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Research Article

Nonlinear Gravitoelectrostatic Sheath Fluctuation in Solar Plasma

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The nonlinear normal mode dynamics is likely to be modified due to nonlinear, dissipative, and dispersive mechanisms in solar plasma system. Here we apply a plasma-based gravitoelectrostatic sheath (GES) model for the steady-state description of the nonlinear normal mode behavior of the gravitoacoustic wave in field-free quasineutral solar plasma. The plasma-boundary wall interaction process is considered in global hydrodynamical homogeneous equilibrium under spherical geometry approximation idealistically. Accordingly, a unique form of KdV-Burger (KdV-B) equation in the lowest-order perturbed GES potential is methodologically obtained by standard perturbation technique. This equation is both analytically and numerically found to yield the GES nonlinear eigenmodes in the form of shock-like structures. The shock amplitudes are determined ($\sim$0.01 V) at the solar surface and beyond at 1 AU as well. Analytical and numerical calculations are in good agreement. The obtained results are compared with those of others. Possible results, discussions, and main conclusions relevant to astrophysical context are presented.

1. Introduction

The Sun, like stars and ambient atmospheres, has exploratively been an interesting area of study for different authors by applying different physical model approaches and observational techniques for years [1–13]. Some of the basic electromagnetic properties of such stars and stellar atmospheres have been reported in hydrostatic equilibrium of the constituent ionized gas [1, 2]. The steady supersonic radial outflow of the ionized gas from the Sun (or star), called solar wind (or stellar wind), has been found to support various nonlinear eigenmodes [4]. Such nonlinear eigenmodes are usually solitons, shocks, and so forth [4, 9–17] found to exist almost everywhere in space including dust-contaminated space plasma [14, 15]. Nonlinear stability analyses of the Sun and its atmosphere have, however, been boldly carried out by many authors applying magnetohydrodynamic (MHD) equilibrium configurations [9–13] by means of standard multiple scaling techniques. Nevertheless the effects of space charge, plasma-boundary wall interaction, and sheath formation mechanism have hardly been addressed in such model stability analyses on the Sun and its atmosphere reported so far.

We are here going to propose a nonlinear stability analysis on the Sun on the basis of the plasma-based gravitoelectrostatic sheath (GES) model [3]. According to this GES model analysis, the solar plasma system divides into two parts: the Sun which is the subsonic solar interior plasma (SIP) on bounded scale and the supersonic or hypersonic solar wind plasma (SWP) on unbounded scale. The solar surface boundary (SSB) couples the SIP (Sun) with the SWP through plasma-boundary wall interaction processes in a self-gravitating equilibrium configuration of hydrodynamic type. The SSB behaves like a spherical electrical grid negatively biased with the equilibrium GES potential $\theta_\Theta(\xi_\Theta) \sim -1$ ($= -1.00$ kV) through the process of the self gravito-electrostatic interaction. Henceforth, for coupled structural information, the terms "GES fluctuation", "SIP fluctuation", "GES perturbation," and "SIP perturbation" will be synonymously used to describe the "nonlinear GES stability on the bounded SIP scale" in this investigation of solar plasma fluctuation dynamics.

The main motivation of this paper is to examine whether the solar plasma system, as a natural plasma laboratory, can support any nonlinear characteristic eigenmode through the GES model with plasma-boundary interaction taken into account. Spacecraft probes and Earth-orbiting satellites have also technically detected many wide-scale nonlinear mode features [4, 12, 13] like nonpropagating pressure-balance structures, collisionless shocks, turbulence-driven instability, soliton, and so forth. These have particularly been applied to probe plasma kinetic effects in the form of collective wave activities in some important parameter regimes [4] experimentally inaccessible to laboratories due to the complex nature of the dynamics of the solar wind particles. Thus a theoretical model analysis is highly needed in support of the description of these experimental observations.

A distinct set of nonautonomous self-consistently coupled nonlinear dynamical eigenvalue equations in the defined astrophysical scales of space and time configuration is accordingly developed. In view of that, a unique form of KdV-Burger (KdV-B) equation [9–11, 14, 15] is methodologically derived on the SIP scale in terms of the lowest order GES potential fluctuation. It is then studied analytically as well as numerically as an initial value problem. The gravito-electrostatic features are asymptotically examined even on the SWP to explore some new observations on the nonlinear eigenmodes of the GES. Apart from the "Introduction" part described in Section 1 above, this paper is structurally organized in a usual simple format as follows. Section 2, as usual, contains physical model of the solar plasma system under investigation. Section 3 contains mathematical formulation and required derived analytical equations and expressions. Section 4 shows the obtained results and discussions in three subsections. Sections 4.1, 4.2, and 4.3 give the analytical, numerical, and comparative results, respectively. Lastly and most importantly, Section 5 depicts the main conclusions of scientific interest and astrophysical applicability.

2. Solar Plasma Model

A very simplified ideal solar plasma fluid model is adopted to study the GES model stability under a global hydrodynamic type of homogeneous equilibrium configuration. Gravitationally bounded quasineutral field-free plasma by a spherically symmetric surface boundary of nonrigid and nonphysical nature is considered. An estimated typical value $\sim 10^{-20}$ of the ratio of the solar plasma Debye length and Jeans length of the total solar mass justifies the quasineutral behavior of the solar plasma on both the bounded and unbounded scales. A bulk nonisothermal uniform flow of solar plasma is assumed to preexist. For minimalism, we consider spherical symmetry of the self-gravitationally confined SIP mass distribution, because this helps to reduce the three-dimensional problem of describing the GES into a simplified one-dimensional problem in the radial direction since curvature effects are ignorable for small scale size of the fluctuations. Thus only a single radial degree of freedom is sufficient for describing the three-dimensional SIP and, hence, the SWP in radial symmetry approximation. This is to elucidate that our plasma-based theory of the GES stability is quite simplified in the sense that it does not include any complicacy like the magnetic forces, nonlinear thermal forces and the role of interplanetary medium or any other difficulties like collisional, viscous processes, and so forth.

