^+) = N_1(t) + \widehat{\nu}_1 \\ N_2(t^+) = N_2(t) + \widehat{\nu}_2 \end{array} \right\} t = nT. \quad (11)$$

The parametric description of the above-mentioned models is given in Table 1.

### 3. Preliminaries

Consider the solution  $X(t) = (P(t), S(t), E(t), I(t), N_1(t), N_2(t))'$  of system (8) & (9) and is a piecewise continuous function  $X: R_+ \rightarrow R_+^6$ , therefore,  $X(t)$  is continuous in  $(nT, (n + 1)T]$ ,  $n$  is a natural number, and  $X(nT^+)$  exists.

Here, we recall some preliminaries and establish results for the following sections.

**Lemma 1** (see [4]): *The left continuous function  $v \in PC'[R^+, R]$  at  $t_k, k \in \mathbb{N}$ , satisfies the inequalities*

$$\begin{cases} v'(t) \leq g(t)v(t) + h(t), & t \geq t_0, \quad t \neq t_k, \\ v(t_k^+) \leq \delta_k v(t_k) + l_k, & t = t_k, \quad k \in \mathbb{N}, \end{cases} \quad (12)$$

where  $g, h \in PC[R^+, R]$  and  $\delta_k \geq 0, l_k$  are constants; thus,

TABLE 1: Parametric values for the models.

Parameter	Description
$\vartheta_1$	Contact number of susceptible pest per unit time for infected pest
$P_1$	Predation rate of plant
$\vartheta_3$	Production rate of virus from infected pest
$k$	Conversion rate of immature to mature natural enemy
$\delta_1$	Death rate of exposed and infected pest population
$\delta_3$	Death rate of immature natural enemies
$\delta_4$	Death rate of mature natural enemies
$\delta$	Death rate of susceptible pest population
$\delta_2$	Death rate of a virus particle
$\widehat{\sigma}$	Half saturation constant
$\widehat{v}_1$	Impulsive releasing amount of immature natural enemies
$\widehat{\eta}_1$	Impulsive releasing amount of infected pest
$\widehat{\vartheta}_2$	Impulsive releasing amount of mature natural enemies
$\widehat{\eta}_2$	Impulsive releasing amount of virus particle
$\omega$	Inverse of a latent period
$v_2$	Predation rate of mature natural enemy

$$\begin{aligned}
 v(t) \leq & v(t_0) \prod_{t_0 < t_k < t} \delta_k \exp\left(\int_{t_0}^t g(u) du\right) + \sum_{t_0 < t_k < t} \left( \prod_{t_k < t_j < t} \delta_j \exp\left(\int_{t_k}^t g(u) du\right) \right) l_k \\
 & + \int_{t_0}^t \prod_{u < t_k < t} \delta_k \exp\left(\int_u^t g(x) dx\right) h(u) du, \quad t \geq t_0.
 \end{aligned}
 \tag{13}$$

If we reverse the directions of (12), we will also reverse the directions of inequality in (13).

**Lemma 2.** Consider  $C > 0$ , with  $P(t) \leq C, S(t) \leq C, E(t) \leq C, N_1(t) \leq C$ , and  $N_2(t) \leq C$  for all solutions  $X(t) = (P(t), S(t), E(t), I(t), N_1(t), N_2(t))$  for large  $t$  in system (8) and (9).

*Proof.* Consider  $(P(t), S(t), E(t), I(t), N_1(t), N_2(t))$  is a solution of (8) and (9).

Assume  $\widehat{V}(t) = P(t), S(t), E(t), I(t), N_1(t), N_2(t)$  and  $\bar{\delta} = \min\{\delta, \delta_1, \delta_3, \delta_4\}$ . Let  $t \neq nT$ . We get

$$D^+ \widehat{V}(t) + \bar{\delta} \widehat{V}(t) \leq (1 + \bar{\delta})P(t) - P^2(t) \leq C_0, \quad C_0 = \frac{(1 + \bar{\delta})^2}{4}.
 \tag{14}$$

Let  $t = nT$  and  $\widehat{V}(t^+) = \widehat{V}(t) + \widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2$ . From Lemma 1 for  $t \in (nT, (n + 1)T]$ , we get

$$\begin{aligned}
 \widehat{V}(t) \leq & \widehat{V}(0) \exp(-\bar{\delta}t) + \int_0^t C_0 \exp(-\bar{\delta}(t-s)) ds + \sum_{0 < nT < t} [\widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2] \exp(-\bar{\delta}(t - nT)) \\
 = & \widehat{V}(0) \exp(-\bar{\delta}t) + \frac{C_0}{\bar{\delta}} [1 - \exp(-\bar{\delta}t)] + \frac{[\widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2] \exp(-\bar{\delta}(t - T)) - \exp(-\bar{\delta}(t - (n + 1)T))}{1 - \exp(-\bar{\delta}T)} < \widehat{V}(0) \exp(-\bar{\delta}t) \\
 & + \frac{C_0}{\bar{\delta}} [1 - \exp(-\bar{\delta}t)] + \frac{[\widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2] \exp(-\bar{\delta}(t - T))}{1 - \exp(-\bar{\delta}T)} + \frac{[\widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2] \exp(\bar{\delta}T)}{\exp(\bar{\delta}T) - 1} \rightarrow \frac{C_0}{\bar{\delta}} + \frac{[\widehat{\eta}_1 + \widehat{v}_1 + \widehat{v}_2] \exp(\bar{\delta}T)}{\exp(\bar{\delta}T) - 1},
 \end{aligned}
 \tag{15}$$

when  $t$  is large enough.

As a result,  $\widehat{V}$  is uniformly bounded; there is a constant

$$C, := \frac{C_0}{\delta} + \frac{[\hat{\eta}_1 + \hat{\nu}_1 + \hat{\nu}_2] \exp(\delta T)}{\exp(\delta T) - 1} > 0, \quad (16)$$

such that  $P(t) \leq C, S(t) \leq C, E(t) \leq C, N_1(t) \leq C,$  and  $N_2(t) \leq C,$  for all  $t$  large enough.  $\square$

**Lemma 3.** Consider  $C > 0,$  with  $P(t) \leq C, S(t) \leq C, E(t) \leq C, V(t) \leq C, N_1(t) \leq C,$  and  $N_2(t) \leq C$  for all solutions  $X(t) = (P(t), S(t), E(t), I(t), V(t), N_1(t), N_2(t))$  for large  $t$  in (10) and (11).

*Proof.* We can easily prove this lemma by similar techniques used in Lemma 2.  $\square$

**Lemma 4.** Let  $\hat{y}(t)$  be a positive periodic solution of the system

$$\begin{cases} y'(t) = k - \nu y(t), & t \neq nT \\ y(t^+) = y(t) + \eta, & t = nT, n \in \mathbb{N}, \end{cases} \quad (17)$$

and for every solution  $y(t)$  of (17), we obtain  $|y(t) - \hat{y}(t)| \rightarrow 0$  as  $t \rightarrow \infty,$  for  $t \in (nT, (n+1)T],$

$$\begin{aligned} \hat{y}(t) &= \frac{k}{\nu} + \frac{\eta \exp(-\nu(t-nT))}{1 - \exp(-\nu T)} \text{ with } \hat{y}(0^+) \\ &= \frac{k}{\nu} + \frac{\eta}{1 - \exp(-\nu T)}. \end{aligned} \quad (18)$$

When pests become extinct, we have

$$\left\{ \begin{array}{l} \frac{dI(t)}{dt} = -\delta_1 I(t), \\ \frac{dN_1(t)}{dt} = -(\delta_3 + \kappa) N_1(t), \\ \frac{dN_2(t)}{dt} = \kappa N_1(t) - \delta_4 N_2(t), \end{array} \right\} t \neq nT, \quad (19)$$

$$\left\{ \begin{array}{l} I(t^+) = I(t) + \hat{\eta}_1, \\ N_1(t^+) = N_1(t) + \hat{\nu}_1, \\ N_2(t^+) = N_2(t) + \hat{\nu}_2 \end{array} \right\} t = nT. \quad (20)$$

For (19) and (20), from Lemma 4,

$$\begin{aligned} \hat{I}(t) &= \frac{\hat{\eta}_1 \exp(-\delta_1(t-nT))}{1 - \exp(-\delta_1 T)} \text{ and } \hat{I}(0^+) = \frac{\hat{\eta}_1}{1 - \exp(-\delta_1 T)}, \\ \hat{N}_1(t) &= \frac{\hat{\nu}_1 \exp(-(\delta_3 + \kappa)(t-nT))}{1 - \exp(-(\delta_3 + \kappa) T)} \text{ and } \hat{N}_1(0^+) = \frac{\hat{\nu}_1}{1 - \exp(-(\delta_3 + \kappa) T)}, \end{aligned} \quad (21)$$

is a positive solution of the system (19) and (20), which is globally asymptotically stable.

