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Research Article

Statistical Design of an Adaptive Synthetic $\overline{X}$ Control Chart with Run Rule on Service and Management Operation

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An improved synthetic $\overline{X}$ control chart based on hybrid adaptive scheme and run rule scheme is introduced to enhance the statistical performance of traditional synthetic $\overline{X}$ control chart on service and management operation. The proposed scientific hybrid adaptive schemes consider both variable sampling interval and variable sample size scheme. The properties of the proposed chart are obtained using Markov chain approach. An extensive set of numerical results is presented to test the effectiveness of the proposed model in detecting small and moderate shifts in the process mean. The results show that the proposed chart is quicker than the standard synthetic $\overline{X}$ chart and CUSUM chart in detecting small and moderate shifts in the process of service and management operation.

1. Introduction

Since control chart was introduced by Walter A. Shewhart in 1924, it has been treated as an important tool to detect the process shifts that may occur in services and managements operation process. In today's service and management operation practice, processes have obtained a low level of nonconformities or defects as a result of technological advancement and automation. Although the traditional Shewhart charts have the advantage of easy setup, they also have the high false alarm rates and inability to detect further process improvement under a low-defect environment [1].

To deal with this situation, different types of control charts have been proposed to obtain good performance for effective detection with a wide range of shift sizes, such as cumulative sum (CUSUM) chart, synthetic $\overline{X}$ chart, and time-between-events (TBE) chart. Among them, the synthetic $\overline{X}$ chart which was introduced by Wu and Spedding (2000) outperformed the traditional $\overline{X}$ chart in terms of smaller average run length under the small shift in process mean. It is the combination of Shewhart $\overline{X}$ chart and conforming run length (CRL) chart used to detect shifts in the process mean. The difference between Shewhart $\overline{X}$ charts and synthetic $\overline{X}$ charts is that the synthetic $\overline{X}$ charts do not send an alarm instantly when a sample falls outside of the limits but inspect the number of the samples taken since the last time that a point fell outside the limits, or since the first sample if there have been no previous points outside the limits. If that number of samples is sufficiently small, then a signal is triggered. The synthetic $\overline{X}$ chart provides a significantly better detection power than Shewhart $\overline{X}$ chart for all levels of mean shifts [2]. Numerous studies and extensions have been performed on the synthetic chart. Among the more recent ones are Chen et al. [3], Zhang et al. [4], Yen et al. [5], and Zhen [6].

On the other hand, to enhance further detection of the power of control charts for better process control, the adaptive control charts in which at least one of input parameters (the sampling interval, the sample size, and the control limits) is allowed to be changed based on the current state of the process are proposed. Some common adaptive control charts are often concerned in SPC studies, such as the variable sample size (VSS) control charts (see [7–10]), the variable sampling interval (VSI) control charts (see [11, 12]), and the variable sample size and sampling interval (VSSI) control charts (see [7, 8, 13, 14]). It is found that totally adaptive

control charts have been shown to detect the process change faster than the corresponding standard Shewhart chart [15]. Qu and Meng [16], Qu et al. [17], Zhen [18], and Zhen [19] used the fundamental diagrams for extreme-scenario analysis on transportation. Among these different types of adaptive charts, the VSSI $\overline{X}$ chart is even quicker than the VSI or VSS $\overline{X}$ charts in detecting moderate shifts in the process [20].

Some researchers tried to combine together the standard synthetic $\overline{X}$ chart with adaptive schemes. Khoo et al. [21] proposed a synthetic double sampling chart that integrated the double sampling (DS) $\overline{X}$ chart and CRL chart and they concluded that the synthetic DS chart is superior to the synthetic or even the DS $\overline{X}$ chart. Chen and Huang [22] considered the variable sampling interval scheme as an enhancement to their proposed synthetic chart in order to further improve the chart's performance. Lee and Lim [23] proposed a VSSI-CRL synthetic control chart and they concluded that it has better detection power than the CRL synthetic chart or the VSSI chart in general. Costa and Rahim [24] considered the synthetic chart based on the statistic T to monitor both the mean and variance. They found that their proposed chart always detected process disturbances faster than the joint $\overline{X}$ and R charts. Machado et al. [25] proposed a synthetic control chart based on two sample variances for monitoring the covariance matrix. The proposed chart was thought to be more efficient than the chart based on the generalized variance $|S|$. In this work, the schemes that consider both variable sampling interval and variable sample size combined with run rules are applied to the standard synthetic $\overline{X}$ chart for obtaining better detection capacity.

