Research Article

Efficient Parallel Sorting for Migrating Birds Optimization When Solving Machine-Part Cell Formation Problems

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The Machine-Part Cell Formation Problem (MPCFP) is a NP-Hard optimization problem that consists in grouping machines and parts in a set of cells, so that each cell can operate independently and the intercell movements are minimized. This problem has largely been tackled in the literature by using different techniques ranging from classic methods such as linear programming to more modern nature-inspired metaheuristics. In this paper, we present an efficient parallel version of the Migrating Birds Optimization metaheuristic for solving the MPCFP. Migrating Birds Optimization is a population metaheuristic based on the V-Flight formation of the migrating birds, which is proven to be an effective formation in energy saving. This approach is enhanced by the smart incorporation of parallel procedures that notably improve performance of the several sorting processes performed by the metaheuristic. We perform computational experiments on 1080 benchmarks resulting from the combination of 90 well-known MPCFP instances with 12 sorting configurations with and without threads. We illustrate promising results where the proposal is able to reach the global optimum in all instances, while the solving time with respect to a nonparallel approach is notably reduced.

1. Introduction

The Machine-Part Cell Formation Problem (MPCFP) is based on the well-known Group Technology (GT) [1] widely used in the manufacturing industry. The goal of the MPCFP is to organize a manufacturing plant in a set of cells that contain a limited number of machines and parts, but minimize the cell-interchange of parts. The purpose is to reduce costs and increase productivity. The MPCFP is known to be NP-Hard, so there is always the challenge of producing high quality solutions in a limited time interval. During the last years, several techniques have successfully been applied to this problem, from mathematical methods such as linear programming [2] to classic metaheuristics such as particle swarm optimization [3] and more modern ones such as artificial fish swarms [4].

In this paper, we present a new and efficient parallel version of the Migrating Birds Optimization metaheuristic

for solving the MPCFP. Migrating Birds Optimization (MBO) is a population-based metaheuristic inspired by the V-shaped flight employed by birds when they migrate. This technique is known to be very effective for energy saving during flying. This interesting approach is enhanced by precisely integrating parallel procedures particularly for the efficient sorting of birds and neighboring solutions. It results in a notable improvement in terms of performance of the whole solving process. We perform computational experiments by using 90 well-known MPCFP instances, 2 well-known sorting algorithms, 1 sequential configuration, and 5 thread configurations (1, 4, 8, 16, and 32) resulting in a total of 1080 benchmarks. The obtained results are encouraging where the proposal is able to reach the global optimum in all instances, while the solving time with respect to a nonparallel approach is notably reduced.

The outline of the study is as follows. In Section 2, we give some information on the related work. Section 3 describes

TABLE 1: Initial incidence matrix $M \times P$.

	Part 1	Part 2	Part 3	Part 4	Part 5	Part 6	Part 7
Machine 1	0	1	0	1	1	1	0
Machine 2	1	0	1	0	0	0	0
Machine 3	1	0	1	0	0	0	1
Machine 4	0	1	0	1	0	1	0
Machine 5	1	0	0	0	0	0	1

and models the MPCFP. Section 4 gives an overview of MBO. Section 5 describes how to resolve the MPCFP using MBO with parallel sort. Section 6 presents and discusses the experimental results. Section 7 concludes and provides guidelines for future work. Finally, the Appendix provides detailed results for the 1080 benchmarks, in which each includes 90 test instances.

2. Related Work

Several investigations that have been carried out for the problem MPCFP are as follows: linear programming [5], a goal programming model [6], a hybrid genetic algorithm [7], boolean satisfiability [8], constraint programming [9], and using an artificial fish swarm algorithm [4]. An analysis of the evolution of cell formation problem can be found in [10, 11]. As MPCFP is NP-Hard problem, many researchers have considered applying approximation methods as metaheuristics. Various research can be found in the field of metaheuristics, in which the search is performed using iterative procedures, which allow moving from one solution to another in the search space. This type of metaheuristic performs movements in the neighborhood of the current solution, which means it has a perturbative nature. As metaheuristics are based on trajectories, among which we have simulated annealing [12–14], tabu search [15–17], local iterated search [18, 19], and local variable search [20], there is also another type of metaheuristics which are based on population. The population metaheuristics have a set of individuals, where each individual encodes a temporary solution. These metaheuristics use an evaluation function fitness of each individual, with the aim of obtaining the best values of the population that solve an optimization problem. In addition, there are disturbances in the algorithm that directed individuals in the population or part of them to possible solutions for better fitness. Among these population-based metaheuristics, ant colony optimization [21, 22], particle swarm optimization [3, 23, 24], cat swarm [25, 26], and Migrating Birds Optimization [27] can be found in problems like a hybrid flowshop scheduling with total flowtime minimisation [28], closed loop layout with exact distances in flexible manufacturing systems [29], and preliminary results for MPCFP [30]. A brief review of nature-inspired algorithms for optimization can be found in [31].

In the area of manufacturing and industrial applications, parallel implementations using metaheuristics have been addressed, such as GRASP and grid computing to solve the location area problem [32], performance analysis of coarse-grained parallel genetic algorithms on the multicore

Sun Ultra SPARC T1 [33], a parallel genetic algorithm for the multilevel unconstrained lot-sizing problem [34], and a memetic algorithm and a parallel hyperheuristic island-based model for a 2D packing problem [35]. We can also find research using parallel metaheuristics as a cooperative parallel metaheuristic for the capacitated vehicle routing problem [36] and optimizing shared-memory hyperheuristics on top of parameterized metaheuristics [37] and, finally, we can find a review of recent progress in implementing parallelism with metaheuristics included in the following work [38].

Different studies have addressed the parallel sorting algorithms as massively parallel sort-merge joins in main memory multicore database systems [39], an efficient parallel merge sort for fixed and variable length keys [40], a randomized parallel sorting algorithm with an experimental study [41], sorting algorithm for many-core architectures based on adaptive bitonic sort [42], performance comparison of sequential quick sort and parallel quick sort algorithms [43], the time profit obtained by parallelization of quick sort algorithm used for numerical sorting [44], an efficient massively parallel quick sort [45], and, finally, a fast parallel implementation of quick sort and its performance evaluation on SUN enterprise 10000 [46].

In this paper, we have chosen metaheuristic MBO because it has a friendly design, composed of two linked lists for each of the sides of the flock of birds and a node as a leader bird. Furthermore, MBO has been applied to various optimization problems with promising results [47–49]. We focus on an efficient parallel sorting for MBO when solving MPCFP, which to our knowledge has not yet been reported. It has been chosen to parallelize the sorting algorithms in MBO because 4 out of the 8 steps of the MBO metaheuristic used sorting algorithms, and therefore we believe that applying parallel sorting in these steps is possible to achieve reduction of the execution time in MBO algorithm.

3. Machine-Part Cell Formation Problems

The Machine-Part Cell Formation Problem (MPCFP) is NP-Hard optimization problem; the main objective is the formation of set of machines and parts in groups so that the number of intercell transportation of parts is minimized. Therefore, the initial matrix (see matrix in Table 1) must be converted into a matrix that has a block diagonal structure (cells are easily visible; see matrix in Table 2). This example corresponds to a MPCFP with the following parameters: 5 machines, 7 parts, and $M_{\max} = 3$ for 2 cells. The optimum value obtained is 0, and the final incidence matrix a_{ij} is

TABLE 2: Rearranged incidence matrix $M \times P$.

	Part 1	Cell 1 Part 3	Part 7	Part 2	Cell 2 Part 4	Part 6	Part 5
Cell 1							
Machine 2	1	1	0	0	0	0	0
Machine 3	1	1	1	0	0	0	0
Machine 5	1	0	1	0	0	0	0
Cell 2							
Machine 1	0	0	0	1	1	1	1
Machine 4	0	0	0	1	1	1	0

TABLE 3: Matrix $M \times C \rightarrow y_{ik}$.

	Cell 1	Cell 2
Machine 1	0	1
Machine 2	1	0
Machine 3	1	0
Machine 4	0	1
Machine 5	1	0

TABLE 4: Matrix $P \times C \rightarrow z_{jk}$.

	Cell 1	Cell 2
Part 1	1	0
Part 2	0	1
Part 3	1	0
Part 4	0	1
Part 5	0	1
Part 6	0	1
Part 7	1	0

constructed from the results of the matrices y_{ik} and z_{jk} (see Tables 3 and 4).

A rigorous mathematical formulation of MPCFP is given by Boctor [2]. The problem is represented by the following mathematical model:

(i) Parameters, index, and sets:

(a) a_{ij} , the $M \times P$ binary machine-part incidence matrix, where

$$a_{ij} = \begin{cases} 1 & \text{if } j_{\text{th}} \text{ part visits } i_{\text{th}} \text{ machine;} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

(b) M , the number of machines,

(c) P , the number of parts,

(d) C , the number of cells,

(e) i , the index of machines ($i = 1, \dots, M$),

(f) j , the index of parts ($j = 1, \dots, P$),

(g) k , the index of cells ($k = 1, \dots, C$),

(h) $M_{\max}$, the maximum number of machines per cell.

(ii) Variables and domains:

(a) y_{ik} , the $M \times C$ machine-cell matrix, where

$$y_{ik} = \begin{cases} 1 & \text{if machine } i \in \text{cell } k; \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

(b) z_{jk} , the $P \times C$ part-cell matrix, where

$$z_{jk} = \begin{cases} 1 & \text{if part } j \in \text{family } k; \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

(iii) Objective function:

$$\text{minimized } \sum_{k=1}^C \sum_{i=1}^M \sum_{j=1}^P a_{ij} z_{jk} (1 - y_{ik}). \quad (4)$$

(iv) Being subject to the following constraints:

$$\begin{aligned} \sum_{k=1}^C y_{ik} &= 1 \quad \forall i, \\ \sum_{i=1}^M y_{ik} &\leq M_{\max} \quad \forall k, \\ \sum_{k=1}^C z_{jk} &= 1 \quad \forall j. \end{aligned} \quad (5)$$

4. Migrating Birds Optimization

Migrating Birds Optimization (MBO) is a nature-inspired population metaheuristic based on the V-shaped flight of the migrating birds [50, 51], which is proven to be an effective formation in energy saving [52]. In the V-formation (shown in Figure 1), some parameters like wing-tip spacing (WTS), maximum width of the wing (w), angle of the V-formation (α), and depth and wing span (b) are important to form an effective V-formation [53, 54]. The conceptual similarity between the parameters of the algorithm of MBO with the actual migration of birds in V-flight formation is investigated in Duman et al. [27]. This conceptual similarity is described in Table 5.

TABLE 5: Conceptual similarity of MBO metaheuristic and V-shape natural migration of birds.

Parameter description	Concept in real migration birds in V-formation
The number of initial solutions of the flock.	Birds in V-formation.
The number of neighboring solutions generated for each initial solution.	The induced power required which is inversely proportional to the speed.
The number of neighboring solutions shared with the next solution.	Wing-tip spacing (WTS).
The number of tours.	The number of wing flaps before a change occurs in the leading bird.
The number of iterations (total number of generated neighbor solutions).	There is no conceptual relationship.

FIGURE 1: The V-shaped flight.

The MBO starts with a number of n initial solutions corresponding to birds in a V-flight formation. These solutions are generated randomly and a solution as the leader bird is chosen; then the remaining birds are alternately distributed to each side of the flock. In the next step, for all tours, improving the leader solution is performed by generating and evaluating its k neighbors. For each solution S_r in the flock (except leader), try to improve S_r by evaluating its $(k - x)$ neighbors and x unused best neighbors from the solution in the front. Move the leader solution to the end and forward one of the solutions following it to the leader position. Finally, when the iterations are over, return the best solution in the flock. Algorithm 1 depicts the classic procedure of MBO algorithm.

5. Solving the MPCFP Using MBO with Parallel Sort

The first step in solving the problem MPCFP is a proper integration with the MBO metaheuristic. In this sense, we conceptualize every bird of the flock (including the leader of the flock) as a possible solution to the problem MPCFP (see Figure 2). Each bird will be composed of a matrix $M \times C$ and matrix $P \times C$. These matrices are those modified while the problem lapsed iterations. In addition, they will be used for the calculation of fitness to problem MPCFP.

5.1. Generate Initial Solutions. For generating initial solutions, consider the following inputs: machines = 4, parts = 5, cells = 2, $M_{\max} = 2$, and a matrix $M \times P$ (see Table 6).

Considering data in Table 6, the first step to generate an initial solution is as follows.

Step 1. Generate random allocations in the matrix $M \times P$ with some random method. This matrix $M \times C$ (see Table 7) satisfies constraints that exists in only one machine in a cell and $M_{\max}$ is less than or equal to 2 in an entire cell.

Step 2. Generate with a manual method the matrix $P \times C$ from the matrix $M \times C$. For this, a connection is established with the matrix $M \times P$ and with the matrix solution $M \times C$. In lightface, machines correspond to the cell number 1 and, in bold, machines correspond to cell number 2 (see Table 8).

The manual method for determining the matrix $P \times C$ involves adding the value 1 out of every position i, j of the $M \times P$ matrix and putting this value in the temporary array of sums according to the corresponding cell. We perform the operation for each value of positions i, j of the matrix $M \times P$ (see Table 9).

Example 1.

- (i) $M \times P[1, 1] = 1 \rightarrow$ is the value that is associated with cell 1; we add this value to the Row Sum of Cell 1.
- (ii) $M \times P[2, 1] = 1 \rightarrow$ is the value that is associated with cell 2; we add to this value to the Row Sum of Cell 2.
- (iii) $M \times P[3, 1] = 0 \rightarrow$ is the value that is associated with cell 2; we add to this value to the Row Sum of Cell 2.
- (iv) $M \times P[4, 1] = 1 \rightarrow$ is the value that is associated with cell 1; we add to this value to the Row Sum of Cell 1.

Step 3. From the results of Table 9, we build a partial solution to the matrix $P \times C$ (see Table 10).

Step 4. Find possible solutions to $P \times C$.

We must choose the values of the positions i, j that belong to the temporary matrix $M \times P$ and have the largest values of each row. Subsequently, we replace larger values assigning them the value 1, and for the other values in the row we assign them the value of 0 (see Table 11). In case of draw, that is, the entire row has equal numbers, we keep unchanged if the value of their positions is 1. In the case that the same numbers are greater than 1, to each number greater than 1, we assign them a value of 1.

We consider rows having values equal to 1 in this case correspond to rows pertaining to machines 3 and 4. From this point, we generate all possible combinations of matrices and randomly choose one (see Table 12).

```

(1) Generate  $n$  initial solutions in V-formation
(2)  $i = 0$ 
(3) while ( $i < I$ ) do
(4)   for int  $j = 0; j < m; j++$  do
(5)     Improve the leading solution by generating and evaluating  $k$  neighbors of it
(6)      $i = i + k$ 
(7)     for each solution  $S_r$  in the flock (except leader) do
(8)       Try to improve  $S_r$  by evaluating  $(k - x)$  neighbors of it and  $x$  unused best
       neighbors from the leader
(9)        $i = i + (k - x)$ 
(10)    end
(11)  end
(12)  Move the leader solution to the end and forward one of the solutions following it
      to the leader position
(13) end
(14) Return the best solution in the flock

```

ALGORITHM 1: Pseudocode of Migrating Birds Optimization.

FIGURE 2: Integration of Migrating Birds Optimization with the Machine-Part Cell Formation Problem.

TABLE 6: Matrix $M \times P$.

	Part 1	Part 2	Part 3	Part 4	Part 5
Machine 1	1	0	0	1	1
Machine 2	1	0	1	0	1
Machine 3	0	1	0	1	1
Machine 4	1	0	1	0	0

TABLE 7: Matrix $M \times C$.

	Cell 1	Cell 2
Machine 1	1	0
Machine 2	0	1
Machine 3	0	1
Machine 4	1	0

5.2. Generating Neighboring Solutions. To generate a neighboring solution, we consider the following inputs: machines = 4, parts = 5, cells = 2, $M_{\max} = 2$, matrix $M \times P$, matrix $M \times C$, and matrix $P \times C$ (see Table 13).

Step 1. Randomly choose a position i, j in the matrix $M \times C$ that has a value of 1 and we work with the machine associated with that row i . This is the same as selecting a machine and working with its row i . In this case, we chose randomly the

TABLE 8: Relationship between the matrix $M \times P$ with the matrix $M \times C$.

	Part 1	Part 2	Part 3	Part 4	Part 5	Cell 1	Cell 2
Machine 1	1	0	0	1	1	1	0
Machine 2	1	0	1	0	1	0	1
Machine 3	0	1	0	1	1	0	1
Machine 4	1	0	1	0	0	1	0

TABLE 9: Generation of temporary matrix sum.

	Part 1	Part 2	Part 3	Part 4	Part 5	Cell 1	Cell 2
Machine 1	1	0	0	1	1	1	0
Machine 2	1	0	1	0	1	0	1
Machine 3	0	1	0	1	1	0	1
Machine 4	1	0	1	0	0	1	0
Sum of Cell 1	2	0	1	1	1		
Sum of Cell 2	1	1	1	1	2		

TABLE 10: Matrix $P \times C$.

	Cell 1	Cell 2
Part 1	2	1
Part 2	0	1
Part 3	1	1
Part 4	1	1
Part 5	1	2

TABLE 11: Matrix $P \times C$.

	Cell 1	Cell 2		Cell 1	Cell 2
Part 1	2	1		1	0
Part 2	0	1		0	1
Part 3	1	1	→	1	1
Part 4	1	1		1	1
Part 5	1	2		0	1

position of the matrix $M \times C[2, 2]$ with a value of 1, which corresponds to machine 2 (see Table 14).

Step 2. Then on machine 2, we randomly pick a cell that has a 0. In this case, we choose the position of the matrix $M \times C[2, 1]$; we accomplish a replacement of the value of 0 to 1 and finally realize a replacement for $M \times C[2, 2]$ value of 1 to 0. You can see the result of this operation in Table 15. In the case that the constraints are inconsistent, you must repeat the procedure from Step 1.

Step 3. We carry out the construction of the matrix $P \times C$ using the manual method explained in Section 5.1.

5.3. Flowchart of Migrating Birds Optimization. This section describes the stages of the algorithm MBO, as it integrates with the problem of MPCFP and in which stages parallel arrangement algorithms are applied. For a better understanding, a flow chart has been included (see Figure 3) which is divided into 4 main phases: algorithm initialization, a tour process, leader replacement, and algorithm finalization.

5.3.1. Algorithm Initialization. *Parameter Initialization (Stage 1 in Figure 3).* At this stage, all the values required by the MBO metaheuristic are initialized; all the values required by the problem of MPCFP and that type of sorting algorithm are to be implemented in the metaheuristic.

Among the metaheuristic values, there are

- (i) number of birds (n),
- (ii) number of neighbors (k),
- (iii) number of tours (m),
- (iv) number of shared solutions (x),
- (v) number of iterations (I).

And between complementary metaheuristic parameters, we can include

- (i) leader exchange mode, where there are 2 types of changes of leader; exchange the leader with the successor and exchange the leader with the best;
- (ii) sort type applied in the initial birds flock, where there is an ordering with random birds in the flock and another with birds ordered according to their fitness.

Among the parameters required for the problem MPCFP, we have

- (i) incidence matrix ($M \times P$),
- (ii) number of machines (M),
- (iii) number of parts (P),
- (iv) number of cells (C),
- (v) maximum number of machines in a cell ($M_{\max}$).

Finally, for the types of sorting applied to MBO, we have the following parameters:

- (i) Type of sorting algorithm: quick sort or merge sort.
- (ii) Sorting technique: normal or with threads.
- (iii) Number of threads if required.

TABLE 12: All possible combinations of matrices $P \times C$.

	Cell 1	Cell 2		Cell 1	Cell 2
Part 1	1	0	Part 1	1	0
Part 2	0	1	Part 2	0	1
Part 3	1	0	Part 3	1	0
Part 4	1	0	Part 4	0	1
Part 5	0	1	Part 5	0	1
	Cell 1	Cell 2		Cell 1	Cell 2
Part 1	1	0	Part 1	1	0
Part 2	0	1	Part 2	0	1
Part 3	0	1	Part 3	0	1
Part 4	1	0	Part 4	0	1
Part 5	0	1	Part 5	0	1

FIGURE 3: Flowchart of Migrating Birds Optimization.

TABLE 13: Initial matrices for generating neighboring solutions.

(a)					
	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5
Machine 1	1	0	0	1	1
Machine 2	1	0	1	0	1
Machine 3	0	1	0	1	1
Machine 4	1	0	1	0	0

(b)		
	Cell 1	Cell 2
Machine 1	1	0
Machine 2	0	1
Machine 3	0	1
Machine 4	1	0

(c)		
	Cell 1	Cell 2
Part 1	1	0
Part 2	0	1
Part 3	0	1
Part 4	1	0
Part 5	0	1

TABLE 14: Machine selection.

(a)		
	Cell 1	Cell 2
Machine 1	1	0
Machine 2	0	1
Machine 3	0	1
Machine 4	1	0

(b)		
	Cell 1	Cell 2
Machine 1	1	0
Machine 2	0	1
Machine 3	0	1
Machine 4	1	0

TABLE 15: Final matrix $M \times C$.

	Cell 1	Cell 2
Machine 1	1	0
Machine 2	1	0
Machine 3	0	1
Machine 4	1	0

Generate Initial Solutions (Stage 2 in Figure 3). At this stage, we generate initial solutions according to the method described in Section 5.1. These solutions must satisfy the three constraints of model MPCFP.

Create Initial Flock (Stage 3 in Figure 3). From the initial solutions, we create an array of birds, and finally ordered

arrangement created the conceptual V-formation birds. For it, we assign the first position (first solution) in the array as the leader of the flock, and we perform an alternate distribution of birds between each side of the flock for the rest of the birds. We must consider that the birds are always sorted by fitness from low to high, the bird with lower fitness being the best solution to the problem (MPCFP is a minimization problem). Furthermore, we must consider that this is a stage in which we incorporate parallel order by quick sort and merge sort.

5.3.2. A Tour Process. This phase has two subphases internally: evolution of the leader bird and the evolution of the other birds. The evolution of the leader bird phase has one stage (generating neighboring solutions for the leader) and the evolution of the other birds phase has two stages (generating neighboring solutions for the other birds and sharing neighboring solutions that are not used by the leader with the other birds).

Generate Neighboring Solutions for the Leader (Stage 4 in Figure 3). This stage consists of generating neighboring solutions according to the steps outlined in Section 5.2. Subsequently, these neighboring solutions are sorted using an algorithm of parallel sort; see line (5) in the algorithm MBO (Algorithm 1).

Generate Neighboring Solutions for the Other Birds (Stage 5 in Figure 3). In this stage, we create $k - x$ neighboring solution to each bird of the flock; then these birds are sorted by an algorithm of parallel sort; see lines (7) and (8) in the algorithm MBO (Algorithm 1).

