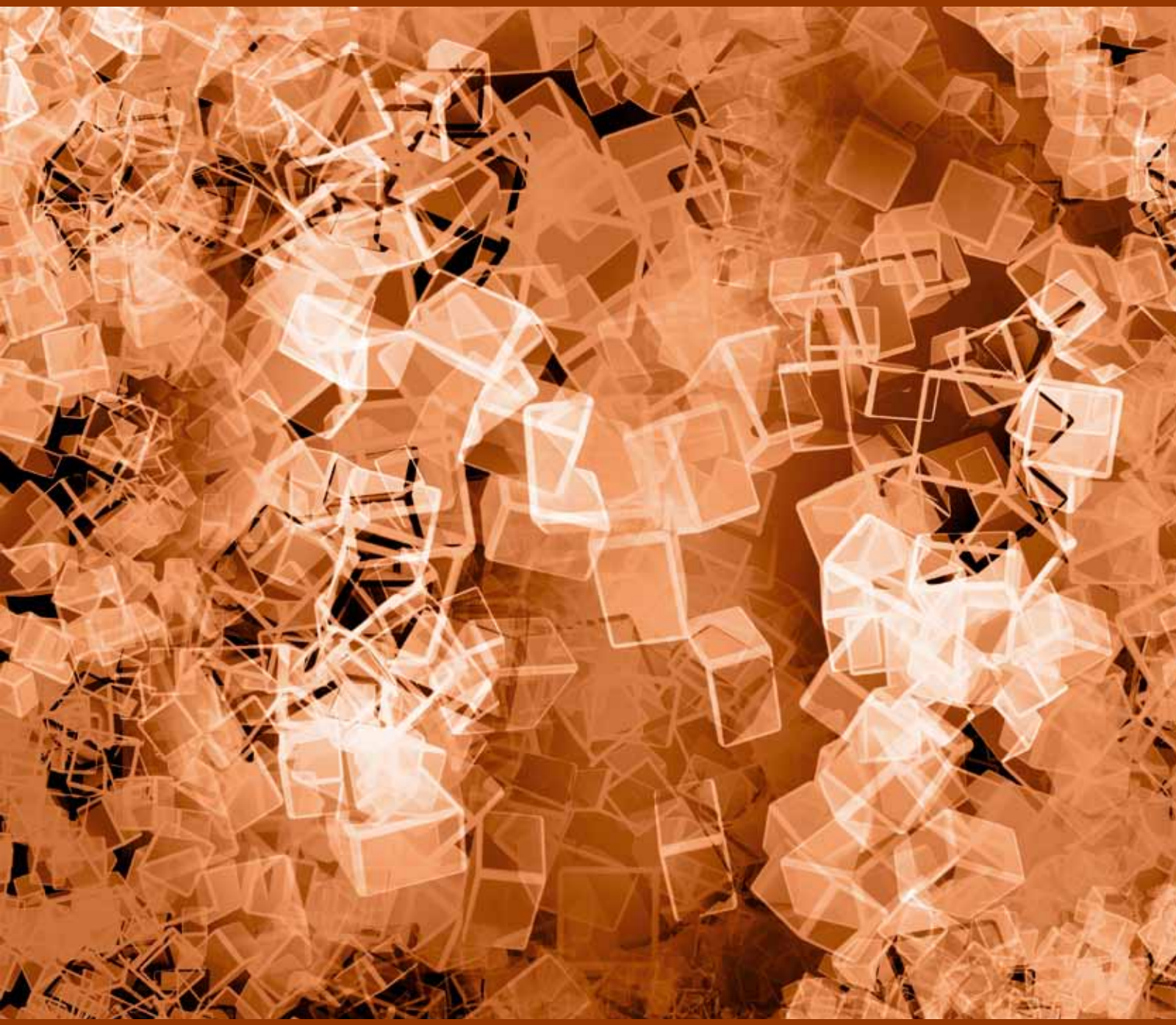
Discrete Dynamics in Nature and Society

Discrete Dynamics in Supply Chain Management

Guest Editors: Tingqui Chen, Kai Huang, and Zhigang Jiang



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Editorial Board

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Editorial

Discrete Dynamics in Supply Chain Management

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Supply chain management (SCM) is considered as the integration of business processes in providing products or services to end customers by establishing a strategic alliance of all parties, which has become a critical means of adding value to products and increasing the company's competitive advantage. However, organizations in the supply chains in real world are always vulnerable to partial or complete disruptions. Various factors such as natural disasters, labor strikes, power outages, parts shortages, quality related recalls, transportation interruptions, and machine breakdowns can lead to performance fluctuations or even disruptions of business processes in organizations, which indicates that supply chain processes are dynamic, discrete, volatile, and unpredictable.

The main objective of this special issue is to present the original research and review articles on the latest theoretical and practical achievements that will contribute to the field of discrete and dynamics supply chain management, in all branches of management science.

The special issue received 52 high quality submissions from different countries all over the world. All submitted manuscripts have followed the same standard (peer-reviewed by at least three independent reviewers) as applied to regular ones to this journal. Due to space limit, only 21 papers could be published (acceptance ratio of 1 : 2.5). Inevitably, difficult decisions had to be made, and some high-quality submissions could not be included. The primary guideline was to demonstrate discrete dynamics in supply chain management. Besides, some novel research questions from different applications that are worth further investigation in the future are also included.

In the paper *"Price and service competition of dual-channel supply chain with consumer returns,"* L. Ren et al. propose dual-channel supply chain models involving consumer returns policies. Also, the price and service competition between retail channel and direct channel is considered in the models. According to the models, they analyze the optimal decisions in both centralized and decentralized scenarios. Then they design a new contract, coordinate the dual-channel supply chain, and enable both the retailer and the manufacturer to be a win-win.

In the paper *"Viability discrimination of a class of control systems on a nonsmooth region,"* N. Zhao et al. consider the viability for both an affine nonlinear hybrid system and a hybrid differential inclusion on a region with subdifferentiable boundary. Based on the nonsmooth analysis theory, they obtain a method to verify the viability condition at a point, when the boundary function of the region is subdifferentiable and its subdifferential is convex hull of many finite points.

In the paper *"Incentive contract in supply chain with asymmetric information,"* Y. Su et al. use principal-agent theory and the theory of regulation to design the contract to realize the maximization of principal's profit on the condition that the contract satisfies the participant and incentive conditions of agent. As a result, it is obvious that the contract achieves the goal of control. In addition, it also can be concluded that the amount of rent that the manufacturer can obtain is up to the value of his information and the condition of his resource.

In the paper entitled *"A constraint programming method for advanced planning and scheduling system with multilevel structured products,"* Y. Peng et al. deal with the advanced

planning and scheduling (APS) problem with multilevel structured products. A constraint programming model is constructed for the problem with the consideration of precedence constraints, capacity constraints, release time, and due date. A new constraint programming (CP) method is proposed to minimize the total cost. This method is based on iterative solving via branch and bound. And, at each node, the constraint propagation technique is adapted for domain filtering and consistency check. Three branching strategies are compared to improve the search speed. The results of computational study show that the proposed CP method performs better than the traditional mixed integer programming (MIP) method. And the binary constraint heuristic branching strategy is more effective than the other two branching strategies.

In the paper *"Multiobjective vehicle routing problem with route balance based on genetic algorithm,"* W. Zhou et al. propose a genetic algorithm to solve the biobjective vehicle routing problem with time windows simultaneously considering total distance and distance balance of active vehicle fleet. A new complex chromosome is used to present the active vehicle route. Through tournament selection, one-point crossover, and migrating mutation operator, the solution of the problem is solved. In experiment on Solomon's benchmark problems, considering the total distance and distance balance, the results are improved in all classes of problems. According to the experimental results, the suggested approach is sufficient and the average GA performance is good.

In the paper entitled *"The game analysis of manufacturers' political connections on product safety in supply chain: evidence from China,"* Z. Na and W. Fusheng study the political connections on product safety in supply chain. In market economy, information asymmetry exists throughout the entirety of supply chains that ought to ensure product safety. Due to the existence of game relations between the government and manufacturers in the aspects of product safety and regulation, the formation of market equilibrium depends on political connections between the government and manufacturers. Based on study and analyses of a static game model and a dynamic game model, this paper reveals that governments and manufacturers must use positive political connections to achieve product protection and supervision of safety throughout the supply chain. On the other hand, negative political connections lead to losses of both governmental credibility and social profits. This study indicates that inherent mechanism of political connections exists in the supply chain; it will help to enrich the theory of supply chain.

The paper of H. Wang et al. entitled *"Optimizing route for hazardous materials logistics based on hybrid ant colony algorithm"* devises an improved hybrid ant colony algorithm (HACA) to deal with optimizing route for hazardous materials logistics (ORHML). To achieve the purpose of balancing risk and cost for route based on the principle of ACA that used to solve TSP, the improved HACA was designed. Considering the capacity of road network and the maximum expected risk limits, a route optimization model to minimize the total cost is established based on network flow theory. Improvement on route construction rule and pheromone updating rule was adopted on the basis of the former algorithm. An example was analyzed to demonstrate the correctness of the application. It is proved that improved HACA is efficient and feasible in solving ORHML.

In the paper entitled *"System dynamics model for VMI&TPL integrated supply chains,"* G. Li et al. establish VMI-APIOBPCS II model by extending VMI-APIOBPCS model from serial supply chain to distribution supply chain. Then TPL is introduced to this VMI distribution supply chain, and operational framework and process of VMI&TPL integrated supply chain are analyzed deeply. On this basis VMI-APIOBPCS II model is then changed to VMI&TPL-APIOBPCS model and VMI&TPL integrated operation mode is simulated. Finally, compared with VMI-APIOBPCS model, the TPL's important role of goods consolidation and risk sharing in VMI&TPL integrated supply chain is analyzed in detail from the aspects of bullwhip effect, inventory level, service level, and so on.

In the paper entitled *"A location-allocation model for seaport-dry port system optimization,"* X. Feng et al. construct a location-allocation model for the regional seaport-dry port network optimization problem and develop a greedy algorithm and a genetic algorithm to obtain its solution. This model is applicable to situations under which the geographic distribution of demand is known. A case study involving configuration of dry ports near the west bank of the Taiwan Strait is conducted, and the model is successfully applied.

In the paper entitled *"An inventory controlled supply chain model based on improved BP neural network,"* Wei He applies the improved BP neural network model to predict the inventory level of an automotive parts company. The results show that the improved algorithm not only significantly exceeds the standard algorithm but also outperforms some other improved BP algorithms both on convergence rate and prediction accuracy.

In the paper entitled *"Tourist behavior pattern mining model based on context,"* D.-s. Liu and S.-j. Fan take the context into consideration and propose an analyzed method to the tourist based on the context. Firstly, they analyze the context which influences the tourist behavior patterns, select the main context factors, and construct the tourist behavior pattern model based on it. Then, they calculate the interest degree of the tourist behavior pattern and mine out the rules with high interest degree with the association rule algorithm. At last, an experiment is given so as to show the feasibility and effectiveness of their method.

In the paper entitled *"Coordination in the decentralized assembly system with dual supply modes,"* X. Guan and M. Liu investigate a decentralized assembly system that consists of one assembler and two independent suppliers, wherein one supplier is perfectly reliable for the production, while the other generates yield uncertainty. Facing the random market demand, the assembler has to order the components from one supplier in advance and meanwhile requires the other supplier to deliver the components under VMI mode. They construct a Nash game between the supplier and the assembler so as to derive their equilibrium procurement/production strategies. The results show that the channel's performance is highly undermined by the decentralization between players

and also the combination of two supply modes. Compared to the centralized system, they propose an advance payment contract to perfectly coordinate the supply chain performance. The numerical examples indicate some management implications on the supply mode comparison and sensitivity analysis.

In the paper *"Dynamic pricing and supply coordination with reimbursement contract under random yield and demand,"* G. Li et al. study the dynamic pricing and supply chain coordination in a decentralized system that consists of one supplier and one manufacturer, in which both the market demand and production yield are stochastic. They show that the centralized expected profit is jointly concave in the production quantity and order quantity when the price is ex ante selected. They also derive the equilibrium strategies in the decentralized system and prove that the entire profit of supply chain is inevitably lower than that under centralized system. Based on this, the authors propose a reimbursement contract to coordinate the decentralized supply chain so as to achieve the maximized profit. It is worth mentioning that, under reimbursement contract, the equilibrium production and order quantities are irrelevant to the manufacturer's risk sharing coefficient but are only determined by the supplier's risk sharing coefficient.

In the paper *"Research on self-organization in resilient recovery of cluster supply chains,"* L. Geng et al. explore dealing with high-risk and low-probability disruptions. First, the paper describes the representation method of cluster supply chain resilience. Second, a cluster supply chain network structure generation model is proposed. And based on cascading effect model, it makes analysis of dynamic evolution process when cluster supply chain failure happens. Then it focuses on the self-organization characteristic, which contributes to cluster supply chain emergence of overall resilient recovery through local self-organization reconstruction behavior. They also make theoretical analysis of cluster supply chain network characteristics and its effect on the resilience, which helps to illustrate that the root of vulnerability lies in cascading failure while self-organization is the key to resilient recovery. Besides, with the study of self-organization characteristic, it provides theoretical guidance for local control and further achievement of overall resilient optimization.

In the paper entitled *"Optimal acquisition and inventory control for a remanufacturing system,"* Z. Jiang et al. propose a method for optimal acquisition and inventory control of a remanufacturing system. The method considers three inventories, one for returned item and the other for serviceable and recoverable items. Taking the holding cost for returns and recoverable and remanufactured products, remanufacturing cost, disposal cost, and the loss caused by backlog into account, the optimal inventory control model is established to minimize the total costs. Finally, a numerical example is provided to illustrate the proposed methods.

In the paper *"Two-level credit financing for noninstantaneous deterioration items in a supply chain with downstream credit-linked demand,"* Y. He and H. Huang assume that the items have the property of noninstantaneous deterioration and the demand is a function of downstream credit. Then, an EOQ model for noninstantaneous deterioration is built based on the two-level financing policy. The purpose of this paper is to maximize the total average profit by determining the optimal downstream credit period, the optimal replenishment cycle length, and the optimal ordering quantity per cycle. Useful theorems are proposed to characterize the method of obtaining the optimal solutions. Based on the theorems, an algorithm is designed, and numerical tests and sensitive analysis are provided. Lastly, according to the sensitive analysis, managerial insights are proposed.

In the paper *"Location optimization of multidistribution centers based on low-carbon constraints,"* P. Zhao et al. analyze the necessity of industrial carbon dioxide emission cost internalization in four aspects and build a model for multidistribution centers location in effort to reduce carbon footprint that can provide optimized strategy support for decision makers and logistic operators. Numerical examples are presented to illustrate the feasibility and effectiveness of the models.

In the paper *"Simulation research of space-time evolution of emergency logistics network reliability based on complex network theory,"* L. Huang et al. propose the conception and evaluation indexes of emergency logistics network connecting reliability to construct evaluation index system of complex network reliability and describe these indexes quantitatively to evaluate the network connecting reliability. Moreover, the network topological model and the simulation methods of reliability measurement when the network is under attack are present. Finally, the authors take three classical emergency logistics networks as examples, and through emulation analysis they obtain the connecting reliability changing situation of these three networks under random attack, the changing curve of the ratio of effective demand nodes, and emergence supply mileage of emergency logistics network with the same network density but different forms and then evaluate the emergency logistics network connecting reliability. This can provide references for the designing of emergency logistics network with high reliability and analysis means for research in other fields.

In the paper entitled *"Risk-averse suppliers' optimal pricing strategies in a two-stage supply chain,"* risk-averse suppliers' optimal pricing strategies in two-stage supply chains under competitive environment are discussed. The suppliers in this paper focus more on losses as compared to profits, and they care about their long-term relationship with their customers. R. Shen et al. introduce for the suppliers a loss function, which covers both current loss and future loss. The optimal wholesale price is solved under situations of risk neutral, risk averse, and a combination of minimizing loss and controlling risk, respectively. Besides, some properties and relations among these optimal wholesale prices are given as well. A numerical example is given to illustrate the performance of the proposed method.

In the paper *"Personal recommendation using a novel collaborative filtering algorithm in customer relationship management,"* C. Xu constructs a user recommendation model containing a new method to compute the similarities among users on bipartite networks. Different from other standard similarities, he considers the influence of each object node

including popular degree, preference degree, and trust relationship. Substituting these new definitions of similarity for the standard cosine similarity, the author proposes a modified collaborative filtering algorithm based on multifactors (CF-M). Detailed experimental analysis on two benchmark datasets shows that the CF-M is of high accuracy and also generates more diversity.

In the paper *"Collaborative policy of supply-hub for assemble-to-order systems with delivery uncertainty,"* G. Li et al. build a collaborative replenishment model in the assembly-to-order system based on supply-hub under delivery uncertainty, transform the original model into a one-dimensional optimization problem appropriately, and then calculate the optimal assembly quantity of the manufacture and the optimal order points of various component suppliers. In order to enable collaborative replenishment, penalties and bonus mechanisms are proposed for supply-hub to coordinate various component suppliers. Based on the analysis in detail, they can conclude that when the penalty coefficient is very large, the suppliers will do their best to deliver on time, and when the reward of the supplier's on-time delivery is great enough, the suppliers will avoid delay delivery. The numerical analyses also show that penalty and bonus mechanisms can significantly improve the supplier's initiative of collaborative replenishment, thus improving the service level in assemble-to-order systems. This paper also provides a theoretical basis as well as useful guidance to the practice of collaborative replenishment in assemble-to-order systems based on supply-hub under delivery uncertainty.

The study of discrete dynamics in supply chain management is still in its early stage. This special issue demonstrates the theoretical and practical importance of further studies in discrete dynamics in supply chain management.

Acknowledgments

We would like to express our gratitude to all of the authors for their contributions and the reviewers for their effort providing valuable comments and feedback. We hope this special issue offers a comprehensive and timely view of the area of discrete dynamics in supply chain management and that it will offer stimulation for further research. The work is also supported by the National Natural Science Foundation of China (51205295) and Wuhan Youth Chenguang Program of Science and Technology (2014070404010214).

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Research Article

Collaborative Policy of the Supply-Hub for Assemble-to-Order Systems with Delivery Uncertainty

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This paper considers the collaborative mechanisms of the Supply-Hub in the Assemble-to-Order system (ATO system hereafter) with upstream delivery uncertainty. We first propose a collaborative replenishment mechanism in the ATO system, and construct a replenishment model with delivery uncertainty in use of the Supply-Hub. After transforming the original model into a one-dimensional optimization problem, we derive the optimal assembly quantity and reorder point of each component. In order to enable the Supply-Hub to conduct collaborative replenishment with each supplier, the punishment and reward mechanisms are proposed. The numerical analysis illustrates that service level of the Supply-Hub is an increasing function of both punishment and reward factors. Therefore, by adjusting the two factors, suppliers' incentives of collaborative replenishment can be significantly enhanced, and then the service level of whole ATO system can be improved.

1. Introduction

The supply disruptions in ATO systems caused by upstream suppliers nowadays happen frequently due to the influences of natural disasters, strikes, terrorist attacks, political instability, and other factors. As shown by Li et al. [1, 17], in 2000 Philips Semiconductor Factory's fire led to Ericsson's supply disruption of the chip, which caused Ericsson a loss of 1.8 billion dollars and 4% of its market share. In July 2010, Hitachi's unexpected shortage of car engine control part resulted in the shutdown of Nissan's plant for 3 days, and the production of 1.5 million cars was affected by this. In March 2011, Japan's 9-magnitude earthquake in northeast devastated the industrial zone. Three major automakers in Japan, Toyota, Honda and Nissan, were affected by supply disruptions, and some Sino-Japanese joint ventures in China also had different levels of supply disruptions. Accordingly, driven by these serious losses caused by supply uncertainty, both scholars and practitioners try to find out effective ways to improve ATO systems' overall performances by handling upstream disruptions. Under this circumstance, Supply-Hub arises.

Supply-Hub, also called VMI (vendor-managed inventory) Hub, is often located near the core manufacturer to integrate the logistics operation of part or all suppliers and mostly managed by the third party logistics operator. Supply-Hub operation mode evolves from the traditional VMI operation mode. In practice, because there exist all sorts of problems in the distributed VMI operation mode, some advanced core manufacturers consider to manage independent warehouses in a centralized way instead of the original decentralized way through resources integration, organization reconstruction, and coordination optimization. This not only helps to reduce the investment cost in fixed facilities, but also can greatly reduce the operation and management cost of the whole supply chain. Gradually, a lot of the third party logistics distribution centers, which mainly focus on integration management service of upstream supply logistics, appear.

In this sense, the Supply-Hub can be viewed as an intermediary between the suppliers and manufacturer, and Indirect Distribution Channel is the intermediary between the manufacturer and retailers. Furthermore, the Supply-Hub can reduce the risk of components shortage caused by desynchronized delivery from different suppliers and improve the efficiency and benefit of supply chain. Nonetheless, although there exist some papers that take the Supply-Hub into consideration, how to coordinate the suppliers by useful policies for the Supply-Hub is still rarely examined explicitly.

To address the gap between the practice and current literature, we investigate the interaction between delivery uncertainty and coordinative policy of the Supply-Hub and mainly address the following questions:

(1) What are the optimal replenishment decisions for the Supply-Hub in ATO systems with multiple suppliers and one manufacturer in case of uncertain delivery time?

(2) How would the Supply-Hub coordinate the suppliers, eliminate the delivery uncertainty, and improve the whole service level?

(3) What are the relations between the two coordinative factors and service level of the Supply-Hub?

To answer these questions, we consider an ATO system with multiple suppliers, one Supply-Hub and one manufacturer. This paper aims to establish a cost model with consideration of the effects caused by each component's delivery time that may be sooner or later than the expected arrival time. The reorder point of each component and assembly quantity are regarded as the decision variables, and we propose an order policy that minimizes the supply chain's total cost. Since the model in this paper can be viewed as a convex programming problem, we provide the unique optimal solution. Finally, we apply the punishment and reward mechanisms to the Supply-Hub for the purpose of coordinating suppliers and improving service level, and through theoretical and numerical analysis we find the relations between the two coordinative factors and service level.

2. Literature Review

Production uncertainty can be attributed to the uncertainty of demand and supply. In recent years, some scholars investigate some secondary factors that cause supply delay in ATO systems, such as Song et al. [2, 3], Lu et al. [4, 5], Hsu et al. [6], Xu and Li [7], Plambeck and Ward [8, 9], Li and Wang [10], Lu et al. [11], Doğru et al. [12], Hoena at al. [13], Bernstein et al. [14],Reiman and Wang [15], Bušić et al. [16], and Li et al. [1, 17].

Song and Yao [3] consider the demand uncertainty in ATO systems under random lead time and expand random lead time into an inventory system assembled by a number of components. By assuming that the demand obeys passion distribution and that the arriving time of different components is independent and identically distributed, they conclude that the bigger the mean of lead time of components is, the higher the safety stock should be set, and they also give definite methods of finding the optimal safety stock under certain constraint of service level. Based on this, Lu and Song [5] consider the inventory system where products are composed of many components with random lead time. They deduce the optimal values that the safety stock should be set under different means of lead time. Hsu et al. [6] explore optimal inventory decision making in ATO systems in the situation that the demand is random, and the cost and lead time of components are sensitive to order quantity. Li and Wang [10] focus on the inventory optimization in a decentralized assembly system where there exists competition among suppliers under random demand and sensitive price. Hoena et al. [13] explore ATO systems with multiple end-products. They devide the system into several subsystems which can be analyzed independently. Each subsystem can be approximated by a system with exponentially distributed lead time, for which an exact evaluation exists. Bušić et al. [16] present a new bounding method for Markov chains inspired by Markov reward theory. With applications to ATO systems, they construct bounds by redirecting selected sets of transitions, facilitating an intuitive interpretation of the modifications of the original system. Li et al. [1, 17] consider an assembly system with two suppliers and one manufacturer under uncertainty delivery time. They prove that a unique Nash equilibrium exists between two suppliers. Decroix et al. [18] consider the inventory optimization problem in ATO systems where the product demand is random and components can be remanufactured. All literature above consider some optimal problems, such as stocks or order quantities in ATO systems from different perspectives. And the common characteristics are as follows: (1) the views expressed in these works are based on a single assembly manufacturing enterprise, rather than the whole supply chain. (2) Most papers assume that the replenishment of components is based on make-to-stock environment, rather than JIT replenishment. Therefore, how to realize the two-dimensional collaborative replenishment of multiple suppliers in JIT replenishment mode is the most urgent problem that needs to be solved currently.

Zimmer [19] studies the supply chain coordination between one manufacturer and multiple suppliers under uncertain delivery. In the worst case, under decentralized decision the optimal decisions of the manufacturer and suppliers are analyzed and in the best situation, under symmetric information, the optimal decision of supply chain is also obtained. Two kinds of coordination mechanisms (punishment and reward) are established, which realize the flexible cost allocation between collaborative enterprises. In the works of Gurnani [20],Gurnani and Gerchak [21], and Gurnani and Shi [22], the two-echelon supply chain is composed of two suppliers and one manufacturer. Under uncertain delivery quantity caused by suppliers' random yield, each side is optimized in decentralized and centralized decision and the total cost of supply chain is lower in centralized decision compared to the decentralized. Finally, the collaborative contract is proposed to coordinate the suppliers and manufacturer. In fact, the above articles study the assembly system based on the whole supply chain under

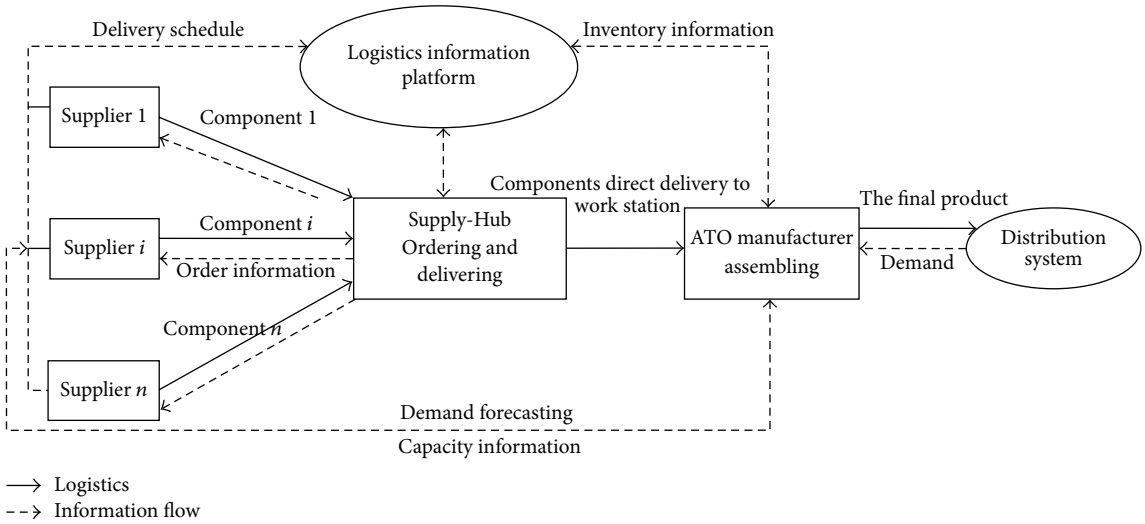

Figure 1: Framework of logistics and information flow based on the Supply-Hub.

random delivery quantity, but they do not take the random delivery time into consideration.

As to the Supply-Hub, Barnes et al. [23] find that Supply-Hub is an innovative strategy to reduce cost and improve responsiveness used by some industries, especially in the electronics industry, and it is a reflection of delaying procurement. Firstly they give the definition of Supply-Hub and review its development, then they propose a prerequisite to establish a Supply-Hub and come up with a way to operate it. Shah and Goh [24] explore the operation strategy of the Supply-Hub to achieve the joint operation management between customers and their upstream suppliers. Moreover, they discuss how to manage the supply chain better in vendor-managed inventory model, and find that the relation between operation strategy and performance evaluation of the Supply-Hub is complex and nonlinear. As a result, they propose a hierarchical structure to help the Supply-Hub achieve the balance among supply chain members.

Based on the Supply-Hub, Ma and Gong [25] develop, respectively, collaborative decision-making models of production and distribution with considering the matching of distribution quantity between suppliers. The result shows that, the total cost of supply chain and production cost of suppliers decrease significantly, but the logistics cost of manufacturers and cost of Supply-Hub operators increase. With the consideration that multiple suppliers provide different components to a manufacturer based on the Supply-Hub, Gui and Ma [26] establish an economical order quantity model in such two ways as picking up separately from different suppliers and milk-run picking up. The result shows that the sensitivity to carriage quantity of the transportation cost per unit weight of components and the demand variance in different components have an influence on the choices of the two picking up ways. Li et al. [27] propose a horizontally dual-sourcing policy to coordinate the Supply-Hub model. They indicate that the total cost of supply chain can be decreased obviously while the service level will not be reduced by using this horizontally collaborative replenishment policy.

However, how the Supply-Hub plays with the consolidation function is rarely discussed in detail, for example, how to improve the service level of upstream assembly system and efficiency of the whole supply chain. This issue is of great practical significance, because a wrong decision of certain component's replenishment will make the right decisions of other components' replenishment in the same Bill of Material (BOM) nonsense, thus leading to the low efficiency of the whole supply chain [27]. After introducing the BOM into consideration of order policy, due to the matching attribution of all materials, calculations of optimal reorder point of each component and assembly quantity are very complex, so we transform the original model into a one dimensional optimization problem and successfully obtain the optimal values of the decision variables. After that we propose a collaborative policy of the Supply-Hub for ATO systems with delivery uncertainty.

3. Model Description and Formulation

3.1. Model Assumptions. The operation framework of this model is illustrated in Figure 1 [25]. Based on this, we propose the following assumptions.

(1) According to the BOM, the manufacturer needs n different kinds of components to produce the final product and each supplier provides one kind of the components, with the premise that the delivery quantity of each component should meet the equation $\text{Item } 1 : \text{Item } 2 : \ldots : \text{Item } n = 1 : 1 : \cdots : 1$. The manufacturer entrusts the Supply-Hub to be in charge of the JIT ordering and delivery service. The supply chain is an ATO system which consists of n suppliers, one manufacturer, and one Supply-Hub. Note that the model in this paper only considers the cost of the two-echelon supply chain that includes the Supply-Hub and manufacturer and omits the suppliers' costs.

(2) The time spent by the manufacture for assembling the components is assumed to be 0, which is appropriate

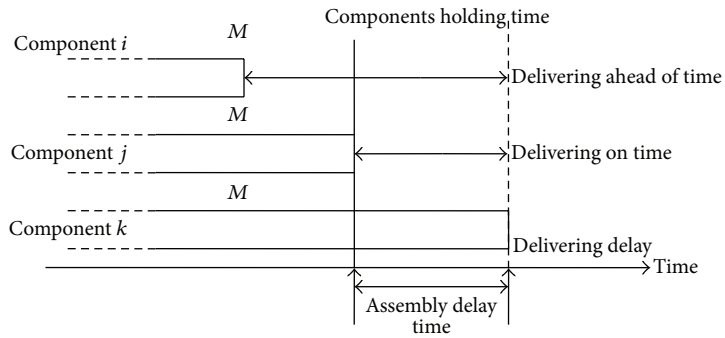

Figure 2: Arrival situations of different components in the Supply-Hub.

when the suppliers are relatively far away from each other. In addition, When the inventory of the final product turns to be used up, the manufacturer begins to assemble the components, and we call it the starting point of the assembly. So in the whole ordering and delivery process, there are two more situations in the Supply-Hub besides all components' arriving on time: (1) if the components arrive sooner than the expected assembly starting point, the Supply-Hub has to hold components until the manufacturer's inventory is used up. In this situation, the holding cost of all components is $\sum_{i=1}^{n} \mathrm{TC}_{h_i}$. (2) In another situation, because the Supply-Hub has to wait for all components' arrival, if there is a delay delivery of certain component, the assembly time will be delayed, resulting in the shortage cost TC_{π} (see Figure 2).

(3) The Supply-Hub delivers components to the manufacturer in certain frequency, such as K times, then Order Quantity = $k \times$ delivery quantity [28]. It may be assumed here that $k = 1$, and the Supply-Hub adopts the lot-for-lot method to distribute the components. If the manufacturer needs to assemble Q final products, the Supply-Hub needs to order Q components from each supplier. The lead time $Y_i (i = 1, 2 \ldots, n)$ of the components is mutually independent random variables, and the probability distribution function and probability density function are, respectively, $F_i(x)$ and $f_i(x)$.

(4) The market demand D per unit time for the final product is fixed, and the backorder policy is adopted to deal with shortage. Without loss of generality, we assume $\pi > h > \sum_{i=1}^{n} h_i$.

Related parameters are defined as follows.

A is unit order cost of components.

π is unit shortage cost of the final product.

Y_i is random lead time of component i.

L_i is late or early arrival period of component i.

h_i is unit holding cost of component i.

h is unit holding cost of the final product.

r_i is reorder point of component i (decision variables).

R_i is distribution parameter of lead time of component i.

Q is assembly quantity (decision variable).

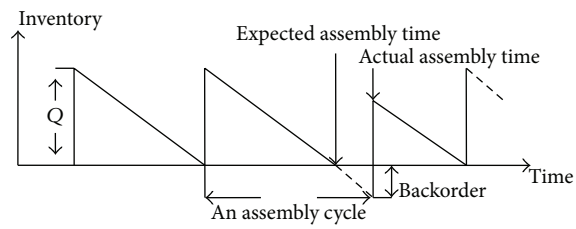

Figure 3: Inventory status of the manufacturer's final product.

3.2. Model Formulation. The period between two adjacent actual assembly starting points can be regarded as a cycle (see Figure 3). In each cycle, the Supply-Hub needs to deliver a collection of Q components directly to the manufacturer's work station. Besides, the purchase order should be issued before the expected assembly starting point, which should be issued at the moment of r_i/D. The late or early arrival period of each component can be expressed as the difference between actual lead time and expected lead time, or $L_i \equiv Y_i - r_i/D$. In addition, we define $L_i^+ \equiv \max\{L_i, 0\}$, which means the delay time of component i, and $L \equiv \max_{1 \le i \le n}\{L_i^+\} \equiv \max_{1 \le i \le n}\{L_i, 0\}$, which means the delay time of the manufacturer's assembling. If L_i is negative, it means component i is delivered before the expected assembly starting point.

Based on the assumptions and definitions above, we can derive the following conclusions.

(1) Average delay time of the manufacturer's assembling per cycle is $E[L]$.

(2) Average shortage quantity of the final product for the manufacturer is $D \cdot E^2[L]/2$ (as shown in Figure 4).

(3) Average holding cost of the final product for the manufacturer per cycle is $(h/2D)(Q - D \cdot E[L])^2$.

(4) Average holding time of component i for the Supply-Hub per cycle is $[L] - E[L_i]$.

Therefore, for the two-echelon supply chain model that consists of the Supply-Hub and manufacturer, the average total cost TC_p per cycle is the sum of the order cost of components, holding cost of the components, holding cost

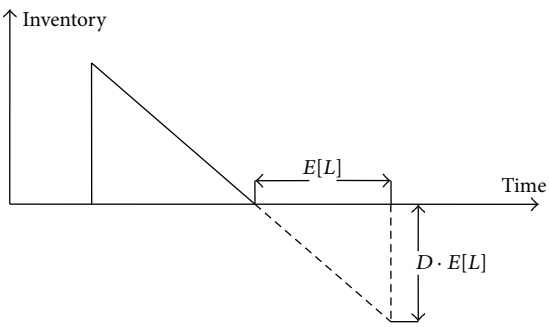

Figure 4: Shortage status of the manufacturer' final product.

and shortage cost of the final product, which can be described as follows:

$$\mathrm{TC}_p = A + \frac{h}{2D}(Q - D \cdot E[L])^2 + Q\sum_{i=1}^{n} h_i \left(E[L] - E[L_i]\right) + \frac{\pi D \cdot E^2[L]}{2}. \tag{1}$$

Furthermore, the delivery frequency is D/Q, so the average total cost per unit time is $\mathrm{TC}(Q, r)$, which can be calculated by the following formula:

$$\mathrm{TC}(Q, r) = \frac{AD}{Q} + \frac{hQ}{2} + D\left(\sum_{i=1}^{n} h_i - h\right) E[L] + \frac{(h+\pi) D^2}{2Q} E^2[L] - D\sum_{i=1}^{n} h_i E[L_i]. \tag{2}$$

In general, the model in the paper can be abstracted as the following nonlinear programming problem (P):

$$\begin{aligned} &\text{Min} \quad \mathrm{TC}(Q, r) \\ &\text{s.t.} \quad Q \geq 0, \quad r_i \geq 0, \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{P}$$

4. Model Analysis and Solution

4.1. Model Analysis. To derive the results, we first take the partial derivatives of $\mathrm{TC}(Q, r)$ with respect to Q and r_i ($i = 1, 2, \ldots, n$):

$$\frac{\partial \mathrm{TC}}{\partial Q} = \frac{-1}{Q^2}\left[\frac{(h+\pi) D^2}{2} E^2[L] + AD\right] + \frac{h}{2}, \tag{3}$$

$$\frac{\partial TC}{\partial r_i} = D\left[\sum_{i=1}^{n} h_i - h + \frac{(h+\pi) D}{Q} E[L]\right] \frac{\partial E[L]}{\partial r_i} + h_i \tag{4}$$

After that, we deduce from formula (3) that $\lim_{Q\to\infty}(\partial \mathrm{TC})/(\partial Q) = h/2 > 0$ and $\lim_{Q\to 0^+}(\partial \mathrm{TC})/(\partial Q) = -\infty < 0$. Moreover, it is easy to know $\partial^2 \mathrm{TC}/\partial Q^2 \geq 0$, so if only $\partial \mathrm{TC}/\partial Q = 0$ and $Q \geq 0$, the extreme value of $\mathrm{TC}(Q, r)$ is unique.

We continue with the terms in formula (4). Here we define that the distribution function and probability density function of $\max_{1\leq i\leq n}\{L_i\}$ are $G(x)$ and $g(x)$, respectively, which can be expressed as follows

$$\begin{aligned} G(x) &= P\left[\max_{1\leq i\leq n}\{L_i\} \leq x\right] = \prod_{i=1}^{n} P[L_i \leq x] \\ &= \prod_{i=1}^{n} P\left[Y_i - \frac{r_i}{D} \leq x\right] = \prod_{i=1}^{n} F_i\left(\frac{r_i}{D} + x\right), \\ g(x) &= \frac{dG(x)}{dx} = \sum_{i=1}^{n}\left[f_i\left(\frac{r_i}{D} + x\right) \prod_{j=1, j\neq i}^{n} F_j\left(\frac{r_j}{D} + x\right)\right]. \end{aligned} \tag{5}$$

By the definition of expectation on a random variable, we can calculate $E[L]$:

$$\begin{aligned} E[L] &= \int_0^\infty x g(x)\, dx \\ &= \sum_{i=1}^{n} \int_0^\infty x f_i\left(\frac{r_i}{D} + x\right) \prod_{j=1, j\neq i}^{n} F_j\left(\frac{r_j}{D} + x\right) dx. \end{aligned} \tag{6}$$

In order to get $\partial \mathrm{TC}/\partial r_i$, we need to compute the first-order partial derivative of $E[L]$ with respect to r_i:

$$\begin{aligned} &\frac{\partial E[L]}{\partial r_i} \\ &= \frac{\partial}{\partial r_i} \lim_{u\to\infty} \int_0^u x g(x)\, dx \\ &= \frac{\partial}{\partial r_i} \lim_{u\to\infty} \left\{[x G(x)]_0^u - \int_0^u G(x)\, dx\right\} \\ &= \frac{\partial}{\partial r_i} \lim_{u\to\infty} \left\{u G(u) - \int_0^u G(x)\, dx\right\} \\ &= \lim_{u\to\infty} \frac{\partial}{\partial r_i} \left\{u G(u) - \int_0^u G(x)\, dx\right\} \\ &= \lim_{u\to\infty} \left\{u \frac{\partial G(u)}{\partial r_i} - \int_0^u \frac{\partial G(x)}{\partial r_i} dx\right\} \\ &= \frac{1}{D} \lim_{u\to\infty} \left\{u f_i\left(\frac{r_i}{D} + u\right) \prod_{j=1, j\neq i}^{n} F_j\left(\frac{r_j}{D} + u\right) \right. \\ &\qquad \left. - \int_0^u f_i\left(\frac{r_i}{D} + x\right) \prod_{j=1, j\neq i}^{n} F_j\left(\frac{r_j}{D} + x\right) dx\right\} \\ &= -\frac{1}{D} \int_0^u f_i\left(\frac{r_i}{D} + x\right) \prod_{j=1, j\neq i}^{n} F_j\left(\frac{r_j}{D} + x\right) dx. \end{aligned} \tag{7}$$

According to formulas (4) and (7), we can deduce

$$\begin{aligned}\lim_{r_i\to\infty}\frac{\partial \mathrm{TC}}{\partial r_i} &= D\left(\sum_{i=1}^{n}h_i - h + \frac{(h+\pi)D}{Q}E[L]\right)\\ &\quad\times \lim_{r_i\to\infty}\frac{\partial E[L]}{\partial r_i} + h_i\\ &= h_i > 0.\end{aligned} \tag{8}$$

Similarly, if $E[L] > Q(h - \sum_{i=1}^{n} h_i)/D(h+\pi)$, we can get $\lim_{r_i\to 0^+}\partial TC/\partial r_i < 0$and $\partial^2 TC/\partial {r_i}^2 \geq 0$.

In summary, there must be a unique global optimal solution for $\mathrm{TC}(Q,r)$.

4.2. Model Solution. According to the above analysis, the optimal values Q^* and r_i^* are interacted, which implies that the simple application of first-order partial derivatives cannot ensure that we can get the two optimal values simultaneously. To solve this problem we will use $E[L]$ as an intermediary to make some appropriate changes on the objective function: firstly transform the original problem into a one-dimensional optimization problem and find the optimal solution of $E[L]$ for problem (P), and then get the optimal values of Q and r_i.

The following steps can be adopted to solve the problem (P).

(1) Define $\Phi_1(z)$ and $\Phi_2(z)$, where $z = E[L]$, and formula (2) can be decomposed into the following two according to decision variables:

$$\begin{aligned}\Phi_1(z) \equiv \min_{Q}\Bigg[\varphi_1(Q) \equiv D\left(\sum_i h_i - h\right)z\\ + \frac{(h+\pi)D^2z^2}{2Q} + \frac{h}{2}Q + \frac{AD}{Q}\Bigg],\\ \text{s.t. } Q > 0,\end{aligned} \tag{9}$$

$$\begin{aligned}\Phi_2(z) \equiv \min_{r_i}\left[\varphi_2(r) \equiv -D\sum_{i=1}^{n}h_iE[L_i]\right],\\ \text{s.t. } E[L_i] \leq z,\ r_i \geq 0,\ i = 1,2,\ldots,n.\end{aligned} \tag{10}$$

Since $\varphi_1(Q)$ is a simple convex function in Q, the optimal value is

$$Q^* = \sqrt{\frac{(h+\pi)D^2z^2 + 2AD}{h}} \tag{11}$$

(2) Substitute Q^* into $\Phi_1(z)$, and the optimal solution of the minimization problem can be expressed as a function in z:

$$\Phi_1(z) = D\left(\sum_i h_i - h\right)z + \frac{(h+\pi)D^2z^2}{2Q^*} + \frac{h}{2}Q^* + \frac{AD}{Q^*}. \tag{12}$$

As $\partial^2\Phi_1(z)/\partial z^2 = 2A(h+\pi)D^3/h{Q^*}^3 > 0$, we can know $\Phi_1(z)$ is a strict convex function in z, where Q^* meets the constraint that it should be larger than 0, then formula (9) can be regarded as a convex programming problem.

As $\varphi_2(r) = \sum_{i=1}^{n} h_ir_i - D\sum_{i=1}^{n} h_iE[Y_i]$ is a linear function in r_i, and $E[L] = E[\max_i\{Y_i - (r_i/D), 0\}]$ is a convex function in r_i, then formula (10) can be also regarded as a convex programming problem, which has a very good feature as shown in the next step.

(3) Define $z'' \equiv \max_{r_i>0, i=1,\ldots,n}E[L] = E[\max_i Y_i]$. $\varphi_2(r)$ increases with the gradual increase of r_i. At the same time, $E[L]$ decreases nonlinearly. So it can be inferred that $E[L]$ will increase to the maximum as r_i decreases to 0 gradually. As a result, we can transform the constraint $r_i \geq 0$ into $E[L] \leq z''$, where $0 \leq z \leq z''$.

From the above analysis, $\mathrm{TC}(Q,r)$ can be decomposed into two functions in Q and r:

$$\mathrm{TC}(Q,r) = \varphi_1(Q) + \varphi_2(r). \tag{13}$$

Furthermore, the original problem (P) can be transformed into the following problem (R):

$$\min_{0\leq z\leq z''}\left[\Phi_1(z) + \Phi_2(z)\right], \tag{14}$$

where $z = E[L]$, $z'' = E[\max_i Y_i]$. As $\Phi_1(z)+\Phi_2(z)$ is a convex function, problem (R) is a one-dimensional search problem about z under the given constraint of $0 \leq z \leq z''$. We can use the one-dimensional search method to find all possible values of z under the constraint and obtain the optimal solution of problem (R), then get the optimal value of Q^*, and finally find r_i^* by solving the equations.

Numerical Analysis. Assume that the Supply-Hub orders components from two suppliers, and the lead time Y_i of the two components obeys exponential distribution with the parameter λ_i $(i = 1,2)$, of which the probability density function is $f(x) = \lambda_i e^{-\lambda_i x}$, moreover $D = 250$ units/year, $A = 800$ units/year, $h = 70$ USD/units$*$year, $h_1 = 30$ USD/units$*$year, $h_2 = 20$ USD/units$*$year, $\pi = 400$ USD/product, and $\lambda_1 = 25$, $\lambda_2 = 20$.

Table 1 shows that the total cost decreases as the value of z increases by 0.001 units from 0. When $z = 0.052$, the total cost reaches the minimum, and after that, it will be greater than the minimum again with the increase of z. Therefore, we obtain $z^* = 0.052$ and $Q^* = 82.75869$. Then by calculating the nonlinear programming of formula (10), we get $r_1^* = 1.81699$, $r_2^* = 5.23407$, and consequently $\mathrm{TC}^*(Q,r) = 5718.469$.

As mentioned above, we only take costs of the Supply-Hub and manufacture into consideration and finally prove that there must be an optimal assembly quantity Q and an optimal reorder point r_i of each component, which can contribute to the lowest cost $\mathrm{TC}^*(Q,r)$ under centralized decision making. Obviously, we also need to talk about the relations between the Supply-Hub and suppliers. In the next section, we will discuss how the Supply-Hub makes use of

Table 1: Optimal solutions of the expected total cost.

z	Q^*	$\varphi_1(Q)$	$\varphi_2(r)$	$TC(Q,r)$
0.045	81.0189	5446.323	283.5354	5729.859
0.046	81.25423	5457.796	268.9064	5726.703
0.047	81.49403	5469.582	254.5106	5724.093
0.048	81.73826	5481.678	240.3141	5721.992
0.049	81.98688	5494.081	226.3121	5720.394
0.05	82.23985	5506.789	212.493	5719.282
0.051	82.49713	5519.799	198.8464	5718.646
0.052	82.75869	5533.108	185.3613	5718.469
0.053	83.02447	5546.713	172.0275	5718.74
0.054	83.29444	5560.611	158.8353	5719.446
0.055	83.56857	5574.8	145.775	5720.575

Punishment and Reward mechanisms to coordinate each supplier and reaches its goal of the expected service level.

5. Collaborative Replenishment Mechanism Based on Punishment and Reward

The above discussion shows that in ATO systems, the uncertainty of suppliers' delivery lead time will inevitably lead to the occurrence of shortages. If the Supply-Hub and suppliers both focus on the elimination of low efficiency, the shortage cost of the Supply-Hub will be higher than suppliers. In this sense, it implies that the Supply-Hub has a higher concern for shortages than suppliers. At the same time, compared with endeavoring to avoid shortages, suppliers are more willing to realize the overall optimization of supply chain with the premise of adding their own profits. Therefore, it is necessary for the Supply-Hub to impose punishment and reward incentives on suppliers, by which we can not only reduce the uncertainty but also increase the efficiency of supply chain.

This part will establish ordering relations between suppliers and the Supply-Hub based on BOM and will explore how to achieve the expected service level with the application of punishment and reward mechanisms. The implementation of the mechanisms can be described as that: if the supplier delay in delivery for a period of t, the Supply-Hub will punish him with a penalty of P_t, and if the supplier deliver on time, he will get a bonus of B. By calculating the Hessian matrix, we can verify the convexity of objective function and get the optimal values of decision variables under the given punishment and reward factors. The relations between the service level of the Supply-Hub and the two factors will be shown in figures.

The lead time of supplier i is a random variable Y_i, and we assume Y_i follows the uniform distribution in the range of $[u_i - R_i, u_i + R_i]$ with mean value $u = u_i$, and variance $\delta = R_i^2/3$, $i = 1, 2, \ldots, n$. Then the service level of the Supply-Hub is the probability that n suppliers deliver on time at the same time, which is

$$\rho = \prod_{i=1}^{n}\left(1 - \frac{(u_i + R_i) - r_i/D}{2R_i}\right). \tag{15}$$

As we know, the variance of actual lead time can be reduced by increasing investment and improving inventory level of raw materials. Here we assume that the investment of reducing the variance of actual lead time to zero is θ_i, and each supplier will only try to reduce unit variance, so the investment cost for supplier i is $C(\delta) = \theta_i/\delta = 3\theta_i/R_i^2$.

5.1. Punishment Coordination Mechanism. We now assume that the Supply-Hub implements the same punishment and reward mechanisms to every supplier.

For supplier i, the expected total cost is the sum of holding cost of the component, penalty and investment cost of reducing lead time variance, which is

$$C(r_i, R_i) = h_i E[L_i^-] + PE[L_i^+] + \frac{3\theta_i}{R_i^2}, \quad i = 1, 2, \ldots, n, \tag{16}$$

where

$$\begin{aligned} E[L_i^-] &= \int_{u_i - R_i}^{r_i/D}\left(\frac{r_i}{D} - x\right) f_i(x)\,dx \\ &= \left[\frac{r_i}{2D} + \frac{r_i^2}{4D^2R_i} + \frac{R_i}{4} - \frac{u_i}{2} - \frac{r_iu_i}{2DR_i} + \frac{u_i^2}{4R_i}\right], \\ E[L_i^+] &= \int_{r_i/D}^{u_i + R_i}\left(x - \frac{r_i}{D}\right) f_i(x)\,dx \\ &= \left[-\frac{r_i}{2D} + \frac{r_i^2}{4D^2R_i} + \frac{R_i}{4} + \frac{u_i}{2} - \frac{r_iu_i}{2DR_i} + \frac{u_i^2}{4R_i}\right]. \end{aligned} \tag{17}$$

For convenience, we replace the expected lead time r_i/D with A_i, then the cost function of supplier i can be expressed as

$$\begin{aligned} C(r_i, R_i) = {} & h_i\left[\frac{A_i}{2} + \frac{A_i^2}{4R_i} + \frac{R_i}{4} - \frac{u_i}{2} - \frac{A_iu_i}{2R_i} + \frac{u_i^2}{4R_i}\right] \\ & + P\left[-\frac{A_i}{2} + \frac{A_i^2}{4R_i} + \frac{R_i}{4} + \frac{u_i}{2} - \frac{A_iu_i}{2R_i} + \frac{u_i^2}{4R_i}\right] + \frac{3\theta}{R_i^2}. \end{aligned} \tag{18}$$

Take the first-order derivatives of cost function $C(r_i, R_i)$ with respect to $A_i$and R_i, respectively, and make them equal to 0 as follows.

$$\frac{\partial C_i}{\partial A_i} = \frac{h_i}{2R_i}(R_i + A_i - u_i) + \frac{P}{2R_i}(-R_i + A_i - u_i) = 0, \tag{19}$$

where

$$A_i(P) = \frac{(P - h_i)R_i}{(P + h_i)} + u_i. \tag{20}$$

Similarly,

$$\begin{aligned} \frac{\partial C_i}{\partial R_i} = {} & \frac{h_i}{4R_i^2}\left(R_i^2 - A_i^2 + 2A_iu_i - u_i^2\right) \\ & + \frac{P}{4R_i^2}\left(R_i^2 - A_i^2 + 2A_iu_i - u_i^2\right) - \frac{6\theta_i}{R_i^3} = 0 \end{aligned} \tag{21}$$

Substitute $A_i(p)$ into the above formula, we can get

$$R_i(P) = \left[\frac{6\theta_i(h_i+P)^2}{h_iP}\right]^{1/3}. \tag{22}$$

Then substitute $R_i(P)$ into formula (20)

$$A_i(p) = \frac{(P-h_i)\left[6\theta_i(h_i+P)^2/h_iP\right]^{1/3}}{(P+h_i)} + u_i. \tag{23}$$

After calculation, Hessian matrix of the binary differentiable function is

$$\begin{vmatrix} \dfrac{h_i+P}{2R_i} & \dfrac{(h_i+P)(u_i-A_i)}{2R_i^2} \\ \dfrac{(h_i+P)(u_i-A_i)}{2R_i^2} & \dfrac{(h_i+P)(A_i-u_i)^2}{2R_i^3} + \dfrac{18\theta_i}{R_i^4} \end{vmatrix} = \frac{9\theta_i(h_i+P)}{R_i^5} > 0. \tag{24}$$

So $C(r_i, R_i)$ is a convex function, and we can know $A_i(P)$ and $R_i(P)$ are the optimal values when P is given.

Based on the above analyses, substitute $A_i(P)$ and $R_i(P)$ into the expression of ρ, and the service level of the Supply-Hub under the given punishment factor P can be obtained:

$$\begin{aligned} \rho &= \prod_{i=1}^{n}\left(1 - \frac{(u_i+R_i)-r_i/D}{2R_i}\right) \\ &= \prod_{i=1}^{n}\left(1 - \frac{(u_i+R_i)-A_i}{2R_i}\right) \\ &= \prod_{i=1}^{n}\frac{P}{h_i+P}. \end{aligned} \tag{25}$$

As $\partial\rho/\partial P = \sum_{i=k}^{n}(h_k/(h_k+P)^2)\prod_{i\neq k}^{n}P/(h_i+P) > 0$, the expected service level is an increasing function in punishment factor P, and only when $P \longrightarrow \infty$, $\rho \longrightarrow 1$, which means when the punishment factor is large enough, the service level will approach illimitably to 100%. In fact, the conclusion is in line with the practical situation. If the punishment factor is very large, the supplier's late delivery will lead to a significant increase of total cost, thus suppliers will avoid delay delivery.

Numerical Analysis. We assume that the Supply-Hub places orders, respectively, to two suppliers. Here we follow the parameters in previous chapter, $h_1 = 30$ USD/unit∗year and $h_2 = 20$ USD/unit∗year. A relational diagram between the expected service level of the Supply-Hub ρ and the value of punishment factor P can be illustrated in Figure 5.

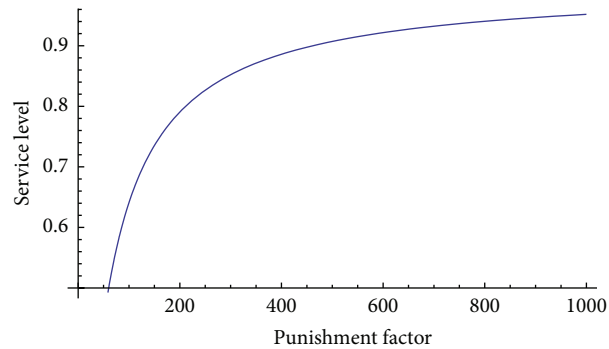

Figure 5: Relation between expected service level and punishment factor.

5.2. Reward Coordination Mechanism. When the Supply-Hub uses reward mechanism to coordinate the JIT operation, for supplier i, the cost function is the sum of holding cost of the component, bonus and actual investment cost of reducing lead time variance, which is

$$\begin{aligned} C(r_i, R_i) &= h_iE\left[L_i^-\right] - B\cdot P(u_i - R_i \le Y_i \le A_i) \\ &\quad + \frac{3\theta_i}{R_i^2}, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{26}$$

Similarly, take the first-order derivatives of cost function $C(r_i, R_i)$ with respect to A_i and R_i, respectively, and make them equal to 0, then we can get the expressions of A_i and R_i

$$\begin{aligned} A_i(B) &= \frac{B}{h_i} - \frac{24h_i\theta_i}{B^2} + u_i, \\ R_i(B) &= \frac{24h_i\theta_i}{B^2}. \end{aligned} \tag{27}$$

After calculation, Hessian matrix of the binary differentiable function is

$$\begin{vmatrix} \dfrac{h_i}{2R_i} & \dfrac{h_i(u_i-A_i)+B}{2R_i^2} \\ \dfrac{h_i(u_i-A_i)+B}{2R_i^2} & \dfrac{h_i(A_i-u_i)^2 - 2B(A_i-u_i)}{2R_i^3} + \dfrac{18\theta_i}{R_i^4} \end{vmatrix} = \frac{B^2}{8R_i^4} > 0. \tag{28}$$

So $C(r_i, R_i)$ is a convex function, then we can know $A_i(B)$ and $R_i(B)$ are the optimal values if B is given.

Based on the above analyses, substitute $A_i(B)$ and $R_i(B)$ into the expression of ρ, we can get the service level of the Supply-Hub under the given B:

$$\begin{aligned} \rho &= \prod_{i=1}^{n}\left(1 - \frac{(u_i+R_i)-r_i/D}{2R_i}\right) \\ &= \prod_{i=1}^{n}\left(1 - \frac{(u_i+R_i)-A_i}{2R_i}\right) \\ &= \prod_{i=1}^{n}\frac{B^3}{48\theta_ih_i^2}. \end{aligned} \tag{29}$$

Table 2: Corresponding reward factor B and expected service level obtained.

B	270.00	263.80	257.43	250.89	244.15
Service level	100.00	95.00	90.00	85.00	80.00

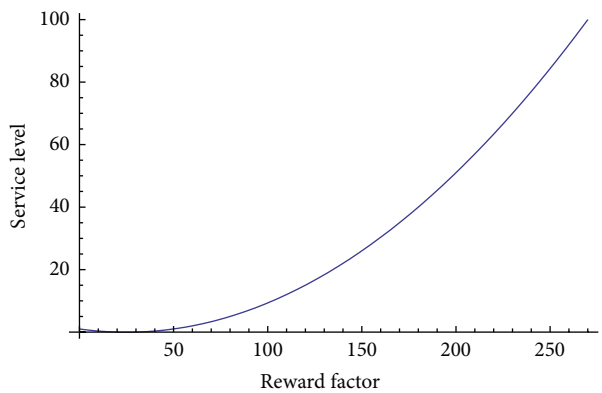

Figure 6: Relation between expected service level and reward factor.

It is easy to see that ρ is an increasing function in B. That is to say, when the bonus is great enough, suppliers will do their best to delivery on time.

Numerical Analysis. Here we assume that the Supply-Hub still places orders, respectively, to two suppliers, with $h_1 = 30$ USD/unit∗year and $h_2 = 20$ USD/unit∗year. The expected service level under corresponding reward factor can be calculated by Mathematica software, as shown in Table 2.

In the table above, when the reward factor is 270.00, the service level gets 100%, which means the bonus that exceeds 270.00 is redundant. To illustrate the changing trend of expected service level ρ caused by the changes of the value of B better, a diagram can be drawn as Figure 6, in which the horizontal axis represents B, and the vertical axis stands for the expected service level ρ.

6. Conclusion

This paper constructs a collaborative replenishment model in the ATO system based on the Supply-Hub with delivery uncertainty. We transform the traditional model into a one-dimensional optimization problem and derive the optimal assembly quantity and the optimal reorder point of each component. In order to enable collaborative replenishment, punishment and reward mechanisms are proposed for the Supply-Hub to coordinate the supply chain operation. The results show that if the punishment factor is very large, suppliers will avoid late delivery, also, if the reward factor is great enough, they will do their best to delivery on time. The numerical analysis also finds that punishment and reward mechanisms can significantly improve the suppliers' initiatives of collaborative replenishment, thereby leading to a higher service level in ATO systems. Overall, this paper provides a theoretical basis and also the useful guidance to the practice of collaborative replenishment in ATO systems based on the Supply-Hub with delivery uncertainty.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Price and Service Competition of Dual-Channel Supply Chain with Consumer Returns

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Products returned by consumers are common in the retail industry and result in additional costs to both the manufacturer and the retailer. This paper proposes dual-channel supply chain models involving consumer returns policies. Also, the price and service competition between retail channel and direct channel is considered in the models. According to the models, we analyze the optimal decisions in both centralized and decentralized scenarios. Then we design a new contract, coordinate the dual-channel supply chain, and enable both the retailer and the manufacturer to be a win-win.

1. Introduction

Purchased products that are returned by consumers to retailers are common in the retail industry; for example, Wal-Mart provides full credit for consumer returns under very broad circumstances. Consumer returns policies are part of the competitive strategy used by most of retailers, many of whom commonly provide full refunds without question. In USA, the value of products returned by consumers exceeds $100 billion each year, and only about 5% of them were truly defective [1, 2]. Related researches show that consumer returns policies are able to strengthen consumer's confidences, stimulate their demands, and improve the market share [3–5]. However, consumer returns policies also bring in some negative impacts on profits, such as increasing cost and malicious return.

The internet has significantly changed the way a consumer consumes as well as the sales model of manufactures and vendors. Nowadays, online shopping became more and more popular, and the electronic channel has become a critical sales channel. Hence, in order to ensure a greater advantage in the market competition, manufacturers and vendors both change the sales model to satisfy the different consumption habits of consumers.

Many papers in dual-channel supply chain address the issue of how manufacturers and retailers could benefit from their own advantages, mainly concentrated on the sales price and service levels. Chen et al. [6] examine how to make the proper decisions of the direct online sales channel together with a retail channel based on the same sales price and the different service levels of dual channel. Dumrongsiri et al. [7] study a dual channel supply chain, which exists in the service and price competition, and analyze how different product, cost, or service characteristics influence the equilibrium behavior of such supply chain. Based on the former research, Hua et al. [8] consider the influence of the delivery time on the price strategy and profits of both manufactures and retailers under coordination of dual channel supply chain which are based on price and delivery time. Cai et al. [9] study the channel choice and coordination of dual channel supply chain. They compare four different channels to illustrate the pros and cons of these channels. Chun et al. [10] analyze optimal channel strategies of a manufacturer when it considers an online store as its new direct channel and discuss some strategic implications of channel strategies from the perspective of consumer heterogeneity and the retail services. Liu et al. [11] study the dual-channel supply chain under information asymmetry. They design two kinds of contracts to coordinate the supply chain and derive the optimal production and the optimal price of dual channel. Chen and Bell [12] bring consumer returns into the dual channel supply chain and study how a firm that faces consumer returns can enhance profit by using different consumer returns policies,

full-refund, and no-returns, as a device to segment its market into a dual-channel structure. Huang et al. [13] develop a two-period pricing and production decision model in a dual-channel supply chain with demand disruption.

Coordination among suppliers and retailers is a very important strategic issue in supply chain management, and there is a vast literature on coordination contracts in one channel supply chain. For example, Lariviere and Porteus [14] show that the simple wholesale-price-only contract is unable to coordinate the supply chain, and Pasternack [15] shows that a returns policy, when the parameters are properly chosen, can coordinate the supply chain. Recently, some papers put forward several new contracts to coordinate the complex supply chain. For example, He et al. [16] consider a condition in which the stochastic market demand is sensitive to both retail price and sales effort. They show that coordination is achieved by using a properly designed returns policy with a sales rebate and penalty (SRP) contract. Chen and Bell [17] investigate a channel that consists of a manufacturer and a retailer where the retailer simultaneously determines the retail price and order quantity while experiencing customer returns and price dependent stochastic demand. They propose an agreement that includes two buyback prices, one for unsold inventory and one for customer returns, and show that this revised returns policy can achieve perfect supply-chain coordination and lead to a win-win situation. He and Zhao [18] employ a properly designed returns policy between the manufacturer and the retailer, combined with a wholesale-price contract between the raw-material supplier and the manufacturer to coordinate a multiechelon supply chain under supply and demand uncertainty. Cachon [19] and Arshinder et al. [20] provide detailed reviews of this literature. But the coordination in dual-channel supply chain has been largely neglected in the research literature. Only limited studies consider this problem. For example, Cai [21] investigated the influence of channel structures and channel coordination on the supplier, the retailer, and the entire supply chain in a dual-channel supply chain. He shows that a combination of a revenue sharing and a linear price relationship between the price in the retail channel and the price in the direct channel can coordinate the supply chain. Chen et al. [22] examine a manufacturer's pricing strategies in a dual-channel supply chain. They show the conditions under which the manufacturer and the retailer both prefer a dual-channel supply chain and examine the coordination schemes for a dual-channel supply chain and find that a manufacturer's contract with a wholesale price and a price for the direct channel can coordinate the dual-channel supply chain, benefiting the retailer but not the manufacturer.

Different from previous research, in this paper we consider the case of a price and service competition that exists in dual-channel supply chain, besides that, the consumer returns are considered in such supply chain. In decentralized supply chain, we use Stackelberg game and Nash equilibrium to derive the equilibrium solutions; then a new contract is designed to coordinate the dual-channel supply chain.

The rest of this paper is organized as follows. Section 2 introduces model assumptions and problem formulation. Section 3 examines a benchmark case of centralized dual-channel supply chain. In Section 4, we analyze the case of decentralized dual-channel supply chain by using Stackelberg game and Nash equilibrium, respectively. In Section 5, we investigate supply chain behavior and find an optimal contract to coordinate the decentralized supply chain. Section 6 uses numerical studies to illustrate our results. Section 7 presents conclusions, insights, and directions for future research.

2. Assumptions and Problem Formulation

In this paper, we assume that both the manufacturer and the retailer provide consumer returns policies and there exists price and service competition between the two channels. Similar to Chen et al. [6], we use the delivery time t to measure the service competition, where $t \in (0, 1)$. Also, following several research studies (such as Chen and Bell [12] and Chiang et al. [23]), we assume that consumers have a consumption value v, which refers to subjective beliefs about desirable ways to attain personal values, and that v is uniformly distributed [0,1] within the population where v captures individual differences in the value of the product: consumer perceives the product to be worth v when sold by retail channel and θv when sold by direct channel. Here, we assume $\theta \in (0, 1)$; it indicates that consumer's valuation parameter to the retail channel is greater than the direct channel.

We use subscripts d and r to denote the direct channel and the retail channel, respectively. Then we can derive the consumer's utility $U_r = v - p_r$ from traditional retail channel or $U_d = \theta v - p_d - t$ from the direct channel, where $p_r \in [0, 1]$ and $p_d \in [0, 1]$ are the sales prices of retail channel and direct channel, respectively.

Usually, if $U_d \geq 0$, consumers choose to purchase products from the direct channel; if $U_r \geq 0$, consumers choose to purchase products from the retail channel. In this paper, we consider the competition between the direct channel and the retail channel. So, if $U_d > U_r$, consumers choose to purchase products from the direct channel; on the contrary, if $U_d < U_r$, consumers choose to purchase products from the retail channel.

Let c be the product cost, let l be the salvage value, and w be the wholesale price. In this paper, we assume c, l, and w are constant and less than 1. The following assumptions are used to formulate the problem.

(a) The products that consumers brought can be returned to the manufacturer who provides full refunds without question. The manufacturer can continue to sell the returned products which have been renewed with small additional costs.

(b) If the products cannot be sold out at the end of sale season, the manufacturer can get salvage value from the unsold products. It is reasonable to assume that $c > l$.

(c) Let α be the return rate in the retail channel and let β be the return rate in the direct channel. It is natural to assume that $0 \leq \alpha < 1$, $0 \leq \beta < 1$, and $0 \leq \alpha + \beta < 1$.

(d) If consumers return the products that were bought from the direct channel, while their utility for the

retail channel is greater than zero, they will choose to purchase the products from the retail channel. Meanwhile if consumers return the products that were bought from the retail channel, while if their utility for the direct channel is greater than zero, they will choose to purchase the products from the direct channel.

(e) The utility of the consumer for the same kind of product is subject to uniform distribution.

And according to Chen and Bell [12] and Assumption (e), we know the demand for the product is $\int_p^1 dv = 1 - p$, where p is the sale price. We denote the indifferent values for each channel as $v_d = (p_d+t)/\theta$ and $v_r = p_r$ and the point where the customer is indifferent between the channels as $v_{rd} = (p_r - p_d - t)/(1-\theta)$.

When $v_r < v_d$, we have $v_d > v_r > v_{rd}$; this implies that no customer will buy from the direct channel and the customer whose reservation price is in the range $[v_r, 1]$ will only purchase a product in the retailer sale channel.

When $v_r > v_d$, we have $v_d < v_r < v_{rd}$; this implies that the customer whose reservation price is in the range $[v_{rd}, 1]$ will purchase from the retailer sale channel, while they will purchase from the direct channel if their reservation price is in the range $[v_d, v_{rd}]$. The customers whose reservation price is in the range $[0, v_d]$ will not buy a product.

Because of the valuation of the customers is uniformly distributed, the demands of the dual-channel can be modeled as follows:

$$\begin{aligned} D_r(p_r, p_d) &= 1 - p_r, \\ D_d(p_r, p_d) &= 0, \end{aligned} \quad \text{if } p_r \le \frac{p_d + t}{\theta}, \tag{1}$$

$$\begin{aligned} D_r(p_r, p_d) &= 1 - \frac{p_r - p_d - t}{1-\theta}, \\ D_d(p_r, p_d) &= \frac{p_r - p_d - t}{1-\theta} - \frac{p_d + t}{\theta}, \end{aligned} \quad \text{if } p_r \ge \frac{p_d + t}{\theta}. \tag{2}$$

Hence, from analysis we have the following.

(I) If $p_r \le (p_d + t)/\theta$, $\alpha D_r(p_r, p_d)$ consumers will return their products in the retail channel, and, among these consumers, $(1-v_d)/(1-v_r)$ ratio of them will buy from the direct channel since $U_d \ge 0$ for these consumers who become the additional demand for the direct channel. The rest, that is, $(v_d - v_r)/(1 - v_r)$ ratio of $\alpha D_r(p_r, p_d)$ consumers, have $U_d < 0$, and they leave the system without buying the product and become the lost demand.

(II) If $p_r \ge (p_d + t)/\theta$, $\beta D_d(p_r, p_d)$, consumers will return their products in the direct channel, and they are lost demand since $U_r < 0$ for these consumers in the retail channel and they leave the system without buying the product.

(III) If $p_r \ge (p_d + t)/\theta$, $\alpha D_r(p_r, p_d)$, consumers will return their products in the retail channel, but these consumers will buy from the direct channel since they have $U_d \ge 0$, and these consumers become the additional demand for the direct channel.

Therefore, when $p_r \le (p_d + t)/\theta$, the direct channel only sells the returned products which are returned from the retail channel and renewed with small additional costs. In this case, the direct channel can be recognized as a second market to sell the remanufactured products.

When $p_r \ge (p_d + t)/\theta$, the new products can be sold in both channels, but only the direct channel can sell the returned products. This phenomenon is very popular in reality.

3. The Centralized Dual-Channel Supply Chain

In this section, we consider a centralized dual-channel supply chain, in which the manufacturer and the retailer are vertically integrated in the traditional channel.

3.1. Scenario 1 ($p_r \le (p_d + t)/\theta$). Under such dual-channel supply chain, the direct channel only sells the returned products. Then the profit of the centralized supply chain Π_T is

$$\begin{aligned} \Pi_T = \max_{p_r, p_d} & p_r(1-\alpha)(1-p_r) - c(1-p_r) \\ & + \alpha \frac{1-v_d}{1-v_r}(1-p_r)p_d + \alpha l \frac{v_d - v_r}{1-v_r}(1-p_r) \\ \text{subject to } & p_r \le \frac{p_d + t}{\theta}. \end{aligned} \tag{3}$$

This optimization leads to the following result.

Proposition 1. *Under a centralized dual-channel supply chain with $p_r \le (p_d + t)/\theta$, there is a threshold delivery time $t_1^* = (\theta(c-\alpha l) - l(1-\alpha))/(1-\alpha)$ such that, if $t < t_1^*$, the optimal prices are*

$$\begin{aligned} p_d^* &= \frac{\theta}{2}\left(1 + \frac{c + \alpha t}{1 - \alpha + \alpha\theta}\right) - t, \\ p_r^* &= \frac{1}{2}\left(1 + \frac{c + \alpha t}{1 - \alpha + \alpha\theta}\right). \end{aligned} \tag{Case A1}$$

If $t \ge t_1^*$, the optimal prices are

$$\begin{aligned} p_d^* &= \frac{\theta - t + l}{2}, \\ p_r^* &= \frac{1}{2}\left(1 + \frac{c - \alpha l}{1 - \alpha}\right). \end{aligned} \tag{Case A2}$$

The optimal policy is continuous at the threshold t_1^*.

Proof. From (3), we know that $\partial^2\Pi_T/\partial p_r \partial p_d = \partial^2\Pi_T/\partial p_d \partial p_r = 0$, $\partial^2 L/\partial p_r^2 = -2(1-\alpha)$, and $\partial^2 L/\partial p_d^2 = -(2\alpha/\theta)$. Let M be the Hessian Matrix; then we obtain that

$$M = \begin{pmatrix} \dfrac{\partial^2 L}{\partial p_r^2} & \dfrac{\partial^2 L}{\partial p_r \partial p_d} \\ \dfrac{\partial^2 L}{\partial p_r \partial p_d} & \dfrac{\partial^2 L}{\partial p_d^2} \end{pmatrix} \tag{4}$$

is negative definite matrix. Hence, (3) is a convex optimization problem. In all proofs, we focus on the Lagrangian multipliers and on the orthogonality conditions of the Karush-Kuhn-Tucker theorem.

Then the Lagrangian and the Karush-Kuhn-Tucker optimality conditions are

$$\begin{aligned} L(p_r, p_d) &= p_r(1-\alpha)(1-p_r) - c(1-p_r) \\ &\quad + \alpha\frac{1-v_d}{1-v_r}(1-p_r)p_d + \alpha l\frac{v_d - v_r}{1-v_r}(1-p_r) \\ &\quad + \lambda\left(\frac{p_d+t}{\theta} - p_r\right), \\ \frac{\partial L}{\partial p_r} &= c + (1-2p_r)(1-\alpha) - \alpha l - \lambda = 0, \\ \frac{\partial L}{\partial p_d} &= \frac{\alpha l - 2\alpha p_d - t\alpha + \alpha\theta + \lambda}{\theta} = 0, \\ \lambda\left(\frac{p_d+t}{\theta} - p_r\right) &= 0. \end{aligned} \tag{5}$$

Since the multiplier can be either zero or positive, we could have two cases to examine.

Case A1 ($\lambda > 0$). The value of λ is

$$\lambda = \frac{-\alpha(t + l - l\alpha - c\theta - \alpha t + l\alpha\theta)}{1-\alpha+\alpha\theta}. \tag{6}$$

Since the multiplier is positive, we have the necessary condition in this case

$$t < \frac{\theta(c-\alpha l) - l(1-\alpha)}{1-\alpha} = t_1^*. \tag{7}$$

Since the condition is satisfied, the prices in each channel are

$$\begin{aligned} p_d^* &= \frac{\theta}{2}\left(1 + \frac{c+\alpha t}{1-\alpha+\alpha\theta}\right) - t, \\ p_r^* &= \frac{1}{2}\left(1 + \frac{c+\alpha t}{1-\alpha+\alpha\theta}\right). \end{aligned} \tag{8}$$

Case A2 ($\lambda = 0$). The prices in each channel are

$$\begin{aligned} p_d^* &= \frac{\theta - t + l}{2}, \\ p_r^* &= \frac{1}{2}\left(1 + \frac{c-\alpha l}{1-\alpha}\right). \end{aligned} \tag{9}$$

Because of $p_r \le (p_d + t)/\theta$, we obtain the necessary condition in this case

$$t > \frac{\theta(c-\alpha l) - l(1-\alpha)}{1-\alpha} = t_1^*. \tag{10}$$

Also, it is simple to show that the policy is continuous in t_1^*, by forcing $t = t_1^*$ in the optimal policy of Case A2. □

From Proposition 1, we know that $dp_d^*/dt < 0$ in both Cases A1 and A2; this indicates that the direct sale price is always decreasing in t. But $dp_r^*/dt > 0$ in Case A1 while $dp_r^*/dt = 0$ in Case A2; this means that the retail sale price is increasing in t when $t < t_1^*$; however, when $t \ge t_1^*$, the retail sale price will not be influenced by t anymore.

3.2. Scenario 2 ($p_r \ge (p_d+t)/\theta$). In such dual-channel supply chain, the direct channel can sell not only the new products but also the returned products. Then the profit of the centralized supply chain is

$$\begin{aligned} \Pi_T &= \max_{p_r, p_d} p_r(1-\alpha)D_r + p_d(1-\beta)D_d - c(D_r + D_d) \\ &\quad + \alpha p_d D_r + l\beta D_d \\ &\quad \text{subject to } p_r \ge \frac{p_d+t}{\theta}. \end{aligned} \tag{11}$$

From (11), we know that when $p_r \ge (p_d + t)/\theta$, taking second partial derivatives with respect to p_r and p_d, the existence condition for a unique optimal solution to p_r and p_d is $4(1-\theta)/\theta > \beta^2/((1-\alpha)(1-\beta))$ or

$$\theta < \overline{\theta} = 1 - \frac{\beta^2}{4(1-\alpha)(1-\beta) + \beta^2}. \tag{12}$$

$\overline{\theta}$ in (12) is a decreasing function of α, β. For instance, when $\alpha = \beta = 0.3$, we have $\overline{\theta} = 0.9561$. So we conclude that optimal prices exist for a broad range of θ, α, and β.

Proposition 2. *Under a centralized dual-channel supply chain with $p_r \ge (p_d+t)/\theta$, there is a threshold delivery time $t_2^* = ((1-\alpha)(2c\theta+2\beta l-2c-\beta\theta)+\beta\theta(2\alpha l-c-\alpha\theta))/(2(1-\alpha)(1-\beta)-\alpha\beta\theta)$ such that, if $t > t_2^*$, the optimal prices are*

$$\begin{aligned} p_d^* &= \frac{\theta}{2}\left(1 + \frac{c+\alpha t}{1-\alpha+\alpha\theta}\right) - t, \\ p_r^* &= \frac{1}{2}\left(1 + \frac{c+\alpha t}{1-\alpha+\alpha\theta}\right). \end{aligned} \tag{Case B1}$$

If $t \le t_2^*$, the optimal prices are

$$\begin{aligned} p_d^* &= l - (1-\alpha)\big((2-\beta)[(2l-\theta)(1-\theta) + (2-\theta)t] \\ &\quad + 2c\theta - 2t - 2c\big) \\ &\quad \times \left(4(1-\alpha)(1-\beta)(1-\theta) - \beta^2\theta\right)^{-1}, \\ p_r^* &= \frac{1}{2} - \left(\left(\alpha\beta - \alpha\beta\theta - \frac{\beta^2}{2}\right)(\theta - 2l) \right. \\ &\quad \left. + (1-\theta)(\beta c - 2c + 2\alpha c) + \beta t(\alpha\theta - 1 + \beta)\right) \\ &\quad \times \left(4(1-\alpha)(1-\beta)(1-\theta) - \beta^2\theta\right)^{-1}. \end{aligned} \tag{Case B2}$$

The optimal policy is continuous at the threshold t_2^*.

Table 1: The Stackelberg equilibriums.

	Case C1: $t \le t_1^S$	Case C2: $t_1^S \le t \le t_2^S$	Case C3: $t \ge t_2^S$
p_d^S	$\frac{1}{2}\left[\frac{M}{(1-\beta)(2-\theta)-\alpha\theta} - t\right]$	$\frac{(1-\theta+w)\theta}{2-\theta} - t$	$\frac{c-w+w\alpha-\alpha t+\alpha\theta}{2\alpha}$
p_r^S	$\frac{1}{4}\left[2(1-\theta+w)+t+\frac{M}{(1-\beta)(2-\theta)-\alpha\theta}\right]$	$\frac{1-\theta+w}{2-\theta}$	$\frac{c-w+w\alpha+\alpha t+\alpha\theta}{2\alpha\theta}$

$M = (\beta-1-\alpha)\theta^2 + (1+2w+\alpha+l\beta-2c-2w\alpha-\beta-w\beta)\theta + 2c - 2l\beta$,
$t_1^S = (2\theta(1-\theta+w))/(2-\theta) - M/((1-\beta)(2-\theta)-\alpha\theta)$, and
$t_2^S = (2\theta(1+w-\theta))/(2-\theta) - (c+w\alpha-w+\alpha\theta)/\alpha$.

Proof. The proof is similar to that of Proposition 1 and is omitted. □

4. The Decentralized Dual-Channel Supply Chain

In this section, we consider a decentralized dual-channel supply chain, in which both the manufacturer and the retailer make their own decisions to maximize their individual profits. For decentralized dual-channel supply chain, the most popular mode is that the manufacturer also sells the new products by direct channel, which will lead to the service and price competition with retailer. Hence, in decentralized supply chain we only consider the scenario of $p_r \ge (p_d+t)/\theta$.

Since $p_r \ge (p_d+t)/\theta$ in this scenario, D_r and D_d satisfy (2). The profit of the retailer is

$$\Pi_r = (p_r - w)(1-\alpha) D_r. \tag{13}$$

The manufacturer's profit is

$$\begin{aligned}\Pi_d = {} & w(1-\alpha)D_r - c(D_r+D_d) + p_d(1-\beta)D_d \\ & + \alpha p_d D_r + l\beta D_d.\end{aligned} \tag{14}$$

Next, we will obtain the equilibrium solutions by using the Stackelberg game and the Nash equilibrium.

4.1. Stackelberg Game. In reality, decisions usually are taken sequentially; some players decide first and the others respond to these. This may be due to the lack of information of a player about the other. With Stackelberg game, we model the decision process as a sequential. The manufacturer can therefore operate as a Stackelberg leader while the retailer would be the follower. The sequence of the events is as follows.

(a) The manufacturer, as the Stackelberg leader, owns the complete information. He determines the direct sale price p_d and the delivery lead time t first based on the retailer's response.

(b) The retailer, knowing the supplier's decisions, determines his retail price.

(c) The selling season starts and demand is observed.

Hence, we know that such a Stackelberg game will lie on the following two-stage process: first, we can obtain the retailer's best response function from the retailer's maximization problem; then, the manufacturer maximizes their profit based on the retailer's best response function.

Proposition 3. *Under the decentralized dual-channel supply chain, the Stackelberg equilibrium prices (p_d^S, p_r^S) can be shown in Table 1.*

From Proposition 3, we know that there are two threshold delivery times t_1^S and t_2^S, and the Stackelberg equilibrium prices are continuous at the thresholds t_1^S and t_2^S.

Proof. First, we derive the retailer's best response to manufacturer's decisions. The profit of the retailer is

$$\begin{aligned}\Pi_r = {} & \max_{p_r} (p_r - w)(1-\alpha)\left(1 - \frac{p_r - p_d - t}{1-\theta}\right) \\ & \text{s.t. } p_r \ge \frac{p_d + t}{\theta}.\end{aligned} \tag{15}$$

Using the Lagrangian duality, we can easily get

$$\begin{aligned} & p_r^S = \frac{1-\theta+p_d+w+t}{2}, \quad p_d \le \frac{(1-\theta+w)\theta}{2-\theta} - t, \\ & p_r^S = \frac{p_d+t}{\theta}, \quad p_d \ge \frac{(1-\theta+w)\theta}{2-\theta} - t.\end{aligned} \tag{16}$$

Hence, the profit of the manufacturer will be divided into two scenarios:

(1)

$$\begin{aligned}\Pi_d = {} & \max_{p_d} [w(1-\alpha)+\alpha p_d]\left[\frac{1}{2} + \frac{p_d+t-w}{2(1-\theta)}\right] \\ & - c\left(1 - \frac{p_d+t}{\theta}\right) \\ & + [p_d(1-\beta)+l\beta]\left[\frac{1}{2} - \frac{p_d+t-w}{2(1-\theta)} - \frac{p_d+t}{\theta}\right] \\ & \text{s.t. } p_d \le \frac{(1-\theta+w)\theta}{2-\theta} - t;\end{aligned} \tag{17}$$

(2)

$$\begin{aligned}\Pi_d = {} & \max_{p_d} [w(1-\alpha)+\alpha p_d - c]\left(1 - \frac{p_d+t}{\theta}\right) \\ & \text{s.t. } p_d \ge \frac{(1-\theta+w)\theta}{2-\theta} - t.\end{aligned} \tag{18}$$

Also, using the Lagrangian duality to solve the above problems, we can get Proposition 3. □

From Table 1, we easily know that p_d^S is always decreasing in t; p_r^S is increasing in t when $t \le t_1^S$ and $t \ge t_2^S$, but it does not have any relationship with t when $t_1^S \le t \le t_2^S$.

4.2. Nash Equilibrium. In the Nash equilibrium version of the competition, the price decisions of two channels are assumed to be given simultaneously. Each player is assumed to know how the other would behave; there is no priority or time sequence among the players' decisions. Hence, the profits of retailer and manufacturer are

$$\begin{aligned}
\Pi_r &= \max_{p_r} (p_r - w)(1-\alpha)\left(1 - \frac{p_r - p_d - t}{1-\theta}\right),\\
\Pi_d &= \max_{p_d} [w(1-\alpha) + \alpha p_d]\left(1 - \frac{p_r - p_d - t}{1-\theta}\right)\\
&\quad - c\left(1 - \frac{p_d + t}{\theta}\right)\\
&\quad + [p_d(1-\beta) + l\beta]\left(\frac{p_r - p_d - t}{1-\theta} - \frac{p_d + t}{\theta}\right)\\
&\quad \text{s.t. } p_r \ge \frac{p_d + t}{\theta}.
\end{aligned} \tag{19}$$

Since each profit function is strictly concave, each player has a unique response for every decision of the competitor. Hence, we can get the following proposition.

Proposition 4. *Under the decentralized dual-channel supply chain, there is a threshold delivery time*

$$\begin{aligned}
t^N = {} & \left(\alpha\theta^3 - (1 - \beta + c - w + 3\alpha w)\theta^2\right.\\
& + (1 - \alpha - \beta + 3c - w - \beta l + 3\alpha w - \beta w)\theta\\
& \left. -2c + 2l\beta\right)((2-\theta)(1 - \alpha\theta - \beta))^{-1}.
\end{aligned} \tag{20}$$

If $t > t^N$, the Nash equilibrium prices (p_d^N, p_r^N) are

$$\begin{aligned}
p_d^N = {} & \left((\beta - \alpha - 1)\theta^2\right.\\
& + (1 + \alpha + t + 3w + \alpha t - \beta t - 3\alpha w - \beta w - \beta - 2c)\theta\\
& \left. + 2\beta t + 2c - 2t - 2l\beta\right)\\
& \times (4 + \beta\theta - 3\alpha\theta - 4\beta - \theta)^{-1},\\
p_r^N = {} & \left(\alpha\theta^2 - (\alpha - 2\beta + c + 2 + \alpha t - w + 3\alpha w)\theta\right.\\
& \left. +2 + c + t + 2w - 2\beta - 2\beta w - l\beta - \beta t\right)\\
& \times (4 + \beta\theta - 3\alpha\theta - 4\beta - \theta)^{-1}.
\end{aligned} \tag{Case D1}$$

If $t \le t^N$, the Nash equilibrium prices (p_d^N, p_r^N) are

$$\begin{aligned}
p_d^N = {} & \left(-\alpha\theta^3 + (2\alpha + w + \alpha t - c - 1 - \alpha w)\theta^2\right.\\
& \left. + (1 + c + t + w - \alpha - l\beta - 2\alpha t - \alpha w)\theta + 2\alpha t - 2t\right)\\
& \times \left(2 + 2\alpha\theta - 2\alpha - \beta\theta - 2\alpha\theta^2\right)^{-1},\\
p_r^N = {} & \left(\left(\left(\frac{\beta}{2} + \alpha + w - c - \alpha t - \alpha w - 1\right)\theta\right.\right.\\
& \left. + c + t + w - l\beta - \beta t - \alpha w\right)\\
& \left. \times \left(2 + 2\alpha\theta - 2\alpha - \beta\theta - 2\alpha\theta^2\right)^{-1}\right) + \frac{1}{2}.
\end{aligned} \tag{Case D2}$$

The Nash equilibrium policy is continuous at the threshold t^N.

Proof. The Lagrangian and the Karush-Kuhn-Tucker optimality conditions are

$$\begin{aligned}
L(p_r) &= (p_r - w)(1-\alpha)\left(1 - \frac{p_r - p_d - t}{1-\theta}\right)\\
&\quad + \lambda\left(p_r - \frac{p_d + t}{\theta}\right),\\
L(p_d) &= [w(1-\alpha) + \alpha p_d]\left(1 - \frac{p_r - p_d - t}{1-\theta}\right)\\
&\quad - c\left(1 - \frac{p_d + t}{\theta}\right) + [p_d(1-\beta) + l\beta]\\
&\quad \times \left(\frac{p_r - p_d - t}{1-\theta} - \frac{p_d + t}{\theta}\right) + \lambda\left(p_r - \frac{p_d + t}{\theta}\right),\\
\frac{\partial L(p_r)}{\partial p_r} &= (1-\alpha)\left(1 - \frac{2p_r - p_d - t - w}{1-\theta}\right) + \lambda = 0,\\
\frac{\partial L(p_d)}{\partial p_d} &= \alpha - \frac{2p_d - c + t + \lambda + l\beta - 2\beta p_d - \beta t}{\theta}\\
&\quad + (2p_d - p_r + t - w + l\beta - 2\alpha p_d + \alpha p_r\\
&\quad -2\beta p_d + \beta p_r - \alpha t - \beta t + \alpha w)\\
&\quad \times (\theta - 1)^{-1} = 0,\\
&\quad \lambda\left(p_r - \frac{p_d + t}{\theta}\right) = 0.
\end{aligned} \tag{21}$$

Since the multiplier can be either zero or positive, we could have two cases to examine. Using the similar analysis of Proposition 1, we can get the result. □

5. Supply Chain Coordination

From the above analysis, we can find that the prices of decentralized supply chain are different with the centralized supply

chain. Hence, the total profit of decentralized supply chain is lower than the centralized supply chain. In this section, we will use resale price maintenance (RPM) to coordinate the supply chain. In an RPM (w, p, Q) contract with quantity fixing, the manufacturer specifies a resale price for the retailer and controls the quantity of goods to be sold in the market [24].

Since the coordination policy can only be used in the Stackelberg game, in order to maximize the total supply chain profit, the manufacturer as the Stackelberg leader should let the prices (p_d^S, p_r^S), equal to the centralized prices (p_d^*, p_r^*). This coordination contract can be recognized as the RPM (w, p_d^*, p_r^*) contract. But only the RPM (w, p_d^*, p_r^*) contract may damage one player's profit and benefit the other. Hence, we will discuss the implementation of the contract (w, p_d^*, p_r^*) with other complementary agreements between the manufacturer and the retailer that can coordinate the dual-channel supply chain and ensure a win-win for both players.

In this paper, we find that a two-part tariff agreement is a useful method to resolve this problem; that is, the retailer can charge a lump sum fee (F) when it accepts the contract (w, p_d^*, p_r^*). Hence, the contract (w, p_d^*, p_r^*, F) can coordinate the dual-channel supply chain and enable both the retailer and the manufacturer to be a win-win, where F satisfies $\Pi_r(p_d^S, p_r^S) \le \Pi_r(p_d^*, p_r^*) + F$ and $\Pi_d(p_d^S, p_r^S) \le \Pi_d(p_d^*, p_r^*) - F$, that is,

$$F \in \left[\Pi_r\left(p_d^S, p_r^S\right) - \Pi_r\left(p_d^*, p_r^*\right), \Pi_d\left(p_d^*, p_r^*\right) - \Pi_d\left(p_d^S, p_r^S\right)\right]. \tag{22}$$

Here, F is positive if the contract (w, p_d^*, p_r^*) damages the retailer's profit; F is negative if the contract (w, p_d^*, p_r^*) damages the manufacturer's profit.

We provide an example to illustrate how to use the coordination contract in practice. We use the same base numbers as the next section and let $t = 0.1$. Then we have $\Pi_r(p_d^S, p_r^S) = 0.0400$, $\Pi_d(p_d^S, p_r^S) = 0.0450$ and $\Pi_r(p_d^*, p_r^*) = 0.0371$, $\Pi_d(p_d^*, p_r^*) = 0.0644$. We can find that the (w, p_d^*, p_r^*) enhances the retailer's profit and coordinates the supply chain but does not benefit the manufacturer. Hence, we use a complementary agreement (F) to adjust the profit allocation and ensure a win-win for both the manufacturer and the retailer. Using (22) we can get the retailer to charge a lump sum fee $F \in [0.0029, 0.0194]$, so the contract (w, p_d^*, p_r^*, F) can coordinate the dual-channel supply chain.

6. Numerical Studies

Our objective in this section is to gain further insights based on a numerical example. Also, in this section we only consider the scenario of $p_r \ge (p_d + t)/\theta$ in both centralized and decentralized supply chains. We use the following numbers as the base values of the parameters: $\alpha = 0.2$, $\beta = 0.25$, $l = 0.2$, $c = 0.3$, $\theta = 0.9$, and $w = 0.4$.

In centralized supply chain, we can get $t > t_2^* = -0.2061$. Hence, we know that the prices satisfy Case B1. Then we can get the optimal prices by changing the delivery time t. This can be shown in Figure 1.

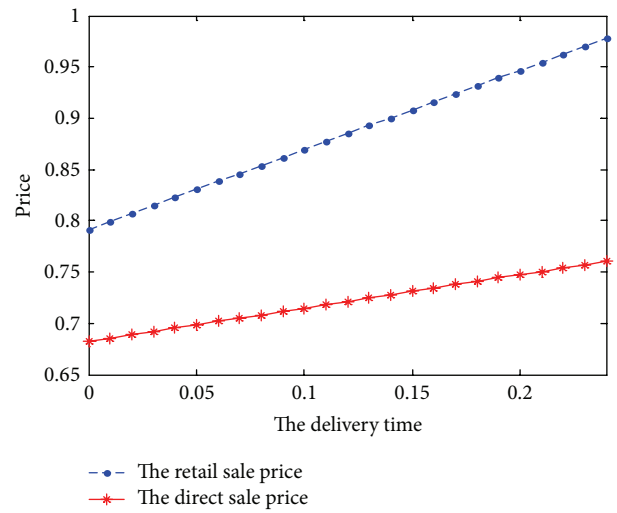

Figure 1: Effect of the delivery time on optimal prices of centralized supply chain.

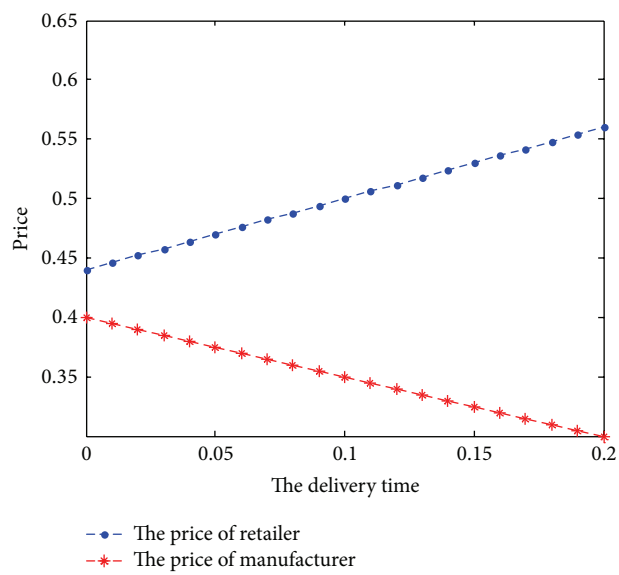

Figure 2: Effect of the delivery time on Stackelberg equilibrium prices.

Figure 1 indicates that the retail sale price is greater than the direct sale price under centralized supply chain. Since both prices are in $(0, 1)$, we can get that the range of the delivery time is $(0, 0.24)$. Also, we can find that the impact of delivery time on the direct sale price is less than on the retail sale price.

In Stackelberg game, similarly we can get the $t_1^S = -0.0575$ and $t_2^S = -0.0186$. Since $p_d > c = 0.3$, we know that the range of t should be $(0, 0.2)$. Hence, we conclude that the Stackelberg equilibrium prices should satisfy Case C3. Then, we can draw the prices curves with the delivery time shown in Figure 2.

From Figure 2, we can see that p_d^S is decreasing in t, while p_r^S is increasing in t. This means that the manufacturer will

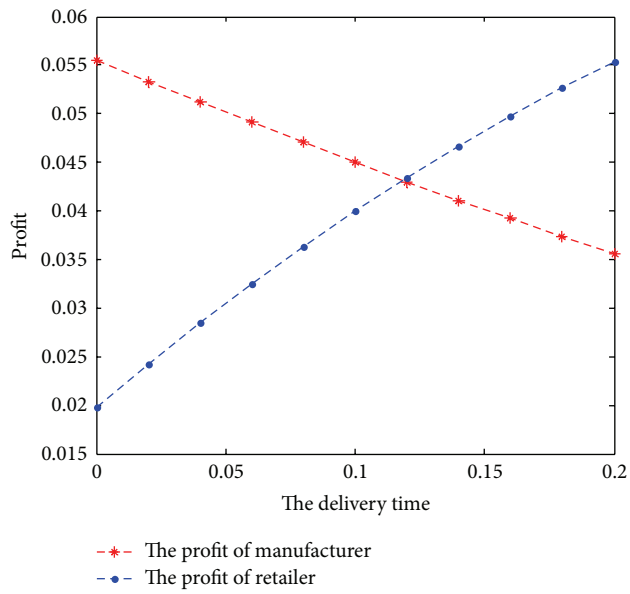

Figure 3: Effect of the delivery time on each player's profit under Stackelberg game.

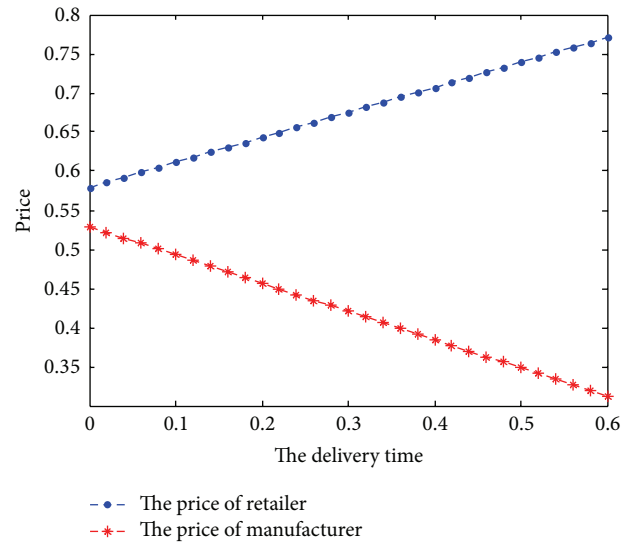

Figure 4: Effect of the delivery time on Nash equilibrium prices.

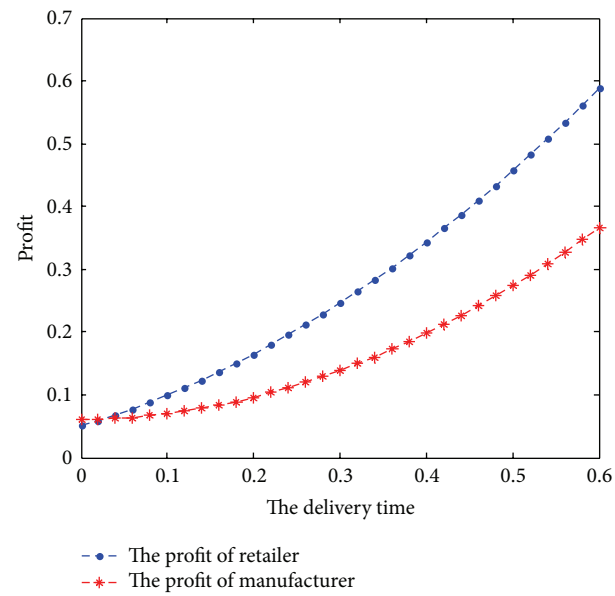

Figure 5: Effect of the delivery time on each player's profit under Nash equilibrium.

have to decrease their direct sale price when their service level is low. This result is caused by the competition from the retailer.

Then, we can get Figure 3 which reflects how the delivery time affects each player's profit under Stackelberg game.

Figure 3 shows that the manufacturer's profit under Stackelberg game is decreasing in t, while the retailer's profit is increasing. Hence, under the Stackelberg game, the manufacturer should improve the service level and increase the direct sale price simultaneously to gain more profit.