In this paper, an adaptive synthetic $\overline{X}$ chart with a joint sampling strategy combining variable sampling interval and variable sample size is developed. A redefined running rule scheme is adopted to further improve the performance of the control chart. Compared with the work of Lee and Lim (2005), different running rules are embedded in the proposed synthetic chart. In our model, the chart sends an alarm not only when the conforming run length is sufficiently small but also when the measuring index outsides the control limits. Undoubtedly the detection capability is enhanced, but the problem becomes even more complicated. In this work, we present a Markov chain model and use it to evaluate the zero-state and steady-state average time to signal (ATS) performance of the proposed chart. The numerical results show that the proposed chart has achieved better detection power than the traditional synthetic $\overline{X}$ chart and CUSUM chart in detecting small and moderate shifts in the process.

The rest of the paper is organized as follows: in the next Section, the formulation of the proposed chart is developed; then the design model is presented. In Section 3, the genetic algorithm is used to solve the statistical design model; and the obtained results are reported and discussed. Finally Section 4 concludes the paper.

2. Description of the Developed Chart

Assuming that the production process starts in in-control (healthy) state, with the in-control mean μ_0 and the in-control standard deviation σ_0, the one key quality characteristic X is assumed to follow an identical and independent normal distribution $N(\mu_0, \sigma_0^2)$. When process shift occurs, the mean μ will change: that is, $\mu = \mu_0 + \delta\sigma_0$ ($\delta \geq 0$), where δ is the magnitude of the process shift. Since quality shifts are not directly observable yet undesirable, the process is monitored by a control chart. At each sampling instance, a sample is taken and the sample statistic, $\overline{X}$, is computed and plotted on the control chart. Then the control chart gives an indication for the actual process condition. In this work, an adaptive synthetic $\overline{X}$ chart is proposed to monitor the process and two alternative combinations of the control chart parameters are considered: a relaxed one and a tightened one. The relaxed scheme uses sampling interval h_L, sample size n_S, while the respective parameter values for the tightened scheme are h_S, n_L, where $h_L > h_S > 0$, $n_L > n_S > 0$.

If we set m as the lower limit of the CRL subchart and set the constant $k_1 > k_2 > 0$ as the parameters of the $\overline{X}$ subchart, then the lower control limit, upper control limit, and the warning control limit of the adaptive synthetic $\overline{X}$ are given by

$$\begin{aligned} \text{UCL} &= \mu_0 + k_1\sigma_{\overline{X}}, \\ \text{UWL} &= \mu_0 + k_2\sigma_{\overline{X}}, \\ \text{LCL} &= \mu_0 - k_1\sigma_{\overline{X}}, \\ \text{LWL} &= \mu_0 - k_2\sigma_{\overline{X}}, \end{aligned} \tag{1}$$

where $\sigma_{\overline{X}}$ is the in-control standard deviation of the sample mean.

By referring to the graphical view of the chart in Figure 1, the operation of the proposed chart is as follows:

(1) Set the control limits of the charts.

(2) A sample is taken and its mean ($\overline{X}$) is measured at each inspection instance.

(3) If a sample produces a value between in the central region, that is, (UWL, LWL), the process is in control. The relaxed scheme used for next sampling is (n_S, h_L).

(4) If the sample mean lies outside the limits, that is, (UCL, $+\infty$) or ($-\infty$, LCL), the control chart gives a signal. The process is declared as out-of-control and an investigation and possible restoration take place.

(5) If the sample mean falls in the warning region, that is, (UWL, UCL) or (LCL, LWL), then the CRL is checked. In this work, CRL is defined as the number of samples since the most recent previous sample mean fell in the warning region or since sampling began if no point fell in the warning region. It is obvious that CRL is a larger-the-better characteristic.

(a) If CRL is larger than the lower control limit m, where m is a specified positive integer, the process is still considered as in-control, but the tightened sampling scheme (n_L, h_S) is used in next sampling.

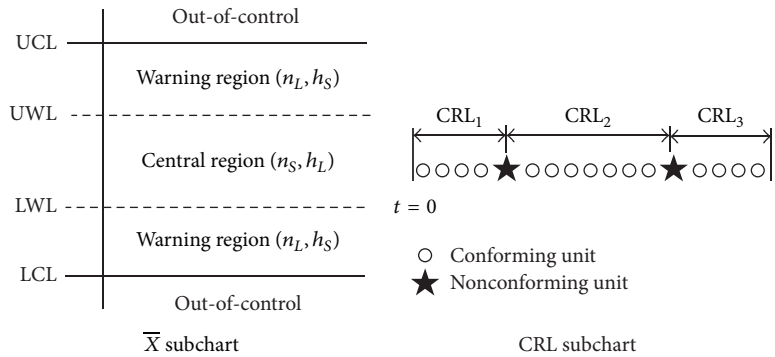

Figure 1: The adaptive synthetic X-bar chart.

