Child Development Research

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Editorial Board

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Editorial

The Development of Attitudes and Emotions Related to Mathematics

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Attitudes and emotions regarding mathematics are an important topic, especially in view of the fact that many people have very negative attitude to mathematics, sometimes to the point of serious mathematics anxiety, which is distressing in itself and also tends to impair mathematical performance [1]. Attitudes toward mathematics, and mathematics anxiety in particular, have been topics of interest to researchers for a long time [2–5]. However, until recently most such studies dealt with adolescents and adults and gave relatively little attention to attitudes in younger children, or to the factors that influence their development. There has been increased emphasis on attitudes to mathematics in elementary school children in recent years [6–8]) but the database has still been small, with far more research needed, especially as the results are somewhat conflicting: some studies suggest that mathematics anxiety is rare in young children, and that attitudes only become seriously negative later on, while others suggest that mathematics anxiety is a very significant problem from an early age.

Moreover, perhaps in part as a result of the paucity of research on the early development of attitudes to mathematics, we have relatively little knowledge as yet about their antecedents or even their correlates. We know a certain amount about what attitudes people have toward mathematics, but not very much about why or how they develop. There are also questions to be asked about the specificity of mathematics anxiety, and the extent to which it may reflect more general academic-related anxieties or cognitive difficulties.

This special issue attempts to examine attitudes to mathematics at different ages, with a particular emphasis on early ages and to investigate some of the factors associated with attitudes and emotional reactions toward mathematics.

Research reported in this special issue indicates that young children's attitudes should indeed receive more study, not only for their own sake, but because they may have an influence on subsequent mathematical development. M. M. M. Mazzocco et al. describe a longitudinal study of primary school children's spontaneous comments about mathematics. The children's likability comments were similar for "math" and "reading," but they were more likely to describe math than reading as difficult. Achievement at Grade 3 was predicted by comments at Grade 2. This indicates that young children's spontaneously expressed attitudes to maths can be a predictor of later achievement.

A. Dowker et al. report a study which gives data about primary school children's attitudes to mathematics and leads to some hypotheses about the development of relationships between attitudes and performance. English primary school children in Grades 3 and 5 took a Mathematics Attitude and Anxiety Questionnaire, using pictorial rating scales to record their self-rating for maths, liking for maths, anxiety about maths, and unhappiness about poor performance in mathematics. They were also given the British Abilities Scales Basic Number Skills test. Anxiety as such was not related to actual performance, but self-rating was. Although a relatively small sample size means a need for caution in drawing conclusions, it is of interest that the relationship between self-rating and

actual performance seemed to develop between Grade 3 and Grade 5, suggesting that attitudes and performance may become increasingly linked with age.

G. Wood et al. extend research on young children's attitude development cross-culturally. They describe a study, where they gave German and Brazilian 7- to 12-year-old school children the same Mathematics Anxiety Questionnaire. They found a similar factor structure in both groups, but much more negative attitudes in the Brazilian group. The Brazilian children liked mathematics less than the German children did, were more anxious about it, and were more unhappier if they could not do a mathematics task. They did not, however, differ in their self-ratings of their own performance. Mathematics anxiety increased with age in both groups. In both groups, attitudes were related to actual mathematics performance.

One question that arises with regard to mathematics anxiety is that of whether it is just one form of academic performance anxiety. It is generally assumed that mathematics anxiety is greater than anxiety about other subjects; but it could be argued that this simply reflects a lack of research on anxiety about other subjects. L. Punaro and R. Reeve report a study that compares mathematics and literacy anxiety in Australian 9-year-olds and relates their anxiety to their actual academic abilities. Although children expressed anxiety about difficult problems in both mathematics and literacy, worries were indeed greater for mathematics than literacy. Moreover, anxiety about mathematics was related to actual mathematics performance, whereas anxiety about literacy was not related to actual literacy performance.

A related issue is whether mathematics anxiety really is specific to mathematics, or is just one consequence of general anxiety and/or of difficulties with attention and executive functions, especially in view of Ashcraft and Krause's [1] findings of important links between mathematics anxiety, working memory, and mathematical performance. V. G. Haase et al. report here that psychosocial competencies (general anxiety and attention deficits) and self-rating in mathematics are independent predictors of children's mathematics performance. Moreover, general psychosocial competencies predict both mathematics and spelling performance, while self-rating in mathematics predicts only mathematics performance. This gives support to the view that, though general anxiety and attentional factors do affect academic skills in general, including mathematics, there is also a more specific relationship between attitudes and performance in mathematics, that cannot be reduced to a more general emotional or cognitive problem.

There are many questions to be asked as to what factors lead to individual and perhaps gender and cultural differences in attitudes to arithmetic. M. L. Mata et al. report a study examining several factors that may influence attitudes to mathematics. They investigated Portuguese fifth to twelfth grade pupils' motivation and their perceptions of teacher and peer support and also assessed their attitudes to mathematics. Most pupils had positive attitudes to mathematics. There was no overall gender effect on attitudes, but there was an interaction between gender and grade, such that girls but not boys showed a steady decline in attitudes to mathematics as grade level increased. A hierarchical analysis using structural equation modelling indicated that motivational variables were the strongest predictors of attitudes to mathematics, but that perceived social support from teachers and peers was also a very important factor.

It is often assumed that the quality of teaching in mathematics has an influence on attitudes, and also that attitudes to mathematics influence reactions to the teaching. There had, however, been few studies of the relationships between attitudes to mathematics and pupils' perception of the quality of their instruction. R. Lazarides and A. Ittell report a study of German secondary school pupils perceptions of the quality of their mathematics instruction. Nearly half of the sample perceived their teaching as poor, and girls were more likely than boys to have this perception. There was a strong relationship between such negative perceptions of teaching and experiencing negative attitudes toward mathematics.

Parents are also regarded as a strong influence on their children's attitudes, and in particular intergenerational transmission of attitudes to mathematics is sometimes postulated as important. S. Sonnenschein et al. report a study of parents' beliefs about children's development and about the extent to which their children engaged in mathematics-related activities at home. The children were preschool or in the early years of elementary school. Parents who considered it important to have their children do math activities at home, saw themselves as role models, and considered it as important to involve children in daily living math activities, also reported that their children were in fact more frequently involved in math activities at home. Parents' own enjoyment of math and perception of their own mathematical skills were not related to the extent of their children's engagement in mathematical activities, suggesting that, at least in this age group, the provision of mathematical activities at home was directly related to whether parents thought it was important to do so, but not to their own attitudes to mathematics.

The consequences of attitudes to mathematics are as important as their causes and are here investigated in terms not just of effects on overall performance, but on particular key aspects of mathematics. S. Chinn reports a study, not of the factors that cause mathematics anxiety, but of an important but often neglected consequence: a tendency to avoid attempting mathematics problems at all out of fear of failure. This paper presents data taken from over 2500 mathematics test papers from both children and adults. A large number of responses to questions were in the "no attempt" category; that is, the problems were avoided. This avoidance strategy was more common for multiplication than addition, and commonest of all for division.

Attitudes can also have an effect on the type of strategy used. There have been a number of studies (e.g., [9, 10]) of children's use of derived fact strategies, where they use a known fact, combined an arithmetical principle such as commutativity, inversion, or associativity, to obtain the answer to another problem without performing a full calculation. Here, K. M. Robinson (gave Canadian elementary school children three sets of three-term addition problems, that could be solved by shortcuts involving associativity or inversion. They

were then given an intervention where they were shown how to use the shortcut strategies and a standard algorithm and asked which they preferred. The intervention increased the use of shortcuts for subsequent problems, but more so if they expressed a preference for the shortcuts to the standard algorithm. This shows that attitudes may influence strategy use.

We hope that this special issue will inspire further research on the nature, causes, and consequences of children's attitudes and emotional reactions toward mathematics.

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Research Article

Math Anxiety Questionnaire: Similar Latent Structure in Brazilian and German School Children

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Math anxiety is a relatively frequent phenomenon often related to low mathematics achievement and dyscalculia. In the present study, the German and the Brazilian versions of the Mathematics Anxiety Questionnaire (MAQ) were examined. The two-dimensional structure originally reported for the German MAQ, that includes both affective and cognitive components of math anxiety was reproduced in the Brazilian version. Moreover, mathematics anxiety also was found to increase with age in both populations and was particularly associated with basic numeric competencies and more complex arithmetics. The present results suggest that mathematics anxiety as measured by the MAQ presents the same internal structure in culturally very different populations.

1. Introduction

Every student knows how unpleasant life can be when the mathematics test is approaching. Although there is no gold standard to measure the levels of math anxiety (MA) that should be considered maladaptive, depending on their intensity and duration, negative physiological reactions, effects, and thoughts regarding mathematics can be considered a form of performance-related phobia [1]. Correlations between MA and math achievement have been reported [2, 3] as well as bidirectional associations between MA and math performance on several time scales going from online or short-term to long-term effects. On the long term, low math achievement is an antecedent of MA [4, 5] but MA also interferes with math performance. MA leads to hastened performance on math tasks and avoidance of math activities and courses, resulting in lower math skills and choice of careers with less demanding curricular requirements regarding mathematics [6, 7]. Besides, successful treatment of MA leads to significant improvements in math performance [2].

Short-term, online effects of MA on math performance have also been described. Negative emotional and math-related primes have been shown to speed up math performance in children with math learning disability [8]. Other studies indicate that MA negatively interferes with math performance. Initial research showed that online effects of MA on math performance were more pronounced for tasks demanding higher levels of working memory resources, such as those involving transfer between columns [9]. Newer findings demonstrate, however, that MA also interferes

with performance in more basic number processing tasks, such as magnitude comparison [10] and counting, but not subitizing [11]. In line with these last results, children with high MA display comparatively lower levels of frontoparietal and higher levels of right amygdala activations performing a simple equation verification task [12]. Moreover, children with high MA also exhibit higher functional correlation levels between right and left amygdala and between right amygdala and ventromedial prefrontal cortex. These results indicate that in children MA is associated to a consistent pattern of activation in brain regions associated to emotion processing and regulation and hypoactivity in brain areas associated with number processing and calculation.

MA is a multilevel construct keeping similarities with disorders such as social phobia and test and computer anxiety [1]. The physiological arousal component of MA has been variously assessed by salivary cortisol measures [13] or experimentally manipulated by breathing of CO_2 enriched air [14]. However, most studies on MA have used self-report scales assessing an affective component of tension, apprehension and fear, and a cognitive component related to negative attitudes, worrisome thoughts, and low self-assessments of performance [1, 4, 15]. The cognitive component of MA is correlated to but psychometrically distinguishable from other self-related constructs such as math self-concept or math self-efficacy [16]. Research investigating the latent structure of MA self-assessments reliably distinguishes the affective from the cognitive component [4]. There is also cross-cultural research with adolescents confirming the differentiation between affective and cognitive components of MA [16, 17].

Although the latent structure of MA has been investigated in adolescents, to our knowledge, there is no research on the latent structure of MA in elementary school children focused on transcultural similarities and differences. This would be important, as evidence suggests that distinct MA components may differentially correlate to math performance and other factors according to age [4, 5]. Besides, although latent structure has been shown to preserve similarity across cultures, there are regional variations in the levels and correlates of different constructs. For instance, students in some Asian countries such as Japan and Korea, exhibit higher levels of math performance, they also endorse lower levels of math self-concept and math self-efficacy [16].

Research on MA has been largely concentrated on high-school and college samples [2, 3]. But MA instruments to assess children of elementary school age are increasingly available [4, 12, 15, 17, 18]. Age differences in the component of MA associated to math performance have been reported by Krinzinger and coworkers [4]. These authors found that lower math performance is a longitudinal predictor of lower math performance self-perceptions but not of negative affective reactions towards math in a sample examined repeatedly from the first to the third elementary school grade. In contrast, Ma and Xu [5] observed that lower math achievement in the 7th grade is predictive of higher levels in the affective component of MA in the 12th grade. These results suggest that correlates of MA may vary according to age but also that more research with elementary school children is still necessary.

Research reported here aimed at comparing MA latent structure, math performance, and sociodemographic correlates in samples of typically developing elementary school children in two countries, Germany and Brazil. While the PISA 2003 math scores of German students are among those with the highest ranks, those of Brazilian students are placed in the lowest ranks [16]. Math anxiety (lower in Germany, higher in Brazil) and math self-efficacy (higher in Germany, lower in Brazil) were found previously to vary accordingly. Interestingly, math concept seems not to be affected by low achievement and high math anxiety in Brazil [16].

More specifically, we aimed at examining if the bifactorial structure described by Krinzinger and coworkers [4, 15] in Germany could be replicated in a Brazilian sample of elementary school children, and if the affective and cognitive MA components varied in level and associations to math performance in the two countries. Based on previous results by Krinzinger and coworkers, we hypothesize that the bifactorial affective-cognitive structure may be reliably identifiable in the two countries and that the cognitive MA component should be more strongly associated to math performance.

2. Materials and Methods

2.1. Participants

2.1.1. Brazilian Sample. The Brazilian sample was constituted by children with ages ranging from 7 to 12 years and attending from 1st to 6th grade. The study was approved by the local research ethics committee (COEP-UFMG). Children participated only after providing informed consent in written form from their parents, and orally from themselves. Children were recruited from schools of Belo Horizonte and Mariana, Brazil. A wide set of evenly geographically distributed schools was sampled. For this reason, the sample is representative of the Brazilian school population, with 80% of children attending public school, and the 20% attending private schools. In a first phase of testing, children with normal intelligence (i.e., who scored above the 16th percentile in the Raven colored matrices test, [19]) were included in the study. These children also solved the Arithmetic and Spelling subtests of the Brazilian School Achievement Test (Teste do Desempenho Escolar, TDE, [20]). Those children scoring above the 25th percentile on both Arithmetic and Spelling subtests of the TDE were assigned to our study. The sample consisted of 171 children (see Table 1 for further details). In a next step, children were evaluated with a neuropsychological battery containing the Digit-span and Corsi-blocks, forward and backward, basic arithmetic operations, addition, subtraction, and multiplication problems, individually.

2.1.2. German Sample. Four hundred fifty children with ages between 6 and 10 years old, attending to grades 1 to 3, who took part in a study aiming to collect norms for the German version of the TEDI-MATH [4, 21] were included in the present study. In this sample, only children without

Table 1: Descriptive data from Brazilian and German children.

	Brazil	Germany
N	171	450
Female (%)	99 (57.9%)	227 (50.4%)
Male (%)	72 (42.1%)	223 (49.6%)
Age (mean (sd))[§]	119.26 (13.2)	96.04 (5.1)

[§]expressed in months.

difficulties in mathematics—as screened by the TEDI-MATH [21]—were selected. Besides the TEDI-MATH, these children have also completed a Digit-span task. Descriptive data are depicted in Table 1. All children come from public schools of the state of North Rhine-Westphalia in Germany. A more detailed description of the sample can be found in Krinzinger et al. [15].

For the purpose of comparing results between the Brazilian and the German samples, a subsample of those children with ages between 7.6 and 10.1 years was selected, which is the age interval common to both samples (Brazilian $n = 101$, German $n = 284$). When subsamples are compared, this will be mentioned explicitly in the results. Importantly, the two subsamples were formed by children attending to 2nd or 3rd grades in school.

2.2. Psychological Instruments. The Math Anxiety Questionnaire (MAQ) was applied both in Brazil as in Germany. Other instruments differed according to country. In Brazil, school achievement was assessed with the TDE and intelligence with the Raven's Colored Progressive Matrices. Besides, basic computation abilities were assessed with Basic Arithmetic Operations, and short-term and working memory with the Digit-span and Corsi-blocks tests. The TEDI-MATH was used in Germany to assess basic math abilities. In the following, these instruments will be described in more detail.

2.3. Brazilian School Achievement Test (TDE) [20]. The TDE is the most widely used standardized test of school achievement with norms for the Brazilian population. It comprises three subtests: Arithmetics, single-word Spelling, and single-word Reading. In the screening phase, the Arithmetics and Spelling subtests were used, which can be applied in groups. Norms are provided for school-aged children between the second and seventh grade. The Arithmetics subtest is composed of three simple verbally presented word problems (i.e., which is the largest, 28 or 42?) and 45 written arithmetic calculations of increasing complexity (i.e., very easy: $4 - 1$; easy: $1230 + 150 + 1620$; intermediate: 823×96; hard: $3/4 + 2/8$). Specific norms for each school grade were used to characterize children's individual performance. The Spelling subtest consists of dictation of 34 words of increasing syllabic complexity (i.e., *toca*; *balanço*; *cristalização*). Reliability coefficients (Cronbach α) of TDE subtests are 0.87 or higher. Children are instructed to work on the problems to the best of their capacity but without time limits.

2.4. Raven's Colored Progressive Matrices. General intelligence was assessed with the age-appropriate Brazilian validated version of Raven's Colored Matrices [19].

Digit-Span (Forward and Backward). Verbal short-term memory was assessed with the Brazilian WISC-III Digits subtest [22]. Performance in the forward order was considered a measure of phonological short-term memory, and the backward order was used to assess verbal working memory.

Corsi Blocks (Forward and Backward). This test is a measure of the visuospatial component of working memory. It is constituted by a set of nine blocks, which are tapped, in a certain sequence by the examiner. The test starts with sequences of two blocks and can reach a maximum of nine blocks. We used the forward and backward Corsi span tasks according to [23]. In the forward condition, the child is instructed to tap the blocks on the same order as the examiner, in the backward condition, in the inverse order. Span is determined by the longest sequence correctly repeated before two successive failures.

Basic Arithmetic Operations. This task consisted of addition (27 items), subtraction (27 items), and multiplication (28 items) operations for individual application, which were printed on separate sheets of paper. Children were instructed to answer as fast and as accurate as they could, time limit per block being 1 minute. Arithmetic operations were organized in two levels of complexity and were presented to children in separate blocks: one consisted of simple arithmetic table facts and the other of more complex ones. Simple additions were defined as those operations with the results below 10 (i.e., $3+5$), while complex additions with the results between 11 and 17 (i.e., $9 + 5$). Tie problems (i.e., $4 + 4$) were not used for addition. Simple subtraction comprised of problems in which the operands were below 10 (i.e., $9 - 6$), while for complex subtractions the first operand ranged from 11 to 17 (i.e., $16 - 9$). No negative results were included in the subtraction problems. Simple multiplication consisted of operations with results below 25 and with the number 5 as one of the operands (i.e., $2 * 7$, $5 * 6$), while for the complex multiplication the result of operands ranged from 24 to 72 ($6 * 8$). Tie problems were not used for multiplication. Reliability coefficients were high (Cronbach's $\alpha > 0.90$).

2.5. TEDI-MATH. The TEDI-MATH is a battery for the assessment of numerical and arithmetic competencies in 4–9-year primary school children. There are norms for the TEDI-MATH in three different languages: French, Flemish, and German. The TEDI-MATH is a multicomponential dyscalculia test based on cognitive neuropsychological models of number processing and calculation. The German version of the TEDI-MATH [4] was translated of the Belgium Neuropsychological Test for a Developmental Dyscalculia diagnostic [24]. The German version offers in addition to the original version an implementation of a core battery with subtests of the first grade elementary school in the two components of number processing (reading and writing

multidigit numbers, size comparison multidigit numbers, recognition of unit and decade, representation of the decimal system), and arithmetic (addition, subtraction, multiplication, word problems, additive decomposition of numbers, understanding of arithmetic concepts).

2.6. Math Anxiety Questionnaire (MAQ). The Math Anxiety Questionnaire is a well-known scale developed by Thomas and Dowker [25] for the assessment of anxiety towards mathematics in primary school children. The present study used a Brazilian Portuguese version developed and standardized by us (this paper). Children answer the questionnaire in individual sessions of 5 to 10 minutes. The Brazilian version of the MAQ contains 24 items that can be answered by children individually or in groups within 5 to 10 minutes. The items can be combined into four basic subscales ("self-perceived performance" (Scale A), "attitudes in mathematics" (Scale B), "unhappiness related to problems in mathematics" (Scale C) and "anxiety related to problems in mathematics" (Scale D)) according to the authors of the original version [25]. Some studies have examined the construct validity of the MAQ [15]. Krinzinger and colleagues [15] have established with help of multidimensional scaling that the latent structure of the MAQ contains two main dimensions. These authors have shown that the four original subscales can be combined into two main scores called "Self-perceived performance and attitudes" and "Mathematics anxiety." The first one, named evaluation of mathematics, includes the first two subscales, while the second one, called math anxiety, combines the last two subscales. Moreover, in a longitudinal study, Krinzinger and colleagues [4] also have shown that the two combined scales of the MAQ show a high stability over time and are useful to predict calculation abilities. Finally, Haase and colleagues [26] showed recently that the different subscales of the MAQ can be differentiated from more general forms of anxiety and are more specifically related to performance in mathematics in school children than general measures of anxiety. The MAQ items have the format of one out of four types of questions: "*How good are you at...*" (Scale A); "*How much do you like...*" (Scale B); "*How happy or unhappy are you if you have problems with...*" (Scale C) and "*How worried are you if you have problems with...*" (Scale D). Each question is to be answered regarding six different categories related to mathematics, namely, mathematics in general (MAQ_G), easy calculations (MAQ_E), difficult calculations (MAQ_D), written calculations (MAQ_W), mental calculations (MAQ_M), and math homework (MAQ_H). Children are encouraged by supportive figures to give their responses according to a Likert scale with 5 points (coded 0 to 4, such as in the study by Krinzinger et al. [15]. The higher the score, the higher is the math anxiety. Reliability coefficient (Cronbach α) of MAQ in the German study ranged between 0.83 and 0.91 for the total scale.

2.7. Testing Procedures. The assessment was performed in an appropriate room in children's schools. Tests as well as their order of application varied in the two countries. In Brazil, the TDE and Raven were applied during the screening phase, while four different pseudo-randomly varying sequences of application were used in the individual testing phase. In Germany, the MAQ was applied immediately after the TEDI-MATH. The data from the Brazilian and German samples were obtained originally for very different purposes and at different time points. The aim of the present study to analyze the latent structure of the MAQ emerged after both data sets have been collected. This is the reason why the choice of measurement instruments was so different in Brazilian and German populations.

2.8. Analyses. The internal consistency of all subscales of the MAQ will be calculated for the first time for the Brazilian version of the MAQ. Because of existing findings in the German population [4], predictive validity over arithmetics achievement will be assessed by means of regression analyses linking basic number magnitude representations and arithmetics performance to the MAQ subscales. The construct validity of the MAQ will be assessed by different methods. To determine the dimensionality of the MAQ, Mokken automatic item classification [27] and multidimensional scaling will be employed. Mokken scaling is based on the Monotone Homogeneity Model (MHM, [27–29]). It tests the assumptions that the traits being measured are unidimensional, that items can be ordered monotonically according to their difficulty, and that responses to single items bear local independence. Mokken analyses provide scalability coefficients H for each item in each scale and for the scales as a whole. Values H can vary from 0 to 1. The higher the H value, the higher is the scalability of an item according to the Monotone Homogeneity Model [27, 28]. To determine the convergent validity of the MAQ, a comparison between a German and a Brazilian versions of the MAQ was carried out.

3. Results

All children in the Brazilian sample reached a score above the 25th percentile in the subtests of the TDE and can be considered as typically achieving children in both arithmetic and spelling abilities. All children in the German sample can be considered as typically achieving children in arithmetics according to their scores in the TEDI-MATH. In the following, results regarding the internal consistency of the MAQ will be presented first. Thereafter, the raw scores of the Brazilian and the German sample will be compared and the predictive validity of the MAQ regarding numeric and arithmetic abilities will be reported. Finally, investigations on the latent structure of the MAQ that employed automatic item classification and multidimensional scaling will be reported.

3.1. Brazilian Sample

Internal Consistency. Means, standard deviations, minimum and maximum and internal consistency (Cronbach's α) of the scores in each MAQ subscale and each composite scale obtained in the Brazilian sample are presented in Table 2. The internal consistency of the different subscales is satisfactory or high in all cases (Table 2). According to the criteria

Table 2: Descriptives of MAQ subscales and composite scales ($n = 171$).

Scales	Mean	Sd	Minimum	Maximum	Cronbach's α
Scale A (6 items)	23.66	3.53	11	30	0.71
Scale B (6 items)	23.00	4.66	8	30	0.71
Scale C (6 items)	17.80	6.13	6	30	0.88
Scale D (6 items)	17.20	5.80	6	30	0.80
Scale AB (12 items)	46.60	7.03	24	59	0.78
Scale CD (12 items)	35.06	10.55	12	58	0.88
Total (24 items)	81.70	14.67	46	116	0.87

Table 3: Regression models for arithmetic abilities.

Task	Adjusted r^2 §	Sample size	Significant predictors in the model
Simple addition	34.7	164	Sex, grade, raven, Corsi backwards, digit-span forward
Complex addition	38.4	164	Grade, Corsi backward, digit-span forward, Corsi backward
Simple subtraction	26.1	164	Sex, grade, Corsi backward, Corsi forward
Complex subtraction	29.9	164	Grade, Corsi backward, digit-span backward, MAQ-scale A
Simple multiplication	53.5	164	Grade, Corsi backward, digit-span forward, MAQ-scale A, MAQ-scale D
Complex multiplication	42.9	164	Corsi forward, digit-span forward, digit-span backward, MAQ-scale A

§ expressed as the proportion of variance in the dependent variable explained by the model.

established by Willmes [30], scales C and D as well as CD and MAQ-total are practically invariant for the sample size $n = 171$ and alpha-error probability of 5%. Composite scale AB is very close to practical invariance. The optimal sample size would be reached with 30 more children in the normative sample (see [30]).

To investigate the predictive validity of the MAQ, regressions analyses entering the four MAQ subscales as predictors of numeric and arithmetic abilities were calculated.

Numeric and Arithmetic Abilities. The impact of math anxiety on simple and complex addition, subtraction, and multiplication tasks was examined in the Brazilian sample. Hundred sixty four children from the Brazilian sample completed all these tasks. In order to ascertain the specificity of the contribution of MAQ scales to explaining variance in the arithmetic tasks, age, sex, grade, general intelligence, verbal and nonverbal short-term memory and working memory (digit span and Corsi span, both of them forward and backward) were entered in the model as well. Age, sex, grade, and general intelligence were entered first in the model using the method "enter," while the measures of short-term memory and working memory were entered using the method "stepwise." This regression method was adopted to ascertain that the impact more general sociodemographic and cognitive functions has been removed before analyzing the impact of math anxiety on numeric and arithmetic abilities and competencies was investigated. A summary containing the adjusted r^2 and significant predictors in each single model is presented in Table 3.

As depicted in Table 3, scale A of the MAQ has a significant impact on more complex arithmetic operations such as complex subtraction, simple, and complex multiplication. Moreover, verbal and visuospatial working memory measures (digit-span and Corsi blocks backwards) are significant predictors of individual differences in performance in addition, subtraction, and multiplication problems but they cannot account for the impact of different aspects of MA on arithmetics performance.

3.2. German Sample

Internal Consistency. The internal consistency of the MAQ in the German population has been reported in detail elsewhere [15]. The Cronbach's alpha coefficient of single subscales ranges between .65 and .86.

To investigate the predictive validity of the MAQ, regressions analyses entering the four MAQ subscales as predictors of seven different subtests of the TEDI-MATH, which measure numeric and arithmetic abilities, were calculated.

Numeric and Arithmetic Abilities. The impact of math anxiety on seven subtests of the TEDI-MATH was examined in the German sample. Between 279 and 284 children from the German sample completed all these tasks. In order to ascertain the specificity of the contribution of MAQ scales to explaining variance in the arithmetic tasks, age, sex, grade, and verbal short-term memory (digit span forward and backward) were entered in the model as well. Age, sex, and grade were entered first in the model using the method "enter," while the measures of short-term memory and were entered using the method "stepwise". This regression method was adopted to ascertain that the impact more general sociodemographic and cognitive functions has been removed before analyzing the impact of math anxiety on numeric and arithmetic abilities and competencies was investigated. A summary containing the adjusted r^2, sample sizes, and significant predictors in each single model is presented in Table 4.

Table 4: Regression models for numeric and arithmetic abilities.

Task	Adjusted r^2 §	Sample size	Significant predictors in the model
Addition decomposition	5	284	Age, MAQ-scale A
Text problems	8	284	Age, grade, MAQ-scale A
Magnitude comparison Arabic	2.6	284	MAQ-scale A
Magnitude comparison number words	3.3	284	MAQ-scale A
Writing number	6.7	284	Age
Reading number	6.8	284	Age
Arithmetic concepts	9.6	279	MAQ-scale A

§expressed as the proportion of variance in the dependent variable explained by the model.

As depicted in Table 4, scale A of the MAQ has a significant—although small—impact on fundamental numeric and arithmetic abilities measured by the TEDI-MATH such as magnitude comparison (Arabic numbers and number words) as well as more complex abilities such as addition decomposition, text problems, and arithmetic concepts.

3.3. Comparisons between Raw Scores. To compare data between the Brazilian and the German sample, a subsample of each group was selected (Brazilian sample, $n = 101$; German sample $n = 284$), which had ages between 7.5 and 10.1 years in both groups. t-tests revealed no difference between both samples in the subscale A "self-perceived performance" ($t(383) = 0.45$; se $= 0.49$; $P = 0.65$; Cohen's $d = 0.05$). The Brazilian sample showed higher scores that the German sample in the subscale B "attitudes in mathematics" ($t(383) = 2.81$; se $= 0.63$; $P = 0.0053$; Cohen's $d = 0.34$), subscale C "unhappiness related to problems in mathematics" ($t(383) = 5.30$; se $= 0.62$; $P = 0.0001$; Cohen's $d = 0.61$) and subscale D "anxiety related to problems in mathematics" ($t(383) = 2.22$; se $= 0.68$; $P = 0.03$; Cohen's $d = 0.26$) although the effect sizes of these differences were small or moderate.

3.4. Automatic Item Classification Analysis [27]. To investigate the latent structure of the MAQ, an automatic item classification analysis was employed [27]. Based on the Loevinger H-index of scalability, items were automatically assigned to a one-dimensional scales [28]. Only items reaching an H-index of at least 0.3 were assigned to a scale, while items with lower scalability are dropped automatically from the analysis [27]. Results were quite similar in both Brazilian and German samples. In both cases, three scales were disclosed by the Mokken analysis (Table 5). MAQ scales C and D were subsumed under a single unidimensional scale in both Brazilian and German samples. However, in both samples subscales A and B could not be assigned to one single composite scale, but to two separate scales in which items from both subscales A and B were mixed. Closer inspection of items being classified in scales 2 and 3 reveals that in the German sample "written calculations" drove the process of item classification in scale 3. In contrast, in the Brazilian sample the items being assigned to scale 3 originate from the original scale B measuring "attitudes towards mathematics".

Three items could not be classified in the Brazilian sample, while all items could be assigned to a scale in the German sample. Results were replicated when considering only children with ages between 7.5 and 10.1 years. For this reason, these analyses will not be reported here.

3.5. Dimensionality of the MAQ. To investigate the construct validity of the Brazilian version of the MAQ, multidimensional scaling was employed. The facets diagram (see Figure 1) depicts the projection of the distances between the different items on a two-dimensional space. As can be easily recognized, Items from scales A and B cluster together as well as items from scales C and D. These results replicate those reported by Krinzinger et al. [15].

4. Discussion and Conclusion

In the present study, the psychometric properties of a Brazilian version of the MAQ were investigated for the first time as well as its transcultural validity in German Brazilian samples. The internal consistency of all subscales and composite scales obtained in the Brazilian sample is throughout satisfactory or even high. A direct comparison of the raw scores obtained in the Brazilian sample with those obtained in the German sample reveal no differences in the subscale representing "self-perceived performance". However, the Brazilian sample showed higher scores in the subscales "attitudes towards mathematics," "unhappiness related to problems in mathematics," and "anxiety related to problems in mathematics" when compared to the German sample. The investigation of the predictive validity of the MAQ revealed that "self-perceived performance" is a significant predictor of basic numeric abilities such as magnitude comparison as well of more complex arithmetic abilities and competencies. Importantly, "self-perceived performance" remains a significant predictor even after removing the specific effects of grade, age, sex, verbal, and nonverbal short-term memory and working memory on these abilities. Finally, automatic item selection as well as multidimensional scaling procedures revealed the similarities in the structure of the MAQ between both Brazilian and German samples. In the following, these results will be discussed in more detail.

Table 5: Results of the automatic classification of items (Mokken analysis).

Scale	Brazilian sample ($n = 171$)		German sample ($n = 450$)	
	Items (scale)[§]	Loevinger's H-score	Items (scale)[§]	Loevinger's H-score
Scale 1	MAQ_M(C)	($H = 0.64$)	MAQ_M(D)	($H = 0.57$)
	MAQ_D(C)	($H = 0.64$)	MAQ_D(D)	($H = 0.57$)
	MAQ_G(C)	($H = 0.62$)	MAQ_H(D)	($H = 0.55$)
	MAQ_H(C)	($H = 0.60$)	MAQ_G(D)	($H = 0.52$)
	MAQ_E(C)	($H = 0.59$)	MAQ_E(D)	($H = 0.51$)
	MAQ_W(C)	($H = 0.58$)	MAQ_W(D)	($H = 0.49$)
	MAQ_H(D)	($H = 0.54$)	MAQ_D(C)	($H = 0.46$)
	MAQ_W(D)	($H = 0.50$)	MAQ_H(C)	($H = 0.44$)
	MAQ_E(D)	($H = 0.48$)	MAQ_E(C)	($H = 0.43$)
	MAQ_M(D)	($H = 0.46$)	MAQ_G(C)	($H = 0.42$)
	MAQ_D(D)	($H = 0.43$)	MAQ_W(C)	($H = 0.40$)
	MAQ_G(D)	($H = 0.42$)	MAQ_M(C)	($H = 0.39$)
Scale 2			MAQ_M(A)	($H = 0.61$)
	MAQ_H(A)	($H = 0.54$)	MAQ_G(A)	($H = 0.61$)
	MAQ_G(A)	($H = 0.54$)	MAQ_D(A)	($H = 0.57$)
	MAQ_D(A)	($H = 0.46$)	MAQ_M(B)	($H = 0.53$)
	MAQ_D(B)	($H = 0.43$)	MAQ_D(B)	($H = 0.51$)
	MAQ_M(A)	($H = 0.41$)	MAQ_G(B)	($H = 0.48$)
	MAQ_M(B)	($H = 0.39$)	MAQ_H(B)	($H = 0.45$)
	MAQ_G(B)	($H = 0.37$)	MAQ_H(A)	($H = 0.43$)
			MAQ_E(B)	($H = 0.40$)
			MAQ_E(A)	($H = 0.39$)
Scale 3	MAQ_W(B)	($H = 0.42$)		
	MAQ_W(A)	($H = 0.42$)	MAQ_W(B)	($H = 0.30$)
	MAQ_E(B)	($H = 0.37$)	MAQ_W(A)	($H = 0.30$)
	MAQ_H(B)	($H = 0.35$)		

[§]Item description is composed of the content of each item, that is: mathematics in general (MAQ_G); easy calculations (MAQ_E); difficult calculations (MAQ_D); written calculations (MAQ_W); mental calculations (MAQ_M); math homework (MAQ_H) and its scale (A), (B), (C), or (D).

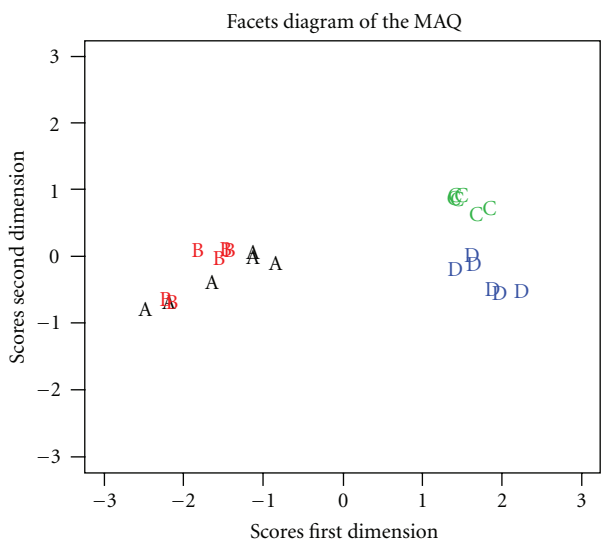

Figure 1: Configuration of MAQ items represented in a two-dimensional space using multidimensional scaling. Symbols A, B, C, and D represent the different scale items according to their scale assignment. The scale of axes x and y is arbitrary.

4.1. Internal Consistency and Diagnostic Properties of the MAQ. In Brazil, a raw sample covering a broad spectrum of ages was investigated. The degree of accuracy to describe MA in children is lower than in the larger German sample, where specific norms for children in first and second halves of each grade were obtained [15]. However, the Brazilian data on the MAQ still have some very useful psychometric properties. The internal consistency was satisfactory for all scales. Especially subscales C and D as well as the composite scales CD and Total were found to be practically invariant. This means, the reliability estimations obtained for these scales are sufficient for the construction of stable confidence intervals on the individual performance and specially for testing intervention-related changes in the levels of MA. Moreover, the composite scale AB presents an internal consistency high enough to be very close to practical invariance as defined by Willmes [30] Since investigations on the dimensionality of the MAQ support the view that scales A and B can be grouped into a composite scale AB it is unproblematic to consider scale AB as equally useful for individual diagnostics purposes.

4.2. Comparisons between the Brazilian and the German Samples. The Brazilian sample showed higher scores on "attitudes towards mathematics," "unhappiness related to problems in mathematics," and "anxiety related to problems in mathematics" when compared to the German sample. These

results reveal higher levels of MA in the Brazilian sample when compared to the German sample. This replicates evidence from the recent literature [16]. Lee [16] reported that Brazilian students present levels of math anxiety much higher than those presented by German students (i.e., Brazilian students report math anxiety levels 0.4 standard deviations above the average of the 41 nations examined in that study, while German students report levels below −.35 standard deviations, [16, Figure 3, p. 361]. Interestingly, these differences were not found in the subscale "self-perceived performance". These results are in line with the study by Lee [16], where the levels of self-concept presented by both Brazilian and German students were comparable [16, Figure 1, p. 360].

One possible interpretation of these results can be derived from the view that there are at least two different ways for MA to impact math performance [31]. Chinn [31] argues that feelings of tension, apprehension, and fear may interfere directly with math performance but the state of discomfort associated to math activities may impact on the more intern driven constructs of self-esteem, self-concept, which also impact on math performance. The higher levels of "unhappiness related to problems in mathematics" and "anxiety related to problems in mathematics" as well as the lower levels of "attitudes towards mathematics" observed in the Brazilian sample can be directly attributed to the less efficient Brazilian educational system. However, independently of the prejudice in educational resources to which Brazilian children are exposed, their self-perceived performance may still be relatively high because of some compensatory factors and coping strategies. This could explain why comparable levels of self-perceived performance were observed in Brazilian and German samples in the present study. According to the framework conceived by Chinn [31], one could argue that particularly MAQ scales B, C, and D reflect more directly environmental influences such as a bad educational system, less motivated teachers on math performance indicate larger differences between countries. In contrast, the scale A seems to reflect the more self-oriented aspects of MA, which reveal no difference between countries because this dimension of MA is more internally regulated and less driven by the environment. In summary, the MAQ offers a fine-grained evaluation of both environment driven and self-oriented aspects of MA, which contribute to refine the diagnostics of MA in its different dimensions and aspects.

4.3. Predictive Validity of the MAQ. Investigation on the predictive validity of the MAQ revealed specific effects of self-perceived performance on basic number processing abilities such as magnitude comparison. These results replicate those obtained by Maloney and colleagues [11] regarding magnitude comparison (Arabic numbers and number words) as well as more complex abilities such as addition decomposition, text problems, and arithmetic concepts. Interestingly, no effect of self-perceived performance on number reading and writing was observed in the present study. Moreover, self-perceived performance also explained variance of simple and complex arithmetics. In the Brazilian sample, the effect of self-perceived performance could be distinguished from the more general factors such as age, general intelligence, short-term memory, and working memory. This is direct evidence on the specificity of the contribution of self-perceived performance to the diagnostics of performance in both simple and complex arithmetics. Interestingly, self-perceived performance contributes to explain variance of complex subtraction as well as simple and complex multiplication problems but is not associated with the performance in addition tasks and simple subtraction. Therefore, one may conclude that the impact of self-perceived performance on arithmetics is more pronounced in more demanding tasks. These findings reflect probably the fact that more demanding tasks may differentiate better children's performance than easier tasks and may reveal more about the self-perceived performance in mathematics than less demanding arithmetics tasks.

In general, these results suggest that the self-perceived performance is to some extent objectively associated to the actual level of performance observed in school children. This is indicative that the self-perceived performance may be assessed and used to complement the diagnostics of difficulties not only with the most elementary abilities in magnitude processing but also in those arithmetics tasks more typical of the academic context.

4.4. The Latent Structure of the MAQ. Automatic item classification after Mokken produced similar results in both samples. Scales A and B, on the one side, and scales C and D, on the other side, can be grouped into scales AB and CD. Items from scales A and B as well as items from scales C and D seem to load in the same latent dimensions in a way that item difficulty and individual competency are sufficient to describe the properties of the scale. The most central evidence provided by the Mokken analysis is that the items of the MAQ measure a broad spectrum of difficulty regarding MA. These findings are quite intuitive and can be related in a very transparent fashion to the contents of the MAQ items. In other words, the kind of question asked in the MAQ is related to a broad spectrum of expressions of the different facets of the construct "mathematics anxiety." While items asking for "easy problems" are easy for everyone and have a high probability of being responded positively even by children with high levels of MA, items representing the more complex categories such as "written calculations," "mental calculations," or "difficult calculations" have a decreasing probability of being answered positively by children with increasing levels of MA. Moreover, the good scalability of most items of the MAQ put in evidence the property of monotonicity found within each the MAQ scales. The high monotonicity found the different scales of the MAQ reflects the fact that only children with low levels of MA respond positively to more difficult items, while all children (those with low levels of MA as well as those with high levels of MA) tend to respond positively to easier items.

Finally, data from the multidimensional scaling analysis revealed clear similarities between the German and the Brazilian version of the MAQ. In both samples, a clear separation

between subscales A and B, on the one side, and C and D, on the other side was observed. Two latent dimensions have been found in the German version of the MAQ by Krinzinger at al. [15] as well as in the present data. For the objectives of the present study, the substantial differences between samples should not be considered a drawback when comparing the latent structure of the MAQ obtained in the two populations but a strength of the study, since they reinforce the conclusions about the invariance of the latent structure of the MAQ even when comparing datasets from Brazilian and German populations obtained under very different circumstances and for very different purposes.

4.5. Final Considerations. The MAQ is a valid and useful scale for measuring mathematics anxiety in children with diverse cultural backgrounds with useful psychometric properties. The MAQ also specifically predicts basic number processing abilities as well as arithmetics performance and should, for this reason, be included in the assessment protocols used in the diagnostics of mathematics difficulties [32]. Studies on the psychophysiological correlates of math anxiety [33] could benefit from the use of the MAQ as well. Moreover, the latent structure of the Brazilian version of the MAQ seems to be two dimensional such as in the German version. Finally, probabilistic analyses revealed that the MAQ shows properties of monotonic organization, which are valuable to characterize a broad spectrum of variation in the different dimension of math anxiety.

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Research Article

Beliefs, Anxiety, and Avoiding Failure in Mathematics

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Mathematics anxiety has been the subject of several books and numerous research papers, suggesting that it is a significant issue for many people. Children and adults develop strategies to cope with this anxiety, one of which is avoidance. This paper presents data taken from over 2500 mathematics test papers in order to compare the levels of accuracy and the frequency of the use of the "no attempt" strategy, that is, avoidance, for arithmetic problems given to children aged from 10 years to adults aged up to 49 years from across the UK.

1. Introduction

The difficulties in learning mathematics are a fascinating and complex area for study. The interactions between factors that can be attributed to the cognitive domain and those that can be attributed to the affective domain are many and varied. For example, anxiety has a negative influence on working memory [1]. Skemp ([2], page 127) suggested that the reflective activity of intelligence is most easily inhibited by anxiety. Lundberg and Sterner [3] claim that "over and above common cognitive demands and neurological representations and functions, performance in reading and arithmetic is influenced by a number of motivational and emotional factors such as need for achievement, task orientation, helplessness, depression, anxiety, self-esteem, self-concept…." Hattie [4] selects a pithy quote from O'Connor and Paunonen [5], "Whereas cognitive ability reflects what an individual can do, personality traits reflect what an individual will do."

The implications on learning of anxiety, motivation, self-worth, self-efficacy and attributional style are significant (e.g., [6–8]) particularly in mathematics where a curriculum may make inappropriate assumptions about how some children learn. Those assumptions may be rooted in beliefs about mathematics and how it can be taught and learnt.

