Research Article

Automatic Search for the Linear (Hull) Characteristics of ARX Ciphers: Applied to SPECK, SPARX, Chaskey, and CHAM-64

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Linear cryptanalysis is an important evaluation method for cryptographic primitives against key recovery attack. In this paper, we revisit the Walsh transformation for linear correlation calculation of modular addition, and an efficient algorithm is proposed to construct the input-output mask space of specified correlation weight. By filtering out the impossible large correlation weights in the first round, the search space of the first round can be substantially reduced. We introduce a concept of combinational linear approximation table (cLAT) for modular addition with two inputs. When one input mask is fixed, another input mask and the output mask can be obtained by the *Splitting-Lookup-Recombination* approach. We first split the n -bit fixed input mask into several subvectors and then find the corresponding bits of other masks, and in the recombination phase, pruning conditions can be used. By this approach, a large number of search branches in the middle rounds can be pruned. With the combination of the optimization strategies and the branch-and-bound search algorithm, we can improve the search efficiency for linear characteristics on ARX ciphers. The linear hulls for SPECK32/48/64 with a higher average linear potential (ALP) than existing results have been obtained. For SPARX variants, an 11-round linear trail and a 10-round linear hull have been found for SPARX-64 and a 10-round linear trail and a 9-round linear hull are obtained for SPARX-128. For Chaskey, a 5-round linear trail with a correlation of 2^{-61} has been obtained. For CHAM-64, 34/35-round optimal linear characteristics with a correlation of $2^{-31}/2^{-33}$ are found.

1. Introduction

The three components: modular addition, rotation, and XOR, constitute the basic operations in ARX cryptographic primitives [1]. In ARX ciphers, modular additions provide nonlinearity diffusion with efficient software implementation and low dependencies on computing resources. Compared with S-box-based ciphers, ARX ciphers do not need to store S-box in advance, which can reduce the occupation of storage resources, especially in resource-constrained devices. In addition, ARX ciphers do not need to query S-boxes in the encryption and decryption process, which can reduce a lot of query operations. Therefore, ARX construction is preferred by many designers of lightweight ciphers. At present, there are many primitives used in this construction, such as HIGHT [2], SPECK [3], LEA [4], Chaskey [5], SPARX [6], and CHAM [7].

Until now, cryptanalysis on ARX ciphers is still not well understood as S-box-based ciphers, and the security analysis on them are relatively lagging behind [8]. Linear cryptanalysis is very important for evaluating the security margin of symmetric cryptographic primitives [9, 10]. The linear approximation tables of S-box-based ciphers mostly can be constructed and stored directly; however, the full linear approximation table of modular addition will be too large to store when the word length of modular addition is large.

For linear cryptanalysis of ARX ciphers, one crucial step is to calculate the linear correlation of modular addition. In [11–14], the linear properties of modular addition have been carefully studied. In [13], a method to calculate the linear correlation of modular addition recursively was proposed, but the calculation process that was based on state transition in bit level leads to high complexity. Based on this method,

only the optimal linear characteristics for the variants of SPECK32 [15] and SPECK32/48 [16] were found.

In 2013, Schulte-Geers used CCZ equivalence to improve the explicit formula for the calculation of linear correlation of modular addition [17]. Based on the improved formula and SAT solver model, Liu et al. obtained better linear characteristics for SPECK [18], the optimal linear trails for SPECK32/48/64 with correlation close to the security boundary ($2^{-n/2}$) were obtained, and the 9/10-round linear hull with a potential of $2^{-29.1}/2^{-32.1}$ for SPECK32 was obtained.

According to the position of the starting round of the search algorithm, there are currently 3 types of automatic search technologies for linear/differential cryptanalysis on ARX primitives. They are bottom-up techniques [15], top-down techniques [19–21], and the method of extending from the middle to the ends [22]. In these methods, the linear correlations are directly calculated based on the input-output masks or by looking up the precomputed partial linear approximation table (pLAT) [23]. For addition modulo 2^n with two inputs, the correlations need to be calculated based on the known input-output masks. However, in the search process for linear characteristics of ARX ciphers, due to the existence of three-forked branches, in most case, for the input-output masks $((v, w) \rightarrow u)$ of modular addition, only one input mask v is determined, and another input mask w and the output mask u are unknown. Although all 2^{2n} space of (w, u) can be traversed in a trivial way, it is very time-consuming.

High efficiency query operations can be achieved by constructing a linear approximation table of reasonable storage size. The pLAT can store the input-output masks whose linear correlation is greater than a certain threshold [15]. When the branches cannot be queried in pLAT and that need to be calculated by the input-output masks, the calculation process will lead to a significant reduction in search efficiency. Although the heuristic method can speed up the search, it cannot guarantee the results will be the best [24].

Therefore, constructing a search model based on the precise correlation calculation formula and realizing an efficient search for linear characteristics on ARX ciphers are still a study worth working on. The motivation of this paper is to investigate how to speed up the search algorithm in order to realize the search for linear (hull) characteristics on typical ARX ciphers.

1.1. Our Contributions. In this paper, we first revisit the linear correlation calculation of modular addition and introduce an algorithm to construct the input-output masks of specific correlation weight. Then, we propose a novel concept of combinational linear approximation table (cLAT) and introduce an algorithm to generate the lookup tables. Combining with these two optimization algorithms, we propose an automatic algorithm to search for the optimal linear characteristics on ARX ciphers. In the first round, we can exclude the search space of the non-optimal linear trails by increasing the correlation weight of

each modular addition monotonically. In the middle rounds, the undetermined masks and the corresponding correlation weight of each modular addition can be obtained by querying the cLAT, and a large number of nonoptimal branches can be filtered out during the recombination phase. Also, the algorithm can be appropriately modified for the heuristic search.

As applications, for SPECK32/48/64, the 9/11/14-round linear hulls are obtained. For SPARX-64, the 11-round linear trail with a correlation of 2^{-28} and a 10-round linear hull with an ALP of $2^{-40.92}$ are found. For SPARX-128, we can experimentally get the optimal linear trails of the first eight rounds, and we get a 10-round linear trail with a correlation of 2^{-23} . For Chaskey, the linear characteristics covering more rounds are updated, and a 5-round linear trail with a correlation of 2^{-61} is found. For CHAM-64, we find a new 34-round optimal linear trail with a correlation of 2^{-31} . A summary table is shown in Table 1.

1.2. Roadmap. This paper is organized as follows. We first present some preliminaries used in this paper in Section 2. In Section 3, we introduce the algorithm for constructing the space of input-output mask tuples, the algorithm for constructing cLAT, and the improved automatic search algorithm for linear cryptanalysis on ARX ciphers. In Section 4, we apply the new tool to several typical ARX ciphers. Finally, we conclude our work in Section 5.

2. Preliminaries

2.1. Notation. For addition modulo 2^n , i.e., $x \boxplus y = z$, we use the symbols $\ll$ and $\gg$ to indicate rotation to the left and right and $\ll$ and $\gg$ to indicate the left and right shift operation, respectively. The binary operator symbols $\oplus$, $\vee$, $\wedge$, $\|$, and $\neg$ represent XOR, OR, AND, concatenation, and bitwise NOT, respectively. For a vector x , $wt(x)$ represents its Hamming weight and x_i is the i^{th} bit of it. 0 is a zero vector.

2.2. Linear Correlation Calculation for Modular Addition. Let $\mathbb{F}_2^n$ be the n dimensional vector space over binary field $\mathbb{F}_2 = \{0, 1\}$; for Boolean function $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ and $h : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$, $x \in \mathbb{F}_2^n$, the linear correlation between f and h can be denoted by

$$\text{Cor}(f, h) = 2 \times \frac{\#\{x \mid f(x) \oplus h(x) = 0\}}{2^n} - 1. \quad (1)$$

For modular addition $x \boxplus y = z$, let (v, w) be the input masks, u be the output mask, and $\cdot$ be the standard inner product. According to the definition of linear correlation, when $(v \cdot x \boxplus w \cdot y) \oplus u \cdot z = 0$, the linear approximation probability is defined as

$$\Pr(u, v, w) = 2^{-3n} \times \#\{(x, y, z) \mid (v \cdot x \boxplus w \cdot y) \oplus u \cdot z = 0\}. \quad (2)$$

Let $\mu(t) = (-1)^t$, then the linear correlation of modular addition can be denoted by Walsh transformation, and thus,

TABLE 1: Summary of the linear characteristics on SPECK, SPARX, Chaskey, and CHAM-64, where “s,” “m,” “h,” and “d” represent seconds, minutes, hours, and days, respectively.

