# Financial Networks
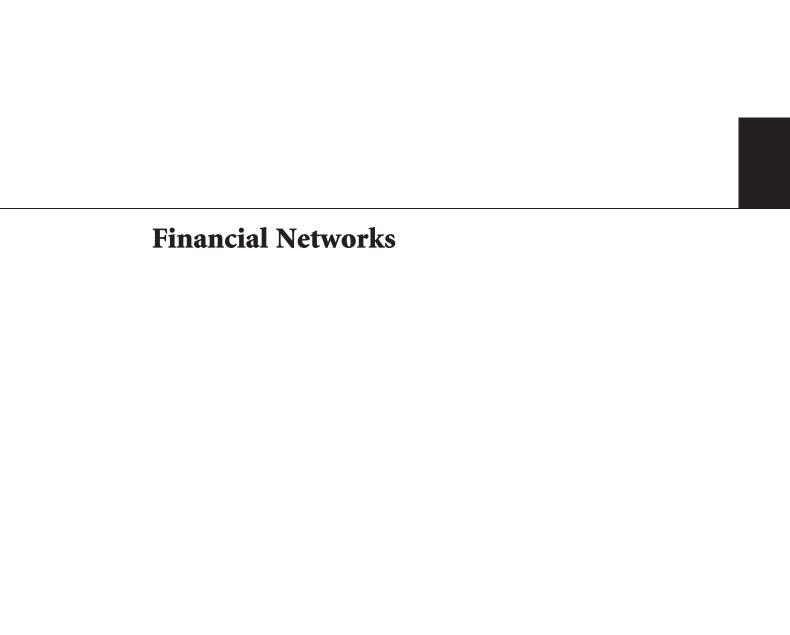
Lead Guest Editor: Benjamin M. Tabak
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# Editorial Board

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*Editorial*

# Financial Networks

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Financial networks have been on the research agenda since the financial crisis of 2008. Today, both regulators and the academia recognize that interconnectedness is a crucial component that had a key role in the amplification of losses in the last crisis. Therefore, understanding the structure of financial networks is important for assessing, monitoring, and regulating financial systems. In addition, it washed away the belief that supervising banks in an individual manner was sufficient to guarantee a safe financial system, as networks can largely amplify negative spillover effects. In this sense, we have seen an increasing effort in designing novel mechanisms for macroprudential regulation that include overseeing aspects of the entire financial system, including its structure.

Though understanding how financial networks amplify shocks is of uttermost importance for policymakers, especially for financial stability and systemic risk issues, the literature is still at its early stages in understanding the role of financial networks as a medium for shock amplification. This mainly occurs because modern financial networks are inherently complex to analyze as economic agents participate in a multiplex of interrelationships in several different markets.

Modeling this heterogeneity of interconnections stands as an important open problem because each connection can potentially create contagion transmission channels that can amplify losses. Another component that further increases the modeling complexity is that each risk channel that arises from this multiplex of interconnections is potentially dependent on each other and thus can additively increase systemic risk in nonlinear ways.

Complex networks evolve rapidly overtime and their topology changes substantially. Understanding this evolution and its impact on systemic risk and financial stability is an important research question. There are many gaps in the literature and we hope to address some of them within this special issue.

The paper by K. Khashanah and T. Alsulaiman introduces an agent-based approach. They contribute to the literature by developing a metamodel of markets that leads to a financial economic environment simulator. They find that the simulated market is driven to instability in a similar manner to patterns observed in a crisis where all agents become homogeneous in information awareness of (negative) events.

The paper by S. Li and S. Wen studies a multiplex network of the guarantee market, which has three layers (different type of guarantee relationships). They provide empirical evidence that central companies in one layer are not necessarily central in another layer. The study contributes to the discussion on how to model multiplex networks.

The paper by C. M. Fan et al. provides a theoretical framework to manage the systemic risk of the banking system in Nigeria based on macroprudential capital requirements, which requires banks to hold capital that is proportional to their contribution to systemic risk. They find that, despite the heterogeneity in macroprudential capital requirements, all risk allocation mechanisms bring a substantial decrease in the systemic risk. The risk allocation mechanism based on $\Delta$CoVaR decreases the average default probability the most.

The paper by L. He and S. Li investigates network entropy of dynamic banking systems. They find that network entropy is positively correlated with the effect of systemic risk in the three kinds of interbank networks and that network entropy

in the small-world network is the largest, followed by those in the random network and the scale-free network.

The paper by H. Fan et al. proposes a theoretical framework to reveal the time evolution of the systemic risk using sequences of financial data and uses the framework to assess the systemic risk of the Kenyan banking system. They study the case of Kenya and find several banks displaying characteristics of systemically important banks (SIB) and a case in which highly unusual interconnectedness may lead to contagion defaults.

The paper by M. Pollak and Y. Guan studies a calibrated network model of heterogeneous interbank exposures. Their results show that trends in capital buffers and the distribution and type of assets have a significant effect on the predictions of financial network contagion models and that the rising trend in ownership of banks by banks amplifies shocks to the financial system.

The paper by J. Eberhard et al. studies how changes in the structure of a brokers' transaction network affect the probability with which the returns and volume of the traded financial assets change significantly. They find that changes of this structure are significantly correlated with variables that describe the local and international economic-financial environments.

The paper by D. Csercsik and H. J. Kiss introduces a new discrete time probabilistic model of depositor behavior, which takes into account the information flow among depositors. They study the role of connections and use variants of a simple example. They find in these examples that connections matter. The more connections the depositor has, the larger the optimal offer from the bank is. Consequently, the denser the connection structure is, the larger the expected payment of the bank to the depositors is.

The paper by I. Anagnostou et al. contributes to the portfolio credit risk literature by introducing a portfolio credit risk model which incorporates both common factors and contagion. They also use a credit stress propagation network constructed from real data to quantify the impact of deterioration of credit quality of the sovereigns on corporates. And they present the impact of accounting for contagion which can be useful for banks and regulators to quantify credit, model, or concentration risk in their portfolios.

The paper by T. Chen introduces an evolving network model of credit risk contagion in the credit risk transfer market. The model contributes to the explicit investigation of the connection between the factors of market behavior and network structure and also provides a theoretical framework for considering credit risk contagion in an evolving network context.

The paper by L. Rossoni et al. shows that the financial cost of capital of companies listed on the Brazilian stock exchange is determined by the social capital of the networks of directors and shareholders they have. The paper contributes to the field of board interlocking and corporate governance by studying the analysis of the social capital of the board and providing empirical evidence of how interlocking influences the cost of capital of these companies.

The paper by Z. Zhang et al. studies dynamic cross-correlations between mass media news, new media news, and stock returns. They provide evidence of the existence of power-law correlations between two types of news and between news and stock returns. They also find a general increasing trend for the cross-correlation between the two types of news and between news and stock returns.

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*Benjamin Miranda Tabak*
*Thiago Christiano Silva*
*Ahmet Sensoy*

*Research Article*

# Optimal Payments to Connected Depositors in Turbulent Times: A Markov Chain Approach

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We propose a discrete time probabilistic model of depositor behavior which takes into account the information flow among depositors. In each time period each depositors' current state is determined in a stochastic way, based on their previous state, the state of other connected depositors, and the strategy of the bank. The bank offers payment to impatient depositors (those who want to withdraw their funds) who accept or decline them with certain probability, depending on the offered amount. Our principal aim is to see what are the optimal offers of the bank if it wants to keep the expected chance of a bank run under a certain level and minimize its expected payments, while taking into account the connection structure of the depositors. We show that in the case of the proposed model this question results in a nonlinear optimization problem with nonlinear constraints and that the method is capable of accounting for time-varying resource limits of the bank. Optimal offers increase (a) in the degree of the depositor, (b) in the probability of being hit by a liquidity shock, and (c) in the effect of a neighboring impatient depositor.

## 1. Introduction

Banks and other financial intermediaries convert short-term liabilities into long-term and often illiquid assets, a process called maturity transformation. Liquidating the assets is generally costly; hence if many depositors or investors attempt to withdraw their funds from the bank or from other forms of financial intermediation, then liquidity problems may arise that may spark a bank run and result in solvency problems through fire sales. If depositors anticipate such potential problems, then it may in turn make them more prone to withdraw. Moreover, during financial crisis it is even more likely that depositors are concerned about the liquidity and solvency of their bank, making bank runs more probable also.

In traditional bank run models [1, 2] banks are supposed to determine payment to those who withdraw as a result of a maximization problem. The bank maximizes the overall expected utility of its depositors. Depending on the specified environment, these models either allow a bank run outcome [1] or do not [2]. In this paper we take a different approach.

In times of crises, it makes sense to assume that the most important objective of the bank is to survive. More precisely, it wants to keep the probability of a potentially devastating bank run very low at a minimum cost. The bank's intention to minimize the cost (in our case the payments to depositors) in times of financial distress is due to the uncertainty about the duration of the crisis and unforeseeable contingencies. Hence, the bank wants to keep as much funds available as possible to be prepared for future potential difficulties. However, it aims also to pay to those who withdraw smoothly so that rumours about problems of receiving a payment from the bank do not set off a bank run.

Our main aim is to understand the optimal payments of the bank during crises when the bank wants to minimize payments but also maintain sufficiently low the probability of bank runs given the connections between depositors. Unlike in other models, depositors' decision is assumed to be determined in the following way. Each bank customer starts as a depositor without urgent liquidity needs (i.e., they are patient in bank run parlance). However, in any period

each depositor may be hit by a liquidity shock, turning a patient depositor into an impatient one. Impatient depositors have immediate liquidity needs so they withdraw money from the bank as soon as they can. When they contact the bank to withdraw money, the bank makes them an offer that they can accept or reject. The probability of accepting the offer depends on the amount that is being offered. The larger the amount, the more likely that the depositor accepts it. Accepting or not does not only affect the bank and the depositor in question, but may have an additional effect. Depositors are connected by an underlying social network and if a patient depositor is connected to an impatient depositor, then the probability to become impatient increases. A possible explanation is that if a depositor notes that the number of those who want to withdraw from the bank increases, then she may interpret it as people trying to get out their funds due to some problem with the bank. In such a case it may be optimal to withdraw as well, because if a depositor waits too long while the rest withdraws, then she may have problems to recover her funds. Once an impatient depositor accepts the offer from the bank, she ceases to be impatient and her effect on other depositors disappears as well. We define a bank run as a situation in which there is no more patient depositor because everybody wants to withdraw or has done so already. We compute how the payments to depositors who want to withdraw should be so as to minimize these payments, but also to keep the probability of a bank run low, taking into account how depositors affect each other through the underlying social network.

Even though bank run models differ in important points (e.g., whether there is aggregate uncertainty about the liquidity needs), almost all models have multiple equilibria, one of which involves bank runs. This paper also admits bank runs as the bank sets payments in a way that the probability of runs is reduced but need not be zero. However, we do not have equilibria as depositors do not make strategic decisions. Depositors in our model react as an automaton to what happens around them, in a nonstrategic way. Our focus is on how banks determine payment optimally in such an environment. We show that the problem can be neatly formalized as an optimization problem. However, the general formulation is too complex to be analyzed. Hence, we focus in detail on a small, tractable problem. We find that depositors with more connections receive larger optimal offers from the bank, *ceteris paribus*, reflecting the idea that the bank attempts to avoid that these depositors increase the probability that the depositors they are connected to become also impatient. We also find that the lower is the probability of bank runs that a bank tolerates, the larger is the optimal offer, all other things held constant. It is intuitive because through larger offers the bank reduces the probability of rejected offers that may lead to a bank run. As expected, the optimal payment to impatient depositors increases as the probability of liquidity shocks increases. In our examples, the optimal offers are almost linear in the probability of being hit by the liquidity shock, but the expected cost increases often nonlinearly in the same parameter. We show also through our example that the larger is the effect that an impatient depositor exerts on her neighbors, the larger is the optimal

offer, *ceteris paribus*. We find also that the sparser is the connection structure between depositors, the less they affect each other; hence the optimal offers from the bank are also smaller. We show also that the same analysis can be carried out in more complicated models that take into account feasibility constraints and allow for time-varying offers. Most importantly, even in these more intricate setups we find that more connected depositors receive larger offers from the bank.

There are several institutions and policies that are designed to handle problems that may arise during crises. The most prominent is deposit insurance that guarantees the recovery of deposits in case the bank has liquidity or other problems. There are several issues that make deposit insurance an imperfect solution. It entails moral hazard since the insurance of deposits may motivate banks to take on excessive risks. The coverage is limited both in size and scope, so depositors with a large deposit and investors with uninsured investments still remain a concern for the bank. For these reasons other ways of coping with financial distress have been used also. The most frequent alternative is liquidity suspension and rescheduling of payments. Our paper can be viewed as an attempt to formalize how this rescheduling should be if connections between depositors matter and the bank aims at minimizing payments to depositors but also wants to keep the probability of bank runs low. In many instances, the renegotiation of payments is done by the banking authority. Ennis and Keister [3] have examples of how such rescheduling occurred in some countries.

Note that many nonbank institutions (like mutual funds) engage also in maturity transformation and in their case short-term liabilities also retain a debt-like structure. Generally, the investments these institutions have are susceptible to investor run as well. In general, consider any firm or financial institution that owes to investors and negotiates with them about the terms of repayment, knowing that those investors may be connected. Our analysis applies to them as well. A further motivating example may be countries that struggle to pay their sovereign debt. Consider for instance Argentina that restructured its debt several times in the last decade. During this process the affected bondholders are offered payments (often in form of longer term bonds) that are lower than the original bonds promised. Obviously, the country that is dealing with the bondholders tries to minimize the payments but wants the bondholders to accept the offers it makes. If a bondholder accepts an offer, then it may influence the willingness of other bondholders to accept the offer as well [4].

Next we show that that depositors react to the decisions of other depositors that they observe and then discuss issues related to the importance of setting the right payment to depositors.

In our model, depositors react to other depositors' observed decisions. More precisely, we assume that the chance that a depositor becomes impatient is growing in the number of impatient depositors that she is linked to. Empirical studies support this idea. Kelly and O'Grada [5] investigate a bank run episode in New York in the 19th century and show that the most important factor determining

whether an individual panicked or not was her county of origin in Ireland. Immigrants from the same country clustered in the same neighborhood and observed each other, so if a depositor saw a large number of individuals trying to withdraw, then she was more likely to do so as well. Iyer and Puri [6] study a bank run that occurred in India in 2001 and demonstrate that observing withdrawals in one's social network increases the probability that the depositor in question follows suit. Starr and Yilmaz [7] show that, during a bank run incident in 2001 that occurred in Turkey, small and medium-sized depositors of an Islamic bank seemed to observe withdrawals of their peers and the larger the mass that withdrew, the more depositors did the same the next day. Experimental evidence also suggests that observability plays an important role in the emergence of bank runs (see, e.g., [8–11]). A common finding of these experimental studies is that if previous withdrawals are observed, then they induce further withdrawals. These empirical and experimental findings make it clear that depositors are affected by the withdrawal decision of depositors they are connected to. That is why we consider it important to introduce this feature into our model.

In the theoretical literature banks are supposed to act in the interest of the depositors and hence they set payments in a way that maximizes the overall utility of depositors. Important for our study, Green and Lin [2] find that the optimal payment depends on the self-reported type of the depositors and their position in the sequence of decision. As a consequence, depositors who withdraw early may end up with quite different payments. Our model has that feature as well, although in our case differences in payments are due to the connection structure between depositors. Ennis and Keister [3] use a Diamond and Dybvig [1] model and show that if the bank realizes that a bank run is underway, then it reoptimizes the payments to the subsequent depositors. Our idea is similar in the sense that the bank adapts to hard times, but the optimization problem in our model is very different. Unlike [3], we do not maximize the utility of the remaining depositors but minimize the payments to them so that they accept it with sufficiently large probability and do not ignite a bank run. Considering that reactions to the first withdrawal may appear within days (or even hours), this seems to be a plausible scenario. According to this, the most straightforward interpretation for the discrete time periods of our model is the scale of days.

Note that as the depositors in our model are automatons that do not decide in a strategic way to maximize their own utilities and neither does the bank set payments in order to maximize the overall utility of the depositors, this study is clearly not a strictly neoclassical economic model for bank runs.

We are aware of only one paper that uses Markov chains to study bank runs. Temzelides [12] studies how depositors are affected by the number of withdrawals from their and other banks in the previous period. He investigates myopic best response in an evolutionary banking setup. In his model there is no underlying social network that determines who affects whom, but each depositor observes other depositors' past action. Neither does he study the optimal payment, so our approaches are quite different.

## 2. Materials and Methods

Assume that there are $n$ depositors that are located at the nodes of a network and connections enable observability. Hence, a link connecting two depositors implies that they can observe each other's action.

To define a Markov chain model, we have to specify the possible states of the model and the state transition matrix $Q$ which contains the state transition probabilities. Since the connections of a certain depositor to another ones matter, we will distinguish them. This means that the set of the possible states of the model ($\mathcal{S}$) can be determined as the Descartes product over the set of possible states of the depositors ($S$). To keep the model as simple as possible, we will assume three possible state for each depositor:

   (i) *Patient* ($P$): this is the basic state of each depositor and following the literature a patient depositor does not have urgent liquidity need.

  (ii) *Impatient* ($I$): if a liquidity shock hits a depositor, her state changes from $P$ to $I$, meaning that she is demanding money from the bank. Furthermore, if two depositors are connected and one is impatient, she increases the probability of the other one becoming impatient. We assume that this effect is additive, so two impatient acquaintances double the chance of a $P \rightarrow I$ transition.

 (iii) *Out* ($O$): we assume that the bank offers a certain amount of money to impatient depositors, who may accept or reject the offer. We suppose that the chance that the impatient depositor accepts the offered quantity increases with the offered sum. If the depositor accepts the offered amount of money, her state will turn from $I$ to $O$ and ceases to affect her neighbors thereafter. If she rejects, then she stays in state $I$ for the next step.

Hence, $S = \{P, I, O\}$. Note that all depositors start being patient and then may turn impatient. Those impatient depositors who accept the offer of the bank enter state $O$. It is not possible to go directly from $P$ to $O$ and any move in the reverse direction (for example from $I$ to $P$) is disregarded also. Given this setup, the total number of states of the system will be $3^n$ where $n$ is the number of depositors. For the sake of tractability we use the following assumptions:

   (i) *Connection structure*: the structure of the links connecting the depositors may be described by a simple undirected graph, whose adjacency matrix is $A$ (a straightforward generalization of the model could be where we assume asymmetric information, and thus a directed $A$). Take any two depositors. If they are linked, then the corresponding entry in the adjacency matrix is 1. If one of them is impatient, while the other one is patient, then the former affects the latter one by increasing the probability that it turns impatient as well. Following the standard language of network analysis, if two depositors are connected, then they are neighbors, and the number of connections a depositor has is called degree. Hereafter, we assume that the

bank knows the connection structure. We admit that this is a strong assumption, but banks may have a lot of information about depositors including information about connections between them. Based on [5, 6], depositors living in the same neighborhood are likely to observe each other. Starr and Yilmaz [7] suggest that deposit size also may be a determinant of which depositor observes which other depositors. Banks may take into account such information.

(ii) *Homogeneity*: depositors are homogeneous in the following senses: (i) the chance of being hit by the liquidity shock is the same for all patient depositors (and independent of the degree); (ii) when offered a certain amount by the bank, the probability of accepting that amount (and hence change into the state $O$) is also the same for all impatient depositors (and again independent of the degree).

(iii) *Degree-dependent payments*: let $o_{i,\deg(i)}(t)$ be the sum offered to depositor $i$ at time $t$ where $\deg(i)$ is the number of neighbors of $i$. The bank distinguishes depositors based only on their degree. In other words, assuming time-independent payments, two depositors with the same degree receive always the same offer from the bank if in any given state and time. Since $o_{i,\deg(i)}(t) = o_{j,\deg(j)}(t)$ if $\deg(i) = \deg(j)$, hereafter we use $o_{\deg(i)}(t)$.

Note that the assumptions on the connection structure and homogeneity imply that depositors in the model differ only in their degree. As a consequence, there are $2^{n-1}$ possible connection structures.

*2.1. State Transition Probabilities.* Now we are ready to define the state transition probabilities. Any state $\sigma$ in $\mathscr{S}$ can be composed as $s_1 s_2 \cdots s_n$ where $s_i \in S = \{P, I, O\}$. Let us furthermore use the following notation convention: $\sigma(t, i)$ denotes $s_i(t)$, the state of depositor $i$ at time $t$.

Within a given time period, we assume that the following events take place simultaneously:

(i) Patient depositors turn impatient with some probability (that is determined by the probability of being hit by the liquidity shock and the number of impatient neighbors).

(ii) Impatient depositors decide if they accept or reject the offers by the bank.

We assume that transition events of the depositors in one step are independent, so the transition probability from state $\sigma(1) = s_1(1)s_2(1) \cdots s_n(1)$ at $t = 1$ to $\sigma(2) = s_1(2)s_2(2) \cdots s_n(2)$ at $t = 2$ can be written as

$$p(\sigma(1) \longrightarrow \sigma(2)) = \prod_{i=1}^{n} p(s_i(1) \longrightarrow s_i(2)), \qquad (1)$$

where $p(s_i(1) \rightarrow s_i(2))$ denotes the probability that depositor $i$ changes her state from $s_i(1)$ to $s_i(2)$ where $s_i(1), s_i(2) \in S$. Next, we determine the single transition probabilities.

(i) The chance of a liquidity shock hitting each patient depositor at each time period is denoted by $p_s$. We assume that impatient depositors affect the behavior of patient neighbors and may induce $P \rightarrow I$ transitions: $\delta$ denotes the level of how much an impatient depositor connected to a patient one increases the $P \rightarrow I$ transition. This way the patient-impatient transition at time $t$ for patient depositor $i$ may be calculated as

$$p_s + k_i^I(t)\delta, \qquad (2)$$

where $k_i^I(t)$ is the number of impatient depositors connected to $i$ at time $t$ (in general we do not assume the connection structure to change, but the model framework is capable of handling such cases). The chance of staying in the $P$ state is $1 - p(P \rightarrow I)$.

(ii) The chance that an impatient depositor $i$ accepts the offered money at time $t$ is denoted by $f(o_{\deg(i)}(t))$ where $f$ is a monotone increasing function, assumed to be the same for each depositor. The chance of staying in the $I$ state is $1 - p(I \rightarrow O)$.

To characterize the evolution of the system we introduce a lexicographic ordering of the states (e.g., $PPP \cdots PP = \sigma_1$; $PPP \cdots PI = \sigma_2$; e.g., see Appendix A). Furthermore, we define the state transition matrix $Q \in \mathscr{R}^{3^n \times 3^n}$. $Q_{i,j}$ is equal to the probability of the transition from state $\sigma_i$ to $\sigma_j$. The probability of state $i$ at time $t$ is given by the $i$th element of the vector $p(t) \in \mathscr{R}^{3^n}$. We will denote the probability of state $j$ ($\sigma = \sigma_j$) at time $t$ shortly with $p_j^t$ ($p_j^t$ equals the $j$th element of $p(t)$). Therefore,

$$p(t) = \left(Q^T\right)^t p(0), \qquad (3)$$

where $p(0) \in \mathscr{R}^{3^n}$ is a vector describing the initial state of the system. Thus, $p(0) = (1, 0, 0, \ldots, 0)$ denotes that the probability of the initial state (in our lexicographic ordering $\sigma_1 = PPP \cdots P$) is 1.

Let us define the cost of a given state $\sigma$ at time $t$ as the sum of offers accepted in the last time period (corresponding to $I \rightarrow O$ transitions from $t - 1$ to $t$)

$$c(\sigma(t)) = \sum_{j:\sigma(t,j)=O,\sigma(t-1,j)=I} o_{\deg(j)}(t-1). \qquad (4)$$

That is, the sum of offers that have been accepted at time period $t$ (but not before) by depositors with degree $j$ that ranges from 0 to $n - 1$. The total (or cumulated) cost ($C$) of $\sigma(t)$ may be defined as

$$C(\sigma(t)) = \sum_{k=2}^{t} c(\sigma_i(k)). \qquad (5)$$

As already pointed out, it is plausible to believe that in turbulent times banks attempt to minimize the payments to depositors who want to withdraw, but at the same time the bank tries to keep the probability of a bank run at a low level. As we will show, with the proposed formalized model we are able to exactly grasp this intuition.

# 3. Results and Discussion

*3.1. Optimization with No Offer Constraints and Time-Independent Offers.* In this section we assume that the offers are time-independent, which means that a depositor with a certain number of neighbors receives the same offer in each time period if in impatient state (and so we omit the argument $t$ in $o_{\deg(i)}(t)$).

Let $E[C(t)]$ denote the expected payments to depositors at time $t$. In general,

$$E[C(t)] = \sum_i C(\sigma_i(t)) p_i(t),\tag{6}$$

where $p_i(t)$ is the probability of state $\sigma_i$ at time $t$, which may be calculated from $x(t)$. However, since the offers are time-independent, we may write

$$E[C(t)] = \sum_{j:O\in\sigma_j} o_{\deg(j)} p_i(t),\tag{7}$$

so we omit the argument $t$ in $o_{\deg(i)}(t)$.

Assuming time-independent offers, the expected cost of the bank $E[C(t)]$ depends on the probability of those states at time $t$, which have at least one $O$, and the position of those $O$-s in $\sigma$. The position determines the degree which determines the payment (since payments only depend on the depositors' degree) to those depositors that were impatient and accepted the offer.

Actual payments are made only to those impatient depositors who accept the offer. Since offers are assumed to be time-independent, in this case it does not matter when the given state changed from $I$ to $O$ (in other words, when the offer was accepted), since amounts offered for a certain (impatient) node are time-independent.

To formulate the optimization problem, we also need to define what we consider a bank run.

*Definition 1.* Any state where no patient depositor is present is considered as a bank run event. The probability of a bank run event at time $t$ is denoted by $P_{BR}(t)$ (note the irreversibility: since there is no $O \rightarrow I$, $O \rightarrow P$, or $I \rightarrow P$ transition, once the system is in a state of bank run event, all following states will be bank run events).

The precise interpretation of the above event is that for a time instance $t$ a bank run is present at time $t$ or it has already occurred before. Intuitively, we assume that the bank run itself means that active depositors are present, who are all impatient. Since the state where all depositors are in the state $O$ also fulfills the definition, it is contra-intuitive in the sense that at that time the bank run is already over. However if all depositors are in the state $O$, it is sure that a bank run has already occurred.

The optimization problem of the bank is the following at time $t$ and given a connection structure $A$:

$$\min_{o_1,\ldots,o_d} \quad E[C(t)]$$
$$\text{subject to} \quad P_{BR}(t) < \overline{P}_{BR},\tag{8}$$
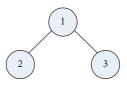


FIGURE 1: Topology 1: connection of the depositors in the case of Example 1.

where $d$ is the maximal degree in the connection structure, and

$$P_{BR}(t) = \sum_{i:P\notin\sigma_i} p_i(t),\tag{9}$$

where $P \notin \sigma_i$ refers to those states which do not include patient ($P$) depositors. $P_{BR}(t) < \overline{P}_{BR}$ denotes that we want to keep the probability of bank run event below some given threshold.

While this is a general optimization, it is too complex to be analyzed. The size of the state transition matrix grows exponentially with the number of the modelled depositors, and the complexity of the resulting expressions may be very high even in the case of quite simple examples that we show next. One may partially overcome this problem by merging states or doing simplifications regarding the Markov chain model. Moreover, nonlinear optimization problems with nonlinear constraints such as, for example, (8), are not easy to handle, and the solvers may run into local extrema (Bertsekas 1999). Furthermore, the needed computing capacity may be also significant due to the complexity of the functions. For these reasons, in the rest of the paper we limit ourselves to small examples to gain insight into how the optimal payments and expected costs vary as the environment changes.

In this section we assume that the quantities offered by the bank to the impatient depositors are not constrained by any consideration regarding their upper bound. In other words, the bank may offer arbitrarily high sums in order to control the expected chance of a bank run event; there is no feasibility constraint.

*3.1.1. Example 1.* Consider an example with three depositors. We fix the connection structure as depicted in Figure 1.

Furthermore, we assume the most simple possible case regarding the function $f$ that determines the probability of accepting an offer, namely, $f(o_{\deg(i)}) = o_{\deg(i)}$. This implies that we assume $o_n \in [0,1)\ \forall n$. We use a lexicographic ordering of the states described in Appendix A.

