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Research Article

High-Order Dimension Synthesis of Planar/Spatial Mechanisms with One-DoF by CAD Variational Geometry

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This paper proposes a (computer aided design) CAD variational geometry approach for the high-order dimension synthesis of one-DoF mechanisms based on the given velocity/acceleration of a moving platform along a prescribed trajectory. The objective of this approach is to determine the reasonable dimensions of the mechanisms when given the velocity or/and the acceleration of the moving platform along a prescribed trajectory. First, some concepts and mathematical foundations are explained for constructing the velocity/acceleration simulation mechanism of a general mechanism. Second, the inverse velocity/acceleration simulation mechanisms of the planar/spatial four-bar mechanisms with one-DoF are constructed by the CAD variational geometry. Third, when given the position and the velocity/acceleration of the coupler along a prescribed trajectory, all the reasonable dimensions of the planar/spatial four-bar mechanisms are solved from their simulation mechanisms.

1. Introduction

In industrial practice, problems are frequently encountered that require the design of mechanisms that will generate a prescribed planar/spatial trajectory [1, 2]. The problems may become further complicated if the velocity and acceleration of the motion are critical, as might be the case where possible damage to the mechanism may result from large velocity and accelerations. In some other cases, the combination of both position and velocity/acceleration are desired [1, 3]. In this aspect, Alba et al. [4] proposed a general analytic method for the synthesis of 2D and 3D mechanisms by considering velocity, acceleration, and jerk. Chang [5] synthesized adjustable four-bar mechanisms generating circular arcs with specified tangential velocities. Avilés et al. [6] proposed second-order methods for the optimum synthesis of multibody systems. Arsenault and Boudreau [7] synthesized planar parallel mechanisms while considering workspace, dexterity, stiffness, and singularity avoidance. Perry et al. [8] conducted 2 position, 2 velocity synthesis of a spherical mechanism with translating center. Using a global optimization algorithm, Prebil et al. [9] studied dimension synthesis of a four-bar mechanism in a hydraulic support. Lin et al. [10] proposed motion-curve synthesis and geometric design of a planar mechanism with intermittent variable speed. Lin and Huang [11] conducted dimension synthesis for higher-order kinematic parameters and self adjustability design of planar linkage mechanisms. In terms of CAD (computer aided design) mechanism synthesis, mainly based on analytical approaches, some suitable programs are studied and compiled [12–18]. By using CAD variational geometry, Lu solved the forward position, forward velocity/accelerations of some parallel mechanisms [19], and developed a planar four-bar simulation mechanism for the approximate dimensional synthesis [20]. However, these analytical methods are not easy to learn. The reasonable solutions are difficulty to determine from multisolutions, the processes of programming compile are complicated, and synthesis results are not straightforward.

In order to overcome these conflicts, this paper focuses on a CAD variational geometry approach for the high-order dimension synthesis of mechanisms based on the given velocity or acceleration along the prescribed trajectory.

All the reasonable dimensions of the planar or spatial four-bar mechanisms with one-DoF are determined by the CAD variational geometry when given the position and the velocity/acceleration of the platform along the prescribed trajectory.

2. Concepts and Mathematical Foundations

2.1. Some Concepts of the CAD Variational Geometry. A simulation mechanism, which includes the vectors of velocity, acceleration, angular velocity, or angular acceleration, is designated as a velocity/acceleration simulation mechanism. The purposes of designing the velocity/acceleration simulation mechanism are to solve all dimensions of links when given the velocity and acceleration of the output link. Some concepts on CAD variationalgeometry are explained as follows.

The dimensions in the simulation mechanisms are classified into the driving dimension, the driven dimension, and the fixed dimension. The driving dimensions are given to some input parameters, such as the initial position/orientation of the input link, the input velocity of the input link, the position/orientation of the output link, the output velocity of the output link, the acceleration of the output link, and its orientation. The driven dimensions are given to some key links. The fixed dimensions are given to the prescribed path and the platform. When varying or modifying some driving dimensions, all the driven dimensions are solved automatically. In addition, when varying or modifying some fixed dimensions, the driven dimensions are varied automatically [1]. Thus, the suitable dimensions of some key links can be solved.

The simulation mechanism constructed by the CAD variational geometry includes three different constraints: the completed constraint, the lacking constraint, and the over constraint. When a mechanism is in completed constraint, its number of DoF is the same as the prescribed value. In this case, when modifying some driving dimensions, all the driven dimensions are varied automatically and can be solved correctly. When a mechanism has a lacking constraint, its number of DoF is larger than the prescribed value. In this case, when modifying some driving dimensions, some driven dimensions are varied automatically, but the correct driven dimensions can not be solved. However, some key dimensions of the mechanism with the lacking constraint can be modified easily by transforming some driven dimensions into driving dimensions. When a simulation mechanism is in over constraint, its number of DoF is less than the prescribed value. In this case, the mechanism becomes a structure, and each of the driving dimensions can not be modified unless deleting one of the driving dimensions. Therefore, based on the 3 different constraints, the number of DoF of the mechanism can be determined.