Share Neighboring Solutions That Are Not Used by the Leader with the Other Birds (Stage 6 in Figure 3). Share the other neighboring solutions of the leader with the other sides of the V-formation of birds; this is done alternately from left to right or from right to left. After sharing the neighboring solutions, we performed a sorting using an algorithm of parallel sort in all neighboring solutions that every bird in the flock has (not including the leader).

Leader Replacement (Stage 7 in Figure 3). At this stage, we performed a replacement of leader; for this we choose the nearest successor; that is, choose one of the birds closer to the leader. The successor will take its place and the leader will be available at the end of the flock. We must take into consideration that if we choose a successor belonging to the left section of the flock, the leader will pass to the end of the left section. The same procedure applies to the right section of the flock (see Algorithm 2).

Algorithm Finalization (Stage 8 in Figure 3). In this step, the fitness of the leader is obtained; in addition, a procedure to remove a response matrix explicitly is made; that is, we construct an incidence matrix $A[M][P]$ according to the order given by the matrices of solutions Y and Z . The order is according to the index of less than most of the machines and parts (see Algorithm 3).


```

(1) int type_of_sort = Get_Type_Of_Sort ()
(2) boolean left_or_right_side = Random_Boolean ()
(3) Main Function Replace_Leader_With_Successor () is
(4)   if left_or_right_side then
(5)     Change_Leader_With_Left_Birds ()
(6)     Sort_Left_Birds (type_of_sort)
(7)   else
(8)     Change_Leader_With_Right_Birds ()
(9)     Sort_Right_Birds (type_of_sort)
(10)  end
(11)  left_or_right_side = !left_or_right_side
(12) end

```

ALGORITHM 2: Replace leader with successor.

```

(1) Main Function Convert_to_final_matrix (int A[i][j], int Y[[]], int Z[[]]) is
(2)  int final_matrix[M][P], temp_matrix[M][P], ii = 0, jj = 0
(3)  /* Sort machines */
(4)  for int k = 0; k < C; k++ do
(5)    for int i = 0; i < M; i++ do
(6)      if Y[i][k] == 1 then
(7)        for int j = 0; j < P; j++ do
(8)          temp_matrix[ii][j] = A[i][j]
(9)        end
(10)       ii = ii + 1
(11)      end
(12)    end
(13)  end
(14)  /* Sort parts */
(15)  for int k = 0; k < C; k++ do
(16)    for int j = 0; j < P; j++ do
(17)      if Z[j][k] == 1 then
(18)        for int i = 0; i < M; i++ do
(19)          final_matrix[i][jj] = temp_matrix[i][j]
(20)        end
(21)        jj = jj + 1
(22)      end
(23)    end
(24)  end
(25)  return final_matrix
(26) end

```

ALGORITHM 3: Convert matrix $M \times C$ and $P \times C$ to final matrix.

5.3.3. Sorting Algorithm. For MBO, we use two sorting algorithms, merge sort and quick sort, in which we executed sequentially and in parallel using threads.

Merge Sort. It was developed in 1945 by John Von Neumann. It is a search algorithm of complexity $O(n \log n)$ (see Table 16). Algorithm 4 illustrates a general pseudocode for merge sort.

Quick Sort. It was developed in 1959 by Charles Richard Hoare. It is a search algorithm of complexity $O(n \log n)$ (see Table 16). Algorithm 5 illustrates a general pseudocode for quick sort.

6. Computational Experiments

6.1. Configuration Benchmarks. The MPCFP performance was evaluated experimentally using 90 test instances from Boctor's experiments, composed of 10 problems considering 5 values of $M_{\max} = 8, 9, 10, 11, 12$ with $C = 2$ and 10 instances considering 4 values of $M_{\max} = 6, 7, 8, 9$ with $C = 3$. The test data are available in [55]. The 90 instances have been executed 31 times using 12 different configurations of sequential and parallel sorting (see Table 18). We must also consider that the sorting algorithms (sequential and parallel) were applied in the following stages (see Figure 3) of MBO:

- (i) Stage 3 $\rightarrow$ create initial flock.
- (ii) Stage 4 $\rightarrow$ generate neighboring solutions for the leader.

```

(1) Main Function MergeSort (Object[] A) is
(2)   Object[] result, left, right
(3)   if length(A) ≤ 1 then
(4)     return A
(5)   else
(6)     int middle = length(A)/2
(7)     for i = 0 to middle do
(8)       left[i] = A[i]
(9)     end
(10)    for i = middle + 1 to length(A) do
(11)      right[i] = A[i]
(12)    end
(13)    left = MergeSort (left)
(14)    right = MergeSort (right)
(15)    result = Merge (left, right)
(16)    return result
(17)  end
(18) end

```

ALGORITHM 4: Merge sort.

```

(1) Main Function QuickSort (Object[] A, int low, int high) is
(2)   if low < high then
(3)     pivot_location = Partition (A, low, high)
(4)     QuickSort (A, low, pivot_location)
(5)     QuickSort (A, pivot_location + 1, high)
(6)   end
(7) end
(1) Function Partition (Object[] A, int low, int high) is
(2)   Object pivot = A[low]
(3)   int leftwall = low
(4)   for i = low + 1 to high do
(5)     if A[i] < pivot then
(6)       Swap (A[i], A[leftwall])
(7)       leftwall = leftwall + 1
(8)     end
(9)   end
(10)  Swap (A[low], A[leftwall])
(11)  return leftwall
(12) end

```

ALGORITHM 5: Quick sort.

(iii) Stage 5 → generate neighboring solutions for other birds.

(iv) Stage 6 → share neighboring solutions that are not used by the leader with the other birds.

The optimization algorithm using parallel sorting was coded in Java version 1.8.0_40 using Eclipse IDE for Java Developers Version Mars.1 Release (4.5.1). For running the MBO with parallel sorting, we use Java(TM) SE Runtime Environment (build 1.8.0_40-b27). The parameter values of MBO are described in Table 17. Finally, the hardware features that have run instances are MacBook Pro computer (Retina, 13-inch, late 2013) with an Intel Core i5 (processor speed = 2,4 GHz; number of processors = 1; total number of cores = 2; L2 cache per core = 256 KB; L3 cache = 3 MB), 4 GB RAM

1600 MHz DDR3, and Video Card Intel Iris 1536 MB running OS X El Capitan version 10.11.2 (15C50).

6.2. *Results.* Tables 19 and 20 represent a summary of the results provided in Appendix A [55]. These summary tables describe the average results of 90 instances. Column 1 (threads) corresponds to the identifier assigned to the number of threads used in the search algorithm. Column 2 (execution time-BET) shows the best execution time obtained in milliseconds. Column 3 (execution time-AVG) describes the average time in milliseconds of the finest time of execution of MBO. Column 4 (execution time-ST) represents the standard deviation of the best times of execution obtained by MBO. Column 5 (cycle-BC) shows the cycle in which the optimal value was found. Column 6 (cycle-AVG)

TABLE 16: Complexity sorting algorithm.

Name	Best	Average	Worst	Memory	Method	Stable
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	Depends	Merging	Yes
Quick sort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	$O(\log n)$	Partition	Depends

TABLE 17: Configuration parameters for MBO.

Name	Parameter	Value
Number of birds	n	5
Number of neighbors	k	3
Number of tours	m	10
Number of shared solutions	x	1
Number of iterations	I	2591
Leader exchange mode	—	Exchange the leader with the successor
Sort type applied in the initial birds flock	—	Birds ordered according to their fitness

TABLE 18: Configuration sorting algorithms.

Algorithm	Sequential	Threads				
		1	4	8	16	32
Merge sort	Yes	Yes	Yes	Yes	Yes	Yes
Quick sort	Yes	Yes	Yes	Yes	Yes	Yes

describes the average cycle. Column 7 (cycle-ST) represents the standard deviation. Column 8 (time cycle-BTC) shows the best cycle time in which the optimal value was found in milliseconds. Column 9 (time cycle-AVG) describes the average cycle in milliseconds. Column 10 (time cycle-ST) represents the standard deviation in milliseconds. Finally, we must consider all tests that have achieved the global optimum.

Execution Time Analysis. The execution time is the total time when the metaheuristic was searching in the search space for the best solution. In this sense, according to what is described in Tables 19 and 20, we can consider that the merge sort algorithm using 8 threads had the lowest average time (152.7333 ms) to run 90 instances (see column 2: execution time-BET in Tables 19 and 20). In addition, when we compare the times of running the metaheuristic sequentially (without threads and using merge sort algorithm), regarding the best time to run the metaheuristic reached with threads (8 threads), we can see that using 8 threads is an improvement in the execution time of the metaheuristic. This behavior can also be seen in Tables 19 and 20 (see column 3: execution time-AVG in Tables 19 and 20).

Cycle Analysis. A cycle is fulfilled after replacing the leader of the flock (stage 7 in Figure 3). Currently with a parameter of 2591 iterations (I), we can achieve 38 cycles. Consequently, the leader of the flock during the algorithm changes its value 38 times. Basically, the cycles, where the best value has been obtained before the execution is finished, provided us with information to reduce the iteration time assigned in the input parameter of the metaheuristic. Considering the values of the cycles (see column 6: cycle-AVG in Tables 19 and 20), we can approximate an average value in 9, for the number of cycles to

find a global optimum, for using both merge sort and quick sort, with threads and without threads. Also, this involves reducing the value of the iteration limit $I = 2591 \rightarrow I = 561$. But we must also consider the random factor having the metaheuristic (see stage 2 in Figure 3, stage 4 in Figure 3, and stage 5 in Figure 3), and this randomness allows the possibility of finding a global optimum with fewer cycles (see column 5: cycle-BC in Tables 19 and 20). Therefore, this is a recommended change, but it is not enough to always find a global optimum. The quick sort algorithm using threads having irregular times is even much worse than the quick sort used sequentially.

Time Cycle Analysis. In relation to the average cycle time where the best optimum was found, we note that, according to the tests, the merge sort algorithm with 8 threads has been the least time used (see column 9: time cycle-AVG in Tables 19 and 20). It should be considered interesting results as it has achieved better times using parallel sort versus sequential sort, mainly by using merge sort with 4, 8, and 16 threads and quick sort with 16 threads (see column 8: time cycle-BTC in Tables 19 and 20).

7. Conclusions

In this paper, we use a Migrating Birds Optimization algorithm by a parallel sorting to solve Machine-Part Cell Formation Problems. Computational experiments using conventional hardware resources have been conducted in 90 test instances, 2 well-known sorting algorithms, 1 sequential configuration, and 5 thread configurations (1, 4, 8, 16, and 32), giving a total of 1080 benchmarks. Among the main results,

TABLE 19: Result of merge sort.

Threads	Execution time (ms)				Cycle		Time cycle (ms)		
	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
Without threads	153.9667	171.1556	21.1539	3.1667	9.0778	4.7263	13.6778	43.3778	23.8747
1	155.5556	168.4111	13.9276	3.0222	9.0778	4.7812	13.2333	42.6556	22.9180
4	153.9333	170.9556	28.8802	2.8556	9.1556	4.9501	12.1444	43.7333	25.6019
8	152.7333	155.8667	3.8451	2.9667	9.2778	4.8785	12.2111	40.2333	20.2717
16	153.1333	161.8222	11.5292	2.9222	9.1222	4.9112	12.2444	40.9000	21.5416
32	153.9222	169.5000	21.8506	3.1889	9.2000	4.8937	13.5667	43.6333	24.7712
Minimum	152.7333	155.8667	3.8451	2.8556	9.0778	4.7263	12.1444	40.2333	20.2717
Maximum	155.5556	171.1556	28.8802	3.1889	9.2778	4.9501	13.6778	43.7333	25.6019

TABLE 20: Result of quick sort.

Threads	Execution time (ms)				Cycle		Time cycle (ms)		
	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
Without threads	153.3333	163.0000	12.7284	2.9889	9.0111	4.9168	12.5222	41.2222	22.1071
1	154.5778	167.9000	15.5926	3.0889	9.0778	4.8703	13.1778	42.2667	22.3487
4	153.3111	163.6778	12.2109	3.1333	9.2000	4.9019	13.3222	41.7444	22.4733
8	169.2889	195.6667	26.3893	2.8667	9.1667	4.8796	13.5667	49.6556	27.5041
16	154.3444	169.7778	18.9543	2.8111	8.9889	4.7619	12.1333	42.2444	23.6231
32	156.0889	176.7222	25.8717	2.9889	9.1111	4.8993	13.4111	45.5889	25.9852
Minimum	153.3111	163.0000	12.2109	2.8111	8.9889	4.7619	12.1333	41.2222	22.1071
Maximum	169.2889	195.6667	26.3893	3.1333	9.2000	4.9168	13.5667	49.6556	27.5041

we highlight a new alignment of the MBO parameters to solve MPCFP. The new configuration consists of the following parameter values: number of birds ($n \rightarrow 5$), number of neighbors ($k \rightarrow 3$), number of tours ($m \rightarrow 10$), number of shared solutions ($x \rightarrow 1$), and number of iterations ($I \rightarrow 2591$). We observed a decrease in cycles from 38 to 9, which means that the leader of the flock reaching the cycle number 9 has a very high probability of finding the global optimum; therefore, we can reduce the number of iteration limit from $I = 2591$ to $I = 561$. This observation of the number of cycles applied to all benchmarks.

In addition, it is important to note that quick sort algorithm has variable times in the execution of MBO (see column 9: time cycle-AVG in Table 20). There have been cases in which quick sort algorithm runs without threads; it is even more favorable than when run with threads. Otherwise, with the merge sort algorithm, we found promising results. We can consider that the merge sort algorithm using 8 threads shows a reduction in execution time compared to using sequential sorting algorithm.

Among the guidelines for future research, we can point to applying different ways of generating initial solutions as well as of generating neighboring solutions. We can also compare them with other sorting algorithms implemented in MBO.

Appendix

A. Details of the Benchmarks

A.1. Bactor's Problems. Table 21 describes the configuration of the 90 instances of tests. The description of the table is

given the following attributes: Column 1 (ID) corresponds to the identifier assigned to each instance. Column 2 (Bactor's problem) represents the identifier of the 10 Bactor's problems. Column 3 (cell) is the number of cells. Column 4 ($M_{\max}$) corresponds to the maximum number of machines per cell. Column 5 (OPT) depicts the optimum global for the given problem.

A.2. Experimental Results. Tables 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, and 33 contrast the times by using different sorting algorithms. These tables have the same headers, which are described below: column 1 (ID) corresponds to the identifier assigned to each instance. Column 2 (OPT) depicts the optimum value for the given problem. Column 3 (optimum-MBO) depicts the best value reached by using Migrating Birds Optimization. Column 4 (optimum-AVG) the average value of 31 executiones is depicted. Column 5 (optimum-SD) represents the standard deviation. Column 6 (optimum-RPD%) represents the difference between the best known optimum value and the best optimum value reached by MBO in terms of percentage. Column 7 (execution time-BET) shows the best execution time obtained in milliseconds. Column 8 (execution time-AVG) describes the average time in milliseconds of the finest time of execution of MBO. Column 9 (execution time-ST) represents the standard deviation of the best times of execution obtained by MBO. Column 10 (cycle-BC) shows the cycle in which the optimal value was found. Column 11 (cycle-AVG) describes the average cycle. Column 12 (cycle-ST) represents the standard deviation. Column 13 (time cycle-BTC) shows the best cycle

TABLE 21: Boctor's problems.

ID	Boctor's problem	Cell	$M_{\max}$	OPT
B01	1	2	8	11
B02	1	2	9	11
B03	1	2	10	11
B04	1	2	11	11
B05	1	2	12	11
B06	2	2	8	7
B07	2	2	9	6
B08	2	2	10	4
B09	2	2	11	3
B10	2	2	12	3
B11	3	2	8	4
B12	3	2	9	4
B13	3	2	10	4
B14	3	2	11	3
B15	3	2	12	1
B16	4	2	8	14
B17	4	2	9	13
B18	4	2	10	13
B19	4	2	11	13
B20	4	2	12	13
B21	5	2	8	9
B22	5	2	9	6
B23	5	2	10	6
B24	5	2	11	5
B25	5	2	12	4
B26	6	2	8	5
B27	6	2	9	3
B28	6	2	10	3
B29	6	2	11	3
B30	6	2	12	2
B31	7	2	8	7
B32	7	2	9	4
B33	7	2	10	4
B34	7	2	11	4
B35	7	2	12	4
B36	8	2	8	13
B37	8	2	9	10
B38	8	2	10	8
B39	8	2	11	5
B40	8	2	12	5
B41	9	2	8	8
B42	9	2	9	8
B43	9	2	10	8
B44	9	2	11	5
B45	9	2	12	5
B46	10	2	8	8
B47	10	2	9	5
B48	10	2	10	5
B49	10	2	11	5
B50	10	2	12	5
B51	1	3	6	27

TABLE 21: Continued.

ID	Boctor's problem	Cell	$M_{\max}$	OPT
B52	1	3	7	18
B53	1	3	8	11
B54	1	3	9	11
B55	2	3	6	7
B56	2	3	7	6
B57	2	3	8	6
B58	2	3	9	6
B59	3	3	6	9
B60	3	3	7	4
B61	3	3	8	4
B62	3	3	9	4
B63	4	3	6	27
B64	4	3	7	18
B65	4	3	8	14
B66	4	3	9	13
B67	5	3	6	11
B68	5	3	7	8
B69	5	3	8	8
B70	5	3	9	6
B71	6	3	6	6
B72	6	3	7	4
B73	6	3	8	4
B74	6	3	9	3
B75	7	3	6	11
B76	7	3	7	5
B77	7	3	8	5
B78	7	3	9	4
B79	8	3	6	14
B80	8	3	7	11
B81	8	3	8	11
B82	8	3	9	10
B83	9	3	6	12
B84	9	3	7	12
B85	9	3	8	8
B86	9	3	9	8
B87	10	3	6	10
B88	10	3	7	8
B89	10	3	8	8
B90	10	3	9	5

time in which the optimal value was found in milliseconds. Column 14 (time cycle-AVG) describes the average cycle in milliseconds. Column 15 (time cycle-ST) represents the standard deviation in milliseconds. Finally, it describes the average time for each column.

Competing Interests

The authors declare that they have no competing interests regarding the publication of this paper.

TABLE 22: MBO using sequential merge sort algorithm.

ID	OPT	Optimum				Execution time			Cycle			Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	158	251	171.6413	4	7	2.4350	17	53	34.2198
B2	11	11	11.0000	0.0000	0.00	157	263	84.9052	1	3	1.1575	4	31	23.2120
B3	11	11	11.0000	0.0000	0.00	153	209	47.3905	2	4	1.4984	9	24	10.2378
B4	11	11	11.0000	0.0000	0.00	155	160	4.7848	2	4	1.9449	8	20	8.5007
B5	11	11	11.0000	0.0000	0.00	155	160	4.4053	2	4	2.0723	8	20	9.2034
B6	7	7	7.0000	0.0000	0.00	165	172	5.9968	2	8	5.4244	10	40	25.3084
B7	6	6	6.0000	0.0000	0.00	158	165	10.2942	2	5	2.6567	9	24	10.9301
B8	4	4	4.1935	0.6011	0.00	154	226	63.7678	3	10	8.5001	13	59	55.3723
B9	3	3	3.1935	0.4016	0.00	152	157	3.4802	2	5	2.5686	8	24	10.5719
B10	3	3	3.0000	0.0000	0.00	152	183	41.7133	2	4	1.5422	8	21	10.5715
B11	4	4	4.1613	0.3739	0.00	155	233	66.3620	2	11	9.9109	9	73	63.6411
B12	4	4	4.0000	0.0000	0.00	153	235	86.2169	2	4	1.6643	11	29	18.5750
B13	4	4	4.0000	0.0000	0.00	152	161	30.0537	3	6	3.6577	12	27	14.8101
B14	3	3	3.3548	0.4864	0.00	152	159	26.0370	2	6	3.7006	8	26	15.6561
B15	1	1	1.6452	0.9504	0.00	152	155	2.6176	2	5	2.3505	8	22	9.6363
B16	14	14	14.0000	0.0000	0.00	155	182	48.3518	3	8	3.5493	12	43	25.2893
B17	13	13	13.0000	0.0000	0.00	152	174	35.1554	2	5	2.1717	8	26	9.4585
B18	13	13	13.0000	0.0000	0.00	152	177	41.1985	2	4	1.8275	8	21	8.6059
B19	13	13	13.0968	0.5388	0.00	153	218	62.5518	1	6	4.0377	4	39	26.7267
B20	13	13	13.0000	0.0000	0.00	155	225	138.3519	3	7	4.5423	13	41	21.4062
B21	9	9	9.1935	0.4016	0.00	153	172	43.6026	2	9	5.2990	8	41	26.7630
B22	6	6	6.1290	0.3408	0.00	152	155	2.0494	1	5	3.1326	4	23	12.9070
B23	6	6	6.1290	0.3408	0.00	152	153	2.0489	2	7	4.8491	8	31	19.7735
B24	5	5	5.2581	0.5755	0.00	152	153	1.6409	2	6	2.4012	8	24	9.6949
B25	4	4	4.1290	0.3408	0.00	152	154	6.0300	3	6	5.4158	12	27	22.1520
B26	5	5	5.0000	0.0000	0.00	154	157	2.3372	2	8	6.0393	8	37	24.7760
B27	3	3	3.0323	0.1796	0.00	152	154	2.2017	2	7	4.8524	8	30	19.3886
B28	3	3	3.2903	0.5884	0.00	152	155	4.7996	1	5	3.2670	4	24	13.9205
B29	3	3	3.0968	0.3005	0.00	152	156	7.3660	3	5	2.0447	12	23	8.2142
B30	2	2	2.4516	0.5680	0.00	152	153	1.7289	2	7	6.5031	8	28	26.3895
B31	7	7	7.0000	0.0000	0.00	153	157	2.7615	4	8	3.2156	19	37	13.2221
B32	4	4	4.2903	0.9016	0.00	152	155	4.1960	3	7	3.7805	13	31	15.0563
B33	4	4	4.1290	0.4995	0.00	152	154	3.7868	2	6	3.0154	8	26	12.7931
B34	4	4	4.3226	0.7478	0.00	152	153	1.2835	2	6	4.2925	8	27	17.1088
B35	4	4	4.7097	0.9727	0.00	152	153	1.6468	1	5	2.0723	4	21	8.5183
B36	13	13	13.0000	0.0000	0.00	154	158	3.8763	3	9	4.9110	13	39	19.3956
B37	10	10	10.3226	0.7478	0.00	152	155	2.5387	2	6	5.4497	9	28	22.2304
B38	8	8	8.0968	0.3005	0.00	152	154	3.3601	2	8	5.6323	8	34	22.8731
B39	5	5	5.1935	1.0776	0.00	152	166	20.9165	2	6	3.5493	8	27	15.7638
B40	5	5	5.0000	0.0000	0.00	152	162	16.8069	2	4	1.8357	8	19	7.9044
B41	8	8	8.0000	0.0000	0.00	155	158	2.9064	1	10	5.2974	4	42	22.2617
B42	8	8	8.0000	0.0000	0.00	152	198	70.2758	1	4	2.0718	8	24	17.2073
B43	8	8	8.0000	0.0000	0.00	152	154	1.7221	1	4	2.2818	4	19	9.3681
B44	5	5	5.9677	1.4256	0.00	152	154	3.8616	2	5	2.5264	8	24	10.1847
B45	5	5	6.1935	1.5366	0.00	152	153	1.3536	2	6	4.6467	8	26	19.1860
B46	8	8	8.0645	0.2497	0.00	154	158	3.4348	3	10	6.7563	12	42	27.9592
B47	5	5	5.0000	0.0000	0.00	152	155	4.1228	2	5	2.9256	8	22	13.0905
B48	5	5	5.4839	2.0145	0.00	152	153	1.3897	2	5	2.8931	8	22	11.7203
B49	5	5	5.1935	0.6011	0.00	152	152	1.1704	3	6	3.1819	12	26	12.7969
B50	5	5	5.3226	0.7478	0.00	152	153	1.4473	2	6	3.4529	8	26	14.0041

