Research Article

To Study the Effect of the Generating Polynomial on the Quality of Nonlinear Components in Block Ciphers

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Substitution box (S-box), being the only nonlinear component, contributes to the confusion creating capability of a cryptosystem. Keeping in view the predominant role of S-box, many design algorithms to synthesize cryptographically stronger S-boxes have gained pivotal attention. A quick review of these algorithms shows that all these ideas mainly concentrate on the choice of bijective Boolean functions, with nonobservance to the irreducible polynomial that generates the Galois field. In this paper, we propose that the selection of irreducible polynomial has a deep influence on the highly desirable features of an S-box such as nonlinearity, strict avalanche, bit independence, linear approximation probability, and differential approximation probability. We underpin our claim by investigating a detailed model, which deploys the same algorithm but different polynomials and produces unusual changes in the results regarding the performance parameters of S-box.

1. Introduction

Electronic exchange of data has undoubtedly revolutionized the communication in recent years but, on the other hand, the secure transfer of confidential material over Internet has become the biggest challenge nowadays. It definitely demands seriously high level of security. The main problem is to avoid unauthorised access to the secret data. To achieve the desired level of security, many techniques such as cryptography, watermarking, and steganography have been the major focus of research for past few years [1–5]. In this paper, we deal with cryptography.

Cryptography is categorized into two types, *symmetric key cryptography* and *asymmetric key cryptography*. The symmetric key cryptography can be further split into two types: block ciphers and the stream ciphers. Advanced Encryption Standard is an example of block cipher that was officially adopted by the US government as the Federal Information Processing Standard (FIPS) in May 2002. AES algorithm [6] is based on four steps: round key addition, byte substitution,

shift row, and mix column, but the most influential of all these is the byte substitution step. This step relies on a substitution box (S-box), which serves as the only nonlinear component in any *substitution-permutation network* (SPN).

It has been established that the substitution box (S-box) is a standout in different block ciphers and is a widely used mechanism in any substitution-permutation network as a source to produce nonlinearity [6]. It renders an absolutely complex, unforeseeable layout to requite various blocks of bits in output data. To extend high resistance against unexpected surveillance, S-box structure is required to fulfil certain standards. The indispensable involvement of S-box to induce complexity and nonlinearity motivates studying the properties and algorithms for safer and more reliable S-boxes. In this regard, many advanced structural developments are witnessed in literature. Khan et al. [7] proposed a technique for S-box construction based on chaotic Lorenz systems. Hussain et al. presented S-box algorithms using generalized Bakers map [8] and projective general linear group [9]. Algebraic, analytical, and chaotic approaches for S-box are

studied in [10–13]. Özkaynak et al. [14] applied fractional-order chaotic Chen system, to develop S-box. Tian and Lu [15] structured dynamic chaos-based S-box in conjunction with DNA sequence operation. Some other more efficient algorithms could be reviewed in [6, 16–20]. In addition to this, applications of S-boxes in digital image encryption, steganography, and watermarking have become quite popular and influential in recent years [4, 11, 13, 21].

The study of innovation in design algorithms for S-boxes witnesses that the change of model and the selection of Boolean function contribute little to the performance indices of an S-box. We, in this paper, propose that the performance of an S-box is highly related to the background Galois field. The fact that finite fields of the same order are isomorphic is definitely of worth but the scrambling effect of a nonlinear Boolean function applied on two different fields of the same order might vary. Since in cryptography, an S-box is the salient component used to produce confusion in the data, it is worth studying that the confusion creating ability is associated with the choice of the irreducible polynomial used to form the background Galois field.

In [9], Hussain et al. presented an algorithm for generating S-box through the application of a linear fractional transformation on the Galois field $\text{GF}(2^8)$, structured by the polynomial $X^8 + X^4 + X^3 + X^2 + 1$. We in the proposed work show that the same algorithm used for a different polynomial exhibits highly improved values of nonlinearity, strict avalanche criterion (SAC), bit independent criterion (BIC), linear approximation probability (LAP), and differential approximation probability (DAP). By comparing the numerical results of these tests, we prove that different polynomials produce significantly different results. This observation leads to revising the existing models by choosing different background polynomials as it could be more influential in improvement of ideas rather changing the whole scheme.

We organize the contents of this paper as follows. In Section 2, we discuss the properties of the background Galois field $\text{GF}(2^8)$. The detailed algorithm for the design of the S-box is presented in Section 3. Section 4 deals with the analyses of S-boxes against several common attacks and the comparison of respective results. We further compare the cryptographic standing of both of the newly synthesized S-boxes with the state-of-the-art AES S-box. Conclusion is presented in Section 5.