2.2. Some Mathematical Foundations. A general one-DoF mechanism includes an actuator, a base B, a moving platform m, and some branches r_i $(i = 1, 2, \ldots, n)$ for connecting m

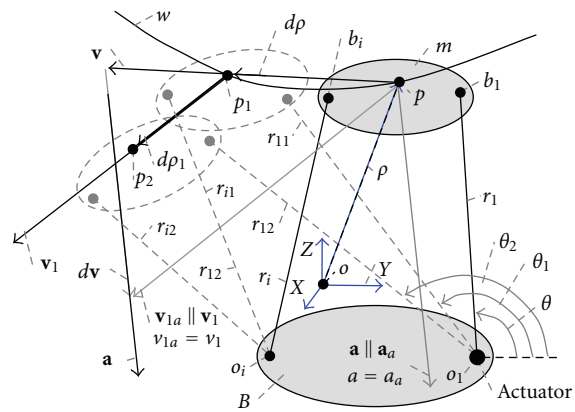

Figure 1: A general one-DoF mechanism.

with B, as shown Figure 1 (solid line). Let $\{B\}$ be a coordinate O-XYZ fixed on ground at O. Let $\parallel$ be a parallel constraint, and $\perp$ a perpendicular constraint. Let p be a point on m, and $w$be a prescribed trajectory in $\{B\}$.

Suppose the actuator drives an input link r_1 to move from an initial position angular θ to an angular θ_1 and brings p to move along w at the given position and velocity $\mathbf{v}$ and acceleration $\mathbf{a}$. In this case, it is a significant and challenging issue to solve all reasonable dimensions of the mechanism.

Since the pose of m is varied continually with time t, a point p on m is moved along w from point p to point p_1 during time increment δt. An average output velocity $\mathbf{v}^*$ of p during δt coincides with a vector $\delta\boldsymbol{\rho}$ from p to p_1. Its formula is derived as below:

$$\mathbf{v}^* = \frac{\delta\boldsymbol{\rho}}{\delta t}. \tag{1}$$

As $\delta t \to 0$ in the velocity/acceleration simulation mechanism, $\delta\boldsymbol{\rho} \to 0$ is satisfied. Thus, the velocity of point p on m can be solved as below:

$$\mathbf{v} = \lim \mathbf{v}^* = \frac{d\rho}{dt}. \tag{2}$$

Similarly, an output velocity v_1 of point p on m at position p_1 during dt_1 coincides with a line $\delta\boldsymbol{\rho}_1$ from point p_1 to point p_2 without coincident with w. The velocity of point p on m at p_1 is derived as

$$\mathbf{v}_1 = \frac{d\rho_1}{dt_1}. \tag{3}$$

In order to solve the acceleration of m at point p, constitute an auxiliary velocity vector $\mathbf{v}_{1a}$, connect its one end to p, set $\mathbf{v}_{1a} \| \mathbf{v}_1$ and $v_{1a} = v_1$. Thus, an average acceleration $\mathbf{a}^*$ of m at p during δt is parallel to a line from the end point of $\mathbf{v}$ to the end point of $\mathbf{v}_{1a}$. Its formula is derived as below:

$$\mathbf{a}^* = \frac{(v_{1a} - v)}{\delta t} = \frac{\delta\mathbf{v}}{\delta t}. \tag{4}$$

As $\delta t \rightarrow 0$ in the velocity/acceleration simulation mechanism, $\delta \mathbf{v} \rightarrow 0$ is satisfied. Thus, the acceleration $\mathbf{a}$ of m at point p can be solved as below:

$$\mathbf{a} = \lim \mathbf{a}^* = \frac{d\mathbf{v}}{dt}. \tag{5}$$

Let $dt_1 = dt$. From (2), (3), it leads to

$$dt = \frac{d\rho}{\mathbf{v}}, \qquad dt = \frac{d\rho_1}{\mathbf{v}_1}. \tag{6}$$

From (2) and (5), it leads to

$$dt = \frac{d\mathbf{v}}{\mathbf{a}}. \tag{7}$$

Based on (2), a forward velocity simulation mechanism can be constructed by the CAD variational geometry. When all the dimensions of links and the input velocity of this mechanism are given, and when gradually reduces dt from an initial value to a value being close to 0, $d\rho$ can be reduced automatically from an initial value to a value being close to 0. Thus, forward velocity v can be solved and its vector is along the tangent lines of w at p [19].

Similarly, based on (6) and (7), an inverse velocity simulation mechanism can be constructed by the CAD variational geometry. When the initial position and the input velocity of input link and the output velocity of m at point p are given, and when gradually reduces $d\rho$ from an initial value to a value being close to 0, dt must be reduced automatically from an initial value to a value being close to 0. Thus, all the driven dimensions of links in length or angular can be solved [19]. After that, the first order (velocity) dimension synthesis of the mechanisms can be conducted based on its output velocity. The first and second order (velocity and acceleration) dimension synthesis can be conducted based on its output velocity and the output acceleration.