We are interested in the bank's optimal strategy. Concretely, let us determine which offers should the bank make to impatient depositors, if the initial state of the system (e.g., $PPP$) is known, and the bank wants to keep the chance of a bank run event ($P_{BR}$) under a certain probability level while also aiming to minimize the expected cost.

At $t = 1$ the probability of a bank run event is independent of the offered sums and is equal to $P_{BR}(1) = p_s^3$. Regarding $t = 2$, $P_{BR}(2)$ is equal to $p_8(2) + p_{15}(2) + p_{16}(2) + p_{17}(2) + p_{18}(2) + p_{19}(2) + p_{20}(2) + p_{21}(2)$, which can be calculated from (3), assuming $x_0 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}^T$ (i.e., at the beginning

each depositor is patient). Note that in Appendix A we define all potential states that may arise with three depositors, and states 8, 15, 16, 17, 18, 19, 20, and 21 are those that do not contain any patient depositor. In Appendix A, we develop the probabilities for all the possible 27 states and it yields that at $t = 2$ the probability of bank run event is

$$P_{BR}(2) = \delta^2 p_s^3 - 2\delta^2 p_s^2 + \delta^2 p_s + 4\delta p_s^4 - 12\delta p_s^3$$
$$+ 8\delta p_s^2 - p_s^6 + 6p_s^5 - 12p_s^4 + 8p_s^3, \tag{10}$$

which is also independent of the offers. This is not surprising. If a depositor becomes impatient at $t = 1$, with the offer we may influence the probability that she changes to $O$ at $t = 2$. However, regarding patient depositors (on whom the bank run event definition is based on), the important thing is how long their impatient neighbors remain in the $I$ state.

Consider the following example. At $t = 1$ the state $IPP$ appears. With the offer at $t = 1$ we may affect the probability of, for example, $IPP$ and $OPP$ at $t = 2$, but it makes no difference in terms of whether the next state will be considered as a bank run event or not. Recall the bank run event definition according to which, for example, $IPP$ and $OPP$ are regarded as the same. Depositors 2 and 3 are affected by the $I$ state of player 1 at the transition from $t = 1$ to $t = 2$, irrespective of the offer to depositor 1 at $t = 1$. On the other hand, at $t = 3$ it matters whether depositor 1 remained in state $I$ at $t = 2$ as well or not (which in turn already depends on the offer).

As expected, the offers first appear in the probability of bank run at $t = 3$ ($P_{BR}(3)$):

$$P_{BR}(3) = 36\delta p_s^2 + 7\delta^2 p_s - 120\delta p_s^3 - 4\delta^3 p_s + 156\delta p_s^4$$
$$+ \delta^4 p_s - 100\delta p_s^5 + 32\delta p_s^6 - 4\delta p_s^7 + 27p_s^3$$
$$- 81p_s^4 + 108p_s^5 - 81p_s^6 + 36p_s^7 - 9p_s^8 + p_s^9$$
$$- 43\delta^2 p_s^2 + 70\delta^2 p_s^3 + 12\delta^3 p_s^2 - 46\delta^2 p_s^4$$
$$- 12\delta^3 p_s^3 - 2\delta^4 p_s^2 + 13\delta^2 p_s^5 + 4\delta^3 p_s^4$$
$$+ \delta^4 p_s^3 - \delta^2 p_s^6 - 6\delta p_s^2 o_1 - 6\delta p_s^2 o_2$$
$$+ 18\delta p_s^3 o_1 - 3\delta^2 p_s o_2 + 18\delta p_s^3 o_2 - 20\delta p_s^4 o_1 \quad (11)$$
$$+ 4\delta^3 p_s o_2 - 20\delta p_s^4 o_2 + 10\delta p_s^5 o_1 - \delta^4 p_s o_2$$
$$+ 10\delta p_s^5 o_2 - 2\delta p_s^6 o_1 - 2\delta p_s^6 o_2 + 8\delta^2 p_s^2 o_1$$
$$+ 18\delta^2 p_s^2 o_2 - 14\delta^2 p_s^3 o_1 - 30\delta^2 p_s^3 o_2$$
$$+ 8\delta^2 p_s^4 o_1 - 12\delta^3 p_s^2 o_2 + 20\delta^2 p_s^4 o_2$$
$$- 2\delta^2 p_s^5 o_1 + 12\delta^3 p_s^3 o_2 + 2\delta^4 p_s^2 o_2$$
$$- 5\delta^2 p_s^5 o_2 - 4\delta^3 p_s^4 o_2 - \delta^4 p_s^3 o_2.$$

Since there are significantly more possible ways for a bank run event to form at $t = 3$ than at $t = 2$, this expression is more complex than the one in (10).
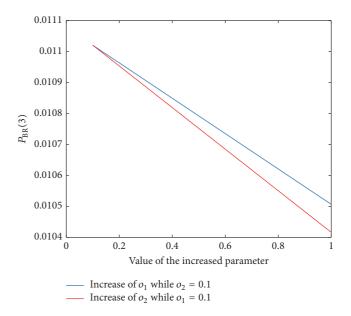


FIGURE 2: $P_{BR}(3)$ as function of the offer. The blue line illustrates the effect when the offer to the degree 1 depositor is increased while the offer to the degree 2 depositor is kept constant at 0.1, while the red line illustrates the dual case.

In our case the expected costs are as follows:

$$E[C(3)] = p_s \left(3o_2^2 - 2o_1^3 + 6o_1^2 - o_2^3 + 2o_1^2\delta + 2o_2^2\delta \right.$$
$$\left. - 2o_1^2 p_s - o_2^2 p_s - 2o_1^2\delta p_s - 2o_2^2\delta p_s \right). \tag{12}$$

To fix ideas, we compute the probability of bank run event and the expected costs at $t = 3$. Without loss of generality, assume that the probability of being hit by a liquidity shock is 7% and having an impatient neighbor increases by 2.5% the chances that a patient depositor turns impatient as well ($p_s = 0.07$, $\delta = 0.025$) and suppose that $f(o_{\deg(i)}) = o_{\deg(i)}$.

Given the payments to depositors with different degree, the probability of bank run event is depicted in Figure 2.

As expected, an increase of the offers implies the decrease of the chance of a bank run event. Second, by increasing the offer to the depositor with the higher degree is more efficient. This highlights how important may be to differentiate between depositors regarding the offers.

To determine the optimal strategy of the bank at $t = 3$, we have to solve the following nonlinear optimization problem with nonlinear inequality constraints:

$$\min_{o_1, o_2} \quad E[C(3)]$$
$$\text{subject to} \quad P_{BR}(3) < \overline{P}_{BR}. \tag{13}$$

Regarding the above problem, the NLOPT function [13] of the MATLAB OPTI toolbox was used [14] with the algorithm LDSLSQP. NLOPT was chosen based on its ability of handling nonlinear objective function and constraints, on its numerical stability, and on its advantageous convergence properties. Considering $\delta = 0.08$, the following figures show how the optimal payments and expected cost depend on
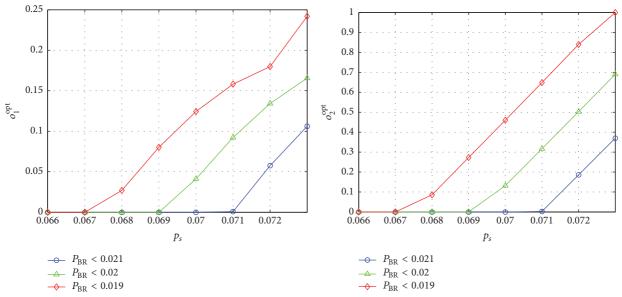
FIGURE 3: The dependence of optimal offers $o_1^{\text{opt}}$ and $o_2^{\text{opt}}$ on $p_s$ at various levels of $\overline{P}_{\text{BR}}$.

the model parameters. In these figures we see that if the chance of a liquidity shock ($p_s$) is low enough, the probability of a bank run event remains under the defined threshold even if we offer 0 return to impatient depositors. However, as the probability of becoming impatient ($p_s$) increases, the importance of these offers becomes significant. Furthermore, if we observe the $y$-axis in the graphs of Figure 3, we see that the optimal offer to a depositor connected to two other depositors is larger, as expected. We defined the function of accepting offer ($f$) as the identity function, and thus the offered sum is equal to the acceptance probability, so we restrict our analysis to the range where $o_1, o_2 < 1$.

Figure 3 shows that the optimal offers to depositors with one and two connected depositors ($o_1^{\text{opt}}$ and $o_2^{\text{opt}}$) increase as the probability of being hit by a liquidity shock ($p_s$) is increased (as expected) and also shows how the maximum probability of bank run events that the bank admits ($\overline{P}_{\text{BR}}$) modulates this increase. The larger is the acceptable probability of bank runs, the lower is the offer, *ceteris paribus*.

Figure 4 shows how the expected cost changes in function of the probability of a liquidity shock ($p_s$) and the acceptable probability of bank run events ($\overline{P}_{\text{BR}}$). Note that while optimal offers are often almost linear in $p_s$, the expected cost increases nonlinearly in $p_s$.

*Finding 1.* Anything else held constant, optimal offers increase in the probability of a liquidity shock and in the number of connections. The larger is the probability of bank run event that a bank tolerates, the lower are the optimal offers and hence the expected costs, *ceteris paribus*.

Figures 5 and 6 show how the sensitivity to neighbor depositors who are impatient ($\delta$) affects the dependence of the optimal offers ($o_1^{\text{opt}}, o_2^{\text{opt}}$) and the expected cost ($E[C(3)]$) on the probability of being hit by a liquidity shock ($p_s$). We assume $\overline{P}_{\text{BR}} = 0.02$ in these cases. As expected, the more
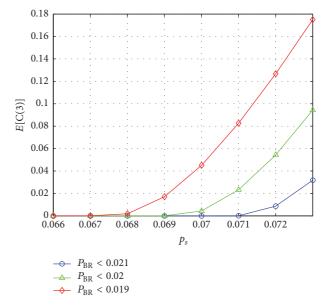


FIGURE 4: The dependence of $E[C(3)]$ on $p_s$ at various levels of $\overline{P}_{\text{BR}}$.

an impatient depositor increases the probability of turning impatient of her neighbor(s) ($\delta$), the larger is the optimal offer to her so that the probability of bank run event can be kept at the desired level. Note also that the expected cost increases nonlinearly in $\delta$ as the probability of being hit by a liquidity shock ($p_s$) grows beyond a certain threshold (in our example it is around 0.07).

The interpretation of the bank run event definition may be subject to different considerations. Here we applied a simple approach; however one may define more complex scenarios (e.g., we may consider a state as a bank run if less than half of the depositors is in $P$ state). Such alternative bank run definitions may be easily interpreted in the proposed framework.

$o_1^{\text{opt}}$ axis (left plot), $o_2^{\text{opt}}$ axis (right plot), $p_s$ horizontal axes.

— $\delta = 0.08$
— $\delta = 0.0775$
— $\delta = 0.075$
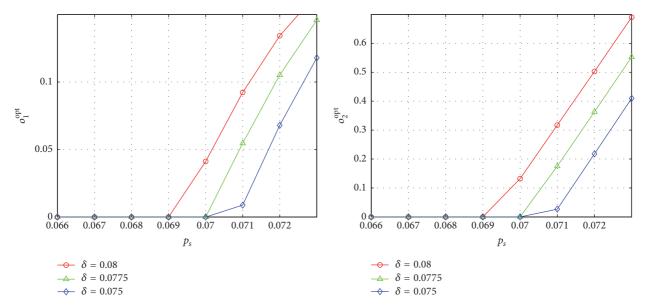
FIGURE 5: The dependence of optimal offers $o_1^{\text{opt}}$ and $o_2^{\text{opt}}$ on $p_s$ at various levels of the parameter $\delta$, describing the strength of the connections.



$E[C(3)]$ versus $p_s$.

— $\delta = 0.08$
— $\delta = 0.0775$
— $\delta = 0.075$
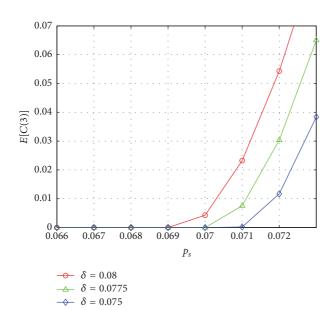
FIGURE 6: The dependence of $E[C(3)]$ on $p_s$ at various levels of the parameter $\delta$, describing the strength of the connections.

*The Role of Connections.* To get an impression how the connections of the depositors affects the results, we modify the connection structure of Example 1. Now, as depicted in Figure 7, depositor 1 is not connected to depositor 2.

In this case we obtain that

$$P_{\text{BR}}(2) = p_s^2 (2 - p_s) \left(2\delta + 4p_s - 2\delta p_s - 4p_s^2 + p_s^3\right), \quad (14)$$

$$P_{\text{BR}}(3) = p_s^2 \left(p_s^2 - 3p_s + 3\right) \left(6\delta + 9p_s - 14\delta p_s - 2\delta o_1\right.$$

$$+ 10\delta p_s^2 + 2\delta^2 p_s - 2\delta p_s^3 + 2\delta^2 o_1 - 2\delta^2 - 18p_s^2 \quad (15)$$

$$+ 15p_s^3 - 6p_s^4 + p_s^5 - 2\delta p_s^2 o_1 - 2\delta^2 p_s o_1 + 4\delta p_s o_1\right).$$
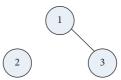


FIGURE 7: Topology 2: Connection of the depositors in the case of example 2.

Moreover,

$$E[C(3)] = p_s \left(6o_1^2 - o_0^3 + 3o_0^2 - 2o_1^3 + 2o_1^2\delta - o_0^2 p_s\right.$$

$$\left. - 2o_1^2 p_s - 2o_1^2 \delta p_s\right). \quad (16)$$

Now we have only depositors with zero or one neighbor. Note that offer to depositors without any connection ($o_0$) does not appear in (14) and (15) since an isolated depositor does not affect anybody else. The offer $o_0$ does not influence the probability of an isolated depositor changing to $I$. Once an isolated depositor is in state $I$, we may enhance her transition to $O$ with a larger $o_0$, but this does not affect the probability of a bank run event. This is the case because an isolated depositor in state $I$ does not influence anybody else, so from the perspective of bank run it does not matter if she is in state $I$ or $O$. Trivially, if we want to minimize the expected cost, we choose $o_0$ to be zero.

Without loss of generality, we consider the following parameters: $\delta = 0.025$, $\overline{P}_{\text{BR}} = 0.02$. We determine now the optimal offer to depositors with one connection ($o_1$). In Figure 8 we see that, in the case of less connections (i.e., compared to topology 1), a larger $p_s$ value is required to trigger the role of the offers (about 0.08 instead of about 0.07). The connection parameter $\delta$ modulates the results similarly to the previous case depicted in Figure 5. As expected, if the connection structure is sparser (topology 2), the depositors
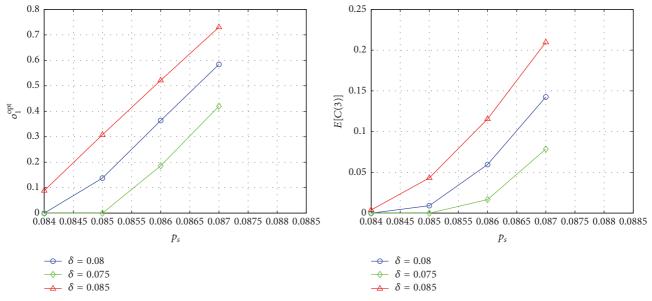
FIGURE 8: The dependence of $o_1^{opt}$ and $E[C(3)]$ on $p_s$ and $\delta$, in the case of topology 2.

influence each other less, so the expected cost to prevent a bank run is always lower. It reflects the intuitive idea that less connection implies fewer channels to affect other depositors, so the peril of contagion between depositors is more limited.

When there is no connection between depositors, the optimal offer to depositors is zero as we explained before. In the case when all depositors are connected to each other (topology 3),

$$
\begin{aligned}
P_{BR}(3) = {} & 54\delta p_s^2 + 21\delta^2 p_s - 180\delta p_s^3 - 18\delta^3 p_s \\
& + 234\delta p_s^4 + 3\delta^4 p_s - 150\delta p_s^5 + 48\delta p_s^6 \\
& - 6\delta p_s^7 + 27p_s^3 - 81p_s^4 + 108p_s^5 - 81p_s^6 \\
& + 36p_s^7 - 9p_s^8 + p_s^9 - 111\delta^2 p_s^2 \\
& + 174\delta^2 p_s^3 + 48\delta^3 p_s^2 - 114\delta^2 p_s^4 \\
& - 42\delta^3 p_s^3 - 6\delta^4 p_s^2 + 33\delta^2 p_s^5 + 12\delta^3 p_s^4 \\
& + 3\delta^4 p_s^3 - 3\delta^2 p_s^6 - 18\delta p_s^2 o_2 - 9\delta^2 p_s o_2 \\
& + 54\delta p_s^3 o_2 + 12\delta^3 p_s o_2 - 60\delta p_s^4 o_2 \\
& - 3\delta^4 p_s o_2 + 30\delta p_s^5 o_2 - 6\delta p_s^6 o_2 \\
& + 60\delta^2 p_s^2 o_2 - 96\delta^2 p_s^3 o_2 - 36\delta^3 p_s^2 o_2 \\
& + 60\delta^2 p_s^4 o_2 + 36\delta^3 p_s^3 o_2 + 6\delta^4 p_s^2 o_2 \\
& - 15\delta^2 p_s^5 o_2 - 12\delta^3 p_s^4 o_2 - 3\delta^4 p_s^3 o_2,
\end{aligned}
\tag{17}
$$

$$
E[C(3)] = 3o_2 p_s \left(3 - p_s + 2\delta - o_2 - 2\delta p_s\right). \tag{18}
$$

In Figure 9 we see that, in the case of full connectedness, a lower $p_s$ value, compared to the previous cases, is required

to trigger the role of the offers (about 0.05), and the expected cost is also higher even in the case of relatively low $p_s$ values (compared to Figure 6).

*Finding 2.* The connection structure between depositors matters. The more connections there are between depositors, the larger are the optimal offers and the expected cost of the bank, *ceteris paribus*.

*3.2. Optimization Problem with Offer Constraints and Time-Dependent Offers.* It is a natural to extend the model in a direction which takes into account the bank's investments, returns, and liquidity, which affects the possible offers. Moreover, in this section we also allow for the case that the offer changes in time. This is consonant with some papers that we cited before, as, for instance, [2, 3] also show that banks adjust the payments to the new situations. However, we assume that the bank does not reevaluate the situation between time periods (in this case the realized states could be taken into account as certain starting state of the model, and the optimization could be performed according to this), all offers are determined prior. While these features make the model more realistic, they make it more complicated also. Hence, in this section we focus on the example that we introduced in the last section.

Consider again the topology depicted in Figure 1 and a time horizon of 4 periods. The state transition matrix $Q$ will be time-dependent, since the offers at $t = 1$ and $t = 2$ may differ. We denote the offer to a depositor with $n$ neighbors at time $t$ as $o_n(t)$.

Since the offers ($o_n$) are time-dependent, the resulting values of the acceptance functions ($f(o_n(t))$) are also time dependent. We assume that they are the same for all depositors with the same degree.

Since the main risk is to leave depositors too long in the impatient state, we expect that earlier offers should be larger.
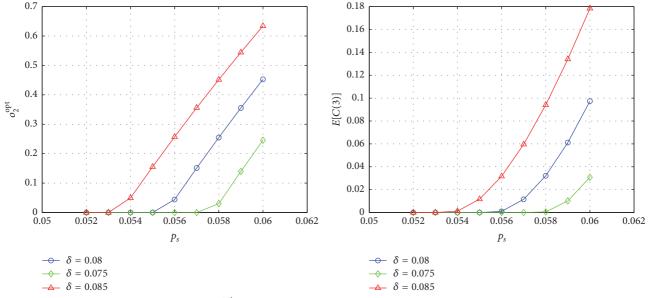
FIGURE 9: The dependence of $o_2^{\text{opt}}$ and $E[C(3)]$ on $p_s$ and $\delta$, in the case of topology 3 (full connectedness).

On the other hand early offers are limited by early returns. In the optimization problem below we take this consideration into account.

In this case the probability of bank run event at $t = 3$ is

$$
\begin{aligned}
P_{\text{BR}}(3) = {} & 36\delta p_s^2 + 7\delta^2 p_s - 120\delta p_s^3 - 4\delta^3 p_s + 156\delta p_s^4 \\
& + \delta^4 p_s - 100\delta p_s^5 + 32\delta p_s^6 - 4\delta p_s^7 + 27 p_s^3 \\
& - 81 p_s^4 + 108 p_s^5 - 81 p_s^6 + 36 p_s^7 - 9 p_s^8 + p_s^9 \\
& - 43\delta^2 p_s^2 + 70\delta^2 p_s^3 + 12\delta^3 p_s^2 - 46\delta^2 p_s^4 \\
& - 12\delta^3 p_s^3 - 2\delta^4 p_s^2 + 13\delta^2 p_s^5 + 4\delta^3 p_s^4 \\
& + \delta^4 p_s^3 - \delta^2 p_s^6 - 6\delta p_s^2 f(o_1(1)) \\
& - 6\delta p_s^2 f(o_2(1)) + 18\delta p_s^3 f(o_1(1)) \\
& - 3\delta^2 p_s f(o_2(1)) + 18\delta p_s^3 f(o_2(1)) \\
& - 20\delta p_s^4 f(o_1(1)) + 4\delta^3 p_s f(o_2(1)) \\
& - 20\delta p_s^4 f(o_2(1)) + 10\delta p_s^5 f(o_1(1)) \\
& - \delta^4 p_s f(o_2(1)) + 10\delta p_s^5 f(o_2(1)) \\
& - 2\delta p_s^6 f(o_1(1)) - 2\delta p_s^6 f(o_2(1)) \\
& + 8\delta^2 p_s^2 f(o_1(1)) + 18\delta^2 p_s^2 f(o_2(1)) \\
& - 14\delta^2 p_s^3 f(o_1(1)) - 30\delta^2 p_s^3 f(o_2(1)) \\
& + 8\delta^2 p_s^4 f(o_1(1)) - 12\delta^3 p_s^2 f(o_2(1)) \\
& + 20\delta^2 p_s^4 f(o_2(1)) - 2\delta^2 p_s^5 f(o_1(1)) \\
& + 12\delta^3 p_s^3 f(o_2(1)) + 2\delta^4 p_s^2 f(o_2(1))
\end{aligned}
$$

$$
\begin{aligned}
& - 5\delta^2 p_s^5 f(o_2(1)) - 4\delta^3 p_s^4 f(o_2(1)) \\
& - \delta^4 p_s^3 f(o_2(1));
\end{aligned}
\tag{19}
$$

the above expression (19) is a slight modification of (11).

$P_{\text{BR}}(4)$, which depends also on $f(o_1(2))$ and $f(o_2(2))$, can be similarly derived; however the expression is too long to be detailed here.

Regarding the expected cost, the derivation is not as simple as in Section 3.1, since if a state ends up in $O$, it does matter when did it change from $I$ to $O$. As detailed earlier, the offers in $t = 3$ do not affect the probability of the bank run event at $t = 4$, so it is enough to derive the expected costs of the states at $t = 3$. Consider a simple example. The expected cost of the state $POO$ at time $t = 3$ may be calculated as

$$
\begin{aligned}
& p_{12}(3)\,(p_5(2)\,(2o_1(2)) + p_{22}(2)\,(o_1(1) + o_1(2)) \\
& \quad + p_{23}(2)\,(o_1(1) + o_1(2)) + p_{12}(2)\,(2o_1(1))),
\end{aligned}
\tag{20}
$$

where $p_{12}(3)$ is the probability of state 12 at time $t = 3$ and so on. Let us discuss this expression a bit more in detail. The expected cost of state $POO$ at $t = 3$ is proportional to its probability $p_{12}(3)$. Furthermore there are 4 ways to get to $POO = \sigma_{12}$:

(i) From $PII = \sigma_5$: in this case both depositor 2 and depositor 3 (depositors with one neighbor) accept the offer at $t = 2$, so the relevant term is $p_5(2o_1(2))$ (remember that $o_1(2)$ refers to offers with one neighbor at time $t = 2$).

(ii) From $PIO = \sigma_{22}$: in this case depositor 3 accepted the offer at $t = 1$ (since he is already in $O$ state), while depositor 2 accepted the offer at $t = 2$ which implies $p_{22}(o_1(1) + o_1(2))$.
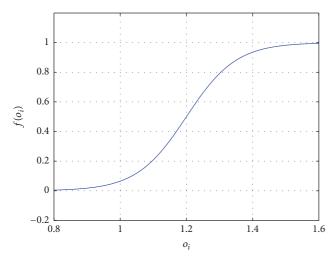
FIGURE 10: $f(o_i)$: the function describing how the probability of acceptance depends on the offered sum.



FIGURE 11: Own liquidity and investment return profile for the bank.

(iii) From $POI = \sigma_{23}$: in this case depositor 3 accepted the offer at $t = 2$, while depositor 2 accepted the offer at $t = 1$ which implies $p_{23}(o_1(1) + o_1(2))$.

(iv) From $POO = \sigma_{12}$: in this case both depositors 2 and 3 accepted the offer at $t = 1$: $p_{12}(2o_1(1))$.

With the summation of all such expressions we can derive the expression for the expected cost at $t = 3$.

We assume the nonlinear offer-acceptance function

$$f(o) = \frac{1}{\left(1 + \exp\left((1.2 - o)/0.075\right)\right)} \tag{21}$$

depicted in Figure 10 for all depositors.

In this case, the resulting optimization problem is as follows:

$$\min_{o_1(1), o_2(1), o_1(2), o_2(2)} \quad E\left[C(3)\right] \tag{22}$$
$$\text{s.t.} \quad o_1(1), o_2(1), o_1(2), o_2(2) > 0.$$

*3.2.1. A Numerical Example with Time-Dependent Offers.* Consider the following numerical parameters:

$$p_s = 0.1,$$
$$\delta = 0.02, \tag{23}$$
$$\overline{P}_{BR} = 0.05.$$

The optimization process returns the solution $\left[o_1(1) \; o_2(1) \; o_1(2) \; o_2(2)\right] = \left[1.127 \; 1.253 \; 0.993 \; 1.068\right]$. In this case $E[C(3)] = 0.0216$. As before, the depositor with more connections receives a larger amount in both periods. Moreover, the degree-dependent offers in period 1 are larger than in period 2. This latter finding is intuitive because the more periods lie ahead, the more risky is to have an impatient depositor that may negatively affect other depositors.

*3.2.2. A Numerical Example Including Offer Constraints.* In the previous example we assumed that the bank has with certainty enough resources to carry out the payments to its impatient customers. However, this need not be the case, that is why we consider here an example where feasibility constraints may bind.

Suppose that there are three depositors characterized by the offer-acceptance function $f(o_i)$ depicted in Figure 10 and each of them places 1 unit of money into the bank before $t = 0$, with an expected return of 1.6 at $t = 4$. The bank has 1 unit of own liquidity and invests the 3 units with an expected return of 4 at $t = 4$. The return profile however is incremental, but nonlinear in time, as depicted in Figure 11. The bank receives 1.2 unit at $t = 1$, an additional 0.4 at $t = 2$, an additional 1.4 at $t = 3$, and finally one more unit at $t = 4$. Once more, considering the bank run event at $t = 4$, the latest offers which matter are given at $t = 2$, since the critical issue is how long impatient depositors stay in impatient state, and so offers at $t = 3$ do not count. The topology that we consider is as in Figure 1.

If any of the depositors is hit by a liquidity shock, she may withdraw from the bank. In this case the return profile affects the possible offers. Naturally, each single offer is limited by the actual liquidity ($o_1(1) < 2.2$; $o_2(1) < 2.2$; $o_1(2) < 2.6$; $o_2(2) < 2.6$). That is, the maximum payment to any depositor is constrained by the available amount in the given period. On the other hand, the following feasibility constraints are to be considered in the case of multiple offers:

(i) $o_1(1) + o_2(1) < 2.2$, $2o_1(1) < 2.2$.

(ii) $o_1(1) + o_1(2) < 2.6$, $o_2(1) + o_1(2) < 2.6$, $o_1(1) + o_2(2) < 2.6$.

(iii) $o_1(2) + o_2(2) < 2.6$, $2o_1(2) < 2.6$.