Applying the spherical capacitor charging model [3], the coulomb charge on the SSB comes out to be $Q_{\mathrm{SSB}} \sim 120\,\mathrm{C}$. More on the basic electromagnetic properties of the Sun and its atmosphere could be understood from the electrical stellar models [1, 2]. Let us approximately take mean rotational frequency of the SSB about the centre of the SIP system to be $f_{\mathrm{SSB}} \sim 1.59 \times 10^{-12}\,\mathrm{Hz}$. Applying the electrical model [2] of the Sun, the mean value of the strength of the solar magnetic field at the SSB in our model analysis is estimated as $\langle |B_{\mathrm{SSB}}| \rangle = 4\pi^2 Q_{\mathrm{SSB}} f_{\mathrm{SSB}} \sim 7.53 \times 10^{-11}\,\mathrm{T}$, which is negligibly small for producing any significant effects on the dynamics of the SIP particles. Thus the effects of the magnetic field are not realized by the solar plasma particles due to the weak Lorentz force, which is now estimated to be $F_L^{\mathrm{SIP}} = e(v_{\mathrm{SIP}} \times \langle |B_{\mathrm{SSB}}| \rangle) \approx 3.61 \times 10^{-33}\,\mathrm{N}$ corresponding to an average subsonic flow speed $v_{\mathrm{SIP}} \sim 3.00\,\mathrm{cm\,s}{-1}$, and hence, neglected. It equally justifies the convective and circulation dynamics being neglected in our SIP model. Therefore our unmagnetized plasma approximation is well justified in our model configuration on the bounded SIP scale. The same, however, may not apply for the unbounded SWP scale description. This is because the Lorentz force for the SWP comes out to be $F_L^{\mathrm{SWP}} = e(v_{\mathrm{SWP}} \times \langle |B_{\mathrm{SWP}}| \rangle) \approx 1.64 \times 10^{-2}\,\mathrm{N}$ to a mean supersonic flow speed $v_{\mathrm{SWP}} \sim 340.00\,\mathrm{km\,s}^{-1}$, thereby showing that $F_L^{\mathrm{SIP}}/S_L^{\mathrm{SWP}} \sim 10^{-31}$. ln addition, the effects of solar rotation, viscosity, nonthermal energy transport, and trapping of plasma particles are as well, for mathematical simplicity, neglected in our idealized plasma-based model approach for the SIP description.

The solar plasma is assumed to consist of a single component of Hydrogen ions and electrons. The thermal electrons are assumed to obey Maxwellian velocity distribution in an idealized situation. In reality, of course, deviations exist, and hence different kinds of exospheric models have already been proposed with special velocity distribution functions kinetically [4, 7, 8]. Again inertial ions are assumed to exhibit their full inertial response of dynamical evolution governed by fluid equations of quasihydrostatic equilibrium. This includes the ion fluid momentum equation as well as the ion continuity equation. The first describes the change in ion momentum under the action of the heliocentric gravito-electrostatic field due to self-gravitational potential gradient and forces induced by thermal gas pressure gradient. The latter is considered as a gas dynamic analog of the solar plasma self-similarly flowing through a spherical chamber of radially varying cross-sectional area with macroscopic bulk uniformity in accordance with the basic rule of idealistic fluid flux conservation.

3. Mathematical Formulation

The basic normalized (with all standard astrophysical quantities) autonomous set of nonlinear differential evolution equations with all the usual notations [1] constituting a closed hydrodynamical structure of the solar plasma system has already been developed in time-stationary form. The same set of the basic structure equations defined in the gravitational scale of space and time is developed in nonautonomous form and enlisted as follows:

$$\frac{dM}{d\tau} + M\frac{dM}{d\xi} = -\alpha\frac{d\theta}{d\xi} - g_s, \tag{1}$$

$$\frac{d\theta}{d\tau} + M\frac{d\theta}{d\xi} + \frac{dM}{d\xi} + \frac{2}{\xi}M = 0, \tag{2}$$

$$\frac{dg_s}{d\xi} + \frac{2}{\xi}g_s = e^{\theta}, \tag{3}$$

$$\frac{d^2\theta}{d\xi^2} + \frac{2}{\xi}\frac{d\theta}{d\xi} = 0. \tag{4}$$

Here $\alpha = 1 + \in_T = 1 + (T_i/T_e)$, T_e is the thermal electron temperature, and T_i is the inertial ion temperature on the bounded SIP scale (each in eV). This should be mentioned that (4) is a nonplaner geometrical outcome of the generalized electrostatic Poisson's equation with global quasineutrality approximation in a spherical symmetric distribution of the solar plasma with all the usual notations [1] given as

$$\left(\frac{\lambda_{\mathrm{De}}}{\lambda_J}\right)^2\left[\frac{d^2\theta}{d\xi^2} + \frac{2}{\xi}\frac{d\theta}{d\xi}\right] = N_e - N_i. \tag{5}$$

Here $\lambda_{\mathrm{De}} = \sqrt{T_e/4\pi n_0 e^2}$ denotes the SIP electron Debye length. For instant information, the solar parameters $M(\xi)$, $g_s(\xi)$, and $\theta(\xi)$ represent the equilibrium Mach number, solar self-gravity, and electrostatic potential, respectively. They are respectively normalized by plasma sound phase speed (c_s), solar free-fall (heliocentric) self-gravity strength (c_s^2/λ_J), and electron thermal potential (T_e/e). Moreover, the independent variables like time (τ) and position (ξ) are normalized with Jeans time (ω_J^{-1}) and Jeans length (λ_J) scales, respectively, as already carried out in our earlier publication [3] too.

In order to get a quantitative flavor for a typical value of $T_e = 10^6$ K, for example, one can estimate the value of $\lambda_{\mathrm{De}}/\lambda_J \approx 10^{-20}$ [3]. This implies that the size of the Debye scale length is quite smaller than that of the Jeans scale length of the solar plasma mass. Thus on the typical gravitational scale length of the inertially bounded solar plasma system, the limit $\lambda_{\mathrm{De}}/\lambda_J \rightarrow 0$ becomes a realistic (*physical*) approximation. By virtue of this limiting scale condition the entire SIP extended up to the SSB and beyond obeys the plasma approximation of global quasineutrality, as $N_e \approx N_i$ from (5), in our defined self-gravitating solar plasma system justifiably.