2. Generating Polynomial and the Galois Field

For any prime p , Galois field $\text{GF}(p^n)$ is expressed as the factor ring $\mathbb{F}_p[X]/(\mu(x))$ where $\mu(x) \in \mathbb{F}_p[X]$ is an irreducible polynomial of degree n . For $\text{GF}(2^8)$ we choose an irreducible polynomial of degree 8 that generates the maximal ideal of the principal ideal domain $\mathbb{F}_2[X]$. We know that the multiplicative group of the resultant field $\text{GF}(2^8)$ is cyclic and hence each nonzero element of the field can be expressed as a power of the generator $\alpha = 00000010$.

In order to support our claim regarding the effect of polynomial, we choose two irreducible primitive polynomials

μ_1 and μ_2 of degree 8, to construct Galois fields $\mathbb{F}_1$ and $\mathbb{F}_2$, respectively, where $\mu_1 = X^8 + X^6 + X^5 + X^4 + 1$ and $\mu_2 = X^8 + X^4 + X^3 + X^2 + 1$, as used in [9]. We may choose other polynomials as well to compare our calculations but the selected pair beautifully serves for the purpose. Let G_i represents the multiplicative group of the Galois field, $\mathbb{F}_i$. The exponential form of elements of the multiplicative group G_1 , along with their inverses, is represented in Table 1; however the elements of G_2 are presented in Table 2 of [9]. In the next section, we use these calculations to develop the corresponding S-boxes.

3. Algorithm for S-Box

An $n \times n$ S-box is defined by a *vector Boolean function* $\mathcal{S}_n : \text{GF}(2^n) \rightarrow \text{GF}(2^n)$, defined as

$$\mathcal{S}_n(v) = (s_1(v), s_2(v), \dots, s_n(v)), \quad (1)$$

where $v = (v_1, v_2, \dots, v_{2^n}) \in \text{GF}(2^n)$ and each of s_i 's is regarded as a component Boolean function.

For a field $\mathbb{F}$, the general linear group $\text{GL}(n, \mathbb{F})$ is a group formed by all $n \times n$ invertible matrices. A projective general linear group of degree n over a field $\mathbb{F}$ is defined to be the quotient group of $\text{GL}(n, \mathbb{F})$ by its center. For this paper, we form the 8×8 S-box by considering the action of the Galois field $\text{GF}(2^8)$ on the projective linear group $\text{PGL}(2, \text{GF}(2^8))$; that is, we take a function $\sigma : \text{PGL}(2, \text{GF}(2^8)) \times \text{GF}(2^8) \rightarrow \text{GF}(2^8)$ defined as follows:

$$\sigma(v) = \frac{\alpha v + \beta}{\gamma v + \delta}. \quad (2)$$

In (2), σ is known as a linear fractional transformation (LFT) with α, β, γ and $\delta \in \text{GF}(2^8)$ satisfying the nondegeneracy condition $\alpha\delta - \beta\gamma \neq 0$. The ease of implementation, lesser computational labour, and high algebraic complexity of an LFT are the prime features that give incentive to employ this map for byte substitution. We may choose any values for LFT parameters that satisfy the aforementioned condition but, for the presented calculations, we, in particular, choose the same values as in [9], so that a comparison could be set easily. We consider $\alpha = 35$, $\beta = 15$, $\gamma = 9$ and $\delta = 5$. The images of the map σ , when applied on $\mathbb{F}_1$ and $\mathbb{F}_2$, produce our S-boxes S_1 and S_2 , respectively, as shown in Tables 2 and 3.

4. Performance Analysis of S-Boxes

The cryptographic strength if the S-boxes, generated in the foregoing section, are examined through the most widely used analysis techniques such as nonlinearity, bit independence, strict avalanche, and linear and differential approximation probabilities. In the following subsections we present all these performance indices one by one and compare the performance of S_1 and S_2 with one another, as well as, with the ever-prevailing algorithm AES.

4.1. Nonlinearity. Nonlinearity analysis measures the distance of the reference function from all of the affine functions.

TABLE 1: Exponential representation and the multiplicative inverses of elements of G_1 .