The first point is clear: if two offers are made at $t = 1$, and both are accepted, the potential payment is limited by the liquidity available for the bank at $t = 1$. The second point refers to the case if one offer is accepted at $t = 1$ and another at

$t = 2$. In this case the first offer is constrained by 2.2, but in the second step, the bank may only make offer from its remaining liquidity. If the offer(s) in the first step is (are) rejected or there is no need for offers at $t = 1$ (because), and in the second step two offers have to be made, the constraints corresponding to the third point describe the effects of limited liquidity.

We do not consider the case when in either $t = 1$ or $t = 2$ three offers are to be made, or when at $t = 2$ one (two) offer is made after two (one) accepted offers, since this would mean that the system is already at a bank run.

If we consider these offer constraints the resulting optimization problem will take the following form:

$$\min_{o_1(1), o_2(1), o_1(2), o_2(2)} E\left[C\left(3\right)\right]$$

$$\text{subject to} \quad P_{\text{BR}}\left(4\right) < \overline{P}_{\text{BR}}$$

$$o_1\left(1\right) + o_2\left(1\right) < 2.2$$

$$2o_1\left(1\right) < 2.2$$

$$o_1\left(1\right) + o_1\left(2\right) < 2.6 \tag{24}$$

$$o_2\left(1\right) + o_1\left(2\right) < 2.6$$

$$o_1\left(1\right) + o_2\left(2\right) < 2.6$$

$$o_1\left(2\right) + o_2\left(2\right) < 2.6$$

$$2o_1\left(2\right) < 2.6.$$

Considering $p_s = 0.1$ $\delta = 0.02$ $\overline{P}_{\text{BR}} = 0.05$, this results in $\left[o_1(1) \; o_2(1) \; o_1(2) \; o_2(2)\right] = \left[1.067 \; 1.133 \; 1.083 \; 1.184\right]$ with $E[C(3)] = 0.027$. Similarly to previous results, in each period the depositor with more connections receives a larger offer from the bank. On the other hand, offers at $t = 1$ are limited by liquidity constraints, and note that now the expected cost is higher than in the previous example (Section 3.2.1) if the objective is to keep the bank run event probability under 5% chance. The reason for this is that the offers cannot be efficiently distributed because of the liquidity constraints. Note that although we considered examples with some parameters, the optimization problem can handle any meaningful parameter constellation.

*Finding 3.* When allowing for time-dependent offers and feasibility constraints on offers we find that connections matter. The more connections a depositor has, the larger is the optimal offer from the bank.

## 4. Conclusions

We set up a model that studies the optimal offer from a bank to a withdrawing depositor if we assume that the bank attempts to minimize the payments, but also the probability of a bank run event that is affected by connections between depositors. More concretely, we assume that a depositor who does not receive an offer that she accepts makes her neighbors more prone to withdraw. We claim that our assumptions

TABLE 1

| State | Notation |
|-------|----------|
| PPP | $\sigma_1$ |
| PPI | $\sigma_2$ |
| PIP | $\sigma_3$ |
| IPP | $\sigma_4$ |
| PII | $\sigma_5$ |
| IPI | $\sigma_6$ |
| IIP | $\sigma_7$ |
| PPO | $\sigma_9$ |
| POP | $\sigma_{10}$ |
| OPP | $\sigma_{11}$ |
| POO | $\sigma_{12}$ |
| OPO | $\sigma_{13}$ |
| OOP | $\sigma_{14}$ |
| OOO | $\sigma_{15}$ |
| IIO | $\sigma_{16}$ |
| IOI | $\sigma_{17}$ |
| OII | $\sigma_{18}$ |
| IOO | $\sigma_{19}$ |
| OIO | $\sigma_{20}$ |
| OOI | $\sigma_{21}$ |
| III | $\sigma_8$ |
| PIO | $\sigma_{22}$ |
| POI | $\sigma_{23}$ |
| IPO | $\sigma_{24}$ |
| IOP | $\sigma_{25}$ |
| OPI | $\sigma_{26}$ |
| OIP | $\sigma_{27}$ |

and setup capture important features on payment decisions during crises. We show that the optimization problem can be formulated neatly in spite of nonlinearities. However, due to the very high number of potential connection structures between depositors finding a general solution proved elusive. Therefore, we use three variants of a simple example to gain some insight into the role of connections. An overarching finding in these examples is that connections matter. The more connections a depositor has, the larger is the optimal offer from the bank. Consequently, the denser is the connection structure, the larger is the expected payment of the bank to the depositors.

One clear limitation of the applicability of the described model corresponds to the number of depositors taken into account. The size of the state transition matrix and so the complexity of probability formulas used in the calculation grow exponentially with the number of depositors. A possible approach to overcome this problem is the application of epidemic models (see [15]) to describe the spreading of impatience among depositors. A somewhat similar methodology in the case of financial contagion has been described in [16]. On the one hand, these methods represent a natural possibility of describing larger networks; on the other hand we see that the approach presented in this paper has its benefits as well. As we distinguish between depositors, individual

characteristics (e.g., individual deposit size and individual risk aversion measures corresponding to the function $f$) and thus more detailed knowledge about depositors may be taken into account. The homogeneity assumption described in Section 2 can be relaxed, and the necessary equations may be similarly derived.

Furthermore, although we have shown small examples with a limited number of depositors, such an analysis may make sense. As already mentioned, Starr and Yilmaz [7] study a bank run episode in Turkey. They group depositors according to the size of their deposits (small, medium-size, and large) and analyze how they reacted to each other's decision. For instance, did small depositors withdraw more after observing a surge in withdrawals by large depositors? They find that while large depositors observe small and medium-size depositors, the latter do not seem to observe the former ones. This suggests an intricate connection structure between these groups (a directed graph may capture the idea that one group observes another one but observability in general is unilateral). If we interpret depositors as representing groups, then our model may help to understand this kind of situations.

## Appendix

### A. Notation of the States Used in the Model

We use notation for the states of Example as shown in Table 1.

### B. Analytical Form of the State Probability Vector

The state probability vector of Example 1 at $t = 2$ is as follows:

$$
\begin{pmatrix}
(p_s - 1)^6 \\
-p_s (p_s - 1)^5 - p_s (p_s - 1)^3 (o_1 - 1)(\delta + p_s - 1) \\
-p_s (p_s - 1)^5 - p_s (p_s - 1)^3 (o_1 - 1)(\delta + p_s - 1) \\
-p_s (p_s - 1)^5 - p_s (p_s - 1)^2 (o_2 - 1)(\delta + p_s - 1)^2 \\
p_s^2 (p_s - 1)^4 + 2p_s^2 (p_s - 1)^2 (o_1 - 1)(\delta + p_s - 1) + p_s^2 (p_s - 1)(o_1 - 1)^2 (2\delta + p_s - 1) \\
p_s^2 (p_s - 1)^4 + p_s (\delta + p_s)(p_s - 1)^3 (o_1 - 1) + p_s (\delta + p_s)(p_s - 1)^2 (o_2 - 1)(\delta + p_s - 1) + p_s^2 (p_s - 1)(o_1 - 1)(o_2 - 1)(\delta + p_s - 1) \\
p_s^2 (p_s - 1)^4 + p_s (\delta + p_s)(p_s - 1)^3 (o_1 - 1) + p_s (\delta + p_s)(p_s - 1)^2 (o_2 - 1)(\delta + p_s - 1) + p_s^2 (p_s - 1)(o_1 - 1)(o_2 - 1)(\delta + p_s - 1) \\
-p_s^3 (p_s - 1)^3 - p_s^3 (o_1 - 1)^2 (o_2 - 1) - 2p_s^2 (\delta + p_s)(p_s - 1)^2 (o_1 - 1) - p_s (\delta + p_s)^2 (p_s - 1)^2 (o_2 - 1) - p_s^2 (2\delta + p_s)(p_s - 1)(o_1 - 1)^2 - 2p_s^2 (\delta + p_s)(p_s - 1)(o_1 - 1)(o_2 - 1) \\
p_s o_1 (p_s - 1)^3 (\delta + p_s - 1) \\
p_s o_1 (p_s - 1)^3 (\delta + p_s - 1) \\
p_s o_2 (p_s - 1)^2 (\delta + p_s - 1)^2 \\
p_s^2 o_1^2 (p_s - 1)(2\delta + p_s - 1) \\
p_s^2 o_1 o_2 (p_s - 1)(\delta + p_s - 1) \\
p_s^2 o_1 o_2 (p_s - 1)(\delta + p_s - 1) \\
p_s^3 o_1^2 o_2 \\
p_s^2 o_1 (\delta + p_s)(p_s - 1)^2 + p_s^3 o_1 (o_1 - 1)(o_2 - 1) + p_s^2 o_1 (\delta + p_s)(p_s - 1)(o_2 - 1) + p_s^2 o_1 (2\delta + p_s)(p_s - 1)(o_1 - 1) \\
p_s^2 o_1 (\delta + p_s)(p_s - 1)^2 + p_s^3 o_1 (o_1 - 1)(o_2 - 1) + p_s^2 o_1 (\delta + p_s)(p_s - 1)(o_2 - 1) + p_s^2 o_1 (2\delta + p_s)(p_s - 1)(o_1 - 1) \\
p_s^3 o_2 (o_1 - 1)^2 + p_s o_2 (\delta + p_s)^2 (p_s - 1)^2 + 2p_s^2 o_2 (\delta + p_s)(p_s - 1)(o_1 - 1) \\
-p_s^3 o_1^2 (o_2 - 1) - p_s^2 o_1^2 (2\delta + p_s)(p_s - 1) \\
-p_s^3 o_1 o_2 (o_1 - 1) - p_s^2 o_1 o_2 (\delta + p_s)(p_s - 1) \\
-p_s^3 o_1 o_2 (o_1 - 1) - p_s^2 o_1 o_2 (\delta + p_s)(p_s - 1) \\
-p_s^2 o_1 (p_s - 1)^2 (\delta + p_s - 1) - p_s^2 o_1 (p_s - 1)(o_1 - 1)(2\delta + p_s - 1) \\
-p_s^2 o_1 (p_s - 1)^2 (\delta + p_s - 1) - p_s^2 o_1 (p_s - 1)(o_1 - 1)(2\delta + p_s - 1) \\
-p_s o_1 (\delta + p_s)(p_s - 1)^3 - p_s^2 o_1 (p_s - 1)(o_2 - 1)(\delta + p_s - 1) \\
-p_s o_1 (\delta + p_s)(p_s - 1)^3 - p_s^2 o_1 (p_s - 1)(o_2 - 1)(\delta + p_s - 1) \\
-p_s^2 o_2 (p_s - 1)(o_1 - 1)(\delta + p_s - 1) - p_s o_2 (\delta + p_s)(p_s - 1)^2 (\delta + p_s - 1) \\
-p_s^2 o_2 (p_s - 1)(o_1 - 1)(\delta + p_s - 1) - p_s o_2 (\delta + p_s)(p_s - 1)^2 (\delta + p_s - 1)
\end{pmatrix}. \quad (\mathrm{B.1})
$$

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# Credit Risk Contagion in an Evolving Network Model Integrating Spillover Effects and Behavioral Interventions

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We introduce an evolving network model of credit risk contagion in the credit risk transfer (CRT) market. The model considers the spillover effects of infected investors, behaviors of investors and regulators, emotional disturbance of investors, market noise, and CRT network structure on credit risk contagion. We use theoretical analysis and numerical simulation to describe the influence and active mechanism of the same spillover effects in the CRT market. We also assess the reciprocal effects of market noises, risk preference of investors, and supervisor strength of financial market regulators on credit risk contagion. This model contributes to the explicit investigation of the connection between the factors of market behavior and network structure. It also provides a theoretical framework for considering credit risk contagion in an evolving network context, which is greatly relevant for credit risk management.

## 1. Introduction

Credit risk is the most important risk in the credit risk transfer (CRT) market, and one of the key issues is dealing with credit risk contagion [1–9]. Modeling credit risk contagion in the CRT network is an important yet challenging problem; credit risk modeling involves examining the role of counterparty risks [2, 4, 6, 9]. If a key investor is in financial distress or default, then any investor who is economically influenced by this given investor will be affected, including the providers and purchasers of credit derivatives and the banks with the investor's credit line. The direct correlation between firms caused by credit contagion leads to further complications in modeling the overall risk level, either portfolio or economy wide [3, 6, 9, 10].

In the CRT market, an intricate web of credit relations links a wide variety of counterparties in a complex system. If a key investor is in financial distress, then credit rating declines, or defaults, which will lead to credit risk contagion. Credit risk will also produce spillover effects of defaults for other investors with indirect correlations. The spillover effects

of credit risk contagion mainly come from the similarities in assets structure and in the effects of some behavior deviations of investors, including credit risk holders and financial market regulators. Thus, the behavioral factors of investors and financial market regulators, particularly investor sentiments, exert important spillover effects of credit risk contagion. The market behavioral approach recognizes that investors are not "rational" but "boundedly rational" and that systematic biases in their beliefs cause them to trade on nonfundamental information called "sentiment" [11]. Several financial economists also recognize that the market exhibits mood swings. The link between asset valuation and investor sentiment will soon become the subject of considerable deliberation among financial economists. Theories departing from rational asset pricing often posit the influence of investor sentiment [12], which leads to price fluctuation, and risk contagion generation. A number of theoretical studies offer models for establishing the relationship between investor sentiment and asset prices [12–15]. In these models, investors are categorized into two types, namely, rational arbitrageurs who are sentiment-free and irrational traders who are prone

to exogenous sentiment [16]. Baker and Stein [16] find that total sentiment, particularly the global component of total sentiment, is a contrarian predictor of country-level market returns. Baker and Wurgler [15, 17] predict that extensive waves of sentiment will exert greater effects on hard-to-arbitrage and hard-to-value stocks, which exhibit high "sentiment beta" [18]. Therefore, given that sentiment influences valuation, taking a position opposite to the prevailing market sentiment can be expensive and risky. Several theoretical studies show that investor sentiment is the most relevant factor in the decision-making domain, which primarily affects an investor's personal investment decisions [19]. Baker and Wurgler [17] pointed out that sentiment-based mispricing is based on the uninformed demand of several investors, noise traders, and a limit to arbitrage. Mispricing can be persistent given that the length of period during which overly optimistic and pessimistic noise traders will continue exerting buying or selling pressures is unknown. Similarly, numerous significant studies in this area are available [20–23]. Recently, theoretical studies have found that investor sentiment is contagious across markets [24], thus providing clues on how investor sentiment induces the spread of risk. The effects of the behaviors of credit risk investors have been a concern of credit risk contagion [4, 8, 9, 25–27]. This concern is also our motivation in considering the effect of the risk sentiment of credit risk investors on the evolving network of credit risk contagion. In addition, the behaviors of regulator and the ability of investors for risk resistance can decrease the influence of credit risk contagion [8, 9, 27, 28]. Thus, we also introduce them to analyze the effect of these factors on credit risk contagion. Our study enhances the understanding of the effects of behaviors of investors and regulators on credit risk contagion.

Given the significant development of complex network theory, a number of scholars have looked for evidence of contagion risk in the financial system which results from complex credit connections. The most well-known contribution to contagion analysis through direct linkages in the financial system is Allen and Gale [29]. This work demonstrates that the spread of contagion depends crucially on the pattern of interconnectedness among banks through a network structure involving four banks. Since the publication of this work, numerous scholars have applied the complex network theory to model the complex structure of the financial system and to analyze risk contagion in the financial system, particularly in banking systems. Several theoretical studies have found that the network structure is crucial to credit risk contagion [10, 30] (Chen et al., 2012), including random [31] (Chen et al., 2012) and tiered structures [30, 32–36]. These theoretical studies examine risk contagion in banking systems via direct linkages among banks, whereas others analyze risk contagion via indirect linkages [30, 37–41] (Jorion and Zhang, 2009). The aforementioned studies show that the network structure can significantly affect credit risk contagion. In our study, we consider the effect of the characteristics of the CRT network structure and behaviors of investors and regulators on credit risk contagion. Our objective is to understand the spillover effects of infected investors, behaviors of investors and regulators, emotional disturbance of investors,

market noise, and CRT network structure on credit risk contagion.

The rest of this paper is organized as follows. Section 2 presents some assumptions and notations for the following investigation. Section 3 defines the contagious process and feature of credit risk and builds an evolving network model of credit risk contagion in the CRT market. Section 4 uses stochastic dominance theory to theoretically study the effects of risk spillover, participants' behavioral factors, and the CRT network structure factors. Section 5 uses numerical simulations to deeply analyze and verify the effects of the aforementioned factors on credit risk contagion. Finally, Section 6 summarizes with some concluding remarks.

## 2. Notations and Assumptions

This study considers a network of credit risk contagion that evolves through the spillover effects of infected investors and behavioral interventions of investors and regulators. To model the evolving mechanism of credit risk contagion during credit risk transfer, we make the following assumptions. We assume that each node represents one individual investor engaged in the dealing of credit derivatives in the CRT market, and these investors are connected to each other. Thus, investors of the CRT market can use social network for representation. We also assume that the number of individuals $N$ is limited in the evolving network, $N = 1, 2, \ldots, n$. In order to simulate the actual situation of the CRT market, we further assume that the number of the direct connection edges of an investor with other investors is not less than 2, namely, the degree $k \geq 2$ of nodes in the CRT network of credit risk contagion.

In addition, we mark the main variables in this paper and describe their economic and financial meanings. Thus, the notation used in this paper has been summarized as follows:

(i) $\phi_k$ is the proportion of nodes infected with credit risk by other nodes in the cluster with the degree of nodes equal to $k$, and $\phi_k \in [0, 1]$.

(ii) $\alpha$ is the degree of the effect of market noises on investors. It is used to depict the influence mechanism of noise attribute on credit risk contagion when market noise attribute is consistent with the emotion, aspirations, or demand of the people. In addition, $\alpha \in [0, 1]$.

(iii) $\rho$ is the malicious attack strength of some institutional investors. $\rho$ indicates that some institutional investors maliciously trigger and strengthen the contagion effects of credit risk by distorting market information and making use of resource advantage. In addition, $\rho \in [0, 1]$.

(iv) $\beta_k$ is the inherent risk preference level of nodes with the degree of nodes equal to $k$, and $\beta_k \in [0, 1]$.

(v) $\theta_k$ is the resistance of nodes for credit risk contagion. $\theta_k$ reflects the risk resistance level and ability of investors under the state of credit risk contagion, and $\theta_k \in [0, 1]$.

(vi) $\delta$ is the supervisor strength of financial market regulators, and $\delta \in [0, 1]$.

(vii) $\eta$ is the initial fitness of credit risk contagion in the network. $\eta$ is chosen from a fitness distribution $f(\eta)$. It mainly refers to the strength of the impact of credit default behavior of one or a class of investors on others. In other words, $\eta$ indicates the contagious capacity of credit risk in the CRT network, and $\eta \in [0, 1]$.

(viii) $c_k$ is the emotional disturbance probability of nodes equal to $k$ for credit risk contagion, and $\eta > c_k \geq 0$, where $\partial c_k / \partial \rho > 0$, $\partial^2 c_k / \partial \rho^2 > 0$.

(ix) $\mu$ is the spillover effect of credit risk contagion of infected nodes. $\mu$ describes the degree of the effect of credit risk of infected investors on other investors that are not directly connected to the infected investors. However, a similar investment assets structure exists between the infected investors and the other investors that are not directly connected to the infected investors, where $\partial \mu / \partial c_k > 0$, $\partial^2 \mu / \partial c_k^2 < 0$, $\partial \mu / \partial \rho > 0$, $\partial^2 \mu / \partial \rho^2 > 0$. And $\mu \in [0, 1]$.

(x) $\lambda$ is the probability of infected nodes by credit risk to restore the health status. $\lambda$ indicates the official rescue strength, and $\lambda \in [0, 1]$.

## 3. Definition of the Evolving Network Model of Credit Risk Contagion

We begin by formally defining a dynamic evolving network model in the CRT market that considers the spillover effects of infected investors, behaviors, and emotional disturbance of investors and regulators, market noise, and the CRT network structure on credit risk contagion. Let $P(k)$ represent the probability distribution of nodes with the degree of nodes equal to $k$ in the dynamic evolving network. Then, the average degree $\langle k \rangle$ of the dynamic evolving network is as follows:

$$\langle k \rangle = \sum_k k P(k), \tag{1}$$

where $0 < k < n$.

In the CRT network, the initial fitness $\eta$ of credit risk contagion mainly depends on the average degree $\langle k \rangle$ of the CRT network, the probability distribution $P(k)$ of nodes that the degree of nodes is equal to $k$, and the proportion $\phi_k$ of infected nodes with credit risk in the cluster with the degree of nodes that is equal to $k$. Thus, the initial fitness $\eta$ of credit risk contagion in the dynamic evolving network is as follows:

$$\eta = \frac{\sum_k \phi_k k P(k)}{\langle k \rangle}. \tag{2}$$

In the actual financial market, investors are not rational but boundedly rational and systematic biases in their beliefs cause them to trade on nonfundamental information [11]. This will lead to credit asset price fluctuation and induce credit risk contagion generation. In fact, many theoretical studies have found that investor behaviors are contagious

across markets [24, 42–44], thus providing clues on how investor behavior induces the spread of risk. In the CRT market, the interactions of credit behavior among investors were more significant [4, 9, 25–27]. Certainly, the regulators' behaviors can restrain irrational behaviors of investors but can also increase the irrational behavior of investors and accelerate its contagion [9, 45–48]. Some literatures of behavioral finance and psychology also show that market noise can also further strengthen the irrational behavior of investors and accelerate its contagion (Aase et al., 2000; Barber and Odean, 2000; Shleifer, 2000; Tumarkin and Whitelaw, 2001; Barber et al., 2009; Gúegan, 2009) [8, 28]. In addition, investors' behaviors can also affect regulators' behaviors and decisions. Thus, in the social network, for investors with degree of nodes equal to $k$, with the increase in the risk preference level $\beta_k$ of investors, the contagion effect of credit risk will be intensified in the CRT network, and the emotional disturbance probability of investors and the spillover effect of credit risk contagion of infected nodes will also increase. However, the regulators' behaviors and the investors' ability of risk resistance can change the evolution trend. In other words, with the increasing supervision strength of financial market regulators and the investors' ability of risk resistance, the emotional disturbance probability of nodes and the spillover effect of credit risk contagion of infected nodes can also be reduced. Thus, we assume the fitness $\eta_k$ of credit risk contagion with the degree of nodes equal to $k$, the effect degree $\rho_k$ of the malicious attack of some institutional investors on other investors with the degree of nodes equal to $k$, and the spillover effect $\mu_k$ of credit risk contagion of infected nodes with the degree of nodes equal to $k$ could been written as follows:

$$\eta_k = \eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}$$
$$\rho_k = \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} \tag{3}$$
$$\mu_k = \mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)},$$

where $\eta_k$ depicts the contagion effect of infected investors on healthy investors and represents the change in the average density of infected investors in the CRT network. $\rho_k$ depicts some institutional investors who maliciously trigger or intensify the contagion effect of credit risk by distorting market information and making use of resource advantage. $\mu_k$ depicts the effect degree of the default behavior of infected nodes with the degree equal to $k$ on the other nodes that are not directly connected to infected nodes. Their parameter values are independent of $k$.

In addition, for the infected nodes by credit risk, the probability to restore the health status is opposite to the mechanism above. Thus, we assume that the probability $\lambda_k$ of nodes infected with credit risk by other nodes to restore the health status can be written as follows:

$$\lambda_k = \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}, \tag{4}$$

where $\lambda_k$ depicts the evolution behaviors that infected investors restored to health status by the effect of their

own internal and external factors. Its parameter value is independent of $k$.

In a recent series of literatures, the mean-field approach as a basic tool of dealing with the Markov process has been used to deal with the influence of different things [27, 28, 49–52]. It can convert a multidimensional problem into a low dimensional problem and is also considered as a very important theoretical analysis method in statistical physics. Eboli [53] shows that the infection mechanism in the financial system is similar to the physical phenomenon of network flow. Lopez [54] shows that this kind of problem can been described using the mean-field method. Based on

the existing literatures, we adopt the mean-field approach to describe the Markov process of credit risk contagion in the CRT network. Thus, we represent the model of credit risk contagion with the spillover effects of infected investors and behavioral intervention of investors and regulators as follows:

$$\frac{\partial \phi_k}{\partial t} = \rho_k \left(1 - \phi_k\right) \left[k\eta_k + \mu_k\right] - \phi_k \lambda_k. \tag{5}$$

For the contagion system of credit risk, represented by (5), let $\partial \phi_k / \partial t = 0$, and we will get the equilibrium point of the contagion system of credit risk as follows:

$$\phi_k = \frac{\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)} \left[k\left(\eta + c_k{}^{(1-\beta_k{}^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right) + \mu^{(1-\beta_k{}^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)}\right]}{\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)} \left[\mu^{(1-\beta_k{}^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)} + k\left(\eta + c_k{}^{(1-\beta_k{}^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right)\right] + \lambda^{(\beta_k{}^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}}. \tag{6}$$

Equation (6) is the equilibrium probability of the contagion system of credit risk, which describes the proportion of infected investors with credit risk by other investors with the degree of nodes equal to $k$ in the CRT network. Equation (6) describes the mechanism of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, the risk resistance ability $\theta_k$ of investors, the supervision strength $\delta$ of financial market regulators, the initial fitness $\eta$ of credit

risk contagion, the emotional disturbance probability $c_k$ of investor, the spillover effect $\mu$ of credit risk contagion of infected nodes, the probability $\lambda$ of infected nodes with credit risk by other nodes restored to the health status, and the degree $k$ of nodes on the proportion $\phi_k$ of infected nodes under the equilibrium status of the credit risk contagion system. Then, incorporating (6) into (2), we can get the following equation.

$$\eta^* = \frac{1}{\langle k \rangle} \sum_k \frac{kP(k) \left[k\left(\eta + c_k{}^{(1-\beta_k{}^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right) + \mu^{(1-\beta_k{}^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)}\right] \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)}}{\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)} \left[\mu^{(1-\beta_k{}^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)} + k\left(\eta + c_k{}^{(1-\beta_k{}^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right)\right] + \lambda^{(\beta_k{}^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}}. \tag{7}$$

Thus, we derive the fitness $\eta^*$ of credit risk contagion as (7) under the equilibrium status of the credit risk contagion system. Equation (7) describes the following factors under the equilibrium status of credit risk contagion system, namely, the effect mechanism of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, the risk resistance ability $\theta_k$ of investors, the supervision strength $\delta$ of financial market regulators, the fitness $\eta$ of credit risk contagion, the emotional disturbance probability $c_k$ of investor, the spillover effect $\mu$ of credit risk contagion of infected nodes, the probability $\lambda$ of nodes infected with credit risk by other nodes restored to the health status, the probability distribution $P(k)$ of nodes that the degree of nodes is equal to $k$, and the average degree $\langle k \rangle$ of the dynamic evolving network on the fitness $\eta^*$ of credit risk contagion in the CRT network.

## 4. Evolving Network Analysis of Credit Risk Contagion with Market Participants' Behavioral Factors and Network Structure in the CRT Market

We provide a theoretical analysis of the evolving network of credit risk contagion to study the effect of the effect degree

$\alpha$ of market noises, the risk preference level $\beta_k$ of investors, the resistance $\theta_k$ of investors for credit risk contagion, the supervision strength $\delta$ of financial market regulators, the initial fitness $\eta$ of credit risk contagion, the emotional disturbance probability $c_k$ of investor, the spillover effect $\mu$ of credit risk contagion of infected nodes, the probability $\lambda$ of nodes infected with credit risk by other nodes restored to the health status, the probability distribution $P(k)$ of nodes that the degree of nodes is equal to $k$, and the average degree $\langle k \rangle$ of the dynamic evolving network on the evolution behaviors of credit risk contagion in the CRT market.

