

## Research Article

# Dual Solutions of Non-Newtonian Casson Fluid Flow and Heat Transfer over an Exponentially Permeable Shrinking Sheet with Viscous Dissipation

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The two-dimensional boundary layer flow of a non-Newtonian Casson fluid and heat transfer due to an exponentially permeable shrinking sheet with viscous dissipation is investigated. Using similarity transformations, the governing momentum and energy equations are transformed to self-similar nonlinear ODEs and then those are solved numerically by very efficient shooting method. The analysis explores many important aspects of flow and heat transfer of the aforesaid non-Newtonian fluid flow dynamics. For the steady flow of non-Newtonian Casson fluid, more amount of wall mass suction through the porous sheet is required in comparison to that of Newtonian fluid flow. Dual similarity solutions are obtained for velocity and temperature. The viscous dissipation effect has major impact on the heat transfer characteristic. In fact, heat absorption at the surface occurs and it increases due to viscous dissipation. For higher Prandtl number, the temperature inside the boundary layer reduces, but with larger Eckert number (viscous dissipation) it is enhanced.

## 1. Introduction

The study of boundary layer flow and heat transfer over a shrinking sheet has received considerable attention of many researchers due to its numerous applications in many technological processes. These situations occur in aerodynamic extrusion of plastic sheets, polymer processing, metal spinning, artificial fibers, glass-fiber production, paper production, and drawing of plastic films. The boundary layer flow of a viscous fluid over a linearly stretching sheet was first investigated by Crane [1]. P. S. Gupta and A. S. Gupta [2] studied the effect of suction/injection with heat and mass transfer towards a stretching sheet. Dutta et al. [3] examined the boundary layer flow over a stretching sheet in the presence of heat flux. Chen and Char [4] investigated mixed convection flow and heat transfer over a stretching sheet with uniform or linear skin friction boundary condition. Ali and Magyari [5] obtained the similarity solution of mixed convection flow

over a nonlinear stretching sheet with skin friction boundary condition. Afterwards many researchers [6–11] extended Crane's work by considering different aspect. In these attempts, the flow over linear stretching/shrinking has been considered. Later, Wang [12] proposed the flow due to shrinking sheet, and the existence, uniqueness, and nonexistence of similarity solution for steady flow with mass suction were demonstrated by Miklavčič and Wang [13]. Also, some other important investigations on the boundary layer flow due to linearly shrinking sheet can be found in literature [14–19].

On the other hand, Magyari and Keller [20] initiated a study of the boundary layer flow with heat transfer over an exponentially stretching sheet. The effect of wall mass suction on the boundary layer flow and heat transfer over an exponentially stretching sheet was studied by Elbashbeshy [21]. Al-Odat et al. [22] considered an exponential temperature distribution on the boundary layer flow towards an exponentially stretching surface. Later, Sajid and Hayat [23] obtained

the series solutions for the boundary layer flow over an exponentially stretching sheet with thermal radiation using homotopy analysis method (HAM). Bidin and Nazar [24] and Ishak [25] numerically investigated the effect of radiation on the boundary layer flow and heat transfer over an exponentially stretching sheet. However, very limited attention has been given to study the boundary layer flow over an exponentially shrinking sheet though it is equally significant in many engineering processes as that of exponentially stretching sheet. The flow and heat transfer due to exponentially shrinking sheet were first discussed by Bhattacharyya [26] and the effect of magnetic field was illustrated by Bhattacharyya and Pop [27]. Rahman et al. [28] showed the effect of nanoparticles on the boundary layer flow past an exponentially shrinking sheet with second-order slip.