Variants	Round	Cor	T_{Cor}	ALP	T_{ALP}	Reference
SPECK32	9	2^{-14}	N/A	$2^{-29.1}$	N/A	[18]
	9	2^{-14}	N/A	2^{-28}	N/A	[16]
	9	2^{-14}	9 s	$2^{-27.78}$	25 s	This paper
SPECK48	10	2^{-22}	N/A	2^{-44}	N/A	[18]
	10	2^{-22}	N/A	2^{-44}	N/A	[16]
	10	2^{-22}	3.2 h	$2^{-43.64}$	157.3 h	This paper
SPECK64	13	2^{-30}	N/A	2^{-60}	N/A	[18]
	13	2^{-30}	N/A	2^{-60}	N/A	[16]
	13	2^{-30}	8.6 h	$2^{-55.29}$	7.3 h	This paper
	14	2^{-33}	25.6 h	$2^{-61.24}$	5.8 h	This paper
SPARX-64	10	2^{-22}	3 d	$2^{-40.92}$	1 h	This paper
	11	2^{-28}	5 m	2^{-56}	—	This paper
SPARX-128	9	2^{-18}	27 m	$2^{-35.22}$	6 h	This paper
	10	2^{-23}	4.4 d	2^{-46}	—	This paper
Chaskey	3	2^{-9}	N/A	2^{-18}	N/A	[18]
	4	2^{-29}	15.7 m	2^{-58}	—	This paper
	5	2^{-61}	6.6 h	2^{-122}	—	This paper
CHAM-64	34	2^{-31}	N/A	2^{-62}	N/A	[7]
	34	2^{-31}	1.1 d	2^{-62}	—	This paper
	35	2^{-33}	4.8 d	—	—	This paper

$$\text{Cor}(u, v, w) = 2^{-3n} \times \sum_{x, y, z \in \mathbb{F}_2^n} \mu((v \cdot x \boxplus w \cdot y) \oplus u \cdot z). \quad (3)$$

Let $\Pr(u, v, w) = (1/2) + \varepsilon$, where ε is the bias. When $(v \cdot x \boxplus w \cdot y) \oplus u \cdot z = 1$, the linear approximation probability is $\Pr(u, v, w) = (1/2) - \varepsilon$. The linear correlation can be denoted by

$$\text{Cor}(u, v, w) = \Pr(u, v, w) - \overline{\Pr}(u, v, w) = 2\Pr(u, v, w) - 1. \quad (4)$$

We call $\text{Cw}(u, v, w) = -\log_2 \text{Cor}(u, v, w)$ as the *correlation weight*, and the linear square correlation can be denoted by

$$\text{LSC}(u, v, w) = \text{Cor}(u, v, w)^2 = 2^{-2 \times \text{Cw}(u, v, w)}. \quad (5)$$

For addition $x \boxplus y$ modulo 2^n , it can be rewritten as $x \boxplus y = x \oplus y \oplus \text{carry}(x, y)$, in which $\text{carry}(x, y)_{i+1} = \text{carry}(x, y)_i \oplus x_i \oplus y_i$ and $\text{carry}(x, y)_0 = 0$ for $0 \leq i \leq n-1$. The first-order approximation is $\text{carry}(x, y) = (x \wedge y) \ll 1$. If all $\text{carry}(x, y)_j = 0$ for $j \leq i$, and $0 \leq i \leq n-1$, the high-order approximation is

$$\begin{aligned} \text{carry}(x, y)_{i+1} &= \frac{1}{2} \left| (-1)^{x_i} + (-1)^{y_i} + \text{carry}(x, y)_i \right. \\ &\quad \left. - (-1)^{x_i + y_i} \text{carry}(x, y)_i \right|. \end{aligned} \quad (6)$$

In [13], Wallén introduced the theorem to calculate the linear correlation by analyzing the carry high-order approximation function recursively. In [12], based on the bit state transformation, the formula to calculate the correlation was given by the following theorem.

Theorem 1 (See [12]). *For addition modulo 2^n , let v and w be the input masks and u be the output mask. Define an auxiliary vector $d = d_{n-1} \cdots d_0$, and each $d_i = u_i \| v_i \| w_i \in \mathbb{F}_2^3$ is an octal word, $0 \leq i \leq n-1$. Then, the linear correlation can be denoted by*

$$\text{Cor}(u, v, w) = L A_{d_{n-1}} A_{d_{n-2}} \cdots A_{d_1} A_{d_0} C, \quad (7)$$

where the row vector $L = (1 \ 0)$, the column vector $C = (1 \ 1)^T$, and each 2×2 matrix A_{d_i} is defined by

$$\begin{aligned} A_0 &= \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, A_1 = A_2 = -A_4 = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \\ A_7 &= \frac{1}{2} \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, -A_3 = A_5 = A_6 = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \quad (8)$$

In [17], Schulte-Geers extended Theorem 1 and derived a fully explicit formula for the linear correlation calculation, given by Theorem 2.

Theorem 2 (See [17]). *For addition modulo 2^n with input-output mask tuple (u, v, w) , a vectorial Boolean function $M : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ denotes the partial sum mapping:*

$$\begin{aligned} x &= (x_0, x_1, \dots, x_{n-1}) \rightarrow M(x) \\ &= (0, x_0, x_0 \oplus x_1, \dots, x_0 \oplus x_1 \oplus \dots \oplus x_{n-2}). \end{aligned} \quad (9)$$

Let $z := M^T(u \oplus v \oplus w) = (0, x_{n-1}, x_{n-1} \oplus x_{n-2}, \dots, x_{n-1} \oplus x_{n-2} \oplus \dots \oplus x_1)$, then the linear correlation can be denoted by

$$\text{Cor}(u, v, w) = 1_{\{u \oplus v \leq z\}} 1_{\{u \oplus w \leq z\}} (-1)^{v \cdot w} 2^{-wt(z)}, \quad (10)$$

where 1_{G_f} is an indicator function for graph $G_f := \{(x, f(x)) \mid x \in \mathbb{F}_2^n\}$; for n -bit vectors a and b , $a \leq b$ represents $a_i \leq b_i$ for $0 \leq i \leq n-1$.

In iterative ciphers, the correlation of a single r -round linear trail is the product of the correlations of each round [25]. Assuming that there are N_A additions modulo 2^n with two inputs in i^{th} round, Γ_{in} and Γ_{out} are the input and output masks of the r -round linear trail, and the correlation of it can be denoted by

$$\text{Cor}(\Gamma_{\text{in}}, \Gamma_{\text{out}}) = \prod_{i=1}^r \prod_{j=1}^{N_A} \text{Cor}(u_{i,j}, v_{i,j}, w_{i,j}). \quad (11)$$

The linear approximation of a linear hull represents the potential of all linear trails with same input-output masks [26]. The averaged linear potential (ALP) can be counted by the following formula:

$$\text{ALP}(\Gamma_{\text{in}}, \Gamma_{\text{out}}) = \frac{1}{|K|} \sum_{k \in K} \text{Cor}(\Gamma_{\text{in}}, \Gamma_{\text{out}})^2. \quad (12)$$

Assuming that the key k is selected uniformly from the key space K , the statistics of ALP can be formulated as (13), where $T[\text{Cw}]$ is the number of trails with *correlation weight* of Cw . Let $C_{\min}$ be the *correlation weight* of the linear trail

whose input-output masks are chosen as the fixed input-output masks of the linear hull. $C_{\max}$ is the upper bound to be searched, which should be chosen by the trade-off between the search time and the accuracy of ALP:

$$\text{ALP}(\Gamma_{\text{in}}, \Gamma_{\text{out}}) = \sum_{Cw=C_{\min}}^{C_{\max}} 2^{-2Cw} \times T[Cw]. \quad (13)$$

2.3. Linear Properties of SPECK, SPARX, Chaskey, and CHAM. The SPECK family ciphers were designed by NSA in 2013 [3]. The SPARX family ciphers were introduced by Dinu et al. at ASIACRYPT'16 [6]. In SPARX, the nonlinear ARX-box (SPECKEY) is obtained by modifying the round function of SPECK32. The linear mask propagation properties of the round function in SPECK and SPECKEY are shown in Figure 1. The rotation parameters $(r_a, r_b) = (7, 2)$ for SPECK32, while $(r_a, r_b) = (8, 3)$ for other variants.

If the input-output masks $(u[i], v[i], w[i])$ and $(u[i+1], v[i+1], w[i+1])$ of the modular additions in the two consecutive rounds of SPECK are known, the input and output masks of these two rounds can be denoted by Property 1.