Therefore, the mathematical foundations for design velocity/acceleration simulation mechanism are the finite differential theory and the differential geometry. When dt is 0.01s or less, the output velocity and the output acceleration solved by the CAD variational geometry are coincident with analytic solutions [19].

3. Order Dimension Synthesis of Planar Four-Bar Mechanism

Based on Section 2, the order (velocity) dimension synthesis of a planar four-bar mechanism (see Figure 2) can be conducted in a 2D sketch. In this case, all dimensions of a planar four-bar mechanism with one-DOF can be solved by CAD variational geometry when given the initial orientation and the input velocity of input link, and the output position and output velocity of output link. The synthesis processes are described as follows.

(1) Construct a horizontal line X and a vertical line Y, connect their one end to original point o. Thus a coordinate frame $\{B\}O\text{-}XY$ is constructed.

(2) Construct a spline w for prescribed trajectory in $\{B\}$.

(3) Construct a coupler by some subprocesses as follows.

  (a) Construct 2 lines l_1 and l_2, connect the one end of l_2 with l_1, and set $l_1 \perp l_2$.

  (b) Give l_1 an initial driving dimension (206) in length, l_2 an initial driving dimension (60) in length, the distance from l_2 to the one end of l_1 an initial driving dimension (70).

  (c) Construct a moving platform (coupler) m from l_1 and l_2 by the block command.

(4) Copy and paste platform m by the block copy command and paste m to form a new platform m_1.

(5) Construct a circle c_1 with central point o_1 and a circle c_2 with central point o_2.

(6) Construct a prescribe path w by spline command and fix it by the fixing command.

(7) Connect the 2 ends of l_1 in m with c_1 at j_1 and c_2 at j_2, respectively.

(8) Connect the free end of l_2 in m with w at point p.

(9) Construct a line r_1, connect its 2 ends to c_1 at j_1 and o_1, respectively. Thus, 2 revolute joints R at points j_1 and o_1 are formed, respectively.

(10) Construct a line r_2, connect its 2 ends to c_2 at j_2 and o_2, respectively. Thus, 2 revolute joints R at points j_2 and o_2 are formed, respectively. Thus, the first simulation mechanism of four-bar mechanism is constructed from (m, r_1, r_2, o_1o_2, and 4 revolute joints R).

(11) Repeat steps 7–10, except that $(p, m, r_1, r_2, j_1, j_2)$ are replaced by $(p_1, m_1, r_{11}, r_{21}, j_{11}, j_{21})$. Thus, the second simulation mechanism of the planar four-bar mechanism is constructed from ($m_1, r_{11}, r_{21}, o_1o_2$, and 4 revolute joints R).

(12) Construct a line v for velocity v, connect its one end to point p, coincide v with point p_1, and give v an initial driving dimension (50) in length.

(13) Give the vector increment of p (the distance between p and p_1) a driving dimension $d\rho = 10.1$.

(14) Construct a line ω for input angular velocity, connect its one end to point o_1, set $\omega \perp X$, give ω an initial driving dimension (20).

(15) Construct a line dt for time increment, connect its one end to point o_1, set $dt \| X$, give dt a driven dimension. Give dt a driven dimension by a constraint equation $dt = d\rho/v$.

(16) Give the angular θ between r_1 and a horizontal line a driven dimension for initial angular of r_1.

(17) Give the angular θ_1 between r_{11} and a horizontal line a driven dimension for the angular of link r_{11} by a constraint equation $\theta_1 = \theta + \omega dt$.

