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Research Article

New Iterative Learning Control Algorithm Using Learning Gain Based on σ Inversion for Nonsquare Multi-Input Multi-Output Systems

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Model inversion Iterative Learning Control (ILC) for a class of nonsquare linear time variant/invariant multi-input multi-output (MIMO) systems is considered in this paper. A new ILC algorithm is developed based on σ-right inversion of nonsquare learning gain matrices to resolve the matrix inversion problems appeared in the direct model inversion of nonsquare MIMO systems. Furthermore, a sufficient and necessary monotonic convergence condition is established. With rigorous analysis, the proposed ILC scheme guarantees the convergence of the tracking error. To prove the effectiveness and to illustrate the performance of the proposed approach for linear time-invariant (LTI) and time-varying nonsquare systems, two illustrative examples are simulated.

1. Introduction

Iterative Learning Control is an intelligent control strategy to deal with repetitive processes. The aim of the ILC is to control systems which execute the same task over a finite duration, such as the industrial robot manipulator [1]. The basic principle behind ILC is that the data generated from previous trial are used to adapt the control input for current trial. The control input in each trial is adjusted by using the tracking error obtained from the previous trial. As the iterations continue, the control system eventually learns the task and follows the desired trajectory with minimized tracking errors. The concept has been well developed in terms of both the fundamental theory and experimental applications that were accomplished in [2–6]. Thus, ILC research has considerably coped with mechanical systems such as multijoint hand-arm robots [7], the station stop control of train [8], and wafer stage motion systems [9]. The most of ILC schemes in the literature focus on proportional type learning law [10] and Optimal Iterative Learning Control (OILC) [11–14] where in [13] the learning approach has been applied to a rapid thermal processing. However, the majority of proposed algorithms were based on the notion of direct model inversion where the learning gain is obtained by Markov matrix inverse which represents the input-output map of the system [15]. However, this kind of ILC cannot be developed for general nonsquare (rectangular) MIMO systems that are systems in which the numbers of inputs and outputs are unequal. Indeed, a problem of the matrix inversion is encountered.

Many industrial processes require nonsquare MIMO mapping, especially chemical plants such as crude distillation process [16], mixing tank process [17], three-tank systems [18], distillation column [19], and chemical mechanical planarization process [20]. Therefore, the need of control schemes treating this type of systems is of major interest. The model inverse design and its related applications in control design have been widely studied in [21–24].

In the literature, there are control works that treated the nonsquare MIMO systems such as a Proportional Integrator Derivator (PID) controller cited in [25] which has been applied to a voltage model of Proton Exchange Membrane Fuel Cell (PEMFC); Model Predictive Control (MPC) was studied in [26] and has been applied to a Shell Heavy Oil Fractionator (SHOF); Minimum Variance Control (MVC) which is an inverse model control was studied in [27] and other examples of control strategies applied to nonsquare systems [28]. In the above cited works [25, 26], the control schemes were achieved without a need to matrix inversion. However, in [27] the synthesis of the inverse model in the control procedure required a matrix inversion based on left and right inverses which are developed in [29–31].

However, all the above mentioned methods could not deal with repetitive systems that require a learning strategy based on inversion model to achieve the control of repetitive nonsquare MIMO systems. Therefore, developing an ILC procedure for nonsquare systems remains an open problem.

Based on nonsquare polynomial matrix inversions [32], we propose the design and the analysis of an ILC scheme based on the model inversion called $I^{(n)}$-ILC for linear time-invariant and varying-time nonsquare MIMO systems. The main contribution of this work is to prove the monotonic convergence of the proposed scheme where the tracking error trial-to-trial will converge to zero even though the system has the initial resetting state. Through the simulation results, we prove the effectiveness of the proposed method.

The rest of this paper is organized as follows: In Section 2, the problem formulation is presented. The proposed inverse learning gain is developed in Section 3. A sufficient and necessary monotonic convergence of the $I^{(n)}$-ILC is established in Section 4. Further, the proposed $I^{(n)}$-ILC law is extended to time-varying systems in Section 5. Simulation results are illustrated in Section 6 to prove the effectiveness of the scheme for nonsquare MIMO systems. Finally, conclusions and an outlook on future work are given in Section 7.

2. Problem Formulation

The state-space representation of an LTI discrete-time system is given by (8):

$$\begin{aligned} x_j(k+1) &= Ax_j(k) + Bu_j(k), \\ y_j(k) &= Cx_j(k), \\ x_{j+1}(0) &= d_{j+1}, \end{aligned} \tag{1}$$

where $j \in \mathfrak{I} \triangleq \{1, 2, \dots, n_j - 1\}$ denotes the iteration numbers and n_j is the total number of trials. $k \in \Lambda \triangleq \{0, 1, \dots, N-1\}$ represents the number of the discrete-time sampling steps and N is the total number of discrete-time steps at each trial. $x_j(k) \in \mathfrak{R}^p$ is the jth iteration vector of system states, $u_j(k) \in \mathfrak{R}^{n_u}$ is the jth iteration vector of the system inputs which will be recursively generated by an iterative learning algorithm, $y_j(k) \in \mathfrak{R}^{n_y}$ is the jth iteration of the system outputs and $x_{j+1}(0)$ is a fixed initial state. Further, A, B, and C are constant matrices with appropriate dimensions, and $CB \neq 0$. Therefore, this state-space system can be described as follows:

$$y_j = Mu_j + d_j, \tag{2}$$

where M is a Markov matrix of rank N and whose terms are Markov parameters of the plant as cited in [33] and

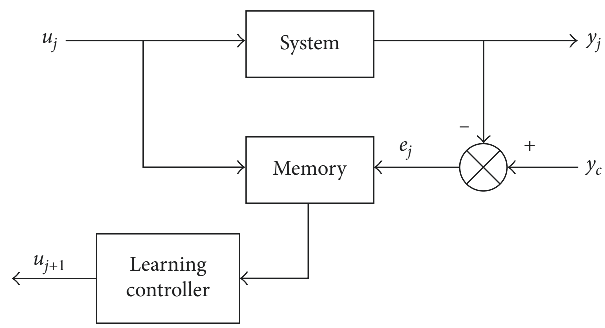

Figure 1: Basic ILC structure.

$$\begin{aligned} d_j(k) &= CA^k x_j(0), \\ M &= \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CB \end{bmatrix} \in R^{n_y N \times n_u N}. \end{aligned} \tag{3}$$

The control objective is to find a control sequence $\{u_j(k)\}$ with the ability to reduce tracking error for the whole trajectory based on the past tracking experiences until $j \to \infty$, the system tracking error limit is

$$\lim_{j\to\infty} \|e_j\| = \lim_{j\to\infty} \|y_c - y_j\|_\infty = 0, \tag{4}$$

where $\|\|_\infty$ is the infinite norm.