There are a number of beliefs about mathematics that are long established and embedded in its culture. This does not necessarily make them helpful in creating a positive student attitude to mathematics, especially for those who have difficulties with learning mathematics or, indeed, mathematics learning difficulties. For example, Mtetwa and Garofalo [9] discuss five beliefs, which include "mathematics problems have only one correct answer" and "computation problems must be solved by using a step-by-step algorithm." The first belief leads children to perceive of mathematics as highly judgmental, that answers are right or they are wrong. The second belief leads children and their teachers to perceive of mathematics as a series of procedures which have to be memorised and not necessarily understood. Ernest [10] reviews the literature on beliefs in his book, "*The Psychology of Learning Mathematics*." Three examples from his review are "some people have a mathematics mind and some do not," "mathematicians do problems quickly in their head," and "mathematics requires a good memory." The first belief permits people to rationalise their inabilities in mathematics and to protect their feelings of self-worth. The second belief sets up children who process some information more slowly, for example dyslexic children, for failure. The last belief is pervasive, for example, Porkess et al. [11] claims that "As with any language, the fundamentals of mathematics (e.g., multiplication tables and number bonds) are most easily learnt when you are young." Unfortunately the reliance on memorising facts and procedures does not stop there.

It could be hypothesised that these beliefs have been, and still are, influential in the way mathematics' curricula are designed and in the way mathematics is taught. For example, the beliefs that surround the task of learning times table facts, where the primary belief is that, providing the child practises enough, then the learning is guaranteed and achieved early in the child's life. Informal surveys of teachers across the UK by the author lead to an estimate of somewhere around 50% of ten-year-old students failing to achieve this goal. Evidence on levels of achievement in retrieving basic facts acquired from a large sample of pupils from across the UK can be found in Chinn [12].

The implications of the experience of failure in learning are succinctly described in the back-cover summary of Covington's [13] book on motivation:

"Achievement behaviour in schools can best be understood in terms of attempts by students to maintain a positive self-image. For many students, trying hard is frightening because a combination of effort and failure implies low ability, which is often equated with worthlessness. Thus many students described as unmotivated are in actuality highly motivated—not to learn, but to avoid failure."

The experience of failure is a consequence of the inherently judgmental nature of arithmetic. For example, the answer for 8×7 is 56. Giving an answer of 54 is rarely judged empathetically as, "That was close. Well done." The "54" answer generates the response, "Wrong." Arithmetic is, unavoidably, the dominant experience of mathematics for young pupils.

Failure and the judgmental nature of mathematics contribute to anxiety. In Chinn's [14] survey of mathematics anxiety in over 2500 secondary students (ages 11 to 15 years) in England, the item "waiting to hear your score on a maths test" was ranked high for anxiety, that is, from second to 6th out of 20 items for all ages and both genders of mainstream students. "Having to take a written maths test" was ranked from second to 4th and "taking an end of term maths exam" was ranked first out of 20 items for all pupils.

The mathematics task which ranked highest was, "doing long division questions without a calculator", ranking from second to 5th. "Long multiplication without a calculator" was ranked less highly, between 9th and 13th for ten of the fifteen subgroups of students.

The ranking of the item, "having to work out the answers to maths questions quickly," which reflects the mathematics belief that computations have to be done quickly, was also ranked high, for both the dyslexic (442 males) and the mainstream school students (2084 male and female) in the sample. The anxiety generated by having to attempt a problem that is a threat to the pupil's confidence is often exacerbated by the need to work out an answer quickly, that is, within an arbitrary time limit.

Studies from many countries over many years have shown that performance in mathematics is related negatively to mathematics anxiety ([15, page 249], [16, page 334], and [17, 18]).

Failure can motivate or undermine ([6, page 5]) depending on whether students' reactions are mastery-oriented or helpless. Dweck uses the term "helpless" to include all the reactions that some students show when they meet failure, including plunging expectations, negative emotions, and deteriorating performance. It seems that the consequence of failure in mathematics is to undermine.

One reaction, or strategy, to deal with failure that Chinn [19] observed in a classroom study on errors in arithmetic is to avoid the challenge and use the "no attempt" [20] error. The study was set up to compare both the performance and the errors for pupils in mainstream schools to those for dyslexic pupils in specialist schools (ages 11 to 13 years). There appeared to be no difference in the frequency of occurrence of any of Engelhardt's other categories of errors, but the one outstanding exception was the "no attempt" error. For example, for 37.6–4, 14.0% of the dyslexic pupils did not attempt the item compared to 2.2% of the mainstream pupils. This compared to the addition item, $12.3 + 5$ (where finger counting forwards is a strategy that is accessible to almost all children) where the no attempt percentages were 2.5% and 0%, respectively. For division the contrast was greater, for the item, $6040 \div 10$, 39.7% of the dyslexic cohort did not attempt the problem in comparison to 5.8% of the mainstream cohort. The study suggested that children with specific learning difficulties, even if of above average intelligence, or perhaps because they are of above average intelligence, will use avoidance rather than risk failure. Data from a 15-minute mathematics test [12], the source of the data for this paper, is given for the bottom and top quartiles of performers to support this conjecture.

Hadfield and McNeil [21] proposed a model of mathematics anxiety, centred on three factors. Environmental factors include classroom issues and the perception of mathematics as a rigid set of rules. Intellectual variables include a mismatch of learning styles and self-doubt. Personality factors include a reluctance to ask questions in class and low self-esteem. Any or all of these three factors could influence a learner and generate a "no attempt" or avoidance attitude.

It is unlikely that this reaction is confined solely to the special needs population, nor just to avoidance of individual questions. Ashcraft and Krause [17] note that "Mathematics anxiety leads to a global avoidance pattern—whenever possible, students avoid taking math classes and avoid situations in which math will be necessary. . ."

The problem of avoidance begins when some children are quite young. An informal survey by the author, taken over the past ten years, of hundreds of teachers from across the UK and abroad indicates that enough children at age 7 years are withdrawing from and giving up on maths for classroom teachers to notice.

The English Government ([22], page 17) has noted that a "minority of pupils" who were making less than expected progress in mathematics shared certain characteristics, including the following.

They would work if the task was straightforward, but would get distracted and might behave badly if they decided they could not do the work.

They did not want to be told they had got something wrong—they were likely to give the impression that they did not care and give up.

They were concerned about how their peers viewed them—for some it was a risk to be seen to try and fail.

Table 1

		34 = 4 + —	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
8 y	225	62.2	10.2
10 y	220	77.7	5.5
12 y	173	84.4	2.9
40–49 y	129	92.2	1.6

Table 2

		9 = — – 4	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
8 y	225	36.9	22.2
10 y	220	72.3	9.5
12 y	173	71.1	9.2
14 y	225	81.3	5.3
40–49 y	129	82.9	4.7

These observations were personalised by a quote (p18) from a Year 9 (13 years old) girl:

> *"I don't get stuck in other subjects—only maths. When I'm doing English, I can always get on with my work. If I'm not sure about a spelling, I can just have a go and still get my work done. But I can't do that in maths. If I'm stuck I can't do anything but wait for help. Then I don't get anything done."*

2. The Sample

1783 school children from age 7 years to 15 years old. 792 people from age 16 years to 59 years old.

The sample was collected from across the UK from over 50 sources, including state schools, independent schools, colleges, a prison, postal workers, and college students. The majority of the data was collected over a period of twelve months, mid 2010 to mid 2011. Only one of the schools that participated was a special school (for dyslexic pupils).

3. The Test/Survey

The data was collected as a consequence of setting up a 15-minute norm-referenced test/survey of mathematics [12]. The "test" was renamed for adults as a "survey" in an attempt to lower anxiety levels and thus to encourage more of them to take part. The tactic was marginally successful.

The test consists of 44 items, ranging from 2 + 5 = —, to "Write forty thousand and seventy as a number," to 20% of 140, to 2y + 5 = 31. It is, however, primarily a test of arithmetic.

4. The Results

A number of hypotheses follow, stimulated by the analysis of the data collected for the standardising procedure combined with reflections on the author's previous research and thirty years of teaching experience with students with specific learning difficulties. This analysis of the data is in terms of the percentage of correct answers and the percentage of "no attempts." A "no attempt" may have been caused by the subject having not yet reached the relevant stage in the curriculum. The National Numeracy Strategy for England was referred to in order to ascertain at what age a topic is included in the curriculum in order to minimise this problem.

4.1. Hypothesis 1. Addition is the default operation. The other operations create less success, more insecurity and more "no attempt" errors (see Table 1).

This item is testing knowledge of place value at a basic level of two-digit numbers. It is less about testing addition skills, although a + sign is present. The percentage of correct answers grows steadily between 8 years and 12 years old and is at a slightly higher level for the 40 to 49 years old group. The percentage of "no attempts" is marginal at 12 years and older.

The problem in Table 2 is a basic fact presented as a subtraction with a missing number and in the reverse order to basic fact format, that is, — − 4 = 9. For those pupils who are heavily dependent on consistency, even this simple change in order may be a challenge. The percentage of pupils aged 10 and 12 years who did not get a correct answer was high at 27.7% and 28.9%, respectively. Almost 1 in 10 pupils of these ages did not attempt the question. This dropped to almost 1 in 20 at 14 years old. These figures suggest that subtraction is perceived as being more difficult than addition.

An item, which requires the subject to cross the tens (see Table 3), provides further evidence to suggest that addition is believed to be easier than subtraction. The percentages of

Table 3

36 +54 Age	Sample	Percentage of correct answers	Percentage of "no attempts"
8 y	225	71.6	7.6
10 y	220	92.7	1.0
13 y	191	92.7	1.6
14 y	225	92.4	0.9
40–49 y	129	94.6	0.8

Table 4

103 –96 Age	Sample	Percentage of correct answers	Percentage of "no attempts"
8 y	225	29.3	34.7
10 y	220	63.6	7.7
13 y	191	69.6	8.9
14 y	225	79.6	5.8
40–49 y	129	93.0	1.6

correct answers are higher and the percentages of "no attempts" are lower than for the subtraction item, 103 – 96, particularly for the school pupils (see Table 4).

The National Numeracy Strategy includes subtraction problems of the type HTU-TU in Year 4 (8 years old).

The results shown in Table 4 indicate that around one-third of pupils aged from 10 to 13 years old cannot obtain a correct answer to this question. A number of the answers given, for example 193, were manifestly incorrect.

The problem shown in Table 5 is about the commutative property. If that is recognised, then the answer is achieved quickly and accurately. If it is not recognised then the method is to add 534 and 185, giving an interim answer of 719, followed by subtracting 185. The item uses three-digit numbers and thus may be perceived as more challenging. Even though the item uses the low anxiety inducing + sign, the percentage of "no attempts" is the highest for the examples listed so far which incorporate the + symbol. It is noteworthy that the percentage of "no-attempts" is now higher than for other addition problems in the test for the 40 to 49 year olds, who may not have the frequent exposure to the commutative property of numbers that schoolchildren do.

The hypothesis may require a caveat, in that once the addition problem becomes more challenging or is perceived as more challenging, the security of "doing" addition is weakened. This caveat may be required for other hypotheses.

It can be tempting for teachers who feel that a lesson is going well to push the concept a step beyond the comfort zone of some learners.

4.2. Hypothesis 2. Multiplication is not widely understood and is judged as difficult, so more learners avoid the risk of being wrong.

Caveat: simply learning the times table facts by rote may not help in building an early understanding of the concepts of multiplication (or division).

The multiplication item in Table 6 was designed to not require the retrieval of any of the basic multiplication facts that are often perceived as difficult (to retrieve). It requires only a retrieval of 2x and 3x facts. Thus, the item is testing the ability to use a procedure. The procedures that can be used all require the ability to organise, spatially and cognitively, a relatively large amount of information on the page.

The percentage of correct answers increases from age 13 years to age 16–19 years old, but remains well under 50%. The percentage of "no attempts" is around 34% for ages 16 to 39 years and then drops to around 15% for ages 40 to 59 years. This could suggest that whatever the methods taught, the "traditional long multiplication" or the more recently favoured grid method, they are not creating enduring security and efficacy for people after-school. Even methods that resulted in a correct answer sometimes indicated a rigid adherence to procedure, for example, a 15 y 9 m student wrote:

$$\begin{array}{r} 541 \\ \underline{\times 203} \\ 1623 \\ 0000 \\ \underline{108200} \\ 109823 \end{array} \tag{1}$$

However, among 15-year-old students this method, which uses partial products selected via the place values of the digits in the multiplying number, was the preferred

Table 5

		534 + 185 = 185 + __	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
10 y	220	52.7	12.3
13 y	191	70.7	12.7
15 y	220	82.3	10.0
40–49 y	129	77.5	15.5

Table 6

541 ×203			
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
13 y	191	15.2	40.8
14 y	225	27.1	22.2
15 y	220	38.2	16.8
16–19 y	307	33.2	32.6
20–29 y	160	33.1	34.4
30–39 y	108	32.4	33.3
40–49 y	129	57.4	16.3
50–59 y	88	52.3	13.6

Table 7

		2)38	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
10 y	220	59.1	30.5
13 y	191	60.7	27.2
15 y	220	75.0	13.2
16–19 y	307	67.8	20.8
40–49 y	129	86.0	8.5

method and usually led to a correct answer. Place value and partial product-based errors included

$$\begin{array}{r} 541 \\ \underline{\times 203} \\ 1043 \end{array} \quad \begin{array}{r} 541 \\ \underline{\times 203} \\ 1003 \end{array}$$

$$\begin{array}{r} 541 \\ \underline{\times 203} \\ 10820 \\ 0000 \\ \underline{1623} \\ 11443 \end{array} \quad \begin{array}{r} 541 \\ \underline{\times 203} \\ 1623 \\ 000 \\ \underline{1082} \\ 2705 \end{array} \qquad (2)$$

The insecurity, fear of failure, and anxiety created by multiplications that are perceived of as complex and demanding are likely to contribute significantly to the high percentage of "no attempts". A 59-year-old, successful, business man who scored 39/44 simply wrote "too hard" next to this item and moved on.

The percentages for correct answers suggest that whatever is happening to teach multiplication in many mathematics classrooms is not successful for a large proportion of learners.

4.3. Hypothesis 3. Division is even less understood than multiplication (and is rarely linked to multiplication in learners' minds) and is judged by learners as being even more difficult, so that more children and adults will avoid the risk of being wrong (see Table 7).

This item is at a low level of complexity. It could be viewed as "halving 38."

The percentage of "no attempts" is high for the 10 years and 13 years cohorts, representing between a third and a quarter who did not take the risk of attempting this item. Even at 40 years, the "no attempt" percentage showed that approximately 1 in 12 did not attempt a division by 2 (see Table 8).

Even though this item is still a division by a single digit number, it did create high percentages of "no attempts" and low percentages of correct answers, data that, when compared with the 541 × 203 item shows that the percentages of "no attempts" for this division item were higher than for the multiplication item (see Tables 9 and 10).

For the latter two items the percentages of "no attempts" are high. Both items could be considered to be, fundamentally, about place value. The "rules" about moving the decimal place when multiplying and dividing by powers of

Table 8

		9)927	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
13 y	191	41.9	36.1
14 y	225	47.6	29.8
15 y	220	45.5	32.3
16–19 y	307	38.1	46.0
20–29 y	160	31.2	45.6
40–49 y	129	55.8	26.4

Table 9

		$10)\overline{6030}$	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
10 y	220	44.5	31.4
11 y	200	44.5	23.0
13 y	191	48.7	27.2
15 y	220	62.3	14.5
16–19 y	307	56.5	22.8
40–49 y	129	77.5	7.8

Table 10

		23 ÷ 1000	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
13 y	191	31.4	38.7
14 y	225	42.2	30.2
15 y	220	46.8	23.6
16–19 y	307	46.9	29.3
40–49 y	129	45.0	20.2

Table 11

		1 km = ___ metres	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
13 y	191	37.7	34.0
14 y	225	50.7	23.1
15 y	220	56.4	20.5
16–19 y	307	53.7	27.0
40–50 y	129	69.0	19.4

10 are still promoted as, for example, in a revision book for mathematics at Key Stage 3 ([23], p 30). Rules and procedures that bypass understanding are less likely to be remembered accurately.

Many procedures that are taught for mathematics are very unforgiving on faulty memories. Often, even one small error in the application of a procedure is enough to generate failure.

Recent research by Siegler et al. [24] found that knowledge at age 10 of division was consistently related to later mathematics proficiency. This is not happy news for the UK, if over 30% of 10-year-old pupils do not attempt to divide 38 by 2 or 6030 by 10.

4.4. Hypothesis 4. Problems that involve the application of multiplication and division generate more "no attempts" than those that are simply computations (see Tables 11 and 12).

The two items mix arithmetic and measurement. Over one-third of 13-year-old pupils and one-fifth of 15-year-old pupils did not attempt to divide a kilometre by 5. The item which involves conversion of a quantity in kilometres to metres, illustrated, via the type of errors it generated, the implications of a partial recall of the rules about "moving the decimal point and adding zeros." The most frequently occurring error was 5067 metres.

4.5. Hypothesis 5. The percentage of pupils using the "no attempt" avoidance strategy is greatest for the bottom quartile of achievers.

Caveat: as questions become "harder" more of the high achievers use the "no attempt" strategy.

Table 12

		5.67 km = __ metres	
Age	Sample	Percentage of correct answers	Percentage of "no attempts"
13 y	191	36.6	43.5
14 y	225	44.4	32.4
15 y	220	50.9	25.5
16–19 y	307	45.0	39.4
40–50 y	129	54.3	27.9

Table 13

	All	BQ	TQ	BQ/All	PC
10 years					
2)38	30.5	18.2	1.0	60.0	59.1
10)6030	31.4	18.2	1.0	58.0	44.5
13 years					
2)38	27.2	17.3	0.5	63.5	60.7
10)6030	27.2	17.8	0	65.4	48.7
15 years					
2)38	13.2	8.2	0	62.1	75.0
10)6030	14.1	10.0	0	71.0	62.3
4.8 + 5.21 + 6	12.3	9.1	0	74.0	55.4
541 × 203	17.3	8.6	1.8	49.7	38.2
(2/5) + (3/8)	29.1	13.2	0.5	45.4	37.3
9)927	33.6	17.7	0.5	52.7	45.5

All: for all students.
BQ: for the bottom quartile of total score.
TQ: for the top quartile of total score.
BQ/All: percentage of no attempts for the bottom quartile compared to all no attempts (for example, for 2)38 for pupils of 10 years, 60.0% of the "no attempts" were from the bottom quartile of students).
PC: percentage of correct answers.

The "no attempts" for two items from the 15-minute test were analysed for the bottom quartile of scores, the top quartile, and for all scores. The main focus was on the division items 2)38 and 10)6030.

The students in the bottom quartile of achievers were contributing a high proportion of the no attempts (see Table 13). When the percentage of that contribution drops below 50% the questions generate correct scores below 40%, suggesting that a larger number of the more able children are being dragged into the "no attempt" behaviour when questions become less straightforward. A related conclusion was reached by Ashcraft and Krause [17] in a study on 80 undergraduates, which investigated the correlation between math performance and math anxiety. The lower achievement of the undergraduates who were math-anxious seemed limited to more difficult math which is described as math taught at or after late elementary school.

5. Summary

Poor achievement levels in mathematics for a significant percentage of the population have been an issue in the UK for decades [25]. The data presented in this paper supports a number of hypotheses, which combine to suggest a final hypothesis, that is, that too many children and adults give up on mathematics learning by withdrawing from any task that is perceived as likely to result in failure, which in turn becomes a pervasive attitude. The withdrawal strategy avoids the learner being judged as wrong and thus adding to their sense of helplessness. Some of the demands of mathematics that contribute to a sense of failure, anxiety, and helplessness are based on beliefs, rather than academic necessity. These beliefs also have an effect on the way mathematics is taught with a focus on the curriculum whilst overlooking the characteristics of the learners.

The combination of "no attempts" and incorrect answers for many of the examples used above is not indicative of a successful outcome for the mathematics curriculum and how it is taught in the UK.

Finally, a 12-year-old student summed up his experiences of mathematics teaching as, "All I hear is talking, talking. It's a river." Perhaps it is time to challenge some of the mathematics beliefs that have a profound influence on the way the subject is perceived and taught and give more focus to how learners learn.

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Research Article

Attitudes to Mathematics in Primary School Children

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44 Grade 3 children and 45 Grade 5 children from English primary schools were given the British abilities scales basic number skills subtest, and a Mathematics Attitude and Anxiety Questionnaire, using pictorial rating scales to record their Self-rating for maths, Liking for maths, Anxiety about maths, and Unhappiness about poor performance in mathematics. There were few year group differences in attitudes. Boys rated themselves higher than girls, but did not differ significantly in actual performance. Overall, Anxiety was not related to actual performance, but Self-rating was. This relationship between Self-rating and actual performance seemed to develop between Grade 3 and Grade 5. Implications of the findings are discussed.

1. Introduction

Mathematics depends not only on cognitive abilities but also on emotional factors and attitudes. Several studies have shown that emotional factors may play a large part in mathematical performance, with mathematics anxiety playing a particularly large role [1–3]. One possible reason for the negative association between mathematics anxiety and actual performance is that people who have higher levels of math anxiety are more likely to avoid activities and situations that involve mathematics and, thus, have less practice [4]. Mathematics anxiety might also influence performance more directly, by overloading working memory. For example, Ashcraft and Krause [5] reported that highly math-anxious individuals had significantly lower working memory capacities in mathematics-related tasks than individuals with low levels of mathematics anxiety.

Relationships between mathematics anxiety and performance may also be in the other direction. Poor mathematical attainment may lead to mathematics anxiety, as a result of repeated experiences of failure. Indeed, there is likely to be a vicious circle, where anxiety and performance affect each other negatively. Maloney and Beilock [6] point out that the development of mathematics anxiety is likely to be due both to social factors, such as exposure to teachers who themselves suffer from mathematics anxiety, and to pre-existing difficulties in numerical cognition, and that those with initial mathematical difficulties are also likely to be more vulnerable to the negative social influences.

Although mathematics anxiety is a major focus for research, not all attitudes to mathematics are negative, and some people express a strong liking for mathematics [7]. This seems to be particularly the case for mathematically gifted people [8] but is not restricted to them.

Most studies of attitudes to mathematics have involved older children and adults. It is particularly important to investigate the early development of attitudes to mathematics, if we are to understand the relationships between these and actual performance, and if possible to prevent the development of strongly negative attitudes. The relatively few studies that have looked at younger children's attitudes to mathematics have usually shown positive attitudes, with most children claiming to like mathematics ([9–11]). However, studies suggest that attitudes may deteriorate with age, especially through the secondary school period [9, 12, 13], but also within the primary age group ([10, 13]). One study by Gierl and Bisanz [14] gave somewhat more ambiguous results, indicating an increase in mathematics anxiety but also an increase in liking for maths over the later elementary school years.

Most studies of secondary school children and adults have been consistent in showing actual performance in mathematics tends to be related positively to liking for mathematics and to high self-ratings in mathematics, and negatively to mathematics anxiety (e.g., [2, 5, 15–17] Lam et al., 2000). This is the case not only for advanced mathematics but for basic magnitude comparison skills [18].

Studies of the relationships between attitudes to maths and performance in younger children have not resulted in as clear results as those for older children. Ma and Kishor [17] found that the relationships between attitudes and performance increase with age. Cain-Caston [19] found little relationship between mathematics performance and attitudes in a sample of 8- to 9-year olds. Thomas and Dowker [20] (also see [10]) and Krinzinger et al. [21] found that 6- to 8-year olds' mathematical performance was related to Liking mathematics and to Self-rating of ability in the subject, but not to anxiety. Krinzinger et al. [21] failed to find either longitudinal or concurrent relationships between anxiety and performance in young children. On the other hand, relationships between mathematics anxiety, other attitudes, and actual performance have been found in primary school children in a white South African community [22], in New Zealand [23], and in several studies in the United States ([24–27]).

It is thus possible that the relationships between attitudes and performance become clearer in the later primary years. The present study aimed to examine relationships between mathematical performance, mathematics anxiety, unhappiness over poor performance, liking for mathematics, and self-rating in mathematics in children of midprimary age (Grade 3; age 7-8) and later primary age (Grade 5; age 9 to 10).

On the basis of the earlier studies using a similar method [20, 21], it was predicted that mathematics performance would be related to liking mathematics and to self-rating of ability in the subject, but not to anxiety. It was also predicted, especially on the basis of Thomas and Dowker's [20] results that there would be specific independent relationships between liking mathematics and self-rating, and between anxiety and performance-related unhappiness, and that otherwise there would be little correlation between the attitude variables.

Another aim was to investigate changes between Grade 3 and Grade 5. It was predicted that older children might show more negative attitudes. We also aimed to look at whether the relationships between the different variables might change between Grade 3 and Grade 5. Although any such analysis would have to be tentative due to reduced numbers, it was predicted that the relationships between mathematics performance and other attitude variables, and especially between mathematics performance and self-rating, might not be noticeable at Grade 3 but might become significant at Grade 5. This prediction is based on the assumption that children's experiences of success and failure might influence their attitudes toward and confidence in mathematics, and that such experiences and their effects would become increasingly salient during the later primary school years.

The study also aimed to look at gender differences in mathematics performance and attitudes. Most current studies suggest that females do not perform lower in mathematics than males, but that they do tend to rate themselves lower, and to experience more anxiety [28, 29]. The study aimed to investigate whether such gender differences would already be present in young children.

2. Method

2.1. Participants. The participants were 89 children taken from two state primary schools in Brentwood, Essex and one in Enfield, London. They were selected randomly from the third- and fifth-grade classes in these schools. These were nonselective schools with varied intakes, but situated in predominantly relatively middle-class areas. They included 44 children (21 boys and 23 girls) in Grade 3 and 45 children (21 boys and 24 girls) in Grade 5. The Grade 3 children had a mean age of 90.1 months (s.d. 3.48) and the Grade 5 children had a mean age of 112.39 months (s.d. 6.51).

2.2. Materials and Procedure. In order to measure their attitudes, children completed the Mathematics Attitudes and Anxiety Questionnaire [20, 30]. The attitude questionnaire consists of 28 questions which focused on 7 areas of maths: maths in general, written sums, mental sums, easy maths, difficult maths, maths tests, and understanding the teacher. For each item, children were asked about their Self-rating ("How good are you?") on a scale consisting of ticks and crosses ("very good" to "very bad"); Liking for the items ("How much do you like it?") on a scale consisting of sweets and wasps ("like very much" to "hate very much"); Anxiety about them ("How worried would you feel?") on a scale of facial expressions based on Roger Hargreaves' "Mr. Men" picture-book characters Mr. Happy and Mr. Worry ("very relaxed" to "very worried"); Unhappiness at poor performance ("How unhappy would you feel if you did badly?") on a scale consisting of faces with frowning or happy faces ("very unhappy" to "very happy"). Possible rating scores for each item ranged from 1 to 5, so the total possible scores on each scale ranged from 7 to 35. Scores were calculated so that a higher score represented a more positive attitude. Thus, the Anxiety and Unhappiness scores represent relative *freedom* from Anxiety and Unhappiness. In places where there may be ambiguity, the paper will refer to "(Non-)Anxiety" and "(Non-)Unhappiness".

3. Results

Table 1 shows the BNS standard score and the attitude scores according to gender and school year.

3.1. Effects of Gender and School Year on Mathematical Performance and Attitude Scores. An ANOVA was carried out with Year Group (Grade 3 versus Grader 5) and Gender (Male versus Female) as the grouping factor, and BNS standard score and the attitude scores of Liking, Self-rating, Anxiety, and Unhappiness as the dependent variables. There

Table 1: Maths performance and attitude scores by year group and gender.

Year group	Gender	BAS Basic Number Skills standard score	Self-rating	Liking	(Non-)Anxiety	(Non-)Unhappiness
Year 3	Boys ($n = 21$)	109.14 (13.42)	28.9 (4.31)	25.62 (7.89)	23.19 (7.57)	22.71 (4.36)
	Girls ($n = 23$)	112.65 (12.8)	27.26 (4.46)	24.43 (5.56)	23.0 (6.33)	21.04 (5.23)
Total year 3 ($n = 44$)		110.98 (13.07)	28.04 (4.42)	25.0 (6.72)	23.09 (6.87)	21.84 (4.86)
Year 5	Boys ($n = 21$)	120.52 (13.61)	29.05 (4.76)	24.62 (8.52)	23.05 (4.86)	21.00 (4.86)
	Girls ($n = 24$)	115.71 (16.59)	25.75 (5.56)	24.46 (5.73)	23.46 (6.21)	19.71 (5.7)
Total year 5 ($n = 45$)		117.96 (15.29)	27.29 (5.4)	24.53 (7.08)	23.27 (5.57)	20.31 (5.3)
	Total boys ($n = 42$)	114.83 (14.54)	28.98 (5.56)	25.12 (8.12)	23.11 (6.28)	21.86 (4.64)
	Total girls ($n = 47$)	114.21 (14.78)	26.49 (5.06)	24.45 (5.58)	23.23 (6.21)	20.36 (5.46)
Total ($n = 89$)		114.51 (14.59)	27.66 (4.93)	24.76 (6.87)	23.18 (6.21)	21.07 (5.11)

Means are given with standard deviations in brackets.

was a significant effect of Year Group on basic number skills standard score ($F(1,85) = 5.7$; $P = 0.019$; $\eta P^2 = 0.063$), indicating higher scores by the Grade 5 children. There was a significant effect of Gender on Self-rating ($F(1,85) = 5.93$; $P = 0.018$; $\eta P^2 = 0.064$), indicating higher self-rating by boys. No other effects were significant.

Because there were year group differences in BNS standard score, and because the year groups differed in both chronological age *and* school experience, an ANCOVA was then carried out including BNS standard score and chronological age in months (Age) as covariates. Again, Year Group (Grade 3 versus Grade 5) and Gender (Male versus Female) were the grouping factors, and BNS standard score and the attitude scores of Liking, Self-rating, Anxiety, and Unhappiness were the dependent variables. BNS standard score made a significant independent contribution to the variance in Self-rating ($F(1,83) = 6.58$; $P = 0.012$; $\eta P^2 = 0.073$). Age made a significant independent contribution to Unhappiness ($F(1,83) = 13.79$; $P < 0.001$; $\eta P^2 = 0.143$). After controlling the effects of the covariates, Gender still had a significant effect on Self-rating ($F(1,83) = 5.64$; $P = 0.02$; $\eta P^2 = 0.064$). Year Group now had a significant effect on Unhappiness ($F(1,83) = 16.38$; $P < 0.001$; $\eta P^2 = 0.165$).

3.2. Correlations between Attitudes and Mathematics Performance. Pearson correlation coefficients were obtained between the attitude scores of Liking, Self-rating, Anxiety and Unhappiness, BNS standard score, and age in months. These are shown in Table 2.

3.3. What Attitudes Predict Mathematical Performance? Table 3 shows entry-type linear regressions on each of the test variables, with the other variables as predictors. First, in order to investigate more directly the influence of attitudes on mathematical performance an entry-type linear regression was carried out with *BNS standard score* as the dependent variable, and Liking, Self-rating, (Non-)Anxiety and (Non-)Unhappiness as the predictors. The only significant predictor was Self-rating.

Similar analyses were then carried out for each of the attitude variables, with BNS standard score and the other attitude variables as predictors.

Self-rating was predicted by *all* the other variables except (Non-)Anxiety; BNS standard score, (Non-)Unhappiness, and Liking were all significant independent predictors of Self-rating.

Liking was independently predicted by Self-rating and (Non-)Anxiety.

(Non-)*Anxiety* was independently predicted only by Liking, though (Non-)Unhappiness almost reached significance as a predictor ($P = 0.058$).

(Non-)*Unhappiness* was independently predicted only by Self-rating, though, as expected from the previous analysis, (Non-)Anxiety almost reached significance as a predictor.

The same analyses were then carried out separately for the group of third-grade children (Table 4) and the group of fifth-grade children (Table 5).

The findings for third-grade children are shown in Table 4.

None of the attitude variables predicted BNS standard score. Liking and (Non)-Unhappiness predicted Self-Rating. Self-Rating and (Non-)Anxiety predicted Liking. Self-Rating and Liking predicted (Non-)Anxiety. Self-rating predicted (Non-)Unhappiness.

The findings for fifth-grade children are shown in Table 5.

Self-rating was the only significant predictor of BNS standard score. Similarly, BNS standard score was the only significant predictor of Self-rating, (Non-)Anxiety was the only significant predictor of Liking, and Liking was the only significant predictor of (Non-)Anxiety. No predictor approached significance for (Non-)Unhappiness.

4. Discussion

The results support previous studies with primary age children in suggesting that attitudes to mathematics are generally positive in the primary age group. All attitude scores, except for (Non-)Unhappiness at poor performance, were considerably higher than the notional neutral score of 3, though questions could be raised as to whether such a rating

Table 2: Correlations between attitude scores, British abilities scales basic number skills standard score, and age in months.

	Self rating	Liking	(Non-)Anxiety	(Non-)Unhappiness	Basic Number Skills: standard score	Age in months
Self-rating	1					
Liking	0.264*	1				
(Non-)Anxiety	0.191	0.413**	1			
(Non-)Unhappiness	0.329**	0.178	0.268*	1		
Basic Number Skills: Standard score	0.255*	−0.1	−0.069	0.021	1	
Age in months	−0.055	−0.083	0.232	0.075	0.231	1

*$P < 0.05$ **$P < 0.01$.

Table 3: Entry-type multiple regressions on British abilities scales basic number skills standard score and the attitude variables: whole group.

	Regression on Basic Number Skills standard score	Regression on Self-rating	Regression on Liking	Regression on (non-)Anxiety	Regression on (non-)Unhappiness
Degrees of freedom	4	4	4	4	4
Residual	84	84	84	84	84
R^2	0.106	0.484	0.226	0.211	0.159
F	2.502	6.414	6.148	5.631	3.96
P	0.048*	0.000**	0.000**	0.000**	0.005**
Predictors:					
Basic number skills	—				
β	—	0.286	−1.415	0.034	−0.047
t	—	2.973	−1.402	−0.333	−0.446
P	—	0.004**	0.164	0.74	0.657
Self-rating					
β	0.333	—	0.236	0.021	0.306
t	2.973	—	2.219	0.191	2.795
P	0.004**	—	0.029*	0.849	0.006**
Liking			—		
β	−0.163	0.234	—	0.368	0.006
t	−1.402	2.219	—	3.591	0.049
P	0.104	0.029*	—	0.001**	0.961
(Non-)Anxiety					
β	0.039	0.021	0.361	—	0.212
t	0.333	0.191	3.591	—	1.919
P	0.74	0.849	0.001**	—	0.058
(Non-)Unhappiness					
β	−0.05	0.278	0.005	0.198	—
t	−0.446	2.973	0.049	1.919	—
P	0.657	0.006**	0.961	0.058	—

*$P < 0.05$ **$P < 0.01$.

really does represent "neutrality". There was little difference in attitude between Grade 3 and Grade 5, contradicting the hypothesis that attitudes might deteriorate during this period.

In this particular group, the standard scores on a standardized arithmetic test were rather higher in Grade 5 than in Grade 3, giving rise to the possibility that group differences in mathematical attainment were masking deterioration in attitudes. However, even after controlling both arithmetic standard score and chronological age, there continued to be little effect of year group on attitude.

The exception was (non-)Unhappiness at poor performance, where the year group effect did become significant. However, this reflected a more positive attitude in Grade 5

Table 4: Entry-type multiple regressions on British abilities scales basic number skills standard score and the attitude variables: Grade 3 alone.

	Regression on Basic Number Skills standard score	Regression on Self-rating	Regression on Liking	Regression on (non-)Anxiety	Regression on (non-)Unhappiness
Degrees of freedom	4	4	4	4	4
Residual	39	39	39	39	39
R^2	0.01	0.318	0.281	0.204	0.264
F	0.095	4.552	3.45	2.503	3.496
P	0.984	0.004**	0.017*	0.058	0.016*
Predictors:					
Basic number skills:					
β	—	0.019	−0.016	−0.011	0.08
t	—	−0.145	−0.112	−0.074	0.583
P	—	0.885	0.911	0.942	0.563
Self-rating					
β	−0.028	—	0.414	−0.054	0.417
t	−0.145	—	2.035	−0.314	2.74
P	0.885	—	0.01*	0.755	0.009**
Liking					
β	−0.021	0.382	—	0.322	−0.121
t	−0.112	2.707	—	2.035	−0.764
P	0.911	0.01*	—	0.049*	0.449
(Non-)Anxiety					
β	−0.013	−0.047	0.298	—	0.288
t	−0.074	−0.314	2.035	—	1.957
P	0.942	0.785	0.049*	—	0.058
(Non-)Unhappiness					
β	0.108	0.387	−0.122	0.311	—
t	0.583	2.74	−0.764	1.957	—
P	0.563	0.009**	0.449	0.058	—

*$P < 0.05$ **$P < 0.01$.

than Grade 3 children. Age was also separately significant as a covariate, and it correlated negatively with this attitude variable. As Unhappiness was not significantly related to either age or year group when they were considered separately, these results should be taken with caution at this point, but attempts should be made to replicate them in future studies.

It should be noted that there may be more ambiguity about the positive or negative nature of Unhappiness at poor performance than over the other attitudes. Certainly, a high level of unhappiness at poor performance is a negative attitude, but at the end of the scale where children are relatively free of such unhappiness, individual differences in scores may be more reflective of a preference for performing well, arguably a positive attitude. Perhaps the somewhat puzzling results here could reflect this ambiguity.

Correlational analyses indicate little relationship between chronological age and the attitude variables, supporting the findings with regard to year group effects. As predicted, they support the earlier results of [10] and Krinzinger et al. [21], in showing no consistent relationship between mathematics Anxiety and performance among primary school children. There was, however, a significant relationship between Anxiety and Liking for mathematics; not surprisingly, a Liking for mathematics was strongly associated with a freedom from Anxiety.

The only attitude variable that correlated significantly with actual performance was Self-rating. Those who rated themselves higher at maths performed better. Self-rating appeared indeed to be the only variable that was strongly related to all the other variables. As well as being related to actual performance, it was related to Liking for maths and to freedom from Anxiety. After controlling for other variables in regression analyses, it was also related to freedom from performance-related Unhappiness.

Somewhat unexpectedly, there was no significant relationship between Liking for maths and actual performance, though Liking for maths was significantly related to other attitude variables.

It had been predicted that Anxiety and Unhappiness at poor performance might be strongly and independently

Table 5: Entry-type multiple regressions on British abilities scales basic number skills standard score and the attitude variables: Grade 5 alone.

	Regression on basic number skills standard score	Regression on self-rating	Regression on liking	Regression on (non)anxiety	Regression on (non)unhappiness
Degrees of freedom	4	4	4	4	4
Residual	40	40	40	40	40
R^2	0.31	0.337	0.275	0.281	0.103
F	4.495	5.084	3.794	3.903	1.154
P	0.004**	0.002**	0.01*	0.009**	0.346
Predictors:					
Basic number skills	—				
β	—	0.522	−0.157	−0.15	−0.05
t	—	3.983	−0.979	−0.939	−0.277
P	—	0.000**	0.334	0.353	0.784
Self-rating					
β	−0.028	—	0.135	0.133	0.243
t	−0.145	—	0.821	0.813	1.354
P	0.885	—	0.416	0.421	0.183
Liking					
β	−0.021	0.123	—	0.436	0.087
t	−0.112	0.821	—	3.082	0.498
P	0.911	0.416	—	0.004**	0.621
(Non)anxiety					
β	−0.013	0.122	0.44	—	0.129
t	−0.074	0.813	3.082	—	0.735
P	0.942	0.421	0.004**	—	0.467
(Non)unhappiness					
β	0.108	0.18	0.021	0.103	—
t	0.583	1.354	0.498	0.735	—
P	0.563	0.183	0.621	0.467	—

*$P < 0.05$ **$P < 0.01$.

correlated with one another but not with the other attitude variables, and that Liking for maths and Self-rating might be strongly and independently correlated with one another but not with the other attitude variables. This prediction was not entirely borne out. Self-rating and Liking for maths did show an independent relationship, but Self-rating was also independently related to the other attitude variables. Liking for maths was correlated with Anxiety as well as Self-rating. There was only a weak relationship between Anxiety and performance Unhappiness, and each correlated more strongly with other factors: Anxiety with Liking for maths, and performance Unhappiness with Self-rating.

The results of separate regression analyses for different year groups suggest that, although there are few age or year group differences in the attitude scores themselves, there may be some differences in the relationships between the variables. A great deal of caution is needed in interpreting these results, because of the reduced sample sizes in these analyses, resulting in relatively low statistical power. It is possible that there were some small effects that would only have become obvious in larger samples. However, the results do support the hypothesis that the relationship between self-rating and actual performance may develop between Grade 3 and Grade 5. There was no relationship between the two variables in Grade 3 children. In Grade 3, none of the attitude variables predicted mathematical performance, and Self-rating was predicted by Liking for maths and freedom from Anxiety, but not by performance. In Grade 5, by contrast, there was a very strong and specific relationship between Self-rating and actual performance. This finding may suggest that children become more aware of their mathematical performance in relation to that of their peers as they become older, either because of generally greater self-awareness, or because of greater experience of tests and teacher assessments. Alternatively, or additionally, it may be that children's self-assessment has an increasing influence on their motivation, and thereby on their performance, as they get older.

The only significant gender difference was in self-rating, where boys rated themselves higher than girls did. There

were no gender differences in actual performance or in other attitudes. This supports findings from previous studies that indicate that males tend to rate themselves higher than females in mathematics, although at least in recent studies the genders usually perform similarly. Future research should investigate whether this finding is specific to self-rating in mathematics, where it may reflect existing stereotypes about the subject, or whether it might indicate generally higher self-ratings by boys in other subjects as well.

One key limitation of this study is of course that the sample is relatively small. It is important for future studies to extend both the number and the range of the sample. For example, the children in the sample were attending schools in relatively middle-class catchment areas and, perhaps linked to this fact, were generally performing at somewhat above average levels in mathematics. The mean standard score on the basic number skills test was 114.51, as compared with an assumed population mean standard score of 100. It is possible that a lower-attaining group, or one including a higher proportion of children from low-SES backgrounds, might show more negative attitudes and/or stronger relationships between attitudes and performance.

Also, it is important to carry out cross-cultural comparisons. This study aligns with most similar non-American studies (e.g., [17, 21]), in suggesting that younger children do not show a significant relationship between mathematics anxiety and actual mathematical performance, but, as indicated in the Introduction, most American studies have given contrasting results (e.g., [24–26]). It would be interesting to investigate whether there is some difference in cultural attitudes or in methods of mathematics education that contributes to this disparity. It would also be desirable to investigate young children's attitudes to mathematics in a wider variety of cultures.

It could be argued that a questionnaire measure is not ideal as a sole measure of attitudes in young children, or perhaps in any group. In future studies, it would be desirable to include other measures: for example implicit attitude tests [31], behavioural measures such as children's responses when given a choice between activities with and without mathematical content, and perhaps physiological indicators of anxiety. It would also be desirable to use a wider variety of mathematics tests, in particular, to use tests of mental as well as written arithmetic, as the former is more likely to be affected by the increased load that anxiety may place on working memory ([5, 25]).

Despite such potential limitations, the study yields some important conclusions. Young children appear to show relatively positive attitudes to mathematics, though a larger sample of different ages would be needed to confirm this. They show little relationship between mathematics anxiety and mathematical performance, but they do show a relationship between self-rating and mathematical performance, which seems to develop during the later primary years. There are also significant relationships between self-rating and other attitudes. It is thus arguable that self-rating, rather than mathematics anxiety, is the key factor in the primary school age group, and that perhaps researchers on younger children's attitudes to mathematics should focus more on self-rating.

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Research Article

Instructional Quality and Attitudes toward Mathematics: Do Self-Concept and Interest Differ across Students' Patterns of Perceived Instructional Quality in Mathematics Classrooms?

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Using a person-centered research approach, the present study explored individual differences in students' perceptions of instructional quality in secondary school mathematics classes and their relations to students' self-concept and interest in mathematics. Drawing on data collected from 425 high school students from ten schools in Berlin, Germany (male: 53.2%; female: 46.3%), latent class analyses (LCA) revealed four distinct patterns of perceived quality of instruction. Almost half of the sample (46%) had a high likelihood of perceiving an overall low quality in mathematics classes. Those students reported particular low self-concept and interest in mathematics. Compared to male students, female students were significantly more likely to belong to this "challenging pattern." Consequences for educational practice are discussed and suggest that instruction in mathematics should take into account learners' highly individual ways of perceiving and evaluating their learning environment.