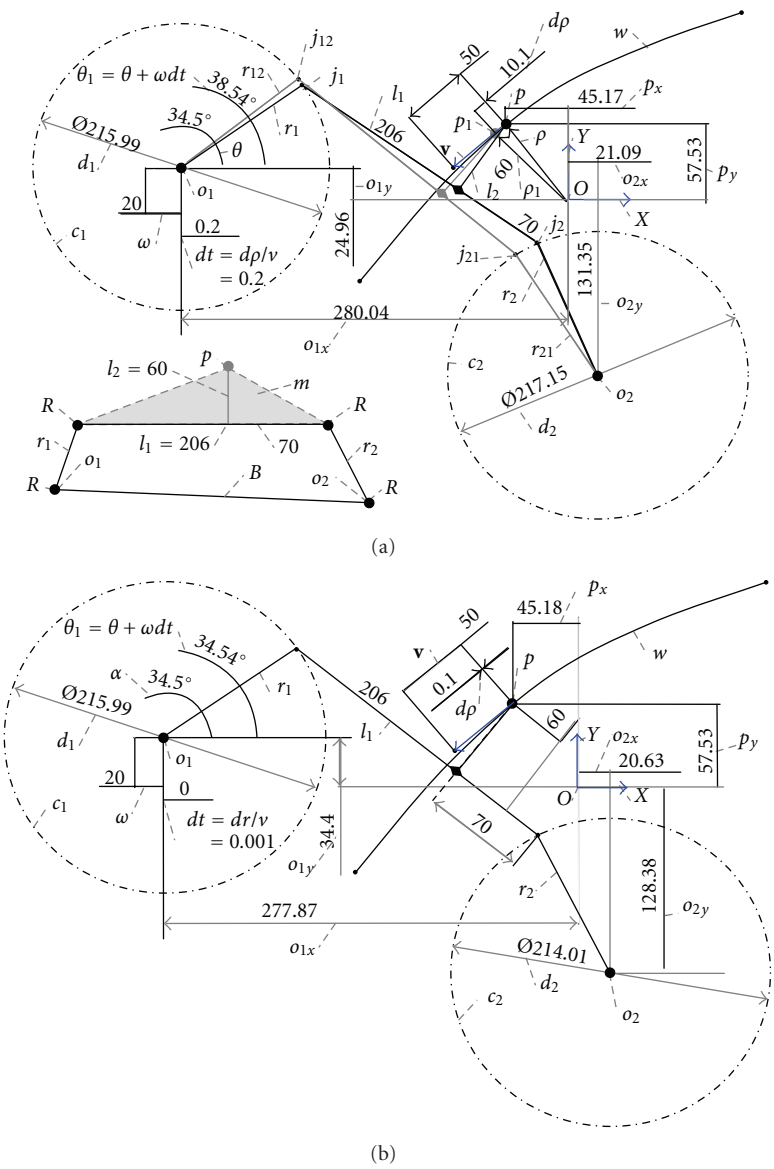

Figure 2: A planar four-bar velocity simulation mechanism versus $d\rho = 10.1$ and $d\rho = 0.1$.

(18) Give the distance from each of points $o_i (i = 1, 2)$ to X a driven dimension o_{ix} for the vertical component of o_i and to Y a driven dimension o_{iy} for the horizontal component of o_i.

(19) Give the distance from point p to y a driving dimension p_x for the horizontal component of p, give the distance from point p to X a driven dimension p_y for the vertical component of p.

(20) Give each of circles $c_i (i = 1, 2)$ a driven dimension d_i for the diameter of c_i.

When given (the output velocity v, the input angular velocity ω, the initial angular θ of link r_1, the platform m parameters l_1, l_2, l_3), and gradually reducing the vector increment $d\boldsymbol{\rho}$ of point p (the distance between p and p_1) from 10.1 to 0.1, dt is reduced automatically from 0.202 to 0.002 t. At the same time, twin simulation mechanisms of planar four-bar mechanism are coincident to each other. Thus, the driven dimensions of $(o_{iy}, o_{ix}, d_i, \theta_1)$ are solved automatically, as shown in Figure 2(b).

The solved results of $(o_{iy}, o_{ix}, d_i, \theta_1)$ corresponding to $d\rho = 10.1$ and 0.1 mm are listed in Table 1 the first and the second rows. When $d\rho = 0.1$ and varying input velocity, the solved results of $(o_{iy}, o_{ix}, d_i, \theta_1)$ corresponding to $v = 60$ and 80 are listed in Table 1 the third and the fourth rows.

When given (the output velocity v, the input angular velocity ω, the initial angular θ of link r_1, the platform m parameters l_1, l_2, l_3), the velocity simulation mechanism of the planar four-bar mechanism may include a lacking constraint. In this case, when transforming one of the driven dimensions $(o_{iy}, o_{ix}, d_i,\ i = 1, 2)$ into a driving dimension, the simulation mechanism with the lacking constraint can be

Table 1: The solved results of the planar four-bar linkage mechanism.

Given values									Solved values						
ω	v	p_x	$d\rho$	dt	l_1	l_2	l_3	θ	θ_1	d_1	d_2	o_{1x}	o_{1y}	o_{2x}	o_{2y}
°/s	mm/s	mm	mm	s	mm	mm	mm	(°)	(°)	mm	mm	mm	mm	mm	mm
20	50	45.17	10.1	0.202	206	60	70	34.5	38.54	215.99	217.15	−280.04	24.96	21.09	−131.35
20	50	45.18	0.1	0.002	206	60	70	34.5	34.54	215.99	214.01	−277.87	34.4	20.63	−128.38
20	60	45	0.1	0.0017	206	60	70	34.5	34.53	215.99	217.15	274.3	45.62	11.91	−131.93
20	80	45	0.1	0.0013	206	60	70	34.5	34.53	215.99	217.15	268.01	60.38	1.67	−134.04

Table 2: Solved results of planar four-bar mechanism as (ω, l_1, l_2, l_3) are the same as that in Table 1.