TABLE 22: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.1613	0.9694	0.00	154	159	4.7774	5	21	9.0993	21	92	39.5406
B52	18	18	18.9355	1.0307	0.00	154	156	2.2785	2	19	9.3341	8	80	38.7324
B53	11	11	11.0000	0.0000	0.00	153	155	1.5889	4	10	2.8815	16	42	12.1075
B54	11	11	11.0000	0.0000	0.00	152	154	1.6277	3	7	2.4596	12	32	10.2627
B55	7	7	7.0000	0.0000	0.00	154	158	4.4897	4	11	5.5967	16	48	22.6364
B56	6	6	6.0000	0.0000	0.00	153	155	1.9233	4	10	3.4112	16	44	13.9193
B57	6	6	6.0968	0.3962	0.00	153	154	1.3552	6	12	5.2644	24	52	21.4015
B58	6	6	6.0645	0.3592	0.00	152	155	4.0377	3	9	3.7643	14	38	14.9752
B59	9	9	9.1290	0.3408	0.00	154	158	2.6825	5	15	6.7165	21	64	28.5839
B60	4	4	4.3871	1.2021	0.00	152	156	2.3145	6	13	5.7136	24	56	23.8186
B61	4	4	4.1290	0.3408	0.00	152	155	2.0394	4	13	6.7101	16	54	27.4017
B62	4	4	4.0000	0.0000	0.00	152	155	4.1824	4	9	3.4273	18	40	13.6191
B63	27	27	27.2581	0.5143	0.00	155	165	11.2005	4	21	11.4243	16	90	48.4598
B64	18	18	18.3871	0.9549	0.00	155	167	14.2569	7	19	9.3371	28	86	39.7379
B65	14	14	14.0645	0.2497	0.00	153	161	9.3472	6	15	7.5537	25	65	31.4611
B66	13	13	13.0000	0.0000	0.00	154	247	97.6943	6	11	3.3020	24	81	67.8854
B67	11	11	11.0323	0.1796	0.00	156	209	33.3034	4	14	7.7241	19	77	43.7733
B68	8	8	8.8065	2.3154	0.00	156	179	30.7013	2	13	7.7239	9	63	40.9466
B69	8	8	8.5161	0.6256	0.00	154	190	35.2266	5	12	5.7914	20	64	35.7448
B70	6	6	6.4839	0.6256	0.00	152	167	16.6121	5	12	5.6220	22	57	26.9089
B71	6	6	6.0000	0.0000	0.00	155	176	15.3473	6	11	5.4370	29	54	24.4164
B72	4	4	4.2258	0.5603	0.00	155	168	21.0614	6	12	4.0783	28	55	23.1119
B73	4	4	4.4516	0.6752	0.00	152	173	15.2415	4	11	6.4611	16	52	28.1919
B74	3	3	3.3226	0.4752	0.00	153	166	14.9468	4	13	8.6221	20	60	35.3722
B75	11	11	11.6774	0.9794	0.00	156	169	8.4222	4	17	10.5949	17	77	47.6167
B76	5	5	5.2581	0.6816	0.00	154	167	23.7204	3	12	5.0513	14	56	26.0949
B77	5	5	5.3871	0.8032	0.00	165	174	8.3753	6	14	8.1313	27	66	37.9381
B78	4	4	4.9355	1.2093	0.00	163	176	12.2235	5	13	4.7399	22	60	25.3983
B79	14	14	14.0323	0.1796	0.00	156	181	28.4157	5	10	3.2965	25	47	16.9919
B80	11	11	11.0000	0.0000	0.00	171	190	20.4805	3	10	4.6704	17	53	23.3860
B81	11	11	11.3871	0.8032	0.00	159	186	24.6086	7	14	7.1159	34	70	35.5031
B82	10	10	10.3548	0.6607	0.00	153	169	25.0386	4	12	6.0875	16	55	29.4125
B83	12	12	12.2258	1.2572	0.00	154	167	8.4371	4	12	6.3860	18	57	29.6714
B84	12	12	12.0000	0.0000	0.00	155	165	13.7350	6	11	3.8852	31	52	17.8962
B85	8	8	8.9355	1.4361	0.00	153	170	23.4146	4	14	7.3518	17	64	33.0030
B86	8	8	8.1613	0.5829	0.00	156	177	15.4220	3	10	3.7814	13	47	18.0706
B87	10	10	10.4839	1.0605	0.00	156	184	20.5574	4	18	9.4481	19	89	50.8878
B88	8	8	8.0000	0.0000	0.00	155	194	109.8234	4	9	3.1995	20	61	96.8101
B89	8	8	8.0000	0.0000	0.00	153	159	8.7550	4	10	3.7275	17	42	15.4842
B90	5	5	5.2903	0.9016	0.00	152	156	2.2509	4	11	5.6435	17	49	22.9895
		AVG	8.1025	0.4576	0.00	153	171	21.1539	3	9	4.7263	13	43	23.8747

TABLE 23: MBO using parallel merge sort algorithm with 1 thread.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	159	206	78.6150	2	6	3.2038	14	43	35.4355
B2	11	11	11.0000	0.0000	0.00	155	173	19.0187	1	4	1.5141	6	21	8.1043
B3	11	11	11.0000	0.0000	0.00	154	172	13.6524	1	3	1.2495	4	16	5.5298
B4	11	11	11.0000	0.0000	0.00	153	172	16.4752	3	5	1.6552	12	24	10.3208
B5	11	11	11.0000	0.0000	0.00	152	155	5.0775	2	5	3.1064	8	24	12.5736
B6	7	7	7.0000	0.0000	0.00	155	185	36.0141	3	10	6.0544	13	48	27.2948
B7	6	6	6.0000	0.0000	0.00	159	202	24.7416	2	6	4.4252	9	35	21.5014
B8	4	4	4.4516	0.8500	0.00	158	197	88.5635	3	12	8.1669	17	58	40.5206
B9	3	3	3.0645	0.2497	0.00	153	172	12.9714	2	5	2.3122	12	23	9.5575
B10	3	3	3.0000	0.0000	0.00	152	168	13.5060	1	4	2.2609	7	20	10.8349
B11	4	4	4.2258	0.4250	0.00	156	186	18.4762	1	10	7.4314	5	53	41.0204
B12	4	4	4.0000	0.0000	0.00	160	178	17.7416	2	4	1.7585	8	21	10.0164
B13	4	4	4.0000	0.0000	0.00	152	171	19.3874	2	4	2.2714	8	21	10.1196
B14	3	3	3.2903	0.4614	0.00	152	169	15.2222	2	6	4.9287	8	28	22.3718
B15	1	1	1.0968	0.5388	0.00	153	165	11.9830	2	6	3.4694	8	29	16.5698
B16	14	14	14.0000	0.0000	0.00	155	163	9.5187	2	7	3.4128	8	33	14.8828
B17	13	13	13.0000	0.0000	0.00	153	155	2.8564	2	5	2.0225	8	21	8.8107
B18	13	13	13.0000	0.0000	0.00	152	155	2.6401	2	5	2.2264	8	24	9.3277
B19	13	13	13.3871	1.0223	0.00	152	155	3.7145	2	7	4.0236	8	30	16.7423
B20	13	13	13.0000	0.0000	0.00	152	154	2.2240	2	6	3.6308	8	28	14.7566
B21	9	9	9.1935	0.4016	0.00	154	161	4.3366	3	11	6.9041	12	47	29.4637
B22	6	6	6.1935	0.4016	0.00	153	156	2.2926	1	5	2.4203	4	23	10.0929
B23	6	6	6.0645	0.2497	0.00	152	155	4.3901	2	7	4.9174	8	31	19.7046
B24	5	5	5.3548	0.6607	0.00	152	155	2.7712	2	6	5.4696	8	28	22.2898
B25	4	4	4.3871	0.6672	0.00	152	156	3.3534	3	7	3.7411	12	29	15.0894
B26	5	5	5.0323	0.1796	0.00	155	162	5.0227	2	7	3.3376	9	34	13.9975
B27	3	3	3.0323	0.1796	0.00	154	157	4.5927	3	6	3.4619	12	27	14.2464
B28	3	3	3.0968	0.3005	0.00	152	156	3.8156	2	6	2.3714	8	25	10.0092
B29	3	3	3.3548	0.4864	0.00	152	155	3.1271	2	4	2.4913	8	19	10.2875
B30	2	2	2.5484	0.6752	0.00	154	157	3.3421	1	8	7.1106	6	33	29.8981
B31	7	7	7.0000	0.0000	0.00	158	166	5.7951	1	10	5.4402	5	44	23.6942
B32	4	4	4.1935	0.7492	0.00	153	159	3.6603	2	7	4.7390	8	31	20.0530
B33	4	4	4.0645	0.3592	0.00	153	158	3.9906	2	6	3.3872	8	25	13.7612
B34	4	4	4.2581	0.6816	0.00	152	158	3.7035	2	5	3.5636	8	25	14.7240
B35	4	4	4.4516	0.8500	0.00	153	158	4.3965	2	5	3.3916	8	22	14.3879
B36	13	13	13.0000	0.0000	0.00	156	166	6.3017	1	10	5.6101	5	44	24.9727
B37	10	10	10.3226	0.7478	0.00	154	157	2.4314	2	8	6.6553	8	34	27.1586
B38	8	8	8.1613	0.4544	0.00	152	156	4.2246	3	10	8.3402	12	44	34.0656
B39	5	5	5.0000	0.0000	0.00	153	159	11.6618	3	5	2.3363	12	23	9.8435
B40	5	5	5.1935	0.6011	0.00	152	160	6.3128	2	5	2.0645	9	25	8.9185
B41	8	8	8.1613	0.8980	0.00	154	169	10.6175	1	8	5.2096	4	36	22.4695
B42	8	8	8.0000	0.0000	0.00	153	166	10.1747	2	4	1.9438	8	18	8.0620
B43	8	8	8.0000	0.0000	0.00	152	165	10.7002	2	5	2.1568	12	24	9.2664
B44	5	5	5.7742	1.3344	0.00	153	161	8.5650	2	4	1.8062	8	21	7.8376
B45	5	5	5.9677	1.4256	0.00	152	159	8.0203	2	5	2.6453	8	24	10.7374
B46	8	8	8.0645	0.2497	0.00	155	172	12.5416	2	9	6.5325	8	44	31.3379
B47	5	5	5.0968	0.5388	0.00	153	162	6.1438	2	6	2.2423	8	26	9.7972
B48	5	5	5.1290	0.4995	0.00	152	157	3.8888	2	5	2.3646	8	23	9.8283
B49	5	5	5.2581	0.6816	0.00	153	166	11.7214	3	5	2.3335	14	25	9.2466
B50	5	5	5.3226	0.7478	0.00	158	297	112.2328	2	5	4.2598	8	46	59.0964

TABLE 23: Continued.

ID	OPT	Optimum				Execution time			BC	Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST		AVG	SD	BTC	AVG	SD
B51	27	27	28.1935	1.0139	0.00	170	210	50.5103	9	21	8.4341	40	120	65.5098
B52	18	18	19.0000	1.2383	0.00	160	174	12.9624	7	21	8.9031	29	96	39.5778
B53	11	11	11.0000	0.0000	0.00	161	178	16.1011	4	10	3.4251	17	49	15.4218
B54	11	11	11.0000	0.0000	0.00	164	225	110.0060	4	9	2.2274	17	53	25.5038
B55	7	7	7.0000	0.0000	0.00	170	227	95.8822	6	12	4.8697	35	73	34.6457
B56	6	6	6.0000	0.0000	0.00	158	176	22.7563	4	10	4.1773	18	46	17.2158
B57	6	6	6.0323	0.1796	0.00	158	164	3.8771	6	11	4.7317	29	49	20.2836
B58	6	6	6.0000	0.0000	0.00	157	167	7.3482	4	10	4.1721	17	47	17.6931
B59	9	9	9.1290	0.3408	0.00	162	176	9.1676	4	12	7.6146	16	60	35.3615
B60	4	4	4.2581	0.9989	0.00	160	167	5.9831	3	13	7.2062	14	62	31.0985
B61	4	4	4.1290	0.3408	0.00	160	165	3.8543	6	12	6.4014	25	53	28.3432
B62	4	4	4.0000	0.0000	0.00	158	164	4.8012	4	10	5.3501	18	43	23.0980
B63	27	27	27.2903	0.4614	0.00	167	175	6.0466	2	16	7.7237	9	76	36.5377
B64	18	18	18.3871	1.0223	0.00	160	168	5.1665	3	19	9.7662	15	86	43.3281
B65	14	14	14.0323	0.1796	0.00	160	165	3.8291	4	16	8.6324	17	71	37.8486
B66	13	13	13.0000	0.0000	0.00	158	164	4.3960	4	9	3.8069	17	42	17.3308
B67	11	11	11.0000	0.0000	0.00	161	169	5.7328	3	13	6.7766	15	60	30.8939
B68	8	8	8.5161	1.3873	0.00	157	166	4.9242	5	12	5.9210	20	55	25.9415
B69	8	8	8.7097	0.8244	0.00	159	186	80.6533	4	10	3.5186	20	59	78.6617
B70	6	6	6.3548	0.6082	0.00	157	164	4.1299	5	11	6.0651	21	49	26.3690
B71	6	6	6.3226	0.9087	0.00	164	173	4.9813	3	13	7.2318	15	64	34.0172
B72	4	4	4.2581	0.5143	0.00	162	169	7.0074	4	13	7.3266	18	61	31.5165
B73	4	4	4.3548	0.5507	0.00	155	163	7.4271	4	10	4.5181	16	46	19.8506
B74	3	3	3.3871	0.6152	0.00	154	159	6.2096	4	12	6.5673	18	54	27.7339
B75	11	11	11.8710	0.9217	0.00	156	166	6.2745	7	19	9.0094	32	84	39.6957
B76	5	5	5.1290	0.4995	0.00	154	159	4.0391	3	12	7.1859	12	53	30.5339
B77	5	5	5.6129	0.9892	0.00	154	158	2.9061	3	12	6.2763	12	52	26.0995
B78	4	4	4.7097	1.1887	0.00	152	157	4.4870	3	15	8.9251	12	64	37.0953
B79	14	14	14.0000	0.0000	0.00	154	160	3.9374	4	10	3.9070	17	46	16.9129
B80	11	11	11.0000	0.0000	0.00	154	159	3.6503	4	10	3.0409	16	43	12.6801
B81	11	11	11.2258	0.7169	0.00	154	157	2.5025	3	12	7.2826	13	51	29.8398
B82	10	10	10.7097	0.7829	0.00	153	160	9.2766	4	10	4.2187	19	45	17.2205
B83	12	12	12.1290	0.7184	0.00	154	161	5.0024	5	13	7.6357	23	59	32.3602
B84	12	12	12.0968	0.5388	0.00	154	159	3.8746	3	11	5.8103	12	49	25.2224
B85	8	8	9.3548	1.4731	0.00	153	157	3.7096	4	13	7.1319	19	57	29.5541
B86	8	8	8.0000	0.0000	0.00	152	157	4.4661	4	11	4.7817	16	46	19.8128
B87	10	10	10.3871	0.9193	0.00	156	165	5.2583	6	18	8.5468	25	81	38.3296
B88	8	8	8.0000	0.0000	0.00	154	159	6.6066	4	11	3.8846	16	47	16.1636
B89	8	8	8.0323	0.1796	0.00	153	158	4.5929	3	10	5.0891	12	42	21.2703
B90	5	5	5.7742	1.3344	0.00	153	157	2.6204	6	11	3.8401	24	46	15.4224
		AVG	8.1065	0.4446	0.00	155	168	13.9276	3	9	4.7812	13	42	22.9180

TABLE 24: MBO using parallel merge sort algorithm with 4 threads.

ID	OPT	Optimum				Execution time				Cycle			Time cycle	
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	158	216	109.1743	2	6	3.6142	9	42	25.9406
B2	11	11	11.0000	0.0000	0.00	152	157	6.6604	2	4	1.2365	10	18	7.4686
B3	11	11	11.0000	0.0000	0.00	152	166	47.4789	1	4	1.2960	4	18	8.4496
B4	11	11	11.0000	0.0000	0.00	152	155	2.5716	1	5	2.6331	4	20	10.8924
B5	11	11	11.3871	1.4984	0.00	153	167	45.7047	1	6	3.3085	4	29	18.6757
B6	7	7	7.0000	0.0000	0.00	154	162	9.6934	2	9	5.7726	8	40	24.3002
B7	6	6	6.0000	0.0000	0.00	154	167	41.5747	3	6	2.6845	12	26	10.8095
B8	4	4	4.3226	0.7478	0.00	153	155	2.5003	2	11	9.7807	8	49	39.5824
B9	3	3	3.0645	0.2497	0.00	152	168	58.7230	2	6	2.9494	8	27	17.1893
B10	3	3	3.0000	0.0000	0.00	152	166	41.0109	2	4	1.9355	8	20	9.1206
B11	4	4	4.1290	0.3408	0.00	155	160	4.3005	2	10	6.9986	8	43	29.6649
B12	4	4	4.0000	0.0000	0.00	153	166	41.1127	1	4	1.8044	4	19	7.2704
B13	4	4	4.0000	0.0000	0.00	152	155	2.5059	2	6	4.9324	8	25	20.2386
B14	3	3	3.1935	0.4016	0.00	152	167	46.3310	1	5	2.2269	4	26	14.4843
B15	1	1	1.3548	0.8386	0.00	152	154	2.6249	2	5	3.1356	8	24	12.8711
B16	14	14	14.0000	0.0000	0.00	154	171	46.8582	3	8	4.1048	12	36	19.3441
B17	13	13	13.0000	0.0000	0.00	154	157	4.7342	2	5	1.9903	8	21	8.5249
B18	13	13	13.0000	0.0000	0.00	152	166	40.2106	3	5	1.9210	12	26	11.5105
B19	13	13	13.1935	0.7492	0.00	152	157	9.6044	2	7	4.3160	8	29	17.8754
B20	13	13	13.0000	0.0000	0.00	152	165	54.4453	2	7	3.8504	8	32	17.1361
B21	9	9	9.2258	0.4250	0.00	154	173	54.5359	2	11	7.4947	10	51	32.7887
B22	6	6	6.0968	0.3005	0.00	154	160	5.8877	3	5	2.8044	12	24	11.6705
B23	6	6	6.0645	0.2497	0.00	152	169	48.5055	2	7	4.5711	8	35	21.3986
B24	5	5	5.2581	0.5755	0.00	152	154	2.3010	2	6	2.7875	8	25	11.3416
B25	4	4	4.1935	0.4016	0.00	152	167	46.4296	2	6	2.6644	8	27	23.6771
B26	5	5	5.0000	0.0000	0.00	154	158	3.7574	3	8	6.3478	15	37	26.7206
B27	3	3	3.0323	0.1796	0.00	154	168	47.1433	2	5	3.5903	9	26	19.3762
B28	3	3	3.3226	0.4752	0.00	152	158	19.6735	1	5	3.6966	4	23	15.3801
B29	3	3	3.1613	0.4544	0.00	152	163	41.5225	2	5	3.3315	8	25	35.7845
B30	2	2	2.4839	0.5699	0.00	152	165	44.6843	2	7	5.1800	8	30	22.1249
B31	7	7	7.0000	0.0000	0.00	153	161	6.9997	3	8	4.2651	12	37	18.4910
B32	4	4	4.2903	0.9016	0.00	155	205	147.1775	1	8	5.0298	5	40	29.8585
B33	4	4	4.0645	0.3592	0.00	153	172	25.2682	2	5	3.2974	8	25	13.9995
B34	4	4	4.3226	0.7478	0.00	152	155	3.4554	1	6	6.1399	4	28	24.5491
B35	4	4	4.3871	0.8032	0.00	152	157	7.2196	2	5	2.8979	8	23	11.7220
B36	13	13	13.0000	0.0000	0.00	158	172	15.0594	3	10	5.7384	12	46	26.7314
B37	10	10	10.4516	0.8500	0.00	154	166	14.1400	2	8	6.8311	8	38	28.5876
B38	8	8	8.0323	0.1796	0.00	155	164	12.4326	2	8	5.6068	8	37	23.8998
B39	5	5	5.3548	1.3796	0.00	154	163	12.5069	2	5	2.0884	8	22	9.4440
B40	5	5	5.2581	0.6816	0.00	153	165	16.5927	2	5	3.6106	8	24	15.9870
B41	8	8	8.1613	0.8980	0.00	156	166	9.9435	1	10	8.3483	4	47	36.1707
B42	8	8	8.0000	0.0000	0.00	155	174	34.2187	2	4	1.6350	8	19	8.1873
B43	8	8	8.0000	0.0000	0.00	153	198	46.1624	1	4	2.1423	4	24	13.0368
B44	5	5	5.7742	1.3344	0.00	156	183	32.3323	2	5	1.6919	10	25	9.0367
B45	5	5	5.8065	1.4005	0.00	152	157	4.5948	2	5	2.8658	9	24	12.1692
B46	8	8	8.0323	0.1796	0.00	157	176	28.1980	3	10	7.1360	14	50	40.1945
B47	5	5	5.1935	0.7492	0.00	155	183	36.2397	2	6	2.5649	8	31	16.0299
B48	5	5	5.1935	0.6011	0.00	152	166	23.6329	2	5	2.2832	8	24	10.1954
B49	5	5	5.2581	0.6816	0.00	152	155	2.8071	2	5	2.4613	8	22	10.1229
B50	5	5	5.1290	0.4995	0.00	152	155	4.6881	2	7	5.3336	8	30	25.5601