In Nash equilibrium, we can get $t^N = -0.325$. Hence, we know that the Nash equilibrium prices satisfy Case D1. Then, we can get the relations of the Nash equilibrium prices and the delivery time shown in Figure 4.

Figure 4 shows that p_d^N is decreasing in t, while p_r^N is increasing in t. Since $p_d^N > c = 0.3$, we can obtain that the valid range of t is $(0, 0.6)$. Then, we can get profit curves shown in Figure 5.

From Figure 5 we can find that both players' profits under Nash equilibrium are increasing in t. From intuition, the manufacturer's profit should decrease when the delivery time increases. However, we find that this intuition is wrong under Nash equilibrium. The main reason is that the source of manufacturer's profit is not only from the direct channel's customers but also from the retailer. Hence, when the delivery time increases, the decrement in direct channel can be offset by increased retailer's order quantity.

From the above analysis, we can find that $t \in (0, 0.2)$ is the common valid range in modes of centralized supply chain, Stackelberg game, and Nash equilibrium. Hence, we will compare the profits under the range of $t \in (0, 0.2)$. We use (Π_d^S, Π_r^S) and (Π_d^N, Π_r^N) to represent the manufacturer's profit and the retailer's profit under Stackelberg game and Nash equilibrium, respectively. Let Π_T^C be the total supply chain's profit under centralized supply chain. Then, we can plot the profit gap between Nash equilibrium and Stackelberg game at the range of $t \in (0, 0.2)$ in Figure 6.

Figure 6 shows that each player's profit under Nash equilibrium is higher than the one under Stackelberg game, and each player's profit gap between the two games is increasing in t.

Figure 7 depicts the total profits of supply chain under three modes with respect to t.

From Figure 7 we can find that the total supply chain's profit under centralized supply chain is larger than the decentralized modes, that is, Stackelberg game and Nash equilibrium. This will make it possible that the decentralized supply

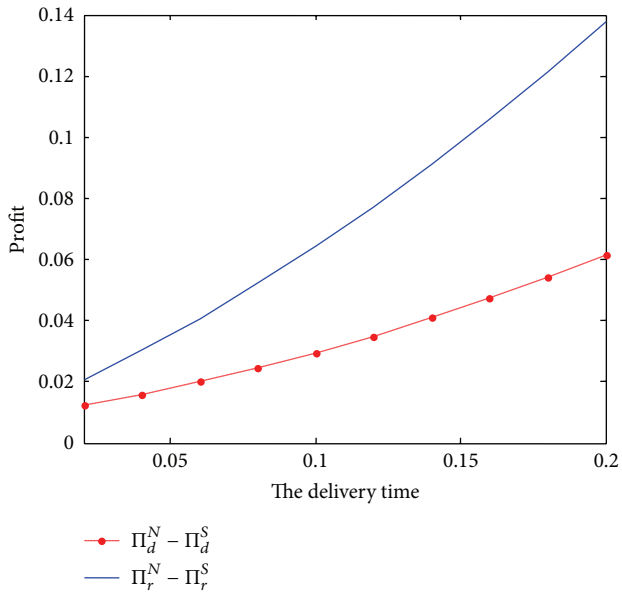

FIGURE 6: Profit gap between Nash equilibrium and Stackelberg game with the delivery time.

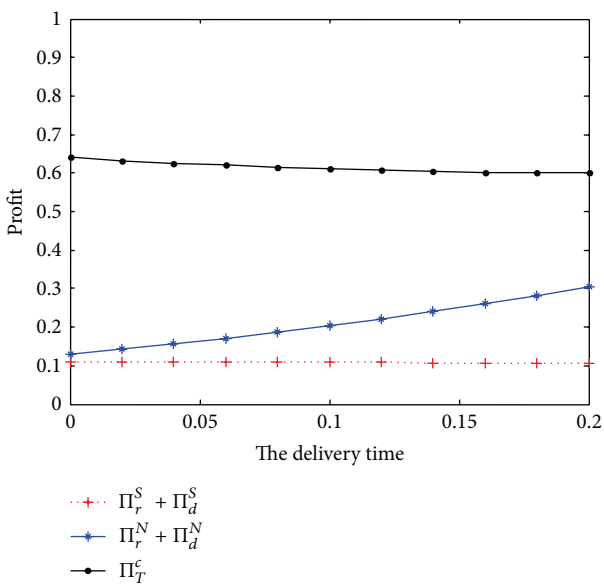

FIGURE 7: Total profits of three modes with respect to the delivery time.

chain can adopt the coordination contract (w, p_d^*, p_r^*, F) to realize the win-win outcome. Also, we can see that the total profit under Nash equilibrium is bigger than the Stackelberg game.

7. Conclusions

Dual-channel supply chain with price and service competition is prevalent in reality. In this paper, considering there exists price and service competition between dual channels, we derive the equilibrium prices under centralized and decentralized supply chains with consumer returns. In decentralized supply chain, we consider the two different competition types, that is, Stackelberg game and Nash equilibrium. Since the total supply chain's profit under decentralized supply chain is less than the centralized supply chain, we put forward a new contract (w, p_d^*, p_r^*, F) to coordinate the decentralized supply chain and gain the win-win outcome.

From numerical studies, we also find some interesting results. For instance, in some cases, both players can get more profit under Nash equilibrium than under the Stackelberg game. In addition, we can find that, under Nash equilibrium, prolonging the delivery time possibly becomes an effective measure to improve each player's profit.

There are several interesting topics for further research. For example, we can extend one retailer to multiple retailers. The competition between the multiple retailers will affect the decisions between channels. We plan to explore this issue in a future study.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Viability Discrimination of a Class of Control Systems on a Nonsmooth Region

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The viability problem is an important field of study in control theory; the corresponding research has profound significance in both theory and practice. In this paper, we consider the viability for both an affine nonlinear hybrid system and a hybrid differential inclusion on a region with subdifferentiable boundary. Based on the nonsmooth analysis theory, we obtain a method to verify the viability condition at a point, when the boundary function of the region is subdifferentiable and its subdifferential is convex hull of many finite points.

1. Introduction

Hybrid systems have been used to describe complex dynamic systems that involve both continuous and discrete systems. Such hybrid systems can be extensively used in robotics, automated highway systems, air traffic management systems, manufacturing, communication networks, and computer synchronization, and so forth. There has been significant research activity in the area of hybrid systems in the past decade involving researchers from several areas [1–8]. In recent years, the viability of systems is an important research topic; it has been widely used in both reach-ability and designing security domain.

In the study of hybrid systems, the concept of viability is more prevalent. The notion of viability was first introduced by Aubin [9]. Viability property provides a very nice theoretical framework for a hybrid controller design problem. Many researchers have considered the problem of viability for the analysis and control of hybrid systems [10–14]. The nonsampling viability problem was examined in the pioneering work of Aubin and coworkers [10] in which impulse differential inclusions are used to describe hybrid behavior.

As an important part of hybrid system, studies in the viability theory include two topics. One is to verify viability condition for a given set. Another one is to design a viable solution within a viable set. Viability conditions for a linear control system have been studied widely in recent years; see [15, 16]. A necessary and sufficient viability condition for a differential inclusion was given in [8, 17], but it is a hard work to check that condition in most applications directly. In the literature [10], the authors give the necessary and sufficient condition of the viability, but it is still very difficult to judge quantitatively. Gao in [18] discusses the viability discrimination for an affine nonlinear control system on a smooth region; it gives some results on continuous system. There is certain limitation in the application of the literature [18]. The limitation is that the region must be smooth; in fact most of the region's boundaries are nonsmooth. Ahmed considers the viability criteria for a hybrid differential inclusions on smooth region in [19]. Gao in [20] gives viability criteria for differential inclusions on a nonsmooth region.

In this paper, we mainly consider the viability condition of a hybrid differential inclusion on a region with subdifferentiable boundary. Based on nonsmooth analysis theory, a method for checking the validity of the viability is given for such case as mapping of the set valued at the right hand of the differential inclusion is a polyhedron, the

boundary function of the region is sub-differentiable, and its sub-differential is a convex hull with finite point set.

The paper is organized as follows. Section 2 states the main assumption, definitions and describes the hybrid dynamics. Section 3 overcomes these limitations in the literature [18]; we deal with the viability criteria for a hybrid system on a region with sub-differentiable boundary. Section 4 considers the viability of a hybrid differential inclusion. Section 5 shows an example.

2. Preliminaries

Consider the general form of nonlinear control system

$$\dot{x}(t) = f(x,u), \quad u \in U, \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state variable, $u \in U$ denotes the control variable, $U \subset \mathbb{R}^m$, and $f(x,u)$ is a Lipschitz function which is from $\mathbb{R}^{m+n}$ to $\mathbb{R}^n$.

Definition 1 (see [8]). Let $W \subset \mathbb{R}^n$ be a subset of $\mathbb{R}^n$, for any initial states $x_0 \in W$, if there exists one solution $x(t)$ of the system (1), such that $x(t) \in W$ for all $t \geq 0$; then we call the subset W viable under the system (1); the solution $x(t)$ is called viable solution.

Definition 2 (see [8]). Let $K \subseteq \mathbb{R}^n$ be a nonempty subset of $\mathbb{R}^n$; the tangent cone of the set K at $x \in K$ is given by the formula

$$T_K(x) = \left\{ v \in \mathbb{R}^n \mid \liminf_{t \to 0^+} d_K(x + tv) = 0 \right\}, \tag{2}$$

where $d_K(y)$ is distance from the point $y \in \mathbb{R}^n$ to the set K.

Definition 3. Let $F(\cdot) : X \to 2^X$ be a set valued map, it is said to be upper semicontinuous if for all $x^0 \in X$ and each $\epsilon > 0$, there exists $\delta > 0$, such that $\|x - x^0\| < \delta$ implies $F(x) \subseteq F(x^0) + \epsilon B$ for all $x \in X$; that is, $F(x) \subseteq B(F(x^0), \epsilon)$.

Definition 4. Let $F(\cdot) : X \to 2^X$ be a set valued map; F is said to be Marchaud if the following conditions hold:

(i) F is upper semicontinuous;

(ii) $F(x)$ is a nonempty convex compact set for all $x \in X$;

(iii) F is linear growth; that is, there exists $\alpha > 0$, such that

$$\sup\{\|v\| \mid v \in F(x)\} \leq \alpha(\|x\| + 1) \tag{3}$$

for all $x \in X$.

Definition 5. Let $F(\cdot) : X \to 2^X$ be a set valued map, if there exists a constant $\lambda > 0$ such that

$$F(x^1) \subseteq F(x^2) + \lambda \left\| x^1 - x^2 \right\| B(0,1) \tag{4}$$

for all $x^1, x^2 \in X$, then F is said to be *Lipschitz*, where $\lambda > 0$ is a *Lipschitz* constant.

Definition 6 (see [10], *hybrid differential inclusion*). A hybrid differential inclusion is a collection $H = (X, F, R, J)$, consisting of a finite dimensional vector space X, a set valued map $F : X \to 2^X$, regarded as a differential inclusion $\dot{x}(t) \in F(x)$, a set valued map $R : X \to 2^X$, regarded as a reset map, and a set $J \subseteq X$, regarded as a forced transition set.

Definition 7 (see [10], *run of a hybrid differential inclusion*). A run of a hybrid differential inclusion $H = (X, F, R, J)$ is a pair (τ, x), consisting of a hybrid time trajectory τ and a map $x : \tau \to X$, that satisfies:

(1) discrete evolution: for all i, $x(\tau_{i+1}) \in R(x(\tau_i'))$;

(2) continuous evolution: if $\tau_i < \tau_i'$, $x(\cdot)$ is a solution to the differential inclusion $\dot{x}(t) \in F(x)$ over the interval $[\tau_i, \tau_i']$ starting at $x(\tau_i)$, with $x(t) \notin J$ for all $t \in [\tau_i, \tau_i')$.

We use $\mathscr{R}_H(x_0)$ to denote the set of all runs of a hybrid differential inclusion $H = (X, F, R, J)$ starting at a state $x(\tau_0) = x_0 \in X$.

Definition 8 (see [10]). Let $H = (X, F, R, J)$ be a hybrid differential inclusion. A set $K \subseteq X$ is called viable under a hybrid differential inclusion H, if for all $x_0 \in K$, there exists an infinite run $(\tau, x) \in \mathscr{R}_H^{\infty}(x_0)$ viable in K. K is called invariant under the hybrid differential inclusion H, if for all $x_0 \in K$, all runs $(\tau, x) \in \mathscr{R}_H(x_0)$ are viable in K.

Proposition 9 (see [8]). *The closed set $W \subset \mathbb{R}^n$ is said to be viable under the system* (1)*, if and only if for any $x \in W$, the following formula is satisfied:*

$$\left(\bigcup_{u \in U} f(x,u) \right) \bigcap T_K(x) \neq \emptyset. \tag{5}$$

For any interior point x in the set W, the tangent cone $T_K(x) = \mathbb{R}^n$, so the above formula is satisfied. Hence, if we want to judge the above formula, we should only consider the boundary point.

3. The Viability of a Hybrid System

To discuss the problem in $\mathbb{R}^n$, we assume $X = \mathbb{R}^n$ in the following paper.

Consider the following hybrid system $H = (X, F, R, J)$, and

$$\dot{x} \in F(x) = f(x) + g(x)u, \quad x \in X, \ u \in U, \tag{6}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{m+n}$ are both Lipschitz functions. $U \subset \mathbb{R}^m$ is a convex set; it denotes

$$U = \{u \in \mathbb{R}^m \mid h_i(u) \leq 0, \ i = 1, 2, \ldots, p\}, \tag{7}$$

where $h_i(u)$ $(i = 1, 2, \ldots, p)$ are convex functions on the $\mathbb{R}^n$. R is a reset map, and J is a forced transition set.

Consider the following region K:

$$K = \left\{ x \in \mathbb{R}^n \mid \varphi_j(x) \leq 0, \ j = 1, 2, \ldots, q \right\}, \tag{8}$$

and $\varphi_j(x)$ $(j = 1, 2, \ldots, q)$ are sub-differentiable functions on $\mathbb{R}^n$. Furthermore, we assume that sub-differential $\partial\varphi_j(x)$ is a convex hull of many finite points.

For hybrid time set $\tau = \{I_i\}_{i=0}^{N}$, where τ is interval sequence. For $i < N$, it has $I_i = [\tau_i, \tau_i']$, for all i, $\tau_i \le \tau_i' = \tau_{i+1}$. $x(\tau_i')$ are the points at which discrete transitions take place, $x(\tau_{i+1})$ are the points after discrete transitions take place; that is,

$$x(\tau_{i+1}) = R\left(x\left(\tau_i'\right)\right), \quad i < N. \tag{9}$$

On the other hand, we assume that the discrete transition does not occur infinite times within the limited time. The set J is a forced transition set; that is, the discrete transition must happen for every point in J. Without generality, we assume that the set K contains the forced transition set J and the set J contains countable transition points. For discussing easily, we still denote by $x(\tau_i')$ $(i = 0, 1, 2, \ldots, N-1)$. In addition, in order to describe the uncertainy in the hybrid differential system and to determine whether discrete transition will happen for every point x in the set

$$R^{-1}(X) = \{x \in X \mid R(x) \neq \emptyset\}, \tag{10}$$

we assume that

$$R^{-1}(X) \subset K, \quad J \subset R^{-1}(X); \tag{11}$$

it can prevent the system from death cycle. Obviously, the points which are in $R^{-1}(X) \setminus J$ may not be jump.

Let

$$\varphi(x) = \max_{1 \le j \le q} \varphi_j(x). \tag{12}$$

Since the point $x \in \mathbb{R}^n$ satisfies

$$\max_{1 \le j \le q} \varphi_j(x) \le 0, \tag{13}$$

which is equivalent to

$$\varphi_j(x) \le 0, \quad (j = 1, 2, \ldots, q), \tag{14}$$

so the set K can be denoted by the following formula:

$$K = \{x \in \mathbb{R}^n \mid \varphi(x) \le 0\}. \tag{15}$$

Because $\varphi_j(x)$ $(j = 1, 2, \ldots, m)$ are sub-differentiable, so $\varphi(x)$ is also sub-differentiable; since $\partial\varphi_j(x)$ is a convex hull of many finite points, the sub-differential of $\varphi(x)$ is also a convex hull of many finite points, marking

$$\partial\varphi(x) = \operatorname{co}\left\{v^1, \ldots, v^r\right\}, \quad v^i \in \mathbb{R}^n, \ i = 1, 2, \ldots, r. \tag{16}$$

Define matrix $B = (v^1, \ldots, v^r)^T$.

In nonsmooth optimization, two frequently used constraint qualifications:

constraint qualification 1 [8]: there exists $y^0 \in \mathbb{R}^n$, such that $\varphi'(x; y^0) < 0$;

constraint qualification 2 [21]: $\operatorname{cl}\gamma(x) = \Gamma(x)$, where

$$\begin{aligned} \gamma(x) &= \left\{y \in \mathbb{R}^n \mid \varphi'(x; y) < 0\right\}, \\ \Gamma(x) &= \left\{y \in \mathbb{R}^n \mid \varphi'(x; y) \le 0\right\}. \end{aligned} \tag{17}$$

Lemma 10 (see [14, 21]). *If the set K satisfied constraint qualification 1 or constraint qualification 2 at $x \in \mathbb{R}^n$, then $T_K(x) = \Gamma(x)$.*

According to [20], we get the following Proposition 11 immediately.

Proposition 11 (see [20]). *Assume that constraint qualification 1 or 2 is satisfied; then $T_K(x) = \{y \in \mathbb{R}^n \mid By \le 0\}$, where $B = (v^1, \ldots, v^r)^T$, $v^i \in \mathbb{R}^n$ $(i = 1, 2, \ldots, r)$, $\partial g(x) = \operatorname{co}\{v^1, \ldots, v^r\}$.*

Lemma 12 (see [10]). *Consider a hybrid system $H = (X, F, R, J)$ such that F is Marchaud, R is upper semicontinuous with closed domain, and J is a closed set. A closed set $K \subseteq X$ is viable under H if and only if*

(1) $K \cap J \subseteq R^{-1}(K)$;

(2) $F(x) \cap T_K(x) \neq \emptyset$, $\forall x \in K \setminus R^{-1}(K)$.

Before we state Theorem 13, we construct the following inequality system:

$$h_i(u) \le 0, \quad i = 1, 2, \ldots, p, \qquad Bf(x) + Bg(x)u \le 0, \tag{18}$$

where $u \in \mathbb{R}^m$ is a variable.

According to [20] and Lemma 12, we get the following theorem immediately.

Theorem 13. *For the above hybrid system $H = (X, F, R, J)$, if*

$$\begin{aligned} K &= \left\{x \in X \mid \varphi_j(x) \le 0, \ j = 1, 2, \ldots, m\right\} \\ &= \left\{x \in X \mid \varphi(x) \le 0\right\} \end{aligned} \tag{19}$$

satisfies constraint qualification 1 or 2, then the set K is viable under the hybrid system H if and only if

(1) *discrete transition (or jump) must take place: $\varphi(x(\tau_{i+1})) \le 0$, $i = 0, 1, 2, \ldots, N-1$.*

(2) *continuous section: for each fixed point $x \in K \setminus R^{-1}(K)$ inequality system (18) is solvable.*

Proof. Under the above assumptions, it is sufficient to show that Theorem 13(1) is equivalent to Lemma 12(1) and Theorem 13(2) is equivalent to Lemma 12(2).

In Lemma 12(1), $K \cap J \subseteq R^{-1}(K)$ is equivalent to the following statement: when discrete transition (or jump) must happen $(x \in K \cap J)$ for every $x \in K$, then the point after the transition (or jump) must be in the set K $(R(x) \cap K \neq \emptyset)$. Based on the aforementioned assumptions, for the jump point $x(\tau_0'), x(\tau_1'), \ldots, x(\tau_{N-1}')$ contained in the set J, we only need to show that the point will still be in K after the jump$(x(\tau_{i+1}) = R(x(\tau_i')))$, $(i = 0, 1, 2, \ldots, N-1)$. That is, $x(\tau_{i+1}) \in K$ $(i = 0, 1, 2, \ldots, N-1)$. Since $K = \{x \in X \mid \varphi(x) \le 0\}$, $\varphi(x(\tau_{i+1})) \le 0$ $(i = 0, 1, 2, \ldots, N-1)$. Hence Theorem 13(1) is equivalent to Lemma 12(1).

The Lemma 12(2) is sufficient to show that the changes is possible $(F(x) \cap T_K(x) \neq \emptyset)$ for continuous section in K,

when discrete transition point (or jump point) ($R(x) \cap K = \emptyset$) will be not in K after the jump. The set K satisfies constraint qualification 1 or 2; then $T_K(x) = \{y \in \mathbb{R}^n \mid By \leq 0\}$. We set $f(x,u) = f(x) + g(x)u$ in Proposition 9; then the set K is viable under the hybrid system H if and only if the following formula is satisfied:

$$\left(\bigcup_{u \in U} (f(x) + g(x)u)\right) \bigcap T_K(x) \neq \emptyset, \tag{20}$$

where x is a fixed point in $K \setminus R^{-1}(K)$. Consider the expressions of the set U and $T_K(x)$; the above expression is equivalent to

$$\begin{aligned} &\{f(x) + g(x)u \mid h_i(u) \leq 0, \\ &\quad i = 1, 2, \ldots, p\} \bigcap \{y \in \mathbb{R}^n \mid By \leq 0\} \neq \emptyset. \end{aligned} \tag{21}$$

Obviously, the above equation is equivalent to following solvable system:

$$\begin{gathered} h_i(u) \leq 0, \quad i = 1, 2, \ldots, p, \qquad By \leq 0, \\ y = f(x) + g(x)u. \end{gathered} \tag{22}$$

In (22), we set that $y = f(x) + g(x)u$ substitute into $By \leq 0$; then we can obtain (18). Also, we can obtain (22) by substituting $y = f(x) + g(x)u$ into (18). This shows that the system (18) is equivalent to the system (22). This completes the proof. □

4. The Viability of a Hybrid Differential Inclusion

Hybrid differential inclusion can describe a hybrid system in a wide range of significance.

Consider the following hybrid differential system $H = (X, F, R, J)$, and

$$\dot{x} \in F(x) = \operatorname{co}\{f_i(x) \mid i = 1, 2, \ldots, p\}, \quad x \in X, \tag{23}$$

where $f_i(x)$ ($i = 1, 2, \ldots, p$) are functions on X. R is a reset map, and J is a forced transition set.

Consider the following region K:

$$K = \{x \in X \mid g_j(x) \leq 0,\ j = 1, 2, \ldots, m\}, \tag{24}$$

where $g_j(x)$ ($j = 1, 2, \ldots, m$) are sub-differentiable functions on X. We further assume that the functions $g_j(x)$ are sub-differentiable, and sub-differential $\partial g_j(x)$ is a convex hull with finite point set. Let

$$g(x) = \max_{1 \leq j \leq m} g_j(x); \tag{25}$$

then the set K can be rewritten as

$$K = \{x \in X \mid g(x) \leq 0\}. \tag{26}$$

Since $g_j(x)$ ($j = 1, 2, \ldots, m$) are sub-differentiable, $g(x)$ is sub-differentiable. Because $\partial g_j(x)$ is a convex hull with finite point set, sub-differential of $g(x)$ is also a convex hull with finite point set, denoted by

$$\partial g(x) = \operatorname{co}\{v^1, \ldots, v^q\}, \qquad v^i \in X \quad (i = 1, 2, \ldots, q). \tag{27}$$

Theorem 14. *For the above hybrid differential inclusion $H = (X, F, R, J)$, if*

$$\begin{aligned} K &= \{x \in X \mid g_j(x) \leq 0,\ j = 1, 2, \ldots, m\} \\ &= \{x \in X \mid g(x) \leq 0\} \end{aligned} \tag{28}$$

satisfies constraint qualification 1 or 2, then the set K is viable under the hybrid differential inclusion H if and only if

(1) *discrete transition (or jump) must take place: $g(x(\tau_{i+1})) \leq 0$, $i = 0, 1, 2, \ldots, N-1$;*

(2) *continuous section: Optimal value of the following linear programming problem (P) is zero for each $x \in K \setminus R^{-1}(K)$. Consider*

$$\begin{aligned} \min \quad & \omega \\ \text{s.t.} \quad & \sum_{i=1}^{p} \lambda_i B f_i(x_i) + (\omega, \ldots, \omega)^T \leq 0, \\ & \sum_{i=1}^{p} \lambda_i = 1, \\ & \lambda_i \geq 0, \quad i = 1, 2, \ldots, p, \\ & \omega \geq 0, \end{aligned} \tag{P}$$

where $B = (v^1, \ldots, v^q)^T$.

Proof. Under the above assumptions, it is sufficient to show that Theorem 14(1) is equivalent to Lemma 12(1) and Theorem 14(2) is equivalent to Lemma 12(2).

In Lemma 12(1), $K \cap J \subseteq R^{-1}(K)$ is equivalent to the following statement: when discrete transition (or jump) must happen ($x \in K \cap J$) for every $x \in K$, then the point after the transition (or jump) must be in the set K ($R(x) \cap K \neq \emptyset$). Based on the aforementioned assumptions, for the jump point $x(\tau_0'), x(\tau_1'), \ldots, x(\tau_{N-1}')$ in the set J, we only need to show that the point after the jump ($(x(\tau_{i+1}) = R(x(\tau_i'))$, $i = 0, 1, 2, \ldots, N-1$)) will be still in K. That is, $x(\tau_{i+1}) \in K$ ($i = 0, 1, 2, \ldots, N-1$). Since $K = \{x \in X \mid g(x) \leq 0\}$, $g(x(\tau_{i+1})) \leq 0$ ($i = 0, 1, 2, \ldots, N-1$). Hence Theorem 14(1) is equivalent to Lemma 12(1).

In Lemma 12(2), we noticed that when discrete transition point (or jump point) after the jump ($R(x) \cap K = \emptyset$) will be not in K, then the changes are possible ($F(x) \cap T_K(x) \neq \emptyset$) for continuous section in K. Since the set K satisfies constraint qualification 1 or 2,

$$\begin{gathered} T_K(x) = \{y \in X \mid By \leq 0\}, \quad B = \left(v^1, \ldots, v^q\right)^T, \\ v^i \in X \quad (i = 1, 2, \ldots, q), \qquad \partial g(x) = \operatorname{co}\{v^1, \ldots, v^q\}. \end{gathered} \tag{29}$$

In addition,

$$F(x) = \mathrm{co}\{f_i(x) \mid i = 1, 2, \ldots, p\}; \tag{30}$$

then the condition

$$F(x) \cap T_K(x) \neq \emptyset \tag{31}$$

and the following problem which has a solution

$$\begin{gathered} B\left(\sum_{i=1}^{p} \lambda_i f_i(x)\right) = \sum_{i=1}^{p} \lambda_i B f_i(x) \leq 0, \\ \sum_{i=1}^{p} \lambda_i = 1, \quad \lambda_i \geq 0, \quad i = 1, 2, \ldots, p, \end{gathered} \tag{32}$$

are equivalent, and also are equivalent to the linear programming problem (P) in which the optimal solution is zero. This completes the proof. □

Lemma 15 (see [10]). *Let hybrid differential inclusion be $H = (X, F, R, J)$ such that F is Marchaud and Lipschitz, and J is a closed set. A closed set $K \subseteq X$ is invariant under H if and only if*

(1) *$R(K) \subseteq K$;*

(2) *$F(x) \subseteq T_K(x)$, for all $x \in K \setminus J$.*

Theorem 16. *$H = (X, F, R, J)$ is a hybrid differential inclusion as above; if the set K satisfies constraint qualification 1 or 2, then the set $K = \{x \in X \mid g(x) \leq 0\}$ is invariant under hybrid differential inclusion H if and only if*

(1) *discrete transition (or jump) must take place: $g(x(\tau_{i+1})) \leq 0$, $i = 0, 1, 2, \ldots, N-1$;*

uncertainty section: $g(R(x)) \leq 0$, for all $x \in K \setminus J$;

(2) *continuous section: $Bf_i(x) \leq 0$, $i = 1, 2, \ldots, p$, for all $x \in K \setminus J$.*

Proof. Under the above assumptions, it is sufficient to show that Theorem 16(1) is equivalent to Lemma 15(1) and Theorem 16(2) is equivalent to Lemma 15(2).

In Lemma 15(1), to verify $R(K) \subseteq K$, we just need to show that the point after the transition (or jump) must be in the set K ($R(K) \subseteq K$), when discrete transition (or jump) must happen ($x \in J$) for every $x \in K$. By the previous assumptions, the jump point contained in the set J should show that the point will be still in K after the jump. That is, there exists $x(\tau_{i+1}) \in R(x(\tau_i'))$ ($i = 0, 1, 2, \ldots, N-1$), such that $x(\tau_{i+1}) \in K$ ($i = 0, 1, 2, \ldots, N-1$); that is, $g(x(\tau_{i+1})) \leq 0$ ($i = 0, 1, 2, \ldots, N-1$). In addition, for each x in $K \setminus J$, the point after the transition (or jump) will still be in K; that is, $g(R(x)) \leq 0$, $x \in K \setminus J$. Hence, Theorem 16(1) is equivalent to Lemma 15(1).

In Lemma 15(2), for $x \in K \setminus J$, $F(x) \subseteq T_K(x)$ is equivalent to the following statement: if the continuous evolution is possible ($x \notin J$), then all solutions of $\dot{x} \in F(x)$ are all in K ($F(x) \subseteq T_K(x)$). The set K satisfies constraint qualification 1 or 2; then

$$T_K(x) = \{y \in X \mid By \leq 0\}. \tag{33}$$

Moreover,

$$F(x) = \mathrm{co}\{f_i(x) \mid i = 1, 2, \ldots, p\} \tag{34}$$

then the condition $F(x) \subseteq T_K(x)$ is equivalent to $Bf_i(x) \leq 0$ ($i = 1, 2, \ldots, p$), which completes the proof of Theorem 16. □

5. Example

We provide here an example that better illustrates the class of hybrid systems where our theoretical framework is relevant.

Consider the differential inclusion $H = (X, F, R, J)$, where

$$\begin{gathered} F(x) = \mathrm{co}\{f_1(x), f_2(x)\}, \quad x \in \mathbb{R}^2, \\ f_1(x) = (x_1 + x_2, x_2 + 1)^T, \qquad f_2(x) = (x_1 + x_2 + 1, x_2)^T, \\ J = \{x \in \mathbb{R}^2 \mid x_1 \geq 0, x_2 \geq 0, x_1 + x_2 - 1 \leq 0\}, \\ K = \{x \in \mathbb{R}^2 \mid g(x) \leq 0\}, \\ g(x) = \max\{-x_1, -x_2, x_1^2 + x_2^2 - 1\}, \\ R(x) = \left(x_1 + \frac{1}{2}, x_2 - \frac{1}{3}\right)^T, \quad x = (x_1, x_2)^T. \end{gathered} \tag{35}$$

We can easily conclude that $g(x)$ is a sub-differentiable function, and the set K is a quarter of the unit circle.

(1) Viability discrimination of the point $x^{(1)} = (0, 1)^T$ under the hybrid differential inclusion H: by the definition and operation of the sub-differential,

$$\partial g\left(x^{(1)}\right) = \mathrm{co}\left\{(-1, 0)^T, (0, 2)^T\right\}. \tag{36}$$

Obviously, the point $x^{(1)}$ is in the set J, so discrete transition (or jump) must take place. The point after jump is $R((0, 1)^T) = (1/2, 2/3)^T$. It can be shown that the point $(1/2, 2/3)^T$ in $K \setminus J$ and it is the interior point of the set $K \setminus J$. Hence the point $x^{(1)} = (0, 1)^T$ in K satisfied the viability condition.

(2) Viability discrimination of the point $x^{(2)} = (1/2, \sqrt{3}/2)^T$ under the hybrid differential inclusion H: Obviously, the point $x^{(2)} = (1/2, \sqrt{3}/2)^T$ is in the set $K \setminus J$ and it is the boundary point. Viability discrimination of the point $x^{(2)} = (1/2, \sqrt{3}/2)^T$

is equivalent to the following linear programming problem (37) in which the optimal solution is zero

$$\begin{aligned} &\min \quad \omega \\ &\text{s.t.} \quad \sum_{i=1}^{p} \lambda_i B f_i(x) + (\omega, \ldots, \omega)^T \leq 0, \\ &\qquad \lambda_i \geq 0, \quad i = 1, 2, \\ &\qquad \lambda_1 + \lambda_2 = 1, \\ &\qquad \omega \geq 0. \end{aligned} \tag{37}$$

We can obtain sub-differential $\partial g(x^{(2)}) = (1, \sqrt{3})^T$; hence, $B = (1, \sqrt{3})$,

$$\begin{aligned} f_1\left(x^{(2)}\right) &= \left(\frac{1+\sqrt{3}}{2}, \frac{2+\sqrt{3}}{2}\right)^T, \\ f_2\left(x^{(2)}\right) &= \left(\frac{3+\sqrt{3}}{2}, \sqrt{3}\right)^T. \end{aligned} \tag{38}$$

Consequently, the linear programming problem reduces to

$$\begin{aligned} &\min \quad \omega \\ &\text{s.t.} \quad \frac{4+3\sqrt{3}}{2}\lambda_1 + \frac{9+\sqrt{3}}{2}\lambda_2 - \omega \leq 0, \\ &\qquad \lambda_i \geq 0, \quad i = 1, 2. \\ &\qquad \lambda_1 + \lambda_2 = 1, \\ &\qquad \omega \geq 0. \end{aligned} \tag{39}$$

Obviously, $\omega = 0$ is not the optimal value of constrained optimization. Hence $x^{(2)} = (1/2, \sqrt{3}/2)^T$ does not satisfy the conditions for the viability of differential inclusion.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Incentive Contract in Supply Chain with Asymmetric Information

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The supply chain always appears inefficient because of the different targets of members and information asymmetry, especially when upstream enterprises not only hide information about their effort levels, but also hide information about their technology level. The paper uses principal-agent theory and the theory of regulation to design the contract to realize the maximization of principal's profit on the condition that the contract satisfies the participant and incentive conditions of agent. As a result, it is obvious that the contract achieves the goal of control. In addition, it also can be concluded that the amount of rent that the manufacturer can obtain is up to the value of his information and the condition of his resource.

1. Introduction

Supply chain is a network which can put suppliers, manufacturers, distributors, retailers, and final users together, which is characterized by the integration of external resources for cooperation networks. Supply chain performance depends on the joint benefit of the enterprises [1]. However, the supply chain always appears inefficient because of the different targets of members and information asymmetry. Therefore, an important issue in supply chain management is how to establish the appropriate coordination mechanism for the independent enterprises in order to achieve the maximization of the overall profit of the supply chain [2–8].

Because of information asymmetry, the difficulty of coordination increases [9]. Many contracts and pieces of literature have studied the coordination of supply chain from perspective of principal-agent problem, while, at the same time, various aspects of coordination in supply chain have been studied too, such as pieces of literature [10–16].

In practice, the effort level of retailers can affect products' demand. Literature [17] puts the effort level and risk preference of decision-making of node enterprise into decision-making model and analyses their impact on the decision and cooperation of supply chain. The literature [18] investigates the issue of channel coordination for a supply chain facing stochastic demand that is sensitive to both sales effort and retail price. In the literature [19], a perishable product's supply chain consisting of a manufacturer and a retailer is considered; on the premise of retailer's effort and return price dependent demand, the mathematical models of quantity flexibility contract are established. The literature [20] analyses retailer's effort level's impact on supply chain revenue-sharing evolvement- contract and gain retailer's effort level's reaction function and its optimal value.

These pieces of literature, for example, literature [19], think that effort level of the upstream agents is private information which is the key to cause motivation. The profit of supply chain increases synchronously when the effort level of upstream enterprises increases, while, at the same time, the cost of effort level of upstream enterprises also increases. What is more, the increased cost of upstream enterprises may be much more than profit of upstream enterprises. Therefore, in order to maximize their own profits, efforts of downstream enterprises may be not in the appropriate level. So, an effective solution to the principal-agent problem between the upstream and downstream enterprises is to stimulate the upstream enterprises to improve their effort levels. However, we think that, in fact, upstream enterprises

not only hide information about their effort levels, but also hide information about their technology level (or resource endowments). If the downstream enterprise has poor knowledge about distribution of technology level to the upstream enterprise, the upstream enterprise may obtain higher rent (e.g., supply chain distribution is carried out in accordance with the amount of resource input. If the downstream enterprise has vaguer understanding of the upstream firm's resource input, the upstream firm's transferring rent may be higher). Therefore, an effective solution to the agency problem between the upstream and downstream enterprises may need not only simulating the effort level of downstream firm, but also taking the effect of technology level to income distribution into account. This paper designs the incentive system based on the supply chain through two kinds of constraints, moral hazard and adverse selection.

2. Assumptions of Model

This paper considers that there are one seller and one manufacturer in a two-lever supply chain. The seller simulates the manufacturer that hides private information. In order to keep generality, Assumptions 5–7 are basic hypothesizes of general information economics. Assumptions 1–4 are the basis of our model.

Assumption 1. When he gets paid from consumers, the seller gives the manufacturer cost compensation $s \cdot C$ (s is compensation rate and $0 < s < 1$) and transferring payment t.

This assumption points out the characteristics of the decision-making. The profit of the manufacturer is made up with the payment of cost compensation and the transferring payment t. Therefore, the key motivation of the seller to the manufacturer is the design of t. It is a common assumption in incentive problem. For example, literature [20] uses the same assumption.

Assumption 2. There are moral hazard and adverse selection in motivation.

In contract, the manufacturer has his private information of technology, but the cost of the activity which is reduced cannot be observed by the seller. Cost function [21] can be written in the following form:

$$C = (\beta - e)\, q + \alpha. \tag{1}$$

In this function, β is a technical parameter (high β means inefficient technical level). The seller does not know the β of the manufacturer, but he knows that β is a continuous parameter which belongs to $[\underline{\beta}, \overline{\beta}]$; $F(\cdot)$ means the absolutely continuous distribution function and its density is $f(\cdot)$; what is more, when $\beta \in [\underline{\beta}, \overline{\beta}]$, $f(\beta) > 0$. In addition, the monotone hazard rate or log-concave $d[F(\beta)/f(\beta)]/d\beta \geq 0$. For most distributions, such as uniform distribution, normal distribution, logarithmic distribution, exponential distribution, and Laplace distribution, they all satisfy the condition of monotone hazard rate [21]. e is the effort or cost reduction activities (the speed of cost reduction activity is decreasing). e is related to the types of the manufacturer; in other words, $e = e(\beta)$. This cannot be observed by the seller. We can define $\psi(e)$ as the cost of effort and it satisfies $\psi'(e) > 0, \psi''(e) > 0, \psi'''(e) \geq 0, \psi(\beta) = +\infty$. q means production. α is the fixed cost which we standardize to zero. At this point, $c = (\beta - e)$ is not only the marginal cost of the manufacturer, but also the average cost of the manufacturer.

This assumption means that this paper involves two kinds of asymmetric information. Generally speaking, most of the pieces of literature about asymmetric information only involve a certain kind. For example, literature [22] provides quantity discount contract in which cost is private information (adverse selection). Literature [23] studies problem about moral hazard in warranty contract. However, a few pieces of literature study two kinds of asymmetric information. For example, literature [24] uses the principal-agent theory and studies the game between supplier and retailer in supply chain with two kinds of asymmetric information—adverse selection and moral hazard.

Assumption 3. The cost of the manufacturer is C and the production is q; what is more, the cost and production can be verified. However, the seller cannot distinguish the various components of the cost.

This assumption indicates that the optimal incentive contract is based on the total cost and demand data. It is the extension of Assumption 2.

Assumption 4. Because the seller cannot observe β, e, when the seller designs the incentive contracts, he considers the design of t through $\widehat{\beta}$ which is announced by the manufacturer.

Assumption 5. If the contract cannot guarantee the lowest expected utility of the manufacturer, the manufacturer can refuse to produce.

This assumption forces the seller to keep to the "participation" constraint of the manufacturer. It can be seen in many pieces of literature about contract design, such as pieces of literature [18–20], and literature [22]. We can define U as the expected utility of the manufacturer and standardize the lowest expected utility to zero. $U \geq 0$ is the rent of the manufacturer or surplus.

Assumption 6. The risks of the seller and manufacturer are neutral.

This assumption not only can decrease the computational difficulty, but also does not lose generality.

Assumption 7. In order to discuss easily, we can define the relationship of the total production and price as

$$q = a - bp. \tag{2}$$

Based on these assumptions, the seller is the designer and executor of the contract. The seller can design the transfer

payment t to motivate the manufacturer, while, at the same time, the action of the manufacturer chooses the lever of effort e and announces the level of technical parameter $\widehat{\beta}$. The design of incentive system has the following game order and strategies.

(1) The seller designs the contract, which is the function of the total output, cost, and $\widehat{\beta}$.

(2) When the manufacturer knows the content of the contract, the manufacturer needs to decide whether to accept the contract, to choose the lever of effort, and to announce the level of $\widehat{\beta}$.

(3) The seller observes the total production, cost, and $\widehat{\beta}$.

(4) According to the observations, the seller executes the contract.

3. Model Analysis and Solution

3.1. Conditions of Participation and Incentive for the Manufacturer. According to Assumptions 2–5, when the manufacturer announces the technical parameter $\widehat{\beta}$, the transfer payment that is received by the manufacturer is $t(\widehat{\beta})$, and the utility of the manufacturer is the function of real technical parameter β and announced technical parameter $\widehat{\beta}$:

$$u\left(\beta,\widehat{\beta}\right)=t\left(\widehat{\beta}\right)-(1-s)\,c\left(\widehat{\beta}\right)q-\psi\left(\beta-c\left(\widehat{\beta}\right)\right). \tag{3}$$

According to Assumption 6, the contract must reach the lowest expected utility level of the manufacturer, or the manufacturer will refuse to cooperate. So,

$$u\left(\beta,\widehat{\beta}\right)\geq 0. \tag{4}$$

Formula (4) embodies the manufacturer's participation constraint. When the seller designs contract, he must keep the "participation" constraint of the seller.

In order to motivate the manufacturer to announce the real technical parameter, the design of the contract must satisfy the following conditions.

For all $\beta_1,\beta_2\in[\underline{\beta},\overline{\beta}]$, if the manufacturer claims that his technology level is higher than the actual level, the seller will pay more rent to motivate. Therefore, in order to stimulate the manufacturer to announce the true technology level, the seller develops the following rules:

$$\begin{aligned}
&t(\beta_1)-(1-s)\,c(\beta_1)\,q-\psi(\beta_1-c(\beta_1))\\
&\qquad\geq t(\beta_2)-(1-s)\,c(\beta_2)\,q-\psi(\beta_1-c(\beta_2)),\\
&t(\beta_2)-(1-s)\,c(\beta_2)\,q-\psi(\beta_2-c(\beta_2))\\
&\qquad\geq t(\beta_1)-(1-s)\,c(\beta_1)\,q-\psi(\beta_2-c(\beta_1)).
\end{aligned} \tag{5}$$

Formula (5) may transform to

$$\int_{\beta_1}^{\beta_2}\int_{c(\beta_1)}^{c(\beta_2)}\psi''(y-x)\,\mathrm{d}x\,\mathrm{d}y\geq 0. \tag{6}$$

Form Assumption 2, we know that $\psi''(\cdot)>0$. So formula (6) indicates that $c(\beta)$ is the nondecreasing function of β; in other words, $\dot{c}(\beta)\geq 0$.

If the manufacturer chooses the best $\widehat{\beta}$ in order to maximize the rent, we can define $U(\beta)=\max_{\widehat{\beta}}u(\beta,\widehat{\beta})$ as the rent of the manufacturer, and $\widehat{\beta}$ is based on the real technical parameter β; in other words, $\widehat{\beta}=\widehat{\beta}(\beta)$. Then the maximum first derivative of formula (3) to β is zero. We use the envelope theorem to maximize formula (3) and announce that the technical parameter $\widehat{\beta}=\beta$; we can get that

$$\dot{U}(\beta)=-\psi'(\beta-c(\beta)). \tag{7}$$

If you meet the first-order condition, and $c(\beta)$ is nondecreasing, then formula (7) is not only the necessary condition, but also the sufficient condition of both formulas of (5), thereby it can satisfies the incentive requirements. Then from "participation" condition (4), we know that $U(\beta)\geq 0$. Because the rent of the seller which is designed to motivate is cost, and from the formula (7), we know that $\dot{U}(\beta)\leq 0$. So, the seller defines $U(\overline{\beta})=0$. We can obtain from formula (7)

$$U(\beta)=\int_{\beta}^{\overline{\beta}}\psi'\left(e\left(\widetilde{\beta}\right)\right)\mathrm{d}\widetilde{\beta}. \tag{8}$$

So, the expected transfer rent of the seller is

$$\begin{aligned}
\int_{\underline{\beta}}^{\overline{\beta}}U(\beta)\,\mathrm{d}F(\beta)&=\int_{\underline{\beta}}^{\overline{\beta}}\int_{\beta}^{\overline{\beta}}\psi'\left(e\left(\widetilde{\beta}\right)\right)\mathrm{d}\widetilde{\beta}\,\mathrm{d}F(\beta)\\
&=\int_{\underline{\beta}}^{\overline{\beta}}\frac{F(\beta)}{f(\beta)}\psi'(e(\beta))\,\mathrm{d}F(\beta).
\end{aligned} \tag{9}$$

The second equation uses integration by parts. Therefore, we can get the following incentive compatibility constraints:

$$\int_{\underline{\beta}}^{\overline{\beta}}U(\beta)\,\mathrm{d}F(\beta)=\int_{\underline{\beta}}^{\overline{\beta}}\frac{F(\beta)}{f(\beta)}\psi'(e(\beta))\,\mathrm{d}F(\beta), \tag{10}$$

$$\dot{c}(\beta)\geq 0. \tag{11}$$

3.2. The Goal of the Seller. The seller can decide $t(\cdot)$ and maximize its expected profit according to the observed contract parameters $c(\cdot),q(\cdot)$ (the cost of the seller can be standardized to 0). Consider

$$\max_{\{C(\cdot),t(\cdot),q(\cdot)\}}\int_{\underline{\beta}}^{\overline{\beta}}\left[p(q)\cdot q-s\cdot c\cdot q-t\right]\mathrm{d}F(\beta). \tag{12}$$

Put formulas (1) and (2) into formula (12), we can get

$$\begin{aligned}
\max_{\{e(\cdot),q(\cdot),U(\cdot)\}}\int_{\underline{\beta}}^{\overline{\beta}}&\left[p(q)\cdot q-(\beta-e(\beta))\cdot q-\psi(e(\beta))\right.\\
&\left.-U(\beta)\right]\mathrm{d}F(\beta).
\end{aligned} \tag{13}$$

At the same time, when the seller designs the contract, he is also restricted by the incentive compatibility constraints, namely, (10) and (11) constraints.

3.3. Determination of the Contract. Put formula (10) into formula (13), we can get the optimal planning of the seller:

$$\max_{\{e(\cdot),q(\cdot),U(\cdot)\}} \int_{\underline{\beta}}^{\overline{\beta}} \Big[p(q)\cdot q - (\beta - e(\beta))\cdot q - \psi(e(\beta)) - \frac{F(\beta)}{f(\beta)}\psi'(e(\beta)) \Big] \mathrm{d}F(\beta). \tag{14}$$

The condition of constraint is

$$\dot{e}(\beta) \le 1. \tag{15}$$

Here, formula (15) is based on the $C(\beta) = (\beta - e(\beta))\cdot q$ of Assumption 2 and formula (11).

From Assumption 2, we can know that $\psi'''(e) \ge 0$, so the integrand function of formula (14) is concave. Let us ignore the constraint of formula (15); find the first-order condition of $e(\cdot), q(\cdot)$ from the integral term of formula (14) and we can obtain

$$q - \psi'(e(\beta)) - \frac{F(\beta)}{f(\beta)}\psi''(e(\beta)) = 0, \tag{16}$$

$$p(q) - (\beta - e(\beta)) + \dot{p}(q)\,q = 0. \tag{17}$$

Through formulas (2) and (17), we can get

$$q = \frac{a}{2} - \frac{b}{2}(\beta - e(\beta)). \tag{18}$$

Then

$$\dot{q}(\beta) = -\frac{b}{2}((\beta - e(\beta)))'_{\beta} = -\frac{b}{2}\dot{c}(\beta) \le 0. \tag{19}$$

Find the derivate of β from formula (16), we can get

$$\dot{q}(\beta) - \psi''(e(\beta))\cdot\dot{e}(\beta) - \frac{d}{d\beta}\left[\frac{F(\beta)}{f(\beta)}\right]\cdot\psi''(e(\beta)) - \frac{F(\beta)}{f(\beta)}\cdot\psi'''(e(\beta))\cdot\dot{e}(\beta) = 0. \tag{20}$$

So

$$\dot{e}(\beta) = -\frac{0.5b\cdot\dot{c}_{\beta} + \psi''(e(\beta))\cdot\{d[F(\beta)/f(\beta)]/d\beta\}}{\psi''(e(\beta)) + \psi'''(e(\beta))\cdot[F(\beta)/f(\beta)]}. \tag{21}$$

From Assumption 2 and formula (19), we can know that the denominator of formula (21) is positive. Then through Assumption 2, the monotone risk rate is positive, and we know that the molecule of formula (21) is positive. So,

$$\dot{e}(\beta) \le 0_{\circ}. \tag{22}$$

It satisfies the condition of formula (20).

From formulas (16) and (17), we can get $e^*(\beta)$ and $q^*(\beta)$; the rent of the manufacturer and transfer payment is

$$U^*(\beta) = \int_{\beta}^{\overline{\beta}} \psi'(e^*(\widetilde{\beta}))\,\mathrm{d}\widetilde{\beta}, \tag{23}$$

$$t^*(\beta) = U^*(\beta) + \psi(e^*(\beta)) + (1-s)C(\beta)_{\circ}. \tag{24}$$

According to Assumption 2, we know that the average cost $c = C/q$ is a strictly increasing function, so the inverse function exists as $\beta = \beta^*(c)$. Put this function into formula (24) we can get that the optimal transfer payment is the function of observed average cost:

$$t^*(c) = U^*(\beta^*(c)) + \psi(e^*(\beta^*(c))) + (1-s)\,c\cdot\left(\frac{a}{2} - \frac{b}{2}(\beta^*(c) - e(\beta^*(c)))\right). \tag{25}$$

When $s = 1$, the optimal contract which can be performed is

$$T(\widehat{\beta}, c) = U^*(\widehat{\beta}^*(c) + \psi(e^*(\widehat{\beta}^*(c)))). \tag{26}$$

4. Propositions Related to the Supply Chain

Proposition 8. *The best incentive program can be performed through the contract which is defined by formula* (27).

When $s = 1, t(\cdot)$ is a convex function, so you can use it to replace its tangent cluster. These tangents mean the contract menu of linear function of cost

$$t^*(\widehat{\beta}, c) = t^*(\widehat{\beta}) - \psi'(e^*(\widehat{\beta}))\cdot(c - c^*(\widehat{\beta})). \tag{27}$$

Formula (27) will induce the manufacturer to tell the truth ($\widehat{\beta} = \beta$) and can induce the appropriate effort level. When the manufacturer faces the menu of linear contracts, the optimal planning of the seller is

$$\max_{\{\widehat{\beta},e\}} \{t^*(\widehat{\beta}) - \psi'(e^*(\widehat{\beta})) \cdot (\beta - e - \widehat{\beta} + e^*(\widehat{\beta})) - \psi(e)\}. \tag{28}$$

We can get the first-order condition of $\widehat{\beta}$ and e from formula (28) and we can find that $e = e^*(\widehat{\beta})$, $\widehat{\beta} = \beta$.

Proposition 9. *The rent of the manufacturer is due to the value of the private information.*

If the seller knows the type of β from the manufacturer, according to Assumption 6, as long as the contract can guarantee the lowest expected utility level of the manufacturer, the manufacturer will cooperate, so the transfer rent of the seller to the manufacturer can be $U(\beta) = 0$. At this point, the rent of the manufacturer is 0. However, in the scene of asymmetric information, the seller does not know the β of the manufacturer, but he knows that β is a continuous parameter which belongs to $[\underline{\beta}, \overline{\beta}]$. In order to ensure that the manufacturer will cooperate, the seller needs to take the

type of $\overline{\beta}$ into consideration and the transfer rent is $U(\overline{\beta}) = 0$. Therefore, from (8) formula, we can know that the rent which the manufacturer will receive is $U(\beta) = \int_{\beta}^{\overline{\beta}} \psi'(e(\widetilde{\beta}))d\widetilde{\beta}$. From the above analysis, we can know that the reason why the manufacturer can receive the rent $U(\beta)$ is that he has private information. So, on the other side, $U(\beta)$ is the value of information β.

Proposition 10. *If the resource endowment of the manufacturer is better (the technical level is higher), the effort will be increasing.*

Form formula (22), we know that the effort of the manufacturer is increasing as β decreases.

Proposition 11. *If the resource endowment of the manufacturer is better, the production will be increasing.*

Form formula (19), we know that the production of the manufacturer is increasing as β decreases.

Proposition 12. *The better the resource endowment of the manufacturer is, the greater the incentive intensity of the seller will be.*

When $s = 1$, by finding the derivative of formula (25), we can get

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -\dot{\psi}\left(\beta^{*}(c) - c\right) \cdot \dot{c}(\beta) < 0. \tag{29}$$

So, if β is lower, the transfer payment will be more.

Proposition 13. *The better the resource endowment of the manufacturer is, the greater the revenue from supply chain will be.*

Putting formulas (2) and (18) into the function $\pi = p(q)\cdot q$ of the revenue chain and finding the derivative of β in π, we can get

$$\dot{\pi}(\beta) = -\frac{b^2}{4}\left(\beta - e(\beta)\right) \cdot \dot{c}(\beta) < 0. \tag{30}$$

Proposition 14. *When the tax rate increases, the production of the manufacturer will reduce.*

Finding the derivative of λ in the formula (18), we can get

$$\dot{q}(\lambda) = -\frac{b}{2}\left(\beta - e(\beta)\right) < 0. \tag{31}$$

5. Numerical Example

In the agrifood supply chain, agricultural cooperatives provide green agricultural products to supermarket. The cost of green agricultural products from agricultural cooperatives can be decreased by agricultural cooperatives effort; however, this effort is somewhat exhausting. Let us give some specific expression.

Table 1: Comparison with profit when a different $\widehat{\beta}$ is announced by the manufacturer.

β	e	$\widehat{\beta}$	$e^{*}(\widehat{\beta})$	U^{*}
2.8	1.4	2.8	1.4	3.744
2.8	1.4	3	1	2.112
2.8	1.4	2.6	1.8	1.344

Demand function is

$$q = 84 - 24p. \tag{32}$$

Effort's cost function is

$$\psi(e) = 2e^{3}. \tag{33}$$

Continuous density function is

$$f(\beta) = \begin{cases} \frac{1}{2} & 2 \le \beta \le 4 \\ 0 & \text{other} \end{cases}. \tag{34}$$

Let $s = 1$, $\beta = 2.8$. If we execute the linear contract expressed by formula (27) we can get agricultural cooperatives' profit in different circumstances as follows.

In Table 1, it is easily concluded that agricultural cooperatives can get optimal profit when agricultural cooperatives announce the true technology level and work as optimal effort lever.

6. Conclusion

This paper discusses the incentive system design between the seller and manufacturer with two types of information constraints. According to Proposition 8 of this paper, the paper provides linear contract. At the same time, by Proposition 9 we find that in the supply chain one of the major reasons why the principal enterprise provides rents, which is higher than the reservation utility of agent enterprise, to agent enterprise is that agent enterprises have some private information. We can also call this kind of rent as value of information.

Known from Proposition 12, when the agent's resource endowment is higher, the strength of motivation to the principal is greater. At the same time, the rent which the agent can obtain is more. In addition, known from Proposition 13, when the resource endowment of enterprises in the supply chain is higher, the benefit of the whole chain will be greater. Therefore, the resource situation of enterprises in this supply chain not only affects the distribution of all members, but also affects the benefits of the supply chain.

Further study is the investment incentive which cannot be contracted between core firms and nondominated firms in the supply chain.

Conflict of Interests

The authors declare that there is no Conflict of Interests regarding the publication of this paper.

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Research Article

A Constraint Programming Method for Advanced Planning and Scheduling System with Multilevel Structured Products

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This paper deals with the advanced planning and scheduling (APS) problem with multilevel structured products. A constraint programming model is constructed for the problem with the consideration of precedence constraints, capacity constraints, release time and due date. A new constraint programming (CP) method is proposed to minimize the total cost. This method is based on iterative solving via branch and bound. And, at each node, the constraint propagation technique is adapted for domain filtering and consistency check. Three branching strategies are compared to improve the search speed. The results of computational study show that the proposed CP method performs better than the traditional mixed integer programming (MIP) method. And the binary constraint heuristic branching strategy is more effective than the other two branching strategies.

1. Introduction

The complexity of planning processes makes most of companies develop the enterprise resource planning (ERP) system to deal with it [1]. However, as the core planning module of ERP system, material requirement planning (MRP) has its limitations. MRP generally makes plan according to finite material requirements and infinite capacity requirements, meanwhile the production lead time which is actually depending on production planning is predetermined. To cope with these limitations, advanced planning and scheduling (APS) has evolved from both software developers and academics. Compared to these traditional planning systems, APS systems offer the advantage that plans can be optimized within the boundaries of material and capacity constraints [2].

Both academicians and commercial APS providers (such as SAP APO, i2, and Asprova) have attempted to construct effective methods to generate detailed production schedules to balance the demand of the marketplace with the resources capacity. Mathematical programming and heuristic algorithms are often used to achieve this balance. Heuristic algorithms usually concentrate on bottleneck resources [3]. For example, Kung and Chern propose a heuristic factory planning algorithm (HFPA) to solve factory planning problem for product structures with multiple final products. It first identifies the bottleneck work, center then sorts jobs according to various criteria, and finally plans jobs in three iterations [4]. Previous studies have often adopted mix integer programming model to represent the planning and scheduling problem. Moon et al. suggested an advanced planning and scheduling model which integrates capacity constraints and precedence constraints to minimize the makespan [5]. Chen and Ji present a mixed integer programming model explicitly considering capacity constraints, operation sequence, lead times, and due dates in a multiorder environment [6]. Örenk et al. extend this model to the situation that an operation can be assigned to alternative machines [7]. The extensions to the basic model include sequence dependent setups and transfer times between machines [8]. Although mathematical programming and heuristic algorithms are widely used, their obstacles are also obvious. Mathematical programming method is too time-consuming when the problem size is large, which makes it not practical, while each heuristic algorithm is only applicable to a specific kind of problem [9].

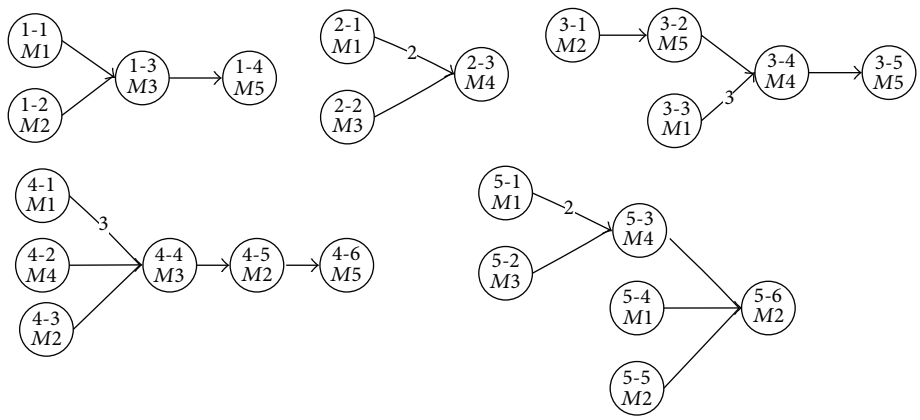

Figure 1: Examples of product tree structures.

Constraint Programming (CP) method is a relatively new technique. It has been identified as a strategic direction and dominant form for the industrial application of production planning and scheduling [10, 11]. It has been proved to be effective in dealing with combined optimization problems because of its broad representational scope and generally applicable solving algorithm. CP was originally developed to solve constraint satisfaction problem (CSP) to find a value for each variable where constraints specify that some subsets of values cannot be used together. And it has been extended to constraint optimized problem (COP) which adds an objective function (such as cost). The optimized solution is achieved by solving a CSP in which the objective function of the problem is rewritten as a constraint that forces it to be equal to a new value. Constraint-based scheduling is the discipline that studies how to solve the scheduling problems by using CP. It is analyzed and discussed by using theory and examples including how objectives, decision-variables, and penalty factors are handled in the literature [12]. The research group also presents an integrated approach that uses the complementary strengths of MILP and CP for solving the combined planning and scheduling problem within an APS system as part of the core optimization engine [13]. A constraint programming technique and a new genetic algorithm are proposed to solve a preemptive and nonpreemptive scheduling model as one of the advanced scheduling problems in the literature [14]. The experiment results show that the proposed method is effective. However, it is only applied to job-shop scheduling problems under a single machine. The literature [15] concentrates on building a constraint programming model in a flexible manufacturing system, but the solving algorithm is not discussed.

In this paper, a constraint programming method for advanced planning and scheduling system with multilevel structured products is presented. The constraint programming model with multilevel structured products was proposed with the consideration of precedence constraints, capacity constraints, release time, and due date. The solving process for the COP combines constraint programming method with branch and bound algorithm. The constraint propagation and branching strategy are discussed to deal with CSP. This paper is organized as follows. Section 2 describes the problem of APS that we studied; Section 3 introduces the constraint programming model and the solving algorithm in detail; a computational study is given to illustrate the effectiveness of this model and algorithm in Section 4; some conclusions and further research direction will be given in Section 5.

2. Problem Description

The advanced planning and scheduling problem defined in this paper is similar to the situation in the literature [6] which deals with it by a mixed integer programming (MIP) method. The products considered in this system have multilevel structures. The product tree structure (bill of material) defines the precedence constraints in scheduling problems (Figure 1). The information in the circle gives the operation number and its processing machines. The number on the arrows shows the quantity of material needed for a final product (the default number is 1). For example, the final product five (with the assembly 5-6) is composed of one subassembly (5-3) and two components (5-4 and 5-5). Meanwhile, the subassembly (5-3) is made up of components (5-1 and 5-2). This multilevel structure is typical in industry production and is often more complex than this example.

Assume that a production system contains M machines and I products. Every product i has n_i operations. Every operation task(i, j) can be processed on a dedicated machine. An optimal schedule for the products should be found to minimize the total cost including tardy cost and early cost. Some conditions are assumed to reduce the complexity of the problem. The product tree structure, release time, and due date are known in advance and similarly for processing time of operations. A lot-for-lot strategy is adopted for making products, while the setup times are negligible.

3. Constraint Programming Approach

The classic definition of a constraint satisfaction problem (CSP) is as follows. A CSP is a triple $P = (X, D, C)$, where X is an n-tuple of variables $X = (x_1, x_2, \ldots, x_n)$, D is a

corresponding n-tuple of domains $D = (D_1, D_2, \ldots, D_n)$ such that $x_i \in D_i$, and C is a t-tuple of constraints $C = (C_1, C_2, \ldots, C_t)$. A solution to the CSP is an n-tuple $A = \{a_1, a_2, \ldots, a_n\}$, where $a_i \in D_i$ and each C_j is satisfied. The algorithm for solving constraint model can be classified into two categories: inference and search [16]. Inference techniques can eliminate large subspaces by local constraint propagation method. Search systematically explores solution, often eliminating subspaces with a single failure. These two basic strategies are usually combined in most applications.

CSP provides a feasible solution satisfying all the constraints. But in real life, we try to find an optimal or relatively better solution with a definite objective such as the minimization of the cost or time. As a result, the emergency of the constraint optimization problem (COP) is extended from CSP. The solving algorithm for COP is based on CSP. The domain of the objective function value is defined by the lower bound (LB) and upper bound (UB) and it is gradually restricted with the calculation process. The objective function is rewritten as a constraint which forces it to be equal to (or less than, or more than) a new value (generally, it is a liner relation with LB and UB). Then the COP is transferred to solve CSP iteratively. Once a feasible solution is found, the LB or the UB is changed and the additional constraint for the objective is restricted. The search terminates when LB equals UB or all the nodes are fathomed.

3.1. The Constraint Programming Model. In order to build the APS constraint programming model, the following notations are used to describe the problem:

i: product index

m: machine index

j: operation index.

Parameters

I: number of products

M: number of machines

N_i: operation number of product i

task(i, j): the jth operation of product i

q_{ij}: the quantity of part which task(i, j) takes

t_{ij}: the processing tome of the operation task(i, j)

v_i: the quantity of product i

r_i: the release time of product i

d_i: the due time of product i

z_{ijk}: 1 if the operation task(i, j) precedes operation task(i, k), 0 otherwise

H: the effective work time per day

TC: cost of tardy products per day per job

EC: cost of early products per day per job.

Variables

$C_{\max}$: production makespan

e_i: early days of product i (real)

l_i: tardy days of product i (real)

E_i: early days of product i (integer)

L_i: tardy days of product i (integer).

The variable of the constraint model for advanced planning and scheduling problem is the start time of each operation task(i, j).start. The variable task(i, j).duration and task(i, j).end, respectively, denote duration time and end time of operation task(i, j). The equation task(i, j).start + task(i, j).duration = task(i, j).end clarifies their relationship. The problem described in Section 2 can be formulated as the following model:

$$\text{Min} \quad Z = \sum_{i=1}^{I} \left(TC \cdot L_i + EC \cdot E_i\right) \tag{1}$$

subject to

$$\text{task}\left(i, j\right).\text{start} \geq r_i \quad \forall i, j \tag{2}$$

$$\text{task}\left(i, j\right).\text{duration} = t_{ij} \cdot q_{ij} \cdot v_i \quad \forall i, j \tag{3}$$

$$\text{task}\left(i, n\left(i\right)\right).\text{end} \leq C_{\max} \quad \forall i, \tag{4}$$

$$\begin{aligned} &\text{task}\left(i, k\right).\text{start} + \text{task}\left(i, k\right).\text{duration} \\ &\quad \leq \text{task}\left(j, p\right).\text{start or} \\ &\text{task}\left(j, p\right).\text{start} + \text{task}\left(j, p\right).\text{duration} \\ &\quad \leq \text{task}\left(i, k\right).\text{start} \\ &\qquad \forall i, j, k, p \in \{i, j, k, p \mid \text{task}\left(i, k\right) \neq \text{task}\left(j, p\right), \\ &\qquad\qquad RES\left(i, k\right) = RES\left(j, p\right)\} \end{aligned} \tag{5}$$

$$\begin{aligned} &\text{task}\left(i, j\right).\text{start} + \text{task}\left(i, j\right).\text{duration} \\ &\quad \leq \text{task}\left(i, k\right).\text{start} \quad i, \quad j, k \in \left\{i, j, k \mid z_{ijk} = 1\right\}, \end{aligned} \tag{6}$$

$$\frac{C_i}{H} - d_i \leq l_i, \quad \forall i, \tag{7}$$

$$d_i - \frac{C_i}{H} \leq e_i, \quad \forall i, \tag{8}$$

$$L_i \geq l_i \quad \forall i, \tag{9}$$

$$E_i \geq e_i - 0.99 \quad \forall i, \tag{10}$$

$$L_i, E_i, l_i, e_i \geq 0 \quad \forall i. \tag{11}$$

The objective is to minimize the total cost which includes the early cost and tardy cost. The penalties on tardiness and earliness mean just-in-time (either early or late delivery results in an increase in the cost).

The multilevel structure of products is defined by the binary parameter z_{ijk}, which can express all the tree structures. Constraint (2) means that the start time of each operation should not be less than the release time of the product. Constraint (3) defines the duration of each operation. Constraint (4) shows that the completion time of

each product should be less than or equal to the makespan. Constraint (5) implies that if two operations require the same machine, then one cannot start before the end of the other operation. Constraint (6) ensures the precedence constraints based on the product tree structure. Constraints (7) and (8) define the early time and tardy time (real type) of each product. Constraints (9) and (10) convert the value of real type time to integer type because the penalty costs are in the unit of days. Constraint (11) indicates the domain of variables.

3.2. Solving Approach. The proposed constraint programming model is a COP which can be transferred to CSPs with the addition of an objective function value constraint

$$Z \leqslant C. \tag{12}$$

We delete the objective function (1) and add constraint (12) to structure a CSP. The optimal solution of the COP can be generated by iteratively solving the CSP and continuously restricting constraint (12). The detailed solving steps can be summarized as follows.

Step 1. Compute the LB and UB of the objective function Z.

Step 2. Add the constraint $Z \leqslant C$, where $C = (\mathrm{LB} + \mathrm{UB})/2$ to the CSP.

Step 3. Solve the CSP and set a fixed time cutoff. If a feasible solution S is found, the UB is updated to the value of the objective function $Z(S)$. Else, update $\mathrm{LB} = C + 1$.

Step 4. Repeat Steps 2 and 3, until LB = UB.

Initially, the *UB* of the objective function Z is calculated by solving the CSP without constraint (12) to find an initial feasible solution S_0. The UB is set to be equal to the value of $Z(S_0)$. While the LB of the objective function Z is calculated by the constraint propagation technique to determine the time window of each operation which will be introduced in Section 3.2.1, LB is formulated as follows:

$$\begin{aligned} LB_z = {} & TC \cdot \sum_{i=1}^{I} \max\left\{0, \left\lceil \frac{C_{i\min}}{H} - d_i \right\rceil\right\} \\ & + EC \cdot \sum_{i=1}^{I} \max\left\{0, \left\lceil d_i - \frac{C_{i\max}}{H} - 0.99 \right\rceil\right\}, \end{aligned} \tag{13}$$

where $C_{i\min}$ is equal to the earliest start time of the last operation task(i, n_i) plus the processing time of the operation. Correspondingly, $C_{i\max}$ equals the latest start time of the last operation task(i, n_i) plus the processing time of the operation. The symbol $\lceil x \rceil$ is the smallest integer greater than or equal to x. Since the starting upper bound is often very poor, we reduce the gap by performing a dichotomic search.

In this paper, the solving process of CSP in Step 3 is based on a depth-first exploration of the search tree (Figure 2). At each node, a propagation phase is triggered in order to detect possible inconsistencies and reduce the search space. If this phase detects an inconsistency, the algorithm backtracks and removes the effects of the previous decision. If no inconsistency is detected, a branching process is applied recursively to the child nodes until a solution is found or until all the search spaces have been explored. So in the next two parts, we will introduce the constraint propagation technique and branching strategies which directly influence the search speed.

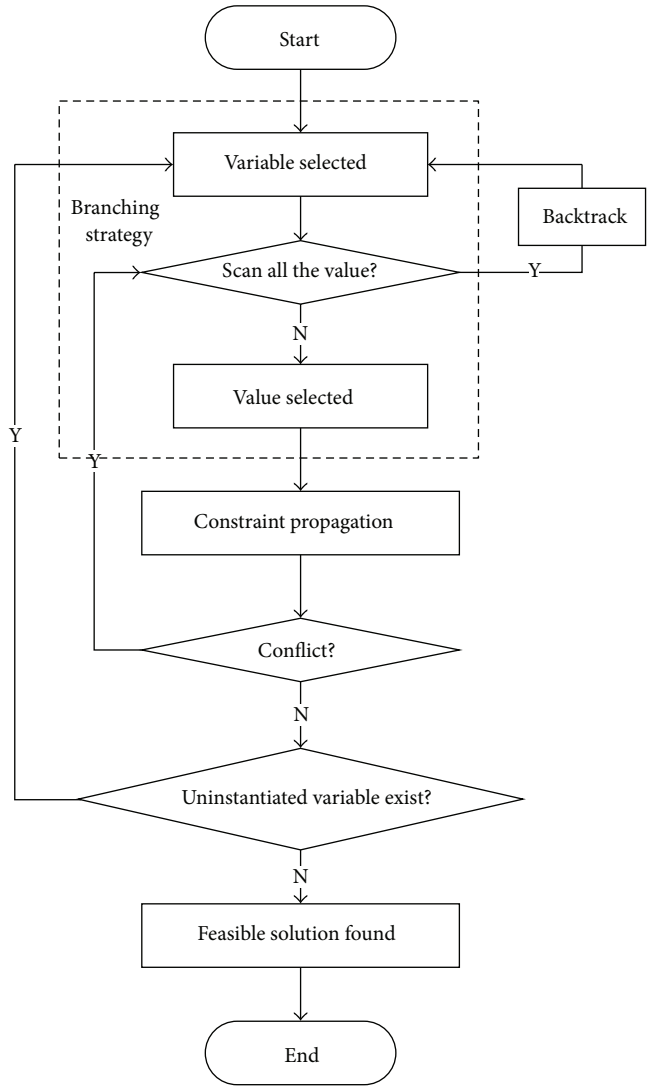

Figure 2: Flow chart of CSP solving process.

3.2.1. Constraint Propagation. The propagation phase is fundamental to reduce the size of the search space and to avoid exploring an exponential size space. The constraint propagation could be based on time or resources. Usually, timetable method uses time window to express the constraints when using constraint propagation based on time. The time window of the start time of an operation task(i, j) is [task(i, j).est, task(i, j).lst], in which task(i, j).est means the earliest start time and task(i, j).lst means the latest start time. The time window will be tightened during the constraint propagation process. Once an operation's time window is

Table 1: Product information.

Products	(v_i, r_i, d_i)	O1	O2	O3	O4	O5	O6
1	(8, 0, 2)	5	4	2	4	/	/
2	(4, 20, 5)	5	10	3	/	/	/
3	(6, 10, 4)	4	5	5	10	5	/
4	(8, 50, 3)	5	4	1	6	1	2
5	(6, 0, 5)	3	5	6	6	5	5

changed, then the time window of succeeding and preceding operations will be also changed. The constraint propagation rule for timetable is as follows:

$$\text{task}(i,j).\text{est} = r_i \quad \forall i,j \in \{i,j \mid z_{ikj} = 0\ \forall k\} \tag{14}$$

$$\text{task}(i,k).\text{est} = \text{task}(i,j).\text{est} + \text{task}(i,j).\text{duration} \quad \forall i,j,k \in \{i,j,k \mid z_{ijk} = 1\} \tag{15}$$

$$\text{task}(i,j).\text{lst} = C_{\max}(S_0) - \text{task}(i,j).\text{duration} \quad \forall i,j \in \{i,j \mid z_{ijk} = 0\ \forall k\} \tag{16}$$

$$\text{task}(i,j).\text{lst} = \text{task}(i,k).\text{lst} - \text{task}(i,k).\text{duration} \quad \forall i,j,k \in \{i,j,k \mid z_{ijk} = 1\}. \tag{17}$$

The earliest start time of each operation can be updated by formulae (14) and (15). Formula (14) is defined for the start operations without any operation preceding them. The earliest start time of succeeding operation is updated based on formulae (15) accompanied with the change of the preceding operations. Correspondingly, the latest start time of each operation can be updated by formulae (16) and (17). Formula (16) is defined for the last operations without any operation succeeding it. The latest start time of preceding operation is updated based on formula (17) accompanied with the change of the succeeding operations.

3.2.2. Branching Strategy. The earliest search method used in CP algorithm is the generate-and-test (GT) algorithm. Its efficiency is poor because of noninformed generator and late discovery of inconsistencies, and consequently the backtracking (BT) algorithm was put forward. Backtrack is the fundamental "complete" search method for constraint satisfaction problems. Basic backtracking search builds up a partial solution by choosing values for variables until it reaches a dead end, where the partial solution cannot be consistently extended.

Several branching strategies have been proposed for the standard job-shop problem [17]. The branching strategy determines the shape of the search tree which directly influences the search speed. In this part, we will consider three heuristic branching strategies.

Strategy 1 (binary constraint heuristic): it creates a binary search tree by branching on the two possibilities defined by a disjunction. Constraint (5) defines two possibilities. Assuming two operations o_{ik} and o_{jp} share the same machine m, the constraint task(i,k).start + task(i,k).duration $\leq$ task(j,p).start is posted to one branch and the constraint task(j,p).start + task(j,p).duration $\leq$ task(i,k).start corresponds to another branch.

Strategy 2 (variable-based heuristic): we use variable ordering heuristic to select the variable with the smallest domain size. The variables in the constraint model are the start times of each operation. The domain of variable is in the interval of the earliest start time and the latest start time. We select the variable with the smallest domain size and set the value with ascending order.

Strategy 3 (task-based heuristic): it consists of the definition of a task selection strategy and a value selection heuristic for the task start times. We select the task with the smallest latest completion time and choose the value with descending order.

4. Computational Study

To illustrate the proposed CP method for advanced planning and scheduling, a simple example is given below which involves five types of products and five types of machines. Figure 1 gives the representation of the product tree structures which shows the processing machines. Table 1 provides more information about these products. A product i is described by the triplet (v_i, r_i, d_i), where v_i is the demand quantity, r_i is the release time, and d_i is the due date. The processing time of operation task(i,j) is also shown in this table. The cost of tardiness and earliness is 25 per day and 5 per day. The effective work time is 80 per day.

The example model was solved by an OR-optimization software tool called Xpress-MP with 2.53 GHz CPU and 4 GB RAM. The Gantt chart is shown in Figure 3 (we add a virtual machine to allocate the virtual tasks). The makespan is 392. Product 4 is delayed for two days. The optimized total cost is 50.

The benchmark does not exist for this problem because it considers all the product tree structures, not only job-shop type. As a result, we generate the testing problems based on the simple example. We extend the data set to three types of problems with different number of products, maximum number of operations, and number of machines (Table 2). Each type contains three instances. Instances a-1 to a-3 are of type 5∗6∗5 (small instances). Instances b-1 to b-3 are of type 10∗6∗5 (medium instances). Instances c-1 to c-3 are of type 5∗21∗5 (large instances).

The MIP and CP models are, respectively, solved by Xpress-mmxprs and Xpress-Kalis modules. Table 3 shows the size of the CP and MIP models in terms of the number of variables and constraints according to the above problem

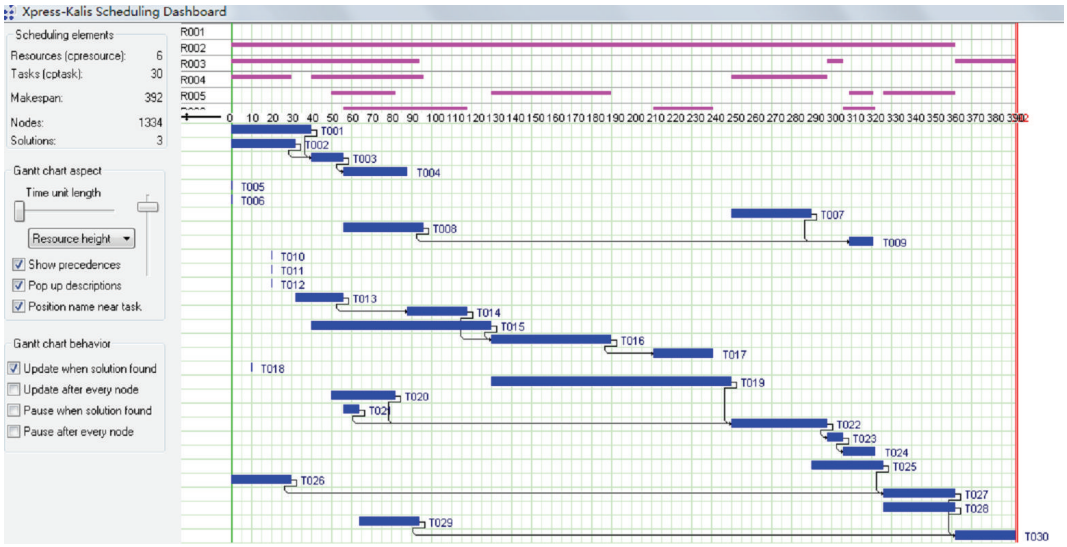

Figure 3: Gantt chart of this example.

Table 2: Data set.

Problem type	No. of products	Max. no. of operations	No. of machines
a	5	6	5
b	10	6	5
c	5	21	5

Table 3: Comparison of CP and MIP models.

Instance	CP				IP			
	No. of variables	No. of constraints	CPU time (s)	Objective	No. of variables	No. of constraints	CPU time (s)	Objective
a-1			0.1	**50**			0.2	**50**
a-2	182	539	0.1	**75**	182	301	0.2	**75**
a-3			0.1	**150**			0.1	**150**
b-1			0.5	**50**			37.4	**50**
b-2	337	1964	1000+	75	657	1190	1000+	75
b-3			3.6	**125**			1000+	175
c-1			16.5	**75**			44.4	**75**
c-2	407	13937	5.7	**125**	4693	9188	1000+	125
c-3			1000+	300			1000+	275

The instance results in bold are proved to be optimal.

Table 4: Comparison of the search results of three branching strategies.

Problem type	Branch strategy					
	1		2		3	
	Computation time (s)	Backtracks	Computation time (s)	Backtracks	Computation time (s)	Backtracks
a-1	0.05	1166	0.122	3601	0.132	3601
a-2	0.02	492	0.08	1762	0.09	1862
a-3	0.03	798	0.099	2499	0.112	2499
b-1	0.488	5369	0.897	7354	10.023	39392
b-2	1000+	100000+	1000+	100000+	1000+	100000+
b-3	3.596	12359	5.691	17316	1000+	100000+
c-1	16.529	16522	26.253	56329	46.639	92853
c-2	5.712	10636	1000	100000+	36.952	89352
c-3	1000+	100000+	1000	100000+	1000+	100000+

instances. The MIP models contain a noticeably larger number of variables in all problems than the CP models, while the number of constraints is less than in the CP models. The computation time and the objective value are also shown in this table. It should be also noted that when the problem size is large, the solving time for MIP is increased fast and cannot even get the optimal solution in acceptable time. The instance results in bold are proved to be optimal. The maximum computation time is set to be 1000 s. The CP method gets 7 optimal solutions in 9 instances, while the MIP method only gets 5. The computation time of CP method (strategy 1) is apparently less than that of the MIP method.

The branching strategies are also compared in our computational study. The computation time and backtrack iterations are shown in Table 4. It is obviously shown that branching strategy 1 is superior to other strategies in terms of high computation speed and less backtrack times.

5. Conclusion and Future Work

We have proposed a constraint programming approach to solve the advanced planning and scheduling problem with multilevel structured products. The cooperation of constraint programming method and branch and bound algorithm is applied to deal with the CP model. And it is proved to be more effective than the MIP model. Moreover, three branching strategies for constraint model are compared. The results have shown that the performance of binary constraint branching strategy is better.

This study shows that constraint programming is effective for advanced planning and scheduling problem. Although we have considered all the types of product structures, there still are some conditions we havenot taken into consideration, such as setup time and alternative operations [18]. In our future research, we will build a more comprehensive model closer to the real-life production and design a hybrid algorithm to solve it.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of the article.

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Research Article

Multiobjective Vehicle Routing Problem with Route Balance Based on Genetic Algorithm

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This study proposes a genetic algorithm to solve the biobjective vehicle routing problem with time windows simultaneously considering total distance and distance balance of active vehicle fleet. A new complex chromosome is used to present the active vehicle route. Through tournament selection, one-point crossover, and migrating mutation operator, the solution of the problem is solved. In experiment on Solomon's benchmark problems, considering the total distance and distance balance, the results are improved in all classes of problems. According to the experimental results, the suggested approach is sufficient and the average GA performance is good.

1. Introduction

The vehicle routing problem (VRP) is one of the most attractive topics in operation research, logistics, and supply chain management. VRP deals with minimizing the total cost of logistics systems. VRPs are well-known combinatorial optimization problems arising in transportation logistic that usually involve scheduling in constrained environments. In transportation management, there is a requirement to provide services from a supply point (depot) to various geographically dispersed points (customers) with significant economic implications. Because of VRP's important applications, many researchers have developed solution approaches for those problems.

Vehicle routing problem with time windows (VRPTW) is a variant of VRP with adding time windows constraints to that model. In VRPTW, a set of vehicles with limited capacity is to be routed from a central depot to a set of geographically dispersed customers with known demands and predefined time windows in order that fleet size of vehicles and total traveling distance are minimized and capacity and time windows constraints are satisfied. Due to its inherent complexities and usefulness in real life, the VRPTW continues to draw the attention of researchers and has become a well-known problem in network optimization, so many authors developed different solution approaches based on exact and heuristics methods.

Many exact optimization approaches have been used to solve the VRPTW which is a well-known NP-hard problem [1]. An exact algorithm [2] of branch and cut techniques is presented. For its complexity, only small scale models can be solved [3] and such methods are inefficient in general [4]. By far Kohl's work [5] is one of the most efficient exact methods for solving 100-customer-size instance. As a result, many researchers have investigated the VRPTW using heuristics and metaheuristics approaches.

In recent years, approximate approaches are used in VRPTW instead of exact methods considering latter's intolerably high cost. Various heuristic methods may be found in literature in [6, 10]. These methods, including simulated annealing [7], and tabu-search [8], were proposed in literature. Genetic algorithm for VRPTW [6, 9–11] maybe the most widely used solution because of its efficiency. Thangiah [12] presents a hybrid using genetic algorithm and local search optimization. Different performance of genetic algorithm, tabu-search, and simulated annealing is studied in [6, 10].

These above pieces of literature focus on the single objective problems of the VRPTW by far. In fact multiobjective problems attract many researchers' attention since

the multiobjective is closer to real environments in these years. Some multiobject VRPs are formulated as a single function using weight parameters determined only experientially. Pareto-based approach is good to solve such problem since the managers can make their own decisions from the Paretooptimal output [13]. A specialized genetic operators and variable-length chromosome representation was used in VRPTW and produced very good result on Solomon's 56 benchmark problems [14].

Different objectives were classified in [15] according to different factors, that is, the tour, the resources, and the node activity. On tour, minimizing distance travelled (or time required) was the most common objective, while reducing the imbalance (or disparity) in the workload of vehicles was studied in [16]. Minimizing the number of vehicles is one of objectives related to resources. Ghoseiri and Ghannadpour [17] studied the multiobjective problem of minimizing the number of vehicles and the travelling distance. However, sometimes in real life the vehicles are often employed by the company and the cost is fixed. That means it is impossible to reduce such cost by reducing the numbers of vehicles, Whereas the total travelling distance is an important economic variable which is related to fuel consumption [18]. Furthermore, the workload balance of vehicles is another important variable because of management requirements.

This paper studies a biobjective VRPTW considering simultaneous minimization of the total traveling distance and workload imbalance of vehicles. Generally, the workload imbalance includes the distance imbalance and the load imbalance. However, in some real life environment, that is, fresh food delivery, the weight of good can be ignored because these orders are not heavy and make no influence on the workload cost. In other words, this paper will consider the multiobjective of the total travelling distance and the distance imbalance of active vehicle fleet. Section 2 describes the formulation of the VRPTW problem. Section 3 discusses the process of genetic algorithm to solve this problem. The experiments and results are analyzed in Section 4. Finally, Section 5 provides the conclusions to this work.

2. Model Formulation

The VRP problem was introduced by [19] and became one of the most widely analyzed NP-hard problems. The single objective VRPTW aims to determine which customers are visited by each vehicle and the route each vehicle follows to serve the assigned customers, while the distances travelled by the vehicles are minimized and the capacity and time windows constraints are satisfied. The VRPTW has been widely studied because it remains one of the most difficult problems in combinational optimization and has a considerable economic impact on all the logistic system [11], especially due to the importance of supply chain operations [20]. Some VRPTW problems were discussed with exact methods, such as Lagrangian relaxation-based methods, column generation, and dynamic programming. However, these exact methods often perform poorly for some intermediate and large problems. In this case, some heuristic and meta-heuristic methods have been proposed to solve these problems. And the results show that these methods obtain feasible solutions in acceptable times.

2.1. Formulation for VRPTW. A nondirected complete graph $G(V, E)$ can be used to model the VRPTW. The vertices $V = \{1, \ldots, N\}$ denote the depot and the customers, and edges $e \in E\{(i, j),\ i, j \in V\}$ correspond to the links between them.

The VRPTW can be formulated as follows.

Notation

a_j: is the earliest time for customer j to allow the service.

b_j: is the latest time for customer to allow the service.

C_{ij}: is the cost for travelling from node i to node j. It is considered as the distance or time required for travelling from node i to node j.

d_j: is the demand at customer j.

K: is the maximum number of vehicles that can be used.

N: is the number of customers plus the depot. The depot is denoted with number 1, and the customers are denoted as $(2, \ldots, N)$.

Q: is the loading capacity of each vehicle.

S_{kj}: is the corresponding time at which vehicle k starts to service customer j.

L: is a given large value.

X_{ij}^k : is the decision variable. It is equal to 1 if vehicle k travels from node i to node j and is equal to 0 otherwise.

$$\text{Minimize TD} = \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^k C_{ij} \tag{1}$$

subject to

$$X_{ii}^k = 0 \quad (\forall i \in \{1, \ldots, N\}, \forall k \in \{1, \ldots, K\}), \tag{2}$$

$$X_{ij}^k \in \{0, 1\} \quad (\forall i, j \in \{1, \ldots, N\}, \forall k \in \{1, \ldots, K\}), \tag{3}$$

$$\sum_{k=1}^{k}\sum_{i=1, i\neq j}^{N} X_{ij}^k = 1 \quad (\forall j \in \{2, \ldots, N\}), \tag{4}$$

$$\sum_{i=2}^{N} d_j \sum_{j=2, i\neq j}^{N} X_{ij}^k \leq Q \quad (\forall k \in \{1, \ldots, K\}), \tag{5}$$

$$\sum_{k=1}^{K}\sum_{j=2}^{N} X_{1j}^k \leq K, \tag{6}$$

$$\sum_{j=2}^{N} X_{1j}^k - \sum_{j=2}^{N} X_{j1}^k = 0 \quad (\forall k \in \{1, \ldots, K\}), \tag{7}$$

$$a_j \leq s_{kj} \leq b_j \quad (\forall i, j \in \{2, \ldots, N\}, \forall k \in \{1, \ldots K\}), \tag{8}$$

$$s_{ki} + C_{ij} - L\left(1 - X_{ij}^k\right) \leq s_{kj} \quad (\forall i, j \in \{1, \ldots, N\}, \forall k \in \{1, \ldots K\}). \tag{9}$$

Equation (1) is the objective function of the single objective problem. Equation (2) denotes that a vehicle must travel from one node to a different one. Equation (3) indicates that variable X_{ij}^k is equal to 1 if vehicle k goes from node i to node j and is equal to 0 otherwise. Equation (4) states that a customer is serviced only once by exactly one vehicle. By specifying the constraint of (5), it is taken into account that the load for a given vehicle k cannot exceed its capacity Q. Equation (6) specifies that there are up to K routes going out of the delivery depot. Equation (7) guarantees that each vehicle departs from and returns to the depot. Equation (8) ensures that time windows are observed. Given a large value, L, the inequality represented in (9) specifies that, if vehicle k is travelling from customer i to customer j, the vehicle cannot arrive at customer j before $s_{ki} + C_{ij}$. The variable s_{kj} corresponds to the time at which vehicle k starts to service customer j. If the vehicle k does not service j, S_{kj} is not calculated.

2.2. Multiobjective VRPTW with Distance Balance. The paper aims to solve the vehicle routing problem with hard time windows and route balance as a multiobject problem, where both the total travelling distance and route imbalance are minimized. The route balance often was related to the following factors:

(1) balancing the number of customers visited by each active vehicle,

(2) balancing the quantity or weight of the good delivered by each active vehicle, sometimes balancing the load rate (BLR), denoted as (10), where LV_i is the exact load of vehicle I and Q_i is the capacity of vehicle i

$$\mathrm{BLR} = \frac{\mathrm{LV}_i}{Q_i}, \tag{10}$$

(3) balancing the time required of the route,

(4) balancing the waiting time required of the route,

(5) balancing the delayed time of the route,

(6) balancing the distance of the route travelled by active vehicles.

In this paper, we consider the imbalance of the distances of the route travelled, which is defined as (11). And (12) is the mean of all distances. In order to describe the balance more clearly, we use the balance factor to represent the degree in (13). Consider the following:

$$\mathrm{BL}_d = \max\left(\sum_{i=1}^{N}\sum_{j=2}^{N} X_{ij}^k C_{ij}\right) - \min\left(\sum_{i=1}^{N}\sum_{j=2}^{N} X_{ij}^k C_{ij}\right) \quad (\forall k \in K), \tag{11}$$

$$\mathrm{BL}_m = \frac{1}{k}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^k C_{ij}, \tag{12}$$

$$\mathrm{BF} = \frac{\mathrm{BL}_d}{\mathrm{BL}_m}. \tag{13}$$

Thus, from (1) and (11), the new multiobjective problem is defined as follows.

Minimize

$$f_{td+bl} = f\left(\left(\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^k C_{ij}\right), \left(\frac{\mathrm{BL}_d}{\mathrm{BL}_m}\right)\right), \tag{14}$$

while the constraints described above ((2)–(9)) are satisfied.

3. Multiobjective Genetic Algorithm

Various heuristic and meta-heuristic approaches have been proposed for solving the VRPTW. GA, compared with other heuristics [21–23], has been widely used to solve this problem, because of its efficiency and flexibility. Three main GA algorithmic operators, namely, selection, crossover, and mutation, can be configured in different ways, resulting in various GA combinations. Thangiah et al. [24] were the first to apply GA to VRPTW. During the past few years, numerous studies have been devoted to developing GAs for solving VRPTW. GA is an adaptive heuristic search method based on population genetics. The genetic algorithm represents the solution space using genetic coding of a feasible solution as a chromosome that defines an individual member of a population. While binary strings have been commonly used in the literature to code chromosomes, we adopt integer strings in the proposed GA, where each gene in a chromosome represents a customer (or a node). In a single objective problem of genetic algorithm, special fitness function is often defined but in MOP application of genetic algorithm the Pareto ranking scheme has often been used [25]. The Pareto ranking process tries to rank the solutions to find the nondominated solutions. Therefore, according to this process each solution gives a rank value in respect of different objective values that shows the quality of the solution compared to the other solutions. It is easily incorporated into the fitness evaluation process within a genetic algorithm by replacing the raw fitness scores with Pareto ranks. These ranks, to be defined later, stratify the population into preference categories. With it, lower ranks are preferable and the individuals within rank 1 are the best in the current population. The idea of Pareto ranking is to preserve the independence of individual objectives. This is done by treating the current candidate solutions as stratified sets or ranks of possible solutions. The individuals in each rank set represent solutions that are in some sense incomparable with one another. Pareto ranking will only differentiate individuals that are clearly superior to others in all dimensions of the problem. This contrasts with a pure genetic algorithms attempt to assign a single fitness score to a MOP, perhaps as a weighted sum. Doing so essentially recasts the MOP as a single objective problem. The difficulty with this is that the weighted sum necessitates the introduction of bias into both search performance and quality of solutions obtained. For many MOP's, finding an effective weighting for the multiple dimensions is difficult and ad hoc and often results in unsatisfactory performance and solutions.

3.1. Chromosome Representation. This paper uses a complex two-part chromosome to represent the solution of VRPTW.

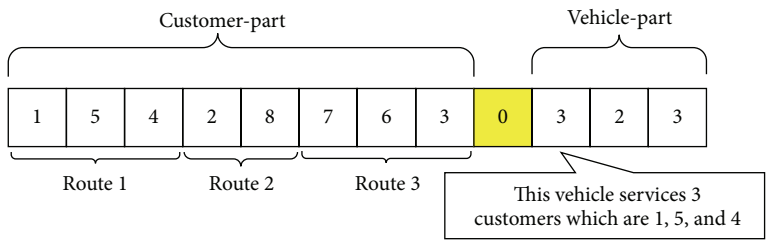

Figure 1: An example of a chromosome.

The chromosome is separated into two parts by a zero number decorated by yellow. The first part of the chromosome is a chain of integers and each of the integers represents a customer. We also call this part customer-part. The customers on it are separated to several routes, each of them representing a sequence of delivers that must be covered by a vehicle. The second part of the chromosome contains vehicles information. We also can call this part vehicle-part. In the vehicle-part the quantity of genes equals the quantity of routes in the customer-part. The number on each of the genes represents the length of its corresponding route. The sum of these numbers in vehicle-part must be equal to the quantity of customers. For example, Figure 1 shows a representation of a possible solution with 8 customers and 3 vehicles. There are 3 genes in vehicle-part which means that those 8 customers are separated to 3 routes. The 3 on the first gene in vehicle-part represents route 1 that services 3 customers which are 1, 5, and 4. The 2 on the second gene means that route 2 services 2 and 8. The 3 on the third gene means that route 3 services 7, 6, and 3.

This design is different from the classical approach, in which the route information is mixed with the customer sequences in a single chromosome. Storing the route information and customer sequences separately can represent the solution more clearly and facilitate the implementation of the algorithm, but its effect on the computational efficiency would not be significant. Without loss of generality, we consider the following implementations of the three operators.

3.2. Selection. There are several commonly used selection operators used in GA selection process. Roulette wheel selection (RWS) is to stochastically select from one generation to create the basis of the next generation. RWS enables the fittest individuals to have a greater chance of survival than weaker ones. This replicates nature in that fitter individuals will tend to have a better probability of survival and will go forward to form the mating pool for the next generation. Weaker individuals are not without a chance. In nature such individuals may have genetic coding that may be proven to be useful for future generations. Unlike RWS, uniform selection (US) assigns the same probability to each chromosome of the population. The US operation proceeds at random and is easy to implement. However, it has been criticized for lacking the spirit of natural evolution compared with RWS. Tournament selection (TS) is the most commonly used operation besides RWS. The TS operator involves running several "tournaments" among a set of chromosomes chosen at random. The one with the largest fitness is selected for crossover in a pair of chromosomes. The tournament size can be used to adjust the selection pressure. If the tournament size is larger, weaker individuals will have a smaller chance to be selected. This process is repeated until the mating pool is full. Since some experiments indicate that the TS operator outperforms the RWS and US, we choose TS as the selection operator. A possible explanation is that TW always selects the best set of individuals to crossover, whereas RWS and TS are probabilistic and hence some good individuals may be lost in the evolutionary process [26].

3.3. Crossover. One-point crossover operator evolves selecting one point randomly which divides a parent into two parts. Each of these points is selected with equal probability. For example, the crossover point is selected at the third gene of parent 1 randomly. The offspring inherits the left side from parent 1 and other genes are inherited from parent 2. Another offspring is produced by exchanging the roles of two parents. Figure 2 illustrates the process of one-point crossover [27]. The cycle crossover can produce offspring through a cycle which is a sequent of the position of the first parents. The partially mapped crossover operator produces the offspring by randomly selecting two crossover points [28]. The linear-order crossover also selects two crossover points from one parent and produces a new offspring with another parent [29]. Some experiments illustrates that the one-point crossover is more efficient than the other tree operators [26]. Thus this paper chooses the one-point crossover operator.

3.4. Mutation. In mutation process, there are also several mutation operators in different literatures. Some of them are very complex. However, some different mutation operators were experimented that they did not make significant difference in GA efficiency. A possible explanation to that maybe the mutation rate is always very small, typically between 0.01 and 0.1. A migrating mutation is adopted to produce heterogeneous chromosomes in the pool to avoid early convergence of the algorithm. This mutation method is to randomly select a chromosome from the pool and then randomly choose a customer from one route. Then the selected customer is tried to be inserted into a new route. If the insertion results produce a feasible route, this mutation operator succeeds. Otherwise, this process is repeated until a feasible solution is achieved. Figure 3 illustrates the migrating mutation process.

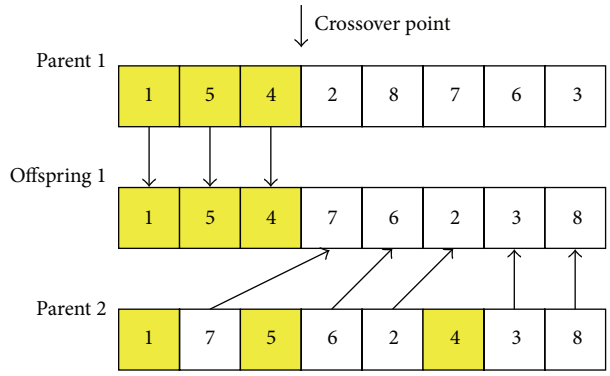

Figure 2: One-point crossover operator.