Property 1. If $(u[i], v[i], w[i])$ and $(u[i+1], v[i+1], w[i+1])$ are given, then $\Gamma X[i] = v[i] \ll r_a$, $\Gamma X[i+1] = v[i+1] \ll r_a$, $\Gamma Y[i+1] = (v[i+1] \ll r_a) \oplus u[i]$, $\Gamma Y[i] = (\Gamma Y[i+1] \gg r_b) \oplus w[i]$, $\Gamma Y[i+2] = (\Gamma Y[i+1] \oplus w[i+1]) \ll r_b$, and $\Gamma X[i+2] = \Gamma Y[i+1] \oplus u[i+1]$.

The linear layer functions $\mathcal{L}/\mathcal{L}'$ [6] for SPARX-64 and SPARX-128 are shown in Figure 2. Due to the existence of the three-forked branches, the masks of the linear transformation layer have the following properties.

Property 2. For SPARX-64, if the masks are transformed by the linear layer function $\mathcal{L}$, let $c = \Gamma X_2 \oplus \Gamma X_3$, $d = c \gg 8$, then $\Gamma X'_0 = \Gamma X_2$, $\Gamma X'_1 = \Gamma X_3$, $\Gamma X'_2 = \Gamma X_0 \oplus d \oplus \Gamma X_2$, and $\Gamma X'_3 = \Gamma X_1 \oplus d \oplus \Gamma X_3$.

Property 3. For SPARX-128, if the masks are transformed by the linear layer function $\mathcal{L}'$, let $e = \Gamma X_4 \oplus \Gamma X_5 \oplus \Gamma X_6 \oplus \Gamma X_7$, $f = e \gg 8$, then $\Gamma X'_0 = \Gamma X_4$, $\Gamma X'_1 = \Gamma X_5$, $\Gamma X'_2 = \Gamma X_6$, $\Gamma X'_3 = \Gamma X_7$, $\Gamma X'_4 = \Gamma X_0 \oplus f \oplus \Gamma X_6$, $\Gamma X'_5 = \Gamma X_1 \oplus f \oplus \Gamma X_5$, $\Gamma X'_6 = \Gamma X_2 \oplus f \oplus \Gamma X_4$, and $\Gamma X'_7 = \Gamma X_3 \oplus f \oplus \Gamma X_7$.

Chaskey is a MAC algorithm introduced by Mouha et al. at SAC'14 [5], and an enhanced variant was proposed in 2015 [27], which increases the number of permutation rounds from 8 to 12. The round function of the permutation $(v'_0, v'_1, v'_2, v'_3) = \pi(v_0, v_1, v_2, v_3)$ is shown in Figure 3. The 4 modular additions are labeled by A0, A1, A2, and A3, respectively. The input mask (a, b, c, d) and the output mask (a', b', c', d') of the first round can be denoted by Property 4.

Property 4. For the permutation of Chaskey, if the input-output masks of each modular addition in the first round are $(u[i], v[i], w[i])$, $0 \leq i \leq 3$, the corresponding correlation weight of each modular addition is c_0, c_1, c_2, c_3 , respectively. Hence, in the first round, $a = v[0]$, $b = w[0] \oplus ((u[0] \oplus (v[3] \gg 16))) \gg 5$, $c = v[1]$, $d = w[1] \oplus (u[1] \oplus v[2]) \gg 8$,

FIGURE 1: Linear mask propagation properties of SPECK and SPECKEY.

$a' = u[3] \oplus (u[1] \oplus v[2] \oplus w[3]) \ll 13$, $b' = (u[0] \oplus w[2] \oplus w[3] \gg 16) \ll 7$, $c' = u[2] \oplus (w[2] \oplus (u[0] \oplus v[3]) \gg 16) \ll 7$, and $d' = (u[1] \oplus v[2] \oplus w[3]) \ll 13$. The corresponding correlation weight of the round function is $Cw = \sum_{i=0}^3 c_i$.

CHAM is a family of lightweight block ciphers that was proposed by Koo et al. at ICISC'17, which blends the good designs of SIMON and SPECK [7]. 3 variants of CHAM have two kinds of block size, i.e., CHAM-64 and CHAM-128. The linear mask propagation for the 4 consecutive rounds of CHAM is shown in Figure 4. If the input-output mask tuples of each modular addition of the first 4 rounds are given, the input and output masks of the first 4 rounds can be deduced by Property 5.

Property 5. For CHAM, if the input-output mask tuples $(u[i], v[i], w[i])$ of each modular addition of the first 4 rounds are given, $1 \leq i \leq 4$, the input and output masks of the first 4 rounds can be deduced as follows. $\Gamma X'_0 = v[1]$, $\Gamma X'_1 = (w[1] \ll 1) \oplus v[2]$, $\Gamma X'_2 = (w[2] \ll 8) \oplus v[3]$, $\Gamma X'_3 = (w[3] \ll 1) \oplus v[4]$; $\Gamma X'_0 = v[2]$, $\Gamma X'_1 = \Gamma X'_0$, $\Gamma X'_2 = \Gamma X'_0$, $\Gamma X'_3 = u[1] \ll 8$; $\Gamma X'_0 = v[3]$, $\Gamma X'_1 = \Gamma X'_1$, $\Gamma X'_2 = \Gamma X'_1$, $\Gamma X'_3 = u[2] \ll 1$; $\Gamma X'_0 = v[4]$, $\Gamma X'_1 = \Gamma X'_2$, $\Gamma X'_2 = \Gamma X'_2$, $\Gamma X'_3 = u[3] \ll 8$; $\Gamma X'_0 = (w[4] \ll 8) \oplus \Gamma X'_1$, $\Gamma X'_1 = \Gamma X'_3$, $\Gamma X'_2 = \Gamma X'_3$, $\Gamma X'_3 = u[4] \ll 1$.

3. Automatic Search for the Linear Characteristics on ARX Ciphers

3.1. Input-Output Masks of Specific Correlation Weight. The number of input-output mask tuples in the first round is closely related to the complexity of the branch-and-bound search algorithm, but traversing all possible input masks of the first round will result in high complexity. An alternative approach is to consider the possible correlation weight corresponding to the input-output masks and exclude those tuples that have a large correlation weight. However, for a fixed correlation weight, it may correspond to multiple input-output mask tuples although the correlation can be calculated by Theorem 2 when the input-output masks are fixed for a modular addition.

For addition modulo 2^n , its maximum correlation weight is $n - 1$, and the size of the total space S of all input-output

FIGURE 2: (a) and (b) represent the linear layer of SPARX-64. (c) and (d) represent the linear layer of SPARX-128.

FIGURE 3: Linear masks in the first round of Chaskey.

FIGURE 4: Linear masks in the first 4 rounds of CHAM.

mask tuples is 2^{3n} . We can rank the correlation weights Cw from 0 to $n-1$ and construct the input-output masks subspace S_{Cw} corresponding to correlation weight Cw , $0 \leq Cw \leq n-1$. Therefore, the total space S can be divided into n subspaces, i.e., $S = \bigcup_{Cw=0}^{n-1} S_{Cw}$.

Definition 1. Let $((v, w) \rightarrow u)$ be the input-output masks for a modular addition with nonzero correlation. Let us

define an octal word sequence $\Phi := \{\xi_{n-1} \cdots \xi_0\}$, where $\xi_i = u_i \| v_i \| w_i \in \mathbb{F}_2^3$, for $0 \leq i \leq n-1$.

Definition 2. Let us define three sets that ξ_i may belong to, i.e., $U_0 = \{1, 2, 4, 7\}$, $U_1 = \{0, 3, 5, 6\}$, and $U_2 = \{0, 7\}$.

In Theorem 2, when the correlation of a modular addition is nonzero, the value distribution of the 3 consecutive bits in z and the 3 consecutive words in

Φ has the following relationships, shown in Observation 1.