Applying the usual standard methodology of reductive perturbation technique [14, 15] over the coupled set of (1)–(4), we want to derive a nonlinear dynamical equation in the lowest-order perturbed GES potential in the SIP scale. Methodologically, the independent variables are thus stretched directly as $\xi = \in^{1/2}(x - \lambda t)$ and $\tau = \in^{3/2}t$. Thus in the newly defined space of stretched variables, the linear differential operators transform as $\partial/\partial\xi \equiv \in^{1/2}\partial/\partial x$, $\partial^2/\partial\xi^2 \equiv \in \partial^2/\partial x^2$, and $\partial/\partial\tau \equiv (\in^{3/2}\partial/\partial t - \lambda\in^{1/2}\partial/\partial x)$, where λ is the phase speed of the GES perturbation and $\in$ is a smallness parameter characterizing the balanced strength of dispersion and nonlinearity. The dimensionless amplitude of the lowest-order fluctuations is usually given by this parameter $\in$ [10, 11].

The nonlinearities in our might have directly come from the large-scale dynamics (in space and time) through the harmonic generation involving fluid plasma convection, advection, dissipation, and so forth. These nonlinearities may contribute to the localization of waves and fluctuations leading to the formation of different types of nonlinear coherent structures like solitons, shocks, vortices, and so forth which have both theoretical as well as experimental importance [14]. The scale size of all the nonlinear fluctuations of current interest is assumed to be much shorter than all the characteristic mean free paths. These scaling analyses have, systematically, been derived under the conditions that the normalized fluctuations in the dependent solar plasma variables are of the same order within an order of magnitude in the astrophysical scale of space and time.

In order to study the GES stability of the present concern, the relevant solar physical variables (M, g_s, θ) are now perturbatively expanded around the respective well-defined GES equilibrium values (M_0, g_{s0}, θ_0) as follows:

$$\begin{pmatrix} M \\ g_s \\ \theta \end{pmatrix} = \begin{pmatrix} M_0 \\ g_{s0} \\ \theta_0 \end{pmatrix} + \in \begin{pmatrix} M_1 \\ g_{s1} \\ \theta_1 \end{pmatrix} + \in^2 \begin{pmatrix} M_2 \\ g_{s2} \\ \theta_2 \end{pmatrix} + \cdots. \tag{6}$$

We now substitute (6) into the basic governing (1)–(4). Equating the terms in various powers of $\in$ from both sides of (1), one gets

$$\in^{1/2}: \ -\lambda\frac{\partial M_0}{\partial x} + M_0\frac{\partial M_0}{\partial x} = -\alpha\frac{\partial\theta_0}{\partial x}, \tag{7}$$

$$\in^{3/2}: \ \frac{\partial M_0}{\partial t} - \lambda\frac{\partial M_1}{\partial x} + M_0\frac{\partial M_1}{\partial x} + M_1\frac{\partial M_0}{\partial x} = -\alpha\frac{\partial\theta_1}{\partial x}, \tag{8}$$

$$\in^{5/2}: \ \frac{\partial M_1}{\partial t} - \lambda\frac{\partial M_2}{\partial x} + M_0\frac{\partial M_2}{\partial x} + M_1\frac{\partial M_1}{\partial x} + M_2\frac{\partial M_0}{\partial x} = -\alpha\frac{\partial\theta_2}{\partial x}, \text{ etc.} \tag{9}$$

Similarly, equating the terms in various powers in $\in$ from (2), one gets

$$\in^{1/2}: \ (M_0 - \lambda)\frac{\partial\theta_0}{\partial x} + \frac{\partial M_0}{\partial x} = 0,$$
$$\in^{3/2}: \ \frac{\partial\theta_0}{\partial t} - \lambda\frac{\partial\theta_1}{\partial x} + M_0\frac{\partial\theta_1}{\partial x} + M_1\frac{\partial\theta_0}{\partial x} + \frac{\partial M_1}{\partial x} = 0, \text{ etc.} \tag{10}$$

The order-by-order analysis in various powers of $\in$ from (3) similarly yields

$$\begin{aligned}
\in^0 &: \left(\frac{2}{\xi}\right) g_{s0} = 1 + \theta_0, \\
\in^{1/2} &: \frac{\partial g_{s0}}{\partial x} = 0, \\
\in^1 &: \left(\frac{2}{\xi}\right) g_{s1} = \theta_1, \\
\in^{3/2} &: \frac{\partial g_{s1}}{\partial x} = 0, \\
\in^2 &: \left(\frac{2}{\xi}\right) g_{s2} = \theta_2, \text{ etc.}
\end{aligned} \tag{11}$$

The same order-by-order analysis in various powers in $\in$ from (4) yields

$$\in^1 : \frac{\partial^2 \theta_0}{\partial x^2} + \left(\frac{2}{x}\right) \frac{\partial \theta_0}{\partial x} = 0, \tag{12}$$

$$\in^2 : \frac{\partial^2 \theta_1}{\partial x^2} + \left(\frac{2}{x}\right) \frac{\partial \theta_1}{\partial x} = 0, \tag{13}$$

$$\in^3 : \frac{\partial^2 \theta_2}{\partial x^2} + \left(\frac{2}{x}\right) \frac{\partial \theta_2}{\partial x} = 0, \text{ etc.} \tag{14}$$

We are involved in the dynamical study of the lowest-order GES potential fluctuation associated with the SIP system. Equation (8), therefore, is now approximately simplified into the following form:

$$M_1 = -\left(\frac{\alpha}{M_0 - \lambda}\right) \theta_1. \tag{15}$$

A little exercise with the substitution of (15) in (9) (under an approximation of equal rate of harmonic covariation) jointly gives

$$\frac{\partial \theta_1}{\partial x} = -\frac{1}{(M_0 - \lambda)} \frac{\partial \theta_1}{\partial t} + \frac{\alpha}{(M_0 - \lambda)^2} \theta_1 \frac{\partial \theta_1}{\partial x}. \tag{16}$$