The main formulation of the ILC design problem is to achieve an update mechanism for the control input trajectory of a new cycle based on the information from previous cycles, so that the output trajectory converges asymptotically to the reference trajectory [34–36]. This idea is depicted in the block diagram form in Figure 1, which shows the next trial's control input u_{j+1} to be calculated from the previous trials control input u_j and transient output error e_j.

In this paper, we focus on the nonsquare systems when the number of input variables n_u is different from the number of output variables n_y. The goal is to make the system outputs track a given desired reference trajectory.

The first-order ILC algorithms update the input trajectory u with the following equation:

$$u_{j+1}(k) = u_j(k) + He_j(k), \tag{5}$$

where we designate by H the learning gain and by e_j the error between the set-point reference y_c and the jth output as

$$e_j = y_c - y_j. \tag{6}$$

The most proposed algorithms were based on the direct model inversion, that is $H = M^{-1}$, assuming that M^{-1} is invertible. M^{-1} represents the input-output map of the process [37, 38]. However, in nonsquare system case, the learning gain based on the model inversion cannot be calculated. In fact, the adaptation law in (5) cannot be satisfied for general nonsquare MIMO processes. To overcome this problem, a new method is proposed to design a realizable controller based on the direct model inversion

I-ILC for $n_u \times n_y$ linear nonsquare system process called $I^{(n)}$-ILC.

Theorem 1 *(stability of ILC). Let an LTI discrete-time system be described by (1) is asymptotically stable if and only if the polynomial matrix M is right invertible [39, 40].*

The ILC law (5) can be immediately written as

$$u_{j+1}(k) = u_j(k) + M^R e_j(k), \tag{7}$$

where M^R is the right inverse of Markov matrix M.

3. Inverses of Nonsquare Polynomial Matrices and $I^{(n)}$-ILC Design

Several methods of right/left inverses of nonsquare polynomial matrices, such as the classical minimum-norm right/left inverse, called T-inverse and τ-σ-S-inverses methods, have been studied in [27, 32]. In this work, we focus on σ-inverse as follows:

Corollary 1. *Let the polynomial matrix $M(q^{-1}) = m_0 + m_1 q^{-1} + \cdots + m_s q^{-s}$ be a full normal rank n_y (or n_u) and let $z^r \psi(z^{-1}) = \psi(z) \in \mathbb{R}^{n_y \times n_u}[z]$ of full normal rank n_y (or n_u) be arbitrary, including an arbitrary order r. The product $\underline{M}(q^{-1})\underline{\psi}^T(q^{-1})$ must be of full normal rank n_y. Then a σ-inverse can be defined as*

$$\underline{M}^R(q^{-1}) = \underline{\psi}^T(q^{-1})\left[\underline{M}(q^{-1})\,\underline{\psi}^T(q^{-1})\right]^{-1}. \tag{8}$$

For right invertible system (1), $\underline{M}^R$ denotes an infinite number of right inverse of M.

Definition 1. Let $M \in \Re^{n_y \times n_u}$ and $\psi \in \Re^{n_y \times n_u}$ are a full rank $n_y < n_u$ where M is right invertible and ψ is selected such that $\psi_{\text{opt}} = \arg\min_{\psi} \sum_{i=0}^{N} [u_{j+1}^T(i) u_{j+1}(i)]$. Then, the right inverse of M is

$$M^R = \psi^T [M\psi^T]^{-1}. \tag{9}$$

Using previous Definition 1, we can employ the σ-inverse in $I^{(n)}$-ILC (7) to obtain

$$u_{j+1}(k) = u_j(k) + \psi^T(q^{-1})\left[M(q^{-1})\psi^T(q^{-1})\right]^{-1} e_j(k). \tag{10}$$

Using (2) and (3), the tracking error (6) can be rewritten as

$$\begin{aligned} e_{j+1}(k) &= y_c(k) - y_{j+1}(k) \\ &= y_c(k) - Mu_{j+1}(k) - CA^k x_{j+1}(0) \\ &= y_c(k) - Mu_j(k) \\ &\quad - M\left(\psi^T[M\psi^T]^{-1}\right)e_j(k) - CA^k x_{j+1}(0) \\ &= y_c(k) - Mu_j(k) - CA^k x_{j+1}(0) \\ &\quad - M\psi^T[M\psi^T]^{-1} e_j(k). \end{aligned} \tag{11}$$

Adding and subtracting $CA^k x_j(0)$ into (11), we get

$$\begin{aligned} e_{j+1}(k) &= y_c(k) - Mu_j(k) - CA^k x_j(0) + CA^k x_j(0) \\ &\quad - CA^k x_{j+1}(0) - M\psi^T[M\psi^T]^{-1} e_j(k) \\ &= y_c(k) - y_j(k) - CA^k\left(x_{j+1}(0) - x_j(0)\right) - MHe_j(k) \\ &= (I_n - MH)e_j - CA^k\left(x_{j+1}(0) - x_j(0)\right) \\ &= G_{\text{ILC}_n} e_j - CA^k\left(x_{j+1}(0) - x_j(0)\right). \end{aligned} \tag{12}$$

4. Convergence Analysis of the Proposed Learning Gain

The control design problem is to determine a new ILC law based on the model inversion such that the trial-to-trial error convergence occurs in j; that is, $\lim_{j\to\infty}\|e_j\| = 0$ [41].