TABLE 24: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.2903	1.0064	0.00	159	164	4.3110	4	22	9.2894	16	98	40.3385
B52	18	18	19.0645	1.0935	0.00	154	159	5.8249	5	22	9.8280	22	95	42.6600
B53	11	11	11.0000	0.0000	0.00	153	178	32.7949	4	11	3.7102	19	51	18.7233
B54	11	11	11.0000	0.0000	0.00	153	164	19.9278	3	7	2.7140	12	34	14.0533
B55	7	7	7.0000	0.0000	0.00	155	160	3.4790	2	10	4.3510	8	46	18.3698
B56	6	6	6.0000	0.0000	0.00	154	156	1.7121	2	10	3.2772	8	42	13.7300
B57	6	6	6.0323	0.1796	0.00	153	160	16.4040	4	10	3.8788	17	45	16.0311
B58	6	6	6.0323	0.1796	0.00	154	168	26.7755	3	10	3.9119	12	45	16.9634
B59	9	9	9.2258	0.4973	0.00	155	194	38.6829	3	15	8.8444	13	76	44.3162
B60	4	4	4.9032	1.7001	0.00	154	169	24.2931	3	12	6.6625	13	54	30.0641
B61	4	4	4.1290	0.5623	0.00	155	268	200.8557	4	12	7.0176	16	109	179.0541
B62	4	4	4.0000	0.0000	0.00	153	157	4.2149	3	9	3.8691	13	40	15.6801
B63	27	27	27.2581	0.5143	0.00	154	165	10.6263	6	16	9.2118	24	71	40.8554
B64	18	18	18.4516	0.8099	0.00	154	178	18.2276	7	19	9.1876	31	91	45.9490
B65	14	14	14.0323	0.1796	0.00	155	161	7.8149	6	15	7.3228	25	68	30.9394
B66	13	13	13.0000	0.0000	0.00	155	194	41.5668	6	10	3.4251	24	52	21.8884
B67	11	11	11.0323	0.1796	0.00	158	210	67.1852	1	13	6.8971	4	72	42.9510
B68	8	8	8.3226	1.2751	0.00	153	197	51.5259	4	13	7.9708	16	68	52.5572
B69	8	8	8.5806	0.6720	0.00	155	208	84.9156	5	11	5.6323	21	60	31.5805
B70	6	6	6.5484	0.8884	0.00	154	231	66.9705	4	15	8.5271	29	91	56.9068
B71	6	6	6.0645	0.3592	0.00	157	217	70.6903	3	12	6.9808	14	71	46.9560
B72	4	4	4.2258	0.4250	0.00	161	189	40.5461	5	13	7.1670	24	68	41.0736
B73	4	4	4.4516	0.6239	0.00	156	177	29.4202	4	13	5.3836	20	59	23.6061
B74	3	3	3.3871	0.5584	0.00	154	161	4.3200	3	13	7.5998	12	55	32.4734
B75	11	11	11.7742	1.0866	0.00	155	172	13.9606	4	15	9.4963	16	69	45.7583
B76	5	5	5.1935	0.6011	0.00	154	159	4.2812	4	13	6.7199	16	55	28.2946
B77	5	5	5.9677	1.1968	0.00	154	157	3.0589	4	12	7.3398	16	54	30.8010
B78	4	4	4.6452	1.1120	0.00	153	156	2.6757	1	13	7.3425	4	56	30.7701
B79	14	14	14.0000	0.0000	0.00	154	161	5.2525	4	10	2.9487	18	43	12.1078
B80	11	11	11.0000	0.0000	0.00	155	202	147.0668	3	11	5.4449	16	50	26.2353
B81	11	11	11.4516	0.9605	0.00	155	208	59.8935	7	13	4.7648	28	74	32.8557
B82	10	10	10.6452	0.7978	0.00	155	190	37.9565	4	13	7.3710	20	68	41.9503
B83	12	12	12.0000	0.0000	0.00	155	162	6.1394	3	10	5.8394	12	48	28.0464
B84	12	12	12.1935	0.6011	0.00	154	158	2.8090	7	12	6.3531	28	53	25.9466
B85	8	8	9.1613	1.7146	0.00	154	156	1.8346	4	13	6.5165	16	57	27.0130
B86	8	8	8.0000	0.0000	0.00	153	157	3.0107	3	11	3.7543	14	47	15.8781
B87	10	10	10.8065	1.0462	0.00	156	168	14.7487	6	17	9.3396	24	78	43.8068
B88	8	8	8.1935	1.0776	0.00	154	157	2.8132	4	10	5.2145	16	44	21.6391
B89	8	8	8.0000	0.0000	0.00	154	157	2.5607	3	10	4.7956	13	44	20.3743
B90	5	5	5.3871	1.0223	0.00	154	156	2.1397	4	11	4.6769	16	46	19.3850
		AVG	8.1183	0.4847	0.00	153	170	28.8802	2	9	4.9501	12	43	25.6019

TABLE 25: MBO using parallel merge sort algorithm with 8 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	153	175	52.9600	2	7	3.4650	10	40	24.1691
B2	11	11	11.0000	0.0000	0.00	152	155	4.2538	1	4	1.3663	4	17	6.3123
B3	11	11	11.0000	0.0000	0.00	152	154	2.2737	2	4	1.2675	8	16	4.9966
B4	11	11	11.0000	0.0000	0.00	152	154	2.1875	3	5	1.6669	12	22	6.6451
B5	11	11	11.0000	0.0000	0.00	152	154	3.5484	1	5	4.6460	4	22	19.1976
B6	7	7	7.0000	0.0000	0.00	154	159	6.9589	3	9	5.6927	12	41	23.3150
B7	6	6	6.0323	0.1796	0.00	152	155	2.6567	2	5	3.3376	8	24	13.6479
B8	4	4	4.2581	0.6816	0.00	152	154	2.2500	2	10	7.8339	8	43	31.7022
B9	3	3	3.0645	0.2497	0.00	152	154	2.8853	1	6	4.2480	4	26	17.5545
B10	3	3	3.0000	0.0000	0.00	152	155	4.3342	2	4	2.2017	8	19	9.0625
B11	4	4	4.1613	0.3739	0.00	154	158	4.0725	3	11	6.8310	12	46	28.0720
B12	4	4	4.0000	0.0000	0.00	152	154	2.2041	1	4	1.7487	4	18	7.3882
B13	4	4	4.0645	0.3592	0.00	152	157	8.3824	2	6	4.0825	8	24	16.4007
B14	3	3	3.4516	0.5059	0.00	152	156	10.8955	2	5	2.0848	8	21	8.3643
B15	1	1	1.2903	0.7829	0.00	152	153	1.8322	2	5	2.4110	8	23	9.8292
B16	14	14	14.0000	0.0000	0.00	155	160	4.5349	2	7	3.6044	10	32	14.9542
B17	13	13	13.0000	0.0000	0.00	152	155	2.2423	2	4	1.6145	8	20	6.7850
B18	13	13	13.0000	0.0000	0.00	152	154	3.8763	1	5	2.1553	4	22	9.3721
B19	13	13	13.0000	0.0000	0.00	152	153	1.4796	2	7	6.3034	8	28	25.4949
B20	13	13	13.0000	0.0000	0.00	152	153	1.4803	2	8	5.1278	8	33	20.5847
B21	9	9	9.1613	0.3739	0.00	154	158	2.8485	2	12	10.3146	8	52	42.6586
B22	6	6	6.2581	0.6308	0.00	152	155	3.1380	2	6	2.8459	8	25	11.7755
B23	6	6	6.1935	0.5428	0.00	152	153	2.0198	2	10	7.3810	8	43	30.1207
B24	5	5	5.2581	0.5755	0.00	152	154	1.9355	1	7	3.5024	4	28	14.1402
B25	4	4	4.2581	0.5143	0.00	152	153	2.0562	2	6	3.2132	8	26	12.9135
B26	5	5	5.0000	0.0000	0.00	154	158	4.3165	2	9	6.1518	8	39	26.6302
B27	3	3	3.0645	0.2497	0.00	152	155	2.3085	2	7	3.3033	8	30	13.5624
B28	3	3	3.2581	0.5755	0.00	152	153	1.9680	2	5	3.0261	8	24	13.0409
B29	3	3	3.2258	0.4250	0.00	152	154	2.4705	2	4	1.8228	8	18	7.6122
B30	2	2	2.5484	0.6239	0.00	152	154	3.7018	2	7	5.7358	8	28	23.3637
B31	7	7	7.0000	0.0000	0.00	154	158	3.0953	2	10	6.2340	8	42	25.8763
B32	4	4	4.3871	1.2826	0.00	152	155	2.7875	2	8	5.7081	8	36	23.1095
B33	4	4	4.1935	0.6011	0.00	152	154	3.7125	2	5	3.5753	8	21	15.1291
B34	4	4	4.1290	0.4995	0.00	152	153	1.5268	2	5	4.2205	8	23	16.8363
B35	4	4	4.6452	0.9504	0.00	152	152	1.0984	1	5	2.2177	4	21	8.8371
B36	13	13	13.0000	0.0000	0.00	154	158	3.6041	4	9	4.6647	16	39	19.2969
B37	10	10	10.3871	0.8032	0.00	152	155	4.3569	1	6	2.7689	4	24	11.4273
B38	8	8	8.2258	0.4973	0.00	152	154	2.2026	2	9	7.5453	8	40	30.0411
B39	5	5	5.0000	0.0000	0.00	152	154	2.2327	2	5	2.9759	9	22	11.9398
B40	5	5	5.1935	0.6011	0.00	152	153	1.9989	2	5	2.7552	8	22	11.5980
B41	8	8	8.0000	0.0000	0.00	154	159	4.8193	3	10	5.9482	14	45	24.6265
B42	8	8	8.0000	0.0000	0.00	152	154	2.3692	2	4	2.3592	8	20	9.9095
B43	8	8	8.0645	0.3592	0.00	152	154	1.8526	1	5	2.9563	4	21	12.2532
B44	5	5	6.3548	1.5176	0.00	152	153	1.1538	2	5	2.9494	8	24	11.9644
B45	5	5	6.1613	1.4854	0.00	152	153	1.5443	2	5	2.6917	8	23	10.8397
B46	8	8	8.0323	0.1796	0.00	155	159	4.2805	2	9	6.9125	8	41	28.3549
B47	5	5	5.0968	0.5388	0.00	152	154	2.0294	2	5	2.0811	8	21	8.4729
B48	5	5	5.1935	0.6011	0.00	152	154	1.6428	2	4	2.2647	8	20	9.1640
B49	5	5	5.2581	0.6816	0.00	151	153	3.5342	2	5	2.6482	8	21	10.6357
B50	5	5	5.1290	0.4995	0.00	152	153	3.1826	2	5	2.8493	8	22	11.5451

TABLE 25: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	27.9355	0.8139	0.00	154	161	3.4660	10	26	8.4370	41	111	35.6271
B52	18	18	18.9355	1.0626	0.00	154	157	3.3758	8	24	9.0310	32	102	36.6151
B53	11	11	11.0000	0.0000	0.00	153	155	1.8334	6	11	4.6911	24	45	19.0585
B54	11	11	11.0000	0.0000	0.00	152	154	3.7637	3	8	2.6941	12	35	11.1208
B55	7	7	7.0000	0.0000	0.00	154	158	3.2984	3	11	5.1034	13	49	21.0208
B56	6	6	6.0000	0.0000	0.00	153	156	3.0093	4	11	3.8401	16	48	15.2743
B57	6	6	6.1290	0.4275	0.00	153	155	2.8387	4	11	5.4081	16	46	22.2407
B58	6	6	6.0645	0.2497	0.00	152	155	4.2566	3	10	3.8702	12	43	15.8509
B59	9	9	9.0323	0.1796	0.00	155	158	3.0526	5	14	6.9780	22	60	29.3752
B60	4	4	4.3871	1.2021	0.00	153	156	2.1687	5	14	7.0227	21	59	29.2113
B61	4	4	4.1613	0.4544	0.00	153	156	4.8604	3	12	6.3903	12	52	27.7761
B62	4	4	4.0323	0.1796	0.00	152	154	2.3678	4	10	4.8644	17	45	20.3279
B63	27	27	27.1935	0.4016	0.00	156	162	4.4719	4	15	7.7450	16	66	32.7914
B64	18	18	18.0968	0.3005	0.00	154	157	2.4722	5	19	8.3824	23	81	35.1227
B65	14	14	14.0323	0.1796	0.00	153	156	4.1228	6	15	8.5828	24	65	35.2348
B66	13	13	13.0000	0.0000	0.00	152	155	2.5179	7	9	2.7072	28	41	11.3806
B67	11	11	11.1290	0.3408	0.00	154	159	4.6810	4	12	6.4386	17	52	27.4067
B68	8	8	8.2258	0.9205	0.00	154	157	3.4830	3	14	9.5942	12	61	40.0795
B69	8	8	8.6129	0.7154	0.00	153	156	4.5924	4	11	5.9648	16	48	24.0260
B70	6	6	6.3548	0.7549	0.00	152	159	9.1831	3	14	6.6604	12	60	29.2535
B71	6	6	6.0000	0.0000	0.00	154	160	5.0885	6	14	6.2436	28	60	26.1248
B72	4	4	4.2581	0.5143	0.00	153	156	2.0956	3	11	5.6317	13	49	23.2173
B73	4	4	4.3548	0.6607	0.00	152	155	4.3015	3	12	7.6978	12	51	31.3144
B74	3	3	3.4839	0.5699	0.00	152	154	2.1038	3	11	6.2472	12	45	25.2912
B75	11	11	11.7419	0.9298	0.00	154	160	4.5345	3	17	8.7525	13	72	37.3021
B76	5	5	5.0645	0.3592	0.00	153	156	2.3192	4	12	6.1097	19	53	25.0673
B77	5	5	5.5161	0.8896	0.00	152	155	4.3891	3	12	7.1977	12	52	29.2321
B78	4	4	4.6774	1.1072	0.00	152	155	2.3908	4	15	6.2862	17	62	25.6253
B79	14	14	14.0000	0.0000	0.00	155	159	3.0324	6	10	4.4923	24	43	18.5246
B80	11	11	11.0000	0.0000	0.00	153	157	3.3665	4	11	3.6697	16	46	15.4853
B81	11	11	11.3871	0.8032	0.00	152	156	4.2659	3	14	8.3982	12	61	34.5279
B82	10	10	10.6774	0.7911	0.00	153	155	1.8613	5	12	5.4156	21	53	22.2498
B83	12	12	12.1935	0.7492	0.00	154	160	4.3436	2	11	5.9686	8	50	25.0351
B84	12	12	12.1290	0.4995	0.00	153	156	2.6567	6	13	6.1890	25	55	25.4035
B85	8	8	9.5806	1.8934	0.00	153	156	3.8652	3	13	6.6922	13	55	27.3698
B86	8	8	8.0000	0.0000	0.00	152	155	2.2504	4	10	4.1195	16	43	17.4584
B87	10	10	10.4839	0.9263	0.00	155	159	3.6559	4	18	8.6221	16	79	36.8990
B88	8	8	8.0000	0.0000	0.00	154	156	2.8530	4	10	3.8077	16	41	15.5335
B89	8	8	8.0000	0.0000	0.00	152	156	4.3232	3	9	3.6948	12	39	15.6479
B90	5	5	5.3871	1.0223	0.00	152	155	3.4501	3	11	4.1108	13	45	16.8234
		AVG	8.1029	0.4293	0.00	152	155	3.8451	2	9	4.8785	12	40	20.2717

TABLE 26: MBO using parallel merge sort algorithm with 16 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	156	174	38.3904	1	6	3.8370	4	33	26.3519
B2	11	11	11.0000	0.0000	0.00	152	170	37.5675	2	4	1.4338	8	21	9.5525
B3	11	11	11.0000	0.0000	0.00	152	153	1.4421	2	4	1.3384	8	18	5.5949
B4	11	11	11.0000	0.0000	0.00	152	184	126.1403	3	5	1.9620	12	25	9.3102
B5	11	11	11.0000	0.0000	0.00	153	176	53.4639	1	5	2.5716	4	25	14.3306
B6	7	7	7.0000	0.0000	0.00	154	170	28.7805	3	10	4.5744	12	47	22.7679
B7	6	6	6.0323	0.1796	0.00	155	178	33.1697	3	5	3.8685	12	28	17.8745
B8	4	4	4.3871	0.8032	0.00	152	177	28.8102	2	12	10.4629	9	57	49.2948
B9	3	3	3.1290	0.3408	0.00	154	177	40.8947	2	6	6.0373	10	33	28.3257
B10	3	3	3.0000	0.0000	0.00	154	200	42.9216	3	4	1.3125	12	24	8.8010
B11	4	4	4.2258	0.4250	0.00	154	208	57.8835	2	11	7.2915	9	61	47.9815
B12	4	4	4.0000	0.0000	0.00	154	161	7.1918	2	3	1.6271	8	16	7.6983
B13	4	4	4.0000	0.0000	0.00	156	241	69.3555	2	5	2.0452	11	34	24.6320
B14	3	3	3.1935	0.4016	0.00	153	180	41.3214	2	5	2.6700	10	28	12.9253
B15	1	1	1.4516	0.9605	0.00	152	167	20.7708	3	6	2.0992	12	26	11.3581
B16	14	14	14.0000	0.0000	0.00	155	193	49.9363	1	8	5.0855	5	47	33.2326
B17	13	13	13.0000	0.0000	0.00	153	179	33.2854	2	4	1.7604	8	23	9.0254
B18	13	13	13.0000	0.0000	0.00	154	176	28.2143	2	5	2.2442	8	26	12.5006
B19	13	13	13.1935	0.7492	0.00	153	205	61.2246	2	6	4.9091	8	38	28.1094
B20	13	13	13.0000	0.0000	0.00	152	155	4.7306	2	8	5.9932	10	35	24.7893
B21	9	9	9.3226	0.4752	0.00	155	167	12.4572	2	8	7.2314	12	39	31.2266
B22	6	6	6.1613	0.3739	0.00	152	156	3.6509	3	7	3.6945	12	29	15.2464
B23	6	6	6.0645	0.2497	0.00	152	155	4.2187	2	8	5.2534	8	34	21.9115
B24	5	5	5.4516	0.7229	0.00	152	153	1.5651	1	5	2.7048	4	24	11.3314
B25	4	4	4.3871	0.7154	0.00	152	153	1.5317	2	6	2.5012	8	26	10.3027
B26	5	5	5.0000	0.0000	0.00	154	157	3.2883	1	8	5.9864	4	36	24.8583
B27	3	3	3.0645	0.2497	0.00	152	156	3.6957	1	5	2.6453	4	23	11.2241
B28	3	3	3.3548	0.5507	0.00	152	154	1.6415	1	6	2.6506	4	25	11.1220
B29	3	3	3.2903	0.5287	0.00	152	153	1.8228	2	5	2.5239	8	21	10.6774
B30	2	2	2.4839	0.6768	0.00	152	154	3.7411	2	9	7.4944	10	36	29.9656
B31	7	7	7.0000	0.0000	0.00	154	158	3.1174	3	9	3.2322	13	39	14.2112
B32	4	4	4.0000	0.0000	0.00	152	155	4.6167	2	7	4.6467	8	31	19.3095
B33	4	4	4.1290	0.4995	0.00	152	154	2.5653	2	4	2.4947	8	20	10.4209
B34	4	4	4.1290	0.4995	0.00	152	154	3.5070	2	8	6.2436	8	36	25.3171
B35	4	4	4.6452	0.9504	0.00	152	153	1.8866	1	5	3.1398	4	22	13.0124
B36	13	13	13.0000	0.0000	0.00	154	157	3.0980	2	7	4.4587	8	32	19.6268
B37	10	10	10.3226	0.7478	0.00	153	156	2.3582	2	6	3.8032	8	28	15.7390
B38	8	8	8.0645	0.2497	0.00	152	154	3.6121	2	11	9.1926	9	47	36.9595
B39	5	5	5.1613	0.8980	0.00	152	153	1.7858	3	5	1.9126	12	22	7.5316
B40	5	5	5.1290	0.4995	0.00	152	153	1.5568	2	6	3.7757	8	26	15.1195
B41	8	8	8.0000	0.0000	0.00	154	158	4.4959	3	8	4.4325	12	37	19.3518
B42	8	8	8.0000	0.0000	0.00	152	154	1.8416	2	4	2.9146	8	19	11.7187
B43	8	8	8.0000	0.0000	0.00	152	153	1.9527	2	4	2.2500	8	20	9.6155
B44	5	5	6.0645	1.4591	0.00	152	153	1.9757	2	5	2.3859	8	23	9.7173
B45	5	5	6.1935	1.5366	0.00	152	154	3.6550	3	7	4.0113	12	29	16.0929
B46	8	8	8.0323	0.1796	0.00	155	158	3.2614	3	8	4.9381	13	35	20.3281
B47	5	5	5.0000	0.0000	0.00	152	155	1.8769	2	5	1.8590	8	22	7.6797
B48	5	5	5.0645	0.3592	0.00	152	154	2.0925	2	5	2.5012	8	22	10.5511
B49	5	5	5.3226	1.1658	0.00	152	155	3.6409	2	6	3.4125	8	26	14.2755
B50	5	5	5.3226	0.7478	0.00	152	157	3.3205	3	6	4.1309	12	28	16.7865