Observation 1. Let $x = u \oplus v \oplus w$ and $z_i = \bigoplus_{j=i+1}^{n-1} x_j$ for $0 \leq i \leq n-2$, $z_{n-1} = 0$; hence, $z_i = z_{i+1} \oplus x_{i+1}$. For $z \in \mathbb{F}_2^n$, assuming when $z_j = 0$ for $n-1 \geq j > i+1$ or $z_{i+2} = 0$, they should have $u_i \oplus v_i \leq z_i$, $u_i \oplus w_i \leq z_i$ on bit level, and it is equivalent to $u_{i-1} \oplus v_{i-1} \leq x_{i+1} \oplus x_i$ and $u_{i-1} \oplus w_{i-1} \leq x_{i+1} \oplus x_i$. Since $x_i = 1$ when $\xi_i \in U_0$ and $x_i = 0$ when $\xi_i \in U_1$,

$$\begin{aligned} \text{if } (z_{i+1}, z_i, z_{i-1}) &= (0, 0, 0), & \text{then } \xi_{i+1} &= 0, \xi_i = 0, \xi_{i-1} = 0; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (0, 0, 1), & \text{then } \xi_{i+1} &= 0, \xi_i = 0, \xi_{i-1} = 7; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (0, 1, 0), & \text{then } \xi_{i+1} &= 0, \xi_i = 7, \xi_{i-1} \in U_0; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (0, 1, 1), & \text{then } \xi_{i+1} &= 0, \xi_i = 7, \xi_{i-1} \in U_1; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (1, 0, 0), & \text{then } \xi_{i+1} &= 7, \xi_i = U_0, \xi_{i-1} = 0; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (1, 0, 1), & \text{then } \xi_{i+1} &= 7, \xi_i = U_0, \xi_{i-1} = 7; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (1, 1, 0), & \text{then } \xi_{i+1} &= 7, \xi_i = U_1, \xi_{i-1} \in U_0; \\ \text{if } (z_{i+1}, z_i, z_{i-1}) &= (1, 1, 1), & \text{then } \xi_{i+1} &= 7, \xi_i = U_1, \xi_{i-1} \in U_1; \end{aligned} \quad (14)$$

Hence, the value of $\xi_i = u_i \parallel v_i \parallel w_i$ depends on whether the bit positions of z_{i+1} and z_i are active. The last significant bits (u_0, v_0, w_0) of the input-output masks construct the value of ξ_0 , which is only related to the Hamming weight of $z_0 = \bigoplus_{j=1}^{n-1} x_j$, i.e., $u_0 \oplus v_0 \leq wt(z_0)$ and $u_0 \oplus w_0 \leq wt(z_0)$. Therefore, if we get the Hamming weight distribution of z , from the LSB to MSB direction, as z_0 is determined, ξ_0 can be obtained. Next, $x_1 = z_1 \oplus z_0$ is determined, and $u_1 \oplus v_1 \leq wt(z_1)$ and $u_1 \oplus w_1 \leq wt(z_1)$ should be satisfied; hence, the possible values of ξ_1 can be obtained. Recursively, all ξ_i values can be constructed as an octal word sequence from the LSB to MSB direction to subject to the above observation. Hence, the tuples of (u, v, w) can be generated from the elements in Φ . The process to construct the subspace S_{Cw} is shown in Algorithm 1, marked as **Const** (S_{Cw}).

3.2. Combinational Linear Approximation Table. For addition modulo 2^n , the full LAT requires a storage size of 2^{3n} ; when n is too large, it will be very difficult to store. To facilitate the storage, an intuitive approach is to store only a part of the full LAT. For a n -bit vector, we can split it into t subvectors of m bits, where $n = mt$. When each m -bit subvector is determined, the n -bit vector can be obtained by concatenating. This idea gives birth to the concept of combinatorial LAT (cLAT).

Property 6. For $u \oplus v \leq z$ and $u \oplus w \leq z$, they are equivalent to $(u \oplus v) \wedge (\neg((u \oplus v) \wedge \vec{z})) = \vec{0}$ and $(u \oplus w) \wedge (\neg((u \oplus w) \wedge \vec{z})) = \vec{0}$.

Corollary 1. Let $u, v, w \in \mathbb{F}_2^n$ be the input-output masks of the modular addition with nonzero correlation, and let $A = u \oplus v$, $B = u \oplus w$, $C = u \oplus v \oplus w$, and $z = M^T(C)$.

Splitting the vectors $A = A^{t-1} \parallel \dots \parallel A^0$, $B = B^{t-1} \parallel \dots \parallel B^0$, $C = C^{t-1} \parallel \dots \parallel C^0$, and $z = z^{t-1} \parallel \dots \parallel z^0$ into t subvectors, respectively, $n = mt$, $A^k, B^k, C^k \in \mathbb{F}_2^m$, $0 \leq k \leq t-1$. Then, the correlation weight of the modular addition can be denoted by

$$-\log_2 \text{Cor}(u, v, w) = \sum_{k=0}^{t-1} wt(z^k) = \sum_{k=0}^{t-1} \sum_{j=0}^{m-1} C_{mk+j+1} \oplus z_{mk+j+1}, \quad (15)$$

when

$$\begin{aligned} A^k \wedge (\neg(A^k \wedge z^k)) &= 0, \\ B^k \wedge (\neg(B^k \wedge z^k)) &= 0. \end{aligned} \quad (16)$$

Proof. $wt(z)$ is the sum of the Hamming weight of each subvector z^k , so $-\log_2 \text{Cor}(u, v, w) = \sum_{k=0}^{t-1} wt(z^k)$. For $C = u \oplus v \oplus w$, the i^{th} bit in z can be denoted by $z_i = \bigoplus_{j=i+1}^{n-1} C_j = z_{i+1} \oplus C_{i+1}$. Let $0 \leq j < i$, $0 \leq i \leq n-1$, and the j^{th} bit in z^k should be $z_j^k = C_{mk+j+1} \oplus z_{mk+j+1}$. Hence, $wt(z^k) = \sum_{j=0}^{m-1} C_{mk+j+1} \oplus z_{mk+j+1}$, when $u_i \oplus v_i \leq \bigoplus_{l=i+1}^{n-1} C_l$ and $u_i \oplus w_i \leq \bigoplus_{l=i+1}^{n-1} C_l$ are satisfied, i.e., $A^k \wedge (\neg(A^k \wedge z^k)) = 0$ and $B^k \wedge (\neg(B^k \wedge z^k)) = 0$ for $0 \leq k \leq t-1$. $\square$

If the m -bit subvector z^{k+1} adjacent to z^k is known, z^k can be calculated by subvector tuple (u^k, v^k, w^k) and the lowest bit of z^{k+1} . We call (u^k, v^k, w^k) as a *subblock*, and we call the bit $z_{(k+1)m} \in \{0, 1\}$ as the *connection status* when used in the calculation of z^k . Splitting the n -bit vector z into t subvectors, there should have $t-1$ *connection status* z_j , $j \in \{(t-1)m, \dots, 2m, m\}$, and for the highest subvector, its *connection status* $b = 0$. Hence, for the highest subblock $(u^{t-1}, v^{t-1}, w^{t-1})$, the Hamming weight of z^{t-1} and the bit $z_{(t-1)m}$ can be obtained, recursively, and the Hamming weight of the remaining subvectors can also be obtained. Therefore, as *connection status* $b \in \{0, 1\}$ and $u^k, v^k, w^k \in \mathbb{F}_2^m$, we can construct a m -bit lookup table for modular addition in advance and query the tables by indexing input-output masks and the *connection status*. In addition, the *connection status* for the next subblock can also be generated.

In the top-down search techniques for the ARX ciphers, for the modular additions in the middle rounds, in most cases, only one input mask is fixed (assuming it is v), and another input mask w and the output mask u are unknown. In the lookup tables, we need to lookup all valid subvectors of (u, w) that correspond to nonzero correlation based on v . The lookup table (called as cLAT) is constructed by Algorithm 2, and it takes about 4 seconds on a 2.5 GHz CPU to generate the table with storage size about 1.2 GByte when $m = 8$.

3.3. Splitting-Lookup-Recombination. Algorithm 2 constructs a m -bit cLAT for the addition modulo 2^n , and this section describes how to use it. When one input mask is fixed, we can get another input mask, the output mask, and the corresponding correlation weight by the *Splitting-Lookup-Recombination* approach, which contains three steps.