In all above investigations, Newtonian fluid flows are discussed. But in modern engineering, many fluids show non-Newtonian behavior; therefore, many researchers are more interested in those industrial non-Newtonian fluids and their dynamics. A single constitutive equation is not sufficient to cover all physical properties of non-Newtonian fluids and thus several non-Newtonian fluid models [29–32] have been introduced to explain all such behaviors. One of such non-Newtonian fluids is Casson fluid. Casson fluid behaves like elastic solids, and, for this kind of fluid, a yield shear stress exists in the constitutive equation. The examples of Casson fluid are as follows: jelly, tomato sauce, honey, soup, and concentrated fruit juices. Human blood can also be treated as Casson fluid. Fredrickson [33] considered the steady flow of a Casson fluid in a tube. Hayat et al. [34] investigated the MHD boundary layer flow of a Casson fluid over stretched sheet. The electrically conducting boundary layer flow of a Casson fluid towards an exponentially shrinking sheet was studied by Nadeem et al. [35]. Bhattacharyya et al. [36] reported analytic solution of a MHD flow of a Casson fluid over a permeable stretching/shrinking sheet. The effect of thermal radiation on MHD stagnation boundary layer flow of a Casson fluid and heat transfer towards a stretching sheet was investigated by Bhattacharyya [37]. Nandy [38] obtained analytic solution of MHD flow and heat transfer of a Casson fluid near a stagnation point towards a stretching sheet in the presence of partial slip. Thiagarajan and Senthilkumar [39] reported similarity solutions of MHD flow of a Casson fluid towards a permeable shrinking sheet. Mukhopadhyay and Gorla [40] investigated the effect of chemical reaction on Casson fluid and mass transfer over an exponentially stretching sheet. Recently, Qasim and Noreen [41] studied boundary layer flow and heat transfer of a Casson fluid over a permeable shrinking sheet.

Fluid viscosity changes some amount of kinetic energy into thermal energy during motion and this effect of viscosity is irreversible. This is known as viscous dissipation and though it is small, it is very important. Brinkman [42] was the first who considered the effect of viscous dissipation. Kishan and Deepa [43] studied the boundary layer flow and heat transfer near a stagnation point immersed in a porous medium in the presence of viscous dissipation. Singh [44] examined the viscous boundary layer flow and heat transfer of an electrically conducting fluid past a moving plate in a porous medium with viscous dissipation and variable

viscosity. The unsteady MHD flow over a permeable stretching sheet with combined effects of viscous dissipation and radiation was investigated by Chand and Jat [45]. Recently, Malik et al. [46] investigated the electrically conducting flow of a non-Newtonian Sisko fluid past a stretching cylinder with viscous dissipation.

To the best of the authors' knowledge, no one yet has considered the Casson fluid and heat transfer over an exponentially shrinking sheet considering the viscous dissipation effect. Therefore, in this research, the boundary layer flow of a Casson fluid and heat transfer due to an exponentially permeable shrinking sheet with viscous dissipation is investigated. The self-similarity ODEs obtained here are solved numerically. The computed results are plotted in graphs and detailed discussion is given.

## 2. Mathematical Formulation

Consider a steady two-dimensional incompressible fluid flow and heat transfer of a Casson fluid over an exponentially permeable shrinking sheet with viscous dissipation. The sheet is situated at  $y = 0$ , with the flow being confined in  $y > 0$ . It is also assumed that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as [47, 48]

$$\tau_{ij} = \begin{cases} \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi > \pi_c, \\ \left( \mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) 2e_{ij}, & \pi < \pi_c, \end{cases} \quad (1)$$

where  $e_{ij}$  is the rate-of-strain tensor,  $\mu_B$  is the Casson coefficient of viscosity,  $p_y$  is the yield stress of fluid,  $\pi$  is the product of the component of deformation rate with itself, and  $\pi_c$  is the critical value of the product of the component of the rate-of-strain tensor with itself. Under these conditions the boundary layer equations for the steady flow of Casson fluid and heat transfer over an exponentially shrinking sheet may be written in usual notation as [36]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2}, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial u}{\partial y} \right)^2, \quad (4)$$

subject to the boundary conditions

$$u = -u_w = -ae^{x/L},$$

$$v = v_0 e^{x/2L},$$

$$T = T_w = T_\infty + e^{2x/L},$$

at  $y = 0$ ,

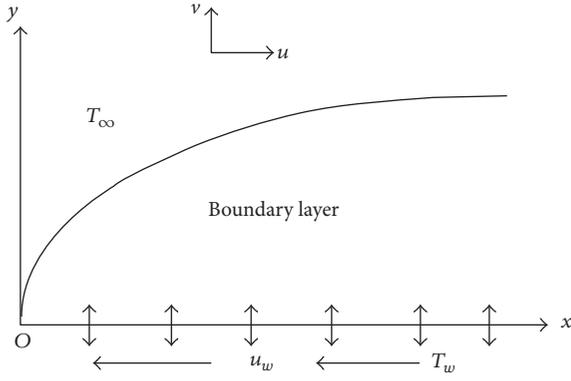


FIGURE 1: Sketch of the physical flow problem.