Given values									Solved values					
v	a	β	p_x	dv	θ	dt	$d\rho$	θ_1	d_1	d_2	o_{1x}	o_{1y}	o_{2x}	o_{2y}
mm/s	mm/s^2	(°)	mm	mm/s	(°)	s	mm	(°)	mm	mm	mm	mm	mm	mm
60	120	140	50	15.1	34.5	0.202	4.97	37.02	201.07	286.17	−280.04	17.22	−44.95	−160.88
60	120	140	50	0.1	34.5	0.0008	0.03	34.52	201.07	185.97	−280.04	17.22	−24.20	−115.11
60	120	120	50	0.1	34.5	0.0008	0.03	34.52	201.07	163.19	−280.04	16.49	18.51	−105.97
60	140	120	50	0.1	34.5	0.0008	0.03	34.52	201.07	191.89	−280.04	16.49	25.68	−118.40
70	140	120	50	0.1	34.5	0.0008	0.03	34.52	201.07	228.89	−280.04	16.49	34.91	−134.43

transformed into one with the completed constraint. Thus, more reasonable dimensions of mechanism can be solved easily.

4. High-Order Dimension Synthesis of Planar Four-Bar Mechanism

Based on Section 2, the high-order (velocity/acceleration) dimension synthesis of a planar four-bar mechanism (see Figure 3) can be conducted. In this case, all the dimensions of the planar four-bar mechanism with one-DoF can be solved by the CAD variational geometry when the input velocity, the output velocity, and the output acceleration are given. The processes of the high-order dimension synthesis are described as follows:

(1) Repeat steps 1–6 in Section 3.

(2) Repeat steps 7–10 in Section 3, except that $(p, m, r_1, r_2, b_1, b_2)$ are replaced by $(p_2, m_2, r_{12}, r_{22}, b_{12}, b_{22})$. In addition, p_2 is not connected with w. Thus, the third simulation mechanism of planar four-bar mechanism is constructed from $(m_2, r_{12}, r_{22}, o_1o_2)$. Point p_2 is attached on m_2.

(3) Construct a line v_1 for velocity $\mathbf{v}_1$, connect its one end to point p_1, coincide v_1 with point p_2.

(4) Construct an auxiliary line v_a, connect its one end to point p, and set $v_a \| v_1$.

(5) Construct a line a for acceleration $\mathbf{a}$, connect its one end to the free end of vector $\mathbf{v}$ at point e, give the angular between line a and line v an initial driving dimen-sion (140°) for orientation, give acceleration line a an initial driving dimension (60) in length for magnitude.

(6) Connect the free end of line v_a to line a at point e_1. Give the distance between e and e_1 an initial driving dimension (15.1) for velocity increment dv.

(7) Give dt a driven dimension by a constraint equation $dt = dv/a$.

(8) Repeat steps 16 to 20 in Section 3.

(9) Give the angular θ_2 between r_{12} and a horizontal line a driven dimension for the angular of link r_{12} by a constraint equation $\theta_2 = \theta + 2\omega dt$.

Thus, when given some driving dimensions (the output acceleration a and it orientation β, the output velocity v, the input angular velocity ω, the initial angular θ of link r_1, and the platform m parameters l_1, l_2, l_3), and gradually reducing the vector increment dv (the distance between e and e_1) from 15.1 to 0.1, dt is reduced automatically from 0.202 to 0.0008. At the same time, three simulation mechanisms of planar four-bar mechanism are coincident to each other. Thus, the driven dimensions of $(d\rho, o_{iy}, o_{ix}, d_i, \theta_i,\ i = 1, 2)$ are solved automatically, as shown in Figure 3(b).

The solved results of $(dt, d\rho, o_{iy}, o_{ix}, d_i, \theta_i, i = 1, 2)$ versus $dv = 15.1$ and 0.1 mm/s are listed in Table 2 the first and the second rows. When varying orientation β of acceleration a, the solved results of $(dt, d\rho, o_{iy}, o_{ix}, d_i, \theta_i,\ i = 1, 2)$ versus $dv = 0.1$ are listed in Table 2 the third rows. When varying a and β, the solved results of $(dt, d\rho, o_{iy}, o_{ix}, d_i, \theta_i,\ i = 1, 2)$ versus $dv = 0.1$ are listed in Table 2 the fourth rows. When varying velocity v, a and β, the solved results of $(dt, d\rho, o_{iy}, o_{ix}, d_i, \theta_i,\ i = 1, 2)$ versus $dv = 0.1$ are listed in Table 2 the fifth rows.