Using  $\hat{N}_1(t)$  on (19) and (20),

$$\frac{dN_2(t)}{dt} = \kappa \hat{N}_1(t) - \delta_4 N_2(t) \quad t \neq nT, \quad (22)$$

$$N_2(t^+) = N_2(t) + \hat{\nu}_2 \quad t = nT.$$

Then,  $t \in (nT, (n+1)T],$

$$N_2(t) = \frac{\kappa \hat{\nu}_1 [\exp(-(\delta_3 + \kappa)(t-nT)) - \exp(-\delta_4(t-nT))]}{(\delta_4 - \delta_3 - \kappa)(1 - \exp(-(\delta_3 + \kappa) T))} + N_2(nT^+) \exp(-\delta_4(t-nT)). \quad (23)$$

We get the following stroboscopic map of (22) by following the periodic discharge of impulses:

$$N_2((n+1)T^+) = \frac{\kappa \hat{\nu}_1 [\exp(-(\delta_3 + \kappa)(t-nT)) - \exp(-\delta_4(t-nT))]}{(\delta_4 - \delta_3 - \kappa)(1 - \exp(-(\delta_3 + \kappa) T))} \quad (24)$$

$$\begin{aligned} &+ N_2(nT^+) \exp(-\delta_4(t-nT)) + \hat{\nu}_2, \\ &= f(N_2(nT^+)). \end{aligned} \quad (25)$$

Then, (24) has a fixed point which is unique and positive

$$N_2^* = \frac{\kappa \hat{v}_1 [\exp(-(\delta_3 + \kappa)(t - nT)) - \exp(-\delta_4 T)]}{(\delta_4 - \delta_3 - \kappa)(1 - \exp(-(\delta_3 + \kappa)T))(1 - \exp(-\delta_4 T))} + \frac{\hat{v}_2}{(1 - \exp(-\delta_4 T))}, \quad (26)$$

which satisfy  $N_2 < f(N_2) < N_2^*$  if  $0 < N_2 < N_2^*$  and  $N_2^* < f(N_2) < N_2$  if  $N_2 > N_2^*$ . By [36], we obtained that  $N_2^*$  is

globally asymptotically stable. Then, the periodic solution of (22) is

$$\hat{N}_2(t) = \frac{\kappa \hat{v}_1}{\delta_4 - \delta_3 - \kappa} \left[ \frac{\exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} \right] + \left[ \frac{(-\kappa \hat{v}_1 / \delta_4 - \delta_3 - \kappa + \hat{v}_2) (\exp(-\delta_4(t - nT)))}{1 - \exp(-\delta_4 T)} \right], \quad (27)$$

with initial value

$$\hat{N}_2(0^+) = \frac{\kappa \hat{v}_1}{\delta_4 - \delta_3 - \kappa} \left[ \frac{1}{1 - \exp(-(\delta_3 + \kappa)T)} \right] + \left[ \frac{(-\kappa \hat{v}_1 / \delta_4 - \delta_3 - \kappa + \hat{v}_2)}{1 - \exp(-\delta_4 T)} \right]. \quad (28)$$

That is globally asymptotically stable.

In the case of system (10) and (11), when pests are extinct, we obtain the (19) and (20) together with the following equations:

$$\frac{dV(t)}{dt} = \vartheta_3 \delta_1 I(t) - \delta_2 V(t) \quad t \neq nT, \quad (29)$$

$$V(t^+) = V(t) + \hat{\eta}_2 t = nT.$$

Substituting  $\hat{I}(t)$  into (29), then we obtain

$$\frac{dV(t)}{dt} = \vartheta_3 \delta_1 I(t) - \delta_2 V(t) \quad t \neq nT, \quad (30)$$

$$V(t^+) = V(t) + \hat{\eta}_2 t = nT.$$

Then,  $t \in (nT, (n+1)T]$ ,

$$V(t) = \frac{\vartheta_3 \delta_1 \hat{\eta}_1 [\exp(-\delta_1(t - nT)) - \exp(-\delta_2(t - nT))]}{(\delta_2 - \delta_1)(1 - \exp(-\delta_1 T))} + V(nT^+) \exp(-\delta_2(t - nT)). \quad (31)$$

We get the following stroboscopic map of (30) by following the periodic discharge of impulses:

$$\begin{aligned} V((n+1)T^+) &= \frac{\vartheta_3 \delta_1 \hat{\eta}_1 [\exp(-\delta_1(t - nT)) - \exp(-\delta_2(t - nT))]}{(\delta_2 - \delta_1)(1 - \exp(-\delta_1 T))} + V(nT^+) \exp(-\delta_2(t - nT)) + \hat{\eta}_2 \\ &= f(V(nT^+)). \end{aligned} \quad (32)$$

Then, (32) has a unique positive fixed point

$$V^* = \frac{\vartheta_3 \delta_1 \hat{\eta}_1}{(\delta_2 - \delta_1)} \left[ \frac{\exp(-\delta_1 T) - \exp(-\delta_2 T)}{(1 - \exp(-\delta_1 T))(1 - \exp(-\delta_2 T))} \right] + \left[ \frac{\hat{\eta}_2}{(1 - \exp(-\delta_2 T))} \right], \quad (33)$$



which satisfy  $V < f(V) < V^*$  if  $0 < V < V^*$  and  $V^* < f(V) < V$  if  $V > V^*$ . By [36], we obtained that  $V^*$  is globally asymptotically stable. Then,

$$\widehat{V}(t) = \left( \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{(\delta_2 - \delta_1)} \right) \left[ \frac{\exp(-\delta_1(t - nT))}{(1 - \exp(-\delta_1 T))} \right] + \left( \widehat{\eta}_2 - \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{(\delta_2 - \delta_1)} \right) \left[ \frac{\exp(-\delta_2(t - nT))}{(1 - \exp(-\delta_2 T))} \right], \quad (34)$$

with initial value

$$\widehat{V}(0^+) = \left( \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{(\delta_2 - \delta_1)} \right) \left[ \frac{1}{(1 - \exp(-\delta_1 T))} \right] + \left( \widehat{\eta}_2 - \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{(\delta_2 - \delta_1)} \right) \left[ \frac{1}{(1 - \exp(-\delta_2 T))} \right]. \quad (35)$$

which is globally asymptotically stable.

After that, we will take a look at the subsystem of (8) and (9),

$$\frac{dP(t)}{dt} = P(t)(1 - P(t)). \quad (36)$$

Then, there exist a  $P = 1$  stable equilibrium which is globally asymptotic and  $P = 0$ , unstable equilibrium. Periodic solutions are as follows:

- (1)  $(0, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$ : plant pest extinction periodic solution
- (2)  $(1, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$ : pest extinction periodic solution

#### 4. Stability Analysis

By Floquet's theory of the linear T-periodic impulsive equation, we are deriving the stability of pest eradication periodic solution and plant pest eradication periodic solution of models with and without virus particles. And also, we give a comparative result in this section, which shows the effectiveness of the model with virus particles.

**Theorem 1.** Let  $(P(t), S(t), E(t), I(t), N_1(t), N_2(t))$  be any solution (8) and (9); the plant pest eradication periodic solution  $(0, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$  is unstable.