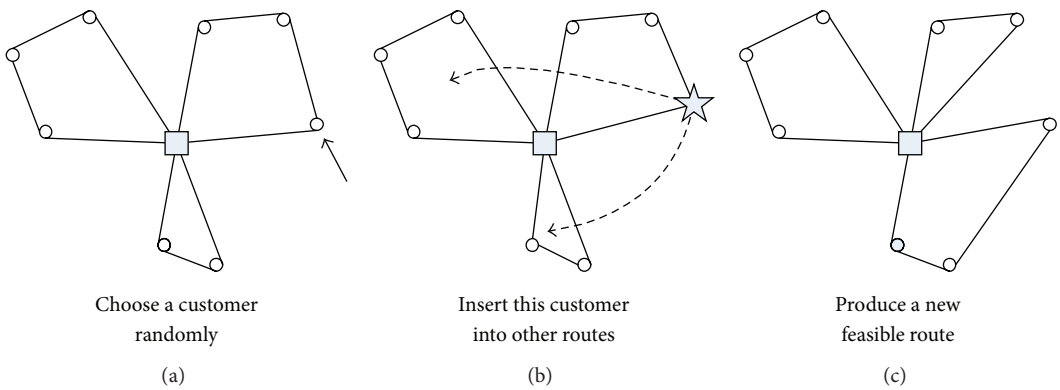

Figure 3: Migrating mutation process.

4. Experimental Results and Comparisons

The Solomon's problems consist of 56 data sets, which have been extensively used for benchmarking different algorithms for VRPTW in literature over the years, since they represent relatively well different kinds of routing scenarios [14]. These problems are different in fleet size, vehicle capacity, traveling time of vehicles, and so on. The customers' details include the sequence of customer index, location in x and y coordinates, the demand for load, the ready time, due date, and the service time required. All the test problems consist of 100 customers and these customers are generally adopted as the problem size for performance comparisons in VRPTW. The traveling time between customers is equal to the corresponding Euclidean distance. This problem data is clustered into six classes, named C1, C2, R1, R2, RC1, and RC2, respectively. And different categories indicate the different customer distribution and different time windows constraints. According to the customer location, problem category C has all customers clustered in groups, problem category R has all customers located randomly, and problem category RC has a mixture of random and clustered customers. In other words, customers are located closer to each other in problem category C than in R and customers in RC are in the middle. That also means that the R is more difficult to be solved. Moreover, the time windows of category 1 (C1, R1, and RC1) is smaller than that of category 2 (C2, R2, and RC2). Smaller time windows mean that some candidate solutions are more likely to become unfeasible after a small change in the sequence of visited customers. Furthermore, for category 1, the time window is also narrow for the depot, which means that only a few customers can be served by one vehicle.

This section describes computational experiments carried out to investigate the efficiency of the proposed GA. The algorithm is coded in JAVA and run on a PC with 2.53 GHz CPU and 2000 MB memory. The standard Solomon's VRPTW benchmark problem instance is used as experimental data [30]. Empirically the computation is based on the following parameters:

Population size = 100,

Generation number = 500,

Crossover rate = 0.9,

Mutation rate = 0.2.

To illustrate the influence of different models with routing balance, we have employed the best solution of the benchmark problems in Table 1, 2, and 3, for the data groups C, R, and RC, respectively. Each table includes the following:

in the first column, the benchmark problem instance according to [30],

in the second column, the best known solution for that problem in literatures,

Table 1: Results of C instances.

No.	Instance	Best known	Suggestive algorithm				
			Single objective			Biobjective	
		Total distance	Distance	Difference (%)	Balance	Distance	Balance
1	C101	828.94	828.94	0.00	42.2%	839	10.3%
2	C102	828.94	828.94	0.00	42.2%	838	4.2%
3	C103	828.06	828.06	0.00	42.3%	1116	1.8%
4	C104	824.78	824.78	0.00	41.2%	1253	2.9%
5	C105	828.94	828.95	0.01	42.2%	969	4.2%
6	C106	828.94	828.95	0.01	42.2%	1098	6.0%
7	C107	828.94	828.95	0.01	42.2%	842	3.9%
8	C108	828.94	828.95	0.01	42.2%	1169	5.8%
9	C109	828.94	828.95	0.01	42.2%	1198	8.9%
10	C201	591.56	591.58	0.01	11.2%	565	2.7%
11	C202	591.56	591.58	0.01	11.2%	792	0.8%
12	C203	591.17	591.18	0.01	10.7%	716	0.8%
13	C204	590.60	590.62	0.01	11.2%	759	2.4%
14	C205	588.16	588.18	0.01	10.7%	784	1.7%
15	C206	588.49	588.51	0.01	10.7%	815	2.2%
16	C207	588.29	588.30	0.01	10.7%	693	3.9%
17	C208	588.32	588.32	0.00	10.7%	839	4.5%

Table 2: Results of R instances.

No.	Instance	Best known	Suggestive algorithm				
			Single objective			Biobjective	
		Total distance	Distance	Difference (%)	Balance	Distance	Balance
1	R101	1645.79	1650.8	0.30	64.6%	1673	8.1%
2	R102	1486.12	1486.12	0.00	62.9%	1746	4.5%
3	R103	1292.68	1292.68	0.00	49.3%	1471	5.1%
4	R104	1007.24	1007.24	0.00	4.5%	1141	3.2%
5	R105	1377.11	1377.11	0.00	50.8%	1332	4.7%
6	R106	1251.98	1251.98	0.00	51.8%	1505	4.5%
7	R107	1104.66	1104.66	0.00	4.5%	1303	1.5%
8	R108	960.88	960.88	0.00	0.9%	1164	0.5%
9	R109	1194.73	1194.73	0.00	30.4%	1518	3.9%
10	R110	1118.59	1118.59	0.00	5.4%	1090	3.7%
11	R111	1096.72	1096.72	0.00	7.3%	1335	1.7%
12	R112	982.14	987.24	0.52	3.7%	992	2.9%
13	R201	1252.37	1252.37	0.00	22.0%	1282	2.5%
14	R202	1191.70	1191.70	0.00	14.9%	1146	2.4%
15	R203	939.54	939.54	0.00	29.1%	1041	0.8%
16	R204	825.52	832.14	0.80	0.7%	847	3.2%
17	R205	994.42	994.42	0.00	4.2%	1138	1.1%
18	R206	906.14	906.14	0.00	7.6%	1054	4.7%
19	R207	890.61	896.88	0.70	0.4%	895	3.4%
20	R208	726.75	726.75	0.00	8.0%	891	0.1%
21	R209	909.16	909.16	0.00	20.8%	1171	5.6%
22	R210	939.34	939.37	0.00	7.7%	1046	2.4%
23	R211	892.71	904.78	1.33	0.7%	925	9.0%

Table 3: Results of RC instances.

No.	Instance	Best known	Suggestive algorithm				
			Single objective			Biobjective	
		Total distance	Distance	Difference (%)	Balance	Distance	Balance
1	RC101	1696.94	1696.95	0.00	51.2%	1963	4.5%
2	RC102	1554.75	1554.75	0.00	21.6%	1894	2.6%
3	RC103	1261.67	1261.67	0.00	28.8%	1669	5.3%
4	RC104	1135.48	1135.48	0.00	21.1%	1447	1.0%
5	RC105	1629.44	1629.44	0.00	30.3%	1956	3.5%
6	RC106	1424.73	1424.73	0.00	17.8%	1858	1.7%
7	RC107	1230.48	1230.48	0.00	22.3%	1764	2.9%
8	RC108	1139.82	1139.82	0.00	11.4%	1545	1.6%
9	RC201	1406.91	1406.94	0.00	16.8%	1646	4.3%
10	RC202	1365.65	1365.65	0.00	8.8%	1439	1.9%
11	RC203	1049.62	1058.33	0.82	10.0%	1243	0.4%
12	RC204	789.41	798.46	1.13	16.3%	936	9.6%
13	RC205	1297.19	1297.65	0.04	14.8%	1485	3.4%
14	RC206	1146.32	1146.32	0.00	17.3%	1187	5.1%
15	RC207	1061.14	1061.14	0.00	15.0%	1327	3.9%
16	RC208	828.14	828.71	0.07	25.0%	1046	8.1%

Table 4: Compared results of all instances.

No.	Instance class	Best known			Suggestive approach			Improved
		$B_{\min}$	$B_{\max}$	B_{avg}	$B_{\min}$	$B_{\max}$	B_{avg}	B_{dec}
1	C1	42.3%	41.2%	42.1%	1.8%	8.9%	5.3%	87.4%
2	C2	11.2%	10.7%	10.9%	0.8%	5.5%	2.4%	78.3%
3	R1	64.6%	0.9%	28.0%	0.5%	8.1%	3.7%	86.8%
4	R2	29.1%	0.4%	10.6%	0.1%	9.1%	3.2%	69.7%
5	RC1	51.2%	11.4%	25.6%	1.0%	5.3%	2.9%	88.7%
6	RC2	25.0%	8.0%	15.5%	0.42%	8.08%	4.6%	73.9%

in the third column, the best solution found by the algorithm of this paper,

in the fourth column, the difference by percent between the best known and the best found,

in the third column from last, the balance rate of single objective search,

the second column from last and the final column, the new distance value and the balance rate of biobjective algorithm of this paper.

From the results of Tables 1 to 3 in single objective approach, it is found that the balance rate of C1, R1, and RC1 is bigger than that of C2, R2, and RC2 when the balance is not considered. In other words, category 1 with wider time windows has more space to improve the balance than category 2 with smaller time windows. For example, the balance rate in C101 is 42.2% but in C201 is 11.2%.

After considering the distance balance, the biobjective solution data illustrated much improvement on the balance rate without much influence on the distance cost. For example, in C101 when the balance decreases from 42.2% to 10.3%, the distance only reduces by about 10 (839 minus 828.94). Not only the instances with wide time windows get a great improvement, but also the ones with the narrow time windows reduce the balance rates.

The comparative data shows that suggestive algorithm of this paper reaches better route balance without significant deterioration of the VRPTW solution, in terms of the active vehicle fleet. From Table 4, the last column (B_{dec}) presents the degree of balance improved.

5. Conclusion

The problem discussed in this paper is of significant practical importance in cases where employee's labor balance is key motivation of vehicle routing. In some situation, the weight is not very important compared with the distances of the active vehicle fleet when considering labor balance.

This paper proposed a genetic algorithm to solve the biobjective vehicle routing problem with time windows simultaneously considering total distance and distance balance of active vehicle fleet. We used a new complex chromosome to present the active vehicle route. We choose Tournament selection, one-point crossover, and migrating mutation operator to solve this GA. After iterator operation, the solution of the problem was solved. In experiment on Solomon's

benchmark problems, we found that this objective is close to best known value in literatures, even though it was not designed for the single objective problems. Considering the distance balance, those instances are imbalanced and have much space to improve. From the results, the distance balance was improved in all classes of problems in the biobjective problem. According to the produced results, the suggested approach was sufficient and the average GA performance was adequate.

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Research Article

The Game Analysis of Manufacturers' Political Connections on Product Safety in Supply Chain: Evidence from China

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This paper studied the political connections on product safety in supply chain. In market economy, information asymmetry exists throughout the entirety of supply chains that ought to ensure product safety. Due to the existence of game relations between the government and manufacturers in the aspects of product safety and regulation, the formation of market equilibrium depends on political connections between the government and manufacturers. Based on study and analyses of a static game model and a dynamic game model, this paper reveals that governments and manufacturers must use positive political connections to achieve product protection and supervision of safety throughout the supply chain. On the other hand, negative political connections lead to losses of both governmental credibility and social profits. This study indicates that inherent mechanism of political connections exists in the supply chain; it will help to enrich the theory of supply chain.

1. Introduction

Product safety is an issue of utmost importance all around the world and it is related to the supply chain in its entirety. The situation brings some new problems to traditional supply chain [1]. As the world's second largest economy, with frequent international business and a large population, China considers product safety a top priority. Product safety affects not only national credibility and the credibility of companies but also the important basic commitments of a nation and its enterprises for users. However, because of information asymmetry, product users require product manufacturers to be highly disciplined and require strict supervision and management of government departments. "Product" refers to everything available in the market that can meet the specific needs of people, including in-kind, service assurance, ideas, and other forms. "Security" refers to the conditions that prevent death, injury, occupational diseases, equipment damage, property damage, and environmental damage.

Ensuring the safety of products first depends on sound laws and regulations. In addition, government departments must work with the law, enforce the law, properly regulate rules, and abandon favoritism or irregularities. Second, the manufacturers should develop and enforce their own techniques for ensuring production quality, sustaining the self-regulation of the industry, and maintaining the professional ethics of the employed. In order to strengthen the safety supervision of special equipment, prevent accidents, ensure the safety of people's lives and property, and promote economic development, China has made related laws to constrain product safety issues, such as Article 5 in the Constitution of the People's Republic of China, Article 146 in the PRC Criminal Law, and Article 14, Article 15, Article 49, and Article 50 in the PRC Product Quality Law. However, laws and regulations are a double-edged sword. When they are supposed to serve a function in binding constraints to protect the majority of people, the government appears to use administrative powers to participate in certain economic activity concerning political connections with enterprises, thus creating opportunities for a few privileged persons to achieve excess revenue. According to the statement of two economists, Buchanan and Krueger, this excess revenue is called "rent". The activities of seeking authority to obtain rent are called "rent-seeking activities" [2, 3].

Therefore, to pursue increased benefits, manufacturers circumvent or address legal and regulatory risks so that

certain people obtain privileges and excess revenue via rent-seeking activities that establish political connections with the government. Political connection is different from the government intervention. It is a dynamic interaction of the relationship between government and enterprises. Generalized political connections: it is described in order to achieve the enterprise or individual interests, and the government directly or indirectly is related to the collective or individual behavior. Narrow political connections: it refers to the existence of a political background of personnel in the enterprise shareholders, board of directors, and management. These political connections link the interests of enterprise, government, and individual. The definition of political connections in this paper takes into account the standard definitions used in the literature. Agrawal and Knoeber show that politically experienced directors might affect their performance in U.S. manufacturing firms [4]. Khwaja and Mian bring to light the role of ex-politicians in providing government bank loans to politically connected firms [5]. Fan et al. define a Chinese firm as being politically connected if the CEO is a current or former officer of the central government, local government, or the military [6]. Ferguson and Voth consider firms to be political connections if the executives and supervisory board members were close to the ruling party [7]. Our definition of political connections uses the special political connections; it covers current and former political connections research. The root of rent originates from the formation of price differences because demand increases for this kind of production factor, whereas supply cannot increase due to various other factors. However, Kruger believes that rent seeking is a dredging activity conducted to obtain licenses and quotas to gain additional revenue. Therefore, political connections will have a positive effect on firm value as a rent-seeking behavior of enterprise [8], it will help enterprises on product safety in supply Chain process.

From the perspective of game theory [9], by building static and dynamic game models, this paper analyzes the game relationship of how political connections between government and manufacturers mutually restrain and make contact with each other on product safety in supply chains. It also reveals that political connections between the government and manufacturers are crucial in product safety in the product supply chain, and both sides need to collaborate with each other and fulfill their duties to better ensure product safety throughout the entire supply chain [10]. This paper for the first time mentions political connections in supply chain and introduced the theory of political connections into supply chain theory. It will not only enrich the research of supply chain and political connections, but also will reveal the internal mechanism about political connections on safety production in supply chain system.

2. Static Game between the Government and Manufacturers

2.1. Establishment of the Static Model. Consider the behavior of political connections on safety production in supply chain. We need to clear the behavior of the participants, namely, the government and manufacturers as well as their behavior in the supply chain production. Static game is the principle that actors either participate in the selection simultaneously or, if they are not in the same selection, the latter actor does not know the specific action that had been taken earlier in the game. The game between the government and manufacturers constitutes a static game with complete information [11].

Table 1: Game matrix of the government and manufacturers.

Manufacturers	Government	
	Supervision p_1	Nonsupervision $1 - p_1$
Value product safety p_2	$M - C_2, -C_1$	$M - C_2, 0$
Neglect product safety $1 - p_2$	$-C_3, C_3 - C_1$	$0, -C_5$

Table 2: Game matrix of the government and manufacturers.

Manufacturers	Government	
	Supervision p_1	Nonsupervision $1 - p_1$
Value product safety p_2	$M - C_2, -C_1$	$-C_3, C_3 - C_1$
Neglect product safety $1 - p_2$	$M - C_2, 0$	$0, -C_4 - C_5$

Assumptions: (a) the cost used by the government to monitor the safe behavior of manufacturers by administrative, economic, legal, and other means is C_1, and its probability is p_1; (b) if manufacturers can recognize the importance of product safety, fulfill quality commitment, and actively cooperate with government regulations, the cost paid should be C_2, and the intangible benefit brought by credibility is M, which has a probability of p_2; (c) if manufacturers focus only on short-term interests and behave contrarily to industry ethics, the cost of compulsory fines, legal responsibilities, and the damage of social image resulting from product safety issues is C_3; (d) the lost social cost when negative political connections are established between the government and manufacturers, which means that governmental officials profit by ignoring product safety issues as companies label substandard products as fine products under the security umbrella of government to obtain excess revenue, should be C_4; the cost of losing the governmental credibility is C_5.

The political game matrix between manufacturers and government is shown in Table 1.

Under the assumption that worldwide concerns on product safety continue to increase, if the government does not handle the product safety issues of manufacturers properly, the government's image will incur damage. When C_5 is smaller than $C_1 - C_3$, Nash equilibrium occurs (ignoring product safety and supervision). The lost social cost C_4 caused by negative political connections should be attributed to the government because the loss of social cost is much greater than the cost when the government fulfills regulatory responsibilities, which is $C_1 - C_3 < C_1 < C_4$. Therefore, social pressure prompts the government to use various means to regulate and constrain product safety issues. The game matrix of the political connections between the government and manufacturers is shown in Table 2.

2.2. Solving the Game Model. The expected revenue function of political connections between manufacturers and the government is shown by the following:

$$\begin{aligned} E_G &= p_1\left[-p_2C_1 + (1-p_2)(C_3 - C_1)\right] \\ &\quad + (1-p_1)(1-p_2)(-C_4 - C_5), \\ E_G &= p_2\left[p_1(M - C_2) + (1-p_1)(M - C_2)\right] \\ &\quad - (1-p_2)\ p_1C_3. \end{aligned} \tag{1}$$

The solved mixed strategy Nash equilibrium [12] is

$$\left(p_1^* = \frac{C_2 - M}{C_2}, p_2^* = 1 - \frac{C_1}{C_3 + C_4 + C_5}\right). \tag{2}$$

The relation between p_1^* and p_2^* is

$$p_2^* = 1 - \frac{p_1^*C_1}{C_2 - M + C_4p_1^* + C_5p_1^*}. \tag{3}$$

The first derivative of p_1^* is

$$p_2^{*\prime} = \frac{C_1(M - C_2)}{\left(C_2 - M + C_4p_1^* + C_5p_1^*\right)^2}. \tag{4}$$

When $M - C_2 > 0$, $M > C_2$, $p_2^{*\prime} > 0$, and p_2^* is an increasing function.

2.3. Results Analysis. Equation (3) shows that the smaller the governmental regulatory cost C_1, the greater the social cost C_4 and the lost credibility C_5 caused by the government defaulting and the greater the p_2 probability of manufacturers valuing product safety.

Equation (4) shows that when the profits a company gains due to probability are greater than its cost of protection, the probability p_2 of valuing the product increases along with the rising of probability p_1 of the supervision of the government.

3. Dynamic Game between the Government and Manufacturers

Dynamic game means that the actions of persons involved follow an order and that the action of the former person can be observed by the latter. Based on the inaccuracy of the information, such as the characteristics of other participants, the strategy space, and payoff function and the principle of order and repeating a game includes, the game between the government and manufacturers constitutes a dynamic game of incomplete information.

3.1. Establishment of the Dynamic Game Model. Assumptions: (a) the cost used by the government to monitor the safe behavior of manufacturers by administrative, economic, legal, regulatory, and other means is C_1, and its probability is p_1; (b) if the manufacturers can recognize the importance of product safety and actively cooperate with government regulations, the cost paid should be C_2, and the intangible benefit brought by credibility is M_1, which has a probability of p_2; if manufacturers focus only on short-term interests and behave contrarily to industry ethics, the cost of compulsory fines and legal responsibilities resulting from product safety issues is C_3, and the cost of the credibility is C_4; (c) in the case that the government abuses authority (with a probability p_3), if the manufacturers do not attach importance to product safety and bribe. φC_2 $(0 < \varphi < 1)$ the relevant government officials in order to avoid punishment, the probability of bribery is p_4. If the relevant government officials take the bribes φC_2 of manufacturers as their own revenue, the loss of credibility is C_5; under the condition that manufacturers neither attach importance to product safety nor to bribing the government, the fine charged from manufacturers by the relevant government officials because of abuse of authority should be γC_3, $\gamma > 1$; (d) if the relevant government officials do not abuse their authority, instead they will submit the bribe φC_2 and the fine C_3 to the treasury on their own initiative, and the government will give proper incentives $\gamma(\varphi C_2 + C_3)$ $0 < \gamma < 1$ for them; (e) relevant government officials do not supervise and manufacturers do not attach importance to product safety, causing serious impact on society. Losses caused by the defaulting of relevant government officials are C_6 (including loss of credibility and the resulting loss of social costs). The game tree of the government and the manufacturers is shown in Figure 1.

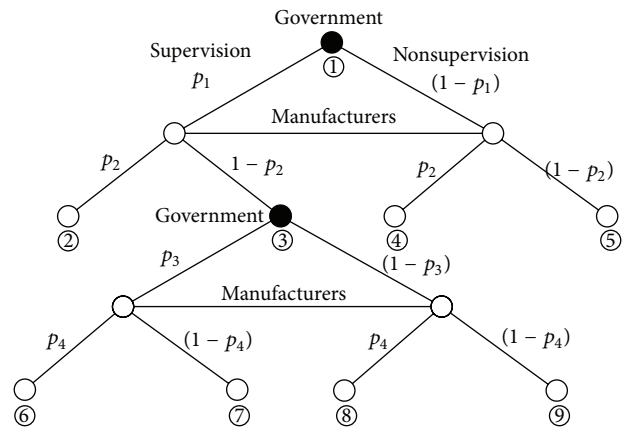

Figure 1: Game tree of government and the manufacturers.

Revenue of each node in the game tree:

① (E_{21}, E_{22})

② $(-C_1, M_1 - C_2)$

③ (E_{11}, E_{12})

④ $(0, -C_2)$

⑤ $(-C_6, 0)$

⑥ $(-C_1 + \varphi C_2 - C_5,\ -\varphi C_2 - C_4)$

⑦ $(-C_1 + \omega C_3 - C_5,\ -\omega C_3 - C_4)$

⑧ $(-C_1 + \gamma(\varphi C_2 + C_3),\ -\varphi C_2 - C_3 - C_4)$

⑨ $(-C_1 + \gamma C_3,\ -C_3 - C_4)$.

3.2. Seeking Nash Equilibrium in Dynamic Game Problem. This backward induction method is for this dynamic game problem to seek the equilibrium value. Firstly, expectation of node ③ is computed. Consider

$$\begin{aligned} E_{11} = {} & p_3 p_4 (-C_1 + \varphi C_2 - C_5) \\ & + p_3 (1 - p_4)(-C_1 + \omega C_3 - C_5) \\ & + (1 - p_3) p_4 [-C_1 + \gamma (\varphi C_2 + C_3)] \\ & + (1 - p_3)(1 - p_4)(-C_1 + \gamma C_3), \end{aligned} \tag{5}$$

$$\begin{aligned} E_{12} = {} & p_3 p_4 (-\varphi C_2 - C_4) \\ & + p_3 (1 - p_4)(-\omega C_3 - C_4) \\ & + (1 - p_3) p_4 (-\varphi C_2 - C_3 - C_4) \\ & + (1 - p_3)(1 - p_4)(-C_3 - C_4). \end{aligned} \tag{6}$$

The Nash equilibrium of formula (5) and formula (6) is sought as follow:

$$\begin{aligned} \frac{\partial E_{11}}{\partial p_3} = {} & p_4 (-C_1 + \varphi C_2 - C_5) + (1 - p_4) \\ & \times (-C_1 + \omega C_3 - C_5) \\ & - p_4 [-C_1 + \gamma (\varphi C_2 + C_3)] \\ & - (1 - p_4)(-C_1 + \gamma C_3) = 0, \\ \frac{\partial E_{12}}{\partial p_4} = {} & p_3 (-\varphi C_2 - C_4) - p_3 (-\omega C_3 - C_4) \\ & + (1 - p_3)(-\varphi C_2 - C_3 - C_4) \\ & - (1 - p_3)(-C_3 - C_4) = 0, \\ & \left(p_3^* = \frac{\varphi C_2}{\omega C_3}, p_4^* = \frac{(\gamma - \omega) C_3 + C_5}{\varphi (1 - \gamma) C_2 - \omega C_3} \right). \end{aligned} \tag{7}$$

Substituting (p_3^*, p_4^*) into formula (5) and formula (6), we can obtain

$$\begin{aligned} E_{11}^* &= -C_1 + \alpha\gamma p_4^* C_2 + \gamma C_3, \\ E_{12}^* &= \frac{\varphi (1 - \omega)}{\omega} C_2 - C_3 - C_4. \end{aligned} \tag{8}$$

Second, the expected value of node ① is calculated as follow:

$$\begin{aligned} E_{21} &= -p_1 p_2 C_1 - (1 - p_1)(1 - p_2) C_6 + p_1 (1 - p_2) E_{11}^*, \\ E_{22} &= -(1 - p_1) p_2 C_2 + p_1 p_2 (M_1 - C_2) + p_1 (1 - p_2) E_{12}^*. \end{aligned} \tag{9}$$

We can seek the Nash equilibrium of the game problem as follow:

$$\begin{aligned} \frac{\partial E_{21}}{\partial p_1} &= -p_2 C_1 + (1 - p_2) C_6 + (1 - p_2) E_{11}^* = 0, \\ \frac{\partial E_{22}}{\partial p_2} &= -(1 - p_1) C_2 + p_1 (M_1 - C_2) - p_1 E_{12}^* = 0. \end{aligned} \tag{10}$$

Nash equilibrium obtained is

$$\left(p_1^* = \frac{C_2}{M_1 - E_{12}^*}, p_2^* = \frac{C_6 + E_{11}^*}{C_1 + C_6 + E_{11}^*} \right). \tag{11}$$

3.3. Results Analysis. (a) The probability of government regulation is related to the costs of valuing the product safety of manufacturers, the profits protecting the credibility, and their expectations. The more the manufacturers value the product safety cost C_2, the greater the probability of government regulation p_1^*. The greater the profit M_1 by protecting the credibility, the smaller the probability of government regulation p_1^*. The larger E_{12}^* is, the greater the positivity of manufacturers' emphasis on product safety and the smaller the probability of government regulation p_1^* will be.

(b) The probability of manufacturers' emphasizing product safety is related to the supervision costs, expected revenue, and credibility losses of the relevant government officials. The higher the government regulatory cost C_1 is, the smaller the p_2^* is. The higher the expected revenue of relevant government officials in the first stage E_{11}^*, the bigger the p_2^*. The higher the loss of government credibility C_6, the bigger the p_2^*.

(c) The probability of relevant government officials abusing power is associated with the following factors: the bigger φ the higher the bribe φC_2 and the greater the temptation of relevant government officials. Thus, p_3^* will be bigger. The greater the fines charged by the abusive government, the smaller the probability p_3^* of abusing power of relevant government officials because the enterprises will begin to attach importance to product safety in order to avoid penalties.

(d) The probability p_4^* of the bribery of manufacturers is connected with the following factors: when the incentive payments given to relevant government departments are disproportionate with the amount they turn into the state treasury, the relevant government officials will have greater tendencies to expend funds. Thus, the formation of rent-seeking behavior between the government and manufacturers will be stimulated. The greater the probability p_4^* of the bribery of manufacturers, the greater the loss C_5 of government credibility resulting from the abuse of power and the smaller the probability of bribery of manufacturers.

4. Conclusion

This paper for the first time mentions political connections in supply chain and introduces the theory of political connections into supply chain theory. It studied political connections of product safety in supply chain. Based on study and theoretical analyses of a static game model and a dynamic game model, first of all, this paper reveals the relationship between the degree of manufacturers who pay attention to product safety and government supervision of product safety cost, expected return, and credit loss size. Namely, both sides need to establish the positive political connection to maintain the credibility and promote the virtuous cycle of the production supply chain. On the other

hand, the paper indicated the probability of a political contact person who breach of privilege depends on the bribes and the fine proportion. However, the probability of a production manufacturer who have offered bribes depends on the ratio of political contacts official income and grey income. Namely, both sides established the negative political ties. Although negative political connections are conducive to short-term interests on the surface, they are not conducive to the long-term development of enterprises and harmed governmental credibility and the social public interest.

This research will not only enrich the research of supply chain and political connections but also will reveal the internal mechanism about political connections on safety production in the research of supply chain system. In this paper, some problems need to be further studied. For example the model parameters need to be confirmed by real statistics data and a part of conditions of model was built on the basis of some rational hypothesis.

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Research Article

Optimizing Route for Hazardous Materials Logistics Based on Hybrid Ant Colony Algorithm

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Optimizing Route for Hazardous Materials Logistics (ORHML) belongs to a class of problems referred to as NP-Hard, and a strict constraint of it makes it harder to solve. In order to dealing with ORHML, an improved hybrid ant colony algorithm (HACA) was devised. To achieve the purpose of balancing risk and cost for route based on the principle of ACA that used to solve TSP, the improved HACA was designed. Considering the capacity of road network and the maximum expected risk limits, a route optimization model to minimize the total cost is established based on network flow theory. Improvement on route construction rule and pheromone updating rule was adopted on the basis of the former algorithm. An example was analyzed to demonstrate the correctness of the application. It is proved that improved HACA is efficient and feasible in solving ORHML.

1. Introduction

Hazardous materials, which have different physical and chemical properties, have high risk during transportation, as a series of problems may arise in this process. Route optimization is a complex combinatorial optimization problem, which is a typical NP-complete problem and difficult to come up with a direct answer. It is a practical problem in urgent need of solution in which we can find the optimal plan under the restrictions quickly, accurately, safely, and economically.

Optimizing Route for Hazardous Materials Logistics (ORHML) can be described as follows. Given a set of hazardous materials and an underlying network consisting of a number of nodes and capacitated arcs, we wish to find an optimal routing plan to ship the hazardous materials through the network at lowest cost without violating the capacity limits. ORHML models also appear as subproblems in more complicated models, such as distribution system design and capacitated network design.

ORHML has attracted the attention of many OR researchers. Kara et al. [1] presented several route planning models of road. Verma and Verter [2] gave a number of route planning models of rail. Iakovou [3] provided route planning models of water. Miller-Hooks [4] modeled ORHML as a path selection problem in a stochastic time-varying network. Dell'Olmo et al. [5] finding a number of spatially dissimilar paths between an origin and a destination can also be considered in this area. Jin and Batta [6] presented a risk model that took into account the dependency to the impedances of preceding road segments. Erkut and Verter [7] proposed a collection of edges in place of an origin-destination route for a hazmat shipment, where travel on this path can be viewed as a probabilistic experiment. It considered minimizing for a given OD pair in a hazmat transport network is a shortest path problem which is solved easily for even large networks. Their work also pointed out that this approximation is likely to result in a very small error in measuring the incidence probability along a hazmat transport route [8].

In general, in spite of their more realistic assumption, most of the exact versions of risk models have some puzzling properties and these models may not be suitable for hazmat transportation planning. We suggest that researchers and practitioners consider the properties of the risk models carefully.

Successful ant colony algorithms have been developed for several combinatorial optimization problems, such as Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP).

Based on the successful application of ant colony algorithm to TSP, a new ACA was adopted in solving ORHML. A collaborative metaheuristic approach is inspired by the foraging behavior of real colonies of ants. The basic ACA application is a large number of simple artificial agents that are able to build good solutions for hard combinatorial optimization problems through low-level based communications. Real ants cooperate in their search for food by depositing chemical traces (pheromones) on the floor.

2. Model Analysis

ORHML optimization model can be demonstrated as follows:

$$\begin{aligned} &\text{Minimize} && \sum_{1\le k\le K}\left(\lambda C^k + (1-\lambda)R\right)x^k \\ &\text{subject to} && \sum_{1\le k\le K} x_{ij}^k \le u \quad \forall (i,j)\in \mathrm{A} \\ & && \mathbb{N}x^k = b^k \quad \forall k\in K \\ & && x_{ij}^k \ge 0 \quad \forall (i,j)\in A,\ k\in K \\ & && 0\le\lambda\le 1, \end{aligned} \tag{1}$$

where $G = (N, A)$ represents the actual road network, in which the set of nodes N represents a road intersection, and arc set A indicates the connection between the intersection of sections. If (i, j) denotes the sections, then obviously $i, j \in N$ and $(i, j) \in A$. K represent a different flow of hazardous materials set $k \in K$; k represents the hazardous materials k. x_{ij}^k represents hazardous materials k using the arc (i, j); C_{ij}^k represents the unit cost when hazardous materials k use arc (i, j); R_{ij}^k represents the unit risk when hazardous materials k use arc (i, j). u represents the capacity constraints of the road. N represents the node-arc incidence matrix λ as the parameter [9].

$b^k(i)$ represents the supply amount or demand amount in point i of hazardous materials k, $b^k(i) > 0$ means that the supply, $b^k(i) < 0$ which means that the demand.

Based on the risk analysis theory, the risk of hazardous materials k in (i, j) can be calculated by the following formula [10]:

$$\begin{aligned} R_{ij}^k &= \sum_{s=1}^{6} p\left(A_{ij}^k, M^k, D^k\right) * R_{ij}\left(d_s^k\right), \\ p&\left(A_{ij}^k, M^k, D^k\right) \\ &= p\left(D^k \mid A_{ij}^k, M^k\right) * p\left(M^k \mid A_{ij}^k\right) * p\left(A_{ij}^k\right), \end{aligned} \tag{2}$$

where A_{ij}^k represents the traffic accident of hazardous materials k in (i, j); M^k represents the leak accident of hazardous materials k in (i, j); D^k represents the accident loss of hazardous materials k in (i, j). D^k= {casualties along the road, casualties in the vehicle, property damage, traffic interruption, evacuation, environmental damage}. $R_{ij}(d_s^k)$ represents the risk of loss caused by the harm s.

(1) Casualties along the road $R_{ij}(d_1^k)$:

$$R_{ij}\left(d_1^k\right) = l_{ij} * r^k * \rho_{ij} * \mathrm{HLV}, \tag{3}$$

where l_{ij} represents the length of the road; r^k represents the radius of the influence area for the accident disaster; ρ_{ij} represents the population density along the road; HLV represents the value of the loss of life.

(2) Casualties in the vehicle $R_{ij}(d_2^k)$:

$$R_{ij}\left(d_2^k\right) = \left(Q_{ij}^1 * N^1 + Q_{ij}^2 * N^2\right) * \mathrm{HLV}, \tag{4}$$

where Q_{ij}^1 represents the traffic flow of bus; Q_{ij}^2 represents the traffic flow of car; N^1 represents the patronage of bus; N^2 represents the patronage of car.

(3) Property damage $R_{ij}(d_3^k)$:

$$R_{ij}\left(d_3^k\right) = Q_{ij}^1 * \mathrm{CLV}^1 + Q_{ij}^2 * \mathrm{CLV}^2, \tag{5}$$

where CLV^1 represents the loss value of bus; CLV^2 represents the loss value of car.

(4) Traffic interruption $R_{ij}(d_4^k)$:

$$R_{ij}\left(d_4^k\right) = \frac{T_1}{T_0} * \left(Q_{ij}^1 * N^1 + Q_{ij}^2 * N^2\right) * \mathrm{TLV}, \tag{6}$$

where T_1 represents the traffic disruption time; T_0 represents per unit time; TLV represents the value of loss time.

(5) Evacuation $R_{ij}(d_5^k)$:

$$R_{ij}\left(d_5^k\right) = l_{ij} * r^k * \rho_{ij} * \mathrm{AEC}, \tag{7}$$

where AEC represents average evacuation costs.

(6) Environmental damage $R_{ij}(d_6^k)$:

$$R_{ij}\left(d_6^k\right) = l_{ij} * r^k * \mathrm{ELV}_{ij}, \tag{8}$$

where ELV_{ij} represents value of environmental damage of unit area.

3. Solving Strategies

HACA goal is to find the shortest tour. In HACA m ants build tours in parallel, where m is a parameter. Each ant is randomly assigned to a starting node and has to build a solution, that is, a complete tour. A tour is built node by node: each ant iteratively adds new nodes until all nodes have been visited. Improvement process of solution strategy is present in the algorithm; this strategy can transform solution of the problem to a single ant path. Given multiple source points, ink points, and the corresponding flow, construct an ant colony; in the initial state, the ants are randomly distributed in each source point; the ants' next point is selected at random. Updated according to the flow from the source point to sink point as weight. When the ant went to the source point and the corresponding sink point, place the ants to another

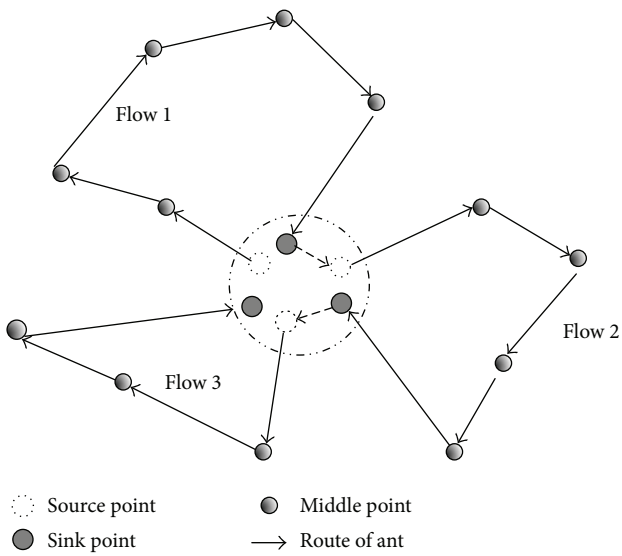

Figure 1: Improvement process of solution strategy.

source point, begin the next search process until all sources and sinks were finished, and then complete a cycle of the ant. As can be seen in Figure 1 [11].

When ant k is located in node i, it chooses the next node j probabilistically in the set of feasible nodes according to $p_{ij}^{k}(t)$.

In the HACA, original version formula for $p_{ij}^{k}(t)$ is

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l\in N_i^k}[\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta}} & j \in N_i^k \\ 0 & \text{else.} \end{cases} \tag{9}$$

The number of next nodes which ant k in node i could select is represented by $\text{win}^k(i)$. In basic ant colony algorithm, $\text{win}^k(i) = \text{cnt}(\text{allowed}_k)$, $\text{cnt}(\text{allowed}_k)$ denotes the number of nodes in set allowed_k. The value of $\text{win}^k(i)$ should be adjusted dynamically:

$$\text{win}^k(i) = \begin{cases} \text{cnt}(\text{allowed}_k) \\ \text{if } (1 - V(i)) * \text{cnt}(\text{allowed}_k) = \text{cnt}(\text{allowed}_k) \\ \lfloor (1 - V(i)) * \text{cnt}(\text{allowed}_k) \rfloor + 1 \\ \text{else.} \end{cases} \tag{10}$$

The sum of ants is represented by M; there are r trails from node i; the number of ants that pass node i is Y_i, and Y_i is distributing in r trails with number $a_1, a_2, \ldots a_r$:

$$V(i) = \begin{cases} \dfrac{Y_i}{M}\left(1 - \dfrac{1}{Y_i}\sqrt{\dfrac{r\sum_{i=1}^{r}(Y_i/r - a_i)^2}{r-1}}\right) & \text{if } Y_i \neq 0 \\ 0 & \text{else.} \end{cases} \tag{11}$$

Using for reference the idea of ant-Q algorithm of Dorigo, the meliorative route construction rule can be expressed as follows

$$j = \begin{cases} \text{According } P_{ij} \text{ select } j, \\ P_{ij} = \dfrac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\mu_{ij}]^{\gamma}[\kappa_{ij}]^{\lambda}}{\sum_{h\in\text{allowed}_k}[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\mu_{ij}]^{\gamma}[\kappa_{ij}]^{\lambda}} \quad j \in \text{allowed}_k, \quad P \le r_0 \\ \text{According } \max_{j\in\text{allowed}_k}\left\{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\mu_{ij}]^{\gamma}[\kappa_{ij}]^{\lambda}\right\} \text{ select } j, \quad P \ge r_1 \\ \text{According } P'_{ij} \text{ select } j, \\ P'_{ij} = \dfrac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\mu_{ij}]^{\gamma}[\kappa_{ij}]^{\lambda}}{\sum_{h\in\text{allowed}'_k}[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}[\mu_{ij}]^{\gamma}[\kappa_{ij}]^{\lambda}} \quad j \in \text{allowed}'_k, \quad r_0 \le P \le r_1 \end{cases} \tag{12}$$

$\mu_{ij} = d_{i0} + d_{0j} - d_{ij}$, called saving; $\kappa_{ij} = (Q_i + q_i)/Q$. Variable induced considering the time constraint.

The algorithm constructs two ant colonies to realize the goal of optimization: minimize the cost and risk of the hazardous materials transportation. These two ant colonies are the HACA-RISK and HACA-COST, which are used to determine the optimal route. In consideration of the two together, we may find better solutions to the problem. After getting the improved solution, the overall pheromone update would be used to get the exchange of information concerning the advantages and disadvantages of the solution [12].

The HACA-RISK optimization process is to get the feasible solution which uses one less risk than ψ^{gb}. HACA-RISK process is used to determine a shorter path under the circumstances. Once a new better feasible solution has been figured out, ψ^{gb} should be updated. These two processes will

Table 1: Parameter value.

	l_{ij} (km)	ρ_{ij} (person/km^2)	Q^1_{ij}	Q^2_{ij}	ELV_{ij}	$p(A^x_{ij})$	$p(A^y_{ij})$
AB	80	500	16	400	19000	0.0000018	0.000002
AC	35	1000	20	730	6000	0.0000055	0.0000069
AD	60	700	34	500	12000	0.0000035	0.0000044
BA	70	400	40	250	18000	0.0000015	0.0000019
BD	50	800	52	670	14000	0.0000035	0.0000044
BE	70	700	18	370	15000	0.0000025	0.0000031
CF	15	800	42	850	5000	0.0000065	0.0000081
CG	30	1200	68	680	9000	0.0000045	0.0000056
DF	20	2000	48	970	0	0.0000075	0.0000094
EF	25	1800	36	750	6000	0.0000055	0.0000069
EH	65	800	44	350	18000	0.0000015	0.0000019
FG	40	900	24	500	12000	0.0000035	0.0000044
FH	50	700	18	380	16000	0.0000025	0.0000031
GH	35	400	16	250	12000	0.0000015	0.0000019
HG	65	300	12	250	18000	0.0000015	0.0000019

come to an end if a solution with the use of fewer cost comes up. At the same time, a new ant colony can be built on the basis of this new solution, which restarts a new interactive optimization process.

If the least cost has been figured out by this algorithm, the HACA-RISK optimization process can be stopped, and then we shift to the HACA-COST optimization process.

In the HACA-COST we introduce an integer vector IN; IN_i means the times that the point has not been covered by the solution. This parameter should be updated correspondingly with the optimal solution. Whenever a new solution has been found, the overall pheromone update should be applied to reset ψ^{gb}.

The HACA-COST optimization process is similar to the traditional HACA optimization process which is to optimize the utilization of the load capacity and volume of RISKs.

The optimizing process can be divided into two parts: first, the ants move between the various points in search of the optimal solution; second, decompose the optimal solution into $V_{\min}$ subsets according to the location of the original node, then allocate goods to the RISKs.

The movement of ants in the HACA-COST and HACA-RISK is similar to each other. At the beginning of the algorithm, ants are randomly distributed among all the nodes. From the initial node, the ants determine the next move within the scope of available points, on the basis of the probability and the restrictions on RISK load capacity and volume. The time and cost of all nodes to the virtual origin node are 0, and no direct connection between the virtual origin nodes is allowed. Once the ants arrive the terminal node, thus begins the construction of the next subset. After traversing all the nodes, the ants return to the initial node to form a loop, that is, feasible solution.

In the HACA-COST optimization process, a feasible solution $\psi^1, \ldots, \psi^m$ is formed through the collaboration of all the ant colonies. Compare $\psi^1, \ldots, \psi^m$ with ψ^{gb}, if better, update ψ^{gb} with the value of $\psi^1, \ldots, \psi^m$.

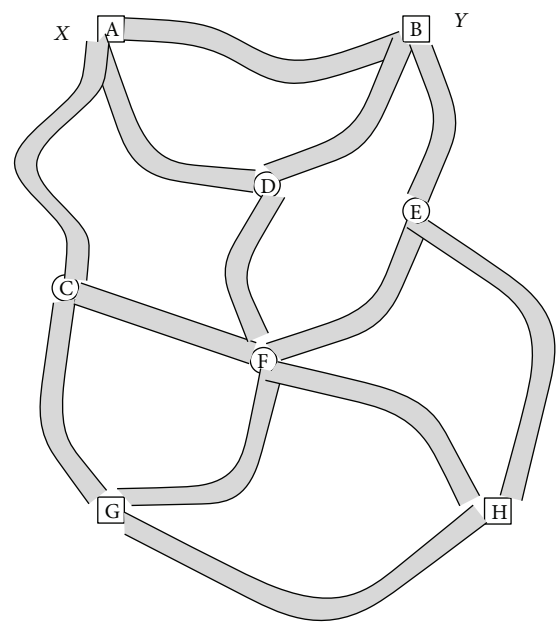

Figure 2: Road network.

When the above process was being completed, the obtained solution may still be incomplete (and may have missed points), so the insertion process is needed as further optimization. Insertion process is mainly to deal with the point that has not been included in the existing solution. The process determines the optimal feasible insertion position (the shortest path) for each point until the plug cannot find a viable location [13].

4. Example and Analysis

As can be seen in Figure 2, there are two kinds of hazardous materials X (flammable liquid) and Y (poisonous gas): X, from note A to note H, flow is 60 units; Y, from note B to note

Table 2: The probabilities of hazard losses.

$p(D^k \mid A_{ij}^k, M^k)$	$R_{ij}(d_1^k)$	$R_{ij}(d_2^k)$	$R_{ij}(d_3^k)$	$R_{ij}(d_4^k)$	$R_{ij}(d_5^k)$	$R_{ij}(d_6^k)$
X	0.4	0.8	0.6	0.4	0.1	0.2
Y	0.7	0.6	0.3	0.4	0.7	0.7

Table 3: Total expected risk.

R_{ij}^k	AB	AC	AD	BA	BD	BE	CF	CG
X	1495.61	2527.264	2448.515	847.7595	2496.6	2256.487	978.3397	2412.688
Y	744.24	1408.428	1373.803	480.7152	1402.368	1250.897	540.918	1337.213
	DF	EF	EH	FG	FH	GH	HG	
X	2880.29	3249.342	1574.389	2098.725	1663.237	349.7931	590.3967	
Y	1590.48	1810.836	892.7568	1177.546	922.467	197.7444	334.7838	

Table 4: The cost.

	AB	AC	AD	BA	BD	BE	CF	CG	DF	EF	EH	FG	FH	GH	HG
C_{ij}	40	17.5	30	35	25	35	7.5	15	10	12.5	32.5	20	25	17.5	32.5

G, flow is 20 units. That is, b(A) = 60, b(H) = −60, b(B) = 20, and b(G) = −20. The related data values are shown in Table 1.

Set freight = 0.5 yuan/km/unit, HLV = 1000000 yuan/person, N^1 = 50 persons/vehicle, N^2 = 3 persons/vehicle, CLV^1 = 500000 yuan, CLV^2 = 100000 yuan, T_0 = 24 h, T_1 = 16 h, AEC = 10000 yuan, and TLV = 20 yuan/h.

$p(M^x \mid A_{ij}^x) = 0.6$, $r^x = 0.3$ km; $p(M^y \mid A_{ij}^y) = 0.3$, $r^y = 2$ km. The probabilities of hazard losses $p(D^k \mid A_{ij}^k, M^k)$ can be seen in Table 2.

Total expected risk R_{ij}^k is shown in Table 3.

Set Fright = 0.5 yuan/km/unit. The cost C_{ij} can be shown in Table 4.

In view of complexity of this problem, the whole level optimization solutions are obtained based on the random searching and evolution process of the improved ant colony algorithm. The solution can be seen in Figure 3.

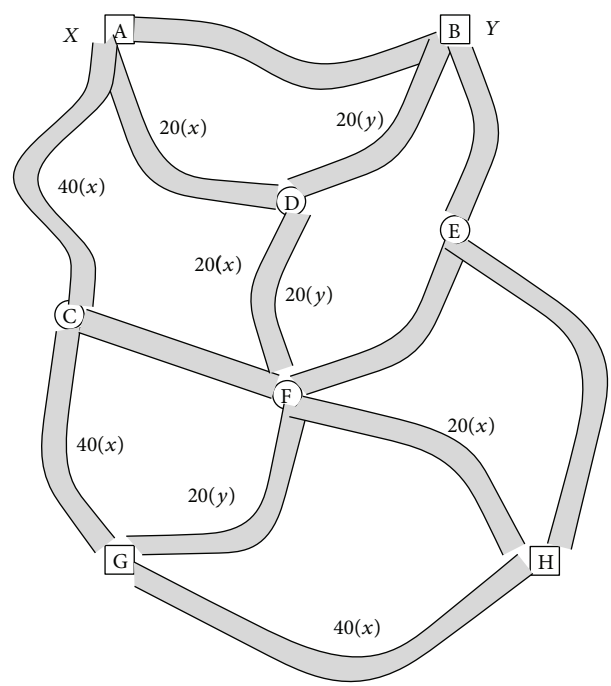

Figure 3: Solution.

5. Summary

Solving the ORHML is to determine route plans that can make a minimum of risk and safety under the premise of restrictions of the placements. That is, to seek a safe routing plan that could obtain the lowest cost with the conditions has been known.

A new ant colony optimization based approach to solve ORHML was introduced. In particular, the algorithm has been designed to solve ORHML with a balance between the safe and the cost. Pinpointing the characteristics of this problem, our algorithm introduces a new methodology for optimizing multiple objective functions. We consider the optimization of the safe and the cost at the same time. This paper analyzes the differences of the ant colony algorithms in solving the restricted ORHML. In order to deal with ORHML, an improved HACA was devised. Pinpointing the characteristics of this problem, we consider the optimization of cost and risk together. The basic idea is to coordinate the activity of different ant colonies, each of them optimizing a different objective. These colonies work by using independent pheromone trails but they collaborate by exchanging information. Furthermore, the integrated use of HACA-SAFE and HACA-COST has been applied as the improvement of the solving strategy.

Finally, the feasibility and effectiveness of this method has been scrutinized with practical examples. From the result on the test problem, we can conclude that the model and the heuristic procedures are quite successful in solving ORHML.

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Research Article

System Dynamics Model for VMI&TPL Integrated Supply Chains

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This paper establishes VMI-APIOBPCS II model by extending VMI-APIOBPCS model from serial supply chain to distribution supply chain. Then TPL is introduced to this VMI distribution supply chain, and operational framework and process of VMI&TPL integrated supply chain are analyzed deeply. On this basis VMI-APIOBPCS II model is then changed to VMI&TPL-APIOBPCS model and VMI&TPL integrated operation mode is simulated. Finally, compared with VMI-APIOBPCS model, the TPL's important role of goods consolidation and risk sharing in VMI&TPL integrated supply chain is analyzed in detail from the aspects of bullwhip effect, inventory level, service level, and so on.

1. Introduction

Under vendor-managed inventory (VMI) operation mode, many suppliers outsource their logistics to third-party logistics (TPL) due to their poor logistics capabilities. So far, that TPL participates in VMI has been widely used in many industries. For example, Dell and Lenovo both chose Burlington Company to help them operate VMI service, and Wuhan Shenlong Automobile Company in China allows GEFCO to provide VMI service with components supply. On the one hand, this integrated operational model combining VMI with TPL ensures that the supply chain information is shared fully on one central platform. On the other hand, it can take full advantage of TPL and reduce the total operational cost of the supply chain.

As to the research of VMI&TPL integrated replenishment and delivery, Çetinkaya et al. [1] take Dell as an example, which outsources VMI business to Burlington Logistics, and analyze TPL replenishment and delivery strategies. They do not only consider the optimal delivery strategies about logistics outsourcing but also find out differences of the optimal delivery strategies before and after outsourcing. Based on the above study, Çetinkaya and Lee [2] consider the time-based delivery policy and obtain the optimal delivery time structure with transportation lot constraints and capability limitations while the demand of retailers obeys the Poisson distribution. Lee et al. [3] assume that the replenishment and delivery can be started at the beginning of each period with determined demand and finite horizon and that the lead time for delivery of the replenishment is zero. They consider the problem of inventory and transportation integration, which is similar to Çetinkaya and Lee [2]. In order to achieve economies of transport scale, TPL implements goods consolidation strategy. As a result, products may be delivered to retailers in an earlier or later time, which would lead to the inventory cost or shortage cost. Their research shows that the problem is NP hard and that the demand in each period must be satisfied in just one delivery, with transportation constraints. Furthermore, they point out that if each delivery can meet demands in several consecutive cycles and the demand in the first and last periods may be met by two deliveries then the optimal replenishment and delivery policy exists. They even propose a polynomial algorithm to solve the above problem of optimal replenishment and delivery. Çetinkaya et al. [4] provide a foundation for a comparison of the impact of time-based versus quantity-based consolidation in the context of integrated inventory and transportation decisions. Numerical and analytical results verify that quantity-based consolidation is superior to the time-based version in terms of the resulting cost. Furthermore, several easily implementable TQ-based policies are proposed and their impacts on cost and service

are compared to those of time-based and quantity-based versions via simulation. Mutlu et al. [5] extend the results in Çetinkaya et al. [4] and develop an analytical model for computing the expected long-run average cost of a consolidation system implementing a TQ-based policy. The presented analytical results prove that (i) the optimal TQ-based policy outperforms the optimal time-based policy and (ii) the optimal quantity-based policy is superior to the other two (i.e., optimal time-based and TQ-based) policies in terms of cost. Considering the expected maximum waiting time as a measure of timely delivery performance, however, they numerically demonstrate that the TQ-based policies improve on the quantity-based policies significantly with only a slight increase in the cost.

Besides, some scholars consider replenishment strategies of TPL in VMI mode under different conditions, such as Çetinkaya et al. [6], Dejonckheere et al. [7], Wikrom et al. [8–11], Lee [12], Hwang [13], and Howard and Marklund [14]. Çetinkaya et al. [6] consider different delivery and replenishment strategies under two kinds of transportation modes. One is self-transportation which is the same as in the above literature. The other one is outsourcing transportation. As transportation providers will take the discount policy to encourage suppliers to transport more, transportation cost may be a piecewise function in this situation. They propose two kinds of delivery and replenishment strategies based on time and quantity under different transportation modes. Dejonckheere et al. [7] investigate the utilization of a linear (Type II) or quadratic (Type III) instead of a constant (Type I) exponential smoothing forecasting mechanism in the continuous-time APIOBPCS model. Mustafa et al. [15] consider the impact of coordinated replenishment and shipment in inventory/distribution systems and analyze a system with multiple retailers and one outside supplier. They present a centralized ordering policy that orders for all retailers and some other well-known policies like (a) can-order policy, (b) echelon inventory policy, and (c) fixed-replenishment interval policy. Leopoldo et al. [16] present an alternative heuristic algorithm to solve the vendor-managed inventory system with multiproduct and multiconstraint based on EOQ with backorders considering two classical backorders costs: linear and fixed. Sadeghia et al. [17] develop a constrained multivendor multiretailer single-warehouse (MV-MR-SW) supply chain, in which both the space and the annual number of orders of the central warehouse are limited. Since the problem is formulated into an integer nonlinear programming model, the metaheuristic algorithm of particle swarm optimization (PSO) is presented to find an approximate optimum solution of the problem. In the proposed PSO algorithm, a genetic algorithm (GA) with an improved operator, namely, the boundary operator, is employed as a local searcher to turn it to a hybrid PSO. Moreover, Harigaa et al. [18] consider a supply chain composed of a single vendor and multiple retailers operating under a VMI contract that specifies limits on retailers' stock levels. They address the problem of synchronizing the vendor's cycle time with the buyers' unequal ordering cycles by developing a mixed integer nonlinear program that minimizes the joint relevant inventory costs under storage restrictions.

As to logistics optimization based on system dynamics, Towill [19] establishes a new inventory-and-order-based production and control system (IOBPCS) by extending production inventory control (PIC) [20] and then optimizes the system by using the coefficient plane model. Sterman [21] constructs a general inventory management model, making use of the system dynamics, and points out that different complexity of feedback in supply chain inventory management system and the pressure of time usually lead decision makers to misunderstand the feedback information and thus make irrational decisions. John et al. [22] introduce WIP feedback control mechanism into IOBPCS model and expand the IOBPCS model into APIOBPCS. Later, Mason-Jones et al. [23] analyze the function of WIP feedback control mechanism in the models by comparing IOBPCS and APIOBPCS. Disney and Towill [24, 25] construct VMI-APIOBPCS model and analyze VMI strategy's effects on the supply chain bullwhip effect, customer service level and inventory costs with the assumption that the enterprises face the obvious fluctuations of demand. Besides, they optimize the VMI-APIOBPCS model and obtain the optimal parameters after considering different weights of production adjustment costs, different proportions of inventory costs, and different coefficients of safety stock. Disney and Towill [24, 25] study a simple vendor-managed inventory (VMI) supply chain consisting of one production unit and one distributor. In VMI systems all supply points in the chain have access to stock positions for setting production and distribution targets. The discrete-time APIOBPCS model is used to describe the dynamics of the manufacturing unit. Pure delay is initially utilized to model the production delay. The only difference to the APIOBPCS structure presented previously is that instead of the demand signal CONS the manufacturing facility receives a "virtual" consumption signal. This is caused by adding in each time period the demand signal received by the distributor to the difference between the current time period and the previous period reorder-point. The system is checked for stability. The stability criteria that are produced are also valid for the standard APIOBPCS model, since the distributor's policy described previously is a stable feed-forward element. One year later, Disney and Towill [26–28] analyze deeply how VMI affects the bullwhip effect in the supply chain and compare the VMI supply chain's expected performance with that of a traditional supply chain. VMI strategy shows that it has better reactions when demand is not steady, and this kind of instability may be caused by discounts available for orders or price's changes. Besides, the restoration of inventory level will be improved dramatically by VMI strategy. Moreover, Disney and Towill [26–28] concentrate on VMI strategy's effects on transport operations in supply chain, especially the batch problem in transportation strategy. By using system dynamics, they establish three different kinds of models—the traditional one, the internal integrated one, and the VMI one. The simulation case shows that VMI model can reduce transportation frequency by adopting a larger batch without influencing the dynamic performance of the entire supply chain. Wikner [29] presents a methodology that introduces structure dependencies of MLMS systems in the IOBPCS production control framework. The methodology

uses matrix representation to account for multiple informational channels. It is shown that for a single-level single-stage system the model is reduced to the standard IOBPCS format. The extended model has the capability to describe the dynamics of both pull-driven (base stock, kanban) and push-driven (MRP) policies.

In the other field, through STELLA/iThink software platform, Chen et al. [30] construct a system dynamics model of inventory management, analyze system structures and operational mechanisms of VMI inventory management and traditional inventory management, and finally compare their operational performance. Yang and Liu [31] extend VMI-APIOBPCS model from one supplier-one retailer supply chain to one supplier-two retailers supply chain and then construct VMI-APIOBPCS II. With the integration of TPL, they establish VMI&TPL-APIOBPCS model and the simulation shows that TPL can help reduce bullwhip effect in the supply chain availably. Cho and Lazaro [32] extend PID controller for just-in-time production scheduling. Lin et al. [33] develop a fuzzy system dynamic to simulate vendor-managed inventory, automatic pipeline, and inventory-and-order-based production control system (VMI-APIOBPCS) model based on fuzzy difference equations, and these operators of difference equations adopt the weakest t-norm (TW) operators. The results of fuzzy VMI-APIOBPCS model can provide the whole extended information regarding the system behavior uncertainties for the decision makers with fuzzy interval.

After that Darya and Martin [34] address the steady-state optimization of a supply chain model that belonged to the class of vendor-managed inventory, automatic pipeline, and inventory-and-order based production control systems (VMI-APIOBPCS). They optimize the supply chain with the so-called normal vector method, which has specifically been developed for the economic optimization of uncertain dynamical systems with constraints on dynamics. Kristianto et al. [35] propose an adaptive fuzzy control application to support a vendor-managed inventory (VMI). The methodology applies fuzzy control to generate an adaptive smoothing constant in the forecast method, production, and delivery plan to eliminate, for example, the rationing and gaming or the Houlihan effect and the order batching effect or the Burbidge effects and finally the bullwhip effect. In order to improve the level of integration in all aspects of supply chain reconfiguration, Kristianto et al. [36] construct an optimum supply chain network by combining optimization at the strategic and tactical level. A system dynamic based computer simulation model is used to validate the operations of the supply chain. The performance of the system is measured in terms of backorders and inventory level. The results and analysis indicate that fewer stockholding points and a shorter review period of demand can improve performance in this respect.

Our work differs from these studies in important aspects. First of all, these models are mainly constructed based on two echelon supply chains, that is, Disney and Towill [24–28], Lin et al. [33], Darya and Martin [34], and Kristianto et al. [35, 36]. In our model, however, VMI&TPL integrated operational model is a relatively complex three-echelon supply chain system. The TPL role integrated with VMI should be explored. Furthermore, we also model VMI&TPL integrated supply chain from aspects of bullwhip effect, inventory levels, service level with stochastic demand, and other uncertainties, compared with VMI-APIOBPCS model. Since system dynamics suit researches of complex systems very well, VMI&TPL-APIOBPCS model is constructed based on system dynamics to simulate and analyze the performance of VMI&TPL integrated operational model.

Table 1: Definitions of parameters and variables.

T_i: inventory adjustment time	T_a: demand smooth time
T_p: production delay time	$T_{\hat{p}}$: estimated value of production delay time
CON: demand rate	ACON: demand rate after forecast
ORT: production rate	CRT: production fulfillment rate
TINV: target inventory level	INV: actual inventory level
EINV: inventory deviation	WIP: work-in-process
TWIP: target work-in-process	EWIP: work-in-process deviation

2. IOBPCS Model Family

2.1. Definitions of Parameters and Variables. Table 1 is the definitions of parameters and variables used in this paper.

2.2. IOBPCS Model. As shown in Figure 1, inventory levels of work-in-process and finished products can be controlled by production order rate in IOBPCS system, and customers' demand is met by inventory.

Figure 2 is the causal relationship in IOBPCS model, including 4 main parts, which are demand forecast feed-forward loop, production delays, inventory feedback loop, and target inventory. Production delays refer to the time from production orders to production fulfillment rate. In series IOBPCS models, it is assumed that production process in production lead time meets a certain order, and orders keep a sequence of events. Demand forecast feed-forward loop refers to the demand forecasting mechanism which is used to predict the demand in and after production lead time. Inventory feedback loop actually is a kind of inventorydeviation adjustment mechanism. It is necessary to produce more goods to adjust inventory deviation when the actual inventory level and target inventory level differ greatly.

The productivity in IOBPCS model is decided by the demand forecasting mechanism and the inventory deviation adjustment mechanism, while the inventory deviation adjustment mechanism is decided by the inventory adjustment time and the target inventory level, and the demand forecasting mechanism is decided by the demand smooth time. Therefore, IOBPCS system optimization includes the definition of two basal parameters, such as demand smooth time and inventory deviation adjustment time. When designing the best production control strategy, cost from two sides should be balanced, including production cost due to production

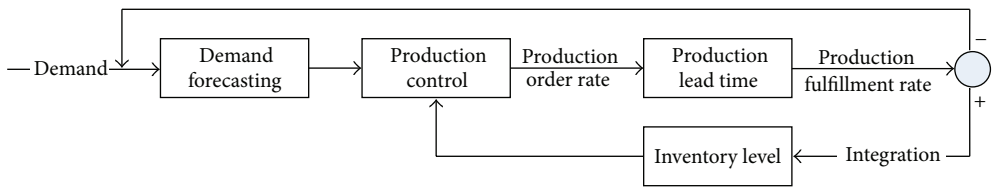

Figure 1: IOBPCS model of production and inventory system.

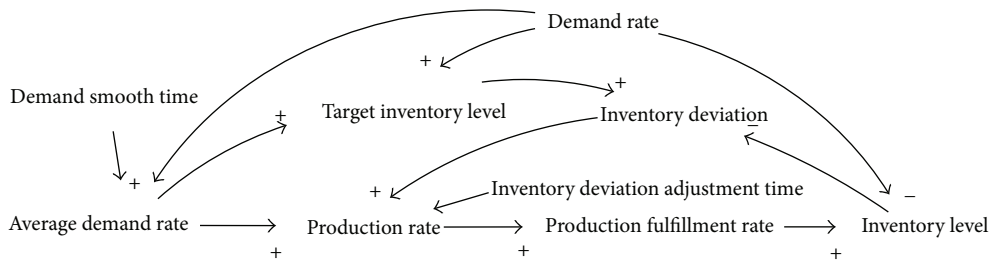

Figure 2: IOBPCS model causal relationship.

fluctuation and the inventory cost (or shortage cost) as the inventory level changes.

2.3. IOBPCS Expansion Model. After many scholars amending and improving the model based on IOBPCS model, now it has been turned into an IOBPCS model family consisting of five parts as Figure 3 shows.

2.3.1. Production Delay. The lead time of production delays can be regarded as production rhythm smooth time, which describes the speed of adjusting production rhythm to production order changes (ORATE). The production delay is one of the system characteristics, which cannot be controlled at random by the system's designers, but designing different delay models will have an important effect on the entire system's performance. Formula (1) is the dynamic behavior of the three delay models:

$$G_p(s) = \frac{1}{\left(\left(T_p/n\right)s + 1\right)^n}, \tag{1}$$

where $n = 1$, first-order delay; $n = 3$, third-order delay; $n = 8$, pure delay; T_p is production delay time.

2.3.2. Target Inventory Level (DINV). In IOBPCS model, target inventory level is a fixed value or the integer multiple of the demand forecasting number (ACON) after smoothing. Target inventory level is variable in VIOBPCS. Compared with IOPBPCS, the width of ORATE is larger, but the inventory adjustment response time is shorter. The only difference between APIOBPCS model and APVIOBPCS model lies in the setting of target inventory. In APIOBPCS model, target inventory level DINV $= k *$ ACON, where k is a positive integer.

2.3.3. Demand Forecasting Mechanism. Demand forecasting mechanism is an important part of the feed-forward loop. Demand forecasting mechanism is measured by exponential smoothing method in most literatures, because exponential smoothing method comprehensively includes all the historical information and is easy to use and to formulate a model. Exponential forecasting method (e.g., single exponential smoothing method, double exponential smoothing method, and triple exponential smoothing method) makes the steady-state error of system inventory in phase step and slope demand zero, but the steady-state error of system inventory becomes larger and larger when the demand function is a parabola.

Single exponential smoothing transfer function in s region is

$$G_a(s) = \frac{1}{T_a s + 1}. \tag{2}$$

Double exponential smoothing transfer function in s region is

$$G_a(s) = \frac{2T_a s + 1}{T_a^2 s^2 + 2T_a s + 1}. \tag{3}$$

Triple times exponential smoothing transfer function in s region is

$$G_a(s) = \frac{3{T_a}^2 s^2 + 3T_a s + 1}{T_a^3 s^3 + 3T_a^2 s^2 + 3T_a s + 1}. \tag{4}$$

2.3.4. Inventory Deviation Adjustment Mechanism. Inventory deviation adjustment mechanism is an inventory feedback loop which controls inventory deviation by controlling productivity. Inventory adjustment mechanism needs to consider production delay effect which means that only after a regular period of time can the controlling decision about productivity adjust the inventory level. The purpose of inventory adjustment is to reach target inventory level in a period of time (T_i). When the adjustment time is shorter,

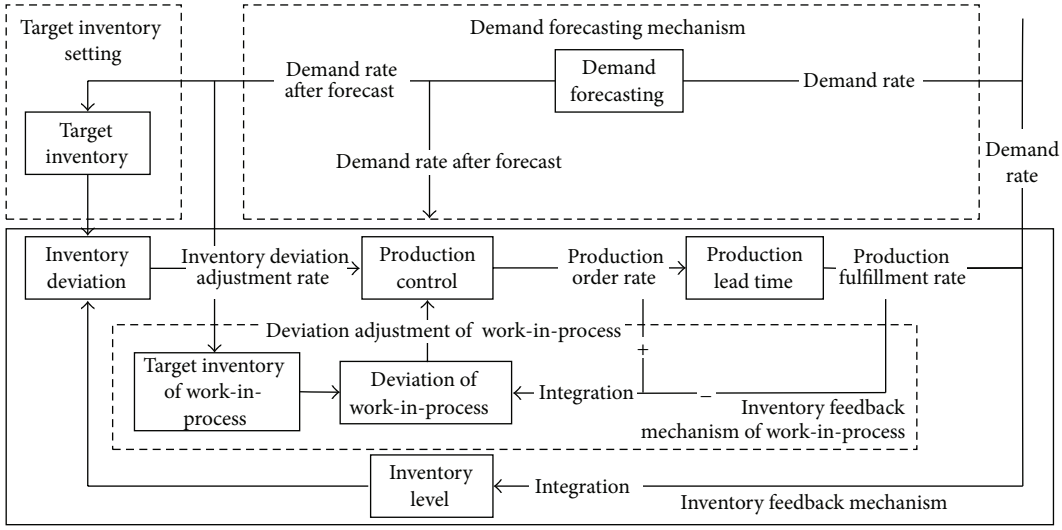

Figure 3: The main parts of expanded IOBPCS model.

the inventory can adjust faster. As a result, shortage cost and risks become smaller. However, it requires manufacturers for a higher production capacity, because the system needs to correct the early inventory deviation in a short time by adjusting productivity, and this can lead to higher cost of production and work-in-process inventory.

2.3.5. WIP Inventory Deviation Adjustment Mechanism. WIP inventory deviation is caused by actual work-in-process inventory in contradiction with target work-in-process inventory when demand changes. WIP adjustment mechanism adjusts inventory deviation by controlling productivity so that it can reach the target value in a period of time (T_w). Therefore, WIP inventory deviation adjustment rate is one of the three parts of the productivity control mechanisms. It is necessary to study and analyze the production process and get a relatively exact estimate value by making statistical analysis of production delay time before designing the productivity control mechanism. If the observed production delay time is different from actual time, inventory level in steady-state will not be in accord with target inventory level, which will cause more risk of inventory or shortage.

2.4. APIOBPCS Model. Figure 4 is a block diagram of API-OBPCS model lying in s region. The purpose of system designing is to find a proper target inventory level and design three optimal control mechanisms (demand forecasting mechanism, inventory deviation adjustment, and WIP inventory deviation adjustment mechanism), in order to minimize system cost, including production cost and inventory cost.

In general, inventory dynamic fluctuation is measured by inventory rising time and adjustment time and overshoot, and productivity dynamic change is analyzed by frequency response method.

Here is the main control mechanism of APIOBPCS model in phase-step demand.

(1) Forecasting mechanism. Formula (5) is the transfer function in s region with single exponential smoothing:

$$G_a(s) = \frac{1}{T_a s + 1}. \tag{5}$$

(2) Target inventory setting. TINV = 0.

(3) Production process. Formula (6) is the transfer function in s region with first-order delay:

$$G_p(s) = \frac{1}{T_p s + 1}. \tag{6}$$

As a result, two important transfer functions about productivity and changes in inventory level can be obtained as follows:

$$\begin{aligned} F_1(s) &= \frac{\mathrm{ORT}(s)}{\mathrm{CON}(s)} \\ &= \frac{T_p T_i s + T_w + (T_a + T_i) T_w s}{(1 + T_a s)\left[T_i T_w T_p s^2 + T_i s\left(T_p + T_w\right) + T_w\right]}, \\ F_2(s) &= \frac{\mathrm{INV}(s)}{\mathrm{CON}(s)} \\ &= \frac{T_i\left(T_{\hat{p}} - T_p\right) - T_i T_p T_w s - T_i T_p T_a s - T_i T_w T_a s - T_a T_i T_w T_p s^2}{(1 + T_a s)\left[T_i T_w T_p s^2 + T_i\left(T_p + T_w\right) s + T_w\right]}. \end{aligned} \tag{7}$$

3. VMI-APIOBPCS Model

3.1. Definitions of Parameters and Variables. Table 2 is the definitions of parameters and variables used in this paper.

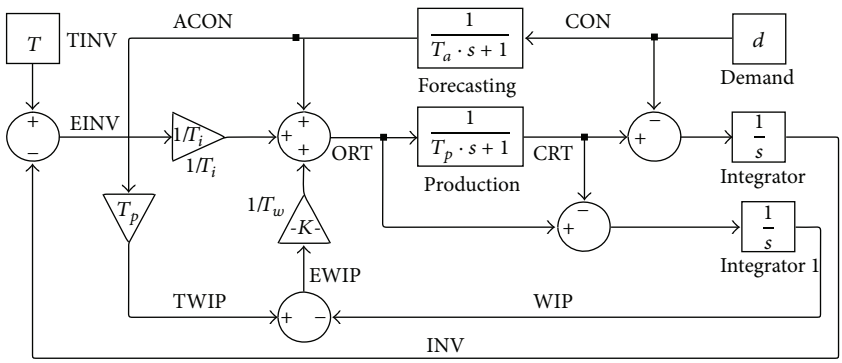

Figure 4: Block diagram of APIOBPCS model.

Table 2: Definitions of parameters and variables.

Parameters and variables of suppliers	
VCON: demand rate of suppliers	AVCON: demand factor of suppliers after smoothing
VINV: inventory level of suppliers	TINV: system target inventory level
WIP: work-in-process inventory level	TWIP: target work-in-process inventory level
EINV: inventory deviation	EWIP: deviation of work-in-process inventory
ORT: productivity of suppliers	CRT: production fulfillment rate of suppliers
T_i: adjustment time of system inventory deviation	T_a: demand rate of suppliers smooth time
T_w: work-in-process inventory deviation adjustment time	T_p: production delay time of suppliers
AEWIP: work-in-process inventory deviation regulation factor	
AEINV: deviation adjustment rate of system inventory	
Parameters and variables of retailers	
CON: demand rate of retailers	ACON: demand rate of retailers after forecasting
RINP: inventory level of retailers (including inventory on the way)	SS: safety inventory level
G: safety inventory factor of retailers	ROP: reorder-point of distributors
SRT: delivering rate to retailers	
DSS: reorder-point of retailers variation	
GIT: inventory of distributors on the way	
RINV: actual inventory level of retailers	
L: transportation time from suppliers to retailers	
ETQ: economic order quantity	

In VMI operation mode, retailers share inventory information and sales information with suppliers dynamically and determine the customer service level together with suppliers. According to the fixed customer service level, suppliers choose quantity-based delivering model, which means that vehicle shipped way is chosen in order to guarantee economical efficiency of transportation when the total inventory level is lower than the reorder-point. Figure 5 is VMI-APIOBPCS system dynamics model.

Here are the relational formulas in VMI-APIOBPCS model. They are as follows.

3.1.1. Suppliers Production or Replenishment Mechanism

(1) Demand forecasting mechanism:

$$\mathrm{AVCON}_t = \mathrm{AVCON}_{t-1} + \frac{\mathrm{VCON}_t - \mathrm{AVCON}_{t-1}}{1 + T_a}. \quad (8)$$

(2) Target work-in-process inventory level:

$$\mathrm{TWIP}_t = \mathrm{AVCON}_t \times T_P. \quad (9)$$

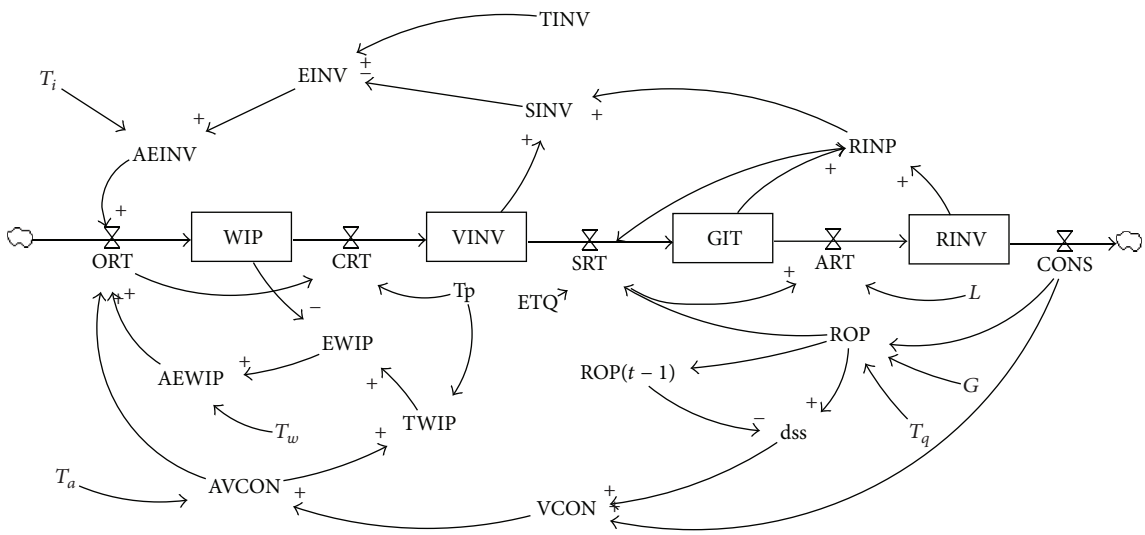

Figure 5: VMI-APIOBPCS system dynamics model.

(3) Work-in-process inventory level:

$$\mathrm{WIP}_t = \mathrm{WIP}_{t-1} + \mathrm{ORT}_t - \mathrm{CRT}_t. \tag{10}$$

(4) Finished goods inventory level:

$$\mathrm{VINV}_t = \mathrm{VINV}_{t-1} + \mathrm{CRT}_t - \mathrm{SRT}_t. \tag{11}$$

(5) Target inventory level:

$$\mathrm{TINV}_t = \mathrm{AVCON}_t \times T_s. \tag{12}$$

(6) System inventory level:

$$\mathrm{SINV}_t = \mathrm{VINV}_t + \mathrm{RINV}_t. \tag{13}$$

(7) Finished goods fulfillment rate:

$$\mathrm{CRT}_t = \mathrm{delay}\left\{\mathrm{ORT}_t, T_p\right\}. \tag{14}$$

(8) Productivity:

$$\mathrm{ORT}_t = \mathrm{AVCON}_t + \mathrm{AEINV}_t + \mathrm{AEWIP}_t. \tag{15}$$

(9) Inventory deviation adjustment rate:

$$\mathrm{AEINV}_t = \frac{\mathrm{TINV}_t - \mathrm{SINV}_t}{T_i}. \tag{16}$$

(10) Work-in-process inventory deviation adjustment rate:

$$\mathrm{AEWIP}_t = \frac{\mathrm{TWIP}_t - \mathrm{WIP}_t}{T_w}. \tag{17}$$

3.1.2. Suppliers Delivering Mechanism

(1) Reorder-point of retailers:

$$\mathrm{ROP}_t = \mathrm{ROP}_{t-1} + \frac{\mathrm{SS}_t - \mathrm{ROP}_{t-1}}{1 + T_q}. \tag{18}$$

(2) Inventory levels of retailers:

$$\mathrm{RINP}_t = \mathrm{RINV}_t + \mathrm{GIT}_t. \tag{19}$$

(3) Order arrival rate:

$$\mathrm{ART}_t = \mathrm{delay}\left\{\mathrm{SRT}_t, L\right\}. \tag{20}$$

(4) Safety inventory setting of retailers:

$$\mathrm{SS}_t = \mathrm{CON}_t \times G. \tag{21}$$

3.1.3. Information Sharing Mechanism in VMI Supply Chain

(1) Suppliers can get customers' demand in time and obtain the actual total customer demand downstream through terminal customer information, including the terminal customer demand and changes of reorder-points of downstream retailers:

$$\mathrm{VCON}_t = \mathrm{CON}_t + \mathrm{dSS}_t = \mathrm{CON}_t + \mathrm{ROT}_t - \mathrm{ROT}_{t-1}. \tag{22}$$

(2) Suppliers can check the inventory level of downstream retailers so that they can get the total supply chain inventory level which can optimize the total supply chain inventory decision:

$$\mathrm{SINV}_t = \mathrm{RINP}_t + \mathrm{VINV}_t. \tag{23}$$

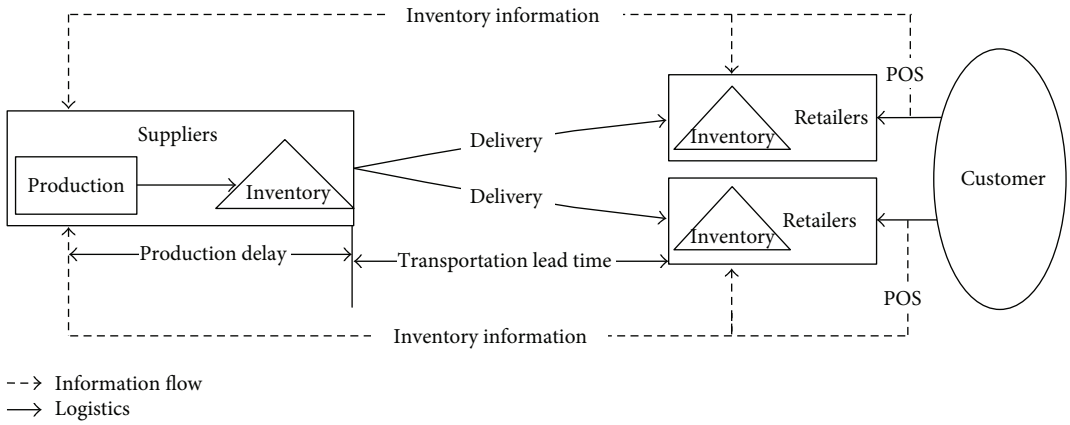

Figure 6: Operation mode of VMI distribution supply chain.

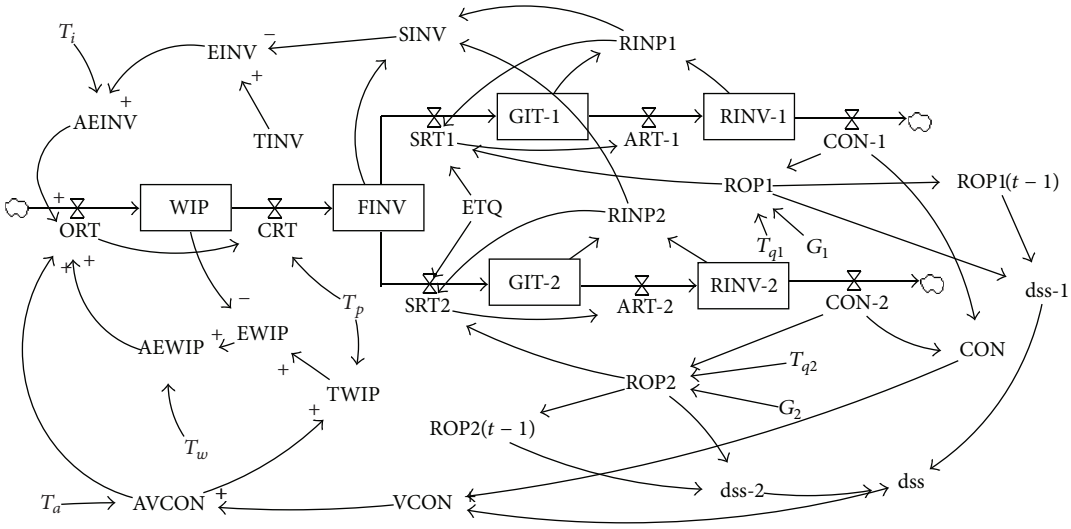

Figure 7: VMI-APIOBPCS II system dynamics model.

3.2. VMI Distribution Supply Chain. In the actual operational process, suppliers can adopt VMI model on multiple downstream retailers. According to the aforementioned VMI serial supply chain when suppliers provide multiple retailers downstream with VMI service, they can get VMI distribution supply chain, as Figure 6 shows, including one supplier and two retailers.

VMI-APIOBPCS II model as shown in Figure 7 is constructed based on the operation mode in Figure 6.

4. VMI&TPL-APIOBPCS Model

4.1. VMI Operational Process Based on TPL

4.1.1. VMI&TPL Integrated Operation Mode. To decrease the logistics cost and avoid the delivery risks in VMI system, in practice an upstream enterprise normally prefers to outsource its purchasing business to the third-party logistics (TPL) and requires his supplier to keep the inventory in the warehouse operated by TPL. For example, BAX Global is responsible for Apple, Dell, IBM, and other IT companies with their supplies in Southeast Asia, and United Parcel Service manages goods and materials procurement for Fender overseas and achieves its integration of process in distribution. Besides, Shanghai Volkswagen and Wuhan Shenlong Automobile adopt VMI&TPL integrated operation mode to effectively support the mixed flow job shop manufacturing with JIT delivering components to the work station directly.

After TPL is introduced into VMI, we consider the supply chain including one supplier (S), one TPL, and two retailers ($R1$ and $R2$). Suppliers give the rights of inventory operation and decision to TPL through a contract. TPL is responsible for replenishment and delivery in the total supply chain which means that TPL stores finished products in the warehouse near suppliers and, meanwhile, builds a district distribution center in order to meet retailers' requirements in time. Besides, considering scale effect of transportation, TPL takes a certain delivery strategy in the district distribution center.

Figure 8 mainly describes VMI&TPL integrated operation mode. Similar to VMI operation mode, the information in the supply chain is shared fully. Retailers share real-time

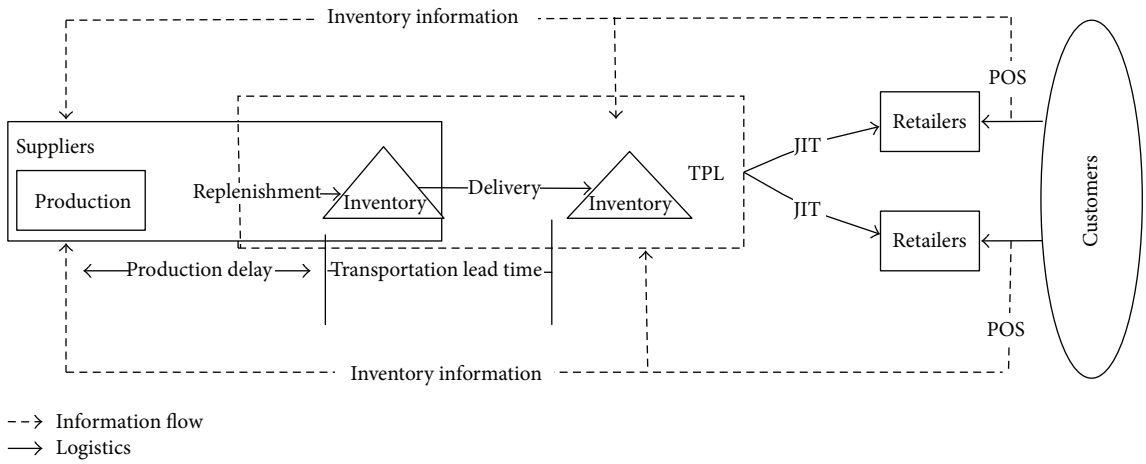

Figure 8: VMI&TPL-APIOBPCS integrated operating model.

sales information with suppliers and TPL. Meanwhile, TPL shares inventory information with suppliers and suppliers provide TPL with production information. It is different from VMI operation mode that TPL is in charge of inventory management in the total supply chain, oversees the inventory level in the whole supply chain, and sends requests for replenishment to suppliers in time so that suppliers can organize capacities to produce according to orders. Besides, TPL organizes transit power and sends products to the district distribution center close to suppliers and to retailers and distributes products to retailers according to sales information and contracts in VMI&TPL integrated operation mode.

4.1.2. VMI&TPL Operational Process. After carrying out TPL and VMI integrated operation, suppliers are not responsible for concrete logistics activities but give rights of inventory operation and decision to TPL through a contract. Therefore, TPL does not only undertake physical distribution business but is also responsible for orders generated in integrated logistics operation. The operational process of the whole system is illustrated in Figure 9.

(1) TPL updates retailers' inventory information everyday according to inventory information provided by retailers.

(2) TPL makes recommended orders according to retailers' inventory level and service level and replenishment point confirmed in advance.

(3) TPL sends orders to retailers and chooses the proper distribution route according to self-inventory level and retailers' demand after retailers confirm their orders. It is necessary to send requests for replenishment to suppliers and ask them to replenish inventory in time if self-inventory reaches replenishment level.

(4) Suppliers know well about the logistics operational situation by information sharing, then replenish inventory according to TPL's demand, and settle accounts in time according to the orders confirmed by retailers.

Table 3: Definitions of parameters and variables.

ORT: productivity	CRT: production fulfillment rate
RPT: replenishment rate	SRT: delivery rate
W-ROP: TPL replenishment point	D-ROP: TPL redelivery point
GIT: TPL transportation inventory	RINV: retailers' temporary inventory level

4.2. VMI&TPL-APIOBPCS Model. The definitions of parameters and variables in VMI&TPL-APIOBPCS model are given in Table 3.

According to the operational structure in Figure 8 and operational process in Figure 9, three subsystems of VMI&TPL-APIOBPCS models, including suppliers' production subsystem, TPL replenishment and delivery subsystem, and retailers' sales subsystem, are analyzed as follows.

4.2.1. Suppliers' Production Subsystem. As shown in Figure 10, suppliers' production decisions are influenced by three aspects, which are demand information (terminal customer requirements), system inventory level, and work-in-process inventory level.

Difference equations of suppliers' production operational process can be obtained according to the causality in Figure 10 as shown in the following formulas.

Work-in-process:

$$\mathrm{WIP}_t = \mathrm{WIP}_{t-1} + \mathrm{ORT}_t - \mathrm{CRT}_t. \tag{24}$$

Work-in-process deviation:

$$\mathrm{EWIP}_t = \mathrm{DWIP}_t - \mathrm{WIP}_t. \tag{25}$$

Productivity:

$$\mathrm{ORT}_t = \mathrm{AVCON}_{t-1} + \frac{\mathrm{EINV}_{t-1}}{T_i} + \frac{\mathrm{EWIP}_{t-1}}{T_w}. \tag{26}$$

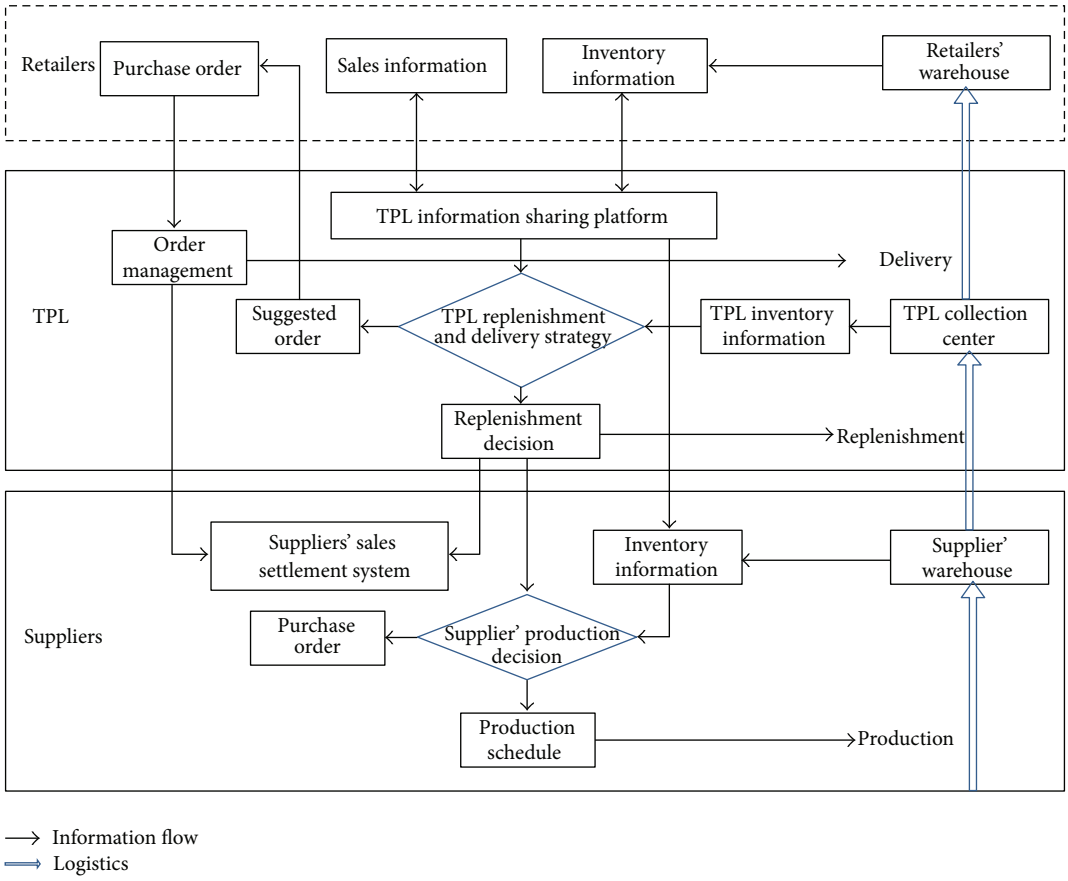

Figure 9: VMI&TPL integrated operation process.

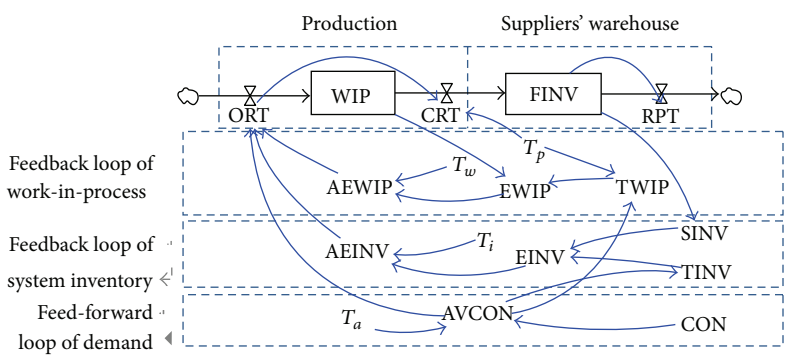

Figure 10: Causality diagram of suppliers' production subsystem.

Productivity fulfillment rate:

$$\mathrm{CRT}_t = \mathrm{ORT}_{t-Tp}. \tag{27}$$

System inventory deviation:

$$\mathrm{EINV}_t = \mathrm{TINV} - \mathrm{SINV}_t. \tag{28}$$

Target system inventory:

$$\mathrm{TINV} = \mathrm{AVCON} * \left(T_p + T_q\right). \tag{29}$$

4.2.2. TPL Replenishment and Delivery Subsystem. TPL is responsible for replenishment and delivery decisions between suppliers and retailers. On the one hand, TPL sends requests for replenishment to suppliers in order to ensure proper inventory of the TPL warehouse (TPL-W). On the other

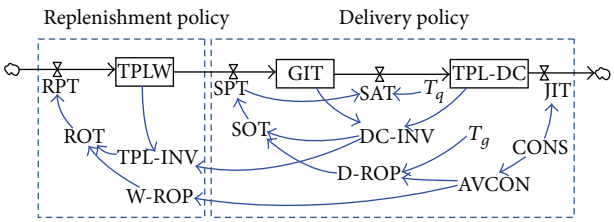

Figure 11: TPL replenishment and delivery subsystem causality diagram.

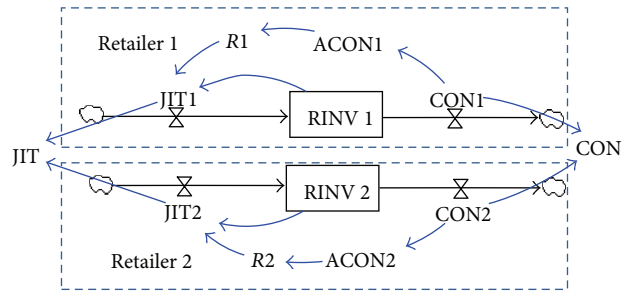

Figure 12: Retailers' sales subsystem causality diagram.

hand, TPL needs to deliver products to the TPL distribution center (TPL-DC) near retailers in order to meet retailers' demand. Figure 11 is TPL replenishment and delivery subsystem causality diagram.

Formulas (30)–(32) are difference equations of TPL replenishment strategy and formulas (33)–(37) are difference equations of TPL delivery strategy.

TPL total inventory level:

$$\text{TPL-INV}_t = \text{DC} - \text{INV}_t + \text{TPLW}_t. \tag{30}$$

TPL-W replenishment point:

$$\text{W-ROP}_t = \text{AVCON}_t \times G_w. \tag{31}$$

TPL replenishment capacity:

$$\text{ROT}_t = \begin{cases} \text{W-ROP}_t - \text{TPL-INV}_t & \text{if TPL-INV}_t < \text{W-ROP}_t \\ 0 & \text{if TPL-INV}_t \geq \text{W-ROP}_t. \end{cases} \tag{32}$$

TPL-DC redelivery point:

$$\text{D-ROP}_t = \text{AVCON}_t \times G_d. \tag{33}$$

TPL-DC inventory level:

$$\text{DC-INV}_t = \text{GIT}_t + \text{TPL-DC}_t. \tag{34}$$

TPL transportation inventory level:

$$\text{GIT}_t = \text{GIT}_{t-1} + \text{SPT}_t - \text{SAT}_t. \tag{35}$$

(T, S) delivery strategy:

$$\text{SOT}_t = \begin{cases} \text{D-ROP}_t - \text{DC-INV}_t & \text{if DC-INV}_t < \text{D-ROP}_t \\ 0 & \text{if DC-INV}_t \geq \text{D-ROP}_t. \end{cases} \tag{36}$$

(R, Q) delivery strategy:

$$\text{SOT}_t = \begin{cases} n * \text{ETQ} & \text{if DC-INV}_t < \text{D-ROP}_t \\ 0 & \text{if DC-INV}_t \geq \text{D-ROP}_t. \end{cases} \tag{37}$$

4.2.3. Retailers' Sales Subsystem. Retailers' sales subsystem is relatively simple. Though TPL can adopt JIT delivery to retailers, as service level should be improved, retailers still need to keep a small quantity of safety inventory and this inventory level is related to the demand fluctuation level. Figure 12 is retailers' sales subsystem causality diagram.

According to the retailer's sales system causal diagram, the difference equations for retailers can be obtained as follows:

$$\begin{gathered} R1 = \text{AVCON1}_t \times G_1, \qquad R2 = \text{AVCON2}_t \times G_2, \\ \text{JIT1}_t = R1_t - \text{RINV1}_t, \qquad \text{JIT2}_t = R2_t - \text{RINV2}_t, \\ \text{CONS}_t = \text{CON1}_t + \text{CON2}_t, \\ \text{JIT}_t = \text{JIT1}_t + \text{JIT2}_t. \end{gathered} \tag{38}$$

4.2.4. VMI&TPL-APIOBPCS System Dynamics Model. According to the aforementioned analysis, VMI&TPL-APIOBPCS system dynamics model is constructed as shown in Figure 13.

5. Simulation Analysis

According to the aforementioned VMI&TPL-APIOBPC system dynamics model, the two different conditions with phase-step and random demands are investigated, respectively, and parameter settings are as follows.

(1) Production subsystem parameters settings. Referring to that of Disney and Towill [24, 25], $T_a = 4$, $T_i = 14$, $T_w = 8$, and $T_p = 5$.

(2) Parameters settings of replenishment and delivery subsystem, $G_w = 8$, $G_d = 5$, ETQ = 10, and $T_q = 5$.

(3) Parameters settings of sales subsystem, $G_1 = 3$, $G_2 = 3$.

Then simulate the two models, VMI-APIOBPCS II and VMI&TPL-APIOBPCS, using Vensim, and run the test for 100 units of time (month).

5.1. Phase-Step Demand Test. The demand test functions CONS1 and CONS2 are both phase-step functions, CONS1 = STEP (5, 0) (STEP ({height}, {stime})), CONS2 = STEP (10, 10).

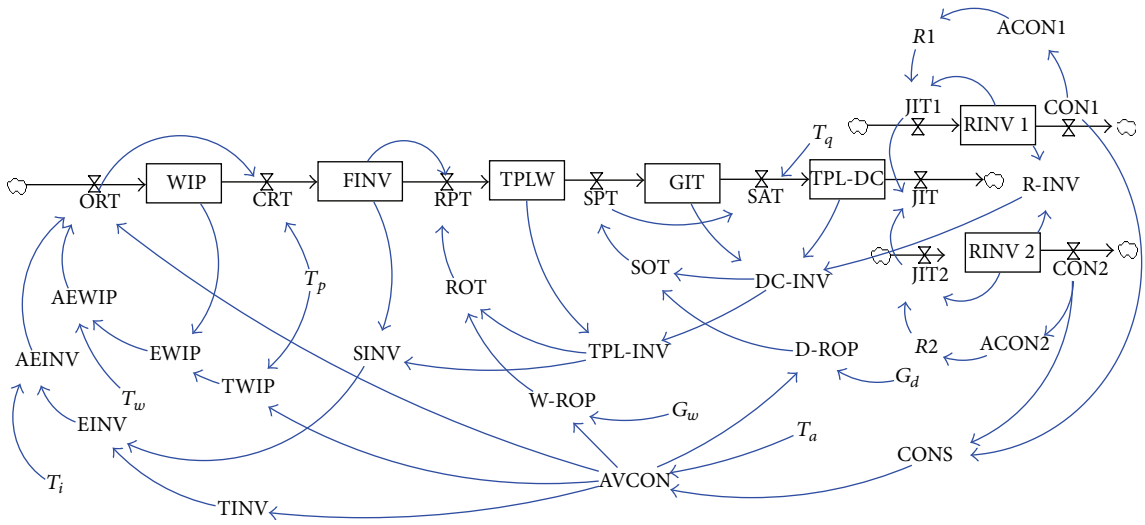

Figure 13: VMI&TPL-APIOBPCS model causality diagram.