5. Order Synthesis of Spatial Four-Bar Mechanism with One-DoF

A spatial four-bar mechanism with one-DoF is shown in Figure 4(a). It includes a base B, a crank link r_1, a coupler (moving platform m), an oscillator r_2, 2 revolute joints R_1 and R_2, a universal joint U, and a spherical joint S. Here, U is composed of 2 crossed revolute joints R_3 and R_4. Some

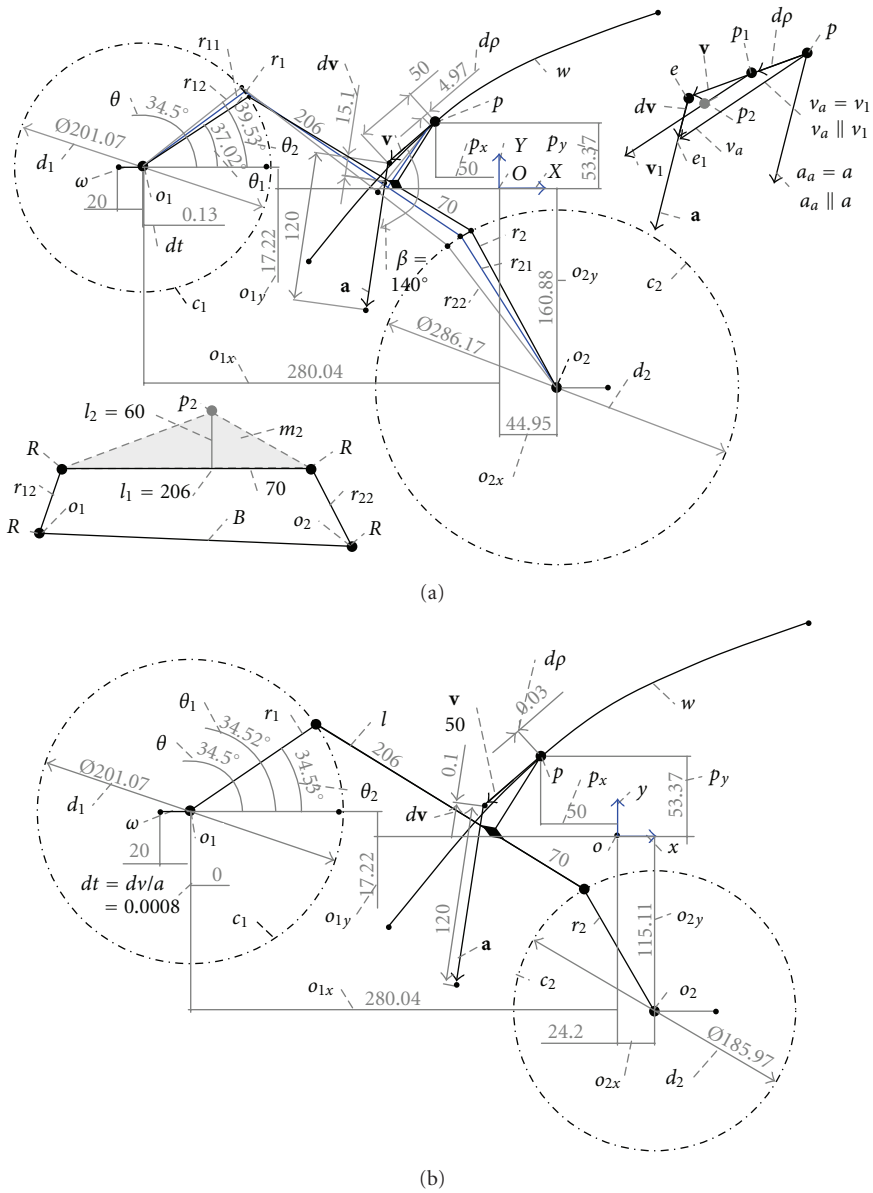

Figure 3: A planar four-bar velocity/acceleration simulation mechanism versus (a) $d\rho = 4.97$, $dv = 15.1$; (b) $d\rho = 0.03$, $dv = 0.1$.

structural constraints ($R_3 \| R_1$, $R_3 \perp R_4$, $l_1 \perp R_4$, $l_2 \perp R_4$, $l_1 = 200$, $l_2 = 50$, $l_3 = 100$) are satisfied.

The order (velocity) dimension synthesis of a spatial four-bar mechanism (see Figure 4(b)) can be conducted in a 3D sketch based on the Section 2. In this case, all the dimensions of the spatial four-bar mechanism with one-DoF can be solved by the CAD variational geometry when the initial orientation and the input velocity of input link, and the output position and output velocity of output link are given. The order dimension synthesis processes of this mechanism are described as follows:

(1) Construct 3 lines for (X, Y, Z), connect their one ends to original point O, set the 3 lines in (X, Y, Z), respectively. Thus a coordinate frame $\{B\}$ O-XYZ is constructed.

(2) Construct a spatial spline w for prescribed trajectory in $\{B\}$ by the spline command.

(3) Construct 2 spatial circles c_i with central point $o_i (i = 1, 2)$, and connect o_1 with O.

(4) Construct a line r_{10}, connect its 2 ends to c_1 and O, set $r_{10} \| Y$ and $r_{10} \perp X$.