*Proof.* Considering the local stability of the periodic solution  $(0, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$ , we have

$$\begin{aligned} P(t) &= \tau_1(t), \\ S(t) &= \tau_2(t), \\ E(t) &= \tau_3(t), \\ I(t) &= \widehat{I}(t) + \tau_4(t), \\ N_1(t) &= \widehat{N}_1(t) + \tau_6(t), \\ N_2(t) &= \widehat{N}_2(t) + \tau_7(t), \end{aligned} \quad (37)$$

where  $\tau_i(t), i = 1, 2, 3, 4, 6,$  and  $7$  are small-amplitude perturbations of the solution. The linearized form of (8) and (9) is

$$\left\{ \begin{array}{l} \frac{d\tau_1(t)}{dt} = \tau_1(t) \\ \frac{d\tau_2(t)}{dt} = -(\vartheta_1 \widehat{I}(t) + \vartheta_2 \widehat{N}_2(t) + \delta) \tau_2(t) \\ \frac{d\tau_3(t)}{dt} = \vartheta_1 \widehat{I}(t) \tau_2(t) - (\omega + \delta_1) \tau_3(t) \\ \frac{d\tau_4(t)}{dt} = \omega \tau_3(t) - \delta_1 \tau_4(t) \\ \frac{d\tau_6(t)}{dt} = \vartheta_2 \widehat{N}_2(t) \tau_2(t) - (\delta_3 + \kappa) \tau_6(t) \\ \frac{d\tau_7(t)}{dt} = \kappa \tau_6(t) - \delta_4 \tau_7(t) \end{array} \right\} t \neq nT, \quad (38)$$

$$\left\{ \begin{array}{l} \tau_1(t^+) = \tau_1(t) \\ \tau_2(t^+) = \tau_2(t) \\ \tau_3(t^+) = \tau_3(t) \\ \tau_4(t^+) = \tau_4(t) \\ \tau_6(t^+) = \tau_6(t) \\ \tau_7(t^+) = \tau_7(t) \end{array} \right\} t = nT. \tag{39}$$

Let  $\tau(t)$  be the fundamental matrix of (38) and (39): where

$$\frac{d\tau(t)}{dt} = A\tau(t), \tag{40}$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\vartheta_1(t)\widehat{I} + \vartheta_2\widehat{N}_2(t) + \delta) & 0 & 0 & 0 & 0 \\ 0 & \vartheta_1\widehat{I}(t) & -(\omega + \delta_1) & 0 & 0 & 0 \\ 0 & 0 & \omega & -\delta_1 & 0 & 0 \\ 0 & \vartheta_2\widehat{N}_2(t) & 0 & 0 & -(\delta_3 + \kappa) & 0 \\ 0 & 0 & 0 & 0 & \kappa & -\delta_4 \end{pmatrix}. \tag{41}$$

Then, linearization of impulsive conditions of (8) and (9) yields

$$\begin{pmatrix} \tau_1(t^+) \\ \tau_2(t^+) \\ \tau_3(t^+) \\ \tau_4(t^+) \\ \tau_6(t^+) \\ \tau_7(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_1(t) \\ \tau_2(t) \\ \tau_3(t) \\ \tau_4(t) \\ \tau_6(t) \\ \tau_7(t) \end{pmatrix}. \tag{42}$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tau(T). \tag{43}$$

We obtain  $\tau(T) = \tau(0)e^{\int_0^T A dt}$ , where  $\tau(0)$  is the identity matrix (38) and (39). The fundamental solution matrix is as follows:

The corresponding monodromy matrix of (8) and (9) is

$$\tau(T) = \begin{pmatrix} e^T & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{-\int_0^T (\vartheta_1\widehat{I}(t) + \vartheta_2\widehat{N}_2(t) + \delta) dt} & 0 & 0 & 0 & 0 \\ 0 & * & e^{-(\omega + \delta_1)T} & 0 & 0 & 0 \\ 0 & 0 & * & e^{-\delta_1 T} & 0 & 0 \\ 0 & * & 0 & 0 & e^{-(\delta_3 + \kappa)T} & 0 \\ 0 & 0 & 0 & 0 & * & e^{-\delta_4 T} \end{pmatrix}. \tag{44}$$

It is not necessary to compute the exact value of (\*) in the following research. This is a list of the eigenvalues of the monodromy matrix  $M$ :

$$\begin{aligned}
 \lambda_1 &= e^T > 1, \\
 \lambda_2 &= e^{-\int_0^T (\vartheta_1 \widehat{I}(t) + \vartheta_2 \widehat{N}_2(t) + \delta) dt} < 1, \\
 \lambda_3 &= e^{-(\omega + \delta_1)T} < 1, \\
 \lambda_4 &= e^{-\delta_1 T} < 1, \\
 \lambda_6 &= e^{-(\delta_3 + \kappa)T} < 1, \\
 \lambda_7 &= e^{-\delta_4 T} < 1.
 \end{aligned} \tag{45}$$

Since  $|\lambda_1| > 1$ , by Floquet's theory of IDE, we found that the (8) and (9)'s plant pest extinction periodic solution is unstable.  $\square$

**Theorem 2.** *( $P(t), S(t), E(t), I(t), N_1(t), N_2(t)$ ) is any solution (8) and (9); the pest eradication periodic solution  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  is locally asymptotically stable iff  $T \leq T_{\max}$ , where*

$$T_{\max} = \frac{\vartheta_1}{(p_1 - \delta)} \left[ \frac{\widehat{\eta}_1}{\delta_1} \right] + \frac{\vartheta_2}{(p_1 - \delta)} \left[ \left( \frac{\kappa \widehat{v}_1}{\delta_4 - \delta_3 - \kappa} \right) \left( \frac{1}{(\delta_3 + \kappa)} \right) + \left( \frac{-\kappa \widehat{v}_1}{\delta_4 - \delta_3 - \kappa} + \widehat{v}_2 \right) \frac{1}{\delta_4} \right]. \tag{46}$$

*Proof.* As in the previous case, we may establish the local stability of the periodic solution  $(1, 0, 0, 0, \widehat{N}_1, \widehat{N}_2)$ . Let

$$P(t) = 1 + \tau_1(t), S(t) = \tau_2(t), E(t) = \tau_3(t), I(t) = \tau_4(t), N_1(t) = \widehat{N}_1(t) + \tau_6(t), N_2(t) = \widehat{N}_2(t) + \tau_7(t), \tag{47}$$

where  $\tau_i(t), i = 1, 2, 3, 4, 6,$  and  $7$  are small-amplitude perturbations of the solution. That is, the linearized form of (8) and (9) is

$$\left. \begin{aligned}
 \frac{d\tau_1(t)}{dt} &= -\tau_1(t) - p_1 \tau_2(t) \\
 \frac{d\tau_2(t)}{dt} &= -(-p_1 + \vartheta_1 \widehat{I}(t) + \vartheta_2 \widehat{N}_2(t) + \delta) \tau_2(t) \\
 \frac{d\tau_3(t)}{dt} &= \vartheta_1 \widehat{I}(t) \tau_2(t) - (\omega + \delta_1) \tau_3(t) \\
 \frac{d\tau_4(t)}{dt} &= \omega \tau_3(t) - \delta_1 \tau_4(t) \\
 \frac{d\tau_6(t)}{dt} &= \vartheta_2 \tau_2(t) \widehat{N}_2(t) - (\delta_3 + \kappa) \tau_6(t) \\
 \frac{d\tau_7(t)}{dt} &= \kappa \tau_6(t) - \delta_4 \tau_7(t)
 \end{aligned} \right\} t \neq nT, \tag{48}$$

$$\left\{ \begin{array}{l} \tau_1(t^+) = \tau_1(t) \\ \tau_2(t^+) = \tau_2(t) \\ \tau_3(t^+) = \tau_3(t) \\ \tau_4(t^+) = \tau_4(t) \\ \tau_6(t^+) = \tau_6(t) \\ \tau_7(t^+) = \tau_7(t) \end{array} \right\} t = nT. \quad (49)$$

If  $\tau(t)$  is the fundamental matrix of (48) and (49), then  $\tau(t)$  holds:

$$\frac{d\tau(t)}{dt} = A\tau(t), \quad (50)$$

where

$$A = \begin{pmatrix} -1 & -p_1 & 0 & 0 & 0 & 0 \\ 0 & -(p_1 + \vartheta_1 \widehat{I}(t) + \vartheta_2 \widehat{N}_2(t) + \delta) & 0 & 0 & 0 & 0 \\ 0 & \vartheta_1 \widehat{I}(t) & -(\omega + \delta_1) & 0 & 0 & 0 \\ 0 & 0 & \omega & -\delta_1 & 0 & 0 \\ 0 & \vartheta_2 \widehat{N}_2(t) & 0 & 0 & -(\delta_3 + \kappa) & 0 \\ 0 & 0 & 0 & 0 & \kappa & -\delta_4 \end{pmatrix}. \quad (51)$$