### 4.1. Influence Mechanism of Market Participants' Behavioral Factors on Credit Risk Contagion

**Theorem 1.** *For the evolving network with degree equal to F, under the equilibrium status of credit risk contagion system, the evolving behavior of credit risk contagion exists with the following properties. (1) If $\mu = 0$ and $\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)}/\lambda^{(\beta_k{}^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} \leq \langle k \rangle/\langle k^2 \rangle$, then the credit risk contagion system exists only at the equilibrium point $\eta^*$, and $\eta^* = 0$. (2) If $\mu = 0$ and $\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k{}^2+\alpha^2)}/\lambda^{(\beta_k{}^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} > \langle k \rangle/\langle k^2 \rangle$, then the credit risk contagion system exists at two equilibrium points $\eta_1^*$*

*and $\eta_2^*$, and $\eta_1^* = 0$, $\eta_2^* > 0$. (3) If $\mu > 0$, then the credit risk contagion system exists only at the equilibrium point $\eta^*$, and $\eta^* > 0$.*

$$G(\eta) = \sum_k \frac{kP(k)\left[k\left(\eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right) + \mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)}\right]\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}}{\langle k \rangle \left[\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}\left[\mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)} + k\left(\eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right)\right] + \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}\right]}. \tag{8}$$

Let $A = (1 - \beta_k^2)(\ln(\delta^3 + 1) + \theta_k)/(1 + \alpha^2)$, $B = (\ln(\delta^3 + 1) + \theta_k)/(\beta_k^2 + \alpha^2)$. And $A > 0$ and $B > 0$, then (8) can be written as

$$G(\eta) = \sum_k \frac{kP(k)\left[k\left(\eta + c_k^A\right) + \mu_k^A\right]\rho_k^B}{\langle k \rangle \left[\rho_k^B\left[\mu_k^A + k\left(\eta + c_k^A\right)\right] + \lambda_k^{1/B}\right]}. \tag{9}$$

We can get $G'(\eta) = \sum_k (k^2 P(k)\rho_k^B \lambda_k^{1/B}/\langle k \rangle [\rho_k^B[\mu_k^A + k(\eta + c_k^A)] + \lambda_k^{1/B}]^2) > 0$; thus $G(\eta)$ is an increasing function of $\eta$. And we can also get $G''(\eta) = -\sum_k (2k^3 P(k)\rho_k^{2B}\lambda_k^{1/B}/\langle k \rangle [\rho_k^B[\mu_k^A + k(\eta + c_k^A)] + \lambda_k^{1/B}]^3) < 0$. Thus $G(\eta)$ is a concave function of $\eta$.

According to the above assumptions $\eta > c_k \geq 0$, we can get $G(0) = \sum_k (kP(k)\mu_k^A \rho_k^B/\langle k \rangle (\rho_k^B \mu_k^A + \lambda_k^{1/B}))$, $G(1) = \sum_k (kP(k)[k(1 + c_k^A) + \mu_k^A]\rho_k^B/\langle k \rangle [\rho_k^B[\mu_k^A + k(1 + c_k^A)] + \lambda_k^{1/B}]) < \sum_k (kP(k)[k(1 + c_k^A) + \mu_k^A]\rho_k^B/\langle k \rangle [\mu_k^A + k(1 + c_k^A)]\rho_k^B) = 1$. Thus when $\mu = 0$, the credit risk contagion system has at least one equilibrium point $\eta^* = 0$, but no more than two.

According to $G'(\eta) = \sum_k (k^2 P(k)\rho_k^B \lambda_k^{1/B}/\langle k \rangle [\rho_k^B[\mu_k^A + k(\eta + c_k^A)] + \lambda_k^{1/B}]^2)$, we can get $G'(\eta = 0)|_{\mu=0} = \sum_k (k^2 P(k)\rho_k/\langle k \rangle \lambda_k) = (\rho_k/\lambda_k)(\langle k^2 \rangle/\langle k \rangle)$. Thus when $\mu = 0$ and $\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}/\lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} \leq \langle k \rangle/\langle k^2 \rangle$, $G'(\eta = 0) \leq 1$, which means that the credit risk contagion system exists only at the equilibrium point $\eta^*$, and $\eta^* = 0$.

In the same way, when $\mu = 0$ and $\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}/\lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} > \langle k \rangle/\langle k^2 \rangle$, $G'(\eta = 0) > 1$. Thus the credit risk contagion system exists at two equilibrium points $\eta_1^*$ and $\eta_2^*$, and $\eta_1^* = 0$, $\eta_2^* > 0$.

According to the above, for $\mu > 0$, we can get $G(\eta = 0) > 0$ and $G(\eta = 1) < 1$. Thus the credit risk contagion system exists only at equilibrium point $\eta^*$, and $\eta^* > 0$. $\square$

**Corollary 2.** *With increasing market noises and risk preferences of investors, the contagion effect of credit risk and its spillover effect will be intensified, but the effectiveness of market supervision and official rescue will be crippled.*

*Proof.* According to (3) and (4), we can get $\partial c_k/\partial\alpha > 0$, $\partial\rho_k/\partial\alpha > 0$, $\partial\mu_k/\partial\alpha > 0$, $\partial\lambda_k/\partial\alpha < 0$. And $\partial c_k/\partial\beta > 0$, $\partial\rho_k/\partial\beta > 0$, $\partial\mu_k/\partial\beta > 0$, $\partial\lambda_k/\partial\beta < 0$. Thus we can get $\partial\phi_k/\partial\alpha > 0$, $\partial\phi_k/\partial\beta > 0$, $\partial G(\eta)/\partial\alpha > 0$, and $\partial G(\eta)/\partial\beta > 0$. Thus Corollary 2 is true. $\square$

**Corollary 3.** *With increasing supervision strength of financial market regulators, the contagion effect of credit risk and its spillover effect, the effect degree of market noises, and the*

*Proof.* Let

*malicious attack strength of institutional investors will be crippled. However, the effectiveness of the market supervision and official rescue will be enhanced, such that the recovery probability of investors who are from the infected status to health status will be enhanced.*

*Proof.* In the same way as Corollary 2, Corollary 3 can be proven. $\square$

*Conclusion 4.* When the degree of the similar investment asset structure among investors in the CRT market is lower, namely, the spillover effect $\mu = 0$ of credit risk contagion, and if the official rescue strength and the supervision strength of financial market regulators are higher, then the effect of credit risk contagion can be quickly controlled, and credit risk will not be contagious and diffusive. However, if the official rescue strength and the supervision strength of financial market regulators are lower, then the contagious and diffusion of credit risk will emerge. When the degree of the similar investment asset structure among investors in the CRT market is higher, namely, the spillover effect $\mu > 0$ of credit risk contagion, then the contagion effect of credit risk will emerge and be difficult to control.

*4.2. Influence Mechanism of Network Structure on Credit Risk Contagion.* In the above, we have analyzed and studied the influence mechanism of investors' behavior and financial market regulators' behavior on credit risk contagion in the CRT market. We obtained meaningful conclusions for controlling the contagion effects of credit risk. However, the different network structures will cause different market behaviors. Let $P(k)$ and $P'(k)$ represent the degree distribution of two different CRT networks. According to network stochastic dominance theory (e.g., [55–58]), if $P(k)$ strict first order stochastically dominates $P'(k)$, it is equivalent to having $\sum_k P(k)f(k) > \sum_k P'(k)f(k)$ for all monotone increasing function $f(k)$. If $P(k)$ strict second order stochastically dominates $P'(k)$, then it is equivalent to having $\sum_k P(k)f(k) > \sum_k P'(k)f(k)$ for all convex function $f(k)$. In addition, according to the network stochastic dominance theory, if $P(k)$ strict first order stochastically dominates $P'(k)$, then the network average degree of the degree distribution $P(k)$ is greater than the network average degree of the degree distribution $P'(k)$. If $P(k)$ strict second order stochastically dominates $P'(k)$, then the heterogeneity of the network of the degree distribution $P(k)$ is higher than the heterogeneity of the network of the degree distribution $P'(k)$. Thus, we assume the contagion fitness $\eta^* > 0$ and the proportion $\phi^* > 0$ of infected investors are at the equilibrium point of the system

of credit risk contagion in the CRT network, then we can get the following theorems by using the network stochastic dominance theory in this work.

**Theorem 5.** *CRT networks A and B have degree distributions equal to $P(k)$ and $P'(k)$, respectively. If $P(k)$ strict first order stochastically dominates $P'(k)$, then $\eta_A^* > \eta_B^*$ in the same conditions.*

*Proof.* We assume Theorem 5 is untenable, then $\eta_A^* \leq \eta_B^*$ is tenable.

Let

$$f(k) = \frac{k\left[k\left(\eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right) + \mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)}\right]\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}}{\langle k\rangle\left[\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}\left[\mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)} + k\left(\eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}\right)\right] + \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}\right]}. \tag{10}$$

Namely,

$$f(k) = \frac{k(k\eta_k + \mu_k)\rho_k}{\langle k\rangle[\rho_k(k\eta_k + \mu_k) + \lambda_k]}. \tag{11}$$

Then, we can obtain $\partial f(k)/\partial k = ((k\eta_k + \mu_k)^2\rho_k^2 + \rho_k\lambda_k(2k\eta_k + \mu_k))/\langle k\rangle[\rho_k(k\eta_k + \mu_k) + \lambda_k]^2 > 0$, and $\partial^2 f(k)/\partial k^2 = 2\eta_k\rho_k\lambda_k(\rho_k\mu_k + \lambda_k)/\langle k\rangle[\rho_k(k\eta_k + \mu_k) + \lambda_k]^3 > 0$. Thus $f(k)$ is a monotone increasing convex function for all $k \geq 2$. According to the network stochastic dominance theory, $\sum_k P(k)f(k) > \sum_k P'(k)f(k)$ when $P(k)$ strict first order stochastically dominates $P'(k)$, namely, for all $\eta > 0$ having

$$G_p(\eta) > G_{p'}(\eta). \tag{12}$$

According to Theorem 1, $G_p(\eta) \in [0, 1]$. We assume $\eta_A^*$ and $\eta_B^*$ are the equilibrium point of the system of credit risk contagion in the CRT networks $A$ and $B$, and $\eta_A^* > 0, \eta_B^* > 0$, thus for all $\eta \in (\eta_A^*, 1]$ having $\eta \geq G_p(\eta)$. According to the assumption $\eta_B^* \geq \eta_A^*$ is tenable, we can get

$$\eta_B^* \geq G_p(\eta_B^*). \tag{13}$$

According to (12), we can get

$$\eta_B^* \geq G_p(\eta_B^*) > G_p'(\eta_B^*). \tag{14}$$

Namely,

$$\eta_B^* > G_p'(\eta_B^*). \tag{15}$$

However, $\eta_B^* = G_p'(\eta_B^*)$ for $\eta_B^* > 0$ is the equilibrium point of the system of credit risk contagion in the CRT network $B$. Thus the assumption $\eta_B^* \geq \eta_A^*$ is untenable; namely, Theorem 5 is tenable.                                  □

**Theorem 6.** *CRT networks A and B have degree distributions equal to $P(k)$ and $P'(k)$, respectively. If $P(k)$ strict first order stochastically dominates $P'(k)$, then $\phi_A^* > \phi_B^*$ in the same conditions.*

*Proof.* According to (6), we can derive the proportion $\phi_k$ of infected investors that the degree of investors is equal to $k$ as follows:

$$\phi_k = \frac{(k\eta_k + \mu_k)\rho_k}{\rho_k(k\eta_k + \mu_k) + \lambda_k}, \tag{16}$$

where $\eta_k = \eta + c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)}$, $\rho_k = \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)}$, $\mu_k = \mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)}$, and $\lambda_k = \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}$.

Then we can obtain $\partial\phi_k/\partial k = \eta_k\rho_k\lambda_k/[\rho_k(k\eta_k + \mu_k) + \lambda_k]^2 > 0$. Thus $\phi_k$ is a monotone increasing function for all $k \geq 2$. Since $P(k)$ strict first order stochastically dominates $P'(k)$, then we can derive

$$\sum_k \phi_k^{B^*} P(k) > \sum_k \phi_k^{B^*} P'(k). \tag{17}$$

In addition, due to $\partial\phi_k/\partial\eta = k\rho_k\lambda_k/[\rho_k(k\eta_k + \mu_k) + \lambda_k]^2 > 0$, thus $\phi_k$ is a monotone increasing function for all $\eta > 0$. According to Theorem 5, if $P(k)$ strict first order stochastically dominates $P'(k)$, then $\eta_A^* > \eta_B^*$. Thus for all $k \geq 2$, we can get

$$\phi_k^{A^*} > \phi_k^{B^*}. \tag{18}$$

Equation (18) is equivalent to having $\sum_k \phi_k^{A^*} > \sum_k \phi_k^{B^*}$ for all $k \geq 2$. Thus we can get

$$\sum_k \phi_k^{A^*} P(k) > \sum_k \phi_k^{B^*} P(k). \tag{19}$$

Thus we can obtain

$$\sum_k \phi_k^{A^*} P(k) > \sum_k \phi_k^{B^*} P'(k). \tag{20}$$

Namely, $\phi_A^* > \phi_B^*$. Thus Theorem 6 is tenable.      □

*Conclusion 7.* Under the same conditions of investor behavior and market supervision, the greater the average degree of CRT network is, the higher the contagion fitness of credit risk and the proportion of infected investors in the CRT network. In other words, the more dense the CRT network is, the higher the similarities are in terms of investment asset structure, the convergence effect of investor behaviors, and

the complexity of market regulation. Thus, the more dense the CRT network is, the greater the influence of the investors' irrational behaviors, the lower the efficiency of the market regulation, and the more significant the contagion effect of credit risk. In addition, the greater the heterogeneity of the CRT network, the higher the contagion fitness of credit risk in the CRT network.

**Theorem 8.** *CRT networks A and B have degree distributions equal to $P(k)$ and $P'(k)$, respectively. If $P(k)$ strict second order stochastically dominates $P'(k)$, then $\xi$ ($0 < \xi < 1$) is obtained as follows. (1) When $\langle k \rangle / \langle k^2 \rangle < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} < \xi$, and $\mu^{(1-\beta_k^2)[\ln(\delta^3+1)+\theta_k]/(1+\alpha^2)} \rightarrow 0$, then $\phi_A^* > \phi_B^*$ for all $k \geq 2$. (2) When $\langle k \rangle / \langle k^2 \rangle < \xi < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}$, then $\phi_A^* < \phi_B^*$ for all $k \geq 2$.*

*Proof.* According to (16), we can get

$$\left( \rho_k \mu_k + \lambda_k \right) \phi_k P(k) = \eta_k \rho_k k P(k) + \rho_k \mu_k P(k) \\ - \eta_k \rho_k k \phi_k P(k). \quad (21)$$

In (16) and (21), we know the variables $\rho_k$, $\mu_k$, $\lambda_k$, and $\eta_k$ are not functions of $k$; that is, these parameter values are independent of $k$. Thus, we can further derive

$$\left( \rho_k \mu_k + \lambda_k \right) \sum_k \phi_k P(k) \\ = \eta_k \rho_k \sum_k k P(k) + \rho_k \mu_k - \eta_k \rho_k \sum_k k \phi_k P(k). \quad (22)$$

Putting (1) and (2) into (22), we can obtain

$$\phi = \frac{\langle k \rangle \rho_k \left( \eta_k - \eta \eta_k \right) + \rho_k \mu_k}{\left( \rho_k \mu_k + \lambda_k \right)}. \quad (23)$$

Thus we can obtain $\partial \phi / \partial \eta$ as follows:

$$\frac{\partial \phi}{\partial \eta} = \frac{\langle k \rangle \rho_k \left( 1 - 2\eta - c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)} \right)}{\left( \rho_k \mu_k + \lambda_k \right)}. \quad (24)$$

Thus $\phi$ is an increasing function of $\eta$ for all $\eta < (1 - c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)})/2$. And $\phi$ is a decreasing function of $\eta$ for all $\eta > (1 - c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)})/2$.

According to (10) $\partial f(k)/\partial k = ((k\eta_k + \mu_k)^2 \rho_k^2 + \rho_k \lambda_k (2k\eta_k + \mu_k))/\langle k \rangle [\rho_k(k\eta_k + \mu_k) + \lambda_k]^2 > 0$, and $\partial^2 f(k)/\partial k^2 = 2\eta_k \rho_k \lambda_k (\rho_k \mu_k + \lambda_k)/\langle k \rangle [\rho_k(k\eta_k + \mu_k) + \lambda_k]^3 > 0$. Thus $f(k)$ is a monotone increasing convex function for all $k \geq 2$. According to the network stochastic dominance theory, if $P(k)$ strict second order stochastically dominates $P'(k)$, then it is equivalent to having $\sum_k P(k) f(k) > \sum_k P'(k) f(k)$ for all convex function $f(k)$. According to Theorem 5, we can get $\eta_A^* > \eta_B^*$.

According to Theorem 1, we know that with $\xi$ ($0 < \xi < 1$), when $\langle k \rangle / \langle k^2 \rangle < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} < \xi$, we can get $\eta < (1 - c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)})/2$. And when $\langle k \rangle / \langle k^2 \rangle < \xi < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}$, we can get $\eta > (1 - c_k^{(1-\beta_k^2)(\ln(\delta^3+1)+\theta_k)/(1+\alpha^2)})/2$. With $\xi$ ($0 < \xi < 1$), when $\langle k \rangle / \langle k^2 \rangle < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} < \xi$, $\phi$ is an increasing function of $\eta$. When $\langle k \rangle / \langle k^2 \rangle < \xi < \rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)}$, $\phi$ is a decreasing function of $\eta$. Thus we can get that Theorem 8 is tenable. □

*Conclusion 9.* In the case of $\rho^{(\ln(\delta^3+1)+\theta_k)/(\beta_k^2+\alpha^2)} / \lambda^{(\beta_k^2+\alpha^2)/(\ln(\delta^3+1)+\theta_k)} > \langle k \rangle / \langle k^2 \rangle$, the effects of network heterogeneity on the contagion scale of credit risk depend on the interaction of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, the resistance $\theta_k$ of investors for credit risk contagion, the supervision strength $\delta$ of financial market regulators, the spillover effect $\mu$ of credit risk contagion of infected nodes, and the probability $\lambda$ of nodes infected with credit risk by other nodes restored to the health status. First, decreasing the similarity of investment asset structure, the contagion scale of credit risk will be reduced. However, network heterogeneity promotes the contagion level and scale of credit risk, whereas network homogeneity can decrease the contagion level and scale of credit risk. Second, the malicious attack and market noises can promote the contagion level and scale of credit risk.

## 5. Simulation Analysis of the Evolving Network Model of Credit Risk Contagion

Given the absence of a large amount of time series data for empirical tests, numerical simulation analysis is the most effective testing method. Such analysis is conducted by considering the different values of the parameters in the evolving network model of credit risk contagion. The following are assumed: the number of investors $N = 10000$ in the CRT market. We choose the WS small world network and the BA network to conduct numerical simulation, where the probability $p = 0.01$ of the long distance connection of investors in the WS small world network and the degree distributions in the BA network are $P(k) = 2m^2/k^3$. Thus we can find the effects of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, the resistance $\theta_k$ of investors for credit risk contagion, the supervision strength $\delta$ of financial market regulators, the initial fitness $\eta$ of credit risk contagion, the probability $c_k$ of a randomly chosen old node being deleted from the network that degree of nodes is equal to $k$, the spillover effect $\mu$ of credit risk contagion of infected investors, the probability $\lambda$ of infected investors with credit risk by other nodes restored to the health status, the degree $k$ of investors, the average degree $\langle k \rangle$ of the dynamic evolving network, and the network structure of credit risk contagion on credit risk contagion in the CRT market. Furthermore, with investor behavior and the financial market
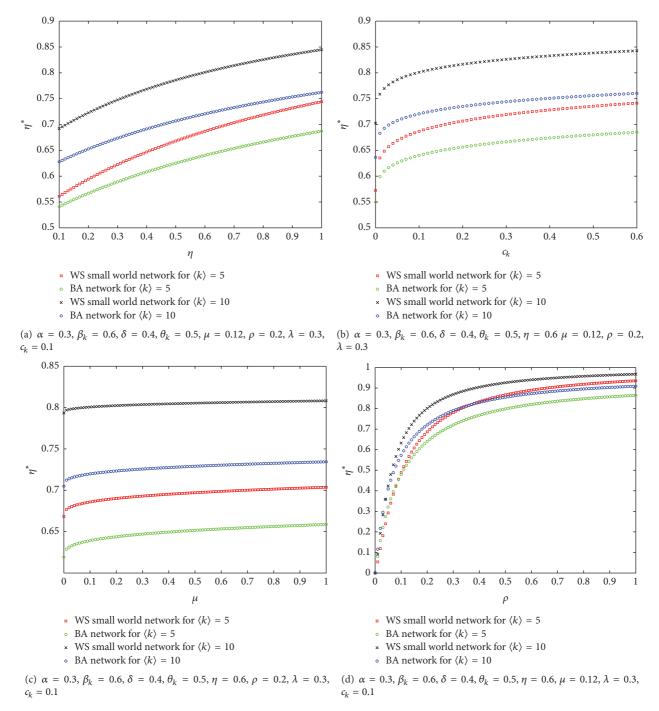
Figure 1: The evolution function of the equilibrium point $\eta^*$ of credit risk contagion system as function in the initial fitness $\eta$ of credit risk contagion, the emotional disturbance probability $c_k$ of investor, the spillover effect $\mu$ of credit risk contagion of infected investor, and the malicious attack strength $\rho$ of some institutional investors under the different network structure.

regulators' behaviors, we analyze the evolving properties of the proportion of infected investors $\phi$, the global fitness $\eta$ of credit risk of the contagion network, and the individual fitness $\eta_k$ of infected investors that the degree of nodes is equal to $k$. In the numerical simulations, we initialize the contagion network with $m_0 = 10$ nodes being infected with credit risk.

In Figure 1, the equilibrium point $\eta^*$ of the credit risk contagion system is a concave function of the initial fitness $\eta$ of credit risk contagion, the emotional disturbance probability $c_k$ of investor, the spillover effect $\mu$ of credit risk contagion of infected investor, and the malicious attack strength $\rho$ of some institutional investors. The malicious attack strength $\rho$ of some institutional investors is more significant. The reason is

that the malicious attack of some institutional investors cause market information confusion. It brings about irrational behavior among the majority of small- and medium-sized investors. It also shows that the disturbance effect of private information is more significant on credit risk contagion in the semi-strong valid market. Second, Figure 1 also shows that the contagion rate has a significant positive correlation with network density and has a significant negative correlation with the network heterogeneity. The reason is that the more dense the credit network is, the more significant the interaction between counterparties is. However, network heterogeneity hampers the interaction between investors. Thus the higher the network heterogeneity is, the more unfavorable it is to the credit risk contagion in the CRT market. In the same case of network density, the effect of the network heterogeneity is the most significant on credit risk contagion by inducing in the spillover effect of credit risk contagion of infected investor. This also confirms Conclusions 4 and 7.

In fact, credit risk contagion is a complex process in the CRT market. The process is mixed with complex interactions of the behaviors of counterparties and financial market regulators, and market noises. This adds to the probability of the uncertainty, unpredictability, and uncontrollability of credit risk contagion. Figure 2 shows the following factors under the different network structure, namely, the interaction mechanism of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, and the supervision strength $\delta$ of financial market regulators on the equilibrium point $\eta^*$ of the credit risk contagion system. First, Figure 1 shows the differential effect of the network heterogeneity. The lower the network heterogeneity, the more significant the reciprocal effects of the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, and the supervision strength $\delta$ of financial market regulators on credit risk contagion. Second, market noises and the risk preference of investors have a strengthening effect, and the supervision behaviors of financial market regulators exert weakening effects. Figures 2(a) and 2(b) show the mutual reinforcing effects between market noises and the risk preference of investors. Namely, with increasing market noises, the effects of the risk preference of investors on the infectious rate of credit risk will be promoted. With increasing risk preference of investors, the effects of the market noises on the infectious rate of credit risk will be also promoted. In Figures 2(c), 2(d), 2(e), and 2(f), we found the supervision behaviors of financial market regulators will reduce the effect of the market noises and the risk preference of investors on credit risk contagion. With increasing supervision strength $\delta$ of financial market regulators, the effects of the market noises and the risk preference of investors on the infectious rate of credit risk will be reduced. Thus, the behaviors of counterparties and the market noises can promote the contagious rate of credit risk and have the mutual reinforcing effects on the contagious speed of credit risk in the CRT market. However, the supervision behaviors of financial market regulators will weaken the effects of the external disturbance factors and reduce the contagious speed of credit

risk in the CRT market. This confirms Corollaries 2 and 3.

Figures 3 and 4 depict the effect mechanism of the initial contagious fitness of credit risk, the spillover effect of credit risk contagion of infected investor, the emotional disturbance probability of investor, the malicious attack strength of institutional investors, and the official rescue strength on the contagious scale of the credit risk in the CRT market under the different network structures. First, Figures 3 and 4 show that the contagious scale of credit risk has a significant positive correlation with network density. The sparser the CRT network, the weaker the effect of the CRT network heterogeneity on the contagious scale of credit risk. However, with increasing average degree of the CRT network, the effect of the CRT network heterogeneity on the contagious scale of credit risk will be significantly promoted. When the average degree of the CRT network is greater than a certain threshold, the higher the CRT network heterogeneity, the greater the contagious scale of credit risk. Second, Figure 4(b) shows that the official market rescue will restrain the contagious scale of credit risk. With increasing official rescue strength, the contagious scale of credit risk will be reduced. On the contrary, with increasing network density, the efficiency of the official market rescue will be reduced. When the average degree of the CRT network is greater than a certain threshold, the higher the CRT network heterogeneity, the lower the efficiency of the official market rescue.

Figure 5 depicts the effects of the risk preference of investors, the effect degree of market noises, the supervisor strength of financial market regulators, and the risk resistance ability of investors on credit risk contagion. Figure 5(a) shows that when the risk preference of investors is infinitesimally small, credit risk contagion will be controlled, and the contagious scale of credit risk will be also infinitesimally small. With increasing risk preference level $\beta_k$ of investors, credit risk contagion presents the concavity evolution of monotone increasing. However, when the risk preference of investors is greater than a certain threshold, credit risk contagion presents the convexity evolution of monotone increasing. Figure 5(b) shows that when the market noise is smaller than a certain threshold, credit risk contagion presents the concavity evolution of monotone increasing along with the increasing of the effect degree of market noises. In contrast, when the market noise is greater than the threshold, credit risk contagion presents the convexity evolution of monotone increasing along with the increasing of the effect degree of market noises. In addition, Figures 5(a) and 5(b) also depict, with increasing network density, the contagion effect of credit risk which is amplified and the effects of the CRT network heterogeneity which will be significantly promoted. When the average degree of the CRT network is greater than a certain threshold, the higher the CRT network heterogeneity, the more significant the effects of the risk preference of investors and the market noises on credit risk contagion. Figures 5(c) and 5(d) depict the inhibiting effect of the supervisor strength of financial market regulators and the risk resistance ability of investors

(a) WS small world network for $\langle k \rangle = 5$

(b) BA network for $\langle k \rangle = 5$

(c) WS small world network for $\langle k \rangle = 5$

(d) BA network for $\langle k \rangle = 5$

(e) WS small world network for $\langle k \rangle = 5$

(f) BA network for $\langle k \rangle = 5$
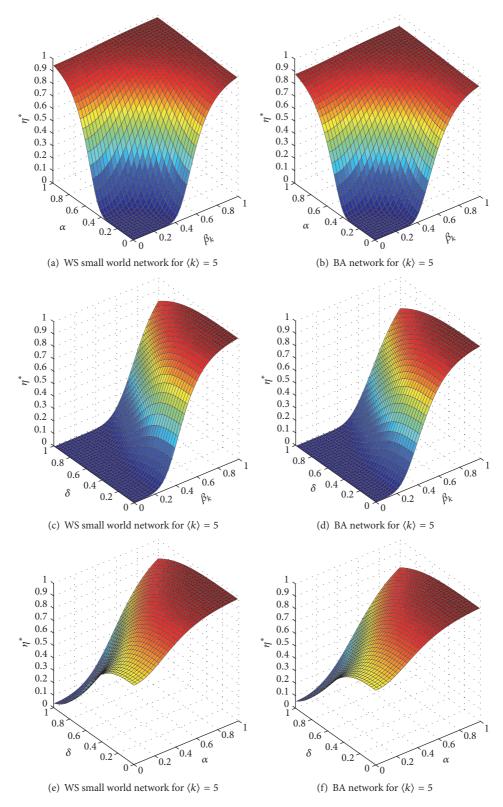
FIGURE 2: The influencing mechanism of the interaction among the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, and the supervision strength $\delta$ of financial market regulators on the equilibrium point $\eta^*$ of the credit risk contagion system under the different network structure. (a) and (b) for $\delta = 0.4, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$; (c) and (d) for $\alpha = 0.3, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$; (e) and (f) for $\beta_k = 0.6, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$.