TABLE 26: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.0323	0.7521	0.00	158	168	4.9579	5	18	10.2663	21	82	45.4441
B52	18	18	18.9032	1.1062	0.00	155	163	4.9174	3	20	9.9102	12	87	41.3697
B53	11	11	11.0000	0.0000	0.00	155	159	3.0438	3	11	5.6777	13	48	23.7862
B54	11	11	11.0000	0.0000	0.00	152	158	3.8968	3	8	2.6384	12	37	11.8709
B55	7	7	7.0000	0.0000	0.00	155	164	4.9862	4	11	5.2009	16	51	23.2484
B56	6	6	6.0000	0.0000	0.00	154	160	5.1881	3	10	3.0761	14	42	12.8828
B57	6	6	6.0000	0.0000	0.00	153	159	7.0420	4	15	8.2207	16	64	35.1398
B58	6	6	6.0000	0.0000	0.00	152	159	8.3060	6	10	3.8362	25	46	17.9194
B59	9	9	9.0645	0.2497	0.00	154	158	4.4036	3	13	6.4481	13	57	25.8823
B60	4	4	4.6452	1.7426	0.00	154	157	3.1356	3	11	6.5308	12	47	26.7605
B61	4	4	4.0645	0.2497	0.00	153	155	2.7337	4	10	4.5938	18	43	18.8219
B62	4	4	4.0000	0.0000	0.00	152	154	2.3592	3	10	4.4376	12	42	18.0429
B63	27	27	27.2258	0.4973	0.00	154	159	4.5781	3	16	8.9346	14	71	38.5607
B64	18	18	18.1613	0.3739	0.00	154	157	2.6036	2	21	10.4142	9	89	43.2948
B65	14	14	14.0645	0.2497	0.00	153	156	2.2977	9	16	7.2749	37	68	30.2108
B66	13	13	13.0000	0.0000	0.00	153	155	4.6939	4	9	4.0798	16	39	16.7346
B67	11	11	11.0323	0.1796	0.00	154	158	3.7491	4	13	6.3256	20	57	27.2279
B68	8	8	8.2581	1.2641	0.00	154	156	2.0780	4	14	8.1412	17	61	33.2352
B69	8	8	8.5161	0.5699	0.00	153	155	1.9806	3	11	6.3845	12	48	25.9051
B70	6	6	6.4839	0.8112	0.00	152	155	4.0201	6	14	7.0459	26	60	28.5073
B71	6	6	6.0968	0.3962	0.00	154	157	2.9403	4	12	6.6669	17	51	26.8024
B72	4	4	4.2258	0.4973	0.00	153	156	2.5425	4	12	4.9425	17	50	19.7463
B73	4	4	4.6129	0.8032	0.00	153	156	4.3359	4	12	7.1703	16	52	29.2444
B74	3	3	3.5806	0.7648	0.00	152	154	2.1597	3	12	6.5769	12	51	26.8259
B75	11	11	11.7097	1.0064	0.00	155	158	2.7669	4	16	8.1914	16	67	34.7627
B76	5	5	5.2581	0.6816	0.00	154	156	2.1938	4	15	8.8187	16	64	36.6178
B77	5	5	5.3226	0.7478	0.00	152	155	3.8935	3	12	5.2079	12	51	21.4117
B78	4	4	4.5806	1.4323	0.00	152	155	2.1292	5	16	7.5307	21	67	29.9967
B79	14	14	14.0000	0.0000	0.00	155	159	3.9738	4	10	4.1530	17	43	16.7283
B80	11	11	11.0000	0.0000	0.00	154	156	1.8293	6	11	4.1832	24	45	17.7170
B81	11	11	11.4516	0.9605	0.00	152	155	3.8187	4	12	6.0628	17	49	24.8217
B82	10	10	10.5484	0.8500	0.00	153	155	2.7044	3	11	6.4430	12	47	26.5292
B83	12	12	12.0000	0.0000	0.00	155	158	2.8105	6	10	3.6928	24	44	15.9765
B84	12	12	12.0000	0.0000	0.00	153	157	5.0225	2	10	4.6216	9	44	18.9497
B85	8	8	9.4839	1.6304	0.00	152	155	1.9587	3	13	8.1050	12	57	33.4782
B86	8	8	8.1290	0.5623	0.00	152	154	1.6611	3	11	4.6358	13	45	18.8382
B87	10	10	10.4194	0.8860	0.00	154	158	2.7913	4	18	9.6760	17	76	41.0048
B88	8	8	8.0000	0.0000	0.00	154	156	3.9358	5	10	3.5021	21	44	14.3271
B89	8	8	8.0323	0.1796	0.00	152	155	2.0294	4	11	5.5903	17	47	22.9967
B90	5	5	5.3871	1.0223	0.00	152	154	1.8334	5	11	5.2521	20	46	21.4332
		AVG	8.1007	0.4296	0.00	153	161	11.5292	2	9	4.9112	12	40	21.5416

TABLE 27: MBO using parallel merge sort algorithm with 32 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	156	202	73.4610	3	7	2.8048	13	41	25.2275
B2	11	11	11.0000	0.0000	0.00	153	159	8.6862	3	4	1.4473	12	22	9.7667
B3	11	11	11.0000	0.0000	0.00	152	155	3.1575	1	4	1.6481	4	17	7.1419
B4	11	11	11.0000	0.0000	0.00	153	159	12.7325	3	5	2.2756	12	23	10.9487
B5	11	11	11.0000	0.0000	0.00	153	159	7.8248	2	5	2.0583	8	24	8.4883
B6	7	7	7.0000	0.0000	0.00	154	168	11.3770	2	10	6.2576	10	44	28.5622
B7	6	6	6.0645	0.2497	0.00	153	162	10.6669	3	6	2.3623	13	29	12.9453
B8	4	4	4.1935	0.6011	0.00	152	157	4.6720	2	12	9.9450	8	51	41.2599
B9	3	3	3.0968	0.3005	0.00	153	157	5.1856	1	5	2.5098	4	25	10.4043
B10	3	3	3.0000	0.0000	0.00	152	159	8.1494	1	3	1.7001	4	16	7.8501
B11	4	4	4.2581	0.4448	0.00	153	164	11.3640	3	12	7.4554	14	52	32.6614
B12	4	4	4.0000	0.0000	0.00	152	157	5.4880	1	4	1.8026	4	17	7.6960
B13	4	4	4.0645	0.3592	0.00	152	159	9.9945	2	5	2.4679	9	22	10.6621
B14	3	3	3.2581	0.4448	0.00	152	157	7.2963	2	5	2.6632	8	23	11.1257
B15	1	1	1.5161	0.8896	0.00	152	162	14.1797	2	6	2.7731	8	25	12.0180
B16	14	14	14.0000	0.0000	0.00	155	178	28.1446	3	6	2.8459	12	32	12.6052
B17	13	13	13.0000	0.0000	0.00	152	163	9.4999	1	4	1.6669	4	19	7.8038
B18	13	13	13.0000	0.0000	0.00	153	176	50.4821	1	5	2.3431	5	22	12.0773
B19	13	13	13.1935	0.7492	0.00	153	167	23.5020	2	7	4.6575	12	34	25.0622
B20	13	13	13.0000	0.0000	0.00	152	196	65.9215	2	7	4.9246	9	39	27.7927
B21	9	9	9.3548	0.4864	0.00	154	197	89.8626	2	11	8.3362	8	56	40.0088
B22	6	6	6.1613	0.5829	0.00	155	213	63.0063	2	5	2.0161	8	32	18.9462
B23	6	6	6.1935	0.4016	0.00	155	241	100.2556	3	7	5.8505	12	48	39.5844
B24	5	5	5.2903	0.5884	0.00	152	164	31.9873	2	6	3.1288	8	28	12.6167
B25	4	4	4.1613	0.3739	0.00	152	155	2.4839	2	6	4.9858	8	26	20.2954
B26	5	5	5.0645	0.2497	0.00	155	162	5.6949	2	7	5.6531	8	34	24.0685
B27	3	3	3.0968	0.3005	0.00	154	158	2.8860	2	6	3.6580	9	28	15.0662
B28	3	3	3.1613	0.4544	0.00	152	156	3.2326	2	7	3.2322	8	30	13.6381
B29	3	3	3.0968	0.3005	0.00	152	155	3.9811	2	5	2.2785	8	23	9.0958
B30	2	2	2.5161	0.6256	0.00	152	156	4.9036	2	7	5.6843	8	30	23.2274
B31	7	7	7.0000	0.0000	0.00	155	164	5.3710	1	7	4.3247	4	33	19.4109
B32	4	4	4.1935	0.7492	0.00	152	157	3.1191	2	6	3.7035	8	28	15.7836
B33	4	4	4.0000	0.0000	0.00	153	156	2.9574	3	5	2.7712	12	23	11.8504
B34	4	4	4.0645	0.3592	0.00	152	156	4.1986	2	7	7.1646	8	32	31.8354
B35	4	4	4.3226	0.7478	0.00	152	155	3.3758	2	5	3.0862	8	22	12.7461
B36	13	13	13.0000	0.0000	0.00	155	162	5.9413	2	7	3.9284	9	34	16.4346
B37	10	10	10.3226	0.7478	0.00	154	157	3.3153	2	7	5.8622	8	29	23.8995
B38	8	8	8.2581	1.2641	0.00	152	156	4.6789	2	10	8.4328	8	42	34.3955
B39	5	5	5.0000	0.0000	0.00	152	159	6.1503	2	5	2.9153	8	25	12.1929
B40	5	5	5.1290	0.4995	0.00	155	168	23.4268	3	5	1.5513	13	23	7.9119
B41	8	8	8.1613	0.8980	0.00	154	161	4.2011	4	9	6.6078	16	42	27.6753
B42	8	8	8.0000	0.0000	0.00	153	157	6.3106	2	4	1.7828	8	18	7.7493
B43	8	8	8.0000	0.0000	0.00	152	160	7.4285	2	5	2.6019	8	23	11.7932
B44	5	5	5.8710	1.3842	0.00	154	161	7.9767	2	5	2.0687	12	22	9.0735
B45	5	5	5.8710	1.3842	0.00	153	156	5.2525	2	5	3.0603	8	25	12.6925
B46	8	8	8.0323	0.1796	0.00	154	165	8.5232	1	9	7.2947	4	40	32.4977
B47	5	5	5.0000	0.0000	0.00	153	163	7.4939	2	6	3.3732	8	27	13.6618
B48	5	5	5.0645	0.3592	0.00	155	190	88.3159	3	5	2.3550	12	26	10.0136
B49	5	5	5.1290	0.4995	0.00	153	167	23.7535	2	6	2.2562	8	25	9.6192
B50	5	5	5.1935	0.6011	0.00	153	206	111.6447	1	5	2.3619	4	30	22.2696

TABLE 27: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.1613	0.7347	0.00	160	233	106.5829	4	19	9.9746	16	113	64.2842
B52	18	18	18.8387	0.7347	0.00	154	164	6.9436	6	21	9.2897	27	91	41.3948
B53	11	11	11.0000	0.0000	0.00	155	164	13.5560	5	11	3.1833	22	47	13.7809
B54	11	11	11.0000	0.0000	0.00	156	163	4.5475	4	8	2.4274	17	37	11.2196
B55	7	7	7.0000	0.0000	0.00	159	170	8.7338	5	10	3.2038	26	47	13.2918
B56	6	6	6.0000	0.0000	0.00	154	166	10.0578	6	10	3.3742	27	47	16.6980
B57	6	6	6.0000	0.0000	0.00	155	169	18.2314	6	11	5.5162	25	52	24.6267
B58	6	6	6.0000	0.0000	0.00	154	162	4.3901	3	10	5.1287	12	44	22.2686
B59	9	9	9.1613	0.5829	0.00	161	176	14.3747	5	14	7.2798	23	70	36.4352
B60	4	4	4.2903	1.1312	0.00	157	173	14.4923	6	14	6.1914	25	64	28.9730
B61	4	4	4.1613	0.5829	0.00	155	163	6.7429	4	9	4.0040	16	42	16.5396
B62	4	4	4.0323	0.1796	0.00	154	160	3.5345	4	10	4.6504	16	43	19.3696
B63	27	27	27.2258	0.4250	0.00	159	172	5.1895	3	20	11.1084	13	91	50.3296
B64	18	18	18.1935	0.4016	0.00	157	173	22.8970	6	21	9.5444	25	102	51.5291
B65	14	14	14.0645	0.2497	0.00	156	229	88.8105	6	16	6.8318	29	96	56.9926
B66	13	13	13.0000	0.0000	0.00	155	198	64.6472	4	10	4.2088	20	55	27.5365
B67	11	11	11.0968	0.3005	0.00	166	269	216.8374	3	15	9.5772	14	118	195.6026
B68	8	8	8.2258	0.9205	0.00	154	170	13.6269	4	11	6.2202	17	53	26.0361
B69	8	8	8.5161	0.7244	0.00	155	168	15.0527	6	13	6.1990	26	59	36.8672
B70	6	6	6.5484	0.8884	0.00	153	156	5.1117	3	14	8.3743	14	62	34.9847
B71	6	6	6.1290	0.4995	0.00	154	160	5.1506	4	11	4.5231	16	49	19.3207
B72	4	4	4.3548	0.5507	0.00	152	157	3.9059	6	16	8.5049	25	67	36.2737
B73	4	4	4.5161	0.6768	0.00	152	166	25.8682	4	13	9.0983	17	57	39.4182
B74	3	3	3.5484	0.7676	0.00	154	169	18.5330	3	13	7.3941	13	57	32.0482
B75	11	11	11.6452	0.9504	0.00	156	166	10.2917	6	14	9.4745	26	64	42.2402
B76	5	5	5.1290	0.4995	0.00	153	202	49.9134	6	13	6.4288	25	73	35.4865
B77	5	5	5.3871	0.8032	0.00	152	161	11.1016	4	13	7.0557	17	56	28.8527
B78	4	4	4.7742	1.3344	0.00	153	159	7.9270	4	13	6.6488	17	55	29.2830
B79	14	14	14.0000	0.0000	0.00	153	166	11.9017	2	9	3.9646	8	41	16.3156
B80	11	11	11.0000	0.0000	0.00	154	162	9.6500	6	11	5.3587	24	50	23.5722
B81	11	11	11.3548	0.8386	0.00	153	158	6.6857	5	13	7.0744	20	56	29.7124
B82	10	10	10.4516	0.7676	0.00	152	160	10.6532	5	13	5.9386	20	55	27.4567
B83	12	12	12.0968	0.5388	0.00	153	170	19.9290	5	11	3.8451	24	51	17.9308
B84	12	12	12.0000	0.0000	0.00	154	161	6.7725	4	13	7.3941	16	55	31.6225
B85	8	8	8.8387	1.2409	0.00	154	164	14.7388	5	14	7.2984	22	62	34.7666
B86	8	8	8.0968	0.5388	0.00	153	162	10.3043	4	10	4.3183	16	44	18.7357
B87	10	10	10.1290	0.4995	0.00	154	166	10.6363	5	18	8.1722	21	80	36.7600
B88	8	8	8.0000	0.0000	0.00	154	160	6.2300	6	11	4.4717	24	50	18.2966
B89	8	8	8.0000	0.0000	0.00	154	181	63.1321	4	10	3.9393	16	50	22.3958
B90	5	5	5.4839	1.1216	0.00	154	158	5.3563	3	10	4.3045	12	41	18.0081
		AVG	8.0756	0.4173	0.00	154	170	21.8506	3	9	4.8937	14	44	24.7712

TABLE 28: MBO using sequential quick sort algorithm.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	156	190	58.2650	2	7	3.8624	11	43	33.3476
B2	11	11	11.0000	0.0000	0.00	153	161	11.6192	2	4	1.4074	8	19	6.0245
B3	11	11	11.0000	0.0000	0.00	152	155	2.5085	2	4	1.5176	8	17	6.5214
B4	11	11	11.0000	0.0000	0.00	152	182	101.4110	2	4	1.8246	8	22	9.4797
B5	11	11	11.0000	0.0000	0.00	152	156	6.3229	2	5	3.4426	9	22	13.9587
B6	7	7	7.0000	0.0000	0.00	154	161	13.5107	3	8	4.1693	12	37	17.3353
B7	6	6	6.0323	0.1796	0.00	153	168	13.8935	3	6	2.9068	12	31	13.1726
B8	4	4	4.0645	0.3592	0.00	152	182	34.6447	2	10	7.8375	8	49	35.8762
B9	3	3	3.1290	0.3408	0.00	152	156	9.6349	2	6	5.9664	8	29	33.1260
B10	3	3	3.0000	0.0000	0.00	152	154	2.1771	2	4	1.1968	8	16	4.7947
B11	4	4	4.1290	0.3408	0.00	154	157	3.1084	1	11	8.3456	4	46	34.7346
B12	4	4	4.0000	0.0000	0.00	152	155	3.0374	1	3	1.5794	5	15	6.4709
B13	4	4	4.2258	0.7169	0.00	152	153	1.8814	2	5	5.3585	8	23	22.5828
B14	3	3	3.3548	0.4864	0.00	152	154	1.6533	2	5	2.3971	8	23	9.8987
B15	1	1	1.1290	0.4995	0.00	152	153	1.5027	2	6	2.2912	8	24	9.3735
B16	14	14	14.0000	0.0000	0.00	154	158	6.5646	2	8	4.7026	8	34	19.1483
B17	13	13	13.0000	0.0000	0.00	153	155	2.0283	2	5	1.8860	9	21	7.4763
B18	13	13	13.0000	0.0000	0.00	152	157	7.3231	1	6	2.4995	4	25	10.3130
B19	13	13	13.0968	0.5388	0.00	152	159	6.4381	2	7	6.0690	8	33	24.6737
B20	13	13	13.0000	0.0000	0.00	153	159	7.5257	2	7	4.6504	8	31	19.5123
B21	9	9	9.2903	0.4614	0.00	154	158	4.8569	2	9	7.8941	8	41	32.4426
B22	6	6	6.1613	0.3739	0.00	152	160	7.6086	3	6	2.7611	12	26	12.0577
B23	6	6	6.1613	0.3739	0.00	152	157	5.0575	4	10	7.1245	16	43	29.3833
B24	5	5	5.2903	0.5287	0.00	152	159	6.8078	2	7	4.8146	9	30	19.7313
B25	4	4	4.1613	0.3739	0.00	152	155	4.9514	2	6	2.5430	8	24	10.4495
B26	5	5	5.0000	0.0000	0.00	154	163	8.4537	2	8	4.4837	8	36	18.4994
B27	3	3	3.0323	0.1796	0.00	153	158	6.3527	1	7	5.5072	5	29	22.6066
B28	3	3	3.2581	0.4448	0.00	152	162	9.1764	2	6	4.4041	8	26	20.0669
B29	3	3	3.1613	0.3739	0.00	152	163	8.9060	1	5	2.9534	4	23	12.2417
B30	2	2	2.6129	0.8032	0.00	152	162	11.2350	2	7	6.3716	8	30	26.7117
B31	7	7	7.0000	0.0000	0.00	155	172	13.0047	3	8	5.2176	14	40	23.0526
B32	4	4	4.0968	0.5388	0.00	154	163	16.2313	2	8	5.0189	8	36	22.6064
B33	4	4	4.0645	0.3592	0.00	153	161	8.5798	2	5	2.5016	8	22	10.6692
B34	4	4	4.1290	0.4995	0.00	152	158	9.0177	2	6	5.0302	8	27	20.6482
B35	4	4	4.6452	0.9504	0.00	152	153	1.5098	1	5	4.4678	4	23	18.0923
B36	13	13	13.0000	0.0000	0.00	154	174	26.3191	3	8	4.2014	12	41	24.8909
B37	10	10	10.5161	0.8896	0.00	153	161	12.2450	2	7	3.9136	8	30	15.9159
B38	8	8	8.0968	0.3962	0.00	152	157	6.5512	2	10	8.7204	8	43	35.7477
B39	5	5	5.0000	0.0000	0.00	152	161	13.6204	3	5	2.3431	12	25	10.6618
B40	5	5	5.0000	0.0000	0.00	152	163	9.7288	2	5	2.1438	8	21	9.9864
B41	8	8	8.0000	0.0000	0.00	155	165	12.7196	1	10	6.2553	5	48	26.9376
B42	8	8	8.0000	0.0000	0.00	152	157	3.5951	2	4	1.7804	8	19	7.5267
B43	8	8	8.0645	0.3592	0.00	152	155	2.4947	2	5	5.3110	8	24	22.0386
B44	5	5	6.0645	1.4591	0.00	152	155	3.3302	2	6	3.3601	8	25	13.7010
B45	5	5	6.4516	1.5240	0.00	152	154	1.4135	2	5	2.2907	8	21	9.2427
B46	8	8	8.1290	0.3408	0.00	154	161	4.7801	3	9	6.7587	14	42	29.0482
B47	5	5	5.0968	0.5388	0.00	153	156	2.5392	1	4	1.8139	4	20	7.8833
B48	5	5	5.1935	0.6011	0.00	152	155	4.2327	2	5	2.4021	8	23	9.6806
B49	5	5	5.1290	0.4995	0.00	152	155	3.8960	3	7	4.9298	12	31	20.3514
B50	5	5	5.1935	0.6011	0.00	152	157	7.6718	2	5	2.2728	8	24	9.2511

TABLE 28: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.2258	0.9205	0.00	155	161	4.3769	4	19	9.2586	20	82	39.4718
B52	18	18	18.9032	1.1062	0.00	154	158	4.4286	7	20	9.4888	29	85	40.2330
B53	11	11	11.0000	0.0000	0.00	154	157	2.8269	4	10	4.1346	16	44	16.8027
B54	11	11	11.0000	0.0000	0.00	153	156	2.3075	4	9	3.2052	16	38	13.3121
B55	7	7	7.0000	0.0000	0.00	154	158	2.9370	4	10	5.4839	16	45	22.7564
B56	6	6	6.0000	0.0000	0.00	154	157	2.2337	4	8	2.6184	16	36	11.1519
B57	6	6	6.0000	0.0000	0.00	153	157	4.3503	6	10	4.0231	24	43	16.3044
B58	6	6	6.1290	0.3408	0.00	154	157	2.8189	5	10	3.8882	20	44	15.7987
B59	9	9	9.1935	0.6011	0.00	154	160	3.4817	3	13	8.1588	12	59	34.8749
B60	4	4	4.5161	1.3631	0.00	154	157	2.8511	3	12	4.7288	12	50	19.8736
B61	4	4	4.2258	0.6170	0.00	153	156	3.6237	3	11	4.2571	17	48	17.2788
B62	4	4	4.0323	0.1796	0.00	152	155	2.5098	4	10	3.6820	16	41	15.5667
B63	27	27	27.2581	0.4448	0.00	155	164	6.3053	4	19	10.1183	19	84	42.8466
B64	18	18	18.3548	0.5507	0.00	154	175	29.3106	7	20	9.2391	37	95	41.8859
B65	14	14	14.1935	0.4016	0.00	154	162	11.5242	5	14	6.2896	22	61	26.2881
B66	13	13	13.0000	0.0000	0.00	154	174	31.3824	6	11	3.7301	24	51	22.7029
B67	11	11	11.0323	0.1796	0.00	155	166	11.5943	6	12	5.7328	24	53	23.5936
B68	8	8	8.5806	1.7469	0.00	154	160	5.9015	3	11	6.2869	12	48	25.7106
B69	8	8	8.5161	0.8112	0.00	153	165	12.9041	4	10	5.4759	16	47	24.7696
B70	6	6	6.6452	0.8386	0.00	153	160	11.3877	2	12	7.7095	8	52	31.6072
B71	6	6	6.1290	0.4995	0.00	157	178	18.6546	2	11	5.3096	11	54	28.3288
B72	4	4	4.2581	0.7732	0.00	154	171	19.4453	4	11	6.3664	16	54	29.4289
B73	4	4	4.4194	0.6204	0.00	154	174	16.4459	5	10	4.0422	21	51	19.9695
B74	3	3	3.5484	0.6239	0.00	156	187	33.9678	5	12	7.8744	20	67	49.6365
B75	11	11	11.5806	1.0255	0.00	159	200	45.3931	4	17	9.9043	17	95	67.7548
B76	5	5	5.0645	0.3592	0.00	154	171	37.0215	4	14	6.9785	17	67	30.3144
B77	5	5	5.3871	0.8032	0.00	154	161	7.2560	4	14	7.2969	17	62	31.3623
B78	4	4	4.5806	1.1188	0.00	153	166	11.1021	8	15	4.9588	32	69	24.4737
B79	14	14	14.0000	0.0000	0.00	155	173	15.1100	5	9	2.9421	20	44	16.3185
B80	11	11	11.0000	0.0000	0.00	154	164	13.3813	4	11	3.7198	17	48	16.2591
B81	11	11	11.5484	1.1787	0.00	155	177	20.2888	2	13	8.5572	8	64	39.3331
B82	10	10	10.7419	0.7732	0.00	154	227	142.7347	3	10	5.1533	13	56	31.9425
B83	12	12	12.3548	1.5176	0.00	154	180	28.3520	3	11	6.7907	13	58	36.4259
B84	12	12	12.0000	0.0000	0.00	155	165	12.8809	2	12	5.0773	9	54	23.5388
B85	8	8	9.4516	1.6899	0.00	154	164	17.4202	3	13	6.9161	13	59	29.5533
B86	8	8	8.1613	0.5829	0.00	153	155	2.0525	4	11	6.3534	16	49	25.9607
B87	10	10	10.3871	0.9549	0.00	154	159	5.2984	5	16	9.5733	20	68	40.5421
B88	8	8	8.0000	0.0000	0.00	154	157	2.8466	5	10	3.0610	21	43	12.7689
B89	8	8	8.0323	0.1796	0.00	154	156	2.3349	4	10	5.9039	16	43	23.9970
B90	5	5	5.3871	1.0223	0.00	153	155	2.5778	3	11	4.7201	13	46	18.9533
		AVG	8.1111	0.4529	0.00	153	163	12.7284	2	9	4.9168	12	41	22.1071