$$\begin{aligned}
 u &\longrightarrow 0, \\
 T &\longrightarrow T_{\infty}, \\
 &\text{as } y \longrightarrow \infty,
 \end{aligned} \tag{5}$$

where  $u$  and  $v$  are velocity components in  $x$  and  $y$  directions, respectively,  $u_w$  is shrinking velocity of the sheet with  $a$  being a positive constant,  $\nu$  is the kinematic fluid viscosity,  $\rho$  is the density,  $\beta = \mu_B \sqrt{2\pi_c}/p_y$  is the Casson parameter,  $\nu_0 (< 0)$  is the strength of the suction velocity,  $T$  is the temperature,  $L$  is the reference length,  $\alpha$  is the thermal diffusivity of the fluid,  $c_p$  is the specific heat,  $T_w$  is the temperature at the sheet, and  $T_{\infty}$  is the free stream temperature assumed to be constant. A physical sketch of the flow problem is given in Figure 1.

The stream function  $\psi$  is defined as

$$\begin{aligned}
 u &= \frac{\partial \psi}{\partial y}, \\
 v &= -\frac{\partial \psi}{\partial x}.
 \end{aligned} \tag{6}$$

Introducing the following transformation [40]

$$\begin{aligned}
 u &= ae^{x/L} f'(\eta), \\
 v &= -\sqrt{\frac{av}{2L}} e^{x/2L} [f(\eta) + \eta f'(\eta)], \\
 \eta &= \sqrt{\frac{a}{2Lv}} e^{x/2L} y, \\
 T &= T_{\infty} + be^{2x/L} \theta(\eta).
 \end{aligned} \tag{7}$$

Using (6) and (7), the equation of continuity (2) is automatically satisfied and the nonlinear partial differential equations (3) and (4) are transformed into the following ordinary differential equations:

$$\begin{aligned}
 \left(1 + \frac{1}{\beta}\right) f'''' + ff'' - 2f'^2 &= 0, \\
 \frac{1}{\text{Pr}} \theta'' + f\theta' - 4f'\theta + \text{Ec} \left(1 + \frac{1}{\beta}\right) f''^2 &= 0.
 \end{aligned} \tag{8}$$

The transformed boundary conditions (6) are

$$\begin{aligned}
 f(0) &= S, \\
 f'(0) &= -1, \\
 \theta(0) &= 1, \\
 f'(\eta) &\longrightarrow 0, \\
 \theta(\eta) &\longrightarrow 0, \\
 &\text{as } \eta \longrightarrow \infty,
 \end{aligned} \tag{9}$$

where prime denotes differentiation with respect to  $\eta$ ,  $\text{Pr} = \nu/\alpha$  is the Prandtl number,  $\text{Ec} = a^2/bc_p$  is the Eckert number, and  $S = -\nu_0 \sqrt{2L/av} > 0$  is the suction parameter.

The quantities of engineering interest are the local skin friction coefficient and the local Nusselt number which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2}, \tag{10}$$

$$\text{Nu}_x = \frac{xq_w}{\kappa(T_w - T_{\infty})},$$

where  $\tau_w$  is the shear stress along the exponentially shrinking sheet and  $q_w$  is the heat flux from the sheet and those are defined as

$$\tau_w = \left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{11}$$

$$q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

Therefore, we get the wall skin friction coefficient  $C_f$  and the local Nusselt number  $\text{Nu}_x$  as follows:

$$C_f \text{Re}_x^{1/2} \sqrt{\frac{2L}{x}} = \left(1 + \frac{1}{\beta}\right) f''(0), \tag{12}$$

$$\text{Nu}_x \text{Re}_x^{-1/2} \sqrt{\frac{2L}{x}} = -\theta'(0),$$

where  $\text{Re}_x = xu_w/\nu$  is the local Reynolds number.