The linearization of impulsive conditions of (8) and (9) gives

$$\begin{pmatrix} \tau_1(t^+) \\ \tau_2(t^+) \\ \tau_3(t^+) \\ \tau_4(t^+) \\ \tau_6(t^+) \\ \tau_7(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_1(t) \\ \tau_2(t) \\ \tau_3(t) \\ \tau_4(t) \\ \tau_6(t) \\ \tau_7(t) \end{pmatrix}. \quad (52)$$

The monodromy matrix that corresponds to (8) and (9) is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tau(T). \quad (53)$$

Then, the eigenvalues are

$$\lambda_1 = e^{-T} < 1,$$

$$\lambda_2 = e^{\int_0^T (p_1 - \vartheta_1 \widehat{I}(t) - \vartheta_2 \widehat{N}_2(t) - \delta) dt},$$

$$\lambda_3 = e^{-(\omega + \delta_1)T} < 1, \quad (54)$$

$$\lambda_4 = e^{-\delta_1 T} < 1,$$

$$\lambda_6 = e^{-(\delta_3 + \kappa)T} < 1,$$

$$\lambda_7 = e^{-\delta_4 T} < 1.$$

The periodic solution of the system (8) and (9) for plant pest extinction is locally asymptotically stable iff  $|\lambda_2| \leq 1$  i.e.,  $T \leq T_{\max}$ . Hence, the proof.

Next, we are going to consider the subsystem of (10) and (11). The two periodic solutions are as follows:

(1)  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$ : pest extinction periodic solution

(2)  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$ : plant pest extinction periodic solution  $\square$

**Theorem 3.** Let  $(P(t), S(t), E(t), I(t), V(t), N_1(t), N_2(t))$  be any solution of the system (10) and (11); then

(i) The pest eradication periodic solution  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  is locally asymptotically stable iff  $T \leq \widehat{T}_{\max}$ , where

$$\begin{aligned} \widehat{T}_{\max} = & \frac{\vartheta_1}{(p_1 - \delta)} \left[ \frac{\widehat{\eta}_1}{\delta_1} \right] + \frac{\vartheta_2}{(p_1 - \delta)} \left[ \left( \frac{\kappa \widehat{\nu}_1}{\delta_4 - \delta_3 - \kappa} \right) \left( \frac{1}{(\delta_3 + \kappa)} \right) + \left( \frac{-\kappa \widehat{\nu}_1}{\delta_4 - \delta_3 - \kappa} + \widehat{\nu}_2 \right) \frac{1}{\delta_4} \right] \\ & + \frac{\vartheta_3}{(p_1 - \delta)} \left[ \left( \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{\delta_2 - \delta_1} \right) \frac{1}{\delta_1} + \left( \widehat{\eta}_2 - \frac{\vartheta_3 \delta_1 \widehat{\eta}_1}{\delta_2 - \delta_1} \right) \frac{1}{\delta_2} \right]. \end{aligned} \tag{55}$$

(ii) The plant pest eradication periodic solution  $(0, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  is unstable.

(i) It is possible to determine the local stability of periodic solution  $(1, 0, 0, 0, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  in a similar way to the earlier study.

*Proof*

Consider  $P(t) = 1 + \tau_1(t), S(t) = \tau_2(t), E(t) = \tau_3(t), I(t) = \tau_4(t), V(t) = \widehat{V}(t) + \tau_5(t), N_1(t) = \widehat{N}_1(t) + \tau_6(t), N_2(t) = \widehat{N}_2(t) + \tau_7(t)$ .

Then, the linearized form of (10) and (11) is

$$\left. \begin{aligned} \frac{d\tau_1(t)}{dt} &= -\tau_1(t) - p_1 \tau_2(t) \\ \frac{d\tau_2(t)}{dt} &= -(-p_1 + \vartheta_1 \widehat{I}(t) + \vartheta_2 \widehat{N}_2(t) + \vartheta_3 \widehat{V}(t) + \delta) \tau_2(t) \\ \frac{d\tau_3(t)}{dt} &= \vartheta_1 \widehat{I}(t) \tau_2(t) - (\omega + \delta_1) \tau_3(t) + \vartheta_3 \widehat{V} \tau_2(t) \\ \frac{d\tau_4(t)}{dt} &= \omega \tau_3(t) - \delta_1 \tau_4(t) \\ \frac{d\tau_5(t)}{dt} &= \vartheta_3 \delta_1 \tau_4(t) - \delta_2 \tau_5(t) \\ \frac{d\tau_6(t)}{dt} &= \vartheta_2 \widehat{N}_2(t) \tau_2(t) - (\delta_3 + \kappa) \tau_6(t) \\ \frac{d\tau_7(t)}{dt} &= \kappa \tau_6(t) - \delta_4 \tau_7(t) \end{aligned} \right\} t \neq nT, \tag{56}$$

$$\left. \begin{aligned} \tau_1(t^+) &= \tau_1(t) \\ \tau_2(t^+) &= \tau_2(t) \\ \tau_3(t^+) &= \tau_3(t) \\ \tau_4(t^+) &= \tau_4(t) \\ \tau_5(t^+) &= \tau_5(t) \\ \tau_6(t^+) &= \tau_6(t) \\ \tau_7(t^+) &= \tau_7(t) \end{aligned} \right\} t = nT. \tag{57}$$

If  $\tau(t)$  is the fundamental matrix of (56) and (57), then  $\tau(t)$  holds:



i.e,  $\widehat{w}_1(t)$  is globally asymptotically stable. When  $t \rightarrow \infty$ , by Lemma 4 and comparison theorem of the IDE,  $I(t) \geq w_1(t)$  and  $w_1(t) \rightarrow \widehat{w}_1(t)$ . Thus, there exist  $m_2 > m_1, t > m_2$  with

$$I(t) \geq w_1(t) > \widehat{I}(t) - \varepsilon_0, \quad (nT, (n+1)T], n \in \mathbb{Z}^+, n > m_2. \tag{66}$$

Using the fifth and twelfth equations of (8) and (9),

$$\begin{cases} \frac{dN_1(t)}{dt} \geq -(\delta_3 + \kappa)N_1(t), & t \neq nT, \\ N_1(t^+) = N_1(t) + \widehat{v}_1, & t = nT. \end{cases} \tag{67}$$

Comparison system is

$$\begin{cases} \frac{dw_2(t)}{dt} = -(\delta_3 + \kappa)w_2(t), & t \neq nT, \\ w_2(t^+) = w_2(t) + \widehat{v}_1, & t = nT. \end{cases} \tag{68}$$

From Lemma 4, we get (68) has a periodic solution

$$\widehat{w}_2(t) = \frac{\widehat{v}_1 \exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} \quad (nT, (n+1)T], n \in \mathbb{Z}^+, \tag{69}$$

i.e,  $\widehat{w}_2(t)$  is globally asymptotically stable. When  $t \rightarrow \infty$ , by Lemma 4 and comparison theorem of the IDE,  $N_1(t) \geq w_2(t)$  and  $w_2(t) \rightarrow \widehat{w}_2(t)$ . Thus, there exist  $m_3 > m_2, t > m_3$  with

$$N_1(t) \geq w_2(t) > \widehat{N}_1(t) - \varepsilon_0, \quad (nT, (n+1)T], n \in \mathbb{Z}^+, t > m_3 \tag{70}$$

By using (70), the sixth and thirteenth of (8) and (9), we get the following subsystem:

$$\begin{cases} \frac{dN_2(t)}{dt} \geq \kappa \left[ \frac{\widehat{v}_1 \exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} - \varepsilon_0 \right] - \delta_4 N_2(t), & t \neq nT, \\ N_2(t^+) = N_2(t) + \widehat{v}_2, & t = nT. \end{cases} \tag{71}$$

Consider the following comparison system:

$$\begin{cases} \frac{dw_3(t)}{dt} = \kappa \left[ \frac{\widehat{v}_1 \exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} - \varepsilon_0 \right] - \delta_4 w_3(t), & t \neq nT, \\ w_3(t^+) = w_3(t) + \widehat{v}, & t = nT. \end{cases} \tag{72}$$