(b) If CRL is smaller than the lower control limit m, then the process is signaled out-of-control, and an investigation and possible restoration take place. After that the control flow goes back to Step (2) and the relaxed sampling plan is used.

It is worth noting that as m increases, the adaptive synthetic $\overline{X}$ control chart behaves more and more like an ordinary VSSI $\overline{X}$ chart. The most important performance metrics of control charts are average run length (ARL) and average time to signal (ATS) in a long-run process [26]. ARL is commonly studied under two cases in literatures: one when process is in control (denoted by ARL_0) and the other when the process is out-of-control (denoted by ARL_1). Usually, the ARL_0 of the chart is preferred to be long to avoid high frequency of false alarms and ARL_1 to be short to reduce the number of produced nonconforming units. ATS is presented by Tagaras [27], which is defined as the expected value of the time from the start of the process to the time when the chart indicates an out-of-control signal. Correspondingly, denote ATS_0 and ATS_1 as the average time to signal when the process is in-control and out-of-control, respectively.

2.1. Computation of ARL and ATS of the Proposed Chart. Assuming that a single assignable cause occurs at a random time and results in a shift in the process mean of a known magnitude δ ($\delta > 0$) so that the out-of-control mean value is $\mu_1 = \mu_0 + \delta\sigma_0$, the occurrence of the assignable cause indicates that the process has gone out of control. The Markov chain approach suggested by Davis and Woodall [28] is used to compute the in-control and out-of-control average run lengths in the process.

Denote p_1^0 (p_1^1) as the probability that a sample falls beyond the control limits of $\overline{X}$ subchart when the last sample falls in the central (warning) region. Then the detecting power, p_1^0 and p_1^1, of $\overline{X}$ subchart can be calculated as

$$\begin{aligned} p_1^0 &= 1 - \Phi\left(k_1 - \delta\sqrt{n_S}\right) + \Phi\left(-k_1 - \delta\sqrt{n_S}\right), \\ p_1^1 &= 1 - \Phi\left(k_1 - \delta\sqrt{n_L}\right) + \Phi\left(-k_1 - \delta\sqrt{n_L}\right), \end{aligned} \tag{2}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal distribution function.

Denote p_2^0 (p_2^1) as the probability that a sample falls in warning region of $\overline{X}$ subchart when the last sample falls in the central (warning) region. p_2^0 and p_2^1 are given by

$$\begin{aligned} p_2^0 &= \Phi\left(k_1 - \delta\sqrt{n_S}\right) - \Phi\left(k_2 - \delta\sqrt{n_S}\right) \\ &\quad + \Phi\left(-k_2 - \delta\sqrt{n_S}\right) - \Phi\left(-k_1 - \delta\sqrt{n_S}\right), \\ p_2^1 &= \Phi\left(k_1 - \delta\sqrt{n_L}\right) - \Phi\left(k_2 - \delta\sqrt{n_L}\right) \\ &\quad + \Phi\left(-k_2 - \delta\sqrt{n_L}\right) - \Phi\left(-k_1 - \delta\sqrt{n_L}\right). \end{aligned} \tag{3}$$

Denote p_3^0 (p_3^1) as the probability that a sample falls in central region of $\overline{X}$ subchart when the last sample falls in the central (warning) region. p_3^0 and p_3^1 can be calculated as

$$\begin{aligned} p_3^0 &= \Phi\left(k_2 - \delta\sqrt{n_S}\right) - \Phi\left(-k_2 - \delta\sqrt{n_S}\right), \\ p_3^1 &= \Phi\left(k_2 - \delta\sqrt{n_L}\right) - \Phi\left(-k_2 - \delta\sqrt{n_L}\right). \end{aligned} \tag{4}$$