Theorem 2. *Assume that $I^{(n)}$-ILC is applied to the linear time-invariant systems (1). Suppose $x_{j+1}(0) = x_j(0)$, for all $j \in \mathfrak{I} \triangleq \{1, 2, \ldots, n_j - 1\}$. Then, the propositions*

$$\sum_{k=0}^{+\infty} \left\|e_{j+1}(k)\right\|^2 < \sum_{k=0}^{+\infty} \left\|e_j(k)\right\|^2, \tag{13}$$

$$\lim_{j\to\infty} \sum_{k=0}^{+\infty} \left\|e_{j+1}(k)\right\|^2 = 0, \tag{14}$$

hold if and only if

$$\left\|I_n - MH\right\|_l = \left\|G_{\text{ILC}_n}\right\|_l < 1. \tag{15}$$

Proof (sufficiency). Based on initial condition, the initial states $x_{j+1}(0) \underset{j\to\infty}{\equiv} x_j(0)$, then (12) becomes

$$e_{j+1}(k) = G_{\text{ILC}_n} e_j(k). \tag{16}$$

Let $S_j = \sum_{k=0}^{+\infty} \|e_j(k)\|^2$. By taking the assumption (13), it is deduced that $0 \le S_{j+1} < S_j < \cdots < S_1$. Then, the sequence $\{S_{j+1}\}$ is strictly decreasing and lower-bounded by zero. This means that $\lim_{j\to\infty} S_{j+1}$ exists. Therefore, the series $S_j = \sum_{k=0}^{+\infty} \|e_j(k)\|^2$ is convergent.

Then, we prove that $\lim_{j\to\infty} S_{j+1} = 0$ by reduction to absurdity. Suppose that $\lim_{j\to\infty} S_{j+1} = S > 0$. Therefore, for $\tau_1 = S/2$, there exists a finite positive integer j_0 so that, for all $j > j_0$, $S_{j+1} > S - \tau_1 > (S/2)$. However, S_j is convergent, so for all $j > j_0$, the limit of the partial sequence $S_{j+1}(r) = \sum_{k=0}^{r} \|e_{j+1}(k)\|^2$ exists. This means that, for a given constant $\tau_2 = S/4$, there exists an integer N_0 so that, for all $r > N_0$, we have

$$S_{j+1}(r) = \sum_{k=0}^{r} \left\|e_{j+1}(k)\right\|^2 > S - \tau_1 - \tau_2, \tag{17}$$

in particular,

$$\sum_{k=0}^{2N_0} \left\| e_{j+1}(k) \right\|^2 > S - \tau_1 - \tau_2, \tag{18}$$

then, for $k_0 \in [0, 2N_0]$, we obtain

$$\sum_{k=0}^{2N_0} \left\| e_{j+1}(k_0) \right\|^2 > S - \tau_1 - \tau_2, \tag{19}$$

thus,

$$2N_0 \left\| e_{j+1}(k_0) \right\|^2 > S - \tau_1 - \tau_2. \tag{20}$$

Hence, we have

$$\left\| e_{j+1}(k_0) \right\|^2 > \frac{S - \tau_1 - \tau_2}{2N_0} = \frac{S}{8N_0} > 0. \tag{21}$$

On the other hand, by recursion, (16) reduces to

$$e_{j+1}(k_0) = G_{\mathrm{ILC}_n}^{j} e_1(k_0). \tag{22}$$

Although, by considering the assumption (15), we obtain

$$\lim_{j\to\infty} e_{j+1}(k_0) = 0. \tag{23}$$

This is contradictory to inequality (19). This contradiction means that the propositions (13) and (14) are true. This proves the sufficiency assumption.

Proof (necessity). We assume that the inequality (15) is not always true. So, there exists at least a number k_0 such that

$$\left\| G_{\mathrm{ILC}_n}(k_0)_l \right\| \geq 1. \tag{24}$$

The equality (16) gives

$$\left\| e_{j+1}(k_0) \right\| = \left\| G_{\mathrm{ILC}_n} \right\|^j \left\| e_1(k_0) \right\| \geq \left\| e_1(k_0) \right\|, \tag{25}$$

thus,

$$\sum_{k=0}^{+\infty} \left\| e_{j+1}(k)^2 \right\| \geq \left\| e_{j+1}(k_0) \right\|^2 \geq \left\| e_1(k_0) \right\|^2. \tag{26}$$

Finally, we obtain

$$\lim_{j\to\infty} \sum_{k=0}^{+\infty} \left\| e_{j+1}(k) \right\|^2 \geq \lim_{j\to\infty} \left\| e_1(k_0) \right\|^2 = \left\| e_1(k_0) \right\|^2. \tag{27}$$

It is evident to choose $u_1(k)$ and $y_c(k)$ such that $\| e_1(k_0) \| > 0$, which contradicts to the assumption (14). This proves the necessity of the assumption.

The sufficient and necessary assumption for the monotonic convergence of the given $I^{(n)}$-ILC algorithm shows that ILC trial-to-trial error convergence requires that all of the initial states and tracking errors are reset.

5. Extension to Time-Varying Systems

In this part, the proposed $I^{(n)}$-ILC scheme is extended to linear time-varying systems as follows:

$$\begin{aligned} x_j(k+1) &= A(k)x_j(k) + B(k)u_j(k), \\ y_j(k) &= C(k)x_j(k), \end{aligned} \tag{28}$$

where $A(k)$, $B(k)$, and $C(k)$ are time-varying matrices with appropriate dimensions and $C(k)B(k) \neq 0$. Therefore, this state-space system can be described as follows:

$$y_j(k) = Mu_j(k) + d_j(k), \tag{29}$$

where

$$M = \begin{bmatrix} C(1)B(0) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ C(N)\prod_{r=1}^{N-1} A(N-r)B(0) & \cdots & C(N)B(N-1) \end{bmatrix}, \tag{30}$$

such that $M \in R^{n_y N \times n_u N}$ and $d_j(k) = C(k)\prod_{r=0}^{N-1} A(N-r)\, x_j(0)$. Thus the output can rewritten as

$$\begin{aligned} y_j(k) &= C(k)\prod_{r=0}^{N-1} A(N-r)x_j(0) \\ &\quad + C(k)\sum_{i=1}^{N}\prod_{r=1}^{i-1} A(N-r)B(N-i)u_j(k). \end{aligned} \tag{31}$$

Using the σ-inverse (8), the $I^{(n)}$-ILC to control time-varying MIMO systems is similar to control law (10).