1. Introduction

After major international large scale assessments on educational performance such as the Programme for International Student Assessment (PISA) revealed low to average performance of German students in mathematics and science compared to other participating countries, the enhancement of learning success and later professional careers in the STEM-disciplines (STEM = *science*, *technology*, *engineering*, and *mathematics*) have been prioritized in educational policy [1, 2]. Attitudinal and affective variables such as mathematics self-concept and interest were shown to be central to students' achievement in mathematics [3]. Mathematics self-concept, which is broadly defined as a persons' self-related perceptions in the area of mathematics that are formed through experience with others and one's own interpretations of their environment [4], is reciprocally related to achievement in mathematics and positively related to course choice in mathematics domains during upper-secondary education [5, 6]. Another key factor for students' learning is students' interest in mathematics, which is shown to be related to achievement goals in mathematics classes and mathematics-related career choices [7, 8]. Students who are interested in mathematics enjoy engaging in math, tend to reengage in mathematical contents, and view mathematics as important for their individual development [9].

Both self-concept and interest in mathematics are influenced by educational settings and teaching styles. Research suggests that particular aspects of instructional quality in mathematics classrooms such as classroom management, classroom climate, and cognitive activation relate to students' attitudes and emotions concerning mathematics [10–12]. Although a vast number of previous studies consider characteristics of instruction and their impact on learning [13–16], individual differences in students' perceptions of instructional quality in their mathematics classrooms are explored less frequently. However, it is important to address individual differences as students' learning success depends highly on the level of adaptability of learning environments to students' individual needs [17, 18]. Subsequently, in

the current study, we explored distinct student patterns of perceived instructional quality in mathematics classes. Thereby, the aim of this research was to analyze whether students' self-concept and interest in math differ across distinct student patterns. As empirical studies suggest that demographic variables such as age and gender are related to students' perceptions of instructional quality in mathematics classrooms [19, 20], we considered in our analysis associations between these demographic characteristics and students' patterns of perceived instructional quality.

2. Characteristics of Instructional Quality Mathematics Self-Concept and Interest

Referring to a cognitive perspective of learning, research on instructional quality conceptualizes learning as a self-determined, constructive, and self-regulated process of conceptual growth, which is supported or undermined by perceived learning conditions and shaped by a dynamic interplay among personal, behavioral, and environmental factors [21–23]. Thus, a high level of instructional quality of learning environments is seen as a prerequisite to enhanced learning outcomes depending on students' subjective perceptions, preknowledge, and internal structures of cognitive processing [24, 25]. Based on related theoretical frameworks such as self-determination theory (SDT; [26]), research concerning relations between learning environments and students' motivation often focuses on students' individual perceptions of their learning environment [13, 27, 28]. SDT suggests that learning conditions that lead to the fulfillment of students' basic psychological needs for autonomy, competence, and social relatedness facilitate students' intrinsic motivation. Thus, related research highlights students' individual experiences in their learning environments. Conversely, research on effective teaching strategies often considers students' perceptions aggregated on class level [16, 29]. Using aggregate measures of classroom context refers to instructional quality as an objective criterion of the learning environment that is perceived by all students within one class in a similar way [30]. However, it highly depends on the aims of research as to which conceptualization of instructional quality is most appropriate. Lütdke et al. [31] point out that assessing characteristics of learning environments with data aggregated at the group level focuses on differences between learning environments, while assessing students' personal perceptions focuses on differences between students. Concerning motivational learning outcomes, it is unclear which conceptualization is most appropriate. Clausen [32] highlights that students' perceptions aggregated at the class level are valid indicators of teaching behaviours and are highly related to students' motivational learning outcomes. However, Kunter et al. [30] highlighted that the validity of mean ratings as descriptions of the learning environment might be questioned. Their results indicated that mean ratings of classroom management strategies were unrelated to the change in students' interest in mathematics from grade 7 to grade 8, while individual perceptions of classroom features were highly related to students' change in interest. Referring to this theoretical and empirical work, the present study aimed to examine how students' individually experience the instructional quality in their classrooms. To this end, we questioned which subtypes of individual perceptions of instructional quality exist, how gender and age are related to potential subtypes, and how the subtypes differ in terms of motivational and attitudinal learning outcomes. Thus, we focused on students' individual perceptions rather than on data aggregated at the class level. Appropriate to the aims of the present study, person-centered research approaches such as Latent Class Analysis (LCA) allow the distinction of specific homogenous subgroups of students within a heterogeneous total population of students [33].

Regarding the question of how instructional quality is defined, three basic dimensions which structure the array of single characteristics of teaching and classroom components have emerged consistently as being crucial for motivational learning outcomes in mathematics [12, 34, 35]. Although the terminology used varies [36], the three basic dimensions are (1) classroom management, (2) supportive climate, and (3) cognitive activation. Our study enlists single characteristics of instruction assigned to these three dimensions as indicators of instructional quality in mathematics classes.

The basic dimension of classroom management typically refers to an efficient classroom and time management or low levels of disruptive student behavior [36, 37]. Recent empirical studies furthermore outline high levels of clarity and structuredness of teachers' instruction as important components of an effective classroom management [12, 38]. An efficient classroom management characterized by high structuredness and clarity of instruction is a salient predictor of students' interest in mathematics classes [16]. Theoretically, research related to self-determination theory [26] revealed that effective classroom management enhances students' experience of intrinsic need satisfaction and thus facilitates students' interest [28, 30, 39].

The basic dimension of supportive climate includes features of teacher-student interaction such as supportive teacher-student relationships, caring and attentive teacher behavior, or constructive feedback [18]. Based on social comparison theory [40], research has shown that students' self-concepts are highly influenced by their social environment and the social comparisons provided by this environment [41]. Langford et al. [42] point out, for example, that perceived social support leads to more accurate perceptions of normative peers [43]. Sarason and colleagues [43] propose that low-perceived social support may encourage downward social comparisons. Perceived social support further leads to the experience of relatedness to others, whereby facilitating students' intrinsic motivation [39, 44]. Thus, studies show that the degree to which students' perceive teacher support in class plays a critical role in the development of students' self-concept and interest in mathematics [13, 14].

The basic dimension of cognitive activation refers to characteristics of instruction, which promote students' conceptual understanding by including, for example, challenging tasks or enhancing different solution strategies and nonroutine problem solving [17]. Additionally, discursive

effectiveness, which refers to students' opportunity to co-construct knowledge by participating in mathematics classroom discourse, is viewed as a mechanism to activate cognitive processing [18, 45]. Participating in cognitively challenging classroom discourse facilitates students' emotional well-being [46] and thus is linked to students' interest in mathematics classrooms [15] and students' mathematics self-concept [47].

Concerning the relation between perceived characteristics of instructional quality in math and science classes and motivational learning outcomes, Eccles and colleagues [20] suggest in their stage-environment-fit approach that the consistent developmental decline of attitudinal and motivational-affective learning outcomes during junior high school [10, 16, 48] results from a mismatch between the needs of developing adolescents and the opportunities afforded to them by their learning environments. However, research has shown that distinct subtypes of students exist despite this general decline, ranging from students with overall positive characteristics (e.g., high self-concept of ability and interest) to students with "challenging" characteristics (e.g., low self-concept of ability and interest) [49]. Seidel [49] reveals that the differences in students' motivational learning outcomes relate to perceived learning conditions in mathematics classrooms such as instructional clarity and interaction with the teacher. Based on a theoretical conceptualization of learning as a self-directed process, in which teachers provide learning opportunities that must be perceived and utilized by students [17, 50], in the present study we expected to find distinct student subtypes with respect to perceived instructional quality corresponding to the subtypes of motivational learning outcomes as shown by Seidel [49]. Thus, we expected to find a subtype of learners with overall positive perceptions of the characteristics of instructional quality in math class. We expected further to find another extreme subtype of students with overall negative perceptions of the instructional quality in math class. As it is well known that single characteristics of instructional quality, such as perceived structuredness and autonomy support, are independent learning factors that can be complementary [51], we expected to find mixed subtypes of students' perceived characteristics of instructional quality *structuredness of teachers' instruction, teachers' social support,* and *discursive effectiveness*.

In the present study, we controlled for gender and age effects in the latent-class models. Previous studies revealed that gender has been shown to be related to learning environment perceptions [52] as well as to mathematics self-concept and interest [10, 53]. Ditton [19], for example, showed that male students evaluate their mathematics teachers' performance more positively than their female classmates. Furthermore, age was shown to be related to students' perceptions of the instructional quality in mathematics classes. Eccles and colleagues [20] suggest that negative learning-related changes in adolescence as in, for example, the decline of students' self-competence beliefs and interest in math and science [54, 55] results from an increasing mismatch between adolescent students' needs and the opportunities afforded to them by their classrooms. Referring to these results, the present study tested for relations between gender and age and students' pattern of perceived quality of instruction in mathematics classes. Instructional quality and attitudinal and affective variables were assessed using student self-report scales. Compared to objective descriptions of instructional quality, student ratings offers a range of conceptual advantages such as a high reliability due to students' extensive experiences with different teachers and experiences with the same teacher in different domains [31]. However, in particular, students' subjective perceptions of their learning environment were shown to be highly predictive for motivational learning outcomes [30]. In her multilevel study, Daniels [16] revealed that students' perceived structuredness in mathematics classrooms influences mathematics interest at the individual and class level. However, the effect was considerably stronger at the individual level. Frenzel et al. [29] conclude from their data that the relationships between perceived quality of instruction in mathematics classrooms and students' emotional experiences in class predominantly function at the individual level and not at the level of averaged classroom experiences. Although a large volume of research examines the relations between instructional quality and factors of students' learning, nearly no empirical studies focus on students' distinct patterns of perceived instructional quality as rated by students themselves. Referring to the high importance of students' subjective experiences in their learning environments for successful learning processes [16, 29, 30] there is an urgent need for *explorative* studies on students' individual patterns of perceived instructional quality and their associations to students' learning.

3. Research Questions

Based on the theoretical state of research and previous empirical results, the present study addressed the following explorative research questions.

(a) What distinct student patterns can be identified with respect to perceived levels of characteristics of instructional quality (*structuredness*, *teachers' social support,* and *discursive effectiveness*) in secondary school mathematics classes?

(b) How do the demographic characteristics *gender* and *age* relate to patterns of instructional quality in math?

(c) Do the attitudinal and affective variables *mathematics self-concept* and *interest* differ significantly across the distinct patterns of instructional quality in mathematics classes?

4. Method

4.1. Participants and Procedure. The sample included 425 high school students (grades 8 through 10) from 21 classrooms from ten schools in Berlin, Germany. Each classroom had a different teacher. The mean age of the participating male (53.2%) and female (46.3%) students was 14.93 years (SD = 1.04; age range: 13–17). The majority of participants (54.6%, $n = 232$) reported that they and both of their parents

were born in Germany. Students' participation was voluntary and required parental consent if students were under 14 years old, following the research principles of the Berlin Senate Administration for Education, Science and Research. The recruitment procedure consisted of sending letters addressed to the students and their parents explaining the aims and procedure of the study and requested students' participation. Trained research assistants introduced the students to the questionnaire, which they completed in approximately 45 minutes during their mathematics class.

4.2. Measures. All items assessing instructional quality, described in more detail below, were divided into binary items to indicate those students' with low to moderate ratings on the perceived instructional quality characteristics versus those children with high ratings. After examining the factor structure using confirmatory factor analysis, items with high loadings on the three factors "structuredness," "social support", and "discursive effectiveness" (<.75) were used as indicators for Latent Class Models.

4.2.1. Structuredness. The scale for "structuredness" assessed the extent to which students perceived their teachers' instructions in mathematics class as well structured. Thus, the study focused on structuredness in terms of "a systematic approach in the design of instruction" [56, page 252]. Perceived structuredness was measured with a 4-item scale derived from Daniels [16] (e.g., "Our teacher in mathematics usually summarizes everything, helping us remembering what we learned in class.") The 4-point Likert scale ranging from 1 (*strongly disagree*) to 4 (*strongly agree*) yielded good internal consistency with Cronbachs' $\alpha = .85$.

4.2.2. Social Support. The participants' sense of their mathematics teachers' social support was measured using a 4-item scale by Daniels [16]. An example item is "Our mathematics teacher makes time for students who want to talk with him/her." The 4-point Likert scale ranging from 1 (*strongly disagree*) to 4 (*strongly agree*) demonstrated adequate internal consistency with Cronbachs' $\alpha = .79$.

4.2.3. Discursive Effectiveness. Students' perception of opportunities to participate in decisions concerning their learning process in mathematics was assessed with the 7-item scale "Feeling of discursive effectiveness" from Steinert et al. [57]. An example item is "Students' opportunities to decide things in class are never seriously considered by our mathematics teacher." Using a 4-point Likert response scale, items ranged from 1 (*strongly disagree*) to 4 (*strongly agree*). All items from the scale were negatively worded and recoded so that a higher score indicated greater discursive effectiveness. The scale demonstrated good internal consistency with Cronbachs' $\alpha = .83$.

4.2.4. Students' Interest in Mathematics Class. Students' interest in mathematics class was measured with a 9-item self-report scale, based on Berger [58]. An example item is "I value mathematics class particularly because of the interesting topics." The 5-point Likert-type scale ranged from 1 (*strongly disagree*) to 5 (*strongly agree*). The scale displayed good internal consistency with Cronbachs' $\alpha = .83$.

4.2.5. Self-Concept in Math Class. Students' self-concept in mathematics class was measured using an established German 4-item self-report scale derived from Bos et al. [59]. An example item is "I' m just not good at math." The 4-point Likert scale ranging from 1 (strongly agree) to 4 (strongly disagree) was recoded so that a higher score indicated greater self-concept in math class. The scale yielded good internal consistency with Cronbachs' $\alpha = 0.81$.

4.3. Data Analysis. All analyses were conducted using Mplus Version 6.12 [60]. Due to the cluster sampling of the data (students' in classrooms), and thus the nonindependence of observations, corrections to the standard errors and chisquare test of model fit were obtained using a maximum likelihood estimator with robust standard errors for all steps in data analysis (Type = Complex [60]).

Steps in Data Analysis.

(1) Separate confirmatory factor analyses (CFAs) were conducted for the 16 items of the self-report scales structuredness, social support, and discursive effectiveness, which were used as indicators for the latent class analysis to examine the adequacy of the factor structure and to identify problematic items (low loadings; cross-loadings).

(2) A series of latent class analysis procedures were conducted. The first step was to choose the optimal number of classes by specifying separate LCA models with various numbers of classes. The appropriate number of latent classes was evaluated based on a comparison between several statistical criteria, including Akaike information criterion (AIC: lowest) [61], Bayesian information criterion (BIC: lowest) [62], sample-size-adjusted Bayesian information criterion (ABIC: lowest), entropy (>.80) [63], and adjusted Lo-Mendell-Rubin Likelihood Ratio Test (LMR LRT: P value is used to determine if the null k-1 class model should be rejected in favor of the k class model). Suggested by a recent simulation study, we gave the most weight to model with the lowest BIC, as BIC value may provide the most reliable indicator of true number of classes [64].

(3) As the current study explored whether gender and age were significantly associated with students' latent class membership, these demographic characteristics were included as covariates in the basic model via multinomial logistic regression.

(4) Mean differences of students' self-reported motivational and cognitive learning outcomes across the latent classes were tested by incorporating the continuous distal outcome variables *students' self-concept* and *interest* in mathematics class into the latent class model with covariates.

5. Results

5.1. Confirmatory Factor Analysis. Based on the established measures, the 16 items of the three self-report scales *structuredness*, *social support,* and *discursive effectiveness* were dichotomized and assigned to three dimensions. After examining this initial model, we identified two items with multiple factor loadings greater than 0.3 and subsequently removed them from the model. The removed items consisted of one item from the *social support* scale ("Our math teacher usually helps us like a friend."; $M = 2.56$, SD $= 0.90$; standardized loading on latent factor "social support" $\lambda = 1.20$ and on latent factor "discursive effectiveness" $\lambda = -.52$) and one item from the *discursive effectiveness* scale ("My mathematics teacher often does not listen to what I say.", $M = 2.00$, SD $= 0.84$; standardized loading on latent factor "structuredness" $\lambda = .36$ and on latent factor "discursive effectiveness" $\lambda = .47$). Furthermore, due to low loadings compared to the other items which were all greater than .80, two items were removed from the model ("Our math teacher often says 'We have to' which means 'You have to'.", $M = 2.33$, SD $= .96$, and $\lambda = .63$; "In the end in our classroom it depends on who is in charge.", $M = 2.13$, SD $= .98$, and $\lambda = .72$). The final model with the remaining 12 items fit the data well, χ^2 (51) $= 63.99$, $P > .05$; CFI $= .990$; TLI $= .987$; RMSEA $= .02$, 90% CI [.00–.02].

5.2. Latent Class Analysis. Although the 5-class model had the lowest AIC- and aBIC-value, results of Nylund et al. [64] reveal that the AIC-value is not a good indicator for class enumeration for LCA models with categorical outcomes. Further, the authors propose that the BIC value more consistently identifies the correct model over the adjusted BIC. Thus, Nylund and colleagues [64] conclude that the BIC is the most reliable information criterion (IC) between AIC and aBIC information criteria for correctly identifying the number of classes in LCA-modeling. In our model, the LMR LRT test statistics showed the first nonsignificant P value for the 3-class model, suggesting that the 2-class model should not be rejected in favor of the 3-class model. However, Nylund and colleagues [64] also emphasize based on results of their Monte Carlo simulation study that the LMR LRT test has inflated Type I error rates for LCA models with categorical outcomes. Thus in our study, we preferred the BIC value as an indicator of the best model fit. As indicated by the lowest BIC-value, the four-class model had the best model fit. Model fit indices for the various latent class models are shown in Table 1. Figure 1 depicts the class profiles. The y-axis indicates the probability of a student rating high on the specific instructional quality items, which are listed on the x-axis. Latent Classes were labeled referring to their most decisive criterion concerning students' ratings on characteristics of instructional quality. Class 1 was best characterized as the "High Quality Pattern" (9.8%) given that students in this subgroup had moderate to high probabilities of reporting high levels of structuredness (.57–.81), high levels of social support by their teacher (.62–.78), and high levels of discursive effectiveness (.68–.86) in their mathematics classes. Class 2 was labeled as "High Structuredness Pattern" (21.9%) and consisted of students who had high probabilities of reporting high to moderate levels of structuredness of instruction in their mathematics classes (.45–.72), moderate to low probabilities of reporting high levels of social support by their mathematics teacher (.22–.43), and low probabilities of reporting high levels of opportunities in their mathematics classes (.00–.17). Class 3 was denoted as "High Social Support Pattern," as members of this subgroup (22.4%) displayed, aside from item "social support 4" (.18), overall moderate to high probabilities of feeling supported by their mathematics teachers (.45–.59), low probabilities of reporting high levels of structuredness (.06–.13), and moderate probabilities of reporting high levels of discursive effectiveness (.23–.43). Class 4 was the most prevalent class (45.9%) and was characterized as "Low Quality Pattern" as students in this subgroup had low probabilities of reporting high levels of structuredness (.00–.04), high levels of social support by their mathematics teacher (.00–.05), and high levels of discursive effectiveness (.00–.08) in their mathematics classes.

In a next step, we tested to which extent the groups corresponded to individual classrooms/teachers. We thereby focused only on both extreme patterns as they can be seen as a struggling group "Low Quality Pattern" and a no-risk group "High Quality Pattern". Results of crosstab analysis revealed significant associations between students' group membership and individual classrooms/teachers [Pearson $\chi^2 = 169.83$, df $= 60$, and $P < .000$; Cramer's $V = 0.37$]. Multinominal logistic regressions were conducted using dummy-coded classrooms/teachers as independent variables and latent class membership as dependent variable. Class 4 "Low Quality Pattern" was set as reference class. Results revealed that students who were in classrooms numbers 3, 4, 5, 15, and 19 were significantly more likely to belong to the "High Quality Pattern" than to belong to the "Low Quality Pattern" ($OR_3 = 4.68$, $P < .001$; $OR_4 = 2.16$, $P < .01$; $OR_5 = 8.69$, $P < .001$; $OR_{15} = 5.59$, $P < .05$; $OR_{19} = 12.94$, $P < .001$). Using Class 1 "High Quality Pattern" as the reference class, results indicated that students from classrooms numbers 3, 4, 5, 15, and 19 were significantly less likely to belong to the "Low Quality Pattern" than to the "High Quality Pattern" ($OR_3 = 0.21$, $P < .001$; $OR_4 = 0.46$, $P < .01$; $OR_5 = 0.12$, $P < .001$; $OR_{15} = 0.18$, $P < .05$; $OR_{19} = 0.08$, $P < .001$).

5.3. Latent Class Analysis with Covariates. The current study explored whether the demographic characteristics gender and age were associated with class membership by incorporating these variables as covariates in the latent class models. Latent class analysis with covariates is analogous to a multinomial logistic regression approach with latent class membership serving as categorical dependent variable and using observed covariates as independent variables [60]. Class 4 "Low Quality Pattern" was set as the reference group. The model fit statistics of the LCA models with demographic covariates are reported in Table 2. Covariates included gender (0 = male; 1 = female; male as referent) and age (centered). In the four-class model with covariates, it was shown that

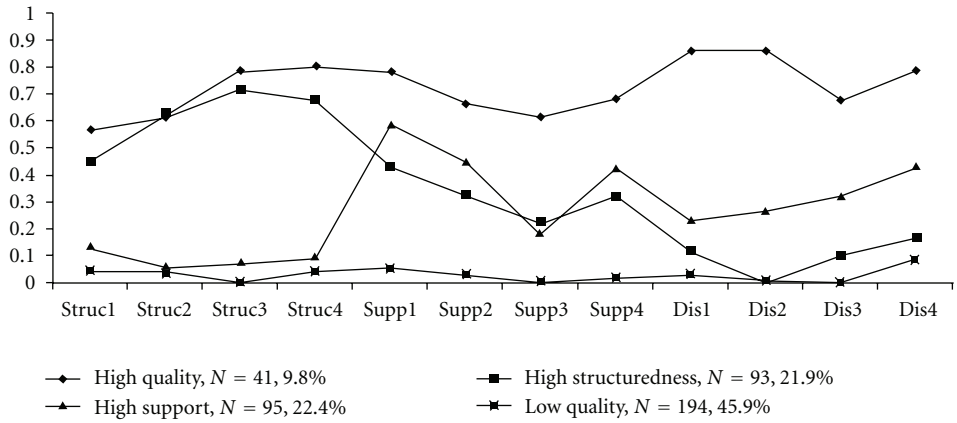

Figure 1: Latent class characteristics and prevalence for the four-class model.

Table 1: Model fit indices for 2–5 class solutions of students' perceived characteristics of instructional quality with and without covariates (gender and age).

Number of classes	Without covariates				With covariates			
	2	3	4	5	2	3	4	5
AIC	4526.28	4392.59	4320.60	4278.15	4505.44	4372.51	4298.91	4260.15
BIC	4627.47	4546.39	4527.02	4537.18	4618.70	4546.44	4533.52	4555.44
ABIC	4548.13	4425.81	4365.17	4334.09	4529.84	4409.99	4349.46	4323.78
Entropy	0.827	0.850	0.822	0.847	0.826	0.848	0.830	0.852
aLMR	0.024	0.379	0.365	0.729	0.040	1.000	1.000	1.000

Note. AIC: Akaike's information criteria. BIC: Bayesian information criteria. ABIC: sample-size-adjusted Bayesian information criterion. aLMR LRT: Lo-Mendell-Rubin adjusted likelihood ratio test.

partial measurement noninvariance needed to be considered. Gender directly influenced the probability to rate high on the item "social support 1" (OR = 0.51; $\beta = -.668$, SE = .257, $z = -2.59$, and $P < .01$). That is, compared to male students, female students were significantly less likely to endorse the item "Our math teacher always has time to speak with his students." Concerning the associations between demographic characteristics and latent class membership, the results of the four-class model demonstrated that age had no significant effect at all. Gender was significantly associated with students' membership in Class 1 "High Quality Pattern". Results demonstrated that the odds of belonging to the "High Quality Pattern" for females were only 0.45 as high as for males (OR = 0.45; $\beta = -.801$, SE = .387, $z = -2.07$, and $P < .05$). Using an alternative parameterization for the categorical latent variable regression with Class 1 "High Quality Pattern" as reference class, it was shown that the chance of belonging to Class 4 "Low Quality Pattern" for female students was twice as high as for male students (OR = 2.74; $\beta = 1.006$, SE = .364, $z = 2.77$, and $P < .01$). Age did not relate significantly to students' pattern of perceived instructional quality.

As shown in preliminary classroom-related analysis, particular classrooms were significantly related to students' group membership in the perceived "High Quality" and perceived "Low Quality" groups. Therefore, examining the question whether female students possibly were on average more likely to attend classrooms which were significantly related to the perceived "Low Quality Pattern," we examined the associations between those classrooms numbers 3, 4, 5, 15, and 19 and gender via crosstab analysis. Only classroom numbers 15 and 19 were significantly associated with gender, indicating less female students than male students in classroom numbers 15 [Pearson $\chi^2 = 4.68$, df = 1, $P < .05$; Phi = 0.105] and 19 [Pearson $\chi^2 = 4.49$, df = 1, $P < .05$; Phi = 0.103]. These results suggest that females' higher chance of belonging to the "Low Quality Pattern" was not consistently caused by attending the classrooms with a high percentage of students who perceived low quality in the sample.

5.4. Mean Differences across the Latent Classes: Motivational Learning Outcomes. Similar to an analysis of covariance (ANCOVA), we tested for mean differences across the latent classes controlling for the covariates gender and age. Thus, we modeled a latent class analysis including the observed variables self-concept and interest as distal outcomes and adding gender and age as covariates to the LCA model to see how class membership predicts the distal outcome when controlling for the influence of covariates on the distal outcome [65]. Further, using the "Model Test" command of M*plus,* the statistical significance of mean differences of self-concept and interest was tested with Wald chi-square tests of parameter equalities by describing equality of means as

Table 2: Class-specific means and standard errors of motivational and cognitive learning outcomes.

	C1: High quality		C2: High structuredness		C3: High teacher support		C4: Low quality	
	M	SE	*M*	SE	*M*	SE	*M*	SE
Self-concept	3.24	0.15	3.05	0.16	2.95	0.12	2.63	0.08
Interest	3.86	0.13	3.42	0.12	3.32	0.18	3.03	0.09

restrictions on the analysis model for the Wald test [60]. The class-specific means and standard errors of the distal outcomes self-concept and interest in mathematics classes are reported in Table 2.

5.4.1. Self-Concept in Mathematics Class. Results revealed that students who had a high probability of belonging to Class 1 ("High Quality Pattern"; $M = 3.24$, SE = 0.13) reported significantly higher self-concepts than students with a high probability of belonging to Class 4 ("Low Quality Pattern"; $M = 2.63$, SE = 0.08; C1 versus C4: Wald test = 15.33, df = 1, and $P > .01$). However, students with a high probability of belonging to Class 4 ("Low Quality Pattern"; $M = 2.63$, SE = .083) reported significantly lower self-concepts in mathematics class than students in any other subgroup (C1 versus C4: Wald test = 15.33, df = 1, and $P < .001$; C2 versus C4: Wald test = 7.56, df = 1, and $P < .01$; C3 versus C4: Wald test = 3.84, df = 1, and $P < .05$).

5.4.2. Interest in Mathematics Class. Results revealed that students who had a high probability of belonging to Class 1 ("High Quality Pattern"; $M = 3.85$, SE = .119) reported significantly higher values of interest in mathematics class than students in any other subgroup (C1 versus C2: Wald test = 6.57, df = 1, and $P = .01$; C1 versus C3: Wald test = 4.73, df = 1, and $P < .05$; C1 versus C4: Wald test = 30.24, df = 1, and $P < .001$). Inversely, students who had a high probability of belonging to Class 4 ("Low Quality Pattern"; $M = 3.85$, SE = .119) reported significantly lower values of interest in mathematics class than students with a high probability of belonging to the "High Quality" (Class 1) subgroup and students with a high probability of belonging to the "High Structuredness" (Class 2) subgroup (C4 versus C1: Wald test = 30.24, df = 1, and $P < .001$; C4 versus C2: Wald test = 6.78, df = 1, and $P < .01$).

5.4.3. Summary of Mean Differences across Classes. The results demonstrated that students with a high probability of belonging to the "High Quality Pattern" (Class 1) showed significantly higher interests than students from any other subgroup, while students' with a high probability of belonging to the "Low Quality Pattern" (Class 4) showed significantly lower self-concepts than students from any other subgroup.

6. Discussion

The aim of the present study was to explore how characteristics of classroom instruction relate to male and female students' attitudinal and affective learning characteristics in mathematics. To probe this question, our investigation focused on students' patterns of perceived instructional quality in secondary school mathematics classes and their role for students' self-concept and interest in math. Results of LCA analyses indicated the existence of four distinct student patterns and, thus, suggested that students' perception of the instructional quality in their classrooms is highly individualized and heterogeneous. Almost half of the students in the sample (about 46%) belonged to a "challenging pattern" perceiving an overall low quality of instruction (low structuredness of content, low social support by the teacher, and low discursive effectiveness). These students also demonstrated lower levels of mathematics self-concept than students from any other subtype. Only a small percentage of students in the sample (about 10%) belonged to a group of students who perceived high quality of instruction and also demonstrated significantly higher levels of mathematics self-concept and interest than students with other patterns of perceived instructional quality. Thus, results suggest that the impact of learning environments on students' learning outcomes depends highly on students' subjective experiences of their learning environments. The existence of distinct patterns of students' perceived instructional quality and their relations to students' mathematics self-concept and interest support the theoretical assumption that learning environments are shaped by the involved actors and their perceptions of the learning context [66]. Thus, our results suggested that learning success relates to these individually different perceptions. Accordingly, results of the present study suggest in line with previous research [25] that learning environments are processed in different ways by each student based on demographic characteristics. When examining demographic characteristics such as gender and age, which are associated with students' perception of their learning environment, results revealed that gender related significantly to students' perceived quality of instruction in mathematics. Age did not relate to students' patterns of perceived instructional quality.

Concerning significant associations between single classrooms and students' group membership, five of the twenty-one classrooms were significantly related to students' group membership. Two of these five classrooms were also significantly associated with students' gender suggesting that female students' higher probability of belonging to the "Low Quality Pattern" was not consistently due to the fact that female students in our sample had on average poorer teachers. Our data did not include more teacher-related information; thus we did not explore particular teacher characteristics, which might be related to students' group membership. However, for future analyses, it will be interesting to use the subgroup-related classrooms for

the study of micro learning environments and to analyze, for instance, characteristics of questions, tasks, and received feedback within these classrooms [49].

The unique value of the present study lies in the highlighting of the particularly high probability of female students to belong to a group of students who, in comparison to male students, were significantly more likely to perceive an overall low quality of instruction in mathematics classes. These results align with previous research, suggesting that female students perceive less supportive feedback by their mathematics teachers [67] and perceive low opportunities to participate in classroom discourse [20]. Students who belonged to this "Low Quality" pattern also reported particular low levels of self-concept in math, which is associated with negative attitudes and emotions concerning their mathematics classes. The findings thus point to the urgent necessity to examine the ways in which the instructional design of mathematics classrooms can be better adapted to more effectively meet the needs of female students.

6.1. Implications for Educational Practice. As the results of the present study indicate that students' perceptions of their learning environments are highly diverse, and that these different perceptions are associated with students' learning outcomes, the findings highlight the necessity of taking into account strategies of differentiated instruction in mathematics education. Differentiated instruction allows teachers to provide choices to the students concerning content, learning processes, and products by, for example, using varied resource materials, tasks, and texts or adapting curriculum to students' learning processes [68]. Specifying these general characteristics of differentiated instruction, the two extreme subtypes as well as the mixed subtypes found in the present study indicate that mathematics teachers should differentiate content, learning processes, and products by considering students' different levels of perceived *structuredness*, *support,* and *discursive effectiveness*. Concretely, these variances can be targeted through the implementation of practices such as diverse feedback strategies and offering differently structured learning materials.

To address students' perceptions of their learning environment effectively, teacher evaluation is highly important. The instruments used in this study might be appropriate evaluation tools for regular student surveys. However, for optimal teacher evaluation, other aspects of instructional quality which relate to achievement and motivation such as task complexity or level of mathematical argumentation should also be taken into account [34].

In the present study, it was shown that female students belong, compared to male students, significantly more often to the subtype of learners who perceive mathematics classes as (a) less structured with respect to the presentation of the discussed content, (b) less supportive concerning teachers' behavior, and (c) allowing low levels of participation in classroom discourse. Thus, gender-appropriate teaching in mathematics is needed to enhance female students' positive evaluation of their learning environment in math. According to the presented results and the current state of research, gender-sensitive teaching strategies include the following.

(a) To facilitate structuredness of presented content in math class by involving different learning styles for different students—research on gender differences in learning styles, for example, suggested that female students tend to be reading- and-writing-based learners [69].

(b) To provide gender-balanced social support by the teacher—it is, for example, well known that, compared to male students, female students receive more negative feedback from their teachers concerning their intellectual abilities [70].

(c) To foster experiences of discursive effectiveness in the classroom by facilitating classroom discussions. Classroom discussions in mathematics can be facilitated by talking about mathematical concepts and procedures, introducing various discourse formats such as extended group discussion and allowing students to talk about their thinking and problem solving [71]. Another important point is to encourage female and male students equally to participate actively in these discussions by, for example, in the classroom puzzle strategy [72]. Hänze and Berger [73], for example, revealed in their study on teaching settings in 12th grade physics classes that critical groups with a low-academic self-concept such as female students felt clearly more competent in learning environments, which were characterized by cooperative instruction than in the traditional teaching setting.

6.2. Limitations. The design of the present study is cross-sectional, and subsequently, the direction of relationships among the variables cannot be determined. Results do not indicate whether students' learning outcomes or other relevant factors impact the pattern of perceived instructional quality to which they belong or vice versa. As suggested by previous longitudinal analyses [10, 16], it was assumed that attitudinal and affective learning characteristics differ depending on students' perceived instructional quality. However, longitudinal studies are needed to conclusively determine the direction of influence. Second, the measurement instruments utilized depict only some aspects of the highly complex construct of instructional quality in mathematics. Further research is needed including additional characteristics of instructional quality, which are related to students' self-concept and interest in math such as challenge, sense of task novelty, or the opportunity to engage with others [9].

6.3. Conclusions. The present study highlights the necessity of instruction in mathematics classes to take into account learners' different ways of perceiving and evaluating their learning environment. Considering almost half of the sample in the present study had a high probability of perceiving low structuredness, low support, and low discursive effectiveness in their math classes and thus, particular low mathematical

self-concept and interest, this research emphasizes that attitudes and emotions toward mathematics should be enhanced through increased adaption of mathematics classroom instruction to students' different learning strategies and by considering students expectations and perceptions of instructional quality in classroom discussions.

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Research Article

Relationships between 9-Year-Olds' Math and Literacy Worries and Academic Abilities

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We investigated whether 9-year-olds experience math and/or literacy worries and, if they do, whether it is related to problem-solving abilities. Fifty-eight children judged the correctness of math, literacy, and mental rotation problems that differed in difficulty and rated their worry level about the correctness of judgments. Nonverbal IQ, general math, and literacy abilities were also assessed. Results showed children's worry ratings varied as a function of task and problem difficulty. Latent class analyses of math and literacy worry ratings revealed high-, moderate- and low-worry subgroups in both domains. The high-worry math subgroup exhibited poorer math performance than the other math subgroups, demonstrating a link between math worry and math performance. No relationship was found between worry literacy subgroups and literacy performance. Moreover, no relationship was found between teachers' rating of children's academic and general worry and children's own worry ratings. The relevance of the findings for understanding math and literacy worry is discussed.

1. Introduction

Insofar as preadolescent children experience academic-related anxieties/worries, it is unclear whether these effects are specific to an academic domain (e.g., math) exists in other domains (e.g., literacy), or, if they exist, affect abilities similarly across domains. Although it is assumed that the trepidation associated with math is greater than other academic domains [1, 2], research into anxiety problem-solving abilities associations in areas other than math is limited, even in older students [1]. Moreover, even though academic anxieties may exist in preadolescent children, it is unclear whether children would be able to accurately describe these experiences because of immature metacognitive abilities. The research reported herein investigates 9-year-olds' academic anxiety/worry-ability relationships in math, reading, and mental rotation (a nonacademic area), where children rate anxiety level immediately after solving a problem (We use the term "worry" rather than anxiety because pilot work suggested that children could readily provide examples of their worry experiences, but had more difficulty providing examples of anxiety experiences. Moreover, the term "worry" has occasionally been used as a way of describing "anxiety" to children (see [2, 42]). Nevertheless, we also use the generic terms "math anxiety" and "literacy anxiety" because they are used in the literature). The aim is to better understand the nature of academic anxiety/worry-ability associations in preadolescent children.

Although most academic anxiety research focuses on math anxiety (MA), literacy anxiety may also exist. In one of the few studies on language anxiety, Carroll et al. [3] found that, compared to their literate peers, 5- to 15-year-olds with poor literacy tended to be more anxious about their language abilities. This is an important issue because MA might be an aspect of academic anxiety more generally. However, the reasons for literacy anxiety are not well understood, and the research considered herein mainly focuses on math anxiety/worry performance relationships.

MA is thought to emerge in the preadolescent years, peak around Grades 9 to 10, and not change thereafter [4, 5]. It is associated with poor math achievement, as well as socioemotional and behavioral difficulties in high school students [1, 5–7]. Surprisingly little is known about the impact of math anxiety/worries on preadolescent children's math abilities [8,

9]. Dowker [1] suggests that MA affects math performance only after Grade 4 (10 years old)—a claim supported by Ma [9] in his meta-analysis of research on the association between MA and math performance (note, however, Ma [9] only included three studies of children younger than 11 years old in his meta-analysis). Moreover, Krinzinger et al. [10] found no association between MA and arithmetic ability in 6- to 9-year-olds and suggested that the way young children are questioned about math-related anxieties may affect their responses. These findings suggest that young children are either not affected by math anxiety/worries, do not understand anxiety-related questions, or are unable to report their anxieties [1, 11, 12].

It is often assumed that children are able to report anxiety states—a claim that overlooks limitations in children's metacognitive capabilities, which may affect their ability to answer questions about their own cognitive or socioemotional states [13]. In questionnaires typically used to assess MA, children are asked to reflect on general events (e.g., imagine being in a math lesson) and rate their anxiety, yet this may be beyond their cognitive capabilities [9, 14, 15].

Some researchers have used simplified MA tasks to probe young children's anxiety states. Thomas and Dowker [16], for example, used a pictorial MA task in which 6- to 9-year-olds pointed to facial expressions to convey their emotional response to math events. The questions focused on children's perceived math ability, how much they enjoyed math tasks, and how worried they felt when they encountered difficult problems. Thomas and Dowker [16] found that their participants did not express anxiety or unhappiness at perceived math difficulties and concluded that MA was unrelated to poor math performance in young children. Gierl and Bisanz [12] also used a pictorial judgment paradigm to assess MA in 9- to 12-year-olds. They found that 9-year-olds reported low levels of anxiety, while 12-year-olds reported higher anxiety levels. They concluded that MA increases as students mature and become more concerned about the consequences of success and/or failure. However, both groups of researchers asked probe questions about general retrospective events (e.g., experiences with mental calculation), rather than a specific problem solving event as they occurred. It is possible that the 9-year-olds in both studies lacked the abilities to reflect on general events [13].

The question of whether MA is a manifestation of general anxiety or is specific to math has been often debated. In a meta-analysis of MA research conducted on high school and college students, Hembree [5] reported moderate correlations (0.35 and 0.40) between MA, general anxiety, trait anxiety, and state anxiety, and a stronger correlation (0.52) between MA and test anxiety. However, stronger correlations (0.5 to 0.8) have been found between MA measures (e.g., Math Anxiety Rating Scale and the Abbreviated Math Anxiety Scale), which suggests that MA is a specific anxiety—a claim supported by findings which show that general anxiety measures account for little of the variance in MA scores [6, 11]. However, these analyses do not address whether MA is unique or a part of academic anxiety more generally. Findings from a study by Faust [17] suggest that MA is unique in older students, at least. He found that high MA students did not show anxiety on a verbal anagram task. A question of some interest is whether preadolescent children exhibit similar across-domain anxieties/worries. Indeed, Gierl and Bisanz [12] found that preadolescent children's MA ratings did not differ from their general school anxiety ratings, which could be interpreted as suggesting that children exhibit anxiety across academic domains.

A key aim of studying MA is to understand its impact on math abilities. Higher levels of MA are often linked to lower levels of math achievement [2, 7, 18]. However, the relationship between MA and math ability is complex. For instance, MA is often associated with avoidance behaviors, less time spent studying, and lower class engagement [10, 19], factors that can impact the acquisition of math skills. Poor performance by high MA students may also reflect the intrusion of worrisome thoughts while solving math problems. A reduced processing efficiencymodel was proposed by Eysenck and Calvo [20] who suggest that worries limit working memory capacity because they take up problem-solving resources [4, 10, 11, 21]. Consistent with this claim, Ashcraft and Faust [22] found that MA had minimal impact on simple arithmetic problems, but had substantial impact on more difficult multiplication problems. However, not all researchers support the cognitive resources hypothesis. For arousal theorist, worry may motivate some students to increase efforts, while extreme worry may impede effort [2, 9]. Also, Wigfield and Meece [2] and Ho et al. [7] suggested that it is the affective-emotional component (i.e., tension, unpleasant physiological reactions) rather than the cognitive component that affects (correlates negatively) with math performance.

In the present study we investigate 9-year-olds' academic worry-performance associations, by probing their "worry" reactions using a "faces worry scale" immediately following problem solving (i.e., deciding whether two problems presented side by side represent the same solution to a problem). We are interested in whether worry judgments vary as a function of to-be-judged problem difficulty and problem correctness for math, literacy and, a nonacademic domain (mental rotation). If they do, it would confirm that children experience worry, whether or not they are able to report it. Of interest is whether math and language tasks elicit distinct patterns of worry judgments compared to the nonacademic task. If academic worries are unique, math and language worry judgments would be expected to differ from nonacademic worry judgments. Insofar as children's patterns of worry judgments differ, of interest is whether judgments are related to ability. On the basis of research with adolescents, it is expected that children's academic worry ratings would be associated with ability.

2. Method

2.1. Participants. Fifty-eight Grade 4 children (32 boys and 26 girls: $M = 117.55$ months, SD = 6.91) from an urban primary school in a large Australian city participated. Common to Australian urban schools, the sample comprised children from diverse multicultural and socioeconomic backgrounds. All children had normal or correct-to-normal vision; and,

according to school personnel, none had known learning difficulties. The research was, approved by, and conducted in accordance with, the authors' university human ethics committee requirements.

2.2. Materials and Procedures. To determine whether worry level differed as a function of math, literacy, or nonacademic (mental rotation) problems, children were presented two problems side by side from the three domains and (1) judged whether the solution to the pair of problems was the same or not and (2) rated their worry about the confidence they had in their decision using a face worry scale (described above) (see Table 1 for examples of pairs of to-be-judged problems for each of the three domains). Children judged 48 separate problems in each of the three domains ($n = 144$ problems across the three domains) comprising pairs of to-be-judged problems that were easy or difficult to judge and for which the solutions were either equivalent or not ($n = 12$ for each difficulty/equivalence combination within each domain: see Table 1 e.g., of these combination for each domain).

Children completed the Ravens Colored Progressive Matrices nonverbal IQ measures two weeks before judgment tasks [23]. They also completed a general numeracy and literacy worry test one week before completing judgment tasks. This task was used to (1) introduce children to the face worry judgment procedure, described above (see Figure 1), and (2) assess children's general retrospective worries. The term worry rather than anxiety was used in discussing faces with children because pilot work revealed it was more meaningful to them. Children were able to point to faces corresponding to "not worried at all," "a little bit worried" and "very worried" without hesitation and grasped the purpose of these procedures without difficulty.

Teachers rated children's general school-related anxiety prior to the judgment tasks. They also rated children's literacy and numeracy competency. Teachers were encouraged to consult children's formal test scores in making competency judgments—Australian children complete national numeracy and literacy tests throughout the primary (elementary) school years.

To avoid fatigue, the three domain judgment tasks were completed in three 15-minute sessions on consecutive days. Task procedures were explained before each session using four practice problems. Children were reintroduced to the faces worry scale (introduced one week earlier for use in the general numeracy and literacy worry judgment task) and encouraged to use the range of anxiety/worry faces to judge worry level if they felt it was appropriate to do so. Task order presentation was randomized across the three days; however within-task problems were presented in a fixed order.