$x \in G_1$	α^n	x^{-1}
1	α^{255}	1
2	α^1	184
3	α^{231}	208
4	α^2	92
5	α^{207}	159
6	α^{232}	104
7	α^{59}	134
8	α^3	46
9	α^{35}	173
10	α^{208}	247
11	α^{154}	139
12	α^{233}	52
13	α^{20}	48
14	α^{60}	67
15	α^{183}	117
16	α^4	23
17	α^{159}	252
18	α^{36}	238
19	α^{66}	83
20	α^{209}	195
21	α^{118}	204
22	α^{155}	253
23	α^{251}	16
24	α^{234}	26
25	α^{245}	181
26	α^{21}	24
27	α^{11}	180
28	α^{61}	153
29	α^{130}	121
30	α^{184}	130
31	α^{146}	73
32	α^5	179
33	α^{122}	232
34	α^{160}	126
35	α^{79}	141
36	α^{37}	119
37	α^{113}	220
38	α^{67}	145
39	α^{106}	248
40	α^{210}	217
41	α^{224}	175
42	α^{119}	102
43	α^{221}	188
44	α^{156}	198
45	α^{242}	108
46	α^{252}	8
47	α^{32}	172
48	α^{235}	13
49	α^{213}	53
50	α^{246}	226
51	α^{135}	84
52	α^{22}	12
53	α^{42}	49
54	α^{12}	90

TABLE 1: Continued.

$x \in G_1$	α^n	x^{-1}
55	α^{140}	148
56	α^{62}	244
57	α^{227}	223
58	α^{131}	132
59	α^{75}	203
60	α^{185}	65
61	α^{191}	224
62	α^{147}	156
63	α^{94}	68
64	α^6	225
65	α^{70}	60
66	α^{123}	116
67	α^{195}	14
68	α^{161}	63
69	α^{53}	157
70	α^{80}	254
71	α^{167}	143
72	α^{38}	131
73	α^{109}	31
74	α^{114}	110
75	α^{203}	154
76	α^{68}	240
77	α^{51}	150
78	α^{107}	124
79	α^{49}	186
80	α^{211}	212
81	α^{40}	196
82	α^{225}	239
83	α^{189}	19
84	α^{120}	51
85	α^{111}	227
86	α^{222}	94
87	α^{240}	193
88	α^{157}	99
89	α^{116}	163
90	α^{243}	54
91	α^{128}	149
92	α^{253}	4
93	α^{205}	158
94	α^{33}	86
95	α^{18}	192
96	α^{236}	190
97	α^{163}	235
98	α^{214}	162
99	α^{98}	88
100	α^{247}	113
101	α^{55}	123
102	α^{136}	42
103	α^{102}	189
104	α^{23}	6
105	α^{82}	135
106	α^{43}	160
107	α^{177}	169

TABLE 1: Continued.

$x \in G_1$	α^n	x^{-1}
108	α^{13}	45
109	α^{169}	199
110	α^{141}	74
111	α^{89}	155
112	α^{63}	122
113	α^8	100
114	α^{228}	215
115	α^{151}	237
116	α^{132}	66
117	α^{72}	15
118	α^{76}	221
119	α^{218}	36
120	α^{186}	152
121	α^{125}	29
122	α^{192}	112
123	α^{200}	101
124	α^{148}	78
125	α^{197}	187
126	α^{95}	34
127	α^{174}	140
128	α^7	200
129	α^{150}	171
130	α^{71}	30
131	α^{217}	72
132	α^{124}	58
133	α^{199}	202
134	α^{196}	7
135	α^{173}	105
136	α^{162}	167
137	α^{97}	176
138	α^{54}	246
139	α^{101}	11
140	α^{81}	127
141	α^{176}	35
142	α^{168}	255
143	α^{88}	71
144	α^{39}	249
145	α^{188}	38
146	α^{110}	183
147	α^{239}	243
148	α^{115}	55
149	α^{127}	91
150	α^{204}	77
151	α^{17}	241
152	α^{69}	120
153	α^{194}	28
154	α^{52}	75
155	α^{166}	111
156	α^{108}	62
157	α^{202}	69
158	α^{50}	93
159	α^{48}	5
160	α^{212}	106
161	α^{134}	168
162	α^{41}	98

TABLE 1: Continued.