From the above manner, we get the system (72) has a periodic solution

$$\begin{aligned} \widehat{w}_3(t) = & \frac{\kappa \widehat{v}_1}{\delta_4 - \delta_3 - \kappa} \left[ \frac{\exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} \right] \\ & + \left[ \frac{(-\kappa \widehat{v}_1 / \delta_4 - \delta_3 - \kappa + \widehat{v}_2)(\exp(-\delta_4(t - nT)))}{1 - \exp(-\delta_4 T)} \right] - \frac{\vartheta_2 \kappa \varepsilon_0}{\delta_4} \quad (nT, (n+1)T], n \in \mathbb{Z}^+, \end{aligned} \tag{73}$$

i.e,  $\widehat{w}_3(t)$  is globally asymptotically stable. When  $t \rightarrow \infty$ , by Lemma 4 and comparison theorem of the IDE,  $N_2(t) \geq w_3(t)$  and  $w_3(t) \rightarrow \widehat{w}_3(t)$ . Then, there exist  $m_4 > m_3, t > m_4$  with

$$N_2(t) \geq w_3(t) > \widehat{N}_2(t) - \varepsilon_0, \quad (nT, (n+1)T], n \in \mathbb{Z}^+, t > m_4. \tag{74}$$

By system (8) and (9) we obtain

$$\frac{dS(t)}{dt} \leq S(t) \left[ p_1(1 + \varepsilon_0) - \frac{\vartheta_1(\widehat{I}(t) - \varepsilon_0)}{1 + \widehat{\sigma}K} - \vartheta_2(\widehat{N}_2(t) - \varepsilon_0) - \delta \right] t \neq nT, \tag{75}$$

$$S(t^+) = S(t) \quad t = nT.$$

Integrating the preceding equation over the pulses,

$$S(t) \leq S(nT^+) \exp \int_{nT}^{(n+1)T} \left[ p_1(1 + \varepsilon_0) - \frac{\vartheta_1(\widehat{I}(t) - \varepsilon_0)}{1 + \widehat{\sigma}K} - \vartheta_2(\widehat{N}_2(t) - \varepsilon_0) - \delta \right] dt. \tag{76}$$

We get the stroboscopic map after the periodic discharge of impulses:

$$S((n+1)T^+) \leq S(nT^+) \exp \int_{nT}^{(n+1)T} \left[ p_1(1 + \varepsilon_0) - \frac{\vartheta_1(\widehat{I}(t) - \varepsilon_0)}{1 + \widehat{\sigma}K} - \vartheta_2(\widehat{N}_2(t) - \varepsilon_0) - \delta \right] dt$$

$$= S(nT^+) \Gamma, \tag{77}$$

where  $\Gamma = \exp \int_{nT}^{(n+1)T} [p_1(1 + \varepsilon_0) - \vartheta_1(\widehat{I}(t) - \varepsilon_0)/1 + \widehat{\sigma}K - \vartheta_2(\widehat{N}_2(t) - \varepsilon_0) - \delta] dt < 1$ , as  $T < T_{\max}$ .

Thus,  $S(nT^+) \leq S(0^+) \Gamma^n$  and so  $S(nT^+) \rightarrow 0$  as  $n \rightarrow \infty$ . Hence,  $S(t) \rightarrow 0$  as  $n \rightarrow \infty$ . Then, there exists an  $\varepsilon_1 > 0$ , sufficiently small, so that  $0 < S(t) < \varepsilon_1$  for all  $t > m_5$ . As a result, we can obtain an integer  $m_6 > 0$  such that  $\vartheta_1 I(t) S(t) / 1 + \widehat{\sigma}K < \varepsilon_2$  for all  $t \geq m_6$ .

Again, in the system (8) and (9), we have  $dP(t)/dt \geq P(1 - p_1 \varepsilon_1 - P)$ ,

That implies  $\liminf_{t \rightarrow \infty} P(t) = 1$ , i.e.,  $P(t) \rightarrow 1$  as  $t \rightarrow \infty$ .

By the third equation of (8) and (9),

$$\frac{dE(t)}{dt} \leq \varepsilon_2 - (\omega + \delta_1)E(t), \tag{78}$$

which implies

$$E(t) \leq E(0^+) e^{-(\omega + \delta_1)t} + \frac{\varepsilon_2}{(\omega + \delta_1)}$$

$$- \frac{\varepsilon_2}{(\omega + \delta_1)} e^{-(\omega + \delta_1)t} \rightarrow 0 \text{ as } t \rightarrow \infty. \tag{79}$$

Thus, there exists an integer  $m_7 > m_6$  such that  $E(t) < \varepsilon_3$  for all  $t \geq m_7$ .

By the fourth and tenth equations of (8) and (9),

$$\frac{dI(t)}{dt} \leq \omega \varepsilon_3 - \delta_1 I(t), \quad t \neq nT. \tag{80}$$

Let us consider

$$\begin{cases} \frac{dw_4^*(t)}{dt} = \omega \varepsilon_3 - \delta_1 w_4^*(t), & t \neq nT, \\ w^*(t) = w_4^*(t) + \widehat{\eta}_1, & t = nT. \end{cases} \tag{81}$$

By Lemma 4, the system (81) has a periodic solution

$$\widehat{w}_4^*(t) = \frac{\omega \varepsilon_3}{\delta_1} + \frac{\widehat{\eta}_1 \exp(-\delta_1(t - nT))}{1 - \exp(-\delta_1 T)} \quad (nT, (n+1)T], n \in \mathbb{Z}^+, \tag{82}$$

i.e.,  $\widehat{w}_4^*(t)$  is globally asymptotically stable. When  $t \rightarrow \infty$ , by Lemma 4 and comparison theorem of the IDE,  $I(t) \leq w^*(t)$  and  $w^*(t) \rightarrow \widehat{w}_4^*(t)$ ; there is an integer  $m_8$ , so,

$$I(t) \leq w_4^*(t) < \widehat{w}_4^*(t) + \varepsilon_0, \quad t \geq m_8. \tag{83}$$

Using the fifth and eleventh equations of (8) and (9) we get

$$\begin{cases} \frac{dN_1(t)}{dt} \leq \vartheta_2 \varepsilon_2 M - (\delta_3 + \kappa) N_1(t), & t \neq nT, \\ N_1(t^+) = N_1(t) + \widehat{\nu}_1, & t = nT. \end{cases} \tag{84}$$

By Lemma 4 and the comparison theorem of the IDE, there is  $\varepsilon_4 > 0$  such that

$$N_1(t) < \frac{\vartheta_2 \varepsilon_1 M}{(\delta_3 + \kappa)} + \frac{\widehat{\nu}_1 \exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} + \varepsilon_4 \tag{85}$$

$$(nT, (n+1)T], n \in \mathbb{Z}^+,$$

By (8) and (9),



$$\begin{cases} \frac{dN_2(t)}{dt} \leq \kappa \left[ \frac{\vartheta_2 \varepsilon_1 M}{(\delta_3 + \kappa)} + \frac{\widehat{v}_1 \exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} - \varepsilon_4 \right] - \delta_4 N_2(t), & t \neq nT, \\ N_2(t^+) = N_2(t) + \widehat{v}_2, & t = nT. \end{cases} \tag{86}$$

Obtaining the previous manner, then there exists  $\varepsilon_5 > 0$  such that

$$\begin{aligned} N_2(t) < \frac{\kappa \widehat{v}_1}{\delta_4 - \delta_3 - \kappa} \left[ \frac{\exp(-(\delta_3 + \kappa)(t - nT))}{1 - \exp(-(\delta_3 + \kappa)T)} \right] + \left[ \frac{(-\kappa \widehat{v}_1 / \delta_4 + \delta_3 - \kappa + \widehat{v}_2) (\exp(-\delta_4(t - nT)))}{1 - \exp(-\delta_4 T)} \right] \\ - \frac{\kappa}{\delta_4} \left[ \frac{\vartheta_2 \varepsilon_1 M}{(\delta_3 + \kappa)} + \varepsilon_4 \right] + \varepsilon_5, \quad (nT, (n + 1)T], n \in \mathbb{Z}^+. \end{aligned} \tag{87}$$