### 3. Results and Discussion

The analysis of the obtained numerical results using shooting method [16, 19] explores the condition for which the steady flow is possible for the non-Newtonian Casson fluid. According to Miklavčič and Wang [13] and Fang and Zhang [14], for Newtonian fluids, the steady two-dimensional flow due to a linearly shrinking sheet with wall mass transfer occurs only when the wall mass suction parameter is greater than or equal to 2. However in case of Newtonian fluid ( $\beta \rightarrow \infty$ ) over an exponentially shrinking sheet the similarity solution is achieved when  $S \geq 2.266684$ , which is consistent with the results obtained by Bhattacharyya [26]. On the other hand,

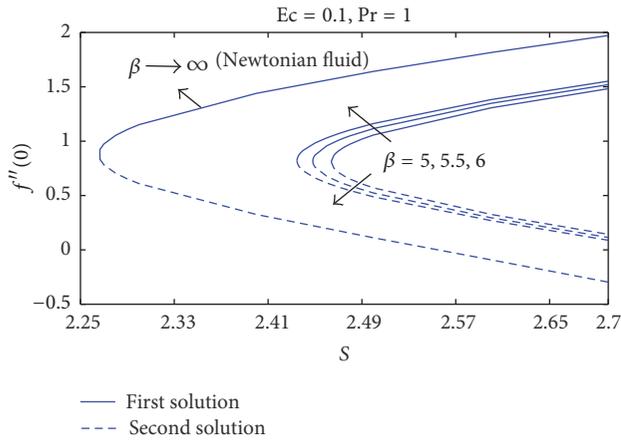


FIGURE 2: Skin friction coefficient  $f''(0)$  with  $S$  for different values of  $\beta$ .

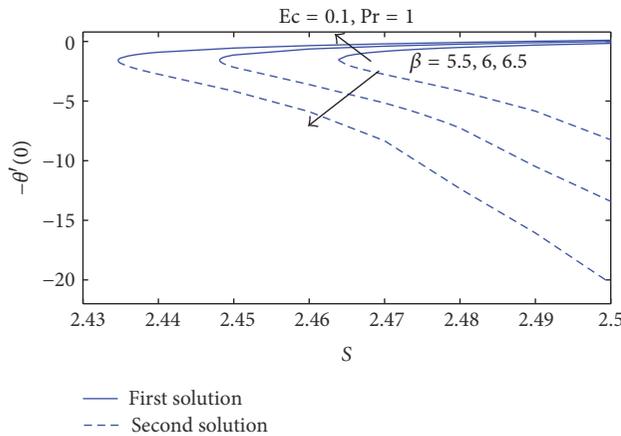


FIGURE 3: Heat transfer coefficient  $-\theta'(0)$  with  $S$  for different values of  $\beta$ .

it is quite different for non-Newtonian Casson fluid. For  $\beta = 6.5$ , the flow has dual similarity solutions for  $S \geq 2.43467$  and consequently for  $S < 2.43467$  no similarity solution exists. For  $\beta = 6$ , the dual similarity solutions exist if the ranges of  $S$  are  $S \geq 2.4481$  and hence no similarity solution exists for  $S < 2.4481$ . Further, it is interesting to note that more increment in Casson parameter  $\beta$  causes more reduction in the solution suction domain. For  $\beta = 5.5$ , the similarity solution exists when  $S \geq 2.46398$  and thus no solution exists for  $S < 2.46398$ . So, for the steady flow of Casson fluid (with decreasing values of  $\beta$ ), more amount of wall mass suction is needed in comparison with that of Newtonian flow. This effect is physical realistic because when the Casson parameter  $\beta$  decreases the yield stress  $p_y$  becomes larger and for this more amount of vorticity generated due to shrinking and to suppressing the vorticity the requirement of mass suction is higher for the Casson fluid.