TABLE 29: MBO using parallel quick sort algorithm with 1 thread.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	156	193	68.9999	2	7	3.1195	9	41	17.8353
B2	11	11	11.0000	0.0000	0.00	153	165	18.8674	2	4	1.1538	8	19	8.8834
B3	11	11	11.0000	0.0000	0.00	153	161	19.3106	2	4	1.3827	8	19	6.5820
B4	11	11	11.0000	0.0000	0.00	153	167	25.8893	2	4	1.8014	8	20	9.8274
B5	11	11	11.0000	0.0000	0.00	154	177	33.0442	2	5	2.6769	9	29	17.8432
B6	7	7	7.0000	0.0000	0.00	154	165	11.4363	1	8	5.7640	4	38	25.6395
B7	6	6	6.0645	0.2497	0.00	153	159	3.8032	2	8	5.1941	9	36	22.4181
B8	4	4	4.2581	0.6816	0.00	152	158	5.4317	3	9	7.4731	12	38	30.7397
B9	3	3	3.0968	0.3005	0.00	153	168	20.1787	2	5	2.6102	10	25	11.7018
B10	3	3	3.0000	0.0000	0.00	152	158	7.1719	2	4	1.5794	8	17	6.8647
B11	4	4	4.1613	0.3739	0.00	154	162	8.1435	4	11	8.0349	16	48	34.9012
B12	4	4	4.0000	0.0000	0.00	152	154	1.6950	2	4	1.7064	8	18	6.9414
B13	4	4	4.0000	0.0000	0.00	152	154	2.0832	2	6	3.3636	8	25	13.6855
B14	3	3	3.4516	0.5680	0.00	152	168	22.8717	2	6	4.3793	8	27	19.2045
B15	1	1	1.4194	0.8860	0.00	153	175	34.8684	3	7	3.2531	12	33	16.1056
B16	14	14	14.0000	0.0000	0.00	159	221	63.7515	2	7	4.2848	11	41	21.9744
B17	13	13	13.0000	0.0000	0.00	154	178	48.9971	2	4	1.7026	9	22	9.6839
B18	13	13	13.0000	0.0000	0.00	152	155	4.3855	3	6	2.3655	12	24	10.3454
B19	13	13	13.0968	0.5388	0.00	152	153	2.1513	2	7	5.6009	8	30	22.7850
B20	13	13	13.0000	0.0000	0.00	152	153	1.5027	2	7	5.0298	8	32	20.2303
B21	9	9	9.0968	0.3005	0.00	154	158	3.0132	1	10	8.6519	4	43	35.6950
B22	6	6	6.1613	0.3739	0.00	152	156	4.0430	2	6	4.0364	8	26	16.4319
B23	6	6	6.0968	0.3005	0.00	152	153	1.4803	3	6	4.5416	12	27	18.2106
B24	5	5	5.3871	0.6152	0.00	152	153	1.5394	2	5	2.4132	8	23	9.6668
B25	4	4	4.4516	0.8500	0.00	152	154	1.8757	2	5	1.8567	8	23	7.7759
B26	5	5	5.0000	0.0000	0.00	154	158	4.2421	2	9	6.0213	8	40	25.2638
B27	3	3	3.0323	0.1796	0.00	152	156	3.4467	1	6	4.3334	4	27	17.5791
B28	3	3	3.3548	0.5507	0.00	152	154	2.4328	3	5	1.9724	12	24	8.1028
B29	3	3	3.2581	0.4448	0.00	152	154	3.9059	2	5	2.6126	8	23	10.6863
B30	2	2	2.2581	0.5143	0.00	152	154	1.8062	4	8	3.8077	16	32	15.4194
B31	7	7	7.0000	0.0000	0.00	154	166	12.3793	3	9	3.5214	12	39	14.8130
B32	4	4	4.1935	0.7492	0.00	154	161	9.6416	3	8	5.6009	12	35	23.3415
B33	4	4	4.1290	0.4995	0.00	152	160	7.4242	1	5	2.9712	4	24	12.6727
B34	4	4	4.0645	0.3592	0.00	152	156	4.2459	3	6	4.2170	12	28	17.9599
B35	4	4	4.9032	1.0118	0.00	153	166	10.7510	2	5	3.7626	8	26	16.9638
B36	13	13	13.0000	0.0000	0.00	155	177	17.3078	3	8	4.6900	12	38	23.3320
B37	10	10	10.5161	0.8896	0.00	153	165	14.0154	2	6	4.9622	8	26	20.6570
B38	8	8	8.0645	0.2497	0.00	152	165	14.2440	2	11	7.5177	8	52	33.8136
B39	5	5	5.3548	1.3796	0.00	154	168	10.6828	2	6	2.8136	8	27	13.6350
B40	5	5	5.1935	0.6011	0.00	152	162	7.7362	2	5	2.0832	8	22	8.8589
B41	8	8	8.0000	0.0000	0.00	154	177	24.0561	1	10	7.6418	4	47	36.2878
B42	8	8	8.0000	0.0000	0.00	153	182	36.9639	2	4	1.4081	8	20	10.2833
B43	8	8	8.0968	0.5388	0.00	162	268	137.6530	2	4	2.1463	12	33	27.5295
B44	5	5	5.8710	1.3842	0.00	155	172	21.8345	2	5	3.8857	8	27	19.0926
B45	5	5	5.8710	1.3842	0.00	153	182	50.6757	2	6	3.1036	8	29	20.3958
B46	8	8	8.1935	0.4016	0.00	172	219	56.1605	3	10	5.7329	15	60	43.2816
B47	5	5	5.0968	0.5388	0.00	166	207	40.9797	2	6	5.1442	10	40	29.2231
B48	5	5	5.0645	0.3592	0.00	163	170	5.2351	2	5	2.6258	8	26	12.1034
B49	5	5	5.1935	0.6011	0.00	160	170	8.4292	2	5	3.0754	9	24	13.6420
B50	5	5	5.5806	0.9228	0.00	160	174	21.8488	1	6	5.7528	4	28	25.8977

TABLE 29: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.3548	0.9504	0.00	167	198	36.1613	3	20	9.4901	14	106	48.8988
B52	18	18	18.7742	0.6688	0.00	155	159	3.0282	4	19	10.1538	17	84	42.9794
B53	11	11	11.0000	0.0000	0.00	154	159	4.8285	5	10	3.0593	20	43	13.3710
B54	11	11	11.0000	0.0000	0.00	152	156	2.9388	3	9	3.0061	12	38	12.4134
B55	7	7	7.0000	0.0000	0.00	154	164	12.3552	4	13	6.6611	16	57	28.3718
B56	6	6	6.0000	0.0000	0.00	154	175	24.3904	5	9	3.9809	22	44	17.8687
B57	6	6	6.0000	0.0000	0.00	153	163	6.9322	4	11	5.2964	16	50	24.4128
B58	6	6	6.0000	0.0000	0.00	153	156	3.5822	5	10	5.6569	21	41	23.9429
B59	9	9	9.0323	0.1796	0.00	155	166	5.9601	2	12	6.4739	8	54	27.6165
B60	4	4	4.5161	1.3631	0.00	154	163	7.2272	5	11	4.4119	21	49	19.4739
B61	4	4	4.2581	0.6308	0.00	154	158	6.1989	2	9	3.6644	8	41	15.6830
B62	4	4	4.1290	0.5623	0.00	153	156	2.4999	5	10	4.2704	20	42	17.9183
B63	27	27	27.2903	0.4614	0.00	155	166	6.3789	5	17	8.5487	26	77	38.2296
B64	18	18	18.2903	0.5287	0.00	157	165	6.0370	7	23	9.6062	28	104	42.0711
B65	14	14	14.1613	0.4544	0.00	154	160	4.3692	4	15	8.1672	17	65	36.0926
B66	13	13	13.0000	0.0000	0.00	154	163	8.8223	5	9	2.8923	20	42	12.0080
B67	11	11	11.0323	0.1796	0.00	159	170	7.5927	4	12	5.8966	17	55	26.7593
B68	8	8	8.5484	1.7481	0.00	154	162	7.1544	3	11	5.3146	12	50	22.5538
B69	8	8	8.3548	0.5507	0.00	153	156	2.4425	4	12	6.3070	16	51	25.7643
B70	6	6	6.2581	0.6308	0.00	153	156	2.2671	3	10	5.7856	12	44	23.9520
B71	6	6	6.0645	0.3592	0.00	156	181	27.7347	2	13	8.4332	11	64	39.5748
B72	4	4	4.1290	0.3408	0.00	155	177	29.3796	8	14	5.3225	36	64	23.1387
B73	4	4	4.5484	0.5680	0.00	153	182	46.2198	5	12	5.6883	20	63	40.8649
B74	3	3	3.4839	0.6768	0.00	153	181	24.0085	5	13	7.7974	22	62	38.6443
B75	11	11	11.7742	1.0866	0.00	164	216	65.0219	7	19	8.5961	30	102	42.8679
B76	5	5	5.1290	0.4995	0.00	154	159	3.2132	4	12	6.9587	19	50	28.7074
B77	5	5	5.9355	1.0626	0.00	154	159	2.5791	3	11	6.9711	13	50	29.2492
B78	4	4	4.7419	1.4135	0.00	153	158	2.8838	4	15	9.7257	20	64	40.5362
B79	14	14	14.0000	0.0000	0.00	156	162	5.3378	4	11	5.1178	18	49	21.6838
B80	11	11	11.0000	0.0000	0.00	153	159	2.9928	4	10	3.3575	16	46	14.4630
B81	11	11	11.2903	1.0064	0.00	153	158	2.8994	5	12	5.9435	21	52	25.2294
B82	10	10	10.6129	0.9892	0.00	154	158	2.6680	2	10	5.5058	8	45	22.8974
B83	12	12	12.0000	0.0000	0.00	156	165	6.5898	4	10	3.6210	20	46	14.5889
B84	12	12	12.0645	0.3592	0.00	155	163	5.1148	4	10	5.6932	18	44	24.3315
B85	8	8	8.9355	1.2893	0.00	155	161	5.5007	4	16	8.2380	16	69	35.8578
B86	8	8	8.0645	0.2497	0.00	153	165	10.2890	3	10	4.3411	15	46	18.2102
B87	10	10	10.5484	0.9605	0.00	157	177	13.8084	5	18	7.6776	23	87	37.4388
B88	8	8	8.2581	1.4368	0.00	154	166	13.5674	5	10	5.8803	24	46	24.4021
B89	8	8	8.0323	0.1796	0.00	156	168	18.2189	6	9	3.7301	24	43	20.6299
B90	5	5	5.3871	1.0223	0.00	153	164	9.5021	4	12	5.7030	18	54	24.8741
		AVG	8.1115	0.4667	0.00	154	167	15.5926	3	9	4.8703	13	42	22.3487

TABLE 30: MBO using parallel quick sort algorithm with 4 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	154	188	70.6184	1	6	4.1450	7	35	21.4910
B2	11	11	11.0000	0.0000	0.00	152	182	34.7551	2	3	1.4591	8	19	11.0777
B3	11	11	11.0000	0.0000	0.00	153	177	33.6228	2	4	1.7083	8	19	7.7936
B4	11	11	11.0000	0.0000	0.00	153	170	28.4944	2	4	1.7314	8	21	7.5054
B5	11	11	11.0000	0.0000	0.00	153	177	33.1479	2	4	2.1347	8	23	13.0829
B6	7	7	7.0000	0.0000	0.00	161	192	19.0742	3	9	5.1148	15	50	25.7463
B7	6	6	6.0323	0.1796	0.00	152	159	9.1029	1	5	2.5413	4	23	12.2419
B8	4	4	4.4516	1.2339	0.00	152	159	12.7028	2	11	8.2441	8	46	33.8427
B9	3	3	3.0968	0.3005	0.00	158	257	84.1696	1	6	3.5033	14	44	31.5700
B10	3	3	3.0000	0.0000	0.00	153	183	28.7203	2	4	2.5145	8	24	18.5823
B11	4	4	4.0968	0.3005	0.00	156	165	7.6151	3	11	6.1848	13	50	27.3824
B12	4	4	4.0000	0.0000	0.00	152	155	2.0000	2	4	1.5596	8	16	6.3920
B13	4	4	4.0645	0.3592	0.00	152	154	1.8538	3	4	2.1089	12	19	8.8271
B14	3	3	3.2903	0.4614	0.00	152	160	9.8399	1	5	2.5222	4	24	10.5106
B15	1	1	1.4516	0.8500	0.00	152	156	3.8373	1	5	3.5542	4	24	14.7010
B16	14	14	14.0000	0.0000	0.00	154	173	25.6756	3	9	4.8887	12	41	27.1373
B17	13	13	13.0000	0.0000	0.00	153	157	6.4014	2	5	1.6950	8	21	7.1785
B18	13	13	13.0000	0.0000	0.00	152	155	5.7326	2	5	2.4926	8	22	10.7346
B19	13	13	13.0000	0.0000	0.00	152	157	7.7789	3	7	3.6761	12	30	15.0969
B20	13	13	13.0000	0.0000	0.00	152	155	4.7308	2	6	3.0810	8	27	12.7058
B21	9	9	9.0968	0.3005	0.00	154	158	2.9072	3	11	7.1601	12	46	30.0206
B22	6	6	6.0645	0.2497	0.00	154	182	76.4377	3	6	2.7909	12	39	63.2356
B23	6	6	6.1290	0.4275	0.00	152	160	13.7304	2	7	4.4337	8	33	23.9618
B24	5	5	5.3871	0.6672	0.00	152	154	2.6893	3	6	2.5123	12	27	10.0762
B25	4	4	4.1935	0.6011	0.00	152	155	3.2505	2	6	2.8128	10	26	11.2697
B26	5	5	5.0000	0.0000	0.00	153	158	4.2021	2	8	6.8480	8	37	28.5317
B27	3	3	3.0968	0.3005	0.00	152	159	7.4229	3	8	6.4966	12	34	27.3041
B28	3	3	3.1935	0.4016	0.00	152	174	36.7644	2	6	4.5661	10	30	20.8310
B29	3	3	3.2258	0.4250	0.00	152	154	1.6224	1	5	1.8526	4	20	7.7248
B30	2	2	2.3871	0.5584	0.00	152	153	2.0651	2	6	3.0268	8	25	12.0320
B31	7	7	7.0000	0.0000	0.00	154	158	3.8702	2	9	5.1157	8	40	21.2321
B32	4	4	4.0000	0.0000	0.00	153	155	2.0925	2	6	3.6732	8	27	15.0795
B33	4	4	4.0645	0.3592	0.00	152	154	2.3150	1	5	2.5089	4	24	10.0413
B34	4	4	4.2581	0.6816	0.00	152	154	3.8401	2	6	4.1587	8	28	17.4530
B35	4	4	4.7097	0.9727	0.00	152	153	1.4701	1	7	5.4404	4	29	21.7851
B36	13	13	13.0000	0.0000	0.00	154	157	3.1167	2	6	3.5128	8	29	14.5407
B37	10	10	10.4516	0.8500	0.00	152	155	2.1840	1	5	3.6124	4	24	14.6059
B38	8	8	8.0323	0.1796	0.00	152	155	3.6706	2	14	9.2075	8	59	37.5165
B39	5	5	5.1613	0.8980	0.00	152	153	1.7339	2	5	2.6929	8	22	10.9139
B40	5	5	5.2581	0.6816	0.00	152	153	1.6091	2	5	2.4279	8	23	10.0457
B41	8	8	8.0000	0.0000	0.00	154	158	4.7956	2	7	4.4959	8	32	18.6759
B42	8	8	8.0000	0.0000	0.00	152	154	1.7469	2	4	2.2418	8	17	9.4866
B43	8	8	8.0000	0.0000	0.00	152	154	2.0064	1	5	2.4552	4	21	10.1844
B44	5	5	6.2581	1.5048	0.00	152	153	1.3647	1	6	3.6715	4	25	14.7284
B45	5	5	6.2903	1.5534	0.00	152	154	3.6805	3	5	2.3316	12	22	9.3036
B46	8	8	8.1290	0.3408	0.00	154	158	2.9719	3	9	6.7561	12	39	28.1927
B47	5	5	5.0968	0.5388	0.00	153	165	8.1313	2	5	2.2322	9	22	9.6494
B48	5	5	5.1935	0.6011	0.00	156	169	11.8787	1	6	4.2139	4	27	18.5138
B49	5	5	5.2581	0.6816	0.00	152	182	30.8552	2	5	2.3787	10	24	10.4171
B50	5	5	5.2581	0.6816	0.00	152	196	63.4998	3	8	5.8501	12	40	28.8074

TABLE 30: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.2581	1.2902	0.00	154	167	15.4226	7	21	7.9024	29	95	36.2584
B52	18	18	18.5806	0.9228	0.00	154	157	4.2457	7	20	9.5797	29	85	41.1011
B53	11	11	11.0000	0.0000	0.00	154	156	2.3477	5	10	3.7096	20	41	15.1854
B54	11	11	11.0000	0.0000	0.00	152	154	2.3408	6	9	2.1973	24	37	9.2985
B55	7	7	7.0000	0.0000	0.00	154	157	2.9607	4	11	4.6251	20	48	20.0461
B56	6	6	6.0000	0.0000	0.00	154	156	3.9259	5	10	3.7273	20	44	15.4734
B57	6	6	6.1290	0.4995	0.00	152	155	1.9394	4	11	3.9784	16	48	16.3756
B58	6	6	6.0000	0.0000	0.00	153	155	1.8439	4	9	3.4241	16	39	13.8646
B59	9	9	9.1290	0.4275	0.00	154	158	5.4091	4	11	4.9422	19	49	20.9249
B60	4	4	4.6452	1.4955	0.00	153	156	2.4622	5	12	6.4630	21	52	27.5544
B61	4	4	4.0323	0.1796	0.00	153	156	2.9838	3	11	7.3924	12	46	30.1892
B62	4	4	4.0323	0.1796	0.00	153	165	11.7614	3	9	4.1137	12	41	18.7758
B63	27	27	27.1935	0.4016	0.00	155	174	17.4625	6	17	9.3253	25	77	41.1960
B64	18	18	18.2903	0.5287	0.00	154	168	12.6685	4	18	9.1350	16	81	44.2824
B65	14	14	14.0323	0.1796	0.00	155	167	9.6409	3	15	7.6449	15	69	33.6520
B66	13	13	13.0000	0.0000	0.00	153	160	6.3507	4	9	3.0157	19	42	13.7272
B67	11	11	11.0000	0.0000	0.00	155	175	15.9737	6	15	8.3154	26	73	40.7381
B68	8	8	8.3226	1.2751	0.00	153	164	16.0762	3	11	6.8135	13	52	31.3006
B69	8	8	8.4839	0.6768	0.00	153	157	4.3202	4	12	6.9641	17	53	28.7297
B70	6	6	6.6452	0.9504	0.00	152	155	4.3401	4	15	8.3663	16	65	33.8478
B71	6	6	6.0645	0.3592	0.00	154	158	3.7631	7	13	6.9158	30	58	28.7037
B72	4	4	4.3226	0.6525	0.00	153	156	2.3215	4	13	8.2617	16	55	33.7554
B73	4	4	4.6774	0.6525	0.00	153	159	6.6099	3	11	5.8756	12	48	25.2215
B74	3	3	3.5484	0.5680	0.00	152	158	6.5660	4	13	7.7735	17	54	32.1404
B75	11	11	11.7419	1.0945	0.00	155	162	9.8424	5	18	7.4919	21	77	31.6711
B76	5	5	5.1935	0.6011	0.00	154	169	17.1585	6	14	6.6203	26	64	36.1670
B77	5	5	5.5161	0.8896	0.00	153	166	17.0023	3	13	6.6431	14	56	28.0002
B78	4	4	4.8387	1.4628	0.00	154	167	16.8144	4	17	8.5824	18	77	36.9169
B79	14	14	14.0000	0.0000	0.00	158	182	31.2523	4	9	6.0580	20	48	28.0606
B80	11	11	11.2581	1.4368	0.00	154	168	12.6654	4	11	4.0099	16	51	21.3734
B81	11	11	11.3871	0.9549	0.00	153	164	10.3919	4	15	9.5243	19	67	42.0640
B82	10	10	10.6452	0.7094	0.00	153	160	9.4966	5	11	5.4876	23	51	27.6218
B83	12	12	12.0000	0.0000	0.00	154	157	2.8341	4	12	4.8057	19	51	19.9222
B84	12	12	12.2903	0.7829	0.00	153	157	5.0586	5	14	8.1952	20	59	34.1199
B85	8	8	8.9677	1.3536	0.00	154	162	10.7207	8	15	6.6079	33	65	31.6611
B86	8	8	8.0323	0.1796	0.00	152	166	10.2565	4	9	3.7700	16	40	17.8689
B87	10	10	10.7097	0.9727	0.00	154	166	10.2930	7	19	8.5170	28	82	37.4828
B88	8	8	8.0000	0.0000	0.00	160	177	8.3988	4	10	4.3527	18	50	20.4859
B89	8	8	8.0323	0.1796	0.00	154	173	15.7080	2	10	8.1272	8	49	40.5965
B90	5	5	5.2903	0.9016	0.00	154	164	9.5502	4	11	5.5830	16	49	26.4783
		AVG	8.1108	0.4544	0.00	153	163	12.2109	3	9	4.9019	13	41	22.4733