As  $t \rightarrow +\infty$ , we get  $I(t) \rightarrow \widehat{I}(t), N(t) \rightarrow \widehat{N}_1(t)$ , and  $N_2(t) \rightarrow \widehat{N}_2(t)$ , accounting that  $\varepsilon_i$  is small enough. Therefore, the pest eradication periodic solution  $(1, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$  is globally attractive.  $\square$

**Theorem 5.** *( $P(t), S(t), E(t), V(t), I(t), N_1(t), N_2(t)$ ) is any solution (10) and (11); the pest eradication periodic solution  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  is globally attractive provided  $T < \widehat{T}_{\max}$ .*

*Proof.* Same as that of Theorem 4.  $\square$

*Remark 1.* If the  $(1, 0, 0, \widehat{I}, \widehat{V}, \widehat{N}_1, \widehat{N}_2)$  of system (10) and (11) and the  $(1, 0, 0, \widehat{I}, \widehat{N}_1, \widehat{N}_2)$  of system (8) and (9) are locally asymptotically stable, then

$$\widehat{T}_{\max} > T_{\max}. \tag{88}$$

### 5. Numerical Simulations and Discussion

This research discusses an integrated eco-epidemiological plant pest natural enemy model with several impulsive methods. The impulsive influence of infected pests, virus particles, and natural enemies on pest depopulation has already been studied. Next, we will calculate the impact of releasing a certain number of infected pests, virus particles, and mature and immature natural enemies on the systems (8) and (9) and (10) and (11). The impact of a variety of impulsive techniques is also explored. The matching parametric values are supplied in Table 2 and Table 3.

Initially, an infected pest and natural enemies are used as control inputs in a pest control model. They are released on the spur of the moment. The Threshold Limit for the Impulsive Period (TLIP) is provided by  $T_{\max} = 9.55$  in this model (8) and (9). If  $T = 9.55$ , the pest extinction periodic solution also is locally and globally stable, according to Theorem 2 and Theorem 4.

We have included an extra impulsive control termed virus particle in our model. The TLIP in this model (10) and (11) is  $\widehat{T}_{\max} = 16.893$ . If  $T < 16.893$ , the pest extinction

periodic solution is both locally and globally stable, according to Theorem 3 and Theorem 4.

There may be a lack of natural enemies, infected pests, or viral particles in some real-life scenarios. This will have an impact on the impulsive period's threshold limit. As can be seen, Table 4 provides a clear image of the impulsive period's threshold limit in various conditions. As a result, the statistics in Table 4 are quite useful in dealing with these crises. It is clear that the model with virus particles is very effective and the period of releasing these impulsive control can be lengthened. Again, we can also conclude that if  $\widehat{v}_1 = \widehat{v}_2 = 0$ , then the pests stay alive and natural enemies become extinct, so we need to release more amount of natural enemies and infected pests. For pest extinction, the impulsive discharging amount of mature natural enemy ( $\widehat{v}_1 > 0$ ) should be increased when there is no impulsive discharging amount of immature natural enemy ( $\widehat{v}_1 = 0$ ). Related outcomes are also obtained when  $\widehat{v}_2 = 0$ . If  $\widehat{\eta}_1 = 0$  and the rest of the parameters are the same, then we need to release natural enemies to make the susceptible and exposed pests extinct. In other words, the virus particle has a noticeable effect on pest population size and influences the size of the pest population to a great extent. The natural enemy can properly control pests only when  $T_{\max}$  is large.

From Table 4, it is evident that the TLIP of the model with virus particle (10) and (11) is always greater than the TLIP of the model without virus particle (8) and (9); that is,  $\widehat{T}_{\max} > T_{\max}$ .

In this study, we focused mainly on TLIP. From the detailed study, it is noticed that changing the parameter creates a great impact on the threshold limit. This is well picturized in the following 3D contour plot in Figures 1–9. In Figures 1–4, the dynamics of  $T_{\max}$  in terms of  $p1, \vartheta_1, \widehat{v}_1, \widehat{\eta}_1$ , and  $\widehat{v}_2$  is shown. And also Figures 5–9 represent the dynamics of  $\widehat{T}_{\max}$  in terms of  $p1, \vartheta_3, \widehat{v}_1, \widehat{\eta}_1, \widehat{v}_2$ , and  $\widehat{\eta}_2$ .

### 6. Comparison with Other Models

This paper considered two models. The first model deals with two pest control techniques and the second deals with three

TABLE 2: Parametric values for numerical.

Parameter	Description	Value per week
$\vartheta_1$	Contact number of susceptible pest per unit time for infected pest	2.6
$k$	Conversion rate of immature to mature natural enemy	0.4
$\hat{\sigma}$	Half saturation constant	0.1
$\delta$	Death rate of susceptible pest	0.2
$\delta_3$	Death rate of immature natural enemies	0.2
$\delta_1$	Death rate of exposed and infected pest population	0.5
$\delta_4$	Death rate of mature natural enemies	0.4
$\hat{v}_1$	Impulsive releasing amount of immature natural enemies	2
$\hat{\eta}_1$	Impulsive releasing amount of infected pest	0.7
$\hat{v}_2$	Impulsive releasing amount of mature natural enemies	4
$\omega$	Inverse of the latent period	0.3
$p_1$	Predation rate of plant	1
$\vartheta_2$	Predation rate of mature natural enemy	0.3

TABLE 3: Parametric values for numerical.

Parameter	Description	Value per week
$\delta_2$	Death rate of the virus particle	0.2
$\hat{\eta}_2$	Impulsive releasing amount of virus particle	2
$\vartheta_3$	Production rate of virus from infected pest	0.5

TABLE 4: Comparison of our models: TLIP.

Parameters	$T_{\max}$	$\hat{T}_{\max}$
$\hat{\eta}_1 > 0, \hat{v}_1 > 0, \hat{v}_2 > 0$	9.55	16.893
$\hat{\eta}_1 = 0, \hat{v}_1 > 0, \hat{v}_2 > 0$	5	12.5
$\hat{\eta}_1 = 0, \hat{v}_1 = 0, \hat{v}_2 > 0$	3.75	10
$\hat{\eta}_1 = 0, \hat{v}_1 > 0, \hat{v}_2 = 0$	1.25	7.5
$\hat{\eta}_1 > 0, \hat{v}_1 = 0, \hat{v}_2 > 0$	8.3	15.643
$\hat{\eta}_1 > 0, \hat{v}_1 = 0, \hat{v}_2 = 0$	4.55	11.893
$\hat{\eta}_1 > 0, \hat{v}_1 > 0, \hat{v}_2 = 0$	5.8	13.143

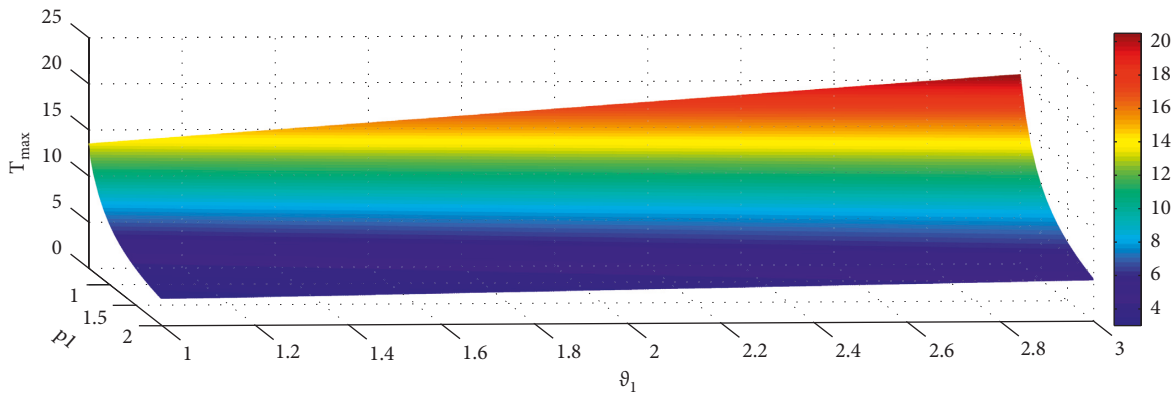


FIGURE 1: The contour plot of  $T_{\max}$  in terms of  $p_1$  and  $\vartheta_1$ .

pest control techniques, respectively. If we consider the other authors' models, they discussed biological controls in different manners. Xiang et al. in 2009 made a study on model (1), in which natural enemy is not considered as an impulsive control. And the TLIP is given by

$$T_{\max} = \frac{\vartheta_1 \hat{\eta}_1}{r \delta_1} = 3.64. \tag{89}$$

Song in 2010 formulated an SEI model using infected pests, whose functioning is the same as a microbial pesticide.