(a) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$

(b) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$

(c) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.3$, $\rho = 0.2$, $\lambda = 0.3$

FIGURE 3: The evolution function of the contagious scale $\phi$ of credit risk as function in the initial fitness $\eta$ of credit risk contagion, the spillover effect $\mu$ of credit risk contagion of infected investor, and the emotional disturbance probability $c_k$ of investor under the different network structure. (a) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$; (b) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$; (c) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$.

on credit risk contagion. With increasing network density, the inhibiting effect of the supervision of financial market regulators and the risk resistance ability of investors on credit risk contagion will be reduced. When the average degree of the CRT network is greater than a certain threshold, the higher the CRT network heterogeneity, the lower the control efficiency of the supervision of financial market regulators and the risk resistance ability of investors on credit risk contagion.

Figure 6 depicts the reciprocal effect of the market noises, the risk preference of investors, and the supervisor strength of financial market regulators on credit risk

(a) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\lambda = 0.3$, $c_k = 0.1$

(b) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $c_k = 0.1$
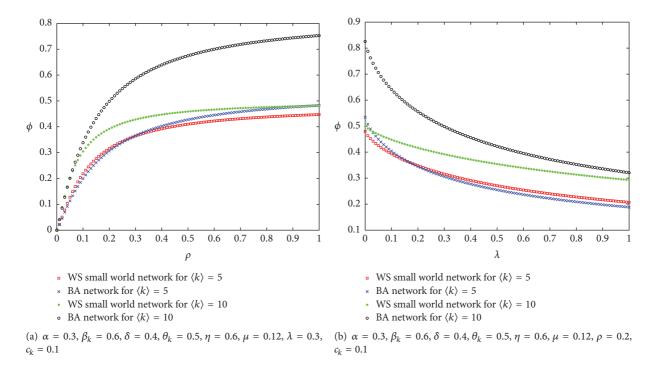
FIGURE 4: The evolution function of the contagious scale $\phi$ of credit risk as function in the malicious attack strength $\rho$ of some institutional investors and the official rescue strength $\lambda$ under the different network structure. (a) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\lambda = 0.3$, $c_k = 0.1$; (b) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $c_k = 0.1$.

contagion. First, Figure 6 shows that the higher the CRT network heterogeneity, the more significant the reciprocal effect of the market noises and the risk preference of investors on credit risk contagion. Second, the reciprocal effect of the market noises and the risk preference of investors will promote the contagious scale of credit risk in the CRT market. However, the supervisor strength of financial market regulators will reduce the effect of the market noises and the risk preference of investors on credit risk contagion.

## 6. Conclusion

In this paper, we design an evolving network model of credit risk contagion that considers the spillover effects of infected investors, behaviors and emotional disturbance of investors and regulators, market noise, and the CRT network structure on credit risk contagion. We use theoretical analysis and numerical simulation to investigate the effect mechanism of the spillover effects and behavioral intervention on credit risk contagion in the CRT market. We find the strengthening effects of the spillover effects of infected investors, the emotional disturbance of investors and the malicious attack behaviors of some institutional investors, the restraining effects of the official market rescue and the risk resistance ability of investors for credit risk contagion, and the density effects and heterogeneous effects of the CRT network on credit risk contagion. In addition, we also investigate the

reciprocal effects of the market noises, the risk preference of investors, and the supervisor strength of financial market regulators on credit risk contagion. We further find the interactive facilitation effect of the market noises and the risk preference of investors on credit risk contagion, and the restraining effects of the supervisor strength of financial market regulators on credit risk contagion. Certainly, we acknowledge several limitations in the modeling method and process, and the testing method and way. Due to these limitations, the investigation results in the paper are considered exploratory and suggestive rather than conclusive. Therefore, future studies can further deepen and expand the results presented in this paper.

## Disclosure

Tingqiang Chen and Binqing Xiao are co-first authors.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Acknowledgments

(a) $\alpha = 0.3$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$

(b) $\beta_k = 0.6$, $\delta = 0.4$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$

(c) $\alpha = 0.3$, $\beta_k = 0.6$, $\theta_k = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$

(d) $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$
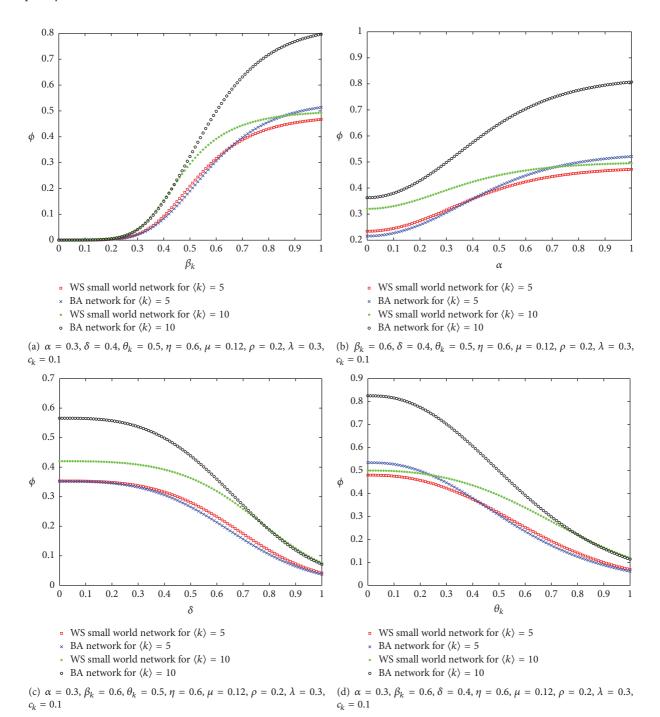
FIGURE 5: The evolution function of the contagious scale $\phi$ of credit risk as function in the risk preference level $\beta_k$ of investors, the effect degree $\alpha$ of market noises, the supervision strength $\delta$ of financial market regulators, and the resistance $\theta_k$ of investors for credit risk contagion. (a) for $\alpha = 0.3$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$; (b) for $\beta_k = 0.6$, $\delta = 0.4$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$; (c) for $\alpha = 0.3$, $\beta_k = 0.6$, $\theta = 0.5$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$; (d) for $\alpha = 0.3$, $\beta_k = 0.6$, $\delta = 0.4$, $\eta = 0.6$, $\mu = 0.12$, $\rho = 0.2$, $\lambda = 0.3$, $c_k = 0.1$.

(a) WS small world network for $\langle k \rangle = 5$

(b) BA network for $\langle k \rangle = 5$

(c) WS small world network for $\langle k \rangle = 5$

(d) BA network for $\langle k \rangle = 5$

(e) WS small world network for $\langle k \rangle = 5$

(f) BA network for $\langle k \rangle = 5$

FIGURE 6: The influencing mechanism of the interaction among the effect degree $\alpha$ of market noises, the risk preference level $\beta_k$ of investors, and the supervision strength $\delta$ of financial market regulators on the contagious scale $\phi$ of credit risk under the different network structure. (a) and (b) for $\delta = 0.4, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$; (c) and (d) for $\alpha = 0.3, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$; (e) and (f) for $\beta_k = 0.6, \theta = 0.5, \eta = 0.6, \mu = 0.3, \rho = 0.2, \lambda = 0.3, c_k = 0.1$.

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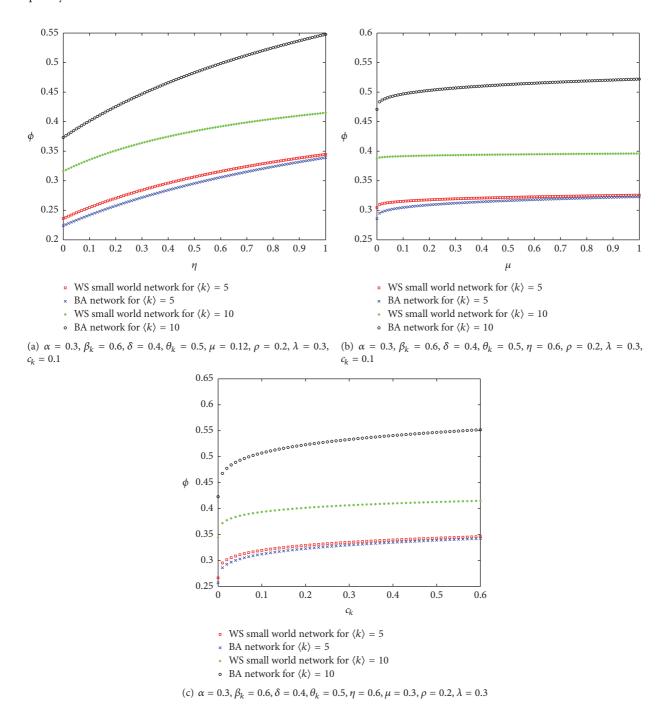
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*Research Article*
# The Dynamic Cross-Correlations between Mass Media News, New Media News, and Stock Returns

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We investigate the dynamic cross-correlations between mass media news, new media news, and stock returns for the SSE 50 Index in Chinese stock market by employing the MF-DCCA method. The empirical results show that (1) there exist power-law cross-correlations between two types of news as well as between news and its corresponding SSE 50 Index return; (2) the cross-correlations between mass media news and SSE 50 Index returns show larger multifractality and more complicated structures; (3) mass media news and new media news have both complementary and competitive relationships; (4) with the rolling window analysis, we further find that there is a general increasing trend for the cross-correlations between the two types of news as well as the cross-correlations between news and returns and this trend becomes more persistent over time.

## 1. Introduction

According to the efficient market hypothesis, the security market could reflect the information instantly. However, there are many anomalies showing that the exogenous information plays an important role in the stock market. News, for example, as one type of the exogenous information, has been intensively investigated for its influence on the stock market, including the relation between the news and stock prices [1, 2], stock returns [3], trading volumes [4], investors' behavior [5, 6], and the correlation between the inherent sentiment behind the news and the stock market [7]. With the development of the Internet, the web has become one of the most important sources of news. Therefore, the news is classified into traditional mass media news and new media news according to the source of the news (as for the definition, see https://en.wikipedia.org/wiki/New_media and https://en.wikipedia.org/wiki/Mass_media). The new source of the news brings new features to the news. Through the Internet, the new media news diffuses more quickly and can be easily obtained more conveniently. Compared to the new media news, the mass media news is more rigorous and we can get more insightful ideas through the mass media news. So the emergence of the new source of news leads

to new directions for the financial research. One could be the correlation of the different news' sources, especially the correlation among them when they are referred to as the factors influencing the stock market, as well as the different roles they play in the stock market. Since the amount of daily news can be seen as the intensity of the information [8], it is meaningful to explore the correlation of the amount of news.

The most commonly used measure of correlation is the Pearson correlation. To use this measure, the time series must be stationary and follow the Gaussian distribution. But the news amount series in stock market are not all the series with stationarity and Gaussian distribution. In order to overcome these limits, existing literatures have engaged in exploring measures to investigate long-range cross-correlation between two time series with nonstationary and non-Gaussian distribution. Based on the method of detrended fluctuation analysis (DFA) introduced by Peng et al. [9] focusing on the long-range autocorrelation of the DNA series, Podobnik and Stanley [10] proposed the method of detrended cross-correlation analysis (DCCA) to explore the long-range cross-correlation between two nonstationary time series. Then Zhou [11] introduced the multifractal detrended cross-correlation analysis (MF-DCCA or called MF-DXA) to investigate the cross-correlation by considering

the multifractal features. With these analyses, this paper is engaged in investigating the cross-correlations between the amount of mass media news and the amount of new media news as well as the cross-correlation of the different types of news and the corresponding stock returns by the methodology of MF-DCCA. Investigations on such relationships are conducive to the understanding of informational efficiency in financial market.

The remainder of the paper is organized as follows. Section 2 reviews the existing literatures about the news and the stock markets as well as the methodology of MF-DCCA. Section 3 describes the sample data used in this paper and its selected process. Section 4 introduced the detailed methodology of MF-DCCA. Section 5 presents the empirical results of this paper and Section 6 draws the conclusion.

## 2. Literature Review

*2.1. News and Stock Market Performances.* Existing literature on the relationships between financial news and stock market behavior can be divided into two categories. The first refers to the investigation on the impacts of mass media news, that is, news extracted from newspapers, television and advertising, returns, trading volume, liquidity, and volatility [1, 4, 7, 12–17]. We mainly find that the majority of these literatures focus on earnings announcement, the number of the headlines, and the expenditures on advertising, but only a few employ the information content. The second refers to the investigation on the relationships between new media news and return predictability as well as market dynamics [18–31]. This line of work has shifted research interests away from building more complicated models to attach more importance to data and its impacts on market dynamics.

*2.2. MF-DCCA.* The existing literatures about the detrended cross-correlation analysis can be approximately divided into two parts. One engaged to extend and modify the methodology, and a series of extended methods have been introduced, such as the methods of MF-X-DMA [32], MF-DHA [33], and MF-X-PF [34]. These methods all improve the efficiency of the method of MF-DCCA to some extent. Another part of literatures concentrated on the cross-correlation between the time series in the financial market or across the markets. Wang et al. [35] found that the multifractal analysis significantly cross-correlates between Chinese A-share and B-share by the method of MF-DCCA. Cao et al. [36] investigated the multifractal detrended cross-correlation between the Chinese exchange market and stock market drawing the similar conclusions. Gu et al. [37] used the MF-DCCA to prove that there is different performance for the cross-correlation between the multifractality and the market efficiency before and after the equity division reform. Lu et al. [38] investigated the dynamic relationship between Japanese Yen exchange rates and market anxiety, finding that the cross-correlation exhibits different volatility. Li et al. [39] studied the cross-correlation between the crude oil and exchange rate markets finding that their cross-correlation are sensitive to sudden events.
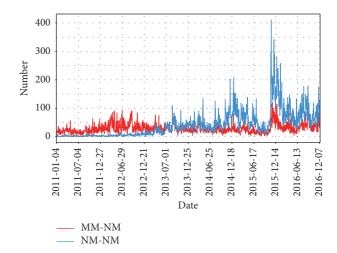


Figure 1: The daily amounts of news from mass media and new media.

## 3. Data Description

We get daily news data from the column of Security Information and News in RESSET financial database in China in this paper. The stocks corresponding to the news sample are the constituent stocks of the Chinese Stock Market Index 50 (SSE 50). The daily returns of SSE 50 are also obtained from the RESSET financial database in China. The stocks of SSE 50 are the most active stocks with the highest market value and the highest liquidity in Chinese stock markets. So they can represent the Chinese stock market well and have enough amount of news to investigate the cross-correlation between the two types of news and the cross-correlation between news and return. Since the constituent stocks are adjusted every half year, we collect all the stocks once they are selected as the constituent stocks of SSE 50. And then we removed the stocks which are delisted. The whole sample period extends from January 1, 2011 to December 31, 2016 with daily observations. And the total number of the stocks used in this paper is 532 across this period.

We classify the news into mass media news and new media news based on the selected process used by Zhang et al. [40] according to the source names of the news. After conducting the selected process, the numbers of mass media sources and new media sources are 265 and 334, respectively. Since the constituent stocks of SSE 50 vary over time and our aim is to investigate the cross-correlations between the amount of news and the return of SSE 50, the daily news used in this paper is just the news of the stocks belonging to SSE 50 on that day. Figure 1 illustrates the daily amount of the two types of news from January 2011 to December 2015. The red line is the amount of daily mass media news (MM) and the blue line is the amount of daily new media news (NM). We can find a stable trend for the amount of mass media news across the sample period, while there is an increasing trend for the amount of new media news on the whole. And the quantitative relationship of the two types of news reverses around the year of 2013. After 2013, the amount of new media news exceeds the amount of mass media news.

## 4. Empirical Methodology

To explore the cross-correlation between two nonstationary time series, various approaches have been introduced in existing literatures. In this paper, We followed the methodology of multifractal detrended cross-correlation analysis introduced by Zhou [11]. Consider any two time series $\{x_i\}$ and $\{y_i\}$, $i = 1, 2, \ldots, N$, that have the equal length $N$. Then the procedure of the MF-DCCA can be described as follows.

*Step 1.* We construct two "profiles" of the two time series and get new series:

$$
\begin{aligned}
X_i &= \sum_{k=1}^{i} (x_k - \overline{x}), \\
Y_i &= \sum_{k=1}^{i} (y_k - \overline{y}),
\end{aligned}
\tag{1}
$$

$$
i = 1, 2, \ldots, N,
$$

where $\overline{x} = (1/N) \sum_{k=1}^{N} x_k$ and $\overline{y} = (1/N) \sum_{k=1}^{N} y_k$.

*Step 2.* Divide the two profiles into $N_s = [N/s]$ nonoverlapping windows of equal length $s$, respectively. If the length $N$ of the time series is not a multiple of the considered time scale $s$, there will remain a short part segment at the end of each profile. In order to avoid this, we repeated the same divided procedure starting from the end of the profile. And then we got $2N_s$ segments of each profile. $N_s$ is set as $10 < N_s < N/4$ and the step is 1 in this paper.

*Step 3.* Estimate the local linear trend of each segment through the OLS method.

$$
F^2(s, \lambda)
$$

$$
= \frac{1}{s} \sum_{j=1}^{s} \left[ X_{(\lambda-1)s+j} - \widetilde{X}_{(\lambda-1)s+j} \right] \left[ Y_{(\lambda-1)s+j} - \widetilde{Y}_{(\lambda-1)s+j} \right]
\tag{2}
$$

for $\lambda = 1, 2, \ldots, N_s$ and

$$
F^2(s, \lambda) = \frac{1}{s} \sum_{j=1}^{s} \left[ X_{N-(\lambda-N_s)s+j} - \widetilde{X}_{N-(\lambda-N_s)s+j} \right]
$$

$$
\cdot \left[ Y_{N-(\lambda-N_s)s+j} - \widetilde{Y}_{N-(\lambda-N_s)s+j} \right]
\tag{3}
$$

for $\lambda = N_s + 1, N_s + 2, \ldots, 2N_s$, where $\widetilde{X}$ and $\widetilde{Y}$ are the local trend.

*Step 4.* Get the $q$th-order fluctuation function by average over all segments:

$$
F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} F^2(s, \lambda)^{q/2} \right\}^{1/q}.
\tag{4}
$$

Particularly, for $q = 0$, the equation is defined by

$$
F_0(s) = \exp \left\{ \frac{1}{4N_s} \sum_{\lambda=1}^{2N_s} \ln \left[ F^2(s, \lambda) \right] \right\}.
\tag{5}
$$

*Step 5.* Analyze the scale behavior of the fluctuation function through observing the log-log plots of $F_q(s)$ versus $s$. If two series are long-range cross-correlated, we could get a power-law relationship as follows:

$$
F_q(s) \propto s^{h_{xy}(q)}.
\tag{6}
$$

$h_{xy}(q)$ can be obtained through the slope of the log-log plots of $F_q(s)$ versus $s$. And we can calculate the slope using the method of OLS. The value of $h_{xy}(q)$ indicates the cross-correlation of the two time series. If $h_{xy}(q) > 0.5$, the cross-correlation between them is persistent (positive). When $h_{xy}(q) < 0.5$, there is an antipersistent (negative) cross-correlation between the two time series. And for $h_{xy}(q) = 0.5$, the two time series are not cross-correlated with each other. In particular, when $q = 2$, the scaling exponent $h_{xy}(q)$ is the generalized Hurst exponents.

## 5. Empirical Results

*5.1. Cross-Correlation Test.* Before investigating the pairwise cross-correlation among the two types of news amount series and the return series of the index, a cross-correlation test using the method proposed by Podobnik and Stanley [10] is conducted firstly. And the test statistic is defined as

$$
Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{xx_i^2}{N - i}.
\tag{7}
$$

Here, $xx_i^2$ is the cross-correlation function and is defined as follows:

$$
xx_i^2 = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}},
\tag{8}
$$

where $\{x_i\}$ and $\{y_i\}$ are two time series with the equal length $N$.

The test statistic $Q_{cc}(m)$ is approximately $\chi^2(m)$ distributed with $m$ degrees of freedom. The null hypothesis of this cross-correlation test is that none of the first $m$ cross-correlations is different from the value of $\chi^2(m)$. Thereby, if the value of the cross-correlation test exceeds the critical value of $\chi^2(m)$, the cross-correlation between the two time series is significant.

Figure 2 shows the result of the test statistic $Q_{cc}(m)$. And the red line is the critical value of $\chi^2(m)$ at 5% significant level. The other lines represent the values of $Q_{cc}(m)$ between the amount of the mass media news and the amount of the new media news, the amount of the new media news and the return of the SSE 50, the amount of total news and the return of SSE 50, and the amount of the mass media news and the return of SSE 50 from the top to the bottom of the figure. It is clear that all the values of $Q_{cc}(m)$ exceed the critical value, so the null hypothesis is rejected and there is a long-range cross-correlation among any pair of these series.

*5.2. Multifractal Detrended Cross-Correlation Analysis.* Since the value of $Q_{cc}(m)$ just proves the existence of the long-range
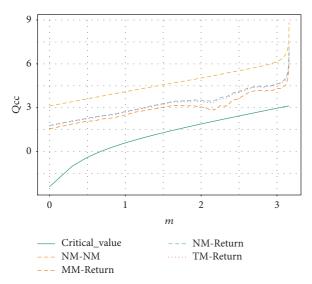
Figure 2: Cross-correlation test ($Q_{cc}(m)$).



Figure 4: Log-log plots of $F_{xyq}(s)$ versus $s$ for amount of MM news and the return of SSE 50.
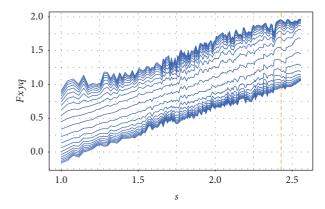


Figure 5: Log-log plots of $F_{xyq}(s)$ versus $s$ for amount of NM news and the return of SSE 50.



Figure 3: Log-log plots of $F_{xyq}(s)$ versus $s$ for amount of MM news and the amount of NM news.

cross-correlation qualitatively, the MF-DCCA is conducted to investigate the cross-correlation quantitatively. The scale $q$ is set from −10 to 10 and the step is 1. Figures 3–6 are the log-log plots of $F_{xyq}(s)$ versus $s$ for the amount of MM news and the amount of NM news, the amount of MM news and the return of SSE 50, and the amount of NM news and the return of SSE 50, as well as the amount of total news and the return of SSE 50. As we can see in Figures 3–6, although there are fluctuations in some lines, they all fit the log-log line of $F_{xyq}(s)$ versus $s$ well at 1% significant level. The significant level can be seen from Table 1. These linear curves provide evidence for the existence of the power-law cross-correlation between pairs of these series.

Figure 7 shows the relationship between the scaling exponents and $q$. The variance of the exponent with $q$ indicates that a multifractality exists in the cross-correlation between the pairwise series among the news and the return. Particularly, for the cross-correlations between the MM news amount and the NM news amount, the MM news amount and the return of SSE 50, and the total amount of news and the return of SSE 50, the scaling exponents for $q < 0$ are larger

than those for $q > 0$. So we can conclude that the cross-correlations for small fluctuations are more persistent than the ones for the large fluctuations. For the cross-correlation between the NM news amount and the return of SSE 50, the "normal" fluctuations display the most persistent cross-correlation.

Table 1 reports the scaling exponent for these time series when $q$ is even (the conclusion is similar when $q$ is odd). The scaling exponents in Table 1 are all larger than 0.5, so the cross-correlations between the amount of MM news, the amount of NM news, the amount of total news, and the return of SSE 50 are persistent. In order to explore the degree of multifractality, the measure, $\Delta H_q$, is introduced as follows [41]:

$$\Delta H_q = \max\left(H_q\right) - \min\left(H_q\right), \tag{9}$$

where the larger the $\Delta H_q$ is, the higher the degree of multifractality is. The last line of Table 1 shows the value of $\Delta H_q$ for the four cross-correlations. $\Delta H_q$ for MM-NM indicates a strong multifractality for the cross-correlation between the amounts of two types of news. The larger $\Delta H_q$ for the cross-correlation of MM-Return than the one of NM-Return shows the stronger multifractal characteristics and more complicated structure for the cross-correlation between the amount of mass media news and the return of SSE 50.

TABLE 1: Results of the MF-DCCA scaling exponent. This table reports the MF-DCCA scaling exponent between the amount of mass media news, the amount of new media news, the amount of total news, and the return of SSE 50. The symbol "MM-NM" denotes the scaling exponent between mass media news amount and new media news amount. The symbol "MM-Return" denotes the scaling exponent between mass media news amount and the return of SSE 50. The symbol "NM-Return" denotes the scaling exponent between new media news amount and the return of SSE 50. The symbol "TM-Return" denotes the scaling exponent between the total news amount and the return of SSE 50. The symbol "$\Delta$" denotes the difference between TM-Return and the sum of MM-Return and NM-return.

| $q$ | MM-NM | MM-Return | NM-Return | TM-Return | $\Delta$ |
|---|---|---|---|---|---|
| −10 | 1.193412*** | 0.904737*** | 0.723414*** | 0.931241*** | 0.696911 |
| | (97.68) | (138.06) | (199.00) | (87.06) | |
| −8 | 1.182705*** | 0.886182*** | 0.71819*** | 0.913263*** | 0.69111 |
| | (98.02) | (145.94) | (209.38) | (87.54) | |
| −6 | 1.169745*** | 0.861594*** | 0.71567*** | 0.889892*** | 0.687372 |
| | (99.44) | (158.73) | (219.38) | (88.13) | |
| −4 | 1.153924*** | 0.83167*** | 0.723031*** | 0.861964*** | 0.692737 |
| | (104.83) | (176.77) | (217.07) | (89.19) | |
| −2 | 1.126114*** | 0.802806*** | 0.768141*** | 0.837509*** | 0.7334348 |
| | (127.66) | (194.62) | (186.21) | (95.46) | |
| 0 | 1.060741*** | 0.78207*** | 0.889481*** | 0.837438*** | 0.834112 |
| | (247.81) | (218.59) | (166.68) | (143.10) | |
| 2 | 1.005653*** | 0.766329*** | 0.874682*** | 0.837032*** | 0.80398 |
| | (81.42) | (196.69) | (193.44) | (189.78) | |
| 4 | 0.915444*** | 0.74425*** | 0.810478*** | 0.797665*** | 0.757063 |
| | (51.71) | (124.14) | (130.09) | (120.17) | |
| 6 | 0.855658*** | 0.718995*** | 0.762753*** | 0.757806*** | 0.723942 |
| | (44.70) | (96.71) | (105.60) | (96.85) | |
| 8 | 0.8204*** | 0.698176*** | 0.731458*** | 0.729416*** | 0.700219 |
| | (41.72) | (86.17) | (94.53) | (86.77) | |
| 10 | 0.797806*** | 0.682709*** | 0.710457*** | 0.709795*** | 0.68337 |
| | (40.08) | (81.03) | (88.40) | (81.31) | |
| $\Delta H_q$ | 0.395606 | 0.222028 | 0.179024 | 0.221446 | — |

∗ ∗ ∗ denotes significance at 1% level.
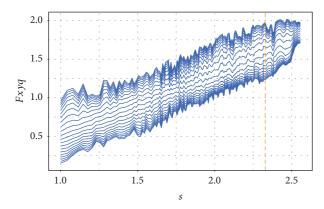


FIGURE 6: Log-log plots of $F_{xyq}(s)$ versus $s$ for amount of total news and the return of SSE 50.

The last column of Table 1 shows the difference between the TM-Return and the sum of MM-Return and NM-Return. The value of the last column that is larger than 0 reveals that the influence of the combination of the mass media news and new media news to the return is smaller than the sum of their respective influences for the return. This leads us to conclude that there is a sharing component between the mass media news and the new media news reflecting a competitive relationship. These findings indicate that there exists over-lapped information conveyed by two information sources and thus investors need to distinguish new information for their decision-making. On the other hand, the value of TM-Return that is larger than the value of either MM-Return or NM-Return reveals the complementary relationship for them. And this result is similar to the conclusions of Zhang et al. [40]. Particularly, the line of TM-Return is closed to the line of MM-Return when $q < 0$ and is closed to the line of NM-Return for $q > 0$. And this reflects the fact that the cross-correlation between the amount of mass media news and the return of SSE 50 leads to the cross-correlation between the news amount and return of SSE 50 for the small fluctuations. The cross-correlation between the amount of new media news and return of SSE 50 is the dominant factor for the big fluctuations. Generally speaking, all these findings suggest that both information sources, that is, mass media and new media, provide useful information to the financial market and influence the variations in the asset prices.