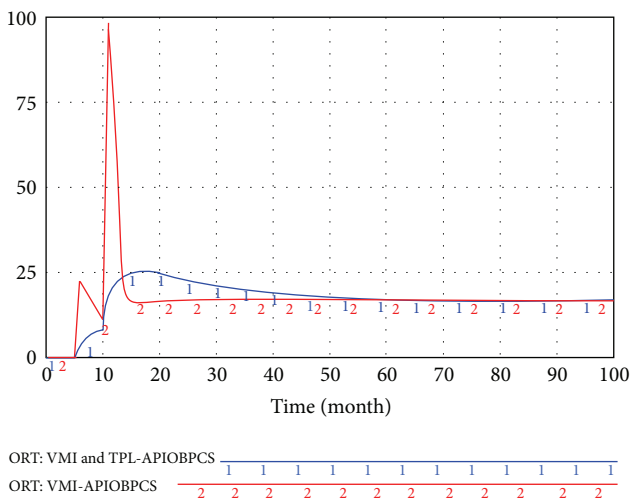

Figure 14: Comparison of production fluctuations in suppliers' production orders.

5.1.1. Fluctuations of Production Order. As shown in Figure 14, when TPL is introduced into VMI, suppliers' productivity becomes smoother, which is mostly due to the smoother system inventory levels. In the meanwhile, the response time of productivity reaching the steady-state in the VMI&TPL-APIOBPCS model is relatively longer.

5.1.2. Fluctuations of Inventory Level

(1) System inventory levels. As shown in Figure 15, in VMI&TPL-APIOBPCS model, the system inventory level significantly decreases and becomes smoother. Under phase-step demand, the two model system inventories in models gradually return to a steady-state value (TINV = 150) and are similar to the suppliers' productivity. The response time of system inventory level reaching the steady-state in VMI&TPL-APIOBPCS model is relatively longer.

(2) Suppliers' inventory level. Figure 16 is the suppliers' inventory level. After introducing TPL, suppliers' inventory level is effectively smoothed and reduced, which is because that the fluctuation of downstream replenishment batch is more consistent compared to that in VMI model; see Figure 17.

(3) Comparison of downstream overall inventory level. In VMI-APIOBPCS model, the downstream inventory mainly includes retailers' inventories. However, in the VMI&TPL-APIOBPCS model, the downstream inventory includes not only the retailers' safety stock but also the inventory in TPL warehouse and distribution center. Figure 18 is the downstream inventory level. After introducing TPL, the downstream inventory level (including the TPL) increases slightly, largely because the downstream structure is added with a new subject (TPL).

From the aforementioned inventory analysis, we can know that after introducing TPL into VMI supply chain, although supply chains add an echelon, the downstream inventory level increases slightly, but the replenishment batch becomes smoother after TPL's participation, thus effectively smoothing suppliers' productivity and reducing supplier inventory levels.

5.1.3. Comparison of Service Level. What defines service level of retailers is the ratio between the retailers' inventory level and customers' demand. Figures 19 and 20 are the service level of the two operation modes, respectively. Since the initial state of the system is zero, the initial service level of the retailers is zero, and the service level gradually improves after a period of time. In VMI operation mode, the average service levels of two retailers are 61.5% and 46.2%, respectively. In VMI&TPL

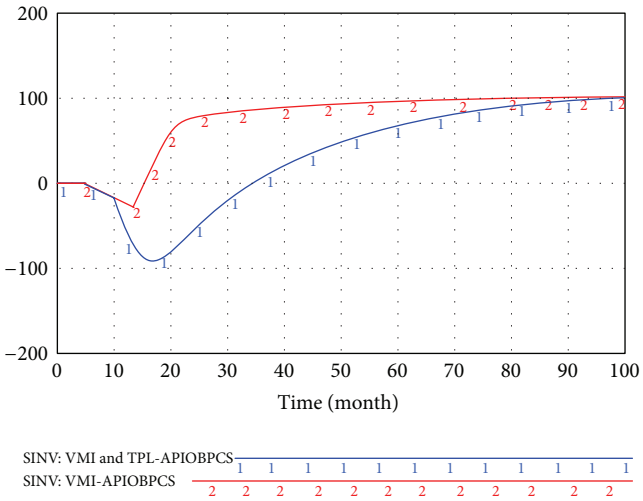

Figure 15: The comparison of system inventory level fluctuation.

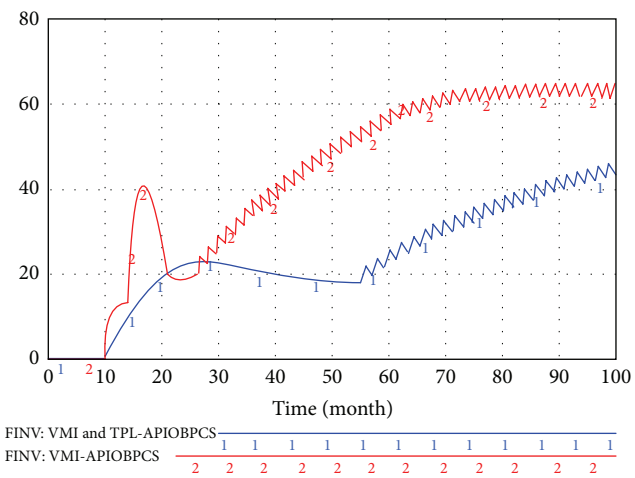

Figure 16: The comparison of supplier inventory level fluctuation.

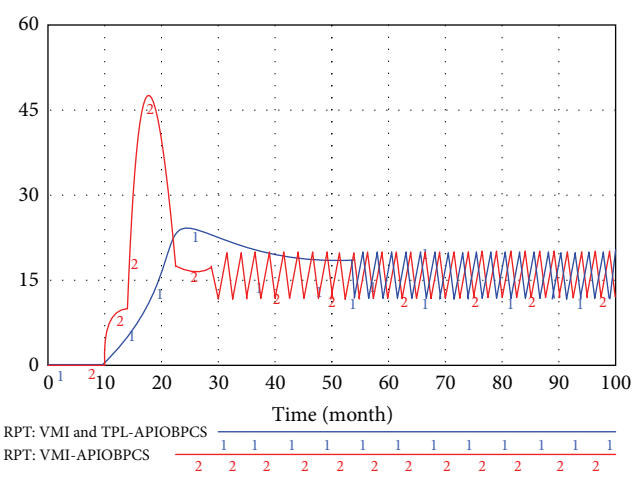

Figure 17: The comparison of replenishment batch fluctuation.

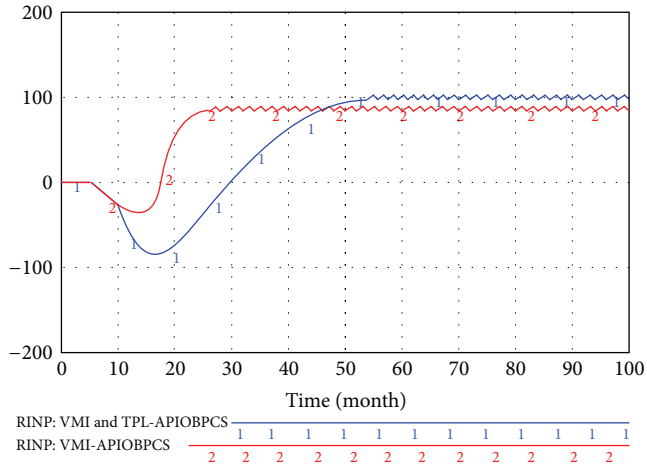

Figure 18: The comparison of downstream inventory level fluctuation.

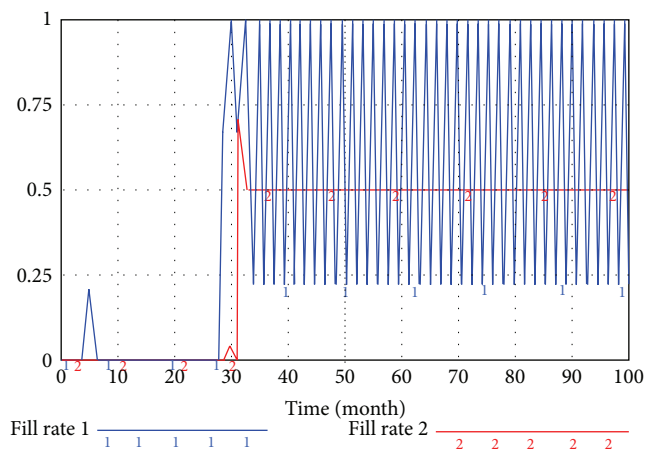

Figure 19: Service levels in the VMI operation mode.

integrated operation mode, the average service level of two retailers is 58.5%.

With the deterministic demand, service level of the first retailer falls slightly, but the second retailer's service level increases substantially, and the overall average service level of retailers improves after introducing TPL. However, the response time of the system increases, because the system service level will not be enhanced till a long period of shortage.

5.2. Random Demand Test. The demand test functions, such as CONS1 and CONS2, are both phase-step functions. CONS1 = RANDOM NORMAL (0, 10, 5, 20, 1) (RANDOM NORMAL ({min},{max}, {mean}, {stdev}, {seed})), CONS2 = RANDOM NORMAL (0, 5, 3, 10, 1).

5.2.1. Suppliers' Productivity. Similar to the phase-step demand situation, supplier's productivity is steadier apparently in VMI&TPL-APIOBPCS mode.

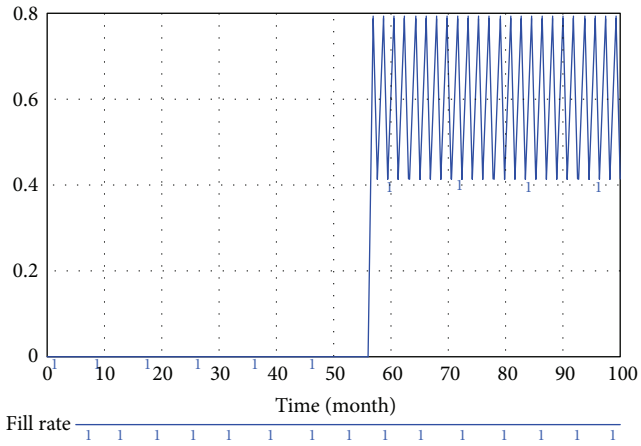

Figure 20: Service levels in the VMI&TPL integrated operation mode.

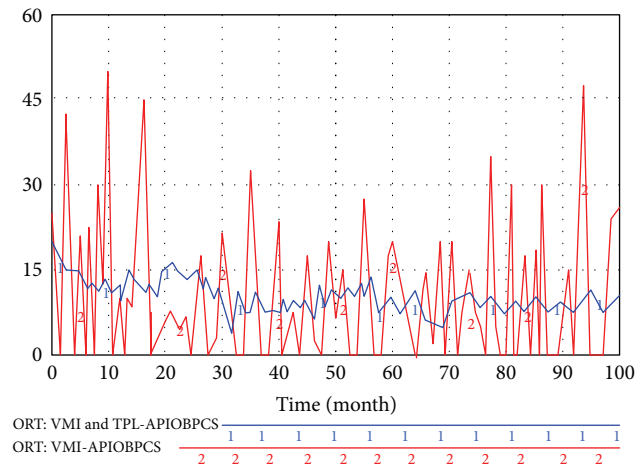

Figure 21: Fluctuation diagram of suppliers' productivity.

5.2.2. Comparison of Inventory Levels

(1) System inventory level. As Figure 22 shows, compared with the VMI model, the system inventory level in VMI&TPL-APIOBPCS model is lower and largely benefits from the dramatical decrease of suppliers' inventory level, as shown in Figure 23.

(2) Suppliers' inventory levels. As Figure 23 shows, in VMI&TPL-APIOBPCS model, suppliers' inventory levels are significantly reduced as suppliers' productivity becomes smoother, and TPL replenishment batch is steadier compared to VMI model.

(3) Downstream inventory level: As Figure 24 shows, in VMI&TPL-APIOBPCS model, the downstream inventory level increases slightly due to an additional echelon of TPL with concentrated restocking and delivery in supply chain.

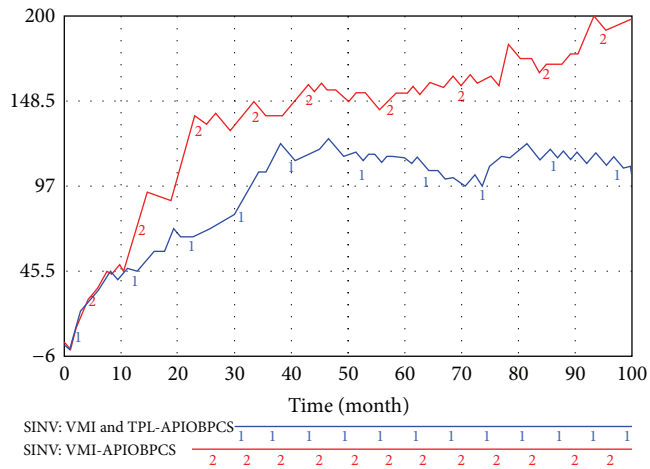

Figure 22: Fluctuation diagram of system inventory level.

5.2.3. Comparison of Service Level. Under the random demand and in the operation mode of VMI the first retailer's average service level is 78.6% and the second retailer's average service level is 40.4%.

In the VMI&TPL integrated operation mode, the overall service levels of the two retailers are 94.8%. It is clear that after introducing TPL the two service levels of retailers are improved under random demand; this benefits from the risk-sharing effect in centralized inventory after the introduction of TPL.

5.3. Simulation Discussion. The complex simulations under phase-step demand and random demand show the following results as this study summarized from the follow-up interviews in practice.

First, VMI&TPL integrated operation mode can smoothen suppliers' productivity under both phase-step demand and random demand as TPL introduced into VMI makes the whole system inventory levels smoother (see Figure 21). However, the response time of reaching the steady-state in the VMI&TPL-APIOBPCS model is relatively

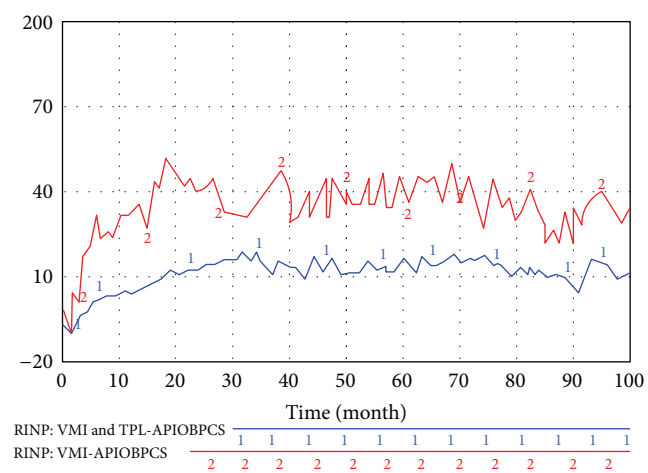

Figure 23: Fluctuation diagram of suppliers' inventory level.

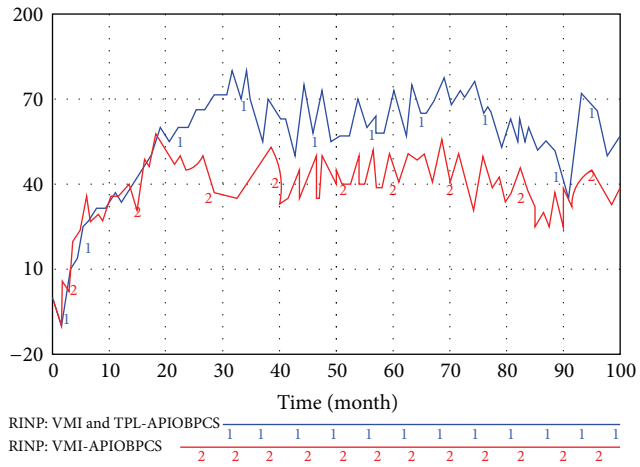

Figure 24: Fluctuation diagram of lower inventory level.

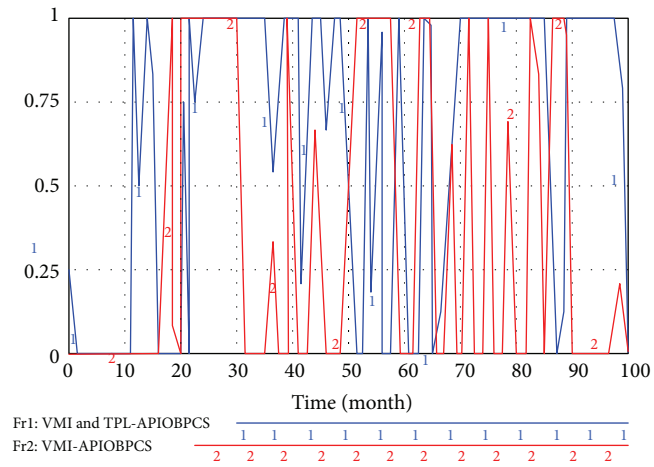

Figure 25: The service level of VMI operational model under random demand.

longer, which may be caused by TPL centralized operation. Particularly, the fluctuation of suppliers' productivity under random demand is more stable than under phase-step demand, as in practice TPL places orders from suppliers periodically, and its response to fluctuation is slow.

Second, VMI&TPL integrated operation mode can reduce the system inventory level significantly. Similarly, the response time of system inventory level reaching the steady-state in VMI&TPL-APIOBPCS model is relatively longer. Compared with under phase-step demand, system inventory level can be lower under random demand. This illustrates that the TPL centralized replenishment has a scale of economics and much risk pooling effects which can decrease the whole system inventory level [37]. Besides, suppliers' inventory level is effectively smoothed and reduced since downstream replenishment batch is more consistent and scale of economics compared to those in VMI model. However, in the VMI&TPL-APIOBPCS model, the downstream inventory includes not only the retailers' safety stock but also the inventory in TPL warehouse and distribution center. As a result, the downstream inventory level rises slightly after introducing TPL.

Third, VMI&TPL integrated operation mode can improve average service level. Under phase-step demand, service level of the first retailer falls slightly, but the second retailer's service level increases substantially. As a contrast, service level of two retailers increases under random demand. On the whole, the service level under random demand is improved significantly than under phase-step demand (see Figures 25 and 26). These may be caused by risk pooling effect, especially under random demand.

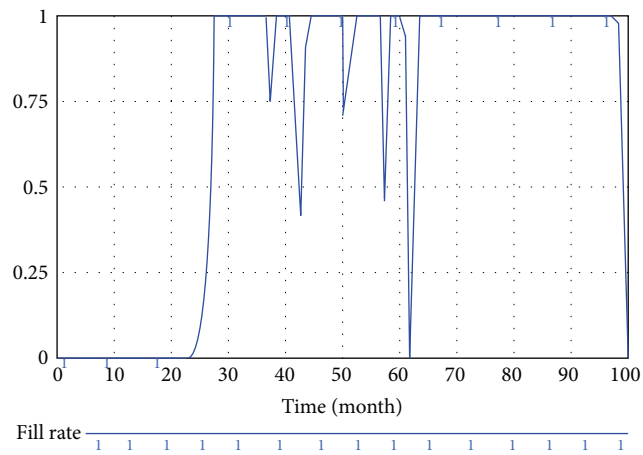

Figure 26: The service level of VMI&TPL integrated operational model under random demand.

6. Conclusion

This paper constructs the VMI&TPL-APIOBPCS model after introducing TPL into VMI distribution based on VMI-APIOBPCS system dynamics model. The system performance of VMI&TPL integrated supply chain under phase-step and random demand is considered. The simulation analysis shows that though the supply chain is turned into a three-echelon structure from a two-echelon one TPL can effectively smooth the replenishment and delivery quantity between suppliers and retailers by goods collection, thus dramatically reducing the inventory level of the suppliers and the whole system, effectively smoothing the production rhythm of suppliers and improving the service level of customers.

Although system dynamics method can describe and simulate VMI&TPL integrated operational model, it lacks the optimization of TPL replenishment and delivery policy in this operation mode. As a result, it is necessary to optimize the TPL replenishment and delivery policy under various demands using mathematical programming and optimization theories, so as to enhance the study of VMI&TPL integrated operational model further.

Acknowledgments

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Research Article

A Location-Allocation Model for Seaport-Dry Port System Optimization

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Seaports participate in hinterland economic development through partnerships with dry ports, and the combined seaport-dry port network serves as the backbone of regional logistics. This paper constructs a location-allocation model for the regional seaport-dry port network optimization problem and develops a greedy algorithm and a genetic algorithm to obtain its solution. This model is applicable to situations under which the geographic distribution of demand is known. A case study involving configuration of dry ports near the west bank of the Taiwan Strait is conducted, and the model is successfully applied.

1. Introduction

Rapid development of seaports and intermodal transportation systems under integrated planning has made it necessary for seaports to dynamically assess what constitutes their hinterlands, and the scramble for hinterlands by seaports is heating up. On the other hand, it is increasingly recognized by hinterlands that seaports guide and support regional economic development, and there is a growing need to perform in hinterland locations seaports' functions except ship loading and unloading. The interactions of these two driving forces have induced rapid development of dry ports as both a means by which seaports vie for hinterland access and a means by which hinterlands stimulate economic growth. Logistics networks, each including a group of seaports and some dry ports, are becoming backbones of regional goods movement. At the end of 2011, there were over 100 dry ports built or being built in China, with the Port of Tianjin leading the development of more than 20 of them. There were also a large number of road and rail transportation hubs which were in many aspects similar to dry ports. The development of dry ports can mitigate problems caused by constraints related to land and others that limit seaports' growth. Dry ports can also coordinate the operation of the port supply chain and support regional economic development. Consequently, dry ports are changing the dynamics of interaction between seaports and hinterlands. This paper studies the location of dry ports from the perspective of seaport-hinterland interaction and optimizes the configuration of the seaport-dry port system, taking into consideration the relationships between dry ports, seaports, and the regional logistics system.

On the evolution of a port, Bird [1] developed the Anyport model describing how port infrastructures develop over time and space and how the relationship between ports and their host cities evolves. Three major steps of port development were identified: setting, expansion, and specialization. Based on his study of East African ports, Hoyle [2] amended the original Anyport model and proposed a six-stage model of port development. Notteboom and Rodrigue [3] proposed that adding to the three stages in Bird's model is the stage of regionalization, during which seaports achieve development mainly through inland expansion. Rodrigue and Notteboom [4] further extended the concept of inland to include both hinterland and foreland. CEMT [5] recognized that hinterland resources would inevitably become indispensable for seaports engaging in intense market competition.

The rapid development of multimodal transportation has driven the movement of containers within inland regions.

Since the early 1980s, operators of containerized transport have built sophisticated networks of inland container transport, and major nodes in these networks become the prototype of dry ports. Roso et al. [6] pointed out that on the backdrop of increased size of container vessels, dry ports play a key role in connecting seaports to the hinterland as they help relieve congestion at seaports while providing the hinterland with improved access to containerized ocean transport. As such, the location of dry ports became an import issue of research. Heaver et al. [7, 8], van Arjen Klink and van den Berg [9], Notteboom [10], Notteboom and Winkelmans [11], and Robinson [12] studied the relationship between ports and dry ports and further proposed different spatial configuration of dry ports. Yang [13] applied the method of multicriteria decision making to the problem of locating dry ports in the state of Texas, USA. Xu [14] developed a discrete choice model for locating an inland container depot, with the objective of profit maximization. In China, Xi et al. [15] proposed developing a dry port in the midwestern city of Xi'an. Guan [16] analyzed problems facing the development of dry ports in China. Cai and Chen [17] proposed 5 codevelopment patterns for seaports and dry ports. Wang [18] compared dry ports with container yards and constructed a discrete choice model for locating dry ports. Ma [19] developed a cellular automata program for locating dry ports that serve the Port of Tianjin. However, none of the existing research took into consideration the complex interaction between the government, the seaports, and the shippers when studying the problem of locating dry ports.

Key factors of a seaport group include port location, capacity, origin of shipments, and the cooperative and competitive relationships. If these factors are known, the task of optimizing the regional seaport-dry port system is to determine for the dynamic hinterland the number of dry ports, their locations and capacities, and the relationships among themselves as well as between them and the seaports. The regional configuration of dry ports is constrained by available candidate locations, transport access to these locations, and the shipping demand within the zone of influence of each location. The demand-supply relationship and the choice behavior of agents (seaports, dry ports, shippers, and carriers) need to be reasonably modeled. This research develops a location-allocation model for seaport-dry port system optimization, characterized by a probabilistic choice for shippers' use of dry ports and a partnership between seaports and dry ports. The research provides both a methodological approach for decision making and new insights on the relationship between seaports and the hinterland.

2. The Location-Allocation Model for Seaport-Dry Port System Optimization

For a regional seaport-dry port system focusing on exporting freight generated in the hinterland through seaports to the outside, there are two essential types of elements: nodes and links. Nodes include seaports, existing and planned dry ports, and hinterland origins of freight. Links connect the origins of freight to seaports, either directly or through dry-ports. This is shown in Figure 1.

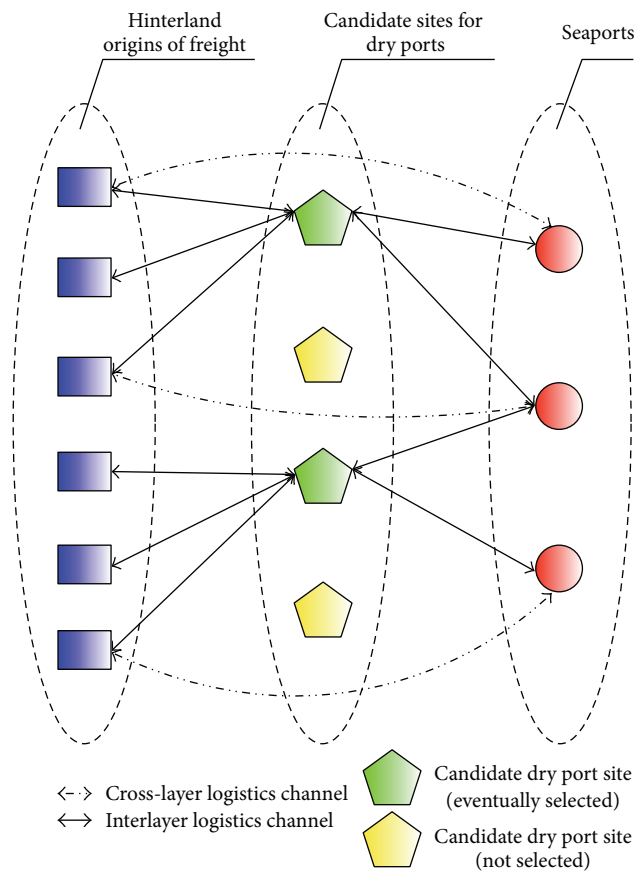


Figure 1: Network flow on a regional seaport-dry port system.

2.1. Model Assumptions. For the abovementioned export-oriented regional seaport-dry port system with a single type of freight, the process of location-allocation is as follows: (1) the government constructs of dry ports and designates for each dry port the seaports it collaborates with; (2) shippers choose to route freight to a seaport, either directly or through a dry port. The government's action in step (1) must anticipate the choice of shippers in step (2), in order to minimize the overall regional logistics cost. Thus we have a location-allocation model, in which the government determines the number and the locations of dry ports, taking into consideration freight allocation by shippers. The objective is to minimize the regional logistics cost.

With regard to the relationship between dry ports and seaports, two scenarios are considered: (1) a dry port can partner with and send freight to any number of seaports and (2) a dry port can send freight to only one seaport. In both cases, a seaport can receive freight from multiple dry ports.

In addition, the following assumptions are made.

(a) The locations of the nodes (freight origins, seaports, and candidate sites for dry ports) are predetermined and known.

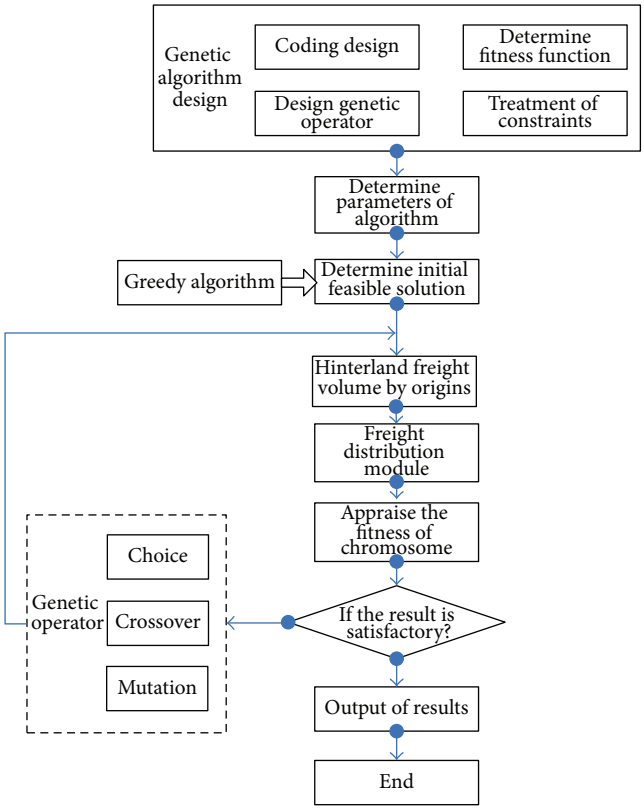


Figure 2: Process of the genetic algorithm.

(b) The transport links between the above nodes are predetermined and known.

(c) The freight volume originating from each hinterland origin is known, and freight must be exported through one of the seaports.

(d) The overall regional logistics cost includes the annual cost of transport, the amortized cost of setting up the dry ports, the cost of maintaining the transport links between dry ports and seaports, and the cost of maintaining the infrastructure at seaports.

(e) The unit transport cost on a link and the cost of setting up a dry port are not dependent on the freight volume, but the cost of maintaining a link or maintaining seaport infrastructure is dependent on the freight volume on the link or through the seaport.

(f) Any freight passes through at most one dry port.

2.2. Model Formulation

2.2.1. When a Dry Port Can Be Shared by Seaports. The programming model we develop consists of a model of system logistics cost minimization though determining from candidate dry port sites a subset to use, picking the collaborating seaports for each dry port, and allocating freight to different routes.

The model is as follows:

$$\min Z\left(\psi_k, \varphi_{jk}\right), \tag{1}$$

where

$$Z = \sum_{i,j}\left\{\left[\sum_{k\geq 1}\psi_k\varphi_{jk}Q_{ijk}\left(C_{i*k}l_{i*k} + \frac{C_{*jk}l_{*jk}}{m_j}\right)\right] + \frac{Q_{ij0}C_{ij0}l_{ij0}}{m_j}\right\} + \sum_k \psi_k b_k + \sum_{j,k}\varphi_{jk}\left(b_{jk} + a_1 Q_{jk}^{\theta_1}\right) + \sum_j a_2 S_j^{\theta_2}, \tag{2}$$

s.t.

$$\psi_k \in \{0,1\}, \quad \forall k \geq 1, \tag{3}$$

$$\varphi_{jk} \in \{0,1\}, \quad \forall j, \forall k \geq 1, \tag{4}$$

$$Q_{jk} = \sum_i Q_{ijk}, \tag{5}$$

$$Q_k = \sum_j Q_{jk}, \tag{6}$$

$$S_j = \sum_k Q_{jk}, \tag{7}$$

$$P_{ijk} = \frac{e^{V_{ijk}}}{\sum_{jk} e^{V_{ijk}}}, \tag{8}$$

$$Q_{ijk} = D_i P_{ijk}. \tag{9}$$

The subscripts i, j, and k denote freight origins, seaports, and dry port candidate sites, respectively.

In the objective function, $\sum_{k\geq 1}\psi_k\varphi_{jk}Q_{ijk}(C_{i*k}l_{i*k} + C_{*jk}l_{*jk}/m_j) + Q_{ij0}C_{ij0}l_{ij0}/m_j$ is the transport cost for freight originating from i and destined to j. $\sum_k \psi_k b_k$ is the amortized cost of setting up the dry ports, $\sum_{j,k}\varphi_{jk}(b_{jk}+a_1Q_{jk}^{\theta_1})$ is the cost for maintaining the transport links between all links from dry ports to seaports, and $\sum_j a_2 S_j^{\theta_2}$ is the cost of maintaining the infrastructure at seaports.

The decision variables are ψ_k and φ_{jk}. ψ_k specifies if candidate site k is used as a dry port. $\psi_k = 1$ if site k is used and 0 if not. φ_{jk} specifies if port j has a partnership with dry port k so j could receive freight from k. $\varphi_{jk} = 1$ if there is a partnership between j and k and 0 if not. Constraints (3) and (4) specify feasible values of ψ_k and φ_{jk}.

Q_{ijk} is the flow of freight from hinterland origin i through dry port k to seaport j. When $k = 0$, Q_{ij0} is the flow of freight from hinterland origin i directly to seaport j. The value of each Q_{ijk} is determined in the expression (8) and (9). D_i is the total volume of freight generated at hinterland origin i, and P_{ijk} is the percentage of freight generated at i that would be routed through dry port k to seaport j. $V_{ijk} = -R_{ijk}$, where R_{ijk} is the transport cost per unit volume unit distance of moving freight from source i to seaport j through dry port k.

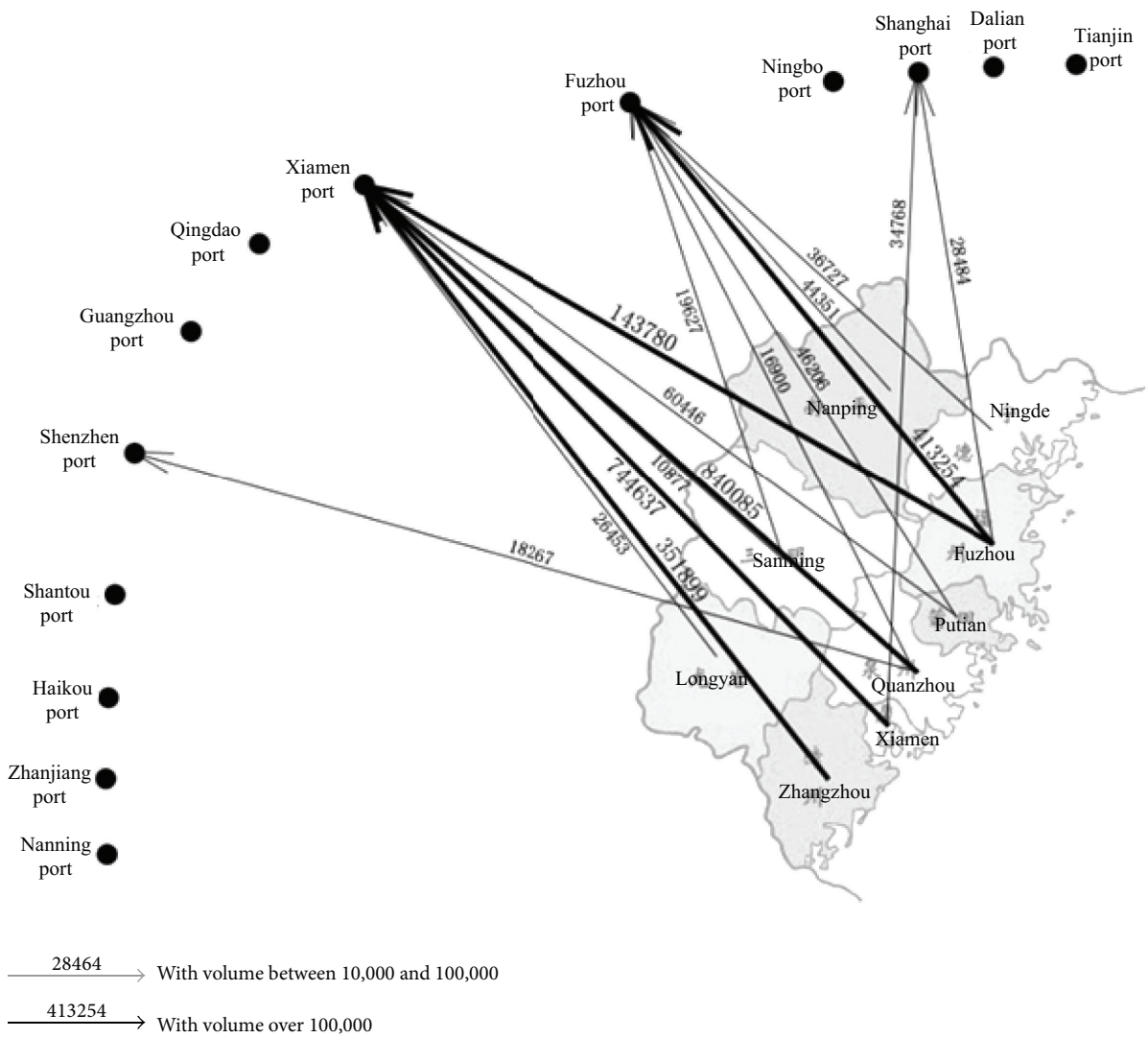


Figure 3: Export freight volumes between cities in Fujian province and various seaports.

Available dry ports and dry port-seaport partnerships given by the values of ψ_k and φ_{jk} are used to define feasible routes.

Q_{jk} is the volume of freight sent by dry port k to seaport j. Q_k is the total volume of freight routed through dry port k. S_j is the total volume of freight sent to seaport j. Constrains (5), (6), and (7) specify these relationships.

l_{i*k} is the transport distance between hinterland origin i and dry port k. l_{*jk} is the transport distance between dry port k and seaport j. l_{ij0} is the direct transport distance between hinterland origin i and seaport j. C_{i*k}, C_{*jk}, and C_{ij0} are the transport cost per unit volume unit distance corresponding to the above three types of distances, respectively. m_j is a parameter indicating seaport j's attractiveness to shippers, as shippers discount unit transport costs to seaports differently due to perceived differences in the levels of service at seaports. A more attractive seaport has a larger m_j.

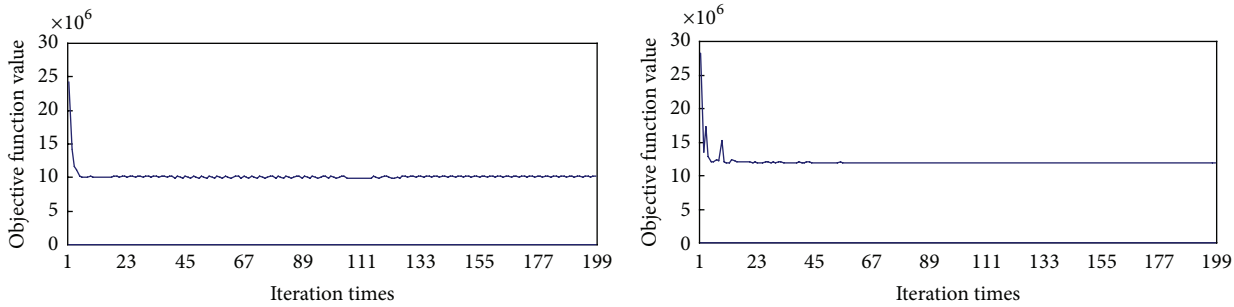
Besides transport costs, the system logistics cost contains also handling costs and the costs related to setting up dry ports and their partnerships. As stated in assumption 7, unit handling cost at dry port k is dependent on Q_{jk}, the freight volume it sends to each seaport, and unit handling cost at a seaport j is dependent on S_j, the overall freight volume of seaport j. a_1, a_2, θ_1, and θ_2 are cost parameters. b_k is the cost of amortized cost of setting up a dry port at candidate site k, and b_{jk} is the cost of maintaining a partnership between dry port k and seaport j. The values of these parameters can be established from expert survey or estimated from field data.

2.2.2. When Each Dry Port Is Dedicated to a Seaport. The model remains essentially the same, with one additional constraint added to the model:

$$\sum_j \varphi_{jk} = \psi_k. \tag{10}$$

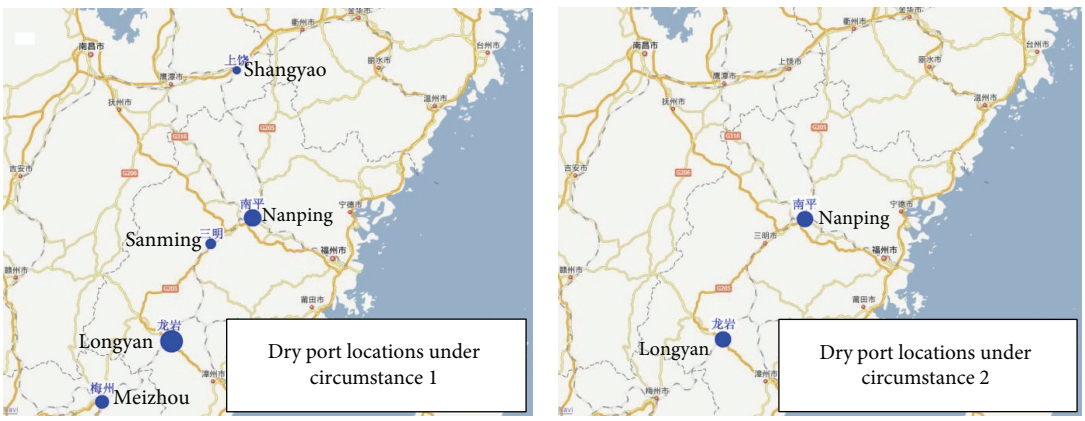
3. Solution Algorithms

The objective of regional seaport-dry port system optimization is to determine the quantity, size, and location of



(a) Shared (b) Dedicated

Figure 4: The iteration process.



(a) Shared (b) Dedicated

Figure 5: Dry port locations.

the dry ports and the transport links between them and seaports so as to minimize region-wide logistics cost. In the programming model formulated above, the objective function is nonlinear, and the decision variables must take binary values. Thus common optimization techniques do not readily apply. Instead, a greedy algorithm and a genetic algorithm are combined to solve the optimization problem.

3.1. Determination of an Initial Feasible Solution with a Greedy Algorithm. A greedy algorithm is adopted to obtain a good feasible solution of the model. The basic idea of the algorithm is to start with the full network (i.e., setting up a dry port at each candidate location and linking each dry port to all seaports in the case of shared dry port or to its nearest seaport in the case of dedicated dry port) and then take out links one at a time to examine if the system logistics cost can be reduced, until no link can be taken out to further reduce the system logistics cost.

When each dry port can be shared by seaports, the steps are the following.

Step 1. Set up m dry ports, where m is the total number of candidate sites for dry ports, and link each dry port with n seaports, where n is the total number of seaports. Denote the current set of dry ports as SM and the set of links between dry ports and seaports as SN. For dry port k in SM, denote the set of links in SN that emanates from dry port k as Sk. The links in Sk are ordered by unit transport cost on the link.

Step 2. Allocate the freight volume generated from hinterland origins to routes according to the expression (8) and (9), and calculate system logistics cost according to the model.

Table 1: Seaports and their attractiveness to shippers.

No.	Name of seaport	Attractiveness to shippers
1	Tianjin	4.5
2	Dalian	4.4
3	Shanghai	10.0
4	Ningbo	4.8
5	Fuzhou	4.0
6	Xiamen	4.3
7	Qingdao	4.4
8	Guangzhou	4.6
9	Shenzhen	4.3
10	Shantou	3.0
11	Haikou	2.0
12	Zhanjiang	3.0
13	Nanning	1.0

Table 2: Hinterland freight origins in Fujian province.

No.	Name of freight origin	Demand volume (tons/year)
1	Fuzhou	597646
2	Xiamen	796707
3	Putian	117423
4	Sanming	33216
5	Quanzhou	886176
6	Zhangzhou	363424
7	Nanping	52748
8	Ningde	49893
9	Longyan	30222

Table 3: Candidate dry ports.

No.	Name of dry port
1	Nanchang
2	Changsha
3	Ganzhou
4	Nanping
5	Sanming
6	Longyan
7	Jiujiang
8	Shangrao
9	Meizhou
10	Yingtan

Step 3. Initialize $k = 0$; denote r as the number of members in set SM.

Step 4. Let $k = k + 1$.

Step 5. Start from the link in Sk with the highest unit transport cost; examine the links in Sk one by one to see if any can be removed. If yes, remove that link from Sk and SN.

Step 6. Examine if the last step removed any link. If yes, go to Step 5.

Step 7. If Sk is empty, mark dry port k from exclusion.

Step 8. If $k < r$, go to Step 4.

Step 9. Remove all dry ports marked for exclusion from SM.

Step 10. Examine if from Step 4 to Step 9 removed any link from SN. If yes, go to Step 3.

Step 11. What remains in SM and SN is an initial feasible solution for the programming problem.

3.2. Genetic Algorithm for the Location-Allocation Model for Seaport-Dry Port System Optimization. With the genetic algorithm to optimize the regional seaport-dry port system, new solution is obtained through random transformation of current solutions. The basic idea is as follows: decide for the decision variables a coding scheme; then generate the initial population, of which individuals shall be corresponding to different dry port-seaport system configurations; use the expression (8) and (9) to obtain network flows and corresponding system logistics costs to evaluate the fitness of each individual. Conduct operations of choice, crossover, and genetic mutation to this population, and after several generations, the algorithm converges, and the resulting system configuration would be adopted as the optimal solution to the programming problem. In this research, real number coding is adopted for coding and the objective function of the problem is adopted as the fitness function. Genetic operations are conducted with the roulette wheel selection based on "ranking," and single point arithmetic crossover is adopted for crossover operations. Penalty factors are added to the fitness function to account for constraint violations.

The detailed process is shown in Figure 2; the steps of the algorithm are shown next.

Step 1 (set the parameters). Set the population size "pop_size," mutation rate p_m, the crossover rate p_c, the maximum number of iterations "Gen," and the initial generation $t = 0$.

Step 2 (initialization). The initial population $P(t)$ with the size of "pop_size" is generated.

Step 3. Apply the allocation expression (8) and (9) to the chromosome in population $P(t)$ to obtain freight flows on the network.

Step 4. Apply the model's objective function to obtain the fitness of the scheme.

Step 5 (conduct genetic operations). Conduct hybrid and mutation operations with specified p_m and p_c values to generate next generation $C(t)$.

Step 6. Apply the allocation expression (8) and (9) to the chromosome in population $C(t)$ to obtain freight flows on the network.

Step 7. Calculate the fitness value of $C(t)$.

Step 8 (execute choice operation). In accordance with the roulette wheel method and the elite preservation strategy, obtain $P(t + 1)$ of population size "pop_size" from $P(t)$ and $C(t)$.

Step 9 (termination conditions). If $t <$ Gen, then $t = t + 1$; return to Step 5 to continue with the evolution operation process; if $t =$ Gen, terminate the algorithm and output the solution.

4. Seaport-Dry Port System Optimization for Fujian Province

Two seaports on the coast of Fujian province, Xiamen and Fuzhou, compete but also cooperate with other major Chinese coastal seaports to serve the hinterland regions of

Table 4: Export volume (in tons/year) between cities in Fujian province and various seaports.

Freight origin	Seaport												
	Tianjin	Dalian	Shanghai	Ningbo	Fuzhou	Xiamen	Qingdao	Guangzhou	Shenzhen	Shantou	Haikou	Zhanjiang	Nanning
Fuzhou	327	124	28484	1117	413254	143780	1546	367	7338	58	11	0	1240
Xiamen	945	186	34768	2775	460	744637	3681	349	8242	8	5	2	650
Putian	28	1	4048	320	46206	60446	983	5	5351	1	0	0	36
Sanming	177	1	882	100	19627	10877	80	0	974	0	0	0	497
Quanzhou	761	141	2932	1430	16900	840085	1047	292	18267	106	2	0	4213
Zhangzhou	126	42	3645	219	268	351899	619	330	3793	2188	2	16	277
Nanping	8	0	3387	278	44351	3657	74	9	829	1	0	0	153
Ningde	290	56	242	1878	36727	9574	115	5	759	167	15	1	64
Longyan	0	0	2148	7	43	26453	15	2	1466	41	0	0	46

Table 5: Unit transport cost (in Yuan/ton) between cities in Fujian province and alternative dry ports.

Freight origin	Dry port									
	Nanchang	Changsha	Ganzhou	Nanping	Sanming	Longyan	Jiujiang	Shangrao	Meizhou	Yingtan
Fuzhou	49	73	50	20	25	32	56	40	44	40
Xiamen	60	71	36	33	27	18	69	62	27	50
Putian	55	81	52	31	26	24	64	49	42	49
Sanming	33	59	32	83	0	26	42	33	26	27
Quanzhou	53	75	41	27	29	24	62	54	35	47
Zhangzhou	52	66	32	33	27	109	61	62	26	46
Nanping	35	61	38	0	83	33	44	31	32	29
Ningde	57	83	60	30	28	38	66	33	54	51
Longyan	44	58	33	25	26	1	53	47	25	38

the Fujian province. For this study, 13 seaports are considered: Tianjin, Dalian Shanghai, Ningbo, Fuzhou, Xiamen, Qingdao, Guangzhou, Shenzhen, Shantou, Haikou, Zhanjiang, and Nanning, as shown in Table 1. There are 9 hinterland freight origins in Fujian province: Fuzhou, Xiamen, Putian, Sanming, Quanzhou, Zhangzhou, Nanping, Ningde, and Longyan, as shown in Table 2. There are 10 candidate sites for dry ports: Nanchang, Changsha, Sanming, Ganzhou, Longyan, Nanping, Jiujiang, Meizhou, Yingtan, and Shangrao, as shown in Table 3. Annual export freight volumes from the hinterland origins are aggregated from customs data on export freight volumes between cities in Fujian province and the seaports (see Table 4 and Figure 3). The minimum unit transport costs between these locations are shown in Tables 5, 6, and 7. These units costs are in the unit of Yuan/ton, and they already account for 3 factors in the objective function: the cost per ton per unit distance, the transport distance, and the seaport attractiveness factor.

We look at both the case of shared dry ports and the case of dedicated dry ports. The values of other parameters in the objective function (i.e., a_1, a_2, θ_1, θ_2, B_k, and B_{jk}) used in this study are taken from a combination of expert survey and estimation of field data. Due to the economy of scale, θ_1 and θ_2 are usually greater than 0 but less than 1.

(1) When the Dry Port Is Jointly Developed and Is Shared by Multiple Seaports. When no dry port is constructed, the total cost is 3.30839873×10^7. The initial solution of the model is obtained with the greedy algorithm. The second column of Table 8 shows the initial solution, under which the total cost is 2.4182488×10^7. For the genetic algorithm, we set the selection probability for the genetic operator at 0.8, the crossover probability at 0.5, and the mutation probability at 0.01. The maximum number of iterations is 200; the iteration process is shown in Figure 4(a), and it converges to the optimum solution after 200 iterations. The fourth column of Table 8 shows the optimal solution; Figure 5(a) shows the resulting dry ports on the map. Under the optimal solution, the total cost is 1.000304×10^7.

(2) When the Dry Port Is Dedicated to a Seaport. When no dry port is constructed, the total cost is 3.30839873×10^7. The initial solution of the model is obtained with the greedy algorithm. The third column of Table 8 shows the initial solution, under which the total cost is 2.827592×10^7. For the genetic algorithm, we set the selection probability for the genetic operator at 0.8, the crossover probability at 0.5, and the mutation probability at 0.01. The maximum number of iterations is 200; the iteration process is shown in Figure 4(b), and it converges to the optimum solution after 200 iterations. The fifth column of Table 8 shows the optimal solution. Figure 5(b) shows the resulting dry ports on the map. Under the optimal solution, the total cost is 1.194766×10^7.

From the above, we can see that development of dry ports dramatically reduces system logistics cost for hinterland origins in Fujian province.

Table 6: Unit transport cost (in Yuan/ton) between cities in Fujian province and various seaports.

Freight origin	Seaport												
	Tianjin	Dalian	Shanghai	Ningbo	Fuzhou	Xiamen	Qingdao	Guangzhou	Shenzhen	Shantou	Haikou	Zhanjiang	Nanning
Fuzhou	152	239	66	45	1	30	187	87	67	41	119	78	130
Xiamen	160	226	83	65	29	1	138	57	48	22	100	142	104
Putian	160	214	70	53	115	20	126	68	59	33	111	143	70
Sanming	137	206	66	58	25	27	117	57	55	38	103	130	105
Quanzhou	157	220	76	58	20	92	132	61	53	26	109	147	109
Zhangzhou	156	220	85	68	33	66	137	54	46	20	98	138	102
Nanping	139	202	60	52	20	33	113	63	61	44	109	132	111
Ningde	136	199	55	38	102	28	111	85	73	47	125	112	129
Longyan	148	217	84	76	32	18	128	49	46	28	96	82	96

Table 7: Unit transport cost (in Yuan/ton) between alternative dry ports and various seaports.

Dry port	Seaport												
	Tianjin	Dalian	Shanghai	Ningbo	Fuzhou	Xiamen	Qingdao	Guangzhou	Shenzhen	Shantou	Haikou	Zhanjiang	Nanning
Nanchang	109	181	62	53	47	60	124	82	74	76	140	111	104
Changsha	136	203	94	91	78	96	130	57	68	99	120	91	78
Ganzhou	148	222	93	91	62	44	155	44	41	43	107	69	108
Nanping	139	202	66	52	20	33	113	63	61	44	109	97	111
Sanming	156	206	75	67	25	27	118	57	55	38	103	91	105
Longyan	148	217	84	76	32	18	128	49	46	27	96	82	96
Jiujiang	100	171	55	54	56	69	83	75	75	73	121	108	108
Shangrao	120	183	41	33	40	59	96	80	80	66	126	113	113
Meizhou	156	225	92	83	44	30	136	32	29	17	78	65	79
Yingtan	113	182	49	41	41	54	93	73	73	58	119	106	106

Table 8: Initial solution and optimization result after 200 iterations.

Dry port	Dry port throughput (shared) (the initial solution)	Dry port throughput (dedicated) (the initial solution)	Dry port throughput (shared) (the optimal solution)	Dry port throughput (dedicated) (the optimal solution)
Nanchang	0	0	0	0
Changsha	0	0	0	0
Ganzhou	0	0	0	0
Nanping	97818.44531	105605.3281	412067.5	891146.75
Sanming	63477.57422	127935.3594	31191.81445	0
Longyan	387477.7813	378427.6563	1083868.75	878881.5
Jiujiang	0	0	0	0
Shangrao	1015.576355	0	72.5763855	0
Meizhou	10148.79297	0.000772247	364213.125	0
Yingtan	0	0	0	0

5. Conclusion

With the increased competition between the regional seaport-dry port networks, optimizing system configuration has attracted attention of many researchers. The regional seaport-dry port system is a complex system. By focusing on the relationship between seaports and dry ports, this paper has developed a location-allocation model for regional seaport-dry port system optimization and has proposed an efficient solution method for the programming problem. This paper

provides justifications for developing dry ports at strategic locations and lays a foundation for future research on regional resource integration.

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Research Article

An Inventory Controlled Supply Chain Model Based on Improved BP Neural Network

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Inventory control is a key factor for reducing supply chain cost and increasing customer satisfaction. However, prediction of inventory level is a challenging task for managers. As one of the widely used techniques for inventory control, standard BP neural network has such problems as low convergence rate and poor prediction accuracy. Aiming at these problems, a new fast convergent BP neural network model for predicting inventory level is developed in this paper. By adding an error offset, this paper deduces the new chain propagation rule and the new weight formula. This paper also applies the improved BP neural network model to predict the inventory level of an automotive parts company. The results show that the improved algorithm not only significantly exceeds the standard algorithm but also outperforms some other improved BP algorithms both on convergence rate and prediction accuracy.

1. Introduction

Inventory control is one of the key topics for supply chain management. Usually inventory takes the form of raw material, work in process (WIP) products, semifinished products, or finished products. Inventory cost is the main cost for supply chain management. A drop of just several percentage points of inventory cost can greatly increase the profits of the whole supply chain. In addition, sound inventory level can prevent shortage of material, maintain the continuity of the production process, and quickly satisfy customers' demand. Thereby, exploring the optimal inventory level is very necessary and valuable for supply chain management.

To date, the following inventory control problems need to be addressed [1, 2].

(1) There are highly nonlinear models which are hard to process.

(2) There are qualitative indicators which are hard to deal with.

(3) The unchangeable indicators of inventory control lack self-adaptation.

(4) Information of inventory control models is always indirect and the collection of information is time-consuming and of low efficiency.

(5) Inventory control models always ignore the influence of uncertain factors, such as lead time, transportation conditions, and change of demand.

Considering the above problems, traditional inventory control theory is hard to meet the requirement posed by the new environment. Thanks to the uncertain feature of inventory control and the strengths of neural network in model prediction, this paper chooses to use BP neural network to establish inventory model and predict inventory level.

BP neural network is a kind of nonlinear feed forward network which has good nonlinear mapping ability. Theories have proved that BP network can approach any nonlinear mapping relationship given enough input and hidden layers while there is no necessity to establish a mathematical model. Furthermore, by learning and training, BP network can store information systematically in weight matrix W. In doing so, it indicates that BP network can memorize the characteristics of inventory information and at the same time can adapt to

the changes of inventory environment. In view of the features of BP neural network, it has great advantages in classification and prediction.

However, it is acknowledged that BP neural network also has such problems as slow convergence and easily converging to local minimum when forecasting. Considering the shortcomings of standard BP algorithm, this paper proposes a new fast convergent BP neural network model for predicting inventory level. By adding an error offset, this paper deduces the new chain propagation rule and the updated weight formula. The application of the improved BP neural network model to predict the inventory level of an automotive parts company shows that the improved algorithm significantly outperforms the standard algorithm and some other improved BP algorithms both on convergence rate and prediction accuracy.

This paper proceeds as follows: Section 2 has a wide review of related literature. Based on the standard BP neural network, Section 3 introduces an improved BP neural network. Section 4 applies the improved BP algorithm to predict the inventory level of an automotive parts company. Section 5 draws some conclusions according to the results.

2. Literature Review

Recently, more and more scholars have applied neural network technique to inventory control. Bansal et al. used a neural network-based data mining technique to solve the problem of inventory of a large medical distribution company [3]. Based on the neural network model described by them, a prototype was conceived with data from a large decentralized organization. The prototype was successful in reducing the total level of inventory by 50% in the organization, while maintaining the same level of probability that a particular customer's demand would be satisfied. Shanmugasundaram et al. [4] discussed the use of neural network-based data mining and knowledge discovery techniques to optimize inventory levels in a large medical distribution company [4]. They identified the strategic data mining techniques used to address the problem of estimating the future sales of medical products using past sales data and used recurrent neural networks to predict future sales. Reyes-Aldasoro et al. adopted neural network technique to create a hybrid framework that could be utilized for analysis, modeling, and forecasting purposes [5]. The framework combined two existing approaches and introduced a new associated cost parameter that served as a surrogate for customer satisfaction. Hong et al. developed an online neural network controller that optimized a three-stage supply chain. With the inventory data feedback from an RFID system, the neural network controller minimized the total cost of the supply chain rapidly while satisfying a target order fulfillment ratio [6]. Some of these studies further proved that the neural network technique exceeded the traditional statistical technique in forecasting inventory level [7]. In fact, comparing with traditional prediction methods, neural network has its own unique advantages in processing prediction problems such as high fault tolerance, fast prediction speed, avoidance of description of complex relation between characteristic factors and object, strong adaptation, and good uncertain information processing ability [8–10].

Although there are various neural network models, BP neural network is the most widely used model because of its simple structure and strong ability to learn. In fact, it has been widely used in inventory control. Zhang et al. used the reinforcement learning technique and the BP neural network to propose a new adaptive inventory control method for supply chain management [11]. Wang proposed a neural network-based classification approach to inventory risk level of spare parts [12]. The BP algorithm for training a neural network is used to decide the weights to connections in the model. Mansur and Kuncoro cooperated to use the market basket analysis (MBA) and artificial neural network (ANN) Back propagation to predict inventory level [13]. In addition, ANN Back propagation is used to predict product inventories requirements/needs for each product. Huang et al. applied back-propagation network (BPN) to evaluate the criticality class (I, II, III, and IV) of spare parts [14]. They found that the proposed BPN could successfully decrease inventory holding costs by modifying the unreasonable target service level setting which was decided by the criticality class.

The BP neural network we mentioned above is referring to the standard BP neural network. The standard BP neural network is based on the Widrow-Hoff rule and uses the gradient descent method to transfer the mapping of a set of inputs to its correct output into nonlinear optimal problems. However, the standard BP algorithm has inherent disadvantages such as slow convergence, problem of converging to local minimum, complication of system, and random network structure selection [15, 16].

Aiming at the weaknesses of standard BP neural network, scholars have made further studies and proposed different improved BP neural network models [17–25]. The improvements of BP neural network mainly incorporate two perspectives: the direct improvements on BP neural network and the improvements based on the proposals of other new theories. Usually the first perspective includes adding the momentum factor [17], varying the learning rate dynamically [18], and introducing resilient back propagation (RPROP) [19]. The second perspective usually includes the introduction of simulated annealing genetic algorithm [24] and introduction of multiple extended Kalman algorithm [25]. All of these improved BP algorithms can reduce the training time to some degree and increase the prediction accuracy. Some have even applied the improved BP algorithms to forecasting inventory level and inventory control [26, 27].

By adding an error offset to the error function, this paper puts forward a direct improvement on standard BP neural network. Based on a dataset of an automotive parts company, it proves that the improved BP algorithm not only exceeds the standard BP algorithm both on convergence rate and prediction accuracy but also outperforms some other improved BP neural networks.

3. Improvement of BP Neural Network

3.1. Standard BP Neural Network. Back-propagation algorithm or BP algorithm, one of the most widely used algorithms in artificial neural network, is a kind of supervised learning algorithm. Its main purpose is to adjust weight matrix according to the squared error between the actual output and target output. The squared error is expressed as follows:

$$E = \frac{1}{2}\sum_{p}\left(d^p - y^p\right)^2. \tag{1}$$

Here, p denotes the pth training sample, d^p denotes the target output of the pth training sample, and y^p denotes the actual output of the pth training sample. The weights to each neuron are revised according to the following delta rule:

$$w_{i,j}^m(p+1) = w_{i,j}^m(p) + \Delta w_{i,j}^m. \tag{2}$$

Here, m denotes the mth layer neural network and $w_{i,j}$ denotes the weight on the connection from the ith neuron in the $(m-1)$th layer to the jth neuron in the mth layer. $\Delta w_{i,j}^m$ is expressed as follows:

$$\Delta w_{i,j}^m = -\eta\frac{\partial E}{\partial w_{i,j}}. \tag{3}$$

Here, η denotes the learning rate. By analyzing the above formula, we know that the key of BP algorithm is the calculation of $\partial E/\partial w_{i,j}$.

Suppose that I_j denotes the input of jth neuron, O_j denotes the output of jth neuron, and O_i denotes the output of ith neuron. Then, $I_j = \sum_i w_{i,j}O_i$, $O_j = f(I_j)$.

When the jth neuron is the output node, we have

$$\frac{\partial E}{\partial w_{i,j}} = -(d-y) * f'\left(I_j\right) * O_i. \tag{4}$$

If the jth neuron is not the output node, it must be the hidden node and we have

$$\frac{\partial E}{\partial w_{i,j}} = -f'\left(I_j\right)\sum_{m}\left(d^k - y^k\right)f'\left(I_m\right)w_{mj}. \tag{5}$$

From the above analysis, we can know that the standard BP algorithm updates the weights of its output layer and hidden layer just according to the above formula. Regarded as a part of the weights, the update of bias is quite similar to that of weights so we will not give further details about its deduction.

3.2. Improved BP Neural Network. To improve the convergence rate of standard BP algorithm, we propose a new algorithm, which can achieve the goal by adding an error offset.

The essence of BP algorithm is the forward propagation of data and backward propagation of errors. The weight value is revised according to the errors in back propagation. However, the convergence rate of standard BP algorithm is slow and often cannot satisfy the requirements when applied. Therefore, we propose a new method: adding an error offset in back propagation to greatly improve the convergence rate. The latter experiment illustrates that its effect is quite outstanding. Here, we redefine the squared error as follows:

$$E_O = \frac{1}{2}\sum_{P}\left(d^p - y^p\right)^2 + \frac{1}{2}\sum_{p}\left(f^{-1}\left(d^p\right) - f^{-1}\left(y^p\right)\right)^2, \tag{6}$$

and $(1/2)\sum_p\left(f^{-1}(d^p) - f^{-1}(y^p)\right)^2$ is the error offset, and what follows next is our deduction of $\partial E_o/\partial w_{i,j}$ from the revised squared error. Consider

$$\frac{\partial E_o}{\partial w_{i,j}} = \frac{\partial}{\partial w_{i,j}}\left(\frac{1}{2}\left(d_j - y_j\right)^2 + \frac{1}{2}\left(f^{-1}\left(d_j\right) - f^{-1}\left(y_j\right)\right)^2\right). \tag{7}$$

For the right-hand side of (7), the first half part is the same with that of standard BP algorithm. What we need to calculate is the second half part. If j is the output node, then $y_j = O_j$, $f^{-1}(y_j) = I_j$. Consider

$$\begin{aligned}
&\frac{\partial}{\partial w_{i,j}}\left(\frac{1}{2}\left(f^{-1}\left(d_j\right) - f^{-1}\left(y_j\right)\right)^2\right)\\
&\quad= -\left(f^{-1}\left(d_j\right) - I_j\right)\frac{\partial}{\partial w_{i,j}}\left(f^{-1}\left(d_j\right) - I_j\right)\\
&\quad= -\left(f^{-1}\left(d_j\right) - I_j\right)\frac{\partial I_j}{\partial w_{i,j}}\\
&\quad= -\left(f^{-1}\left(d_j\right) - I_j\right)O_i,\\
&\frac{\partial E_o}{\partial w_{i,j}} = -(d-y) * f'\left(I_j\right) * O_i - \left(f^{-1}\left(d_j\right) - I_j\right)O_i.
\end{aligned} \tag{8}$$

The new weight formula is

$$\begin{aligned}
w_{i,j}^m(p+1) = w_{i,j}^m(p)&\\
&+ \eta\left[(d-y) * f'\left(I_j\right) * O_i\right.\\
&\left.+ \left(f^{-1}\left(d_j\right) - I_j\right)O_i\right].
\end{aligned} \tag{9}$$

If jth neuron is not the output node, then it must be the hidden node. To avoid confusion, we suppose that kth is the output layer and we have

$$\begin{aligned}
&\frac{\partial}{\partial w_{i,j}}\left(\frac{1}{2}\left(f^{-1}\left(d_k\right) - f^{-1}\left(y_k\right)\right)^2\right)\\
&\quad= -\left(f^{-1}\left(d_k\right) - f^{-1}\left(y_k\right)\right)\frac{\partial}{\partial w_{i,j}}\left(f^{-1}\left(d_k\right) - f^{-1}\left(y_k\right)\right)\\
&\quad= -\left(f^{-1}\left(d_k\right) - f^{-1}\left(y_k\right)\right)\frac{\partial}{\partial I_j}f^{-1}\left(y_k\right) * \frac{\partial I_j}{\partial w_{i,j}}
\end{aligned}$$

$$= -\left(f^{-1}\left(d_j^p\right) - I_k\right)\frac{\partial I_k}{\partial I_j} * O_i$$
$$= -\left(f^{-1}\left(d_j^p\right) - I_k\right)\sum_m f'\left(I_j\right) w_{mj} * O_i. \tag{10}$$

The new weight formula is

$$\begin{aligned} w_{i,j}^m(p+1) &= w_{i,j}^m(p) \\ &+ \eta\left[\begin{array}{l} f'\left(I_j\right)\sum_m\left(d^k - y^k\right) f'\left(I_m\right) w_{mj} \\ \quad + \left(f^{-1}\left(d_j^p\right) - I_k\right)\sum_m f'\left(I_j\right) w_{mj} * O_i \end{array}\right]. \end{aligned} \tag{11}$$

4. Model Construction

This paper uses the dataset of an automotive parts company to train the improved BP neural network. As we know, nowadays automobiles are comprised of lots of parts. These parts are produced on the demand of automobile manufacturers and then are sent to assembly factories to form a complete product. In this way, the whole production process of an automobile exists in the form of a supply chain. To realize the highest overall efficiency, it needs cooperation of all the suppliers, manufacturers, wholesalers, and retailers. Inventory control is an important aspect which reflects such kind of cooperation. In the following part, this paper will use the improved BP neural network to forecast the inventory level of bearings—one of the components for an automobile.

4.1. Factors Influencing Inventory Control and Selection of Sample. Usually accurate inventory level is the precondition for good inventory management. For inventory management, inventory controlling cost and customers' service levels as well as inventory controlling quality are the main factors to estimate the inventory level. Therefore, in the design of inventory control system, we mainly use these factors to predict. They are described as follows [2].

(1) Various Costs. They are one of the main indicators to evaluate inventory control strategy. The costs mainly include all the expenses in product purchase and production as well as sales. For enterprises, analyzing inventory controlling cost can effectively reduce the overall cost of enterprises. However, inventory controlling costs include many aspects and these aspects can influence each other. Therefore, dividing inventory controlling cost in details and analyzing the accumulated data of business systems to find the main factors will be helpful for enterprises to make corresponding decisions and control all kinds of costs. The costs mainly include ordering cost, storage cost, transportation cost, and shortage cost.

(2) Demand Level. The purpose of inventory control is to best satisfy the demands. Therefore, demand is another important factor influencing inventory control. However, demand may be certain but also may be stochastic or seasonal. Demand level is positively proportional to inventory level.

(3) Supply Level. It refers to supply level of finished products of producers. It is positively proportional to inventory level.

(4) Quantity of Substitutes. It refers to the types of other parts which can substitute for the parts used. It is negatively proportional to inventory level.

(5) Lead Time. It refers to the period of time from sending the order to being ready for production. It includes the time for ordering, waiting time, preparatory time for suppliers to deliver goods, time on transportation, time for check and acceptance for warehouse entry, and time for preparation for use. It is positively proportional to inventory level.

(6) Customer Service Level. It refers to the possibility for enterprises to satisfy customers' needs after customers propose the ordering requirements. It is negatively proportional to inventory level. The higher the customer service level goes, the lower the inventory level will be. In this case, we use 2 (very good), 1 (general), and 0 (poor) to represent the extent of the customer service level.

This paper chooses the historical data of factors which influence the safety inventory level and inventory data of bearing of an automotive parts production company in one of the middle provinces of China from March 2012 to March 2013 as a sample to train the improved BP neural network. We mainly choose 100 groups of the data to train the network and then check its prediction ability. The number of training samples cannot be too small; otherwise, the network cannot learn enough which may result in low prediction ability. However, too large samples will lead to redundancy. At this time, the network will be overfitted. Therefore, this paper chooses 100 groups of data as input to train and predict and chooses inventory level as output to establish the BP neural network model. In this case, because the system is nonlinear, the initial value plays very important role in achieving local minimum. Therefore, the input sample needs to be normalized and the purpose is to make the big input values also fall in the range with large gradients of activation function.

Before network training, we normalized the training data according to $D_o = (\mathrm{Max}_D - D_i)/(\mathrm{Max}_D - \mathrm{Min}_D)$ and made them within $[0, 1]$ (see Table 1).

4.2. Network Variables. Any continuous function can be realized by a three-layer artificial neural network. Therefore, this paper adopts the three-layer BP neural network structure. When all information is input into the network, the information starts by being transmitted from input layer to hidden layer. With the work of activation function, the information is then transmitted to output layer. There are 9 input factors and the output is inventory level. The selection of variables of the network is as follows.

(1) Input Layer. The input layer includes 9 factors: storage cost (X1), ordering cost (X2), shortage cost (X3), transportation

Table 1: Normalized data of stock-influencing factor.

Data	Storage cost	Ordering cost	Shortage cost	Transportation cost	Demand level	Supply level	Quantity of substitutes	Waiting time	Service level	Actual inventory level
1	1.0	0.88	0.94	1	0.65	0.8	0.25	0.00	0	0.33
2	0.7	1.00	1.00	0	1.00	1.0	1.00	0.25	0	0.38
3	0.5	0.40	0.31	0	0.22	0.0	0.25	0.58	0	0.50
4	0.0	0.08	0.13	0	0.43	0.0	0.00	0.50	1	0.13
5	0.5	0.40	0.38	1	0.65	0.4	0.00	0.54	0	0.25
6	0.7	0.60	0.31	1	0.74	0.4	0.00	0.87	1	0.38
7	0.4	0.32	0.13	1	0.30	0.2	0.00	0.37	1	0.50
8	0.3	0.20	0.00	0	0.00	0.0	0.00	0.79	1	0.00
9	0.3	0.00	0.13	0	0.13	0.0	0.25	1.00	1	1.00
10	0.5	0.78	0.63	1	0.43	0.4	0.25	0.08	1	0.13

Table 2: Comparison of training convergence rate among standard algorithm, other improved algorithms, and improved algorithm of this paper.

Parameter depiction	Standard BP algorithm	Improved BP algorithm [26]	Improved BP algorithm [27]	Improved BP algorithm of this paper
Maximum iteration times	9897	6245	4268	4432
Minimum iteration times	1456	841	985	756
Average iteration times	5423.4	2315.8	2013.9	1968.7

cost (X4), demand level (X5), supply level (X6), quantity of substitutes (X7), waiting time (X8), and service level (X9).

(2) Hidden Layer. Usually when there are one or two hidden layers, it has the best convergent attributes. If there is no hidden layer or there are too many hidden layers, the convergent effect is not so good. Theories have proved that networks which have deviations and at least one S-type hidden layer and one linear output layer can approach any nonlinear function. That is, a three-layer BP network with a hidden layer can approach any nonlinear function.

According to empirical formula $h = \log_2 I$, h is the number of nodes of hidden layer and I is the number of nodes of input layer. We suppose $h = 4$.

(3) Output Layer. The number of nodes of output layer is the number of system objects. We choose one node as the inventory level of March 2013 to be measured.

(4) Selection of Initial Value and Threshold Value. Because both of them are two random groups of value, we choose a random value between $[-1, 1]$.

(5) Selection of Expected Error and Number of Iterations. We choose 10000 as the number of iterations and the expected error is 0.1.

5. Training Process and Experimental Result

This paper uses the neural network tool package of MATLAB 7.6 to program the model for safety inventory level based on BP neural network. In the BP neural network model established in this paper, there are 9 inputs and the number of neurons is relatively large. We preliminarily set the training variables as follows: times of training are 10000, training target is 0.01, and learning rate is 0.1. The code and training result is as follows:

net. trainParam. Epochs = 10000;

net. trainParam. goal = 0.1;

LP. lr = 0.1;

net-train(net, P, T);

after 1000 trainings, the training is finished.

After network finishes training, the network gets tested. We use the data of March 2013 to test. The code of prediction is as follows:

*P*_test = [0.5 0.78 0.63 1 0.43 0.4 0.25 0.08 1]:

Out = sim (net, *P*_test);

By comparing Figures 1 and 2, we can clearly see that the convergence rate of the improved algorithm is significantly faster than that of standard algorithm. We select the data from February 1, 2013, to February 20, 2013, to test. The result is as follows.

From Table 2, we can know that the improved BP algorithm is significantly better than that of standard BP algorithm on convergence rate. In addition, we also compare our improved BP algorithm with some other improved BP algorithms. The result shows that our BP algorithm also outperforms the other two improved BP algorithms mentioned in the literature review on convergence rate.

Table 3: Comparison of error among standard algorithm, other improved algorithms, and improved algorithm of this paper.

Parameter depiction	Standard BP algorithm	Improved BP algorithm [26]	Improved BP algorithm [27]	Improved BP algorithm of this paper
Prediction set error	0.002687	0.000938	0.000921	0.000780

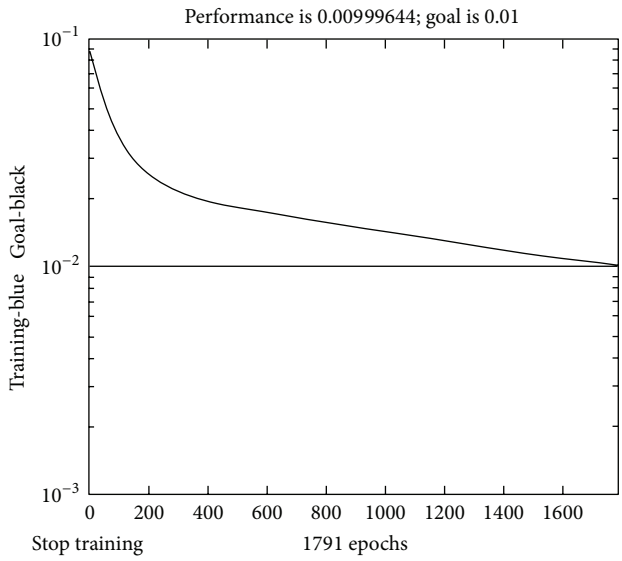

Figure 1: Training convergence effect of improved algorithm.

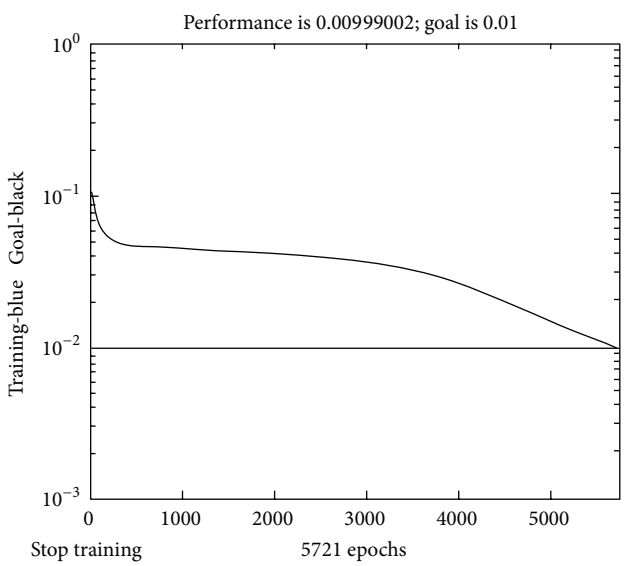

Figure 2: Training convergence effect of standard algorithm.

As prediction accuracy is concerned, from Figure 3, we can know that our improved BP algorithm exceeds significantly the standard BP algorithm.

Suppose $E = (1/2)\sum_p (d^p - y^p)^2$ is the prediction set error. From Table 3 we can clearly see that our improved BP algorithm not only exceeds the standard BP algorithm but also outperforms the other two improved BP algorithms mentioned in the literature review on prediction effect.

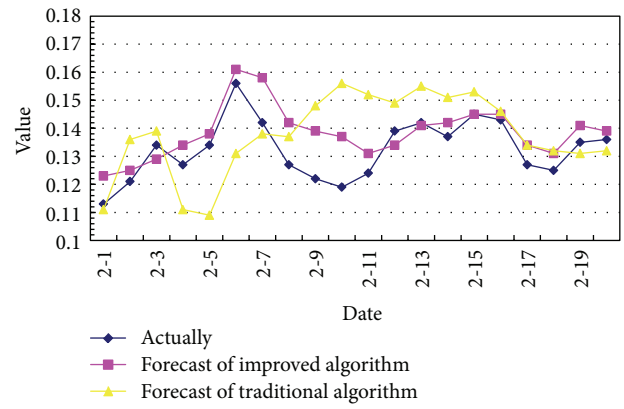

Figure 3: Prediction effect of improved algorithm.

6. Conclusions

We conclude the following with the practical importance of our findings. First, this paper proposes a new, fast convergent BP algorithm and deduces new chain propagation rules of neural network by introducing an error offset. Secondly, this paper applies it to the prediction of inventory level of an automotive parts company and achieves good effect. From the experimental results, we can see that using neural network to predict inventory is effective. The improved BP algorithm not only significantly exceeds the standard algorithm both on convergence time and prediction effect but also outperforms some other improved BP algorithms on these two main indicators. In this sense, this paper provides a valuable reference for inventory control of supply chain. However, this paper also has limitations. There are still some problems that need to be solved such as how to decide the number of nodes of hidden layer and the optimization of whole structure of network. Apart from that, the introduction of the error offset is based on experiences. The theoretical explanation for it still needs to be further discussed. All these problems wait to be further explored in future research.

Acknowledgments

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Research Article

Tourist Behavior Pattern Mining Model Based on Context

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Personalized travel experience and service of tourist has been a hot topic research in the tourism service supply chain. In this paper, we take the context into consideration and propose an analyzed method to the tourist based on the context: firstly, we analyze the context which influences the tourist behavior patterns, select the main context factors, and construct the tourist behavior pattern model based on it; then, we calculate the interest degree of the tourist behavior pattern and mine out the rules with high interest degree with the association rule algorithm; we can make some recommendations to the tourist with better personalized travelling experience and services. At last, we make an experiment to show the feasibility and effectiveness of our method.

1. Introduction

With the development of economy and the improvement of people's living standard, more and more people pay more attention to the quality of personalized travelling experience and service. In recent years, there has emerged more and more personalized ways to travel in tourism, such as FIT travel and independent travel. The traditional mode of travel service limits the diversity of service options, and it cannot fully meet the personalized needs of tourists. How to find the laws and the features of the tourist behavior through mining tourist behavior patterns and offer them better services has been a problem in the tourism service supply chain.

There are many researches concentrating on the tourist behavior pattern. Qing analyzed the characteristics of tourism services and the structural properties, constituent elements, and operation mechanism of tourism service supply chain in the context of modern information technology, and he put forward a new tourism service supply chain conceptual model based on tourist personalized demand [1]. Farmaki took the Troodos (Cyprus) as a case to research on the tourist motivation [2]; Martin and Witt proposed tourism demand forecasting model to represent tourists' cost of living [3]; Smallman and Moore studied on the tourists' decision making [4]; Kim et al. has worked on the Japanese tourists' shopping preference with the decision tree analysis method [5].

These studies only analyzed the tourist from the view point of the psychology and behavioral science of the tourist and have not considered the context set which will influence the tourist behavior patterns. So, in this paper, we take the context into consideration and propose an analyzed method to the tourist based on context to find out the relationship between services in the travel and the context and analyse the important contexts which will influence the tourist behavior. To mine out rules with high interest degree with the association rule algorithm and do some recommendations to the tourist with better personalized travelling experience and services, we propose a method based on network diagram, and it can reflect the relationship of the contexts which influence the tourist behaviour clearly. Through this method, we can delete the low interest degree of tourist behavior patterns; then, we use the Apriori algorithm to mine the association rules of tourist behavior which have high interest degree. Finally, we take an experiment to show the feasibility and effectiveness of our method.

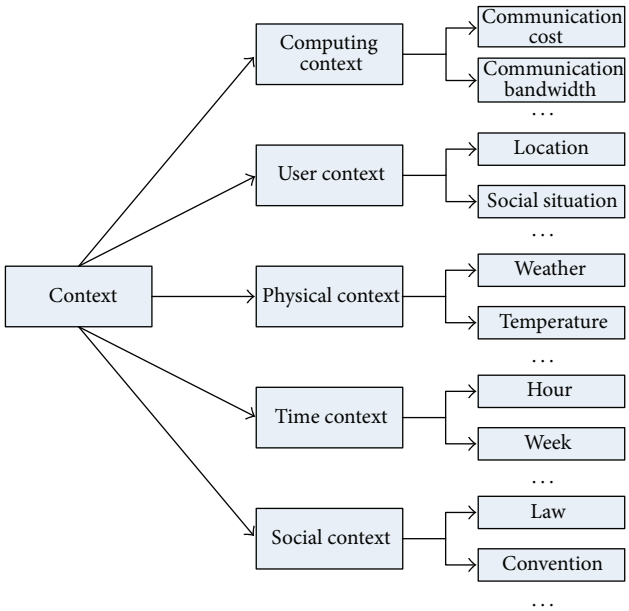

Figure 1: Context spectrum.

2. Related Works

2.1. Context. There are many definitions on the context and many researchers work on it. Schilit et al. defined the context as identifications and change of location, people, and objects around them [6]. Brown et al. thought that the context should be defined as the symbols around people or other objects such as location, time, season, temperature, and so on [7]. In paper [8], the definition of context would be extended to the feature information of some objects' situation, such as people, location, and so on. Snowdon and Grasso defined the context as the multilevel structure, mainly including the individual layer, the project layer, the group layer, and the organization layer [9]; Gu thought that the context would respond to the transformation based on the computers which are used as the centers to the people; in fact, he defined the context as a spectrum in his paper, as shown in Figure 1. He divided the context into computing context (such as communication bandwidth), user context (such as location), physical context (such as weather, temperature), time context (such as hour), and social context (such as law) [10].

In this paper, we think that the context is the influence factors of the tourist behavior pattern; different contexts will lead the tourist to different behavior patterns. We may take the following contexts into consideration: user, location, time, and device, and service type.

2.2. Association Rule and Apriori Algorithm. There are many association rule algorithms, and these algorithms can be divided into two classes: the first one is mainly focused on improving the analytical efficiency of the association rules; the other one pays more attention to the application of association rule algorithm and how to deal with value type variables and promotes the association of the single concept layer to multiple concept layers include and further reveals the inner structure of objects.

Apriori algorithm is one of the classical association rule algorithms; the earliest Apriori algorithm was proposed by Agrawal et al. [11]. The algorithm mainly including two parts: producing frequent item sets and producing association rules according to the frequent item sets. The algorithm scans data base, accumulates each item count, collects the items which meet the minimum support (min_sup), finds out the frequent 1-itemsets, and named it L_1. Then, the algorithm uses L_1 to find out the frequent 2-item sets L_2 and uses L_2 to find out the frequent 2-item sets L_3 and so on and keeps doing these until it cannot find out the frequent k-item sets. In these frequent item sets, it will be defined as a strong-association rule if it reaches the minimum confidence [12]. Since the association rule algorithm was proposed, it has been improved and applied in many fields. For example, Kang et al. applied the association rule algorithm in the Smart home [13], and Zhang et al. used the improved association rule algorithm in the university teaching managements [14].

3. Modeling and Mining Method for Tourist Behavior Pattern Based on Context

3.1. The Context Influence Factors Analysis of Tourist Behavior Pattern. We can consider a tourist as a mobile customer because the tourist moved anytime and anywhere. Presently, there are only a few researchers who work on the mobile customer behavior pattern. Tseng and Lin thought that the service and location are the influence factors of customer behavior in mobile service environment; they proposed a method named SMAP-Mine to mine customer behaviors [15]. Ma et al. took the time context into consideration and constructed a temporal sequence mobile access patterns mining model based on context awareness [16]. Chen et al. studied in the terms of the problem of mining matching mobile access patterns based on joining the following four kinds of characteristics: user, location, time, and service [17]. So in this paper, we think that the context influence factors of mobile customer behavior pattern includes mobile user, location, time, and service type.

At the same time, we take different capabilities of the mobile devices that the customer use, such as screen size, battery durability, and access bandwidth, into consideration. We consider that these capabilities will influence the mobile customer behavior pattern directly or indirectly. To prove that, we make an experiment as as follows. In the particular context, we observed behavior patterns of three customers who used different equipments and recorded the service types, the trajectory at which they moved, and time and type of service. Finally we got the customer movement trajectories as shown in Figure 2 and the service request information table as shown in Table 1. We can conclude from Figure 2 that customers have different behavior patterns when they use different mobile devices. For example, when the user u_1 used the device d_1, his movement trajectory was $l_2 \rightarrow l_6 \rightarrow l_8 \rightarrow l_9$; when he used the device d_2, his movement trajectory changed to $l_2 \rightarrow l_6 \rightarrow l_9$. Then, we can conclude form

Table 1: Customer service information table.

Time instances	Users and devices					
	(u_1, d_1)	(u_2, d_1)	(u_3, d_1)	(u_1, d_2)	(u_2, d_2)	(u_3, d_2)
TI_1	(l_2, t_1, s_1)	(l_9, t_5, s_1)	(l_5, t_{13}, s_1)	(l_2, t_1, s_1)	(l_9, t_1, s_1)	(l_5, t_1, s_1)
TI_2	(l_6, t_2, s_2)	(l_4, t_6, s_2)	(l_6, t_{14}, s_3)	(l_6, t_2, s_3)	(l_4, t_4, s_2)	(l_6, t_4, s_3)
TI_3	(l_8, t_3, s_3)	(l_1, t_8, s_2)	(l_4, t_{15}, s_2)	(l_9, t_3, s_4)	(l_7, t_5, s_4)	(l_3, t_5, s_2)
TI_4	(l_9, t_4, s_4)	(l_9, t_9, s_3)	(l_3, t_{16}, s_4)		(l_1, t_6, s_5)	(l_9, t_7, s_6)
TI_5		(l_7, t_{10}, s_4)	(l_9, t_{17}, s_6)		(l_3, t_8, s_6)	
TI_6		(l_1, t_{11}, s_5)				
TI_7		(l_3, t_{12}, s_6)				

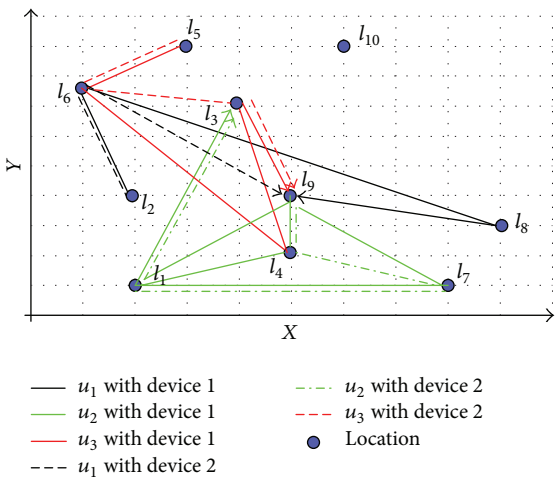

Figure 2: Movement trajectories of customers when they use different devices.

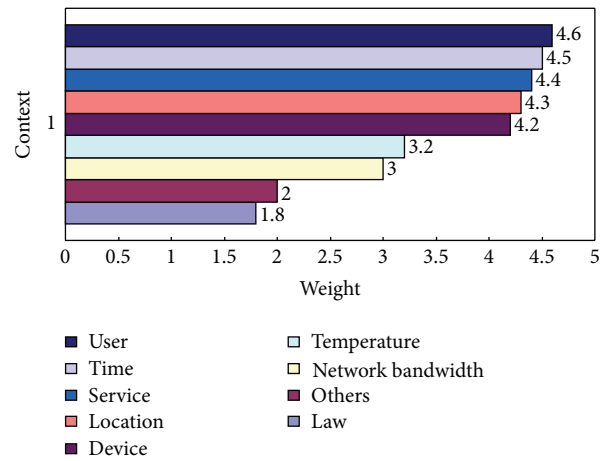

Figure 3: The results of the questionnaire.

Table 1 that the customer requested different services when he used different devices in the same time or requested the same service in different times. For example, when the user u_1 used device d_1 at time t_3; he requested the service s_3; when he used device d_2 at time t_3, he requested the service s_4; the user u_3 requested the service s_2 at the location l_4 when he used device d_1; he requested the service s_2 at the location l_3 when he used device d_2. Through these analyses, we can conclude that the mobile customer has different movement trajectories, request different services at the same times and requests the same service in different places when he or she uses different devices. So we take the mobile device as a context influence factor of mobile customer behavior pattern.

There are other context factors which influence the mobile customer behavior pattern, such as the physically environmental condition in which the customer stays, including weather, temperature, humidity, and so on; and the social situations in which the customer is involved (e.g., manners and customs and laws) will influence the mobile customer behavior pattern.

We use the form of the questionnaire to determine the main context factors. In this questionnaire, we design nine questions. Each of the nine questions involves a context factor which will influence the tourist behavior pattern. From these questions, we can study which contexts will influence the tourist behavior pattern most. A total of 102 individuals participate in the survey; they are all tourists. After stating these questionnaires, we use SPSS to analyze the results. We set that different option to different weight (1–5), and then statistically averaging, what are the context weights influence the behavior. We can get the results as shown in Figure 3. So in this paper, we choose the following five context factors as the main context factors: tourist (user), device, location, time, and service.

3.2. Modeling the Tourist Behavior Pattern Based on Context. The preceding part of this paper has a brief analysis on the context factors which influence the tourist behavior pattern, and then we will build a model based on these context factors. In the following part of this paper, we will give relational definitions about the tourist behavior patterns firstly and construct a model of the tourist behavior pattern based on context latterly.

Definition 1 (tourist user). $U = \{u_1, u_2, u_3, \ldots, u_i\}$ is the set of all the users; every user denotes a person who uses the mobile device to request mobile service messages from the mobile service supplier when he or she was travelling.

Definition 2 (devices of the tourist use). The device of the user use is a set of the devices of the user use to request mobile services and can be defined as $D = \{d_1, d_2, d_3, \ldots, d_h\}$.

Table 2: Timestamp table.

Timestamps	Time intervals
t_1	0:00–1:00
t_2	1:00–2:00
t_3	2:00–3:00
$\vdots$	$\vdots$
t_{22}	21:00–22:00
t_{23}	22:00–23:00
t_{24}	23:00–24:00

Definition 3 (location). Location denotes a set of places in which the tourist moves some times, and we use the set $L = \{l_1, l_2, l_3, \ldots, l_j\}$ to represent it.

Definition 4 (service). Service is a set of messages in which the tourist requests tourism services from the suppliers, and we use the set $S = \{s_1, s_2, s_3, \ldots, s_n\}$ to represent it.

Definition 5 (timestamp, sojourn time and service request time). To represent the time quantum of the forming of the tourist behavior pattern approximately, this paper divides a day's 24 hours into 24 time intervals simply, as shown in Table 2; every time interval denotes one hour, and the hour denotes one timestamp; sojourn time t_s denotes the time in which the user sojourns at somewhere; service request time t_r denotes the time in which the tourist requests some tourism services.

According to the previous definitions, this paper assumes $p = \{u, d, t, l, t_s, s, t_r\}$ as one tourist behavior, where u is an element of the tourist user set U, d is an element of the device of the user use set D, t is an element of the time set T, l is an element of the location set L, t_s is the time in which the tourist sojourns at location l, s denotes an element of service messages set S, and t_r denotes the time in which the tourist requests for tourism services.

In the graph theory, there is a structure called network whose structure is composed of nodes and edges. Every edge has its quantitative index related to the nodes or edges; this quantitative index is normally called weight which could denote distance, expenses, carrying capacity, and so on [18]. Namely, the structure of the network is composed of nodes and edges involving weight; taking this advantage of the network, this paper makes the context factors which influence the tourist behavior pattern as the nodes of the network, the connected relationship among the context factors as the edge of the network, and the connect coefficient among different context factors as the weight of the edge (the specific connect relationship and the connect coefficient will be demonstrated in detail in the following part of this paper). Like this, the behavior pattern of a tourist can be clearly portrayed in the network. Figure 4 illustrates the network structure of the behavior patterns of two different mobile users; we use $p_1 = \{u_1, d_2, t_1, l_2, 5, s_2, 2\}$ and $p_2 = \{u_2, d_1, t_2, l_1, 4, s_1, 3\}$ to represent their behavior patterns, respectively.

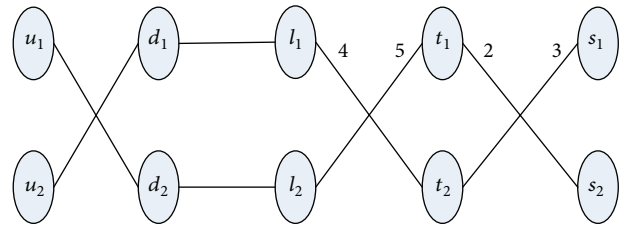

Figure 4: The network model of the tourist behavior pattern.

3.3. Tourist Behavior Pattern Mining Method Based on the Network. The preceding part of this paper has a model analysis on the structure of the network of the tourist behavior; in the following part of this paper, we will give out the related definitions firstly and the specific procedures of the tourist behavior mining pattern based on the network latterly.

3.3.1. Basic Definitions. To explain the content of the mining method more clearly, we will give relational definitions firstly.

Definition 6 (connect coefficient). Connect coefficient denotes the connection relationship between two different attributes; the specific connect coefficients are $U \bowtie D$, $D \bowtie L$, $L \bowtie T$, and $T \bowtie S$. The connect coefficient of $U \bowtie D$ denotes the connection times between a mobile user u and a device d. The connect coefficient of $D \bowtie L$ is LC_{nu} which denotes the connection times of a device d with a location l. The connect coefficient of $L \bowtie T$ is t_s which denotes the time in which a user sojourns at location l. The connect coefficient of $T \bowtie S$ is t_r which denotes the time in which a mobile user requests for services.

Definition 7 (interesting locations and interesting services). When the length of time in which a tourist sojourns somewhere is larger than the threshold value we set, we think that the tourist is interested in this place. Similarly, when the length of time in which a mobile user requests for a service s_j is larger than the threshold value or the connection times is larger than a threshold value, we think that the mobile user is interested in this service. Usually the length of time will be set up to 30 minutes and the connection times will be set up to 10 times.

Definition 8 (repeated edge). For a tourist, he may have the same connection edge in two different behavior patterns; such edge will be called repeated edges in this paper. For example, in the following behavior patterns $p_1 = \{u_1, d_1, l_3, t_{17}, s_3\}$ and $p_2 = \{u_1, d_2, l_3, t_{17}, s_3\}$, they have two repeated edges, namely,$l_3 t_{17}$ and $t_{17} s_3$.

Definition 9 (connect edge value). Connect edge value is a standard value obtained with standardizing the connect coefficient (Definition 6) in the case where the different quantity levels of input variables affect the final mining result. In this paper we use "$\bowtie$" to present the connect relationship between different attributes, and specific weights are $U \bowtie D$,

$D \bowtie L$, $L \bowtie T$, and $T \bowtie S$; the computational formulas of every edge weight are as follows.

Connect Edge Value of $U \bowtie D$. The connect edge value of mobile user u_i and device d_j equals the ratio of the connect times between user u_i and device d_j to the sum times between user u_i and device set; the specific formula is

$$w_{ij} = \frac{\text{count}\left(u_i d_j\right)}{\sum_{j=1}^{n} \text{count}\left(u_i d_j\right)}, \tag{1}$$

where n denotes the amount of devices. Similarly, the connect edge value of $D \bowtie L$ is as follows:

$$w_{jk} = \frac{LC_{nu}\left(jk\right)}{\sum_{k=1}^{m} LC_{nu}\left(jk\right)}, \tag{2}$$

where m denotes the amount of locations and $LC_{nu}(jk)$ and $\sum_{k=1}^{m} LC_{nu}(jk)$ denote the connect times between devices and locations in the same behavior pattern of a mobile user.

The Connect Edge Value of $L \bowtie T$

$$w_{kh} = \frac{t_s\left(kh\right)}{\sum_{h=1}^{24} t_s\left(kh\right)}, \tag{3}$$

where $t_s(kh)$ and $\sum_{h=1}^{24} t_s(kh)$ denote the time in which a mobile user requests services at somewhere in his behavior pattern.

The Connect Edge Value of $T \bowtie S$

$$w_{hz} = \frac{t_r\left(hz\right)}{\sum_{z=1}^{n} t_r\left(hz\right)}, \tag{4}$$

where n denotes the amount of the connect service set and $t_r(hz)$ and $\sum_{z=1}^{n} t_r(hz)$ denote the time in which a mobile user requests for services in his behavior pattern. An edge will be deleted if its connection edge value is smaller than a threshold value. A behavior pattern will not be involved in the calculation of the connect edge value if it contains interesting locations or interesting services.

Definition 10 (connect edge coefficient e). When a repeated edge appears, this edge value constitutes of several behavior patterns; connection edge coefficient e denotes the incidence a behavior pattern has on this edge. Its value equals ratio of the connect edge coefficient of this behavior pattern to the sum of all the connect edge coefficients of the same mobile user at this edge.

Definition 11 (interesting degree id). Interesting degree id is an index to reflect the degrees of interests of the mobile user behavior pattern. Specifically, it equals the value that the sum of all the tuple (u, d, t, l, s) weight, the formula of interesting degree id $= e_{ij} * w_{ij} + e_{jk} * w_{jk} + e_{kh} * w_{kh} + e_{hz} * w_{hz}$. If the value of interesting degree id is smaller than a threshold value th_1, we will regard the degree of interests of this mobile user pattern as low interest level and delete this pattern from the network. If the value of interesting degree id is larger than another threshold value th_2, we will regard the degree of interests of this mobile user pattern as high interest level. Like this, we divide mobile user behavior patterns into three parts, namely, low level of interest, common level of interest, and high level of interest. We can set them by our need; the larger the value is, the higher degree of interest the rules of the results will have. As is illustrated in Figure 5, we can use it as a behavior prediction model to predict the behavior pattern of a mobile user in the future. If a behavior pattern contains interesting locations or interesting services, we will regard it as the high interesting level behavior pattern without calculating the specific value of its interestingness.