Input: Cw and $\Lambda = \{\lambda_{Cw}, \dots, \lambda_1\}$. Each pattern of the Hamming weight distribution of z can be calculated by the combination

algorithm in [28], which is the combination pattern of $\binom{n-1}{Cw}$, where $\xi_i = u_i \| v_i \| w_i$ for $0 \leq i \leq n-1$.

```

(1) Func_LSB:  $i = 0$  //constructing the LSBs of  $u, v$ , and  $w$ .
(2) if  $Cw = 0$  then
(3)   Output the tuple of  $(u, v, w)$  with  $(1, 1, 1)$  or  $(0, 0, 0)$ ;
(4) end if
(5) if  $\lambda_1 \neq 0$  then
(6)   For each  $\xi_i \in U_2$ ,  $c = 1$ , and  $Fw = 0$ , call Func_Middle  $(i + 1, c, Fw)$ ;
(7) else
(8)   For each  $\xi_i \in \mathbb{F}_2^3$ ,  $c = 2$ , and  $Fw = 1$ , call Func_Middle  $(i + 1, c, Fw)$ ;
(9) end if
(10) Func_Middle  $(i, c, Fw)$ : //constructing the middle bits of  $u, v$ , and  $w$ .
(11) if  $c = Cw$  then
(12)   call Func_MSB  $(i, c, Fw)$ ;
(13) end if
(14) if  $\lambda_c \neq i$  then
(15)   if  $Fw = 0$  then //Fw recorded whether the value of  $\lambda_{i-1}$  is 1 or not.
(16)     For each  $\xi_i = 0$  and  $Fw' = 0$ , call Func_Middle  $(i + 1, c, Fw')$ ;
(17)   else
(18)     For each  $\xi_i = 7$  and  $Fw' = 0$ , call Func_Middle  $(i + 1, c, Fw')$ ;
(19)   end if
(20) else //  $\lambda_c = i$ . The value of  $Fw$  determines whether  $\xi_i$  belongs to  $U_0$  or  $U_1$ .
(21)   if  $Fw = 0$  then
(22)     For each  $\xi_i \in U_0$  and  $Fw' = 1$ , call Func_Middle  $(i + 1, c + 1, Fw')$ ;
(23)   else
(24)     For each  $\xi_i \in U_1$  and  $Fw' = 1$ , call Func_Middle  $(i + 1, c + 1, Fw')$ ;
(25)   end if
(26) end if
(27) Func_MSB  $(i, c, Fw)$ : //constructing the bits of  $u, v, w$  with position higher than  $\lambda_{Cw}$ .
(28) if  $\lambda_c \neq i$  then //the value of  $Fw$  determines whether  $\xi_i$  equals to 0 or 7.
(29)   if  $Fw = 0$  then
(30)     Let  $\xi_i = 0$  and  $Fw' = 0$ , call Func_MSB  $(i + 1, c, Fw')$ ;
(31)   else
(32)     Let  $\xi_i = 7$  and  $Fw' = 0$ , call Func_MSB  $(i + 1, c, Fw')$ ;
(33)   end if
(34) else //  $\lambda_c = i$ .
(35)   if  $Fw = 0$  then
(36)     For each  $\xi_i \in U_0$  and  $\xi_{i+1} = 7$ , output each tuple of  $(u, v, w)$ ;
(37)   else
(38)     For each  $\xi_i \in U_1$  and  $\xi_{i+1} = 7$ , output each tuple of  $(u, v, w)$ ;
(39) end if
(40) end if

```

ALGORITHM 1: **Const** (S_{Cw}): constructing the input-output mask tuples with linear correlation weight of Cw for modular addition, $0 \leq Cw \leq n-1$.

3.3.1. Splitting. For addition modulo 2^n , $n = mt$, if one of the two input masks v is fixed, then split v into tm -bit subvectors. The larger the m , the fewer the times to lookup cLAT and the fewer the number of bit concatenation operations, but the more space the memory takes up, and after the trade-offs, we choose $m = 8$.

3.3.2. Lookup. From the MSB to the LSB direction, query the subvectors of (u, w) that correspond to each subvector of v and the corresponding correlation weights. For the highest m -bit subvector v^{t-1} , its *connection status* $b = 0$, looking up cLAT to get w^{t-1} and u^{t-1} , the corresponding correlation weight $c[t-1]$, and the *connection status* for the subvector

v^{t-2} . Similarly, other subvectors of u and w and the corresponding correlation weights can be obtained.

3.3.3. Recombination. All subvectors of u and w can be obtained by lookup tables, and the n -bit u and w can be obtained by bit concatenation. The correlation weight of the modular addition is the sum of the weight of each subblock, i.e., $Cw = \sum_{k=0}^{t-1} c[k]$.

When there are multiple modular additions in the round function, i.e., $N_A > 1$, for each modular addition, its undetermined input mask and output mask need to be obtained by the *Splitting-Lookup-Recombination* approach, respectively. In the lookup phase, a total of tN_A lookup

```

(1) for each  $b \in \{0, 1\}$  and input mask  $v \in \mathbb{F}_2^m$ 
(2)    $\text{cLAT}_{\min}[v][b] = m$ , let  $M^T[k] = 0$  and  $\text{cLAT}_N[v][b][k] = 0$ , for  $0 \leq k \leq m-1$ ;
(3)   for each input mask  $w \in \mathbb{F}_2^m$  and output mask  $u \in \mathbb{F}_2^m$  do
(4)      $A = u \oplus v$ ,  $B = u \oplus w$ ,  $C = u \oplus v \oplus w$ ,  $Cw = 0$ ;
(5)     for  $j = 0$  to  $m-1$  do
(6)        $C_b[j] = (C \gg (m-1-j)) \wedge 1$ ;
(7)     end for
(8)     if  $b = 1$  then //determining the connection status generated by the upper subblock.
(9)        $Cw++$ ,  $M^T[0] = 1$ ,  $Z = 1 \ll (m-1)$ ;
(10)    else
(11)       $M^T[0] = 0$ ,  $Z = 0$ ;
(12)    end if
(13)    for  $i = 1$  to  $m-1$  do //determining the correlation weight.
(14)       $M^T[i] = (C_b[i-1] + M^T[i-1]) \wedge 1$ ;
(15)      if  $M^T[i] = 1$  then
(16)         $Cw++$ ,  $Z = Z \vee (1 \ll (m-1-i))$ ;
(17)      end if
(18)    end for
(19)     $F_1 = A \wedge (\neg(A \wedge Z))$ ,  $F_2 = B \wedge (\neg(B \wedge Z))$ ;
(20)    if  $F_1 = 0$  and  $F_2 = 0$  then //judgment conditions  $u \oplus v \leq z$  and  $u \oplus w \leq z$ .
(21)       $\text{cLAT}_w[v][b][\text{cLAT}_N[v][b][Cw]] = w$ ;
(22)       $\text{cLAT}_u[v][b][\text{cLAT}_N[v][b][Cw]] = u$ ;
(23)       $\text{cLAT}_N[v][b][Cw]++$ ; //the number of tuples correspond to  $v$  and  $b$ .
(24)       $\text{cLAT}_b[u][v][w][b] = (M^T[m-1] + C_b[m-1]) \wedge 1$ ; //connection status.
(25)      if  $\text{cLAT}_{\min}[v][b] > Cw$  then
(26)         $\text{cLAT}_{\min}[v][b] = Cw$ ; //the minimum correlation weight corresponds to  $v$  and  $b$ .
(27)      end if
(28)    end if
(29)  end for
(30) end for.

```

ALGORITHM 2: Constructing the m -bit cLAT for modular addition.

operations are required. And the correlation weight of the round function is $Cw = \sum_{j=1}^{N_A} \sum_{k=0}^{t-1} c_j[k]$.

For each subvector v^k , the possible minimum linear correlation weight corresponding to it can be calculated in advance by Algorithm 2, that is,

$$c[k]_{\min} = \min(\text{cLAT}_{\min}[v^k][0], \text{cLAT}_{\min}[v^k][1]). \quad (17)$$

During the *Recombination* phase, the correlation boundary can be constructed by the associated weights that have been obtained and the possible minimum correlation weights, shown in Corollary 2.

Corollary 2. For addition modulo 2^n , one of the input mask $v = v^{t-1} \parallel \dots \parallel v^0$ is fixed, $n = mt$, $v^k \in \mathbb{F}_2^m$, and $0 \leq k \leq t-1$. For any $u, w \in \mathbb{F}_2^n$ of nonzero correlation, the correlation boundary should have

$$\begin{aligned} \text{Cor}(u, v, w) \leq & \text{Cor}(u^{t-1} \parallel \dots \parallel u^k, v^{t-1} \parallel \dots \parallel v^k, w^{t-1} \parallel \dots \parallel w^k) \\ & + 2^{-\sum_{j=0}^{k-1} c[j]_{\min}}. \end{aligned} \quad (18)$$

Proof. The correlation of modular addition is the product of the correlation of each subblock after splitting, i.e., $\text{Cor}(u, v, w) = \prod_{k=0}^{t-1} \text{Cor}(u^k, v^k, w^k)$. Let $-\log_2 \text{Cor}(u^{t-1}$

$\parallel \dots \parallel u^k, v^{t-1} \parallel \dots \parallel v^k, w^{t-1} \parallel \dots \parallel w^k) = \sum_{l=k}^{t-1} c[l]$ be the correlation weight of the subvector tuples that are obtained by lookup tables. The sum of the correlation weights of the subvector tuples have not been looked up yet, which should s.t. $\sum_{j=0}^{k-1} c[j]_{\min} \leq -\log_2 \text{Cor}(u, v, w) - \sum_{l=k}^{t-1} c[l]$. $\square$

Assuming the number of (u^k, w^k) corresponding to each subvector v^k is X_k , hence, the number of mask branches corresponding to the modular addition is $\prod_{k=0}^{t-1} X_k$. Corollary 2 can be used to filter out (u, w) of large correlation weight.