The three domain judgment task problems were presented on a 15″ laptop screen, and children judged whether to-be-judged problem pairs were equivalent by pressing the left-shift key (for equivalent/similar) or the right-shift key (for not equivalent/not similar). Task problems remained on the screen for a fixed time (math problems = 16 seconds; language problems = 11 seconds; mental rotation problems = 5 seconds). Presentation times were based on pilot data from a separate group of 10-year-olds and reflected the average time taken to complete easy and difficult problems. It was expected that children would have sufficient time to complete easy, but not hard problems, before problems disappeared from the screen. The perception of insufficient time to complete tasks is known to increase worry in children [24]. Although children could make a judgment before problems disappeared, few did so, and thus it was not possible to assess the function of response time.

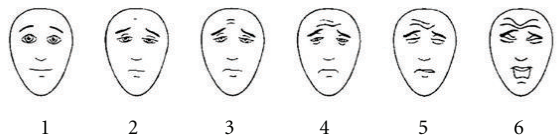


Figure 1: The faces worry scale.

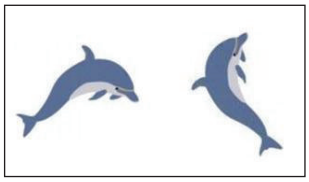
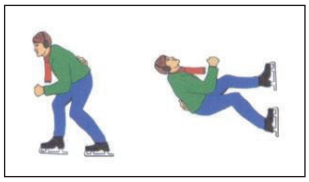
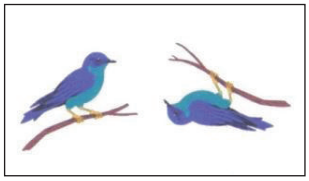
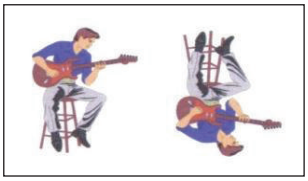
As soon as children made a judgment, the faces worry scale appeared on the screen (see Figure 1 and description below). Children judged how worried they felt about the "correctness" of their equivalence judgment by pressing the numeric computer key corresponding to their worry face choice. Of interest were (1) judgment correctness and (2) worry ratings.

2.3. Judgment Tasks. For the math judgment task children judged whether two addition equations were equivalent. The problems were instances of associativity problems (e.g., 4 + 3 = 3 + 2 + 2) [25]. Difficulty was determined by the number of terms in the equation. Easy problems comprised equations of the form of "a + b = b + c + d" and difficult problems comprised equations of the form of a + b + c = c + d + e + f. Research shows that 8-year-olds are able to solve the *easy* associativity problems, and 10-year-olds are able to solve difficult problems [25]. Equations, terms comprised single-digit numbers between 2 and 9 (excluding doubles: e.g., 6 + 6) and summed to 20 or less. Children's task was to judge whether both sides of the equal sign were equal or unequal. The task comprised 12 easy/same (correct), easy/different (incorrect), difficult/same, and difficult/different problems (see Table 1, e.g., of each).

The language judgment task comprised a synonym judgment test [26], designed to assess literacy (verbal reasoning, reading ability, vocabulary knowledge, and comprehension). Children were presented with two sentences, which differed in a single word and were asked to decide whether the two sentences were equivalent in meaning (e.g., "Elephants are larger than mice/Elephants are bigger than mice" and asked: "Do these sentences mean the same thing?"). Level of difficulty was determined by selecting synonym pairs that were appropriate for Grade 2 (easy) and Grade 4/5 (difficult). Example stimuli are presented in Table 1.

Quaiser-Pohl's [27] mental rotation test was used as the *nonacademic* task—we regarded it as a nonacademic measure because it requires skill not taught in school. It comprised a spatial ability measure designed for primary school aged children. Stimuli comprised colored pictures of two animals or humans, one of which was rotated around a central axis. Children judged whether the rotated figure was the same as

Table 1: Example stimuli used in the math, literacy, and mental rotation tasks.

	Task		
	Math	Literacy	Mental rotation
Easy/Same	4 + 2 + 3 = 6 + 3 3 + 7 = 2 + 3 + 5	Peter RAISED/LIFTED the trophy after winning the Grand Final. Mark is STRONG/POWERFUL enough to lift a truck.	
Easy/Different	3 + 4 + 1 = 2 + 5 4 + 1 + 6 = 8 + 6	Mum's dinner was a major FAILURE/SUCCESS. Kate's pet turtle was very LIGHT/HEAVY.	
Difficult/Same	4 + 3 + 6 = 2 + 7 + 3 + 1 2 + 4 + 9 = 3 + 7 + 4 + 1	The policeman PURSUED/CHASED the thief through the alley. Frank's CONDUCT/BEHAVIOUR in the playground was atrocious.	
Difficult/Different	4 + 8 + 5 + 3 = 3 + 9 + 5 3 + 5 + 9 = 6 + 2 + 3 + 4	Steven waited PATIENTLY/RESTLESSLY for his mother. Roger was HOPEFUL/PESSIMISTIC about getting the job.	

the target figure. Difficulty was determined by the rotation angle between the comparison pictures; *easy* problems were rotated less than 90 degrees (45 or 90 degrees), and difficult items were rotated more than 90 degrees (135 or 180 degrees).

Children's worry ratings for the math, literacy, and mental rotation tasks were computed for the problem difficulty and equivalence conditions (i.e., mean ratings were computed for the four within-task conditions for each of the tasks).

2.3.1. Worry Judgments. Worry judgment was assessed using the Faces Pain Anxiety Scale [28] (see Figure 2). The "faces scales" have been used in medical contexts to measure anxiety/worry [28–30]. The faces depict six different facial expressions in a graded sequence from nonanxious "1" to very anxious "6". The faces are claimed to represent approximately equal intervals in anxiety representations [28]. The Faces Anxiety Scale has a test-retest reliability of 0.79 [28] over a two-week period and correlates with similar scales $r = 0.8$ on average [31]. The scale has been used with 3-year-olds to older adults.

2.3.2. General Math and Literacy Worry. To assess children's general math and literacy worry, they were asked to rate their level of worry for math and literacy. Children rated "How worried do you usually feel when you are doing x''? (x = working on computers, sport, playing music, addition, subtraction, multiplication, reading, and writing stories). These questions were developed by the authors and served to (1) introduce children to the "faces worry scale" and (2) assess possible differences in children's general retrospective math and literacy worries. Children were introduced to the faces worry scales by judging their worry about working on computers, sports, and playing music. The purpose of using these neutral questions was to familiarize children with the worry judgment procedure and to answer questions if they arose. Following the so-called neutral questions, children made worry judgments about the addition, subtraction, multiplication, reading, writing stories questions. Children's math worry ratings were averaged across the three math items and literacy ratings were averaged across the two literacy ratings.

2.3.3. Teacher Judgment of Children's General Anxiety. Teachers completed the modified version of the *School Anxiety Scale Teacher Report* [32], which examines children's general and social anxiety in the school environments. This test has good test-retest reliability ($r = 0.93$) over an eight-week period), and is correlated ($r = 0.83$) with the Strengths and

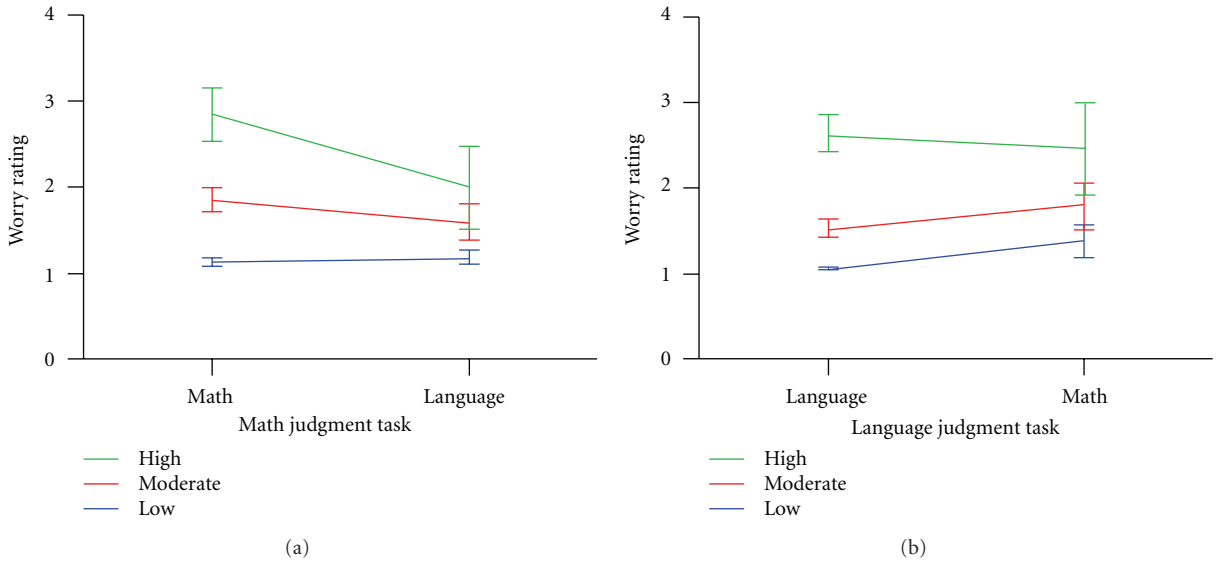


Figure 2: The relationships between math and literacy subgroup membership and literacy (a) and math (b) worry ratings, respectively.

Difficulties Questionnaire Internalizing Symptoms Scale, a widely used measure of teacher-rated anxiety. Seven general anxiety items were administered (e.g., "This child is afraid of making mistakes"; "This child worries about things"; "This child worries that (s)he will do badly at school"). Teachers rated anxiety on a four-point scale (0: never, 1: sometimes, 2: usually, 3: always). Teachers' ratings were averaged across the seven items.

2.3.4. Teacher Judgment of Children's Competency. Teachers rated children's ability in three areas—math, literacy (reading and writing), and general problem solving—on a five-point scale (1: poor, 2: not good, 3: average, 4: very good, and 5: superior). As noted earlier, teachers were encouraged to consult children's formal test results in rating their competencies.

2.4. Analytic Approach. Because we were not confident that the measurement scale properties of the math, literacy, and mental-rotation judgments tasks were sufficiently similar to combine them in analyses, we used separate ANOVAs to identify judgment correctness and worry rating for each task separately. Because we were also interested in identifying different patterns of worry as a function of within-task manipulations, we used latent class methods to isolate possible subgroup patterns of performances. Latent class (LC) methods [33] are a specific form of finite mixture modeling [34] approaches, which aim to uncover two or more subgroups of individuals characterized by distinct patterns of responding. A fundamental aspect of such models is the assumption of a categorical latent variable. The latent (unobserved) categories represent qualitatively different states of the underlying latent variable. Models are estimated with successively more latent classes, with the final model best capturing the structural representation of covariance between the indicator (observed) variables. Information criteria indices (e.g., AICc [35], CAIC [36]) were used to determine the best fitting model to the data, penalising increasingly complex specifications.

Unlike proximity-based cluster approaches (such as K-means), LC approaches offer a formal, model-based approach to the classification of individuals to unique clusters [37, 38]. An important outcome of using this approach is probabilistic estimates of every individual's likelihood of belonging to each class of the latent variable, in this case, levels of worry.

Latent profile analysis (LPA) is a particular type of LC modelling [39]. What separates LPA from other LC models is that the profiles are formed based on continuous indicator variables; in this case, online worry judgments. Models were successively estimated from a two- to four-class specification. The relatively small sample size resulted in idiosyncratic classes beyond this range.

3. Results

Preliminary analyses showed no relationship between gender, Ravens measures, and any of the other measures of interest and so they were not included in analyses reported herein.

Three separate ANOVAs examined judgment accuracy of the to-be-judged problem pairs for the three tasks (math, literacy, and nonacademic tasks) as a function of the difficulty (easy or difficulty) and the similarity (same or different) of the two problems (see Table 2 for the means and standard deviations associated with these analyses). For math judgment accuracy, main effects of problem difficulty ($F(1,57) = 80.83$, $P < 0.001$, $\eta^2 = 0.59$) and problem similarity ($F(1,57) = 23.38$, $P < 0.001$, $\eta^2 = 0.29$) were found. In addition, the interaction between them was also significant ($F(1,57) = 8.30$, $P < 0.01$, $\eta^2 = 0.13$). For literacy judgment accuracy, main effects of problem difficulty ($F(1,57) = 349.00$, $P < 0.001$, $\eta^2 = 0.86$) and problem

Table 2: Mean worry ratings and problem correctness (and standard deviations) for the math, literacy, and mental rotation tasks as a function of the difficulty and similarity of the to-be-judged problem pairs.

Easy		Difficult	
Same	Different	Same	Different
Math problems			
1.48 (0.56)	1.55 (0.67)	1.83 (0.79)	1.93 (0.83)
89.22 (11.47)	92.96 (10.79)	70.69 (22.03)	83.19 (13.92)
Literacy problems			
1.21 (0.40)	1.34 (0.44)	1.73 (0.80)	1.61 (0.70)
95.98 (8.58)	93.97 (6.58)	60.78 (18.67)	73.85 (15.88)
Mental rotation problems			
1.22 (0.35)	1.35 (0.51)	1.35 (0.46)	1.38 (0.46)
88.51 (23.04)	86.06 (13.00)	79.60 (28.02)	81.18 (17.49)

similarity ($F(1,57) = 7.93$, $P < 0.001$, $\eta^2 = 0.12$) were found. In addition, the interaction between them was also significant ($F(1,57) = 23.21$, $P < 0.01$, $\eta^2 = 0.29$). For mental rotation nonacademic accuracy, only a main effect of difficulty was found ($F(1,57) = 22.69$, $P < 0.001$, $\eta^2 = 0.29$).

Three separate ANOVAs also examined judgment worry ratings for the three tasks as a function of the problem difficulty (easy or difficulty) and the similarity (same or different) of the to-be-judged problem pairs. For math judgment worry ratings, main effects of difficulty ($F(1,57) = 50.72$, $P < 0.01$, $\eta^2 = 0.47$) and problem similarity ($F(1,57) = 8.10$, $P < 0.01$, $\eta^2 = 0.12$) were found. For literacy judgment worry ratings, only a main effect of difficulty was found ($F(1,57) = 42.72$, $P < 0.01$), $\eta^2 = 0.43$). In addition, the interaction between them was also significant ($F(1,57) = 18.38$, $P < 0.001$, $\eta^2 = 0.24$). For mental rotation-nonacademic worry ratings, main effects of difficulty ($F(1,57) = 7.92$, $P < 0.01$, $\eta^2 = 0.12$) and problem similarity ($F(1,57) = 11.94$, $P < 0.01$, $\eta^2 = 0.17$) were found. In addition, the interaction between them was also significant ($F(1,57) = 4.19$, $P < 0.05$, $\eta^2 = 0.07$).

These findings show that manipulating the difficulty and similarity of the to-be-judged problem pairs affected judgment accuracy and worry ratings for the math and literacy tasks; however, only task difficulty affected accuracy judgments in the nonacademic task.

The next set of analyses investigated the relationship between the math and literacy worry ratings and accuracy measures. Because of the relatively large math and literacy worry SD's, two separate latent profile analysis [39] were employed to identify possible worry subgroups in the two domains. Specifically, mean worry ratings for problem similarity and problem difficulty were used to identify cluster membership. Different criteria were used to identify the best fitting model (see Table 3). The AICc and CAIC values indicated a three-cluster solution for both the patterns of math and literacy worry judgments (see Table 3). In particular, the solutions revealed low-, moderate- and high-worry subgroups for both domains. Classification errors, defining the degree of uncertainty between probabilistic cluster proportions and cluster membership using maximum class assignment, were acceptably small, indicating that the subgroups were valid.

Table 3: Latent class analysis of the math and literacy worry ratings.

Cluster	LL	AICc	CAIC	BIC	CE (%)
Math worry clusters					
2-cluster	−104.83	224.95	295.68	278.68	0.62
3-cluster	**−68.75**	**182.78**	**269.07**	**243.07**	**2.14**
4-cluster	−49.98	214.50	277.07	242.07	1.99
Literacy worry clusters					
2-cluster	−38.02	91.34	162.07	145.07	0.36
3-cluster	**25.40**	**−5.51**	**80.78**	**54.78**	**0.76**
4-cluster	45.81	22.92	85.50	50.50	1.87

Table 4: Worry subgroups membership cross-classification across the math and literacy domains.

Literacy worry subgroup	Math worry subgroup		
	Low	Moderate	High
Low			
Count	18	8	1
Expected count	12.6	9.8	4.7
Adjusted residual	2.9*	−1	−2.5*
Moderate			
Count	9	9	5
Expected count	10.7	8.3	4
Adjusted residual	−0.9	0.4	0.7
High			
Count	0	4	4
Expected count	3.7	2.9	1.4
Adjusted residual	−2.8*	0.9	2.6*

* $P < 0.05$.

The relationship between low, moderate, and high subgroups across the math and language domains was significant ($\chi^2(4, N = 58) = 15.26$, $P < 0.001$), with a significant ordinal-by-ordinal association ($\gamma = 0.67$, $P < 0.001$: see Table 4 for the cross-classification of the math and literacy subgroups). Table 4 reports differences between expected and observed cell frequencies, and the associated adjusted standardized residuals for significant differences among cells. The information in Table 4 could be interpreted as suggesting that there is a tendency to belong to similar worry groups in both the math and literacy domains; however, the information in Table 4 does not provide information about the within-domain differences on across-domain measures.

Figure 2 shows the mean worry ratings for math and literacy, respectively, as a function of the math and literacy subgroups. Figure 2 suggests a differential impact of subgroup membership on across-domain measures, which was confirmed by analyses of the across-domain worry ratings. Specifically, for the math worry subgroups, a significant interaction between math subgroup and literacy worry rating was found ($F(2,55) = 12.47$, $P < 0.001$, $\eta^2 = 0.31$). However, for the literacy worry subgroups, no significant

interaction between literacy subgroup and math worry ratings was found ($F(2, 55) = 2.26$, $P > 0.05$, $\eta^2 = 0.08$). (Note, only interactions are important here). These results suggest that literacy subgroup worry ratings might reflect general worry ratings, whereas the math worry subgroup ratings reflect unique within-domain differences in worry ratings.

The analyses reported thus far do not address directly the question of whether worry subgroup memberships predict judgment accuracy. These relationships are reported in Table 5, which appears to show that judgment accuracy is indeed associated with worry. In the next set of analyses we report findings from regression analyses in which within-domain worry subgroup membership respectively predicted within-domain judgment accuracy over and above the effect of teachers competence ratings and children's own general worry ratings. The latter two factors are included in the analyses to determine whether teachers' assessment of children's competence and children's general retrospective worry judgments moderate the relationship between worry subgroup membership and judgment accuracy.

Two separate multiple regression analyses were conducted predicting math and literacy correctness judgments, respectively, subgroup membership was entered as a dummy variable (the low-worry subgroup was used as the reference category in both analyses). (Note: we only used the accuracy for difficult judgments because of ceiling effects associated with easy accuracy judgments—see Table 5).

VIF scores were low, suggesting that multicollinearity was not an issue in the analyses. The overall model for math problem judgments was significant ($F(4, 53) = 5.50$, $P < 0.01$, $R^2 = 0.29$) (see Table 6 for the regression parameters). Two significant predictors emerged: teacher rating of math competency and worry subgroup membership. Specifically, teacher math competency ratings predicted math task judgment performance ($\beta = 0.46$, $P < 0.01$) and membership of the high worry subgroup predicted poorer math task judgment performance, relative to belonging to the low-worry subgroup ($\beta = -0.33$, $P < 0.05$). Retrospective worry ratings did not predict difficult math judgment performance. The overall model for literacy problem judgments was significant ($F(4, 53) = 2.97$, $P < 0.05$, $R^2 = 0.18$) (see Table 7 for the regression parameters). Only children's own retrospective literacy worry rating predicted literacy problem judgment correctness ($\beta = 0.46$, $P < 0.01$). It is evident that different factors predict math and literacy correctness judgments.

4. Discussion

The aim of the research was twofold. The first was to determine whether 9-year-olds report differences in math, literacy, and nonacademic worry immediately after solving a problem using the "faces worry scale." The second was to see whether differences in worry ratings were related to problem solving. Three findings are of note. First, worry ratings were higher for more difficult math, language, and mental rotation problems, indicating that children were sensitive to task demands. Second, overall, worry levels were higher for the math problems, compared to language and nonacademic (mental rotation) problems. Three meaningful worry subgroups were identified from worry ratings for both the math and the language tasks (but not for the nonacademic mental rotation task). Importantly, children with high math worry were less successful in making correct math judgments compared to children with low math worry. As noted earlier, neither gender nor general ability effects were observed.

The faces worry scale was used (1) immediately after children made judgment about the equivalence of pairs of math, literacy, and mental rotation problems and (2) to assess children's general math and literacy worries. Children's worry ratings increased as math and literacy judgment task difficulty increased, which shows that children were sensitive to task difficulty (however, worry ratings did not increase as the difficulty of the mental rotation problems increased). This particular finding stands in contrast to Thomas and Dowker's [16] and Gierl and Bisanz's [12] findings that most 6- and 9-year-olds' did not exhibit math performance-related anxiety. Our finding adds weight to claims that math anxiety symptoms may be displayed in primary-aged children [9, 10, 40] and is inconsistent with claims that math anxiety symptoms only appear in late primary/elementary school students [1, 11]. It is possible of course that the latter studies may have underestimated young children's academic anxieties because of difficulties associated with assessing affective states retrospectively [13]. In the present study, this difficulty was minimized by asking children about their worry immediately after solving a problem. Nevertheless, it is also evident that children in the current study were able to use the "faces worry scale" to rate general math and literacy worries.

MA is considered a unique anxiety linked specifically to math [5]. Math is thought to elicit greater worry than other academic domains [1, 17]. Wigfield and Meece [2] suggest that math is particularly anxiety provoking because of its emphasis on precision, logic, and problem solving. While our findings support this claim (worry ratings were higher for math than language), it is evident that 9-year-olds also experienced language worries. This finding suggests that both math and language elicit worry reactions. Interestingly, a similar differential in worry ratings was found in the general academic worry ratings: math problems were rated as more worrisome than literacy problems.

It is noteworthy that associations were found between the math and language subgroups and other tasks. It is evident that (1) children in the low math anxiety/worry subgroup showed equally low levels of worry on all three judgment tasks; (2) children in the high math anxiety/worry subgroup found the math task more worrisome than the language task; (3) some children only showed high levels of worry on the math task, but not on other tasks. The latter finding is important because it provides support for the specificity of math anxiety.

The fact that some children reported high levels of worry in the literacy judgment task suggests literacy per se may evoke anxiety. Indeed, the link between reading difficulties and anxiety symptoms has been noted by researchers [41]. It is assumed that anxiety arises in response to experiences

Table 5: Worry ratings and problems corrects as a function of math and literacy worry groups.

	Math worry group					
	Low (n = 27)		Moderate (n = 21)		High (n = 10)	
	Mean	SD	Mean	SD	Mean	SD
Mean worry rating						
Math easy	1.08	0.09	1.57	0.28	2.60	0.49
Math difficult	1.22	0.20	2.15	0.39	3.10	0.57
Percent correct						
Math easy	90.43	11.25	92.46	8.80	90.00	7.91
Math difficult	80.56	15.33	76.79	13.34	67.50	13.15
	Literacy worry group					
	Low (n = 27)		Moderate (n = 23)		High (n = 8)	
Worry rating						
Literacy easy	1.03	0.04	1.27	0.18	2.12	0.33
Literacy difficult	1.10	0.11	1.82	0.39	3.14	0.41
Percent correct						
Literacy easy	95.99	4.40	95.11	5.43	91.15	9.03
Literacy difficult	68.98	12.73	66.67	10.59	63.54	13.50

Table 6: Predicting math judgment correctness: multiple regression parameters.

	B	Std. Error	β	t	Sig.
(Constant)	57.09	8.89		6.42	0.000
Moderate worry subgroup[1]	−0.09	4.20	−0.00	−0.02	0.983
High worry subgroup[1]	−12.71	5.13	−0.33	−2.48	0.016
Teacher rating math competency	6.73	1.86	0.46	3.61	0.001
Children's retrospective math worry	0.11	2.79	0.01	0.04	0.968

[1] Reference category is low subgroup.

Table 7: Predicting literacy judgment correctness: multiple regression parameters.

	B	Std. Error	β	t	Sig.
(Constant)	45.65	8.91		5.13	0.000
Moderate worry subgroup[1]	−2.14	3.18	−0.09	−0.67	0.505
High worry subgroup[1]	−2.86	4.62	−0.08	−0.62	0.539
Teacher rating literacy competency	1.61	2.73	0.08	0.59	0.559
Child retrospective reading worry	6.01	1.89	0.42	3.19	0.002

[1] Reference category is low subgroup.

of failure rather than literacy anxiety per se. Our results suggest that literacy anxiety is not necessarily a reflection of poor ability, but can arise in children with age-appropriate reading skills. Indeed, our results showed that children in the high language worry subgroup also found the math task worrisome (see Figure 2). However, it should be noted that children in the high math worry subgroup did find the language task worrisome. The difference in the direction of these relationships suggests that language worry might reflect general academic-related worries, whereas math worry is unique to math.

Interestingly, teachers' ratings of children's general anxiety (as assessed on the School Anxiety Scale) were unrelated to children's immediate or general worry ratings. This finding is consistent with previous research with adolescent and adult samples [4, 5, 11].

Some researchers have identified math anxiety in young children that was unrelated to math performance [10, 40]. The failure to find a relationship has led some researchers to question the existence of a math anxiety performance link in children [1, 9]. However, Krinzinger et al. [10] suggest that the failure to find a math anxiety-performance link might be due to methodological issues rather than an absence of math anxiety. In the present study, we found meaningful relationships between worry ratings and problem solving judgments. Similar to Krinzinger et al. [10, 42] we assessed "worry" using a faces rating scale. However, there are two key differences between our scale and the scale used by Krinzinger et al. [10, 42]. First, children rated worry immediately after making problem solving judgments. Second, our worry scale was unipolar in nature (neutral face to worried face); in contrast, Krinzinger et al.'s [10, 42] scale was bipolar in nature (from happy to neutral to worried). Tomlinson et al. [43] suggest that showing a range of emotions on the same scale that require anxiety judgment maybe difficult for children to understand. Nevertheless, it is evident that the children in our study were also able to use our faces worry scale to rate their worry about general math and literacy abilities.

It has been claimed that worry interferes with cognition because it focuses attention on intrusive thoughts rather than the task at hand [10, 11, 21]. Further, MA is claimed to

reduce working memory capacity, which in turn affects math performance. Two findings from the present study are relevant here. First, a relationship between worry subgroup and performance emerged in comparing performance among the three math worry subgroups. Children in the high-worry group made fewer correct judgments than children in the low-worry group judging difficult problems. In the literacy task, overall worry groups differed in judgment accuracy, although no specific subgroup differences were identified. However, recent research suggests that the relationship between numerical cognition and MA is complex. For example, individuals with high MA also possess poor (1) spatial processing ability [42, 44], (2) dot enumeration [45], and (3) symbolic number comparison abilities [46]. These findings suggest that much more work needs to be undertaken to unpack the relationships between MA, numerical cognitive abilities, and math performance.

The present research findings have implications for educational practitioners. In particular, our findings show that it is possible to identify "at risk worry"-children earlier than previously thought. Moreover, it might be valuable to inform teachers of the consequences of academic-related worry on performance so that they can help at risk children minimize worry. Future research should focus on the impact on children's academic performance of reducing worry (see [47], e.g., on reducing, alleviating, and managing anxiety in students).

Although the current research findings provide evidence for a relationship between academic-related worry and the associated academic competencies, it is limited in at least three respects. First, the sample size ($n = 58$) was relatively small and limited possible analyses. In particular, with a larger sample we would have been able to investigate the direction of causal effects more explicitly. Second, it is evident the future research should examine the relationships between general cognitive factors (e.g., different measures of working memory), specific aspects of numerical cognition (e.g., dot enumeration, number comparison), problem solving, and worry. Third, in the present study, we were unable to examine the sequelae associated with response times. Studies have shown that time pressure affects anxiety ratings, and manipulating time requirements might shed more information about the relationships between worry, problem solving, and cognition.

Interestingly, similar to other researchers who have examined MA in preadolescent children, we found no sex differences in worry judgments [2, 12, 40]. The absence of a gender effect stands in contrast to research with adolescent samples [9, 48] in which gender differences have been found in MA. The developmental context in which MA gender effects emerge requires research.

The present study investigated 9-year-olds' worries using a sensitive measure in two academic domains—math and language—and the relationship between worry and problem solving ability. Results showed that children's worry varied with problem difficulty, and that math and literacy worry differed from nonacademic worry. Moreover, math worry predicted math ability, and a negative association was observed between high math worry and math performance. It is evident that the faces worry scale is a sensitive worry measure for assessing both immediate problem solving and more general academic worries. Nevertheless, more research is needed to investigate worry ratings across a broader range of academic-related problems that differ in difficulty, as well as to investigate how worry ratings change over time.

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Research Article

Primary School Age Students' Spontaneous Comments about Math Reveal Emerging Dispositions Linked to Later Mathematics Achievement

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To longitudinally explore children's developing beliefs towards mathematics, we asked 207 children to define "math" and "reading" at grades 2 and 3 and coded for spontaneous references to likability or difficulty of math (or reading) in their definitions. We found that children attributed more difficulty to math than to reading despite their relatively neutral comments on the likability of either subject. Children described math and reading with comparable degrees of specificity, but girls' definitions were more specific than boys'. Relative to their peers, children with mathematics learning disability (MLD) provided less specific definitions overall, were more likely to describe math as more difficult than reading, and were more likely to show a decrease in likability ratings of math (but not reading) from grades 2 to 3. Grade 2 ratings predicted math ability at grade 3, more so than predictors from grade 3. These findings, although based on informal analyses not intended to substitute for validated assessments of disposition, support the notions that distinct aspects of dispositions towards math emerge in early childhood, are revealed through casual discourse, and are predictive of later math achievement outcomes. This further supports current interests in developing formal measures of academic disposition in early childhood.

1. Introduction

A productive disposition towards mathematics is an essential component of mathematics proficiency [1]. Like many elements of successful mathematics outcomes, the construct of a "productive disposition" is multifaceted. The National Research Council [1] defines it as "the tendency to see sense in mathematics, to perceive math as both useful and worthwhile, and to believe that steady effort in learning mathematics pays off." At a minimum, this description captures features of positive attitudes about mathematics, seeing sense in mathematics, the belief that effort is needed to support math learning, and a perspective that math is useful within and beyond school experiences—regardless of whether it is liked or disliked or achieved with minimal or great effort. Associations between these and other distinct features of a productive disposition are likely to be dynamic, but the nature and emergence of the elements themselves are not yet fully understood, particularly in early childhood. Here we report on a brief, exploratory study of whether second and third graders' definitions of mathematics shed light on their emerging dispositions and are related to their later mathematics achievement.

The recent interest in measuring beliefs about mathematics in early childhood (e.g., [2, 3]) has yielded evidence of a reciprocal relationship between math interest or beliefs and math ability, even in preschoolers [4]. Accordingly, in this study we ask whether children's spontaneous conversations

about math reveal tendencies in their beliefs. Specifically, we ask if children's definitions of "math" reveal children's beliefs about whether math is easy or difficult, liked or disliked, or useful. We ask whether these potential disposition features apply to math specifically or to school subjects in general by asking our participants to also define "reading." By comparing definitions of math (and reading) among children with a wide range of math achievement levels (including children with mathematics learning disability (MLD)), we test the plausibility that early indicators of children's beliefs about math contribute to predicting their future mathematics outcomes. We base this hypothesis on prior research demonstrating that elementary school students' beliefs about their ability and subjective task values in mathematics predict their current and future activity choices [5, 6].

Until recently, dispositions towards math and related constructs, such as math anxiety, were often not evaluated before middle school [7] or high school [8]. This is troubling in light of evidence of age-related changes in children's motivational patterns. Whereas most students begin schooling enthusiastically [1], a shift from intrinsic to extrinsic motivational orientations emerges at about grade 3 (approximately 8 years of age) [9, 10]. This type of progression has been reported for measures of attitudes and curiosity [11, 12], perceived competence [13–15], and beliefs about intelligence [16], and suggests that early childhood (prior to and up to grade 3) may be an essential period for establishing, maintaining, or at least initiating the foundation of a positive disposition towards mathematics. Moreover, while developing productive dispositions is important for children across a wide range of achievement levels, this may be especially important for students at risk for low mathematics achievement and/or MLD.

To explore how productive learning dispositions are cultivated in early childhood, we utilized an expectancy-value model of achievement motivation [17] which asserts a proportional relationship between task engagement, expectations for success, and subjective task values. Specifically, the amount of effort children invest in an activity (e.g., mathematics engagement) is a product of their expectations for success and the extent to which they value the task. These components are influenced by children's perceptions of competence, task difficulty, and affect, and have been shown to directly affect achievement-related behaviors (as reviewed elsewhere [17]). In the present study, we did not objectively measure students' efforts nor did we rely on self-report measures related to subjective task value. Rather, we used open-ended questioning to specifically test whether indicators of beliefs and values related to mathematics emerge from spontaneous conversations with young children. In particular, we focused on the extent to which children's responses reflect affect (likability), perceived task difficulty, and values reflected in the specificity with which their responses relate to the usefulness of mathematics. The expectancy-value model provided a useful framework for considering the types of beliefs on which to focus in coding children's responses, and for interpreting our findings given the similarities between our variables of interest and those proposed as part of this theoretical model.

What is the relevance of spontaneous conversations of this type in the context of evaluating children's beliefs about math? First, early conversations about math provide a mechanism by which adults can deliberately attend to the potential influences they have on their children's or students' math-oriented beliefs. Second, in addition to using conversations to nurture the development of a healthy math disposition, conversations can be monitored for indicators of a child's emerging beliefs. In other words, early conversations are useful as a platform for adults' messages to children, but also for attending to children's comments—spontaneous or structured.

Why are children's early comments about math potentially important? In view of emerging evidence that early beliefs about mathematics are related to later achievement outcomes, efforts to steer children from paths towards negative outcomes should begin in early childhood, when children's beliefs may be more malleable rather than deeply rooted. Consequences of negative beliefs towards math are frequently described in terms of secondary-school behaviors, such as avoidance of elective mathematics courses after grade 10 (age 15 years) [18]. But the antecedents of such avoidance likely occur much earlier. For example, a negative relationship between math anxiety and math achievement has been observed among elementary school aged children with high working memory capacity [19, 20], and among elementary aged students who believe math-gender stereotypes with female students showing weaker identification with mathematics than their male counterparts despite similar levels of achievement [21, 22]. Gender stereotypes have been shown to influence mathematics achievement gains in first or second grade (ages 6 and 7 years), at least among young girls who conform to the belief that females underperform in math relative to males, and particularly among girls who receive instruction from a teacher with elevated levels of anxiety about mathematics [2]. These early associations warrant attention; information from informal conversation may be among the earliest indicators to direct adults' attention to students' emerging dispositions.

Children's math ability self-perceptions become well established in elementary school (e.g., [6]), but parent expectancies and attributions may influence children's math beliefs [23, 24] before and after the onset of formal schooling. Parents who believe that math and science are male-oriented domains tend to overestimate their sons' math and science performance and underestimate their daughters' performance in math and science at least in mid- to late-elementary school [25]. In middle school students, this underestimation may impact self-concepts and self-underestimation of math ability [26, 27]. These findings suggest that teacher and parent behaviors *can* and *do* play a major role in student learning, and that efforts to counter such effects on negative beliefs about math and achievement-related perceptions should begin in early childhood [23]. Listening to children's dialogues about math may contribute to these efforts and provide opportunity for intercession.

There are a number of ways to evaluate math beliefs. Ratings of children's math self-perception and their enjoyment of mathematics are objectively measureable in elementary

school aged children [13–15, 28] and earlier [4]. Here, we propose that discourse may serve not only as a means for parents and teachers of young children to help shape a positive disposition towards math through adult-initiated "math talk," but also as a means for adults to glean information about a child's emerging disposition, despite the fact that dialogues are no substitute for formal screening and diagnostic assessments of risk for poor math outcomes.

What aspects of children's early dispositions towards mathematics are revealed during open-ended discussions, and are these beliefs related to formal math outcomes? To address these questions, in the present study we used a simple and straightforward approach to elicit children's spontaneous comments about math. Importantly, we avoided explicit prompting for information about beliefs about math in order to focus on children's spontaneous comments. Although findings to emerge from this approach do not reveal causal pathways to successful mathematics, evidence of the mere presence of relevant information from spontaneous speech has implications for the role of early conversations about mathematics among children and their parents, teachers, and care providers in shaping or supporting a child's disposition towards mathematics.

2. Method

2.1. Participants. Participants were drawn from a larger longitudinal study of mathematics achievement and mathematics learning disabilities during the primary school aged years, described elsewhere in more detail [29]. Briefly, the participants from the larger study were recruited from 23 kindergarten classrooms across seven schools in one large metropolitan public school district. The participating schools were selected based on their relatively low rates of mobility and free or reduced lunch participation, to diminish attrition and decrease the likelihood that poor math performance observed among participants was linked to low socioeconomic status. Of the 445 kindergartners from these classrooms invited to participate, 249 enrolled (129 girls). Participation in the present study was limited to the 207 children (107 girls) who completed annual assessments during both grades 2 and 3. Of the 207 included, most (86.5%) participants were white, 8% were black, 3% were Asian, and the remaining 2.5% represented other or mixed ethnicities.

2.2. Materials and Procedures. During each year of the study, children were tested individually by one of five female examiners. Testing occurred during two or three sessions up to 45 minutes each, in a room separate from the classroom or other distracting activities. The testing battery included a range of standardized and experimental assessments. The measures relevant to the present report are two measures of mathematical ability, and a vocabulary probe administered to glean information about each participant's beliefs about mathematics.

2.2.1. Test of Early Math Ability-Second Edition (TEMA-2, [30]). The TEMA-2 is a standardized measure of formal and informal mathematical ability normed for use with children ages 2 to 8 years, 11 months. Items from the TEMA-2 include basic knowledge items such as counting principles, calculation and fact retrieval items, and items testing place value concepts or word problem solving. Total correct scores are converted to age-referenced standard scores for which test-retest reliability is 0.94 [30].

We administered the TEMA-2 to participants in the longitudinal study, from kindergarten through grade 3. In the present study, we used the standard scores from all four grades to assign children to one of three groups: children with mathematical learning disability (MLD), children with low mathematics achievement (LA), and children with age appropriate math achievement (typically achieving or TA). For the participants who exceeded the upper age level for the TEMA-2 ceiling at grade 3, we calculated prorated standard scores using regression-models to predict age 9 years and age 9.5 years outcomes, based on data from over 200 children who completed the TEMA-2 during all four years of the study. The criteria for these participant groups, described elsewhere in detail [31], reflect the growing consensus that MLD classification is strengthened by considering scores over time, rather than one score at a single assessment. Children with TEMA-2 scores below the 10th percentile of the study sample were classified as having MLD ($n = 18$) and children with scores between the 11th to 25th percentile were classified as having low achievement in mathematics (LA; $n = 26$). These criteria were met during at least two of the four grades tested, provided that scores during any remaining years fell within the 95th percentile confidence interval of the 10th or 25th percentile cut off, respectively. Children who did not meet these criteria and who scored above the 25th percentile (or within the 95th percentile confidence interval for this range) were classified as typically achieving (TA; $n = 163$).

2.2.2. Woodcock Johnson—Revised Calculations (WJ-R Calc). The Woodcock Johnson Psycho-Educational Battery-Revised [32] is a widely used standardized test of formal academic skills and is normed for use with children ages 2 to 21 years. The WJ-R Calculation subtest, normed for ages 6 and older, is an untimed paper-and-pencil test that includes a range of arithmetic problems presented in order of increasing difficulty. We administered this subtest at grades 1 and 3. Due to floor effects, we did not administer the test at kindergarten; due to time constraints, it was not administered at grade 2. Internal consistency reliability for the WJ-R Calc subtest ranges from $r = .89$ to $r = .93$ across ages 6–9 years [33]. In the present study, we used the standard scores from the WJ-R Calc subtest at grade 3 as the dependent variable in select analyses, controlling for earlier math performance (WJ-R Calc performance at grade 1).

2.2.3. Defining "Mathematics." During select years of the longitudinal study, the test battery included a standardized expressive vocabulary test from the Stanford Binet Fourth Edition [34] (at grades K to 2) or the Wechsler Abbreviated Scale of Intelligence [35] (at grade 3). These subtests involved

asking children to define a word presented orally by the examiner, beginning and ending at standardized basals and ceilings based on the examinee's age. In accordance with these standardized protocols, we asked each participant, "what is math?" followed by, "what is reading?" without any prompting for any specific or additional information. Only if a child responded with the equivalent of, "I do not know," we added the following prompt: "Just tell me what you think (math/reading) is." These definitions were elicited after a child reached a performance ceiling on the standardized vocabulary subtest, so that the point at which a definition was requested occurred at comparable levels of performance fatigue for all participants. This procedure also avoided unintentional alteration of the standardized vocabulary subtest. Also, by requesting a definition of math at the end of the expressive vocabulary test, we implicitly established expectations for the amount of information expected in response, and could indirectly convey comparable expectations about length of definitions for both "math" and "reading." Definitions of math and reading were obtained during grades 2 and 3.

Children's responses were recorded verbatim and transcribed into a central database with child identifiers removed. This ensured that coders were blind to the children's identity, gender, or math achievement status. Each response was coded independently by two trained coders. Coding responses were compared, and discrepancies were resolved in a coding meeting with the study PI and both coders present. Therefore, all definitions were double coded and, if there was disagreement, by three coders. Coding reliability is reported subsequently.

2.3. Coding Variables and Guidelines. Each definition was coded separately for three attributes: the extent to which the child's definition spontaneously reflected a like or dislike for math or reading (*likability* code), whether the child's definition made reference to ease or difficulty associated with math or reading (*difficulty* code), and how specifically math or reading concepts, procedures, or uses were described (*specificity* code). One set of scoring criteria was developed for each of the likability and difficulty coding protocols (Tables 1 and 2, resp.), each of which was applied to definitions of math and reading. Two parallel sets of criteria were established for coding the specificity of math (Table 3) and reading (Table 4) definitions.

The full range of possible *likability* codes was −2 (extremely dislikes math or reading) to 2 (extremely likes math or reading), with a code of 0 reflecting a neutral response (Table 1). Children's responses were assigned codes based on whether they were moderate (1) to strong or exaggerated (2) statements; these were further coded as positive (1 or 2) or negative (−1 or −2). Neutral responses included either no negative or positive comments, or included both (e.g., "it is kind of fun, but sometimes it is boring"). Of the four sets of 207 valence codes assigned, there were 2 disagreements among the 2 coders (99.9% agreement), which were resolved with a third coder/arbitrator. No child received a likability code of "2" for math at either grade, although at both grades, codes of "2" were assigned for reading.

The full range of possible *difficulty* codes was −2 (very difficult) to 2 (very easy), with a code of 0 reflecting no reference to either the difficulty or ease associated with mathematics or reading (Table 2). Of the four sets of 207 difficulty codes, there were 19 disagreements among the 2 coders (99.97% agreement), which were all resolved with a third coder/arbitrator. No child received a code of "2" (very easy) for either math or reading, at either grade.

Specificity codes ranged from 1 to 5, reflecting irrelevant or otherwise uninformative responses (1) to responses reflecting mathematics (or reading) as a useful tool (5). A summary of these coding criteria and sample responses appears in Tables 3 and 4. Of the four sets of 207 specificity codes, there were 58 disagreements among the 2 coders (93.0% agreement), which were resolved with a third coder/arbitrator.

All data were double entered into two excel spreadsheets, subtracted, and reviewed for discrepancies that were then corrected until the subtraction comparisons yielded no errors.

3. Results

3.1. Defining Math and Reading. First we asked whether qualitative features of children's definitions of math and reading differ as a function of gender or MLD status. We carried out three analyses of variance (ANOVAs), each based on a 2 (subject area: math versus reading) × 2 (grades: 2 and 3) × 2 (gender) × 3 (MLD status: TA, LA, and MLD) design, with repeated measures of the first two factors. The outcome variables were likability codes, difficulty codes, or specificity codes from children's spontaneous definitions of math and reading. Note that the means reported in the text are marginal estimated means whereas means reported in the figures are observed means.