$x \in G_1$	α^n	x^{-1}
163	α^{139}	89
164	α^{226}	207
165	α^{74}	231
166	α^{190}	177
167	α^{93}	136
168	α^{121}	161
169	α^{78}	107
170	α^{112}	201
171	α^{105}	129
172	α^{223}	47
173	α^{220}	9
174	α^{241}	216
175	α^{31}	41
176	α^{158}	137
177	α^{65}	166
178	α^{117}	233
179	α^{250}	32
180	α^{244}	27
181	α^{10}	25
182	α^{129}	242
183	α^{145}	146
184	α^{254}	2
185	α^{230}	209
186	α^{206}	79
187	α^{58}	125
188	α^{34}	43
189	α^{153}	103
190	α^{19}	96
191	α^{182}	234
192	α^{237}	95
193	α^{15}	87
194	α^{164}	205
195	α^{46}	20
196	α^{215}	81
197	α^{171}	213
198	α^{99}	44
199	α^{86}	109
200	α^{248}	128
201	α^{143}	170
202	α^{56}	133
203	α^{180}	59
204	α^{137}	21
205	α^{91}	194
206	α^{103}	230
207	α^{29}	164
208	α^{24}	3
209	α^{25}	185
210	α^{83}	251
211	α^{26}	228
212	α^{44}	80
213	α^{84}	197
214	α^{178}	236
215	α^{27}	114
216	α^{14}	174
217	α^{45}	40

TABLE 1: Continued.

$x \in G_1$	α^n	x^{-1}
218	α^{170}	219
219	α^{85}	218
220	α^{142}	37
221	α^{179}	118
222	α^{90}	245
223	α^{28}	57
224	α^{64}	61
225	α^{249}	64
226	α^9	50
227	α^{144}	85
228	α^{229}	211
229	α^{57}	250
230	α^{152}	206
231	α^{181}	165
232	α^{133}	33
233	α^{138}	178
234	α^{73}	191
235	α^{92}	97
236	α^{77}	214
237	α^{104}	115
238	α^{219}	18
239	α^{30}	82
240	α^{187}	76
241	α^{238}	151
242	α^{126}	182
243	α^{16}	147
244	α^{193}	56
245	α^{165}	222
246	α^{201}	138
247	α^{47}	10
248	α^{149}	39
249	α^{216}	144
250	α^{198}	229
251	α^{172}	210
252	α^{96}	17
253	α^{100}	22
254	α^{175}	70
255	α^{87}	142

Nonlinearity criterion outlines the total number of bits that must be altered in the truth table of a Boolean function to get close to the nearby affine function [22].

Table 4 shows that, for S_1 , the average nonlinearity measure is 112., which is the highest figure attained by the AES S-box. Figure 1 shows the comparison which clearly depicts outstanding performance of S_1 as compared to S_2 .

4.2. Linear Approximation Probability. The measure of unevenness of an event is determined by linear approximation probability. This analysis is used to evaluate the maximum imbalance of the outcome. Mathematically, the linear approximation probability for a given S-box is defined as follows:

FIGURE 1: Nonlinearity of different S-boxes.

FIGURE 2: LP of different S-boxes.

$$LP = \max_{\Gamma_x, \Gamma_y \neq 0} \left| \frac{\#\{x \mid x \cdot \Gamma_x = S(x) \cdot \Gamma_y\}}{2^n} - \frac{1}{2} \right|, \quad (3)$$

where X represents the set of all possible inputs and Γ_x and Γ_y are the input and output masks, respectively. Numerical results presented in Table 5 and compared in Figure 2 show that the linear approximation probability of S_1 is much better than S_2 .

4.3. Differential Approximation Probability. For further analysis, we use the differential approximation probability, which determines the differential uniformity demonstrated by an S-box. The mathematical expression for DP is given by the following:

$$DP = \left\lceil \frac{\#\{x \in X \mid S(x) \oplus S(x \oplus \Delta x) = \Delta y\}}{2^n} \right\rceil. \quad (4)$$

In the above-mentioned expression input and output differentials are represented by Δx and Δy , respectively. The smaller the differential uniformity, the stronger the S-box. It is evident from Table 5 and Figure 3 that in terms of the differential approximation probability S_1 is much stronger than S_2 .

4.4. Strict Avalanche Criterion. This criterion examines the changes in the output bits caused as a result of single input

TABLE 2: S-box S_1 .

3	214	37	74	126	4	18	26	219	62	45	226	50	136	104	148
154	38	200	199	185	228	170	245	177	114	137	231	139	35	8	134
158	27	223	232	17	157	217	49	83	141	171	42	47	206	64	194
106	29	30	110	14	40	72	236	105	221	202	87	241	41	11	71
186	28	253	175	67	31	23	33	66	189	117	118	94	149	135	252
235	43	124	125	90	229	204	215	218	100	101	249	243	54	173	166
138	234	244	201	167	44	250	25	16	187	207	246	107	103	161	1
183	99	179	240	129	123	188	193	20	143	155	174	7	220	213	239
108	84	113	184	5	57	208	153	75	112	223	178	180	150	65	24
224	248	102	89	70	111	59	172	95	131	198	163	93	164	55	209
86	132	6	225	51	79	53	34	97	48	197	142	182	210	91	247
195	15	10	144	85	63	168	238	196	162	98	32	251	254	203	156
80	152	237	24	127	78	165	12	52	222	122	58	211	36	140	191
216	146	109	96	147	73	116	190	128	68	56	77	115	160	19	92
69	2	192	121	145	21	76	61	60	181	133	151	159	0	88	205
13	230	22	82	39	119	9	255	120	46	81	212	227	130	176	169

TABLE 3: S-box S_2 .