TABLE 31: MBO using parallel quick sort algorithm with 8 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	190	260	90.7097	2	6	3.5636	12	55	39.8897
B2	11	11	11.0000	0.0000	0.00	205	231	16.0884	1	3	1.3408	5	27	11.3853
B3	11	11	11.0000	0.0000	0.00	201	218	12.8483	2	4	1.2487	15	25	8.4055
B4	11	11	11.0000	0.0000	0.00	197	224	13.2765	2	4	1.6198	9	30	10.1960
B5	11	11	11.0000	0.0000	0.00	203	226	16.9506	1	5	2.5425	5	31	15.3820
B6	7	7	7.0000	0.0000	0.00	179	209	26.6540	3	8	4.1551	16	46	20.1586
B7	6	6	6.0645	0.2497	0.00	159	170	8.6506	2	5	2.3958	9	26	10.8302
B8	4	4	4.5806	1.2852	0.00	161	208	24.9863	2	9	8.2168	11	49	44.8206
B9	3	3	3.0000	0.0000	0.00	156	169	9.0874	1	5	2.9975	4	25	14.9742
B10	3	3	3.0000	0.0000	0.00	160	187	29.3461	1	4	1.9143	4	21	10.0919
B11	4	4	4.2581	0.4448	0.00	221	265	38.7226	3	12	7.1297	18	92	57.9354
B12	4	4	4.0000	0.0000	0.00	189	244	25.8799	2	4	1.8917	9	30	13.7061
B13	4	4	4.0000	0.0000	0.00	175	212	26.0266	2	5	2.0609	9	29	12.7301
B14	3	3	3.5161	0.5080	0.00	191	233	25.7782	2	5	2.1251	10	32	17.8553
B15	1	1	1.4839	0.9263	0.00	158	172	15.6618	2	5	2.1266	8	25	10.8872
B16	14	14	14.0000	0.0000	0.00	219	241	17.5973	2	8	4.4454	10	51	28.6909
B17	13	13	13.0000	0.0000	0.00	155	178	22.8931	2	5	1.4142	8	23	6.1196
B18	13	13	13.0000	0.0000	0.00	157	170	16.5604	2	5	1.9620	8	23	10.1844
B19	13	13	13.2903	0.9016	0.00	184	212	13.5299	2	6	2.9024	10	38	15.5537
B20	13	13	13.0000	0.0000	0.00	179	195	11.0056	2	8	5.7083	8	44	29.5659
B21	9	9	9.1613	0.3739	0.00	173	223	67.1284	4	10	8.0616	19	60	50.9887
B22	6	6	6.1613	0.3739	0.00	193	273	88.1693	3	6	3.0764	12	46	37.1433
B23	6	6	6.2258	0.4973	0.00	187	232	20.6745	2	6	4.7992	14	44	30.3885
B24	5	5	5.4516	0.7229	0.00	185	216	28.3318	2	5	2.5046	14	32	12.6408
B25	4	4	4.3548	0.7094	0.00	180	220	38.7354	2	7	3.3575	11	41	21.8570
B26	5	5	5.0000	0.0000	0.00	222	267	65.9248	2	9	6.2550	11	63	40.3980
B27	3	3	3.0968	0.3005	0.00	163	187	72.6372	1	6	4.0640	4	32	19.3312
B28	3	3	3.2903	0.4614	0.00	161	188	57.0519	2	5	2.3773	8	26	13.4644
B29	3	3	3.1290	0.3408	0.00	153	176	30.2510	2	5	1.9615	9	24	10.0626
B30	2	2	2.3871	0.5584	0.00	153	175	21.4547	3	9	6.7079	12	41	29.4979
B31	7	7	7.0000	0.0000	0.00	161	211	43.1659	2	10	4.5541	10	58	30.4089
B32	4	4	4.3871	1.0223	0.00	155	191	40.5896	1	7	6.1083	4	38	31.5310
B33	4	4	4.1935	0.6011	0.00	156	214	47.1663	2	6	3.0204	12	37	18.9796
B34	4	4	4.2581	0.6816	0.00	156	169	21.7325	2	5	2.2855	10	26	10.2277
B35	4	4	4.4516	0.8500	0.00	155	184	36.3288	1	5	2.4861	4	25	14.0484
B36	13	13	13.0000	0.0000	0.00	159	172	12.9491	2	9	5.4378	8	44	24.1031
B37	10	10	10.5806	0.9228	0.00	154	165	12.0246	2	7	6.6159	9	35	31.7865
B38	8	8	8.0968	0.3005	0.00	155	162	8.0036	2	11	9.7551	8	50	41.9069
B39	5	5	5.0000	0.0000	0.00	154	162	6.2843	2	5	2.1109	8	23	9.1885
B40	5	5	5.1290	0.4995	0.00	160	185	31.2620	2	5	1.9898	9	27	11.3643
B41	8	8	8.1613	0.8980	0.00	157	191	35.3240	3	9	5.6883	12	52	34.1054
B42	8	8	8.0000	0.0000	0.00	153	191	35.1184	2	4	2.0557	8	25	14.0413
B43	8	8	8.0000	0.0000	0.00	153	195	46.0170	2	4	1.8163	8	26	13.6305
B44	5	5	5.9677	1.4256	0.00	155	180	30.1549	1	5	2.3372	4	23	10.0189
B45	5	5	5.8710	1.3842	0.00	154	162	10.2862	1	5	2.5661	4	24	11.2124
B46	8	8	8.0645	0.2497	0.00	159	197	41.1302	2	10	6.8400	8	53	42.0245
B47	5	5	5.0968	0.5388	0.00	156	185	49.5067	2	6	2.5703	9	34	21.4733
B48	5	5	5.1290	0.4995	0.00	154	183	34.0827	2	5	2.5493	9	28	13.3359
B49	5	5	5.2581	0.6816	0.00	155	171	26.4985	2	6	4.9605	8	27	22.1096
B50	5	5	5.1935	0.6011	0.00	159	293	137.7965	2	6	4.3729	10	44	30.5390

TABLE 31: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.2903	0.8638	0.00	237	261	14.2311	4	21	9.5117	22	144	67.3566
B52	18	18	18.7742	0.8835	0.00	219	233	14.9013	3	21	10.1319	19	133	65.4978
B53	11	11	11.0000	0.0000	0.00	201	230	12.1596	5	11	3.8566	28	66	25.8455
B54	11	11	11.0000	0.0000	0.00	202	217	12.3461	3	8	2.6401	21	48	16.3395
B55	7	7	7.0000	0.0000	0.00	214	240	12.0357	4	11	4.1032	23	74	26.0514
B56	6	6	6.0000	0.0000	0.00	204	224	12.4441	4	11	4.5166	22	66	28.6479
B57	6	6	6.0323	0.1796	0.00	208	228	12.0374	4	11	3.5765	24	67	21.0756
B58	6	6	6.0000	0.0000	0.00	206	224	11.6898	4	9	4.5009	16	56	28.0782
B59	9	9	9.1290	0.4275	0.00	174	241	39.2589	5	16	9.8178	23	103	60.2449
B60	4	4	4.7419	1.7883	0.00	154	159	3.9053	3	12	5.6472	12	51	24.0830
B61	4	4	4.1613	0.3739	0.00	154	157	4.0611	3	12	7.1637	12	51	29.2026
B62	4	4	4.0000	0.0000	0.00	154	157	4.8422	4	10	5.3876	17	43	22.6520
B63	27	27	27.1935	0.4016	0.00	158	169	5.2271	3	14	7.9795	14	66	35.6641
B64	18	18	18.2903	0.6426	0.00	155	159	2.8364	3	19	10.2648	14	84	42.7305
B65	14	14	14.0968	0.3005	0.00	154	163	8.1524	3	14	7.8763	13	62	35.1641
B66	13	13	13.0000	0.0000	0.00	154	166	13.0996	7	12	3.6136	28	54	17.3759
B67	11	11	11.0968	0.3005	0.00	166	263	83.3463	3	15	9.6607	17	109	100.7377
B68	8	8	8.1935	0.6011	0.00	155	184	23.4952	4	12	6.7003	19	60	35.4280
B69	8	8	8.5806	0.7199	0.00	158	211	64.9205	5	12	6.2117	22	75	65.2722
B70	6	6	6.7097	0.8638	0.00	155	231	135.2276	3	13	6.6360	18	83	66.1519
B71	6	6	6.2258	0.6170	0.00	162	200	53.9548	5	12	8.4985	27	67	46.9196
B72	4	4	4.3226	0.5993	0.00	154	160	4.7503	6	12	5.8701	24	53	24.2337
B73	4	4	4.4839	0.6256	0.00	154	181	34.0598	3	13	7.3084	14	63	39.7601
B74	3	3	3.3871	0.5584	0.00	154	163	4.2860	3	12	6.5822	18	53	27.1781
B75	11	11	11.6129	0.9892	0.00	168	183	13.2764	4	18	9.8581	18	89	45.1418
B76	5	5	5.1935	0.6011	0.00	163	193	35.1540	4	13	6.0851	20	70	40.9876
B77	5	5	5.2581	0.6816	0.00	154	186	23.2987	4	15	8.8359	24	73	39.9299
B78	4	4	4.7097	1.1887	0.00	156	170	10.0336	5	15	6.8506	23	69	31.6560
B79	14	14	14.0000	0.0000	0.00	156	164	6.0066	3	10	3.5484	12	46	15.1387
B80	11	11	11.0000	0.0000	0.00	153	158	6.2756	5	11	3.4811	20	48	14.9292
B81	11	11	11.4839	0.9263	0.00	154	159	5.5222	3	12	7.9278	16	54	31.9301
B82	10	10	10.5484	0.8500	0.00	154	156	2.3344	4	12	5.7675	16	51	24.5353
B83	12	12	12.0000	0.0000	0.00	157	164	4.1408	4	13	6.6827	16	60	29.7937
B84	12	12	12.0645	0.3592	0.00	154	158	5.3961	4	11	5.1974	16	49	22.7398
B85	8	8	8.9032	1.4687	0.00	154	157	2.9860	5	11	4.7201	21	47	19.4751
B86	8	8	8.2258	0.8835	0.00	154	156	2.0765	3	8	4.5371	12	34	18.4101
B87	10	10	10.5161	0.9957	0.00	162	171	4.9756	7	19	8.8350	29	87	40.9288
B88	8	8	8.0000	0.0000	0.00	154	157	2.9647	4	10	4.8012	17	43	20.1409
B89	8	8	8.0000	0.0000	0.00	154	157	3.8529	4	10	4.7071	18	45	19.3560
B90	5	5	5.1935	0.7492	0.00	154	156	1.4398	3	10	4.2256	12	43	17.4126
		AVG	8.1079	0.4472	0.00	169	195	26.3893	2	9	4.8796	13	49	27.5041

TABLE 32: MBO using parallel quick sort algorithm with 16 threads.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	157	177	45.0254	2	6	3.3999	12	34	19.8172
B2	11	11	11.0000	0.0000	0.00	152	157	6.9088	2	3	1.0555	8	16	5.8123
B3	11	11	11.0000	0.0000	0.00	152	157	6.2524	2	4	1.4760	8	20	7.8748
B4	11	11	11.0000	0.0000	0.00	154	178	34.0738	2	4	1.8198	9	22	13.7301
B5	11	11	11.0000	0.0000	0.00	155	176	26.4524	1	5	2.4065	4	25	11.7152
B6	7	7	7.0000	0.0000	0.00	155	197	56.6853	2	9	5.5449	10	47	28.6512
B7	6	6	6.0000	0.0000	0.00	152	203	48.1049	3	7	3.5948	16	41	32.0212
B8	4	4	4.1935	0.6011	0.00	157	173	22.7143	2	9	7.1292	8	42	31.6330
B9	3	3	3.0645	0.2497	0.00	154	166	16.3615	2	6	5.1249	9	27	21.8324
B10	3	3	3.0000	0.0000	0.00	156	184	33.1923	2	4	1.4323	8	24	19.4711
B11	4	4	4.2258	0.4250	0.00	163	232	56.6515	4	10	5.7893	22	62	40.3968
B12	4	4	4.0000	0.0000	0.00	157	187	39.6154	2	5	1.9962	8	27	14.4440
B13	4	4	4.0000	0.0000	0.00	154	168	11.7917	1	6	3.1890	4	28	13.8900
B14	3	3	3.4516	0.5059	0.00	153	196	78.8003	2	5	2.7549	8	31	37.8248
B15	1	1	1.5806	1.0255	0.00	152	175	23.7062	2	5	2.9523	8	29	17.7501
B16	14	14	14.0000	0.0000	0.00	154	191	21.0426	3	8	3.8362	12	43	22.4885
B17	13	13	13.0000	0.0000	0.00	155	177	30.7922	2	4	1.3190	8	22	5.8902
B18	13	13	13.0000	0.0000	0.00	154	167	11.8313	2	5	2.6011	10	26	10.8408
B19	13	13	13.0968	0.5388	0.00	154	170	17.4841	3	8	4.9782	12	36	23.2831
B20	13	13	13.0000	0.0000	0.00	153	166	11.7813	1	6	4.9294	7	31	23.0983
B21	9	9	9.0968	0.3005	0.00	157	183	16.9218	1	9	5.6618	5	46	26.0856
B22	6	6	6.0968	0.3005	0.00	154	177	24.5003	2	5	2.2590	8	25	13.5459
B23	6	6	6.2258	0.4250	0.00	154	165	14.6940	3	8	6.6832	13	36	30.1027
B24	5	5	5.2903	0.5884	0.00	154	168	13.6450	3	6	4.7176	12	29	19.0212
B25	4	4	4.3226	0.6525	0.00	152	173	25.1826	1	5	2.3233	4	24	11.7187
B26	5	5	5.0000	0.0000	0.00	155	177	24.4236	2	7	4.0783	8	32	18.5792
B27	3	3	3.0968	0.3005	0.00	153	165	10.0336	1	6	2.6303	4	28	11.0112
B28	3	3	3.1613	0.4544	0.00	154	175	22.3457	1	6	3.1084	4	27	13.0248
B29	3	3	3.0323	0.1796	0.00	156	166	14.2496	3	5	2.0457	12	23	9.0459
B30	2	2	2.4839	0.6256	0.00	153	162	10.4239	2	8	7.0603	8	34	28.7763
B31	7	7	7.0000	0.0000	0.00	163	175	10.1719	2	8	4.4158	11	37	20.2809
B32	4	4	4.0968	0.5388	0.00	154	171	16.5987	2	6	3.3376	8	28	16.8551
B33	4	4	4.2581	0.6816	0.00	154	165	13.2649	2	5	3.3237	8	22	13.7882
B34	4	4	4.1290	0.4995	0.00	154	175	30.9978	1	5	4.1265	4	27	22.0238
B35	4	4	4.5806	0.9228	0.00	153	178	96.2795	2	5	2.2337	8	23	9.5409
B36	13	13	13.0000	0.0000	0.00	154	167	13.6572	2	9	4.7026	8	40	19.6464
B37	10	10	10.6452	0.9504	0.00	152	156	2.9487	1	5	3.7108	4	23	15.0990
B38	8	8	8.0645	0.2497	0.00	152	155	2.0182	3	11	8.6129	12	45	34.9003
B39	5	5	5.0000	0.0000	0.00	152	156	2.2874	2	4	2.1677	8	20	9.0137
B40	5	5	5.1290	0.4995	0.00	152	155	2.8021	3	6	4.3584	12	25	17.7059
B41	8	8	8.0000	0.0000	0.00	155	160	4.9095	3	9	5.2548	12	39	21.6216
B42	8	8	8.0000	0.0000	0.00	153	157	4.8928	2	4	2.8744	8	20	12.4631
B43	8	8	8.0000	0.0000	0.00	152	163	9.2213	1	5	3.8633	4	24	16.0156
B44	5	5	6.4516	1.5240	0.00	154	171	24.9946	2	5	2.4132	8	23	14.3787
B45	5	5	5.9677	1.4256	0.00	154	168	16.9289	2	6	2.4323	8	26	10.5386
B46	8	8	8.0323	0.1796	0.00	160	192	33.4808	2	10	8.0861	10	56	44.0563
B47	5	5	5.1935	0.7492	0.00	155	176	28.5692	2	5	2.5539	8	27	19.4449
B48	5	5	5.1935	0.6011	0.00	153	166	16.1625	2	5	2.4569	8	25	12.9483
B49	5	5	5.2581	1.1245	0.00	154	185	24.4041	2	6	4.2487	12	34	26.5968
B50	5	5	5.2581	0.6816	0.00	152	163	12.7533	2	5	2.7309	8	24	12.5672

TABLE 32: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.1290	1.1472	0.00	155	163	4.5195	2	18	8.4909	9	81	36.9657
B52	18	18	18.9032	0.9436	0.00	155	159	3.0589	5	20	9.0487	22	86	37.0107
B53	11	11	11.0000	0.0000	0.00	154	158	3.9611	4	10	3.8532	17	42	15.9685
B54	11	11	11.0000	0.0000	0.00	152	156	2.7739	3	8	2.9461	12	36	12.0974
B55	7	7	7.0000	0.0000	0.00	154	159	4.2006	4	9	3.7540	16	40	15.8327
B56	6	6	6.0000	0.0000	0.00	154	159	4.3319	2	11	4.5788	8	46	19.1962
B57	6	6	6.0645	0.2497	0.00	153	159	9.3087	5	12	5.6773	20	51	24.6464
B58	6	6	6.0645	0.2497	0.00	153	157	3.1748	5	10	5.7940	20	44	23.7433
B59	9	9	9.0645	0.2497	0.00	155	160	3.7944	3	13	6.6326	14	58	27.4709
B60	4	4	4.6452	1.7233	0.00	154	158	5.7524	6	14	5.8481	25	58	25.0316
B61	4	4	4.0645	0.2497	0.00	153	157	3.0000	3	11	4.8558	12	48	20.1127
B62	4	4	4.0000	0.0000	0.00	152	155	3.5305	2	10	5.8902	8	44	25.2071
B63	27	27	27.2581	0.4448	0.00	157	162	3.2847	4	17	10.0560	19	75	43.2526
B64	18	18	18.4516	0.9946	0.00	154	157	4.0473	4	20	8.9582	16	86	36.8209
B65	14	14	14.1290	0.3408	0.00	154	156	2.2562	5	18	8.3264	21	75	34.2576
B66	13	13	13.0000	0.0000	0.00	154	156	2.8368	5	10	3.5281	23	45	14.0460
B67	11	11	11.0968	0.3005	0.00	155	159	2.1357	3	11	5.8249	13	50	24.4252
B68	8	8	8.1290	0.3408	0.00	154	158	4.0166	3	13	6.0748	13	56	25.8166
B69	8	8	8.3226	0.5408	0.00	154	157	3.5042	3	10	3.6715	13	44	15.2921
B70	6	6	6.6129	0.6672	0.00	153	156	2.5003	3	12	5.7179	12	52	23.5929
B71	6	6	6.1290	0.4995	0.00	154	158	3.0494	4	12	6.1740	16	53	25.9316
B72	4	4	4.2258	0.4973	0.00	153	158	4.4504	4	13	7.0095	16	54	29.2744
B73	4	4	4.4839	0.7244	0.00	154	157	3.1088	5	15	8.0446	20	62	33.2164
B74	3	3	3.6129	0.6672	0.00	153	156	2.3601	4	14	7.6069	16	59	31.1370
B75	11	11	11.6452	0.9848	0.00	155	162	8.1721	3	16	9.6780	16	71	41.5330
B76	5	5	5.0645	0.3592	0.00	154	168	15.3240	5	12	4.8560	20	56	23.5850
B77	5	5	5.5806	0.9228	0.00	154	171	25.1381	3	15	9.7619	12	66	41.9013
B78	4	4	4.8065	1.2495	0.00	159	182	29.0268	4	12	6.2212	17	60	27.7959
B79	14	14	14.0000	0.0000	0.00	157	169	15.0903	3	9	3.8852	13	44	15.6725
B80	11	11	11.0000	0.0000	0.00	154	162	10.4361	4	11	3.9841	23	47	16.2254
B81	11	11	11.3548	0.8386	0.00	154	172	19.7290	7	15	7.2530	33	71	32.6199
B82	10	10	10.4839	0.7690	0.00	156	199	119.2439	5	12	6.1247	20	80	132.6427
B83	12	12	12.0000	0.0000	0.00	157	188	28.7156	3	11	5.8672	13	56	32.9586
B84	12	12	12.1935	0.6011	0.00	155	180	30.9175	6	11	5.1700	25	54	26.6157
B85	8	8	9.0645	1.4127	0.00	154	162	8.6101	3	12	5.6041	12	54	23.3782
B86	8	8	8.1290	0.7184	0.00	155	178	29.0973	2	10	5.2824	9	47	28.5301
B87	10	10	10.6452	1.0503	0.00	157	180	28.1470	4	15	6.8785	16	72	29.6574
B88	8	8	8.0000	0.0000	0.00	153	194	35.9039	2	9	3.5636	8	51	20.8887
B89	8	8	8.0323	0.1796	0.00	155	183	37.7474	3	10	4.7424	14	52	34.4475
B90	5	5	5.5806	1.2048	0.00	152	178	32.5974	4	10	5.5323	22	51	24.4181
		AVG	8.1072	0.4325	0.00	154	169	18.9543	2	8	4.7619	12	42	23.6231

TABLE 33: MBO using parallel quick sort algorithm with 32 threads.