3.1.1. Children's Likability for Math and Reading Definitions. We found no main effects of grade, MLD status, or gender on likability codes, $P > .437$. Although mean codes for reading were slightly more positive than those for math, the effect of subject area was not statistically significant, $F(1, 201) = 3.86$, $P = .051$, partial $\eta^2 = 0.019$. Group means for likability codes were essentially neutral for math (−0.011) and reading (0.048) when collapsed across grade and MLD status.

Two-way interactions emerged for grade × MLD status, $F(2, 201) = 3.86$, $P < .03$, partial $\eta^2 = 0.037$; and grade × subject area, $F(1, 201) = 6.67$, $P < 0.02$, partial $\eta^2 = 0.032$, but these effects were also very small. From grade 2 to 3, valence codes did not change among children from the TA group (estimated marginal means = 0.032 and 0.023, resp.), became slightly more positive among children in the LA group (means = −0.017 and 0.095, resp.), and slightly more negative among children with MLD (means = 0.013 and −0.036, resp.). From grade 2 to 3, valance codes did not change as much for math (means = 0.013 and −0.035, resp.) as they did for reading (means = 0.006 and 0.090,

Table 1: Likability scores: coding criteria and real examples of math and reading definitions.

Score	Definition	Math examples	Reading examples
−2	Extremely disliked, hated, or dreaded. Includes reference to words such as hate, worst, awful, and so forth.	n/a[a]	"When you sit and read a book bored out of your mind."
−1	Disliked or avoided. Includes reference to words such as do not like, not fun, boring, bad, and so on.	"Math is something I do not like."	"Something you do only when you need to do it, you have to read to figure out information."
0	Neutral feeling or tolerated. Includes no reference to an emotion.	"When you learn math problems."	"Like when you read a book in school or at home."
1	Liked or enjoyed. Includes reference to words such as play, fun, like, good, and so forth.	"Math is fun, you could do math at school—you could do it anywhere!"	"You can read for information or just for the fun of it."
2	Extremely liked, loved, or favored. Includes reference to words such as love, favorite, best, and so forth.	n/a[a]	"Oh I love to read."

Note: [a]No responses were coded at this value.

Table 2: Difficulty scores: coding criteria and real examples of math and reading definitions.

Score	Definition	Math examples	Reading examples
−2	Really hard. Includes reference to words such as *difficult*, *hard*, or *complex*.	"Something you do in school. Something that's very, very hard."	"Like you are reading and you are bad at reading and you read something like hard words."
−1	Kind of hard. Includes reference to words such as the noun form of *work*, words like *learn*, phrases that imply exertion zero, or implicit or explicit reference to cognitive processes.	"It is like problems that you have to solve."	"It is when there is like lots of sentences and you read it to get information."
0	Neutral difficulty. Includes no reference to difficulty, refers to verbs such as work, do, use, teach, or "work with" without implying exertion.	"Like math projects. Doing math homework."	"You have a book and you read the words."
1	Kind of easy. Includes reference to words such as "not hard" or words implying ease.	"It is easy and you have to do it in your homework."	"To just lay back and read a book."
2	Really easy. Includes reference to words such as simple, effortless, or speed.	n/a[a]	n/a[a]

Note: [a]No responses were coded at this value.

resp.), but the direction of these changes differed with math scores becoming slightly more negative, and reading scores becoming slightly more positive (Figure 1).

Even the strongest interaction to emerge from this analysis was associated with a small effect size, for a three-way interaction between subject area, grade, and MLD status, $F(2, 201) = 5.53$, $P < .01$, partial $\eta^2 = 0.052$. Pairwise comparisons indicated that children in the TA group made relatively neutral references to both math and reading at both grades (means = 0.012 for math at grades 2 and 3; and means = .051 and .034 for reading at grades 2 and 3, resp.), with little change from grades 2 to 3. Among children in the LA group, valence codes for definitions of math became slightly more negative over time (means = .000 to −0.045 at grades 2 to 3, resp.), and slightly more positive for reading during the same time period (means = −0.033 to 0.236). Children with MLD made neutral references when defining reading at both grades (both means = 0.000), whereas the

Table 3: Specificity scores: coding criteria and real examples of math definitions.

Score	Definition	Examples
1	No response, a circular response, a response unrelated to math, or another uninformative response.	"You do your math." "I cannot explain it." "I do not know."
2	Response is unspecific or only indirectly related to math as a primary school subject. The response may include references to activities performed in relation to math or in a math class but with no discernible reference to math concepts or procedures.	"You play games and stuff." "Science." "Like you do something. . .and you have pennies and money."
3	Unelaborated basic concepts or mechanics of math. Includes reference to real numbers, operations, math problems, or learning math.	"It has to do with numbers and sizes and fractions." "It means taking away, subtracting, and multiplication." "When you do all sorts of problems like divide, multiply, fractions, decimals." "In my class we do like a drill so it like refreshes our memory from the other day."
4	Elaborated concepts of math.	"Like if you have a word problem, like Jim has 18 apples and eats 3, you use math to solve it."
5	Concept of math as a useful tool.	"Math is something that people do and they have to know math to be able to get a job and do other stuff."

largest difference over time was the shift towards more negative ratings of math among the MLD group (means = 0.026 to −.071 from grades 2 to 3, a difference of −0.097).

In summary, at grades 2 and 3, boys and girls do not spontaneously make reference to extremely positive or negative sentiments about math or reading when defining either term, but there is a slight tendency for more negative comments about math (versus reading) among children with MLD, and, to a lesser degree, among children with low achievement in math.

3.1.2. Children's Reference to the Difficulty of Math and Reading. We also evaluated spontaneous comments about the difficulty versus easiness of math or reading in our second set of repeated measures ANOVAs. Here, the main effect of subject area was significant, $F(1, 201) = 16.02$, $P < .0001$, partial $\eta^2 = 0.074$. Children's definitions of math included more references to difficulty (mean code = −0.347) than did their definitions of reading (mean code = −0.171). The main effect of MLD status reflected significantly more negative references (i.e., more ratings reflecting difficulty) to math *and* reading in the MLD group (mean rating across subject areas = −0.378), relative to the LA and TA groups (whose means ratings were −0.174 and −0.225, resp.), $F(2, 201) = 3.40$, $P < .05$. Still, the effect was small, partial $\eta^2 = 0.033$. Pairwise comparisons showed that math and reading ratings combined were significantly more negative among the MLD versus LA or TA groups, $P < .03$, but did not differ between the TA and LA groups, $P = .361$.

There was no main effect of grade or gender on difficulty ratings, $P > .87$. No interactions were significant, with the exception of a weak three-way interaction between subject area × grade × gender, $F(1, 201) = 4.26$, $P < .05$, partial $\eta^2 = .021$. Here, both boys and girls reported slightly greater difficulty for math than reading at both grades. This difference was more pronounced for girls than for boys at grade 2 but not at grade 3. Over time, boys showed a slight shift towards reporting greater difficulty from grades 2 to 3, for both math and reading; among girls, there was a slight shift towards reporting less difficulty for math (and no change in reports of difficulty for reading) from grades 2 to 3 (Figure 2).

In sum, at both grades 2 and 3, both boys and girls make references to math or reading as difficult (or as requiring "work" or exertion), but there is a tendency for math to be described as slightly more difficult than reading, and a tendency for children with MLD to rate both math and reading as more difficult compared to ratings assigned by their non-MLD peers.

3.1.3. Likability and Difficulty as Distinct Constructs. It is possible that likability and difficulty ratings simply represent

Table 4: Specificity scores: coding criteria and real examples of reading definitions.

Score	Definitions	Examples
1	No response, a circular response, or a response unrelated to reading or otherwise uninformative.	"Reading means you read a lot." "I do not know that one."
2	Response related to unspecific reading activities, or indirectly related to reading activities. Includes reference to activities that may be performed in relation to reading or for a reading class but without a clear reference to the concept of reading.	"It means that you do reading projects." "It is if you have a book report you can report."
3	Unelaborated basic principles or mechanics of reading. Includes reference to sounding out and/or to reading materials or to learning to read.	"To like sound out something." "You read a book, you read a piece of paper." "When you like read words and books and sound out letters." "Something you do to get smarter at knowing words and learning what the words mean and how to pronounce the words and how to say them. That is why you have teachers to help you pronounce, read, and spell the words."
4	Elaborated concepts of reading. Includes reference to the extraction of meaning from written material through the act of reading.	"Like if you have a book, it has words, and you read the words not to just look and say, but to know what the words say and what the story is about."
5	Concept of reading as a useful tool. Must demonstrate usage of reading or usage of meaning derived from reading.	"To like look at books and some books help you make things and repair your house and some people just read for fun."

Table 5: Observed means or definition codes among total study sample ($n = 207$).

	Math		Reading	
	Mean (SD)	Range	Mean (SD)	Range
Grade 2				
Likability	0.01 (0.14)	−1 to 1	0.03 (0.30)	−2 to 2
Difficulty	–0.29 (0.52)	−2 to 0	–0.11 (0.33)	−2 to 0
Specificity	2.85 (0.473)	1 to 4	2.96 (0.63)	1 to 5
Grade 3				
Likability	0.00 (0.14)	−1 to 1	0.06 (0.39)	−2 to 2
Difficulty	–0.32 (0.50)	−2 to 0	−0.21 (0.43)	−2 to 1
Specificity	3.03 (0.66)	1 to 5	2.97 (0.61)	1 to 5

Note: Standard deviations shown in parentheses.

a general reporting tendency, rather than specific constructs. For example, each measure may reflect a positive or negative disposition towards academic subjects in general, in which case likability codes for math and reading should be positively correlated, and difficulty codes for math and reading should also be correlated. Alternatively, the measures may reflect an even broader tendency for positive or negative reporting in general, in which case likability and difficulty codes should be correlated with each other. Finally, if the codes represent stable, subject-domain sentiments, then math (or reading) likability scores should be correlated across grades.

To explore which of these alternatives is supported, we ran three sets of four correlations, using 12 Spearman rank tests, with alpha adjusted to 0.004 based on multiple correlations (.05/12). With respect to the three alternatives posed above, we found only weak, partial support for the notion of a general academic valence bias, based on the weak positive correlation between likability codes for math and reading observed at grade 2 (Spearman *Rho* = .150,

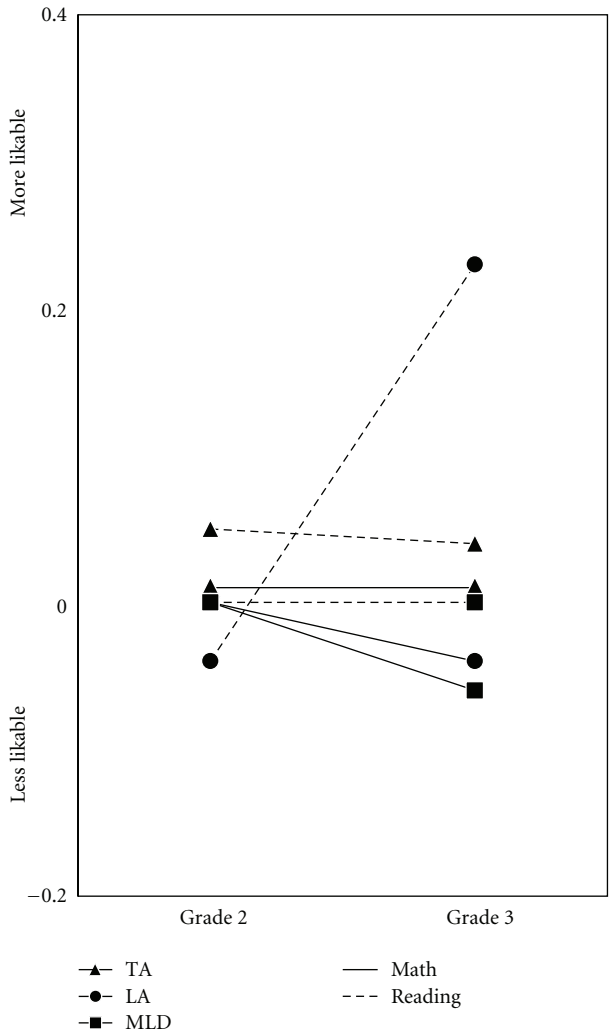

Figure 1: Observed means for likability scores, from grades 2 and 3, depict the three following interactions: grade × MLD status, grade × subject area, and grade × MLD status × subject area.

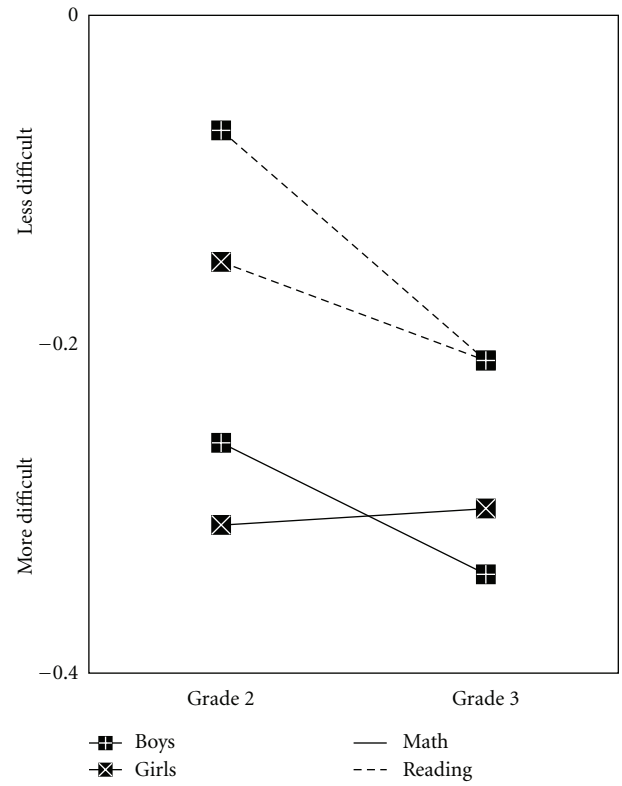

Figure 2: Observed means for difficulty scores in grades 2 and 3 depict the significant main effects of subject area and MLD status, and the significant three way interaction between grade, gender, and subject area.

$P = .031$) but not at grade 3 (Spearman *Rho* = .0, $P = 1.0$); and positive correlations between difficulty codes for math and reading at Grades 2 (Spearman *Rho* = .160, $P = .022$) and 3 (Spearman *Rho* = .311, $P = .0001$). Only the latter correlation at grade 3 met our adjusted significance criteria for multiple correlations.

There was less support for a broad reporting disposition bias, because likability and difficulty codes were not correlated with each other, P ranging from .096 to .594, with the exception of the association between math likability and difficulty at grade 3 only, −.148, $P = .033$, which did not meet criteria adjusted for multiple correlations.

There was far more support for subject-specific likability (or difficulty) over time, suggesting that our codes are indicative of stable, and subject-domain beliefs, at least over the short term (from grade 2 to 3): reading likability at grade 2 was correlated with reading likability at grade 3 (Spearman *Rho* = .239, $P = .001$), and reading difficulty at grades 2 and 3 were also correlated with each other (Spearman *Rho* = .210, $P = .002$). Likewise, math likability at grades 2 and 3 were correlated with each other, (Spearman *Rho* = .251, $P < .0003$), although math difficulty scores were not correlated across grades ($P = .816$). All three of the significant results met our criteria adjusted for multiple correlations. Grade level means are reported in Table 5.

3.1.4. Subject-Area Specificity of Children's Definitions of Math and Reading. The third set of repeated measures ANOVAs concerned the specificity of children's descriptions of math or reading, with codes reflecting noninformative to elaborate descriptions (exemplified in Tables 3 and 4). There was no main effect of subject area on this aspect of children's definitions, $P = .49$. Responses ranged from 1 to 5 for both math and reading, with estimated marginal mean ratings of 2.86 for math and 2.90 for reading, collapsed across all other variables. There was no effect of grade on the specificity of definitions (means = 2.85 to 2.91, at grades 2 and 3, resp.), $P = .29$.

Gender and MLD status each contributed significantly to variance in the specificity of definitions. Girls gave slightly more specific definitions of mathematics or reading (mean = 3.00) than did boys (mean = 2.75), $F(1, 201) = 10.53$, $P = .001$, partial $\eta^2 = 0.050$. The main effect of MLD status, $F(2, 201) = 4.30$, $P < .02$, partial $\eta^2 = 0.041$, reflected more specific definitions of math or reading by children in the TA

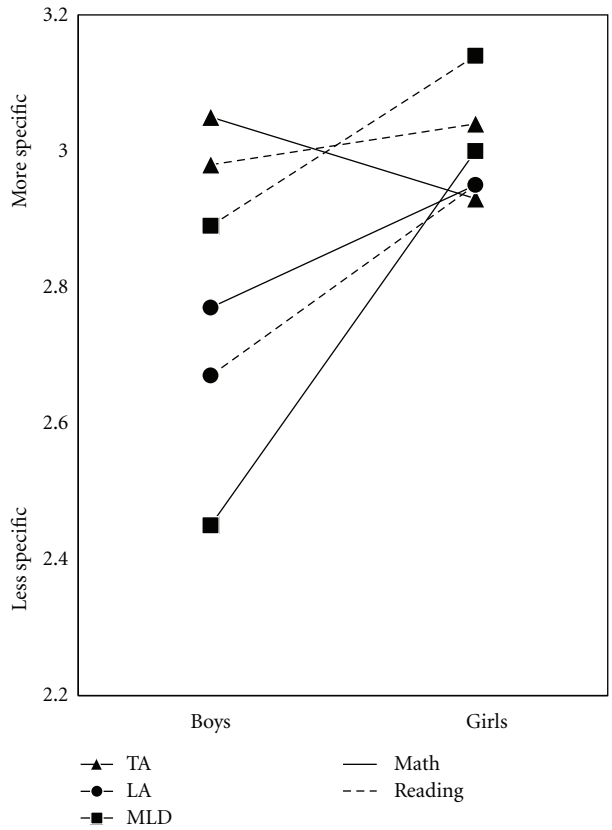

Figure 3: Observed means for specificity scores depict significant main effects of MLD status and gender, and the significant MLD status × gender interaction. Subject area is not significant.

group (mean = 3.00), relative to the MLD or LA groups ($P <$ .02), which did not differ from each other, $P = .68$ (means = 2.84 and 2.80, resp.).

The only interaction to reach statistical significance was a two way interaction between MLD status and gender, $F(2, 201) = 5.96$, $P < .004$, partial $\eta^2 = 0.056$. This interaction reflects that girls' ratings did not differ across MLD groups (with estimated marginal mean ratings of 2.99, 2.96, and 3.07 across the TA, LA, and MLD groups, resp.), whereas boys' ratings did (means = 3.02, 2.72, and 2.52, resp.; Figure 3).

3.2. Predicting Mathematics Achievement. Do these exploratory measures predict future or concurrent math performance? We carried out two regression models, each comprised of three predictors of grade 3 WJ-R math Calculation score as the outcome variable of interest. These models were based on predictors obtained at grade 2 or grade 3 (grade level predictors examined separately). When combined, grade 2 likability, difficulty, and specificity codes accounted for approximately 5% of the variance in WJ-R Calculation scores obtained at grade 3, $F(3, 206) = 5.03$, $P <$ .003, adjusted $r^2 = .055$. However, likability and difficulty codes did not contribute significantly to the model, $P > .12$, whereas specificity codes did, $t(206) = 3.11$, *beta* = .212, $P < .01$.

Model strength improved significantly when grade WJ-R Calculation score was included as a predictor, $F(4, 206) = 32.47$, $P < .0001$, adjusted $r^2 = .379$. Still, the small contribution made by the specificity score remained significant, $t(206) = 2.40$, *beta* = .134, $P < .02$.

To examine whether *concurrent* predictors would be stronger than predictors obtained one year prior, the analysis was repeated using grade 3 likability, difficulty, and specificity codes to predict grade 3 WJ-R Calculation scores. The model strength for the grade 3 predictor model was weaker than the aforementioned grade 2 predictor model, $F(3, 206) = 3.00$, $P < .04$, adjusted $r^2 = .028$. Here, both the specificity and likability codes predicted grade 3 WJ-R Calculation scores, but effects were small ($t(206)s$ (*beta*) = 2.12 (.147), and 1.99 (.138), resp., $P < .05$), and these effects disappeared when the grade 2 WJ-R Calculation scores were added to the model, $P > .31$

To evaluate whether the effect to emerge from grade 2 predictors was specific to math disposition, we carried out parallel regression models using grade 2 or grade 3 likability, difficulty, and specificity codes from *reading* definitions, to predict WJ-R Calculation scores. Neither model accounted for significant variation in grade 3 WJ-R Calculation scores, adjusted $r^2 < .02$, $P > .10$.

4. Discussion

Our research questions examined primary school aged students' spontaneous comments about mathematics and specifically tested whether these comments reveal emerging dispositions linked to students' later mathematics achievement. Additionally, we tested the notion that beliefs about math may differ among children with versus without MLD. For comparative purposes, children's spontaneous comments about reading were also obtained. Our approach involved a straightforward design to elicit spontaneous comments of the kind that may emerge during casual verbal dialogue between young children and their parents or teachers. Finally, by repeating this procedure during two consecutive grades, we were able to test short term stability or reliability of the codes we collected.

Despite the subtlety of our approach, our findings indicate that children's spontaneous comments about math are informative even if they are no substitute for a structured and validated assessment of disposition. Specifically, we found that subject-specific comments about likability (or difficulty) appear to be stable beliefs, at least from grade 2 to 3. Lower likability ratings (i.e., more negative comments) were evident in definitions of math (versus reading) among children with MLD, and, to a lesser degree, among children with LA in math; but overall, reference to liking or disliking mathematics was absent from most of the participants' spontaneous comments of math and variability was greater among definitions of reading.

With regard to *difficulty*, children made more references to difficulty in definitions of math versus reading, collapsed

across grades, groups, and gender. Children rarely, if ever, spontaneously remarked that math was easy but this was also true of reading. Children with MLD were more likely to make references to difficulty of math than their peers, but they also made more references to reading as difficult. This finding is consistent with reports of high co-occurrence of math and reading difficulties [36] and with reports that children with MLD recognize their academic weakness in mathematics, as determined by ratings on self-perception measures [37]. However, Hanich and Jordan [37] found that, on measures of reading, children with math difficulties and normal reading achievement do not rate their competence lower than children without math difficulties, although children with co-morbid math and reading difficulty do. They hypothesized that children in the math and reading difficulty group in their study were more accurate at evaluating their reading performance than math performance given that many of the children in this group were receiving special education services for reading related problems, which may have contributed to the formation of children's achievement-related beliefs.

Analyses related to *specificity* of children's definitions of math and reading showed that, in general, children report unelaborated conceptions regarding the usefulness or importance of both math and reading as a tool. The only contributions of MLD status to the *specificity* rating from children's definitions of math concerned slightly less specific definitions by the MLD and LA groups, relative to the TA group, more so for boys versus girls; but this finding also emerged from definitions of reading. Greater specificity in definitions for math and reading were also observed among girls versus boys, which may simply reflect girls' superior expressive vocabulary during the school age years.

Of the four main predictor variables examined across these analyses—MLD status, subject area, gender, and grade—the only variable to account for variability in all three outcome variables was MLD status. Main effects or interactions involving MLD status emerged for likability, difficulty, and specificity. Although it is erroneous to infer that brief, open-ended questions like the ones used in our study are appropriate for diagnosis of MLD, future and ongoing work is needed to address the dynamic role(s) of a productive disposition and long term math achievement outcomes.

There were no gender differences in likability ratings for math or reading, but boys and girls reported slightly greater difficulty for math than reading at both grades. From grades 2 to 3, boys shifted towards reporting slightly greater difficulty for both math and reading over time, whereas girls shifted towards reporting slightly less difficulty for math over time. This may reflect differences in shift towards behavioral compliance rather than a specific shift in beliefs about mathematics, but neither explanation is supported by our data. The effect was small and warrants more in depth evaluation, especially in light of research that has found primary school aged boys identify more strongly with mathematics and have higher self-concepts than girls despite similar levels of mathematics achievement [22]. Other research conducted with eighth graders found an absence of gender differences in attitudes towards mathematics and achievement, although boys were more likely to attribute their math achievement to their intellectual abilities than girls [38].

It is quite clear that the effects that emerged in this exploratory study are small. Yet the fact that *any* such findings would emerge given the simplistic nature of our questioning is somewhat intriguing. Our ratings were based on children's unstructured responses to brief, open-ended questions, "what is math?" and "what is reading?" and their definitions hint at potentially distinct elements of a productive disposition towards math, evident at second and third grades. This finding is consistent with factors to emerge from formal assessments based research. For instance, Adelson and McCoach [28, 39] found evidence of two factors from their math attitudes survey given to upper elementary students (grades 3 to 5), reflecting an "enjoyment of math" construct (similar to our "likability" codes) and a "math self-perceptions" construct (similar to both our "difficulty" and "specificity" codes). Their development of a structured survey supports the notion that individual differences in dispositions towards math exist and can be measured at grade 3 and above; we add that this work can likely be extended downward to grade 2 and below. As our work and that of others indicates, some dispositional factors may be math-specific ("math is difficult") whereas others may generalize to academic subjects.

4.1. Implications for Further Studies. Although our findings do not indicate causal pathways, they are consistent with evidence that primary school aged students' interests or beliefs about mathematics affect their achievement level or reflect risk status for future math outcome [2, 4]. The predictive contribution of grade 2 specificity ratings to math scores at grade 3 suggests associations emerging in early childhood. One possibility is that *opinions* about the importance or challenges of math precede and direct learning or performance success, as has been implicated in work with adults [40]; alternatively, earlier ability levels may influence emerging opinions, such that these features operate dynamically throughout schooling. There is some evidence that students' perceived competence ratings and ratings of enjoyment of mathematics are independent of achievement in the early years, but are related to math performance at the end of elementary school [41]. Our findings run counter to this conclusion, and thus support the contention that research on dispositional contributions to mathematics achievement must include studies of early childhood.

These findings in early childhood have implications for teachers, parents, and care providers of young children. In general, children in our study expressed relative flat affect with regard to their enjoyment of mathematics, did not hold very elaborate conceptions regarding the usefulness of mathematics, and attributed more difficulty to mathematic than reading. Despite the small effect size that emerged from these findings, these suggest potential negative outcomes when considered in the context of expectancy-value theory. According to the model, these dispositions run counter to facilitating engagement in mathematics activities given

the proportional relationship among components of the model (i.e., engagement = success expectancies × task value). However, the "neutral" codes for likability were based on spontaneous comments, and children may have responded very differently to prompts eliciting remarks about liking or disliking mathematics. To foster adaptive motivational behaviors, parents and teachers should provide feedback to young children about the task values associated with mathematics, help students develop positive but accurate perceptions of their abilities, and teach children to effectively appraise learning to modify their achievement behaviors.

4.2. Limitations and Conclusions. There are several limitations to our exploratory study. Although our sample size was large and data collection over time was longitudinal, the time period over which we examined definitions of math and reading was limited to two consecutive years, and the number of students with MLD and LA was limited by the defining criteria of these constructs (e.g., MLD occurring in only ∼6–10% of the population, as it was in our study). The deliberately simplistic nature of our data collection was consistent with our goal to evaluate more naturalistic and spontaneous versus prompted comments about mathematics likability, difficulty, and specificity, but children's comments could be elaborated upon through structured probes and conversations. For instance, most children's definitions received "neutral" codes for likability, but this does not mean that children did not have beliefs about liking or disliking mathematics.

Our research question did not concern the strength of children's beliefs about math so much as the likelihood of their emergence during conversation. In addition to expressing (or not expressing) beliefs about math and reading, the children in this study remind us of the importance of listening to what they say, even during casual discourse. That is, adult-child discourse provides a means for parents and teachers to both nurture a child's positive disposition towards math, and monitor the child's emerging disposition.

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Research Article

Math Self-Assessment, but Not Negative Feelings, Predicts Mathematics Performance of Elementary School Children

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Mathematics anxiety has been associated to performance in school mathematics. The association between math anxiety and psychosocial competencies as well as their specific contribution to explain school mathematics performance are still unclear. In the present study, the impact of sociodemographic factors, psychosocial competencies, and math anxiety on mathematics and spelling performance was examined in school children with and without mathematics difficulties. The specific contributions of psychosocial competencies (i.e., general anxiety and attentional deficits with hyperactivity) and math anxiety (i.e., self-assessment in mathematics) to school mathematics performance were found to be statistically independent from each other. Moreover, psychosocial competencies—but not math anxiety—were related also to spelling performance. These results suggest that psychosocial competencies are more related to general mechanisms of emotional regulation and emotional response towards academic performance, while mathematics anxiety is related to the specific cognitive aspect of self-assessment in mathematics.

1. Introduction

Negative feelings about mathematics are usually associated with low mathematics achievement both in children and adults [1]. Mathematics anxiety (MA) is a feeling of tension, apprehension, or fear that interferes with mathematics performance, or as a state of discomfort in response to mathematics which is perceived as threatening to self-esteem [2]. As math anxious individuals avoid engagement in math tasks, they dedicate less time and effort to learn mathematics, reach lower attainment levels, enroll less in mathematics courses from high school onwards, and eventually select majors with lower mathematics requirements [3]. MA is thus of potential social and economic relevance in a globalized culture that imposes greater and greater demands on science and technology abilities of individuals in career as well as in everyday life.

MA is related to more general forms of anxiety, but can be distinguished from them. For instance, Young et al. [4] found brain activity patterns specific of MA to be unrelated to general anxiety, intelligence, working memory, or reading ability. In that study, higher levels of activation were observed in the amygdala and other regions associated with emotional processing [4], while lower levels of activation were found in areas associated to number processing and working memory [5]. MA has been linked to performance-related anxiety disorders such as test anxiety and social phobia [6–8]. Research also indicates that the genetic etiology of anxiety disorders is unspecific, being shared by different anxiety syndromes [9]. However, the correlations between

behavioral measurements of MA and other forms of anxiety have been found to range in the interval between .35 for general anxiety, .38 for trait anxiety, and .52 for test anxiety [3]. Data suggesting specificity of MA were obtained by Dew et al. [10, 11]. In that study, intercorrelations between three MA measures varied from .50 to .80, while correlations between MA and other forms of anxiety were substantially lower (r's from .30 to .50).

Aversive experiences with mathematics as well as the interaction between the individual's experience and vulnerability have been related to the occurrence of MA [8, 9, 12]. Two main vulnerability factors related to MA are female gender and low math achievement. There is a trend in females to report higher levels of MA [3, 13] despite equivalent achievement [14]. This has been associated with higher propensity of females to report feelings [8] and to social stereotyping [1, 14–16]. In an fMRI study, Krendl et al. [17] showed that in a neutral control condition, female participants activated usual frontoparietal math-related networks in response to math tasks. Under conditions of gender-stereotyped threat, math-related areas were inhibited, whereas the ventral anterior cingulate cortex was activated. Moreover, female reports on MA have been reported to be mediated by spatial cognitive ability, which is knowingly dependent on fetal androgen levels more than on social stereotyping [18].

Low mathematics achievement and developmental dyscalculia are other risk factors for MA. Results from two meta-analyses indicate that the correlations between MA and IQ are positive but low ($r = .17$). The association of IQ with math achievement is slightly stronger (r's between .27 and .34) [3, 13]. Several studies focused on the reversed causality direction in the assessment of the association between MA and achievement, but the results are inconsistent. Ma and Xu [19] observed that low math achievement at Grade 7 predicted high MA six years later, while the reverse was not true: initial MA was not statistically predictive of later low achievement. Krinzinger et al. [20] showed that children being assessed from the 1st to 3rd Grade do not present an effect of "math anxiety" [1] (Krinzinger et al. [20] used a two-factor model of math anxiety on the Mathematics Anxiety Questionnaire (MAQ, Krinzinger et al. [21]. The first factor describes self-evaluation and math-related attitudes ("evaluation of mathematics") while the second factor comprises negative emotions and worries concerning mathematics ("math anxiety").) on calculation ability or vice versa. In that study, initial low calculation ability was predictive of "evaluation of mathematics." Moreover, Rubinsten and Tannock [22] compared the performance of a group of dyscalculics with that of typically achieving children in an affective arithmetic priming task. Negative affect and arithmetic stimuli induced comparable priming effects in dyscalculics. In typically achieving children arithmetic priming was not observed.

Relationships between MA and mathematics achievement are complex and probably of two ways. For this reason, it is paramount to look not only at studies reporting effects of arithmetics achievement on anxiety levels but also at studies reporting effects of anxiety levels on arithmetics achievement. There is an evidence that MA may moderate the relationship between working memory and mathematics achievement in mathematics disabled undergraduates [23]. Moreover, in real-life situations, the impact of MA on mathematics achievement is mediated by cortisol levels associated with stress responses and anxiety [24]. Moreover, cognitive-behavioral interventions specifically addressing anxiety symptoms in mathematics disabled children have been shown to contribute to substantial improvement in mathematics achievement [25].

As MA probably originates from the interaction between individual vulnerability and experience in the early school years, it is necessary to investigate the specificity of the presumed vulnerability. Three main issues are addressed in this study. First, we investigated whether children with mathematics difficulties (MD) present higher levels of MA compared to typically achieving children (TA), as has been suggested by previous results [22]. Concretely, we expect children with MD to present higher levels of mathematics anxiety in the MAQ [21] when compared to matched TA children. Second, we tested whether MA can be dissociated from general anxiety proneness and psychosocial competencies in children, as suggested by previous studies with adults [3, 10, 11]. We expect that a significant effect of MA would still be present even after removing the impact of general anxiety proneness and psychosocial competencies on mathematics performance. Finally, we explored in more detail the specificity of MA in mathematics achievement comparatively to domains such as sociodemographic characteristics, psychosocial competencies, general cognitive abilities, and other nonmathematic academic subjects such as spelling ability. To examine the specificity of this relation, the impact of MA on domain-related arithmetics achievement as well as domain-unrelated spelling achievement was examined. We expect that MA should be specifically related to mathematics, but not to spelling achievement. This hypothesis will be tested by comparing the predictive strength of MA over performance in arithmetics and spelling tests. An effect of MA on arithmetics is expected to be significant while no significant effect of MA on spelling achievement should be detectable.

2. Materials and Methods

2.1. Participants. Samples were constituted by children with ages ranging from 7 to 12 years and attending from first to sixth grade. No differences between the performance-stratified groups were found regarding sex, age, and grade (Table 1). The study was approved by the local research ethics committee (COEP-UFMG). Only after giving informed consent in written form from their parents and orally from themselves, children were allowed to take part in the study.

Children were recruited from schools in Belo Horizonte and Mariana, Brazil. The proportion of children attending to private and public schools is representative of the sociodemographic characteristics of the population. In the first phase of group testing, only those children with normal intelligence (i.e., who scored above the 16th percentile in the Raven Colored Matrices Test) [26] were included in

Table 1: Descriptive data of TA and MD groups.

	TA		MD					
N	171		36		x^2	*df*	*P*	*w*
Sex (%male)	42.1		52.8		1.38	1	0.241	0.006
	Mean	SD	Mean	SD	*t*	*df*	*P*	*d*
Age (years)	9.5	1.14	9.28	1.27	1.05	205	0.293	0.190
Grade	3.39	1.15	3.08	1.05	1.45	205	0.148	0.270
Raven *z*-score	0.69	0.70	0.28	0.58	3.29	205	0.001	0.604
TDE arithmetic *z*-score	0.53	0.76	−0.90	0.47	14.66	79.7	<.001	2.690
TDE spelling *z*-score	0.67	0.56	0.05	0.58	5.99	205	<.001	1.100

TA: typically achieving; MD: mathematical difficulties; TDE: Brazilian school achievement test.

the study. These children also solved the arithmetic and spelling subtests of the Brazilian School Achievement Test (Teste do Desempenho Escolar, TDE) [27]. Those children scoring above the 25th percentile on both arithmetic and spelling subtests of the TDE were assigned to the typically achieving group (TA). The TA group consisted of 171 children. Children performing below the 25th percentile on arithmetics were assigned to the mathematical difficulties group (MD). Thirty-six children took part in the MD group (Table 1).

In a next step, psychosocial competencies and mathematical anxiety were individually assessed using the Child Behavior Checklist (CBCL) [28] and the Math Anxiety Questionnaire (MAQ) [29], see also [30], respectively. Children answered the MAQ individually, in an appropriate room on their schools. Parents filled out the CBCL in group also in their childrens' school.

2.2. Psychological Instruments

2.2.1. Brazilian School Achievement Test (TDE) [27]. The TDE is the most widely used standardized test of school achievement with norms for the Brazilian population [28, 29]. It comprises three subtests: arithmetics, single-word spelling, and single-word reading. In the screening phase, we used the arithmetics and spelling subtests, which can be applied in groups. Norms are provided for school-aged children between the first and sixth grade. The arithmetics subtest is composed of three simple verbally presented word problems (i.e., which is the largest, 28 or 42?) and 45 written arithmetic calculations of increasing complexity (i.e., very easy: $4 - 1$; easy: $1230 + 150 + 1620$; intermediate: 823×96; hard: $3/4 + 2/8$). Specific norms for each school grade were used to characterize children's individual performance. The spelling subtest consists of dictation of 34 words of increasing syllabic complexity (i.e., *toca*; *balanço*; *cristalização*). The spelling subtest was chosen as a marker for literacy because it can be performed in groups. Reliability coefficients (Cronbach α) of TDE subtests are .87 or higher. Children are instructed to work on the problems to the best of their capacity but without time limits.

2.2.2. Raven's Colored Progressive Matrices. General intelligence was assessed with the Brazilian version of Raven's Colored Matrices [26]. Children with general intelligence below the 16th percentile were not included in the sample.

2.2.3. Math Anxiety Questionnaire (MAQ). The math anxiety questionnaire is a well-known scale developed by Thomas and Dowker [31] for the assessment of anxiety towards mathematics in primary school children. The present study used a Brazilian Portuguese version of the MAQ that was developed and standardized by Wood and colleagues [32]. The Brazilian version of the MAQ contains 24 items that can be answered by children individually or in groups within 5 to 10 minutes. The items can be combined into four-base subscales ("self-perceived performance," "attitudes in mathematics," "unhappiness related to problems in mathematics," and "anxiety related to problems in mathematics") according to the authors of the original version [31]. Moreover, Krinzinger et al. [21] have shown that the four original subscales can be combined into two main scores called "self-perceived performance and attitudes" and "mathematics anxiety." The first one, named evaluation of mathematics, includes the first two subscales, while the second one, called math anxiety, combines the last two subscales. The MAQ items have the format of one out of four types of questions: "How good are you at ...;" "How much do you like ...;" "How happy or unhappy are you if you have problems with ...;" "How worried are you if you have problems with ...." Each question is to be answered regarding six different categories related to math, namely, mathematics in general; easy calculations; difficult calculations; written calculations; mental calculations; math homework. Children are encouraged by supportive figures to give their responses according to a Likert scale with 5 points. The higher the score, the higher the math anxiety. Reliability coefficients (Cronbach α) of MAQ in the German study range between .83 and .91 for the total scale, while in the Brazilian study reliability coefficients are .88 for the total scale; .74 for the "self-perceived performance in mathematics" subscale; .75 for the "attitudes in mathematics" subscale; .85 for the subscale "unhappiness related to problems in mathematics;" finally, .81 for the subscale "anxiety related to problems in mathematics" (see [32], this issue).

2.2.4. Child Behavior Checklist 6/18 (CBCL) [30]. The CBCL is a screening instrument answered by parents that is widely

Table 2: Regression coefficients for mathematics performance (after stepwise regression).

Predictor	Regression analysis for TDE arithmetics (r^2 = .31)					
	Unstandardized coefficient		Std. coefficient	Partial t	sig	% change in explained variance
	B	Std. error	Beta			
Intercept	−0.594	0.81		−1.14	0.26	
Age	0.03	0.01	0.41	3.26	<.001	0.00
Sex	0.06	1.05	0.03	0.53	0.59	0.00
School grade	−0.26	0.10	−0.33	−2.62	0.01	3.00
Intelligence (Raven)	0.43	0.08	0.33	5.49	<.001	15.40
Attention deficit/hyperactivity scale*	−0.02	0.01	−0.18	−2.52	0.01	2.80
MAQ self-perceived**	−0.08	0.01	−0.35	−5.87	<.001	12.40

*CBCL's subscales guided by the DSM-IV; **MAQ's factor. TDE: brazilian school achievement test; MAQ: Math Anxiety Questionnaire; CBCL: children behavior checklist.

used in research and clinical sets. The CBCL is strongly associated with diagnoses guided by international diagnostic manuals such as DSM-IV [33]. The CBCL is divided into two independent parts. The first part consists of a range of psychosocial competencies and the second part consists of 113 items in which the parents answer about behavioral, emotional, and social adjustment of the child. Eight syndrome scales derived from exploratory and confirmatory factor analysis are extracted. In addition to the syndrome scales, there are the DSM-oriented scales, built through a clinical consensus of the items by experienced psychiatrists and psychologists, among them the scales "anxiety problems" and "attention deficit/hyperactivity problems." CBCL data were compared with international norms obtained in a multicultural study (group 3 in [34]), for which there is data on the comparability of the Brazilian population [35].

3. Results

3.1. Single Comparisons. t-tests were used to compare TA and MD groups regarding general anxiety and math anxiety. Degrees of freedom were corrected for in homogeneities of variance when the associated Levene's test was significant.

The typically achieving group presented substantially higher levels of performance in arithmetics and spelling than MD children. MD children presented spelling abilities in the normal range (all children >25th percentile). Moreover, no difference between TA versus MD children was observed in CBCL subscales (all P values higher than .124). Finally, the self-perceived performance subscale of the MAQ revealed group differences between TA and MD children. Group comparison showed substantially more positive self-evaluation in the TA group when compared to MD children ($t(205) = -3.64$; $P \leq .001$; $d = -.067$). Other factors of the MAQ were nonsignificant, all P values higher than .136.

To assess more precisely the specific contribution of psychosocial competencies and mathematics anxiety on arithmetics and spelling abilities, a range of regression models was calculated.

3.2. Regression Analysis. In a second analysis, the specific impact of the different scales of general anxiety as well as of math anxiety on school performance was evaluated in a range of hierarchical multiple-regression models. Separate models were calculated for mathematics and spelling performance. Modelling the impact of general anxiety and mathematics anxiety on measures of spelling performance served to test the specificity of the effect of math anxiety on mathematics performance. While measures of general anxiety should contribute to explain the variance in both mathematics and spelling performance, MAQ scales should be specifically related to mathematics performance and not to spelling performance. For this reason, the last range of models serves to investigate the specificity of the association between math anxiety and school performance on mathematics. Data from TA and MD groups were combined in this analysis. In the first step, general factors predicting individual differences in school performance were included in the regression models. These were sociodemographic factors (gender, school grade, and age) as well as general intelligence. In a second step, those among the six scales of the CBCL which explained specific variance were added to the model (stepwise method). The same procedure was adopted in the third step, when the four MAQ scales were added to the model. The stepwise method was used in the second and third steps to avoid redundant predictors entering the model and producing overfitting.

Table 2 shows the results from the final stepwise regression model calculated for mathematics performance. Although the amount of explained variance in arithmetics achievement was low, regression models revealed that age, school grade, general intelligence as well as attentional deficits and hyperactivity, and self-perceived performance in mathematics contributed independently to explain mathematics performance in TA and MD children.

Table 3 shows the results from the final stepwise regression model calculated for spelling performance. Although the amount of explained variance in spelling performance was low, regression models revealed that general intelligence as well as attentional deficits and hyperactivity contributed independently to explain spelling performance in TA and MD children.

3.3. Path-Analysis Models. As a complement to the results obtained in the regression analyses, path-analyses including

Table 3: Regression coefficients for spelling performance (after stepwise regression).

Predictor	Regression analysis for TDE spelling (r^2 = .14)					
	Unstandardized coefficient		Std. coefficient	Partial t	sig.	% change in explained variance
	B	Std. error	Beta			
Intercept	0.94	0.63		0.49	0.14	
Age	0.001	0.01	0.02	0.11	0.92	0.00
Sex	0.09	0.08	0.07	0.11	0.27	0.50
School grade	0.03	0.08	0.05	0.33	0.74	0.00
Intelligence (Raven)	0.21	0.06	0.24	0.45	0.001	7.50
Attention deficit/hyperactivity scale*	−0.02	0.00	−0.21	3.15	<.001	4.30

* CBCL's subscales guided by the DSM-IV. TDE: Brazilian School Achievement Test; CBCL: children behavior checklist.

Table 4: Fit statistics for path models regarding arithmetics and spelling performance.