(5) Construct 2 lines (r_{20}, n), connect their one ends to o_2, the other end of r_{20} to c_2, and set $n \perp r_{20}$.

(6) Construct 2 lines $r_i (i = 1, 2)$, connect their 2 ends to c_i at j_i and o_i, set $r_1 \perp X$ and $r_2 \perp n$. Thus, 2 revolute joints R_i at point o_i are constructed.

(7) Construct a line b_1 for axis of R_4, connect its one end with point j_1, and set $b_1 \perp X$.

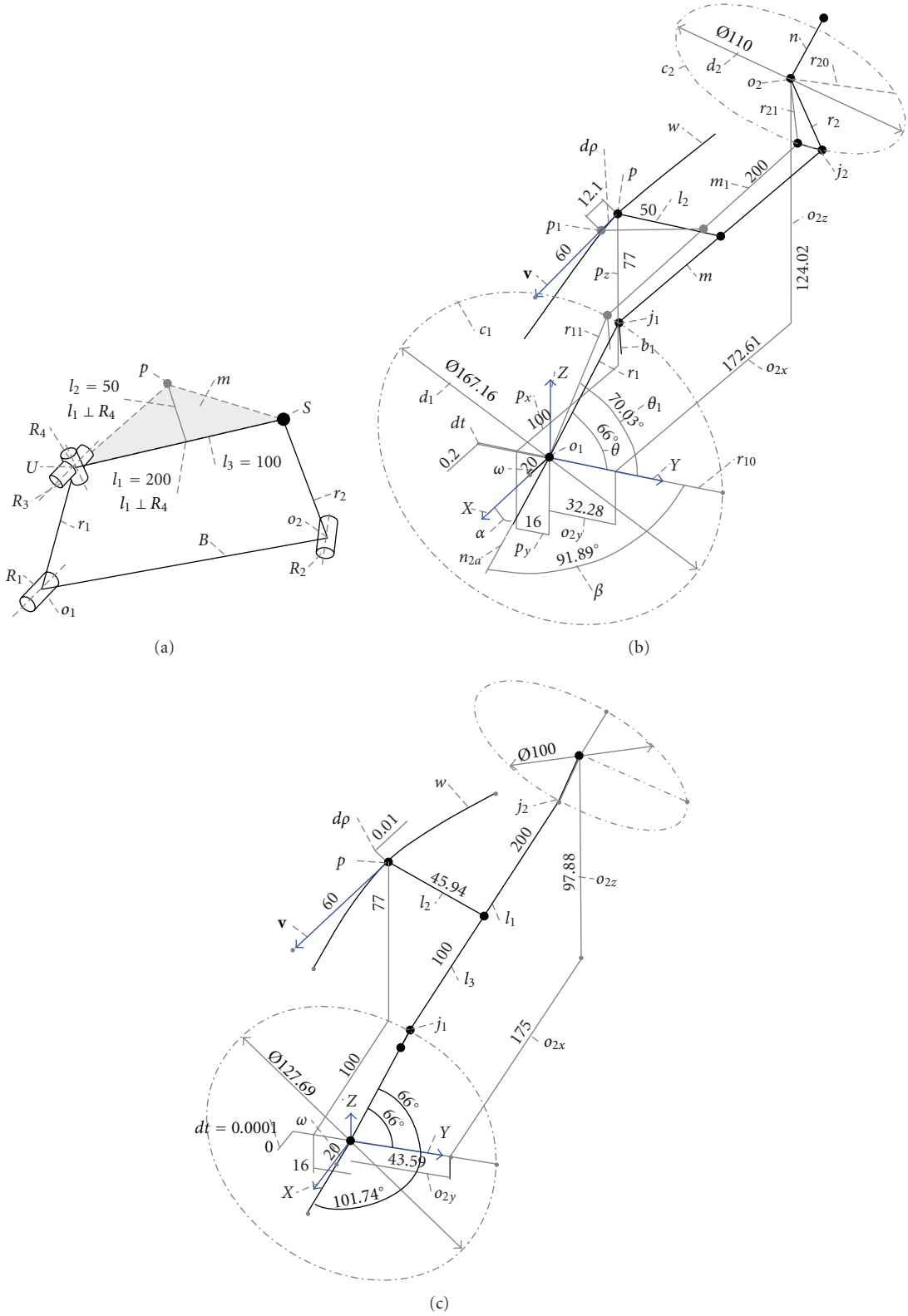

Figure 4: A spatial four-bar mechanism with one-DOF and its velocity simulation mechanism versus $d\boldsymbol{\rho} = 12.1, 0.01$.

Table 3: The solved results of a spatial four-bar linkage mechanism with one-DoF.