Again spatially differentiating (13), one gets

$$\frac{\partial^3 \theta_1}{\partial x^3} + \left(\frac{2}{x}\right) \frac{\partial^2 \theta_1}{\partial x^2} - \left(\frac{2}{x^2}\right) \frac{\partial \theta_1}{\partial x} = 0. \tag{17}$$

Now coupling (16) and (17) dynamically with no variation of the equilibrium parameters, one gets easily the following modified form of KdV-Burger (KdV-B) equation [9–11, 14, 15] for the description of the nonlinear GES fluctuations in terms of θ_1 as follows:

$$\begin{aligned}
&\left(\frac{2}{M_0 - \lambda}\right) \frac{\partial \theta_1}{\partial t} - \left[\frac{2\alpha}{(M_0 - \lambda)^2}\right] \theta_1 \frac{\partial \theta_1}{\partial x} \\
&\quad + x^2 \frac{\partial^3 \theta_1}{\partial x^3} + 2x \frac{\partial^2 \theta_1}{\partial x^2} = 0.
\end{aligned} \tag{18}$$

This is clear from (18) that the temporal part (1st term) and convective part (2nd term) have constant coefficients in a given plasma configuration bounded quasihydrostatically. The dispersive part arising due to the deviation from global plasma quasineutrality (3rd term) and dissipative part arising due to various internal loss processes (4th term), however, have variable coefficients. We are interested in time-stationary structures of dynamical fluctuations, and hence, (18) is transformed into an ordinary differential equation (ODE) with the transformation $\xi \equiv (x - \lambda t)$ so that the operational equivalence $\partial/\partial t \equiv -\lambda \partial/\partial \xi$ and $\partial/\partial x \equiv \partial/\partial \xi$ hold good. Equation (18), therefore, with $A = -2\lambda/(M_0 - \lambda)$ and $B = -2\alpha/(M_0 - \lambda)^2$ gets transformed into a stationary form as

$$A \frac{\partial \Theta}{\partial \xi} + B \Theta \frac{\partial \Theta}{\partial \xi} + \xi^2 \frac{\partial^3 \Theta}{\partial \xi^3} + 2\xi \frac{\partial^2 \Theta}{\partial \xi^2} = 0, \tag{19}$$

where $\Theta = \theta_1(\xi)$ denotes the lowest-order fluctuation in the GES potential.

Equation (19) clearly shows the possibility for the existence of some shock-like structures (due to energy dissipation) in addition to soliton-like structures (due to energy dispersion). The first class of structures realistically arises when the effect of dissipation is significant in comparison with the joint effect of the nonlinearity and dispersion, whereas for the second class, the effect of dissipation is insignificant in comparison with that produced jointly by the nonlinearity and dispersion [14]. Being of nonlinear type, the exact solution of (19) is difficult without any asymptotic approximation. The approximate solutions are obtained analytically by the method of integration with some boundary conditions like $\Theta \to 0$, $\partial\Theta/\partial\xi \to 0$, $\partial^2\Theta/\partial\xi^2 \to 0$ at $\xi \to \infty$ as done by others [14]. The explicit form of the analytical solution (traveling wave) of (19) with all the usual notations is derived and presented as follows:

$$\Theta(x,t) = \frac{\lambda(\lambda - M_0)}{\alpha} \left[1 + \tanh\left\{\frac{\lambda}{2x(M_0 - \lambda)}(x - \lambda t)\right\}\right]. \tag{20}$$

Equation (20), in fact, represents the asymptotic form of a monotonic shock structure (laminar type) with shock speed $U_{sh} = \lambda$, shock amplitude $A_{sh} = \lambda(\lambda - M_0)/\alpha$, and shock front thickness $\Gamma_{sh} = 2x(M_0 - \lambda)/\lambda$. The fundamental difference of the solution (20) with those obtained analytically by others [14] is that the present solution represent shocks in the perturbed GES potential in the self-gravitating hydrodynamic SIP with plasma-boundary interaction taken into account. The other reported solutions [14], on the other hand, represents shocks in the perturbed electrostatic potential and density in dusty plasma in hydrostatic equilibrium, but in the absence of plasma-boundary wall interaction processes, self-gravity, and gravito-electrostatic coupling effects. This, interestingly, is noticed here that the shock width of (20) here is a function of the independent position coordinate x alone in the defined self-gravitating solar plasma configuration. Eventually, this impulsive character of the SIP blast wave is realistically justifiable due to infinite thermal pressure at the core of the Sun [3] generated by the effect of the strong self-gravitational collapse leading to thermonuclear fusion responsible for tremendous amount of energy production.

Equation (19) is furthermore numerically solved (by Runge-Kutta IV method) to get a detailed picture of the basic features of the GES fluctuations on astrophysical scale under different realistic initial values of the relevant solar physical variables. These realistic initial values are analytically arrived at as a natural outcome of the nonlinear dynamical stability analyses around fixed points, as carried out in our earlier work [3], over the coupled dynamical equations of the two-layer GES model description.

4. Results and Discussions

4.1. Analytical Results. A theoretical model analysis is carried out to study the GES fluctuation in a simplified field-free quasineutral solar plasma model in quasihydrostatic type of homogeneous equilibrium configuration. A distinct set of nonautonomous self-consistently coupled nonlinear dynamical eigenvalue equations in a defined astrophysical space and time configuration are developed. Applying the standard methodology of reductive perturbation technique over the defined GES equilibrium [3], a modified form of KdV-Burger (KdV-B) equation in terms of the lowest-order perturbed GES potential is obtained. An explicit form of the approximate analytical solution (shock family) only is derived with the help of conventional method of integration [14] by imposing asymptotic boundary conditions. Similar analytical results exist in the literature [14, 15], but in terms of shocks in the perturbed electrostatic potential and density in dust-contaminated plasma in hydrostatic equilibrium in absence of the effects of all plasma-boundary wall interaction, self-gravity, and gravito-electrostatic coupling mechanisms as already mentioned above in the previous section.