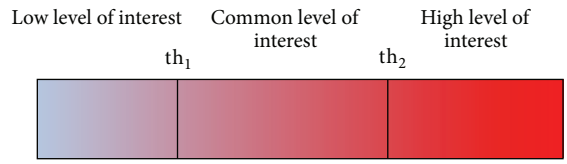

Figure 5: Two thresholds and three degrees.

Table 3: Tourist information.

U_i	d_h	L_j	LC_{nu}	T_m	T_s	S_n	T_r
u_1	d_1	l_1	1	t_{15}	8	s_4	6
u_1	d_2	l_1	1	t_{15}	8	s_2	2
u_2	d_1	l_4	2	t_{16}	9	s_3	9
u_2	d_2	l_3	2	t_{17}	9	s_3	9
u_3	d_1	l_5	6	t_{17}	5	s_5	5
u_3	d_2	l_5	6	t_{18}	12	s_5	8
u_4	d_3	l_7	5	t_{14}	8	s_6	7
u_5	d_2	l_6	3	t_{16}	10	s_8	9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

3.3.2. Mining Steps

First Step (collecting data). To mine tourist behavior pattern, we must collect data about the tourist. We can get the information table as is shown in Table 3 through collecting user data, mainly including tourist information (U_i), mobile device (d_h), location (L_j), collecting times (LC_{nu}), time (T_m), time of the user stay the location (T_s), the service type the user request (S_n), and time of the user request the service (T_r).

Second Step. Extracting context attribute number of the context set which influences the mobile customer behavior pattern and design corresponding layers of the network diagram; in this paper, we should design a network diagram with five layers, each layer corresponds to all nodes of U_i, D_h, L_j, T_m, and S_n, respectively, and the number of the layer nodes corresponds to each attribute value number, as shown in Figure 6.

Third Step. Collecting the adjacent nodes, each connection coefficient should be marked as Definition 7; we need to add the connection coefficient of the side when it repeats several

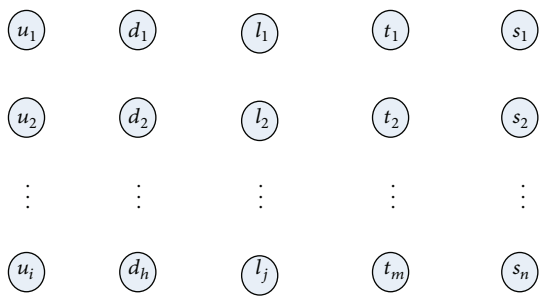

Figure 6: Network diagram model.

times. For example, there are two situations when the device d_1 collects the location l_3: one is 4 and the other is 2, so the connection coefficient of $d_1 l_3$ equals $4 + 2 = 6$.

Forth Step. Considering different customers have different behavior patterns, we classify each user into a group and calculate the collection weight according to Definition 8; when the collection weight is lesser than the threshold, the edge will be deleted.

Fifth Step. Calculating the remaining customer interest degree according to Definition 11 and set the low interest degree th_1 and the high interest degree th_2 value. When the customer interest degree is lesser than the low interest degree th_1, this customer behavior pattern will be unconcerned, and the general interest degree and the high interest degree pattern will be conducted in the next step.

Sixth Step. Using Apriori algorithm to mine the frequent pattern to the general interest degree and the high interest degree pattern, mine out the association rules with higher degree value on support and confidence; we can use these rules to forecast the customer' behaviors in future or recommend some services to mobile customers.

In order to show the availability of our method, we propose the concept of "coverage," which means the ratio of the number of the same rules that are produced by our model to the number of rules that produced directly. If the coverage is larger than a threshold, we say that the method we proposed is available. Generally, the larger the threshold is, the more the availability of the method is. In this paper, we set the threshold to be equal to 80%.

4. Experiment and Analysis

4.1. Example and Analysis. We take the West Lake of Hangzhou, for example, to illustrate the application of the model, via GPS and RFID provide personalized services to users combined with requirements and preference of the user. So we select part of the information data about tourist behavior from West Lake of Hangzhou Scenic Area Management Committee as is shown in Table 4.

To verify the effects of the proposed method, we use two standard metrics: interest degree and coverage.

Table 4: Tourist behavior information table.

P	U_i	d_h	L_j	LC_{mu}	T_m	T_s	S_n	T_r
p_1	u_1	d_1	l_1	1	t_{15}	8	s_4	6
p_2	u_1	d_2	l_1	1	t_{15}	8	s_2	2
p_3	u_1	d_1	l_1	1	t_{16}	11	s_2	3
p_4	u_1	d_2	l_1	1	t_{16}	11	s_2	4
p_5	u_1	d_1	l_3	4	t_{17}	21	s_3	17
p_6	u_1	d_2	l_3	4	t_{17}	4	s_3	3
p_7	u_1	d_1	l_2	11	t_{16}	35	s_4	30
p_8	u_2	d_1	l_4	2	t_{16}	9	s_3	9
p_9	u_2	d_2	l_3	2	t_{17}	9	s_3	9
p_{10}	u_2	d_1	l_3	2	t_{17}	2	s_3	1
p_{11}	u_2	d_2	l_2	2	t_{17}	28	s_5	15
p_{12}	u_2	d_2	l_2	8	t_{16}	38	s_4	31
p_{13}	u_3	d_1	l_5	6	t_{17}	5	s_5	5
p_{14}	u_3	d_2	l_5	6	t_{18}	12	s_5	8
p_{15}	u_3	d_1	l_5	6	t_{18}	12	s_4	4
p_{16}	u_3	d_1	l_5	6	t_{18}	18	s_3	13
p_{17}	u_3	d_2	l_5	6	t_{18}	18	s_3	5
p_{18}	u_3	d_2	l_5	10	t_{16}	32	s_4	30
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

We can conclude from Table 4 that the patterns p_7, p_{12}, and p_{18} are the patterns with interested locations or interested services, and we think that these patterns are the behavior patterns with high interest degree patterns. To show the processes of our method, we choose three users' patterns in this paper and design a network diagram with five layers and collect the adjacent layers and then calculate the connection coefficients according to Definition 7, as shown in Figure 7.

Then, we divide each user into a group and calculate the collection weight according to Definition 8; we set that the edges whose weight is lesser than 0.2 would be deleted, so the edges d_1l_1, d_2l_1, $t_{15}s_2$, $t_{15}s_2$, $t_{15}s_4$, l_4t_{15}, l_5t_{17}, $t_{17}s_5$, and $t_{18}s_4$ will be deleted and the patterns with edges d_1l_1, d_2l_1, $t_{15}s_2$, $t_{15}s_2$, $t_{15}s_4$, l_4t_{15}, l_5t_{17}, $t_{17}s_5$, and $t_{18}s_4$ will be deleted too, as shown in Figures 8, 9, 10.

So remain following patterns: $p_5 = \{u_1, d_1, l_3, t_{17}, s_3\}$, $p_6 = \{u_1, d_2, l_3, t_{17}, s_3\}$, $p_{10} = \{u_2, d_1, l_3, t_{17}, s_3\}$, $p_{11} = \{u_2, d_2, l_2, t_{17}, s_5\}$, $p_{14} = \{u_3, d_2, l_5, t_{18}, s_5\}$, $p_{16} = \{u_3, d_1, l_5, t_{18}, s_3\}$, $p_{17} = \{u_3, d_1, l_5, t_{18}, s_3\}$; then we calculate the interesting degree of these patterns according to Definition 11, as is shown in the following expressions:

$$\begin{aligned}
\text{id}_{p_5} &= e_{11} * w_{u_1d_1} + e_{13} * w_{d_1l_3} \\
&\quad + e_{3-17} * w_{l_1t_{17}} + e_{17-3} * w_{t_{17}s_3} \\
&= \frac{1}{3} * 0.5 + 1 * 0.333 + \frac{21}{25} * 0.397 \\
&\quad + \frac{17}{20} * 0.571 \approx 1.305; \\
\text{id}_{p_6} &= e_{12} * w_{u_1d_2} + e_{23} * w_{d_2l_3} \\
&\quad + e_{3-17} * w_{l_3t_{17}} + e_{17-3} * w_{t_{17}s_3}
\end{aligned}$$

$$= \frac{1}{3} * 0.5 + 1 * 0.333 + \frac{4}{25} * 0.397 + \frac{3}{20} * 0.571 \approx 0.647;$$

$$\mathrm{id}_{p_9} = e_{22} * w_{u_2 d_2} + e_{23} * w_{d_2 l_3} + e_{3-17} * w_{l_3 t_{17}} + e_{17-3} * w_{t_{17} s_3} = 0.5 * \frac{1}{2} + 1 * 0.25 + 0.229 * \frac{9}{11} + 0.294 * \frac{9}{10} \approx 0.952;$$

$$\mathrm{id}_{p_{10}} = e_{21} * w_{u_2 d_1} + e_{13} * w_{d_1 l_3} + e_{3-17} * w_{l_3 t_{17}} + e_{17-3} * w_{t_{17} s_3} = 0.5 * \frac{1}{2} + 1 * 0.25 + 0.229 * \frac{2}{11} + 0.294 * \frac{1}{10} \approx 0.571;$$

$$\mathrm{id}_{p_{11}} = e_{22} * w_{u_2 d_2} + e_{22} * w_{d_2 l_2} + e_{2-17} * w_{l_2 t_{17}} + e_{17-5} * w_{t_{17} s_5} = 0.5 * \frac{1}{2} + 1 * 0.25 + 1 * 0.583 + 1 * 0.441 \approx 1.524;$$

$$\mathrm{id}_{p_{14}} = e_{3-2} * w_{u_3 d_2} + e_{25} * w_{d_2 l_5} + e_{5-18} * w_{l_5 t_{18}} + e_{18-5} * w_{t_{18} s_5} = 0.4 * \frac{1}{2} + 0.4 * \frac{1}{2} + 0.923 * \frac{12}{60} + 1 * 0.229 \approx 0.815;$$

$$\mathrm{id}_{p_{16}} = e_{31} * w_{u_3 d_1} + e_{15} * w_{d_1 l_5} + e_{5-18} * w_{l_5 t_{18}} + e_{18-3} * w_{t_{18} s_3} = 0.6 * \frac{1}{3} + 0.6 * \frac{1}{3} + 0.923 * \frac{18}{60} + 0.514 * \frac{13}{18} \approx 1.05;$$

$$\mathrm{id}_{p_{17}} = e_{31} * w_{u_3 d_1} + e_{15} * w_{d_1 l_5} + e_{5-18} * w_{l_5 t_{18}} + e_{18-3} * w_{t_{18} s_3} = 0.4 * \frac{1}{2} + 0.4 * \frac{1}{2} + 0.923 * \frac{18}{60} + 0.514 * \frac{5}{18} \approx 0.820. \quad (5)$$

In this paper, we set the low interesting degree th_1 value to be equal to 0.8 and the high interesting degree th_2 value to be equal to 1. So the patterns whose interesting degrees are lesser than 0.8 interesting degree are the low interesting patterns, the patterns whose interesting degrees are higher than 1 interesting degree are the high interesting patterns, and the patterns whose interesting degree between 0.8 and 1 are the common patterns. So $p_6 = \{u_1, d_2, l_3, t_{17}, s_3\}$ and $p_{10} = \{u_2, d_1, l_3, t_{17}, s_3\}$ are the low interesting degree patterns, $p_9 = \{u_2, d_2, l_3, t_{17}, s_3\}$, $p_{14} = \{u_3, d_2, l_5, t_{18}, s_5\}$, and $p_{17} = \{u_3, d_1, l_5, t_{18}, s_3\}$ are the common interesting degree patterns, and $p_5 = \{u_1, d_1, l_3, t_{17}, s_3\}$, $p_{11} = \{u_2, d_2, l_2, t_{17}, s_5\}$, and $p_{16} = \{u_3, d_1, l_5, t_{18}, s_3\}$ are the high interesting degree patterns. We delete the low interesting degree patterns and get the patterns with high interesting degree as is shown in Table 5.

Table 5: Patterns with high interesting degree.

P_i	U_i	d_h	L_j	T_m	S_n
p_5	u_1	d_1	l_3	t_{17}	s_3
p_7	u_1	d_1	l_2	t_{16}	s_4
p_9	u_2	d_2	l_3	t_{17}	s_3
p_{11}	u_2	d_2	l_2	t_{17}	s_5
p_{12}	u_2	d_2	l_2	t_{16}	s_4
p_{14}	u_3	d_2	l_5	t_{18}	s_5
p_{16}	u_3	d_1	l_5	t_{18}	s_3
p_{17}	u_3	d_2	l_5	t_{18}	s_3
p_{18}	u_3	d_2	l_5	t_{16}	s_4

Then, we use the Apriori algorithm to mine rules on the high interesting degree patterns; we set the minimum support to 20% and the minimum confidence to 80%, then we can get the results as follows.

The lift denotes the ratio of the confidence to the support of the consequent item; the computational formula is followed: $L_{x \to y} = C_{x \to y}/S_y$. The lift reacts the influence degree of the antecedent item X to the consequent item Y appears. Generally, the lift value should be larger than 1, and it means that the antecedent item X has a positive influence on the consequent item Y appears. The larger the life value is, the better the rule is.

From Table 6, we can conclude that we can get 39 association rules when we use the method we proposed in this paper. These rules were obtained from the high interesting pattern; we thought that these rules were interesting rules. Then we observe the rule with the maximum lift, time = t_{17}, and service = $s_3 \to$ location = l_3. The value is 4.5. It means that this association rule has the highest realistic guidance. So this rule will be firstly considered when we use the rules of the result. We can use these association rules to recommend some services to tourist to offer them better services; for example, using the rule location =l_2 and time = $t_{16} \to$ service = s_4, we can recommend the s_4 to the tourist when the tourist stays in the context with location = l_2 and time = t_{16}. In this paper, the service s_4 is the tourism route guide, so we can send the tourism route guide to the tourist as is shown in Figure 11.

4.2. Comparison and Discussion. To verity the effects of the method we proposed in this paper, we use the Apriori

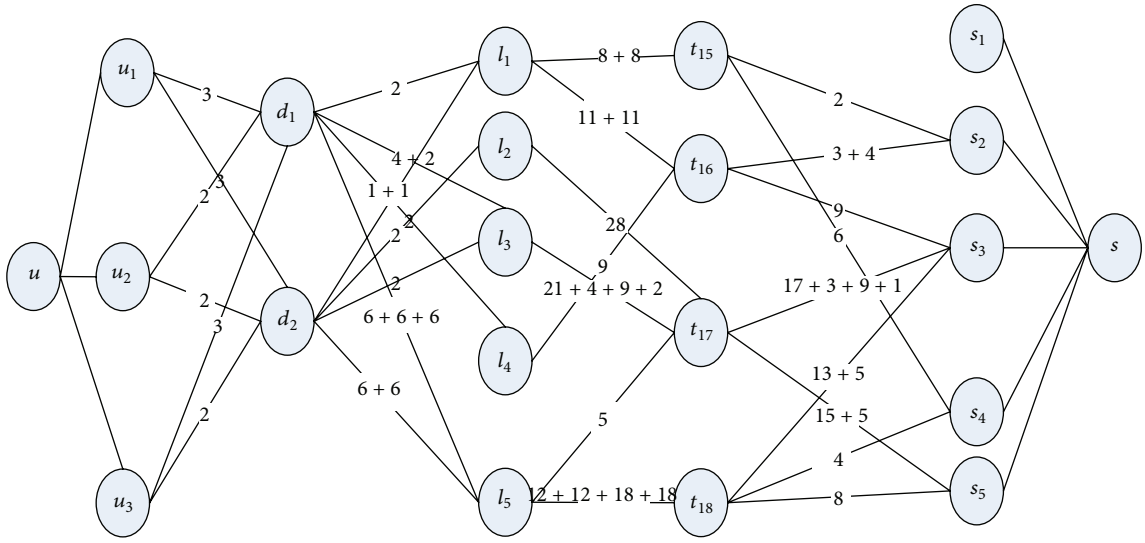

Figure 7: The network we constructed.

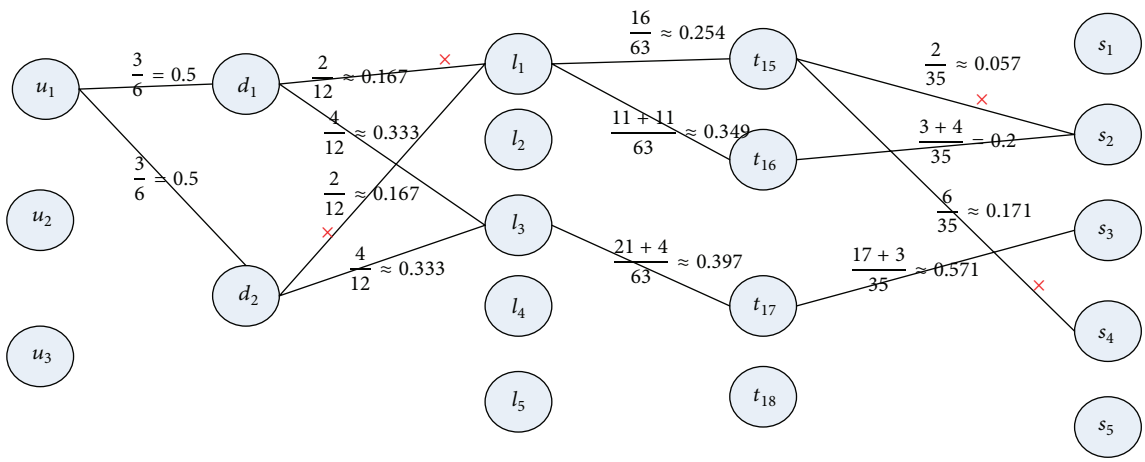

Figure 8: Calculate the tourist u_1 collection weight.

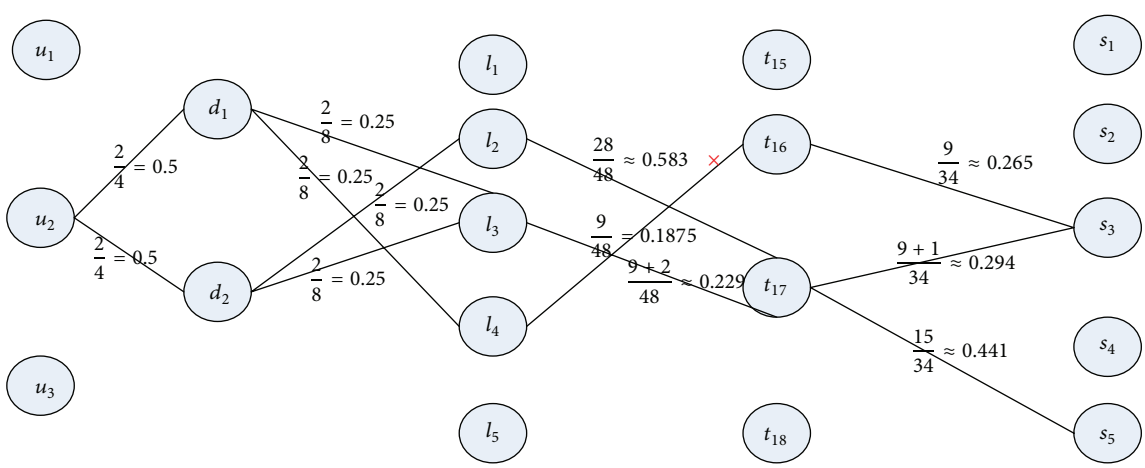

Figure 9: Calculate the tourist u_2 collection weight.

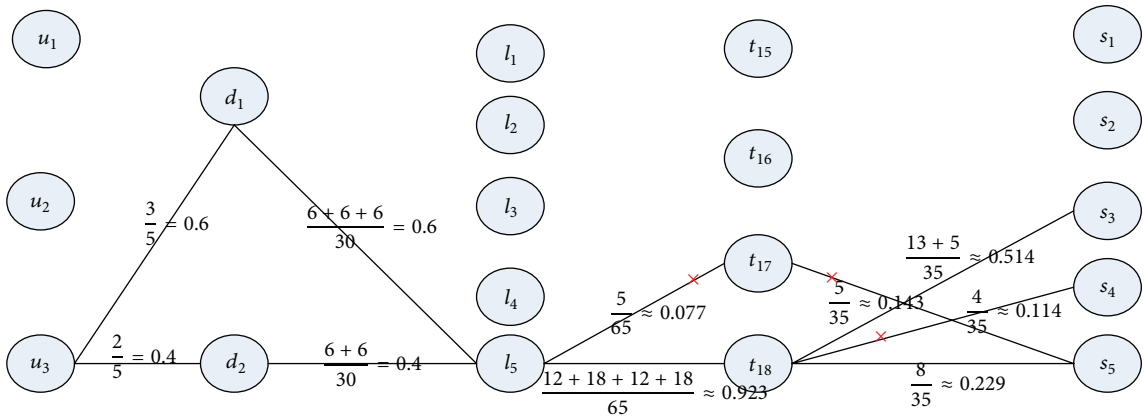

FIGURE 10: Calculate the tourist u_3 collection weight.

FIGURE 11: Tourism route guide.

algorithm, the GRI algorithm, the CARAMA algorithm and Predictive-Apriori algorithm on the original data (here we set the minimum support equals to 20% and the minimum confidence equals to 80%; too), and we get following rules as is shown in Tables 7, 8, 9, and 10.

Comparing Table 6 with Tables 7 and 8, there are 11 rules from Table 7 which have been emerged in Table 6 (the rules marked with yellow as is shown in Table 6), and all rules are in Table 8 have been emerged in Table 6. So we think that the method we proposed to mine the mobile customer behavior pattern has the merit of effectiveness; in this experiment the validity of the method is about 91.67% (11/12) to the Apriori algorithm and 100% (6/6) to the GRI algorithm, which means the coverage values are 91.67% and 100%, which are larger than the threshold we set before. It means that the method we proposed is feasibile and effective. Excluding the 11 rules in Table 7, Table 6 has other 28 rules and these rules have the feature of high interest, so they will provide more choices to the service provider and more services to the mobile customer. Then we observe the rule which has the maximum value of lift from Tables 6 and 7, the rule is time = t_{17} and service = $s_3 \rightarrow$ location = l_3; it means that the method we proposed is similar to the classical Apriori algorithm. At last, the rule whose ID = 1 in Table 7: location = $l_1 \rightarrow$ user = u_1,It is the only rule that is not included in Table 6, although this rule meets the minimum support and the minimum confidence; the pattern with $\{l_1, u_1\}$ is a low interesting pattern as we definite before, and the rule location = $l_1 \rightarrow$ user = u_1 is an uninteresting rule. In our method, we can reject uninteresting rules like this. Through the analysis, the method we proposed in this paper is more feasible and advanced when being compared with the Apriori algorithm.

Table 6: Mining results.

ID	Rules	Count	Lift
1	User = u_1 → device = d_1	2	3
2	Location = l_3 → time = t_{17}	2	3
3	Location = l_3 → service = s_3	2	2.25
4	Service = s_5 → device = d_2	2	1.5
5	User = u_2 → device = d_2	3	1.5
6	Time = t_{16} → service = s_4	3	3
7	Service = s_4 → time = t_{16}	3	3
8	Time = t_{18} → user = u_3	3	2.25
9	Time = t_{18} → location = l_5	3	2.25
10	User = u_3 → location = l_5	4	2.25
11	Location = l_5 → user = u_3	4	2.25
12	Location = l_3 and time = t_{17} → service = s_3	2	2.25
13	Location = l_3 and service = s_3 → time = t_{17}	2	3
14	Time = t_{17} and service = s_3 → location = l_3	2	4.5
15	User = u_2 and location = l_2 → device = d_2	2	1.5
16	Location = l_2 and device = d_2 → user = u_2	2	3
17	User = u_2 and time = t_{17} → device = d_2	2	1.5
18	Time = t_{17} and device = d_2 → user = u_2	2	3
19	Location = l_2 and time = t_{16} → service = s_4	2	3
20	Location = l_2 and service = s_4 → time = t_{16}	2	3
21	Time = t_{16} and device = d_2 → service = s_4	2	3
22	Service = s_4 and device = d_2 → time = t_{16}	2	3
23	Time = t_{18} and user = u_3 → location = l_5	3	2.25
24	Time = t_{18} and location = l_5 → user = u_3	3	2.25
25	Time = t_{18} and service = s_3 → user = u_3	2	2.25
26	User = u_3 and service = s_3 → time = t_{18}	2	3
27	Time = t_{18} and device = d_2 → user = u_3	2	2.25
28	Time = t_{18} and service = s_3 → location = l_5	2	2.25
29	Location = l_5 and service = s_3 → time = t_{18}	2	3
30	Time = t_{18} and device = d_2 → location = l_5	2	2.25
31	User = u_3 and service = s_3 → location = l_5	2	2.25
32	Location = l_5 and service = s_3 → user = u_3	2	2.25
33	User = u_3 and device = d_2 → location = l_5	3	2.25
34	Location = l_5 and device = d_2 → user = u_3	3	2.25
35	Time = t_{18}, user = u_3 and service = s_3 → location = l_5	2	2.25
36	Time = t_{18}, location = l_5 and service = s_3 → user = u_3	2	2.25
37	User = u_3, location = l_5 and service = s_3 → time = t_{18}	2	3
38	Time = t_{18}, user = u_3 and device = d_2 → location = l_5	2	2.25
39	Time = t_{18}, location = l_5 and device = d_2 → user = u_3	2	2.25

5. Conclusion

In this paper we considered the context factors which influence the tourist behavior pattern comprehensively, such as the device the tourist use, time, location, and service types, and got the context set which influences the tourist behavior pattern. Then we proposed a method to mine tourist behavior patterns based on the network diagram; this method constructed a network diagram firstly. Then, we got the behavior patterns with high interesting degree and did association rule mining in the patterns and got the rules; at last, we made an experiment to show the feasibility and effectiveness of our method. In our experiment, we set the low interest degree th_1 value to be equal to 0.8 and the high interest degree th_2 value to be equal to 1 and deleted the low interest pattern; then we did association mining with Apriori algorithm to the remainder of the patterns and got 39 rules; we can do some recommendations to the tourist with these high interest rules. Compared to the results which do not use this method, it has the following advantages: (1) it can keep the interest rules and delete the uninterested rules in the results; (2) it can produce many other interest rules, which

Table 7: The results based on Apriori algorithm.

ID	Rules	Count	Lift
1	Location = l_1 → user = u_1	4	2.57
2	Time = t_{18} → user = u_3	4	3.00
3	Time = t_{18} → location = l_5	4	3.00
4	Location = l_3 → time = t_{17}	4	3.00
5	Location = l_3 → service = s_3	4	2.57
6	User = u_3 → location = l_5	6	3.00
7	Location = l_5 → user = u_3	6	3.00
8	Time = t_{18} and user = u_3 → location = l_5	4	3.00
9	Time = t_{18} and location = l_5 → user = u_3	4	3.00
10	Location = l_3 and time = t_{17} → service = s_3	4	2.57
11	Location = l_3 and service = s_3 → time = t_{17}	4	3.00
12	Time = t_{17} and service = s_3 → location = l_3	4	4.50

Table 8: The results based on GRI algorithm.

ID	Rules	Count	Lift
1	Location = l_1 → user = u_1	4	2.5
2	Location = l_3 → service = s_3	4	2.5
3	User = u_3 → location = u_3	6	3
4	Location = l_5 → user = u_3	6	3
5	Location = l_3 → time = t_{17}	4	3
6	Time = t_{17} and service = s_3 → location = l_3	4	4.5

Table 9: The results based on CARMA algorithm.

ID	Rules	Count	Lift
1	User = u_3 → location = l_5	6	3
2	Location = l_5 → user = u_3	6	3
3	Location = l_1 → user = u_1	4	2.5
4	Location = l_3 → time = t_{17}	4	3
5	Location = l_3 → service = s_3	4	2.5
6	Location = l_3 → time = t_{17} and service = s_3	4	4.5
7	Time = t_{18} → user = u_3	4	3
8	Time = t_{18} → location = l_5	4	3
9	Location = l_3 and time = t_{17} → service = s_3	4	2.5
10	Location = l_3 and service = s_3 → time = t_{17}	4	3
11	Time = t_{17} and service = s_3 → location = l_3	4	4.5

we can use them to make more recommendations for the tourist; (3) it can produce the same rule which has the highest lift compared to the result that does not use this method. That is, the method we used in this paper is feasible and superior.

The future work will be further researching on the context factors which influence the tourist behavior pattern and expanding the context set; also we will analyze the performance of the method we proposed and optimize the method and so on.

Table 10: The results based on Predictive-Apriori algorithm.

ID	Rules
1	User = u_3 → location = l_5
2	Location = l_5 → user = u_3
3	Location = l_1 → user = u_1
4	Location = l_3 → time = t_{17} and service = s_3
5	Time = t_{18} → user = u_3 and location = l_5
6	Time = t_{17} and service = s_3 → location = l_3
7	Service = s_2 → user = u_1 and location = l_1
8	Time = t_{15} → user = u_1
9	User = u_1 and service = s_3 → location = l_3
10	User = u_2 and device = d_1 → service = s_3
11	User = u_3 and service = s_3 → location = l_5 and time = t_{18}
12	Location = l_2 and time = t_{16} → service = s_4
13	Location = l_2 and service = s_4 → time = t_{16}
14	Location = l_5 and service = s_3 → user = u_3 time = t_{18}
15	User = u_1 and device = d_2 and location = l_1 → service = s_2

Appendix

Questionnaire

Your age: —

Gender: male/female

(1) To what extent do you think the user will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(2) To what extent do you think the location will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(3) To what extent do you think the time will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(4) To what extent do you think the device will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(5) To what extent do you think the service will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(6) To what extent do you think the network bandwith will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(7) To what extent do you think the temperature will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(8) To what extent do you think the law will influence the behavior?

(A) Strongly disagree
(B) Disagree
(C) Neutral
(D) Agree
(E) Strongly agree

(9) Others factors will influence the behavior, such as —.

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Research Article

Coordination in the Decentralized Assembly System with Dual Supply Modes

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This paper investigates a decentralized assembly system that consists of one assembler and two independent suppliers; wherein one supplier is perfectly reliable for the production, while the other generates yield uncertainty. Facing the random market demand, the assembler has to order the components from one supplier in advance and meanwhile requires the other supplier to deliver the components under VMI mode. We construct a Nash game between the supplier and the assembler so as to derive their equilibrium procurement/production strategies. The results show that the channel's performance is highly undermined by the decentralization between players and also the combination of two supply modes. Compared to the centralized system, we propose an advance payment contract to perfectly coordinate the supply chain performance. The numerical examples indicate some management implications on the supply mode comparison and sensitivity analysis.

1. Introduction

Assembly systems have been widely applied in automobile, electronics, and many other manufacturing industries. One fundamental advantage of this system is to help the core assembler/manufacturer take the full advantages (e.g., low procurement cost) from his suppliers. Therefore, we have seen that many famous manufacturers (e.g., Toyota and HP) prefer to outsource their components production process to those external suppliers who locate in Asia, so as to significantly reduce their procurement cost and improve the production efficiency. Nonetheless, the accompanying challenge is how to ensure that the independent suppliers move cooperatively in the system. Difficulties arise from two aspects. First, because a single firm normally cares for its own profit, its decisions inevitably go against the channel's overall efficiency. Second, there are multiple uncertain factors that lie in the system, for example, delivery time, customer demand, and production yield, which subsequently undermine the system's operational performance. Therefore, to mitigate the decentralization and improve the system's performance, scholars have made extensive researches on the optimization mechanism in the decentralized assembly systems [1–4].

Nonetheless, in this rich stream of literature most scholars assume that the suppliers have to adopt the same strategies, in which all of them are either enrolled in the *ordering mode* or in the *VMI mode* (vendor managed inventory). Ordering mode is defined as were the assembler orders the components from all his suppliers before the demand is realized. As a result, suppliers just need to follow the assembler's instructions. For example, Yano [1] studies an assembler's optimal order time, wherein both suppliers have stochastic supply lead times. In contrast, under VMI mode all the suppliers have to personally determine when and how to produce the components to the assembler after knowing the demand information. For this issue, Gerchak and Wang [5] investigate how to achieve the coordination with both revenue-sharing contract and buy-back contract under VMI mode.

Differently, in this paper we will combine these two supply modes into a single assembly system, wherein one supplier exerts the ordering mode, while the other supplier has to accept the VMI mode. To our knowledge, this combination is rarely discussed in the prior literature but is prevalent in practice. For example, in China's auto industry, the assembler needs to order and pay for the key components, such as engine, from the overseas suppliers in advance. Meanwhile,

the of the rest components such as windshield wipers are normally provided by the domestic suppliers under VMI mode. Also in the electronics industry, assembler always places orders in advance from key components' provider (e.g., Intel) while requires the local suppliers to produce the other components under VMI arrangement. Given this gap between the practice and the literature, in this paper we will derive some important implications for the firms' equilibrium strategies under the combined supply modes.

To this end, we construct a decentralized assembly system that contains two sources of uncertainty. On one hand, the market demand is stochastic so that the assembler and the supplier have to make their procurement/production decisions before the exact demand is realized. On the other hand, we assume that supplier 1's production is perfectly reliable, while supplier 2 has a stochastic production yield rate. The assembler has to preorder the components from supplier 1 under the ordering mode but is also able to exert the VMI mode on supplier 2. Consequently, the assembler and supplier 2 have to simultaneously choose their production and procurement quantities in consideration of the other's response. This setting helps us to solve the following questions. First, what are the firms equilibrium production and procurement strategies in this decentralized assembly system with dual supply modes? Second, compared to the centralized system how to achieve the supply chain coordination? Third, what are the differences among different supply modes, for example, ordering mode versus VMI mode versus combined mode?

Given the equilibrium strategies, our analysis has the following observations. First, double marginalization significantly undermines the channel's efficiency, in which case both suppliers produce fewer components than that in the centralized system. In particular, the supplier with random yield produces more components than his partner's. Second, to improve the channel efficiency, we incorporate an advance payment contract. When the sharing parameter (donated as λ) falls into a rational range, the suppliers achieve perfect coordination. Moreover, we show that from the channel's perspective, ordering mode dominates both the combined mode and VMI mode. This result coincides with the supplier's interest but goes against the assembler's profit. The intuition is that for any firm, the better operational mode always mitigates its inventory risk. Overall, this paper speaks to the interactions between the assembler and the suppliers under dual supply modes in a decentralized system.

Besides the literature reviewed above, another stream of the related research is random yield and random demand. Note that this issue has been extensively studied by many scholars [6–10]. However, as aforementioned the majority of these models have been developed under the condition that all the suppliers generate either production yield uncertainty *or* demand uncertainty, while in this paper, we assume that one supplier is perfectly reliable while the other supplier has a random yield issue. This setting is prevalent in both literature [11, 12] and practice. For example, the key component providers in assembly system (e.g., Intel) are always stable in their production capacity and can operate quite well. Therefore, they are nearly perfectly reliable. In contrast, the other component providers may suffer yield uncertainty, because they are usually in small size and unreliable in managing the production. One notable paper is Pan and So [13], in which they analyze an assemble-to-order system with these two types of supply. In their paper, assembler determines both kinds of suppliers' production input quantity in the interest of maximizing system's profit when facing price-dependent demand. Different from them, in our research there are multiple decision makers: the assembler decides the production quantity, while the supplier determines his production quantity.

The remainder of this paper is organized as follows. In Section 2, we describe the model setting. The equilibrium decisions of both decentralized and centralized systems are derived in Section 3. Section 4 investigates supply chain coordination and numerical examples. Section 5 concludes the paper and discusses some future researches.

2. Model Descriptions

Consider a decentralized assembly system that consists of two suppliers and one assembler, in which all the participants are risk-neutral and in purpose of maximizing their own profits. The final product consists of two components, as we define them as the key component and the matching component. The key component is provided by supplier 1 under ordering mode, and its unit production cost is c_1 and the unit wholesale price is w_1 ($w_1 \geq c_1$). Differently, supplier 2 produces the matching component under VMI mode with unit production cost c_2, and the unit wholesale price for component 2 is w_2 ($w_2 \geq c_2$). We assume that supplier 1's production yield is perfectly reliable, while supplier 2 has a random production yield. In particular, if supplier 2's initial input quantity is q, finally the output quantity of the components (that meet the quality level) is θq. θ is a random variable that falls into $[0, 1]$ with probability distribution function $g(\cdot)$ and cumulative distribution function $G(\cdot)$ [13–15]. On the other hand, the market demand D is also stochastic and follows the probability distribution function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ in $[0, \infty)$. Without loss of generality, we assume that the final product's unit price is p ($p \geq w_1 + w_2$) and the assembly cost equals 0 (actually, if the cost $c_a > 0$, the final product's unit price can be modified to $p' = p + c_a$).

In a single-period setting, facing a random market demand D, the assembler determines the order quantity Q_1 from supplier 1 and takes the corresponding cost of overstock and shortage. This is the interaction between supplier 1 and assembler under ordering mode. In contrast, under the VMI contract with supplier 2, supplier 2 is the only decision maker that determines the input quantity Q_2 of matching component and will not receive the payment until the component is consumed. After both components' production and delivery are finished, the assembler assembles the components together and sells them to the customer. Once the overall product quantity is lower than the market demand ($\min E(Q_1, \theta Q_2) < D$), the assembler will be penalized by the customer for shortage. The unit penalty cost is β. At the same time, if supplier 2's production quantity θQ_2 is insufficient ($\theta Q_2 < D$), he will also receive the punishment from the

assembler. The unit penalty cost is β_2 ($\beta_2 \leq \beta$). Besides, we normalize the salvage value of mismatched components to zero.

In summary, in this assembly system the assembler and supplier 2 simultaneously decide the production input quantity of the key component and the matching component. Therefore, we can develop a static Nash game model between the assembler and supplier 2 to derive their equilibrium strategies. In the following section, we first set up a benchmark by investigating the centralized system. After that, we focus on the decentralized system with dual supply modes. As a matter of convenience, we use X^d to denote the condition in the decentralized system and X^c to represent the condition in the centralized system.

3. System's Optimal Decisions

In this section, we first set up a benchmark by studying the centralized system. Afterwards, we study the Nash game between the assembler and supplier 2 in the decentralized system. Comparing the channel's performances under these two scenraios, we finally discuss the supply chain coordination mechanism in the system.

3.1. Centralized System. In a centralized system, the assembler and the two suppliers will cooperate as one to achieve the system's highest performance. Therefore, we first formulate a centralized system's (denoted as B) expected profit function as follows:

$$\begin{aligned}\Pi_B^c(Q_1,Q_2) = {} & pE\left[\min\left(Q_1,\theta Q_2,D\right)\right] - \left(c_1Q_1 + c_2Q_2\right) \\ & - \beta E\left[D - \min\left(Q_1,\theta Q_2\right)\right]^+.\end{aligned} \tag{1}$$

Note that if the channel is centralized, its payoff is entirely determined by the production quantities of two components. As in (1), the first term is the final product's expected sales revenue. The second term is the production cost of both the key and the matching components. The third term is the expected penalty when stock-out occurs. Fixing either Q_i ($i = 1,2$), the maximization of $\Pi_B^c(Q_1,Q_2)$ becomes a news-vendor problem. Thus, we can use the following lemma to characterize the property of $\Pi_B^c(Q_1,Q_2)$. Note that throughout paper, all the proofs of lemmas and theorems are given in the appendix.

Lemma 1. *The centralized system's expected profit function, $\Pi_B^c(Q_1,Q_2)$, is jointly concave in $Q_1 \in [0,\infty)$ and $Q_2 \in [0,\infty)$. The unique optimal input quantity of both the key and the matching components, Q_1^{c*} and Q_2^{c*}, meets the F.O.Cs:*

$$\begin{aligned}&\frac{\partial \Pi_B^c(Q_1,Q_2)}{\partial Q_2} \\ &\quad = (p+\beta)\int_0^{Q_1/Q_2}\int_{\theta Q_2}^{\infty}\theta f(D)\,g(\theta)\,dD\,d\theta - c_2 = 0,\end{aligned} \tag{2}$$

$$\begin{aligned}&\frac{\partial \Pi_B^c(Q_1,Q_2)}{\partial Q_1} \\ &\quad = (p+\beta)\int_{Q_1/Q_2}^{1}\int_{Q_1}^{\infty} f(D)\,g(\theta)\,dD\,d\theta - c_1 = 0.\end{aligned} \tag{3}$$

From Lemma 1, we can always find the unique pair of (Q_1^{c*}, Q_2^{c*}) that maximizes $\Pi_B^c(Q_1,Q_2)$. Moreover, we show that (2) and (3) are symmetric and irrespective to the parameter θ. Given so, we can derive the relationship of Q_1^{x*} and Q_2^{x*} (x represents d and c). The following theorem states that $Q_2^{x*} > Q_1^{x*}$ always holds.

Theorem 2. *In centralized system, the optimal input quantity of the matching component is strictly larger than the optimal input quantity of the key component, that is, $Q_2^{c*} > Q_1^{c*}$.*

The intuition behind Theorem 2 can be explained as follows. Consider that with random yield, if the input quantity of matching component is fewer than that of the key component, the output θQ_2 must be eternally fewer than Q_1. This implies that the penalty cost of matching component's shortage can be reduced by enlarging its input quantity, which is also beneficial to supplier 2 and the entire system. Therefore, only if Q_2 exceeds Q_1, they can reach an equilibrium. Now, we have derived the equilibrium decisions in the centralized system. This will be compared with the following scenario wherein the two players make their decisions independently.

3.2. Decentralized System. In the decentralized system, assembler (denoted as A) will pay supplier 1 (denoted as S_1) immediately after he finishes the key component's delivery. Therefore, his profit function is $\Pi_{S_1}^d = (w_1 - c_1)Q_1$, which is nonnegative and meets the participant constraint.

As to supplier 2 (denoted as S_2), he will not receive the payment until the matching component is consumed. Besides, supplier 2 will be penalized by the assembler if his output is less than the customer's demand. Thus, supplier 2's expected profit function can be formulated as

$$\Pi_{S_2}^d = w_2E\left[\min\left(Q_1,\theta Q_2,D\right)\right] - c_2Q_2 - \beta_2E\left[D - \theta Q_2\right]^+. \tag{4}$$

In (4), the first term is the expected revenue. The second term is the production cost, which is based on the input quantity. The third term stands for the expected penalty when the stock out of the matching component occurs. Similarly, we have the assembler's expected profit function as follows:

$$\begin{aligned}\Pi_A^d = {} & (p - w_2)\,E\left[\min\left(Q_1,\theta Q_2,D\right)\right] - w_1Q_1 \\ & - \beta E\left[D - \min\left(Q_1,\theta Q_2\right)\right]^+ + \beta_2E\left[D - \theta Q_2\right]^+.\end{aligned} \tag{5}$$

Note that in the above equation, the first two terms are the final product's expected sales revenue minus the procurement cost of the key and the matching components. The third term is the penalty penalized by the customer when the stock-out occurs. The fourth one is the compensation from supplier 2

when his yield cannot meet the customer's demand. The following lemma demonstrates that the objective functions, $\Pi_{S_2}^d$ and Π_A^d, are concave in supplier 2's and supplier 1's production (input) quantity, respectively.

Lemma 3. *Let $Q_i^{d*}(Q_j)$ denote the optimal production input quantity of the key/matching component for a given Q_j ($i, j = 1, 2$; $i \neq j$). Then, we have the following.*

(i) *Supplier 2's profit function, $\Pi_{S_2}^d$, is concave in $Q_2 \in [0, \infty)$. And the optimal input quantity $Q_2^{d*}(Q_1)$ meets the first-order condition:*

$$\frac{\partial \Pi_{S_2}^d (Q_1, Q_2)}{\partial Q_2} = w_2 \int_0^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) \, dD \, d\theta + \beta_2 \int_0^1 \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta) \, dD \, d\theta - c_2 = 0. \tag{6}$$

(ii) *The assembler's profit function, Π_A^d, is concave in $Q_1 \in [0, \infty)$. And, the optimal input quantity $Q_1^{d*}(Q_2)$ meets the first-order condition:*

$$\frac{\partial \Pi_A^d (Q_1, Q_2)}{\partial Q_1} = (p + \beta - w_2) \int_{Q_1/Q_2}^1 \int_{Q_1}^{\infty} f(D) g(\theta) \, dD \, d\theta - w_1 = 0. \tag{7}$$

With Lemma 3, we can characterize the optimal production quantity of both key and matching components through their first-order-conditions (F.O.Cs) and the static Nash game equilibrium. In particular, we have the following theorem.

Theorem 4. *There is a unique static Nash equilibrium solution $D(Q_1^{d*}, Q_2^{d*})$ to the assembler's and supplier 2's decisions of the key and the matching components' production quantity. The solution (Q_1^{d*}, Q_2^{d*}) meets the combination of* (6) *and* (7).

Theorem 4 derives the unique optimal production quantity of the key component (Q_1^{d*}) and the matching component (Q_2^{d*}), which are given under the exogenous operational parameters, for example, the wholesale price w_1 and w_2. These indicate the highest payoffs that the assembler and suppliers can achieve in the decentralized system, which are certainly lower than those in the centralized system. Therefore, to identify the gap (inefficiency) between these two scenarios (decentralized versus centralized), we have the following theorem.

Theorem 5. *In the decentralized system, the summation of all the participants' optimal profit is strictly less than the centralized system's optimal total profit, that is,*

$$\left(\Pi_{S_1}^d + \Pi_{S_2}^d + \Pi_A^d\right)\Big|_{Q_1 = Q_1^{d*}, Q_2 = Q_2^{d*}} < \Pi_B^c \left(Q_1^{c*}, Q_2^{c*}\right). \tag{8}$$

From Theorem 5, we can see that although there exists a unique static Nash equilibrium in the decentralized system, it is necessary to introduce a proper contract to coordinate the supply chain to move like a centralized system. Therefore, we propose an advance payment contract to achieve the supply chain coordination.

3.3. Supply Chain Coordination. Among the previous literature, a number of different contract types aiming at coordinating the supply chain are discussed, for example, the revenue sharing contract, the buy-back contract, and so forth. For a detailed review, please refer to Cachon [16]. Differently, in this paper we focus on another contract type: *advance payment contract*. That is, at the beginning of a production period, the assembler pays $(\lambda_1 \Pi_B^{c*} + c_1 Q_1^{c*})$ to supplier 1 and $(\lambda_2 \Pi_B^{c*} + c_2 Q_2^{c*})$ to supplier 2. After receiving the customer's order, the assembler announces the production input quantity of both the key and the matching components, Q_i^{c*} ($i = 1, 2$). Afterwards, the suppliers carry out the production according to their announcement. Finally, the assembler assembles the components into the products and sells them to the customer. Accordingly, he obtains the profit of $(1 - \sum_{i=1}^2 \lambda_i)\Pi_B^{c*}$, where $\lambda_i \in [0, 1]$ and $\sum_{i=1}^2 \lambda_i \leq 1$.

Note that the major feature of the above contract is that the assembler should pay the suppliers in advance. Thus, if supplier i's profit $(\lambda_i \Pi_B^{c*})$ is less than what he can gain in the decentralized system, the contract will fail in practice. Therefore, we have the following theorem to state that there always exists a proper pair of (λ_1, λ_2) which can successfully implement the contract.

Theorem 6. *The decentralized assembly system with dual supply modes always can be coordinated through the advance payment contract. In which, the contract parameter λ_i meets the following conditions:*

$$\frac{\Pi_{S_i}^{d*}}{\Pi_B^{c*}} \leq \lambda_i \leq 1 - \frac{\Pi_A^{d*} + \Pi_{S_j}^{d*}}{\Pi_B^{c*}}, \qquad \sum_{i=1}^2 \lambda_i \leq 1 - \frac{\Pi_A^{d*}}{\Pi_B^{c*}}, \tag{9}$$

wherein $\Pi_y^{d*} = \Pi_y^d(Q_1^{d*}, Q_2^{d*})$, $y \in \{S_1, S_2, A\}$, and $i, j = 1, 2$, $i \neq j$.

With Theorem 6, we can draw the insight that the advance payment successfully makes the suppliers operate as the assembler's subsidiaries, so as to achieve the supply chain coordination. Besides, under this contract the assembler actually changes the business scenario with supplier 2 by deciding the input quantity of the matching component himself. This implies that when facing two kinds of supply, the VMI arrangement with the supplier who generates yield uncertainty is not beneficial for the entire supply chain. To this point, we further study the comparison of the decentralized and centralized systems and investigate the effectiveness of the advance payment contract in the following section.

4. Numerical Analysis

In this section, we conduct two numerical examples. In the first example, we make a comparison between components'

Table 1: Performance with different penalty, price, and demand.

Demand	x	(Q_1^{x*}, Q_2^{x*})		$\Pi_{S_1}^{x\,*}$		$\Pi_{S_2}^{x\,*}$		$\Pi_A^{x\,*}$		$\sum_y \Pi_y^{x*}$	
		Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
$N(40, 1.5)$	d	(39.5, 56.1)	(39.8, 60.1)	79.00	39.80	32.46	54.15	184.05	162.60	295.51	256.54
	d'	(40.6, 70.9)	(40.5, 64.0)	81.20	40.50	70.90	96.00	175.54	128.07	327.64	264.57
	c	(41.3, 82.0)	(41.0, 78.8)			/				333.63	274.89
$N(35, 2)$	d	(34.3, 48.6)	(34.8, 52.9)	68.60	34.80	24.44	42.23	147.40	134.52	240.44	211.55
	d'	(35.8, 62.8)	(35.6, 56.5)	71.60	35.60	62.80	84.75	139.21	98.96	273.61	219.31
	c	(36.7, 73.4)	(36.5, 70.1)			/				279.39	229.05
$N(45, 1)$	d	(44.6, 62.6)	(44.9, 68.4)	89.20	44.90	40.10	63.72	213.29	191.80	342.59	300.42
	d'	(45.4, 79.4)	(45.3, 71.7)	90.80	45.30	79.40	107.56	208.46	153.89	378.66	306.75
	c	(45.8, 92.0)	(45.7, 88.2)			/				384.89	318.10

Case 1: $c_1 = 4$, $w_1 = 6$, $c_2 = 3$, $w_2 = 8$, $\beta_2 = 4$, $p = 25$, and $\beta = 15$.
Case 2: $c_1 = 4$, $w_1 = 5$, $c_2 = 3$, $w_2 = 9$, $\beta_2 = 5$, $p = 23$, and $\beta = 13$.

equilibrium quantities and firms' expected payoffs under three alternative scenarios: the decentralized system with dual supply modes, the decentralized system with ordering mode [1, 17], and the centralized system. This helps us to identify the magnitude of decentralization in the assembly system with dual supply modes. Second, we examine the robustness of advance payment contract on the supply chain coordination by making the sensitivity analysis of sharing rate λ_i $(i = 1, 2)$.

4.1. Comparison of the Alternative Scenarios. In Section 3.3, we have shown that dual supply modes is never beneficial to the entire supply chain, which is caused by the conflict between two supply modes: ordering mode and VMI mode. To better distinguish the difference between these two mode, we next introduce another scenario that the assembly system only contains ordering mode (denoted as d'), wherein both the key and the matching components' input quantity are selected by the assembler. Intuitively, under such a circumstance the supply chain's performance should be better than that with dual supply modes; however, it is still worse than the centralized system since the decentralization still exists.

With only ordering mode, the assembler independently decides both the input quantities of key component and matching component. Therefore, each party's expected profit functions change to

$$\Pi_{S_1}^{d'} = (w_1 - c_1) Q_1, \qquad \Pi_{S_2}^{d'} = w_2 E[\theta Q_2] - c_2 Q_2, \tag{10}$$

$$\begin{aligned}\Pi_A^{d'} &= pE[\min(Q_1, \theta Q_2, D)] - w_1 Q_1 - w_2 E(\theta Q_2) \\ &\quad - \beta E[D - \min(Q_1, \theta Q_2)]^+.\end{aligned} \tag{11}$$

Following the similar principle of Lemma 1, we can easily find that (11) is jointly concave in Q_1 and Q_2, and the optimal $(Q_1^{d'*}, Q_2^{d'*})$ meets the following F.O.Cs:

$$\begin{aligned}\frac{\partial \Pi_A^{d'}(Q_1, Q_2)}{\partial Q_1} &= (p + \beta) \int_0^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D) g(\theta)\, dD\, d\theta \\ &\quad - w_2 \int_0^1 \theta g(\theta)\, d\theta = 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_A^{d'}(Q_1, Q_2)}{\partial Q_1} &= (p + \beta) \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} f(D) g(\theta)\, dD\, d\theta - w_1 = 0.\end{aligned} \tag{12}$$

Due to the F.O.Cs and the profit functions' complexity, we now make the comparison of the three alternative scenarios via numerical approaches. In the following numerical example, we conduct several comparisons under different groups of parameters.

Example 1. Assume that the customer's demand follows the normal distribution and the production random variable of supplier 2 follows the uniform distribution. See Table 1.

Note that Table 1 represents the key and the matching components' optimal input quantity, the expected profits of two suppliers and the assembler, and the expected total profit of the supply chain for three alternative scenarios. As observed from the above table, we have $\sum_y \Pi_y^{d*} < \sum_y \Pi_y^{d'*} < \Pi_B^{c*}$, which demonstrates that the decentralized systems generate incoordination, and the decentralized system only with ordering mode performs better than the one with dual supply modes. Specifically, both the key and the matching components' optimal input quantity in the decentralized systems are less than those in the centralized system, that is, $Q_1^{d*} < Q_1^{d'*} < Q_1^{c*}$ and $Q_2^{d*} < Q_2^{d'*} < Q_2^{c*}$, which directly makes the performance of decentralized systems become worse than the centralized system. Besides, there exist $\Pi_{S_1}^{d'*} > \Pi_{S_1}^{d'*}$, $\Pi_{S_2}^{d'*} > \Pi_{S_2}^{d'*}$, and $\Pi_{S_2}^{d'*} < \Pi_{S_2}^{d*}$. This implies that from the supplier's perspective, they prefer to providing components with the ordering mode. In contrast, under the decentralized situation, the assembler will certainly choose the VMI mode with the supplier who generates yield uncertainty, even though this option significantly undermines the channel's performance.

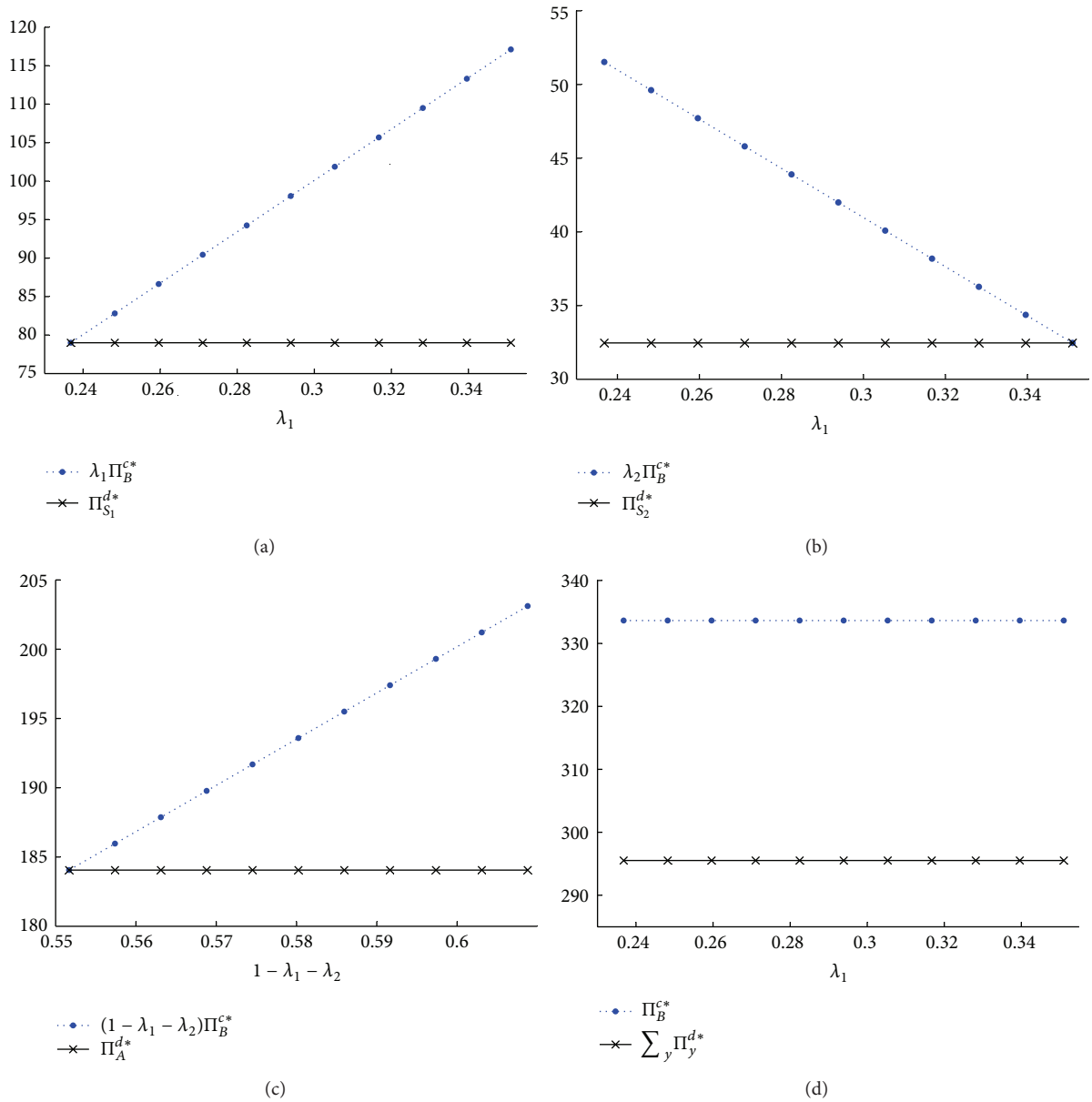

Figure 1: Effectiveness of the advance payment contract under different λ_i.

4.2. Sensitivity Analysis on λ_i. In the above discussion, we have shown that the advance payment contract can perfectly coordinate the decentralized supply chain by setting the appropriate λ_i. We next identify how the firms' equilibrium payoffs react to the variance of λ_i.

Example 2. Given the following data: $c_1 = 4$, $w_1 = 6$, $c_2 = 3$, $w_2 = 8$, $\beta_2 = 4$, $p = 25$, $\beta = 15$; $D \backsim N(40, 1.5)$, $\theta \backsim U(0, 1)$, let λ_1 take the value uniformly distributed over $[(\Pi_{S_1}^{d*}/\Pi_B^{c*}), 1 - ((\Pi_A^{d*} + \Pi_{S_2}^{d*})/\Pi_B^{c*})]$ and λ_2 take the value of $[(1 - \lambda_1 - (\Pi_A^{d*}/\pi_B^{c*})) + (\Pi_{S_2}^{d*}/\Pi_B^{c*})]/2$, which is in $[(\Pi_{S_2}^{d*}/\Pi_B^{c*}), 1 - ((\Pi_A^{d*} + \Pi_{S_1}^{d*})/\Pi_B^{c*})]$. Figure 1 shows the effectiveness of the advance payment contract.

In Figure 1, we can see that as long as the λ_i is in its valid interval, all the suppliers and the assembler can be better off and the decentralized system can be perfectly coordinated with the advance payment contract. Therefore, it is effective and practicable.

5. Conclusion

In this paper, we investigate the equilibrium production and procurement strategies in a decentralized assembly system consisting of a single assembler and two suppliers. In particular, the assembler orders the key component from supplier 1 who is perfectly reliable, while supplier 2 provides the matching component under VMI mode with yield uncertainty. We derive the components' optimal production input quantity in

a static Nash game model and also set up a benchmark case by identifying the centralized system. Given the comparison between these two scenarios, we propose an advance payment contract to achieve supply chain coordination. We also make two numerical examples and find that (1) the less components' optimal production input quantity makes the decentralized systems perform worse than the centralized system, (2) the assembler prefers VMI mode in practice so as to maximize his own expected profit, and (3) advance payment contract is effective regardless the sharing rate.

One extension of our model is to consider a more general assembly system with one assembler and N (≥ 3) suppliers, where the decisions of those suppliers with yield uncertainty can affect each other. Besides, it can be extended to other random yield models. For example, in semiconductor industry, the output of chips $Y(q)$ is a nonlinear function of the input of silicon wafers q, and other related factors. These extensions certainly have the potential to be better explored in the future.

Appendix

Proof of Lemma 1. Equation (1) can be rewritten as $\Pi_B^c(Q_1, Q_2) = pC - \beta D - (c_1 Q_1 + c_2 Q_2)$, wherein

$$\begin{aligned} C &= \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} Q_1 f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta Q_2 f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \int_{0}^{Q_1} \int_{D/Q_2}^{1} D g(\theta)\, f(D)\, d\theta\, dD, \\ D &= \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} (D - Q_1)\, f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} (D - \theta Q_2)\, f(D)\, g(\theta)\, dD\, d\theta. \end{aligned} \tag{A.1}$$

Taking first- and second-order derivatives with respect to Q_1 and Q_2 in (1) separately, we get

$$\begin{aligned} \frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_2} &= (p+\beta) \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D)\, g(\theta)\, dD\, d\theta - c_2, \\ \frac{\partial \Pi_B^c(Q_1, Q_2)}{\partial Q_1} &= (p+\beta) \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} f(D)\, g(\theta)\, dD\, d\theta - c_1, \\ \frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_2^2} &= -(p+\beta)\left(E + \frac{Q_1^2}{Q_2^3} F\right), \\ \frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1^2} &= -(p+\beta)\left(G + \frac{1}{Q_2} F\right), \\ \frac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1 \partial Q_2} &= (p+\beta) \frac{Q_1}{Q_2^2} F. \end{aligned} \tag{A.2}$$

Here, $E = \int_0^{Q_1/Q_2} \theta^2 f(\theta Q_2) g(\theta) d\theta > 0$, $F = \int_{Q_1}^{\infty} f(D) g(Q_1/Q_2) dD > 0$, and $G = \int_{Q_1/Q_2}^{1} f(Q_1) g(\theta) d\theta > 0$. Then, we derive the Hessen Matrix as follows:

$$\begin{aligned} H &= \begin{vmatrix} \dfrac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_1 \partial Q_2} \\ \dfrac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_2^2} & \dfrac{\partial^2 \Pi_B^c(Q_1, Q_2)}{\partial Q_2 \partial Q_1} \end{vmatrix} \\ &= (p+\beta) \begin{vmatrix} -\left(G + \dfrac{1}{Q_2} F\right) & \dfrac{Q_1}{Q_2^2} F \\ \dfrac{Q_1}{Q_2^2} F & -\left(E + \dfrac{Q_1^2}{Q_2^3} F\right) \end{vmatrix}. \end{aligned} \tag{A.3}$$

In (A.3), the value of the first-order determinant is $-(p+\beta)(G + (1/Q_2)F) < 0$ and the value of the second-order determinant is $(p+\beta)(EG + (Q_1^2/Q_2^3)FG + (1/Q_2)EF) > 0$. As such, the Hessen Matrix is negative definite. Therefore, $\Pi_B^c(Q_1, Q_2)$ is joint concave in $Q_1 \in [0, \infty)$ and $Q_2 \in [0, \infty)$. And the optimal production input quantity (Q_1^{c*}, Q_2^{c*}) meets the F.O.Cs, which consists of $(\partial \Pi_B^c(Q_1, Q_2))/\partial Q_2 = 0$ and $(\partial \Pi_B^c(Q_1, Q_2))/\partial Q_1 = 0$. □

Proof of Lemma 3. Equation (4) can be rewritten as $\Pi_{S_2}^d = w_2 A - \beta_2 B - c_2 Q_2$, wherein

$$\begin{aligned} A &= \int_{Q_1/Q_2}^{1} \int_{Q_1}^{\infty} Q_1 f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta Q_2 f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \int_{0}^{Q_1} \int_{D/Q_2}^{1} D g(\theta)\, f(D)\, d\theta\, dD, \\ B &= \int_{0}^{1} \int_{\theta Q_2}^{\infty} (D - \theta Q_2)\, f(D)\, g(\theta)\, dD\, d\theta. \end{aligned} \tag{A.4}$$

Fix Q_1, taking first- and second-order derivatives with respect to Q_2 in (4), we get

$$\begin{aligned} \frac{\partial \Pi_{S_2}^d}{\partial Q_2} &= w_2 \int_{0}^{Q_1/Q_2} \int_{\theta Q_2}^{\infty} \theta f(D)\, g(\theta)\, dD\, d\theta \\ &\quad + \beta_2 \int_{0}^{1} \int_{\theta Q_2}^{\infty} \theta f(D)\, g(\theta)\, dD\, d\theta - c_2, \\ \frac{\partial^2 \Pi_{S_2}^d}{\partial Q_2^2} &= -w_2 \left\{ \int_{0}^{Q_1/Q_2} \theta^2 f(\theta Q_2)\, g(\theta)\, d\theta \right. \\ &\quad \left. + \int_{Q_1}^{\infty} \frac{Q_1^2}{Q_2^3} f(D)\, g\left(\frac{Q_1}{Q_2}\right) dD \right\} \\ &\quad - \beta_2 \int_{0}^{1} \theta^2 f(\theta Q_2)\, g(\theta)\, d\theta. \end{aligned} \tag{A.5}$$

It is easy to find that $\partial^2\Pi^d_{S_2}/\partial Q_2^2 < 0$. Therefore, $\Pi^d_{S_2}$ is concave in Q_2. Besides, we observe that $\partial\Pi^d_{S_2}/\partial Q_2|_{Q_2=0} = w_2 + \beta_2\overline{\theta} - c_2 > 0$ and $\partial\Pi^d_{S_2}/\partial Q_2|_{Q_2\to\infty} = -c_2 < 0$. As such, there must exist a $Q_2 \in [0,\infty)$ that meets the first-order condition $\partial\Pi^d_{S_2}/\partial Q_2 = 0$.

Similarly, fix Q_2, taking first- and second-order derivatives with respect to Q_1 in (5), we get

$$\frac{\partial\Pi^d_A}{\partial Q_1} = (p+\beta-w_2)\int_{Q_1/Q_2}^{1}\int_{Q_1}^{\infty} f(D)\,g(\theta)\,dD\,d\theta - w_1,$$
$$\frac{\partial^2\Pi^d_A}{\partial Q_1^2} = -(p+\beta-w_2) \times \left\{\int_{Q_1/Q_2}^{1} f(Q_1)\,g(\theta)\,d\theta + \frac{1}{Q_2}\int_{Q_1}^{\infty} f(D)\,g\left(\frac{Q_1}{Q_2}\right)dD\right\}. \tag{A.6}$$

We can easily derive $\partial^2\Pi^d_A/\partial Q_1^2 < 0$, $\partial\Pi^d_A/\partial Q_1|_{Q_1=0} = p+\beta-w_2-w_1 > 0$, and $\partial\Pi^d_A/\partial Q_1|_{Q_1\to\infty} = -w_1 < 0$. Therefore, we can claim that Π^d_A is concave in Q_1 and the optimal $Q_1^{d*}(Q_2)$ meets the first-order condition $\partial\Pi^d_A/\partial Q_1 = 0$. □

Proof of Theorem 2. Simplifying (7) and (3), we haveing

$$\left[1-F\left(Q_1^{d*}\right)\right]\times\left[1-G\left(\frac{Q_1^{d*}}{Q_2^{d*}}\right)\right] = \frac{w_1}{p+\beta-w_2}, \tag{A.7}$$

$$\left[1-F\left(Q_1^{c*}\right)\right]\times\left[1-G\left(\frac{Q_1^{c*}}{Q_2^{c*}}\right)\right] = \frac{c_1}{p+\beta}. \tag{A.8}$$

Consider the value of $G(Q_1^{x*}/Q_2^{x*})$. If $Q_1^{d*} \geq Q_2^{d*}$ or $Q_1^{c*} \geq Q_2^{c*}$, $G(Q_1^{x*}/Q_2^{x*}) = 1$. As such, the left parts of (A.7) and (A.8) both equal 0. Meanwhile, the right part of (A.7) and (A.8) both are larger than 0. Thus, both (A.7) and (A.8) cannot be balanced unless $Q_1^{x*} < Q_2^{x*}$. □

Proof of Theorem 4. Considering the profit curves of supplier 2 and the assembler in (4) and (5), we let

$$N_1(Q_1,Q_2) = \frac{\partial\Pi^d_A(Q_1,Q_2)}{\partial Q_1},$$
$$N_2(Q_1,Q_2) = \frac{\partial\Pi^d_{S_2}(Q_1,Q_2)}{\partial Q_2}. \tag{A.9}$$

Taking first-order derivative with respect to Q_1 in $N_1(Q_1,Q_2)$ and $N_2(Q_1,Q_2)$, we have

$$\frac{\partial N_1(Q_1,Q_2)}{\partial Q_1} = \frac{\partial^2\Pi^d_A(Q_1,Q_2)}{\partial Q_1^2} = -(p+\beta-w_2)\left\{\int_{Q_1/Q_2}^{1} f(Q_1)\,g(\theta)\,d\theta + \frac{1}{Q_2}\int_{Q_1}^{\infty} f(D)\,g\left(\frac{Q_1}{Q_2}\right)dD\right\} < 0,$$
$$\frac{\partial N_2(Q_1,Q_2)}{\partial Q_1} = \frac{\partial^2\Pi^d_{S_2}(Q_1,Q_2)}{\partial Q_2\partial Q_1} = w_2\int_{Q_1}^{\infty}\frac{Q_1}{Q_2^2} f(D)\,g\left(\frac{Q_1}{Q_2}\right)dD > 0. \tag{A.10}$$

Then, we get $(\partial N_1(Q_1,Q_2)/Q_1) - (\partial N_2(Q_1,Q_2)/Q_1) < 0$, for all (Q_1,Q_2). As such, the players' profit functions, $\Pi_{S_2}(Q_1,Q_2)$ and $\Pi_A(Q_1,Q_2)$, will meet once at most [3, Proposition 2.1]. Also, referring to Theorem 2.4 in Friedman [18], there is always a Nash equilibrium existing for concave-payoff-functions game like ours. Therefore, the static Nash equilibrium of both component's production input quantity is unique and the solution, (Q_1^{d*},Q_2^{d*}), meets the combination of (6) and (7). □

Proof of Theorem 5. Sum $\Pi^d_{S_1}$, (4), and (5), we get the decentralized system's total profit function:

$$\Pi^d_{S_1} + \Pi^d_{S_2} + \Pi^d_A = pE\{\min(Q_1,\theta Q_2,D)\} - (c_1Q_1 + c_2Q_2) - \beta E[D - \min(Q_1,\theta Q_2)]^+. \tag{A.11}$$

Notice that the formulation is the same as the one of the centralized system. As such, we can compare their optimal performance through the components' optimal production input quantity. Considering (A.7) and (A.8), we have

$$\frac{w_1}{p+\beta-w_2} - \frac{c_1}{p+\beta} = \frac{(p+\beta)(w_1-c_1)+c_1w_2}{(p+\beta)(p+\beta-w_2)} > 0. \tag{A.12}$$

As a result, we can derive that $(Q_1^{d*},Q_2^{d*}) \neq (Q_1^{c*},Q_2^{c*})$. Otherwise, if $Q_1^{d*} = Q_1^{c*}$ and $Q_2^{d*} = Q_2^{c*}$, (A.7) and (A.8) cannot hold simultaneously. Then, plug (Q_1^{d*},Q_2^{d*}) and (Q_1^{c*},Q_2^{c*}) into the system's profit function separately; we can easily get

$$\left(\Pi^d_{S_1} + \Pi^d_{S_2} + \Pi^d_A\right)\Big|_{Q_1=Q_1^{d*},Q_2=Q_2^{d*}} < \Pi^c_B\left(Q_1^{c*},Q_2^{c*}\right), \tag{A.13}$$

since (Q_1^{c*},Q_2^{c*}) is the unique optimal decision. □

Proof of Theorem 6. Under the advance payment contract, each party's expected profit (denoted as Π_y^{co}) can be formulated as follows:

$$\Pi_{S_1}^{co} = \lambda_1 \Pi_B^{c*}, \qquad \Pi_{S_2}^{co} = \lambda_2 \Pi_B^{c*}, \qquad \Pi_A^{co} = \left(1 - \sum_{i=1}^{2} \lambda_i\right) \Pi_B^{c*}. \tag{A.14}$$

To ensure the contract's success, one must satisfy the following constraints:

$$\Pi_y^{co} \geq \Pi_y^{d*}, \quad \forall y \in \{S_1, S_2, A\}. \tag{A.15}$$

Substituting λ_i $(i = 1, 2)$ into (A.15) and combining with $\Pi_B^{c*} > \sum_x \Pi_x^{d*}$, we can easily derive that λ_i meets equation (9). □

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Research Article

Dynamic Pricing and Supply Coordination with Reimbursement Contract under Random Yield and Demand

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This paper investigates the dynamic pricing and supply chain coordination in a decentralized system that consists of one supplier and one manufacturer, in which both the market demand and production yield are stochastic. We show that the centralized expected profit is jointly concave in the production quantity and order quantity when the price is ex-ante selected. We also derive the equilibrium strategies in the decentralized system and prove that the entire profit of supply chain is inevitably lower than that under centralized system. Based on this, we propose a reimbursement contract to coordinate the decentralized supply chain so as to achieve the maximized profit. It is worth mentioning that, under reimbursement contract, the equilibrium production and order quantities are irrelevant to the manufacturer's risk sharing coefficient but are only determined by the supplier's risk sharing coefficient.

1. Introduction

The intense competition in the semiconductor and electronics industry pushes the manufacturers to make substantial effort to reduce the production cost. For example, many manufacturers (e.g., Dell) may cut down the sales representatives or adopt the direct delivery. On the other hand, since the customer demand is getting more variable and price sensitive (e.g., purchasing laptops), the manufacturer also needs to carefully design the order quantity and pricing dynamically so as to extract more profit from the market.

In recent years, supply chain becomes more vulnerable to the influence from natural disasters, strikes, terrorist attacks, political instability, and other factors. In particular, after US "911" incident supply chain uncertainty and its associated potential losses are becoming much significant [1]. Some studies suggest that the frequency of catastrophic events is increasing year by year, and its harms are gradually rising up. Consequently, supply chain risk management raised attentions from both researchers and practitioners in operations management, for example, the multisourcing [2–5], alternative supply sources and backup production options [6–9], flexibility [10, 11], and supplier selection [12–14]. For a recent review of the supply-risk literature, please see the works by Tang and Abhijeet et al. [15, 16]. Generally, after investigation of 800 companies' disruption cases, Hendricks and Singhal find that the firms which experienced supply glitches suffer from declining operational performance and eroding shareholder value (e.g., the abnormal return on stock of such firms is negative 40% over three years) [17, 18].

The issue of linking risk assessment with risk mitigation for low-probability but high-consequence events such as disruptions of supplies is discussed by Kleindorfer and Saad. They provide 10 principles for specifying the sources of risk and assessment and mitigation of risk [19]. In addition, supply chains are also vulnerable to high-likelihood, low-impact operational risks that may arise from problems in supply and production process [20]. One common manifestation of this supply risk is random yield, in which the firm receives a random portion of the order placed with a supplier [21]. Though the production is strictly controlled, yield of the components can be uncertain due to the characteristics of process engineering or uncontrolled operations [22, 23]. For example, it is quite common in the LCD manufacturing industry in which the production yield is normally less than 50%. In other words, manufacturers have to face both

the random yields and random demand in today's competing environment.

Yano and Lee first give thorough review about single item single stage and multiitem multistage in an assembly system with lot sizing [24]. From then on, many scholars considered different random-yield problems from different aspects, and representatives referred to Gurnani et al. [25, 26], Gerchak and Wang [27], Li [28], Güler [29, 30], Tomlin [10], Giri [7], Tang et al. [31], and Gurnani et al. [25] explore a centralized assembly system facing random demand and random yield due to production yield losses. They formulate the exact cost functions with target level of finished products to assemble and the order quantity of the components from suppliers as the decision variables. In a multiperiod case, it is found that it might be optimal to order extra components for future use. Also the optimal ordering policy and assembly target level policy are shown to be an order-up-to type [25]. Gerchak and Wang consider coordination in a decentralized assembly systems with random demand. But they do not consider dynamic pricing and random yield [27]. Gurnani and Gerchak propose coordination in a decentralized assembly system with two suppliers and one manufacturer under uncertain component yield and determined demand. They consider that the component suppliers and manufacturer choose their production quantities and order quantities separately based solely on their own profit structure, but the selling price of final product is determined [26]. Based on Gurnani, Güler and Bilgiç and Keskin (2008, 2013) examine a decentralized assembly system with multisuppliers and one manufacturer under uncertain yield and demand. They compare wholesale price, buy-back, revenue share, quantity discount, and quantity flexibility contracts and propose two combined contracts to coordinate the assembly system. They also illustrate that the randomness in the yield does not change the coordination ability of the contracts but affects the values of the contract parameters [29, 30].

As to the dynamic pricing under random yield, there are few papers related to it. Li studies the joint inventory replenishment and pricing problem for production systems with random demand and yield [28]. Bakal and Akcali consider the effects of recovery yield rate on pricing decisions in reverse supply chains and determine the optimal acquisition price for the end-of-life products [32]. Tomlin and Wang investigate the production, pricing, down conversion, and allocation decisions in a two-class, stochastic-yield coproduction system. They establish that down conversion will not occur if prices are set optimally [10]. Giri considers a single-product single-period inventory model in which the retailer can source from one unreliable supplier with yield uncertainty and the other supplier which is more reliable but expensive [7]. Tang et al. consider a newsvendor problem with random demand and random yields, in which the price decision will be postponed and determined upon recognition of random yield and prior to realizing demand uncertainties [31].

To the best of our knowledge, most literature under random component yield has focused on centralized system where the price is ex-ante selected [33–37]. Some have studied establishing properties of the profit function of the chain and found the optimal order quantity but without pricing control [38]. Few have concentrated on dynamic pricing under random yield, but they consider different aspects from ours.

Consider that a manufacturer in a decentralized system faces the two-sided uncertainty: random yield and demand. Since lot sizing with uncertain yields is an important area in production/manufacturing systems [24], it is necessary to consider a decentralized system with lot sizing and dynamic pricing under both random yield and demand. To address these gaps between practice and the literature, we investigate the interactions among supply uncertainty, dynamic pricing, and supply chain coordination. In particular, we address the following questions.

(1) What are the optimal decisions at an exogenous price in the centralized system in case of uncertain yield and demand?

(2) What is the best solution to dynamic pricing in the centralized system?

(3) What are the manufacturer and supplier's best decisions in the decentralized system? How do they react to each other?

(4) How does the manufacturer in the decentralized system eliminate the uncertainty, achieve supply coordination, and maximize its profit?

To answer these questions, we construct a supply chain that consists of a single supplier and a single manufacturer. In Section 2, we investigate the basic model that is set as our benchmark. Section 3 provides the optimal decentralized decision for the supplier and manufacturer. Section 4 proposes the reimbursement contract to coordinate the supplier and manufacturer in the decentralized system. Section 5 reports the numerical study to illustrate our optimal decision and performance of our useful model. Section 6 concludes the paper with a summary of results.

2. Model Formulation and Analysis

2.1. Model Description. Consider a two-layer supply chain that contains one manufacturer and one supplier, who are both risk neutral and in purpose of maximizing the profit. The market demand is stochastic and price sensitive. Therefore, we use $y(p)\cdot\varepsilon$ to denote the demand function, in which $y(p)$ is the function of selling price p and ε is the random variable. Also, for simplicity we assume that $y(p)\cdot\varepsilon$ is equal to $y\varepsilon$. The manufacturer decides the selling price p and also the order quantity of size Q from the supplier. Given this order quantity Q, the supplier decides the production quantity x. However, since the production yield is also stochastic, the realized production quantity is αx, wherein α is a random variable in $[0, 1]$. Thus, the final size that will be delivered to the manufacturer is $\min(\alpha x, Q)$. The related parameters are denoted as follows.

p: manufacturer's unit selling price for the final product,

π_m: manufacturer's unit shortage cost for the final product,

c_s: supplier's unit production cost,

w: supplier's wholesale price,

Π_s, Π_m, Π_{sc}: profit of supplier, manufacturer and supply chain, respectively,

μ_α: mean value of α,

$f(\cdot)$, $F(\cdot)$: probability density function and cumulative density function of ε, respectively,

$g(\cdot)$, $G(\cdot)$: probability density function and cumulative density function of α, respectively.

Since $y(p)$ is sensitive to the final price, we use dynamic pricing to derive the optimal price. The manufacturer's profit function can be established as

$$\begin{aligned}\Pi_m = {} & p \cdot \min(\alpha x, Q, y\varepsilon) \\ & - \pi_m [y\varepsilon - \min(\alpha x, Q)]^+ \\ & - w \cdot \min(\alpha x, Q).\end{aligned} \tag{1}$$

The first term stands for the sales revenue, and the second term represents the penalty cost when the random demand exceeds the minimum value of random yield and order quantity. The third term is the purchasing cost for the manufacturer.

Similarly, the profit function for the supplier can be formulated as

$$\Pi_s = w \cdot \min(\alpha x, Q) - c_s x. \tag{2}$$

Note that in (2) the first term stands for sales revenue for the supplier and the second term is production cost.

Combining (1) and (2), we can obtain the profit function of supply chain, where

$$\begin{aligned}\Pi_{sc} = {} & p \cdot \min(\alpha x, Q, y\varepsilon) \\ & - \pi_m [y\varepsilon - \min(\alpha x, Q)]^+ - c_s x.\end{aligned} \tag{3}$$

The basic models formulate a supply chain that is dominated by the manufacturer, in which the manufacturer will maximize supply chain profit and personal own profit under random demand and yield.

2.2. Optimal Decision and Dynamic Pricing for Centralized System. The expected profit for the centralized model is determined by (Q, x), in which the selling price of final product is exogenously determined. The expected profit for the centralized system can be rewritten as follows:

$$\begin{aligned}& E(\Pi_{sc}) \\ & = p \cdot \left[\int_0^{Q/x} \int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right. \\ & \quad + \int_{Q/x}^{1} \int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\ & \quad \left. + \int_0^{Q/y} \int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right] \\ & \quad - \pi_m \left[\int_0^{Q/x} \int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right. \\ & \quad \left. + \int_{Q/x}^{1} \int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right] - c_s x.\end{aligned} \tag{4}$$

We subsequently obtain the optimal (Q^*, x^*), namely, $(Q^*, x^*) = \arg\max E[\Pi_{sc}(Q, x)]$.

Proposition 1. *The expected profit $E[\Pi_c(Q, x)]$ is jointly concave in (Q, x). The optimal order quantity and production quantity are the unique solution to the following two equations:*

$$(p + \pi_m) \int_{Q^*/x^*}^{1} \int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon = 0 \tag{5}$$

$$(p + \pi_m) \int_0^{Q^*/x^*} \int_{\alpha x^*/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon = c_s. \tag{6}$$

Proof. Consider the following:

$$\frac{\partial^2 E(\Pi_{sc})}{\partial Q^2} < 0, \qquad \frac{\partial^2 E(\Pi_{sc})}{\partial x^2} < 0. \tag{7}$$

The Hessian matrix is as follows:

$$H = \begin{pmatrix} \dfrac{\partial^2 E(\Pi_{sc})}{\partial Q^2} & \dfrac{\partial^2 E(\Pi_{sc})}{\partial Q\, \partial x} \\ \dfrac{\partial^2 E(\Pi_{sc})}{\partial x\, \partial Q} & \dfrac{\partial^2 E(\Pi_{sc})}{\partial x^2} \end{pmatrix} > 0. \tag{8}$$

See Appendix for details. □

Obviously (5) and (6) are solutions to (4), which are facilitated by the concavity of expected profit $E[\Pi_c(Q, x)]$. Based on the observation of (5) and (6), a proposition can be drawn as follows.

Proposition 2. *The unique solution to* (5) *must satisfy* $Q^* = x^*$.

Proof. While $(p + \pi_m) \int_{Q^*/x^*}^{1} \int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon) d\alpha\, d\varepsilon \geq 0$, then $\int_{Q^*/x^*}^{1} \int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon) d\alpha\, d\varepsilon \geq 0$.

Only if $Q^* = x^*$, then $\int_{Q^*/x^*}^{1} \int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon) d\alpha\, d\varepsilon = \int_1^1 \int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon) d\alpha\, d\varepsilon = 0$.

Note that $Q^* = x^*$, and (6) can be rewrite in another form as follows:

$$\begin{aligned}& \int_0^{Q^*/x^*} \int_{\alpha x^*/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\ & = \int_0^1 \int_{\alpha Q^*/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\ & = \frac{c_s}{p + \pi_m}.\end{aligned} \tag{9}$$

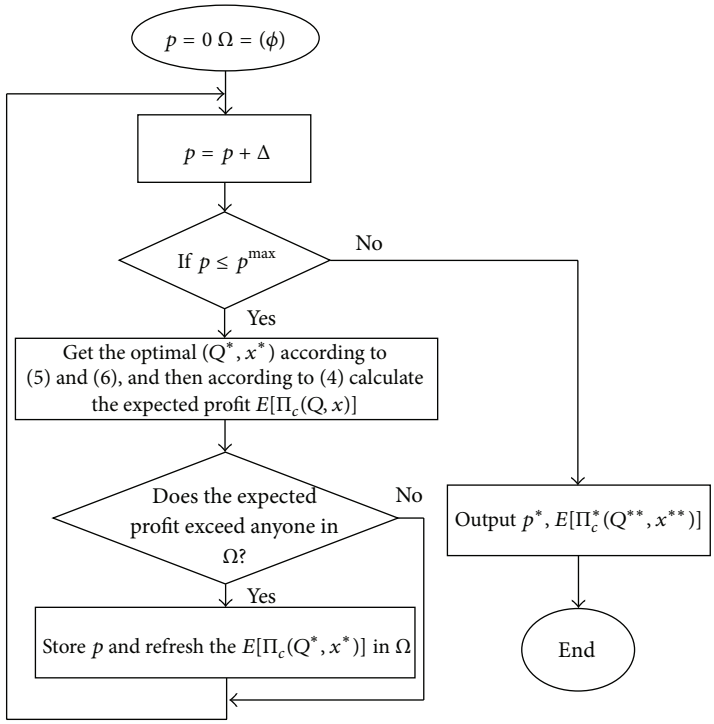

Figure 1: The algorithm process for dynamic pricing. $p^{\max}$, maximized price for a kind of product; Δ, price markup coefficient; $E[\Pi_c^*(Q, x)]$, optimal expected profit of centralized system at the determined price p; $E[\Pi_c^*(Q^{**}, x^{**})]$, optimal expected profit of centralized system at the determined price p^*, while $p^* \in (0, p^{\max}]$.

This is similar to the equation developed by Shih [39] so that the optimal Q^* can be obtained to satisfy (9).

As analyzed above, the expected profit $E[\Pi_c(Q, x)]$ for the centralized system is jointly concave in (Q, x). Since the retail price is exogenously determined, we can derive a different expected profit $E[\Pi_c(Q, x)]$ under a different price p. In the next part, we will discuss the optimal expected profit $E[\Pi_c(Q, x)]$ under the optimal retail price. Therefore, we propose an algorithm to derive the optimal selling price p that can maximize the expected profit $E[\Pi_c(Q, x)]$.

Step 1. Let $\Omega = (\phi)$ and $p = 0$.

Step 2. Set $p = p + \Delta$ (Δ is sufficiently small).

Step 3. If $p \le p^{\max}$, get the optimal (Q^*, x^*) according to (5), and then calculate the expected profit $E[\Pi_c(Q, x)]$ according to (4). If not, then output the optimal p^* and $E[\Pi_c^*(Q^{**}, x^{**})]$.

Step 4. If the new expected profit $E[\Pi_c(Q^*, x^*)]$ exceeds the former one, store p and refresh the $E[\Pi_c(Q^*, x^*)]$ in Ω. Then go to Step 2. If not, return to Step 2 directly.

Step 5. Output the (Q^{**}, x^{**}) which are corresponding to the optimal p^* and $E[\Pi_c^*(Q^{**}, x^{**})]$.

After Step 5, we can derive the optimal expected profit $E[\Pi_c^*(Q^{**}, x^{**})]$ and the optimal price p^*. It also shows that the precision of p^* can be ensured as long as Δ is sufficiently small. The detailed process is also illustrated in Figure 1. □

3. Decentralized Decision for Manufacturer and Supplier

We then consider the decentralized system. As above mentioned, the manufacturer will adopt an optimal p^* to maximize his expected profit $E[\Pi_m(Q)]$, and we start from the manufacturer's decision.

3.1. The Manufacturer's Decision for Order Quantity. According to (1), the manufacturer's expected profit $E[\Pi_m(Q)]$ under the optimal p^* can be explained as follows:

$$\begin{aligned} &E\left[\Pi_m^d(Q)\right] \\ &= p^* \cdot \left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right. \\ &\qquad + \int_{Q/x}^{1}\int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\ &\qquad \left. + \int_0^{Q/y}\int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right] \\ &\quad - \pi_m \left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x)\, g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \right. \end{aligned}$$

$$+\int_{Q/x}^{1}\int_{Q/y}^{\infty} Qg(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\Bigg] - w\left[\int_{0}^{Q/x} \alpha x g(\alpha)\, d\alpha + \int_{Q/x}^{1} Qg(\alpha)\, d\alpha\right]. \tag{10}$$

Then take derivation of $E[\Pi_m^d(Q)]$ in Q, whose process is similar to Appendix. Let us assume that $\partial E[\Pi_m^d(Q)]/\partial Q = 0$; then

$$(p^* + \pi_m)\int_{Q^*/x}^{1}\int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - w\int_{Q^*/x}^{1} g(\alpha)\, d\alpha = 0. \tag{11}$$

Equation (11) is the solution to the manufacturer's expected profit $E[\Pi_m(Q)]$ when the supplier's production quantity is certain.

3.2. The Supplier's Decision for Production Quantity. According to (2), the supplier's expected profit $E[\Pi_s(x)]$ can be explained as follows:

$$E\left[\Pi_s^d(Q)\right] = w\left[\int_{0}^{Q/x} \alpha x g(\alpha)\, d\alpha + \int_{Q/x}^{1} Qg(\alpha)\, d\alpha\right] - c_s \cdot x. \tag{12}$$

$E[\Pi_s^d(x)]$ is differential in x, where

$$\begin{aligned}\frac{\partial E\left[\Pi_s^d(x)\right]}{\partial x} &= w\left[\int_{0}^{Q/x} \alpha g(\alpha)\, d\alpha + Q\cdot g\left(\frac{Q}{x}\right)\left(-\frac{Q}{x^2}\right) - Q\cdot g\left(\frac{Q}{x}\right)\left(-\frac{Q}{x^2}\right)\right] - c_s \\ &= w\int_{0}^{Q/x} \alpha g(\alpha)\, d\alpha - c_s.\end{aligned} \tag{13}$$

Let us assume that $\partial E[\Pi_s^d(x)]/\partial x = 0$; then

$$w\int_{0}^{Q/x^*} \alpha g(\alpha)\, d\alpha - c_s = 0. \tag{14}$$

Equation (14) is the solution to maximizing the supplier's expected profit $E[\Pi_s(Q)]$.

3.3. Analysis of Nash Equilibrium between Manufacturer and Supplier. From (11) and (14), we can derive (Q^{ne}, x^{ne}) so as to derive the following results.

Proposition 3. *There exists a unique Nash equilibrium solution (Q^{ne}, x^{ne}) to the decentralized system between the manufacturer and the supplier.*

Proof. Consider two differentiable functions $f(x)$ and $g(x)$. If $f'(x) - g'(x) > 0$ or $f'(x) - g'(x) < 0$, then there exists x^* which makes $f(x^*) = g(x^*)$ for all x (see Gurnani and Gerchak, [26, Proposition 2.1]).

Therefore,

$$\begin{aligned}&\text{set } f_1(Q,x) = (p^* + \pi_m)\int_{Q^*/x}^{1}\int_{Q^*/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - w\int_{Q^*/x}^{1} g(\alpha)\, d\alpha, \\ &f_2(Q,x) = w\int_{0}^{Q/x} \alpha g(\alpha)\, d\alpha - c_s, \\ &\frac{\partial f_1(Q,x)}{\partial x} = -(p^* + \pi_m)\int_{Q/y}^{\infty} g\left(\frac{Q}{x}\right)\cdot\left(-\frac{Q}{x^2}\right) f(\varepsilon)\, d\varepsilon - w\cdot g\left(\frac{Q}{x}\right)\cdot\left(-\frac{Q}{x^2}\right), \\ &\frac{\partial f_2(Q,x)}{\partial x} = w\cdot\frac{Q}{x}\cdot g\left(\frac{Q}{x}\right)\left(-\frac{Q}{x^2}\right), \\ &\frac{\partial f_1(Q,x)}{\partial x} - \frac{\partial f_2(Q,x)}{\partial x} = (p^* + \pi_m)\int_{Q/y}^{\infty} \frac{Q}{x^2}\cdot g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon + w\cdot\frac{Q}{x^2}\cdot g\left(\frac{Q}{x}\right) + w\cdot\frac{Q^2}{x^3}\cdot g\left(\frac{Q}{x}\right) > 0.\end{aligned} \tag{15}$$

We show that Nash equilibrium always exists for concave-payoff-functions games like ours (see Friedman, [40, Theorem 2.4]); the reaction curves will intersect exactly once and hence the Nash equilibrium is unique. □

According to Yano and Lee [24], the expected profit of supply chain under this Nash equilibrium, namely, $E[\Pi_{sc}^d(Q^{ne}, x^{ne})]$, is less than centralized expected profit of supply chain $E[\Pi_c^*(Q,x)]$. So it is necessary for the decentralized system to achieve the coordination and obtain the same profit as the centralized system. Later, a mechanism will be proposed for the decentralized system to achieve the same profit as the centralized system.

4. Reimbursement Contract

As analyzed above, the Nash game between the manufacturer and the supplier in the decentralized system undermines the supply chain's performance, which is caused by the double marginalization. Therefore, to achieve the profit of centralized system, an effective contract should be proposed to coordinate the supplier and the manufacturer in the decentralized system.

Note that there are many contracts which can coordinate supply chain, such as buy-back contract, revenue-sharing contract, quantity discount contract, and sales rebate contract, in which most of them are used by the supplier to encourage the retailers to order more. Therefore, these are not applicable to our case that contains both random yield and demand. In contrast, we propose a new contract to share the risk of random yield and demand and define it as reimbursement contract. That is, if the supplier's random yield exceeds the manufacturer's order quantity, the manufacturer shares its risk and gives reimbursement θ_1 to the supplier. When the manufacturer's random demand exceeds the order quantity, the supplier share its risk and give reimbursement θ_2 to the manufacturer. Therefore, θ_1 and θ_2 can be defined as

$$\begin{aligned} \theta_1 &= m[\alpha x - Q]^+ \\ \theta_2 &= s[y\varepsilon - \alpha x]^+. \end{aligned} \tag{16}$$

We define m as the manufacturer's risk sharing coefficient for the supplier's excessive production, and s stands for the supplier's risk sharing coefficient for the manufacturer's excessive random demand. It is obvious to show that $m < w$, $s < \pi_m$. Thus, the extra transfer payment under reimbursement contract is T^r; then

$$\begin{aligned} T^r &= \theta_1 - \theta_2 \\ &= m[\alpha x - Q]^+ - s[y\varepsilon - \alpha x]^+. \end{aligned} \tag{17}$$

The expected profit of supplier and manufacturer can be rewritten as follows:

$$\begin{aligned} E(\Pi_m^r) &= p^* \cdot E[\min(\alpha x, Q, y\varepsilon)] \\ &\quad - E\left\{\pi_m[y\varepsilon - \min(\alpha x, Q)]^+\right\} \\ &\quad - w \cdot E[\min(\alpha x, Q)] - E(T^r) \\ E(\Pi_s^r) &= w \cdot E[\min(\alpha x, Q)] - c_s x + E(T^r). \end{aligned} \tag{18}$$

Under this reimbursement contract, the manufacturer will use the optimal price of the centralized supply chain as the selling price, and the supplier will also maximize his expected profit as

$$\begin{aligned} \frac{\partial E[\Pi_m^r]}{\partial Q} &= (p^* + \pi_m) \int_{Q^{**}/x^{**}}^{1} \int_{Q^{**}/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\ &\quad - w \int_{Q^{**}/x^{**}}^{1} g(\alpha)\, d\alpha \\ &\quad - m \int_{Q^{**}/x^{**}}^{1} (\alpha x^{**} - Q^{**}) g(\alpha)\, d\alpha, \\ \frac{\partial E[\Pi_s^r]}{\partial x} &= w \int_0^{Q^{**}/x^{**}} \alpha g(\alpha)\, d\alpha \\ &\quad + \widehat{m} \int_{Q^{**}/x^{**}}^{1} \alpha g(\alpha)\, d\alpha \\ &\quad + \widehat{s} \int_0^1 \int_{\alpha x^{**}/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - c_s. \end{aligned} \tag{19}$$

Let $\partial E[\Pi_m^r]/\partial Q = 0$, $\partial E(\Pi_s^r)/\partial x = 0$, $Q^{**} = x^{**}$ from Proposition 2. Then (19) can be simplified as

$$w\mu_\alpha + \widehat{s} \int_0^1 \int_{\alpha x^{**}/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - c_s = 0 \tag{20}$$

$$\text{s.t.} \quad E(\Pi_m^r) \geq E(\Pi_m^{ne}), \quad E(\Pi_s^r) \geq E(\Pi_s^{ne}). \tag{21}$$

Equation (20) is the solution to coordinate the decentralized supply chain and $\widehat{s}$ should be adjusted to satisfy the previous equation. From (20), there exists unique s^* to satisfy the equation. Though (20) does not contain $\widehat{m}$, $\widehat{m}$ is also effective and its value changes the final profit between the supplier and the manufacturer. Under this reimbursement contract, the decentralized supply chain can achieve the performance of centralized supply chain.

5. Numerical Analysis

In this section, we provide numerical illustrations of optimal lot-sizing, production quantity, and dynamic pricing under different Δ. We assume that random variable ε of demand obeys the normal distribution with $\mu_\varepsilon = 1$, $\sigma_\varepsilon = 0.25$. The random variable α of supply has a uniform distribution of yield taking values in $(0, 1)$. Then the demand function can be assumed as $y(p) = a \cdot p^{-b}$, while $b > 1$. This means that the demand for the final product is elastic. First we will consider optimal decision and dynamic piecing for centralized system, and then supply coordination will be analyzed.

5.1. Dynamic Pricing for Centralized Supply Chain. According to (4), (5), and (6), the optimal order quantities, production quantities, and the expected profit of centralized supply chain are depicted in Table 1 for 12 cases under determined price.

Table 1 shows under different prices that there exists the optimal order quantity, production quantity, and the expected profit in the centralized supply chain. As shown in the Figure 2, when price rises up to 5 the profit of centralized supply chain is maximized. One might wonder whether the price 5 is the optimal price for this system. Therefore, we then explore the optimal price for this centralized supply chain by using our algorithm put forward above. The algorithm can be programmed in MATLAB 7. As in this specific case, the optimal expected profit can be calculated, that is the 559.08, and the error deviating from standard expected profit can be estimated.

From Table 2, when Δ becomes smaller, the error also becomes smaller from the whole view. When $\Delta = 0.1$, the error of the optimal expected profit for centralized system

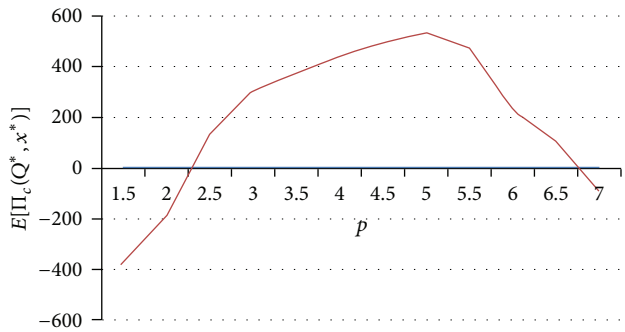

Figure 2: Optimal profits in the centralized system with different determined prices.

Table 1: Centralized solution under different determined prices.