3.4. Improved Automatic Search Algorithm. In this section, we adopt a top-down technique [19–21], taking the first round as the starting point of the search process. In the first/second rounds, the input-output mask tuples of each modular addition with correlation weight increasing monotonically can be obtained by Algorithm 1. In the middle rounds, for each modular addition, u and w can be obtained by the *Splitting-Lookup-Recombination* approach. Algorithm 3 takes SPECK as an example.

Let the optimal correlation weight of the $(r-i)^{\text{th}}$ round that has been obtained be Bc_{r-i} , $1 \leq i \leq r-1$, and let the expected r -round correlation weight be $\overline{Bc}_r$. The correlation weight of the first two rounds should subject to Matsui's

pruning condition, i.e., $Cw_1 + Bc_{r-1} \leq \overline{Bc_r}$ and $Cw_1 + Cw_2 + Bc_{r-2} \leq \overline{Bc_r}$.

Let $\sum_{i=1}^{N'_A} Cw_i = \overline{Bc_r} - Bc_{r-1}$, N'_A is the number of additions in the first two rounds, and then the search space need to be constructed is no more than (19). When the block size of a ARX cipher is large, let n be the word size of the modular addition, the total input-output masks of all N'_A modular additions in the first two round is $S' = 2^{3n \times N'_A}$. Therefore, when the value of $\overline{Bc_r} - Bc_{r-1}$ is small, the search space S will be much smaller than S' intuitively:

$$S = \prod_{i=1}^{N'_A} \sum_{c=0}^{Cw_i} \# \{ (u, v, w) \mid -\log_2 \text{Cor}(u, v, w) = c \}. \quad (19)$$

3.5. Flexible Search Scenario Settings. Combining Algorithm 1, cLAT, pruning conditions, and the properties of the target ciphers in Section 2.3, Algorithm 3 can be adapted to the following search scenarios with appropriate modifications:

Scenario 1: for some ARX ciphers, such as SPARX and Chaskey, the number of modular additions in the round function is more than 1, $N_A > 1$. Hence, the correlation weight of each round is $Cw_i = \sum_{j=1}^{N_A} Cw_i^j$, and Cw_i^j is the correlation weight of the j^{th} addition in the i^{th} round. In the first/second round, the input-output masks of each modular addition should be generated by Algorithm 1. In the middle rounds, the tuple (u, w) of each modular addition should be obtained by calling $LR(v)$ multiple times.

Scenario 2: the linear hulls can also be searched by simply modifying Algorithm 3. For an obtained optimal linear trail, fix the input mask of the first round and the output mask of the last round, call round- r (i, x, y) directly, and modify $\overline{Bc_r}$ to the expected maximum statistical correlation weight $C_{\max}$. Therefore, for ALP, all linear trails with linear correlation weight between the optimal correlation weight $C_{\min}$ and $C_{\max}$ can be counted.

Scenario 3: when the number of rounds of a linear trail or the block size is large, the linear correlation tends to be very small, and the search process will be very time-consuming. Hence, good linear characteristic results under certain conditions can be explored by the heuristic search settings. We can exclude a large number of search branches and reduce the search complexity by these methods, such as starting the search from a desired large correlation weight $\overline{Bc_r}$, fixing the input mask of a certain round, and limiting the correlation weight of a round or a certain modular addition.

4. Applied to SPECK, SPARX, Chaskey, and CHAM-64

4.1. Linear Hulls for SPECK32/48/64. Applying Algorithm 3, the optimal linear trails for SPECK32/48/64 with correlation close to $2^{-n/2}$ can be obtained, shown in Table 2. Fixing the

input and output masks, the ALP of the linear hulls obtained by the cluster experiment are given in Table 3. For SPECK64, new 14-round linear hull with an average linear potential of $2^{-61.24}$ has been found.

When searching for the linear hulls, we need to modify Algorithm 3 to adapt to *Scenario 2*. We use formula (13) to count ALP, where $C_{\min} \leq Cw \leq C_{\max}$, $C_{\min}$ is the correlation weight of the linear trail we choose to use to pin the input and output masks, and $C_{\max}$ is the maximum correlation weight we limit our search. In the middle rounds, we adopt $C_{\max}$ to instead $\overline{Bc_r}$ for filtering out those trails that contribute less to the ALP.

4.2. Linear Characteristics for SPARX. Searching for the optimal linear trails of SPARX variants, Algorithm 3 needs to be modified to fit *Scenario 1*. There are multiple modular additions in each round, $N_A = 2$ for SPARX-64 and $N_A = 4$ for SPARX-128. Hence, for all $2N_A$ additions modulo 2^{16} in the first/second rounds, call Algorithm 1 for each addition to generate its input-output mask tuples, and then apply Property 3/4 to obtain the masks of the first two rounds for SPARX-64 and SPARX-128, respectively. In the middle rounds, the *Splitting-Lookup-Recombination* method is applied to obtain possible input masks and output masks of each addition, as well as correlation weight. The 10-round optimal linear trails for SPARX-64 are listed in Table 4, and the 11-round linear trail is obtained by limiting the correlation weight of each modular addition less than 3 in the first two rounds. Fixing the input and output mask of the 10-round optimal linear trail, a 10-round linear hull with an ALP of $2^{-40.92}$ is obtained.

For SPARX-128, the first 8-round optimal linear trails can be derived from the first 8-round optimal linear trails of SPECK32 when considering the minimum active ARX-box. The experiment results in Table 5 confirmed the derivation. Based on *Scenario 3*, we limit the correlation weight of each modular addition in the first/second round to less than 2, then we get the 9/10-round linear trails with a correlation weight of 18/23. Although the 9/10-round linear trail cannot be guaranteed as the best, they can still be used to get the 9/10-round linear hulls with the corresponding ALP of $2^{-35.22}/2^{-46}$.

The 11/10-round linear trails for SPARX-64/SPARX-128 are given by Table 6. c_j^r represents the correlation weight of the j^{th} modular addition in the r^{th} round, and Cw_r represents the correlation weight of the r^{th} round, $0 \leq j < N_A$.

4.3. Linear Characteristics for Chaskey. From Table 7, the correlation weights of the first 3-round optimal linear trails of Chaskey we have found are 0/2/9. For one-round optimal linear trail with a correlation weight of 0, input-output masks represented in hexadecimal are (1, 1, 0, 0) and (0, 80, 800000, 0). To find the linear trails with longer rounds, we use a heuristic approach in *Scenario 3*, limiting the correlation weight of each modular additions in the first round to less than 2 and setting the correlation weight

Input: the cLAT is precomputed and stored by Algorithm 2. $Bw_1, \dots, Bw_{r-1}$ have been recorded