The variations of local skin friction coefficient  $f''(0)$  and local heat transfer coefficient  $-\theta'(0)$  (which are proportional to the wall skin friction coefficient and the local Nusselt number or the rate of heat transfer, resp.) with suction  $S$  for

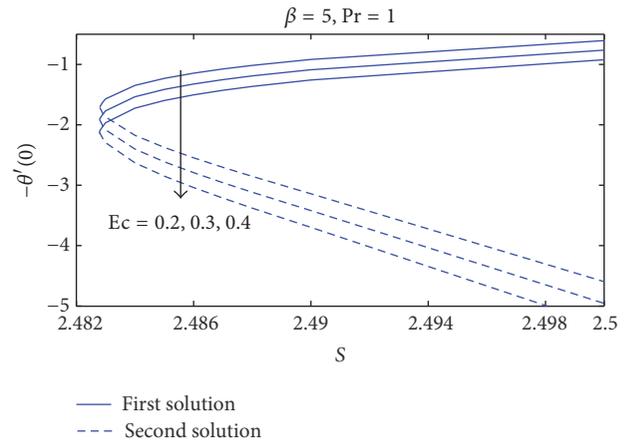


FIGURE 4: Heat transfer coefficient  $-\theta'(0)$  with  $S$  for different values of  $Ec$ .

several values of Casson parameter  $\beta$  are shown in Figures 2 and 3. From Figures 2 and 3, it is observed that the skin friction coefficient and the heat transfer rate increase with an increase in the values of  $\beta$  for the first solution while for the second solution the values decrease. Hence, for non-Newtonian Casson fluid, the local skin friction coefficient is less compared to that of Newtonian fluid case for first solution and reverse result shows in case of second solution. It is also observed from these figures that the values of skin friction coefficient  $f''(0)$  are always positive, which implies that the fluid exerts a drag force on the sheet and the heat transfer coefficient  $-\theta'(0)$  is negative which indicates the heat absorption at the sheet; that is, the heat flows from the ambient fluid to the sheet. Also, the values of temperature gradient at the sheet  $-\theta'(0)$  for different values of the Eckert number  $Ec$  and Prandtl number  $Pr$  are plotted in Figures 4 and 5, respectively. From Figure 4 it is observed that for both the first and second solutions the heat transfer rate decreases with increasing values of  $Ec$ . Thus, more heat is generated in the boundary layer region due to the viscous dissipation and hence it reduces the heat transfer rate from the sheet; that is, it enhances the heat absorption, whereas, with the increase in  $Pr$ , the value of  $-\theta'(0)$  (Figure 5) increases for both solutions and for higher values of  $Pr$  it becomes positive, which implies that heat transfers from the hot sheet to the ambient fluid. In addition, to provide a clear view of the flow field the streamlines are plotted for both solutions for fixed values of suction parameter and Casson parameter in Figures 6 and 7.

Figure 8 shows the effect of Casson parameter on the velocity field. It reveals that due to the increase of Casson parameter the boundary layer thickness reduces for the first solution and increases for the second solution. Physically, the Casson parameter produces a resistance in the fluid flow and consequently the boundary layer thickness decreases for higher value of a Casson parameter. It is quite obvious that the magnitude of the velocity is greater in non-Newtonian Casson fluid when compared with the Newtonian fluid. On the other hand, the effects of all physical parameters on temperature field for the flow of Casson fluid are also significant. Therefore, the dimensionless temperature profiles for

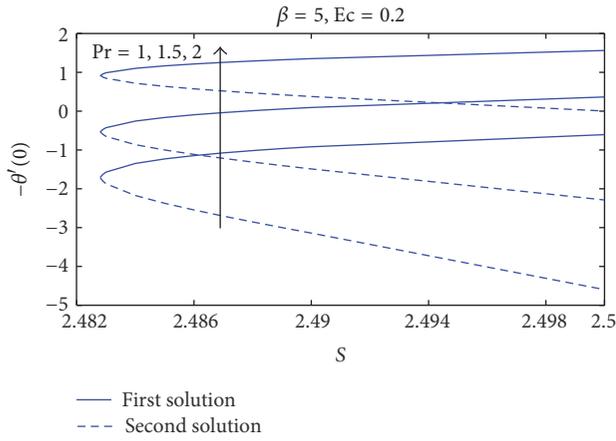


FIGURE 5: Heat transfer coefficient  $-\theta'(0)$  with  $S$  for different values of  $Pr$ .