A Markov chain $\{N(i),\ i \geq 1\}$ is constructed, where $N(i)$ is the number of samples which fall in the central region between the ith and the $(i-1)$th sample which falls in the warning region of the $\overline{X}$ subchart. Then the state spaces of $\{N(i),\ i \geq 1\}$ are $\{0, 1, 2, \ldots, m-1, m, S\}$, where state S represents that the control chart sends out out-of-control signal. Therefore, we can model the adaptive synthetic $\overline{X}$ chart using a $\{m+2, m+2\}$ transition probability matrix P having the following structure:

$$\begin{aligned} P &= \begin{pmatrix} 0 & p_3^1 & 0 & \cdots & \cdots & 0 & p_2^1 + p_1^1 \\ 0 & 0 & p_3^0 & \cdots & \cdots & 0 & p_2^0 + p_1^0 \\ \vdots & & \ddots & \ddots & & \vdots & \vdots \\ \vdots & & & \ddots & p_3^0 & 0 & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & p_3^0 & p_2^0 + p_1^0 \\ p_2^0 & 0 & \cdots & \cdots & 0 & p_3^0 & p_1^0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} Q & r \\ 0^T & 1 \end{pmatrix}, \end{aligned} \tag{5}$$

where $0 = (0, 0, \ldots, 0)^T$ and Q is the $(m+1, m+1)$ matrix of transient probabilities. The $(m+1, 1)$ vector $r = 1 - Q \cdot 1$ (i.e., the row probabilities must sum to 1) with $1 = (1, 1, \ldots, 1)^T$. The expected average run length is given by

$$\mathrm{ARL}_1 = q^T (I - Q)^{-1} 1, \tag{6}$$

where q is a $(m+1, 1)$ vector of initial probabilities associated with the transient states, with 1 for the initial state and 0 elsewhere, that is, $q = (1, 0, \ldots, 0)^T$, and I is a $(m+1, m+1)$ identity matrix.

The steady-state probability of the process is required due to the uncertainty of the instantaneous probability of the process in each state. $\pi = \{\pi_0, \pi_1, \ldots, \pi_m\}$ is represented as the corresponding steady-state probability of the state space. According to the Markov theory, spreading the above matrix, we obtain the following results:

$$\begin{aligned} \pi_0 &= \frac{1 - p_3^0}{1 - p_3^0 + p_3^1}, \\ \pi_i &= \frac{p_3^1 \left(p_3^0\right)^{i-1} \left(1 - p_3^0\right)}{1 - p_3^0 + p_3^1}, \quad i = 1, 2, \ldots, m-1, \\ \pi_m &= \frac{p_3^1 \left(p_3^0\right)^{m-1}}{1 - p_3^0 + p_3^1}. \end{aligned} \tag{7}$$

Then when the magnitude of the process shift is δ (≥ 0), the expected average sampling interval $E_h[\delta]$ and the expected average sample size $E_n[\delta]$ are calculated as

$$\begin{aligned} E_h[\delta] &= \pi_0 h_S + \left(\pi_1 + \pi_2 + \cdots + \pi_m\right) h_L, \\ E_n[\delta] &= \pi_0 n_L + \left(\pi_1 + \pi_2 + \cdots + \pi_m\right) n_S. \end{aligned} \tag{8}$$

The total expected average sample size of the process can be expressed as

$$E_n = p_c E_n[\delta = 0] + \left(1 - p_c\right) E_n[\delta > 0], \tag{9}$$

where p_c is the probability of the process in control at arbitrary time.

The out-of-control average run length, ARL_1, is calculated by substituting $\delta > 0$, while the in-control average run length, ARL_0, is computed by substituting $\delta = 0$. The ATS of the adaptive synthetic $\overline{X}$ chart is given by

$$\mathrm{ATS} = \mathrm{ARL} \times E_h[\delta]. \tag{10}$$

Similarly, ATS_1 is calculated by substituting $\delta > 0$, while ATS_0 is computed by substituting $\delta = 0$.

2.2. Design Model. The statistical design of the proposed synthetic $\overline{X}$ chart can be conducted using the following optimization model:

Objective function is

$$\mathrm{Min}\ \mathrm{ATS}_1. \tag{11}$$

Table 1: The optimal parameters and the values of ATS_1.