Theorem 3. *For the discrete linear varying-time system (28), the $I^{(n)}$-ILC is chosen such that, for any constant $0 \leq \delta \leq 1$,*

$$\sup_{k\in[1,\dots,N]} \left\| I_n - H(k)M \right\| \leq \delta. \tag{32}$$

Therefore, the tracking errors $e_j(k)$ will converge to zero where $j \to \infty$ if and only if the previous conditions (13) and (14) are proved.

Proof. The proof of Theorem 3 can be completed identically as in the proof of Theorem 2 such that the tracking error is given by

$$e_{j+1}(k) = G_{\mathrm{ILC}_n}(k)e_j(k). \tag{33}$$

Following Proof 1, we can conclude that $\lim_{j\to\infty} e_j = 0$ where all of initial states $x_j(0)$ are reset.

6. Illustrative Examples

In order to show the effectiveness of the proposed ILC based on the direct model inversion to deal with nonsquare systems, two examples are considered.

Example 1: Time-Invariant System. Consider a three-output two-input discrete-time Linear Time-Invariant (LTI) system:

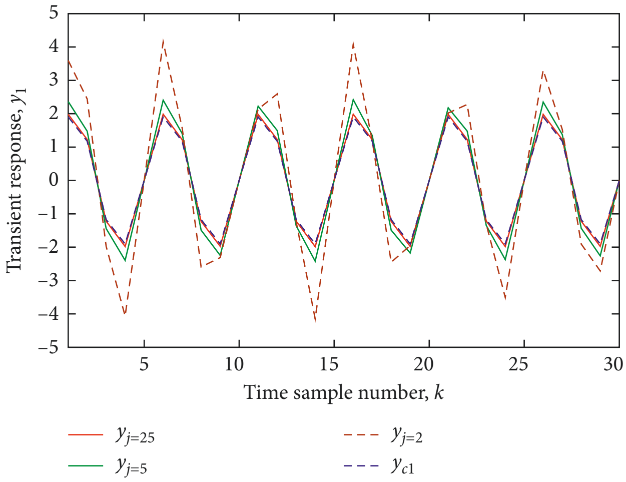

Figure 2: Outputs y_1 at the 2nd, 5th, and 25th iterations.

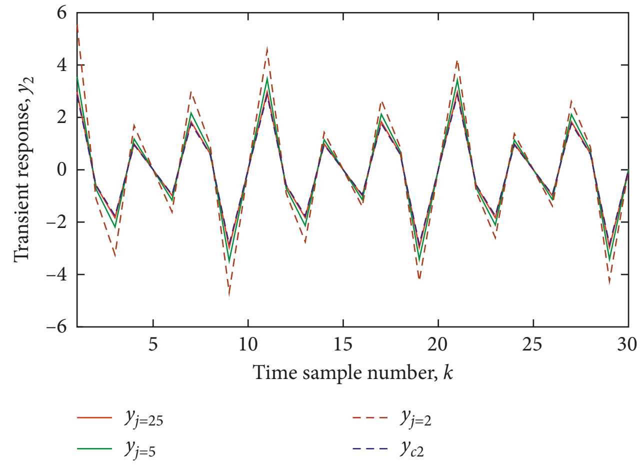

Figure 3: Outputs y_2 at the 2nd, 5th, and 25th iterations.

$$
\begin{aligned}
A &= \begin{pmatrix} 1.1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.9 \end{pmatrix}, \\
B &= \begin{pmatrix} 0.5 & 0.5 & 1 \\ 1 & 1.5 & 0.5 \\ 0.2 & 0.3 & 0.3 \\ 0.2 & 1.5 & 0.3 \\ 0.1 & 0.3 & 0.1 \\ 0.2 & 0.6 & 0.1 \end{pmatrix}, \\
C &= \begin{pmatrix} 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \end{pmatrix}, \\
D &= 0,
\end{aligned}
\tag{34}
$$

where $x_j(0) = [0.1, 0.2, 0, 0.2, 0.1, 0]^T$, $j \in \mathfrak{T} \triangleq \{1, 2, \ldots, 25\}$.

The reference trajectories are chosen as

$$
\begin{aligned}
y_{c1} &= 2\sin\left(\frac{2\pi k}{5}\right) + \sin(50\pi k), \\
y_{c2} &= \sin\left(\frac{2\pi k}{5}\right) + 2\sin\left(\frac{3\pi k}{5}\right) + \sin(50\pi k),
\end{aligned}
\tag{35}
$$

where $k \in \Lambda \triangleq \{1, 2, \ldots, 30\}$; thus $N = 30$ and the initial input vector is chosen as $u_0 = [0, 0, 0]^T$.

By applying the proposed $I^{(n)}$-ILC law (10), the evolutions of transient outputs y_1 and y_2 profiles with different iteration numbers $j = 2$, $j = 5$, and $j = 25$ are given in Figures 2 and 3, respectively. The evolutions of terminal inputs are shown in Figure 4. It is clear that the proposed scheme is well to be used for linear nonsquare MIMO systems to track time-varying reference. The $I^{(n)}$-ILC approach converges quickly and the performance keeps well even the reference changes. Furthermore, the performance of the absolute initial and terminal tracking error profiles $e_{1,j}$ and $e_{2,j}$ versus the iteration numbers are illustrated in Figures 5 and 6. It shows that the tracking $e_{1,j}$ and $e_{2,j}$ will converge within 15 and 20 iterations. Moreover, from Figures 7 and 8 which illustrated the first and the second components of tracking error profiles at 5th, 10th, 15th, and 25th iterations. It is clear that the fast convergence of errors in the iteration domain is obvious for the proposed algorithm.