Arithmetic performance	CMIN	df	P	RMR	GFI	AGFI	CFI	RMSEA
Null model	95	10	0	1.915	0.899	0.638	0.816	0.203
Raven-MAQ (sp)	88	9	0	1.462	0.916	0.664	0.829	0.207
Raven-Arith	53	8	0	1.421	0.942	0.737	0.904	0.164
CBCL (anx)-Arith	52	7	0	1.415	0.943	0.709	0.903	0.176
CBCL (ADH)-Arith	45	6	0	1.401	0.953	0.719	0.915	0.178
MAQ (sp)-Arith	10	5	0.08	1.391	0.989	0.919	0.990	0.067
CBCL (anx)-MAQ (sp)	7.5	4	0.110	1.148	0.991	0.92	0.992	0.066
CBCL (ADH)-MAQ (sp)	7.5	4	0.110	1.148	0.991	0.92	0.992	0.066
Spelling performance	CMIN	df	P	RMR	GFI	AGFI	CFI	RMSEA
Null model	48	10	0	1.874	0.944	0.800	0.907	0.136
Raven-MAQ (sp)	41	9	0	1.408	0.954	0.817	0.921	0.132
Raven-Spell	25	8	0.001	1.399	0.970	0.864	0.958	0.102
CBCL (anx)-Spell	25	7	0.001	1.400	0.970	0.845	0.956	0.112
CBCL (ADH)-Spell	13	6	0.049	1.387	0.985	0.912	0.984	0.073
MAQ (SP)-Spell	10	5	0.086	1.386	0.989	0.919	0.989	0.067
CBCL (anx)-MAQ (sp)	7.5	4	0.110	1.144	0.991	0.920	0.991	0.066
CBCL (ADH)-MAQ (sp)	7.5	4	0.110	1.144	0.991	0.920	0.991	0.066

CMIN: minimum value; RMR: root mean square residual; GFI: goodness of fit index; AGFI: adjusted goodness of fit index; CFI: comparative fit index; RMSEA: root mean square error of approximation. MAQ (sp): self-perceived performance factor of MAQ; arith: arithmetic subtests of TDE; Spell: spelling subtests of TDE CBCL (anx): anxiety subscale of CBCL; CBCL (ADH): attention deficit hyperactivity subscale of CBCL.

demographic and cognitive factors as well as general anxiety and math anxiety were calculated separately for spelling and mathematics performance. These models included only the subscales among those subscales of the CBCL and MAQ, which exerted a specific contribution to explaining variance in mathematics or spelling performance in the regression models. As regarding regression models, path models evaluating the impact of MAQ on spelling performance served as a control measure for unspecific associations between the MAQ and school performance. To estimate the strength of the effects of general anxiety and math anxiety on mathematics and spelling performance, a range of path models was calculated and compared. As indexes of model quality regarding the fit of single models, the Chi-square and the approximate fit indexes *RMR*, *GFI*, *AGFI*, *CFI*, and *RMSEA* [36] were used. In this context, a nonsignificant Chi-square reveals that the discrepancy between data and model specifications are negligible. The *RMR* evaluates the proportion of residuals in comparison to the covariances accounted for by the models. Values smaller than .1 are considered adequate. *GFI*, *AGFI*, and *CFI* evaluate, respectively, the degree of misspecification present on the model. Generally, values over .95 and .90 are accepted as good. A value above .95 is considered good for the *CFI* as well. Finally, the Root mean square error of approximation, or *RMSEA*, considers the model complexity when evaluating model fit. The *RMSEA* is considered acceptable, when it is lower than .05. The Chi-square difference between models was employed to compare models with an increasing amount of free parameters. Models were calculated in the software AMOS v.19 using the maximum likelihood estimation function.

Fit statistics of path models are shown in Table 4. The null models described in Table 4 only included covariances between sociodemographic variables, general cognitive abilities, and psychosocial competencies, but no directed path from a variable to another. Further path models are designated by the paths added to them, which were not present in the previous models in the row.

Table 4 reveals that very satisfactory model fit was reached both in the case of mathematics (model MAQ (self-perceived)-Arith, Figure 1(a)) and spelling abilities (model CBCL (ADH)-Spell, Figure 1(b)). Model MAQ (self-perceived)-Arith depicts the effects of general intelligence on mathematics self-perceived performance and mathematics achievement. A large amount of residuals as indicated by the *RMR* cannot be accounted by the variables inserted in the model; however, the nonsignificant Chi-square and the other indices of approximate fit yielded acceptable values for both models MAQ (self-perceived)-Arith and CBCL (ADH)-Spell. Although in both cases the values obtained in the *RMSEA* are slightly above the acceptable levels (Table 3), it may be associated to the low number of degrees of freedom in the model and does not invalidate the results [36]. Importantly, the nonsignificant Chi-square cannot be attributed to a lack of power to detect discrepancies between data and model structure, since the sample size $n = 207$ employed in the present study is sufficient to guarantee enough statistical power.

In the model MAQ (self-perceived)-Arith, attentional deficits and hyperactivity as well as mathematics self-perceived performance also have specific effects on arithmetics performance. Finally, mathematics self-perceived performance is statistically independent from general anxiety and attentional deficits and hyperactivity. Most importantly, however, is that MAQ scores were important in predicting the arithmetic scores, and not at all important for spelling scores.

Model CBCL (ADHD)-Spell depicts the effects of general intelligence on mathematics self-perceived performance and spelling performance. Moreover, attentional deficits and hyperactivity (but not mathematics self-perceived performance) showed specific effects on spelling performance. MAQ self-perceived performance scores were not relevant for predicting the spelling scores. This is indicative that this MAQ subscale is not evaluating self-perceived performance in an unspecific way but rather seems to be specifically related to the domain of mathematics.

4. Discussion

In the present study the association of math anxiety and more general forms of anxiety as well as their specific contribution to arithmetics achievement were examined. Group comparisons revealed that the children between 7 and 12 years old with and without mathematics difficulties show comparable levels of psychosocial competencies and general anxiety, as assessed by the CBCL. As expected, children with mathematics difficulties also present much lower levels of self-perceived performance for mathematics than typically achieving children. Moreover, in typically achieving children, age, school grade, general intelligence, attentional deficits, and hyperactivity as well as self-perceived performance in mathematics contributed independently to explain mathematics achievement. A different pattern of results was observed for spelling performance. Here, general intelligence, attentional deficits and hyperactivity, and general anxiety contributed independently to explain spelling performance. These results indicate that both hyperactivity/inattention as well as mathematics specific forms of self-assessment predict mathematics performance. Moreover, these results show that self-perceived performance for mathematics as measured by the MAQ is specific for mathematics abilities since it does not predict spelling performance. In summary, the results suggest that mathematics performance is associated with different forms of anxiety. Some more general anxiety, which impair performance not only in mathematics but also in other academic abilities such as spelling, and others more specific, which are directly related to self-perceived performance in mathematics. In the following, these results will be discussed in more detail.

MD children show lower levels of self-perceived performance in mathematics when compared to TA. Following the rationale examined by Krinzinger et al. [21], this scale is more associated to self-assessment of and attitudes towards mathematics. Interestingly, the subscales of the MAQ reflecting unhappiness and anxiety for mathematics presented no significant differences between groups. Some recent studies suggest the presence of such differences [37, 38]. Those authors found associations of MA and math performance in early graders with combined measures of MA. Nevertheless, in these studies it is impossible to dissociate between the specific impact of cognitive and affective components of MA on the reported associations such as in the present study, where four separated dimensions of MA can be distinguished. For this reason, the positive results by Ramirez et al. [37] and Wu et al. [38] should be interpreted cautiously, since it is impossible to determine whether the positive differences obtained should be attributed to cognitive, affective, or both components of MA.

None of the CBCL subscales revealed any significant difference between MD and TA children. Moreover, comparatively to TA individuals, children with MD in our study do not exhibit performance differences in aspects of psychosocial functioning such as attention deficit/hyperactivity and general anxiety and math anxiety levels. One possibility is that our participants are still of a relatively young age for these effects to take over. Literature indicates that it takes some time to build up the psychosocial negative consequences of persistent math failure [39]. For instance, Auerbach and colleagues [39] assessed dyscalculic children from 10/11 years of age to the age of 16/17 years. At the age of 16/17 years, dyscalculics exhibited higher mean levels of maladjustment in several CBCL and Youth Self-Report scales, most notably those pertaining to both internalizing and externalizing disorders, while no significant statistical differences between persistent and non persistent dyscalculics were present in the initial assessment at 10/11 years old. These results suggest that at the assessed ages children with mathematics difficulties are not necessarily less adjusted in behavior, cognition, or affect than the average TA children. These findings are in line with fMRI evidence on specific patterns of neural activation related to math anxiety but not to other more general processes [4].

Moreover, a significant association between self-perceived performance and math attainment must be analyzed in the context of the development of self-perceptions in school children. Children may have some difficulty making

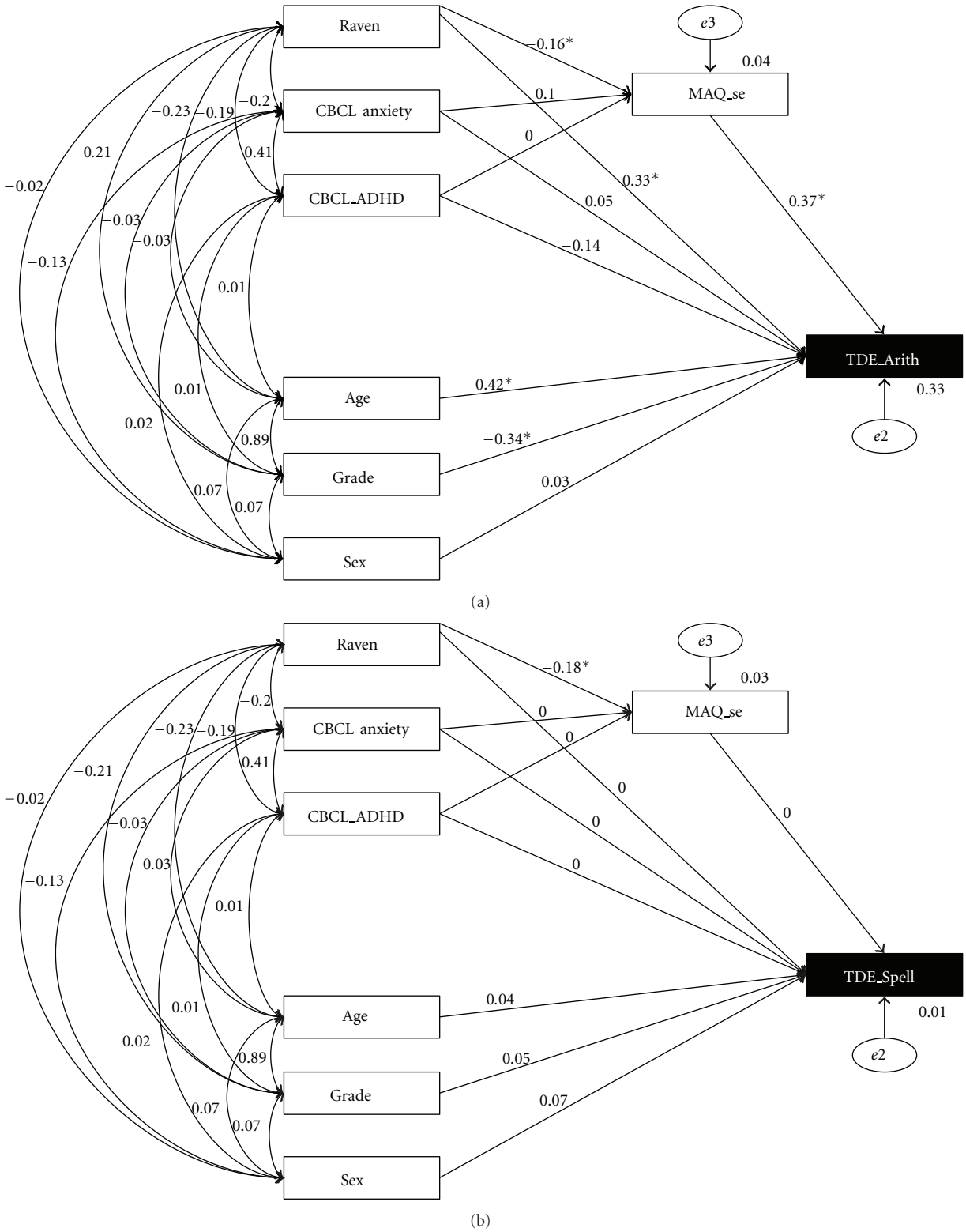

Figure 1: Path-analyses models describing the effects of sociodemographic factors, psychosocial competencies as well as self-perceived performance in mathematics on arithmetics, and spelling performance. Paths marked with * are statistically significant.

realistic assessment of their performance early in the elementary school [40]. For example, in a study by Nicholls [41], correlations between self-assessments and reading attainment raised from virtually nonexistent in the first grade to around 0.70 in the sixth grade (see also [42]). Other evidence indicates that it is not until 11–13 years old that performance begins to systematically benefit from error-related feedback [43]. Our results are, however, more in line with other research showing that from 7 or 8 years onward, children are able to make self-assessments of performance which are more similar to those of their teachers [44, 45]. These results have an important implications for the motivation to learn mathematics. Self-assessment is an important motivational factor [46], which may be an antecedent to more affective aspects of MA observable in high school and college [19, 47], presumably as the math curriculum is more demanding and self-assessments more unfavorable to many students.

The present results regarding the ability of young children to assess their own performance in mathematics are in line with those presented by Krinzinger and coworkers [20]. These authors showed that low calculation ability in the first grade was predictive of "evaluation of mathematics" in the third grade, a composite measure of two MAQ scales: self-perceived performance and attitude. Ma and Xu [19] also found that prior low math achievement in adolescents is predictive of higher levels of MA assessed by questions tapping on more affective components. This discrepancy suggests that it may take some time until the affective consequences of low performance on MA accrue.

Regression analyses conducted in the present study complemented the pattern of results by looking at the predictive value of sociodemographic and general cognitive factors, psychosocial competencies, and mathematics anxiety on arithmetics and spelling achievement. After removing the effects of age, general intelligence, and school grade from data, one still is able to detect specific effects of attentional deficits and hyperactivity as well as of mathematics self-perceived performance on arithmetics performance. This suggests that the impact of hyperactivity and mathematics self-perceived performance on arithmetics performance cannot be reduced to general aspects of cognition, age, gender, or sociodemographic differences in TA children (see also [37, 38]).

The impact of hyperactivity symptoms and mathematics self-perceived performance on arithmetics achievement has very different meanings. Attentional deficits and hyperactivity are associated with lower achievement not only in arithmetics but also in spelling. In the literature, many reports support these results [48, 49]. Interestingly, affective problems and general anxiety also have been associated with lower spelling performance in the present study. These results can be interpreted as a less-specific effect of psychosocial competencies on school performance, since spelling abilities are necessary in almost all topics taught in school. However, we cannot offer a more definitive answer to this question based on our data alone. Moreover, mathematics self-perceived performance impacts on arithmetics performance in a very specific way. The regression analysis of spelling performance revealed no effect of mathematics self-perceived performance on spelling performance. The comparison of this result with the significant effect of mathematics self-perceived performance on arithmetics reveals that there is an effect of mathematics anxiety on arithmetics performance that cannot be reduced to the impact of more general level of anxiety or psychosocial competencies.

Path-analysis models also have corroborated the results of regression analyses that there is an effect of mathematics self-perceived performance on arithmetics achievement. Another important aspect of path-analyses is that the construct of mathematics self-perceived performance was found to be statistically independent from psychosocial competencies in 7–12 years old children. Among the different facets of mathematics anxiety, only the particular aspect of mathematics self-perceived performance was associated to arithmetics performance in a specific way. The perception of cognitive resources, knowledge, and competencies to solve arithmetics problems explained individual differences in arithmetics performance, which cannot be accounted by any specific aspect of general psychosocial competencies. Absence of effects of general anxiety on math performance suggests that the MA is a subject-specific phenomenon, in line with previous research in adolescents and college students [3, 13].

The MA construct as assessed by the MAQ is a complex one, composed of several cognitive and affective components. Lack of association between math achievement and performance on the scale assessing the more affective or dysphoric component of MA deserves explanation. Children with lower achievement in math may be using a coping strategy of "insulating" or detaching themselves from their difficulties in order to resolve internal conflict. Testing of this hypothesis would require further studies focusing on the coping processes employed by MD children. Nevertheless, it is positive news that low mathematics performance may not immediately elicit negative feelings towards mathematics. This may favor early training and support procedures, which may prevent that low mathematics performance triggers a circle of anxiety and avoidance towards mathematics.

5. Conclusions

MA can be reliably assessed in elementary school children and its correlates are specific as no evidence for an early association between general anxiety, literacy, and math performance is present in 7–12 years old children. Moreover, MA comprises both cognitive and affective components. Low mathematics performance may be detected by children already in the early school grades. However, it may take a considerable time for that to increase so much anxiety towards mathematics that more general psychosocial competencies also would be compromised.

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Research Article

Parents' Beliefs about Children's Math Development and Children's Participation in Math Activities

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This study explored associations between parents' beliefs about children's development and children's reported math activities at home. Seventy-three parents were interviewed about the frequency of their children's participation in a broad array of math activities, the importance of children doing math activities at home, how children learn math, parents' role in their children's math learning, and parents' own math skills. Although the sample consisted of African Americans, Chinese, Latino, and Caucasian parents in the United States, the majority were Chinese or Caucasian. Several important findings emerged from this study. Parents' beliefs about math development and their role in fostering it were significantly related to children's math activities. There was important variability and relatively limited participation of children in math activities at home. There were age-related differences in children's engagement in math activities. Chinese and Caucasian parents showed somewhat similar beliefs about how children developed math. Although further research is needed to confirm the findings with a larger sample and to include measures of children's math competencies, these findings are an important step for developing home-based interventions to facilitate children's math skills.

1. Introduction

Many children in the United States do not exhibit adequate math skills [1]. On average, US children earn significantly lower math scores on international assessments such as PISA and TIMSS than children from other industrialized nations [2, 3]. The importance of children's early math skills is well documented. Early math skills continue to have long-lasting effects as children progress through school [1, 4–7]. And math skills have been recognized as important for individual upward mobility and advancing U.S. standing in the global market [1]. Given that there is significant variability in the math skills with which children enter school [4] and that early math skills predict later ones [6], it is critical to identify mechanisms to improve young children's math understanding. One important but relatively understudied mechanism to improve children's math skills during preschool and the early school years is the home-math environment.

Young children's home-based math experiences, including playing games and engaging in everyday math activities, positively predict their math skills ([8–12]; see also [13], for further discussion of the relation between home learning environment and children's academic development). For example, LeFevre and colleagues [10, 11] demonstrated that the frequency with which children in kindergarten through second grade engaged in home-based math activities such as playing board and card games, cooking, and shopping positively predicted their scores on measures of math knowledge and fluency. Ramani and Siegler [14] found that playing board games at home positively predicted children's scores on a measure of number sense.

Although the frequency of engagement in home-based math activities predicts children's math skills, a nontrivial percentage of children reportedly do not engage in much, if any, math activities at home [14, 15]. Ramani and Siegler [14], for instance, found that 20% of middle-income

children and 53% of low-income children did not report playing any math games at home. Tudge and Doucet [15] found 60% of the low- and middle-income preschoolers in their study had no involvement with math-related activities at home. In order to understand the variability in children's engagement in math activities, it is necessary to understand factors that predict such engagement.

This study investigates the relations between parental beliefs about how to foster young children's math development and the frequency of children's reported math activities at home. We provide descriptive information of parental beliefs about the importance of children doing math activities at home, beliefs about how children learn, parents' role in their children's math learning, and parents' own math skills as well as the types of math activities children do at home. We also document similarities and differences in parents' beliefs and children's math activities for younger (prekindergarten and kindergarten) and older children (early elementary school), and for Chinese and Caucasian parents. We then examine the association between the frequency of children's participation in math activities and parents' beliefs.

Our conceptual framework reflects ecological and sociocultural theories, which emphasize the importance of considering the individual and overlapping contexts within which children's development occurs (e.g., [16, 17]), to understand the importance of the home-environment for math development.

A key aspect that influences the context of children's lives according to Super and Harkness [18] is parental beliefs. Parents have specific beliefs about child development that predict the experiences they make available to their children [19] which, in turn, predict children's math development (see also [20–22]). Parents also have ideas about their role in their children's development [19, 23, 24]. These ideas or beliefs reflect parents' cultural heritage and experiences and predict their behavior and practices [21, 25, 26]. Many current theories of children's development emphasize the importance of parental beliefs [27, 28]; however, very little research, with the exception of the following two studies, has investigated the beliefs that parents have about their children's math development and how such beliefs relate to children's engagement in math activities at home.

Simpkins et al. [29] found that parents' socialization of their children's math behavior in elementary school (2nd–5th grades) was related to their children's engagement in out of school activities in math, science, and computer usage. Skwarchuk [12] demonstrated that parents' prior experiences with math were positively related to the frequency of their preschoolers' math activities and math knowledge. Prior experiences were assessed with two questions asking parents to rate how good at math they were in school and whether they found math enjoyable.

An important criticism of Skwarchuk's [12] study is the measure of math activity involved an intervention component; it was not a measure of normative (or daily/routine) math activity because parents were given a bag of math materials and were told to play math-related activities for 10–15 minutes each day. We add to findings from these studies by examining the confluence of different indicators of parental beliefs.

In contrast to research on children's math development, research has examined the relation between children's home-environment and their reading development. Parents who emphasize the importance of engaging their young children's interest in reading have children who choose to read more frequently which, in turn, positively predicts their reading development [30].

The Early Childhood Project [23] was a longitudinal investigation of literacy development with children from diverse income and ethnic/racial backgrounds in prekindergarten through third grade. It provides an impetus for how parents' beliefs about children' math development were assessed in this study. Parents in the Early Childhood Project were asked about the best way to foster their young children's literacy development. Parents' responses were coded for three possible orientations towards literacy development: engaging children's interest/making reading entertaining or enjoyable for the child, inculcating skills, and using daily living activities. Parents' orientation towards engaging young children's interest in reading was related to the types of activities in which children engaged in prekindergarten through third grade. An orientation towards engaging children's interest was also positively related to early literacy and reading competencies. In contrast, an orientation focused on skills inculcation was either not related or negatively related to children's literacy development (see also [31]). Although it seems reasonable to assume that findings of the relation between parents' beliefs and children's reading development will extend to children's math development, it is possible that home environments could have a differential impact on the two domains, given that reading and math require different skill sets.

Findings from the Early Literacy Project also showed the changing nature of children's home-based activities as they get older and become better readers [23]. Research also shows that parents' beliefs assessed when children are in preschool continue to predict their development in elementary school [13, 23]. However, to our knowledge, no research on math development has specifically investigated differences in parents' beliefs about their children's learning or their role in such learning for different age groups of children. This study compares similarities and differences in parents' beliefs and children's activities for younger and older age groups of children.

Another important issue when studying the home-learning environment of children is to take into account the interaction between cultural manifestations and responses to the larger social structure. As Super and Harkness [32] argue, the home-learning environment that parents create for their children includes both culturally defined and transmitted messages. Harkness and Super [33] argue that culture impacts family functioning and child development at the proximal and distal levels through 3 mechanisms: *physical and social settings* (structure and organization of the home plus individuals who interact with child), *culturally specific childcare customs* (common parenting behaviors that are well integrated into the larger structure), and *ethnotheories*

of parenting (specific beliefs about significance and role, expectations about behaviors and development, and nature and needs of children). These three factors, physical and social settings, culturally specific child care customs, and ethnotheories of parenting comprise the developmental niche [18]. Individual differences in the developmental niche reflect differences in ethnic/cultural background and income and are related to differences in children's development [34].

Particularly relevant for this study are the cultural differences between Asian (Chinese) and Western (Caucasian) parents' beliefs and socialization practices. Such differences may account, at least in part, for differences in children's developmental outcomes. It is a well-documented finding that Chinese children display higher math skills from the outset of schooling than Caucasian children (e.g., [35, 36]). Asian parents consider schooling one of the most important responsibilities of parents, strongly believe that without a solid education a person cannot be successful in life [37], and clearly articulate their educational expectations [38]. Such a strong emphasis on education may not be found among Western parents.

Another important belief commonly found among Asian parents is related to the relation between academic success/learning and children's ability/effort. Asian and Asian American parents consider hard work and effort as the key for academic success [39]. In contrast, Caucasian parents consider ability as a very important determinant of academic success, which implies a more deterministic view of academic success [40]. These beliefs have important consequences for Asian parents' socialization practices. This study compares and contrasts Chinese and Caucasian parents' socialization beliefs and practices.

In short, the present study investigates relations between parents' beliefs about how to facilitate their children's math development and the frequency of children's math activities at home. We considered four sets of parental beliefs: the importance of children doing math activities at home, how children learn math, parents' role in their children's math learning, and parents' own math skills. How children learn math included three possible orientations: engaging child's interest, inculcating skills, and using daily living activities. Note that we assess parents' beliefs in a more comprehensive manner than has been done in the limited prior research and with a more ethnically/racially diverse sample. We also assess relations among parents' beliefs and children's involvement in math activities.

2. Method

2.1. Participants. Families were recruited from local schools and preschools, Chinese schools and churches, and community centers in a large city in the Middle Atlantic section of the United States. Data were collected in two waves, during the spring and summer of 2010 (Wave 1) and during the fall of 2011 and spring of 2012 (Wave 2). Data from the first wave included families from four ethnic/racial groups, data from Wave 2 were collected to allow comparisons between Chinese and Caucasian families. On average, participants in both waves of data were comparable in their demographic characteristics except for their average levels of education. As expected, Wave 2 participants were significantly more educated (Mean = 5.50, SD = 0.71; college completion/graduate school) than Wave 1 participants (Mean = 4.30, SD = 1.89; associate degree), $F(1, 52) = 11.68$, $p = .001$, partial eta squared = .183. Accordingly, data from both waves of participants were combined in all analyses.

Table 1 shows demographic characteristics of the 73 participants in this study. All participants were mothers of children in prekindergarten through fourth grade; the majority of the children were in prekindergarten through second grade. About 68% of the children lived in dual parent households. Participants came from four racial/ethnic groups: African American, Chinese, Latino, and Caucasian. The mean age of the participants was 36.58 years (SD = 8.06).

All of the Chinese parents and 80% of the Latinos in our sample were first generation (foreign-born) immigrants. Chinese and Latino immigrants had lived in the Unites States for an average of 9.01 years (SD = 5.39). Consistent with U.S. population statistics, Chinese parents were the most educated; about 93% completed at least college in contrast to 44% of African American, 62% of Caucasian, and 20% of Latino parents.

2.2. Task and Procedure. The Parents' Conceptions of Math Development (PCMD) questionnaire was used to assess parents' beliefs and reported frequency of children's engagement in math activities. It included open-ended questions and rating scales about parents' metacognitions about math, the importance of home-based math engagement for children, parents' views of how children learn math, parents' roles in such learning, parents as role models of math engagement, and the frequency of children's math engagement across a broad base of theoretically-relevant math activities. Versions of the questionnaire were available in English, Mandarin, and Spanish. Mandarin and Spanish questionnaires were first prepared in English, and then translated and back-translated to ensure linguistic validity.

A trained graduate or advanced undergraduate research assistant administered the questionnaire in the parent's preferred language. Completion of the questionnaire took about 30–40 minutes. The research assistant took notes and audio-taped the interviews.

Tapes were transcribed and then reviewed by the interviewer and another member of the research team. Questionnaires conducted in Mandarin or Spanish were transcribed in that language, translated into English, and then back-translated by a third assistant. The two versions were compared and any inconsistencies were discussed among the transcriber and translators.

Coding was based on the written transcript with the oral tapes and field notes consulted as needed.

2.3. Key Variables. The key variables of interest in this study were parents' beliefs and frequency of children's math activities.

Table 1: Demographic characteristics of sample.

	Overall $N = 73$	Chinese $N = 28$	Caucasian $N = 26$	African American $N = 9$	Latinos $N = 10$
Boys (%)	56.2	60.7	65.4	33.3	40
Mean age (SD)	6.12 (1.60)	6.26 (1.84)	5.83 (1.83)	6.10 (1.28)	6.24 (1.62)
Children's grade (%)					
Prekindergarten	23.3	21.4	26.9	22.2	20
Kindergarten	31.5	42.9	19.2	33.3	30
First grade	26.0	17.9	38.5	22.2	20
Second grade	6.8	3.6	11.5	11.1	0
Third grade	11.0	14.3	0	11.1	30
Fourth grade	1.4	0	3.8	0	0
Two-parent homes (%)*	68.1	85.7	61.5	33.3	66.7
Number of other children in home	1.07 (0.93)	0.75 (0.65)	1.27 (0.96)	1.44 (1.51)	1.10 (0.74)
Mean (SD)					
0%	27.4	35.7	19.2	33.3	20
1	49.3	53.6	50	33.3	50
2	13.7%	10.7	15.4	0	30
3	8.2	0	15.4	22.2	0
4	1.4	0	0	11.1	0
Child fluent in English (%)	56.5	44.4	100	100	50
Parent education	4.55 (1.72)	5.54 (0.74)	4.54 (1.68)	3.78 (1.92)	2.50 (1.58)
Mean level (SD)					
Less than HS (%)	8.2	0	11.5	0	30
High school	12.3	0	3.8	44.4	40
Some college	4.1	3.6	3.8	11.1	0
Associate	9.6	3.6	19.2	0	10
Bachelor	23.3	28.6	23.1	11.1	20
Postgraduate	42.5	64.3	38.5	33.3	0
Parent U.S. born (%)	49.3	0	96.2	100	20

Note. *One Hispanic parent did not provide data for this question.

2.3.1. Parents' Beliefs. Four types of beliefs were assessed: importance of children doing math activities at home, beliefs about how children learn, parents' role in their children's math learning, and parents' own math skills.

Importance of Children Doing Math Activities at Home. Parents were asked, "How important is it that your child does math activities at home?" Responses could range from 1 (not very) to 5 (very important).

Beliefs about How Children Learn Math. Parents were asked, "What is the best way to help your child learn math?" Responses to the open-ended question were reliably coded as focusing on entertaining or engaging the child ("to play math games with him" or "do an activity that will hold her attention. . ."), developing skills ("count numbers with him" or "sitting down and showing her the numbers on paper"), or using daily living activities ("taking advantage of everyday activities that incorporate math in them" or "counting cars on the road when we drive to the store"). Each of these beliefs are binary variables (1 = it was mentioned during the conversation, 0 = was not mentioned). These beliefs were not mutually exclusive; parents' responses could include more than one focus. Reliability of coding was based on two raters independently coding 16 of the interviews. Kappas ranged from .875 to 1.00.

In reviewing the transcripts of the interviews, it became apparent that parents' discussion of their beliefs about how children learn extended beyond their response to a specific question. Therefore, we complemented response to the previous question with information from the entire transcript. We call responses that emerged from review of the entire transcript "themes" to contrast them with responses to specific questions. We coded a theme of how children learn math with the same coding scheme (described in prior paragraph) as for the specific question. All responses were reviewed by two coders. These coders had established adequate reliability for the themes using a subset of 20 interviews. Kappas for each of the orientation themes were 1.00. Because the themed responses were based on more

information than the response to a specific question, and because themed responses were correlated with responses to a specific question (.33 skills, .53 daily living, .58 engagement), the analyses presented in the paper include only the themed responses.

Parents' Role in Their Children's Math Learning. There were three separate roles parents could take: they serve as role models of math engagement, they provide children with artifacts, and they provide instructions. These roles were not mutually exclusive; parents could play more than one role.

(1) *Serve as role model of math engagement.* Parents were asked to rate on a 4-point scale (1 = never/almost never through 4 = everyday/almost every day) "How often does your child see you engage in math activities?"

(2) *Provide artifacts.* Responses from the entire transcript were reviewed to identify whether parents discussed providing artifacts for their children as a part of the way they socialized their children's math skills. Two examples of remarks consistent with this role include, "We got him a math book," and "She has a counting book." A parent received a code of 1 if she mentioned the theme and 0 if she did not. The kappa for this theme was 1.00.

(3) *Active involvement in children's math learning.* Parents were asked to rate, "How important is it that you help your child with math?" Responses could range from 1 (not very) to 5 (very).

Parents' Own Math Skills. Parents were asked, "How good at math are you?" and "How much do you enjoy math?". Ratings on each question ranged from 1 (not good at all) to 5 (very good).

2.3.2. Frequency of Children's Math Activities. The frequency of children's math activities was assessed using two indicators, overall math activity and a composite indicator based on specific math activities.

Overall Math Activity. Parents were asked about their children's overall math activity, "How often does your child engage in math activities?" Ratings were 0 (never, almost never), 1 (less than once a week), 2 (once a week to several times a week), and 3 (everyday/almost every day).

Specific Math Activities. Parents were asked to rate the frequency of children's participation in a broad-range of 27 specific math-relevant activities (e.g., counting, playing board games, and playing jigsaw puzzles). We used the same rating scale (from 0 to 3) as with the previous indicator of overall math activity. Scores on these specific math items were averaged. The items comprising the specific math activities composite showed good internal consistency (Cronbach's alpha of .80).

2.3.3. Covariates. Mothers' educational level, a proxy for socioeconomic status, was included as a control variable in the statistical analyses. Educational levels were coded into six categories: 1 = less than high school degree, 2 = high school graduate, 3 = some college, vocational or technical school, 4 = associate degree, 5 = bachelor degree, and 6 = postgraduate degree. Although income and maternal education are separate factors, they are highly related [41]. In fact, several researchers note that much of the differences in children's educational outcomes or factors related to their education are predicted by maternal education rather than income. For example, Suizzo and Stapleton [42], using a large U.S. nationally representative sample, found that maternal education was the strongest predictor of parental involvement. Income was not a significant predictor when maternal education was included as a predictor (see also [43], for further analysis of the role of income and education).

The data in this study revealed systematic group-based differences in parents' education. Chinese parents had significantly higher educational levels than Caucasians who, in turn, had significantly higher educational levels than Latinos, $F(1,71) = 12.48$, $p = .001$. African Americans' educational levels fell between Caucasians and Latinos but did not differ significantly from either group ($p > .10$). There were no significant differences in the educational levels between parents of younger and older children, $F(1,71) = 1.56$, $p = .216$. Preliminary correlational analyses found that parents' educational level was significantly related to several parental beliefs: importance of assisting child with math activities, $r(72) = .29$, $p = .013$, and children learn math through a skills-based focus at home, $r(72) = -.24$, $p = .04$. There also was a borderline effect for parents' self-reported math skills, $r(72) = .21$, $p = .069$.

3. Results

This section begins with unadjusted descriptive information (no controls for mother's educational level) of parents' beliefs about their children's math development for the entire sample. We then compare beliefs of parents of younger (prekindergarten, kindergarten) and older children (first grade and older). We also present ethnic/racial comparisons between the Chinese and Caucasian parents; comparisons for African Americans and Latinos are not included because of their relatively small sample size. We next present information about children's reported math activities following the same strategy as we did for the analysis of parental beliefs. Finally, we analyze relations between parents' beliefs and the frequency of children's math activities for the entire sample. We control for mother's educational level in all analyses (correlations, analysis of covariance, and regressions) except for descriptive information.

3.1. Parents' Beliefs about Children's Math Development

3.1.1. Importance of Children Doing Math Activities at Home. On average, parents highly endorsed the importance of children doing math at home (Mean = 4.51, SD = .77). Eighty-six percent gave scores of 4 or 5 suggesting that

parents considered doing math at home important (or very important); only 14% considered doing math at home as somewhat or not important (scores of 2 or 3).

3.1.2. Beliefs about How Children Learn Math. We asked parents about the best ways to help their children learn math. Most parents reported engaging children's interest (73%) and skills inculcation (77%) as the best way to help their children learn math. About half (56%) mentioned involvement in daily living activities.

3.1.3. Parents' Role in Their Children's Math Learning. As noted below, parents generally reported being actively involved in their children's math learning.

Role Models. There was variability in how often children reportedly observed their parents do math activities. Although about half the parents (48%) reported that their children observed them engage in math activities every day or almost every day, 29% of the children observed their parents do math activities less than once a week (Mean = 3.10, SD = 1.03). Of those children who did observe their parents engage in math activities, they typically observed them participate in daily living activities, such as cooking, paying bills and bank-related matters, and going food shopping.

Provide Artifacts. Seventy-four percent of the parents said they provided their children with math artifacts. Artifacts included various math books or workbooks, games, and calendars.

Active Involvement. Almost all parents reported that it was important to assist their children with math. Eighty-six percent gave ratings of 4 or 5 to this question; 12% gave ratings of 3 (Mean = 4.53, SD = .77).

3.1.4. Parents' Own Math Skills. Almost half of the parents (46.5%) considered themselves good or very good at math (scores of 4 or 5); a similar percentage rated themselves as "ok" (score of 3), (Mean = 3.55, SD = 0.99). Similarly, almost half the parents rated themselves as enjoying math (scores of 4 or 5) and about a third (38.4%) reported only enjoying math somewhat (score of 3) (Mean = 3.41, SD = 1.15). Parents' rating of their enjoyment of math was strongly correlated with their ratings of how good at math they were, $r(70) = .63$, $p = .001$.

3.1.5. Confluence of Parents' Beliefs. Parents have a set of beliefs about how children develop and their role in such development. Therefore it is important to consider interrelations among the different types of beliefs. All of the correlational analyses controlled for maternal level of education.

Parents who emphasized the importance of children doing math at home endorsed the importance of helping their children with math, $r(70) = .54$, $p = .001$. Parents who emphasized children's engagement in learning reported providing artifacts, $r(67) = .44$, $p = .001$.

Parents who emphasized the importance of daily living activities for their children's learning also reported that their children more frequently saw them do math, $r(70) = .34$, $p = .003$, emphasized the importance of helping their children with math, $r(70) = .24$, $p = .045$, and gave higher ratings to enjoying math (borderline significance), $r(70) = .23$, $p = .056$.

Parents who endorsed a skill's orientation also endorsed the importance of their children doing math at home, $r(67) = .29$, $p = .017$ and helping their children with math, $r(67) = .33$, $p = .006$. There was a negative relation between endorsing a skill's orientation and how good at math parents reported they were, $r(67) = -.26$, $p = .033$, and a borderline negative relation between a skill's orientation and how much parents enjoyed math, $r(67) = -.22$, $p = .073$.

3.2. Parents' Beliefs about Math Development, by Children's Age Group. A series of analyses of covariance (ANCOVAs) and logistic regressions were conducted to compare the four categories of parents' beliefs (the importance of children doing math activities at home, how children learn, parents' role in their children's math learning, and parents' own math skills) between parents of younger and older children. ANCOVAs were conducted when the dependent measures were rating scales and logistic regressions were used with binary outcomes. All analyses included mothers' educational level as a control variable.

As Table 2 indicates none of the analyses were statistically significant suggesting that parents of children in prekindergarten through early elementary school have similar beliefs about how children learn math and their role in such learning.

3.3. Chinese and Caucasian Parents' Beliefs about Children's Math Development. Percentages included in this section are unadjusted; ANCOVAS and logistic regression analyses include mothers' educational level as a control variable. Therefore, all reported means are adjusted for mothers' educational level. Table 3 reports coefficients and analyses for comparisons between Chinese and Caucasian parents' beliefs.

3.3.1. Importance of Children Doing Math Activities at Home. Approximately 79% of Chinese parents and 88% of Caucasian parents reported that doing math activities at home was important or very important. Although both Chinese and Caucasian parents emphasized the importance of children doing math activities, Chinese parents (Mean = 4.19, SE = .16) gave significantly lower ratings than Caucasian parents, (Mean = 4.72, SE = .16).

3.3.2. Beliefs about How Children Learn Math. There were no statistically significant differences between Chinese and Caucasian parents in the endorsement of engaging their children's interest, Wald $\chi^2(N = 73, 1) = 0.47$, $p = .495$, involvement in daily living activities Wald $\chi^2(N = 73, 1) = 0.21$, $p = .648$ and skills inculcation as the best way to

Table 2: Beliefs of parents by age group of children.

	B	Statistical test	P
Importance of children doing math at home[1]	0.09	$F(1, 70) = 0.23$	.632
Beliefs about how children learn math[2]			
Engagement	1.13	Wald $\chi^2(N = 73, 1) = 0.05$	.818
Skills	2.13	Wald $\chi^2(N = 73, 1) = 1.52$	.218
Daily living	0.76	Wald $\chi^2(N = 73, 1) = 0.32$	.572
Parents' role in their children's math learning			
Role models[1]	−0.08	$F(1, 70) = 0.10$	.750
Provide artifacts[2]	1.26	Wald $\chi^2(N = 73, 1) = 0.21$	.649
Active involvement[1]	0.27	$F(1, 70) = 2.44$	.122
Parents' own math skills[1]			
How good parents are at math	0.11	$F(1, 70) = 0.23$	.634
How much parents enjoy math	0.09	$F(1, 70) = 0.10$	.756

Note. Parents' highest level of education was used as a covariate in all analyses. There were 40 parents of younger and 33 parents of older children included in analyses.
[1]ANCOVA parameter estimates reported for comparisons between groups.
[2]Logistic regression odds ratios reported for group predicting endorsement of each belief.

Table 3: Statistical analyses for chinese and caucasian parents' beliefs about children's math development.

	B	Statistical test	P
Importance of children doing math at home[1]	0.52	$F(1, 51) = 5.06$	.029
Beliefs about how children learn math[2]			
Engagement	1.58	Wald $\chi^2(N = 54, 1) = 0.47$	.495
Skills	2.74	Wald $\chi^2(N = 54, 1) = 2.12$	.146
Daily living	0.76	Wald $\chi^2(N = 54, 1) = 0.21$	.648
Parents' role in their children's math learning			
Role models[1]	0.64	$F(1, 51) = 5.77$	.020
Provide artifacts[2]	2.56	Wald $\chi^2(N = 54, 1) = 2.16$	.142
Active involvement[1]	−0.12	$F(1, 51) = 0.24$	.625
Parents' own math skills[1]			
How good parents are at math	0.18	$F(1, 51) = 0.49$	.487
How much parents enjoy math	0.20	$F(1, 51) = 0.39$	.533

Note. Parents' highest level of education was used as a covariate in all analyses. There were 28 Chinese and 26 Caucasian parents included in analyses.
[1]ANCOVA parameter estimates reported for comparisons between groups.
[2]Logistic regression odds Ratios reported for group predicting endorsement of each belief.

help their children learn math Wald $\chi^2(N = 73, 1) = 2.12$, $p = .146$. On average, parents regardless of ethnicity/race endorsed these orientations.

3.3.3. Parents' Role in Their Children's Math Learning

Role Models. Caucasian parents (Mean = 3.46, SE = .19) were significantly more likely than Chinese parents (Mean = 2.82, SE = .18) to report that their children saw them engage in math activities, $F(1, 51) = 5.77$, $p = .02$, partial eta squared = .10.

Provide Artifacts. Differences between Chinese and Caucasian parents in providing artifacts to their children were not statistically significant, Wald $\chi^2(N = 73, 1) = 2.16$, $p = .142$.

Active Involvement. There were no statistically significant differences in how strongly Chinese (Mean = 4.47, SE = .17) and Caucasian parents (Mean = 4.35, SE = .17) rated the importance of assisting their children with math at home, $F(1, 51) = .24$, $p = .625$, partial eta squared = .005.

3.3.4. Parents' Own Math Skills. There were no significant differences in how good Chinese (Mean = 3.43, SE = .17) and Caucasian (Mean = 3.61, SE = .16) parents believed they were in math, $F(1, 51) = 0.49$, $p = .487$, partial eta squared = .01. There also were no significant differences in how much Chinese (Mean = 3.25, SE = .22) and Caucasian parents (Mean = 3.49, SE = .22) reported enjoying math, $F(1, 51) = 0.39$, $p = .533$, partial eta squared = .008.

3.4. Frequency of Children's Engagement in Math Activities. As Table 4 indicates, on average, parents' reported that their children engaged in math activities between several times a week and almost every day (Mean = 2.49; SD = .60) based on an overall measure of math activities. About half

Table 4: Mean frequency of children's math activities by age group.