ID	OPT	Optimum				Execution time			Cycle			Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B1	11	11	11.0000	0.0000	0.00	157	193	67.2126	4	8	3.8529	19	47	23.5622
B2	11	11	11.0000	0.0000	0.00	153	160	6.2348	2	4	1.3190	9	19	7.0225
B3	11	11	11.0000	0.0000	0.00	153	158	4.9071	1	4	1.8020	4	19	7.5696
B4	11	11	11.0000	0.0000	0.00	152	156	2.5745	2	4	1.6428	8	16	6.8760
B5	11	11	11.0000	0.0000	0.00	154	160	5.0292	2	6	3.4962	8	29	14.8304
B6	7	7	7.0000	0.0000	0.00	160	195	70.3551	3	11	7.8781	12	60	43.3428
B7	6	6	6.0000	0.0000	0.00	155	160	3.6423	2	5	2.2538	9	24	9.5139
B8	4	4	4.1935	0.6011	0.00	154	160	4.4473	2	13	10.0692	8	56	41.9610
B9	3	3	3.0323	0.1796	0.00	153	158	3.1302	2	5	2.6901	8	22	11.0625
B10	3	3	3.0000	0.0000	0.00	153	161	5.0956	2	4	1.9092	8	19	8.2968
B11	4	4	4.1613	0.3739	0.00	159	174	8.7291	2	11	9.6157	10	55	44.6467
B12	4	4	4.0000	0.0000	0.00	154	158	3.0515	2	4	2.0483	8	20	8.5759
B13	4	4	4.0645	0.3592	0.00	153	157	3.8356	2	4	1.7677	8	18	7.3472
B14	3	3	3.4516	0.5059	0.00	152	156	3.0789	2	5	3.0164	9	24	12.6226
B15	1	1	1.2581	0.6816	0.00	152	158	7.3659	2	6	2.6069	8	25	11.2249
B16	14	14	14.0000	0.0000	0.00	154	168	8.2101	2	7	3.0670	10	35	13.2306
B17	13	13	13.0000	0.0000	0.00	154	159	3.4867	2	5	1.9389	8	22	8.2489
B18	13	13	13.0000	0.0000	0.00	152	156	2.6437	2	4	2.0288	8	20	8.4173
B19	13	13	13.0000	0.0000	0.00	152	159	6.3924	2	6	2.6567	8	27	11.0241
B20	13	13	13.0000	0.0000	0.00	152	158	8.2973	2	7	3.5360	8	30	14.4425
B21	9	9	9.1290	0.3408	0.00	163	180	10.9634	2	9	6.9766	9	47	33.4055
B22	6	6	6.0645	0.2497	0.00	153	159	3.6133	1	6	3.4591	4	29	14.6736
B23	6	6	6.1935	0.4016	0.00	154	159	5.3000	2	7	5.8411	12	32	24.2679
B24	5	5	5.4516	0.6752	0.00	152	157	3.6320	2	6	2.7658	8	25	11.7582
B25	4	4	4.2258	0.4250	0.00	152	157	3.4273	2	7	4.7573	8	29	19.6816
B26	5	5	5.0323	0.1796	0.00	155	167	7.9712	1	8	5.1750	4	38	22.6477
B27	3	3	3.0645	0.2497	0.00	154	160	5.4513	2	6	4.0284	8	28	17.1102
B28	3	3	3.1290	0.3408	0.00	152	157	3.9081	3	5	1.7096	12	23	7.4421
B29	3	3	3.0645	0.2497	0.00	152	157	4.1860	1	5	2.5911	4	21	10.8956
B30	2	2	2.5806	0.5642	0.00	152	157	3.6694	2	7	6.6775	8	31	28.3555
B31	7	7	7.0000	0.0000	0.00	155	165	6.5364	1	8	4.6596	4	37	21.1130
B32	4	4	4.1935	0.7492	0.00	153	159	4.2621	1	8	5.6591	4	34	23.4602
B33	4	4	4.1290	0.4995	0.00	152	157	3.4093	3	6	3.2015	12	25	13.2534
B34	4	4	4.3226	0.7478	0.00	153	157	3.5469	2	5	4.2812	9	24	17.1984
B35	4	4	4.3871	0.8032	0.00	153	159	6.8737	1	5	2.7591	4	24	11.7173
B36	13	13	13.0000	0.0000	0.00	162	191	55.9882	1	8	5.3864	4	42	27.3086
B37	10	10	10.3226	0.7478	0.00	154	163	7.1865	1	8	7.0555	5	35	30.6683
B38	8	8	8.0000	0.0000	0.00	152	158	3.9570	3	9	6.1737	12	38	25.0810
B39	5	5	5.0000	0.0000	0.00	154	159	4.1968	2	5	2.3047	8	22	10.0090
B40	5	5	5.1935	0.6011	0.00	154	162	8.1181	3	5	1.7506	12	22	7.9040
B41	8	8	8.3226	1.2487	0.00	156	172	11.4272	3	11	7.4741	14	55	38.2555
B42	8	8	8.0000	0.0000	0.00	155	163	9.3587	2	4	2.2855	8	21	10.4073
B43	8	8	8.0000	0.0000	0.00	155	215	74.3419	2	5	2.0483	9	29	16.7208
B44	5	5	5.8710	1.3842	0.00	155	208	74.1396	2	6	3.7788	12	35	20.5935
B45	5	5	6.3548	1.5176	0.00	152	174	54.0282	1	5	2.9910	12	26	14.9598
B46	8	8	8.0323	0.1796	0.00	159	266	116.1782	1	10	5.7489	5	65	52.7578
B47	5	5	5.0968	0.5388	0.00	166	197	82.7696	1	5	2.1287	7	25	11.5699
B48	5	5	5.2581	0.6816	0.00	164	241	91.5422	2	5	2.6392	8	40	34.3044
B49	5	5	5.1935	0.6011	0.00	169	281	103.5425	2	7	4.7853	10	49	38.4886
B50	5	5	5.3226	0.7478	0.00	157	196	37.8474	2	5	3.0547	9	29	15.1232

TABLE 33: Continued.

ID	OPT	Optimum				Execution time				Cycle		Time cycle		
		MBO	AVG	SD	RPD%	BET	AVG	ST	BC	AVG	SD	BTC	AVG	SD
B51	27	27	28.0645	0.8920	0.00	158	207	50.7988	3	20	9.7651	15	115	63.2991
B52	18	18	18.7742	1.0555	0.00	160	232	64.3241	4	20	9.6569	17	124	71.2839
B53	11	11	11.0000	0.0000	0.00	161	174	14.6625	6	11	4.6327	26	50	22.0361
B54	11	11	11.0000	0.0000	0.00	158	193	115.3675	4	8	2.3550	18	40	12.4287
B55	7	7	7.1290	0.7184	0.00	169	180	6.2545	4	12	5.4335	21	59	26.9609
B56	6	6	6.0000	0.0000	0.00	163	174	13.8653	4	9	2.4705	18	42	12.1870
B57	6	6	6.2258	0.6688	0.00	161	178	20.6817	5	11	6.1501	26	55	26.7029
B58	6	6	6.0323	0.1796	0.00	161	166	5.6791	2	9	5.4692	10	40	22.9230
B59	9	9	9.0968	0.3005	0.00	170	178	4.8449	4	12	5.8635	19	60	27.2347
B60	4	4	4.2581	0.9989	0.00	162	172	7.4626	3	11	6.1011	14	54	28.2474
B61	4	4	4.0645	0.2497	0.00	159	167	5.3539	4	11	5.2357	18	49	23.0532
B62	4	4	4.0968	0.3005	0.00	159	174	11.7428	2	11	5.9301	9	51	28.0555
B63	27	27	27.2258	0.4250	0.00	176	183	4.3125	5	18	10.3340	24	89	50.4055
B64	18	18	18.2581	0.6308	0.00	163	197	50.6838	4	16	9.1136	19	82	44.4143
B65	14	14	14.0000	0.0000	0.00	155	165	15.1038	6	16	8.3361	24	69	36.6212
B66	13	13	13.0000	0.0000	0.00	155	164	9.6089	4	9	3.0533	19	42	14.9758
B67	11	11	11.1290	0.3408	0.00	154	172	13.8726	3	13	8.6648	13	62	38.9473
B68	8	8	8.5484	1.4338	0.00	154	161	8.0448	2	13	6.7357	8	57	28.8067
B69	8	8	8.5806	0.7199	0.00	152	189	51.5676	5	11	4.3643	20	55	26.5377
B70	6	6	6.3871	0.6152	0.00	153	177	47.0718	6	13	5.1690	24	66	39.7750
B71	6	6	6.0000	0.0000	0.00	161	218	88.2505	7	12	4.1688	34	75	43.2647
B72	4	4	4.1935	0.4774	0.00	154	171	22.0755	4	13	7.8702	17	64	42.7791
B73	4	4	4.6774	0.5408	0.00	156	207	66.9800	4	15	7.8781	16	90	80.4632
B74	3	3	3.4839	0.6768	0.00	158	225	117.9903	4	13	8.8637	18	69	43.8618
B75	11	11	11.6774	0.8713	0.00	175	251	58.8080	5	17	7.5933	34	110	56.0249
B76	5	5	5.1290	0.4995	0.00	155	179	27.7539	4	13	7.2367	22	63	29.9982
B77	5	5	5.5484	0.9605	0.00	152	159	5.4345	5	12	7.0432	20	51	28.9970
B78	4	4	4.6452	1.1120	0.00	152	163	20.6672	3	11	6.2579	12	49	26.2351
B79	14	14	14.0000	0.0000	0.00	154	165	8.7211	3	9	3.3344	15	42	15.1123
B80	11	11	11.0000	0.0000	0.00	152	214	74.0166	6	10	3.1398	26	58	30.2040
B81	11	11	11.5484	1.0595	0.00	152	192	74.2480	5	16	9.2194	20	85	61.0593
B82	10	10	10.4839	0.7244	0.00	152	163	8.3600	5	11	5.7070	22	50	25.8347
B83	12	12	12.0000	0.0000	0.00	153	159	4.4325	4	13	7.7335	16	55	32.6401
B84	12	12	12.0645	0.3592	0.00	154	162	6.8866	4	11	5.2595	20	51	22.9838
B85	8	8	9.1290	1.4774	0.00	153	160	4.7076	4	13	7.8904	18	56	32.4863
B86	8	8	8.0645	0.2497	0.00	152	173	39.7751	6	11	3.4938	24	53	24.8335
B87	10	10	10.3871	0.8032	0.00	154	247	122.0646	6	18	8.3162	32	127	103.6875
B88	8	8	8.0000	0.0000	0.00	152	160	7.8272	4	9	3.9459	16	40	16.1653
B89	8	8	8.0000	0.0000	0.00	154	159	3.6638	6	10	3.9273	24	42	16.1224
B90	5	5	5.5806	1.2048	0.00	153	158	5.3959	4	10	3.8771	16	45	16.0706
		AVG	8.0953	0.4219	0.00	156	177	25.8717	3	9	4.8993	13	46	25.9852

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References

- [1] C. Mosier and L. Taube, "The facets of group technology and their impacts on implementation—a state-of-the-art survey," *Omega*, vol. 13, no. 5, pp. 381–391, 1985.
- [2] F. F. Boctor, "A linear formulation of the machine-part cell formation problem," *International Journal of Production Research*, vol. 29, no. 2, pp. 343–356, 1991.
- [3] O. Durán, N. Rodríguez, and L. A. Consalter, "Collaborative particle swarm optimization with a data mining technique for manufacturing cell design," *Expert Systems with Applications*, vol. 37, no. 2, pp. 1563–1567, 2010.
- [4] R. Soto, B. Crawford, E. Vega, and F. Paredes, "Solving manufacturing cell design problems using an artificial fish swarm algorithm," in *Advances in Artificial Intelligence and Soft Computing*, pp. 282–290, Springer, 2015.
- [5] G. F. K. Purcheck, "A linear-programming method for the combinatorial grouping of an incomplete power set," *Journal of Cybernetics*, vol. 5, no. 4, pp. 51–76, 1975.
- [6] S. M. Shafer and D. F. Rogers, "A goal programming approach to the cell formation problem," *Journal of Operations Management*, vol. 10, no. 1, pp. 28–43, 1991.
- [7] J. A. Joines, M. G. Kay, R. E. King, and C. T. Culbreth, "A hybrid genetic algorithm for manufacturing cell design," *Journal of the Chinese Institute of Industrial Engineers*, vol. 17, no. 5, pp. 549–564, 2000.
- [8] R. Soto, H. Kjellerstrand, O. Durán, B. Crawford, E. Monfroy, and F. Paredes, "Cell formation in group technology using constraint programming and boolean satisfiability," *Expert Systems with Applications*, vol. 39, no. 13, pp. 11423–11427, 2012.
- [9] R. Soto, H. Kjellerstrand, J. Gutiérrez, A. López, B. Crawford, and E. Monfroy, "Solving manufacturing cell design problems using constraint programming," in *Advanced Research in Applied Artificial Intelligence*, H. Jiang, W. Ding, M. Ali, and X. Wu, Eds., vol. 7345 of *Lecture Notes in Computer Science*, pp. 400–406, Springer, 2012.
- [10] H. M. Selim, R. G. Askin, and A. J. Vakharia, "Cell formation in group technology: review, evaluation and directions for future research," *Computers and Industrial Engineering*, vol. 34, no. 1, pp. 3–20, 1998.
- [11] G. Papaioannou and J. M. Wilson, "The evolution of cell formation problem methodologies based on recent studies (1997–2008): review and directions for future research," *European Journal of Operational Research*, vol. 206, no. 3, pp. 509–521, 2010.
- [12] S. Kirkpatrick, J. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, no. 4598, pp. 671–680, 1983.
- [13] V. Černý, "Thermodynamical approach to the traveling salesman problem: an efficient simulation algorithm," *Journal of Optimization Theory and Applications*, vol. 45, no. 1, pp. 41–51, 1985.
- [14] T.-H. Wu, C.-C. Chang, and S.-H. Chung, "A simulated annealing algorithm for manufacturing cell formation problems," *Expert Systems with Applications*, vol. 34, no. 3, pp. 1609–1617, 2008.
- [15] F. Glover, "Future paths for integer programming and links to artificial intelligence," *Computers and Operations Research*, vol. 13, no. 5, pp. 533–549, 1986.
- [16] S. Lozano, B. Adenso-Díaz, I. Eguia, and L. Onieva, "A one-step tabu search algorithm for manufacturing cell design," *Journal of the Operational Research Society*, vol. 50, no. 5, pp. 509–516, 1999.
- [17] T.-H. Wu, C. Low, and W.-T. Wu, "A tabu search approach to the cell formation problem," *The International Journal of Advanced Manufacturing Technology*, vol. 23, no. 11–12, pp. 916–924, 2004.
- [18] H. R. Lourenço, O. C. Martin, and T. Stützle, *Iterated Local Search*, Springer, 2003.
- [19] H. R. Lourenço, O. C. Martin, and T. Stützle, "Iterated local search: framework and applications," in *Handbook of Metaheuristics*, pp. 363–397, Springer, 2010.
- [20] P. Hansen, N. Mladenović, and J. A. M. Pérez, "Variable neighbourhood search: methods and applications," *Annals of Operations Research*, vol. 175, no. 1, pp. 367–407, 2010.
- [21] A. Colomi, M. Dorigo, and V. Maniezzo, "Distributed optimization by ant colonies," in *Proceedings of the First European Conference on Artificial Life (ECAL '91)*, vol. 142, pp. 134–142, Paris, France, 1991.
- [22] X. Li, M. F. Baki, and Y. P. Aneja, "An ant colony optimization metaheuristic for machine-part cell formation problems," *Computers & Operations Research*, vol. 37, no. 12, pp. 2071–2081, 2010.
- [23] J. Kennedy, "Particle swarm optimization," in *Encyclopedia of Machine Learning*, pp. 760–766, Springer, 2010.
- [24] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," in *Proceedings of the IEEE World Congress on Computational Intelligence, IEEE International Conference on Evolutionary Computation*, pp. 69–73, May 1998.
- [25] S.-C. Chu, P.-W. Tsai, and J.-S. Pan, "Cat swarm optimization," in *PRICAI 2006: Trends in Artificial Intelligence*, Q. Yang and G. Webb, Eds., vol. 4099 of *Lecture Notes in Computer Science*, pp. 854–858, Springer, 2006.
- [26] P.-W. Tsai and V. Istanda, "Review on cat swarm optimization algorithms," in *Proceedings of the 3rd International Conference on Consumer Electronics, Communications and Networks (CEC-Net '13)*, pp. 564–567, IEEE, Xianning, China, November 2013.
- [27] E. Duman, M. Uysal, and A. F. Alkaya, "Migrating birds optimization: a new metaheuristic approach and its performance on quadratic assignment problem," *Information Sciences*, vol. 217, pp. 65–77, 2012.
- [28] Q.-K. Pan and Y. Dong, "An improved migrating birds optimisation for a hybrid flowshop scheduling with total flowtime minimisation," *Information Sciences*, vol. 277, pp. 643–655, 2014.
- [29] S. Niroomand, A. Hadi-Vencheh, R. Şahin, and B. Vizvári, "Modified migrating birds optimization algorithm for closed loop layout with exact distances in flexible manufacturing systems," *Expert Systems with Applications*, vol. 42, no. 19, pp. 6586–6597, 2015.
- [30] R. Soto, B. Crawford, B. Almonacid, and F. Paredes, "A migrating birds optimization algorithm for machinepart cell formation problems," in *Advances in Artificial Intelligence and Soft Computing*, vol. 9413, pp. 270–281, Springer, Berlin, Germany, 2015.

- [31] I. Fister Jr., X.-S. Yang, I. Fister, J. Brest, and D. Fister, "A brief review of nature-inspired algorithms for optimization," *Electrotechnical Review*, vol. 80, no. 3, pp. 116–122, 2013.
- [32] S. M. Almeida-Luz, M. M. Rodríguez-Hermoso, M. A. Vega-Rodríguez, J. Gómez-Pulido, and J. M. Sánchez-Pérez, "GRASP and grid computing to solve the location area problem," in *Proceedings of the World Congress on Nature & Biologically Inspired Computing (NaBIC '09)*, pp. 164–169, Coimbatore, India, December 2009.
- [33] J.-H. Byun, K. Datta, A. Ravindran, A. Mukherjee, and B. Joshi, "Performance analysis of coarse-grained parallel genetic algorithms on the multi-core Sun UltraSPARC T1," in *Proceedings of the IEEE (SOUTHEASTCON '09)*, pp. 301–306, IEEE, Atlanta, Ga, USA, March 2009.
- [34] J. Homberger, "A parallel genetic algorithm for the multilevel unconstrained lot-sizing problem," *INFORMS Journal on Computing*, vol. 20, no. 1, pp. 124–132, 2008.
- [35] C. Leon, G. Miranda, and C. Segura, "A memetic algorithm and a parallel hyperheuristic island-based model for a 2D packing problem," in *Proceedings of the 11th Annual Genetic and Evolutionary Computation Conference (GECCO '09)*, pp. 1371–1378, ACM, July 2009.
- [36] J. Jin, T. G. Crainic, and A. Løkketangen, "A cooperative parallel metaheuristic for the capacitated vehicle routing problem," *Computers & Operations Research*, vol. 44, pp. 33–41, 2014.
- [37] J.-M. Cutillas-Lozano and D. Giménez, "Optimizing shared-memory hyperheuristics on top of parameterized metaheuristics," *Procedia Computer Science*, vol. 29, pp. 20–29, 2014.
- [38] E. Alba, G. Luque, and S. Nesmachnow, "Parallel metaheuristics: recent advances and new trends," *International Transactions in Operational Research*, vol. 20, no. 1, pp. 1–48, 2013.
- [39] M.-C. Albutiu, A. Kemper, and T. Neumann, "Massively parallel sort-merge joins in main memory multi core database systems," *Proceedings of the VLDB Endowment*, vol. 5, no. 10, pp. 1064–1075, 2012.
- [40] A. Davidson, D. Tarjan, M. Garland, and J. D. Owens, "Efficient parallel merge sort for fixed and variable length keys," in *Proceedings of the Innovative Parallel Computing (InPar '12)*, pp. 1–9, IEEE, San Jose, Calif, USA, May 2012.
- [41] D. R. Helman, D. A. Bader, and J. Jájá, "A randomized parallel sorting algorithm with an experimental study," *Journal of Parallel and Distributed Computing*, vol. 52, no. 1, pp. 1–23, 1998.
- [42] H. Peters, O. Schulz-Hildebrandt, and N. Luttenberger, "A novel sorting algorithm for many-core architectures based on adaptive bitonic sort," in *Proceedings of the IEEE 26th International Parallel & Distributed Processing Symposium (IPDPS '12)*, pp. 227–237, IEEE, 2012.
- [43] I. S. Rajput, B. Kumar, and T. Singh, "Performance comparison of sequential quick sort and parallel quick sort algorithms," *International Journal of Computer Applications*, vol. 57, no. 9, pp. 14–22, 2012.
- [44] V. Prifti, R. Bala, I. Tafa, D. Saatu, and J. Fejzaj, "The time profit obtained by parallelization of quicksort algorithm used for numerical sorting," in *Proceedings of the Science and Information Conference (SAI '15)*, pp. 897–901, London, UK, July 2015.
- [45] P. Sanders and T. Hansch, "Efficient massively parallel quicksort," in *Solving Irregularly Structured Problems in Parallel*, vol. 1253 of *Lecture Notes in Computer Science*, pp. 13–24, Springer, Berlin, Germany, 1997.
- [46] P. Tsigas and Y. Zhang, "A simple, fast parallel implementation of quicksort and its performance evaluation on sun enterprise 10000," in *Proceedings of the 11th Euromicro Conference on Parallel, Distributed and Network-Based Processing*, pp. 372–381, IEEE, Genova, Italy, February 2003.
- [47] E. Duman and I. Elikucuk, "Solving credit card fraud detection problem by the new metaheuristics migrating birds optimization," in *Advances in Computational Intelligence*, vol. 7903 of *Lecture Notes in Computer Science*, pp. 62–71, Springer, Berlin, Germany, 2013.
- [48] K. Gao, P. Suganthan, and T. Chua, "An enhanced migrating birds optimization algorithm for no-wait ow shop scheduling problem," in *Proceedings of the IEEE Symposium on Computational Intelligence in Scheduling (SCIS '13)*, pp. 9–13, IEEE, 2013.
- [49] L. W. Shen, H. Asmuni, and F. C. Weng, "A modified migrating bird optimization for university course timetabling problem," *Jurnal Teknologi*, vol. 72, no. 1, 2014.
- [50] P. B. S. Lissaman and C. A. Shollenberger, "Formation flight of birds," *Science*, vol. 168, no. 3934, pp. 1003–1005, 1970.
- [51] J. P. Badgerow and F. R. Hainsworth, "Energy savings through formation flight? A re-examination of the vee formation," *Journal of Theoretical Biology*, vol. 93, no. 1, pp. 41–52, 1981.
- [52] I. L. Bajec and F. H. Heppner, "Organized flight in birds," *Animal Behaviour*, vol. 78, no. 4, pp. 777–789, 2009.
- [53] D. Hummel and M. Beukenberg, "Aerodynamische interferenzeffekte beim formationsflug von Vögeln," *Journal of Ornithology*, vol. 130, no. 1, pp. 15–24, 1989.
- [54] J. Rayner, "A new approach to animal flight mechanics," *Journal of Experimental Biology*, vol. 80, no. 1, pp. 17–54, 1979.
- [55] B. Almonacid, *Dataset—Efficient Parallel Sorting for Migrating Birds Optimization when Solving Machine-Part Cell Formation Problems*, Pontificia Universidad Católica de Valparaíso, Valparaíso, Chile, 2016, <http://www.inf.ucv.cl/~balmonacid/MPCFP/ParallelMBO2016>.