In Figures 3–6, we can find a turning point for the linear trend of the curves. As is suggested by Podobnik et al. [42],

TABLE 2: Short term and long term scaling exponents between news and return series. This table reports the short term and long term scaling exponents for the series between the mass media news and return of SSE 50, new media news amount and return of SSE 50, and total news amount and return of SSE 50. The signal "MM-Return" denotes the scaling exponent of mass media news amount and return of SSE 50, "NM-Return" is the scaling exponent of new media news amount and return of SSE 50, and "TM-Return" denotes the scaling exponent of total news number and return of SSE 50. $\Delta H_q$ is the multifractality degree. "Short" denotes the term $s < S^*$; "long" is the term $s > S^*$.

| $q$ | MM-Return | | NM-Return | | TM-Return | |
|---|---|---|---|---|---|---|
| | Short | Long | Short | Long | Short | Long |
| −10 | 0.878 | 1.261 | 0.726 | 0.905 | 0.841 | 2.172 |
| −8 | 0.859 | 1.234 | 0.719 | 0.905 | 0.826 | 2.114 |
| −6 | 0.833 | 1.200 | 0.715 | 0.896 | 0.805 | 2.030 |
| −4 | 0.801 | 1.152 | 0.720 | 0.864 | 0.782 | 1.909 |
| −2 | 0.774 | 1.072 | 0.755 | 0.929 | 0.766 | 1.700 |
| 0 | 0.769 | 0.923 | 0.857 | 1.747 | 0.796 | 1.276 |
| 2 | 0.794 | 0.691 | 0.889 | 1.111 | 0.869 | 0.797 |
| 4 | 0.815 | 0.495 | 0.848 | 0.599 | 0.875 | 0.559 |
| 6 | 0.811 | 0.397 | 0.807 | 0.355 | 0.850 | 0.455 |
| 8 | 0.799 | 0.353 | 0.777 | 0.215 | 0.826 | 0.403 |
| 10 | 0.786 | 0.332 | 0.757 | 0.123 | 0.808 | 0.373 |
| $\Delta H_q$ | 0.110 | 0.929 | 0.175 | 1.624 | 0.114 | 1.799 |
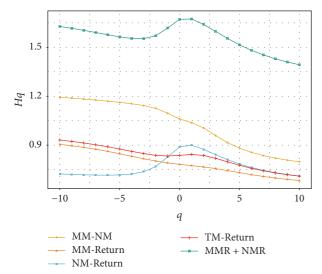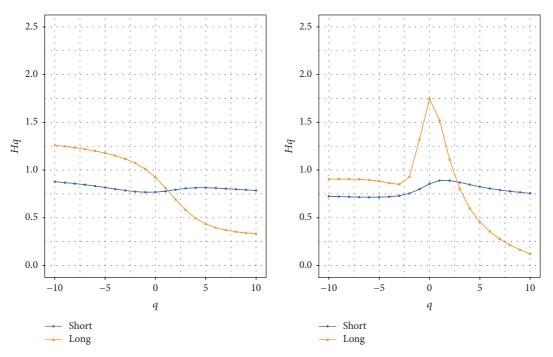


FIGURE 7: MF-DCCA scaling exponent for pairwise time series among two types of news and return of SSE 50.

the point $S^*$ refers to the "crossover." Through the crossover, the scale exponents reflect different features in short term ($s < S^*$) and long term ($s > S^*$). In this paper, the crossovers of the amount of mass media news and the amount of new media news, the amount of mass media news and return of SSE 50, and new media news amount and return SSE 50 as well as total news amount and return of SSE 50 are $\log(s^*) = 2.38$ (about 190 days), $\log(s^*) = 2.33$ (about 213 days), $\log(s^*) = 2.43$ (about 268 days), and $\log(s^*) = 2.29$ (about 195 days), respectively. Table 2 reports the short and long even scale exponents for the series of news and returns. For short term, the scale exponents are all larger than 0.5 for three pairs of series, which reflects a strong persistent cross-correlation for them, while the long term scaling exponents show a different picture. The scaling exponents are larger than

0.5 for the small fluctuations denoting a strong persistent cross-correlation, whereas for the big fluctuations the scaling exponents are smaller than 0.5, reflecting an antipersistent cross-correlation (when $q > 4$ for MM-Return, $q > 6$ for NM-Return, and $q > 6$ for TM-Return). The last row denotes the multifractality of the short and long term cross-correlations for MM-Return, NM-Return, and TM-Return. For any type of the three pairs of cross-correlation, the short term $\Delta H_q$ is smaller than the long term ones significantly. Also, in Figures 8–10 that compare the short and long term cross-correlations for any pairwise cross-correlation of the three types of cross-correlations, there is a steady trend for all the short term scaling exponents and the lines of the short term are nearly parallel with the $x$-axis. So, in long term the multifractality of the scaling exponents is larger than the short term ones and the cross-correlation of the news and return of SSE 50 is more stable. Particularly, by comparing the short term scaling exponents for any pair of cross-correlation, we can find that the values of the short term scaling exponents of MM-Return, NM-Return, and TM-Return are nearly equal to each other at the same scale $q$, whereas for the long term, the scaling exponents of TM-Return are larger than any of the two other scaling exponents but smaller than the sum of them in any scale $q$. We attribute this to the competitive relationship in short term and complementary relationship in long term for the two types of news. So the influence of the mass media news for the return is the same as the influence of the new media for the return and the overlay of them does not increase the cross-correlations for the news and return, while for the long term, the complementary relationship for the two types of news increases the influence of the total news to the return.

5.3. Dynamic Analysis of Cross-Correlation. In this part, we conduct the rolling window method to explore the dynamic features of the cross-correlation. And the length of rolling

Figure 8: Short and long scaling exponents for MM-Return and NM-Return.
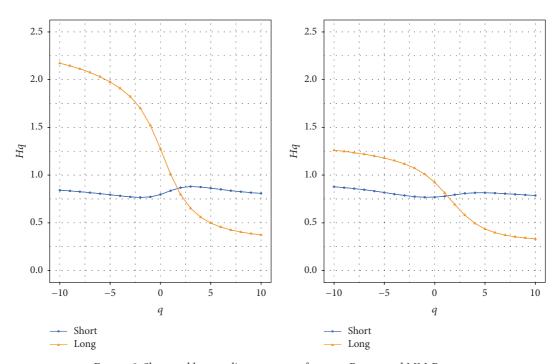


Figure 9: Short and long scaling exponents for news-Return and MM-Return.

windows selected in this paper is 250 trading days (approximately 1 year) [35]. Figure 11 shows the time varying scaling exponents for $q = 2$ for the amount of mass media news and the amount of new media news. Although the process is fluctuating, the general trend of $H_q$ is increasing especially for the period after 2013 when the amount of new media news exceeds the mass media news. And the value of $H_q$ is larger than 0.5 throughout the period. So there is a persistent

cross-correlation between the amount of mass media news and the amount of new media news and it becomes more and more persistent over time. Similarly, Figure 12 shows time varying scaling exponents for $q = 2$ for news and return. There is also a general increasing trend for the news amount and return and the value of $H_q$ is larger than 0.5 over time. The cross-correlations between news amount and return of SSE 50 are more and more persistent. We attribute this
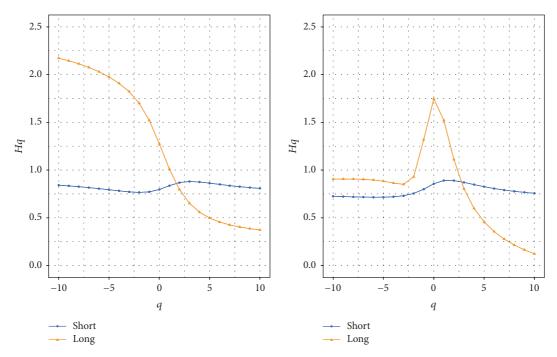
FIGURE 10: Short and long scaling exponents for TM-Return and NM-Return.



FIGURE 11: Time varying scaling exponents for $q = 2$ for mass media news amount and new media amount.



FIGURE 12: Time varying scaling exponents for $q = 2$ for news and return.

to the inefficient market. According to the efficient market hypothesis, the securities' price will reflect all the information [43]. So if the market is not efficient, the return of SSE 50 will depend on the amount of news. To further prove this, we introduce the concept of the market inefficiency index [37]:

$$EI = |H - 0.5|, \tag{10}$$

where $H$ is the Hurst exponent calculated by the detrended fluctuation analysis (DFA).

In the existing literatures, the Hurst exponent is regarded as the measure of the market efficiency [44–47]. When Hurst exponent is equal to 0.5 (the corresponding inefficient indices (EI) in this paper are equal to 0), the market is considered to be efficient. So the larger the value of EI, the more inefficient

the market is. Figure 13 shows the time varying EI with the rolling window of 250 days. In Figure 13, the value of EI is larger than 0 over time, indicating the inefficiency of the market. Moreover, the period after 2013 has the larger EI than the time before, which is also the period that has the larger scaling exponent for news amount and return of SSE 50. So we conduct the MF-DCCA to investigate the relationship between the cross-correlation for news and return and the inefficient indices. Figure 14 shows scaling exponents between cross-correlation exponent for $q = 2$ for news and return and market inefficiency indices. In Figure 14, the scaling exponents for cross-correlations of
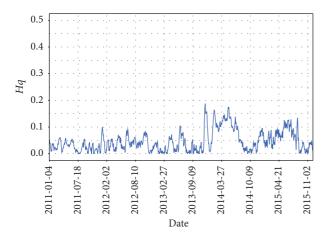
FIGURE 13: Time varying market inefficiency indices.
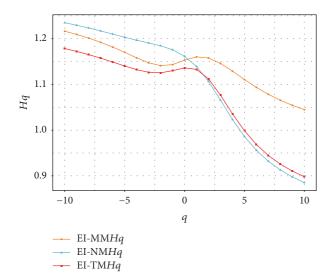


FIGURE 14: Scaling exponents between cross-correlation exponent for $q = 2$ for news and return and market inefficiency indices.

MM-Return, NM-Return, and TM-Return are all larger than 0.5 at any scales, which indicates a persistent cross-correlation between pairwise cross-correlations of them. Therefore, we can conclude that the news, no matter whether it is the mass media news or the new media news, plays an important role in the inefficient market of China now.

## 6. Conclusion

In this paper, we investigate the cross-correlations between two types of news (mass media news and new media news) with the stocks of SSE 50 as well as the corresponding return of SSE 50. By conducting the MF-DCCA method, we can draw the following conclusions.

First, there are power-law cross-correlations between the amount of two types of news as well as between news and its corresponding returns. For the cross-correlation of two types of news and the cross-correlation between mass media news and return of SSE 50, the cross-correlations for small fluctuations are more persistent than the ones for the large fluctuations, while for the cross-correlation between the amount of NM news and the return of SSE 50, the "normal" fluctuations display the most persistent cross-correlation. The cross-correlations all perform multifractality, but the cross-correlation between mass media news and return of SSE 50 shows larger multifractality and more complicated structure.

Second, the value of TM-Return that is smaller than the value of the sum of MM-Return and NM-Return reflects the fact that there is a sharing component between the mass media news and the new media news; hence there is a competitive relationship between them. On the other hand, the value of TM-Return is larger than the value of either MM-Return or NM-Return, which reveals the complementary relationship for them. And this result is similar to the conclusion of Zhang et al. [40]. In addition, the short term multifractality of the cross-correlation between news and return is smaller than the long term one. Moreover, the short term values of the scaling exponents of MM-Return, NM-Return, and TM-Return are close to each other, while the long term value of TM-Return is larger than the value of either MM-Return or NM-Return. This leads us to conclude that there is a competitive relationship in short term and complementary relationship in long term for the two types of news. So the influence of the mass media news for the return is the same as the influence of the new media on returns and the sharing part of them does not increase the cross-correlation for the news and returns while for the long term, the complementary relationship for the two types of news increases the influence of the total news on the return.

Third, by conducting the rolling window method, we find that there is a general increasing trend for the cross-correlation between the two types of news as well as the cross-correlation between news and return and the cross-correlations are more and more persistent over time. We attribute this to the inefficient market. And the persistent cross-correlations between the inefficient indices and the time varying scaling exponents of the news and return indicate that the news, no matter whether it is the mass media news or the new media news, plays an important role in the inefficient market of China now.

Admittedly, the above results cannot completely reveal the relationship between the two types of news as well as the relationship between news and stock returns. To explore the role the mass media news and new media news play in the stock market more precisely, some more work, for example, herding behavior and structural breaks between two types of news [48, 49], needs to be done in the future. Besides, as suggested by Cajueiro and Tabak [50], examination on the role of these two types of news on information efficiency is also a promising research direction. We leave these for future research.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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*Research Article*

# The Complexity of Social Capital: The Influence of Board and Ownership Interlocks on Implied Cost of Capital in an Emerging Market

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Starting from sociological perspectives on complexity, we show how the social capital of boards and owners networks affects the implied cost of capital of companies listed on Brazilian stock exchange. We specifically show arguments and evidence that the effect of the relational resources found in the direct, indirect, and heterogeneous board's ties reduces the cost of capital while relational resources embedded in shareholder networks increase the cost of capital. Our results show that while the increase in the relational resources of the board reduces the implied cost of capital, an increase in these shared resources in the ownership relationships of the firm increases the cost of financial capital.

## 1. Introduction

In this paper we are interested in the effect of two different kinds of relevant complex networks in finance [1–3] on implied cost of capital for the listed companies, particularly in a prominent emerging market, Brazil, which have been the subject of studies on complexity in other disciplines such as epidemiology [4], geophysics [5], ecology [6, 7], and information science [8, 9].

In this sense, our paper discusses the relationship between networks of boards of directors of listed companies and the networks of shareholders in the stock market with the implied cost of capital in an emerging market context. Usually, the cost of capital is considered one of the main aspects of a financial decision, not only for investors but also for chief financial officers. Simultaneously, firms in emerging markets work under the high cost of capital related to firms listed in developed markets. In turn, strategies that could reduce the cost of capital are relevant for the whole capital market.

The problem of the cost of capital can be investigated under the logic of complexity [1, 10]. And looking at complexity in a substantive way, financial market problems should be analyzed by *"the relationships, connectivity and interdependence between the internal and external actors, and the various structural influences on the environment in which they operate [. . .] that we will gain a deeper understanding of the evolution of governance frameworks, and reveal new insights regarding their effectiveness"* [10, p. 2].

About this, we used sociological perspectives on complexity to understanding how organizations acquire and manage their relational resources, such as the means by which the social structure has an influence on the allocation of resources [11]. Among these resources, one of the most important is the financial capital, especially for public corporations, whose financing strategies are fundamental to their survival and growth [12].

Such a statement is even more valid in current times since there are those who say that we no longer live in a society of

organizations: we are now living in a society of investors [13]. For these reasons, the investigation into corporate finance, including the financial cost of capital, has been gaining in prominence in the field of organizational strategy, economic sociology, and corporate governance [14, 15].

In view of the above, in this article we analyze the cost of capital by Brazilian publicly quoted companies, accepting that the assessment of such firms and their access to credit takes place against the background of the concrete social relationships in which they are embedded [12, 15, 16]. We therefore seek to assess the cost of capital based on an estimate known as implied, or ex ante, which is based on the forecasts of analysts [17, 18], while other studies, like the one by Uzzi [12] and Uzzi and Gillespie [11], were based on ex post estimates, which used the history of the returns achieved.

The ex post estimate has been criticized because of its potential inaccuracy. There are three risks associated with such estimates [19]: (1) a difficulty when it comes to identifying the pricing model of the assets; (2) inaccuracy in estimating factor loadings; (3) inaccuracy in estimating the risk factor premiums. For these reasons, Espinosa and Trombetta [17], starting with the work produced by Gebhardt et al. [20] and Easton and Monahan [21], investigated various ways of estimating the implied cost of capital and reached four plausible dimensions: RIV (the residual income valuation model), PEF (the price to forward earnings model), PEG (the price to earnings growth model), and MPEG (the modified price to earnings growth model). The authors concluded that the use of any of these measures would not alter the result, which led us to use the implied cost of capital measure, estimated by the PEG (price to earnings growth model), because of data availability.

As far as the social relationships that condition the cost of capital are concerned, we first studied the relationships of companies established by their directors, a situation known as board interlock, which occurs when a director or officer of one company has also a seat on the board of another company.

Secondly, we investigated the effect of the relationships established between companies because they have owners in common (ownership interlock). This is because there is a whole range of studies that deal with the board and director interlock [15, 22–26] and ownership interlock [27–29], but there are few studies that consider these two types of relationship jointly [30], especially in emerging markets like Brazil.

In the case of the board networks, Davis [31] and Mizruchi [32] point out that the benefits are obvious for corporations because good directors tend to participate in a greater number of organizations, as well as in different groups, and their centrality and position are indicative of their prestige. Furthermore, they tend to suffer greater social pressure to act in a responsible way [31].

Studies also indicate that the best-positioned board directors in the network tend to have a greater capacity for receiving information, resources, and knowledge by way of their privileged access to different, unconnected groups [31]; in other words, they have greater social capital [26, 33, 34].

Despite the fact that such arguments in favor of the greater centrality and privileged position of the board are

convincing, there are controversies as to whether relationships have a positive or negative effect on the conduct and performance of firms [35]. There is evidence of positive results in some studies [15, 36, 37], just as there is evidence of negative results [23, 38].

However, the limitation of these studies lies in the fact that they identify the effects of the privileged position of the boards and their directors by basically using indicators of network centrality and position. What these studies left to one side were the number, type, and availability of the relational resources that these board directors bring to organizations [39].

It is regarding this point that we mainly seek to contribute to the studies of corporate relationship networks: we are interested in analyzing the potential effect of the economic capital—relational resources—that is dispersed among the relationship networks of board directors, since the structure of relationships is not sufficient to explain the effects of the latter on the cost of capital.

In doing so we hope to go further than the investigation undertaken by Uzzi [12] and Uzzi and Gillespie [11], who considered social embeddedness in terms of size, duration, and the complementary nature of the relationships between companies and banks. We also evaluate the effects of the relational resources found in direct, indirect, and heterogeneous relationships, since in the latter there is an assumption that nonredundant ties provide additional advantages [34], without mentioning that the information benefits of the network go beyond its direct ties.

Finally, we also investigated what is called the dark side of networks, in which we present arguments and evidence that the overlapping social capital in ownership relationships, instead of reducing the cost of capital, increases it. This is because cross-ownership relationships may mean that companies form part of a conglomerate with interests and controllers in common. As a negative consequence for investors and creditors, these relationships may be a means by which corporations become involved in dysfunctional transactions between related parties (tunneling), during which both opportunism and conflicts may emerge [40–42].

Given the above, our objective in this study was to analyze how the social capital of boards and owners networks affects the financial cost of capital of companies listed on Brazilian stock exchange, the *B3*. More specifically, we show arguments that the effect of the relational resources found in the direct, indirect, and heterogeneous board's ties reduces the cost of capital while relational resources embedded in shareholder networks increase the cost of capital.

## 2. The Social Capital of Corporations by Board and Ownership Interlocks

Social capital can be understood as that type of capital that is not owned by an individual player but exists as a potential resource because it is embedded in the networks of relationships and is captured and used by way of social exchange [43, 44]. Looking at it instrumentally, social capital refers to the idea that an investment in relationships can lead to greater access to a wide variety of resources [43].

Therefore, any connection with different networks or groups increases the chances of acquiring advantages [33, 34]. As Nahapiet [44] argues, "those that do best, do so by way of their connections and relationships [...] they are more capable of accessing and benefiting from a range of opportunities and resources that affect their performance" (p. 580). In a simple way Portes [45] points out that "economic capital is in the bank accounts of people, human/cultural capital is in their heads and social capital is in the structure of their relationships" (p. 7).

The social mechanisms that support the advantages of social capital involve [43] greater facility in the flow of information, the influence that the social ties exercise over the agents who take decisions, the accreditation and social support given by relationships, and the strengthening of identity and recognition.

Their dimensions, on the other hand, refer to the structure of social capital per se, in which the embeddedness of the players in the network, the psychosocial and institutional apparatus underlying the relationships, among which are the bases of trust, and finally the volume of economic, cultural, and symbolic capital that is accessible to the players in the network are all considered individually [43, 44, 46].

To understand the social capital of corporations we should start with the assumption that there is separation between ownership and control, in which the former is represented by their owners or shareholders and the latter by their directors and officers. In the first case, social capital is embedded in the relationship formed between two companies for having joint ownership (interlocking ownership), while in the latter case this capital is accessible from shared officers and directors (interlocking directors and officers).

These different types of the tie can be shared by the same set of companies, which perhaps may have led to literature not differentiating between the effects and consequences in the conduct and performance of corporations. However, does the social capital embedded in these different types of relationship operate in the same way? Our arguments indicate it does not, and that is why we hypothesize about its effects on the cost of capital in a different way.

As far as the board social capital is concerned, Mizruchi [32] found that studies referring to interlocking and the performance of companies give a variety of results, with some authors finding positive associations and others negative, albeit to a slight degree. This was corroborated by a recent review by Johnson et al. [35], which highlights the complexity and contingent character of the phenomenon. Even though there is a great variety of such companies what they have in common are the ways by which their social capital tends to be operationalized.

For example, Davis and Mizruchi [47] analyzed how the restructuring of industry in the United States affected the position of banks in the network. To corroborate their hypotheses, these authors used the network centrality concepts of Freeman and Bonacich to determine the position of each company. He and Huang [48], on the other hand, used the centrality of the directors to generate a differentiation coefficient (Gini coefficient) with the aim of identifying how broad the informal hierarchy is. The authors found that the broader the informal hierarchy was, the better the financial performance was.

In Brazil, Mendes-Da-Silva et al. [36] founded a significant relationship between having a privileged position (network centrality, density, and cohesion) on the board of directors of companies traded on *B3's* New Market and both market value and the indebtedness of the listed companies. Mendes-Da-Silva [49] subsequently investigated the existence of associations between the positioning of the firm in the network of relationships and the value of the companies.

The author used measures of network centrality (degree, "betweenness," and eigenvector) and found evidence that regarding the value of companies there are optimal levels of centrality. Also in Brazil, Rossoni and Machado-Da-Silva [37] investigated the legitimacy arising from the boards of directors of companies listed on the *B3*. The authors observed that among the companies listed on the Traditional Market, where there are low levels of governance, the bigger the proportion of structural holes is, the greater the market value is. Conversely, however, the authors found that the greater the board network cohesion by way of the clustering coefficient is, the smaller the market value is. Rossoni and Machado-Da-Silva [37] justified such negative results on the basis that greater cohesion between company boards can increase the risk of opportunism on the part of their directors.

This was not the only negative result found in Brazil: Santos et al. [38] found that the value of companies was jeopardized by high levels of interlocking, particularly in companies in which half or more of their directors are on three or more boards. This was also true for those companies where the CEO sits on other boards. This result is corroborated by Fracassi and Tate [23] in their analysis of 1500 companies that comprise the Standard & Poor's index. According to the authors, the ties of the CEO reduce the value of the firm, especially in the absence of other governance mechanisms.

Despite the contradictory results, we still insist that relationships between boards can help explain the effectiveness of corporations, especially their cost of capital. First, because we have elements for this, as Mizruchi et al. [15] point out, there is historical evidence among American corporations that relationships between executives and members of the boards of companies are associated with the debt levels of the latter.

In line with such findings, Uzzi [12] showed how small firms can benefit from close relational ties with banks when the size and complementary nature of the undertaking's network are associated with a greater facility when it comes to borrowing capital. Uzzi [12] and Uzzi and Gillespie [11] also found that time, multiplexity, and a complementary nature reduce the cost of capital to a great degree.

Secondly, because we believe that due attention has not been paid to a fundamental dimension of social capital, the resources exist in social relationships [39]. As Lin [43] advocates, the social capital is present in the exchanges that arise from relationships between players by way of the resources that are mobilized and available in such relationships. In other words, if social capital is made up of resources and relationships, it makes no sense to pay attention to only one dimension and ignore the other. Thus, we propose that access

to and the use of the resources found in the networks between boards and directors can lead to better conditions for raising capital when compared with those that are lacking in such relational resources, which in turn reduces the cost of capital [50].

Looking at the information mechanisms underlying this process, there are indications that the relationships between firms by way of their directors and officers may facilitate, for example, access to lower capital interest rates. In many cases, these firms look for direct financing from sources of funds or credit markets that are little known [12]. There are also motivational benefits embedded in these relationships, in which the ties of trust enable exchange and reciprocity, which are not available in simple market interactions.

Furthermore, having directors who are part of very valuable companies may lead to a firm having greater bargaining power with its creditors, without considering the fact that this may generate a greater number of alternatives, in which these two elements, in both cases, affect the probability of a firm managing to get hold of cheaper financial capital, which implies a lower cost of capital [11].

Finally, knowing that the advantages of relational resources in terms of information and influence can go beyond the direct relationships [34], we advocate that indirect relationships may also be potential catalysts of the benefits we have here described. Such advantages may also reduce the cost of capital of the companies investigated. This being so and given these arguments, we state the following.

*Hypothesis 1a.* The larger the amount of relational resources available in the direct relationships of the board is, the smaller the company's cost of capital is.

*Hypothesis 1b.* The larger the amount of relational resources available in the indirect relationships of the board is, the smaller the company's cost of capital is.

In addition to the relational resources present in direct and indirect relationships between boards, we also advocate that those present in weak, less redundant, and more heterogeneous ties also affect the reduction in the cost of capital [50]. This is because, as Burt [33, 34] stresses, structural holes or nonredundant ties are opportunities to broker the flow of information and resources between players that are on opposite sides or at little-connected points in the network, and who may have information that has greater innovative content.

As the literature about company boards points out, such characteristics can be of advantage to publicly quoted companies. For example, Kim [26] analyzes the effects of the proportion of the number of outside board members as vectors of the social capital on market value. He points out that such ties have the capacity to extract valuable resources from the environment. In Brazil, Rossoni and Mendes-Da-Silva [51] and Rossoni and Machado-Da-Silva [37] used the proportion of nonredundant ties as one of the indicators of the legitimacy of the board. Rossoni et al. [50], using board's social capital ideas, showed that heterogeneous relational resources have a stronger and more significant influence than the resources available from board's direct ties. In the first case, the data suggest that companies that have boards with a greater proportion of structural holes tend to have less systematic risk in the presence of other governance mechanisms (premium listing: New Market). In the second case, nonredundant ties were associated with greater market value only if there were special levels of governance safeguard in place. In the third case, relational resources present in heterogeneous relations have a greater impact on market value.

With regard to the effects on the cost of capital specifically, there is no evidence of this in literature, although Uzzi [12] indicated that an optimal level between transactions with a greater number of banks (opening) and a smaller number (closing) reduces the cost of capital. Given such results, even though some of them are conditioned by institutional factors, we believe that the mechanisms operating in the structural holes are also valid for the cost of capital, especially considering the value of the resources dispersed in these heterogeneous relationships. For these reasons we consider the following.

*Hypothesis 2.* The larger the heterogeneity of relational resources available in the direct relationships of the board is, the smaller the company's cost of capital is.

All the previous arguments could be valid too for the effects of social capital on the cost of capital arising from ownership relationships because such relationships are associated with those of the board [30]. However, if the relationships are of another type, there is evidence that they operate differently [15, 35]. In our case, we argue that the overlapping social capital in ownership relationships presents another side, which is, in fact, contrary to the side of the relationships between boards: the dark side of networks.

To make the link with such a statement we need to understand the nature of such relationships. First of all, as Kim [29] points out that the interlinking of owners may mean that the companies form part of the same group, or a conglomerate, which has interests and controllers in common even though they are different companies. This can create the opportunity for the corporations to carry out dysfunctional transactions between related parties (tunneling), in which both opportunism of the controller and conflicts may emerge, especially when there are minority shareholders in some units, but not in others [40–42].

There is also the fact that the diversification of property, especially nonrelated property, is associated with the growth of the firm beyond the point where it maximizes the value of its shares, which offers opportunities for the controllers to misuse the resources [52]. Furthermore, some of these tactics may complicate the effectiveness of corporate governance, by compromising the monitoring of managers and controllers, which might lead to acquisitions and contracts that give a lower return or have obscure interests.

There is a vast amount of documentation of cases of firms whose assets are undervalued due to the discount given by investors because they are part of a conglomerate [13, 53]. For these reasons, we understand that investors and creditors, in addition to negatively assessing the assets of companies that

establish such relationships, also see that the cost of capital is greater, given the enhanced underlying risk of operating with such companies. Add to this the fact that the greater the resources shared by a network of owners are, the greater the chances of expropriation or tunneling are. Thus, we then have our final hypothesis.