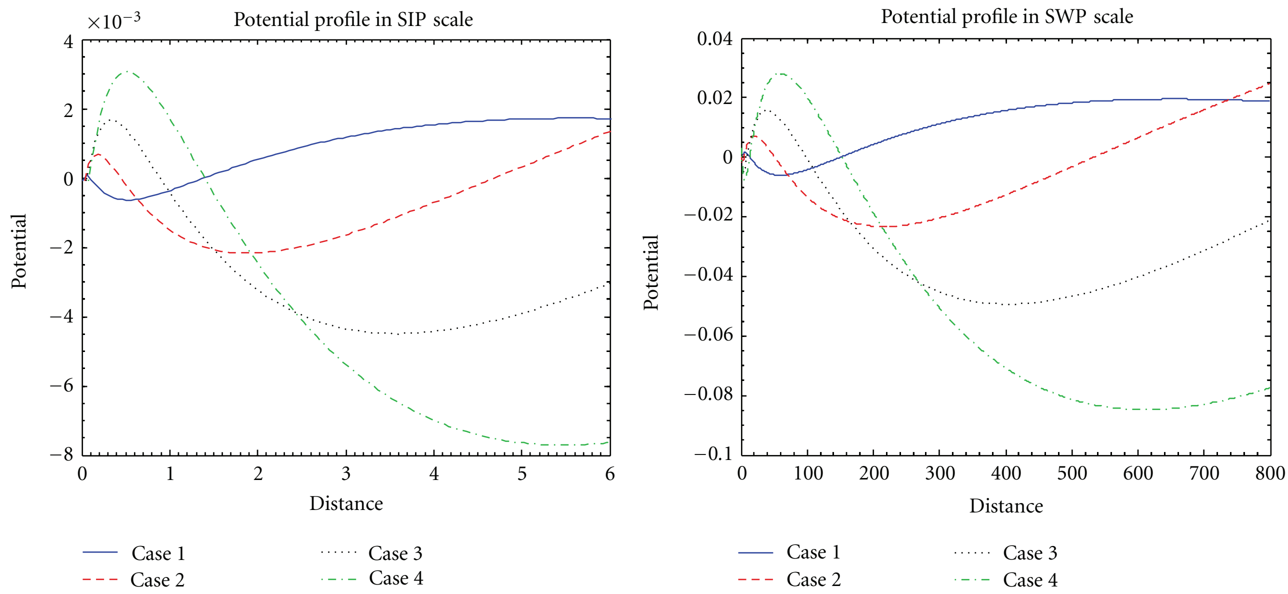
Let us now estimate the physical values of the main shock characterization parameters represented by (20) at the SSB. At the SSB, $(x - \lambda t) = \xi \sim 3.5, M_{\mathrm{SSB}} \sim 10^{-7}$, and $\in_T \sim 0.4$ [3]. Typically, the smallness parameter is $\in \approx 10^{-2}$ [10] for solar plasma atmosphere. Now for $\lambda = U_{\mathrm{sh}}^{\mathrm{SSB}} = 0.1$ and $\alpha^{\mathrm{SSB}} = (1 + \in_T) = 1.4$, we can analytically estimate the physical value of the GES potential shock amplitude $A_{\mathrm{sh,phys}}^{\mathrm{SSB}} = 7.1 \times 10^{-5}$ (=7.10×10^{-2} volts, or 2.36×10^{-4} statvolts) and shock front thickness $\Gamma_{\mathrm{sh,phys}}^{\mathrm{SSB}} = 7.0$ ($\sim 7.00 \times 10^{8}$ m, or 7.0×10^{10} cm). Thus gravito-electrostatic shocks in the SIP system carry relatively lower energy of the GES-induced mode.

The reductive perturbation method is, however, not very popular as a mathematically rigorous perturbation method, even conditionally. It, nevertheless, is a convenient, approximate, and easy way to produce certain mathematically interesting paradigm of nonlinear equations. By the free ordering, we may get almost any explicit form of results as expected. In particular, the ordering required to get, for example, a shock-like solution is now known. Numerical solutions will subsequently show that there are many other shock-like structures that do not satisfy the "required ordering". In fact, although shocks are frequently found experimentally, so far a shock that satisfies the KdV-B ordering has never been found, whether in neutral fluid, lattice, or plasma. Thus, analytically, it provides a new mathematical stimulus scope for future interest to derive analytical results with greater accuracy with newer mathematical techniques so as to get more detailed picture of the self-gravitational fluctuations like in the Sun.

4.2. Numerical Results. Our theoretical GES model analysis shows that the solar plasma system supports shock formation governed by KdV-B (19). This is again integrated numerically too so as to get some numerical profiles for different initial values on a more detailed grip. The main features of our observations based on our numerical analyses may be discussed as follows. Figure 1(a) shows the $\Theta(\xi)$-profile on the SIP scale with $\in_T = 0.4, \lambda = 0.8, M_0 = 10^{-8}$, and $\Theta_i = -0.0001$. The various lines correspond to Case (1) $\xi_i = 0.01$, Case (2) $\xi_i = 0.03$, Case (3) $\xi_i = 0.05$, and Case (4) $\xi_i = 0.07$, respectively. Figure 1(b) similarly depicts the same, but on the SWP scale. It is clear that the GES perturbation excited on the SIP scale gets propagated even at and up to an asymptotically large distance due to space charge polarization effects boosted up by the supersonic SWP flow. The fluctuation assumes various nonlinear forms of shock family sensitive to input initial position values relative to the heliocentric origin. The $\Theta(\xi)$ amplitude is found to vary from -7.5×10^{-3} to $+1.5 \times 10^{-3}$ at the SSB ($\xi = 3.5\lambda_J$), the SWP base. It is again on the order of 10^{-3} at 1 AU ($\xi = 750\lambda_J$) approximately. All these observations are relative to the unperturbed GES potential value at the SSB. The SSB is already reported to act as a spherically symmetric electrical grid which is gravitoelectrostatically negatively biased with a normalized value of the equilibrium GES potential $\theta_\Theta(\xi_\Theta) \sim -1$ ($= -1.00$ kV) [3]. The SWP flow dynamics is the natural outcome of the solar plasma leakage process through this electrical grid.