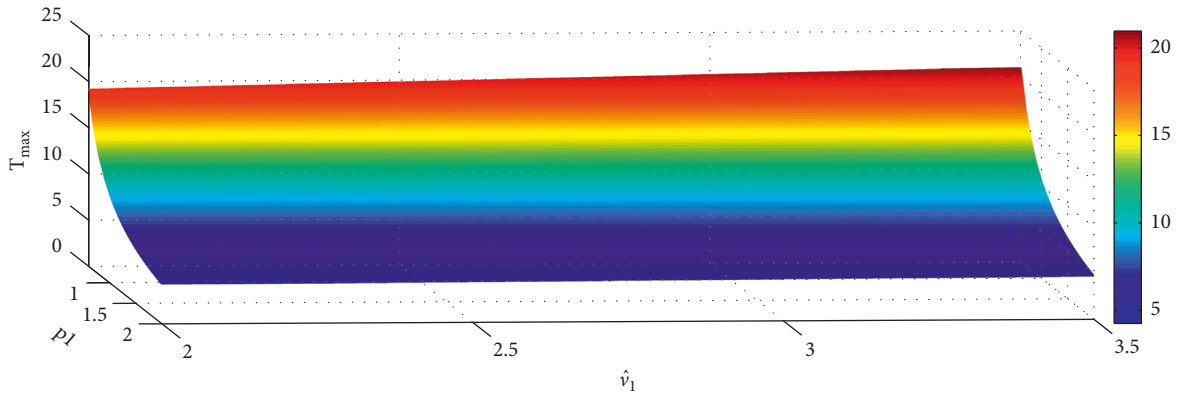


FIGURE 2: The contour plot of  $T_{\max}$  in terms of  $p_1$  and  $\hat{v}_1$ .

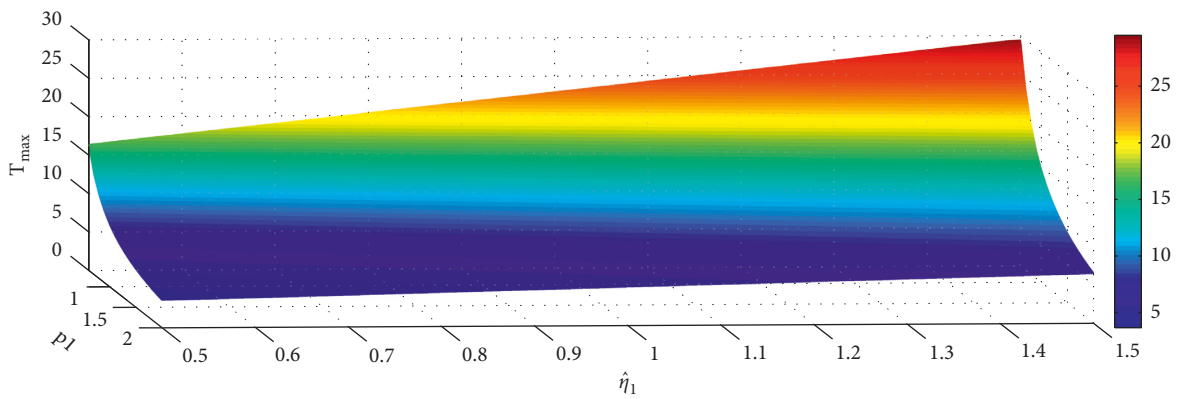


FIGURE 3: The contour plot of  $T_{\max}$  in terms of  $p_1$  and  $\hat{\eta}_1$ .

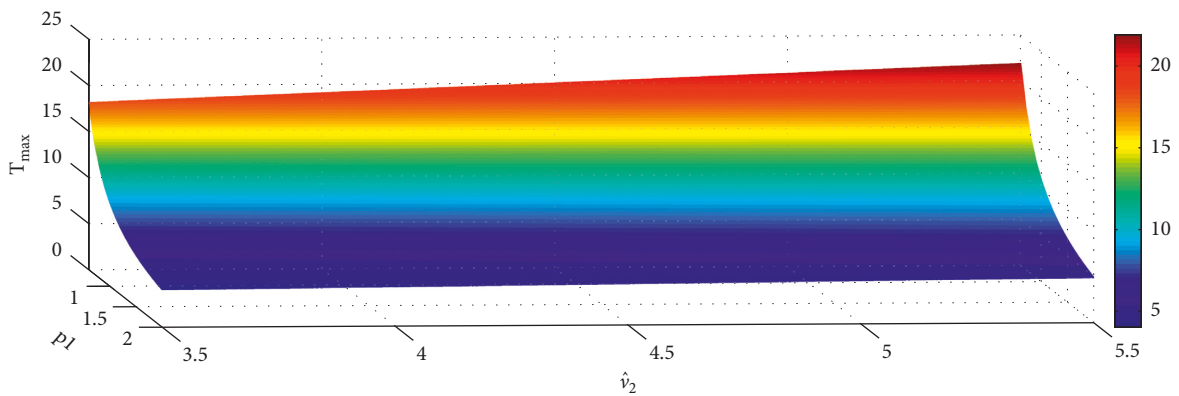


FIGURE 4: The contour plot of  $T_{\max}$  in terms of  $p_1$  and  $\hat{v}_2$ .

These infected pests can also attack healthy pests. For the extinction of pests, the authors only considered the impulsive releasing of infected pests and did not consider the importance of natural enemies. Mathur et al. in 2016 formulate an eco-epidemiology mathematical model (2) and (3)

which includes natural enemies also as a control input. And the TLIP is

$$T_{\max} = \frac{1}{r} \left[ \frac{\vartheta_1 \hat{\eta}_1}{\delta_1} + \frac{\vartheta_2 \hat{v}_2}{\delta_4} \right] = 6.64. \tag{90}$$

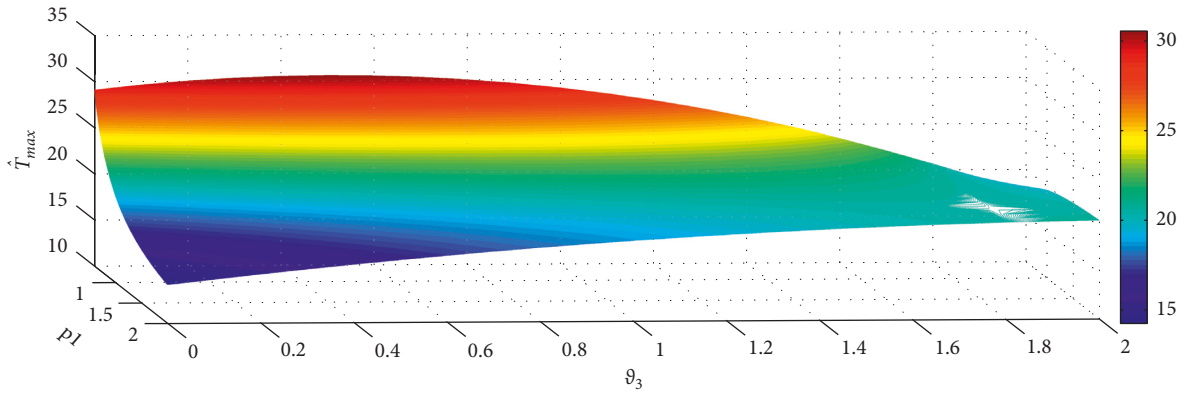


FIGURE 5: The contour plot of  $\hat{T}_{max}$  in terms of  $p1$  and  $\vartheta_3$ .