δ	(n_S, n_L)	(h_L, h_S)	(k_1, k_2)	m	ATS_1
0.1	(12, 14)	(6.5031, 3.3638)	(7.1368, 1.9826)	9	193.6936
0.3	(11, 12)	(2.5894, 0.6658)	(8.1090, 2.1697)	8	25.5154
0.5	(11, 15)	(2.8284, 0.3193)	(6.2454, 2.0591)	5	4.8198
0.6	(12, 14)	(2.5849, 0.2366)	(7.9035, 2.1941)	9	2.7954
0.8	(10, 13)	(1.3651, 0.1589)	(3.9953, 2.2805)	7	0.7449
1.0	(8, 12)	(1.4749, 0.1375)	(7.7022, 2.1316)	3	0.2966
1.5	(12, 14)	(5.7517, 0.1009)	(6.4527, 2.2539)	23	0.1031

Constraints function is

$$\begin{aligned} \mathrm{ATS}_0 &\geq \tau; \\ h_L &> h_S > 0; \\ n_L &> n_S > 0; \\ k_1 &> k_2 > 0; \\ n_{\max} &\geq E_n > 0; \\ n_S, n_L, m &\in \mathrm{IN}. \end{aligned} \tag{12}$$

Design variable is

$$k_1, k_2, h_L, h_S, n_L, n_S, m, \tag{13}$$

where τ and $n_{\max}$ are, respectively, the allowed minimum in-control average run length to signal and the maximum sample size. The value of τ is usually decided by the quality assurance (QA) engineer with regard to the false alarm rate.

3. Numerical Analysis

3.1. Optimization Algorithm. The established design model for the control chart is a nonlinear programming model with mixed continuous discrete variables, which is too complex to be solved in optimality. Therefore, metaheuristic methods, especially genetic algorithms (GA), were commonly used to solve the problem. GA has been commonly used for its adaptiveness and effectiveness. Successful applications of GA in the designs of control charts can be found in Aparisi and García-Díaz [29] and He and Grigoryan [30]. In this study, GA toolbox of the University of Sheffield is developed to solve the optimal statistic designs of the proposed chart.

3.2. The Statistical Performance. In this section, we evaluate the statistical performances of the proposed chart. Table 1 shows the optimal ATS_1 of the proposed synthetic $\overline{X}$ chart; moreover, each row in the table shows the optimal design parameters.

From Table 1, it is found that the value of ATS_1 of the proposed chart decreases along with an increase in the magnitude of the shift (δ). A significant change occurs when the value of δ changes from small to moderate. That is to say, the proposed chart is sensitive to the change of mean shift when the process is out-of-control. On the other hand, when the mean shift changes from moderate to large, the detection

Table 2: Comparison among the optimal ATS_1 corresponding to the two charts.

	Optimal value of ATS_1 for each case					
Cases	$E_n = 3$		$E_n = 5$		$E_n = 9$	
δ	Adap-Syn	Trad-Syn	Adap-Syn	Trad-Syn	Adap-Syn	Trad-Syn
0.1	375.3452	847.1256	300.2754	700.4593	255.5710	603.5124
0.3	115.4521	211.3375	78.0624	144.7012	43.6432	61.6817
0.5	48.2578	103.8345	28.1089	50.0896	12.1607	16.0863
0.6	33.7869	67.7789	15.7966	21.2451	4.3920	8.0664
0.8	10.3149	42.0031	3.6467	10.0180	2.1909	4.7890
1.0	5.0379	20.1121	2.3129	3.4568	1.1089	1.2143
1.5	1.5287	4.2857	0.5903	0.6752	0.1151	0.2137

Notes: Adap-Syn and Trad-Syn represent the proposed adaptive synthetic $\overline{X}$ chart and the traditional synthetic $\overline{X}$ chart, respectively.

Table 3: Results of the proposed chart and CUSUM chart for comparison.

	Optimal value of ATS_1 for each case					
Cases	$E_n = 3$		$E_n = 5$		$E_n = 9$	
δ	Adap-Syn	CUSUM	Adap-Syn	CUSUM	Adap-Syn	CUSUM
0.1	375.3452	415.2016	300.2754	325.4593	255.5710	275.5124
0.3	115.4521	121.3375	78.0624	85.7012	43.6432	48.6817
0.5	48.2578	52.8345	28.1089	42.0896	12.1607	13.0863
0.6	33.7869	38.7789	15.7966	21.2451	4.3920	5.0664
0.8	10.3149	12.0031	3.6467	4.0180	2.1909	2.0890
1.0	5.0379	6.1121	2.3129	2.8568	1.1089	0.8143
1.5	1.5287	2.2857	0.5903	0.6752	0.1151	0.1037

power of the proposed chart is improved slightly, which is in line with the actual situation.