Given values										Solved values							
ω	v	p_x	p_y	p_z	$d\rho$	l_1	l_3	θ	dt	l_2	θ_1	β	d_1	d_2	o_{2x}	o_{2y}	o_{2z}
°/s	mm/s	mm	mm	mm	mm	mm	mm	(°)	s	mm	(°)	(°)	mm	mm	mm	mm	mm
20	60	−100	−16	77	12.1	200	100	66	0.202	50	70.03	91.89	167.16	110	−172.61	32.28	124.02
20	60	−100	−16	77	0.01	200	100	66	0.00016	45.94	66.003	101.74	127.69	100	−175	43.59	97.88
20	100	−100	−16	77	0.01	200	100	66	0.0001	45.93	66.002	101.73	127.67	100	−175	43.57	97.88

(8) Construct a platform m. Subprocesses are:

(a) Construct 2 lines l_1 and l_2, connect the one end of l_2 with l_1, and set $l_1 \perp l_2$.

(b) give l_1 an initial driving dimension (200) in length, l_2 a driven dimension in length, the distance from l_2 to the one end of l_1 a driven dimension.

(9) Connect the end point of l_2 with spline w at point p.

(10) Connect the 2 ends of l_1 to c_1 at point j_1 and point j_2, set $l_1 \perp b_1$ and $l_2 \perp b_1$. Thus, a universal joint U at point j_1 and a spherical joint S at j_2 are constructed. Finally, the first spatial four-bar mechanism with one-DOF is constructed from $(m, r_1, r_2, Oo_2, R_1, R_2, U, S)$.

(11) Repeat steps 6–10, except that the constraint equations ($l_{11} = l_1$, $l_{21} = l_2$, $l_{31} = l_3$) are satisfied and $(p, m, b_1, r_1, r_2, j_1, j_2)$ are replaced by $(p_1, m_1, b_{11}, r_{11}, r_{21}, j_{11}, j_{21})$, respectively. Thus, the second simulation mechanism of spatial four-bar mechanism is constructed from $(m_1, r_{11}, r_{21}, Oo_2, R_1, R_2, U_1, S_1)$.

(12) Construct an auxiliary line n_a, connect its one end to O, and set $n_a \| n$.

(13) Give the angular β between line n_a and X a driven dimension.

When given some driving dimensions (the output velocity v, the input angular velocity ω, the initial angular $\theta$of link r_1, the platform m parameters l_1, l_3), and gradually reducing the vector increment $d\rho$ of point p (the distance between p and p_1) from 12.1 to 0.1, dt is reduced automatically from 0.202 to 0.00016. At the same time, twin simulation mechanisms of spatial four-bar mechanism are coincident to each other. Thus, the driven dimensions of $(o_{iy}, o_{ix}, d_i, \theta_1, l_2)$ are solved automatically, as shown in Figure 4(c). The solved results of $(o_{iy}, o_{ix}, d_i, l_2, \theta_1)$ versus $d\rho = 12.1$ and 0.01 are listed in Table 3.

In fact, when given (the output velocity v, the input angular velocity ω, the initial angular $\theta$of link r_1, the platform m parameters l_1), the velocity simulation mechanism has a lack constraint. In this case, one of $(l_2, l_3, o_{iy}, o_{ix}, d_i, i = 1, 2)$ can be given a driving dimension. Thus, more reasonable dimensions of mechanism can be solved easily.

Similarly, an acceleration simulation mechanism of the spatial four-bar linkage mechanism with one-DOF can be constructed by using the processes in Section 4. Finally, when given some driving dimensions (the output velocity v, output acceleration, the input angular velocity ω, the initial angular θ of link r_1, the platform m parameters l_1, l_3), the driven dimensions of $(l_2, o_{iy}, o_{ix}, d_i, i = 1, 2)$ can be solved automatically.

6. Conclusions

The high-order dimension synthesis of one-DoF mechanisms can be conducted by the CAD variational geometry when the output velocity and acceleration are given.

The reasonable dimensions of the planar four-bar mechanism with one-DoF can be solved by the CAD variational geometry when the position, the velocity, and the acceleration of the coupler along prescribed trajectory are given.

The reasonable dimensions of the spatial four-bar mechanism with one-DoF can be solved by the CAD variational geometry when the position and the velocity of the coupler are given along prescribed trajectory.

When the mechanism is in lacking constraint, one of the driven dimensions can be modified by transforming it into a driving dimension. Thus, more reasonable dimensions may be solved easily.

This approach has not only the merits of fairly quick and straightforward, but also the advantages of accuracy and repeatability. It can be applied for the high-order dimension synthesis of other mechanisms and the spatial parallel mechanisms.

Comparing with the existing analytical methods, the proposed CAD variational geometry approach for high-order dimension synthesis of mechanism has its special merits as follows.

(1) The whole dimension synthesis processes are simple, straightforward, and easy to learn.

(2) Since no any mathematic equations are required, the complicate processes of the programming and determining reasonable solutions from multisolutions of velocity or acceleration are avoided.

(3) When given velocity and acceleration, the reasonable results of dimensional synthesis of mechanisms can be obtained and inspected easily by modifying one or some driving dimensions.

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