*Hypothesis 3. The larger the amount of relational resources available in firms with cross-ownership with the company is, the greater the cost of capital is.*

## 3. Method

*3.1. Data and Sample.* The population of this study comprises companies listed on the *B3*, the Brazilian stock exchange, in the years 2010 and 2011 (after 2008-2009 American subprime crises). Although B3 has more than 300 companies with market liquidity, only a small part is covered by market analysts. For this reason, our sample involved 62 companies totaling 114 valid cases, organized into an unbalanced panel.

The dependent variable, the implied cost of capital, that was collected considering $t + 1$ years, is formed from the prediction of these analysts, in which not all companies are relevant to the rating agencies. The coverage by analysts refers to costs that are not always offset when the companies are smaller or when the volume of the shares traded is lower. After we had identified the companies covered by market analysts and organized the market previsions by company, we consolidated the financial data obtained from the Economatica® database with the other sources of register data: *CVM's disclosure system (Brazil's SEC)*; information from *B3*, the Brazilian stock exchange; and the reference forms from the companies.

*3.2. Board and Cross-Ownership Networks.* We put together the network of company boards based on information available in the reference forms of the listed companies identified in the *CVM* system. First, we listed all the firms and their respective directors and officers individually for the years 2010 and 2011, which allowed us to generate an incidence network in the *2*-mode format (companies versus directors) for each year. We then used PAJEK software to create the relationship networks between boards (*1*-mode format), in which two firms that were directly linked shared at least one director or officer, a phenomenon known as board interlocking.

These data about board networks were exported to UCINET software, which was used to generate the relational indicators at the firm level, which were subsequently incorporated into the panel. To create the cross-ownership company networks, the same sequence of steps described above was used, but we considered that two companies are interrelated if they have the same owners in the list of shareholders available on the *CVM* system or if one of them holds shares in the other.

### 3.3. Dependent Variable

*3.3.1. Implied Cost of Capital.* In line with a study by Espinosa and Trombetta [17], this measure was operationalized in the estimate of the implied cost of capital, or cost of capital ex ante, for the years 2011 and 2012, using

$$rPEG = \frac{\hat{x}_{t+2} - \hat{x}_{t+1}}{\left(r_e\right)^2}, \tag{1}$$

with $\hat{x}_{t+r}$ being the forecasts of the consensus analysts of the EPS (earnings per share) for the following first, second, and third years. Beyond year $t + 3$, gains are estimated by the linear disappearance of the true ROE (return on equity) for a measure of market ROE for year $t + 12$ [17, 20, 21]. In our sample the cost of capital varied between 0.033 (3.3% a year) and 0.378 (37.8%), with an average of 0.148 (14.8%). Table 1 lists descriptive statistics and correlations of all variables used in this study.

### 3.4. Independent Variables

*3.4.1. Social Capital of the Board.* Using the same strategy that Rossoni et al. [50], we operationalized the social capital by the sum of the relational resources present in three types of relationship: direct, indirect, and heterogeneous. Thus, to arrive at the *social capital of the direct relationships*, we first identified for each year the direct ties that each firm had with the others by way of board networks, which are known in the vocabulary of social network analysis as "ego-networks." After this, we identified the market value (total value of the shares traded on the stock exchange) of each of the firm's relationships, which is what we call relational resources. Finally, we added the value of these relational resources for each tie, which generated our social capital indicator of the company's direct relationships. To get a better idea of this, the average value of the direct relational resources was around R$ 21 billion (nearly U$ 9 billion). Companies like Vale (biggest metal and mining Brazilian company), for example, had a value of R$ 120 billion (U$ 52 billion), while the biggest social capital of the board's direct relationships was that of the board of Embraer, the Brazilian aircraft manufacturer (R$ 374 billion or U$ 163 billion).

The *social capital of the indirect relationships* of each firm, on the other hand, as the term itself implies, was obtained by way of the sum of the relational resources (market value) of the first-degree indirect relationships. Despite the similarity with the previous measure, social capital by indirect relationships was generated estimating the geodesic distance between the firms and saved it in a distance matrix. We then encoded the first-degree indirect relationships network (value two in the network) on a binary code with the value 1, while all the other distances were codified as zero. Finally, we add the market value of each indirect relationship with each company to obtain the social capital proxy of the indirect ties.

Finally, the *social capital of heterogeneous relationships* was generated in the following way: First, we ran the structural holes procedure in UCINET and saved the DR (dyadic redundancy) matrix, which gives the degree of redundancy of each *alter* (direct relationship) in relation to each of the *egos* (firms) in the network. Redundancy indicates the percentage of ties that the *ego* and *alter* have in an *ego* network, in which the greater the value is, the more redundant the tie is [33].

TABLE 1: Descriptive statistics.

|  | Mean | SD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) Cost of capital | 0.148 | 0.071 |  |  |  |  |  |  |  |  |  |
| (2) Social capital of the board: direct relationships[a] | 21.966 | 50.982 | −0.078 |  |  |  |  |  |  |  |  |
| (3) Social capital of the board: indirect relationships[a] | 73.244 | 12.192 | 0.052 | 0.301** |  |  |  |  |  |  |  |
| (4) Social capital of the board: heterogeneous relationships[a] | 14.421 | 35.257 | −0.050 | 0.935** | 0.323** |  |  |  |  |  |  |
| (5) Social capital: cross-ownership[a] | 42.639 | 102.803 | 0.014 | 0.404** | 0.285** | 0.401** |  |  |  |  |  |
| (6) Board size | 8.230 | 4.921 | −0.061 | 0.253** | 0.383** | 0.305** | 0.272** |  |  |  |  |
| (7) Outsiders | 0.849 | 0.185 | −0.045 | 0.125** | 0.227** | 0.144** | 0.143** | 0.377** |  |  |  |
| (8) Company age (ln) | 2.442 | 1.137 | 0.065 | −0.029 | −0.052 | −0.021 | 0.067 | −0.008 | −0.171** |  |  |
| (9) Size (ln of assets) | 14.427 | 2.273 | 0.002 | 0.182** | 0.314** | 0.212** | 0.227** | 0.445** | 0.260** | −0.048 |  |
| (10) Leverage | 63.354 | 601.471 | 0.338** | −0.027 | −0.035 | −0.024 | −0.025 | −0.065 | −0.066 | 0.054 | −0.190** |

**Correlation is significant at the 0.01 level (1-tailed). Correlation is significant at the 0.05 level (1-tailed). $N = 114$. [a]Amounts in billions (R$).
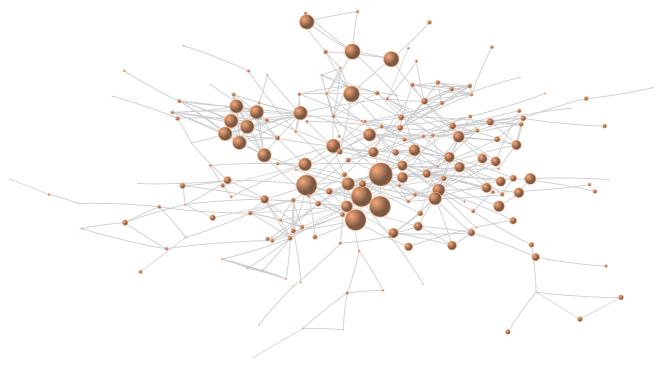


FIGURE 1: *Main component of board interlocking between Brazilian companies.* Main component ($n = 187$). Data for the 2010 year. The size of vertices corresponds to the social capital value of companies: the sum of relational resources. Each vertex is one company, and edges indicate board interlocking. Visualization algorithm: Kamada-Kawai, *Note*. Developed by the authors based on the data collected.

Second, we subtracted the value one from the redundancy score of each valid *alter*, thus obtaining a heterogeneity score for the *alters*, which was tabulated in new matrixes. Third, we multiplied the market value of each existing relationship (*alter*) by its respective heterogeneity score. Finally, we added the product of the relationships of each firm to arrive at the *social capital of the heterogeneous relationships* of the company.

To illustrate the relationship between the board of directors and the presence of sum of relational resources (a proxy of board's social capital), we put the main component of board interlocking network in Figure 1.

*3.4.2. Social Capital of Cross-Ownership.* We also considered the social capital coming from the interlinking between owners since we started from the premise that it might have a different or complementary effect to that of the social capital of the board [30]. Therefore, two firms have a cross-ownership relationship when at least one of them has a shareholding in the other [29]. To arrive at the social capital of the ownership relationships for each of the companies, we identified those that had owners in common or that were shareholders of others [50].

We then identified the market value (total value of the shares traded on the stock exchange) of each of these companies. Finally, we added the market value of each of these companies with cross-ownership, thus forming the social capital proxy of the ownership relationships.

### 3.5. Control Variables

*3.5.1. Board Size.* Recent studies showed that board size impacts financial variables as market value, risk, and ROA [38, 54]. Also, it is a relevant characteristic of the board's structure [23, 55]. Therefore, a higher number of interlocks is more likely on bigger boards. Therefore, we included as control variable the number of board members.

*3.5.2. Outsiders.* We also controlled the percentage of outside directors between board members. External directors can generate more status, knowledge, and information without interlocking with another company [35, 38, 56].

*3.5.3. Company Age.* As other studies have done [57, 58], we operationalized the age of the company by way of the natural logarithm of its time of operation on the Brazilian stock exchange, and more specifically on the *B3*.

*3.5.4. Company Size.* We used the accounting value of total assets as a proxy for the size of the firm, such information was collected from the Economatica database. Moreover, with the aim of reducing problems of symmetry and kurtosis, we use natural logarithm, as was also done in other similar studies [36–38, 59].

*3.5.5. Financial Leverage.* This is the amount of the total financial debt of the company, divided by its total asset value [60]. We operationalized this measure in accordance with Uzzi [12] and Uzzi and Gillespie [11] because we believe there is a suspicion that indebted companies tend to have a greater cost of capital due to their payment needs.

*3.5.6. Year.* To avoid problems related to seasonality, such as temporary trends, we controlled the time effect using dummy variables, which is common practice in panel data [60, 61]. The 2010 year was considered as the reference category and the 2011 year was identified in the model by way of a dummy variable.

*3.5.7. Industry.* We controlled the sector effect since there is evidence that it precedes market value [59–61]. To this end we created $s - 1$ dummy variables, in which $s$ is the number of players identified in Economatica, considering the "others" sector as a reference category because the former has a larger number of observations.

*3.6. Econometric Model.* We analyze the influence of board and owners' social capital on the implied cost of capital by panel data analysis, in which the variables were hierarchically incorporated into eight models. We chose the econometric panel model because we had various cases ($N$) with many observations in time ($T$), for $N \times T$ observations. We tested three alternatives to evaluate which of them was the most appropriate: (1) grouping the cut-off data based on ordinary least squares (pooled OLS); (2) fixed effects (FE); (3) random effects (RE). According to Greene [62], the choice of the most adjusted model depends on confronting three test hypotheses: (a) the existence, or otherwise, of a single intercept of the transversal cut units (evaluated by way of the Chow's $F$ test); (b) if the variance of the intercept is equal to zero (Lagrange multiplier modified by Breusch and Pagan's proposition); (c) if the estimators are consistent, based on an estimation of the generalized least squares (Hausman test). Based on our evaluation of these hypotheses we chose the best model for each of the relationships between variables.

Then we looked for greater robustness in the results in six different ways. First, we assessed if the dependent variable had serious distribution problems. Second, we assessed if the independent variables had a linear functional form relative to the dependent variable. Third, we assessed if the models had problems of heteroskedasticity using the White test; if they had, we would treat this problem by using robust standard errors. Fourth, we checked for the existence of collinearity problems between the independent variables.

As the social capital of direct relationships was highly correlated with the heterogeneous relationships, we did not regress these variables in the same models. Fifth, we observed if the results remained consistent when we regressed the models without the insignificant control variables, just as we checked if they had the same tendency without the outliers. Sixth, as far as endogeneity is concerned, even if the literature indicated such a problem with indicators related to corporate governance [63] and especially because these elements have both characteristics of self-selection and reverse causality [64], we did not use instrumental variables or simultaneous equations [60, 65]. This is because there is no clear evidence in the literature about the endogeneity of relational variables [32, 35] and because it is not possible to trace a reverse causal nexus between future predictions of analysts, which made up the implied cost of capital, and the independent variables.

## 4. Results

In Table 2 we show the effects of the variables on the implied cost of capital. After the adjustment tests of the panel data (Chow, Breusch-Pagan, and Hausman), the fixed model was the most suitable for all models. The White test indicated problems with heteroskedasticity in all models also, which led us to use the robust standard error.

Following a strategy of hierarchical estimation, in model 1 we show the effect of the control variables on the implied cost of capital. The only control variable that was significant is a

Table 2: Influence of Social Capital on the Implied Cost of Capital (Fixed Panel Data Models).

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|---|---|---|---|---|---|---|---|---|
| Board social capital: Direct relationships[a] | | −0.532*** (0.164) | | | | −0.555** (0.276) | | |
| Board social capital: Indirect relationships[a] | | | −0.165** (0.073) | | | | −0.162** (0.072) | |
| Board social capital: Heterogeneous relationships[a] | | | | −0.538** (0.215) | | | | −0.585* (0.326) |
| Social capital: Cross-ownership[a] | | | | | 0.119* (0.071) | 0.115*** (0.047) | 0.109** (0.054) | 0.124** (0.056) |
| *Control variables* | | | | | | | | |
| Board size | −0.001 (0.002) | −0.002 (0.003) | −0.001 (0.002) | −0.002 (0.003) | −0.002 (0.002) | −0.002 (0.002) | −0.002 (0.002) | −0.002 (0.002) |
| Outsiders | −0.336** (0.150) | −0.222 (0.150) | −0.405*** (0.122) | −0.232 (0.158) | −0.341** (0.138) | −0.224** (0.104) | −0.408*** (0.111) | −0.230** (0.108) |
| Company age (ln) | 0.039 (0.039) | 0.056 (0.044) | 0.041 (0.037) | 0.053 (0.046) | 0.051 (0.039) | 0.071* (0.036) | 0.051 (0.038) | 0.040 (0.040) |
| Size (ln of assets) | 0.028 (0.034) | 0.005 (0.036) | 0.047 (0.046) | 0.011 (0.037) | 0.012 (0.032) | −0.003 (0.027) | 0.040 (0.044) | −0.005 (0.029) |
| Leverage | 0.001 (0.001) | 0.001 (0.001) | 0.001 (0.001) | 0.001 (0.001) | 0.001 (0.001) | 0.002* (0.001) | 0.001 (0.001) | 0.002* (0.001) |
| Constant | −0.073 (0.498) | 0.131 (0.526) | −0.281 (0.639) | 0.071 (0.546) | 0.004 (0.477) | 0.238 (0.370) | −0.206 (0.617) | 0.330 (0.433) |
| White's test | 41.138*** | 47.366*** | 46.774*** | 45.202** | 52.862*** | 57.124** | 58.893*** | 56.957*** |
| Chow's $F$ test | 53.498*** | 54.663*** | 51.044*** | 53.745*** | 53.879*** | 53.198*** | 50.997*** | 74.955*** |
| Breusch-Pagan test | 12.578*** | 13.821*** | 12.646*** | 13.239*** | 12.520*** | 12.100*** | 12.6561*** | 13.341*** |
| Hausman's test | 70.758** | 12.121** | 10.786* | 10.014* | 11.114* | 13.522* | 175.47*** | 10.974* |
| Akaike's criterion | 353.33 | 374.68 | 362.91 | 365.91 | 354.20 | 378.34 | 363.599 | 366.62 |
| $F$ | 3.24*** | 4.03*** | 3.58*** | 3.69*** | 3.27*** | 4.17*** | 3.60*** | 3.75*** |
| $R^2$ overall | 0.837 | 0.868 | 0.853 | 0.857 | 0.841 | 0.874 | 0.856 | 0.862 |

Standard error in parentheses. Dummies of industry and years omitted. $N = 114$; *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$. [a]Amounts in billions (R$).

percentage of outsiders' directors ($p < 0.05$): a 1% increase in this variable is related to a 0.33% reduction in the cost of capital. This result is coherent with other studies about board structure, evidencing that the bigger the degree of external financing is, the lower the cost of acquiring financial capital is.

Regarding our hypotheses, in models 2 and 6 we find that the larger the number of relational resources (social capital) in relationships directly established by the board is, the lower the cost of capital is, which corroborates *hypothesis 1a*. The social capital coefficient of direct relationships in model 7 indicates that every R$ 10 billion (U$ 3.3 billion) increase in social capital present in the direct relationship leads to a reduction of 0.55% in the cost of capital. The social capital deriving from indirect relationships also gave a similar effect on the cost of capital; the results were significant both in model 3 ($p < 0.05$) and in model 7 ($p < 0.05$), leading us to accept the validity of *hypothesis 1b*.

About the social capital embedded in heterogeneous relationships (structural holes), the data indicate a significant reduction in the cost of capital ($p < 0.05$, *model 4*; $p < 0.05$, *model 8*), which leads us to corroborate *hypothesis 2*. Among the three measures of board's social capital, that one composed by the heterogeneous ties had the highest coefficients. Results reinforce the argument that nonredundant links can bring additional benefits in company networks.

Finally, we analyzed the effect of the social capital of cross-ownership in models 5 to 8. The effect was significant in all models, leading us to accept *hypothesis 3*. The data indicate that an increase of R$ 10 billion (U$ 3.3 billion) in relational resources coming from cross-ownership is associated with an increase of 0.11% in the cost of capital. This might seem to be a small percentage, but if we consider the average value of these relational resources we are talking about, for example, 0.56% of current liabilities, which are around R$ 5 billion (U$ 1,65 billion), this would give an average increase in the cost of capital of around R$ 27 million (U$ 8.2 million).

## 5. Discussion and Conclusion

In this study, we show that while board's social capital reduces the cost of capital, ownership interlocks' social capital increases the cost of capital of companies listed on the

Brazilian stock exchange. To this end, we used a different measure to capture companies' social capital founded in the relational resources of direct, indirect, and heterogeneous board ties and in the relational resources embedded in ownership networks.

Regarding the social capital of the board, this study indicated that the resources present in direct, indirect, and heterogeneous relationships significantly reduce the cost of capital of publicly quoted companies listed on the Brazilian stock exchange, showing that the greater the relational resources available via the board are, the lower the cost of capital of the companies is. Therefore, our hypotheses were in line with the perspectives of Chalupnicek [39] and Flap and DeGraaf [66]. We advocated earlier measures of social capital that do not put the relational resources as an essential element of social capital [39, 50]. Indeed, it is not enough to explain the complexity of the relational phenomenon in financial networks [1, 2].

Looking at the heterogeneity of the relationships, we found that structural holes are means for obtaining information and various resources, enabling firms to have a greater competitive advantage, including a bigger market value. However, as we reported previously, more studies will be necessary to assess how much impact the social capital deriving from structural holes has since it was highly correlated with the social capital inherent in direct relationships.

With regard to the social capital present in relationships between owners, our study corroborated the other studies that provide evidence of the agency problems inherent to cross-ownership and their respective conglomerates, whether formal or informal [13, 27, 29, 52]. Our data tend to support the statement that firms with cross-ownership are viewed with reservation by investors and creditors because market analysts tend to presume they have a greater cost of capital. Suspicions that there is expropriation or tunneling in the transactions between companies seem feasible. It is worth emphasizing, also, that this effect was only visible when the social capital of the board was included in the model, which highlights the complementary nature between these variables in the investigation of the precedents of the cost of capital, as strongly suggested by Bohman [30].

*5.1. Theoretical and Practical Implications.* The empirical results of this research make it possible to develop some theoretical and practical implications. The first theoretical understanding has to do with the positioning of the board and its effects on the performance of companies. Various studies use positioning measures of a player in the network to test their research hypotheses or questions, such as small world studies, developed by Brookfield et al. [27], Mendes-Da-Silva [49], and Pusser et al. [67], who studied interlocking between company boards and American universities, or Rossoni and Machado-Da-Silva [37] and Rossoni and Mendes-Da-Silva [51], in their research into legitimacy and market value in companies quoted on the *B3*. This study produces evidence that the board and owners relational resources have more effect than only board relational position on the cost of capital [39, 50].

The second theoretical implication is related to the concept of board interlocking and corporate governance. Davis and Mizruchi [47] state that interlocking can have an economic and social influence on organizations, and Davis [31] identified that interlocking has a social influence on governance practices in the United States. Rossoni and Mendes-Da-Silva [51] found that companies with better quality corporate governance, organizational reputation, and board legitimacy have an influence on the risk of shares traded on the stock exchange. Indeed, this study contributes to the field of board interlocking and corporate governance by studying the analysis of the social capital of the board and providing empirical evidence of how interlocking influences the cost of capital of these companies.

The third theoretical implication is linked to the analysis of the relational resources of the board and their effect on performance. Finegold et al. [68] found evidence that the social capital of the board guarantees that companies receive resources, advice, and better monitoring and are assured of a better performance. Kim [26], using Tobin's Q indicator, produces empirical evidence that the social capital of external board members is positively associated with the market value. Like Rossoni et al. [50], this study contributes to the research into social capital by evaluating the relational resources of the board using the market value (capital) embedded in the network of relationships of companies and innovates by measuring the cost of capital by ex ante estimates. This goes further than other studies, like the one by Kim [26], who measured social capital by way of affiliations in company boards and university governing boards, and by Stevenson and Radin [69], who measured social capital by way of surveys with CEOs.

Finally, as Bohman [30] and Mizruchi et al. [15] advocate, the fourth and last theoretical implication of this study refers to the need to consider that different types of tie have different implications; in the case of ownership relationships, for example, there was indeed an increase in the cost of capital. This highlights the fact that relationships may not always have a positive potential but may also have a negative side [70].

In practical terms, this research is enlightening for investment fund managers, the managers of companies listed on stock exchange in emerging markets like Brazil, and individual investors. By analyzing the components of the board, the network of company owners and the cost of capital fund managers and creditors can evaluate strategies for seeking better composition for their share and debenture portfolios in order to reduce the risk of the funds they manage.

For the managers of companies listed on the stock exchange, the data indicate that board members can be used as the drivers of information about the best capital financing rates. The research draws the attention of individual investors to their need to consider the board in their assessment of a firm, before deciding where to invest their capital.

*5.2. Recommendations for Future Studies.* The first recommendation for future work is to analyze publicly quoted companies with a view to looking at outsiders, following the concept of Portes [45], to identify how they are affected because of interlocking boards. Corporate governance is

a topic of great interest and we recommend that a more in-depth evaluation is carried out of social capital at the various levels of corporate governance to assess how the characteristics of each level of governance affect companies.

The network of owners should also be analyzed up to the final level in the structure, in other words, as far as private individual owners. We recommend a more in-depth analysis into the behavior of organizations that, when they experience some difficulty, seek to strengthen their boards by bringing in directors with a better reputation or those who are on boards of companies with a lower cost of capital, with the objective being to provide their organization with greater credibility in the short term, especially if these ties occur between investment funds and banks.

The behavior of independent board members, who leave organizations when the latter get into some difficulty, is also worth analyzing. Thus, their image will not be affected by the companies' problems. Finally, it is also worth using spatial regression and more recent relational statistics methods to evaluate the individual effect of each relational resource and not the sum of their added value at the level of the firm.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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*Research Article*

# The Assessment of Systemic Risk in the Kenyan Banking Sector

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The present paper aims to assess the systemic risk of the Kenyan banking system. We propose a theoretical framework to reveal the time evolution of the systemic risk using sequences of financial data and use the framework to assess the systemic risk of the Kenyan banking system that is regarded as the largest in the East and Central African region. Firstly, we estimate the bilateral exposures matrix using aggregate financial data on loans and deposits from annual reports and analyze the interconnectedness in the market using network centrality measures. Next, we extend the Eisenberg–Noe method to a multiperiod setting to the systemic risk of the Kenyan banking system, in which the multiperiod includes the dynamic evolutions of the Kenyan banking system of every bank and the structure of the interbank network system. We apply this framework to assess dynamically the systemic risk of the Kenyan banking system between 2009 and 2015. The main findings are the following. The theoretical network analysis using network centrality measures showed several banks displaying characteristics of systematically important banks (SIBs). The theoretical default analysis showed that a bank suffering a basic default will trigger a contagious default that caused several other banks in the sector to go bankrupt. Further stress test proved that the KCB bank theoretically caused a few contagious defaults due to an unusually high interconnectedness. This methodology can contribute by being part of monitoring system of the Central Bank of Kenya (regulatory body) as well as the implementation of policies (such as bank-internal stress tests) that assist in preventing default contagion.

## 1. Introduction

Events beginning in the United States in 2008, including the collapse of some major financial institutions and the rescue of others, ideally depict the effect that a systemic crisis can have on an economy. Many financial institutions around the world felt the impact of the default of some of these institutions in the United States. Thus, systemic risk emerged as one of the most challenging aspects. Previous to this occurrence, there was a limited knowledge of how systemic risk affected institutions and how to assess the systemic risk. Therefore, it is essential to have an effective assessment of systemic risk exposure to an institution.

A few measures of systemic risk have been proposed in recent empirical studies. Among them, De Jonghe [1] uses the extreme-value analysis to measure the contribution of each single financial institution to systemic risk. Using CDS (Credit Default Swap) of financial firms and correlations between their stock returns, Huang et al. [2, 3] and Segoviano Basurto [4] propose portfolio credit risk measurement methods, such as CoPoD and CIMDO methods. The conditional

Value at Risk (CoVaR) is proposed as measure of systemic importance of financial institutions proposed by Adrian and Brunnermeier [5]. The CoVaR can capture how much the distress of one institution can affect another institution and it provides a clear direction on the relation between the risks involved between two financial institutions. Another measure that was proposed and used as an indicator to measure the systemic risk is the System Expected Shortfall (SES). It was introduced to measure listed financial institutions' contributions to systemic risk. SES is defined as an institution's tendency to be undercapitalized when the financial system as a whole is undercapitalized. Marginal Expected Shortfall (MES) was also introduced as a measure of an institution's loss in the tail of the system's loss distribution. MES is a systemic fragility metric that can also be used to determine an optimal taxation policy based on systemic risk [6]. Both the SES and the MES methods were proposed by Acharya et al. [7].

However, a limitation of those approaches is that they measure a financial institution's loss only if the system is in normal time and only indirectly take into account the size, the probability of default, and the correlation of each

financial institution. Furthermore, the correlation does not capture the interconnectedness adequately because it does not consider the various interactions (such as contagious defaults) or the relationship between interconnectedness and systemic importance in a financial system.

In normal times, the interbank market ensures efficient liquidity redistribution from banks with surplus liquidity to banks with a shortage of liquidity and thus serves as an absorber of idiosyncratic liquidity shocks. In turbulent times, however, interbank markets can become a channel for liquidity contagion due to liquidity hoarding by banks and/or credit risk contagion due to credit losses on interbank exposures. Interbank market contagion is more likely to occur in banking sectors that are highly dependent on wholesale financing [8]. In an extensive study of the US financial system, Hautsch et al. [9] show that it is mainly the interconnectedness within the financial sector that increases the risk of failure of the entire system, denoted as systemic risk [10].

The intricate structure of linkages can be naturally captured via a network representation of the financial system. Such a network models the interlinking exposures between financial institutions and can thus assist in detecting important shock transmission mechanisms. The use of network theories can enrich our understanding of financial systems, helping to answer questions related to how resilient financial networks are to contagion and how financial institutions form connections when exposed to risk of contagion [11]. Two types of sources for the risk of contagion have been studied in the literature. One is the network of banks investing in similar types of assets, in which one bank failure can lead to a fall in the price of its assets and then affect the solvency of other banks that hold the same assets [12, 13]. The other is the risk of contagion in the interbank market, which concerns the liquidity risk of contagion at a form of interlocking exposure; such exposure is very short term, mainly overnight. The focus of the present paper is on the risk of contagion in the interbank market. The empirical studies of the risk of contagion in the interbank market have been done in those references [14–17]. In the theoretical studies of the risk of contagion in the interbank market, Allen and Gale [18] studied the effect of the static network structure on the risk of contagion. Their results found that an interbank network system with a complete network structure is more stable than that with an incomplete network structure. Besides Allen and Gale, many researchers theoretically study the effect of the network structure on the risk of contagion [19–24]. Although studies on the risk of contagion in the interbank network systems have made the great progress, there are some limitations in existing research. Most of the studies are based on static network structures (fixed bank lending matrixes) and static bank systems (fixed the bank balance sheets). However, the reality of an interbank network system is with high complex dynamics.