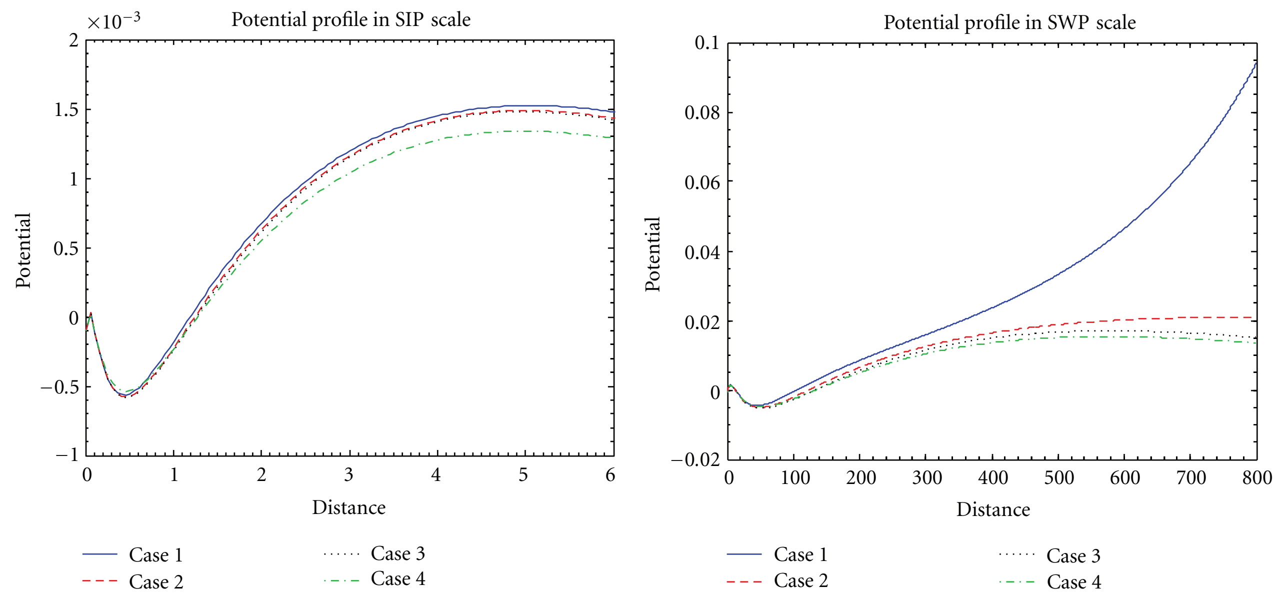
Again Figure 2(a) similarly depicts the $\Theta(\xi)$ profile on the SIP scale with $\in_T = 0.4, \xi_i = 0.01, M_0 = 10^{-8}$, and $\Theta_i = -0.0001$. The various lines are for Case (1) $\lambda = 0.1$, Case (2) $\lambda = 0.2$, Case (3) $\lambda = 0.8$, and Case (4) $\lambda = 5.1$, respectively. Figure 2(b) gives the same, but on the SWP scale. It is clear that, as in Figure 1, different shocklike structures arise from different flow velocities with $\Theta(\xi)$ amplitude lying on the order of 10^{-3} on both the SIP and SWP scales.

Lastly, Figure 3(a) shows the $\Theta(\xi)$-profile on the SIP scale with $\in_T = 0.4, \xi_i = 0.01, M_0 = 10^{-8}$, and $\lambda = 0.8$. The various lines specify Case 1: $\Theta_i = -0.0001$, Case 2: $\Theta_i = -0.003$, Case 3: $\Theta_i = -0.01$, and Case 4: $\Theta_i = -0.02$, respectively. Likewise, Figure 3(b) gives the same, but on the SWP scale. It is found that the $\Theta(\xi)$-amplitude varies from -5.0×10^{-3} to $+15 \times 10^{-3}$ in the SIP scale. Its order is the same even at 1 AU. This is interestingly in totality found numerically (Figures 1–3) that the GES potential fluctuation propagates as shock-like structures with amplitude $\Theta(\xi) \sim 10^{-3}$ at both the SSB and at 1 AU, approximately. Thus for a typical value of the smallness parameter $\in \approx 10^{-2}$ [10] for solar plasma configuration, the physical value of the GES potential shock amplitude can throughout the numerical analyses be calculated as



(a) (b)

FIGURE 1: Profile of the lowest order GES potential fluctuation $\Theta(\xi)$ on the (a) SIP scale and (b) SWP scale with $\in_T = 0.4, \lambda = 0.8, M_0 = 10^{-8}$, and $\Theta_i = -0.0001$. The various lines correspond to Case 1: $\xi_i = 0.01$, Case 2: $\xi_i = 0.03$, Case 3: $\xi_i = 0.05$, and Case 4: $\xi_i = 0.07$, respectively.

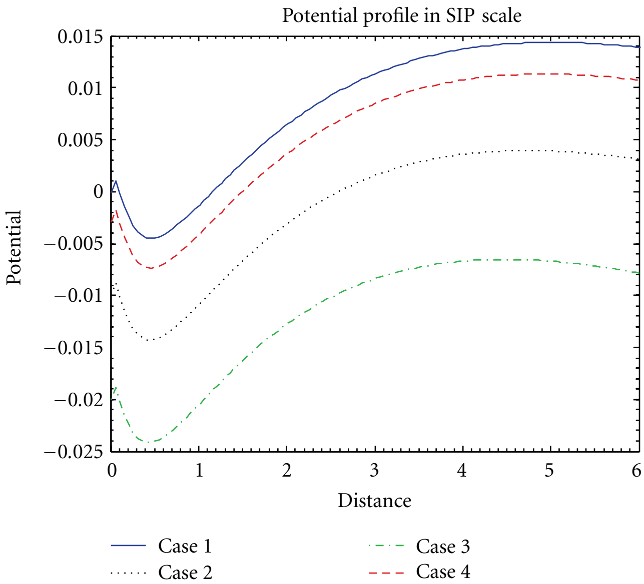


(a) (b)