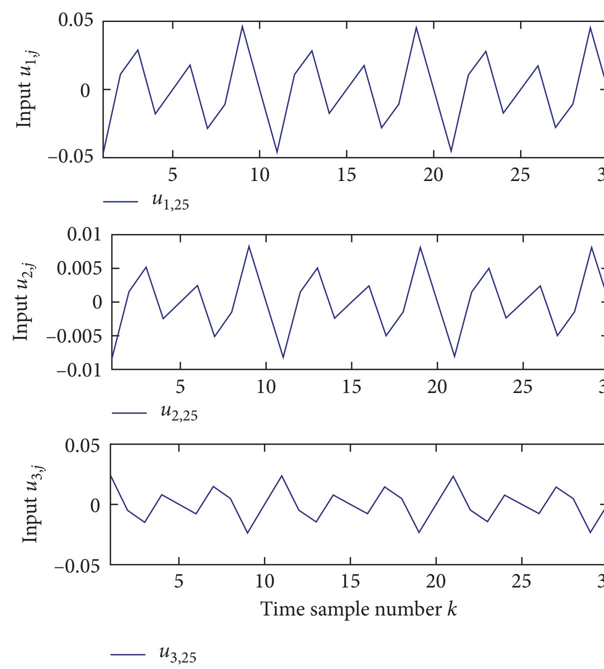

Figure 4: Evolutions of all inputs with trial length $n_j = 25$.

Example 2: Time-Variant System. In order to demonstrate the effectiveness of our proposed algorithm, let the discrete-time linear time-varying system as follows.

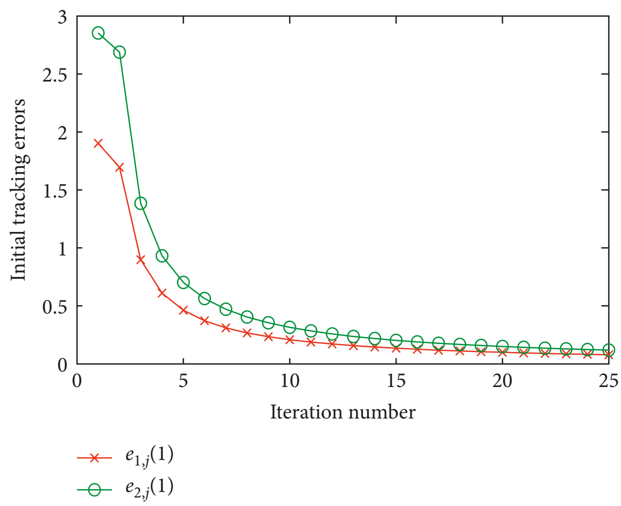

Figure 5: Initial tracking errors $e_{1,j}$ and $e_{2,j}$ versus the iteration number j.

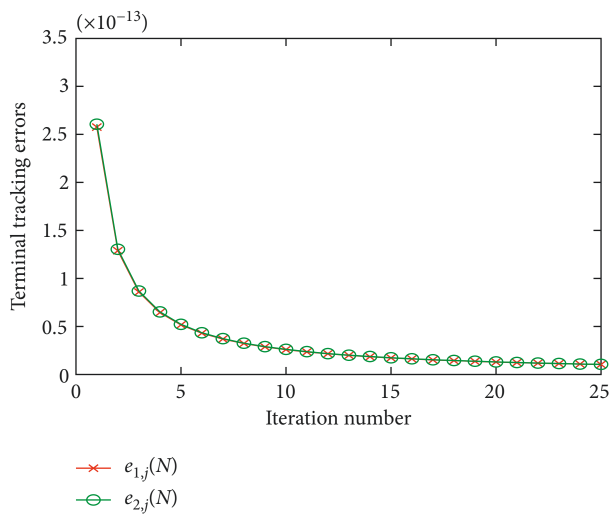

Figure 6: Terminal tracking errors $e_{1,j}$ and $e_{2,j}$ versus the iteration number j.

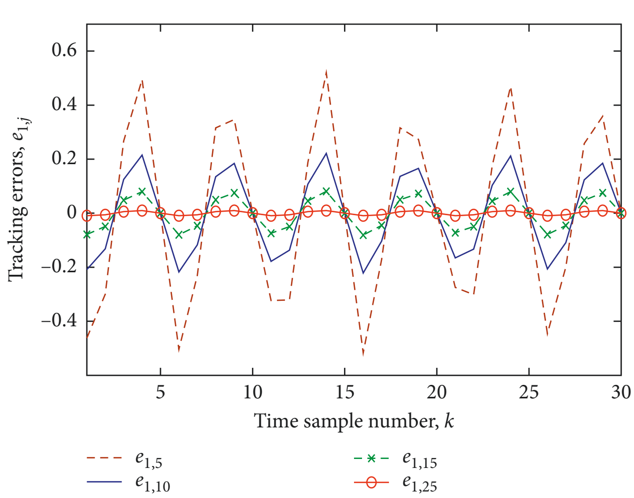

Figure 7: Tracking errors profiles of ILC $e_{1,j}$.

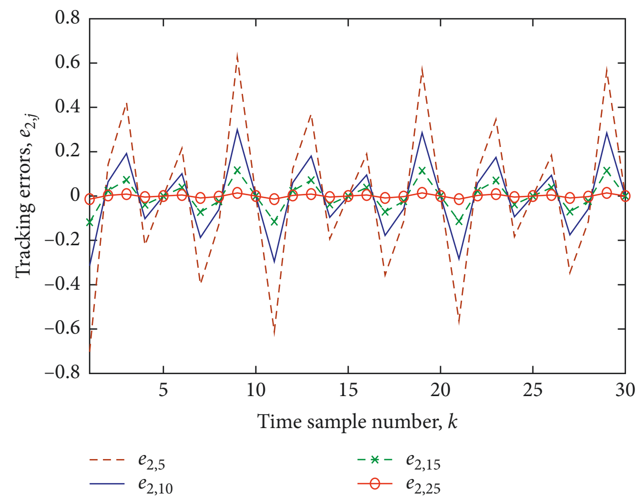

Figure 8: Tracking errors profiles of ILC $e_{2,j}$.