	Overall $N = 73$	Younger $N = 40$	Older $N = 33$	P
Composite math activities				
Overall indicator	2.49	2.57	2.40	.222
Average based on specific math activities	1.48	1.42	1.55	.138
Specific math activities				
Count objects	2.42	2.72	2.07	<.001
Answer/ask questions "How many are things are there?"	2.21	2.31	2.08	.324
Write numbers	2.18	1.99	2.42	.047
Use TV remote	2.07	1.87	2.31	.088
Add/subtract objects	2.01	1.68	2.41	<.001
Match/identify shapes	2.01	2.38	1.57	<.001
Tell time on a clock	1.86	1.59	2.20	.021
Play with blocks/construction toys	1.73	1.87	1.56	.210
Use a computer	1.73	1.38	2.15	.003
Make patterns with beads/blocks	1.50	1.61	1.39	.356
Play with money	1.49	1.50	1.48	.924
Play with puzzles	1.42	1.73	1.05	.001
Do math homework	1.41	0.82	2.12	<.001
Put objects in order	1.38	1.51	1.24	.260
Use math workbooks	1.36	1.01	1.78	.001
Dial telephone	1.33	1.40	1.25	.568
Play math games/board games	1.29	1.21	1.39	.374
Play video games	1.26	0.79	1.83	<.001
Use calendars	1.25	0.82	1.77	<.001
Play card games	1.22	1.22	1.21	.964
Measure things	1.18	1.08	1.29	.344
Watch math TV programs	1.18	1.48	0.81	.008
Keep score in games	1.15	0.92	1.43	.042
Look at math books	1.12	1.19	1.05	.583
Jump rope/play hop scotch games	1.00	0.97	1.04	.762
Use math flashcards	0.69	0.74	0.63	.632
Use maps	0.42	0.35	0.52	.314

Note. Younger group includes children going into prekindergarten and kindergarten; the older group includes children going into first grade and above. Means for younger and older age group are adjusted for mothers' educational level. Overall means are not adjusted. Significance is based on results of an ANCOVA, controlling for mothers' education level.

of parents reported that their children participated in math activities every day or almost every day.

In contrast to parents' reports of the frequency of overall math activity, when we averaged the frequency of involvement across specific math activities, parents reported lower levels of children engagement (between less than once a week and one to several times a week; Mean = 1.48; SD = .39). Very few (about 1.5%) of the children reportedly engaged in math activities every day. As is apparent from the two means, parents' estimates of the frequency of their children's overall engagement in math activities was significantly higher than the average of reported engagement in specific activities, $t(72) = 15.46$, $p = .001$. Results presented in the following sections are based on the average of the specific math activities composite rather than the overall composite question because we think it is a more accurate reflection of children's frequency of engagement.

Table 4 shows important variability in the level of children's reported engagement across math activities. The most commonly occurring activities (mean of 2.00 or higher, once to several times a week) are counting objects, asking/answering questions about quantity, using the television remote, and writing numbers. Lowest levels of reported engagement are found for playing board or card games or watching math video games.

3.5. Younger and Older Children's Reported Engagement in Math Activities. There were some significant differences between children in the two age groups in the frequency of engagement in specific activities (see Table 4). As expected, younger children were more likely to be involved than older children in basic math activities. The specific activities that younger children more frequently reportedly did were count objects, $F(1,70) = 13.34$, $p = .001$, partial eta squared = .168; match or identify shapes, $F(1,70) = 14.13$, $p = .001$, partial eta squared = .168; play with puzzles, $F(1,70) = 12.01$, $p = .001$, partial eta squared = .147; and watch math television programs, $F(1,70) = 7.43$, $p = .008$, partial eta squared = .096. In contrast, older children significantly more frequently engaged in activities such as add/subtract things, $F(1,70) = 14.64$, $p = .001$, partial eta squared = .173; write numbers $F(1,70) = 4.09$, $p = .047$, partial eta squared = .058, do homework (assigned by teachers), $F(1,70) = 24.33$, $p = .001$, partial eta squared = .266; use math workbooks, $F(1,70) = 12.03$, $p = .001$, partial eta squared = .149 keep score in games, $F(1,70) = 4.27$, $p = .042$, partial eta squared = .057; use calendars, $F(1,70) = 13.52$, $p = .001$, partial eta squared = .162; tell time, $F(1,70) = 5.61$, $p = .021$, partial eta squared = .074; play video games, $F(1,70) = 19.22$, $p = .001$, partial eta squared =.215; use a computer, $F(1,670) = 9.72$, $p = .003$, partial eta squared = .122.

3.6. Chinese and Caucasian Children's Reported Frequency of Engagement in Math Activities. There were significant differences between Chinese and Caucasian children's reported frequency of engagement for some math activities (see Table 5). Caucasian children reportedly engaged more frequently in the following math-related activities: counting objects, $F(1,51) = 4.43$, $p = .040$, partial eta squared = .080; playing with or using money, $F(1,51) = 10.27$, $p = .002$, partial eta squared = .168; matching or identifying shapes, $F(1,51) = 4.27$, $p = .044$, partial eta squared = .077; ordering objects, $F(1,51) = 6.01$, $p = .018$, partial eta squared = .105; measuring things, $F(1,51) = 7.81$, $p = .007$, partial eta squared = .133; playing math games/board games, $F(1,51) = 4.76$, $p = .034$, partial eta squared = .085; keeping score in games, $F(1,51) = 9.59$, $p = .003$, partial eta squared = .158.

Table 5: Mean frequency of children's math activities.

	Overall $N = 73$	Chinese $N = 28$	Caucasian $N = 26$	P
Composite math activities				
Overall indicator	2.49	2.41	2.64	.154
Average based on specific math activities	1.48	1.34	1.53	.091
Specific math activities				
Count objects	2.42	2.10	2.58	.040
Answer/ask questions "How many are things are there?"	2.21	2.16	2.02	.628
Write numbers	2.18	2.48	2.00	.057
Use TV remote	2.07	1.83	2.03	.551
Add/subtract objects	2.01	1.91	2.22	.215
Match/identify shapes	2.01	1.73	2.30	.044
Tell time on a clock	1.86	1.63	1.78	.655
Play with blocks/construction toys	1.73	1.65	1.95	.288
Use a computer	1.73	1.90	1.76	.669
Make patterns with beads/blocks	1.50	1.38	1.52	.634
Play with money	1.49	0.95	1.86	.002
Play with puzzles	1.42	1.46	1.20	.239
Do math homework	1.41	1.27	1.64	.309
Put objects in order	1.38	1.00	1.62	.018
Use math workbooks	1.36	1.54	1.04	.082
Dial telephone	1.33	1.16	0.98	.507
Play math games/board games	1.29	1.16	1.64	.034
Play video games	1.26	1.30	1.26	.911
Use calendars	1.25	1.09	1.40	.374
Play card games	1.22	1.08	1.45	.127
Measure things	1.18	0.73	1.41	.007
Watch math TV programs	1.18	1.01	0.99	.937
Keep score in games	1.15	0.68	1.54	.003
Look at math books	1.12	1.13	0.90	.424
Jump rope/play hop scotch games	1.00	0.89	0.89	.991
Use math flashcards	0.69	0.51	0.75	.373
Use maps	0.42	0.44	0.60	.481

Note. Means for Chinese and Caucasian parents are adjusted for mothers' educational level. Overall means are not adjusted. Overall means represent means for all ethnic/racial groups.

Chinese children reportedly engaged more frequently (borderline effects) in writing numbers, $F(1, 51) = 3.80$, $p = .057$, partial eta squared $= .071$, and using workbooks, $F(1, 51) = 3.15$, $p = .082$, partial eta squared $= .058$.

3.7. Associations between Parents' Beliefs and Frequency of Children's Math Activities. We first calculated partial correlations between parents' beliefs and children's math activities to determine which beliefs to include in a regression model. Given the small sample size, it was important to limit the number of predictors in the regression models. Accordingly, we included only those predictors that were significantly correlated with frequency of children's math activities. Correlation and regression analyses controlled for mothers' educational level.

Table 6: Summary of multiple regression analysis for parent beliefs predicting frequency of math activities at home.

Variable	B	SE (B)	β	t	P
Intercept	0.67	0.27		2.46	.016
Mothers' education level	−0.04	0.02	−.16	−1.54	.135
Importance of children doing math at home	0.13	0.06	.25	2.35	.022
Parents' role—role models	0.10	0.04	.26	2.28	.026
How children learn math—daily living	0.16	0.09	.21	1.82	.073

Note. $R^2 = .262$, $F(4, 68) = 6.05$, $P < .001$.

The frequency of children's math activities (composite measure based on average of specific math activities) was significantly related to three parent beliefs: the importance of children doing math activities at home, $r(70) = .35$, $p = .003$; the importance of involving children in daily living activities, $r(70) = .34$, $p = .003$; the frequency with which they saw their parents do math activities (parents as role models), $r(70) = .38$, $p = .001$. On average, children of parents who consider it important to have their children do math activities at home, see themselves as role models, and involve children in daily living math activities are more frequently involved in math activates at home.

It is noteworthy that neither parents' enjoyment of math nor how good they are at math was significantly related to the frequency with which children did math at home, $r(70) = .15, .18$, $p = .21, .14$, respectively. Skills inculcation for helping their children learn math was not significantly related to the frequency with which children participated in math activities at home, $r(70) = .001$, $p = .993$.

Table 6 shows the regression analysis with parents' beliefs as predictors and the composite measure of engagement in math activities as the dependent variable. Parents' ratings of the importance of children doing math at home, and parents' ratings of the frequency with which their child sees them do math activities were both significantly related to the frequency with which children reportedly did math at home. Parents' endorsement of involving their children in daily living activities as a means of learning math showed a borderline relation with children's engagement in math activities. These results show the importance of parents' beliefs for increasing the frequency of children's engagement in math activities.

4. Discussion

Far too many children in the United States do not become competent in math [1]. This exploratory study examined the math home-environment of children by analyzing parents' beliefs and children's involvement in math activities. Given the strong connection between children's involvement in

math activities at home and their math learning [8, 10, 11], we wanted to provide some insights into the basis of the variability in children's home-based involvement in math activities. Understanding the reason for the variability in the frequency with which children participate in math activities at home is an important first step towards neutralizing the math disadvantages experienced by a significant number of children.

Five important findings emerged from this study: the association between parents' beliefs about children's math development and the frequency of children's engagement in math activities at home, the relation among parents' beliefs about children's math development, the frequency and nature of children's math activities, age-related patterns in children's engagement in math activities, and comparisons between Chinese and Caucasian parents' beliefs and children's math activities.

One, these results highlight the importance of considering parents' beliefs in any model of children's math development [19]. The frequency with which children reportedly participated in math activities was related to the frequency with which children observed their parents do math activities [29]. It also was related to parents' beliefs about using daily living activities to foster math learning, and to parents' beliefs about the importance of children doing math at home.

There is an interesting difference between parents' beliefs about math, as documented in this study, and reading development, as documented in Serpell et al. [23]. In contrast to Serpell et al.'s [23] findings, where the importance of using daily living activities to help their children learn to read was not mentioned, more than half of the parents in this study mentioned that involvement in daily living activities is an important means of fostering children's math development. The difference in patterns between reading and math underscores the need to more fully investigate children's home-math environments. We cannot just assume that how parents foster their children's reading competencies will apply to how they foster math competencies.

Two, this study demonstrates the importance of considering the relations among different dimensions of parents' beliefs. We considered four sets of parental beliefs: the importance of children doing math activities at home, how children learn math (through engaging child's interest, inculcating skills, and using daily living activities), parents' role in their children's math learning (serve as role models of engagement, provide artifacts, and active involvement in children's math learning), and parents' own math skills (self-rated skills and enjoyment). By considering different aspects of parents' beliefs, we were able to document relations among the beliefs. Such documentation is necessary to obtain a more valid understanding of parents' beliefs about children's math development. Only with a valid understanding will we be able to design interventions that are effective for a diverse population of children.

Note that parents' enjoyment of math and their self-rated ability in math were not significant predictors of the frequency with which children engaged in math activities. The lack of association between mothers' perceptions of their own ability and enjoyment and children's math engagement may reflect the simplicity of math activities that children, on average, are involved in at this age. As children become involved with more complicated math activities, these particular beliefs may become relevant.

Consider the associations among the beliefs held by parents. Parents who emphasized the importance of children doing math at home also emphasized the importance of helping their child with math, and, in particular, that parents teach their children math. Parents who emphasized the importance of daily living activities for their child's learning also reported that their child more frequently saw them do math. Parents who emphasized the importance of helping their children with math tended to report that they enjoyed math.

Three, this study confirmed the variability and relatively limited participation of children in math activities at home [15]. About 44% reported mean engagement in math activities (based on the average of activities) between once and several times a week. About 53% reported less frequent engagement in math activities. Although we do not know what is the minimum involvement needed to acquire or improve math skills, we suspect that the limited participation reported by the children in this study is not optimal.

Four, these results also add to the literature by showing age-related differences in children's reported engagement in math activities. Differences between younger and older children in the frequency of reported engagement were found in specific activities. Not surprisingly, younger children were more likely to reportedly engage in activities likely to foster counting and basic math skills; older children reportedly engaged in activities that fostered more advanced skills (addition and subtraction) and/or required more advanced competencies (using calendars and playing video games). However, parents' beliefs about how to foster math were not sensitive to the differences in the two age groups of children. It is possible that differences would have emerged if we had used a wider age span. Alternatively, the beliefs tapped in this study may be stable and not subject to change as children go from prekindergarten to elementary school.

Five, these findings also add to the body of research on Chinese and Caucasian children's math development. In contrast to previous findings [44]; (Li [45]), Chinese and Caucasian parents in this study showed somewhat similar beliefs about how children's developed math. When there were differences between groups, Caucasian parents reported higher frequency of being role models for their children and placed more emphasis on the importance of their children doing math at home.

We propose five possible explanations for these findings with Chinese and Caucasian parents in this study. The first three explanations focus on changing acculturation patterns. The next two focus on potential limitations of what was explored in this study. First, Chinese immigrants in this study mainly lived in the suburbs and did not benefit from living in strong ethnic enclaves (and sources of educational support) unlike other Chinese immigrants in the United States who are more likely to live in cities [46]. Consistent with the first option, the Chinese parents included in this study may be more culturally assimilated

to the Caucasian mainstream culture than were previous generations of Chinese immigrants or those who participated in other studies. Third, given globalization trends, it may be possible that the beliefs and practices of parents from different countries are becoming more homogenous over time. Fourth, Chinese and Caucasian parents may differ on other pertinent beliefs not explored in this study. For example, we did not explore parents' expectations for children's academic accomplishments and progress, areas of documented differences across ethnic/racial groups [39]. Fifth, differences in Chinese and Caucasian children's math skills may be due to factors other than parents' beliefs. Further empirical analyses should be conducted to study the relative importance of these explanations.

4.1. Limitations and Future Directions. Because the sample size in this study, particularly for African Americans and Latinos, was fairly small, the findings are exploratory in nature. Future studies should use a larger sample to capture a more representative portrait of the math home-environments of children from diverse ethnic/racial backgrounds. The important variability in ethnic/racial/income patterns of math development [7, 47, 48] highlights the need to examine group-specific family environments. The nature of this study limited any such exploration to only Chinese and Caucasian families residing in the mid-Atlantic region of the United States.

The theoretical model guiding this research is that parents' beliefs are related to children's activities which, in turn, are related to children's math skills. However, we were unable to include a measure of children's math skills in this study. We expect that children's math activities are related to children's math skills (e.g., [8, 10, 11]). Nevertheless, future research should investigate the full model. It is also important to note that our measures of children's math activities are based on parents' reports. Future research should attempt to confirm these findings with some form of direct measure or with corroboration, if possible, with reports by children.

This study investigated a small portion of children's home environments and parents' math-related beliefs and practices. Although such a directed focus appears to us to be valid, it is important for future research to compare parents' beliefs and practices across different academic and nonacademic domains to get a fuller picture of home learning opportunities available to different groups of children.

Regardless of the limitations of this study, our analysis provides new exploratory evidence of the importance of understanding children's math home-environments. These exploratory findings can inform the development of home-based interventions to improve math outcomes of children from different ethnic/racial groups. For example, these results show that certain beliefs of parents, but not others, predicted children's engagement in math activities. Educating parents about the importance of having their children engage in math activities at home, serving as role models of such engagement, and using appropriate daily living activities involving math are important avenues for facilitating children's increased engagement with math at home. Recall that only about half the parents (56%) mentioned involvement in daily living activities as a way to facilitate children's math development. Clearly, some parents are not aware of the positive implications of involving their children in frequent math-related activities. Parents can also be encouraged to give increased attention to being role models of engagement for their children. Only about half (49%) the parents reported that their children saw them do math activities every day and 10% noted their children never saw them engaged in math activities. In fact, taking advantage of the many daily living activities that involve math (e.g., cooking and paying bills) may be an excellent way for parents to serve as role models of math engagement for their children. In contrast, neither parents' own enjoyment of math nor their self-rated skills predicted whether this age group of children engaged in math at home.

5. Conclusion

This study documented aspects of children's home math environments to explore the relation between parents' beliefs and children's participation in math activities. Although research shows that children's math activities at home are related to their math skills, a significant portion of children have limited math involvement at home [8, 10, 11, 15]. Parents' beliefs about children's math development and their role in fostering such development was significantly related to children's math activities. Although further research is needed to confirm the findings with a larger sample, particularly for analyses focusing on ethnic/racial comparisons, and to include measures of children's math competencies, these findings are an important first step that will aid in the development of home-based interventions to facilitate children's math skills.

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Research Article

Children's Use of Arithmetic Shortcuts: The Role of Attitudes in Strategy Choice

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Current models of strategy choice do not account for children's attitudes towards different problem solving strategies. Grade 2, 3, and 4 students solved three sets of three-term addition problems. On inversion problems (e.g., 4 + 8 − 8), if children understand the inverse relation between the operations, no calculations are required. On associativity problems (e.g., 5 + 27 − 23), if children understand the associative relation between the operations, problem solving can be facilitated by performing subtraction before addition. A brief intervention involving demonstrations of different problem solving strategies followed the first problem set. Shortcut use increased after the intervention, particularly for students who preferred shortcuts to the left-to-right algorithm. In the third set, children were given transfer problems (e.g., 8 + 4 − 8, 4 − 8 + 8, 27 + 5 − 23). Shortcut use was similar to first set suggesting that transfer did occur. That shortcut use increased the most for students who had positive attitudes about the shortcuts suggests that attitudes have important implications for subsequent arithmetic performance.

1. Children's Use of Arithmetic: The Role of Preference in Strategy Choice

Children's understanding of the relations between addition and subtraction is considered integral for later mathematical skills [1]. Part of the research agenda needs to understand and account for the large individual differences in children's conceptual knowledge [2]. While many researchers have focussed on factors such as general mathematics skills [3] or working memory [4] to account for individual differences in understanding the relationship between addition and subtraction, one overlooked factor may be students' attitudes and beliefs towards mathematics. Attitudes are positive or negative evaluations (e.g., "I like addition") and beliefs are "thoughts" based on experience (e.g., "division is difficult") [5] and both are considered to be integral in determining success in mathematics [6]. These attitudes and beliefs could be integral in the conceptual knowledge that children use and understand when making problem solving strategy choices.

Researchers have often considered children's use of the inversion shortcut strategy on inversion problems of the form $a+b-b$ as an indicator of understanding of the inverse relation between the two operations [3]. The inversion shortcut involves stating that the answer is a without performing any calculations as the two $b$terms cancel each other out. Children's understanding of the relationship between addition and subtraction can also be assessed via children's use of the associativity shortcut on problems of the form $a + b - c$, which were originally included in studies of children's inversion as control problems [7]. When children understand that addition and subtraction are associative, they can use that information to simplify and speed up problem solving by first subtracting and then adding (e.g., 3 + 29 − 27 would be solved by calculating 29 − 27 and then adding 3) [4]. Both inversion and associativity three-term problems are novel and therefore children must implement spontaneous problem solving strategies [8]. Researchers have taken advantage of this novelty to gain an understanding of children's knowledge of the relations between addition and subtraction by utilizing children's use of conceptually-based shortcuts as an indicator of conceptual understanding because understanding the inverse or associative relations between addition and subtraction is required to implement the inversion and associativity shortcuts [9].

Given the importance of understanding the relations between addition and subtraction, some studies have gone beyond assessing children's spontaneous use or evaluation of the inversion shortcut by trying to promote conceptually-based shortcut use focussing exclusively on inversion problems. In these studies, 8-year-old children's conceptual understanding of the inverse relation between addition and subtraction, as assessed via shortcut use, increased after either 2 or 3 training sessions with concrete objects [10, 11] or after repeated weekly exposure to inversion problems [12].

A possible alternative to the longer interventions used in previous studies comes from a study with Grade 2, 3, and 4 students by Robinson and Dubé [13] who asked children to solve inversion and associativity problems and then had the children complete an evaluation of procedures task [9] as a second measure of conceptual understanding. In the task, for both inversion and associativity problems, the shortcut and a standard left-to-right algorithm (i.e., adding the first two numbers and then subtracting the third number) were demonstrated to participants and participants were asked if they approved of each approach. Bisanz et al. [9] argued that the evaluation of procedures task is a good measure of conceptual understanding as it permits children to demonstrate their understanding which they may have implemented during problem solving. However, in an earlier pilot study, we found that participants were very likely to approve of both the shortcut and the left-to-right algorithm for both inversion and associativity and thus a more stringent measure was needed. To gain more useful information about children's understanding of arithmetic concepts, the participants in Robinson and Dubé [13] were also asked to compare the two solution procedures for each problem type and decide which was better and why. Explicitly justifying a strategy suggests conceptual understanding because the ability to justify typically develops after the ability to assess a strategy's worth and even use the strategy during problem solving [14]. Robinson and Dubé [13] found that most of the participants (80% or more) approved of both the shortcut and the left-to-right procedure for both the inversion and associativity problems. The more stringent measure of choosing the better strategy was more sensitive as a clear majority of participants preferred the shortcut to the left-to-right algorithm for inversion problems (78%) but not for associativity problems (59%) with older students in Grades 3 and 4 more likely to prefer the associativity shortcut. Effects of the evaluation of procedures task on subsequent problem solving were not examined in that study.

The study by Robinson and Dubé [13] provided two conclusions regarding strategy instruction and attitudes about problem solving strategies. The first conclusion is that the children had distinctly positive or negative attitudes about the appropriateness of the shortcuts. For example, some participants commented on the cleverness of the shortcuts (e.g., "that's so easy, I wish I'd thought of doing it that way" while others strongly disapproved of the shortcuts "e.g., it's cheating not to do all the math"). This variability in attitudes across individuals suggests that some children may be open and flexible about learning new ways to problem solve while others may be rigid and inflexible. For example, across a series of studies, Torbeyns and colleagues [15–17] have shown that children are often resistant to using a more efficient arithmetic strategy, even when explicitly encouraged and/or instructed to use that strategy. Previous research has suggested that children's attitudes and beliefs about mathematics are critical to success in mathematics and to enrollment in advanced mathematics courses and therefore engaging children with mathematics is of critical importance [18]. During schooling, children begin to have strong attitudes and beliefs about mathematics [19]. Children who are intrinsically motivated [20], are mastery-oriented [21] or who have low levels of mathematics anxiety [22], are more likely to be successful in mathematics. Other factors relating to mathematics success include the role of peer pressure [23] and disengagement [24].

Although several studies have investigated children's attitudes and beliefs about mathematics, few studies have investigated children's attitudes about specific problem solving strategies. Ellis [25] and Verschaffel et al. [26] note that much of the research on the factors involved in children's strategy choices has focussed on the roles of speed and accuracy and has ignored other factors such as individual and sociocultural influences. The findings of Robinson and Dubé [13] suggest that children's attitudes towards the use of problem solving strategies that involve simplifying and reducing solution times are already entrenched by Grade 2 and that these attitudes may become even more firmly held by the end of the middle school years [27]. Current models of children's strategy choices emphasize the tendency for children to want to use the most efficient problem solving strategies [28, 29] and yet the findings of Robinson and Dubé [13] support Ellis' and Verschaffel et al.'s [26] proposal that other factors, in this case attitudes, may also impact strategy choices.

The second conclusion drawn from Robinson and Dubé [13] was that comparison of problem solving strategies yielded more information about children's understanding of arithmetic concepts than only asking them to evaluate strategies. Research on comparison of solution methods has been encouraging as it appears to be an effective learning tool [30–32]. In two studies, participants in Grades 7 and/or 8 were given the opportunity to compare different solution methods for solving algebraic equations [30, 31] and in Star and Rittle-Johnson [32], participants in Grades 5 and 6 compared different solution methods for computational estimation. In all of the studies, the opportunity to compare different methods led to greater increases in procedural and conceptual knowledge than simply being presented with different methods. None of the studies examined whether there were developmental changes in the effectiveness of comparison and no studies of the effects of comparison have been conducted with younger students. Extending the use of comparisons even further, Yakes and Star [33] found that teachers who participated in a workshop on how to use comparison as an instructional tool for teaching algebra changed their teaching practices. The teachers used comparison of different solution methods as a teaching tool and also began to recognize and emphasize the importance of procedural flexibility in their teaching.

Based on the findings of Robinson and Dubé [13] that there are marked individual differences in children's attitudes about the inversion and associativity shortcuts and that presenting solution methods concurrently improves comparison and is a useful learning technique for older children (e.g., [30, 31]), the first two goals of this study were to determine whether children's attitudes about the solution methods they were asked to evaluate would impact subsequent problem solving strategies and to examine whether presenting solution methods concurrently improves comparison and is an effective learning tool on inversion and associativity problems. The final two goals of the study were to examine whether attitudes and the impact of comparison would change across development and to assess the strength of the intervention task by investigating whether children would transfer their new knowledge about the shortcuts to transfer problems. When concepts such as inversion or associativity are effectively learned, then that knowledge should be applicable or transferable to new problem types [34]. Rittle-Johnson and Star [30, 31] found that comparison of solution methods promoted flexibility and Rittle-Johnson [35] proposed that explaining why a procedure works is an effective learning tool and can result in the ability to transfer knowledge to new situations. Children do not necessarily understand a concept that has been taught to them but the ability to generalize a problem-solving strategy based on conceptual knowledge is an indicator that learning of that concept has occurred [36].

2. Method

2.1. Participants. Twenty-four Grade 2 students (12 boys, 12 girls) (mean age = 7 years, 0 months), thirty-five Grade 3 students (14 boys, 21 girls) (mean age = 7 years, 11 months), and forty-three Grade 4 students (20 boys, 23 girls) (mean age = 8 years, 10 months) participated in the study. The age groups were selected to correspond with previous research [13] and to correspond to the time in which children are learning about addition and subtraction [18]. Participants were from a large Canadian city, were predominantly Caucasian, and from middle SES families. The study took place in the first half of the school year.

2.2. Design and Intervention. The same pretest-intervention-posttest design was used as in the studies by Rittle-Johnson and Star [30, 31] but the participants were tested individually throughout the study rather than being paired up for the intervention, and the posttest was divided into the first posttest with familiar problems and a second posttest with transfer problems to assess strength of learning from the intervention. Half the students per grade were randomly assigned to either the sequential condition ($n = 51$) or the concurrent condition ($n = 51$). The intervention was the evaluation of procedures task. For both inversion and associativity problems, two strategies were demonstrated. In the sequential group, for each problem type: (1) the first strategy was demonstrated and then participants were asked if they approved of the strategy; (2) the second strategy was demonstrated and then participants were asked if they approved of the second strategy; (3) participants were asked which of the two strategies they preferred. In the concurrent group, for each problem type: (1) both strategies were demonstrated consecutively; (2) participants were asked if they approved of both strategies at the same time; (3) participants were asked which of the two strategies they preferred. Problem type and strategy orders were counterbalanced across boys and girls as closely as possible within each grade. For the inversion problems, on one problem, participants were told how a fictitious child had solved the problem by using the inversion shortcut ("When X solved this problem, s/he said that the answer would be the first number because when you add and subtract by the same number, the answer is always the first number"). On the other problem, participants were told how a fictitious child had solved the problem using a left-to-right algorithmic approach ("When X solved this problem, she/he added the first two numbers together and then subtracted the third number from that answer"). For the associativity problems, on one problem, participants were told how a fictitious child had solved the problem by using the associativity shortcut ("When X solved this problem, she/he subtracted the third number from the second number and then took the answer and added it to the first number"). On the other problem, the same left-to-right algorithmic approach used on the inversion problem was used again.

2.3. Materials and Procedure. There were three sessions, which took place within a 7 day period with at least one day between each session. In the first session which was the pretest, participants solved 16 three-term addition and subtraction problems. Half the problems were inversion problems of the form $a + b - b$ and half were associativity problems of the form $a+b-c$. Half the problems of each type were small ($a, b, c < 10$) (e.g., $3+6-6$ and $3+6-4$) and half were large ($a < 10, b$ and c between 21 and 29, $b > c$) (e.g., $7+23-23$ and $7+23-21$). No more than two problems of each type or each size were presented consecutively. Problems were presented on a laptop screen using e-prime. Participants did not have the aid of paper and pencil. Solution latencies (measured from the time the problem appeared on the screen until the participant stated the answer and pressed the space bar), accuracy, and immediately retrospective verbal reports of problem solving strategy (i.e., "how did you solve that problem?") were collected for each problem. Participants who were unable to provide an answer within 30 seconds were given a "cut-off protocol" in which they were asked to report how they were trying to solve the problem before moving on to the next problem. Cut-off problems were coded as incorrect and no solution latencies were recorded.

In the second session which was the intervention and first posttest session, participants were randomly assigned to the concurrent or sequential conditions and completed the evaluation of procedures task and then solved the first posttest of a new set of 16 three-term addition and subtraction problems. The same measures and the same problem parameters were used as in the first session.

In the third session which was the second posttest session, participants solved 16 novel transfer problems. Half of the problems were inversion problems and the other half were associativity problems using the same parameters and measures as in the first and second sessions except that half of the inversion problems were of the form $b+a-b$ (e.g., 4 + 9 − 4) and half of the form $a-b+b$ (e.g., 7 − 23 + 23) and the associativity problems were of the form $b+a-c$ (e.g., 28 + 6 − 25).

3. Results

Data from boys and girls were collapsed together as no significant results involving gender were found. Tukey's Honestly Significant Difference test was used to examine post-hoc effects and the alpha level was .05 or less for all significant results. Analyses of verbal report data only are reported but accuracy, solution latency, and cutoff data matched expected patterns (higher accuracy, shorter solution latencies, and fewer cutoffs when shortcuts were used, see Table 1). Although differences between the concurrent and sequential conditions were expected, no differences were found (see Table 2) so the analyses reported below are all collapsed across condition. This lack of condition effect may have been a result of such a brief intervention or of the nature of the task. Both conditions included exposure to the same information and the same problems and the children in both conditions were asked to decide which strategy they preferred (the conceptually-based shortcut or the left-to-right strategy) yielding comparable data across conditions. This data allows the investigation of how children's attitudes towards different problem solving strategies impacts subsequent conceptually-based shortcut use.

3.1. Promotion of Conceptually-Based Shortcuts across Development. As can be seen in Table 3, some children were already using the inversion and associativity shortcuts in the pretest. Shortcut use between the pretest and the first posttest (familiar problems) was compared to determine if shortcut use on familiar problems increased after the evaluation of procedures task. Two separate 3 (Grade) × 2 (Size) × 2 (Session) analyses of variance were conducted on inversion and associativity shortcut use. Shortcut use increased from the pretest to the first posttest (43.6% versus 63.2% for inversion, 20.9% versus 35.9% for associativity), $F(1, 99) = 39.75$, $MSE = 929.93$, $\eta_p^2 = .29$ and $F(1, 99) = 17.43$, $MSE = 1231.66$, $\eta_p^2 = .15$ for inversion and associativity respectively. There was less shortcut use on small than large problems (50.7% versus 56.2% for inversion, 23.86% versus 32.86% for associativity), $F(1, 99) = 11.17$, $MSE = 259.68$, $\eta_p^2 = .10$ and $F(1, 99) = 15.67$, MSE = 496.78, $\eta_p^2 = .14$ for inversion and associativity respectively. For inversion, no grade differences were found (44.0%, 62.9%, and 53.5%, for Grades 2, 3, and 4) but for associativity (17.71%, 39.46%, and 27.91% for Grades 2, 3 and 4), there was more shortcut use by Grade 3 than Grade 2 students and Grade 4 students did not differ from Grade 2 or 3 students, $F(2, 99) = 3.92$, MSE = 882.88, $\eta_p^2 = .07$, and HSD = 17.7.

If the evaluation of procedures task was successful at promoting shortcut use, then participants should also use the shortcuts on the novel transfer problems in the second posttest. Shortcut use on typical problems before and after the evaluation of procedures task and shortcut use on transfer problems after the evaluation of procedures task were analyzed using two repeated measures ANOVAs. After the evaluation of procedures task, shortcuts were used less frequently on transfer problems in the second posttest than on typical problems in the first posttest (43.8% versus 65.0%, for inversion, 16.8% versus 37.6% for associativity) but were used just as frequently as shortcuts before the evaluation of procedures task (44.0% for inversion, 21.3% for associativity), $F(2, 202) = 25.68$, MSE = 588.35, $\eta_p^2 = .20$, HSD = 14.4, $F(2, 202) = 21.24$, MSE = 576.48, $\eta_p^2 = .17$, and HSD = 14.3 for inversion and associativity problems, respectively. This suggests that participants had greater difficulty applying the shortcuts to transfer problems than to typical problems after the evaluation of procedures task. However, shortcut use on the transfer problems after the intervention was the same as shortcut use on the typical problems before the intervention suggesting that the intervention may have promoted shortcut use on the transfer problems.

Shortcut use on the transfer problems in the second posttest was analyzed in greater detail to determine the effects of grade, problem size, and transfer type. For inversion problems, two types of transfer problems were possible, and a 3 (Grade) × 2 (Size) × 2 (Transfer type) ANOVA was conducted (see Figure 1). Inversion shortcut use was higher on large problems (32.7% versus 49.9% for small and large), $F(1, 99) = 34.32$, MSE = 828.65, $\eta_p^2 = .26$. There was a two-way interaction between transfer type and grade, $F(2, 99) = 5.22$, MSE = 1119.79, and $\eta_p^2 = .10$, with no grade differences on the first type of inversion transfer problem ($a-b+b$) but higher shortcut use by the Grade 4 than the Grade 2 students on the second type of transfer problem ($b+a-b$). There were no differences between transfer type in Grades 2 or 3 but in Grade 4, shortcut use was higher on the second type. This interaction was mediated by a three-way interaction between type, grade, and size, $F(2, 99) = 3.23$, MSE = 615.43, and $\eta_p^2 = .06$. There was a main effect of problem size with more shortcut use on large problems in all grades and both types of transfer problems, except for Grade 4 students who had no size difference on the first type of problem although the means were in the expected direction.

For associativity problems, a 3 (Grade) × 2 (Size) ANOVA was conducted on the transfer problems in the second posttest (see Figure 2). Associativity shortcut use was lower on small problems (12.4% versus 18.5% for small and large), $F(1, 99) = 5.89$, MSE = 303.04, and $\eta_p^2 = .06$, and there was a two-way interaction between grade and size, $F(2, 99) = 6.35$, MSE = 303.04, and $\eta_p^2 = .11$. On small problems, Grade 2 students used the associativity shortcut less frequently than the Grade 3 students, and on large problems the Grade 2 students used the shortcut less than both the Grade 3 and 4 students. There was no main effect of problem size in Grades 2 or 3, but Grade 4 participants used the shortcut more on large problems.

Table 1: Accuracy (%), solution latencies, and proportion of cutoffs (%) in the pretest, first posttest, and second posttest on inversion and associativity problems.

	Session		
	1	2	3
Accuracy		Inversion	
Shortcut	98.3 (2.1)	99.4 (1.7)	98.8 (2.1)
Algorithm	40.8 (2.2)	42.0 (3.0)	56.9 (2.6)
Solution latency			
Shortcut	5174 (293)	3216 (240)	4830 (298)
Algorithm	12407 (484)	114449 (646)	10346 (480)
Cutoffs			
Shortcut	1.4 (1.9)	.4 (1.6)	.9 (2.0)
Algorithm	37.0 (2.1)	36.8 (2.8)	17.2 (2.4)
Accuracy		Associativity	
Shortcut	65.0 (3.4)	76.2 (2.3)	65.0 (3.4)
Algorithm	48.8 (1.6)	47.2 (1.8)	48.8 (1.6)
Solution Latency			
Shortcut	11927 (524)	9389 (361)	10479 (585)
Algorithm	13396 (347)	12358 (373)	13359 (327)
Cutoffs			
Shortcut	16.1 (2.8)	9.1 (2.1)	10.2 (3.2)
Algorithm	33.3 (1.5)	36.3 (1.7)	28.8 (1.5)

Note. Standard errors are in parentheses.

Table 2: Percentage shortcut use in the pretest, first posttest (familiar problems), and second posttest (transfer problems) for the sequential and concurrent groups in each grade.

		Session					
		Pretest	1st posttest	2nd posttest	Pretest	1st posttest	2nd posttest
Grade 2	*n*		Inversion			Associativity	
Sequential	13	42.3 (11.1)	51.0 (11.1)	42.3 (8.6)	15.4 (8.8)	22.1 (11.1)	6.7 (4.8)
Concurrent	11	31.8 (12.0)	50.0 (12.1)	29.5 (7.3)	11.4 (8.8)	21.6 (12.1)	5.6 (3.9)
Grade 3							
Sequential	16	57.0 (10.0)	75.8 (10.0)	53.9 (10.1)	35.2 (7.2)	48.4 (10.0)	29.7 (10.0)
Concurrent	19	51.3 (9.1)	68.4 (9.2)	40.8 (7.5)	30.9 (6.7)	44.1 (9.2)	12.5 (4.8)
Grade 4							
Sequential	23	37.0 (8.3)	70.1 (8.3)	48.9 (6.3)	18.5 (6.1)	41.3 (8.3)	21.7 (5.4)
Concurrent	20	42.5 (8.9)	64.4 (9.0)	41. (7.8)	13.8 (6.5)	37.5 (8.9)	17.5 (5.5)

Note. Standard errors are in parentheses.

3.2. The Effects of Attitudes about Conceptually-Based Shortcuts across Development. In the evaluation of procedures task, participants were asked which they preferred: the shortcut or the left-to-right algorithm for both inversion and associativity problems. For inversion problems, participants were more likely to prefer the shortcut than the algorithm (71.8% versus 28.2%), $\chi^2(1, N = 103) = 19.66$, and that preference was similar across grade (58.3%, 77.1%, and 76.7% for Grades 2, 3, and 4). For associativity problems, participants, however, were not more likely to prefer the shortcut (58.3% versus 41.7%) but older children were more likely to prefer the shortcut (29.2%, 62.9%, and 72.1% for Grades 2, 3, and 4), $\chi^2(2, N = 103) = 12.08$. Thus, the success of the evaluation of procedures task may vary depending on whether participants "bought in" to the shortcuts.

To determine whether increases in shortcut use between sessions were attributable to participants "buying in" to the training provided by the evaluation of procedures task or whether they were attributable to practice or exposure effects, changes in inversion and associativity shortcut use within the pretest and the first posttest were compared to changes in shortcut use between the pretest and the first posttest using two 4 (1st half of pretest, 2nd half of pretest, 1st half of first posttest, 2nd half of first posttest) × 2 (Preference: shortcut, algorithm) ANOVAs (see Figure 3).

For inversion, shortcut use did not change within the pretest of the first posttest (34.6% versus 42.7%, 56.6% versus 58.8% for the 1st and 2nd half of the prestest and first posttest, resp.) but did increase between the 2nd half of the pretest and the first half of the first posttest, $F(3, 300)$

Table 3: Percentage shortcut use in the pretest and first posttest on small and large problems in each grade.

	Session	
	Pretest	First posttest
Grade 2	Inversion	
Small	37.5 (7.8)	49.0 (8.7)
Large	37.5 (8.7)	52.1 (8.2)
Grade 3		
Small	51.4 (6.4)	69.3 (7.2)
Large	56.4 (7.3)	74.3 (6.8)
Grade 4		
Small	34.3 (5.9)	62.8 (6.5)
Large	44.8 (6.5)	72.1 (6.1)
Grade 2	Associativity	
Small	11.5 (5.9)	18.8 (8.1)
Large	15.6 (7.0)	25.0 (9.0)
Grade 3		
Small	30.7 (4.9)	42.1 (6.7)
Large	35.0 (5.8)	50.0 (7.4)
Grade 4		
Small	8.7 (4.4)	31.4 (6.1)
Large	23.8 (6.7)	47.7 (6.7)

Note. Standard errors are in parentheses.

= 20.8, MSE = 516.29, η_p^2 = .17, HSD = 13.3. Participants who preferred the inversion shortcut used the shortcut more frequently than participants who preferred the algorithm (62.2% versus 34.2%, resp.), $F(1, 100) = 13.2$, MSE = 4826.17, and $\eta_{p2} = .11$.

For associativity, shortcut use did not change within the pretest of first posttest (17.3% versus 23.9%, 32.9% versus 36.8% for the 1st and 2nd half of the pretest and first posttest, resp.) nor did it change between the 2nd half of the pretest and 1st half of the first posttest, $F(3, 300) = 14.552$, MSE = 526.11, $\eta_p^2 = .13$, and HSD = 12.1. Participants who preferred the associativity shortcut used the shortcut more frequently than participants who preferred the algorithm (37.6% versus 17.9%, resp.), $F(1, 100) = 11.38$, MSE = 3387.69, and η_p^2 = .10. There was a two-way interaction between session and preference, $F(3, 300) = 9.38$, MSE = 526.11, η_p^2 = .09, and HSD = 14.2. For participants who preferred the algorithm, shortcut use did not change within the pretest or the first posttest nor did it change between the two. For participants who preferred associativity, shortcut use did not change within the pretest or the first posttest but did increase between the 2nd half of the pretest and the 1st half of the first posttest. Furthermore, the difference in shortcut use between participants who preferred the associativity shortcut and participants who preferred the algorithm was not significant in the pretest but was significant in the first posttest.

To determine if participants "buying in" to the training affected shortcut use on transfer problems, two independent sample t-tests were conducted comparing shortcut use in Session 3 between participants who preferred the shortcut and participants who preferred the algorithm. For inversion problems, participants who preferred the shortcut used shortcuts more frequently (M = 48.0%, SE = 3.8%) than participants who preferred the algorithm (M = 32.6%, SE = 5.8%), $t(100) = -2.15$. For associativity problems, participants who preferred the shortcut used the shortcut just as frequently (M = 20.2%, SE =3.6%) as participants who preferred the algorithm (M = 11.9%, SE = 3.6%), $t(100) = -1.58$, $P = .17$.

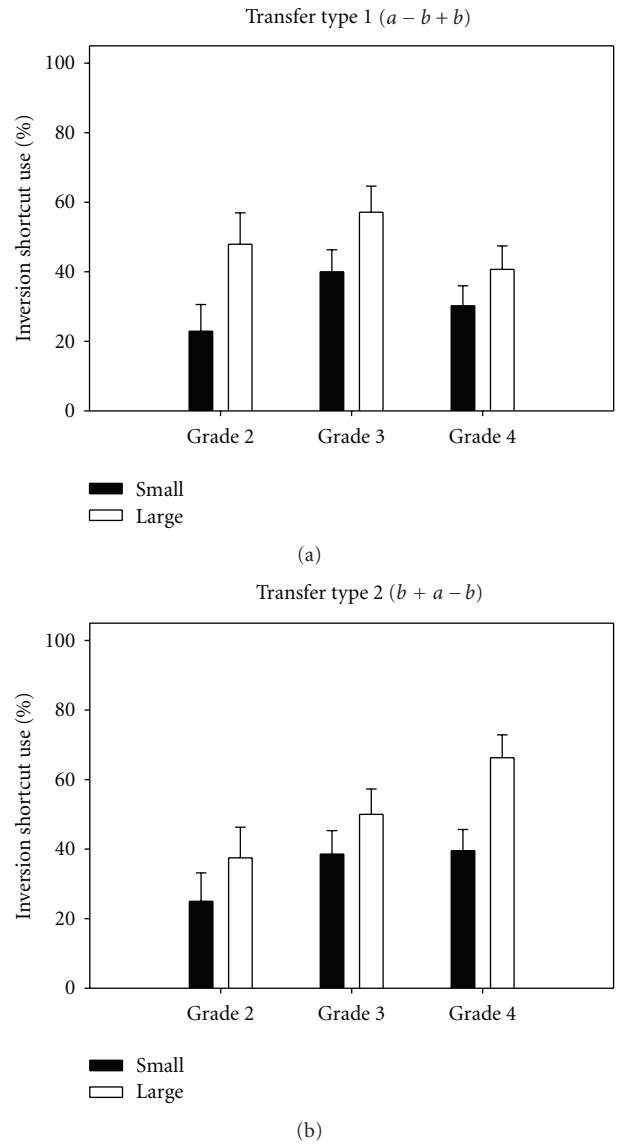

Figure 1: Inversion shortcut use on transfer problems in the second posttest for each grade and problem size Type 1 (a) problems ($a - b + b$) and Type 2 (b) problems ($b + a - b$). Error bars are SE.