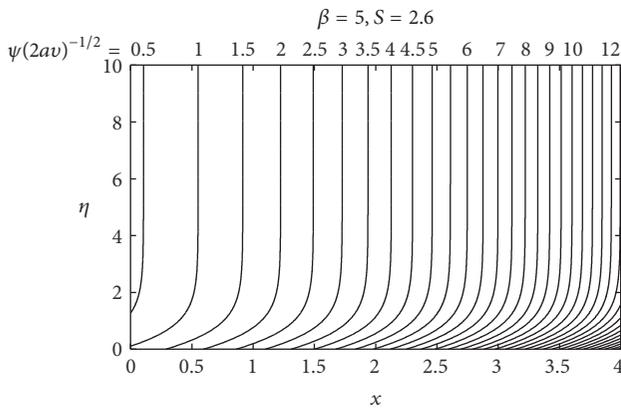


FIGURE 6: The streamline of the flow for first solution with  $L = 1$ .

various values of Casson parameter, viscous dissipation, and Prandtl number are plotted in Figures 9–11. Examining the dual temperature profiles (Figure 9) for various  $\beta$ , it is noted that the temperature of fluid decreases for first solution and consequently reduces the thermal boundary layer thickness while for second solution it increases. This is due to the fact that the introduction of tensile stress due to elasticity leads to contraction in the boundary layer thickness. Figure 10 shows that the thermal boundary layer thickness increases with an increase in the value of Eckert number for both solutions with thermal overshoot in all cases. On the other hand, the temperature of fluid decreases with increasing Prandtl number for both solutions as shown in Figure 11. Physically, an increase in the value of Prandtl number implies the reduction of fluid thermal conductivity increases which in turn causes a decrease in the thermal boundary layer thickness.

The suction is very important to maintain the steady flow near the sheet by delaying the separation. Since the suction is necessary, the effects of suction parameter  $S$  on the velocity and temperature profiles are important in analytical as well as practical point of view. Figures 12 and 13 demonstrate the velocity and temperature profile for different values of suction  $S$ . For first solution, velocity boundary layer thickness

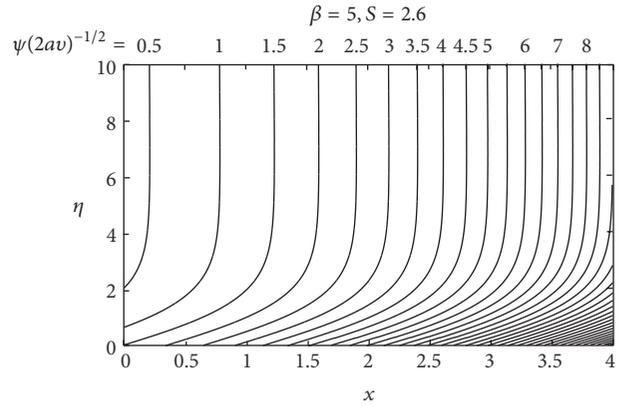


FIGURE 7: The streamline of the flow for second solution with  $L = 1$ .

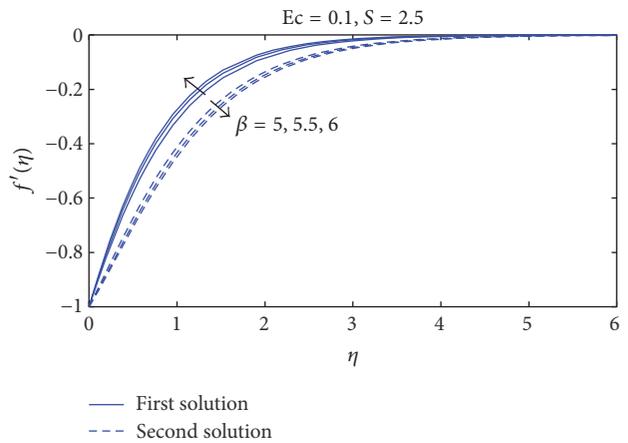


FIGURE 8: Velocity profiles for different values of  $\beta$ .

decreases with increasing values of suction and it increases for second solution. The temperature profiles for different values of  $S$  are illustrated in Figure 13 and from this figure it is clear that the temperature decreases with increasing  $S$  for both solutions. It is also noted that in second solution heat absorption at the surface is found for  $S = 2.5$  and high heat transfer from the sheet is observed for  $S = 2.6$  and  $2.7$ .