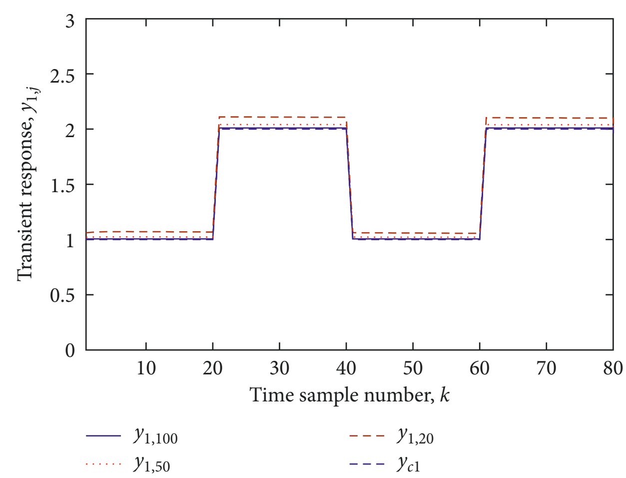

Figure 9: Output y_1 at the 20th, 50th, and 100th iterations.

$$A = \begin{pmatrix} 0.2\sin(k) & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-(k/100)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.5 & 0.5 & 1 \\ 1 & 1.5 & 0.5 \\ 0.2 & 0.3 & 0.3 \\ 0.2 & 1.5 & 0.3 \\ 0.1 & 0.3 & 0.1 \\ 0.2 & 0.6 & 0.1 \end{pmatrix}, \tag{36}$$

$$C = \begin{pmatrix} 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \end{pmatrix},$$

$$D = 0,$$

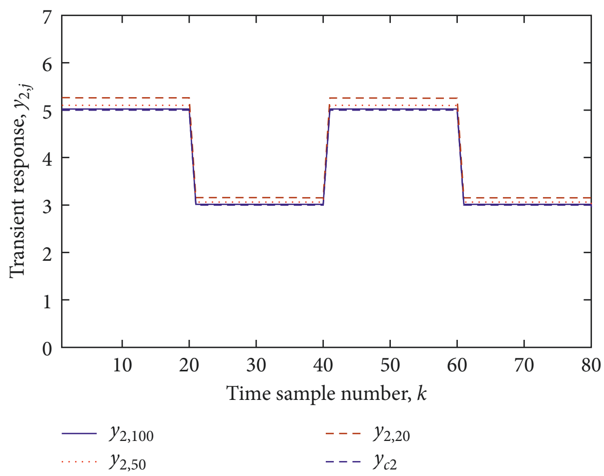

Figure 10: Output y_2 at the 20th, 50th, and 100th iterations.

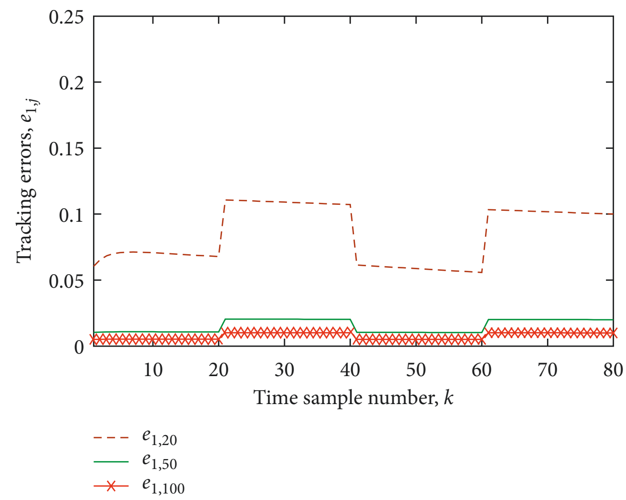

Figure 12: Tracking errors profiles of ILC $e_{1,j}$.

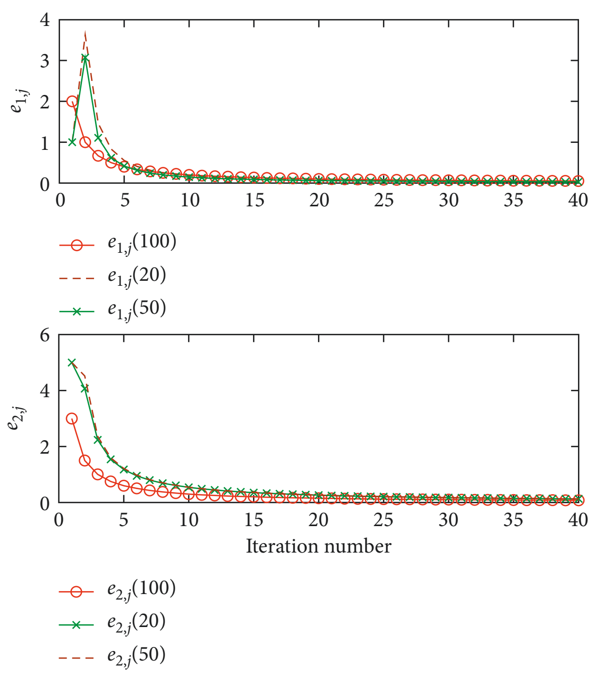

Figure 11: Absolute values of errors $e_{1,j}$ and $e_{2,j}$ versus the iteration number j.