Case	p	π	c	Q^*	x^*	$E[\Pi_c(Q^*, x^*)]$
1	1.5	1	1	231.43	231.43	−379.3
2	2	1	1	351.17	351.17	−193.59
3	2.5	1	1	424.86	424.86	132.84
4	3	1	1	498.57	498.57	295.71
5	3.5	1	1	538.72	538.72	373.83
6	4	1	1	623.47	623.47	441.16
7	4.5	1	1	638.31	638.31	495.93
8	5	1	1	659.29	659.29	532.46
9	5.5	1	1	677.51	677.51	472.17
10	6	1	1	693.22	693.22	230.96
11	6.5	1	1	721.83	721.83	105.58
12	7	1	1	749.54	749.54	−93.07

Table 2: The optimal price under different Δ.

Case	Δ	p	Q^*	x^*	$E[\Pi_c^*(Q^{**})]$	Error
1	0.1	4.8	647.52	647.52	534.83	4.34%
2	0.05	4.85	652.68	652.68	552.14	1.25%
3	0.001	4.854	654.41	654.41	556.83	0.40%
4	0.0005	4.8535	655.11	655.11	558.26	0.15%
5	0.00001	4.85346	655.25	655.25	559.01	0.01%
6	0.000005	4.853458	655.36	655.36	559.09	0.0003%

is 4.34%. As Δ decreases, the error of the optimal expected profit for the centralized system also drops. When $\Delta = 0.000005$, the error of the optimal expected profit drops to 0.0003%, which is fairly small. The result can be drawn that the precision of this algorithm can be ensured as long as Δ are sufficiently small. Therefore the optimal price for this centralized supply chain can be obtained through this algorithm.

5.2. Supply Coordination Based on Reimbursement Contract. As supply chain is dominated by the manufacturer, he will use the optimal price for decentralized supply chain. Suppose that the optimal price is $p^* = 4.85346$ (see Table 2, case 5). Then different coordinated ways and parameters are compared in Table 3.

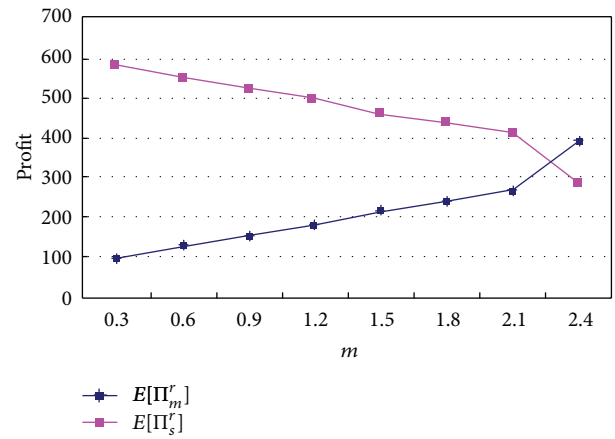

Figure 3: The relation between m, $E[\Pi_s^r]$, and $E[\Pi_m^r]$.

As illustrated in Table 3, decentralized supply chain can be coordinated based on reimbursement contract and achieve the same performance as the centralized supply chain. It should be pointed out that $E(\Pi_m^r) \geq E(\Pi_m^{ne})$, $E(\Pi_s^r) \geq E(\Pi_s^{ne})$. So cases 1 and 8 in Table 3 are not acceptable by the supplier or the manufacturer because under reimbursement contract his profit is less than that under Nash equilibrium. It can be observed that s is unique and determined but m can be changed. Then the relation between m and $E[\Pi_s^r]$, $E[\Pi_m^r]$ can be shown in Figure 3.

From Figure 3, m has its range to ensure $E(\Pi_m^r) \geq E(\Pi_m^{ne})$, $E(\Pi_s^r) \geq E(\Pi_s^{ne})$. And this case shows that this contract can coordinate supply chain properly.

6. Conclusion

In this paper, we study the coordination mechanism in a decentralized system that consists of one manufacturer and one supplier under both random yield and random demand. We consider two scenarios in which the retail price can be either exogenously or endogenously determined. In particular, we propose an algorithm to derive the optimal price in the centralized model. This sets our benchmark to compare the decentralized decisions for the supplier and the manufacturer. Based on this, we finally propose the reimbursement contract to coordinate decentralized supply chain. The major implications in this paper are concluded as follows.

(1) The expected profit $E[\Pi_c(Q, x)]$ is jointly concave in (Q, x) when the price is exogenously determined. Moreover, the optimal production quantity can be equal to the optimal order quantity when the system is integrated.

(2) There exists a unique Nash equilibrium solution (Q^{ne}, x^{ne}) for the manufacturer and supplier in the decentralized system between. Thus, double marginalization significantly undermines the channel's efficiency compared to the centralized system.

(3) A reimbursement contract is proposed to perfectly coordinate the channel. In particular, parameter $\hat{s}$ should be adjusted to coordinate the decentralized

Table 3: Comparison of supply chain under different conditions.

p^*	Centralized supply chain				Supply chain under Nash equilibrium					Coordinate supply chain based on reimbursement contract						
	Q^{**}	x^{**}	$E[\Pi_{sc}^*]$	w	Q^{ne}	x^{ne}	$E[\Pi_s^{ne}]$	$E[\Pi_m^{ne}]$	$E[\Pi_{sc}^{ne}]$	m	s	Q^{**}	x^{**}	$E[\Pi_s^r]$	$E[\Pi_m^r]$	$E[\Pi_{sc}^r]$
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	0.3	0.79	655.25	655.25	97.34	579.79	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	0.6	0.79	655.25	655.25	130.19	546.94	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	0.9	0.79	655.25	655.25	156.24	520.89	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	1.2	0.79	655.25	655.25	182.74	494.39	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	1.5	0.79	655.25	655.25	216.11	461.02	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	1.8	0.79	655.25	655.25	240.85	436.28	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	2.1	0.79	655.25	655.25	266.36	410.77	677.13
4.85346	655.25	655.25	677.13	2.5	662.94	737.41	136.87	292.96	429.83	2.4	0.79	655.25	655.25	392.17	284.96	677.13

supply chain and there exists unique s^*. In contrast, $\widehat{m}$ cannot coordination channel directly, and it becomes effective and changes the profit ratio between the supplier and manufacturer.

Appendix

Proof of Proposition 1

$$\begin{aligned}
&\frac{\partial E(\Pi_{sc})}{\partial Q} \\
&= p \cdot \left\{ \frac{\partial}{\partial Q}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \right. \\
&\qquad + \frac{\partial}{\partial Q}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&\qquad \left. + \frac{\partial}{\partial Q}\left[\int_0^{Q/y}\int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right]\right\} \\
&- \pi_m \left\{ \frac{\partial}{\partial Q}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \right. \\
&\qquad \left. + \frac{\partial}{\partial Q}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} (y\varepsilon - Q) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right]\right\},
\end{aligned} \tag{A.1}$$

where

$$\begin{aligned}
&\frac{\partial}{\partial Q}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&= \int_{Q/y}^{\infty} \frac{Q}{x} \cdot g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial Q}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&= \int_{Q/x}^{1}\int_{Q/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\
&\quad - \int_{Q/x}^{1} \frac{Q}{y} \cdot f\left(\frac{Q}{y}\right) g(\alpha)\, d\alpha \\
&\quad - \int_{Q/y}^{1} \frac{Q}{x} \cdot g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon,
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial Q}\left[\int_0^{Q/y}\int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&= \int_{Q/x}^{1} \frac{Q}{y} \cdot f\left(\frac{Q}{y}\right) g(\alpha)\, d\alpha,
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial Q}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&= \int_{Q/y}^{\infty} (y\varepsilon - \alpha x) \cdot \frac{1}{x} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon,
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial}{\partial Q}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} (y\varepsilon - Q) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] \\
&= -\int_{Q/x}^{1}\int_{Q/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\
&\quad - \int_{Q/y}^{\infty} (y\varepsilon - \alpha x) \cdot \frac{1}{x} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon,
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial E(\Pi_{sc})}{\partial Q} \\
&= (p + \pi_m)\int_{Q/x}^{1}\int_{Q/y}^{\infty} g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon \\
&= (p + \pi_m)\,\overline{G}\left(\frac{Q}{x}\right)\overline{F}\left(\frac{Q}{y}\right),
\end{aligned}$$

$$\frac{\partial^2 E(\Pi_{sc})}{\partial Q^2} = -(p+\pi_m) \times \left[\frac{1}{x} g\left(\frac{Q}{x}\right)\overline{F}\left(\frac{Q}{y}\right) + \frac{1}{y} f\left(\frac{Q}{y}\right)\overline{G}\left(\frac{Q}{x}\right)\right] < 0, \tag{A.2}$$

$$\begin{aligned}\frac{\partial E(\Pi_{sc})}{\partial x} &= p \cdot \left\{\frac{\partial}{\partial x}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] + \frac{\partial}{\partial x}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] + \frac{\partial}{\partial x}\left[\int_0^{Q/y}\int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right]\right\} \\ &\quad - \pi_m \left\{\frac{\partial}{\partial x}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] + \frac{\partial}{\partial x}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} (y\varepsilon - Q) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right]\right\} - c_s,\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}\frac{\partial}{\partial x}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha x g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] &= \int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - \int_0^{Q/x} \frac{\alpha^2 x}{y} f\left(\frac{\alpha x}{y}\right) g(\alpha)\, d\alpha - \int_{Q/y}^{\infty} \frac{Q^2}{x^2} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon, \\ \frac{\partial}{\partial x}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} Q g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] &= \int_{Q/y}^{\infty} \frac{Q^2}{x^2} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon, \\ \frac{\partial}{\partial x}\left[\int_0^{Q/y}\int_{y\varepsilon/x}^{1} y\varepsilon g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] &= \int_0^{Q/y} \frac{y^2\varepsilon^2}{x^2} f\left(\frac{y\varepsilon}{x}\right) f(\varepsilon)\, d\varepsilon, \\ \frac{\partial}{\partial x}\left[\int_0^{Q/x}\int_{\alpha x/y}^{\infty} (y\varepsilon - \alpha x) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] &= -\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - \int_{Q/y}^{\infty} \frac{Q}{x^2}(y\varepsilon - Q) g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon, \\ \frac{\partial}{\partial x}\left[\int_{Q/x}^{1}\int_{Q/y}^{\infty} (y\varepsilon - Q) g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon\right] &= \int_{Q/y}^{\infty} \frac{Q}{x^2}(y\varepsilon - Q) g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon, \\ \frac{\partial E(\Pi_{sc})}{\partial x} &= (p+\pi_m)\int_0^{Q/x}\int_{\alpha x/y}^{\infty} \alpha g(\alpha) f(\varepsilon)\, d\alpha\, d\varepsilon - c_s, \\ \frac{\partial^2 E(\Pi_{sc})}{\partial x^2} &= -(p+\pi_m) \times \left[\int_{Q/y}^{\infty} \frac{Q^2}{x^3} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon + \int_0^{Q/x} \frac{\alpha^2}{y} f\left(\frac{\alpha x}{y}\right) g(\alpha)\, d\alpha\right] < 0, \\ \frac{\partial^2 E(\Pi_{sc})}{\partial x\, \partial Q} &= \frac{\partial^2 E(\Pi_{sc})}{\partial Q\, \partial x} = (p+\pi_m)\int_{Q/y}^{\infty} \frac{Q}{x^2} g\left(\frac{Q}{x}\right) f(\varepsilon)\, d\varepsilon.\end{aligned} \tag{A.4}$$

Hessian matrix is as follows:

$$H = \begin{pmatrix} \dfrac{\partial^2 E(\Pi_{sc})}{\partial Q^2} & \dfrac{\partial^2 E(\Pi_{sc})}{\partial Q\, \partial x} \\ \dfrac{\partial^2 E(\Pi_{sc})}{\partial x\, \partial Q} & \dfrac{\partial^2 E(\Pi_{sc})}{\partial x^2} \end{pmatrix} > 0. \tag{A.5}$$

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Research Article

Research on Self-Organization in Resilient Recovery of Cluster Supply Chains

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An effective way to deal with high-risk and low-probability disruptions is to create a resilient cluster supply chain, in which the study of resilience lies in its recovery mechanism when failures occur. First, the paper describes the representation method of cluster supply chain resilience. Second, a cluster supply chain network structure generation model is proposed. And based on cascading effect model, it makes analysis of dynamic evolution process when cluster supply chain failure happens. Then it focuses on the self-organization characteristic, which contributes to cluster supply chain emergence overall resilient recovery through local self-organization reconstruction behavior. We also make theoretical analysis of cluster supply chain network characteristics and its effect on the resilience, which helps to illustrate that the root of vulnerability lies in cascading failure while self-organization is the key to resilient recovery. Besides, with the study of self-organization characteristic, it provides theoretical guidance for local control and further achievement of overall resilient optimization.

1. Introduction

With economic globalization, customer needs become more diverse and personalized, and economic operation is facing more challenges and uncertainties, all of which make it a difficult problem for the enterprises to operate. As a new supply chain network organization, cluster supply chain can effectively solve this problem since it combines characteristics of both industry cluster and supply chain. Besides, the network structure of cluster supply chain contributes to its generation of positive coordination effect and innovation effect. However, with the scale increasing, the cluster supply chain needs to suffer more risks to improve efficiency.

Cluster supply chain is faced with internal uncertainty and external unpredictable events. Therefore, cluster supply chain risks can be divided into two categories. (1) The first category is the risk of supply and demand, which is a kind of operational risk caused by internal conflict, such as machine failure, demand uncertainty, human changes, and mainly solved by internal coordination. The frequency of such risks is usually high, while the impact is relatively small. (2) The second one is the risk caused by external interrupts [1], such as strikes, natural disasters, and even terrorist attacks. It affects the normal flow of the logistics, information, and capital in cluster supply chain or even makes some links cannot normally operate. Under emergencies, a small perturbation or a minor fault in the network can give rise to a pass-through effect on the whole network or even causes cascading failure of the entire network system, resulting in serious consequences. The frequency of such risk is low, while the impact is very serious. In recent years, with such events occurring frequently, supply chain suffers from disruptions or even collapse, which brings about huge economic losses to enterprises. For example, in March 11, 2011, a Richter 8.9 earthquake hit Japan, bringing about more than $ 4 billion direct economic losses to Toyota. Therefore, when facing such interrupts caused by emergencies, the cluster supply chain system is rather vulnerable. In order to overcome the vulnerability, the supply chain system should combine the advantages of both robust strategy and flexible strategy, which means that the system should possess robustness and adaptability at the same time. Furthermore, resilience should

get more attention. Therefore, the concept of supply chain resilience came into being, based on which the concept of creating resilient supply chain is proposed.

The key to study cluster supply chain resilience is to comprehend recovery mechanism. As the premise of recovery is the occurrence of failures, we should first understand how the failure occurs. Under the disruptions environment, the occurrence of cascading effect is mainly due to the failure of some critical nodes, which brings about serious losses to the cluster supply chain structure or even cripples the whole network operation. At this point, as a complex adaptive system, the resilient cluster supply chain can make quick adjustments and respond to the existing resources according to the current logical constraint conditions of the remaining nodes, which is in accordance with the given principle of self-organization repair, and both new generated nodes and remaining nodes reconstruct logical relationship between each other. Owing to the local self-organization repair behavior of old and new nodes, the supply chain can make a new network structure emerge and restore its overall function quickly. This self-organization repair ability adequately reveals the resilient response strategy of cluster supply chain.

The rest of this paper is organized as follows. Section 2 presents the descriptive approach of cluster supply chain resilience based on the literature review. Section 3 defines a network structure model based on the characteristics of cluster supply chain network. In Section 4, we analyze the causes of cascading failure and the impact of network structure on resilience. In Section 5, we find out that the resilient recovery mechanism of cluster supply chain network lies in its self-organization. It is proved that the key to resilient recovery of cluster supply chain is self-organization with reference to a simulation case in Section 6. Finally, conclusions and possible future research extensions comprise Section 7.

2. Literature Review

The concept of "resilience" is first derived from the papers of Holling [2], who defines the resilience as the ability of a system to absorb disturbance before its equilibrium changes. Anderson [3] pointed out that resilience is not just about recovery; it means that the system is flexible enough to adapt to positive or negative impacts. Later, scholars of different disciplines gradually adopt the concept of resilience to describe the key features of complex dynamic systems that they have researched, and different scholars have different definition of the supply chain resilience. Christoper and Pack [4] deemed that the resilience refers to the ability of a system to recover to the original (or better) state after an interruption. Fiksel [5] thought that the resilience is the ability of a system to survive, adapt, and develop when facing disturbance. Rice and Caniato [6] considered that resilience is the ability to deal with undesirable disruptions and restore normal operations. Sheffi [7] thought that resilience is the ability to restrain disruption and recover from it. Based on the above scholars' points, the supply chain resilience just as the word resilience emphasizes is the ability to quickly recover, and also includes both the word elasticity which means adaptability and the word flexibility which means flexibility and antidisruption.

Some scholars have already carried out some qualitative researches on the key theory of the supply chain resilience. For example, Sheffi [8] is an early scholar who has made significant achievements in the aspect of supply chain resilience. He thought the most direct and effective way is to improve supply chain structure. Recently some scholars make quantitative study on supply chain resilience and analyze internal mechanism in-depth, which provides managers with more direct help. Zhao et al. [9], for example analyzed resilience of supply network topology structure under random attacks and attempted attacks from the aspects of availability, connectivity, accessibility, and so forth, which focused on military security network. Huang et al. [10] tried to measure the vulnerability of supply chain nodes with degree centrality, betweenness centrality, and network factions through the use of social network analysis method. Yan et al. [11] studied emergency management strategy of resilient supply chain based on node failure. The research mentioned above studies space resilience of supply chain from topology structure, and the importance of the nodes is equal to node destruction based on the thought of deleting node. Without considering node repair ability, it is unable to reflect time resilience of supply chain, not to mention the recovery ability that resilience owns.

Based on the synthesis of existing literature [12], resilience can be expressed from various aspects as follows.

2.1. Space Resilience. The space resilience of supply chain is defined as a measure of the overall supply chain effect ability in the spatial structure. When resilient supply chain faces an internal or external disruption, there may be great change in its structure, such as node failure, edge failure. In addition, the resilient supply chain should have a self-restoring property, thus active nodes in the network constantly vary with failure and recovery. We adopt the total load change of the active nodes in the network to measure the space resilience of supply chain.

2.2. Time Resilience. The time resilience of resilient supply chain is the measure of the resilient supply chain ability to response and restore in the time dimension. When the resilient supply chain is faced with disruptions, it should be able to recover its response to user and performance level of the supply chain within a certain time range and reach a steady state in a short time. We adopt the total load of the active nodes in the network to express the network performance and use the changes of the performance over time to measure the time resilience of supply chain.

2.3. Resilience Description. The description method of supply chain resilience that presented in this paper is shown in Figure 1, in which time period 0–T_s reflects response delay time, time period T_s–T_r reflects destruction spread time, and time period T_r–T_e reflects recovery time. With next round of failure and recovery going on until the system reaches

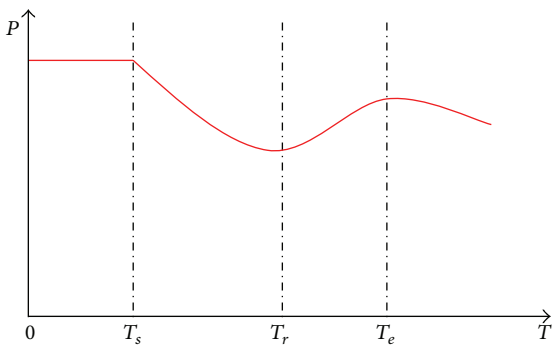

Figure 1: Resilience description of cluster supply chain network.

a steady state, the height of the curve reflects the cluster supply chain network performance at time t. This kind of cluster supply chain network resilience includes characters of three aspects, which are absorption capacity, adaptability, and recovery ability.

Absorption capacity exhibits anti-interference, which is able to automatically absorb the impact of disturbance at a small price, making the impact of damage achieve minimization and not affect system overall performance, shown as time 0–T_s. Although cascading effect has already began during this period, because of the complementarities of cluster supply chain network, the decline in overall performance does not immediately appear.

Adaptability shows active response, which is the self-organization capacity of each member in the supply chain network and also the self-adjusting ability during damage period and recovery period. Adaptability is shown as curvature degree of the curve during the time period T_s–T_e. The actual supply chain system consists of nodes with self-learning ability [13], which can optimally distribute its load to its adjacent nodes when it fails and also can optimally reobtain load after recovery.

Recovery ability is a kind of ability that is easy to recover to a new stable state, shown as the time period T_r–T_e. The system begins to recover at T_r, and the change of the network performance within the time period T_r–T_e reflects recovery ability. Recovery ability of cluster supply chain network members consists of two parts, which are their own recovery ability and recovery ability related to other members.

Such description of cluster supply chain network resilience not only clearly outlines the space characteristics and time characteristics of the cluster supply chain network but also meets the several major features that supply network resilience should have, such as self-recovery, antidisruption, and active response. The resilience framework makes quantitative assessment of supply chain's absorption capacity, adaptability, and recovery ability.

2.4. Definition of Resilience. Cluster supply chain resilience is defined as follows: cluster supply chain network suffers from cascading failure when dealing with undesirable disruption, but it can conduct self-repair through adaptability and make it fast recover to a new stable state.

3. Cluster Supply Chain Network

3.1. Concept of Cluster Supply Chain. Cluster supply chain is a kind of supply chain coupling with industrial cluster, and its effective operation plays an important role in the promotion of industrial clusters. Because of the physical proximity and industry relevancy, cluster enterprises have the characteristics of flexibility, specialization, trust, and cooperation. They cooperate and compete with each other in the single-chain as well as among different cross-chains, which makes them adaptive to the rapid change of market demand and enhance international competitiveness. Based on this, supply chains that cause cross-industry competition and cooperation are called "cluster supply chain," the core of which is to enhance the competitiveness of industry cluster.

Cluster supply chain promotes internal labor division and external collaboration. Enterprises provide a variety of products and services and strengthen regional economy through collaboration. Cluster supply chain possesses resource aggregation effect, which can gather some economic resources, such as technology, capital and labor, and promote resource aggregation effectively as well as industrial structure adjustment.

Cluster supply chain has the following advantages [14–16]: (1) strengthen mutual trust based on the common interests; (2) strengthen conscious cooperation; (3) closely complement each other; (4) be closely organized, of low cost, lower information communication, and coordination cost; (5) have stronger ability of learning and innovation. For example, the accumulation of specialized suppliers and skilled labor can reduce the factors that affect production costs. With relatively lower cost and risk, cluster supply chain enterprises enhance their pursuit of technology innovation in order to obtain high profits and achievements. Because of mutual trust, any technology innovation success of the member in cluster supply chain will result in the whole cluster supply chain faster response to market changes than outside enterprises. Cluster can generate high levels of innovation.

3.2. Characters of Cluster Supply Chain. The current supply chain research evolved from research about "single supply chain" to research about "cross-network based on multiple supply chain," focusing on the "supply chain network" that centers on core business and its multidistributors as well as its multicustomers. Cluster supply chain is a network of organizations involved in the different processes producing value in the form of products and services for the ultimate consumer. There are many suppliers, manufacturers, and distributors around the same industry or related industry value chain. Cluster supply chain networks are sequentially arranged based on the vertical ties between enterprises in different layers. Network analysis explicitly differentiates between horizontal ties (transactions in the same layer) and vertical ties (transactions between layers) [17], as shown in Figure 2, mapping how enterprises in one layer are related to each other and to enterprises in other layers.

One of the most important features of cluster supply chain is the interaction among enterprises which may take place between similar firms (horizontal) and, to a greater

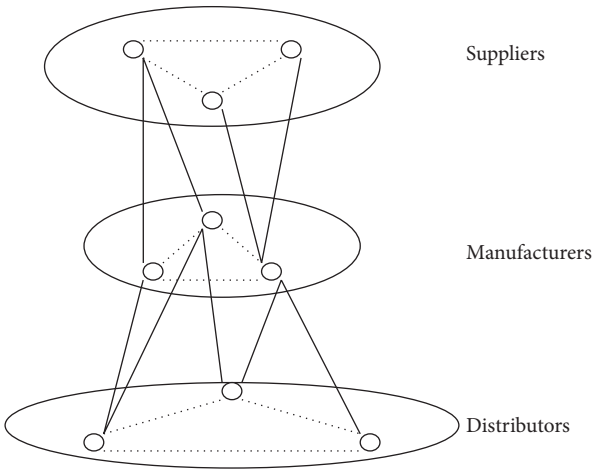

Figure 2: Network structure of cluster supply chain.

extent, between enterprises involved in different phases of the production process (vertical). These interactions are manifested in interaction of logistics, capital flow, and information flow between manufacturers, suppliers, and distributors. This interaction makes enterprises in the cluster supply chain technological spillovers: the increase of workers' production skill, operational management and equipment improvement of enterprises, and continuous improvement of market information transfer rate and efficiency. Each enterprise in the cluster can collaborate from the two-dimension (the vertical and the horizontal) to maximize its efficiency.

Cluster supply chain network possesses the characteristic of dynamic evolution. In the fierce competition, it is possible for each node enterprise to be eliminated or rejoin the cluster supply chain network.

Cluster supply chain network is provided with the characteristic of preferential attachment. New connection between nodes in the network is continuously formed and the old connection constantly disappears. The new entrant will select partners based on its demand for resources according to the merits of alternative nodes. Each node enterprise status is arranged in a hierarchy, and it is usually easier for the core enterprise to obtain relatively abundant resources in information, knowledge, reputation, and so forth than those enterprises on the edge of the network. In addition, the core enterprise has an advantage when attracting new entrants because of its relatively higher degree.

The characteristics of cluster supply chain network mentioned above bring an opportunity of resource integration effect for the node enterprise, which also bring risks for the cluster supply chain because of the complexity of the network structure and the complexity of the relationship between cooperative enterprises. The inherent root cause of cluster supply chain network risks lies in the complexity of cluster supply chain topology [18].

This paper presents a new selection mechanism to simulate real situations about cluster supply chain network.

3.3. Structure Model of Cluster Supply Chain Network. Cluster supply chain contains a number of nodes and edges, which form a network structure together. We can use an undirected graph $G = (V, E)$ to represent the network, where $V = \{v_i\}$ is a nonempty finite set of node enterprises in the network, which includes a number of supplier nodes S_i $(i = 1, 2, \ldots, I)$, a number of manufacturer nodes M_j $(j = 1, 2, \ldots, J)$, and a number of distributor nodes D_k $(k = 1, 2, \ldots, K)$, $E = \{e_{ij}\}$ is a finite set of edges in the network, which represents the various logistics; information flow, capital flow between each enterprise, and the dynamic connections established between each node enterprise are based on the various flows. Adjacency matrix $A_{N\times N} = [a_{ij}]_{N\times N}$ expresses the business relationship between each logistic node, in which if there is business relationship between any node v_i and v_j, set $a_{ij} = 1$, otherwise set $a_{ij} = 0$. Degree k_i represents the number of other nodes that node i connects, which means the enterprise establishes a relation with other k_i enterprises.

Whenever adding a new node enterprise, there will be m old node enterprises to establish business relationship with it. The first step when a node chooses partners is to consider the business connectivity, which means to choose its "local world." And the second step is to make selection in the "local world" formed by business relationships that can be established and establish business relationship.

When a new node enterprise adds into the cluster supply chain network, we should first determine its local world, which means the node enterprise scope that it can establish business relationship with, based on the node enterprise property. If the new node enterprise that joins the network is a supplier, it can establish business relationship with suppliers and manufacturers. If the new node enterprise that joins the network is a manufacturer, it can establish business relationship with suppliers, manufacturers, and distributors. And if the new node enterprise that joins the network is a distributor, it can only establish business relationship with manufacturers.

When choosing partners, mostly prefer to select the enterprise with stronger capacity and the node degree can reflect the node capacity. Suppliers with larger degree reflect that they have a relatively strong supply capacity, manufacturers with larger degree reflect that they have more supply channels and target customers, and distributors with larger degree reflect that they have more purchasing channels and relatively strong purchasing power. So the way of connection is based on the existing nodes and the probability which is proportional to its degree to connect the new node. In other words, the new enterprise tends to establish relationship with large-scale enterprises (nodes with larger degree values).

The new added node selects the appropriate node within its local world according to its preference, and the specific modeling process is as follows:

(1) at the initial time $t = 0$, given a randomly generated network with m_0 nodes and random connections;

(2) add a new node j at each time interval according to a uniform distribution, randomly determine the category of this node, and then select an already existing m nodes to establish connection ($m < m_0$), where the m nodes must be within the local world A_j

of new node j. Use the preferential rules to calculate the probability of connection:

$$\prod_{\text{local}} = \frac{k_i(t)}{\sum_{l\in A_j} k_l(t)}; \tag{1}$$

(3) select m nodes from A_j to connect the new node j according to the connection probability that is calculated in step (2);

(4) loop steps (2), (3) to the preset network size and finally obtain a network with N nodes.

In the evolutionary model, the evolution of the degree k_i of node enterprise i over time can be calculated by mean-field approximation method [19]. k_i satisfies the following dynamic equation:

$$\frac{\partial k_i}{\partial t} = m\prod{}'\left(i \in A_j\right)\prod_{\text{local}} = m\frac{M_t}{m_0+t}\frac{k_i(t)}{\sum_{l\in A_j} k_l(t)}, \tag{2}$$

where m is the number of edges brought by new node, M_t is the number of nodes in the local world of the new node that joins the network at time t, $m_0 + t$ is the sum of nodes in the network at time t, $\prod'(i \in A_j)$ is the probability of node i to be selected into the local world, and $\prod_{\text{local}}$ is the probability for the new node to connect node i in the local world.

To simplify the following analysis, we assume that $\sum_{l\in A_j} k_l(t) = \langle k_i\rangle M_t$, where the average degree is $\langle k_i\rangle = 2(mt + e_0)/(m_0 + t)$

$$\begin{gathered}\sum_{l\in A_j} k_l(t) = 2\left(mt+e_0\right)\frac{M_t}{m_0+t} \approx 2mM_t,\\ \frac{\partial k_i}{\partial t} \approx \frac{mM_t}{m_0+t}\frac{k_i(t)}{2mM_t} \approx \frac{k_i}{2t}.\end{gathered} \tag{3}$$

Then we can get

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{1/2}, \tag{4}$$

$$P(k) \sim 2m^2k^{-3}. \tag{5}$$

This shows that the network model established possesses the characteristic of scale-free and can accurately reflect the realistic cluster supply chain network, in which most of the nodes have just a few adjacent nodes associated with businesses while the minority core enterprise nodes bear large business capacity and need to maintain business contact with a large number of nodes.

4. Cascading Failure of Cluster Supply Chain

In the face of unexpected disruptions, the greatest risk of the cluster supply chain is the trigger of cascading effect, which may lead to the collapse of the whole line. The existing research lacks in-depth study of such microscopic mechanism, which is the key to ensure the normal operation of cluster supply chain. Although some scholars have studied cascading failure of the network, most of the works in their research refer to a generic complex network [20] or a specific network, such as the power grid [21], urban infrastructure networks [22], and lacking specific study of the cluster supply chain network. Based on the network model of cluster supply chain, this section proposed a definition method of the initial load based on the degree distribution, using the node load capacity as the load redistribution standards for failing nodes, and further researched the effect of the cluster supply chain network structure on the robustness to resist cascading failure.

4.1. Cascading Failure Mode of Cluster Supply Chain. We proposed a model to represent cascading failures of cluster supply chain. We show that the breakdown of a single node, which causes load redistribution to other nodes, is sufficient to result in the whole system failure.

As we can know from the characteristics of the cluster supply chain network, the load of the cluster supply chain network refers to the operational capacity of the nodes in the network, and the maximum volume of business that each node can carry is the node's load capacity. Load transfer refers to the fact that the business of some nodes transfers to other nodes, including both the transfer of resources and the transfer of the ability.

Different nodes in cluster supply chain network have different loads and different load capacities, and they have different degrees of influence on the overall network when they fail. Therefore, the primary key issue is how to measure the node's load. Node degree is not only able to reflect its network importance but also can reflect the operational capacity of the node, which is the load capacity.

In most previous cascading failure models, the load on a node was generally determined by its degree [23], which may result in the loss of some information. Consider that the adjacent nodes of a node, which have business relationship with it, also affect its load except its own impact. Assume that the initial load $L_i(0)$ of node i being dependent on the degree of node i and the degrees of its neighbor nodes, the expression of which is defined as follows:

$$\begin{gathered}L_i(0) = k_i^{\alpha}(0)\left(\sum_{v_j\in\Gamma_i} k_j(0)\right)^{1-\alpha}\\ \left(0 \le \alpha \le 1,\ v_i, v_j \in V,\ i \ne j\right),\end{gathered} \tag{6}$$

where $k_i(0)$ is the degree of node v_i at initial time, Γ_i is the set of adjacent nodes of node v_i. Parameter α is used to adjust the influence weight of node v_i on its own initial load, and $1 - \alpha$ is used to accordingly adjust the influence degree of adjacent nodes on the initial load of node v_i.

We know that each node of cluster supply chain has a limited capacity, which is the largest load that the node can handle. We assume that the load capacity C_i of node i is proportional to its initial load:

$$C_i = \beta L_{i0} \quad (\beta \ge 1), \tag{7}$$

where β is a parameter to adjust the network's overall load bearing capacity. As the parameter value increases, the given load capacity of each node becomes stronger, and the ability to resist the shock that is brought by additional load distribution of the failing node grows stronger. However, because the entity network in actual environment is subject to cost constraints, the greater the load capacity nodes have, the larger the total cost of network investment is. The parameter value of β, at which the whole system achieves best load bearing capacity and least investment cost, is called critical threshold and assumed as β_θ. When $\beta = \beta_\theta$, the whole network achieves optimal investment cost and optimal network survivability.

As the business capacity of the adjacent nodes has a direct impact on the load redistribution after the node fails, this paper constructed the following load redistribution rules for the failure nodes based on the size of the node degree. Assume that node v_i fails, its load will be allocated to its adjacent node v_j according to the following proportional function and the proportion is $P_j(t)$:

$$P_j(t) = \frac{k_j^\alpha(t)\left(\sum_{v_\mu \in \Gamma_j} k_\mu(t)\right)^{1-\alpha}}{\sum_{v_\eta \in \Gamma_i}\left[k_\eta^\alpha(t)\left(\sum_{v_\varphi \in \Gamma_\eta} k_\varphi(t)\right)^{1-\alpha}\right]}. \quad (8)$$

According to the above load redistribution rule, any adjacent node v_j will receive a proportion of the failure load distribution $\Delta L_{i\to j}(t)$, which is

$$\Delta L_{i\to j}(t) = \delta L_i(t) P_j(t) \quad (0 \le \delta \le 1). \quad (9)$$

The real-time load of the adjacent node v_j is

$$L_j(t+1) = L_j(t) + \sum_{v_i \in \Gamma_j} \Delta L_{i\to j}(t). \quad (10)$$

Due to the constraints of node load capacity in cluster supply chain, when the real-time load of a node exceeds its load capacity, this node will result in its own collapse failure, which will lead to a new round of node failure and load redistribution. To effectively describe the dynamic propagation process, the failure propagation function is defined as follows:

$$L_j(t+1) > C_j. \quad (11)$$

In order to show the cascading phenomenon of cluster supply chain, here we focus on cascades triggered by the removal of a node. Assume that any logistics node v_a collapses and fails, while its adjacent nodes are v_{bx} $(x = 1, 2, 3)$. Then part of the business load of v_a is redistributed to these adjacent nodes (as shown in dashed red arrows). At the same time, remove the edges between node v_a and its adjacent nodes v_{bx}. This is the first round of node failure and load distribution, as shown in Figure 3.

When the adjacent node v_{b1} meets the failure transfer function, then the node also collapses and fails, resulting in a new round of failure load redistribution (as shown in dashed green arrows). Meanwhile, remove the edges between node v_{b1} and its adjacent node v_{cx}, which is the second round of node failure and load distribution, as shown in Figure 4. As the process carries on, finally the dynamic reaction process of cascading failure will be formed.

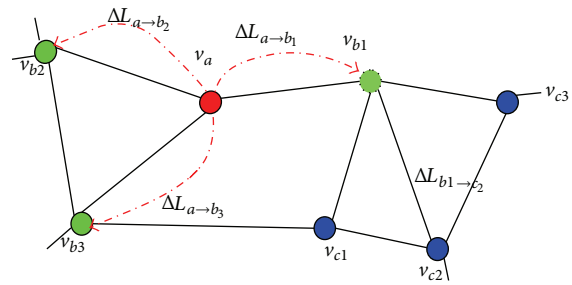

Figure 3: The first round of node failure and its load distribution.

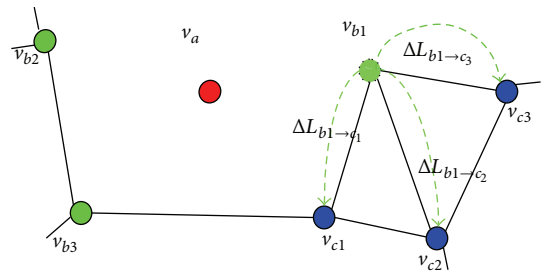

Figure 4: The second round of node failure and its load distribution.

4.2. The Impact of the Network Structure on Resilience. Now, we will analyze this model from the perspective of theoretical analysis. First, the given conditional probability $P(k' \mid k_i)$ represents the probability that the node of k' degree connects the node of k_i degree in the network. Then the sum of the degree of any node v_i's adjacent nodes satisfies the following formula:

$$\sum_{v_j \in \Gamma_i} k_j = \sum_{k'=k_{\min}}^{k_{\max}} k_i P(k' \mid k_i) k', \quad (12)$$

where $k_{\min}$ and $k_{\max}$ are, respectively, the maximum and minimum degree of the nodes in the network. And the nodes degree in the complex network meets the normalization condition:

$$\sum_k P(k) = \sum_{k'} P(k' \mid k) = 1. \quad (13)$$

Furthermore, consider that the amount of edges from the node of k degree connects to the node of k' degree, which will be certainly equal to the amount of edges from the node of k' degree that connects to the node of k degree. The degree between nodes should satisfy the following equilibrium conditions:

$$kP(k'k)P(k) = k'P(kk')P(k'). \quad (14)$$

Because the nodes in scale-free network are provided with characteristics of independence on the node, it should meet the following formula:

$$P\left(k'k\right) \cong P\left(k'\right). \tag{15}$$

Hence, combining formula (13)~(15), we can get

$$\sum_k kP\left(k' \mid k\right)P\left(k\right) = \sum_k k'P\left(k \mid k'\right)P\left(k'\right). \tag{16}$$

Through simplification, we can get

$$\begin{aligned} P\left(k'\right)\sum_k kP\left(k\right) &= k'P\left(k'\right)\sum_k P\left(k\right) \\ &\Longrightarrow P\left(k'\right)\langle k\rangle \\ &= k'P\left(k'\right). \end{aligned} \tag{17}$$

Combining formula (14) and formula (17), we can get

$$P\left(k'k\right) = \frac{k'P\left(k'\right)}{\langle k\rangle}. \tag{18}$$

As we can know from formula (12) and formula (18)

$$\begin{aligned} \sum_{v_j\in\Gamma_i} k_j &= \sum_{k'=k_{\min}}^{k_{\max}} k_i P\left(k' \mid k_i\right)k' \\ &= k_i \sum_{k'=k_{\min}}^{k_{\max}} \frac{k'P\left(k'\right)k'}{\langle k\rangle} \\ &= \frac{k_i\left\langle k^2\right\rangle}{\langle k\rangle}. \end{aligned} \tag{19}$$

Therefore, we can derive the following formula:

$$\begin{aligned} \sum_{v_j\in\Gamma_i} k_j^\alpha &= k_i \sum_{k'=k_{\min}}^{k_{\max}} \frac{k'P\left(k'\right)k'^\alpha}{\langle k\rangle} \\ &= \frac{k_i\left\langle k^{1+\alpha}\right\rangle}{\langle k\rangle}. \end{aligned} \tag{20}$$

According to the rules of the cascading failure function, if and only if the condition (11) is satisfied, can the network control the occurrence of cascading failure. Combining formula (6), formula (7), and formula (11), therefore

$$\begin{aligned} &\left[k_i^\alpha\left(\sum_{v_j\in\Gamma_i} k_j\right)^{1-\alpha}\right]\cdot\frac{k_j^\alpha\left(\sum_{v_\mu\in\Gamma_j} k_\mu\right)^{1-\alpha}}{\sum_{v_\eta\in\Gamma_i}\left[k_\eta^\alpha\left(\sum_{v_\varphi\in\Gamma_\eta} k_\varphi\right)^{1-\alpha}\right]} \\ &\quad + \left[k_j^\alpha\left(\sum_{v_\mu\in\Gamma_j} k_\mu\right)^{1-\alpha}\right] \\ &\quad < \beta\left[k_j^\alpha\left(\sum_{v_\mu\in\Gamma_j} k_\mu\right)^{1-\alpha}\right]. \end{aligned} \tag{21}$$

Through simplification, we can get

$$1 + \frac{k_i^\alpha\left(\sum_{v_j\in\Gamma_i} k_j\right)^{1-\alpha}}{\sum_{v_\eta\in\Gamma_i}\left[k_\eta^\alpha\left(\sum_{v_\varphi\in\Gamma_\eta} k_\varphi\right)^{1-\alpha}\right]} < \beta. \tag{22}$$

Therefore, according to the above formula (20), simplify formula (22) to

$$\begin{aligned} &1 + \frac{k_i^\alpha\left(\sum_{v_j\in\Gamma_i} k_j\right)^{1-\alpha}}{\sum_{v_\eta\in\Gamma_i}\left[k_\eta^\alpha\left(\sum_{v_\varphi\in\Gamma_\eta} k_\varphi\right)^{1-\alpha}\right]} \\ &\quad = 1 + \frac{k_i^\alpha\left(k_i\left\langle k^2\right\rangle/\langle k\rangle\right)^{1-\alpha}}{\sum_{v_\eta\in\Gamma_i}\left[k_\eta^\alpha\left(k_\eta\left\langle k^2\right\rangle/\langle k\rangle\right)^{1-\alpha}\right]} \\ &\quad = 1 + \frac{k_i}{\sum_{v_\eta\in\Gamma_i} k_\eta} = 1 + \frac{k_i}{k_i\left\langle k^2\right\rangle/\langle k\rangle} \\ &\quad = 1 + \frac{\langle k\rangle}{\left\langle k^2\right\rangle} < \beta. \end{aligned} \tag{23}$$

As we can know from the above mentioned, taking the cost factor into account, the smaller the value of β, the better. Thus, the optimal load bearing threshold value of β_θ of the whole network is shown in the following formula:

$$\beta_\theta = 1 + \frac{\langle k\rangle}{\left\langle k^2\right\rangle}. \tag{24}$$

Due to the network structure model of cluster supply chain, the degree distribution of the nodes approximately meets the following formula (5), where m is the minimum degree of the nodes in the network; that is, $m = k_{\min}$. Therefore, we can get the following formula:

$$\begin{aligned} \left\langle k^2\right\rangle &= \int_{k_{\min}}^{k_{\max}} P\left(k\right)k^2dk \\ &= 2m^2\left(\ln k_{\max} - \ln k_{\min}\right) \\ &= 2k_{\min}^2\left(\ln k_{\max} - \ln k_{\min}\right), \end{aligned} \tag{25}$$

$$\begin{aligned} \int_{k_{\min}}^{\infty} P\left(k\right)dk &= \frac{1}{N} \\ &\Longrightarrow \int_{k_{\max}}^{\infty} 2m^2k^{-3}dk = \frac{1}{N} \\ &\Longrightarrow m^2k_{\max}^{-2} = k_{\min}^2k_{\max}^{-2} = \frac{1}{N} \\ &\Longrightarrow k_{\max} = \sqrt{N}k_{\min}. \end{aligned} \tag{26}$$

Hence, combining formula (25) and formula (26), we can get

$$\begin{aligned} \left\langle k^2\right\rangle &= 2k_{\min}^2\left(\ln k_{\max} - \ln k_{\min}\right) \\ &= k_{\min}^2\ln N. \end{aligned} \tag{27}$$

Therefore, based on the above formula (27), we can simplify formula (25):

$$\beta_\theta = 1 + \frac{4}{\langle k \rangle \ln N}. \tag{28}$$

It is clear that as node average degree $\langle k \rangle$ of cluster supply chain network increases, the optimal load threshold β_θ of the whole network gradually decreases instead. Namely, the more complex the structure of cluster supply chain network is, the better robustness the network has to resist cascading failure and the stronger survivability the network has.

5. The Process of Self-Organization Recovery

This section introduced self-organization competition artificial neural networks into the cluster supply chain resilient recovery. Through adaptive learning, the restored nodes make preferential attachment among the old nodes and join the new round of load distribution again, which shows that the cluster supply chain resilience can achieve self-organization recovery under cascading effect.

Due to disruption, a large number of the nodes in the cluster supply chain fail and become vacant, which makes the system cannot operate normally with the combination of the remaining limited nodes. At this moment, resilient cluster supply chain as a complex adaptive system should generate new nodes on the basis of self-maintenance principle. Under the premise of effectively meeting customers' needs, the system reconstructs the logical relationship between the original remaining nodes and repaired nodes through self-organization according to the inherent constraints of supply chain and the reconstruction principle of maximizing profit, which restores the supply chain operation and ensures the system's overall profit maximization.

The intricate interaction in cluster supply chain compels them to constantly compete and collaborate, which is the basic impetus for self-organization evolution. The operation of each enterprise in cluster supply chain meets self-organization phenomenon. As there is neither centralized control mechanism nor unified command sent after the system integration, each enterprise can only consider its own survival and development and take action just based on microindividual local information. However, the spontaneous behavior of the cluster supply chain can lead to the emergence of the global structure, which makes the whole system flexible.

Neural network learning refers to the process that the network adjusts its parameters under stimulus of the external environment to make the network respond to the external environment in a new way. And its adaptability is achieved through learning, which can adjust the weights according to the environmental changes to improve system behavior.

Self-organization competition neural network integrates learning stage and working stage without providing prior standard sample in a kind of unsupervised manner. The variation of learning laws complies with the evolution equation of connection weights. It guides network learning and work based on simulating the dynamics principle where the biological neural systems process information is dependent on the action of excitement, coordination, inhibition, and competition between neurons. Through its own training, the network automatically classifies the input mode.

Fundamentals are as follows. Self-organization competition neural network has a strong self-organization adaptive learning ability. The neurons in competitive layer compete for the opportunity of response to the input mode vector, and finally only one of them becomes winner of the competition. And the winning neuron represents the classification of input mode vector. For those connection weights relevant to the winning neuron tend to be adjusted in a direction that is more conducive to competition, and the winning neuron represents the classification of input mode.

Self-organization competition neural network generally consists of input layer and competitive layer. Input layer is responsible for receiving information from outside and inputting mode and then pass them to competitive layer, which plays a role as "observation." Competitive layer is responsible for making "analysis and comparison" for the mode and finding out the law to identify the correct classification. This function is realized through the following learning mechanisms.

The learning steps of self-organization competition neural network are as follows.

(1) *Initialization*. Give weight vector ω_{ij} a random value within the range of $[0, 1]$ according to the constraint $\Sigma\omega_{ij} = 1$.

(2) Select the Kohonen learning rule $\Delta\omega_{ij} = \mathrm{lr}(p_j - \omega_{ij})$, which corrects weights, where p is the input vector, lr is the learning rate.

(3) Look for the winning neuron. The weight vector that is mostly similar to the input vector is set as the competition winning neuron.

(4) Revise each connection weight that connects the winning neuron, $\omega_{ij}(t+1) = \omega_{ij}(t) + \Delta\omega_{ij}$, and other connection weights remain unchanged.

After an interval time T, the failure nodes are restored as new generated nodes, which select old nodes to establish connection and rejoin the load distribution according to self-organization competitive neural network learning methods. As shown in Figure 5, we assume that the failure node v_a is restored after an interval time T and the history load of all nodes as input vector. It selects the winning node from the old nodes to establish connection through using self-organization competition neural network to make classification. The nodes that already failed will naturally be eliminated in the competitive process. At this moment, select v_{b2} from adjacent node v_{bx} $(x = 2, 3)$ to establish connection through self-organization adaptive learning ability. Meanwhile, part of the initial load is restored:

$$L_i(T) = \gamma L_i(0) \quad (0 \le \gamma \le 1). \tag{29}$$

Then this node joins the next round of cascading failure reaction process.

The ability to recover results from two parts: the characteristics that the system owns and the outside environment.

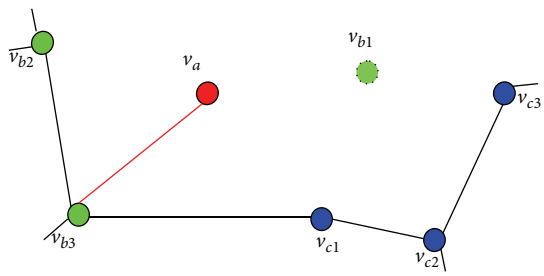

Figure 5: Load distribution when node is restored.

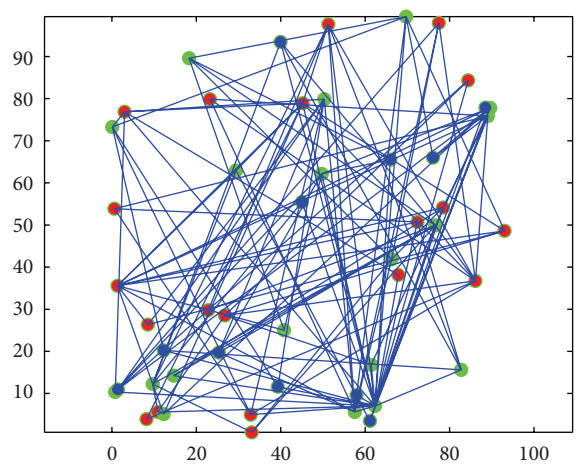

Figure 6: Topology structure of cluster supply chain network.

When rejoin the network, the recovery part is due to the characteristics of the system. When participating in the next round of load redistribution, the recovery part owes to the outside.

Through self-organization function of complex adaptive system, newly restored nodes reconnect the remaining nodes. Thus, cluster supply chain keeps evolving and emerges new network structure.

6. Simulation Analysis

Based on the cluster supply chain network structure model in Section 3.3, establish a cluster supply chain network with nodes $N = 50$, and $m_0 = 5$, $m = 3$, as shown in Figure 6.

Take this cluster supply chain network as an example, first study the cluster supply chain network resilience without recovery mechanism, where take a node of degree 2 as the initial failure node and parameters are given as follows: $\alpha = 0.2$, $\beta = 1.05$, and $\delta = 1$, which means that the whole load is distributed to adjacent nodes after the node fails. As shown in Figure 6, we can see that there are multiple occurrences of horizontality in the curve, which reflects the absorption capacity that the cluster supply chain network has. Although cascading reaction carries on continuously, owing to the complementarities of cluster supply chain network that adjacent nodes can share the capacity of the failure node, the overall performance does not decline within a period. But this method, which only considers the cascading failure without

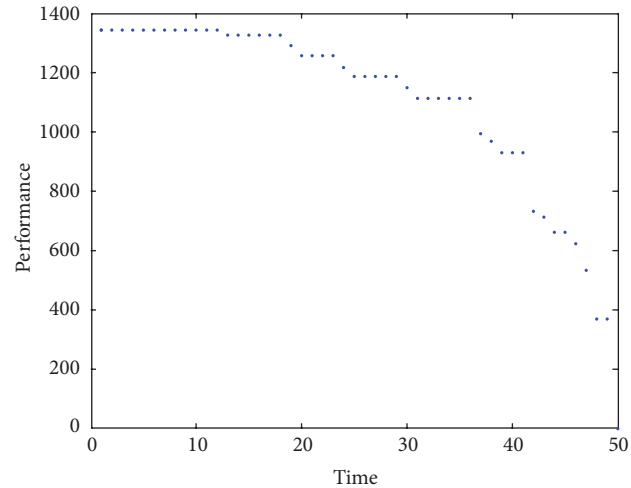

Figure 7: Resilience of cluster supply chain without recovery mechanism.

considering node recovery, will make the entire cluster supply chain network eventually collapse.

Adopt the approach with self-organization recovery mechanism to study the same cluster supply chain network resilience, where $\gamma = 0.6$. When a node is restored, as shown in Figure 7, the situation depicted in Figure 1 will occur repeatedly. Meanwhile, the network performance will undulate down over time and ultimately reach a new steady state. The forward process with undulating style reflects the adaptability and recovery ability that the cluster supply chain network possesses. The node in the network can actively respond to its failure and transfer its load to the neighbors. That behavior belongs to self-organization ability, which makes the network performance decline slowly. The failure nodes are restored after a period of time and take the use of self-organization competition neural network to make classification and select the winning node in the old nodes to establish a connection and rejoin the network. At this moment, the ability of the failure nodes is partly restored but can once again participate in sharing the load distributed by the failure adjacent nodes. The recovery ability that combines both the internal and external part makes the network performance rebound in a period of time.

Comparing Figure 7 to Figure 8, we can get Figure 9. It is clear that the cluster supply chain with self-organization recovery mechanism is more coincident with actual situation and can ultimately reach a steady state through this dynamic process. In order to achieve the original state or better state, it needs more investment than simple repair cost. The reasonable selection of parameters δ, γ reflects network adaptability and is also the key to the degree of network resilience.

Emergence is a kind of transition from low level to high level based on the evolution of the microbody, referring to the mutation of the macrosystem in performance and structure. The phenomenon of emergence is centered on the interaction and is more complex than simple summation of individual behavior. Cluster supply chain failure and recovery have the characteristics of emergence.

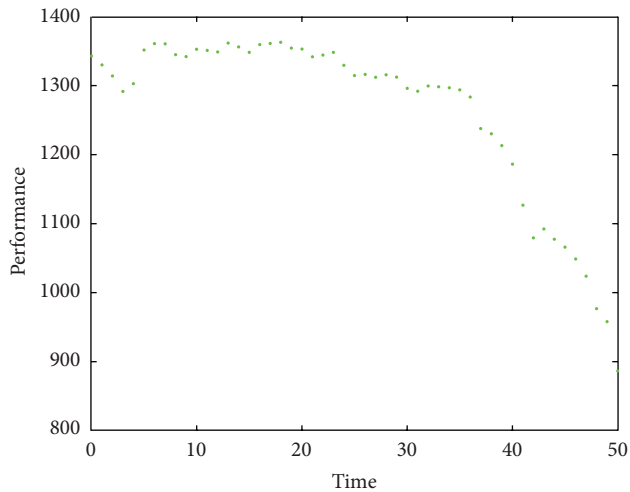

Figure 8: Resilience of cluster supply chain with self-organization recovery mechanism.

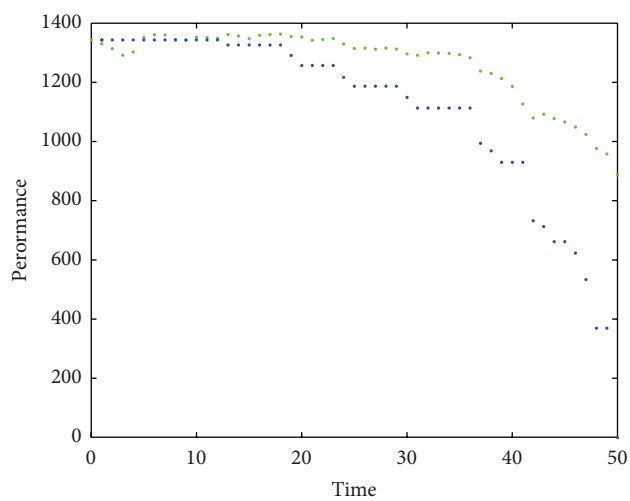

Figure 9: Comparison of whether recovery mechanism exits or not.

By contrast, we can observe the evolution phenomenon of local effect and the emergence of structure and function. Each node enterprise can make the entire cluster supply chain achieve optimization through local self-organization learning, which is reason why the cluster supply chain has resilience.

7. Conclusions

With the rapid economic development, the changes of both cluster supply chain external environment and its internal operations become intense, which leads to the frequent occurrence of interrupt events. The cluster supply network with better resilience can withstand various interrupts, and it has a relatively shorter interrupt recovery time and stronger competitiveness. The major contributions of the research are as follows.

(1) Based on the literature review, the concept of cluster supply chain resilience is put forward, which includes such basic points as absorption capacity, adaptability, and recovery ability. In addition, the expression of resilience is also developed.

(2) The generation of cluster supply chain network structure is well illustrated, which takes its scale-free property into account.

(3) The cascading failure model is developed to illustrate the dynamic evolution process of failure under interrupt environment. Besides, we make full analysis of how this network structure affects anti-disruption, which shows that the greater the node average degree is, the stronger anti-disruption the network has.

(4) The self-organization property of cluster supply chain resilience is fully elaborated. With respect to the self-organization recovery ability, it is shown through simulation. Meanwhile, it is found out that cascading failure is the root of cluster supply chain vulnerability while self-organization is the key to cluster supply chain resilient recovery. In addition, the emergence property that failure and recovery owns is clearly identified, which can serve as theoretical guidance and reference for achieving overall resilience optimization through local control.

The future study will focus on how to improve the adaptability that each node possesses. After that, how to optimize the cluster supply chain resilience so as to provide favorable basis for dealing with supply chain network interrupt also needs further study.

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Research Article

Optimal Acquisition and Inventory Control for a Remanufacturing System

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Optimal acquisition and inventory control can often make the difference between successful and unsuccessful remanufacturing. However, there is a greater degree of uncertainty and complexity in a remanufacturing system, which leads to a critical need for planning and control models designed to deal with this added uncertainty and complexity. In this paper, a method for optimal acquisition and inventory control of a remanufacturing system is presented. The method considers three inventories, one for returned item and the other for serviceable and recoverable items. Taking the holding cost for returns, recoverable and remanufactured products, remanufacturing cost, disposal cost, and the loss caused by backlog into account, the optimal inventory control model is established to minimize the total costs. Finally, a numerical example is provided to illustrate the proposed methods.

1. Introduction

With the increasing awareness of environmental protection worldwide, the green trend of conserving the Earth's resources and protecting the environment is overwhelming. The conversation of resources is being considered from many aspects of product development and use, such as redesign, reuse, recycle, and remanufacture of products and components. Remanufacturing is a powerful product recovery option which generates products as good as new ones from old discarded ones [1]. This technique can also help to reduce the environmental impact of the product in its final disposal [2]. Growing concern for resource conservation and waste reduction led to the augmentation of remanufacturing.

Various strategic and operational aspects of used products remanufacturing have been investigated in the last decades. However, for remanufacturing, the main problem is the collection of used products with good quality at the right time and at the right inventory level [3]. One of the concerns for collection, therefore, is the design of acquisition quantity based on the remanufacturing cost and capability. Dowlatshahi [4] elaborated on how to overcome obstacles within the reverse logistics network in order to ensure a steady supply of cores and presented a framework for effective design and implementation of recycling operations. Ostlin et al. [5] identified different sources of cores and explored how to take back used products for remanufacturing from a customer-supplier relationship perspective. Korchi and Millet [6] presented a detailed practical framework for designing the reverse logistics channel for supplying reusable used modules to the production chain.

With the used product collection available, another important decision in remanufacturing is how many quantities of the available cores should be determined. Jayaraman [7] proposed an analytical approach to capture the variability of the returning condition of used products through a discrete distribution of nominal quality. Kim et al. [8] proposed a mixed integer programming model to maximize the total cost savings of a group of remanufacturing facilities by optimally deciding on the quantity of worn parts to be processed at each facility and the number of parts to purchase from a subcontractor. Li et al. [9] presented a stochastic dynamic programming based model for uncertain production planning of a remanufacturing system. Shah et al. [10] presented an optimal strategy of switching between the "cores" and virgin components with a reinforcement learning algorithm.

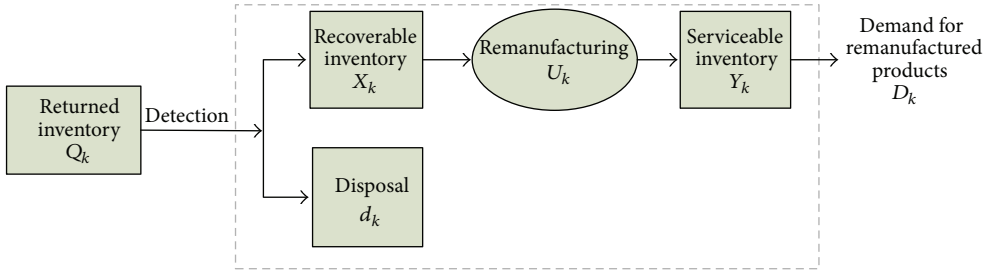

Figure 1: Inventory control framework of a remanufacturing system.

Denizel et al. [11] formulated a stochastic programming model for a multiperiod remanufacturing planning to determine the quantity of returned products at each quality level to be remanufactured. Karamouzian et al. [12] provided an analytical model using queuing network theory to obtain the best policy for accepting returned products. Of course, there have been many other excellent contributions in terms of the used product acquisition and quantity planning.

Used product acquisition and quantity planning have been widely recognized as an efficient tool of remanufacturing [13]. However, the uncertainty and complexity of return and demand make the production planning more complicated. Mukhopadhyay and Ma [14] studied the impact of stochastic quality of returns on remanufacturing. Tang and Li [15] presented a synopsis of ongoing research in uncertainty management of remanufacturing and pointed out that integrated methodologies and their evaluations in real industry practices deserve future research efforts. By matching the demand for used products with potential supply in a systematic way, an effective model is needed and crucial for maximizing the profitability of a remanufacturing system [16].

Motivated by the foregoing discussion, this paper presents an optimal method for used product acquisition and inventory control. The state variable is defined by the recoverable inventory and the serviceable inventory of remanufactured products, the decision variable is defined by the number of returned products per stage, and the Bellman equation is constructed by minimizing the expected cost during the finite stages. And the optimal acquisition and inventory control levels of the remanufacturing system can be obtained with the policy iteration method. Following the presentation of the proposed method, the method is demonstrated via a numerical example.

2. Development of Model

In this section, the framework, assumptions, and the proposed model for used products acquisition and inventory management are presented.

2.1. Modeling Framework. A remanufacturing system begins with the collection of the used product or parts, also named as core, followed by its remanufacturing and delivery of the remanufactured product to the client [17]. All returned used products (Q_k) need to be detected. If the used products cannot be recoverable, then they would be disposed of by material recycling (d_k). Of course, the used products that successfully complete the inspection phase may be placed into a recoverable inventory, and then the best quality returns (U_k) need to be determined and remanufactured as good as new products. Meanwhile, the rest of recoverable products (X_k) need to be stored for the next period. The remanufactured products (Y_k) are stored as serviceable inventory, which in turn is used to feed into sales. Therefore, there are three inventories in a remanufacturing system: the returned inventory, recoverable inventory, and the serviceable inventory of remanufactured products, as depicted in Figure 1.

There are several unique characteristics which predominantly and naturally occur in the remanufacturing environment that further complicate the remanufacturing management [18], for instance, uncertainties in supply sources, uncertainties in the timing and the quantity of returns, uncertainty in quality of used products, and uncertainty in varieties of recycled used products. Due to the uncertainties in arriving timing, quality, and quantity of reverse logistics, the returned and recoverable inventory must be large so as to ensure the smoothness and continuity of remanufacturing production, and then the cost of recovery inventory will increase. For the sake of maximization of profits, the quantity of returned products and the demands of remanufactured products must be balanced in remanufacturing system [19]. In addition, the remanufacturing cores are the used parts and components, which are different in types and specifications or are variable in size changes and material performance changes, so the remanufacturing process and the process parameters are uncertain, and the lead time varies greatly. The uncertainties of remanufacturing process result in the uncertainties of the total remanufacturing cost, including purchase cost, testing cost, disassembly cost, and recovery cost.

Mindful that these problems add complexity to this field of research, the objective of this paper is to determine the optimal acquisition and inventory control to guarantee a required service level and to minimize the total costs.

2.2. Model Assumptions. Several operations, such as collection and inspection of used products, remanufacturing of as-good-as-new products, and disposal of used products unsuitable for the remanufacturing process, are considered within the system. However, a number of assumptions are made throughout this analysis in order to simplify the system

and facilitate the modeling process by helping to focus on the most important factors. The assumptions are summarized as follows:

(1) Market demands of remanufactured products are stochastic variables.

(2) Taking into account that used products should be tested after collection and then be remanufactured, capacity of collection and inspection activities is considered to be infinite.

(3) Remanufacturing cost is a linear function of the state of returned products, and remanufacturing quantity is a linear function of batch quantity of return.

(4) The proportion of returned products which are disposed is stochastic, and the impact of used products disposal on remanufacturing production planning is not considered.

(5) Remanufacturing activity starts and ends in the same stage and ignores the uncertainties of production lead time.

2.3. Model Formulation. As described above, the N stages production planning is formulated in a single used product remanufacturing system. Once a demand of the remanufacturing market is analyzed, the optimal acquisition and inventory control to minimize the total costs of a whole remanufacturing cycle need to be determined. Total costs include return cost, remanufacturing cost, disposal cost, and costs of returned, recoverable, and serviceable inventory. Due to the market demand uncertainty, there may be a backlog cost. Notations used in this paper are as follows:

N: the total stages of the production plan,

Q_k: the return of used products of stage k (the decision variable),

d_k: the disposal quantities of used products of stage k,

X_k: the recoverable inventory of stage k,

Y_k: the serviceable inventory of remanufactured products of stage k,

X_1: the initial recoverable inventory,

Y_1: the initial serviceable inventory of remanufactured products,

M_x: the maximum recoverable inventory,

M_y: the maximum serviceable inventory of remanufactured products,

U_k: the quantity of remanufacturing processing of stage k,

D_k: the demand of remanufactured products of stage k,

m_k: the state variable of returned products,

a_k: the fixed cost of returned products of stage k,

u: the return cost of used products per unit,

s: the disposal cost of used products per unit,

C_h: the holding cost of recoverable inventory per unit,

C_H: the holding cost of serviceable inventory per unit,

C_p: the fixed cost of remanufacturing processing,

V_k: the variable cost of remanufacturing processing,

C_b: the backlog cost per unit,

W_k: the output ratio of returned products of stage k.

The state variable is defined by the recoverable inventory and the serviceable inventory of remanufactured products

$$S_k = (X_k, Y_k). \tag{1}$$

The state transition equation is

$$\begin{aligned} X_{k+1} &= X_k + Q_k - d_k - U_k, \\ Y_{k+1} &= Y_k + U_k - D_k. \end{aligned} \tag{2}$$

As the remanufacturing processing quantity is a linear function of batch quantity of retur, W_k is the yield of returned products of stage k, and then U_k can be calculated by $U_k = W_k Q_k$, where $\max(0, X_k + Q_k - d_k - M_x) \le U_k \le \min((M_y - Y_k), (X_k + Q_k - d_k))$, $Q_k \ge D_k$, $0 < W_k < 1$. The action space of Q_k is

$$\begin{aligned} R_{k(S_k)} = \Bigg\{ Q_k \Big| \max\left(D_k, \frac{d_k - X_k}{1 - W_k}\right) &\le Q_k \\ &\le \min\left(\frac{M_x + d_k - X_k}{1 - W_k}, \frac{M_y - Y_k}{W_k}\right)\Bigg\}. \end{aligned} \tag{3}$$

The total cost of stage k includes the return cost of used products (i.e., the fixed cost of transportation and the total purchase cost), inventory cost (i.e., the holding cost of recoverable and serviceable inventory), remanufacturing cost (i.e., the fixed cost of disassembly, cleaning, the variable cost of recovery, and purchase of new parts), backlog cost (i.e., cost allowance for unsatisfying the demand), and disposal cost. So, the total cost of stage k can be defined as

$$\begin{aligned} C_k(S_k, Q_k) = a_k &+ uQ_k + C_h[X_k + Q_k - d_k - U_k]^+ \\ &+ C_H[Y_k + U_k - D_k]^+ + (C_p + V_k m_k) U_k \\ &+ C_b[D_k - Y_k - U_k]^+ + s d_k \end{aligned} \tag{4}$$

subject to

$$\begin{aligned} (X)^+ &= \max\{0, x\}, \\ 0 &\le X_k \le M_x, \\ 0 &\le Y_k \le M_y. \end{aligned} \tag{5}$$

In this model, the state variable m_k of returned products is subject to uniform distribution $(0, D_k/(2Q_k - D_k))$ according to the statistical analysis of returned products where $m_k = 0$ represents that the best state of the returned products, and $m_k = 1$ indicates the worst state of the returned products.

With the m_k available, the expectation $E(m_k)$ can be calculated by $E(m_k) = D_k/2(2Q_k - D_k)$, and then (4) can be written as

$$\begin{aligned} E\left[C_k\left(S_k, Q_k\right)\right] &= a_k + uQ_k + C_h\left(X_k + Q_k - d_k - W_kQ_k\right) \\ &\quad + C_H\left(Y_k + W_kQ_k - D_k\right) \\ &\quad + \left(C_p + V_k\frac{D_k}{2\left(2Q_k - D_k\right)}\right)W_kQ_k \\ &\quad + C_b\left(D_k - Y_k - W_kQ_k\right) + sd_k, \\ \frac{dE\left[C_k\left(S_k, Q_k\right)\right]}{dQ_k} &= u + C_h - W_kC_h + W_kC_H + W_kC_p \\ &\quad - W_kC_b - \frac{W_kV_kD_k^2}{2\left(2Q_k - D_k\right)^2}. \end{aligned} \tag{6}$$

For $dE[C_k(S_k, Q_k)]/dQ_k = 0$,

$$Q_k = \frac{1}{2}D_k\left(1 + \sqrt{\frac{V_k}{2\left(\left(u + C_h\right)/W_k - C_h + C_H + C_p - C_b\right)}}\right). \tag{7}$$

Therefore, the optimal acquisition quantity

$$\begin{aligned} Q_k = \max\Bigg(D_k, &\frac{1}{2}D_k\left(1 + \left(V_k \times \left(2\left(\frac{u + C_h}{W_k} - C_h + C_H\right.\right.\right.\right. \\ &\left.\left.\left.\left.\left. + C_p - C_b\right)\right)^{-1}\right)^{1/2}\right)\right)\Bigg). \end{aligned} \tag{8}$$

The objective is to determine the optimal inventory control $S_k (k = 1, 2, \ldots, N)$ while minimizing the total expected cost during N stages, and the objective function is defined as

$$f_k\left(S_k\right) = \min_{Q_k \in R_k(S_k)} \sum_{i=1}^{N} E\left\{C_i\left(S_i, Q_i\right)\right\}, \tag{9}$$

where $E()$ represents the expected cost.

According to the theory of dynamic programming, the Bellman equation of the objective function, which can represent the recursive relation, is shown as follows:

$$\begin{aligned} f_k\left(S_k\right) &= \min_{Q_k \in R_k(S_k)} E\left\{C_k\left(S_k, Q_k\right) + f_{k+1}\left(S_{k+1}\right)\right\} \\ &= \min_{Q_k \in R_k(S_k)} E\left\{C_k\left(X_k, Y_k, Q_k\right) + f_{k+1}\left(X_{k+1}, Y_{k+1}\right)\right\} \\ &= \min_{Q_k \in R_k(S_k)} E\left\{C_k\left(X_k, Y_k, Q_k\right)\right. \\ &\qquad + f_{k+1}\left(\left[X_k + Q_k - d_k - U_k\right]^+,\right. \\ &\qquad \left.\left.\left[Y_k + U_k - D_k\right]^+\right)\right\}, \end{aligned} \tag{10}$$

where

$$\begin{aligned} &k = 1, \ldots, N, \\ &f_{N+1}\left(N + 1\right) = 0. \end{aligned} \tag{11}$$

So, the above discussed model can be formulated as follows:

$$\begin{aligned} &\min_{Q_k \in R_k(S_k)} \quad E\left\{C_k\left(X_k, Y_k, Q_k\right)\right. \\ &\qquad + f_{k+1}\left(\left[X_k + Q_k - d_k - U_k\right]^+,\right. \\ &\qquad \left.\left.\left[Y_k + U_k - D_k\right]^+\right)\right\}, \\ &\text{subject to} \quad X_{k+1} = X_k + Q_k - d_k - U_k, \\ &\qquad Y_{k+1} = Y_k + U_k - D_k, \\ &\qquad (X)^+ = \max\left\{0, x\right\}, \\ &\qquad R_{k(S_k)} \\ &\qquad = \left\{Q_k \middle| \max\left(D_k, \frac{d_k - X_k}{1 - W_k}\right) \le Q_k \right. \\ &\qquad \left. \le \min\left(\frac{M_x + d_k - X_k}{1 - W_k}, \frac{M_y - Y_k}{W_k}\right)\right\} \\ &\qquad 0 \le X_k \le M_x, \quad 0 \le Y_k \le M_y, \\ &\qquad k = 1, \ldots, N, \\ &\qquad U_k = W_kQ_k, \quad 0 < W_k < 1, \\ &\qquad d_k = Q_k - U_k - X_k. \end{aligned} \tag{12}$$

This model formulation can be divided into two phases for production planning. Firstly, the optimal acquisition quantity may be determined, and then the optimal inventory control model needs to be established to minimize the total costs.

2.4. Solution Algorithm. The proposed model is a dynamic programming model, which is difficult to solve, especially

that the state at next period is not completely determined by the state and policy decision at current period (i.e., need to obtain the optimal decisions per period). We present a policy iteration approach to obtain near optimal solution for the problem. The approach starts with an arbitrary policy (an approximation to the optimal policy works best) and carries out the following steps: firstly, S_k need to be determined. The definition of S_k is a set of inventories, and there are equations for each state. These equations which have been presented in the model formulation can be solved iteratively, so that we can obtain the expected cost of every state; secondly, when the algorithm has converged, the solution procedure starts at the end and moves backward stage by stage each time to find the optimal policy for that stage, and it should only change the policy if the new action for some state improves the expected value; in the end, stop if it finds the optimal policy at the initial stage. We will use a numerical example to illustrate the iteration processing in the numerical study.

3. Numerical Study

It is assumed that the total stages number N is 2. The maximum number of inventory M_x is 2 and M_y is 2; the initial number of inventory X_1 and Y_1 is 0, respectively. Parameter settings for the model are shown in Tables 1 and 2.

From the history data, the probability distribution of demands can be forecast for each stage. So, we can obtain the expectation of demand $E(D_k)$ and then replace actual products demand with the expectation demand. The distributions of the demands of remanufactured products are given in Tables 3 and 4.

According to Tables 3 and 4, the expectation of demand $E(D_1)$ is 1.8 and $E(D_2)$ is 2.2. In fact, we consider them as actual products demand. Associated with (8), the optimal acquisition quantity can be calculated: Q_1 is 2.5 and Q_2 is 3.3. According to $U_k = W_k Q_k$, the quantity of remanufacturing processing U_1 and U_2 is 1.25 and 1.65, respectively. Then, through statistical analysis of large state data of return products, we can obtain the probability distribution of m_k, owing to $E(m_k) = D_k/2(2Q_k - D_k)$, so $E(m_1)$ is 0.28, and $E(m_2)$ is 0.25.

In the second stage, we can list all the probable inventory states, and according to the optimal inventory control model established in Section 2.3, using (12), the minimum expected cost of this remanufacturing system is shown in Tables 5 and 6. The solution procedure starts at the end and moves backward stage by stage each time to find the optimal policy for that stage, and the optimal inventory arrangement can be obtained along with the minimum expected cost that comes up.

In the above analysis, the parameters are modified by the data from real remanufacturing works of used machine tools. The uniform distribution of the state variable m_k of returned products is very important to determine the optimal acquisition quantity. To ensure the effectiveness of the model, the uniform distribution must be studied before the production preparation. When the costs and the distribution of demands are deterministic, the optimal production plan of period k is completely determined by its state, which is defined by the recovery inventory and the serviceable inventory in this model. From (8), the probable optimal quantity of returned products is 2.5 in stage 1 and 3.3 in stage 2, respectively. From Tables 5 and 6, when (0, 0) is the inventory state of stage 1 and (0, 1) is the inventory state of stage 2, the minimum expected cost during the two periods is 55.055.

Table 1: Parameter settings of constants for the model.

u	C_h	C_H	C_b	s	C_p
3	1.5	1.5	11	0.5	3

Table 2: Parameter settings of variables for the model.

	a_k	V_k	W_k
$k = 1$	8	6	0.5
$k = 2$	10	8	0.5

Table 3: The distribution of the demands of remanufactured products in stage 1.

Demands of remanufactured products D_1	1	2	3
Distribution $P(D_1)$	0.4	0.4	0.2

Table 4: The distribution of the demands of remanufactured products in stage 2.

Demands of remanufactured products D_2	1	2	3
Distribution $P(D_2)$	0.1	0.6	0.3

4. Summary and Conclusions

The success of a remanufacturing business is very dependent on the acquisition and inventory management in order to satisfy the demand for remanufactured products. In this paper, optimal acquisition and inventory control method is proposed for a remanufacturing system with uncertain demand. The state variable is defined by the recoverable inventory and the serviceable inventory of remanufactured products, and the objective is to determine the quantities that have to be remanufactured in these periods in order to minimize the total cost. This method can be used for remanufacturing enterprise to make the production plan in the uncertain environment.

With the development of remanufacturing, the returns amount will be large and the state of returned products will be more and more unpredictable. Thus, this model has much to be desired; in a further research, the sensibility of the optimal production-inventory policy on changes of quantity of return products should be examined. The interaction between the state of recycling products and the cost of remanufacturing processing should also be given deeper consideration.

Table 5: The probable state and its expected cost of stage 2.

State ($k = 2$) $S_2 = (X_2, Y_2)$	Disposal quantity $d_2 = Q_2 - U_2 - X_2$	Expected cost of stage 2 f_2
(0, 0)	1.65	32.406
(0, 1)	1.65	27.030
(0, 2)	1.65	28.531
(1, 0)	0.65	35.406
(1, 1)	0.65	29.531
(1, 2)	0.65	31.031

Table 6: The probable state and its expected cost of stage 1.

State ($k = 1$) $S_1 = (X_1, Y_1)$	Disposal quantity $d_1 = Q_1 - U_1 - X_1$	Expected cost of stage 1 f_1	Accumulative total expected cost $f_1 + f_2$
(0, 0)	1.65	28.025	55.055

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Research Article

Two-Level Credit Financing for Noninstantaneous Deterioration Items in a Supply Chain with Downstream Credit-Linked Demand

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Trade credit financing is a useful tool in business today, which can be characterized as the agreement between supply chain members such as permissible delay in payments. In this study, we assume that the items have the property of noninstantaneous deterioration and the demand is a function of downstream credit. Then, an EOQ model for noninstantaneous deterioration is built based on the two-level financing policy. The purpose of this paper is to maximize the total average profit by determine the optimal downstream credit period, the optimal replenishment cycle length, and the optimal ordering quantity per cycle. Useful theorems are proposed to characterize the method of obtaining the optimal solutions. Based on the theorems, an algorithm is designed, and numerical tests and sensitive analysis are provided. Lastly, according to the sensitive analysis, managerial insights are proposed.

1. Introduction

Deteriorating products are prevalent in our daily life. According to Shah et al. [1], deterioration is defined as decay, change, or spoilage through which the items are not the same as its initial conditions. There are two categories of deteriorating items. The first one is about the items that become decayed, damaged, or expired through time, such as meat, vegetables, fruits, and medicine. The second category is about the items that lose part or total value through time, such as computer chips, mobile phones, and seasonal products. Both of the two kinds of items have short life cycle. After a period of existence in the market, the items lose the original economic value due to consumer preference, product quality or other reasons.

Early research on deteriorating items can be dated back to 1963. An EOQ model with exponentially decaying inventory was initially proposed by Ghare and Schrader [2]. In their study, they show that the inventory level is not only related to the market demand but also to the deteriorating rate, which is a negative exponential function of time. They proposed the deteriorating items inventory model in which $dI(t)/dt + \theta I(t) = -f(t)$. In the function, $f(t)$ is the demand rate at time t, $I(t)$ stands for the inventory level and, θ refers to the deteriorating rate. Based on the assumption of Ghare and Schrader [2], many researchers extended the model by making different assumptions for the deteriorating rate, the demand rate, or shortages allowed, such as Covert and Philip [3], Philip [4], Y. He and J. He [5], He et al. [6], Yang et al. [7], and He and Wang [8]. They all proposed instructive conclusions for real practice. Recently, Goyal and Giri [9] and Li et al. [10] made excellent and detailed review of deteriorating inventory works.

In the above mentioned works, most researchers assume that the deterioration of the products in inventory starts from the arrival in stock. But, in practice, some kinds of products may maintain the original condition for a short time, which means during that time, there is no deterioration. For example, the stock of the firsthand vegetables or fruits has a high quality at the beginning time, in which there is no spoilage. Afterward, the stock starts to perish and induce the deterioration cost. Under this circumstance, it is obvious that the assumption that the deterioration starts from the beginning can make the retailer overestimate the inventory cost, which leads to uneconomical inventory policies. This phenomenon is first proposed by Wu et al. [11] as "noninstantaneous deterioration". They proposed the optimal

replenishment policy of an EOQ model when the demand is inventory level dependent and can be partially backlogged. Based on this assumption, there were several interesting and relevant works such as Ouyang et al. [12, 13], Sugapriya and Jeyaraman [14], Chung [15], Yang et al. [16], Wu et al. [17], Geetha and Uthayakumar [18], Singh et al. [19], Chang and Lin [20], Chang et al. [21], Maihami and Nakhai Kamalabadi [22], and Shah et al. [1].

Furthermore, different from traditional EOQ models, in which payment should be made to the supplier after the retailer receiving the stock, many researches are focusing on the application of trade credit financing tools to improve profits or reduce cost of the supply chain. Actually, it is more practical that the supplier/retailer allows for a fixed period to settle the payment without penalty for its retailer/customer to induce its demand rate or reduce on hand inventory. This permissible delay in payment can reduce the capital investment of stock amount, thus reducing the holding cost of inventory. Besides, during the credit period, the retailer can gain interest profit of his sales revenue by the investing or banking business. Over the years, research on this part is prevalent in many works. Goyal [23] was the first to study the EOQ model with permissible delay in payment. Then Aggarwal and Jaggi [24], Jamal et al. [25], Chang and Dye [26], and Teng [27] extended Goyal's [23] model for deteriorating items, allow for shortages, and so forth. A lot of useful and interesting managerial insights were proposed in their papers. More research on this part can be found in Chung et al. [28], Teng et al. [29], Jaber and Osman [30], and Chung and Liao [31].

The above mentioned works are all assumed one-level credit financing, but sometimes this assumption is unrealistic in real business. For car companies, like TATA (India) and TOYOTA (Japan), they not only delay the payment of the purchasing cost until the end of the credit period to their suppliers, but also provide a credit period to their customers. This kind of business style is called two-echelon (two-level or two-part) credit financing. Huang [32] first proposed an EOQ model with two-level credit financing, and the retailer's credit period is longer than the customer's. Till now, researches on two-level financing can be seen in Ho et al. [33], Liao [34], Thangam and Uthayakumar [35], Chen and Kang [36], Min et al. [37], Ho [38], Urban [39], and Chung and Cardenas-Barron [40].

Although the credit financing problems for EOQ or EPQ models have been studied by many researchers, in most works, it is assumed that the credit period offered by the supplier/retailer to the retailer/customer is a constant parameter. Actually, in real business, supplier/retailer can decide the credit period by himself to minimize inventory cost or maximize total profit. Su et al. [41] studied the EOQ problem of a two-echelon supply chain under two-echelon trade credit financing, where the demand rate is assumed to be dependent on credit period offered by retailer to the customers. The demand rate is an exponential function of the credit period. The similar assumption can be seen in Ho [38]. Besides, Jaggi et al. [42] made a detailed explanation for the property of the credit-lined demand. They show that demand function for any credit period can be represented as the differential equation: $D(N+1) - D(N) = r(S - D(N))$ where S is the maximum demand, and N is the credit period. Thangam and Uthayakumar [35] then proposed a more general continuous differential equation of the demand function based on Jaggi et al. [42] and extended their model to deteriorating items.

These effects of the situations imposes us to establish an EOQ model for noninstantaneous deteriorating items with credit-linked demand under two-echelon financing policy, which can be treated as a general framework for several papers such as Ouyang et al. [12] and Jaggi et al. [42]. There are several useful theorems proposed to illustrate the optimal solution for the model in different conditions. Here we take into account the following factors: (1) noninstantaneous deterioration items; (2) two-level credit financing is considered; (3) credit-linked demand rate; (4) the credit period offered by the supplier is not necessarily shorter than that offered by the retailer to the customer.

The remainder of this paper is organized as follows. In Section 2, assumptions and notations are described in detail. In Section 3, the EOQ model for noninstantaneous deterioration items under two-level credit financing is made. In Section 4, solutions for the model are proposed and useful theorems are presented. In Section 5, two special cases are discussed. In Section 6, numerical examples and sensitive analysis are made, and managerial insights are proposed. Conclusions and future research are given in Section 7.

2. Notations and Assumptions

The following notations and assumptions are adopted throughout this paper.

(1) The annual demand rate for the item, $D(N)$, which is sensitive to the credit offered by the retailer to customers and is a marginally increasing function w.r.t. N. N, is an integer ($N = 1, 2, 3, \ldots$) and a decision variable throughout this paper. $D(N)$ and D can be used interchangeably in this paper.

(2) Replenishment rate is infinite.

(3) Shortage is not allowed.

(4) The product life (time to deterioration) has a probability density function $f(t) = \theta e^{-\theta(t-\gamma)}$ for $t > \gamma$, where γ is the length of time in which the product has no deterioration and θ is a parameter. The cumulative distribution function of t is given by $F(t) = \int_{\gamma}^{t} f(t) = 1 - \theta e^{-\theta(t-\gamma)}$ for $t > \gamma$, so that the deterioration rate is $r(t) = (f(t))/(1 - F(t))$ for $t > \gamma$.

(5) The length of time in which the product has no deterioration, γ, can be estimated by utilizing the random sample data of the product during the past time and statistical maximum method. For simplifying, it is assumed to be a constant.

(6) M is the permissible delay period in payment for the retailer offered by the supplier (upstream credit). During the period, the retailer can use sales revenue to earn the interest with an annual rate I_p up to

the end of M. At time $t = M$, the credit is settled and the retailer has to pay the interest at rate I_c for the items in stock. N is the permissible delay period in payment for the customer offered by the retailer (downstream credit). During the period, the retailer has to bear the opportunity cost of the revenue which is not settled in time N at the rate of I_p.

(7) Time horizon is infinite.

(8) T is the length of replenishment cycle. Q is the replenishment quantity per cycle. T and Q are decision variables.

(9) A, h, c, and p denote the ordering cost per order, the holding cost per time per item excluding interest charges, the purchasing cost per item, and the selling price per item, respectively. All these parameters are constant and positive.

(10) For $\gamma < T$, there is no deterioration in the time 0 to T, where the inventory level is $I_1(t)$.

(11) For $T > \gamma$, there is no deterioration in the time 0 to γ, where the inventory level is $I_{21}(t)$; there is deterioration in the time γ to T, where the inventory level is $I_{22}(t)$.

(12) $Z_i(T)$ is the total average profit which consists of (a) sales profit (SP), (b) the cost of ordering (OC), (c) cost of holding inventory (HC) (excluding interest charges), (d) cost of deterioration (DC), (e) capital opportunity cost (IC), (f) interest earned from the sales revenue (IE), $i = 1, 2, 3$, where $i = 1$ indicates $M \le N$, $i = 2$ indicates $N \le M \le N + \gamma$, and $i = 3$ indicates $M \ge N + \gamma$.

(13) T^* is the optimal replenishment cycle length. Q^* is the optimal replenishment quantity. Z_i^* is the minimum of the total annual cost; that is, $Z_i^* = Z_i(T^*)$.

3. Model Formulation

First, we model the demand rate $D(N)$ w.r.t. N. According to Jaggi et al. [42] and Thangam and Uthayakumar [35], the marginal effect of credit period on sales is proportional to the unrealized market demand without any delay. Under the assumption, demand can be defined in the following two ways.

(1) The demand function of demand can be represented as a differential difference equation

$$D(N+1) - D(N) = r(\beta - D(N)). \tag{1}$$

(2) The demand rate can be depicted by the partial differential equation

$$\frac{\partial D(N)}{\partial N} = r(\beta - D(N)). \tag{2}$$

In both (1) and (2), $0 \le r < 1$, β is the maximum value of demand rate over the planning horizon. Boundary conditions are $D(N \to 0) = \alpha$, $D(N \to \infty) = \beta$. The solutions for (1) and (2) are

$$\text{Type 1: } D(N) = \beta - (\beta - \alpha) e^{-rN}, \tag{3}$$

$$\text{Type 2: } D(N) = \beta - (\beta - \alpha)(1 - r)^N. \tag{4}$$

The two types of demand functions are adopted in the following analysis.

The inventory system evolves as follows: Q units of the items arrived at the warehouse at the beginning of each cycle.

When $T \le \gamma$, there is no deterioration in a single cycle. The inventory system is depicted in Figure 1.

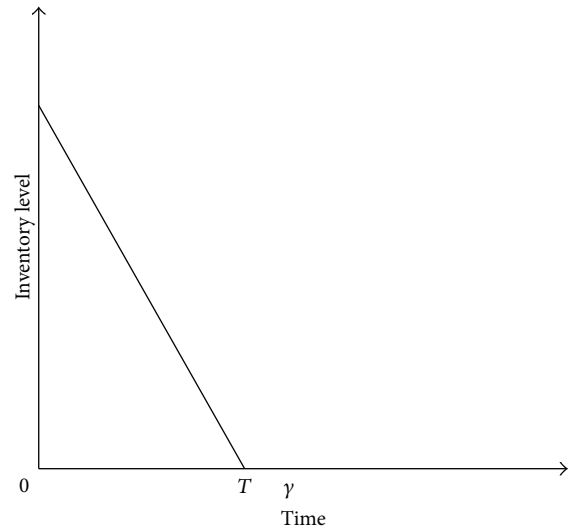

Figure 1: Inventory system for $T \le \gamma$.

The inventory level decreases owing to the constant demand rate during the whole cycle. It is given that

$$I_1(t) = Q - Dt, \quad T \le \gamma. \tag{5}$$

When $T \ge \gamma$, there is no deterioration in the time interval $[0, \gamma]$. After that, in the time interval $[\gamma, T]$, items deteriorates at a constant deterioration rate θ, which is shown in Figure 2.

The inventory level decreases owing to the demand rate in time $[0, \gamma]$, which is given by

$$I_{21}(t) = Q - Dt, \quad 0 \le t \le \gamma. \tag{6}$$

In time $[\gamma, T]$, the inventory level declines owing to both the demand rate and the deterioration. Thus, the inventory level is represented by the partial differential equation

$$\frac{\partial I_{22}(t)}{\partial t} = -D - \theta I_{22}(t), \quad \gamma \le t \le T, \tag{7}$$

with the boundary condition $I_{22}(T) = 0$.

The solution of (7) is

$$I_{22}(t) = \frac{D}{\theta}\left[e^{\theta(T-t)} - 1\right], \quad \gamma \le t \le T. \tag{8}$$

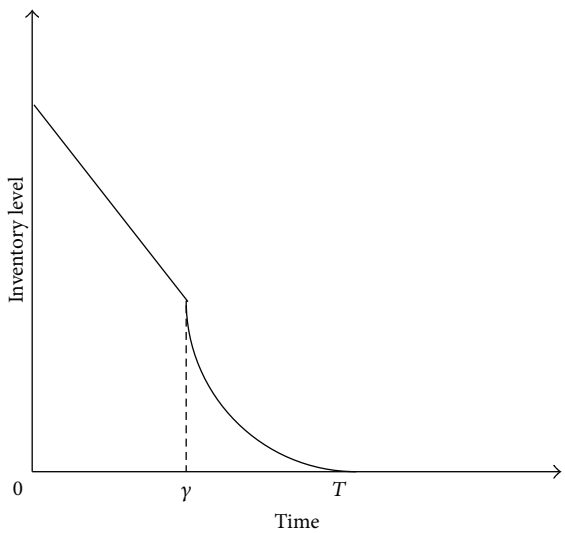

FIGURE 2: Inventory system for $T \geq \gamma$.

Considering the continuity of $I_{21}(t)$ and $I_{22}(t)$ at time $t = \gamma$, it follows from (5) and (7) that

$$I_{21}(\gamma) = I_{21}(\gamma) = Q - D\gamma = \frac{D}{\theta}\left[e^{\theta(T-\gamma)} - 1\right], \tag{9}$$

which implies that the ordering quantity per cycle is

$$Q = D\gamma + \frac{D}{\theta}\left[e^{\theta(T-\gamma)} - 1\right]. \tag{10}$$

Then substitute (10) into (6), we have

$$I_{21}(t) = D(\gamma - t) + \frac{D}{\theta}\left[e^{\theta(T-\gamma)} - 1\right], \quad 0 \leq t \leq \gamma. \tag{11}$$

The total annual relevant cost consists of the following five parts.

(a) Sales profit (SP):

$$\mathrm{SP} = (p - c) D. \tag{12}$$

(b) Cost of ordering cost per year (OC):

$$\mathrm{OC} = \frac{A}{T}. \tag{13}$$

(c) Cost of holding inventory (HC): There are two possible situations based on the value of T and γ. When $T \leq \gamma$, the inventory system is the first type shown in Figure 1. When $T \geq \gamma$, the inventory system is the second type depicted in Figure 2. Consequently, the inventory holding cost is given by

$$\mathrm{HC} = \begin{cases} \dfrac{h}{T}\displaystyle\int_0^T I_1(t)\,dt & T \leq \gamma \\ \dfrac{h}{T}\left(\displaystyle\int_0^\gamma I_{21}(t)\,dt + \int_\gamma^T I_{22}(t)\,dt\right) & T \geq \gamma. \end{cases} \tag{14}$$

(d) Cost of deterioration items (DC): For $T \leq \gamma$, there is no deterioration. For $T \geq \gamma$, the cost of deteriorated items is $c(Q - DT)/T$. So, the deterioration cost is given by

$$\mathrm{DC} = \begin{cases} \dfrac{c(Q - DT)}{T} & T \geq \gamma \\ 0 & T \leq \gamma. \end{cases} \tag{15}$$

(e) Opportunity cost (IC) and interest earned from sales revenue (IE): In order to establish the total relevant inventory cost function, we consider three cases: Case 1. $M \leq N$; Case 2. $N \leq M \leq N + \gamma$; and Case 3. $M \geq N + \gamma$.

Case 1 ($M \leq N$). In this case, there are two circumstances: $T \leq \gamma$ and $T \geq \gamma$. And, when $M \leq N$, there is no interest earned by the retailer.

(1) $T \leq \gamma$. The inventory system is depicted in Figure 3. The retailer has the opportunity cost and has no interest earned. The opportunity cost is calculated as

$$\mathrm{IC}_{11} = \frac{cI_c}{T}\left((N - M)Q + \frac{DT^2}{2}\right). \tag{16}$$

The total average profit function is

$$\begin{aligned} Z_{11}(T, N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} + \mathrm{IC}_{12}) \\ &= (p - c) D - \frac{A}{T} - \frac{(h + cI_c) DT}{2} - cI_c (N - M) D. \end{aligned} \tag{17}$$

(2) $T \geq \gamma$. The inventory system is depicted in Figure 4. The retailer has the opportunity cost and has no interest earned. The opportunity cost is calculated as

$$\mathrm{IC}_{12} = \frac{cI_c}{T}\left((N - M)Q + \int_0^\gamma I_{21}(t)\,dt + \int_\gamma^T I_{22}(t)\,dt\right). \tag{18}$$

The total average profit function is

$$\begin{aligned} &Z_{12}(T, N) \\ &\quad = \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} + \mathrm{IC}_{12}) \\ &\quad = (p - c) D - \frac{A}{T} + \frac{(h + cI_c) D}{T} \\ &\qquad \times \left(\frac{\gamma^2}{2} + \frac{\gamma}{\theta}\left(e^{\theta(T-\gamma)} - 1\right) + \frac{1}{\theta^2}\left(e^{\theta(T-\gamma)} - \theta(T - \gamma) - 1\right)\right) \\ &\qquad - \frac{(c + cI_c(N - M)) D}{T}\left(\gamma + \frac{1}{\theta}\left(e^{\theta(T-\gamma)} - 1\right)\right) + cD. \end{aligned} \tag{19}$$

The problem of Case 1 is to maximize the function

$$Z_1(T, N) = \begin{cases} Z_{11}(T, N) & T \leq \gamma \\ Z_{12}(T, N) & T \geq \gamma. \end{cases} \tag{20}$$

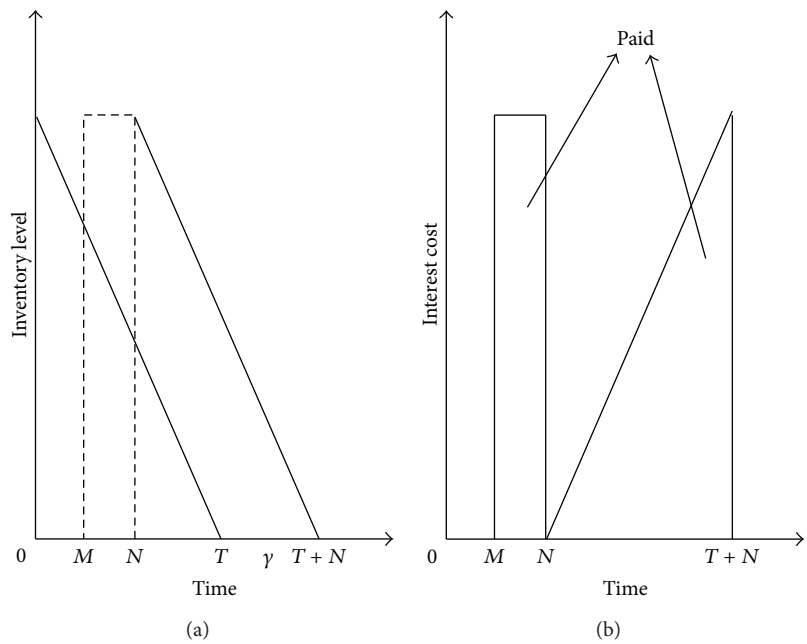

Figure 3: Inventory system for Case 1 when $T \leq \gamma$.

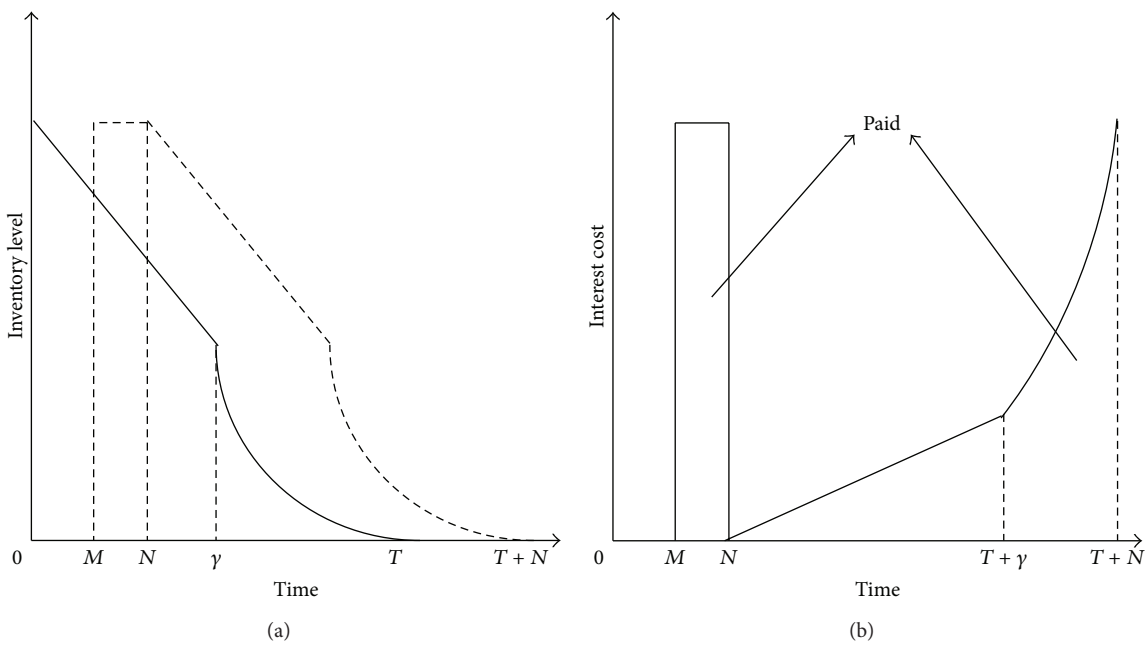

Figure 4: Inventory system for Case 1 when $T \geq \gamma$.

Case 2 ($N \leq M \leq N + \gamma$). In this case, there are three circumstances: $T \leq M - N$, $M - N \leq T \leq \gamma$, and $T \geq \gamma$.