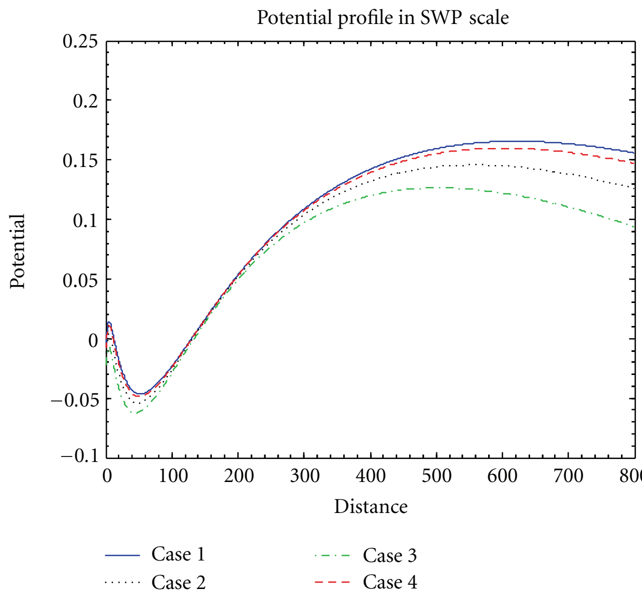
FIGURE 2: Profile of the lowest order GES potential fluctuation $\Theta(\xi)$ on the (a) SIP scale and (b) SWP scale with $\in_T = 0.4, \xi_i = 0.01, M_0 = 10^{-8}$, and $\Theta_i = -0.0001$. The various lines correspond to Case 1: $\lambda = 0.1$, Case 2: $\lambda = 0.2$, Case 3: $\lambda = 0.8$, and Case 4: $\lambda = 5.1$, respectively.

$\Theta_{\text{Phys}}(\xi) = \in \Theta_\Theta(\xi) \approx 10^{-5} = 10^{-2}$ volts. In all the cases, the $\Theta(\xi)$ amplitude is usually found to go more and more negative near the heliocentre ($\xi \sim 0.5\lambda_J$) due to strong self-gravitational effects, acting even on the plasma thermal electrons and thereby, tending to prevent them to escape through solar self-gravitational potential barrier. The reversibility of the magnitudes of the GES shock-like structures from negative to positive values is ascribed due to the solar plasma leakage process through the SSB grid.

4.3. Comparative Results. This has been recognized years ago that the compressional plasma in the solar atmosphere is a perfect medium for magnetohydrodynamic (MHD) waves. Applying the MHD model analyses [9–11, 16, 17], several authors have investigated on the corresponding nonlinear eigenmodes in the compressional solar plasma in presence of magnetic field. Some of the important distinctions between our GES stability analysis and MHD stability analyses on the solar plasma fluctuation dynamics reported so far in



(a)



(b)

Figure 3: Profile of the lowest order GES potential fluctuation $\Theta(\xi)$ on the (a) SIP scale and (b) SWP scale with $\in_T = 0.4, \xi_i = 0.01, M_0 = 10^{-8}$, and $\lambda = 0.8$. The various lines correspond to Case 1: $\Theta_i = -0.0001$, Case 2: $\Theta_i = -0.003$, Case 3: $\Theta_i = -0.01$, and Case 4: $\Theta_i = -0.02$, respectively.

the literature are worth mentioning here. The following tabulation, Table 1, shows the main distinctions between them.

We scientifically admit that the neglect of collisional dissipations and deviation from Maxwellian velocity distributions of the plasma particles is not quite realistic. But our GES stability analyses even under some simplified and idealized approximations may provide quite interesting results for the solar physics community. The main points based on our analyses are summarized as follows.

(1) The GES fluctuations appear in the form of various nonlinear structures (of shock-family eigenmodes) governed by a new analytic form of KdV-Burger (KdV-B) equation. Here the terminology "new analytic form" refers to the appearance of the new type of characteristic coefficients in the dispersive and dissipative terms in it.

(2) The influence and presence of such eigenmodes are also experienced asymptotically even in the SWP scale. The structural modification here is due to the background initial conditions under which being excited. Such blast wave structures arise mainly due to violent disturbances of self-gravitational type. Their front thickness may, however, be a consequence of the homogeneous balance between self-gravitating solar plasma nonlinear compressibility and dissipative mechanisms like viscosity, heat conduction, and so forth. Similar observations in the Sun have also been reported by MHD-community under Hall-MHD approximation [9–11] in terms of the lowest-order perturbed solar density fluctuation. In addition, such structures have also been experimentally observed and reported in a dust-contaminated plasma system [14, 15] with different grain population density in both oscillatory as well as laminar forms, but in absence of gravito-electrostatic effects.

(3) Self-gravity of the SIP mass distribution is normally found to have a tendency to depress (due to dissipation and dispersion) the nonlinear structures in the interior (subsonic SIP flow) and steepens (due to nonlinearity) them in the exterior (supersonic SWP flow) in our two-scale GES stability analyses.

(4) Last but not least, the $\Theta(\xi)$ fluctuations become almost uniform at an asymptotically large distance relative to the heliocentre. This physically means that the SWP flow is a uniform one asymptotically, and net electric current, contributed jointly by solar thermal electrons and inertial ions, will remain conserved (divergence-free current density in a steady-state description) which is in good agreement with the already reported results [18].

This analysis, moreover, is carried out in a homogeneous kind of field-free quasihydrostatic equilibrium configuration under quasineutral plasma approximation. However, even in spite of these limitations, it may perhaps be useful for further investigation of dynamical stability on a nonlinearly coupled system of the SIP and SWP as an interplayed flow dynamics of heliocentric origin in presence of all the possible realistic agencies [16, 17] like collision, viscosity, and so forth. This is speculated that the normal mode behaviors

Table 1: GES versus MHD stability analyses.