In actual production, 100% sampling is not possible and the assignable causes are not self-announced; therefore, the average sample size, E_n, is usually an important matter to QA engineer. In Table 2, we compare the optimal value of ATS_1 between the proposed synthetic $\overline{X}$ chart and traditional synthetic $\overline{X}$ chart in three cases ($E_n = \{3, 5, 9\}$). It is reasonable that the optimal value of ATS_1 decreases along with an increase in the average sample size (E_n). At the same time, it can be noted that the proposed chart generally achieved shorter ATS_1 than the traditional synthetic chart, showing that the variable sample size and sampling interval scheme is helpful in improving the statistical performance of the chart.

3.3. Comparison to CUSUM Chart. In this section, the proposed chart is compared with the cumulative sum (CUSUM) chart in terms of the average time to signal when process is out-of-control. Woodall and Adams [31] recommended use of the ARL approximation given by *Siegmund* for designing a CUSUM chart. For a one-sided CUSUM with parameters l and k, *Siegmund* gives

$$\mathrm{ARL}_U = \frac{e^{-2\Delta b} + 2\Delta b - 1}{2\Delta^2} \tag{14}$$

for $\Delta \neq 0$, where $\Delta = (\mu_1 - \mu_0)/\sigma - k$ for the upper one-side CUSUM, and $b = l + 1.166$. Similarly, ARL_L can be obtained for the lower one-sided CUSUM. Then we can achieve the ATS_1 of a two-sided CUSUM chart as follows:

$$\mathrm{ATS}_1 = \frac{\mathrm{ARL}_U \cdot \mathrm{ARL}_L}{\mathrm{ARL}_U + \mathrm{ARL}_L} \cdot h, \tag{15}$$

where h is the sampling interval.

Table 3 displays the comparison between the proposed chart and CUSUM chart, and we can see that the proposed chart always has the shorter ATS_1 when the mean shift is small; see $\delta = 0.1\sim0.6$. However, the opposite result is achieved when the mean shift is large enough and simultaneously the sample size is great; see $\delta = 0.8\sim1.5$ and $E_n = 9$. In other words, the proposed chart is the better choice for QA engineer if the production process is fragile and samples are difficult to obtain; conversely, the CUSUM chart is superior to the proposed chart.

A specific example to display the result can be found in Figure 2. We can see that the traditional synthetic $\overline{X}$ chart is always the worst of the three to QA engineer. The gap between the three is getting smaller and smaller with the increase of δ. When the value of δ exceeds a certain value, the CUSUM chart is the better choice than the adaptive synthetic $\overline{X}$ chart. The results mean that, when the mean shift of process is small, the detection power of the proposed chart is always superior to traditional synthetic chart and the CUSUM chart; however, when the mean shift of process changes from moderate to large, CUSUM chart would be the better one.

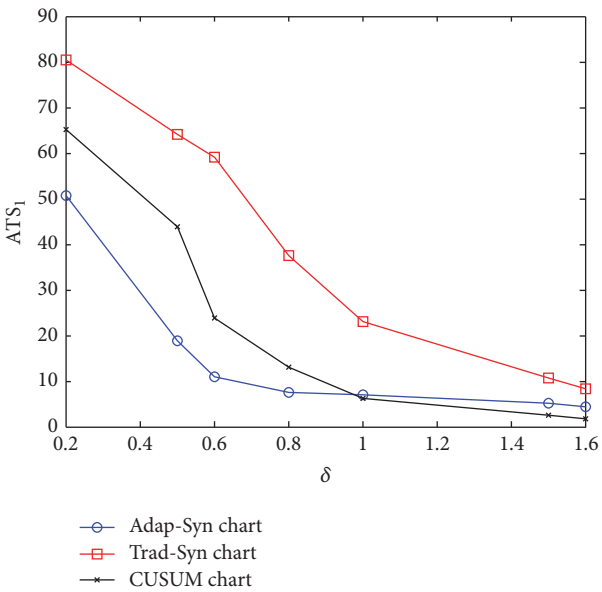

FIGURE 2: Comparison among ATS_1 corresponding to the three charts.

4. Conclusion

In this research, adaptive synthetic $\overline{X}$ charts which integrated variable sample size and sampling interval $\overline{X}$ charts and CRL charts have been developed to control the state of statistical control in service and management operation process. The performances of these charts were evaluated by determining their optimal statistical design and comparing it with tradition synthetic $\overline{X}$ chart and CUSUM chart schemes commonly used in the literature. The optimal design was obtained by genetic algorithm, which works to determine the minimum ATS_1 under the set of selected constraints. The obtained results show that the proposed charts work better than the tradition synthetic $\overline{X}$ chart for all levels of mean shifts and better than CUSUM chart when small to moderate shifts in the mean of the controlled parameter are expected.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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