(1) $T \leq M - N$. The inventory system is depicted in Figure 5. The retailer has no opportunity cost and only has the interest earned. The interest earned per cycle is calculated as

$$\mathrm{IE}_{21} = \frac{pI_p}{T} \cdot \left(\frac{DT^2}{2} + DT(M - T - N) \right). \tag{21}$$

The total average profit function is

$$\begin{aligned} Z_{21}(T, N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} - \mathrm{IE}_{21}) \\ &= (p - c) D - \frac{A}{T} - \frac{hDT}{2} + \frac{pI_p}{T} \\ &\quad \cdot \left(\frac{DT^2}{2} + DT(M - T - N) \right). \end{aligned} \tag{22}$$

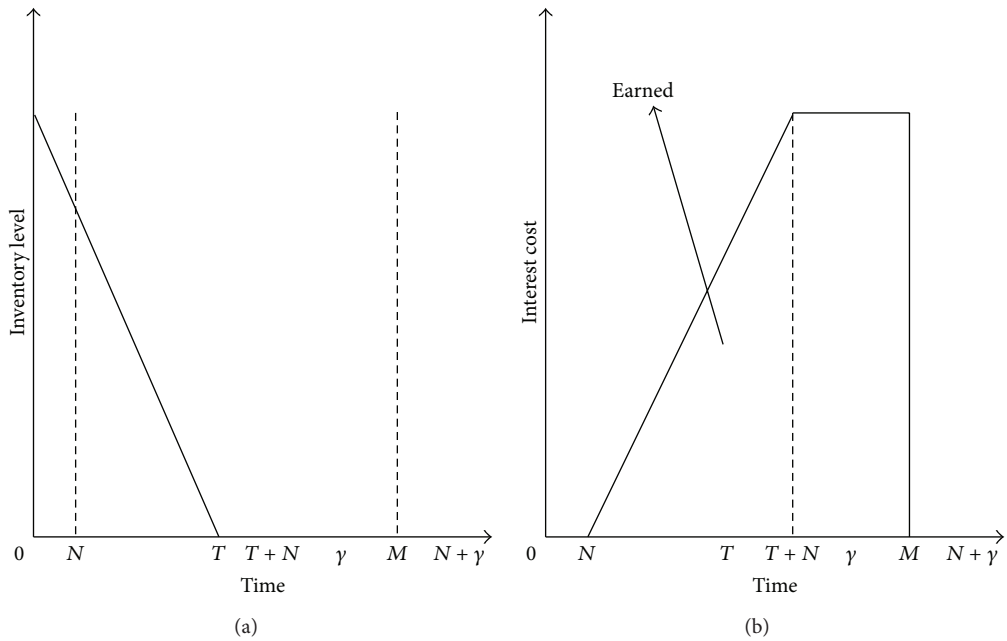

Figure 5: Inventory system for Case 2 when $T \leq M - N$.

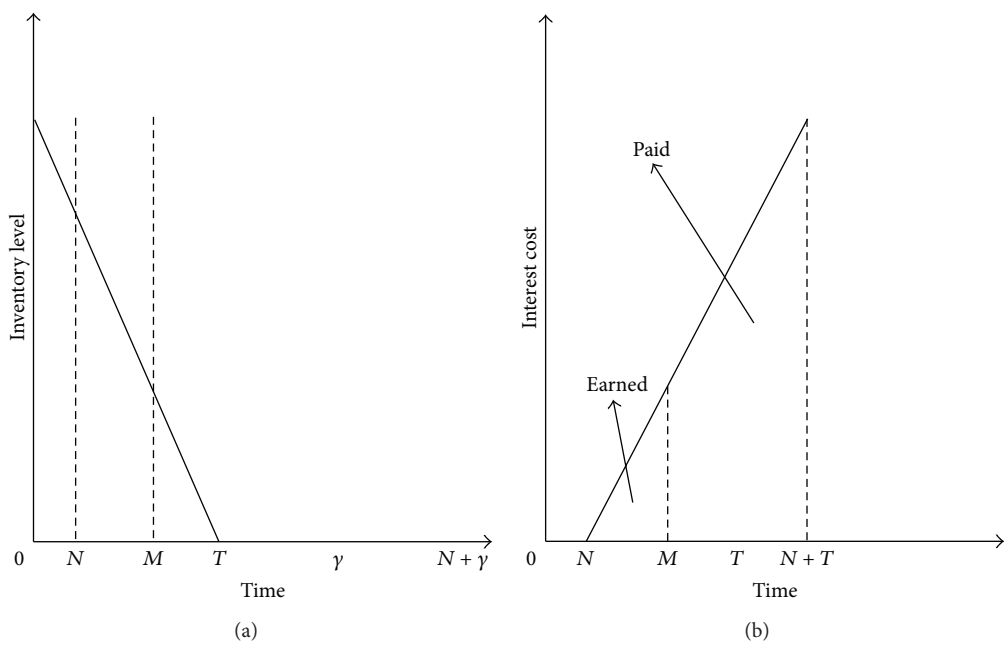

Figure 6: Inventory system for Case 2 when $M - N \leq T \leq \gamma$.

(2) $M - N \leq T \leq \gamma$. The inventory system is depicted in Figure 6. The opportunity cost per cycle is calculated as

$$\mathrm{IC}_{22} = \frac{cI_c}{T} \cdot \frac{D(T + N - M)^2}{2}. \tag{23}$$

The interest earned per cycle is calculated as

$$\mathrm{IE}_{22} = \frac{pI_p}{T} \cdot \frac{D(M - N)^2}{2}. \tag{24}$$

The total average profit function is

$$\begin{aligned} Z_{22}(T, N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} + \mathrm{IC}_{22} - \mathrm{IE}_{22}) \\ &= (p - c) D - \frac{A}{T} - \frac{hDT}{2} - \frac{cI_c}{T} \\ &\quad \cdot \frac{D(T + N - M)^2}{2} + \frac{pI_p}{T} \cdot \frac{D(M - N)^2}{2}. \end{aligned} \tag{25}$$

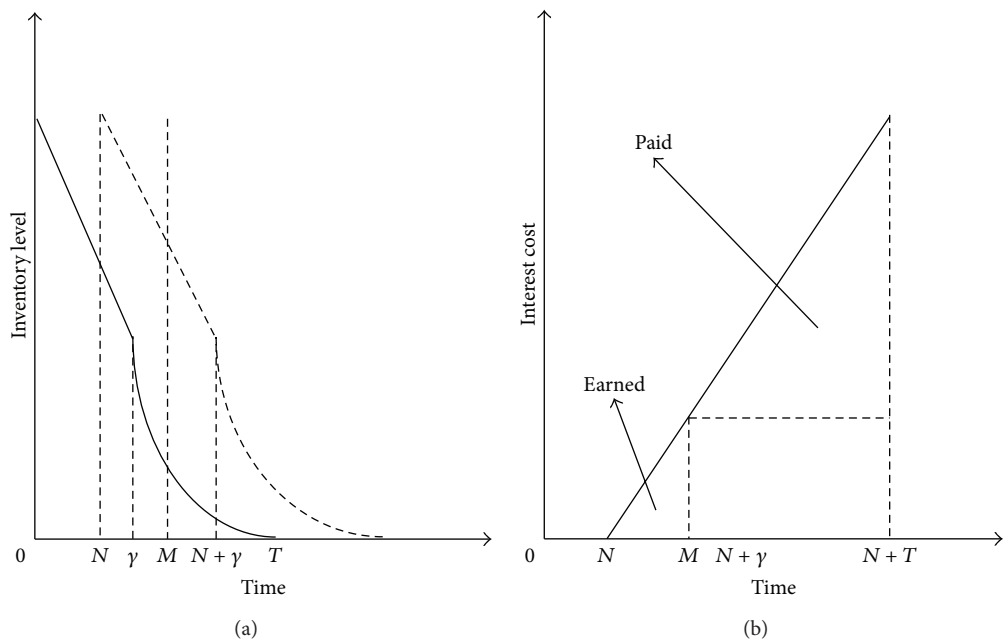

Figure 7: Inventory system for Case 2 when $T \geq \gamma$.

(3) $T \geq \gamma$. The inventory system is depicted in Figure 7. The opportunity cost per cycle is calculated as

$$\mathrm{IC}_{23} = \frac{cI_c}{T}\left(\int_{M-N}^{\gamma} I_{21}(t)\,dt + \int_{\gamma}^{T} I_{22}(t)\,dt\right). \tag{26}$$

The interest earned per cycle is calculated as

$$\mathrm{IE}_{23} = \frac{pI_p}{T}\cdot\frac{D(M-N)^2}{2}. \tag{27}$$

The total average profit function is

$$\begin{aligned} Z_{23}(T,N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} + \mathrm{IC}_{23} - \mathrm{IE}_{23}) \\ &= (p-c)D - \left(A + \frac{(h+cI_c)D\gamma^2}{2} - cI_c(M-N)D\gamma \right. \\ &\qquad + \frac{cI_c(M-N)^2D}{2} \\ &\qquad \left. + cD\gamma - \frac{pI_p(M-N)^2D}{2}\right)\times T^{-1} \\ &\quad - \left(\frac{h\gamma D + cI_c(\gamma - M + N)D + cD}{\theta}\right)\frac{e^{\theta(T-\gamma)}-1}{T} \\ &\quad - \frac{(h+cI_c)D}{\theta^2}\frac{e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1}{T} + cD. \end{aligned} \tag{28}$$

The problem of Case 2 is to maximize the function

$$Z_2(T,N) = \begin{cases} Z_{21}(T,N) & T \leq M-N \\ Z_{22}(T,N) & M-N \leq T \leq \gamma \\ Z_{23}(T,N) & T \geq \gamma. \end{cases} \tag{29}$$

Case 3 ($M \geq N + \gamma$). In this case, there are three circumstances: $T \leq \gamma$, $\gamma \leq T \leq M - N$, and $T \geq M - N$.

(1) $T \leq \gamma$. The inventory system is depicted in Figure 8. There is no opportunity cost under this circumstance. The interest earned per cycle is calculated as

$$\mathrm{IE}_{31} = \frac{pI_p}{T}\cdot\left(\frac{DT^2}{2} + DT(M-T-N)\right). \tag{30}$$

The total average profit function is

$$\begin{aligned} Z_{31}(T,N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} - \mathrm{IE}_{31}) \\ &= (p-c)D - \frac{A}{T} - \frac{hDT}{2} + \frac{pI_p}{T} \\ &\quad \cdot\left(\frac{DT^2}{2} + DT(M-T-N)\right). \end{aligned} \tag{31}$$

(2) $\gamma \leq T \leq M - N$. The inventory system is depicted in Figure 9. There is no opportunity cost under this circumstance. The interest earned per cycle is calculated as

$$\mathrm{IE}_{32} = \frac{pI_p}{T}\cdot\left(\frac{DT^2}{2} + DT(M-T-N)\right). \tag{32}$$

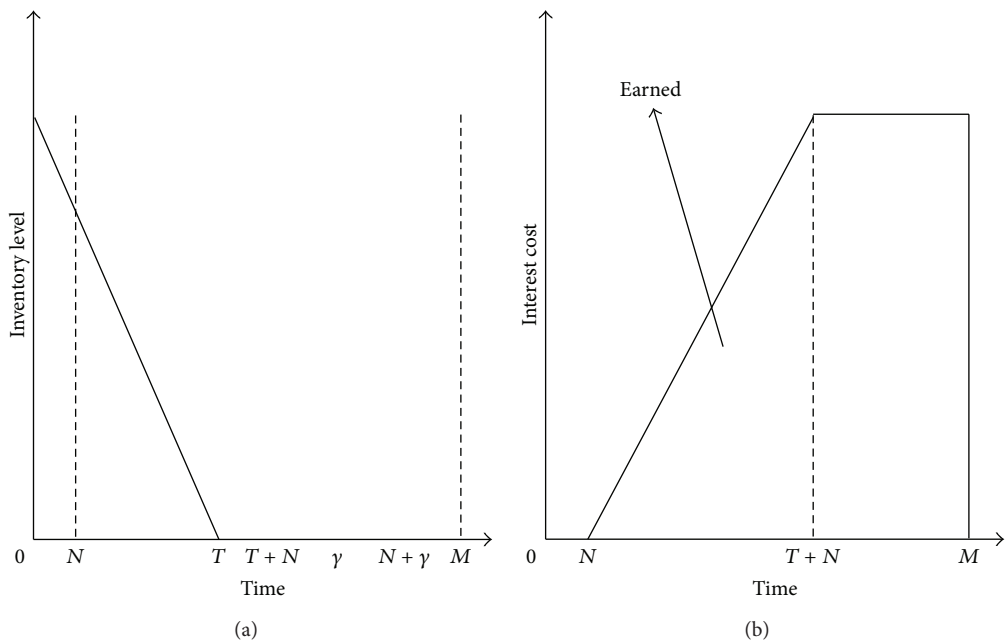

Figure 8: Inventory system for Case 3 when $T \le \gamma$.

The total average profit function is

$$\begin{aligned} Z_{32}(T,N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} - \mathrm{IE}_{32}) \\ &= (p-c)D - \frac{A}{T} - \frac{hD}{T} \\ &\quad \times \left(\frac{\gamma^2}{2} + \frac{\gamma}{\theta}\left(e^{\theta(T-\gamma)} - 1\right) + \frac{1}{\theta^2}\left(e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1\right)\right) \\ &\quad - \frac{cD}{T}\left(\gamma + \frac{1}{\theta}\left(e^{\theta(T-\gamma)} - 1\right)\right) + cD + \frac{pI_p}{T} \\ &\quad \cdot \left(\frac{DT^2}{2} + DT(M-T-N)\right). \end{aligned} \tag{33}$$

(3) $T \ge M - N$. The inventory system is depicted in Figure 10. The opportunity cost per cycle is calculated as

$$\mathrm{IC}_{33} = \frac{cI_c}{T} \cdot \int_{M-N}^{T} I_{22}(t)\,dt. \tag{34}$$

The interest earned per cycle is calculated as

$$\mathrm{IE}_{33} = \frac{pI_p}{T} \cdot \frac{D(M-N)^2}{2}. \tag{35}$$

The total average profit function is

$$\begin{aligned} Z_{33}(T,N) &= \mathrm{SP} - (\mathrm{OC} + \mathrm{HC} + \mathrm{DC} + \mathrm{IC}_{31} - \mathrm{IE}_{31}) \\ &= (p-c)D - \frac{A}{T} - \frac{hD}{T} \\ &\quad \times \left(\frac{\gamma^2}{2} + \frac{\gamma}{\theta}\left(e^{\theta(T-\gamma)} - 1\right) + \frac{1}{\theta^2}\left(e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1\right)\right) \\ &\quad + \frac{cD}{T}\left(\gamma + \frac{1}{\theta}\left(e^{\theta(T-\gamma)} - 1\right)\right) \\ &\quad + cD - \frac{cI_c}{T}\left(\frac{e^{\theta(T-M+N)} - 1}{\theta^2} - \frac{T+M-N}{\theta}\right) \\ &\quad + \frac{pI_p(M-N)^2 D}{2T}. \end{aligned} \tag{36}$$

The problem of Case 3 is to maximize the following function:

$$Z_3(T,N) = \begin{cases} Z_{31}(T,N) & T \le \gamma \\ Z_{32}(T,N) & \gamma \le T \le M-N \\ Z_{33}(T,N) & T \ge M-N. \end{cases} \tag{37}$$

4. Solution Procedure

Now, we shall determine the optimal cycle length and downstream credit period for the three cases under maximizing the total average profit function. To find the optimal solution, say (T^*, N^*), for $Z_i(T,N)$ $(i = 1,2,3)$, the following procedures

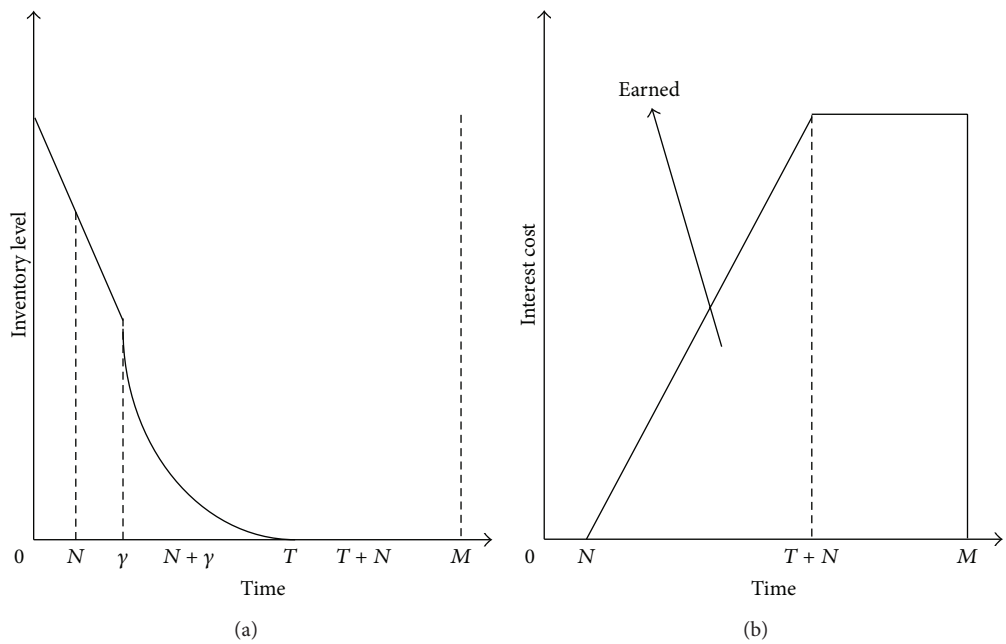

Figure 9: Inventory system for Case 3 when $\gamma \leq T \leq M - N$.

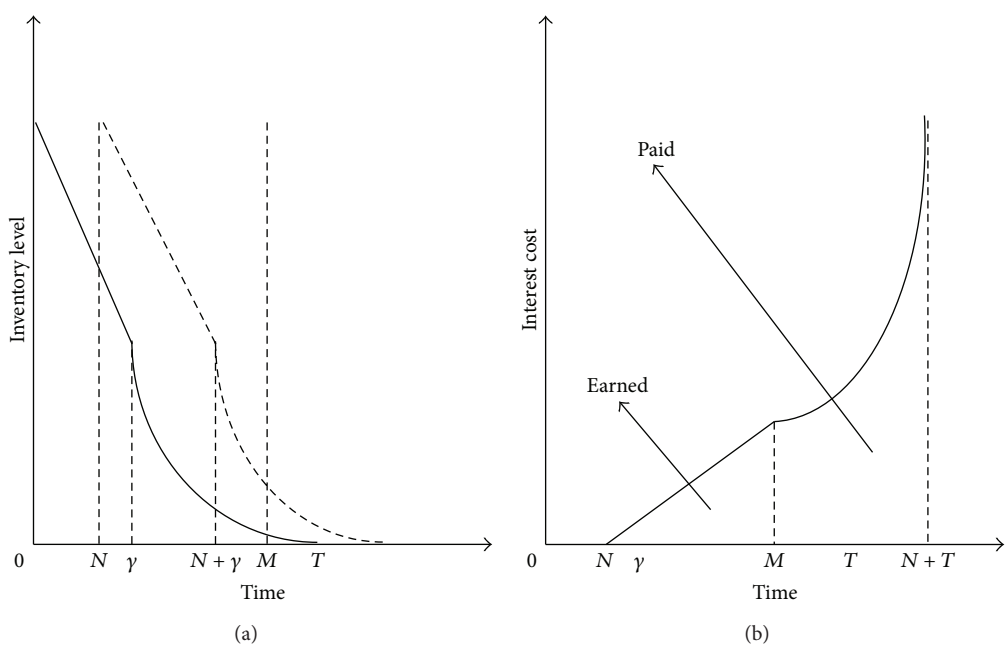

Figure 10: Inventory system for Case 3 when $T \geq M - N$.

are considered. We first analyze the property of the optimal replenishment cycle length for any fixed N ($N = 1, 2, 3, \ldots$).

Case 1 ($M \leq N$). The problem is to minimize function (20). It can be calculated that $Z_{11}(\gamma) = Z_{12}(\gamma)$. So function (20) is continuous at point $T = \gamma$. The first-order necessary condition for $Z_{11}(T)$ in (13) to be maximized is

$$\frac{\partial Z_{11}(T \mid N)}{\partial T} = \frac{A}{T^2} - \frac{(h + cI_c) D}{2} = 0. \tag{38}$$

The second-order sufficient condition is

$$\frac{\partial^2 Z_{11}(T \mid N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{39}$$

Consequently, $Z_{11}(T \mid N)$ is a concave function of T. Thus, there exists a unique value of T (say T_{11}) which minimize $Z_{11}(T \mid N)$ as

$$T_{11} = \sqrt{\frac{2A}{(h + cI_c) D}}. \tag{40}$$

To ensure $T \le \gamma$, we substitute (40) into inequality $T \le \gamma$ and we obtain

$$0 < 2A \le (h + cI_c) D\gamma^2. \tag{41}$$

Likewise, the first-order necessary condition for $Z_{12}(T \mid N)$ in (19) to be maximized is

$$\begin{aligned}
&\frac{\partial Z_{12}(T \mid N)}{\partial T} \\
&= \frac{\left[A + (h + cI_c) D\gamma^2/2 + cD\gamma + cI_c (N - M) D\gamma\right]}{T^2} \\
&\quad - \left[\frac{(h + cI_c) D\gamma}{\theta} + \frac{c + cI_c (N - M)}{\theta} D\right] \\
&\quad \times \left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1}{T^2}\right) \\
&\quad - \frac{(h + cI_c) D}{\theta^2} \left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1 - \theta\gamma}{T^2}\right) = 0.
\end{aligned} \tag{42}$$

By using the analogous arguments, we can easily obtain that

$$\begin{aligned}
&\left[A + \frac{(h + cI_c) D\gamma^2}{2} + cD\gamma + cI_c (N - M) D\gamma\right] \\
&\quad - \left[\frac{(h + cI_c) D\gamma}{\theta} + \frac{c + cI_c (N - M)}{\theta} D\right] \\
&\quad \times \left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \\
&\quad - \frac{(h + cI_c) D}{\theta^2} \left(\theta e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) = 0.
\end{aligned} \tag{43}$$

It is not easy to find a closed form solution of T from (43). But we can show that the value of T satisfy (43) not only exists but also is unique. So we have the following lemma.

Lemma 1. *For a given N, when $M \le N$,*

(a) *if $2A \ge (h + cI_c)D\gamma^2$, then the solution of $T \in [\gamma, +\infty)$ (say T_{12}) in (43) not only exists but also is unique.*

(b) *if $0 < 2A < (h + cI_c)D\gamma^2$, then the solution of $T \in [\gamma, +\infty)$ in (43) does not exist.*

Proof. See Appendix A. □

According to Lemma 1, we have the following result.

Lemma 2. *For a given N, when $M \le N$,*

(a) *if $2A \ge (h+cI_c)D\gamma^2$, then the total relevant cost $Z_{12}(T \mid N)$ has the global maximum value at point $T = T_{12}$, where $T_{12} \in [\gamma, +\infty)$ and satisfies (43).*

(b) *if $0 < 2A < (h + cI_c)D\gamma^2$, then the total relevant cost $Z_{12}(T \mid N)$ has maximum value at the boundary point $T = \gamma$.*

Proof. See Appendix A. □

For notational convenience, we mark that $\Delta_1 = (h + cI_c)D\gamma^2$. Combining the above mentioned inequality (41), Lemmas 1 and 2 and the assumption $M \le N$, we can obtain the following theorem.

Theorem 3. *For a given N, when $M \le N$,*

(a) *if $2A < \Delta_1$, then $Z_1(T^* \mid N) = Z_{11}(T_{11} \mid N)$ and $T^* = T_{11}$;*

(b) *if $2A \ge \Delta_1$, then $Z_1(T^* \mid N) = \max(Z_{12}(T_{12} \mid N), Z_{12}(\gamma \mid N))$. Hence T^* is T_{12} or γ associated with lower total average profit.*

Case 2 ($N \le M \le N + \gamma$). The problem is to maximize function (29). It can be calculated that $Z_{21}(M - M \mid N) = Z_{22}(M - M \mid N)$, $Z_{22}(\gamma \mid N) = Z_{23}(\gamma \mid N)$. So function (29) is continuous at point $T = M - N$ and $T = \gamma$.

The first-order necessary condition for $Z_{21}(T \mid N)$ in (22) to be minimized is

$$\frac{\partial Z_{21}(T \mid N)}{\partial T} = \frac{A}{T^2} - \frac{(h + pI_p) D}{2} = 0. \tag{44}$$

The second-order sufficient condition is

$$\frac{\partial^2 Z_{21}(T \mid N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{45}$$

Consequently, $Z_{21}(T \mid N)$ is a concave function of T. Thus, there exists a unique value of T (say T_{21}) which minimize $Z_{21}(T \mid N)$ as

$$T_{21} = \sqrt{\frac{2A}{(h + pI_p) D}}. \tag{46}$$

To ensure $T \le M - N$, we substitute (46) into inequality $T \le M - N$ and obtain

$$0 < 2A \le (h + pI_p) D(M - N)^2. \tag{47}$$

The first-order necessary condition for $Z_{22}(T \mid N)$ in (25) to be maximized is

$$\begin{aligned}
&\frac{\partial Z_{22}(T \mid N)}{\partial T} \\
&= \frac{A}{T^2} - \frac{(h + cI_c) D}{2} - \frac{pI_p (M - N)^2 D}{2T^2} + \frac{cI_c (M - N)^2 D}{2T^2} \\
&= 0.
\end{aligned} \tag{48}$$

The second-order sufficient condition is

$$\frac{\partial^2 Z_{22}(T \mid N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{49}$$

Consequently, $Z_{22}(T \mid N)$ is a concave function of T. Thus there exists a unique value of T (say T_{22}) which maximizes $Z_{22}(T \mid N)$ as

$$T = \sqrt{\frac{2A + (cI_c - pI_p)(M - N)^2 D}{(h + cI_c) D}}. \tag{50}$$

To ensure $M - N < T \le \gamma$, we substitute (50) into inequality $M - N < T \le \gamma$ and obtain

$$\begin{aligned}&\left(h + pI_p\right) D(M-N)^2 \\ &\quad < 2A \le \left(h + cI_c\right)\gamma^2 D - \left(cI_c - pI_p\right)(M-N)^2 D.\end{aligned} \tag{51}$$

Likewise, the first-order necessary condition for $Z_{23}(T \mid N)$ in (28) to be maximized is

$$\begin{aligned}&\frac{\partial Z_{23}(T \mid N)}{\partial T} \\ &= \left[A + \frac{(h+cI_c)D\gamma^2}{2} - cI_c(M-N)D\gamma \right. \\ &\quad \left. + \frac{cI_c(M-N)^2 D}{2} + cD\gamma - \frac{pI_p(M-N)^2 D}{2}\right] \times \left(T^2\right)^{-1} \\ &\quad - \left[\frac{h\gamma D + cI_c(\gamma - M + N)D + cD}{\theta}\right] \\ &\quad \times \left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1}{T^2}\right) \\ &\quad - \frac{(h+cI_c)D}{\theta^2}\left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1-\theta\gamma}{T^2}\right) = 0.\end{aligned} \tag{52}$$

By using the analogous arguments, we can easily obtain that

$$\begin{aligned}&\left[A + \frac{(h+cI_c)D\gamma^2}{2} - cI_c(M-N)D\gamma \right. \\ &\quad \left. + \frac{cI_c(M-N)^2 D}{2} + cD\gamma - \frac{pI_p(M-N)^2 D}{2}\right] \\ &\quad - \left[\frac{h\gamma D + cI_c(\gamma - M + N)D + cD}{\theta}\right] \\ &\quad \times \left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \\ &\quad - \frac{(h+cI_c)D}{\theta^2}\left(\theta e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) = 0.\end{aligned} \tag{53}$$

So, we have the following lemma.

Lemma 4. *For a given N, when $N \le M \le N + \gamma$,*

(a) *if $2A \ge (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M-N)^2 D$, then the solution of $T \in [\gamma, +\infty)$ (say T_{23}) in (53) not only exists but also is unique.*

(b) *if $2A < (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M-N)^2 D$, then the solution of $T \in [\gamma, +\infty)$ in (53) does not exist.*

Proof. See Appendix B. □

According to Lemma 4, we have the following result.

Lemma 5. *For a given N, when $N \le M \le N + \gamma$,*

(a) *if $2A \ge (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M-N)^2 D$, then the total average profit $Z_{23}(T \mid N)$ has the global maximum value at point $T = T_{23}$, where $T_{23} \in [\gamma, +\infty)$ and satisfies (52);*

(b) *if $2A < (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M-N)^2 D$, then the total average profit $Z_{23}(T \mid N)$ has maximum value at the boundary point $T = \gamma$.*

Proof. See Appendix B. □

For notational convenience, we mark that

$$\begin{aligned}\Delta_2 &= \left(h + pI_p\right) D(M-N)^2, \\ \Delta_3 &= \left(h + cI_c\right)\gamma^2 D - \left(cI_c - pI_p\right)(M-N)^2 D.\end{aligned} \tag{54}$$

Combining the forementioned mentioned equations (47) and (51), Lemmas 4 and 5, and the assumption $N \le M \le N + \gamma$, we can obtain the following theorem.

Theorem 6. *For a given N, when $N \le M \le N + \gamma$,*

(a) *if $0 < 2A < \Delta_2$, then $Z_2(T^* \mid N) = \max(Z_{21}(T_{21} \mid N), Z_{21}(M - N \mid N))$. Hence T^* is T_{21} or $M - N$ associated with higher total average profit;*

(b) *if $\Delta_2 \le 2A < \Delta_3$, then $Z_2(T^* \mid N) = \max(Z_{22}(T_{22} \mid N), Z_{22}(\gamma \mid N))$. Hence T^* is T_{23} or γ associated with higher total average profit.*

(c) *if $2A \ge \Delta_3$, then $Z_2(T^* \mid N) = Z_{23}(T_{23} \mid N)$, $T^* = T_{23}$.*

Case 3 ($M \ge N + \gamma$). The problem is to maximize function (37). It can be calculated that $Z_{31}(\gamma \mid N) = Z_{32}(\gamma \mid N)$ and $Z_{32}(M - N \mid N) = Z_{33}(M - N \mid N)$. So function (37) is continuous at point $T = \gamma$ and $T = M - N$.

The first-order necessary condition for $Z_{31}(T \mid N)$ in (31) to be minimized is

$$\frac{\partial Z_{31}(T \mid N)}{\partial T} = \frac{A}{T^2} - \frac{hD}{2} - \frac{pI_p}{2} = 0. \tag{55}$$

The second-order sufficient condition is

$$\frac{\partial^2 Z_{31}(T \mid N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{56}$$

Consequently, $Z_{31}(T \mid N)$ is a convex function of T. Thus, there exists a unique value of T (say T_{31}) which minimizes $Z_{31}(T \mid N)$ as

$$T_{31} = \sqrt{\frac{2A}{\left(h + pI_p\right)D}}. \tag{57}$$

To ensure $T < \gamma$, we substitute (57) into inequality $T < \gamma$ and obtain

$$0 < 2A < \left(h + pI_p\right) D\gamma^2. \tag{58}$$

Likewise, the first-order necessary condition for $Z_{32}(T \mid N)$ in (33) to be maximized is

$$\begin{aligned}\frac{\partial Z_{32}(T \mid N)}{\partial T} &= \frac{\left(A + hD\gamma^2/2 + cD\gamma\right)}{T^2} - \left(\frac{hD\gamma + cD}{\theta}\right) \\ &\times \left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1}{T^2}\right) \\ &- \frac{hD}{\theta^2}\left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1-\theta\gamma}{T^2}\right) - \frac{pI_pD}{2} = 0.\end{aligned} \tag{59}$$

By using the analogous arguments, we can easily obtain that

$$\begin{aligned}&\left(A + \frac{hD\gamma^2}{2} + cD\gamma\right) - \left(\frac{hD\gamma + cD}{\theta}\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \\ &- \frac{hD}{\theta^2}\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) - \frac{pI_pDT^2}{2} = 0.\end{aligned} \tag{60}$$

Let

$$\begin{aligned}\Delta_4 &= \left(hD + pI_pD\right)\gamma^2, \\ \Delta_5 &= -\left(hD\gamma^2 + 2cD\gamma\right) + 2\left(\frac{hD\gamma + cD}{\theta} + \frac{hD}{\theta^2}\right) \\ &\times \left[\theta(M-N)e^{\theta(M-N-\gamma)} - e^{\theta(M-N-\gamma)} + 1\right] \\ &- \frac{2hD\gamma}{\theta} + pI_pD(M-N)^2.\end{aligned} \tag{61}$$

Then, we have the following lemma.

Lemma 7. *For a given N, when $M \geq N + \gamma$,*

(a) *if $\Delta_4 \leq 2A \leq \Delta_5$, then the solution of $T \in [\gamma, M-N]$ (say T_{32}) in (60) not only exists but also is unique;*

(b) *if $2A < \Delta_4$ or $2A > \Delta_5$, then the solution of $T \in [\gamma, M-N]$ in (60) does not exist.*

Proof. See Appendix C. □

According to Lemma 7, we have the following result.

Lemma 8. *For a given N, when $M \geq N + \gamma$,*

(a) *if $\Delta_4 \leq 2A \leq \Delta_5$, then the total average profit $Z_{32}(T \mid N)$ has the global maximum value at point $T = T_{32}$, where $T_{32} \in [\gamma, M-N]$ and satisfies (60);*

(b) *if $2A < \Delta_4$, then the total average profit $Z_{32}(T \mid N)$ has the maximum value at the boundary point $T = \gamma$;*

(c) *if $2A > \Delta_5$, then the total average profit $Z_{32}(T \mid N)$ has the maximum value at the boundary point $T = M-N$.*

Proof. See Appendix C. □

Likewise, the first-order necessary condition for $Z_{33}(T \mid N)$ in (36) to be maximized is

$$\begin{aligned}\frac{\partial Z_{33}(T \mid N)}{\partial T} &= \frac{\left(A + hD\gamma^2/2 + cD\gamma - cI_c(M-N) - pI_p(M-N)^2D/2\right)}{T^2} \\ &- \left(\frac{hD\gamma + cD}{\theta}\right)\left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1}{T^2}\right) \\ &+ \frac{hD}{\theta^2}\left(\frac{\theta e^{\theta(T-\gamma)}}{T} - \frac{e^{\theta(T-\gamma)}}{T^2} + \frac{1-\theta\gamma}{T^2}\right) \\ &- \frac{cI_c}{\theta^2}\left(\frac{\theta e^{\theta(T-M+N)}}{T} - \frac{e^{\theta(T-M+N)}}{T^2} + \frac{1}{T^2}\right) = 0.\end{aligned} \tag{62}$$

By using the analogous arguments, we can easily obtain that

$$\begin{aligned}&\left(A + \frac{hD\gamma^2}{2} + cD\gamma - cI_c(M-N) - \frac{pI_p(M-N)^2D}{2}\right) \\ &- \left(\frac{hD\gamma + cD}{\theta}\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \\ &- \frac{hD}{\theta^2}\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) \\ &- \frac{cI_c}{\theta^2}\left(\theta T e^{\theta(T-M+N)} - e^{\theta(T-M+N)} + 1\right) = 0.\end{aligned} \tag{63}$$

Also, we have the following lemma.

Lemma 9. *For a given N, when $M \geq N + \gamma$,*

(a) *if $2A \geq \Delta_5$, then the solution of $T \in [M-N, +\infty)$ (say T_{33}) in (63) not only exists but also is unique;*

(b) *if $0 < 2A < \Delta_5$, then the solution of $T \in [M-N, +\infty)$ in (63) does not exist.*

Proof. See Appendix D. □

According to Lemma 9, we have the following result.

Lemma 10. *For a given N, when $M \geq N + \gamma$,*

(a) *if $2A \geq \Delta_5$, then the total average profit $Z_{33}(T \mid N)$ has the global maximum value at point $T = T_{33}$, where $T_{33} \in [M-N, +\infty)$ and satisfies (63);*

(b) *if $0 < 2A < \Delta_5$, then the total average profit $Z_{33}(T \mid N)$ has the global maximum value at point $T = M-N$, where $T_{33} \in [M-N, +\infty)$.*

Proof. See Appendix D. □

Combining the forementioned mentioned equation (58), Lemmas 7–10, and the assumption $M \geq N + \gamma$, we can obtain the following theorem.

Theorem 11. *For a given N, when $M \geq N + \gamma$,*

(a) *if $0 < 2A < \Delta_4$, then $Z_3(T^* \mid N) = \max(Z_{31}(T_{31} \mid N), Z_{31}(\gamma \mid N))$, Hence T^* is T_{31} or γ associated with higher total average profit;*

(b) *if $\Delta_4 \leq 2A < \Delta_5$, then $Z_3(T^* \mid N) = \max(Z_{32}(T_{32} \mid N), Z_{32}(M - N \mid N))$. Hence T^* is T_{21} or $M - N$ associated with higher total average profit;*

(c) *if $2A \geq \Delta_5$, then $Z_3(T^* \mid N) = Z_{33}(T_{33} \mid N)$, and $T^* = T_{33}$.*

For the downstream credit is an integer according to the assumptions, and interactive algorithms can be used to find the optimal solutions for our model. By summarizing the results in Theorems 3, 6, and 11, an algorithm to illustrate the optimal solution for the model is proposed as follows.

Algorithm 12. Consider the following:

(1) Let $N = 1$.

(2) Compare the value of M, N, γ. If $M \leq N$, then go to step 3; If $N \leq M \leq N + \gamma$, then go to step 5; If $M \geq N + \gamma$, then go to step 7.

(3) Calculate $\Delta_1(N)$,

(1) If $2A < \Delta_1$, then $T^* = T_{11}$ and $Z_1(T_N^*, N) = Z_1(T^* \mid N) = Z_{11}(T_{11} \mid N)$.

(2) If $2A \geq \Delta_1$, and

(i) $Z_{12}(T_{12} \mid N) \geq Z_{12}(\gamma \mid N)$, then $T^* = T_{12}$, $Z_1(T_N^*, N) = Z_1(T^* \mid N) = Z_{12}(T_{12} \mid N)$;

(ii) $Z_{12}(T_{12} \mid N) < Z_{12}(\gamma \mid N)$, then $T^* = \gamma$, $Z_1(T_N^*, N) = Z_1(T^* \mid N) = Z_{12}(\gamma \mid N)$.

(4) If $Z_1(T_{N-1}^*, N - 1) \geq Z_1(T_N^*, N)$, then the optimum solution, say (T^*, N^*), is $(T_{N-1}^*, N - 1)$ and $Z^* = Z_1(T^*, N^*)$. Otherwise, $N = N + 1$, go to step 2.

(5) Calculate $\Delta_2(N)$ and $\Delta_3(N)$.

(1) If $2A < \Delta_2$, then $T^* = T_{21}$ and $Z_2(T_N^*, N) = Z_2(T^* \mid N) = Z_{21}(T_{21} \mid N)$.

(2) If $\Delta_2 \leq 2A < \Delta_3$, and

(i) $Z_{22}(T_{22} \mid N) \geq Z_{22}(M - N \mid N)$, then $T^* = T_{22}$, $Z_2(T_N^*, N) = Z_2(T^* \mid N) = Z_{22}(T_{22} \mid N)$;

(ii) $Z_{22}(T_{22} \mid N) < Z_{22}(M - N \mid N)$, then $T^* = M - N$, $Z_2(T_N^*, N) = Z_2(T^* \mid N) = Z_{22}(M - N \mid N)$.

(3) If $2A \geq \Delta_3$, and

(i) $Z_{23}(T_{23} \mid N) \geq Z_{23}(\gamma \mid N)$, then $T^* = T_{23}$, $Z_2(T_N^*, N) = Z_2(T^* \mid N) = Z_{23}(T_{23} \mid N)$;

(ii) $Z_{23}(T_{23} \mid N) < Z_{23}(\gamma \mid N)$, then $T^* = M - N$, $Z_2(T_N^*, N) = Z_2(T^* \mid N) = Z_{23}(\gamma \mid N)$.

(6) If $Z_2(T_{N-1}^*, N - 1) \geq Z_2(T_N^*, N)$, then the optimum solution, say (T^*, N^*), is $(T_{N-1}^*, N - 1)$ and $Z^* = Z_2(T^*, N^*)$. Otherwise, $N = N + 1$, go to step 2.

(7) Calculate $\Delta_4(N)$ and $\Delta_5(N)$.

(1) If $2A < \Delta_4$, then $T^* = T_{31}$ and $Z_3(T_N^*, N) = Z_3(T^* \mid N) = Z_{31}(T_{31} \mid N)$.

(2) If $\Delta_4 \leq 2A < \Delta_5$, and

(i) $Z_{32}(T_{32} \mid N) \geq Z_{32}(\gamma \mid N)$, then $T^* = T_{32}$, $Z_3(T_N^*, N) = Z_3(T^* \mid N) = Z_{32}(T_{32} \mid N)$;

(ii) $Z_{32}(T_{32} \mid N) < Z_{32}(\gamma \mid N)$, then $T^* = \gamma$, $Z_3(T_N^*, N) = Z_3(T^* \mid N) = Z_{32}(\gamma \mid N)$.

(3) If $2A \geq \Delta_5$, and

(i) $Z_{33}(T_{33} \mid N) \geq Z_{33}(M - N \mid N)$, then $T^* = T_{33}$, $Z_3(T_N^*, N) = Z_3(T^* \mid N) = Z_{33}(T_{33} \mid N)$;

(ii) $Z_{22}(T_{22} \mid N) < Z_{22}(M - N \mid N)$, then $T^* = M - N$, $Z_3(T_N^*, N) = Z_3(T^* \mid N) = Z_{33}(M - N \mid N)$.

(8) If $Z_3(T_{N-1}^*, N - 1) \geq Z_3(T_N^*, N)$, then the optimum solution, say (T^*, N^*), is $(T_{N-1}^*, N - 1)$ and $Z^* = Z_3(T^*, N^*)$. Otherwise, $N = N + 1$, go to step 2.

After obtaining the optimal replenishment cycle T^*, the optimal order quantity can be determined by (10), which is given that $Q^* = D\gamma + (D/\theta)[e^{\theta(T^* - \gamma)} - 1]$.

5. Special Cases

In this section, two special cases are discussed (i.e., [12, 42]) and descriptions are made.

Special Case 1 (Ouyang et al. [12]). In this model, they consider an one-level credit financing problem for noninstantaneous deteriorating items with a constant demand, which means, in our model $N \to 0$, $r \to 0$ and $\lim_{r \to 0} D(N) = \alpha$.

If we set $N \to 0$ and $r \to 0$, then for Cases 2 and 3 in our paper, the problem is

(1) for $M \leq \gamma$

$$\lim_{N \to 0} \lim_{r \to 0} Z_{21}(T) = (p - c)\alpha - \left[\frac{A}{T} + \frac{h\alpha T}{2} - \frac{pI_p}{T} \cdot \left(\frac{\alpha T^2}{2} + \alpha T(M - T)\right)\right]$$

$$\lim_{N \to 0} \lim_{r \to 0} Z_{22}(T) = (p - c)\alpha - \left[\frac{A}{T} + \frac{h\alpha T}{2} + \frac{cI_c}{T} \cdot \frac{\alpha(T - M)^2}{2} - \frac{pI_p}{T} \cdot \frac{\alpha M^2}{2}\right]$$

$$\lim_{N\to 0}\lim_{r\to 0} Z_{23}(T)$$
$$= (p-c)\alpha - \left[\left(A + \frac{(h+cI_c)\alpha\gamma^2}{2} - cI_c M\alpha\gamma + \frac{cI_c M^2\alpha}{2} + c\alpha\gamma - \frac{pI_p M^2\alpha}{2}\right)\times T^{-1} + \left(\frac{h\gamma\alpha + cI_c(\gamma - M)\alpha + c\alpha}{\theta}\right) \times \frac{e^{\theta(T-\gamma)}-1}{T} + \frac{(h+cI_c)\alpha}{\theta^2} \times \frac{e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1}{T} - c\alpha\right],$$

$$Z_2(T) = \begin{cases} Z_{21}(T) & T \le M \\ Z_{22}(T) & M \le T \le \gamma \\ Z_{23}(T) & T \ge \gamma, \end{cases} \tag{64}$$

(2) for $M \ge \gamma$

$$\lim_{N\to 0}\lim_{r\to 0} Z_{31} = (p-c)\alpha - \left[\frac{A}{T} + \frac{h\alpha T}{2} - \frac{pI_p}{T}\cdot\left(\frac{\alpha T^2}{2} + \alpha T(M-T)\right)\right]$$

$$\lim_{N\to 0}\lim_{r\to 0} Z_{32}(T) = (p-c)\alpha - \left[\frac{A}{T} + \frac{h\alpha}{T} \times\left(\frac{\gamma^2}{2} + \frac{\gamma}{\theta}\left(e^{\theta(T-\gamma)}-1\right) + \frac{1}{\theta^2}\left(e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1\right)\right) + \frac{c\alpha}{T}\left(\gamma + \frac{1}{\theta}\left(e^{\theta(T-\gamma)}-1\right)\right) - c\alpha - \frac{pI_p}{T}\cdot\left(\frac{\alpha T^2}{2} + \alpha T(M-T)\right)\right],$$

$$\lim_{N\to 0}\lim_{r\to 0} Z_{33}(T) = (p-c)\alpha - \left[\frac{A}{T} + \frac{h\alpha}{T} \times\left(\frac{\gamma^2}{2} + \frac{\gamma}{\theta}\left(e^{\theta(T-\gamma)}-1\right) + \frac{1}{\theta^2}\left(e^{\theta(T-\gamma)} - \theta(T-\gamma) - 1\right)\right) + \frac{c\alpha}{T}\left(\gamma + \frac{1}{\theta}\left(e^{\theta(T-\gamma)}-1\right)\right) - c\alpha + \frac{cI_c}{T}\left(\frac{e^{\theta(T-M)}-1}{\theta^2} - \frac{T+M}{\theta}\right) - \frac{pI_p M^2\alpha}{2T}\right],$$

$$Z_3(T) = \begin{cases} Z_{31}(T) & T \le \gamma \\ Z_{32}(T) & \gamma \le T \le M \\ Z_{33}(T) & T \ge M. \end{cases} \tag{65}$$

In this condition, the relevant cost function is the same as the problem in Case 1 (12)–(14) and Case 2 (15)–(17) of Ouyang et al. [12]. So Ouyang et al. [12] is a special case of our model.

Special Case 2(Jaggi et al. [42]). In this model, they consider the tow-level financing problem for items without deterioration and with a credit dependent demand rate, which means that $\theta \to 0$ and $\gamma \to \infty$ in our model. If we set $\theta \to 0$ and $\gamma \to \infty$, for Cases 1 and 2 in our paper, the problem is

(1) for $M \le N \le \gamma$, according to Case 1 in our paper,

$$\lim_{\gamma\to\infty}\lim_{\theta\to 0} Z_{11}(T,N) = (p-c)D - \left[\frac{A}{T} + \frac{(h+cI_c)DT}{2} + cI_c(N-M)D\right] \equiv Z_1(T,N), \tag{66}$$

(2) for $N \le M \le \gamma$, according to Case 2 in our paper,

$$\lim_{\gamma\to\infty}\lim_{\theta\to 0} Z_{21}(T,N) = (p-c)D - \left[\frac{A}{T} + \frac{hDT}{2} - \frac{pI_p}{T}\cdot\left(\frac{DT^2}{2} + DT(M-T-N)\right)\right] \equiv Z_2(T,N)$$

$$\begin{aligned}
&\lim_{\gamma\to\infty}\lim_{\theta\to 0} Z_{22}(T,N)\\
&= (p-c)D\\
&\quad -\left[\frac{A}{T}+\frac{hDT}{2}+\frac{cI_c}{T}\cdot\frac{D(T+N-M)^2}{2}\right.\\
&\qquad \left.-\frac{pI_p}{T}\cdot\frac{D(M-N)^2}{2}\right]\\
&\equiv Z_3(T,N),
\end{aligned} \tag{67}$$

which can be reduced as follows:

$$Z(T,N)=\begin{cases} Z_1(T,N) & M\le N\le T+N\\ Z_2(T,N) & N\le T+N\le M\\ Z_3(T,N) & M\le N\le T+N.\end{cases} \tag{68}$$

In this condition, the total average profit functions (66)–(67) are consistent with functions (11), (13), and (15), which are the same as the problem in Jaggi et al. [42]. So Jaggi et al. [42] is also a special case of our model.

6. Numerical Analysis

To gain further insights, we conduct the following numerical analysis.

Example 13. We consider the first type of demand rate function: $D(N)=\beta-(\beta-\alpha)e^{-rN}$, in which $\alpha=3600$, $\beta=10800$ and $r=20$. The values of other parameters are $h=4\,\$$ per unit year, $p=30\,\$$ per unit, $c=20\,\$$ per unit, $\theta=0.05$, $\gamma=20/365$ year, $M=30/365$ year, $A=75\,\$$, $I_c=15\%$ per year, and $I_p=20\%$ per year. According to our analysis of the solution procedure and the algorithm, we run the interactive numerical results with the value of $N=1,2,\ldots,90$.

There are three conditions for $M=30/365$ year and $\gamma=20/365$ year.

(a) $N\le 10/365$; (b) $11/365\le N\le 30/365$; (c) $N\ge 31/365$.

When $N\le 10/365$, say $N=5/365$, we have $\Delta_4=159.89$, $\Delta_5=258.94$, and $2A=150<\Delta_4$. Hence, $T^*=T_{31}=0.0531$ year, and $Z^*=Z_{31}(T^*)=53465\,\$$.

When $11/365\le N\le 30/365$, say $N=20/365$, we have $\Delta_2=63.00$, $\Delta_3=195.31$, and $\Delta_2<2A<\Delta_3$. Hence, $T^*=T_{32}=0.0472$ year, and $Z^*=Z_{32}(T^*)=80088\,\$$.

When $N\ge 31/365$, say $N=45/365$, we get $\Delta_1=214.13$ and $2A=150<\Delta_1$. Hence, $T^*=T_{11}=0.0459$ year, and $Z^*=Z_{11}(T^*)=97357\,\$$.

Finally, for every constant $N=1/365,2/365,\ldots,90/365$, we can get the optimal result which is depicted in Figure 11.

From Figure 11 we know that the maximum obtained by the algorithm in this model is indeed the global optimum solution. And the optimal credit offered by the retailer to customers is $N^*=69/365$ year, the optimal length of replenishment cycle is $T^*=16.39/365$ year, and the maximum total average profit is $Z^*=99607\,\$$. Here, we show that the optimal replenishment cycle length is less than nondeterioration period, and the retailer has to pay for the opportunity cost and not the interest earned.

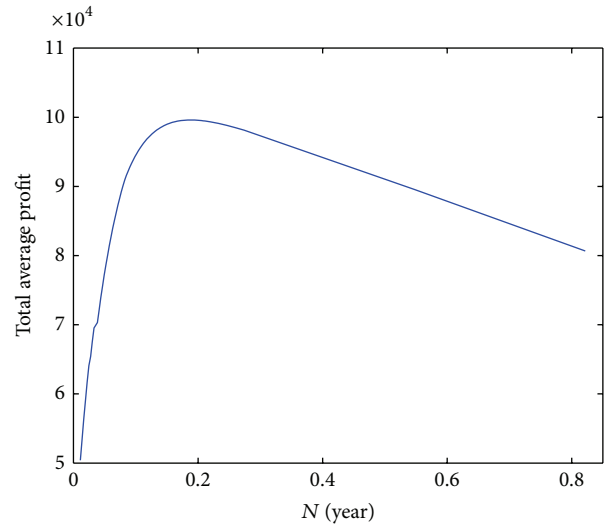

Figure 11: Optimal total average profit w.r.t. downstream credit period length of the first type demand.

Example 14. We also consider the first type of demand rate function: $D(N)=\beta-(\beta-\alpha)e^{-rN}$, in which $\alpha=3600$, $\beta=10800$, and $r=20$. The values of other parameters are the same except that $M=70/365$ year and $A=150\,\$$. Based on the algorithm, the optimal credit offered by the retailer to customers is $N^*=67/365$ year, the optimal length of replenishment cycle is $T^*=22.74/365$ year, and the maximum total average profit is $Z^*=101720\,\$$. In this example, there is interest earned and paid and deterioration cost.

Example 15. Here, we consider the second type of demand rate function: $D(N)=\beta-(\beta-\alpha)(1-r)^N$, in which $\alpha=3600$, $\beta=10800$, and $r=0.995$. The values of other parameters are the same to Example 13. The result is shown as Figure 12.

From Figure 12, we know that the global optimum not only exists but also is unique. Based on the algorithm, the optimal credit offered by the retailer to customers is $N^*=132/365$ year, the optimal length of replenishment cycle is $T^*=16.61/365$ year, and the maximum total average profit is $Z^*=91429\,\$$.

6.1. Sensitive Analysis. Here, we consider the first type of demand rate. Initial parameters are the same to these in Example 13. By varying different values for the parameters, we have the results in Table 1.

Comments can be obtained from Table 1 as follows.

(1) It can be observed that as A increases, T^* and Q^* increases and Z^* decreases. The optimal downstream credit N^* remains at the threshold. It shows that for a higher ordering cost, the retailer should replenish less frequently and should stock more items at one cycle

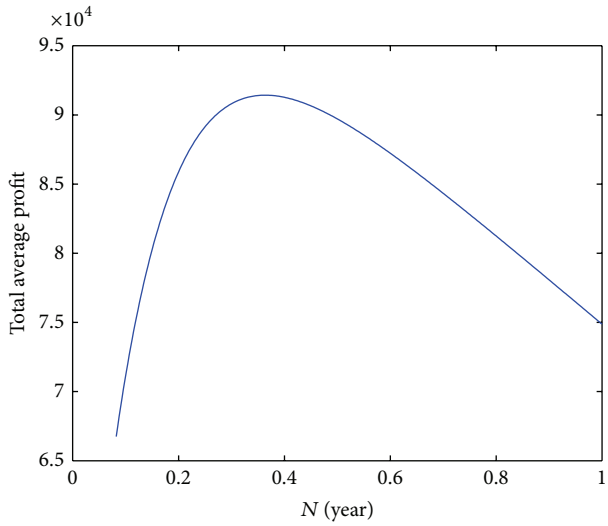

Figure 12: Optimal total average profit w.r.t. downstream credit period length of the second type demand.

to avoid the high ordering cost. As a result, for higher ordering cost, the total average profit decreases.

(2) When h increases, T^*, Q^*, and Z^* all decrease. The optimal downstream credit N^* remains at the threshold. It indicates that for a higher holding cost, the retailer should replenish more often and should reduce the ordering quantity per cycle. Obviously, higher holding cost leads to a lower total average profit.

(3) As the purchasing cost per unit c decreases, N^*, T^*, Q^*, and Z^* all increase, which indicates that if the purchasing cost is lower, the retailer should give more credit to customers to induce the market demand, and set a longer replenishment cycle length, a larger ordering quantity. All these lead to the rise of total average profit.

(4) When selling price p increases, N^*, Q^*, and Z^* increase while T^* decreases. Hence, for a higher selling price, the retailer should give customers more credit to induce market demand. At the same time, he should shorten the replenishment cycle length and order more items to satisfy the demand. As a result, the retailer can earn more due to a higher selling price.

(5) As the interest rate for the interest charged I_c increases, N^*, T^*, Q^*, and Z^* all decrease. It indicates that to avoid the interest cost, retailer has to shorten the downstream credit, shorten the replenishment cycle length and reduce the ordering quantity.

(6) For a higher changing saturation rate of demand r, N^*, T^*, and Q^* decrease while Z^* increases. It means that the retailer can induce the demand by setting a longer downstream credit. At the same time, to reduce the holding cost, he may order more frequently and reduce the items ordered per cycle. Anyway, the increase of the saturation rate brings more profit to the retailer.

(7) As the deterioration starting point γ increases, T^*, Q^*, and Z^* increase. It shows that deterioration can cause cost for retailer. If the items are more unwilling to deteriorate, he can earn more by ordering less often and ordering more items per cycle.

Table 1: Sensitive analysis of Example 13 for parameters A, h, c, p, I_c, r, γ, M.

Parameter		N^* (Days)	T^* (Days)	Q^* (Units)	Z^* (\$)
	65	69	15.26	202.3	99838
	70	69	15.84	210.0	99720
A	75	69	16.39	217.3	99607
	80	69	16.94	224.6	99433
	85	69	17.44	231.2	99391
	3.6	69	16.86	223.6	99704
	3.8	69	16.61	220.2	99655
h	4.0	69	16.39	217.3	99607
	4.2	69	16.17	214.4	99560
	4.4	69	15.95	211.5	99513
	16	79	17.08	234.0	143216
	18	74	16.72	225.4	121352
c	20	69	16.39	217.3	99607
	22	63	16.10	209.1	78019
	24	56	15.84	200.7	56654
	26	59	16.46	210.8	57267
	28	64	16.43	214.2	78381
p	30	69	16.39	217.3	99607
	32	72	16.35	219.0	120903
	34	75	16.35	221.1	142246
	0.11	75	17.37	234.9	100769
	0.13	71	16.86	225.1	100169
I_c	0.15	69	16.39	217.3	99607
	0.17	66	15.95	209.3	99077
	0.19	64	15.55	202.7	98575
	16	81	16.43	226.5	98158
	18	74	16.39	221.0	98951
r	20	69	16.39	217.3	99607
	22	64	16.39	213.6	100160
	24	60	16.35	210.1	100633
	10	70	15.70	208.9	99573
	15	69	16.24	215.4	99599
γ	20	69	16.39	217.3	99607
	25	69	16.39	217.3	99607
	30	69	16.39	217.3	99607
	26	69	16.39	217.3	99257
	28	69	16.39	217.3	99432
M	30	69	16.39	217.3	99607
	32	69	16.39	217.3	99782
	34	69	16.39	217.3	99957

Table 2: Sensitive analysis of Example 14 for parameters I_p and θ.

Parameter		N^* (Days)	T^* (Days)	Q^* (Units)	Z^* (\$)
I_p	0.16	69	22.78	669.2	101715
	0.18	68	22.78	668.9	101717
	0.20	67	22.74	667.4	101719
	0.22	65	22.60	662.5	101726
	0.24	62	22.12	647.1	101743
θ	0.03	67	22.89	671.8	101721
	0.04	67	22.81	669.4	101720
	0.05	67	22.74	667.4	101719
	0.06	67	22.67	665.3	101718
	0.07	67	22.59	663.0	101718

(8) If M increases, N^*, T^*, and Q^* stay at the same threshold while Z^* increases. Obviously, if the supplier offers the retailer a longer credit, he can earn more from the interest earned.

(9) Because there is no deterioration cost and interest earned in Example 13, so there is no influence of parameter I_p and θ.

To better illustrate the sensitive of parameter I_p and θ, we make another sensitive analysis based on Example 14. The results are shown as Table 2.

We also conclude that,

(1) As the interest earned rate I_p increases, N^*, T^*, and Q^* decrease while Z^* increases. It means that the retailer should shorten the downstream credit length and the replenishment cycle length, and reduce the ordering quantity per cycle.

(2) As the deterioration rate θ increases, T^*, Q^*, and Z^* decrease. It shows that to avoid the deterioration cost, the retailer tries to keep a lower stock level and orders more frequently.

7. Conclusions and Future Research

Financing tools play a more and more important role in business today, which provide us with a new method to study the inventory problems. In the inventory problems, credit can have significant influence on the inventory decisions, that is, ordering quantity and ordering cycle length. In this study, we propose an EOQ model of a kind of noninstantaneous deterioration items with two-level credit and credit-dependent demand rate. The purpose of this research is to help the retailer determine the optimal replenishment cycle length, optimal ordering quantity, and optimal credit period offered to customers under different situations. It is also a general frame work for many researches, such as Ouyang et al. [12] and Jaggi et al. [42].

In future research, our model can be extended to more general supply chain structures, for example, decentralized and centralized supply chain. Also, we can regard the price as a decision variable, or we can set assumptions for partial credit and advanced payment discounts.

Appendices

A.

Proof of Lemma 1, Part (a). Motivated by (43), we define a new function $F_1(x)$ as follows:

$$\begin{aligned} F_1(x) = {} & \left[A + \frac{(h+cI_c)D\gamma^2}{2} + cD\gamma + cI_c(N-M)D\gamma\right] \\ & - \left[\frac{(h+cI_c)D\gamma}{\theta} + \frac{c+cI_c(N-M)}{\theta}D\right] \\ & \times \left(\theta T e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1\right) \\ & - \frac{(h+cI_c)D}{\theta^2}\left(\theta e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1 - \theta\gamma\right), \end{aligned} \tag{A.1}$$

for $x \in [\gamma, +\infty)$.

Since the first derivative of $F_1(x)$ with respect to $x \in [\gamma, +\infty)$ is

$$\begin{aligned} F'(x) = {} & -\left[h\theta\gamma + c\theta I_c(N-M) + c\theta I_c\gamma + c\theta + h + cI_c\right] \\ & \times Dxe^{\theta(x-\gamma)} < 0, \end{aligned} \tag{A.2}$$

we obtain that $F_1(x)$ is a strict decreasing function of x in the interval $[\gamma, +\infty)$. Moreover, we have $F_1(x)|_{x\to\infty} = -\infty$, and

$$F_1(x)|_{x\to\gamma} = A - \frac{(h+cI_c)D\gamma^2}{2}. \tag{A.3}$$

Therefore, if $2A \geq (h+cI_c)D\gamma^2$, then $F_1(x)|_{x\to\gamma} \geq 0$. According to the intermediate value theorem, there exists a unique $T_{12} \in [\gamma, +\infty)$ such that $F_1(T_{12}) = 0$. □

Proof of Lemma 1, Part (b). If $0 < 2A < (h+cI_c)D\gamma^2$, then from (A.3), $F_1(\gamma) < 0$. Since $F_1(x)$ is a strict decreasing function of x in the interval $[\gamma, +\infty)$; thus, there is no value of $T \in [\gamma, +\infty)$ such that $F_1(T) = 0$. □

Proof of Lemma 2, Part (a). When $2A \geq (h+cI_c)D\gamma^2$, T_{12} is the unique solution of (43) from Lemma 1(a). Taking the second derivative of $Z_{12}(T)$ with respect to T and finding the value of the function at the point T_{12}, we obtain

$$\begin{aligned} & \frac{\partial^2 Z_{12}}{T^2} \\ & = -\frac{\left[h\theta\gamma + cI_c(N-M) + c\theta I_c\gamma + c\theta + h + cI_c\right]De^{\theta(T_{12}-\gamma)}}{T_{12}} \\ & < 0. \end{aligned} \tag{A.4}$$

Thus, T_{12} is the global maximum point of $Z_{12}(T)$. □

Proof of Lemma 2, Part (b). From the proof of Lemma 1(b), we know that if $0 < 2A \le (h + cI_c)D\gamma^2$, then $F_1(x) < 0$, for all $x \in [\gamma, +\infty)$. Thus we have

$$\begin{aligned}\frac{\partial Z_{12}}{\partial T} &= \frac{\left[A + \left((h + cI_c) D\gamma^2/2\right) + cD\gamma + cI_c (N - M) D\gamma\right]}{T^2} \\ &\quad - \left[\left(\frac{(h + cI_c) D\gamma}{\theta}\right) + \left(\frac{(c + cI_c (N - M))}{\theta}\right) D\right] \\ &\quad \times \left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \times \left(T^2\right)^{-1} \\ &\quad - \frac{\left((h + cI_c) D/\theta^2\right)\left(\theta e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right)}{T^2} \\ &= \frac{F_1(T)}{T^2} < 0,\end{aligned} \tag{A.5}$$

for all $T \in [\gamma, +\infty)$, which implies that $Z_{12}(T)$ is a strict decreasing function of $T \in [\gamma, +\infty)$. So, $Z_{12}(T)$ has a maximum value at the boundary point $T = \gamma$. □

B.

Proof of Lemma 4, Part (a). Motivated by (53), we define a new function $F_2(x)$ as follows:

$$\begin{aligned}F_2(x) = &\left[A + \frac{(h + cI_c) D\gamma^2}{2} - cI_c (M - N) D\gamma \right. \\ &\left. + \frac{cI_c(M - N)^2 D}{2} + cD\gamma - \frac{pI_p(M - N)^2 D}{2}\right] \\ &- \left[\frac{h\gamma D + cI_c (\gamma - M + N) D + cD}{\theta}\right] \\ &\times \left(\theta T e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1\right) \\ &- \frac{(h + cI_c) D}{\theta^2}\left(\theta e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1 - \theta\gamma\right). \\ &\text{for } x \in [\gamma, +\infty).\end{aligned} \tag{B.1}$$

Since the first derivative of $F_2(x)$ with respect to $x \in [\gamma, +\infty)$ is

$$\begin{aligned}F_2'(x) = &-\left[h\theta\gamma + c\theta I_c (\gamma + N - M) + c\theta + h + cI_c\right] \\ &\times Dxe^{\theta(x-\gamma)} < 0,\end{aligned} \tag{B.2}$$

we obtain that $F_2(x)$ is a strict decreasing function of x in the interval $[\gamma, +\infty)$. Moreover, we have $F_2(x)|_{x\to\infty} = -\infty$, and

$$F_2(x)|_{x\to\gamma} = A - \frac{(h + cI_c)\gamma^2 D - \left(cI_c - pI_p\right)(M - N)^2 D}{2}. \tag{B.3}$$

Therefore, if $2A \ge (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M - N)^2 D$, then $F_2(x)|_{x\to\gamma} \ge 0$. According to the intermediate value theorem, there exists a unique $T_{23} \in [\gamma, +\infty)$ such that $F_2(T_{23}) = 0$. □

Proof of Lemma 4, part (b). If $2A < (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M - N)^2 D$, then from (A.3), $F_2(\gamma) < 0$. Since $F_2(x)$ is a strict decreasing function of x in the interval $[\gamma, +\infty)$; thus, there is no value of $T \in [\gamma, +\infty)$ such that $F_2(T) = 0$. □

Proof of Lemma 5, Part (a). When $2A \ge (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M - N)^2 D$, T_{23} is the unique solution of (53) from Lemma 4(a). Taking the second derivative of $Z_{23}(T)$ with respect to T and finding the value of the function at the point T_{23}, we obtain

$$\begin{aligned}\frac{\partial^2 Z_{23}}{T^2} &= -\frac{\left[h\theta\gamma + cI_c (N - M) + c\theta I_c\gamma + c\theta + h + cI_c\right] De^{\theta(T_{23}-\gamma)}}{T_{23}} \\ &< 0.\end{aligned} \tag{B.4}$$

Thus, T_{23} is the global minimum point of $Z_{23}(T)$. □

Proof of Lemma 5, Part (b). From the proof of Lemma 4(b), we know that if $0 < 2A < (h + cI_c)\gamma^2 D - (cI_c - pI_p)(M - N)^2 D$, then $F_2(x) > 0$, for all $x \in [\gamma, +\infty)$. Thus we have

$$\begin{aligned}\frac{\partial Z_{23}}{\partial T} &= \left[A + \frac{(h + cI_c) D\gamma^2}{2} - cI_c (M - N) D\gamma \right. \\ &\quad \left. + \frac{cI_c(M - N)^2 D}{2} + cD\gamma - \frac{pI_p(M - N)^2 D}{2}\right] \times \left(T^2\right)^{-1} \\ &\quad - \left[\frac{h\gamma D + cI_c (\gamma - M + N) D + cD}{\theta}\right] \\ &\quad \times \left(\theta T e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1\right) \times \left(T^2\right)^{-1} \\ &\quad - \frac{\left((h + cI_c) D/\theta^2\right)\left(\theta e^{\theta(x-\gamma)} - e^{\theta(x-\gamma)} + 1 - \theta\gamma\right)}{T^2} \\ &= \frac{F_2(T)}{T^2} < 0, \quad \forall T \in [\gamma, +\infty),\end{aligned} \tag{B.5}$$

which implies that $Z_{23}(T)$ is a strict decreasing function of $T \in [\gamma, +\infty)$. So, $Z_{23}(T)$ has a maximum value at the boundary point $T = \gamma$. □

C.

Proof of Lemma 7, Part (a). Motivated by (60), we define a new function $F_3(x)$ as follows:

$$F_3(x) = \left(A + \frac{hD\gamma^2}{2} + cD\gamma\right) - \left(\frac{hD\gamma + cD}{\theta}\right) \times \left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) - \frac{hD}{\theta^2}\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) - \frac{pI_pDT^2}{2} \quad \text{for } x \in [\gamma, M-N]. \tag{C.1}$$

Since the first derivative of $F_3(x)$ with respect to $x \in [\gamma, M-N]$ is

$$F_3'(x) = -\left(\frac{hD\gamma + cD}{\theta} + \frac{hD}{\theta^2}\right)\theta T e^{\theta(T-\gamma)} - pI_pDT < 0, \tag{C.2}$$

we obtain that $F_3(x)$ is a strict decreasing function of x in the interval $[\gamma, M-N]$. Moreover,

$$F_3(x)\big|_{x\to\gamma} = A - \frac{hD\gamma^2}{2} + \frac{pI_pD}{2}, \tag{C.3}$$

$$\begin{aligned} F_3(x)\big|_{x\to M-N} &= \left(A + \frac{hD\gamma^2}{2} + cD\gamma\right) - \left(\frac{hD\gamma + cD}{\theta} + \frac{hD}{\theta^2}\right) \\ &\quad \times \left[\theta(M-N)e^{\theta(M-N-\gamma)} - e^{\theta(M-N-\gamma)} + 1\right] \\ &\quad + \frac{hD\gamma}{\theta} - \frac{pI_pD(M-N)^2}{2}. \end{aligned} \tag{C.4}$$

According to the intermediate value theorem, when $\Delta_4 \le 2A \le \Delta_5$, then $F_3(x)|_{x\to M-N} \le 0$ and $F_3(x)|_{x\to\gamma} \ge 0$, so there exists a unique $T_{32} \in [\gamma, M-N]$ such that $F_3(T_{32}) = 0$. □

Proof of Lemma 7, Part (b). If $2A < \Delta_4$ or $2A > \Delta_5$, then $F_3(x)|_{x\to\gamma} < 0$ or $F_3(x)|_{x\to M-N} > 0$. Since $F_3(x)$ is a strict decreasing function of x in the interval $[\gamma, M-N]$. Thus, there is no value of $T \in [\gamma, M-N]$ such that $F_3(T) = 0$. □

Proof of Lemma 8, Part (a). When $\Delta_4 \le 2A \le \Delta_5$, T_{32} is the unique solution of (60) from Lemma 7(a). Taking the second derivative of $Z_{32}(T)$ with respect to T and finding the value of the function at the point T_{32}, we obtain

$$\begin{aligned} \frac{\partial^2 Z_{32}(T)}{\partial T^2}\bigg|_{T=T_{32}} &= -\frac{\left((hD\gamma + cD)/\theta + hD/\theta^2\right)\theta e^{\theta(T_{32}-\gamma)}}{T_{32}} - \frac{pI_pD}{T_{32}} \\ &< 0. \end{aligned} \tag{C.5}$$

Thus, T_{32} is the global maximum point of $Z_{32}(T)$. □

Proof of Lemma 8, Part (b). From the proof of Lemma 7(b), we know that if $2A < \Delta_4$, then $F_3(x) < 0$ for all $x \in [\gamma, M-N]$. Thus, we have

$$\begin{aligned} \frac{\partial Z_{32}}{\partial T} &= \frac{\left(A + hD\gamma^2/2 + cD\gamma\right)}{T^2} - \frac{\left((hD\gamma + cD)/\theta\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right)}{T^2} \\ &\quad - \frac{\left(hD/\theta^2\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right)}{T^2} - \frac{pI_pD}{2} \\ &= \frac{F_3(T)}{T^2} < 0, \quad \forall T \in [\gamma, M-N], \end{aligned} \tag{C.6}$$

which implies that $Z_{32}(T)$ is a strict decreasing function of $T \in [\gamma, M-N]$. So, $\max Z_{32}(T) = Z_{32}(\gamma)$. □

Proof of Lemma 8, Part (c). From the proof of Lemma 7(b), we know that if $2A > \Delta_5$, then $F_3(x) > 0$ for all $x \in [\gamma, M-N]$. Thus, we have

$$\begin{aligned} \frac{\partial Z_{32}}{\partial T} &= \frac{\left(A + hD\gamma^2/2 + cD\gamma\right)}{T^2} - \frac{\left((hD\gamma + cD)/\theta\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right)}{T^2} \\ &\quad - \frac{\left(hD/\theta^2\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right)}{T^2} - \frac{pI_pD}{2} \\ &= \frac{F_3(T)}{T^2} < 0, \quad \forall T \in [\gamma, M-N], \end{aligned} \tag{C.7}$$

which implies that $Z_{23}(T)$ is a strict decreasing function of $T \in [\gamma, M-N]$. So, $\max Z_{32}(T) = Z_{32}(M-N)$. □

D.

Proof of Lemma 9, Part (a). Motivated by (53), we define a new function $F_2(x)$ as follows:

$$\begin{aligned} F_4(x) &= \left(A + \frac{hD\gamma^2}{2} + cD\gamma - cI_c(M-N) - \frac{pI_p(M-N)^2D}{2}\right) \\ &\quad - \left(\frac{hD\gamma + cD}{\theta}\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right) \\ &\quad - \frac{hD}{\theta^2}\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right) \\ &\quad - \frac{cI_c}{\theta^2}\left(\theta T e^{\theta(T-M+N)} - e^{\theta(T-M+N)} + 1\right) \\ &\quad \text{for } x \in [M-N, +\infty). \end{aligned} \tag{D.1}$$

Since the first derivative of $F_4(x)$ with respect to $x \in [M - N, +\infty)$ is

$$F_4'(x) = -\left(\frac{hD\gamma + cD}{\theta} + \frac{hD}{\theta^2}\right)\theta T e^{\theta(T-\gamma)} - \frac{cI_c}{\theta} T e^{\theta(T-M+N)} < 0, \tag{D.2}$$

we obtain that $F_4(x)$ is a strict decreasing function of x in the interval $[M-N, +\infty)$. Moreover, we have $F_4(x)|_{x\to\infty} = -\infty$, and

$$F(x)|_{x\to M-N} = \left(A + \frac{hD\gamma^2}{2} + cD\gamma\right) - \left(\frac{hD\gamma + cD}{\theta} + \frac{hD}{\theta^2}\right) \times \left[\theta(M-N)e^{\theta(M-N-\gamma)} - e^{\theta(M-N-\gamma)} + 1\right] + \frac{hD\gamma}{\theta} + \frac{pI_p(M-N)^2 D}{2}. \tag{D.3}$$

Therefore, if $2A \geq \Delta_5$, then $F_4(x)|_{x\to M-N} \geq 0$. According to the intermediate value theorem, there exists a unique $T_{33} \in [M-N, +\infty)$ such that $F_4(T_{33}) = 0$. □

Proof of Lemma 9, Part (b). If $0 < 2A < \Delta_5$, then from (D.3) $F_4(M-N) < 0$. Since $F_4(x)$ is a strict decreasing function of x in the interval $[M-N, +\infty)$. Thus, there is no value of $T \in [M-N, +\infty)$ such that $F_4(T) = 0$. □

Proof of Lemma 10, Part (a). When $2A \geq \Delta_5$, T_{33} is the unique solution of (63) from Lemma 9(a). Taking the second derivative of $Z_{33}(T)$ with respect to T and finding the value of the function at the point T_{33}, we obtain

$$\frac{\partial^2 Z_{33}}{T^2} = -\frac{\left((hD\gamma + cD)/\theta + hD/\theta^2\right)\theta e^{\theta(T-\gamma)}}{T_{33}} - \frac{cI_c e^{\theta(T-M+N)}}{\theta T_{33}} < 0. \tag{D.4}$$

Thus, T_{33} is the global maximum point of $Z_{33}(T)$. □

Proof of Lemma 10, Part (b). From the proof of Lemma 9(b), we know that if $0 < 2A < \Delta_5$, then $F_4(x) < 0$, for all $x \in [M-N, +\infty)$. Thus we have

$$\begin{aligned}\frac{\partial Z_{33}}{\partial T} &= \frac{\left(A + hD\gamma^2/2 + cD\gamma - cI_c(M-N) - pI_p(M-N)^2 D/2\right)}{T^2} \\ &\quad - \frac{\left((hD\gamma + cD)/\theta\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1\right)}{T^2} \\ &\quad - \frac{\left(hD/\theta^2\right)\left(\theta T e^{\theta(T-\gamma)} - e^{\theta(T-\gamma)} + 1 - \theta\gamma\right)}{T^2} \\ &\quad - \frac{\left(cI_c/\theta^2\right)\left(\theta T e^{\theta(T-M+N)} - e^{\theta(T-M+N)} + 1\right)}{T^2} \\ &= \frac{F_4(T)}{T^2} < 0, \quad \forall T \in [M-N, +\infty),\end{aligned} \tag{D.5}$$

which implies that $Z_{33}(T)$ is a strict decreasing function of $T \in [M-N, +\infty)$. $Z_{23}(T)$ has a maximum value at the boundary point $T = M - N$ for $T \in [M-N, +\infty)$. □

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Research Article

Location Optimization of Multidistribution Centers Based on Low-Carbon Constraints

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Location optimization of distribution centers is a systematic and important task in logistics operations. Recently, reducing carbon footprint is becoming one of the decision-making factors in selecting the locations for distribution centers. This paper analyzes the necessity of industrial carbon dioxide emission cost internalization in four aspects and builds a model for multidistribution centers location in effort of reducing carbon footprint that can provide optimized strategy support for decision makers and logistic operators. Numerical examples are presented to illustrate the feasibility and effectiveness of the models.

1. Introduction

Location selection is an important and systematic problem in logistics operations, and it is also a key component of a corporation's strategic management. Optimizations of a distribution center location can improve the center's operational cost and service level, as well as rationalizing the entire logistic systems. This problem has attracted wide attention from managers and scholars over the past years, and there are extensive researches done on this matter. Most of the previous studies focused on location selection based only on the cost of distribution centers, such as fixed cost, transportation cost, and storage cost. Very few of them considered low-carbon footprint factors [1–3]. In an effort of reducing carbon footprint globally, location selection of distribution centers should take the cost of carbon dioxide emission and lowering carbon footprint into account.

Among the few studies of location selection based on carbon footprint, Huang [4] suggested a multibusiness trade of distribution center location selection model that limits carbon footprint without considering the cost of carbon dioxide emission. Yang et al. [5] built a model for carbon tax-constrained capacitated cold chain logistics to mitigate the cost burden resulting from the carbon emission tax; Yang and Lin [6] offered a new model targeting two separate scenarios, carbon trading and carbon tax, and compared the effects of these two mechanisms. These models did not consider the carbon dioxide emission quotas and the cost of carbon dioxide emission simultaneously. This kind of problems will become more and more important with the global low-carbon footprint requirement. To solve this problem, this paper is offering a model that considers not only the cost of carbon footprint emissions but also having emission quotas for each distribution center of a multidistribution center. The rest of the paper is organized as follows: Section 1 explains the indispensability of internalizing carbon emission cost; Section 2 describes a model based on carbon emission quotas in multiple centers of a multidistribution center; Section 3 validates the model and discusses its effectiveness using numerical methods; Section 4 concludes the study.

2. Internalization of Carbon Emission Cost

There are not many research articles about carbon dioxide emission cost because it is difficult to estimate and quantify its cost. For instance, Sadegheih [7] describes a methodology developed for designing an optimal configuration for system transmission planning with carbon emissions costs; Kneifel [8] estimates life-cycle energy savings, carbon emission reduction, and cost effectiveness of energy efficiency

measures in new commercial buildings using an integrated design approach and estimates the implications from a cost on energy-based carbon emissions. The literature on carbon footprint management in supply chain is also very sparse. Some studies focus on the measurement method of carbon emissions in supply chains; Sundarakani et al. [9] examine the carbon footprint across supply chains and thus contribute to the knowledge and practice of green supply chain management; Kannan et al. [10] develop a mixed integer linear model for a carbon footprint based reverse logistics network design; the proposed model aims at minimizing the climate change (specifically, the CO_2 footprint), and it employs reverse logistics activities to recover used products, hence combining the location/transportation decision problem; Piecyk and McKinnon [11] report on research undertaken to determine the baseline trends in logistics and supply chain management and associated environmental effects up to 2020. In this paper, the carbon dioxide emission cost is defined as the financial compensation paid by a corporation for the environment pollutions and damages caused by the carbon dioxide emission during the course of its production processes.

During the time of free carbon emission, it was the entire society which pays for the environment pollutions caused by corporations' production. Corporations were producing carbon dioxide without financial penalties (i.e., the carbon emission cost was considered as an external cost). With the recent global efforts of reducing carbon emission, carbon emission cost needs to be included as a part of the corporations' internal cost to minimize the carbon footprint.

Internalizing carbon emission cost is to quantify the amount of carbon emission as an operational cost in the accounting system. In the era of low-carbon economy, the necessity of internalizing carbon emission cost can be revealed from four perspectives. (i) It can give both pressure and motivations to corporations to reduce their carbon footprint. Without internalizing carbon emission cost, corporations do not have financial motivations to reduce carbon footprint actively. If the carbon emission cost is a part of corporations' internal cost, the balance between carbon footprint and operational revenues will be considered. (ii) Internalizing carbon emission cost is the key problem to maintain corporations' core competences. In reality, different corporations have different carbon emission costs. For 2 enterprises in the same industry, the one that has higher carbon emission cost will be eliminated by the market. This will encourage corporations to include lowering the carbon footprint in their strategic management. (iii) Internalizing carbon emission cost will serve as a catalyst in the process of corporations' development models. Corporations with high carbon footprint emission and high energy consumptions need to develop new technologies to increase its competence, so their production model can be transitioned successfully from resource dependent to technology dependent. (iv) Internalizing carbon emission cost will increase the carbon emission revenue. Carbon trading mechanism is a method to adjust carbon footprint in the market. Carbon trading is actually carbon emission-right trading. Each corporation has a carbon footprint quota, and, during the actual production, corporations that exceeded the quotas would have to purchase quotas from other corporations that had quota surpluses. From the opportunity point of view, the carbon emission quota can be considered as a product, and it will encourage every corporation to emit as little carbon as possible for more profit.

3. Low-Carbon Constraint Model

In this paper, we assume there are m source points, n distribution centers, and p demand points in a 3-level logistic distribution network. The purpose of the research is to solve the following problems: the proper number of distribution centers that does not exceed the given number; the final lowest total cost based on each center's cost and the allocation of customers to distribution centres. In order to build a mathematical model and to find reasonable solutions, we have the following assumptions.

(i) Choose distribution centers that will be built from the candidate distribution centers; the number of the selected distribution centers is bounded. The locations of all candidate available distribution centers are known.

(ii) Every customer's demand is known. The storage capacity of each distribution center is bounded, but it is able to meet customers' demands.

(iii) The flow cost that occurred in each distribution center is known.

(iv) Only a single product's distribution is considered. One distribution center can serve multiple customers, and one customer can be served by multiple distribution centers. There is no product transfer among distribution centers.

(v) From a depot to a distribution center, or from a distribution center to a demand point (customer), only one transportation mode is allowed (i.e., using the same model of vehicle) over the entire system. The unit energy consumptions and road conditions are the same everywhere and are known.

(vi) The equivalent cost of unit mass of carbon dioxide is known.

(vii) The construction cost for each distribution center is the same, so it is not considered in the model, nor the storage cost for each distribution center.

(viii) Only the carbon emission that occurred in the distribution centers' operating processes is considered; the emission that occurred during construction is not included in the total costs.

(ix) Every distribution center's carbon quota is known. The carbon trading mechanism is not considered. Distribution centers can exceed the emission quota but with penalties. The penalty coefficient is known and is constant (i.e., the coefficient does not change with the over-quota amount).

The variables and parameters that will be used in the model are defined as follows:

a_{ij}: transportation rate from the depot i to the distribution center j (\$/t);

b_{jk}: transportation rate from the distribution center j to the demand point k (\$/t);

g_{ij}: energy consumption for unit product transported from the depot i to the distribution center j (L/t);

h_{jk}: energy consumption for unit product transported from the distribution center j to the demand point k (L/t);

d_j: the flow cost for unit product in the distribution center j (\$/t);

f_j: the flow energy consumption for unit product in the distribution center j (L/t);

Q_i: the supply capacity of the depot i (t/year);

R_j: the storage capacity of the distribution j (t/year);

T_k: the yearly demand amount from the demand point k (t/year);

U_j: the carbon dioxide emission quota for the distribution center j (t/year);

M: the maximum number of distribution centers that can be selected;

ε: the carbon dioxide emission coefficient for gasoline (t/L);

μ: the equivalent cost of unit carbon dioxide emission (t/L);

ρ: the penalty coefficient (\$/t);

Z: the total cost (\$);

M, n, p: the numbers of depots, potentially available distribution centers, and demand points, respectively;

X_{ij}: the logistic quantity from the depot i to the distribution center j;

Y_{jk}: the logistic quantity from the distribution center j to the demand point k.

The multidistribution centers location selection model based on low-carbon constraints is as follows:

$$\begin{aligned}\min Z = {} & \sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}+\mu\varepsilon g_{ij}\right)+\sum_{j=1}^{n}\sum_{k=1}^{p}\left(b_{jk}+\mu\varepsilon h_{jk}\right)Y_{jk} \\ & +\rho\left\{\sum_{j=1}^{n}V_j\max\left[\varepsilon\left(\sum_{i=1}^{m}X_{ij}g_{ij}+\sum_{k=1}^{p}Y_{jk}h_{jk}\right.\right.\right. \\ & \left.\left.\left.+\sum_{i=1}^{m}X_{ij}f_j\right)-U_j,0\right]\right\} \\ & +\sum_{i=1}^{m}\sum_{j=1}^{n}\left(d_j+\mu\varepsilon f_j\right)X_{ij},\end{aligned} \tag{1}$$

such that

$$\sum_{j=1}^{n}X_{ij}\le Q_i,\quad i=1,2,\ldots,m, \tag{2}$$

$$\sum_{i=1}^{m}X_{ij}\le R_jV_j,\quad j=1,2,\ldots,n, \tag{3}$$

$$\sum_{j=1}^{n}Y_{jk}\ge T_k,\quad k=1,2,\ldots,p, \tag{4}$$

$$\sum_{i=1}^{m}X_{ij}=\sum_{k=1}^{p}Y_{jk},\quad j=1,2,\ldots,n, \tag{5}$$

$$\sum_{j=1}^{n}V_j\le M, \tag{6}$$

$$\begin{gathered}X_{ij}\ge 0,\quad Y_{jk}\ge 0,\\ i=1,2,\ldots,m;\quad j=1,2,\ldots,n;\quad k=1,2,\ldots,p,\end{gathered} \tag{7}$$

$$V_j=\begin{cases}1, & \text{if the distribution centre } j \text{ is selected}\\ 0, & \text{otherwise.}\end{cases} \tag{8}$$

In the objective function, the total cost is comprised of four components. The first part is the total cost of transporting products from depots to distribution centers, including transportation cost and the carbon emission cost due to transportation. The second part is the total cost of transporting products from distribution centers to demand points, including transportation cost and the carbon emission cost due to transportation. The third part is the penalty cost of all distribution centers that exceeded carbon emission quotas. In the small brackets, the first term is the energy consumption from all depots to the distribution center j, the second term is the energy consumption from the distribution center j to all demand points, and the third term is the flow energy consumption of the distribution center j. The fourth part is the total cost of products routing through distribution centers, which includes the flow cost and the carbon emission cost due to routing through distribution centers. The flow cost is mainly caused by loading and unloading, handling, and processing products in the distribution centers.

In the constraint conditions, (2) is the supply constraint, which is the total supplied product amount that cannot exceed the depot's total supply capacity; (3) is the storage constraint, which is the total supplied amount from each depot to the distribution center j that cannot exceed the distribution center's total construction storage; (4) is the demand constraint, which is the amount that a distribution center supplies to a demand point that should meet the demand point's needs; (5) is the balance constraint, which controls the amount of products flowing into a distribution center that equals the amount of flowing out of it; (6) is the number constraint, which is the total number of selected distribution centers that cannot exceed M; (7) and (8) are the nonnegative constraint and 0-1 constraint, respectively.

Table 1: Supply capacities and transportation cost rate of depots.

Depot	Supply capacity (t/year)	Transportation cost from depot to distribution centers ($/t)				
		N_1	N_2	N_3	N_4	N_5
E_1	800	140	120	135	120	115
E_2	1000	125	130	110	135	120

Table 2: Energy consumption coefficients from depots to distribution centers.

Depot	Energy consumption coefficients (L/t)				
	N_1	N_2	N_3	N_4	N_5
E_1	18	20	23	16	17
E_2	15	22	18	18	19

Table 3: Demands and transportation costs.

Demand point	Demand (t)	Transportation costs to demand points ($/t)				
		N_1	N_2	N_3	N_4	N_5
P_1	150	60	65	55	70	75
P_2	130	55	70	60	60	50
P_3	200	60	55	70	80	65
P_4	110	75	80	65	50	60
P_5	140	70	55	60	65	75
P_6	100	65	80	65	75	60
P_7	125	50	60	60	65	70
P_8	165	60	65	60	70	60

4. Model Solutions and Numerical Examples

Assume that a corporation plans to build a certain number of distribution centers in district S to extend its product's supply and sales. The corporation has 2 production plants, E_1 and E_2, 8 demand points, and 3 potentially available distribution centers. Considering the cost factor, the newly constructed distribution centers will not exceed 3. Tables 1, 2, 3, 4, and 5 showed conditions and parameters that are used in this model.

This model assumes a single type of vehicle for transportation, a single type of energy (gasoline), and a single transportation mode (road). The carbon dioxide emission factor is given by IPCC2006 as 2.26×10^{-3} t/L [12]. Let the equivalent cost of unit carbon dioxide emission be 90 $/t. The penalty coefficient is 200 $/t when a distribution center exceeds its carbon dioxide emission quota.

From the model description, we have $m = 2$, $n = 5$, $p = 8$, $\varepsilon = 0.00226$, $\mu = 90$, and $\rho = 200$. Combining the given conditions from Tables 1 to 5, numerical results can be obtained by LINGO software, and the results are shown in Figure 1. We can see that $V(N_1) = 1$, $V(N_2) = 0$, $V(N_3) = 1$, $V(N_4) = 0$, and $V(N_5) = 1$. Therefore, the selected three distribution centers are N_1, N_3, and N_5, and they will service 8 demand points. The minimum total cost is $268, 586.3. Z_1 is the penalty cost for all the selected distribution centers due to exceeding the emission constraint. In the numerical program, $S(j)$ represents the amount of carbon dioxide exceeded by the jth distribution center. From Figure 1, one can see that $S(N_3)$ and $S(N_5)$ are both larger than 0, which means the distribution centers N_3 and N_5 both exceeded their carbon dioxide quotas. Z_2 is the total cost of transportation from 2 depots to 3 distribution centers. Z_3 is the total cost of transportation from 3 distribution centers to 8 demand points. Z_4 is the flow cost of the products going through the distribution centers. Tables 6 and 7 showed the optimal plan for each depot and each distribution center, respectively.

Table 4: Energy cost from distribution centers to demand points.

Distribution center	Energy cost from distribution centers to demand points (L/t)							
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
N_1	11	12	10	9	10	10	9	10
N_2	9	7	9	10	11	9	10	11
N_3	11	12	12	11	10	12	11	10
N_4	8	8	9	10	11	9	10	11
N_5	10	11	8	8	9	8	8	9

Table 5: Parameters of distribution centers.

	N_1	N_2	N_3	N_4	N_5
Built capacity (t)	550	600	500	650	700
Unit product's flow cost ($/t)	50	60	55	70	50
Unit product's flow energy consumption ($/t)	5	6	5	7	6
CO_2 emission quota (t/year)	10	9	8	10	9

Table 6: The amount that is supplied to distribution centers (t/year).

	Distribution center N_1	Distribution center N_3	Distribution center N_5
Depot E_1			509.2
Depot E_2	155.8	455	

A comparison study is also provided to compare the results between taking and not taking the low-carbon factor into account. All assumptions remain the same for the ordinary case, and the objective function is

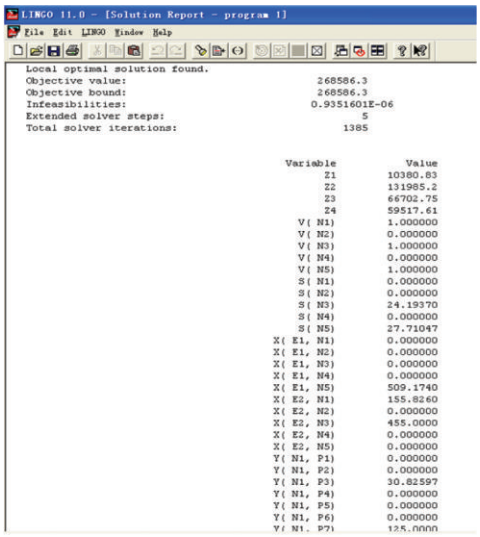
$$\min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}X_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{p} b_{jk}Y_{jk} \sum_{i=1}^{m}\sum_{j=1}^{n} d_j X_{ij}. \quad (9)$$

Figure 2 showed the results $V(N_1) = 0$, $V(N_2) = 0$, $V(N_3) = 1$, $V(N_4) = 0$, and $V(N_5) = 1$, which means two distribution centers (N_3 and N_5) are selected for servicing 8 demand points. The minimum total cost is $250,950.

Z_2 is the total cost of transportation from 2 depots to 2 distribution centers, Z_3 is the total cost of transportation from 2 distribution centers to 8 demand points, and Z_4 is the flow cost of the products going through the distribution centers.

Table 7: The amount that is supplied by distribution centers (t/year).

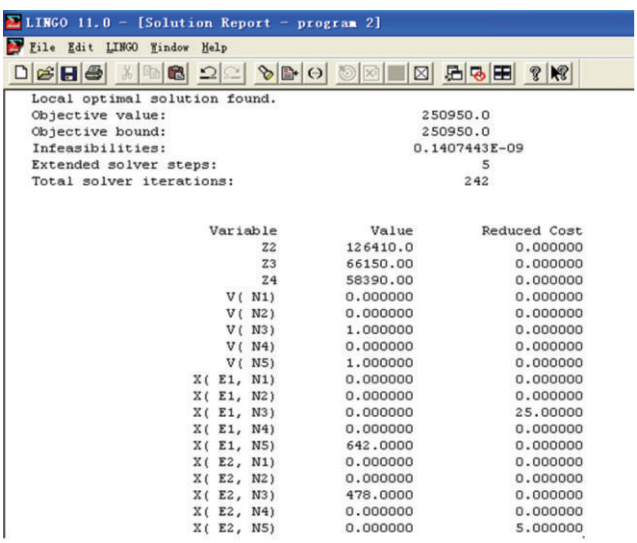
	Demand point P_1	Demand point P_2	Demand point P_3	Demand point P_4	Demand point P_5	Demand point P_6	Demand point P_7	Demand point P_8
Distribution center N_1			30.8				125	
Distribution center N_3	150				140			165
Distribution center N_5		130	169.2	110		100		



```
LINGO 11.0 - [Solution Report - program 1]
File Edit LINGO Window Help
Local optimal solution found.
Objective value:                          268586.3
Objective bound:                          268586.3
Infeasibilities:                        0.9351601E-06
Extended solver steps:                          5
Total solver iterations:                     1385

                      Variable           Value
                            Z1        10380.83
                            Z2        131985.2
                            Z3        66702.75
                            Z4        59517.61
                        V( N1)        1.000000
                        V( N2)        0.000000
                        V( N3)        1.000000
                        V( N4)        0.000000
                        V( N5)        1.000000
                        S( N1)        0.000000
                        S( N2)        0.000000
                        S( N3)        24.19370
                        S( N4)        0.000000
                        S( N5)        27.71047
                    X( E1, N1)        0.000000
                    X( E1, N2)        0.000000
                    X( E1, N3)        0.000000
                    X( E1, N4)        0.000000
                    X( E1, N5)        509.1740
                    X( E2, N1)        155.8260
                    X( E2, N2)        0.000000
                    X( E2, N3)        455.0000
                    X( E2, N4)        0.000000
                    X( E2, N5)        0.000000
                    Y( N1, P1)        0.000000
                    Y( N1, P2)        0.000000
                    Y( N1, P3)        30.82597
                    Y( N1, P4)        0.000000
                    Y( N1, P5)        0.000000
                    Y( N1, P6)        0.000000
                    Y( N1, P7)        125.0000
```

Figure 1: Results of the low-carbon model.



```
LINGO 11.0 - [Solution Report - program 2]
File Edit LINGO Window Help
Local optimal solution found.
Objective value:                          250950.0
Objective bound:                          250950.0
Infeasibilities:                        0.1407443E-09
Extended solver steps:                          5
Total solver iterations:                      242

                      Variable           Value        Reduced Cost
                            Z2        126410.0            0.000000
                            Z3        66150.00            0.000000
                            Z4        58390.00            0.000000
                        V( N1)        0.000000            0.000000
                        V( N2)        0.000000            0.000000
                        V( N3)        1.000000            0.000000
                        V( N4)        0.000000            0.000000
                        V( N5)        1.000000            0.000000
                    X( E1, N1)        0.000000            0.000000
                    X( E1, N2)        0.000000            0.000000
                    X( E1, N3)        0.000000            25.00000
                    X( E1, N4)        0.000000            0.000000
                    X( E1, N5)        642.0000            0.000000
                    X( E2, N1)        0.000000            0.000000
                    X( E2, N2)        0.000000            0.000000
                    X( E2, N3)        478.0000            0.000000
                    X( E2, N4)        0.000000            0.000000
                    X( E2, N5)        0.000000            5.000000
```

Figure 2: Results of the ordinary model.

From the comparison of the above results, one can see that one more distribution center is selected in the low-carbon model, as compared to the ordinary model that does not consider low-carbon constraint, and the distribution plans are different as well. The total carbon dioxide emission in the low-carbon model is 67.30 t with one more distribution center, and, in the traditional model, the total emission is 68.25 t. Although the difference in carbon emission is small, it will make a significant difference in reality.

5. Conclusion

Distribution center selection is a strategic key component in logistic management. This paper analyzed multidistribution center location selection with low-carbon constraints. The results showed one can reduce the system's global carbon emission by introducing low-carbon component to the model. However, the total cost of the low-carbon model results is higher than the traditional models. If one also considers the construction cost of the distribution centers, the low-carbon model might not be energy efficient and cost efficient. Therefore, in reality, corporations should compare results produced by these two models (traditional and low carbon) to make the best decision for both the development of corporations and the sustainability of the environment. Future research direction should take the carbon tax and carbon trading policy factors into account to advance the low-carbon distribution center selection model.

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Research Article

Simulation Research of Space-Time Evolution of Emergency Logistics Network Reliability Based on Complex Network Theory

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We propose the conception and evaluation indexes of emergency logistics network connecting reliability to construct evaluation index system of complex network reliability, and describe these indexes quantitatively to evaluate the network connecting reliability. Moreover, the network topological model and the simulation methods of reliability measurement when the network is under attack are present. Finally, we take three classical emergency logistics networks as examples, and through emulation analysis we obtain the connecting reliability changing situation of these three networks under random attack, the changing curve of the ratio of effective demand nodes and emergence supply mileage of emergency logistics network with same network density but different forms, and then evaluate the emergency logistics network connecting reliability. This can provide references for the designing of emergency logistics network with high reliability and analysis means for research in other fields.

1. Introduction

Emergency logistics refer to special logistic activities with emergence material supply for sudden incidents arising from natural disasters, public health incidents, and so forth as its goals and the optimization of time efficiency and minimization of damages as its objectives and features emergence, uncertainty, weak economy, and unconventionality [1]. Whether emergence materials can be transferred to demand nodes on time upon the arising of emergence incidents will affect results of the whole rescue activities directly. It is of vital importance to design emergency logistics network scientifically for the efficiency of the network which lies on its scale, structure, and layout. As a typical complex network, it may fall as the victim of attacks and damages of different degrees arising from such uncertainties as randomness, diffusivity, and aftermath of the outbreak of emergencies, which may cause local failure or paralysis of the network and affect emergence material supply. It is difficult to work out necessary and universal rules of connecting status in the situation of local failure of the network, so it is a necessity to deepen emergency logistics network reliability research.

The existing huge amount of logistics research mainly focuses on normal logistics activities, but research on abnormal situations of disasters, natural or manmade, is little. Many American researchers studied the logistics supply of America in the Second World War after the end of the war and issued their views on emergency logistics; then the concepts of emergency logistics began to spread in Europe and Japan [2]. Some researchers conducted pilot studies on emergency logistics network optimization. Most of these studies are based on logistics status which is known or can be predicted, and optimization models focus on time, distance, or cost; it is presumed that the road is always in service with utmost transportation capacity, and network structures are fixed in these models; however, little attention is paid to logistics network reliability.