4. Discussion

The brief evaluation of procedures task appears to have been successful in promoting conceptually-based shortcut use and decreasing the use of the less efficient left-to-right problem solving algorithm in many children in Grades 2,

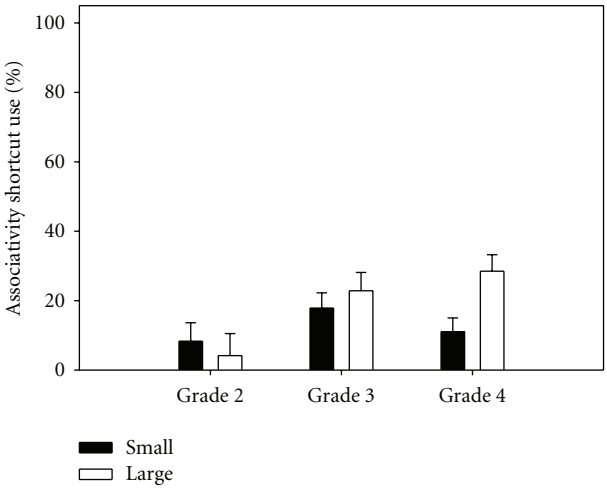

Figure 2: Associativity shortcut use on transfer problems in the second posttest for each grade and problem size. Error bars are SE.

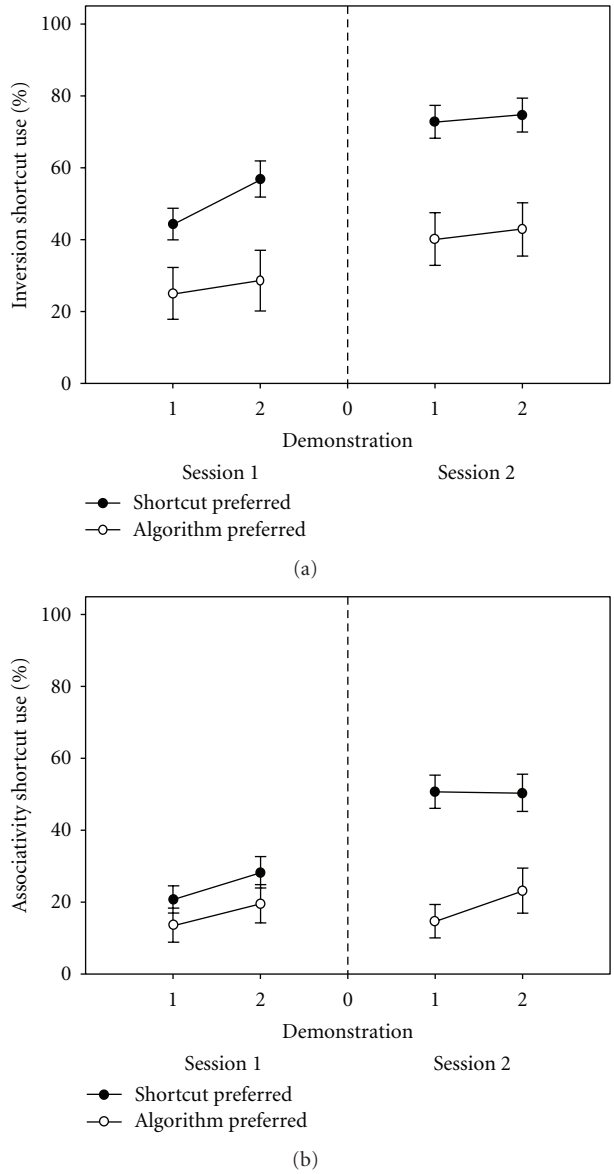

Figure 3: Shortcut use in 1st and 2nd half of the pretest and the first posttest on inversion (a) and associativity (b) problems by preference. Error bars are SE.

3, and 4. Children who use a conceptually-based shortcut are implementing an efficient problem solving strategy or procedure that is dependent on the inverse or associate relation between addition and subtraction, that is, they are using a procedure that is based on an arithmetic concept. The successful promotion of shortcuts that arise from arithmetic concepts is a finding consistent with a recent study with slightly older children showing that the task promoted more conceptually-based shortcut use in a condition similar to the present study's concurrent condition than in a control condition [37]. To learn and implement new problem solving strategies such as the inversion and associativity shortcuts is integral to increasing mathematical knowledge [8] as children with stronger conceptual knowledge of arithmetic will be able to represent problems in arithmetic and algebra more accurately [14, 38]. Children are expected to understand and discover the inversion and associativity concepts and shortcuts on their own [8] and few studies have attempted to teach children about the inverse relationship between addition and subtraction [11] and none have been conducted on teaching children about the associative relationship between addition and subtraction. The results of the current study support both the ability of many children in Grades 2, 3, and 4 to spontaneously discover and apply conceptually-based shortcuts during problem solving and the capacity of children to learn how to use these shortcuts immediately after a brief intervention where children are given demonstration of the shortcuts. Interestingly, children's attitudes towards the shortcuts were much more critical for children adopting a shortcut during subsequent problem solving. In contrast, being in the concurrent or sequential condition had little impact [30–32].

The lack of grade differences in spontaneous inversion shortcut use (see also [3]), the higher use of the inversion shortcut for all students after the evaluation of procedures task, and the more frequent use of the inversion shortcut than the associativity shortcut all suggest that the inversion shortcut is easier to discover, learn, and apply than the associativity shortcut. The associativity shortcut, on the other hand, was more likely to be used by the Grade 3 than the Grade 2 and 4 students. The shortcut requires calculation and therefore the advantage of using the shortcut may not be clear to Grade 2 students struggling with calculation, whilst the Grade 4 students may be good enough calculators that the shortcut does not seem superior. Demonstrating the conceptually-based shortcuts may be most suitable for Grade 3 students who have just enough calculation skills to be able to cope with the shortcut but not enough that the shortcut and the algorithm seem equally efficient. When typical solution methods are too difficult or inefficient, students are more likely to use alternative methods [39], which is consistent with the finding in the current study

that students in all grades had higher use of shortcuts on problems with larger numbers.

When children have good conceptual knowledge of inversion and associativity, they should be able to appropriately transfer the conceptually-based shortcuts to solve novel problems [1]. Our findings suggest that the evaluation of procedures task promoted the use of conceptually-based shortcuts on similar problems and that students were also able to apply the shortcuts to transfer problems. Consistent with the findings of Siegler and Stern [12], inversion shortcut use was lower on transfer problems than in the previous session with typical inversion problems, and the pattern was similar for associativity shortcut use. However, shortcut use on the transfer problems was similar to that of spontaneous shortcut use in the first session. It is possible that, without the intervention, use of the shortcuts on the transfer problems might have been even lower and therefore future studies that investigate spontaneous use of the shortcuts on transfer problems before and after an intervention are needed. Children in all grades were able to spontaneously apply their knowledge of the shortcuts to the novel problems in the first session indicating a strong schema or strong understanding of inversion and associativity [40]. After the very brief intervention, shortcut use may not have transferred because children had learned to use the shortcut rather than understanding the concept behind the shortcut.

Contrary to expectations, the presentation format for the evaluation of procedures task, either sequential or concurrent, made little difference to the promotion of conceptually-based shortcuts. Rittle-Johnson and Star's [30–32] studies differed from the present task in that our task was briefer, the problems and solution procedures in our task were simpler, and our participants were not required to provide as much information about their mathematical reasoning. Future work could use an intervention containing more problems and include problems that are similar in structure but to which the shortcuts do not apply—this could improve the effectiveness of the instruction by showing children the limitations of the shortcuts. Our results indicate that providing multiple problem solving approaches was effective in promoting shortcut use and the presentation of the approaches and the opportunity for comparison was not critical as it was in the Rittle-Johnson and Star studies. However, Rittle-Johnson, Star, and Durkin [41] found that comparison of solution strategies was not always effective. Based on children's shortcut use before and after the evaluation of procedures task, we propose that the evaluation task used in the present study provides a quick and effective instructional tool that can be implemented in the classroom to promote children's understanding of the relations between addition and subtraction.

Children's attitudes about the demonstrated shortcuts, compared to a left-to-right algorithm on inversion and associativity problems, had a clear effect on subsequent problem solving procedures. Inversion and associativity shortcut use was higher for participants who preferred the shortcut, with associativity shortcut use significantly increasing after the demonstration for participants who preferred the shortcut. On transfer problems, preferring the inversion shortcut also led to higher shortcut use and preferring the associativity shortcut did not. This latter finding provides further evidence that the associativity shortcut is more difficult for students perhaps because it can overtax cognitive resources. The associativity transfer problems require students to pay attention to all three numbers and notice that the third number can be subtracted from the first number and then to subsequently perform that subtraction and then add the middle number while keeping track that all of the numbers have been added or subtracted. However, associativity shortcut use was similar on the transfer problems as it was in the pretest so the intervention may still have had a beneficial effect.

As most intervention studies involve providing students with information or instruction without assessing children's attitudes or beliefs about that information or instruction, the finding that children's views of problem solving procedures impact their subsequent procedure or strategy choices also needs to be further investigated and current theories of strategy choice need to include children's attitudes as a factor in strategy choice [27]. The current study used a brief intervention task and children had the opportunity to only provide brief information about their strategy preferences. As Verschaffel et al. [26] propose, more extensive interviews with children to gain deeper insight into their strategy choices and the influences on their strategy choices are needed. However, this is the first study to examine the consequences of children being more or less receptive to conceptually-based shortcuts and the consequences, particularly for the associativity shortcut, were marked. In areas outside of mathematics, research has shown that children's attitudes significantly contribute to their actions [5]. The present study indicates that children's attitudes towards mathematics, even simple preferences between two strategies, can be the basis for action during problem solving.

Researchers are increasingly focussing attention on how attitudes impact children's mathematics performance, decisions to take advanced mathematics courses, and career choices (e.g., [24]). However, there is also a need to investigate how attitudes impact smaller components of mathematics learning such as strategy choice. Conceptually-based shortcut use has important advantages. First, using shortcuts frees up cognitive resources [42] that can then be used to deal with more complex problems such as algebra problems [38]. Second, using shortcuts may make children more likely to pay attention to problem characteristics to guide strategy choices [39, 42] rather than being "mindless" problem solvers who are rigid rather than flexible [43]. Further research on why some children have more positive attitudes towards accepting strategies that are highly efficient but are novel to their current strategy repertoire of algorithmic approaches may help explain some of the large individual differences found in children's procedural and conceptual knowledge of arithmetic.

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Research Article

Attitudes towards Mathematics: Effects of Individual, Motivational, and Social Support Factors

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This paper aims to understand how certain different but interrelated variables such as background, motivation, and social support could lead to an explanation of student attitudes towards math and to an understanding of the defining characteristics of these attitudes in the school environment. Participants consisted of 1719 Portuguese students, from fifth-to-twelfth grade. The study utilizes an adaptation of the "Intrinsic Motivation Inventory" assessing main determinants of intrinsic motivation. One section of the questionnaire—"In my Math Class"—also assesses student perceptions of teacher and peer support as well as student attitudes. The results revealed that, in general, students held positive attitudes towards mathematics and also highlighted the main effects of grade and math achievement on these attitudes. No gender effect was identified although the girls showed a continuous decline in attitudes the further they progressed in school. A hierarchical analysis using structural equation modeling showed that motivation-related variables are the main predictors of attitudes towards mathematics and that teachers and the social support of peers are also highly significant in understanding these attitudes.

1. Introduction

Proficiency in languages, science, and mathematics is seen as an essential precursor to success in modern society. In Portugal, recent guidelines, set by the Ministry of Education regarding Mathematics and Portuguese Language curricula, tasks, evaluation, and workload, reflect this concern as these subjects are cross-curricular and are used in daily life. Comparative international evaluations [1] revealed that Portuguese students did not perform as well as expected, and that they underachieved in mathematics and languages when compared to students from other countries in the OECD. In mathematics, results showed that whilst there was an improvement in mathematics performance by Portuguese students from 2003 to 2009, in 2009, on a scale of six levels, Portugal still has in the region of 25% of their students at level 2 or below [1].

These results give impetus to the development of further research that seeks to characterize and understand different variables which may influence student performance. This will help to make possible strategies for future action in schools, families, and communities, in order to bring about an improvement in the failure rate in math.

The complexity of factors that can influence math performance is demonstrated by Singh, Granville, and Dika [2] when they show that high achievement in mathematics is a function of many interrelated variables related to students, families, and schools. Among student variables, attitudes are regarded by several researchers, as an important/key factor to be taken into account when attempting to understand and explain variability in student performance in maths [3–6].

Mobilizing a set of different definitions concerning attitudes presented since 1935, Eshun [7, page 2] defines an attitude towards mathematics as "a disposition towards an aspect of mathematics that has been acquired by an individual through his or her beliefs and experiences but which could be changed." When emphasizing the importance of individual experiences, the contexts where students interact with others and with mathematics become important focal points. Fraser and Kahle [8] have also highlighted this aspect in research which shows that learning environments at home, at school, and within the peer group accounted for

a significant amount of variance in student attitudes and, furthermore, that class ethos had a significant impact on the scores achieved by students for these attitudes.

In addition, Mohamed and Waheed [5] when reviewing literature aimed at understanding attitudes and the influences on their development in relation to differences between students, identified three groups of factors that play a vital role in influencing student attitudes: factors associated with the students themselves (e.g., mathematical achievement, anxiety, self-efficacy and self-concept, motivation, and experiences at school); factors associated with the school, teacher, and teaching (e.g., teaching materials, classroom management, teacher knowledge, attitudes towards maths, guidance, beliefs); finally factors from the home environment and society (e.g., educational background, parental expectations).

Attitudes can be seen as more or less positive. A positive attitude towards mathematics reflects a positive emotional disposition in relation to the subject and, in a similar way, a negative attitude towards mathematics relates to a negative emotional disposition [9]. These emotional dispositions have an impact on an individual's behavior, as one is likely to achieve better in a subject that one enjoys, has confidence in or finds useful [7]. For this reason positive attitudes towards mathematics are desirable since they may influence one's willingness to learn and also the benefits one can derive from mathematics instruction [7].

1.1. Attitudes and School Grades. Nicolaidou and Philippou [6] showed that negative attitudes are the result of frequent and repeated failures or problems when dealing with mathematical tasks and these negative attitudes may become relatively permanent. According to these authors when children first go to school they usually have positive attitudes towards mathematics. However, as they progress their attitudes become less positive and frequently become negative at high school. Köğce et al. [3] found significant differences between younger and older students' attitudes towards mathematics with 8th graders having lower attitudes than 6th graders.

There are a number of factors which can explain why attitudes towards mathematics become more negative with the school grade, such as the pressure to perform well, over demanding tasks, uninteresting lessons and less than positive attitudes on the part of teachers [6].

1.2. Gender and Attitudes towards Maths. Gender differences are a recurrent theme throughout the literature in academic studies in general and in math studies in particular. Math is often considered to be a domain in which boys are higher achievers, both in terms of attitudes and self-concept. Contrary to this, findings show that math school achievement and grades do not differ significantly between boys and girls (e.g., [10, 11]). This similarity in performance between males and females is clear in the meta-analysis conducted by Lindberg et al. [11] with data from 242 studies representing 1.286.350 people, indicating no gender differences ($d = 0.05$) and nearly equal male and female variances.

There are, however, noticeable differences in the beliefs held by boys and girls. Research has consistently shown that girls have lower math self-concept than boys (e.g., [12]). Results concerning gender differences in attitudes are less consistent than those in self-concept. Some studies have reported significant differences when we compare girls and boys attitudes towards mathematics [7, 13–15], nevertheless there are a number of studies where these differences are not identified [3, 5, 6, 16, 17]. A meta-analysis conducted by Etsey and Snetzler [17] taking into consideration 96 studies ($n = 30490$) concluded that gender differences in student attitudes toward mathematics do exist but are small. The results indicate that males show more positive attitude. However in elementary school studies the effect size was about .20 in favor of females and for grades 9 to 12 the effect size was similar, .23, but in favor of males. Also Hyde et al. [18] in their meta-analysis confirm small gender effects, which increase among older students (high school and college), with females holding more negative attitudes. Although these meta-analyses were developed in the 1990s, there is recent research which confirms these results [13, 14] and attempts to provide a justification for it. Asante [13] states that, when compared with boys, "girls lacked confidence, had debilitating causal attribution patterns, perceived mathematics as a male domain, and were anxious about mathematics" [13, page 2]. The research carried out by this author in Ghana, showed that boys had more positive attitudes towards mathematics than girls. Also Sanchez et al. [14] in a study with North American students found significant gender differences in eighth grade students' attitudes towards math. American boys showed more interest in math than girls, but girls perceived math as more important than boys. Girls also presented higher scores on items with regard to difficulties with math. According to Asante [13] school environment, developmental changes in gender identity, and teacher and parent attitudes and beliefs towards mathematics are factors that may contribute to the differences identified between boys and girls in their attitudes towards mathematics.

Nonetheless there is research which concludes that gender does not affect attitudes towards mathematics [3, 5, 6, 15, 16]. The meta-analysis conducted by Ma and Kishor [15] which looks at 113 studies ($n = 55265$), when studying the effects of gender, concludes that this variable did not have a significant effect on the relationships between attitudes and performance in mathematics because separate analysis by gender demonstrated similar significant effect sizes. Georgiou et al. [16] showed that there was no difference either in math achievement or in math attitudes between boys and girls. However, high achieving boys and girls, despite both considering math as an attractive subject, differed in the explanations they gave for their performance. Since the ability attributions of boys were higher, they believed that their grades were due to their intelligence more consistently than girls did.

1.3. Achievement in Mathematics and Attitudes. Several studies have been undertaken to try to reach an understanding of the relationship between student attitudes towards

mathematics and academic achievement [4–6, 8, 15, 19]. In Ma and Kishor meta-analysis [15] only weak correlations between these variables were identified and these relationships were dependent on several variables (e.g., grade, sample size, ethnic background). With regard to grade, these associations become stronger among older students (7th to 12th grade).

However, more recent studies point to a positive correlation between student attitudes towards mathematics and student academic achievement. Along these lines are the results obtained by Nicolaidou and Philippou [6] which reveal significant correlations between attitudes and performance. Students having positive attitudes achieved better. Mato and De La Torre [4] in a study with secondary school students also showed that those with better academic performance have more positive attitudes regarding math than those with poorer academic performance. These results were confirmed in wider research, concerning math study attitudes among the secondary school students of nine countries, developed by Sanchez et al. [14].

Lipnevich et al. [20] in a study developed with USA and BieloRussian middle school students highlighted the importance of attitudes in predicting academic achievement, when it showed that mathematics attitudes explained a variance of 25% to 32% in mathematics achievement, with much of the explained variance independent of ability in math.

Nevertheless, Georgiou et al. [16] showed that high achievement could serve to predict a positive attitude towards math, but such an attitude could not predict stronger achievement. However, these authors emphasize the role of teachers and schools in changing attitudes stating that, math achievement could be improved by, for example, better teaching methods, more motivated teachers or better course books, which has as its corollary the improvement of attitudes towards math.

1.4. Mathematics Learning Environments and Attitudes. Akey's [21] work showed that several aspects of school context (e.g., teacher support, student-to-student interaction, and the academic and behavior expectations of the teacher) were significantly related to student attitudes and behaviors. Akey [21] concluded that the class environment where teachers who students see as supportive promote student feelings of control and confidence in their ability to succeed. The way students perceive teacher characteristics will affect their attitudes towards mathematics [22]. Maat and Zakaria and Vaughan [22, 23] identified a significant relationship between learning environment and attitude towards mathematics. Students with a higher perception of the learning environment and a more positive perception of their teachers have more positive attitudes towards mathematics [22]. Rawnsley and Fisher [24] also found that students had more positive attitudes toward mathematics when their teacher was perceived to be highly supportive.

1.5. Motivation and Attitudes. A number of authors have shown that the relationship between aspects of the social environment and student emotional aspects may be mediated by other variables such as control-related appraisals and values-related appraisals [25, 26]. Therefore, competence support, autonomy support, expectations, and feedback that students receive from others have an impact on their cognitive appraisals and these are the main sources of their emotional dispositions. When studying attitudes, it is important to take into consideration the role of these mediated variables where we can include the motivation features of each student. In this sense, Wigfield [27], in reading specific domain, maintains that attitudes, realized as the individual's feelings towards reading, could be related to the motivation of the individual concerned because they influence how much individuals involve themselves in reading activities. Attitudes are affective responses that accompany a behavior initiated by a motivational state [28]. Attitudes can therefore be linked directly to motivation and provide key information to a better understanding of attitudinal and motivational processes. In the domain of maths there is little research that studies the relationships between motivation and attitudes. However, a number of studies have highlighted some specific associations. Singh et al. [2] used two sets of items to tap motivation, one related to attendance of school and classes and another to participation and preparedness for math classes. The authors concluded that mathematics attitude was affected by motivational factors since significant direct effects of .19 and .21, of these two motivation components were identified in student attitudes. Students who displayed school behavior associated with low motivation (e.g., coming late to school, skipping classes, coming unprepared without books and homework) had a more negative attitude toward mathematics. Other authors have taken into consideration Effort as an indicator of motivation [29, 30]. Reynolds and Walberg [30] using structural equation modeling to analyze diverse factors effects on math's performance and attitudes with 11th grade students, identify a significant effect on motivation in math attitudes. Hemmings and Kay [29] in a study with 10th grade students also verified that Effort was positively and significantly related to math attitudes.

1.6. Objectives. This study has two main objectives: firstly, to analyze the effects on math attitudes of factors usually analyzed in the literature (gender, grade, and achievement) among Portuguese school students; secondly, to analyze the effect on attitudes in this group of other factors that have been less well researched, associated with learning environment (e.g., perceived math's teacher and peers' support) and with the motivational characteristics of students.

2. Method

2.1. Participants. 1719 fifth-to-twelfth grade Portuguese students from different schools in Lisbon and the surrounding area participated in the study. They were from a wide range of social and economic backgrounds. There were 869 boys and 850 girls. In Portugal, school grades are organized in four cycles: 1st Cycle (1st to 4th grade), 2nd Cycle (5th and 6th grades), 3rd Cycle (7th to 9th grade), and Secondary (10th

Table 1: Distribution of participants by study cycle.

Level	*n*	%
2nd Cycle (5th and 6th grades)	451	26.2
3rd Cycle (7th, 8th, and 9th grades)	847	49.3
Secondary (10th, 11th, and 12th grades)	421	24.5

to 12th grade). In this research participants were from the 2nd and 3rd Cycles and Secondary. The student distribution according to study cycle is presented in Table 1.

In terms of achievement at mathematics, Portuguese students from the 2th and 3rd Cycles have been evaluated on a five point scale, where 1 and 2 are negative marks, 3 is medium, and 4 and 5 are good or very good marks. Secondary students are assessed on a scale ranging from 0 to 20 that we converted into a 5-point scale similar to those of other cycles. Based on a mean of the two last mathematics evaluations, students in this research were organized in three different achievement groups: Low—with marks lower than 3 (27.4%), Medium—with marks of 3 or 3,5 (30.1%), and Good—whose marks ranging from 4 to 5 (42.5%)

2.2. Procedure. Data used in this study was collected at school. Letters describing the study were sent to parents who gave their written consent to the head teacher.

Questionnaires were administered in the classroom under the supervision of a member of our research team. Questionnaires were read aloud for younger students whenever it was thought necessary.

2.3. Instruments. The motivation towards math was measured through a version of IMI Intrinsic Motivation Inventory, directed towards Mathematics, taking into consideration three dimensions: Perceived Competence, Perceived Choice, and Value/Utility [31]. This instrument is conceptualized to take into consideration the main constructs of the Self-Determination Theory (SDT) [32, 33]. Therefore, the Perceived Choice and Perceived Competence are theorized as positive predictors of intrinsic motivation and are related to the SDT innate psychological needs of autonomy and competence [34]. The Value/Usefulness subscale embodies the idea that people internalize and develop more self-regulatory activities when experience is considered as valuable and useful for them [34].

The questionnaire comprised 14 items, distributed over three dimensions: Perceived Competence—four items (e.g., "I think I am pretty good at Math's activities"); Value/Utility—five items (e.g., "Math's activities are valuable to me"); Perceived Choice—five items (e.g., "I only do the Math's tasks because the teacher orders me to"). All items were scored on a 6-point scale ranging from 1("Never") to 6 ("Always"). Items from the Perceived Choice dimension were reversed due to their negative formulation. In this instrument higher scores are related to intrinsic motivation characteristics.

In order to verify the adequacy of item inclusion in the correspondent dimension, reliability has been analyzed using Cronbach's Alpha. Reliability scores for our sample in the three subscales can be considered adequate as Cronbach's Alpha values were .80 for Perceived Competence, .84 for Perceived Choice, and .93 for Value/Utility [35].

Student perceptions of classroom support (teacher and students support) and attitudes towards mathematics were measured by 16 items extracted from the classroom climate scale "In my Math class" [36]. Teacher Social Support (TSS) is a six item dimension looking at the extent to which students feel that their math teacher supports them (e.g., "My math teacher cares about how much I learn"). Students Social Support (SSS) is an index calculated through five items related to how classmates care and support them (e.g., "In math class students want me to do my best in math work"). Based on the conceptualization of attitudes as positive or negative affect associated with a certain subject [9], in this research Student Attitudes towards math is a five item index relating to how students feel in math class and when performing math school tasks (e.g., "When the teacher questions me about math I feel good"). All items were scored on a 6-point scale ranging from 1 ("Never") to 6 ("Always").

Reliability coefficients were .81 for SSS, .95 for TSS and .84 for Attitudes.

3. Results

The results presented focus on two main vectors. Firstly the descriptive statistics of participants' attitudes, motivation and perceived social support towards mathematics, and also their differences considering gender, grade, and math performance were analyzed. Secondly a hierarchical analysis using structural equation modeling, utilizing three blocks of variables, was carried out. These blocks were background, motivation, and support-related variables.

Descriptive statistics of students' perceived social support, motivations, and attitudes toward mathematics are presented in Table 2 in relation to gender, grades, and math performance. Attitude scores vary between 3.37 and 4.11 when different groups organized according to gender, cycle and math performance are considered. These results are above the middle point of the scale, which show that, overall, those students present positive attitudes towards mathematics.

When considering motivation dimensions we can see that scores are near the midpoint of the scale, and some differences are introduced when considering gender, study cycle, and achievement. These scores show that younger students (2nd Cycle) present higher motivational scores when compared with their older colleagues. The same pattern is presented with achievement, since students with low marks present lower scores in all dimensions when compared to medium and good achievers, and good achievers have the highest scores.

With regard to social support, the results are almost similar across gender. In looking at grade and achievement one can see that younger students perceived greater support from their math teachers. Better achievers also seem to feel

Table 2: Descriptive statistics for attitudes towards mathematics by gender, study cycle and math achievement.

		Gender				Study cycle						Math achievement					
		Boys		Girls		2nd Cycle		3rd Cycle		Secondary		Low		Fair		Good	
		M	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD	*M*	SD
Motivation	Attitudes	3.72	1.25	3.70	1.26	4.11	1.30	3.67	1.28	3.37	1.04	3.38	1.30	3.62	1.17	3.99	1.23
	Value/utility	4.16	1.48	4.46	1.35	4.96	1.13	3.84	1.54	4.43	1.09	4.0	1.32	4.32	1.34	4.43	1.51
	Perceived choice	3.67	1.31	3.90	1.33	4.0	1.44	3.69	1.36	3.72	1.07	3.52	1.27	3.65	1.28	4.04	1.34
	Perceived competence	3.65	1.27	3.42	1.16	3.82	1.17	3.5	1.3	3.3	0.86	3.07	1.18	3.47	0.91	3.88	1.25
Social support	Teacher social support	4.33	1.52	4.60	1.43	5.03	1.12	4.06	1.65	4.69	1.19	4.31	1.52	4.53	1.37	4.52	1.54
	Students social support	3.45	1.14	3.62	1.09	3.32	1.1	3.67	1.12	3.49	1.10	3.51	1.15	3.52	1.14	3.56	1.1

that they have more support from their teachers. Students social support scores are very similar when comparing students according to gender, cycle, or achievement.

In order to clarify differences in means, with regard to attitudes toward mathematics (Table 2) we performed a univariate analyses of variance (ANOVA) with gender, grades, and math achievement as fixed factors. No gender effect was identified, although Cycle ($F(2,1701) = 41.904$, $P < .001$), and math achievement ($F(2,1701) = 61.075$, $P < .001$) introduce significant effects in attitudes towards mathematics. Interaction effects were also identified for Gender*Cycle ($F(2,1701) = 5.999$, $P = .003$) and Cycle*achievement ($F(2,1701) = 14.441$, $P < .001$). Multiple comparisons with a Tukey post hoc test show that attitudes towards mathematics became less positive as schooling continues ($P < .001$, from 2nd to 3rd Cycles; $P < .001$, 3rd Cycle to Secondary school). A similar lowering tendency with regard to Math achievement was confirmed, with lower achieving students having less positive attitudes than Medium and Good students, and also with Medium students differing from Good students in their attitudes (low/medium $P < .005$; low/good $P < .001$; medium/good $P < .001$).

An interaction effect between gender and Cycle on attitudes towards mathematics (Figure 1) is evidenced by the different pattern presented by girls through schooling, relative to boys. Whereas girls present a systematic decline in attitudes toward mathematics with their progression through schooling (post hoc comparisons with Tuckey test $P < .001$), boys present less positive attitudes towards mathematics in the 3rd Cycle and Secondary school level than in the 2nd Cycle ($P < .001$) but boys in the 3rd Cycle and Secondary present similar attitudes towards mathematics.

The interaction effect between Cycle and achievement on attitudes towards mathematics is presented in Figure 2. Low achievers show an inverted U shape curve, becoming less positive in their attitudes towards mathematics from the 3rd Cycle to the Secondary school level ($P = .002$), whereas medium achievers adopt more negative attitudes towards mathematics the further they advance in their schooling ($P < .001$ for the differences between Secondary school level and 2nd and 3rd Cycles; $P = .08$ for the difference between 2nd and 3rd Cycles). Good achievers show a greater decrease, between the 2nd and 3rd Cycles ($P < .001$), in attitudes towards mathematics and then a small rise when they are at Secondary school (albeit not significant). From

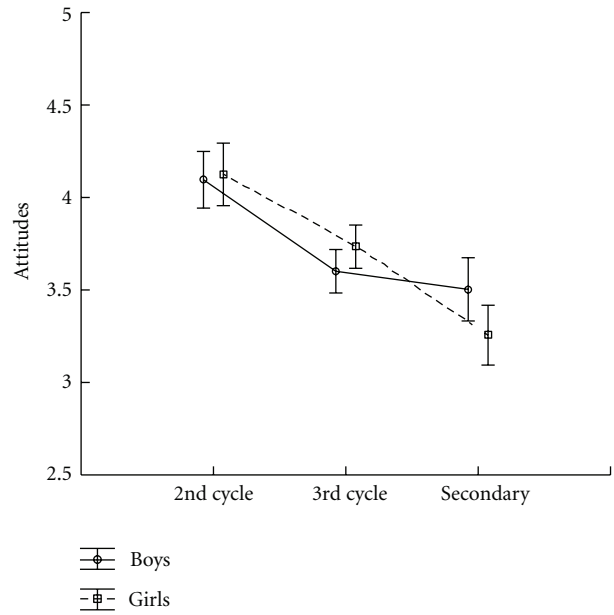

Figure 1: Interaction effect between gender and cycle on attitudes towards mathematics.

the point of view of study cycle, this interaction effect is evidenced by the fact that 3rd Cycle students present similar levels of attitudes towards mathematics, independent of their achievement status, which does not happen in the other cycles of Studies ($P < .001$ for the differences between the three levels of achievement at the 2nd Cycle, and between good achievers and the other two groups at Secondary school level).

Bivariate correlations were computed to examine relations among background, motivational, attitudes and social support variables and are presented in Table 3.

The correlation scores are in general positive, significant, but not very strong. The most positive highlights the associations between attitudes and motivation or support variables. Negative correlations were found when considering Cycle pointing to the idea that older students present less positive attitudes lower levels of motivation and feel less support from their teachers.

In order to test the relationships between background, motivational and social support-related variables with attitudes towards mathematics, we carried out a hierarchical

Table 3: Correlations between study variables.

	Gender	Math achievement	Cycle	Value/ Utility	Perceived Choice	Perceived Competence	Teacher social support	Students social support
Gender	—							
Math achievement	.028	—						
Cycle	.055*	.013	—					
Value/utility	.086**	.116**	−.140**	—				
Perceived choice	.087**	.178**	−.079**	.453**	—			
Perceived competence	−.096**	.289**	−.157**	.431**	.326**	—		
Teacher social support	.090**	.040	−.087**	.706**	.319**	.257**	—	
Students social support	.076**	.020	.055*	.228**	.127**	.121**	.287**	—
Attitudes	−.007	.212**	−.209**	.549**	.562**	.529**	.494**	.299**

* Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level.
Gender codification: (1) boys; (2) girls.

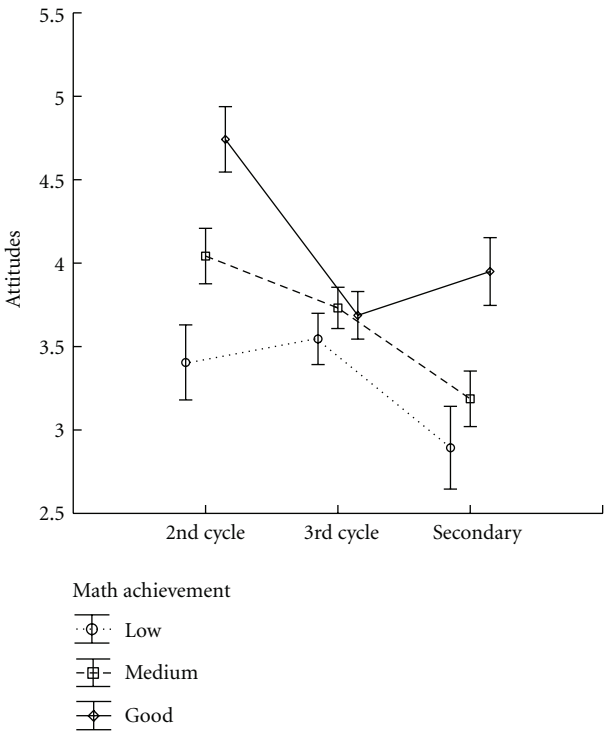

Figure 2: Interaction effect between math achievement and cycle on attitudes towards mathematics.

analysis using structural equation modeling with Amos 20.0. In this analysis, we have taken three blocks of variables into consideration (Table 4). The first block, named background variables, includes gender, grade, and performance in mathematics; the second block includes mathematics motivation dimensions and the third block looks at social support variables. Because all the variables in the second and third block are self-reported, potentially introducing bias with spurious relations, the recommendations of Podsakoff et al. [37] were followed in the treatment. Thus, in order to control for method bias due to the fact that all measures are self-reported, in models 2 and 3 we added a common latent factor with all the items of all measures loading in this factor. In the three models tested we only considered the relationships between each variable and attitudes, discarding other possible relationships among them. All the models tested presented acceptable fit (Model 1: AGFI −.98, CFI −.96, TLI −.94, RMSEA −.06; Model 2: GFI −.92, CFI −.92, TLI −.89, RMSEA −.076; Model 3: GFI −.91, CFI −.93, TLI −.92, RMSEA −.06).

In all models the three blocks of variables (background, motivation, and social support) are significant contributors to Attitudes. Background variables explained 8.9% of the variance and Math Motivation variables explained a further 31.3%. When adding Support variables (teacher and students) the model gains a further 24.1% in the variance explained. Therefore the total amount of variation in Attitudes explained by all the variables included in the final model is 64.3%. We can also verify in Model 3 that among Background variables only Cycle has a significant effect on Attitudes ($\beta = -.055$ $P < .001$). Among motivational variables Perceived Choice and Perceived Competence relate significantly to attitudes ($\beta = .451$ and $\beta = .495$ resp.), whereas both dimensions of social support are positively related to attitudes towards Maths. The contribution of both is also significant for this model ($P < .001$), with a higher contribution of the perception of teacher social support ($\beta = .307$), when compared with the perception of support provided by peers ($\beta = .246$).

4. Discussion

This paper first sought to characterize attitudes towards mathematics in students from 5th to 12th grade and to analyze the effects of gender, cycle, and math performance on these attitudes. Results showed that, in general, the students had positive attitudes towards mathematics, although scores were not very high and distributed mostly around the midpoint. Despite this overall positive attitude towards mathematics the scenario changes when we consider Cycle. Cycle effects are significant and mean that during schooling, attitudes towards mathematics become less positive. In fact, students in the 2nd Cycle present attitudes towards

Table 4: Parameter estimates and associated standard errors hierarchical analysis on attitudes towards mathematics.

	Model 1 Background		Model 2 Background and motivation		Model 3 Full model	
	SE	β	SE	β	SE	β
Gender	.059	−.011	.045	−.017	.041	−.036
Cycle	.042	−.227**	.032	−.146**	.029	−.176**
Math achievement	.034	.194**	.026	.015	.024	.022
Value			.032	.029	.094	.087
Perceived Choice			.029	.452**	.029	.451**
Perceived Competence			.035	.420**	.029	.495**
Teacher social support					.039	.307**
Students social support					.031	.246**
R^2	.089		.402		.643	
ΔR^2(from previous model)			.313**		.241**	

* $P < 0.01$, ** $P < 0.001$.

mathematics which are clearly positive, whereas students in Secondary school exhibit values below the midpoint of the scale. Considering that attitudes towards learning can be related to motivation [38] the decrease in attitudes towards mathematics can be associated with the overall decrease in intrinsic motivation, competence-related beliefs, interest and task values that occur during adolescence [39–41]. This decline is experienced in math in particular [41–43]. Motivation theorists have argued that during adolescence interests are directed towards other fields of experience which could explain the fall in school-related attitudes and interest. An additional explanation is related to the organization of the math curriculum which becomes more demanding as students move through grade levels, requiring increasingly abstract levels of understanding [44]. However, challenge is also an important feature of motivation [45] inasmuch as the challenge is not that great that it would be experienced as overwhelming, leading to feelings of helplessness. Interaction effects between Cycle and Math Achievement seem to provide some support to this explanation. Medium achievers show a gradual decrease in attitudes towards mathematics across the school years whereas low achievers present a slight increase from 2nd to 3rd Cycles and a decline in the transition from 3rd Cycle to Secondary school. An inverse pattern is presented by good achievers. The differences in the pattern of good and low achievers seems to support the hypothesis of differences in the way the challenges set by math learning are experienced by students belonging to different achievement groups. For good achievers mathematical tasks are likely faced as real challenges which could increase intrinsic motivation, raising the sense of competence when the tasks are solved, and leading to the development of positive attitudes towards math. Conversely, for low achievers math tasks are likely experienced as unsurpassable obstacles that will be won infrequently, producing low self-belief in competence and negative attitudes towards mathematics.

Gender-related attitudes towards mathematics seem to be identical. This finding corroborates the results of other research that claims that boys and girls present very similar attitudes towards mathematics [3, 5, 6, 15, 16]. However, results also show an interaction effect between gender and study cycle that results in a systematic decline in attitudes towards mathematics along schooling. This counteracts the pattern presented by boys which showed a decrease in their attitudes from the 2nd to the 3rd Cycles but which then stabilized. This progressive decline in attitudes by girls can be explained with reference to gender stereotypes [46]. Traditionally math is viewed as a male-dominated domain which is evident in career choices and jobs [47]. Studies in stereotyping and development in adolescence support the idea of gender intensification during middle and late adolescence accompanied by less flexibility to stereotyping [46]. This leads to the assumption of roles according to gender, assuming gender-type interests which could explain the less positive attitudes towards mathematics exhibited by girls at Secondary school.

Our findings concerning the relationship between math achievement and attitudes towards mathematics are consistent with research showing that good achievers develop more positive attitudes than lower achievers [2, 4, 14, 16, 20]. Achievement is usually related to self-belief in competence [41, 48] and self-belief in competence can be related to attitudes towards math [38], which suggests that when students succeed at a math task, it increases their sense of competence and this may promote more positive attitudes.

The hierarchical analysis using structural equation modeling extends the findings of this study. The first model tested accounts for less than 9% of variance in attitudes, showing that the effects of Gender, Cycle and Math Achievement are relatively small. Moreover, the contribution of Cycle and achievement for attitudes towards mathematics diminished when psychological variables of students were added. In fact, motivational variables increase the amount of variance explained in attitudes, showing a close relationship between key features of intrinsic motivation and attitudes towards mathematics. Authors involved in intrinsic motivation research have shown that students learn more effectively when they are interested and when they enjoy what they are learning [6, 32]. According to these authors students who feel competent and self-determinate in an ongoing, continuous

way, increase their intrinsic motivation. Likewise, positive attitudes towards mathematics may also increase since they have been conceived as positive or negative emotional dispositions toward a subject and positive emotion is, in general, perceived as pleasurable [9]. Our findings support these ideas revealing that the positive predictors of intrinsic motivation (Perceived Competence and Perceived Choice) and are more strongly related to attitudes towards mathematics. Adding learning environment-related variables to the model also produces an increase in the explanation of the variability of attitudes.

Among the variables considered (teacher support and peer support), teacher support shows closer relationships to attitudes. In the final model, this is the third strongest relationship which shows the importance of teachers in the development of positive attitudes towards math.

Despite the importance of teacher support as a significant predictor of attitudes, we cannot neglect the effect of peer support identified in our data and in research by Fraser and Kahle [8]. However, the improvement in attitudes is likely to be more significant when taking into consideration different environments, but the main contribution is determined in the class environment. Research on this topic has shown that teacher support with regard to autonomy affected student motivation, among other aspects, [49, 50] and that different pedagogical goals also explained variations in student math motivation [51]. Taken together, these findings highlight the role of the teacher in supporting student learning, attitudes and even motivation and have some implications for education and instructional practices. As Aunola et al. [51] have shown, teacher goals may influence child motivation and attitudes not only through their instructional practices and the tasks they propose to students, but also through the messages they send out about learning in general. In this sense if teachers create situations that promote pleasure, are seen as self-determinate and students feel competent; intrinsic motivation can increase [32, 33], and this may also promote positive attitudes towards mathematics. A teacher who is supportive to students [52], who shapes student expectations about learning in a positive way [53], who sets meaningful tasks which are somewhat, but not excessively challenging [54, 55], and promotes cooperative learning environments [23, 56] will probably stimulate intrinsic motivation in their students and, as a corollary, may contribute to the development of more positive attitudes towards math.

The results presented here suggest strong relationships between motivation and support related variables with attitudes. However, the cross-sectional natures of the design used do not allow us to infer causal relationships between the variables. Therefore, for the interpretation of the results we cannot say that motivational and social context factors influence student's attitudes, although we cannot rule out that variables closely related to motivation and social support in the classroom have a significant relationship to attitudes. The use of multiple assessments over time could provide a clearer view of the causal relations between variables. Furthermore, a longitudinal design could help us to achieve a better understanding not only of the changes in time, but also of the development of attitudes toward mathematics and the effects of other variables such as background, motivation, and social support. Likewise, the use of a small group study and the monitoring of the group including multiple assessments over time, also using qualitative data, could provide a deeper comprehension of the heterogeneity of attitudes among subjects and allows for the researcher to take into account the effects of aging on the group. As suggested by Hannula [57] qualitative data collected from observations, interviews, and case studies can be useful in studying attitudes more accurately and to recognize possible factors behind the characteristics of attitudes and any changes that take place.

Another limitation of this study is the fact that only self-report measures were used. It is possible to reduce the bias introduced by single-source data using appropriate statistical procedures [37] as we have done in this research. Although, a worthwhile addition to a future study to overcome this problem would be to incorporate more than one data source, namely, data collected through classroom observations and teacher-reports.

A good deal of research has been conducted on attitudes towards mathematics, but most of the analyses used have focused on how specific variables influence or are related to attitudes, considering these variables in an isolated way. In order to have a more complex perspective towards this topic, the present research has attempted to examine the combined effects of the individual, social contextual, and motivational variables on attitudes toward mathematics. The results of the integration of data provide a more in-depth understanding of the different variables which allow for the exploration of different routes in promoting positive attitudes toward mathematics. On the other hand, researches concerning the relationship between motivation, social support from teachers and peers, and attitudes are scarce. In this regard, our data emphasizes the importance of these variables when trying to understand attitudes toward mathematics.

Lack of student motivation and engagement in academic work is an issue of concern amongst teachers. Since our findings confirm that attitudes are deeply related to motivation and social support, we believe that developing strategies in educational contexts, to improve teacher support and student engagement could be of vital importance in improving not only attitudes but also mathematical performance among students throughout their schooling.

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