198	214	241	163	130	165	217	127	179	123	111	197	43	141	237	3
168	201	17	121	142	101	232	174	11	249	16	156	10	50	183	65
72	184	200	132	58	47	27	159	231	189	8	18	206	194	177	31
193	92	122	192	85	137	243	49	178	170	36	135	230	95	100	128
13	109	227	0	224	144	208	78	173	32	139	234	107	82	172	81
51	233	12	154	94	161	244	55	7	34	251	225	153	93	254	138
102	240	115	242	110	134	124	79	157	160	90	238	73	53	169	250
136	118	112	48	40	114	22	246	46	131	23	69	52	235	248	2
116	91	117	26	166	25	219	59	54	229	120	245	89	185	99	226
105	45	60	199	164	191	228	202	37	104	143	209	220	147	44	186
145	125	203	29	38	41	215	108	64	88	119	74	213	96	211	83
218	146	196	205	67	152	129	175	84	158	207	176	80	62	150	86
57	155	195	216	75	19	1	87	33	68	71	236	239	255	35	212
148	188	133	15	204	187	42	182	97	56	24	221	252	30	77	181
4	247	167	21	9	222	180	190	151	140	39	171	14	126	66	253
103	223	70	98	28	20	63	162	61	113	149	210	106	5	6	76

bit change. This is one of the most desirable features of any cryptographic design that when we change a single input bit, changes must occur in half of the output bits. In other words an S-box, $\mathcal{S}_n : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ is said to satisfy SAC if, for a change in an input bit, the probability of change in the output bit is $1/2$. The results are shown in Table 5 and Figure 4.

4.5. Bit Independence Criterion. The independent behavior of the pair of variables and the variations of input bits are considered as important factors of bit independence criterion. In bit independence criterion, input bits are transformed exclusively, and then output results are scrutinized for their independency [23]. Bit independence has great worth in cryptographic structures. The goal of reaching the maximum complexity and perplexity in a system can be achieved through this property of increasing independence between the bits. In cryptographic systems, the increased independence between bits is an essential requirement as it

makes harder to understand and forecast the design of the system.

The numerical results of BIC when applied to the proposed S-box are given in Table 5 and are compared in Figure 5. It can be observed that according to these results our S-box S_1 is pretty similar to the AES S-box and is much better than S_2 .

One can observe that overall performance of S_1 is much better than that of S_2 . The performance parameters for S_1 seem to be pretty close to that of AES S-box. The algorithm used for both S_1 and S_2 is the same but the primitive polynomial selected to generate the Galois field is different, which really contributes to the outputs.

5. Conclusion

The kernel of the presented work lies in the fact that the choice of the background Galois field and its generating primitive

TABLE 4: Performance Indices for new S-box.

Analysis	Max.	Min.	Average	Square deviation	DP	LP
Nonlinearity	113	111	112			
SAC	0.546875	0.429688	0.498291	0.0157537		
BIC		111	111.751	0.6227		
DP					0.015625	
LP						0.064063

TABLE 5: Comparison of performance indices for different S-boxes.

S-box	Nonlinearity	SAC	BIC	DP	LP
AES	112	0.5058	112.0	0.0156	0.062
S_1	112	0.498291	111.751	0.015625	0.064063
S_2	105.5	0.507	106	0.0242	0.140

FIGURE 3: DP of different S-boxes.

FIGURE 4: SAC of different S-boxes.

polynomial matters to the function and performance of the substitution boxes. This fact leads to the fascinating idea that, rather than the development of new algorithms, the improvement of the existing algorithms is worth studying as its least laborious but most effective. We propose, on the basis of the example discussed, that the effect of the choice of generating polynomial may lead to an intensive research in future to modify the design models of S-boxes. It will definitely affect the applications of S-boxes in other branches of the digital

FIGURE 5: BIC of different S-boxes.

communication, such as steganography, watermarking, and image encryption.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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