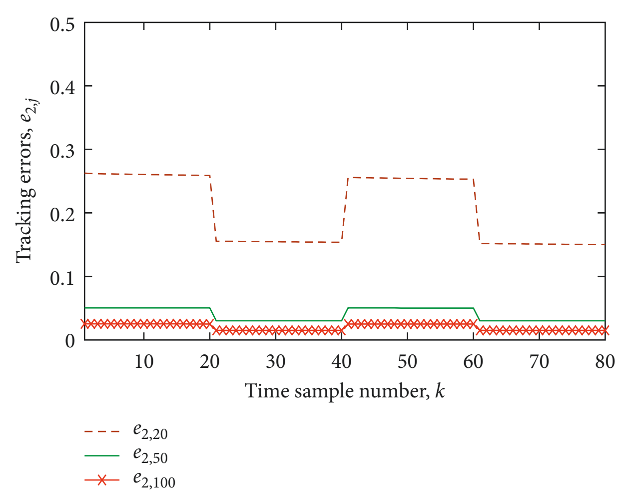

Figure 13: Tracking errors profiles of ILC $e_{2,j}$.

where $x_j(0) = [0, 0, 0, 0, 0, 0]^T$, $j \in \mathfrak{J} \triangleq \{1, 2, \ldots, 100\}$. $k \in \Lambda \triangleq \{1, 2, \ldots, 80\}$; thus $N = 80$ and the initial input vector is chosen as $u_0 = [0, 0, 0]^T$. y_{c1} (dashed line) and the first component of output y_1 (solid line) are depicted in Figure 9. y_{c2} (dashed line) and the first component of output y_2 (solid line) are depicted in Figure 10. The performance of the absolute tracking errors $\|e_{1,j}\|$ and $\|e_{2,j}\|$ is illustrated in Figure 11, where the tracking errors will converge within 20 and 30 iterations. Moreover, Figures 12 and 13 show the tracking error profiles for 20th, 50th, and 100th iterations. It is clear that the tracking errors converge to zero in more than 70 iterations, where $x_j(0) = [0, 0, 0, 0, 0, 0]^T$, $j \in \mathfrak{J} \triangleq \{1, 2, \ldots, 100\}$.

7. Conclusion

In this paper, a new model inversion Iterative Learning Control called $I^{(n)}$-ILC is proposed to deal with nonsquare MIMO systems where the numbers of inputs and outputs are unequal. The proposed scheme is based on σ-right-inverse learning gain in order to resolve a major problem appeared in ILC based on inversion model (I-ILC). The convergence condition of the learning algorithm has been derived. It is shown that under some given conditions, the tracking error of $I^{(n)}$-ILC law converges to zero through a sufficient and necessary stability condition. Then, the convergence properties are established. Through simulation results, we proved that the performances offered by the proposed method in terms of tracking error convergence after few trials are achieved. Therefore, robustness of the $I^{(n)}$-ILC scheme to perturbations and parametric uncertainties, especially with less knowledge of plant model, remains a challenging topic which will be addressed in future works.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] S. Kawamura and N. Sakagami, "Analysis on dynamics of underwater robot manipulators based on iterative learning control and time-scale transformation," in *Proceedings of IEEE International Conference of Robotics and Automation*, pp. 1088–1094, Washington, DC, USA, May 2002.

[2] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[3] H. S. Ahn, Y. Chen, and K. L. Moore, "Iterative learning control: brief survey and categorization," *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1099–1121, 2007.

[4] S. Jin, Z. Hou, and R. Chi, "Optimal terminal iterative learning control for the automatic train stop system," *Asian Journal of Control*, vol. 17, no. 5, pp. 1992–1999, 2015.

[5] A. Mohammadpour, S. Mishra, and L. Parsa, "Fault-tolerant operation of multiphase permanent-magnet machines using iterative learning control," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 2, no. 2, pp. 201–211, 2014.

[6] W. Chen and M. Tomizuka, "Dual-stage iterative learning control for MIMO mismatched system with application to robots with joint elasticity," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 4, pp. 1350–1361, 2014.

[7] S. Arimoto, M. Sekimoto, and S. Kawamura, "Iterative learning of specified motions in task-space for redundant multi-joint hand arm robots," in *Proceedings of IEEE International Conference on Robotics and Automation*, pp. 2867–2873, Roma, Italy, 2007.

[8] Z. S. Hou, Y. Wang, C. K. Yin, and T. Tang, "Terminal iterative learning control based station stop control of a train," *International Journal of Control*, vol. 84, no. 7, pp. 1263–1274, 2011.

[9] D. D. Roover and O. H. Bosgra, "Synthesis of robust multivariable iterative learning controllers with application to a wafer stage motion system," *International Journal of Control*, vol. 73, no. 10, pp. 968–979, 2000.

[10] L. Noueili, W. Chagra, and M. Ksouri, "New architecture of iterative learning control for coupled MIMO systems," in *Proceedings of the 3rd International Conference on Automation, Control, Engineering and Computer Science (ACECS)*, pp. 888–894, Sousse, Tunisia, 2016.

[11] L. Noueili, W. Chagra, and M. Ksouri, "Optimal iterative learning control for square MIMO linear systems," in *Proceedings of the 7th International Conference on Modelling, Identification and Control*, Sousse, Tunisia, December 2015.

[12] L. Noueili, W. Chagra, and M. Ksouri, "Optimal iterative learning control for a class of non-minimum phase systems," *International Journal of Modelling, Identification and Control*, vol. 28, no. 3, pp. 284–294, 2017.

[13] D. R. Yang, K. S. Lee, H. J. Ahn, and J. H. Lee, "Experimental application of a quadratic optimal iterative learning control method for control of wafer temperature uniformity in rapid thermal processing," *IEEE Transactions of Semiconductor Manufacturing*, vol. 16, no. 1, pp. 36–44, 2003.

[14] P. Janssens, G. Pipeleers, and J. Swevers, "A data-driven constrained norm-optimal iterative learning control framework for LTI systems," *IEEE Transactions on Control Systems*, vol. 21, no. 2, pp. 546–551, 2013.

[15] D. H. Owens, E. Rogers, and K. L. Moore, "Analysis of linear iterative learning control schemes using repetitive process theory," *Asian Journal of Control*, vol. 4, no. 1, pp. 68–89, 2002.

[16] K. L. N. Sarma and M. Chidambaram, "Centralized PI/PID controllers for nonsquare systems with RHP zeros," *Journal of the Indian Institute of Science*, vol. 85, no. 4, pp. 201–214, 2005.