- (1) **Program entry:**
- (2) Let $\overline{Bc_r} = Bc_{r-1} - 1$, and $Bc_r = \text{null}$ // Bw_1 can be derived manually for most ARX ciphers.
- (3) **while** $\overline{Bc_r} \neq Bc_r$ **do**
- (4) $\overline{Bc_r} + +$; //the expected r -round correlation weight increases monotonously.
- (5) Call Procedure Round-1;
- (6) **end while**
- (7) Exit the program.
- (8) **Round-1:**/exclude the search space with correlation weights larger than $\overline{Bc_r} - Bc_{r-1}$.
- (9) **for** $Cw_1 = 0$ to $n - 1$ **do** // Cw_1 increases monotonously.
- (10) **if** $Cw_1 + Bc_{r-1} > \overline{Bc_r}$ **then**
- (11) Return to the upper procedure with FALSE state;
- (12) **else**
- (13) Call Algorithm 1 **Const** (S_{Cw_1}), and traverse each output tuple (u_1, v_1, w_1) ;
- (14) **if** call Round-2(u_1, v_1, w_1) and the return value is TRUE, **then**
- (15) Stop Algorithm 1 and return TRUE; /record the optimal linear trail be found.
- (16) **end if**
- (17) **end if**
- (18) **end for**
- (19) Return to the upper procedure with FALSE state;
- (20) **Round-2**(u_1, v_1, w_1): /exclude the correlation weights larger than $\overline{Bc_r} - Bc_{r-1} - Cw_1$.
- (21) **for** $Cw_2 = 0$ to $n - 1$ // Cw_2 increases monotonously.
- (22) **if** $Cw_1 + Cw_2 + Bc_{r-2} > \overline{Bc_r}$ **then**
- (23) Return to the upper procedure with FALSE state;
- (24) **else**
- (25) Call Algorithm 1 **Const** (S_{Cw_2}), and traverse each output tuple (u_2, v_2, w_2) ;
- (26) $y = (u_1 \oplus (v_2 \lll r_a) \oplus w_2) \lll r_b$, $x = u_2 \oplus y$; // (r_a, r_b): rotation parameters.
- (27) **if** call Round- r ($3, x, y$) and the return value is TRUE, **then**
- (28) Stop Algorithm 1, compute the masks of the first/second round and return TRUE;
- (29) **end if**
- (30) **end if**
- (31) **end for**
- (32) Return to the upper procedure with FALSE state;
- (33) **Round- r** (i, x, y): //middle rounds, $3 \leq i \leq r$.
- (34) $v = x \ggg r_a$, and let $v = v^{t-1} \parallel \dots \parallel v^0$ and $v^k \in \mathbb{F}_2^m$, $0 \leq k \leq t - 1$; //splitting v .
- (35) Call $LR(v)$, traversing each u and w ; //where $Cw_i = \text{Cor}(v, w, u)$.
- (36) **if** $i = r$ and $Cw_1 + \dots + Cw_{i-1} + Cw_i = \overline{Bc_r}$ **then** //the last round.
- (37) Let $Bc_r = \overline{Bc_r}$, break from $LR(v)$ and return TRUE;
- (38) **end if** // r -round optimal linear trail of expected correlation weight $\overline{Bc_r}$ have been found.
- (39) $y' = (y \oplus w) \lll r_b$, $x' = y' \oplus u$;
- (40) **if** call Round- r ($i + 1, x', y'$) and the return value is TRUE, **then**
- (41) Break from $LR(v)$ and return TRUE; //record the masks of each round and return.
- (42) **end if**
- (43) Return to the upper procedure with FALSE state;
- (44) **LR**(v): //looking up cLAT and recombining another input mask w and the output mask u .
- (45) Let $c[k]_{\min} = \min(\text{cLAT}_{\min}[v^k][0], \text{cLAT}_{\min}[v^k][1])$, and $b[k] = 0$, for $0 \leq k \leq t - 1$;
- (46) **for** $k = t - 1$ to 0 **do** //from MSB to LSB direction.
- (47) **for** $c_i[k] = \text{cLAT}_{\min}[v^k][b[k]]$ to $\overline{c_i[k]}$ **do** // $\overline{c_i[k]} = m - 1$ and $\overline{c_i[k]} = m$ for $0 \leq k \leq t - 2$.
- (48) **if** $\sum_{g=1}^{i-1} Cw_g + \sum_{j=0}^{k-1} c[j]_{\min} + \text{then } \sum_{l=k}^{t-1} c_i[l] + Bc_{r-i} \leq \overline{Bc_r}$
- (49) **for** $X_k = 0$ to $\text{cLAT}_N[v^{t-1}][b[k]][c_i[k]] - 1$ **do**
- (50) $u^k = \text{cLAT}_u[v^k][b[k]][X_k]$; //querying u^k and w^k .
- (51) $w^k = \text{cLAT}_w[v^k][b[k]][X_k]$;
- (52) $b[k - 1] = \text{cLAT}_b[u^k][v^k][w^k][b[k]]$; //record the next connection status.
- (53) **if** $k = 0$ **then** //recombining u and w .
- (54) Output each $u = u^{t-1} \parallel \dots \parallel u^0$, $w = w^{t-1} \parallel \dots \parallel w^0$, and $Cw_i = \sum_{k=0}^{t-1} c_i[k]$;
- (55) **end if**
- (56) **end for**
- (57) **end if**
- (58) **end for**
- (59) **end for**

ALGORITHM 3: Automatic search for the optimal linear trails of ARX ciphers. Take the application to SPECK as an example, where $n = mt$.

TABLE 2: The 9/10/13 round optimal linear trails for SPECK32/48/64.

r	SPECK32		SPECK48		SPECK64	
	ΓX_r	Cw_r	ΓX_r	Cw_r	ΓX_r	Cw_r
0	00A0062 F	1	000180B80001	1	0100012014010021	2
1	78B818B9	4	000000C00001	0	0001810020000101	1
2	00906021	1	00000E00000 E	2	0000010000000001	0
3	60804081	1	7A0070700070	5	0000000100000000	1
4	00800001	0	C2C080829380	6	0D0000000C000000	2
5	00010000	1	D00000108300	2	60610000606C0000	3
6	0B000800	3	800000809800	1	00024D0300620C03	6
7	20402050	2	00000400C004	1	181070141B107358	6
8	008380C3	1	200020260020	2	0013001818031840	3
9	170B130A	—	013100310100	2	1818000000181200	2
10			8800A8880109	—	0018000000001000	1
11					0000100000000000	1
12					0000009800000080	2
13					5000040480000404	—

TABLE 3: Linear hulls for SPECK32/48/64.

$2n$	r	Γ_{in}	Γ_{out}	C_{min}	C_{max}	ALP	No. of trails	Time	Reference
32	9	0010, 1400	0B00, 0800	15	25	$2^{-29.1}$	69737	N/A	[18]
	9	0380, 5224	066A, 0608	14	14	2^{-28}	1	N/A	[16]
	9	00A0, 062F	170B, 130A	14	20	$2^{-27.78}$	14	25 s	This paper
48	10	000131, 050021	2484F2, 2480F6	22	N/A	2^{-44}	1	N/A	[16]
	10	800121, 158021	DE84DC, C684DC	22	N/A	2^{-44}	1	N/A	[18]
	10	000180, B80001	8800A8, 880109	22	28	$2^{-43.64}$	50	157.3 h	This paper
64	13	18600010, 10724800	00024982, 00420802	30	30	2^{-60}	1	N/A	[16]
	13	00101800, 00001812	00006065, 00006068	30	30	2^{-60}	1	N/A	[18]
	13	01000120, 14010021	50000404, 80000404	30	32	$2^{-55.29}$	178	7.3 h	This paper
	14	01000120, 14010021	26902000, 20802006	33	35	$2^{-61.24}$	194	5.8 h	This paper

All experiment code in this work are run on a single high-performance server with Intel(R) Xeon(R) CPU E5-2680 v4 @ 2.40 GHz. All masks are represented in hexadecimal.

TABLE 4: Linear characteristics for SPARX-64.

r	Cw	T_{trail}	Γ_{in}				Γ_{out}				C_{min}	C_{max}	ALP	No. of trails	T_{ALP}
1	0	0 s	0000	0000	0080	4001	0000	0000	0000	0001	0	0	1	1	0 s
2	0	0 s	0000	0000	0080	4001	0000	0000	0004	0004	0	0	1	1	0 s
3	1	0 s	0000	0000	0080	4001	2C10	2010	2C1C	201C	1	10	2^{-2}	1	0 s
4	3	0 s	0090	6021	0000	0000	0000	0000	0B00	0800	3	10	2^{-6}	1	0 s
5	5	0 s	00A0	3021	0000	0000	0000	0000	85C2	8442	5	10	$2^{-9.98}$	3	3 s
6	7	0 s	0080	5021	0000	0000	4285	4284	4385	4384	7	15	$2^{-13.91}$	11	22 s
7	11	3.2 m	0090	6021	0000	0000	170B	130A	17CF	130F	11	17	$2^{-21.87}$	5	9 s
8	15	4.6 h	0880	503D	0090	6021	0000	0000	0E81	1EB0	15	20	$2^{-29.73}$	22	2 m
9	19	48.6 h	0080	5021	2000	0058	2058	2040	3858	3840	19	25	$2^{-35.97}$	1215	28.7 h
10	22	3 d	0080	4001	00C0	F001	0205	0204	0205	0204	22	25	$2^{-40.92}$	144	1 h
11	≤ 28	5 m	0080	4001	00C0	F001	38A8	2080	2058	2040	28	28	2^{-56}	1	—

to the expected values to start the heuristic search. The 4/5-round linear trails with correlation weights of 29/61 are obtained. The details of the linear trails are listed in Table 8.