Dual solutions are categorized as first solution (upper branch solution) and second solution (lower branch solution). Weidman et al. [49], Weidman and Ali [50], Roşca and Pop [51], Mahapatra and Nandy [52], Nazar et al. [53], and Rahman et al. [28] have performed stability analysis to determine which solution is stable and physically realizable. They have established that the first solution is the stable solution and second one is unstable [28]. However, the physical importance of the second in these cases is less but the situations in some other problem may arise where this has more physical significance. Also, it is worth mentioning that both solutions satisfied the far field boundary conditions asymptotically, which are supporting the validity of the obtained numerical results. In addition, a graphical comparison of velocity profile for Newtonian fluid ( $\beta \rightarrow \infty$ ) with the published results of Bhattacharyya [26] is made in Figure 14 and those are found in excellent agreement.

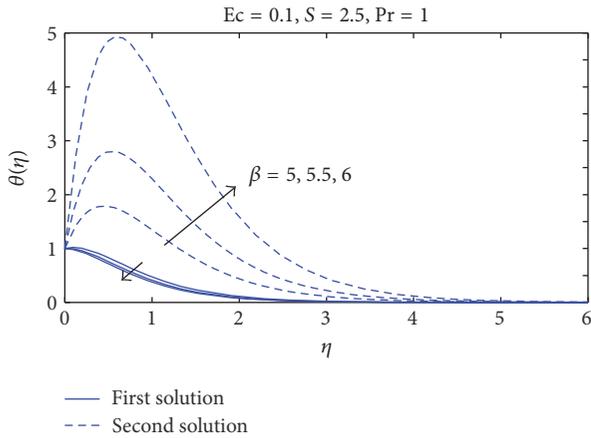


FIGURE 9: Temperature profiles for different values of  $\beta$ .

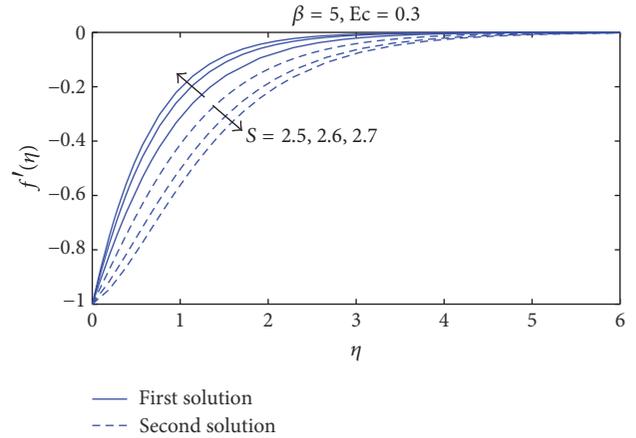


FIGURE 12: Velocity profiles for different values of  $S$ .

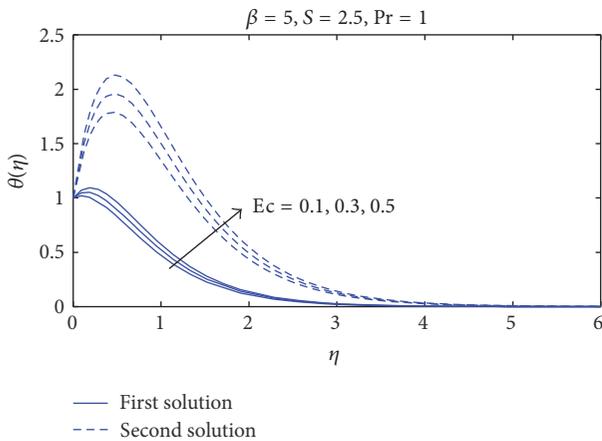


FIGURE 10: Temperature profiles for different values of  $Ec$ .