[17] E. J. Loh and M. S. Chiu, "Robust decentralized control of non-square systems," *Chemical Engineering Communications*, vol. 158, no. 1, pp. 157–180, 1997.

[18] H. Salhi and F. Bouani, "Nonlinear parameters and state estimation for adaptive nonlinear MPC design," *Journal of Dynamic Systems, Measurement, and Control*, vol. 138, no. 4, 2016.

[19] P. Chen, L. Ou, D. Gu, and W. Zhang, "Robust analytical scheme for linear non-square systems," in *Proceedings of 48th IEEE Conference on Decision and Control Held Jointly with 28th Chinese Control Conference*, pp. 1890–1895, Shanghai, China, December 2009.

[20] P. K. M. Rao, B. S. Bhushan, T. S. Bukkapatnam et al., "Process-machine interaction (PMI) modeling and monitoring of chemical mechanical planarization (CMP) process using wireless vibration sensors," *IEEE Transactions on Semiconductor Manufacturing*, vol. 27, no. 1, pp. 1–15, 2014.

[21] L. Chai, J. Zhang, C. Zhang, and E. Mosca, "Optimal noise reduction in oversampled PR filter banks," *IEEE Transactions on Signal Processing*, vol. 57, no. 10, pp. 3844–3857, 2009.

[22] K. L. Law, M. R. Fossum, and M. N. Do, "Generic invertibility of multidimensional FIR filter banks and MIMO systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 11, pp. 4282–4291, 2009.

[23] Q. B. Jin, W. S. Liu, L. Quan, and L. T. Cao, "Internal model control based on singular value decomposition and its application to non-square processes," *Acta Automatica Sinica*, vol. 37, no. 3, pp. 354–359, 2011.

[24] A. E. Frazho, A. M. Kaashoek, and A. C. M. Ran, "Right invertible multiplication operators and stable rational matrix solutions to an associate bezout equation, I: the least squares solution," *Integral Equations and Operator Theory*, vol. 70, no. 3, pp. 395–418, 2011.

[25] Z. Lei, H. Yuchun, H. Hongfei, and T. Wen, "PID control of non-square systems and its application in the fuel cell voltage," in *Proceedings of 24th Chinese Control and Decision Conference*, Taiyuan, China, May 2012.

[26] L. R. E. Shead, C. G. J. Anastassakis, and A. Rossiter, "Steady-state operability of multi-variable non-square systems: application to model predictive control (MPC) of the shell heavy oil fractionator (SHOF)," in *Proceedings of Mediterranean Conference on Control and Automation*, Athens, Greece, June 2007.

[27] W. P. Hunek, K. J. Latawiec, P. Majewski, and P. Dzierwa, "An application of a new matrix inverse in stabilizing state-space perfect control of nonsquare LTI MIMO systems," in *Proceedings of the 19th International Conference on Methods and Models in Automation and Robotics (MMAR)*, Międzyzdroje, Poland, September 2014.

[28] S. Kolavennu, S. Palanki, and J. C. Cockburn, "Nonlinear control of nonsquare multivariable systems," *Chemical Engineering Science*, vol. 56, no. 6, pp. 2103–2110, 2001.

[29] G. Gu and E. F. Badran, "Optimal design for channel equalization via the filterbank approach," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 536–545, 2004.

[30] L. Li and G. Gu, "Design of optimal zero-forcing precoders for MIMO channels via optimal full information control," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3238–3246, 2005.

[31] S. Li and J. Zhangt, "Some properties of generalized inverse of non-square systems," in *Proceedings of 52nd IEEE Conference on Decision and Control*, Firenze, Italy, December 2013.

[32] W. P. Hunek and K. J. Latawiec, "A study on new right/left inverses of nonsquare polynomial matrices," *International Journal of Applied Mathematics and Computer Science*, vol. 21, no. 2, pp. 331–348, 2011.

[33] H. Q. Z. Sun, S. D. Hou, and Y. Li, "Coordinated iterative learning control schemes for train trajectory tracking with overspeed protection," *IEEE Transactions on Automation Science and Engineering*, vol. 10, no. 2, pp. 323–333, 2013.

[34] M. Rzewuski, E. Rogers, and D. H. Owens, "A comparison of optimal iterative learning control schemes," in *Proceedings of IFAC Workshop Adaptation Learning Control Signal Processing*, pp. 77–82, Cernobbio, Italy, 2001.

[35] J. Kang, "A new iterative learning control scheme with global convergence for MIMO dynamic systems," in *Proceedings of International Conference of Intelligent Computation Technology and Automation*, Changsha, China, May 2010.

[36] J. Ding, B. Cichy, K. Galkowski, E. Rogers, and H. Z. Yang, "Robust fault-tolerant iterative learning control for discrete systems via linear repetitive processes theory," *International Journal of Automation and Computing*, vol. 12, no. 3, pp. 254–265, 2015.

[37] J. H. Lee, K. S. Lee, and W. C. Kim, "Model-based iterative learning control with a quadratic criterion for time-varying linear systems," *Automatica*, vol. 36, no. 5, pp. 641–657, 2000.

[38] J. X. Xu and X. Jin, "State-constrained iterative learning control for a class of MIMO systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 5, pp. 1322–1327, 2013.

[39] R. W. Longman, "Iterative learning control and repetitive control for engineering practice," *International Journal of Control*, vol. 73, no. 10, pp. 930–954, 2012.

[40] D. Meng, Y. Jia, J. Du, and F. Yu, "Necessary and sufficient stability condition of LTV iterative learning control systems using a 2-D approach," *Asian Journal of Control*, vol. 13, no. 1, pp. 25–37, 2011.

[41] X. E. Ruan, Z. Z. Li, and Z. Z. Bien, "Discrete-frequency convergence of iterative learning control for linear time-invariant systems with higher-order relative degree," *International Journal Control of Automation and Computing*, vol. 12, no. 3, pp. 281–288, 2015.


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