4.4. Linear Characteristics for CHAM-64. In the first 4 rounds of CHAM, the input-output mask tuples of each modular addition can be constructed by Algorithm 1, and

all input and output masks for the first 4 rounds are deduced by Property 5. In the forward search process, the *Splitting-Lookup-Recombination* approach is adopted to determine the possible unknown input masks and output masks for the modular addition in each round. In [7], for CHAM-64, a 34-round linear trail with a bias of $\varepsilon = 2^{-31}$ was given. The correlation weights of the optimal linear trails obtained by us are shown in Table 9. The correlation of the 34-round optimal linear trail we find is 2^{-31} , and the

TABLE 5: Linear characteristics for SPARX-128.

r	Cw	T_{trail}	Γ_{in}	Γ_{out}	C_{min}	C_{max}	ALP	No. of trails	T_{hull}
1	0	0 s	0000000000000000 0000000000804001	0000000000000000 0000000000000001	0	0	1	1	0 s
2	0	0 s	0000000000000000 0000000000804001	0000000000000000 0000000000040004	0	0	1	1	0 s
3	1	0 s	0000000000000000 0000000000804001	0000000000000000 000000002C102010	1	3	2^{-2}	1	0 s
4	3	0 s	0000000000000000 0000000000906021	000000000B000800 0B03000300030803	3	10	2^{-6}	1	0 s
5	5	2 s	0000000000A03021 0000000000000000	0000000000000000 0000000085C28442	5	10	$2^{-9.98}$	3	0 s
6	7	1 s	0000000000805021 0000000000000000	0000000000000000 0000000042854284	7	12	$2^{-13.91}$	3	1 s
7	9	4 s	0000000000906021 0000000000000000	0000000000000000 00000000170B130A	9	15	$2^{-17.78}$	5	13 s
8	12	13 d	0000000000811381 0000000000000000	00000000170B130A 160F01040104120E	12	18	$2^{-23.65}$	11	101 s
9	≥ 14	—	0000000000004020	0000000002150012					
	≤ 18	27 m	0000000000000000	081308140A060806	18	22	$2^{-35.22}$	53	6 h
10	≥ 18	—	0000000000804001	000000000B810800					
	≤ 23	4.4 d	0000000000000000	09840A0402050204	23	23	2^{-46}	1	—

TABLE 6: The 11/10-round linear trails for SPARX-64/SPARX-128.

11-round trail for SPARX-64					10-round trail for SPARX-128						
r	$\Gamma X_0^r \parallel \dots \parallel \Gamma X_0^r$	c_0^r	c_1^r	Cw_r	r	$\Gamma X_0^r \parallel \Gamma X_1^r \parallel \dots \parallel \Gamma X_6^r \parallel \Gamma X_7^r$	c_0^r	c_1^r	c_2^r	c_3^r	Cw_r
1	0080400100C0F001	0	1	1	1	00000000000402000000000000000000	0	0	0	0	0
2	000000010000C001	0	0	0	2	00000000081008100000000000000000	0	1	0	0	1
3	0004000400070007	1	2	3	3	000000000C010E000000000000000000	0	3	0	0	3
$\mathcal{L}$	2C1020102C1C201 C	—	—	—	4	00000000364834500000000000000000	0	5	0	0	5
4	2C1C201C00000000	4	0	4	$\mathcal{L}'$	00000000008A90E20000000000000000	—	—	—	—	—
5	3140013000000000	3	0	3	5	000000000000000000000000008A90E2	0	0	0	2	2
6	858D05CF00000000	4	0	4	6	0000000000000000000000000078F138E	0	0	0	4	4
$\mathcal{L}$	60117B1800000000	—	—	—	7	000000000000000000000000000E1600	0	0	0	2	2
7	0000000060117B18	0	4	4	8	000000000000000000000000000001800	0	0	0	0	0
8	0000000000812261	0	3	3	$\mathcal{L}'$	00000000000000000000000060006000	—	—	—	—	—
9	0000000080018280	0	1	1	9	00000000600060006000000000006000	0	1	1	0	2
$\mathcal{L}$	0000000000020202	—	—	—	10	00000000828182010280020080018001	0	2	1	1	4
10	0002020200000200	1	0	1	11	0000000002150012081308140A060806	—	—	—	—	—
11	1E08180808000800	3	1	4							
12	38A8208020582040	—	—	—							

TABLE 7: Correlation of the linear trails for Chaskey.

Round	1	2	3	4	5	Reference
Correlation	2^{-1}	2^{-2}	2^{-9}	—	—	[18]
Correlation	1	2^{-2}	2^{-9}	$\geq 2^{-29}$	$\geq 2^{-61}$	This paper

details of the 35-round optimal linear trail with a correlation of 2^{-33} are listed in Table 10.

5. Conclusions

In this paper, we have improved the automatic search algorithm for the linear characteristics on ARX ciphers. Combining with the optimization strategies of constructing the input-output masks corresponding to specific correlation weight and the novel concept of cLAT, this search

tool enables an efficient search for the linear characteristics on typical ARX ciphers. Applying this tool, we get new 9/10/14-round linear hulls for SPECK32/48/64, and the ALP is $2^{-27.78}$, $2^{-43.64}$, and $2^{-61.24}$, respectively. For SPARX-64, a 10-round optimal linear trail with a correlation of 2^{-22} and a 11-round good linear trail with a correlation of 2^{-28} have been obtained. For SPARX-128, a 10-round linear trail with a correlation of 2^{-23} is obtained. The linear cryptanalysis results on SPARX are presented for the first time so far. For Chaskey, the linear characteristic results have been

TABLE 8: Linear trails for Chaskey.

r	2 rounds with $\text{Cor} = 2^{-2}$					3 rounds with $\text{Cor} = 2^{-9}$				
	a, b, c, d	c_0^r	c_1^r	c_2^r	c_3^r	a, b, c, d	c_0^r	c_1^r	c_2^r	c_3^r
0	2000001, B100001, 1, 1	1	0	0	0	1, 8100001, 201, 303	0	1	0	1
1	0, 1, 0, 0	0	0	0	1	300, 1, 0, 0	5	0	0	0
2	300004, 1000, 10000000, 4	—	—	—	—	0, 10000, 1, 0	0	0	1	1
3						240030, 10008080, 81011000, 240000	—	—	—	—
r	4 rounds with $\text{Cor} = 2^{-29}$					5 rounds with $\text{Cor} = 2^{-61}$				
	a, b, c, d	c_0^r	c_1^r	c_2^r	c_3^r	a, b, c, d	c_0^r	c_1^r	c_2^r	c_3^r
0	0, 800, 90001, 10D0801	0	2	1	0	1, 8100001, 201, 303	0	1	0	1
1	0, 0, 1, 1	0	0	0	0	300, 1, 0, 0	5	0	0	0
2	0, 80, 810000, 0	0	2	3	1	0, 10000, 1, 0	0	0	1	1
3	18001A20, 80880040, 400189, 1A20	5	3	7	5	240030, 10008080, 81011000, 240000	4	4	6	6
4	2D224005, A83F0DA8, 2DA1C86D, 25004107	—	—	—	—	50E73286, 8241A0, 5161469B, 40D436A6	10	12	4	6
5						BAAE7E16, 76224512, 65104022, 3EA61E37	—	—	—	—

TABLE 9: Correlation weights of the optimal linear trails for CHAM-64.

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Cw	0	0	0	0	0	0	0	1	2	3	3	4	5	6	7	9	10	12	14	16	17	18	20	21	22	23	25	26	26	26	27	28	29	31	33

TABLE 10: 35-round optimal linear trail for CHAM-64.

r	$\Gamma X_0^r \parallel \dots \parallel \Gamma X_3^r$	Cw_r	r	$\Gamma X_0^r \parallel \dots \parallel \Gamma X_3^r$	Cw_r
1	0000000000018000	0	19	0007010380C08000	1
2	0000000180000000	0	20	810080C080000700	2
3	0001800000000000	0	21	0001800007000201	0
4	0000000000000100	0	22	0000070002010100	0
5	0000000001000000	0	23	0700020101000000	2
6	0000010000000000	0	24	0001010000000006	0
7	0100000000000000	1	25	0000000000060002	0
8	00C0000000000001	1	26	0000000600020000	0
9	8000000000010100	1	27	0006000200000000	1
10	40000001010000C0	1	28	0000000000000400	0
11	0061010000C08000	1	29	0000000040000000	0
12	812000C080004100	4	30	0000040000000000	0
13	0041800041008201	1	31	0400000000000000	1
14	0030410082014100	3	32	0200000000000006	1
15	6500820141000060	3	33	0002000000060600	1
16	A0C1410000600047	3	34	0001000606000200	0
17	C080006000470103	2	35	0106060002000002	2
18	60000047010380C0	1	36	0682020000028401	—

updated, which cover more rounds than the existing results. For CHAM-64, the linear characteristics we obtained are the first third-party linear cryptanalysis results. In addition, we believe that these improved optimization strategies can also be achieved by linear cryptanalysis on other ARX ciphers.

Data Availability

The data obtained to support the findings of this work are available from the authors upon request. An extended version of this article can be found at <https://eprint.iacr.org/2019/1319>.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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