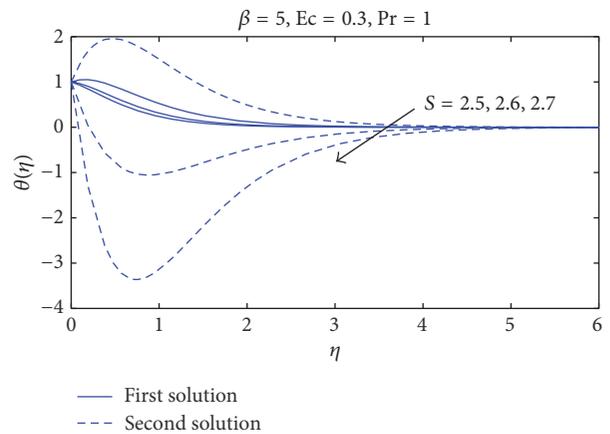


FIGURE 13: Temperature profiles for different values of  $S$ .

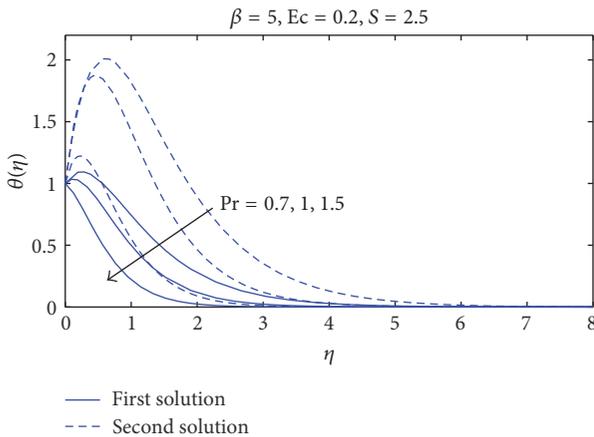


FIGURE 11: Temperature profiles for different values of  $Pr$ .

### 4. Conclusions

The steady boundary layer flow of Casson fluid and heat transfer over a permeable exponentially shrinking sheet with viscous dissipation are studied. The governing equations are

transformed and solved numerically using shooting method. The study reveals that the steady flow of Casson fluid due to exponentially shrinking sheet requires some more amount of mass suction than the Newtonian fluid flow. In all cases, when similarity solution exists, it is found to be dual solutions for velocity and temperature distributions. For first solution, the skin friction coefficient and heat transfer rate decrease with decreasing values of Casson parameter and opposite behavior is observed for second solution. In many cases, heat absorption at the sheet occurs. Moreover, due to the viscous dissipation effect, the heat absorption increases. The temperature inside the boundary layer increases with Eckert number and also thermal overshoot is observed. For less amount of mass suction heat absorption is found for both solutions, but for large mass suction heat transfer from the sheet occurs in both cases.

### Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

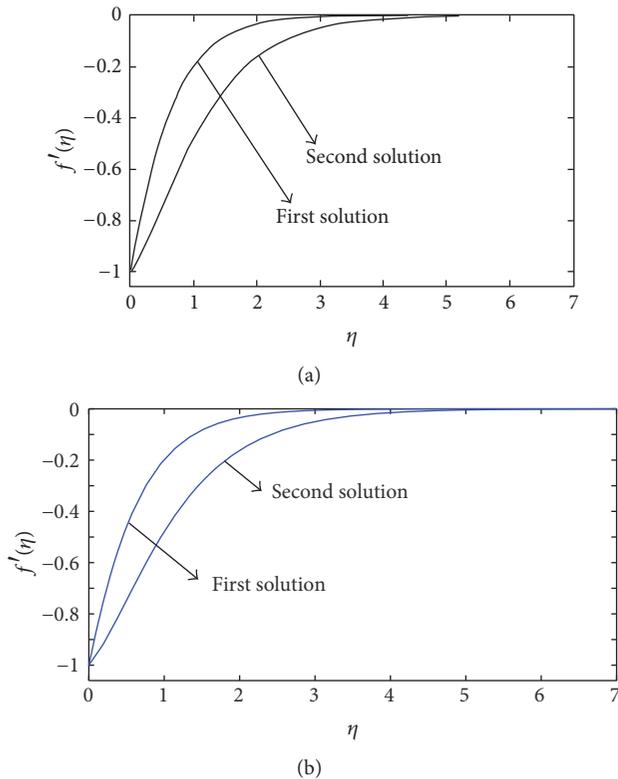


FIGURE 14: A comparison of velocity profiles for Newtonian fluid ( $\beta \rightarrow \infty$ ) with  $S = 2.4$  between (a) Bhattacharyya [26] and (b) present study.

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