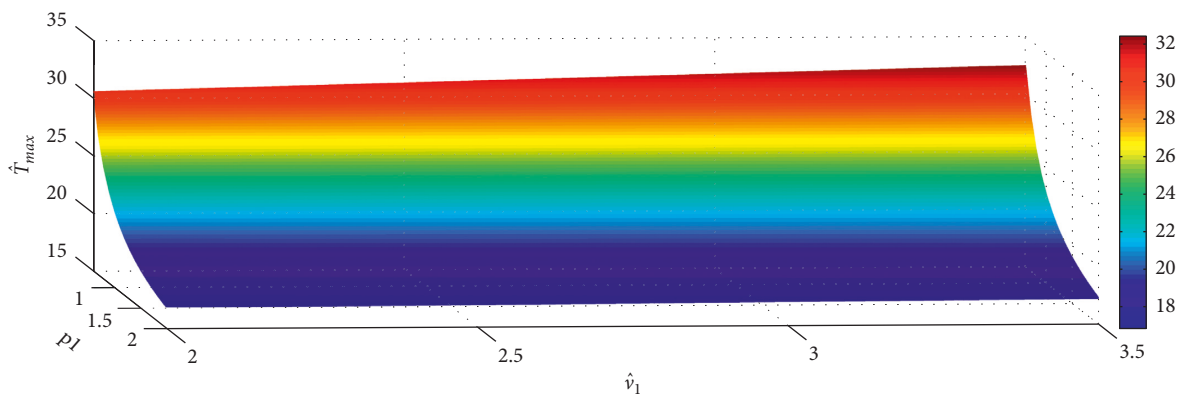


FIGURE 6: The contour plot of  $\hat{T}_{max}$  in terms of  $p1$  and  $\hat{v}_1$ .

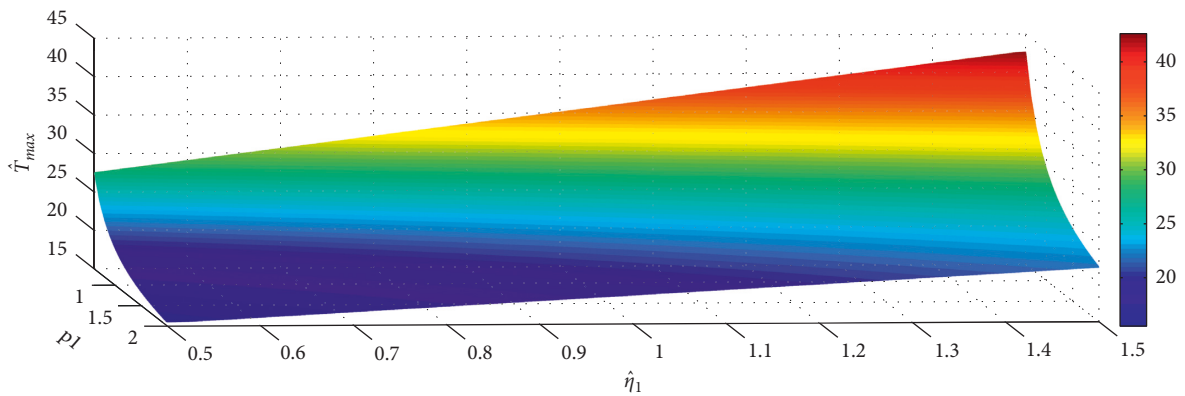


FIGURE 7: The contour plot of  $\hat{T}_{max}$  in terms of  $p1$  and  $\hat{\eta}_1$ .

The impulsive releasing of both natural enemies and infected pests affects the extinction period. And in case of Xiang et al. and Song, releasing the number of infected pests only matters. In our model, we consider natural enemies has two phases of life and this gives a more accurate result and also introduces one more control input as virus particle. If there is a shortage in the availability of infected pest, we manage the pest by spraying virus particle or by releasing

natural enemies in large quantity. Even then, it is not possible to optimize the pest population after a limit without natural enemies. This forces us to produce large amounts of virus particles and infected pests. That is to say, we need to introduce more biological controls to make models more practical. Also, the TLIP is more in our model and impulsive strategy with virus particle gives more fruitful results than the other models. This study can be described as in Table 5.

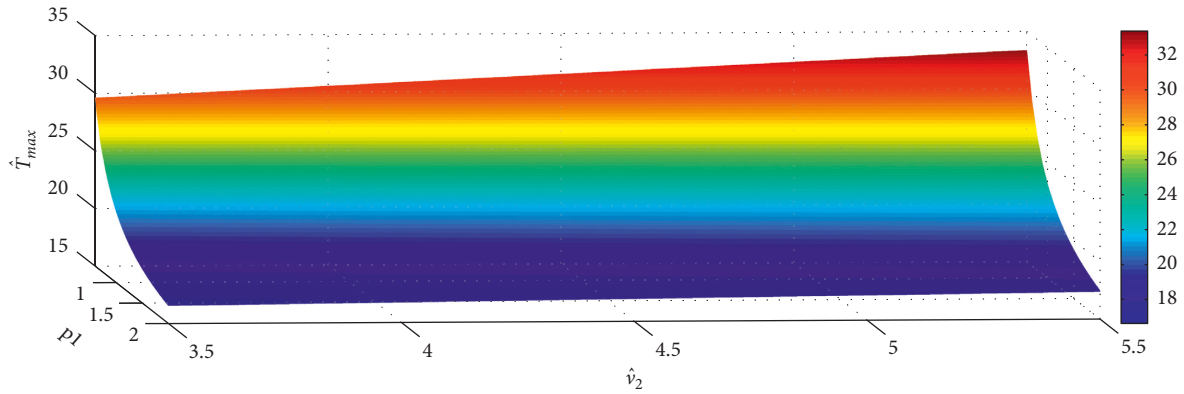


FIGURE 8: The contour plot of  $\hat{T}_{max}$  in terms of  $p_1$  and  $\hat{v}_2$ .

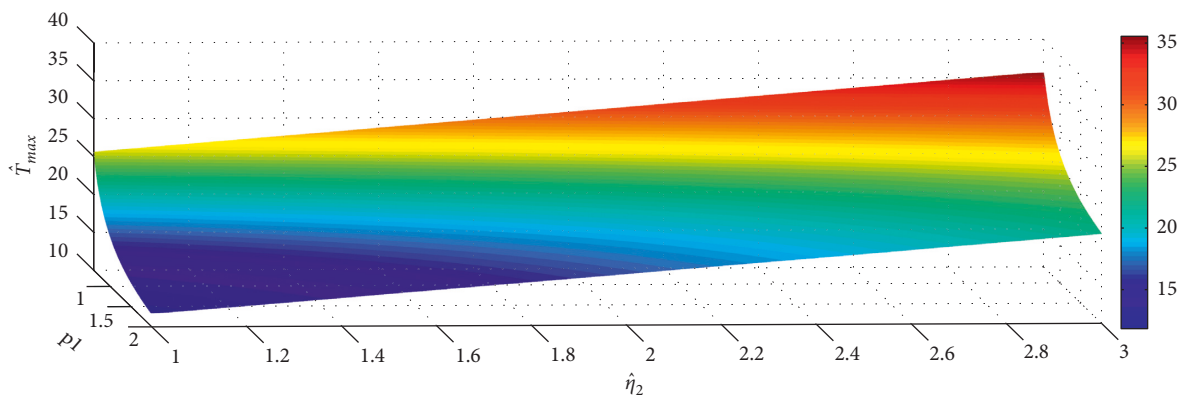


FIGURE 9: The contour plot of  $\hat{T}_{max}$  in terms of  $p_1$  and  $\hat{h}_2$ .

TABLE 5: Comparison with other models: TLIP.

Model	Xiang et al. (2009)	Mathur et al. (2016)	(8) and (9)	(10) and (11)
$T_{max}$ approx. (week)	3.64	6.64	9.55	16.893

### 7. Conclusion

This paper deals with the dynamic behavior of two integrated pest management models with and without periodic releasing of virus particles at fixed times. This work is attempting for a comparative analysis of models which provide pest control methods in the sense of IPM. IPM constitutes an eco-friendly blend of several control methods like incorporating infected pests, virus particles, and predation through natural enemies. Microbial pesticides and virus spray have the same functions. They can affect healthy pests and weaken this pest function till death. The behavior of models is analyzed using numerical methods. The influences of natural enemy populations, infected pests, and virus particles are measured numerically. For instance, a higher rate of predation decreases the size of the population of pests. The numerical analysis shows that the virus particle is more effective and this is because of the interaction between the pest and virus particle.

Further research on our models will contribute to the improvement and evaluation of pest control methods in

ecosystems. How does the stochastic release of natural enemies and infected pests affect the model dynamics? Answering these questions will give good results.

### Data Availability

No datas were used in this proposed research work.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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