S. no.	Items	GES stability analysis	MHD stability analyses
1	Model	Ideal hydrodynamic	MHD
2	Plasma-boundary wall interaction and sheath formation mechanism	Included	Neglected
3	Effect of charge separation	Considered	Not considered
4	Floating surface (at which no net electric current)	Involved	Not involved
5	Magnetic field	Not considered ($\langle \lvert B_{\mathrm{SSB}} \rvert \rangle \sim 7.53 \times 10^{-11}$ T)	Considered ($\langle \lvert B_{\Theta} \rvert \rangle \sim 1.30 \times 10^{-6}$ T)
6	Description	Two-scale (SIP and SWP)	One scale (SWP)
7	Sonic range	Subsonic (SIP) and supersonic (SWP)	Supersonic (SWP)
8	Self-gravity (SIP) and external gravity (SWP)	Considered	Not considered
9	Transonic transition (subsonic to supersonic)	Involved (through SIP and SSB interaction process and thus transformed into SWP)	Not involved
10	Analytical solution	Bounded (SIP)	Unbounded (SWP)
11	Thermal species	Maxwellian	Single fluid (MHD)
12	Surface description and specification	Yes (at $\xi = 3.5\,\lambda_J$) and it is negatively biased (with $\theta_s \sim -1.00\,\mathrm{kV}$) at the cost of thermal loss of SIP electrons	Not precisely, but the diffused surface is electrically uncharged and unbiased
13	Source of nonlinearity	Plasma fluidity	Large-scale dynamics
14	Source of dispersion	Deviation from quasineutrality and self-gravity	Geometrical effect (also, some part of physical effect)
15	Source of dissipation	Weak collisional effects	Viscosity and magnetic diffusion
16	Sun and SWP coupling	Considered	Not considered
17	Nature of solutions	Mainly shocklike structures in the lowest ordered perturbed GES potential	Soliton and shocklike structures in the lowest ordered perturbed density and velocity
18	Solar atmosphere	Not stratified	Stratified (into a number of heliocentric layers)
19	Adopted technique	Standard reductive perturbation technique (about the GES equilibrium)	Standard multiple scaling technique (about the MHD equilibrium)
20	Convection and circulation dynamics	Not treated (for idealized simplicity)	Treated
21	Leakage process	Taken into account	Not taken into account
22	Main application	Surface origin of the subsonic SWP and its transonic flow dynamics	Solar chromospheric and coronal heating

of the global SSB oscillations could also be analyzed in terms of both the local as well as global gravito-electrostatic plasma sheath-induced oscillations with such techniques. Additionally, gaseous phase of the solar plasma is reported to contain solid phase of dust matter [19]. In the SWP scale of uniform flow, application of the inertia-induced acoustic excitation mechanism [20] may further be carried out for further stability analyses. The basic principles of the nonlinear pulsational mode [21] of the self-gravitational collapse model of charged dust clouds by applying the presented methodology may be another important future application in the self-gravitating solar plasma system.

5. Conclusions

The dynamical stability of the GES model, although simplified through idealistic approximations, is analyzed in both analytical and numerical forms with standard perturbation formalism. It provides an idea into the interconnection between the SIP (Sun-) stability in terms of the lowest-order GES fluctuation appearing as various nonlinear structures (shock like) and their asymptotic propagation in the SWP scale as an integrated model approach. This is conjectured that the fluctuations are jointly governed by a new form of KdV-Burger type of nonlinear evolution equation having some characteristic model coefficients. Both analytical and numerical solutions are in qualitative and quantitative agreement. The main conclusions of scientific interest drawn from our present contribution are summarized as follows.

(1) Nonlinear fluctuations of the GES in the SIP scale are governed by a KdV-Burger (KdV-B) type of equation with characteristic coefficients dependent on the solar plasma GES model.

(2) Different forms of nonlinear eigenmodes exist in the SIP scale in different situations. Their presence, pretriggered strongly due to self-gravity on the SIP scale origin, is also experienced at asymptotically large distances beyond the SSB. It goes in qualitative conformity with those reported with different methodologies [2, 12, 13].

(3) The structures are contributed mainly due to gravitoelectrostatically coupled self-gravity fluctuation of the solar plasma inertial ions under an integrated interplay of diverse nonlinear (hydrodynamic origin) and dispersive (self-gravitational origin) effects in the solar plasma system in presence of some internal dissipation.

(4) The SIP is found to be more unstable (more fluctuation gradient) than the SWP (less fluctuation gradient) asymptotically. This is because of the GES fluctuation in presence of strong self-gravity in the bounded SIP scale and weak external gravity in the unbounded SWP scale.

(5) Our two-scale theory of the GES is found to give two-scale dynamical variation of the GES stability as a gravito-electrostatically coupled system of the SIP (subsonic flow) and the SWP (supersonic flow) through the interfacial SSB.

Finally and additionally, the modified GES mode kinetics as a self-gravitationally triggered instability in an intermixed state of the gaseous phase of plasma and solid phase of dust grain-like impurity ions (DGIIs), by using a dissipative multi-fluid colloidal or dusty plasma model with dust scale size distribution power law taken into account, may be another interesting investigation to study DGII-behavior in an SWP-like realistic situation on a global scale. This is because the interplay between gravitational and electrostatic forces in the dynamics of such grains is responsible for many interesting phenomena in the terrestrial and solar environment (like rings of Saturn and Jupiter, satellites' spoke formation, etc.). It eventually may have some useful characteristic implications of acoustic spectroscopy ([20] and references therein) as well on the basis of dispersion wave analyses to be characterized with different scale-sized inertial species (DGIIs) in different realistic astrophysical conditions. The mathematical methodology adopted may also be extensively applicable to other types of nonlinear waves, wherever all being considered as derivatives of shocks in presence of nonlinearity, dispersion, and dissipation, by applying kinetic exospheric model approaches [7, 8] with the more realistic SWP exobases taken into concern. These mathematical analyses may be extended for further investigation of fluctuation and stability with more realistic assumptions like grain rotations, spatial inhomogeneities, different gradient forces, and so forth, taken into account in other astrophysical and space environments. These calculations, although tentative for any concrete application to any sharply specified stellar formation mechanism, may be widely useful in the study of fluctuation-induced dynamics with electrostatic charge fluctuation of dust grains in astrophysical environment of dusty plasmas in the complex form of self-gravitationally collapsing dust cloud [21].

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