Logistics network reliability reflects the adaptability, anti-jamming capacity, stability, and recovery capability of emergency logistics network and is of vital importance in ensuring normal function and well performance of the system. Due to the complexity and particularity of emergency logistics network, it is difficult to have quantitative analysis in research, to ascertain problems and optimize the network; at present,

the research base is very weak and at initial stage, with its definition, methods, models, and theories unshaped [3] and at developing stage. Logistics network is usually constructed with a set of routes and a set of nodes; there are multiple carriers to provide transportation service on each route. Logistics network according to the carrier selection is a typical stochastic-flow network [4] and called a stochastic logistics network (SLN) herein. The network reliability is a performance index of the freight delivery for supply chain management and is defined as the probability that d units of commodity can be transmitted from a supplier (origin O) to a customer (destination D) through a logistics network [5]. Several researchers, such as Aven, Jane, and Lin, have evaluated the network reliability in terms of minimal paths (MPs) or minimal cuts (MCs) [6, 7] without considering unreliable nodes, and Lin has evaluated the network reliability with unreliable nodes in terms of MPs [8] and MCs [9], respectively. Gong et al. construct the operation model and its evaluation index system for emergency logistics system [10].

From the viewpoint of optimization, Levitin and Lisnianski [11] classified the network reliability optimization problems into two categories: minimizing the resources required for providing a specific network reliability level and achieving the maximal network reliability subject to various constraints. Many researchers have discussed the issues involving the network structure, the flow assignment, and the commodity allocation [11–14]. Xu et al. developed a flow assignment strategy with the minimal time elapse and the maximal network reliability, where the carriers on all routes are fixed [15]. Chang et al. formulated the flood emergency logistics problem with uncertainty as two stochastic programming models that allow for the determination of a rescue resource distribution system for urban flood disasters [16]. Lin and Yeh discusse the optimal carrier selection problem based on network reliability criterion [17]. Reliability and spare parts logistics are optimized using heuristic algorithms [18]. Vahdani et al. develop a biobjective model for designing a reliable network of bidirectional facilities in CLSCs under uncertainties [19]. Sheu presents a dynamic relief-demand management model for emergency logistics operations under imperfect information conditions in large-scale natural disasters [20]. Peng et al. model a reliable logistics network design problem with p-robustness constraints and propose a hybrid metaheuristic algorithm to efficiently solve the problem; then substantial improvements in reliability are achievable with minimal increases in cost [21].

Many foreign researchers have studied the supply chain and its measurement methods and got some results. There are mainly two kinds of methods of supply chain reliability measurement, one is experience index measurement, and the other is application reliability engineering index measurement [22]; for example, Hong-Minh et al. studied abrupt logistics by emulation, and Thomas applied reliability engineering in supply chain for the first time and issued the definition of supply chain reliability [23]. Lin developed a MP approach to measure the network reliability for two-commodity supply chain network [24]. Given the fact that emergency logistics can be taken as a kind of special supply chain, the measurement of supply chain reliability is of great value for emergency logistics reliability measurement [25, 26].

With the application of complex network theory, the power failure in North America was examined accurately, which provides a brand new idea for reliability research of local emergency logistics. This essay, based on complex network theories, introduces reliability assessment indexes to assess emergency logistics network reliability and obtains, through emulation attack program development, the changing regularity of emergency logistics network reliability of different types under random attacks. Emulation and analysis methods used in this essay can provide theoretic base and methods for the design and construction of emergency logistics networks and improve the reliability and efficiency of existing emergency logistics networks at a lower cost.

2. Connecting Reliability of Emergency Logistics Network and Its Assessment Indexes

The connecting reliability of emergency logistics network refers to the connecting state between supply nodes and demand nodes when suffering from outer interference. Nodes in the network are divided into supply nodes which send emergence materials in response to emergence demand and demand nodes which need these materials according to features of the network. The connecting reliability reflects whether emergence materials can reach demand nodes from supply nodes when such demand rises; obviously, it is closely related to the topological structure of the network except for attack influence. The attention of this essay mainly goes to emergence supply mileage and the ratio of effective demand nodes, two main indexes in the assessment of connecting reliability of the network.

2.1. Emergence Supply Mileage L. Emergence supply mileage, a measurement of timeliness of the whole network fulfilling emergency logistics supply, is defined as the arithmetic mean value of the mileage of all demand nodes in the network, among which the mileage of a demand node refers to the minimum value it needed for obtaining emergence materials:

$$L = \frac{\sum_t \min\left(D(1,t), D(2,t), \ldots, D(s,t), \ldots, D(n_s,t)\right)}{n_d}, \tag{1}$$

where n_d is all demand node number, n_s is all supply node number, and $D(s,t)$ is the mileage from demand node t ($t = 1, 2, \ldots, n_d$) to supply node s ($s = 1, 2, \ldots, n_s$).

2.2. Effective Demand Node Ratio P. Effective demand node ratio refers to the ratio of effective demand node number among all demand node number, in which effective demand nodes refer to demand nodes with direct or indirect links to supply nodes, that is, demand nodes which can obtain emergency supply in time:

$$P = \frac{n_d'}{n_d}, \tag{2}$$

where n_d is all demand node number, n'_d is effective demand node number.

3. Emulation Methods of Emergency Logistics Network Attack

3.1. Topological Model of Emergency Logistics Network. Based on the physical structure of actual emergency logistics network, its topological model consists of nodes abstracted from supply nodes and demand nodes in the network, edges abstracted from zones with direct links by real transportation means, and weight of edges which can be seen as transportation mileage or time. In Figure 1, node 1 in the topological structure represents a supply node and the rest represent demand nodes; the figure shows that supply node 1 connects with demand nodes 2, 3, and 5 and but not with demand node 4.

The link between a supply node and a demand node includes direct link (e.g., demand nodes numbers 2 and 3 link up supply node 1 directly) or indirect link (e.g., demand node number 5 links the supply node via demand node number 2). Demand nodes in both the above-mentioned cases are effective nodes. However, those demand nodes with no links, direct or indirect, to supply nodes are not effective demand nodes (e.g., demand node number 4). The weight on edges between nodes represents transport distance or time between nodes.

Now the network can be shown by an adjacent matrix:

$$D(n) = \left[d_{ij}\right]_{n\times n}. \tag{3}$$

d_{ij} is the actual distance between node i and node j if there is a direct link between node i and node j, and in case of no direct link between node i and node j, then $d_{ij} = \infty$; n is the total number of nodes ($n = n_s + n_d$).

A topological model of emergency logistics network consisting of supply nodes, demand nodes, and edges is built up. The model keeps topological features of emergency logistics networks, and via the analysis of its connecting reliability the connecting state of the network under attack can be judged.

3.2. Attack Types of Emergency Logistics Network. This essay only studies the situation of nodes in the network under attack. The failure of a node means failure of all zones connecting to it at the same time, and all roads bypassing the node close.

According to the importance of places under attack and attack sequence, attacks can be divided into random attack and target attack.

(1) *Random Attack.* Attacks happen at all nodes (including supply nodes and demand nodes), for instance, under the situation of natural disasters, incidents, local failures, and so forth.

(2) *Target Attack.* Target attacks happen in the sequence of nodes with more connections to fewer connections, for instance, in case of terrorist attack and blocking at important nodes.

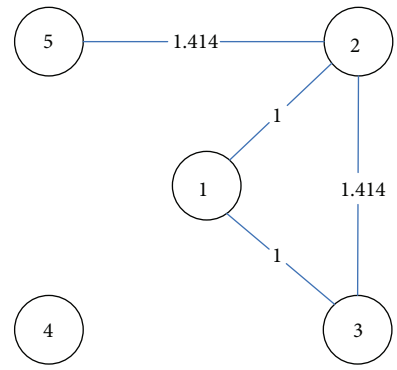


Figure 1: Topological model of emergency logistics network.

Given that natural disasters happen randomly, this essay only analyzes the changing regularity of emergency logistics network reliability under random attack.

3.3. Emulation Analysis of Random Attack

Random Attack Emulation. Random attack targets a certain node in a network randomly, and edges linking up to it lose efficiency at the same time. The shortest supply mileage from supply nodes to demand nodes is computed according to Dijkstra [27]. Those demand nodes with supply mileage lower than L_0 (maximal supply mileage) are defined as effective demand nodes in developing emulation programs, and the number and ratio of effective demand nodes in residual network are obtained; emergence supply mileage L of the network is figured up by calculating the arithmetic mean value of supply mileage of all demand nodes. Next, by choosing and attacking one of the nodes in residual network randomly, the effective demand nodes ratio P and emergence supply mileage L of residual network are calculated. These procedures are repeated again and again until all nodes are attacked. The following are steps of emulation computation.

(1) Initialize an adjacent matrix: $D(n) = [d_{ij}]_{n\times n}$.

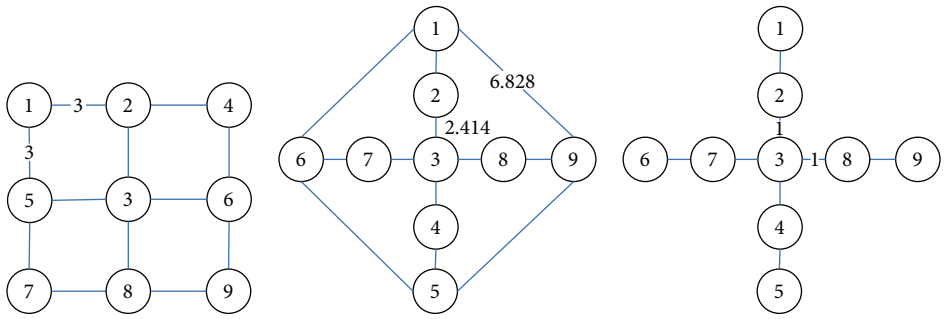
d_{ij} is the actual distance between node i and node j if there is a direct edge between the two, and in case of no direct link between the two $d_{ij} = L_0$.

(2) Generate an integral numeral r_1 in $[1, n]$; then

$$D(r_1, j) = D(j, r_1) L_0 \quad (j = 1, 2, \ldots, n). \tag{4}$$

(3) Programs for counting shortest supply mileage are developed on Visual Basic 6.0 platform according to Dijkstra, and the shortest supply mileage $c[k(m)]$ from supply nodes to all demand nodes is counted out, in which m represents the times taken to count the node of shortest mileage in Dijkstra starting from supply nodes outward step by step, and $k(m)$ is the serial number in the search.

(4) If the supply mileage of a certain demand node $c[k(m)] < L_0$, then the node can be marked as an effective demand node, and the number and ratio P of effective demand nodes can be obtained; emergence



(a) Pane (b) Radial with loop (c) Radial without loop

Figure 2: Three forms of heuristic emergency logistics network.

supply mileage L of the network is figured up by calculating the arithmetic mean value of minimum supply mileage $c[k(m)]$ of all demand nodes.

(5) Generate another integral number r_2 in $[1, n]$ after removal of r_1 randomly, and $D(r_2, j) = D(j, r_2) = L_0$.

(6) Return to step (3) until all integral numbers in $[1, n]$ are taken.

4. Examples of Heuristic Emulation Analysis

In order to study effects of emergency logistics network topological structure on its reliability, three classical forms of emergency logistics network are built (shown in Figure 2). All of these emergency logistics networks consist of 9 nodes, one is a supply node, and the rest are demand nodes. The weight on an edge between two nodes represents the mileage of the two nodes; here suppose $L_0 = 120$. Though with different structures (pane, radial with loop, and radial without loop), the three forms of network have same network density, that is, the same ratio of total passage mileage between network nodes to network coverage; here suppose that all the network density is 1.

The reliability of every form of emergency logistics network is studied by emulation methods proposed above, and emulation results are shown in Tables 1 and 2. Among which letters in network numbering represent network forms, and numerical parts represent the number of supply nodes; for instance, "a1" shows the situation of number 1 supply node in a-form (pane) network.

In order to show the change regularities of emergence network reliability of different forms more visually, comparison is made between emulation results of supply nodes with network number 1 in three forms of emergency logistics network a (pane), b (radial with loop) and c (radial without loop). Details are shown in Figures 3 and 4.

The above emulation result shows the following.

(1) Network efficiency changes in line with the selection of different supply nodes in the same network form. Network efficiency stands the highest with supply node number 3, followed by supply node number 2, and supply node number 1 the lowest in all the three forms of network from the comparison of emergence

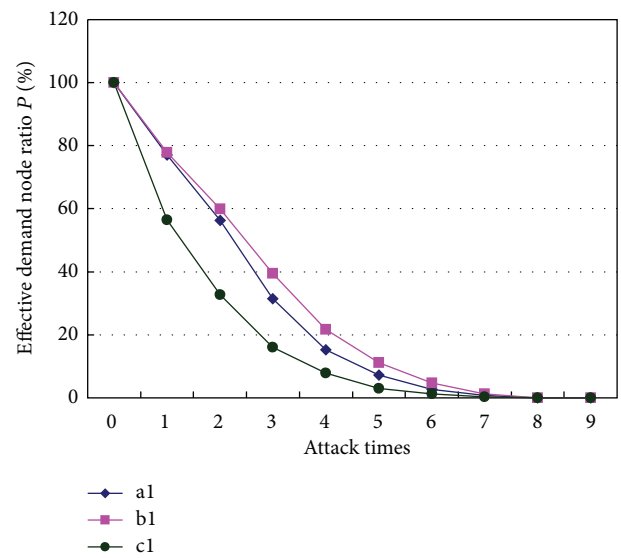


Figure 3: Comparison of effective demand node ratio changes of three different forms of network.

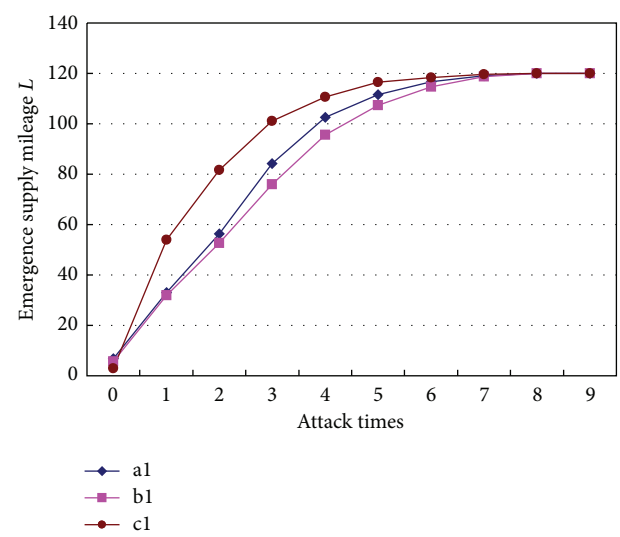


Figure 4: Comparison of emergence supply mileage changes of three different forms of network.

Table 1: Changes of effective supply node ratio P.

Network form	Supply node number	Attack times									
		0	1	2	3	4	5	6	7	8	9
a-form (pane)	1	1.0000	0.7700	0.5638	0.3144	0.1519	0.0725	0.0275	0.0081	0.0000	0.0000
	2	1.0000	0.7613	0.5469	0.3363	0.1988	0.0981	0.0413	0.0100	0.0000	0.0000
	3	1.0000	0.7744	0.5581	0.3669	0.2194	0.1238	0.0500	0.0144	0.0000	0.0000
b-form (radial with loop)	1	1.0000	0.7788	0.5988	0.3944	0.2175	0.1119	0.0475	0.0119	0.0000	0.0000
	2	1.0000	0.7963	0.5575	0.3400	0.1794	0.0819	0.0313	0.0044	0.0000	0.0000
	3	1.0000	0.7569	0.5813	0.3975	0.2381	0.1169	0.0431	0.0138	0.0000	0.0000
c-form (radial without loop)	1	1.0000	0.5644	0.3275	0.1606	0.0794	0.0300	0.0131	0.0038	0.0000	0.0000
	2	1.0000	0.6575	0.4144	0.2213	0.1138	0.0469	0.0175	0.0063	0.0000	0.0000
	3	1.0000	0.7219	0.4694	0.2981	0.1913	0.1006	0.0444	0.0106	0.0000	0.0000

Table 2: Changes of emergence supply mileage L.

Network form	Supply node number	Attack times									
		0	1	2	3	4	5	6	7	8	9
a-form (pane)	1	6.75	33.03	56.39	84.21	102.59	111.57	116.76	119.03	120.00	120.00
	2	5.63	33.03	57.62	81.43	97.07	108.58	115.15	118.80	120.00	120.00
	3	4.50	30.62	55.40	77.65	94.57	105.62	114.16	118.28	120.00	120.00
b-form (radial with loop)	1	5.75	31.88	52.68	75.93	95.59	107.42	114.60	118.65	120.00	120.00
	2	4.25	28.80	56.30	81.32	99.44	110.49	116.33	119.48	120.00	120.00
	3	3.00	32.04	52.47	74.20	92.38	106.31	114.91	118.35	120.00	120.00
c-form (radial without loop)	1	3.00	53.92	81.56	101.07	110.59	116.43	118.43	119.55	120.00	120.00
	2	2.13	42.56	71.03	93.81	106.54	114.42	117.90	119.25	120.00	120.00
	3	1.50	34.27	64.42	84.74	97.32	107.98	114.68	118.73	120.00	120.00

supply mileage emulation results when attack time is 0.

(2) Effective demand ratio of c-form (radial without loop) network falls the fastest (the average value of c-form (radial without loop) network decreases by 35.21%, 37.98%, and 44.68% during the first three attacks, well above 22.27%, 25.43%, and 34.92% of b-form (radial with loop) network and 23.15%, 27.62%, and 39.01% of a-form (pane) network), and its emergence supply mileage increases the fastest, which shows that the reliability of this form of network is relatively low under attack with the same network density.

(3) Generally speaking, the reliability of both b-form (radial with loop) network and a-form (pane) network is good, with the former a little better than the later. (In a pecking order from top to bottom after three attacks the rank of effective demand node ratio is b3[0.3975], b1[0.3944], a3[0.3669], b2[0.3400], a2[0.3363], and a1[0.3144].)

(4) Network efficiency shows different changing regularities in line with the selection of different supply nodes in the same form of network. In a-form (pane) network and c-form (radial without loop) network, from fast to slow the rate of decay of effective reliability ratio P is, in order, 1, 2, and 3, and from slow to fast emergence supply mileage is, in order, 1, 2, and 3, which shows that when demand node 3 is taken as the supply node, the network has the highest reliability, followed by demand node 2, and demand node 1 is the lowest. In b-form (radial with loop) network, from fast to slow the rate of decay of effective reliability ratio P is, in order, 2, 1, and 3, and from slow to fast emergence supply mileage is, in order, 2, 1, and 3, which shows that when demand node 3 is taken as the supply node, the network has the highest reliability, followed by demand node 1, and demand node 3 is the lowest.

(5) There is an inverse correlation between effective demand ratio and emergence supply mileage; the lower the effective demand nodes are, the farther the emergence supply mileage is.

Taken together, the selection of network forms and supply nodes affect their corresponding emergency logistics network reliability greatly. According to different network forms, the reliability of b-form (radial with loop) network is superior to a-form (pane) network, and that of c-form (radial without loop) network is the worst; in line with the selection supply nodes, those nodes, which have high degree and wider radiation and are near the network center, are more reliable.

5. Conclusions and Prospects

The topological model of emergency logistics network and its emulation methods are proposed in the essay first; then

with three typical forms of emergency logistics network as examples, emulation analyses of the three forms of emergency logistics with same density are conducted; and finally, the effective demand node ratio and changing curve of emergence supply mileage of every form of the network under different selection of supply nodes are obtained. The result shows that emergency logistics network reliability is largely affected by network forms and the selection of supply nodes. Emergency logistics network is a hotspot research subject both home and abroad in recent years; and thus, the emulation analysis thought of this thesis can inspire a new thinking in this field, and methods issued and results obtained in this essay may provide guidance and are of enlightening significance for designing an emergency logistics network with high reliability.

It is presumed that once there are connections between demand nodes and supply nodes the task of emergence material supply can be fulfilled, without considering the volume of emergence material demand and actual available supply, which simplifies actual situations. Meanwhile, the attacks of network nodes are considered, but not the attacks of edges which may bring local failure or paralysis to the network. Therefore, the emulation results will be closer to actual situations if actual freight volume and attacks of edges are considered in reliability emulation analysis of emergency logistics network, which is the emphasis of further studies.

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Research Article

Risk-Averse Suppliers' Optimal Pricing Strategies in a Two-Stage Supply Chain

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Risk-averse suppliers' optimal pricing strategies in two-stage supply chains under competitive environment are discussed. The suppliers in this paper focus more on losses as compared to profits, and they care their long-term relationship with their customers. We introduce for the suppliers a loss function, which covers both current loss and future loss. The optimal wholesale price is solved under situations of risk neutral, risk averse, and a combination of minimizing loss and controlling risk, respectively. Besides, some properties of and relations among these optimal wholesale prices are given as well. A numerical example is given to illustrate the performance of the proposed method.

1. Introduction

In traditional researches about pricing strategies of suppliers in two-stage supply chains, suppliers were always supposed to take predominant positions in deciding wholesale prices to maximize their profits. However, changes brought by globalization and standardization have forced suppliers to face competition from other suppliers with the same or similar products or services. Therefore suppliers have to allow retailers to negotiate for lower wholesale prices. So in recent years researchers turned their attention to pricing strategies of suppliers under competitive circumstances. For example, Dai et al. [1] studied the pricing strategies of multiple firms providing the same service in competition for a common pool of customers in a revenue management context. Sohn et al. [2] derived a dynamic pricing model and then a pricing policy for a mobile phone manufacturer to maximize his/her profit under the competitive environment. Xiao and Qi [3] developed an adverse selection model for a two-stage supply chain consisting of a manufacturer, a retailer, and a potential outside entrant manufacturer and obtained the optimal decisions for all players. Li et al. [4] investigated two-supplier pricing strategies and derived the sufficient condition of an equilibrium price under the environment where the two suppliers compete with each other. Wang et al. [5] studied the markup pricing strategies in a supply chain with one dominant retailer and multiple competitive manufacturers. To find a mutually beneficial relation, Voeth and Herbst [6] investigated the opportunities for a manufacturer and a retailer to collaborate on pricing, and demonstrated that it is attained in some cases. Arcelus et al. [7] analyzed the manufacturer's optimal wholesale pricing strategy facing a profit-maximizing retailer when the manufacturer possesses the full information about the cost and the functional relationship between demand and price. Lu et al. [8] highlighted the importance of services from manufacturers in the interactions between two competing manufacturers and one common retailer and proposed a game-theoretic framework for obtaining the equilibrium solutions for each entity. Xia [9] studied the competitive strategies between two coexisting suppliers in a two-echelon supply chain. Pricing strategies for different retailer groups are suggested to the competitive suppliers accordingly. Chen et al. [10] presented a review of the manufacturer's pricing strategies in a two-stage supply chain where the retailer's demand is warranty-period dependent. Sana [11] investigated the issue of channel coordination between a manufacturer and a retailer both facing stochastic demand that is sensitive

to promotional efforts and provided an analytical method to determine the optimal contract parameters of the channel.

However, in recent years, unpredictable disasters like earthquakes and economic crisis have disrupted the operation of supply chains, which further have also brought great losses to agents in supply chains. So agents in supply chains nowadays pay more attention to minimizing their losses than maximizing the profits, but few research paid attention to this. The suppliers in such environment may become risk averse. They must decide a suitable wholesale price, which is neither very low nor very high. Obviously were it very low, suppliers would suffer losses (called current loss) in profits that were supposed to be earned. Were it very high, retailers may turn to other suppliers, which hurts the suppliers long-term profits and results in future loss. In this paper, we are to find such a suitable wholesale price that takes both the current loss and future loss into consideration. But before doing this, we first introduce the following loss function for suppliers:

$$L(x) = A(\xi - x)^{+} + Bq(x - \xi)^{+}. \tag{1}$$

Here, x is the wholesale price offered by a supplier; q is the order quantity from a retailer; ξ, a random variable, is the market wholesale price for a product; A is the shortage penalty coefficient of the supplier for his/her giving a very low wholesale price, and B is the excess penalty coefficient of the supplier for his/her giving a very high wholesale price. In the right hand of (1), the first item represents the current loss, and the second item represents the future loss. In the current literature, the suppliers are supposed to give such wholesale prices so as to maximize their profits or minimize their costs in a deal, while the influence of these decisions on their future profits or costs is neglected. However, (1) considers the influence of the supplier's decision both on the current profit and the future profit. It implies that the supplier must obtain a balance between the current profit and the future profit in deciding the wholesale price. Then by minimizing the expected loss $E[L(x)]$, adopting the CVaR measure, and combining the previous two, we obtain three different optimal solutions to the wholesale price under different situations, which are risk neutral, risk averse, and a combination of minimizing loss and risk control, respectively. It is easily checked that the optimal wholesale price obtained under the risk neutral situation by minimizing the expected loss $E[L(x)]$, where E is the expectation operator, increases with the growth of the shortage penalty coefficient A and decreases with the excess penalty coefficient B and the order quantity q, respectively. Then, by adopting the CVaR measure of risk management, we obtain an optimal wholesale price under the risk-averse situation, which equals the optimal solution to the following problem:

$$\min_{x\in[c,m]} \mathrm{CVaR}_{\alpha}(x) = E\left[L(x) \mid L(x) \geq \mathrm{VaR}_{\alpha}(x)\right]. \tag{2}$$

Finally, in order to balance the expected loss and the CVaR loss of the supplier, which implies the supplier considers both the minimization of expected loss and the risk control, we define the following objective function with a combination of the two objectives:

$$\lambda E[L(x)] + (1-\lambda)\,\mathrm{CVaR}_{\alpha}(x), \tag{3}$$

where $\lambda \in [0,1]$ is the weight of the expected loss, which represents the relative importance of the expected loss as compared to the CVaR loss for the supplier, and by minimizing the previous function the optimal wholesale price is obtained.

The rest of this paper is organized as follows. Section 2 introduces the preliminaries about VaR and CVaR. Section 3 studies the optimal decisions of the supplier under different objectives and the properties of these optimal decisions. Section 4 gives some numerical examples, with conclusions given in Section 5.

2. Some Preliminaries about CVaR

In this section, we give some preliminaries about VaR and CVaR.

Value-at-Risk (VaR) is a popular risk measure that has achieved high status of being written into industry regulations. For a decision x, let $l(x,\xi)$ be the loss associated with x and the random variable ξ. The α-VaR with x is defined as

$$\mathrm{VaR}_{\alpha}(x) = \min\left\{y \in R \mid \Pr\left\{l(x,\xi) \leq y\right\} \geq \alpha\right\}, \tag{4}$$

where $\Pr\{l(x,\xi) \leq y\}$ denotes the probability of $l(x,\xi)$ not exceeding the value y. The value of $\mathrm{VaR}_{\alpha}(x)$ represents the minimal loss of decision x with the confidence level α. Artzner et al. [12] and Mauser and Rosen [13] pointed out that VaR has some undesirable mathematical characteristics such as nonsubadditivity and nonconvexity, which always hinders its efficient usage. Therefore, Rockafellar and Uryasev [14] and Rockafellar and Uryasev [15] introduced another risk measure: Conditional Value-at-Risk (CVaR), which is defined as the expected value of loss exceeding the $\mathrm{VaR}_{\alpha}(x)$. CVaR has some attractive properties such as coherence and convexity, which makes it widely used in risk management as compared to VaR. The CVaR of x with a confidence level α can be defined as

$$\begin{aligned}\mathrm{CVaR}_{\alpha}(x) &= E\left[l(x,\xi) \mid l(x,\xi) \geq \mathrm{VaR}_{\alpha}(x)\right] \\ &= \frac{1}{1-\alpha}\int_{l(x,z)\geq \mathrm{VaR}_{\alpha}(x)} l(x,z)\,\phi(z)\,dz,\end{aligned} \tag{5}$$

where $\phi(z)$ is the probability density function of ξ, and $\mathrm{VaR}_{\alpha}(x)$ is defined by (4). To compute, Rockafellar and Uryasev [14] introduced the function of $F(x,u)$:

$$F(x,u) = u + \frac{1}{1-\alpha}E\left[(l(x,\xi) - u)^{+}\right], \tag{6}$$

and proved that the minimum of $\mathrm{CVaR}(x)$ can be obtained by minimizing the function $F(x,u)$, and the corresponding $\mathrm{VaR}_{\alpha}(x)$ can be reached simultaneously in this way.

3. Main Results

For suppliers, it is supposed that the market wholesale price ξ is a random variable, and let $f(\cdot)$ and $F(\cdot)$ be its probability density function and cumulative distribution function, respectively. Without loss of generality, it is assumed that the

inverse of $F(\cdot)$ exists, and $\xi \in [c, m]$ where $0 < c < m$. For a given order quantity q from a retailer, the loss $L(x)$ of the supplier is given by (1). Now, let us discuss the optimal decisions of the supplier under the previous three different situations as to loss $L(x)$.

3.1. Optimal Wholesale Price in Minimizing Expected Loss. Evidently, to find the optimal wholesale price that minimizes the expected loss of the supplier, we need to obtain the optimal solution to the following problem:

$$\min_{x\in[c,m]} E\left[L(x)\right]. \tag{P}$$

Then, we have the following result about (P).

Theorem 1. *For the supplier, the optimal solution to (P) is given by*

$$x_0^* = F^{-1}\left(\frac{A}{A+Bq}\right). \tag{7}$$

Proof. For a wholesale price x given by the supplier, we have

$$L(x) = A(\xi - x)^+ + Bq(x-\xi)^+. \tag{8}$$

It follows from $(\xi - x)^+ = (\xi - x) + (x - \xi)^+$ that

$$L(x) = A\xi - Ax + (A+Bq)(x-\xi)^+. \tag{9}$$

Then the expected loss $\mu(x) = E[L(x)]$ is given by

$$\mu(x) = A\left(E[\xi] - x\right) + (A+Bq)\int_c^x (x-t)\,dF(t), \tag{10}$$

which implies

$$\mu'(x) = (A+Bq)F(x) - A. \tag{11}$$

Then the solution to problem (P) is obtained by solving the $\mu'(x) = 0$ which gives

$$x_0^* = F^{-1}\left(\frac{A}{A+Bq}\right). \tag{12}$$

□

Corollary 2. *For the supplier, the optimal wholesale price x_0^* decreases with the growth of the excess penalty coefficient B.*

In fact, if the excess penalty coefficient B improves, which implies that the supplier pays more attention to the long-term cooperation with the retailer, the supplier will decrease the wholesale price to maintain the cooperation with the retailer in the future.

Corollary 3. *For the supplier, the optimal wholesale price x_0^* increases with the growth of shortage penalty coefficient A.*

In fact, if the shortage penalty coefficient A improves, that is, the supplier pays more attention to the loss of the current transaction and ignores the future one, then he/she will increase the wholesale price to lessen his/her loss in the current transaction.

Corollary 4. *For the supplier, the optimal wholesale price x_0^* decreases with the growth of the order quantity q from the retailer.*

By Corollary 4, the wholesale price will decrease with the growth of the order quantity q from the retailer.

3.2. Optimal Wholesale Price in Minimizing CVaR Loss. In the above subsection, we discuss how to decide the optimal wholesale price that minimizes the expected loss of the supplier, but this approach may lead to an unpredictable large loss since risks are not considered. To control risks that may lead to possible losses for the supplier, we now consider minimizing the CVaR loss, which can determine the minimal loss of the supplier for a given confidence level.

For the confidence level α, the α-VaR of the supplier with respect to the wholesale price x is given by

$$\mathrm{VaR}_\alpha(x) = \min\left\{y \in R \mid \Pr\left\{L(x) \le y\right\} \ge \alpha\right\}, \tag{13}$$

where $\Pr\{L(x) \le y\}$ denotes the probability of $L(x)$ not exceeding the value y. Then the CVaR loss of the supplier about x can be defined as

$$\mathrm{CVaR}_\alpha(x) = E\left[L(x) \mid L(x) \ge \mathrm{VaR}_\alpha(x)\right]. \tag{14}$$

Evidently, we need to find the optimal solution to the following problem:

$$\min_{x\in[c,m]} \mathrm{CVaR}_\alpha(x). \tag{P_1}$$

Then we have the following result.

Theorem 5. *For the risk-averse supplier, the optimal solution to (P_1) is given by*

$$x_1^* = \frac{1}{A+Bq}\left[AF^{-1}\left(\frac{A+Bq\alpha}{A+Bq}\right) + BqF^{-1}\left(\frac{A(1-\alpha)}{A+Bq}\right)\right]. \tag{15}$$

Proof. By (9), we have $L(x) = A\xi - Ax + (A+Bq)(x-\xi)^+$. Now, we define a convex function

$$\begin{aligned} h(x,v) &= v + \frac{1}{1-\alpha}E[L(x)-v]^+ \\ &= v + \frac{1}{1-\alpha}\int_c^m \left[At - Ax + (A+Bq)(x-t)^+ - v\right]^+ dF(t) \\ &= v + \frac{1}{1-\alpha}\int_c^x \left[Bqx - Bqt - v\right]^+ dF(t) \\ &\quad + \frac{1}{1-\alpha}\int_x^m \left[At - Ax - v\right]^+ dF(t). \end{aligned} \tag{16}$$

Based on the result in Section 2, the optimal solution to (P_1) equals the optimal solution to the following problem:

$$\min_{x\in[c,m]}\left[\min_{v\in R} h(x,v)\right]. \tag{17}$$

Then, for any fixed x, we distinguish the following different cases.

(i) $v \geq Bqx$.

In this case, by (16), we have

$$\begin{aligned} h(x,v) &= v + \frac{1}{1-\alpha}\int_c^x 0\,dF(t) \\ &\quad + \frac{1}{1-\alpha}\int_{x+(v/A)}^m [At - Ax - v]\,dF(t), \end{aligned} \tag{18}$$

$$\frac{\partial h(x,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\left[1 - F\left(x + \frac{v}{A}\right)\right]. \tag{19}$$

Obviously, when v is sufficiently large ($v \geq A(m-x)$), it follows from (19) that $\partial h(x,v)/\partial x = 1 > 0$ holds. Then if it satisfies $(\partial h(x,v)/\partial x)|_{v=Bqx} = 1 - (1/(1-\alpha))[1 - F(x + (Bqx/A))] < 0$, which implies $x < (A/(A+Bq))F^{-1}(\alpha)$, it follows from (19) and $\partial h(x,v)/\partial v = 0$ that the optimal solution v^* to $\min_{v\in R} h(x,v)$ solves

$$\frac{\partial h(x,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\left[1 - F\left(x + \frac{v}{A}\right)\right] = 0. \tag{20}$$

That is

$$v^* = A\left[F^{-1}(\alpha) - x\right]. \tag{21}$$

(ii) $0 < v < Bqx$.

In this case, by (16), we have

$$\begin{aligned} h(x,v) &= v + \frac{1}{1-\alpha}\int_c^{x-(v/Bq)} [Bqx - Bqt - v]\,dF(t) \\ &\quad + \frac{1}{1-\alpha}\int_{x+(v/A)}^m [At - Ax - v]\,dF(t), \end{aligned} \tag{22}$$

$$\frac{\partial h(x,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\left[1 + F\left(x - \frac{v}{Bq}\right) - F\left(x + \frac{v}{A}\right)\right]. \tag{23}$$

Obviously, it satisfies

$$\left.\frac{\partial h(x,v)}{\partial v}\right|_{v=0} = 1 - \frac{1}{1-\alpha} < 0. \tag{24}$$

Then if it satisfies $(\partial h(x,v)/\partial x)|_{v=Bqx} = 1 - (1/(1-\alpha))[1 - F(x + (Bqx/A))] \geq 0$, which implies $x \geq (A/(A+Bq))F^{-1}(\alpha)$, it follows from (23) and $\partial h(x,v)/\partial v = 0$ that the optimal solution v^* to $\min_{v\in R} h(x,v)$ solves

$$\frac{\partial h(x,v)}{\partial v} = 1 - \frac{1}{1-\alpha}\left[1 + F\left(x - \frac{v}{Bq}\right) - F\left(x + \frac{v}{A}\right)\right] = 0. \tag{25}$$

(iii) $v \leq 0$.

In this case, by (16), we have

$$\begin{aligned} h(x,v) &= v + \frac{1}{1-\alpha}\int_c^x [Bqx - Bqt - v]\,dF(t) \\ &\quad + \frac{1}{1-\alpha}\int_x^m [At - Ax - v]\,dF(t), \\ \frac{\partial h(x,v)}{\partial v} &= 1 - \frac{1}{1-\alpha} < 0. \end{aligned} \tag{26}$$

Based on the previous analysis, it is clear that for any fixed x, $h(x,v)$ attains minimum when $v > 0$. Further, for any fixed x, the optimal solution to $\min_{v\in R} h(x,v)$ is given by

$$v^* = \begin{cases} A\left[F^{-1}(\alpha) - x\right] & x < \dfrac{A}{A+Bq}F^{-1}(\alpha), \\ v^1 & x \geq \dfrac{A}{A+Bq}F^{-1}(\alpha), \end{cases} \tag{27}$$

where v^1 is given by (25).

Thus, to solve the problem $\min_{x\in[c,m]}[\min_{v\in R} h(x,v)] = \min_{x\in[c,m]}[h(x,v^*)]$, we distinguish the following cases.

(a) $x < (A/(A+Bq))F^{-1}(\alpha)$.

In this case, it follows from (27) that the optimal solution to the problem $\min_{v\in R} h(x,v)$ is given by

$$v^* = A\left[F^{-1}(\alpha) - x\right]. \tag{28}$$

Then by (16), we have

$$\begin{aligned} h(x,v^*) &= A\left[F^{-1}(\alpha) - x\right] + \frac{1}{1-\alpha}\int_c^x 0\,dF(t) \\ &\quad + \frac{1}{1-\alpha}\int_{F^{-1}(\alpha)}^m A\left(t - F^{-1}(\alpha)\right)dF(t), \\ \frac{\partial h(x,v^*)}{\partial x} &= -A < 0. \end{aligned} \tag{29}$$

(b) $x \geq (A/(A+Bq))F^{-1}(\alpha)$.

In this case, it follows from (27) that the optimal solution to the problem $\min_{v\in R} h(x,v)$ is given by $v^* = v^1$, where v^1 satisfies (25). By (25), we have

$$F\left(x + \frac{v^1}{A}\right) = F\left(x - \frac{v^1}{Bq}\right) + \alpha. \tag{30}$$

Then by (16), we have

$$h\left(x, v^1\right) = v^1 + \frac{1}{1-\alpha}\int_c^{x-(v^1/Bq)}\left[Bqx - Bqt - v^1\right]dF(t) + \frac{1}{1-\alpha}\int_{x+(v^1/A)}^{m}\left[At - Ax - v^1\right]dF(t), \tag{31}$$

$$\frac{\partial h\left(x, v^1\right)}{\partial x} = \frac{1}{1-\alpha}\left[BqF\left(x - \frac{v^1}{Bq}\right) - A\left(1 - F\left(x + \frac{v^1}{A}\right)\right)\right]. \tag{32}$$

It follows from (32) and $\partial h(x, v^1)/\partial x = 0$ that the optimal solution x_1^* to $\min_{x\in[c,m]}[h(x, v^*)]$ solves

$$\frac{1}{1-\alpha}\left[BqF\left(x - \frac{v^1}{Bq}\right) - A\left(1 - F\left(x + \frac{v^1}{A}\right)\right)\right] = 0. \tag{33}$$

Then it follows from (30) and (33) that

$$x_1^* = \frac{1}{A+Bq}\left[AF^{-1}\left(\frac{A+Bq\alpha}{A+Bq}\right) + BqF^{-1}\left(\frac{A(1-\alpha)}{A+Bq}\right)\right]. \tag{34}$$

It is easily checked that the optimal solution to (P_1) is more complicated than that of (P). Similar to Corollaries 2–4, we have the following results. □

Corollary 6. *For the risk-averse supplier, the optimal solution x_1^* to (P_1) decreases with the growth of the excess penalty coefficient B.*

Proof. By Theorem 5, the optimal solution to (P_1) is given by

$$x_1^* = \frac{1}{A+Bq}\left[AF^{-1}\left(\frac{A+Bq\alpha}{A+Bq}\right) + BqF^{-1}\left(\frac{A(1-\alpha)}{A+Bq}\right)\right]. \tag{35}$$

For simplicity, we denote

$$\frac{A+Bq\alpha}{A+Bq} = M, \qquad \frac{A(1-\alpha)}{A+Bq} = N. \tag{36}$$

It is obvious that

$$M - N = \alpha \geq 0, \tag{37}$$

which implies

$$M \geq N. \tag{38}$$

Thus we have

$$\begin{aligned}
\frac{\partial x_1^*}{\partial B} &= \frac{\left(\left(\partial\left[AF^{-1}(M) + BqF^{-1}(N)\right]\right)/(\partial B)\right)(A+Bq)}{(A+Bq)^2} - \frac{\left[AF^{-1}(M) + BqF^{-1}(N)\right]q}{(A+Bq)^2} \\
&= \left(\left(\left(-Aq(1-\alpha)\left[A\left(f\left(F^{-1}(M)\right)\right)^{-1} + Bq\left(f\left(F^{-1}(N)\right)\right)^{-1}\right]\right)\times(A+Bq)^{-1}\right) + (A+Bq)qF^{-1}(N)\right)\left((A+Bq)^2\right)^{-1} \\
&\quad - \frac{\left[AF^{-1}(M) + BqF^{-1}(N)\right]q}{(A+Bq)^2} \\
&= -\left(Aq(1-\alpha)\left[A\left[f\left(F^{-1}(M)\right)\right]^{-1}\right] + Bq\left[f\left(F^{-1}(N)\right)\right]^{-1}\right)\times\left((A+Bq)^3\right)^{-1} \\
&\quad - \frac{Aq}{(A+Bq)^2}\left[F^{-1}(M) - F^{-1}(N)\right].
\end{aligned} \tag{39}$$

It follows from $M \geq N$ that $F^{-1}(M) - F^{-1}(N) \geq 0$ holds, which by (39) implies $\partial x_1^*/\partial B \leq 0$, and x_1^* decreases with the growth of the excess penalty coefficient B. □

Corollary 7. *For the risk-averse supplier, the optimal solution x_1^* to (P_1) increases with the growth of shortage penalty coefficient A.*

Proof. By Theorem 5, we have

$$\begin{aligned}
\frac{\partial x_1^*}{\partial A} &= \frac{\left(\left(\partial\left[AF^{-1}(M) + BqF^{-1}(N)\right]\right)/(\partial A)\right)(A+Bq)}{(A+Bq)^2} - \frac{\left[AF^{-1}(M) + BqF^{-1}(N)\right]}{(A+Bq)^2} \\
&= \left(\left(\left(Bq(1-\alpha)\left[A\left(f\left(F^{-1}(M)\right)\right)^{-1}\right] + Bq\left(f\left(F^{-1}(N)\right)\right)^{-1}\right)(A+Bq)^{-1}\right) + (A+Bq)F^{-1}(M)\right)\times\left((A+Bq)^2\right)^{-1} \\
&\quad - \frac{\left[AF^{-1}(M) + BqF^{-1}(N)\right]q}{(A+Bq)^2}
\end{aligned}$$

$$= \frac{Bq(1-\alpha)\left[A\left(f\left(F^{-1}(M)\right)\right)^{-1} + Bq\left(f\left(F^{-1}(N)\right)\right)^{-1}\right]}{(A+Bq)^3} + \frac{Bq}{(A+Bq)^2}\left[F^{-1}(M) - F^{-1}(N)\right]. \tag{40}$$

It follows from $M \geq N$ that $F^{-1}(M) - F^{-1}(N) \geq 0$ holds. Then it concludes from (40) that $\partial x_1^*/\partial A \geq 0$, which implies that the optimal solution x_1^* to (P_1) increases with the growth of shortage penalty coefficient A. □

Corollary 8. *For the risk-averse supplier, the optimal solution x_1^* to (P_1) decreases with the growth of the order quantity q from the retailer.*

Proof. By Theorem 5, we have

$$\begin{aligned}
\frac{\partial x_1^*}{\partial q} &= \frac{\left(\left(\partial\left[AF^{-1}(M) + BqF^{-1}(N)\right]\right)/(\partial q)\right)(A+Bq)}{(A+Bq)^2} - \frac{B\left[AF^{-1}(M) + BqF^{-1}(N)\right]}{(A+Bq)^2} \\
&= \left(\left(\left(AB(1-\alpha) \times \left[Af\left(F^{-1}(M)\right) + Bq\left(f\left(F^{-1}(N)\right)\right)^{-1}\right]\right) \times (A+Bq)^{-1}\right) + B(A+Bq)F^{-1}(N)\right) \times \left((A+Bq)^2\right)^{-1} - \frac{B\left[AF^{-1}(M) + BqF^{-1}(N)\right]}{(A+Bq)^2} \\
&= -\frac{AB(1-\alpha)\left[A\left[f\left(F^{-1}(M)\right)\right]^{-1} + Bq\left[f\left(F^{-1}(N)\right)\right]^{-1}\right]}{(A+Bq)^3} - \frac{AB}{(A+Bq)^2}\left[F^{-1}(M) - F^{-1}(N)\right].
\end{aligned} \tag{41}$$

It follows from $M \geq N$ that $F^{-1}(M) - F^{-1}(N) \geq 0$ holds. Then it concludes from (41) that $\partial x_1^*/\partial q \leq 0$, which implies that the optimal solution x_1^* to (P_1) decreases with the growth of the order quantity q from the retailer.

By Corollaries 6–8, the optimal wholesale price that minimizes CVaR loss decreases with the growth of the excess penalty coefficient B and the order quantity q from the retailer and increases with the growth of shortage penalty coefficient A. □

Remark 9. For $\alpha \in (0,1)$, x_1^* may not be monotone with α. By Theorem 5, the optimal solution to (P_1) is given by

$$x_1^* = \frac{1}{A+Bq}\left[AF^{-1}\left(\frac{A+Bq\alpha}{A+Bq}\right) + BqF^{-1}\left(\frac{A(1-\alpha)}{A+Bq}\right)\right]. \tag{42}$$

Thus we have

$$\frac{\partial x_1^*}{\partial \alpha} = \frac{ABq}{(A+Bq)^2 f\left[F^{-1}(M)\right] f\left[F^{-1}(N)\right]} \times \left[f\left(F^{-1}(N)\right) - f\left(F^{-1}(M)\right)\right]. \tag{43}$$

Obviously, the previous inequality shows that the sign of $\partial x_1^*/\partial\alpha$ is the same as that of $f(F^{-1}(N)) - f(F^{-1}(M))$, which maybe positive or negative. Specially, if $f(\cdot)$ increases monotonically, it concludes from $M \geq N$ that $f[F^{-1}(M)] \geq f[F^{-1}(N)]$, which implies $\partial x_1^*/\partial\alpha \leq 0$, and then the optimal wholesale price x_1^* decreases with the growth of α. Otherwise, if $f(\cdot)$ decreases monotonically, it concludes from $M \geq N$ that $f[F^{-1}(M)] \leq f[F^{-1}(N)]$, which implies $\partial x_1^*/\partial\alpha \geq 0$, and then the optimal wholesale price x_1^* increases with the growth of α.

Example 10. For a two-stage supply chain, suppose that the market wholesale price ξ subjects to exponential distribution $e(0.25)$, uniform distribution $U(3,5)$, and normal distribution $N(4, 0.5^2)$, respectively. Let $A = 100$, $B = 2$, and $q = 100$. For different confidence level α, we compute the optimal wholesale price of the risk-averse supplier with CVaR measure of loss, with the results listed in Table 1.

As shown in Table 1, if the market wholesale price ξ subjects to the exponential distribution $e(0.25)$, then the optimal wholesale price x_1^* of the supplier increases with the growth of α; if the market wholesale price ξ subjects to the uniform distribution $U(3,5)$, then the optimal wholesale price x_1^* of the supplier stays the same; if the market wholesale price ξ subjects to the normal distribution $N(4, 0.5^2)$, then the optimal wholesale price x_1^* of the supplier decreases with the growth of α.

3.3. Optimal Wholesale Price in Balancing Expected Loss and CVaR Loss. Evidently, the CVaR approach is too conservative for some suppliers, who pay great attention to the loss above the VaR while the part below the VaR is ignored. Therefore, we intend to find a wholesale price that balances the expected loss and CVaR loss. For this aim, we propose the following problem:

$$\min_{x\in[c,m]}\left[\lambda E(L(x)) + (1-\lambda)\,\mathrm{CVaR}_\alpha(x)\right], \tag{P_2}$$

where $\lambda \in [0,1]$ is the weight of the expected loss, which represents the relative importance of the expected loss as compared to the CVaR loss. This utility function reflects the fact that suppliers concern both minimization of expected loss and risk control. Then our objective is to find the optimal solution to Problem (P_2).

Table 1: Optimal wholesale prices for different distributions of ξ.

	α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\xi \sim e(0.25)$	q_1^*	1.632	1.665	1.725	1.817	1.951	2.144	2.427	2.781	3.701
$\xi \sim U(3,5)$	q_1^*	3.667	3.667	3.667	3.667	3.667	3.667	3.667	3.667	3.667
$\xi \sim N(4,0.5^2)$	q_1^*	3.783	3.778	3.771	3.761	3.749	3.734	3.713	3.685	3.639

Theorem 11. *For the supplier, if it satisfies*

$$\lambda \geq \frac{A}{(A+Bq)\,F\left[\left(A/\left(A+Bq\right)\right)F^{-1}\left(\alpha\right)\right]}, \tag{44}$$

then the optimal solution to (P_2) *is given by*

$$x_2^* = F^{-1}\left[\frac{A}{\lambda\left(A+Bq\right)}\right]; \tag{45}$$

otherwise, the optimal solution to (P_2) *solves*

$$\begin{aligned} x_2^* = \Bigg(\Bigg[& AF^{-1}\left(\left(\alpha\left(1-\lambda\right)\left(A+Bq\right)+\left(1-\alpha\right)\right.\right. \\ & \left.\times\left(A-\lambda\left(A+Bq\right)F\left(x_2^*\right)\right)\right) \\ & \times\left(\left(1-\lambda\right)\left(A+Bq\right)\right)^{-1}\Bigg) \\ & +BqF^{-1}\left(\frac{\left(1-\alpha\right)\left(A-\lambda\left(A+Bq\right)F\left(x_2^*\right)\right)}{\left(1-\lambda\right)\left(A+Bq\right)}\right)\Bigg]\Bigg) \\ & \times\left(A+Bq\right)^{-1}. \end{aligned} \tag{46}$$

Proof. Similar to the proof of Theorem 5, let us define a convex function

$$\begin{aligned} k\left(x,v\right) = {} & \lambda\left[AE\xi - Ax + \left(A+Bq\right)\int_c^x \left(x-t\right)dF\left(t\right)\right] \\ & + \left(1-\lambda\right)\left[\, v + \frac{1}{1-\alpha}\int_c^x \left(Bqx - Bqt - v\right)^+ dF\left(t\right)\right. \\ & \left. + \frac{1}{1-\alpha}\int_x^m \left(At - Ax - v\right)^+ dF\left(t\right)\right]. \end{aligned} \tag{47}$$

Obviously, the optimal solution to (P_2) equals the optimal solution to the following problem:

$$\min_{x\in[c,m]}\left[\min_{v\in R} k\left(x,v\right)\right]. \tag{48}$$

Note that the first item in the right hand of (47) has nothing to do with v; then for any fixed x, it is concluded from the proof of Theorem 5 that the optimal solution to $\min_{v\in R} k(x,v)$ is given by

$$v^* = \begin{cases} A\left[F^{-1}\left(\alpha\right)-x\right] & x < \dfrac{A}{A+Bq}F^{-1}\left(\alpha\right), \\ v^1 & x \geq \dfrac{A}{A+Bq}F^{-1}\left(\alpha\right), \end{cases} \tag{49}$$

where v^1 solves

$$F\left(x+\frac{v^1}{A}\right) = F\left(x-\frac{v^1}{Bq}\right)+\alpha. \tag{50}$$

Thus, we solve $\min_{x\in[c,m]}[\min_{v\in R} k(x,v)] = \min_{x\in[c,m]} k(x,v^*)$ in the following two different cases.

(a) $x < (A/(A+Bq))F^{-1}(\alpha)$.

In this case, it follows from (49) that the optimal solution to the problem $\min_v k(x,v)$ is given by

$$v^* = A\left[F^{-1}\left(\alpha\right)-x\right]. \tag{51}$$

Then by (47), we have

$$\begin{aligned} k\left(x,v^*\right) = {} & \lambda\left[AE\xi - Ax + \left(A+Bq\right)\int_c^x \left(x-t\right)dF\left(t\right)\right] \\ & + \left(1-\lambda\right)\left[\, A\left(F^{-1}\left(\alpha\right)-x\right) + \frac{1}{1-\alpha}\int_c^x 0\,dF\left(t\right)\right. \\ & \left. + \frac{1}{1-\alpha}\int_{F^{-1}(\alpha)}^m \left(A\left(t-F^{-1}\left(\alpha\right)\right)\right)dF\left(t\right)\right], \end{aligned} \tag{52}$$

$$\frac{\partial k\left(x,v^*\right)}{\partial x} = \lambda\left(A+Bq\right)F\left(x\right)-A. \tag{53}$$

Obviously, it satisfies

$$\left.\frac{\partial k\left(x,v^*\right)}{\partial x}\right|_{x=c} = -A < 0. \tag{54}$$

If it satisfies $(\partial k(x,v^*)/\partial x)|_{x=(A/(A+Bq))F^{-1}(\alpha)} \geq 0$, that is, $\lambda \geq A/(A+Bq)F[(A/(A+Bq))F^{-1}(\alpha)]$, then it concludes from (53) and $\partial k(x,v^*)/\partial x = 0$ that the optimal solution x^* to $\min_{x\in[c,m]} k(x,v^*)$ is given by

$$x_2^* = F^{-1}\left[\frac{A}{\lambda\left(A+Bq\right)}\right]. \tag{55}$$

Otherwise, we consider the case of $x \geq (A/(A+Bq))F^{-1}(\alpha)$.

(b) $x \geq (A/(A+Bq))F^{-1}(\alpha)$.

In this case, it follows from (49) that the optimal solution to the problem $\min_v k(x,v)$ is given by $v^* = v^1$.

Then by (47), we have

$$\begin{aligned} k(x,v^*) &= k(x,v^1) \\ &= \lambda\left[AE\xi - Ax + (A+Bq)\int_c^x (x-t)\,dF(t)\right] + (1-\lambda) \\ &\quad \times\left[v^1 + \frac{1}{1-\alpha}\right. \\ &\quad \times \int_c^{x-(v^1/Bq)} \left(Bqx - Bqt - v^1\right) dF(t) \\ &\quad \left. + \frac{1}{1-\alpha}\int_{x+(v^1/A)}^m \left(At - Ax - v^1\right) dF(t)\right], \end{aligned} \tag{56}$$

$$\begin{aligned} \frac{\partial k(x,v^1)}{\partial x} &= \lambda(A+Bq)F(x) - \lambda A \\ &\quad + \frac{1-\lambda}{1-\alpha}\left[BqF\left(x - \frac{v^1}{Bq}\right) + AF\left(x + \frac{v^1}{A}\right) - A\right]. \end{aligned} \tag{57}$$

Obviously, it follows from (50) and (57) that

$$\begin{aligned} &\frac{\partial k(x,v^1)}{\partial x} \\ &\quad = \lambda(A+Bq)F(x) - \lambda A \\ &\qquad + \frac{1-\lambda}{1-\alpha}\left[(A+Bq)F\left(x - \frac{v^1}{Bq}\right) - A(1-\alpha)\right] \\ &\quad = \lambda(A+Bq)F(x) - A + \frac{1-\lambda}{1-\alpha}(A+Bq)F\left(x - \frac{v^1}{Bq}\right) \end{aligned} \tag{58}$$

$$\begin{aligned} &= \lambda(A+Bq)F(x) - A + \frac{1-\lambda}{1-\alpha}(A+Bq) \\ &\quad \times\left[F\left(x + \frac{v^1}{A}\right) - \alpha\right]. \end{aligned} \tag{59}$$

It follows from (58) and $\partial k(x,v^1)/\partial x = 0$ that

$$v^1 = Bqx - BqF^{-1}\left[\frac{(1-\alpha)(A - \lambda(A+Bq)F(x))}{(1-\lambda)(A+Bq)}\right]. \tag{60}$$

Thus, it concludes form (59), (60), and $\partial k(x,v^1)/\partial x = 0$ that the optimal solution x_2^* to $\min_{x\in[c,m]} k(x,v^*)$ solves

$$\begin{aligned} x_2^* = \Bigg(&AF^{-1}\Bigg[(\alpha(1-\lambda)(A+Bq) + (1-\alpha) \\ &\times(A - \lambda(A+Bq)F(x_2^*))) \\ &\times((1-\lambda)(A+Bq))^{-1}\Bigg] \\ &+BqF^{-1}\left[\frac{(1-\alpha)(A - \lambda(A+Bq)F(x_2^*))}{(1-\lambda)(A+Bq)}\right]\Bigg) \\ &\times(A+Bq)^{-1}. \end{aligned} \tag{61}$$

In fact, Problems (P) and (P_1) can be seen as the special cases of Problem (P_2) when $\lambda = 1$ and $\lambda = 0$, respectively. By Theorem 11, we have the following conclusions.

(i) For $\lambda = 1$, Problem (P_2) is reduced to problem (P). If it satisfies

$$1 = \lambda \geq \frac{A}{(A+Bq)F[(A/(A+Bq))F^{-1}(\alpha)]}, \tag{62}$$

which implies that (44) holds, then the optimal solution to (P_2) is given by

$$x_2^* = F^{-1}\left[\frac{A}{\lambda(A+Bq)}\right]. \tag{63}$$

It follows with $\lambda = 1$ that

$$x_2^* = F^{-1}\left[\frac{A}{\lambda(A+Bq)}\right] = F^{-1}\left(\frac{A}{A+Bq}\right), \tag{64}$$

which is the same as the optimal solution x_0^* to (P); otherwise, if it satisfies

$$\lambda = 1 < \frac{A}{(A+Bq)F[(A/(A+Bq))F^{-1}(\alpha)]}, \tag{65}$$

then the optimal solution x_2^* to (P_2) is given by (46). By (58) in the proof of Theorem 11, x_2^* solves

$$\begin{aligned} \frac{\partial k(x,v^1)}{\partial x} &= \lambda(A+Bq)F(x) - A \\ &\quad + \frac{1-\lambda}{1-\alpha}(A+Bq)F\left(x - \frac{v^1}{Bq}\right) = 0, \end{aligned} \tag{66}$$

which follows with $\lambda = 1$ that

$$\frac{\partial k(x,v^1)}{\partial x} = (A+Bq)F(x) - A = 0, \tag{67}$$

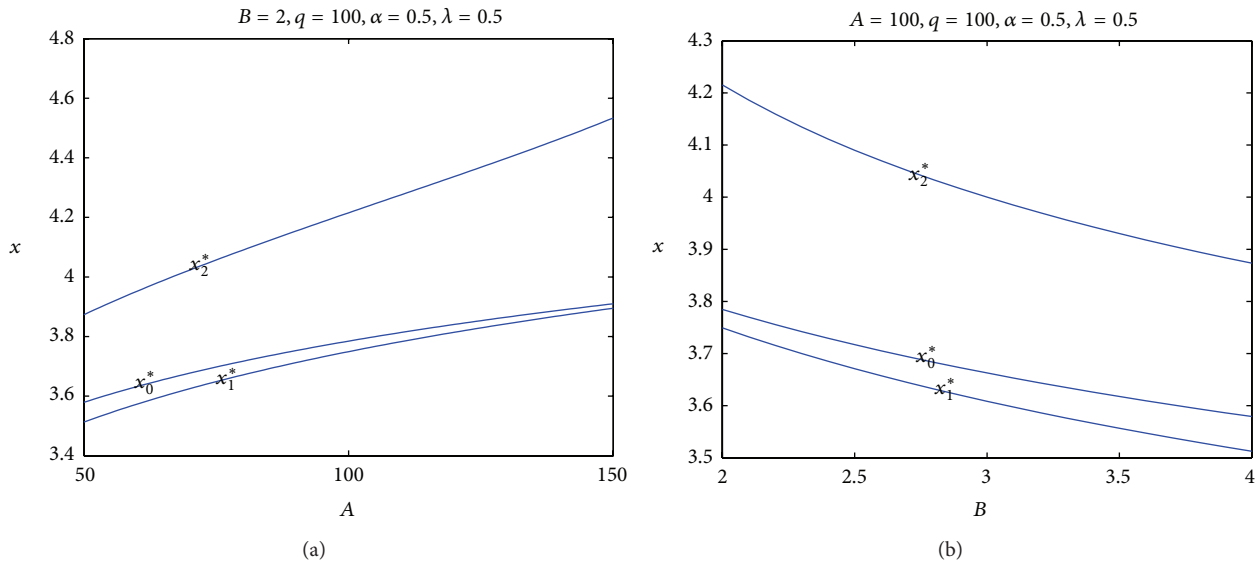

Figure 1: Optimal wholesale prices x_0^*, x_1^*, and x_2^* for different shortage penalty coefficient A and excess penalty coefficient B.

which implies

$$x_2^* = F^{-1}\left(\frac{A}{A+Bq}\right), \tag{68}$$

which is the same as the optimal solution x_0^* to (P), too.

(ii) For $\lambda = 0$, Problem (P_2) is reduced to Problem (P_1). In this case, since

$$\frac{A}{(A+Bq)\,F\left[\left(A/\left(A+Bq\right)\right)F^{-1}(\alpha)\right]} > 0 = \lambda, \tag{69}$$

by Theorem 11, it is obvious that the optimal solution x_2^* to (P_2) is given by

$$\begin{aligned} x_2^* = \Bigg(\Bigg[&AF^{-1}\Big(\Big(\alpha(1-\lambda)(A+Bq)+(1-\alpha) \\ &\times\left(A-\lambda(A+Bq)F(x_2^*)\right)\Big) \\ &\times\left((1-\lambda)(A+Bq)\right)^{-1}\Big) \\ &+BqF^{-1}\left(\frac{(1-\alpha)\left(A-\lambda(A+Bq)F(x_2^*)\right)}{(1-\lambda)(A+Bq)}\right)\Bigg]\Bigg) \\ &\times(A+Bq)^{-1}, \end{aligned} \tag{70}$$

which follows with $\lambda = 0$ that

$$x_2^* = \frac{1}{A+Bq}\left[AF^{-1}\left(\frac{A+Bq\alpha}{A+Bq}\right)+BqF^{-1}\left(\frac{A(1-\alpha)}{A+Bq}\right)\right], \tag{71}$$

which is the same as the optimal solution x_1^* to (P_1).

Based on Corollaries 2–8, it is obvious that the optimal solution x_1^* to (P_2) decreases with the growth of the excess penalty coefficient B and the growth of the order quantity q from the retailer and increases with the growth of shortage penalty coefficient A. □

Remark 12. By Remark 9, the optimal solution to (P_2) may not be monotone with α.

Remark 13. For the supplier, the optimal solution to (P_2) may not be monotone with λ. In fact, by Theorem 11, we have the following conclusion: if $\lambda \geq A/((A+Bq)F[(A/(A+Bq))F^{-1}(\alpha)])$, we have $x_2^* = F^{-1}[A/\lambda(A+Bq)]$, and it is obvious that x_2^* decreases with the growth of λ; otherwise, x_2^* is given by (46). Let

$$\begin{gathered} \left[\frac{(1-\alpha)\left[A-\lambda(A+Bq)F(x_2^*)\right]}{(1-\lambda)(A+Bq)}\right] = H, \\ \frac{\alpha(1-\lambda)(A+Bq)+(1-\alpha)\left[A-\lambda(A+Bq)F(x_2^*)\right]}{(1-\lambda)(A+Bq)} = G. \end{gathered} \tag{72}$$

Then, we have

$$x_2^* = \frac{AF^{-1}(G)+BqF^{-1}(H)}{A+Bq}, \tag{73}$$

and it is obvious

$$G = H + \alpha \geq H. \tag{74}$$

Differentiating x_2^* with respect to λ, we have

$$\begin{aligned}\frac{\partial x_2^*}{\lambda}&\left[(A+Bq)+\left(\frac{A}{f\left(F^{-1}(G)\right)}\right.\right.\\&\left.\left.+\frac{Bq}{f\left(F^{-1}(H)\right)}\right)\frac{\lambda(1-\alpha)f\left(x_2^*\right)}{1-\lambda}\right]\\&=\left[\frac{A}{f\left(F^{-1}(G)\right)}+\frac{Bq}{f\left(F^{-1}(H)\right)}\right]\\&\quad\times\frac{(1-\alpha)\left[A-(A+Bq)F\left(x_2^*\right)\right]}{(1-\lambda)^2(A+Bq)}.\end{aligned}\tag{75}$$

The previous equality shows that the sign of $\partial x_2^*/\partial\lambda$ is the same as that of $A-(A+Bq)F(x_2^*)$, which may be positive or negative.

4. Numerical Example

In this section, we will give an example to show the results obtained in Section 4.

Example 1. For a two-stage supply chain, suppose that the market wholesale price ξ subjects to uniform distribution $U(3,5)$. Let us compute the optimal wholesale prices x_0^*, x_1^*, and x_2^* for the supplier and give some sensitivity analysis.

Let $q=100$, $\alpha=0.5$, and $\lambda=0.5$, we compute the optimal wholesale prices x_0^*, x_1^*, and x_2^* and illustrate the changes of these optimal wholesale prices with different parameters A and B in Figure 1. By Figure 1, it is easily checked that x_0^*, x_1^*, and x_2^* are increasing in shortage penalty coefficient A and decreasing in excess penalty coefficient B, respectively.

Moreover, for $A=100$, $B=2$, $q=100$, and $\lambda=0.5$, we compute the optimal wholesale prices x_1^* and illustrate the changes of x_1^* with confidence level α in Figure 2. By Figure 2, it is easily found that the optimal wholesale prices x_1^* are decreasing in the confidence level α.

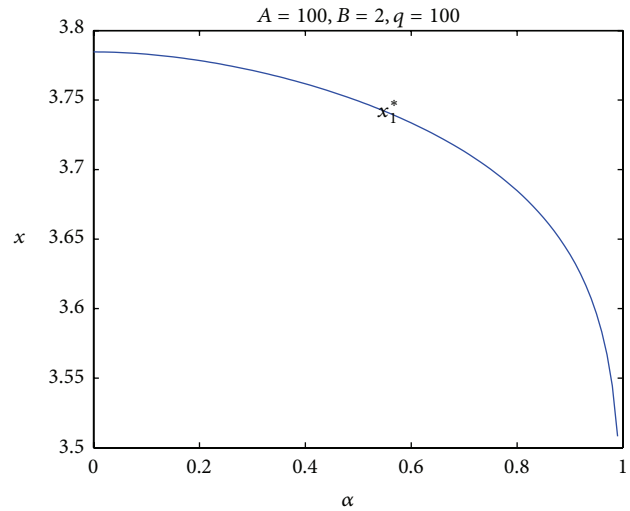

Figure 2: Optimal wholesale prices x_1^* for different confidence level α.

5. Conclusions

With the growing emphasis on globalization, suppliers in two-stage supply chains encounter competitions from counterparts that provide the same products or services. A lower wholesale price certainly can attract customers, but it apparently reduces the profits of the suppliers. Thus, how to decide a wholesale price to coordinate/balance the two aspects—loss and risk—is very important. In this paper, we investigate the optimal pricing strategies of the suppliers in competitive circumstances. We introduce a new loss function for the suppliers, which considers the influence of wholesale prices to both the current loss and the future loss. Some optimal wholesale prices under different objectives are obtained. Further, the properties of these optimal wholesale prices are also studied. We think this paper provides some help for suppliers in deciding the wholesale price for their products.

Several extensions of this paper are possible. A further research is to consider the case where the order quantity of the retailer is wholesale price dependent. In such, the decision of an incumbent supplier as to wholesale price also influences the order quantity of the retailer, which has a more significant impact on his/her present and long-term profits.

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Research Article

Personal Recommendation Using a Novel Collaborative Filtering Algorithm in Customer Relationship Management

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With the rapid development of customer relationship management, more and more user recommendation technologies are used to enhance the customer satisfaction. Although there are many good recommendation algorithms, it is still a challenge to increase the accuracy and diversity of these algorithms to fulfill users' preferences. In this paper, we construct a user recommendation model containing a new method to compute the similarities among users on bipartite networks. Different from other standard similarities, we consider the influence of each object node including popular degree, preference degree, and trust relationship. Substituting these new definitions of similarity for the standard cosine similarity, we propose a modified collaborative filtering algorithm based on multifactors (CF-M). Detailed experimental analysis on two benchmark datasets shows that the CF-M is of high accuracy and also generates more diversity.

1. Introduction

With the great development of enterprise informatization, customer relationship management (CRM) has been an indispensable part of supply chain. More and more entrepreneurs and scholars generate the growing interests in applications of CRM. This trend is partly attributable to the availability of an overwhelming amount of customer transaction data and the necessary data-mining tools to obtain managerially useful insights [1]. CRM is a model for managing company's interactions with current and future customers. It aims to maximize the benefits gained from relationships with customers and enhance the enterprises competitive power. The most important expected outcomes of CRM can be listed as follows[2]: improvements in efficiency, cost reduction, improved profitability, increases in sales, enhanced customer value, customer satisfaction, and improved customer loyalty.

Nowadays, customer satisfaction becomes more crucial among researchers and practitioners alike. How to enhance the customer satisfaction, there are many approaches such as setting lower price and better quality of products, providing better service for customers. Specifically, with the mushroomed development of E-commerce applications, the size and complexity of business websites grow rapidly. For the users of these websites it becomes increasingly difficult and time consuming to find the information or products they are looking for. As a consequence, how to efficiently help users filter out the unwanted information and find what is really useful for them is a challenging problem for customer service. Recommendation technologies are used to provide individual marketing decisions for each user. The main task of them is to recommend good products to users, and their performance metric is the number of recommendations made to users until good products are recommended, as well as the number of users that are eventually satisfied. Crucially, they do not require detailed keywords provided by users. Instead, they use the users' historical activities and possible personal profiles to uncover their preferences or potential interests. Today, some good recommendation technologies have been used to recommend books and CDs at Amazon.com, movies at Netflix.com, and news at VERSIFI Technologies [3].

With the development of the recommendation technologies, various kinds of approaches are proposed, including collaborative filtering (CF) [4, 5], content-based filtering [6, 7],

K-Nearest Neighbor (K-NN) [8–11], diffusion approach [12–14], and spectral analysis [15, 16]. CF is a class of information filtering technique which can predict what users will like according to their similarity to other users based on collecting and analyzing a large amount of information on users' behaviors, activities, or preferences. Content-based filtering method selects items based on the correlation between the content of the items and the users' preferences as opposed to a collaborative filtering system that chooses items based on the correlation between people with similar preferences. K-NN is one of the most commonly used algorithms for classifying objects based on the properties of its closest neighbors in the feature space. In K-NN, an object is classified through a majority vote of its neighbors, with the object being assigned to the class most common amongst its k nearest neighbors. Diffusion approach applies the three-step mass diffusion starting from the target user on a user-object bipartite network. Spectral analysis is a new recommendation algorithm that relies on the singular value decomposition (SVD) of the rating matrix.

Due to the fact that the CF method has been the most widely and successfully used in many applications, more and more scholars devote themselves to improve this technology. In this paper, we construct a user recommendation model and present a novel method to compute the similarities among users on bipartite network. Comparing the proposed method with other standard similarity computation methods, the advantage of our method is that it takes into account the influence of each object node including popular degree, preference degree, and trust relationship. Detailed numerical analysis on two benchmark datasets, *MovieLens* and *Book-Crossing*, indicates that our modified collaborative filtering based on multifactors (CF-M) outperforms other algorithms. Specifically, it can not only provide more accurate recommendations, but also generate more diverse recommendations by precisely recommending less popular objects.

The main contribution of this paper is that we provide a better recommendation algorithm to improve the quality of individual customer service. This proposed algorithm meets high accuracy and certain diversity and also can solve the problems existed in the current CF algorithms which consider little factors of objects in the process of similarity computation. Through detailed experiment, the proposed algorithm demonstrates its superiority. A review of related work is given in Section 2. In Section 3, our recommendation model and modified collaborative filtering algorithm based on multifactors (CF-M) are described. Section 4 provides experimental results and analysis of the CF-M algorithm on two benchmark datasets. Finally, we draw conclusions in Section 5.

2. Related Work

Collaborative filtering is the most widely used technique to produce user-specific recommendations of items based on patterns of ratings or usage without need for exogenous information about either items or users. Since the method was proposed, many scholars attempt to improve it to enhance the quality of recommendation results. As a result, it generates lots of modified CF algorithms.

Liu et al. [17] proposed a novel method to compute the similarity between congeneric nodes on bipartite network. They considered the influence of a node's degree and then presented a modified collaborative filtering (MCF) to substitute the standard cosine similarity. Yang et al. [18] proposed an approach based on the fact that any two users might have some common interest genres as well as different ones. Different from most existing methods, this approach introduced a more reasonable similarity measure metric, considering users' preferences and rating patterns. Zhao et al. [19] presented a shared collaborative filtering approach to alleviate the sparse problem. The proposed approach leveraged the data from other parties to improve CF performance and did not compromise the privacy of other parties. Bobadilla et al. [20] provided a detailed formulation of the method proposed and an extensive set of experiments and comparative results which showed the superiority of designed collaborative filtering compared to traditional collaborative filtering in (a) the number of recommendations obtained, (b) quality of the predictions, and (c) quality of the recommendations. Liu et al. [21] proposed a sequence-based trust model based on users' sequences of ratings on documents. The model considered two factors in computing the trustworthiness of users. It also enhanced the similarity of user profiles and was incorporated into a standard collaborative filtering method to discover trustworthy neighbors for making predictions. Kim et al. [22] proposed a collaborative approach to user modeling for enhancing personalized recommendations to users. Their approach first discovered some useful and meaningful user patterns and then enriched the personal model with collaboration from other similar users. López-Nores et al. [23] presented a new strategy called property-based collaborative filtering (PBCF) to address problems of recommender systems by introducing a new filtering strategy, centered on the properties that characterized the items and the users. Tsai and Hung [24] assessed the applicability of cluster ensembles to collaborative filtering recommendation. They used two well-known clustering techniques and three ensemble methods. The experimental results based on the *MovieLens* dataset showed that cluster ensembles could provide better recommendation performance than single clustering techniques in terms of recommendation accuracy and precision. Choi et al. [25] proposed a hybrid online-product recommendation method combining implicit rating-based collaborative filtering and sequential pattern analysis. They considered the objective of their research by two ways: one was to derive implicit ratings so that CF could be applied to online transaction data even when no explicit rating information was available, and the other was to integrate CF and SPA for improving recommendation quality. Dao et al. [26] proposed a new recommendation model called Context-Aware Collaborative Filtering using genetic algorithm (CACF-GA) for location-based advertising (LBA) based on both users' preferences and interaction's context. They first defined discrete contexts and then applied the concept of context similarity to conventional CF to create the context-aware recommendation model. Eckhardt [27]

proposed a collaborative filtering model which could provide clear information about preferences and then used this model as user similarity measure instead of traditional ratings-based similarity. Kant and Bharadwaj [28] developed an effective content-based filtering (CBF) by introducing an item representation scheme and fuzzy similarity measures and incorporating collaborative diverse predictions for alleviating its recommendation diversity. Lai et al. [29] proposed a hybrid personal trust model which adaptively combined the rating-based trust model and explicit trust metric to resolve the drawback caused by insufficient past rating records; after that, they presented a recommendation method based on a hybrid model of personal and group trust to improve recommendation performance. Choi and Suh [30] proposed a new similarity function in order to select different neighbors for each different target item. In the new similarity function, the rating of a user on an item was weighted by the item similarity between the item and the target item.

The target of our work is to construct a recommendation model containing an effective method to give users high accuracy and certain diversity recommendation results and also improve the quality of customer service. Finally, the experimental results show that our method is better than many other recommendation algorithms. In addition, our research result can be applied to CRM improvement or electronic commerce construction.

3. Modified Collaborative Filtering Recommendation Model

In this section, we introduce the similarity computation of the traditional collaborative filtering algorithm and then construct an effective recommendation model and derive out our modified collaborative filtering based on multifactors (CF-M).

Figure 1 shows the framework of our proposed recommendation model. In this model, we use a new similarity computation method considering more factors to compute the similarity between target user and other users and then obtain the objects collected by the similar users but not by the target user. Finally, we generate a recommendation list made up of these objects and then recommend them to the target user.

There are several phases in this framework.

(1) *Data Preprocessing*. Data preprocessing is an important step in recommendation model. As far as we know, data-gathering methods are often loosely controlled, resulting in out-of-range values, impossible data combinations, missing values, and so forth. Analyzing data that has not been carefully screened for such problems can produce misleading results. So, in this phase, we process the primary data to fulfill the requirement of the recommendation method before similarity computation. For example, we normalize the users' ratings in order that our proposed recommendation method can compute these data directly. Since the existence of some implicit evaluations does not indicate users' preferences but meanwhile they cannot be ignored as well, we assign some suitable values to them.

(2) *Similarity Computation*. This phase is the key procedure. First, we give some definitions. We assume that there is a recommendation model which consists of m users and n objects, and each user has selected some objects. The relationship between users and objects can be described by a bipartite network. Let $U = \{u_1, u_2, \ldots, u_m\}$ denote users set and $O = \{o_1, o_2, \ldots, o_n\}$ denote objects set; the recommendation model can be fully described by an $m \times n$ adjacency matrix $A = \{a_{ij}\}$, where $a_{ij} = 1$ when object j is selected by user i; otherwise, $a_{ij} = 0$. After that we use CF-M to compute the similarity between two users. The detailed process of this algorithm will be described in Section 3.2.

(3) *Recommendation*. In the previous phase, we use the CF-M algorithm to compute the similarity between target user and others based on the influence of each object node including popular degree, preference degree, and trust relationship.

In this step, we calculate the comprehensive preference degree of each product unselected by the target user. Finally, the products with high comprehensive preference degree are used to compile a recommendation list in descending order. At last we recommend top L products to the target user. In general, the number L is no more than 100.

3.1. Similarity Computation of Traditional Collaborative Filtering. Traditional collaborative filtering method usually adopts the standard cosine similarity or Pearson correlation to compute the similarity between two users. For arbitrary users u_i and u_j, the number of common objects shared by them can be expressed as

$$c_{ij} = \sum_{l=1}^{n} a_{li} a_{lj}. \tag{1}$$

Generally, for standard cosine similarity computation, let s_{ij} denote the similarity between u_i and u_j and let $k(u_i)/k(u_j)$ denote the degree of the user u_i/u_j; namely, how many objects are collected by this user? So we can formulate the expression as

$$s_{ij} = \frac{c_{ij}}{\sqrt{k(u_i)\,k(u_j)}} = \frac{\sum_{l=1}^{n} a_{li} a_{lj}}{\sqrt{k(u_i)\,k(u_j)}}. \tag{2}$$

The problem of (2) is that it has not taken into account the influence of an object's degree, so that objects with different degrees have the same contribution to the similarity. If users u_i and u_j both have selected object o_l, then they have a similar preference for object o_l.

3.2. Modified Collaborative Filtering Recommendation Method. As we know, in real recommender system, the similarity computation between two users is not simple but influenced by many factors. So we need to improve the traditional

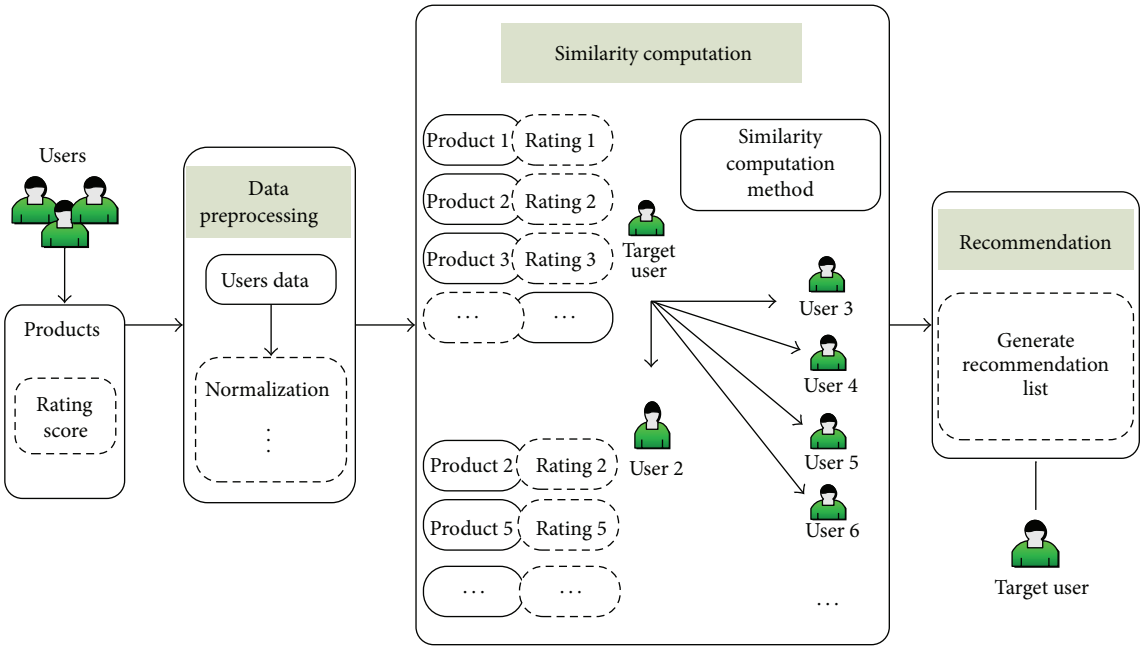

Figure 1: The framework of recommendation model based on CF-M.

collaborative filtering method in order to fit the complex conditions. Generally, the similarity between two users should be somewhat relative to their degrees, preference degree, and trust relationship. According to these features, we propose a modified collaborative filtering algorithm based on multifactors (CF-M) including all factors mentioned previously to increase the accuracy and certain diversity of the recommendation results.

Through analyzing these factors, we can make the conclusion that each object node's degree and preference degree are related to its popular degree and corresponding users' comments or ratings, respectively. For preference degree computation, we need to make quantification of them when encountering users' comments and then distinguish the degrees of preferences. In other cases, we can divide the degrees of preferences directly. Trust relationship is derived from two users' past ratings on corated products by adopting Hwang and Chen's [31] trust computation method. In other words, trust relationship relates to users' evaluations or scores. Generally, a recommender is more trustworthy if he or she has contributed more precise predictions than other users do.

We assume that the similarity computation on user-object bipartite networks is affected by an influence degree that is proportional to $[(1-(|v_{li}-v_{lj}|/M))/k(o_l)]^{\alpha}$, with α being a freely adjustable parameter. Accordingly, the contribution of object o_l to the similarity s_{ij} should be negatively correlated with its degree $k(o_l)$ and positively correlated with its preference degree and trust relationship. It means that it is not very meaningful if two users both select a popular object, while if a very unpopular object is simultaneously selected by two users, there must be some common tastes shared by these two users.

So we suppose that the object o_l contribution to s_{ij} is inversely proportional to $k(o_l)$ and directly proportional to $|v_{li}-v_{lj}|$. The formulation of s_{ij} can be expressed as

$$s_{ij}=\frac{1}{\sqrt{k(u_i)\,k(u_j)}}\sum_{l=1}^{n}a_{li}a_{lj}\left[\frac{\left(1-\left(\left|v_{li}-v_{lj}\right|/M\right)\right)}{k(o_l)}\right]^{\alpha}. \quad (3)$$

M is the range of the rating score, which equals the difference of the maximum and minimum rating scores. v_{li}/v_{lj} represents the preference degree that object o_l obtained from user u_i/u_j. $k(o_l)$ denotes the degree of object o_l; namely, how many users select this object? $k(u_i)$ denotes the degree of the user u_i; namely, how many objects are selected by this user?

Despite the users similarity computation, we can find the products unselected by target user but selected by test users who have much similarity to target user. Then we may predict the comprehensive preference degree of these products. Let p_{ij} represent the comprehensive preference degree of object o_j obtained from target user u_i. The formulation of p_{ij} can be expressed as

$$p_{ij}=\sum_{l=1,l\neq i}^{m}s_{il}a_{jl}. \quad (4)$$

In the process of recommendation, we get the elements of p_{ij} uncollected by target user and then sort them in descending order, as target user prefers the objects in the top, so we recommend top L objects to this user.

The pseudocode of the modified collaborative filtering algorithm based on multifactors (CF-M) algorithm is shown in Pseudocode 1.

```
Algorithm CF-M: Calculating the similarity between users
begin
   get A;
   n = size(A, 2), m = size(A, 1);
   parameter ar;
   preference degree v();
   range of the rating score M;
   S = zeros(m, m), o = sum(A), u = sum(A');
   for i = 1 : m
      for j = 1 : m
         x = (u(i) * u(j)) ∧ (−0.5);
         for z = 1 : n
            y = y + a(i, z) * a(j, z) * ((1 − abs(v(i, z) − v(j, z))/M)/o(z)) ∧ ar,
         end
         S(i, j) = x * y;
         x, y = 0;
      end
   end
End
```

Pseudocode 1

3.3. Recommendation Performance Metrics. In this paper, we adopt some standard metrics to measure the accuracy and diversity of the proposed method, in which accuracy is the most important aspect in evaluating the recommendation algorithmic performance.

Five metrics: ranking score, precision, recall, intrasimilarity, and Hamming distance. The first three are used to test accuracy and the rest are used to test diversity. The detailed descriptions of these metrics are as follows.

(1) Ranking score is used for an arbitrary user u_i if the recommendation o_j is in the test set (according to the training set, o_j is an unselected object for u_i) and ranked in R_{ij} position in the ordered recommendation list L_i. We can formulate the expression as $r_{ij} = R_{ij}/L_i$. For example, if the length of L is 200, namely, there are 200 unselected objects for u_i and o_j is the 10th from the top, we say that the position of o_j is 10/200, denoted by $r_{ij} = 0.05$. The average of r_{ij} of the overall user-object pairs in the test set defines the average ranking score $\langle r \rangle$, which can be used to evaluate the algorithmic accuracy. The smaller the ranking score is, the higher the algorithmic accuracy is.

(2) Precision is defined as the ratio of the number of recommended objects collected by users appearing in the test set to the total number of recommended objects. This measure is used to evaluate the effectiveness of a given recommendation list. The precision can be formulated as a/L, in which a represents the number of recommended products collected by users appearing in test set and L is the total number of recommended products.

(3) Recall is defined as the ratio of the number of recommended objects collected by users appearing in the test set to the total number of objects actually collected by users. The larger recall corresponds to the better performance. The recall can be formulated as a/M, in which a represents the number of recommended products collected by users appearing in test set and M is the total number of users' actual buying.

(4) Intrasimilarity evaluates the similarities between objects inside users' recommendation lists. A good recommendation algorithm is expected to give fruitful recommendation results and has the ability to guide or help the users to exploit their potential interest fields. Therefore, it calls for a lower intrasimilarity. There are many similarity metrics between objects. Here we adopt the widely used one, that is, cosine similarity to measure objects' similarity. For two objects o_t and o_k, their similarity is defined as

$$S_{tk} = \frac{1}{\sqrt{k(o_t)\,k(o_k)}} \sum_{l=1}^{m} a_{tl} a_{kl}. \tag{5}$$

For an arbitrary user u_l, the number of recommendation objects is L. Firstly, we need to calculate $L(L-1)/2$ couple of objects' similarity and then average these values to get $I_l = \langle S_{tk} \rangle$. Finally, we use the mean value of I of the overall users to measure the diversity in recommendation lists.

(5) Hamming distance can measure the strength of personalization. If the overlapped number of objects in u_i and u_j's recommendation lists is Q, their Hamming distance is

$$H_{ij} = 1 - \frac{Q}{L}. \tag{6}$$

Generally speaking, a more personalized recommendation list should have long Hamming distances to other lists. Accordingly, we use the mean value

of Hamming distance $S = \langle H_{ij} \rangle$, averaged over all the user-user pairs, to measure the strength of personalization.

4. Experimental Results and Analysis

To test the recommendation algorithmic performance, we use two benchmark datasets. The *MovieLens* [32] dataset consists of 1682 movies and 943 users. Each user has rated at least 20 movies by using a discrete number in the scale of 1 to 5. The original data contains 100,000 ratings. In the dataset, there are three kinds of information tables: demographic information about the users, information about the items (movies), and the score about the movies. The *Book-Crossing* dataset [33] contains 278,858 users (anonymized but with demographic information) providing 1,149,780 ratings (explicit/implicit) about 271,379 books. Ratings (Book-Rating) are either explicit, expressed on a discrete number in the scale of 1 to 10 (higher values denoting higher appreciation), or implicit, expressed by 0.

In our experiment, we need to preprocess the datasets. For *MovieLens*, only the links with ratings no less than 3 are considered and $v_{li} = \{3, 4, 5\}$. For *Book-Crossing*, only the links with ratings no less than 5 or equal to 0 are considered and $v_{li} = \{10, 9, 8, 7, 6, 5, 1\}$. We divide each processed dataset into two parts: the training set which contains 80% of the data and the remaining 20% of the data for the test.

Firstly, we need to predict the range of the optimal values of α in order to reduce the computational costs in determining the optimal value through our approach. According to some works in the literature on CF approaches, we predict that our optimal values of α are located in the range of 1 to 2. To find the optimal value of parameter α rapidly, we execute the iterative computation based on the strategy of binary search. In the process of iterative computation, we set the interval between α as 0.01. All these computational definitions and steps lead to lower computational costs. Figure 2 shows the algorithmic accuracy, measured by the ranking score, as a function of α. We note that for the two benchmark datasets the best performance of this algorithm occurs around $\alpha = 1.86$. Certainly, people can adjust the parameter's value by themselves in practice.

Recall and precision can be used to realize the balance of two competitive factors: cost and efficiency. The efficiency can be improved by increasing the number of recommended products; however, the cost is increasing at the same time. That is to say, the cost can be decreased by reducing the recommendations, while the efficiency may be decreased correspondingly. At a certain length of recommendation list L, precision tests whether the cost is deserved or necessary, while recall tests whether the efficiency is sufficient. Based on these two measures, one can find a certain L as a tradeoff for cost and efficiency. In general, the number of recommended L is no more than 100.

Figure 3 shows the precision and recall in different value of parameter α. What we can know from this figure is that the algorithm reaches the highest precision and maximum recall when the parameter $\alpha = 1.86$ for the *MovieLens* dataset.

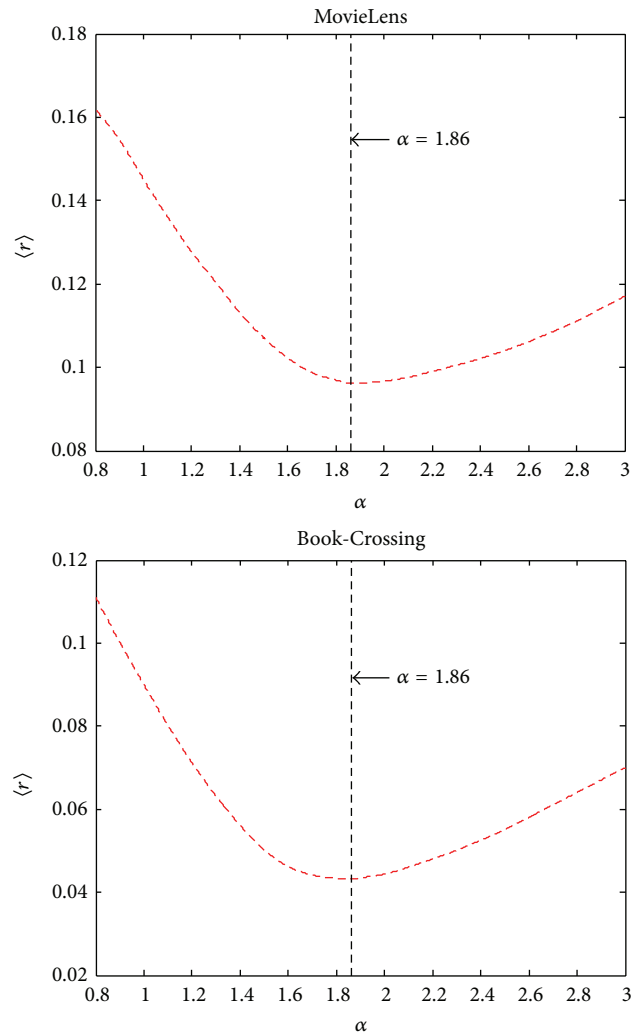

Figure 2: The effect of parameter α on this algorithmic accuracy. The optimal parameter α for *MovieLens* and *Book-Crossing*, corresponding to the minimal $\langle r \rangle$, is equal to 1.86. Recommendation number $L = 50$. Present results are obtained by averaging over five independent 80% versus 20% divisions.

In addition, the precision reaches a good level if $\alpha \in (1.84, 1.88)$ and so does the recall.

For *Book-Crossing* dataset, what we can know from Figure 4 is that the algorithm reaches the highest precision and maximum recall when the parameter α is also equal to 1.86. Furthermore, the precision and recall reache a good level if $\alpha \in (1.84, 1.88)$.

After that, we compare CF-M with three other widely used recommendation algorithms: CF, MCF, and NBI [34] in all five metrics. Different from CF and MCF, NBI is a diffusion algorithm based on homogeneous diffusion process on networks; that is, each object distributes its resource to its neighbors equally. In addition, it has been demonstrated to be more accurate than the classical CF algorithm, with lower computational complexity. We summarize the algorithmic performance in Table 1 for *MovieLens* and Table 2 for *Book-Crossing*.

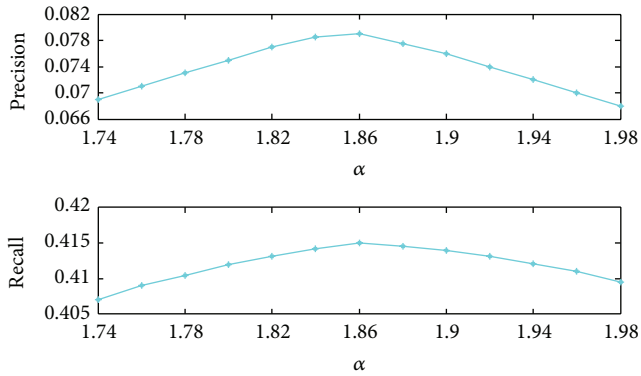

Figure 3: The effect of parameter α on CF-M with recommendation number $L = 50$. For *MovieLens*, the precision is at its highest at about $\alpha = 1.86$, at almost the same point where the recall achieves its maximum. Present results are obtained by averaging over four independent 80% versus 20% divisions.

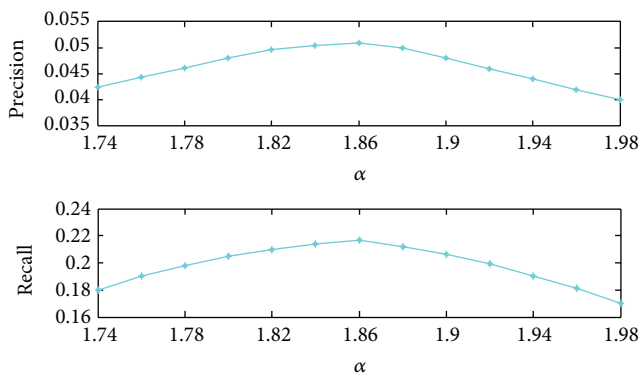

Figure 4: The effect of parameter α on CF-M with recommendation number $L = 50$. For *Book-Crossing*, the precision is at its highest at about $\alpha = 1.86$, at almost the same point where the recall achieves its maximum. Present results are obtained by averaging over four independent 80% versus 20% divisions.

Comparing CF-M with the standard CF, as is seen in Table 1 in the condition of recommendation number $L = 50$, the ranking score can be further reduced by 23.2%, and with MCF the ranking score can be reduced by 12.7%. Similarly, our algorithm has lower ranking score than NBI algorithms. For the rest of metrics, our algorithm is also the best. Although *Book-Crossing* dataset is similar to *MovieLens*, it is much sparse. So we set the number of recommended products no less than 50.

Table 2 shows that our algorithm exceeds the other three algorithms in all the five criterions: lower ranking score, higher precision, bigger recall, lower intrasimilarity, and larger Hamming distance.

CF-M algorithm adjusts the accuracy via parameter α. When $\alpha < 0$, the comprehensive preference degree of each product is inversely proportional to $|v_{li} - v_{lj}|$ and directly proportional to $k(o_l)$. Thus, the algorithm tends to recommend popular products to users. But it is not what people want. On the other hand, when $\alpha > 0$, the comprehensive preference degree of each product is inversely proportional to $k(o_l)$ and directly proportional to $|v_{li} - v_{lj}|$. In this case, the algorithm tends to recommend unpopular products to users. The experimental results show that it is more suitable

Table 1: Algorithmic performance for *MovieLens* dataset. The ranking score, precision, recall, intrasimilarity, and hamming distance are corresponding to $L = 30, 40, 50$. And the value of parameter α is 1.86. Each number presented in this table is obtained by averaging over five runs, each of which has an independently random division of training set and test.

Algorithms	Ranking score	Precision	Recall	Intrasimilarity	Hamming distance
$L = 30$					
CF	0.148	0.077	0.321	0.316	0.704
MCF	0.131	0.087	0.360	0.304	0.751
NBI	0.120	0.089	0.379	0.291	0.778
CF-M	0.116	0.097	0.387	0.275	0.798
$L = 40$					
CF	0.137	0.071	0.332	0.328	0.698
MCF	0.121	0.080	0.373	0.317	0.743
NBI	0.112	0.081	0.392	0.303	0.771
CF-M	0.107	0.088	0.401	0.286	0.790
$L = 50$					
CF	0.125	0.066	0.343	0.342	0.692
MCF	0.110	0.072	0.385	0.330	0.735
NBI	0.101	0.073	0.406	0.315	0.762
CF-M	0.096	0.079	0.415	0.297	0.781

Table 2: Algorithmic performance for *Book-Crossing* dataset. The ranking score, precision, recall, intrasimilarity, and hamming distance are corresponding to $L = 50, 60, 70$. And the value of parameter α is 1.86. Each number presented in this table is obtained by averaging over five runs, each of which has an independently random division of training set and test.

Algorithms	Ranking score	Precision	Recall	Intrasimilarity	Hamming distance
$L = 50$					
CF	0.056	0.039	0.181	0.374	0.519
MCF	0.052	0.044	0.192	0.338	0.547
NBI	0.049	0.047	0.204	0.316	0.613
CF-M	0.043	0.051	0.217	0.263	0.764
$L = 60$					
CF	0.046	0.037	0.201	0.395	0.511
MCF	0.044	0.042	0.214	0.352	0.536
NBI	0.042	0.045	0.233	0.331	0.604
CF-M	0.037	0.048	0.248	0.278	0.752
$L = 70$					
CF	0.042	0.032	0.228	0.421	0.497
MCF	0.040	0.036	0.243	0.375	0.522
NBI	0.034	0.041	0.262	0.349	0.587
CF-M	0.030	0.044	0.296	0.293	0.731

to recommend unpopular and reliable products to users in fact. Finally, for an online recommender system, we need to consider the processing time and memory consumption of its recommendation algorithm. If we denote $\langle k_u \rangle$ and $\langle k_o \rangle$ by the average degree of users and objects on the bipartite network, the computational complexity of CF-M is $O(m^2 \langle k_u \rangle + mn\langle k_o \rangle)$ and the memory store is m^2, which is the same as CF and MCF. For NBI algorithm, its computational complexity is $O(m\langle k_u^2 \rangle + mn\langle k_u \rangle)$ and the memory store is n^2. When the number of objects is much larger than the number of users, the CF-M may be more practicable; otherwise NBI may be more practicable. Although the accuracy and diversity of the proposed method CF-M are increased, it is still confronted with the cold-start problem.

5. Conclusions

Recommendation model predicts users' potential future likes and interests by using users' past preferences data. An excellent recommendation algorithm meets high accuracy and certain diversity and can enhance the quality of personalized service. Since the collaborative filtering approach was proposed, it has attracted much attention for its convenience, high accuracy, and certain diversity as well as low computational complexity.

In this paper, we construct an effective recommendation model in order to improve the current recommender system for better customer service. We analyze the collaborative filtering algorithm and propose a modified one based on multiple influence factors. We compute the similarity between two users on bipartite network. Comparing the proposed method with other standard similarity computation methods, the key feature or superiority is that our method takes into account the influence of each object node including popular degree, preference degree, and trust relationship. All the factors are governed by a parameter which is derived from optimal value calculation. Certainly, people can adjust the parameter's value by actual requirement. Detailed numerical analysis on two benchmark datasets, *MovieLens* and *Book-Crossing*, indicates that the presented algorithm is of high accuracy and also generates certain diversity.

Concerning future work, we will improve our recommendation model and pay more attention to algorithmic structure. The research covers the following aspects: how to compute the similarity while engaging in the user context factors; how to introduce other technologies to improve the accuracy and diversity; how will the recommendation algorithm keep its robustness when meeting hostile attack; and how to alleviate the influence of the cold-start problem.

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