This is where the present paper is useful in assessing systemic risk in relation to dynamic financial networks. In theory, the Eisenberg–Noe framework [25] describes simultaneous defaults for one period and not for a dynamic multiperiod scenario which applies to our case. We have to mention that we extend the framework to a multiperiod

setting, which borrows from the framework of Kanno [26] and theoretically analyze the Kenyan interbank market using aggregate data on loans and deposits from the portal-African Markets. Kanno [26] and Lehar [27] considered the multiperiod scenario on the systemic risk; however, they did not consider the dynamic change of the interbank network. Besides, Kanno only uses the maximum entropy algorithms to estimate the topology of the interbank system. Here, in the present paper, we use two methods (the maximum entropy algorithms and the minimum density approach) to estimate the network structure and make the structure of the network system change with time. Therefore, the present paper proposes a theoretical framework to reveal the time evolution of the systemic risk using sequences of financial data and uses the framework to assess the systemic risk of the Kenyan banking system which is the largest in East and Central Africa. This study will help us understand the impact of systemic risk in the Kenyan banking sector. The theoretical frame work we proposed merges existing algorithms, maximum entropy estimation method [14], the minimum density approach [28], the asset value estimation algorithm [26, 27], and obligation clearing algorithm [25], seamlessly to calculate the time evolution of the systemic risk.

The key results of the present paper are summarized as follows. First, we explain the network structure of the Kenyan interbank market and theoretically examine its structure using the estimated bilateral exposures matrix. Based on the measures on in- and out-strength, we show that most of the banks designated as systematically important banks (SIBs) are quite significant in the role they play in the Kenyan interbank market. Second the contagious defaults are then modeled in the Kenyan interbank market to then analyze the mechanism behind contagious defaults. Thirdly stress tests are conducted to analyze the possibility of contagious defaults conditional on a banks basic default at an evaluation time point.

The present paper is organized as follows. Section 2 proposes a theoretic framework of the systemic risk. In the theoretic framework, Section 2.1 first proposes the method of measuring systemic risk that includes basic defaults and contagious defaults, then Section 2.2 deliberates the methodology of the bilateral exposures matrix, and Section 2.3 explains the estimation methodology for the market values of assets. Section 3 describes the data used in this study. Section 4 presents the results of the risk analysis and finally Section 5 concludes the paper.

## 2. The Dynamic Theory Framework of Assessing Systemic Risk

We first construct a dynamic theory framework of assessing systemic risk, which is described as Figure 1. Figures 1(a) and 1(b) describe the network structure of the interbank system in Kenyan, which can be estimated by the methods described in Section 2.1. Figure 1(a) shows the complete network, which can be estimated by the Maximum Entropy Method, while Figure 1(b) represents the sparse network structure that can be estimated by the minimum density approach, which is first proposed by Anand et al. [28]. Figure 1(c) is the dynamic
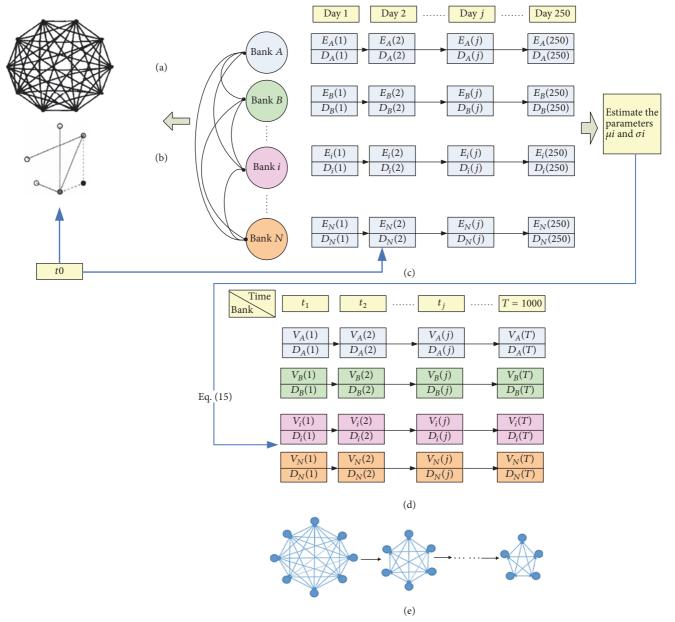
Figure 1: The dynamic theory framework of assessing systemic risk.

estimation method of banks' assets, which is described in Section 2.2. When we get the parameters $u_i$ and $\sigma_i$, then, according to (7), we get the evolution of banks' assets like Figure 1(d). In Figure 1(d), in each time step, we first calculate the basic default of banks; then, due to the connection of banks like Figures 1(a) and 1(b), the basic default of banks, namely, total assets smaller than the total liabilities, cause the default of other banks that are connected to the basic default banks. The method of computing losses from basic default and contagion default is described in Section 2.3. At time $t = 0$, the structure of the interbank network system is estimated by the balance sheet of end of each year. However, due to the time evolution, banks may be defaulted by basic default or by contagion, then banks that are default will be removed from the network, and, therefore, the structure of network

will change with time, which is showed in Figure 1(e), the evolution equations of which will be described in Section 2.4.

### 2.1. Estimation of Bilateral Exposures Matrix.

The lending relationship in the Kenyan interbank market is represented by the following nominal interbank matrix $X$:

$$X = \begin{bmatrix} x_{11} & x_{1j} & x_{1N} \\ x_{i1} & x_{ij} & x_{iN} \\ x_{N1} & x_{Nj} & x_{NN} \end{bmatrix} \begin{matrix} l_1 \\ l_i \\ l_N \end{matrix},$$

$$\sum_i x_{ij} \quad a_1 \; \cdots \; a_j \; \cdots \; a_N$$

$$\sum_j x_{ij}$$

(1)

where $x_{ij}$ denotes the amount of money that bank $i$ borrows from bank $j$. $a_j = \sum_i x_{ij}$ denotes the total value of bank $j$'s interbank assets and $l_i = \sum_j x_{ij}$ denotes bank $i$'s total liabilities. It has to hold that

$$\sum_j a_j = \sum_i l_i = X^\Sigma, \tag{2}$$

where $X^\Sigma$ is size of the interbank market. Next we adopt two methods to estimate the matrix $X$. One is the method of the maximum entropy estimation [14] in Section 2.1.1, and the other is the minimum density approach [28] in Section 2.1.2.

*2.1.1. Method of the Maximum Entropy Estimation.* We know that the diagonal elements of $X$ have to be zero. Therefore, we set the prior matrix of $X^0$ as follows:

$$X^0 = \begin{cases} 0 & \text{for any } i = j \\ a_i l_j & \text{otherwise.} \end{cases} \tag{3}$$

Matrix $X^0$ violates the summing constraints expressed in (2). Consequently, a new matrix $X$ must be found to satisfy the constraints. The solution is provided by solving the optimization problem as follows:

$$\min \quad \sum_{i=1}^N \sum_{j=1}^N x_{ij} \ln\left(\frac{x_{ij}}{x_{ij}^0}\right)$$

$$\text{Subject to} \quad \sum_{j=1}^N x_{ij} = l_i, \tag{4}$$

$$\sum_{i=1}^N x_{ij} = a_j,$$

$$x_{ij} \geq 0.$$

*2.1.2. The Minimum Density Approach.* The minimum density approach minimizes the network's density, the share of actual to potential bilateral links. It minimizes the total number of linkages necessary for allocating interbank positions, consistent with total lending and borrowing observed for each bank. Let $C$ represent the fixed cost of establishing a link. Then the minimum density approach can be formulated as a constrained optimization problem as follows:

$$\min_x \quad C\sum_{i=1}^N \sum_{j=1}^N 1_{[x_{ij}>0]}$$

$$\text{Subject to} \quad \sum_{j=1}^N x_{ij} = l_i, \tag{5}$$

$$\sum_{i=1}^N x_{ij} = a_j,$$

$$x_{ij} \geq 0,$$

where the integer function 1 equals one only if bank $i$ lends bank $j$.

*2.2. Estimation Methodology of Market Values of Assets.* Asset value $V_i(t)$ is not daily observable. However, we can get the asset value in the bank balance sheet at the end of each year, while the equity market price of banks can be observed by stock price on each day. The time $t$ is measured in units of day in the present paper. Next we will give a method to estimate asset values of each day (time evolution of asset value) according to the equity market data of banks. Assume that the asset value $V_i(t)$ of bank $i$ follows a geometric Brownian motion with drift $u_i$ and volatility $\sigma_i$:

$$\frac{dV_i}{V_i} = u_i dt + \sigma_i dW(t), \tag{6}$$

where $W(t)$ is the standard Brownian motion. The solution to this equation is obtained as

$$V_i(t) = V_i(0) e^{(u_i-(\sigma_i^2/2))t+\sigma_i\sqrt{t}z}, \tag{7}$$

where $z$ is a standard normal random variable. If we know the parameters $u_i$ and $\sigma_i$, then, according to (7), we can get the evolution of $V_i(t)$. Next we can observe a time series of equity price $E_i(t)$ from the stock market; then we can use the Black-Scholes model to estimate the parameters $u_i$ and $\sigma_i$ as follows:

$$E_i(t) = V_i(t) N(d_t) - D_i(t) N\left(d_t - \sigma_i\sqrt{T}\right), \tag{8}$$

where $T = 365$ days, $t$ represents the evolution of days, and

$$d_t = \frac{\ln\left(V_i(t)/D_i(t)\right) + \left(\sigma_i^2/2\right)T}{\sigma_i\sqrt{T}}. \tag{9}$$

In the stock market, one can observe a time series of $E_i(t)$ and read the face value of bank debt $D_i(0)$ from the balance sheet. We assume that all bank debt is insured and will therefore grow at the risk-free rate $r$ (the interest rates used have been obtained from the Central Bank of Kenya website, for the relevant years (2009–2015) (https://www.centralbank.go.ke/statistics/interest-rates/)). Then, $D_i(t) = D_i(0)e^{rt}$. Given the initial data of $V_i(0)$, time series data of $E_i(0), E_i(1), \ldots, E_i(T), D_i(0), D_i(1), \ldots, D_i(T)$, and the arbitrary initial value of $u_i(0), \sigma_i(0)$, we can get the estimation of $\widehat{V}_i(1), \widehat{V}_i(2), \ldots, \widehat{V}_i(T)$ according to (8) and (9). Then, we use the following maximization likelihood function to estimate the parameters $u_i$ and $\sigma_i$, which is proposed by Duan et al. [29–31]:

$$L\left(u_i, \sigma_i; \widehat{V}_i(1), \widehat{V}_i(2), \ldots, \widehat{V}_i(T)\right)$$

$$= -\frac{T}{2}\ln\left(2\pi\sigma_i^2 h\right)$$

$$- \frac{T}{2}\sum_{k=1}^T \frac{\left(R_i(k) - \left(u_i - \sigma_i^2/2\right)h\right)^2}{\sigma_i^2 h} \tag{10}$$

$$- \sum_{k=1}^T \ln\widehat{V}_i(k),$$

where $R_i(k) = \ln(\widehat{V}_i(t)/\widehat{V}_i(t-1))$ and $h = 1/365$. Here, $h$ represents business days instead of calendar days. According to the stock market data source, $h$ may equal about 250, which will be different at each year.

The estimated parameters are denoted as $\widehat{u}_i, \widehat{\sigma}_i$ respectively. Then, we compare the estimated parameters $\widehat{u}_i, \widehat{\sigma}_i$ with the initial values $u_i(0), \sigma_i(0)$; if $\widehat{u}_i, \widehat{\sigma}_i$ do not equal $u_i(0), \sigma_i(0)$, we replace the initial values $u_i(0), \sigma_i(0)$, and $V_i(0)$ with $\widehat{u}_i, \widehat{\sigma}_i$, and $\widehat{V}_i(0)$, and then we repeat the estimation method from (8), (9), and (10) once again until the estimated parameters $\widehat{u}_i, \widehat{\sigma}_i$ equal $u_i(0), \sigma_i(0)$. Accordingly, we get the estimated parameters $u_i$ and $\sigma_i$. Then, according to (7), we get the evolution of $V_i(t)$. The estimation method of $V_i(t)$ is the same as [27].

### 2.3. Method of Measuring Systemic Risk.

Here, we study how to assess the systemic risk in the financial network. We extend a fundamental framework proposed by Eisenberg and Noe [25] to a multiperiod setting. There is a clearing payment system that deals with the interbank payment amounts of all the banks in the system daily.

Let us look at a set of banks $N = \{1, \ldots, N\}$ at time $t$. The interbank structure is represented as $(X(t), e(t))$, where

$X$ is a $(N \times N)$ nominal interbank liabilities matrix and $e$ is the noninterbank net claims which is the difference between the market value of assets and the value of liabilities, namely, $V_i(t) - D_i(t)$.

If the total value of a bank becomes negative for a pair $(X(t), e(t))$, then the bank becomes bankrupt. Let $x_i(t) = \sum_{j=1}^{N} x_{ij}(t)$ represent the total interbank liability of bank $i$ to all banks $j$ of the system. Furthermore, we consider a matrix $\Pi$, which is brought about by normalizing the entries to the total claims:

$$\Pi_{ij}(t) = \begin{cases} \dfrac{x_{ij}(t)}{x_i(t)}, & \text{if } x_i(t) > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

A banking system is designated as a tuple $(\Pi(t), e(t),$ and $X(t))$, for which we describe a clearing payment vector $p^*(t)$. The clearing payment vector represents the limited liabilities of the banks and the proportional distribution in the event of a collapse.

A payment vector $p_i^*(t)$ is a clearing payment vector subject to the following happening:

$$p_i^*(t) = \begin{cases} x_i(t) & \sum_{j=1}^{N} \Pi_{ji}(t) p_j^*(t) + e_i(t) \geq x_i(t), \\ \sum_{j=1}^{N} \Pi_{ji}(t) p_j^*(t) + e_i(t) & 0 \leq \sum_{j=1}^{N} \Pi_{ji}(t) p_j^*(t) + e_i(t) < x_i(t), \\ 0 & \sum_{j=1}^{N} \Pi_{ji}(t) p_j^*(t) + e_i(t) < 0. \end{cases} \tag{12}$$

Therefore, bank $j$ is solvent in case 1 and insolvent in cases 2 and 3.

We identify an insolvent bank $j$ under the condition $(p_i^*(t) < x_i(t))$, which holds for cases 2 and 3.

We implement the default algorithm established by Eisenberg and Noe [25] to find a clearing payment vector. They demonstrate that, under mild regulatory conditions, a unique clearing payment vector always exists for $(\Pi(t), e(t),$ and $X(t))$. These regularity conditions refer to properties that the network structure must have in order that there is a unique clearing vector.

These results apply to our multiperiod setting. The number of defaulted banks is computed by comparing the clearing payment vector to the nominal liability vector. A theoretical default algorithm is applied to compute the clearing payment vector and is summarized as follows.

#### Type 1: Basic Default

$$V_i(t) - D_i(t) \leq 0, \tag{13}$$

where $V_i(t)$ is the market value of the total assets of bank $i$ at time $t$ (days) and $D_i(t)$ is the total face value of the interest-bearing debt of bank $i$ at time $t$. The basic default

is an idiosyncratic default, caused by the condition of the defaulting node itself. According to Section 2.3, $V_i(t)$ is a random walk variable causing the idiosyncratic default at time step $t$.

#### Type 2: Contagious Default

$$\left( \sum_{j=1}^{N} \Pi_{ji}(t) x_j(t) - x_i(t) \right) + e_i(t) > 0, \tag{14}$$

$$\left( \sum_{j=1}^{N} \Pi_{ji}(t) p_j^*(t) - x_i(t) \right) + e_i(t) \leq 0. \tag{15}$$

If the claims of bank $i$ are positive, but its obligations banks pay less liability to bank $i$, which results in the fact that the net claim of bank $i$ is negative, then a contagious default occurs on bank $i$.

### 2.4. The Evolution of the Bilateral Exposures $x(t+1), a_i(t+1),$ and $l_i(t+1)$.

After calculating a clearing payment vector at time step $t$, we can calculate the new matrix of $X$ at the time step $t+1$. We should note that when bank $i$ defaults, bank

$i$ can pay only part of their liabilities to other banks. $\chi_i$, the ratio, is defined as follows:

$$\chi_i = \frac{\sum_{j=1}^{N} \Pi_{ji}(t) \, p_j^*(t) + e_i(t)}{l_i(t)}. \tag{16}$$

The total assets and liabilities of bank $j$ from time steps $t + 1$ to $T$ will be updated as follows:

$$V_j(t+1:T) = V_j(t) - x_{ji},$$

$$D_j(t+1:T) = D_j(t) - x_{ji}, \tag{17}$$

$$V_j(t+1:T) = V_j(t) - (1 - \chi_i) x_{ij}.$$

When bank $i$ defaults, we set $x_{i,j}(t) = 0$ and $x_{j,i}(t) = 0$ and clear out bank $i$ from the network bank system. In real world, the interbank exposures may change from day to day. And, in the present paper, due to the default banks, the number of banks decreases, which causes the interbank exposures to change by time step. Note that if there is no default bank in $t$, the network estimated does not change, because the interbank totals remain the same in $t$ and $t + 1$.

Then, we need to recalculate the bilateral exposures matrix $X_{t+1}$ according to the algorithm in Section 2.1. Thus, the evolution of $a_j(t + 1)$ and $l_i(t + 1)$ is described as follows:

$$a_j(t+1) = \sum_{i=1}^{N} x_{i,j}(t+1),$$

$$l_i(t+1) = \sum_{j=1}^{N} x_{i,j}(t+1). \tag{18}$$

## 3. Data

In the present paper, we use data from the portal of African Markets. The portal has historical share price data and has annual reports of the listed companies of the biggest economies on the African continent. The data we have used from this site was obtained from annual reports from the years 2008 to 2015 from eight listed banks, as well as market data (share prices) for the same. We also obtained monthly interest rates from the Central Bank of Kenya for the same banks. The banks selected for this research are big banks in the Kenyan banking sector and they essentially have big market share of banking clients in the market. Financial sector in the Kenyan banking system includes the Central Bank of Kenya (CBK), the primary regulator of the banking industry; 28 domestic and 14 foreign commercial banks with branches, agencies, and other outlets throughout the country; one mortgage finance company; eight representative offices of foreign banks; eleven licensed deposit taking microfinance institutions. However, the banking sector is essentially dominated by four major commercial banks, namely, Equity Bank, Kenya Commercial Bank, Barclays Bank of Kenya, and Standard Chartered. In addition, smaller banks have emerged and experienced tremendous growth in recent years. According to the Central Bank of Kenya, 66.7 percent of the adult population in 2013 had formal access to financial services through commercial banks and the government-owned Post Bank. With the advent of mobile money and its recent linkages to the formal banking system, however, the number of Kenyans with access to electronic financial services has grown rapidly. Kenya has now become a leader in financial inclusion and its example is being replicated in countries around the world.

To be able to achieve the objectives of our research, we need to identify the interbank exposures and noninterbank exposures (net claims cash flow $e$ for each bank). The interbank exposures considered in the analysis are interbank loans and advances to banks. These items are yearly interbank transactions in the interbank markets. We do not consider interbank transfers between parent firms and subsidiaries. There are plenty of interbank transactions in this market; therefore, we can estimate the bilateral exposures matrix from the data available in the annual reports. Because we also need to estimate the market value of the assets from equity data, we only consider publicly traded banks. We therefore acquired daily market share price data of the said eight banks. The data of end-of-year period for 2009–2015 in the Kenya banking system is listed in Table 1.

## 4. Results

This section describes our analysis results. In Section 4.1, we discuss the significance of the bilateral exposures matrix estimation and the network analysis. Section 4.2 deals with the default analysis. Finally, in Section 4.3 we report on the results of the stress test.

*4.1. Network Analysis and Estimation of Bilateral Exposures Matrix.* We estimate the bilateral exposures matrix $X$ stated in (1) and use the matrix to examine the network structure of the Kenyan interbank market. We investigate the global interbank network using network centrality measures. Usually the degree of a node is considered as a proxy variable for interconnectedness and explains the number of edges connected to a node. In the present paper, we define the in-strength that shows the ratio of the money lent to all the other banks to the total money. For simplicity, the in-strength of bank $i$ is $a_i / \sum a_i$. Similarly, the out-strength of bank $i$ is $l_i / \sum l_i$. The total strength of a bank is the summation of its in-strength and out-strength. These measures, hence, give a sense of investment and funding diversifications. Figure 2 highlights the time variations of the in-strength (red line) and out-strength (blue line) for eight listed banks. The bigger the increase in the in-strength, the more the debtors the bank would have. In contrast, the bigger the increase in the out-strength, the more the creditors the bank would have. Therefore, in terms of contagious default, the out-strength is more important than the in-strength. Figure 2 depicts a group of banks that borrow more than they lend and others that lend more than they borrow. Banks like NIC Bank, Diamond Trust Bank, National Bank of Kenya, Cooperative Bank, and Barclays Bank lend more than they borrow to other banks by averages of 55.3%, 53.3%, 46.2%, 38.9%, and 35.8%, respectively, from 2008 to 2015.

Meanwhile, banks like Kenya Commercial Bank and Equity Bank lend more than they borrow by an average of

TABLE 1: Data of the end-of-year period for the years 2009–2015 in the Kenya banking system. We use the letters s1, s2, s3, s4, s5, s6, s7, and s8 to stand for Barclays Bank, Coop, Diamond Trust, Equity Bank, HFCK, KCB, NBK, and NIC respectively. The unit of currency is Shs.

| Year 2009 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| --- | --- | --- | --- | --- | --- |
| S1 | Barclays Bank | 155151000 | 1061000 | 140941000 | 262000 |
| S2 | Coop | 110531373 | 4642338 | 94428114 | 1010216 |
| S3 | Diamond Trust | 47146767 | 5638340 | 40883446 | 1310761 |
| S4 | Equity Bank | 96512000 | 2022000 | 73174000 | 6114000 |
| S5 | HFCK | 18280761 | 2106419 | 14196530 | 1700000 |
| S6 | KCB | 172384128 | 5936128 | 149986213 | 9309706 |
| S7 | NBK | 51404408 | 1154271 | 43496716 | 50290 |
| S8 | NIC | 47558241 | 4936616 | 40765987 | 56767 |
| Year 2010 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| S1 | Barclays Bank | 171151000 | 935000 | 140818000 | 92000 |
| S2 | Coop | 153983533 | 6671257 | 134396691 | 5348291 |
| S3 | Diamond Trust | 58605823 | 4404753 | 50548446 | 1584787 |
| S4 | Equity Bank | 133890000 | 1238000 | 105583000 | 7464 |
| S5 | HFCK | 29325842 | 7866266 | 25056311 | 1573369 |
| S6 | KCB | 223024556 | 3959963 | 182148111 | 10807301 |
| S7 | NBK | 60026694 | 1408699 | 50097083 | 780203 |
| S8 | NIC | 54776432 | 4611935 | 46880578 | 101479 |
| Year 2011 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| S1 | Barclays Bank | 166269000 | 913000 | 139276000 | 123000 |
| S2 | Coop | 167772390 | 7437716 | 147395935 | 1881284 |
| S3 | Diamond Trust | 77453024 | 9452751 | 67086550 | 1658627 |
| S4 | Equity Bank | 176911000 | 1094000 | 141864000 | 13769000 |
| S5 | HFCK | 31972113 | 4724183 | 27189895 | 847507 |
| S6 | KCB | 282493553 | 17648880 | 237330895 | 11502571 |
| S7 | NBK | 5564998 | 3388191 | 58208042 | 155613 |
| S8 | NIC | 73581321 | 4486475 | 63681461 | 206149 |

Table 1: Continued.

| Year 2012 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
|---|---|---|---|---|---|
| S1 | Barclays Bank | 185100000 | 1282000 | 155514000 | 1724000 |
| S2 | Coop | 199662956 | 8886167 | 171270727 | 1065302 |
| S3 | Diamond Trust | 94511818 | 4404676 | 79633326 | 1002394 |
| S4 | Equity Bank | 215829000 | 983000 | 173158000 | 25755000 |
| S5 | HFCK | 40685928 | 6395958 | 35540111 | 10212633 |
| S6 | KCB | 304751807 | 449151 | 250870082 | 8261878 |
| S7 | NBK | 67154805 | 1356954 | 56704829 | 254694 |
| S8 | NIC | 101771705 | 6569964 | 86707098 | 3044959 |
| Year 2013 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| S1 | Barclays Bank | 207011000 | 1386000 | 174639000 | 4738000 |
| S2 | Coop | 228874484 | 10040915 | 193776707 | 5462337 |
| S3 | Diamond Trust | 114136429 | 4120113 | 95568152 | 3734712 |
| S4 | Equity Bank | 238194000 | 1063000 | 187508000 | 25612000 |
| S5 | HFCK | 46755111 | 6878601 | 41073258 | 10212469 |
| S6 | KCB | 322684854 | 5222915 | 260921815 | 5516617 |
| S7 | NBK | 92493034 | 8283461 | 80644743 | 824858 |
| S8 | NIC | 112916814 | 4189408 | 95285885 | 4910171 |
| Year 2014 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| S1 | Barclays Bank | 226616000 | 1207000 | 187931000 | 121000 |
| S2 | Coop | 282689098 | 12814861 | 240337979 | 3241726 |
| S3 | Diamond Trust | 141175794 | 7413484 | 115391380 | 2176219 |
| S4 | Equity Bank | 276115727 | 16904704 | 236382382 | 199136 |
| S5 | HFCK | 60490883 | 8583509 | 54214800 | 0 |
| S6 | KCB | 376969401 | 2577662 | 304802062 | 8733510 |
| S7 | NBK | 122864886 | 2033662 | 110750974 | 5077625 |
| S8 | NIC | 137087464 | 7843903 | 113829447 | 4625406 |
| Year 2015 | Bank name | Total assets | Total interbank lending | Total liabilities | Total interbank borrowing |
| S1 | Barclays Bank | 241152698 | 252867 | 201436326 | 187385 |
| S2 | Coop | 339549808 | 13869273 | 290239033 | 2905405 |
| S3 | Diamond Trust | 190947903 | 4973737 | 160951702 | 8149404 |
| S4 | Equity Bank | 341329318 | 16554308 | 293889704 | 565208 |
| S5 | HFCK | 68808654 | 5517670 | 59718700 | 0 |
| S6 | KCB | 467741173 | 9254721 | 386855215 | 14759625 |
| S7 | NBK | 117789712 | 1850368 | 104345832 | 6806967 |
| S8 | NIC | 156762225 | 5464120 | 130307839 | 7734613 |
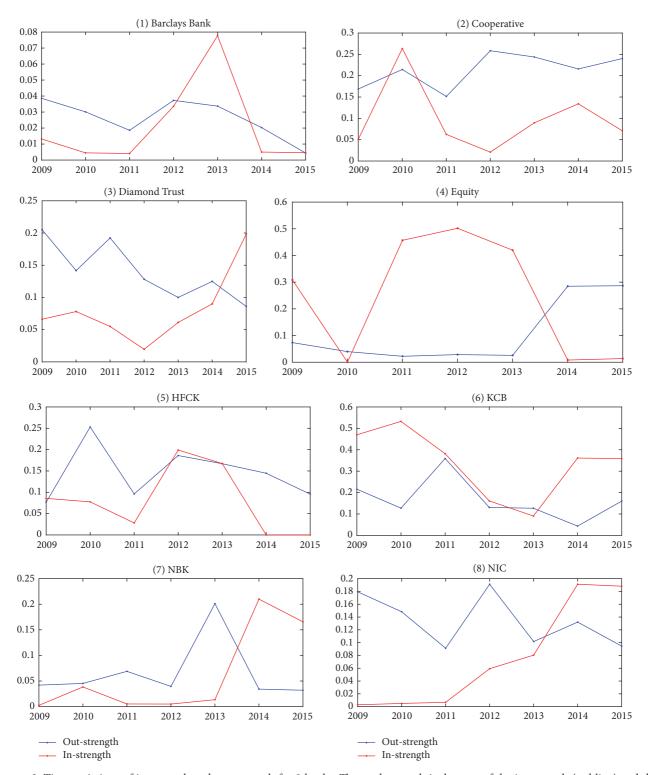
FIGURE 2: Time variations of in-strength and out-strength for 8 banks. The total strength is the sum of the in-strength (red line) and the out-strength (blue line). The vertical axis (the strength) shows the percentage of the amount of money borrowed or lent by a bank while horizontal axis represents time in years.
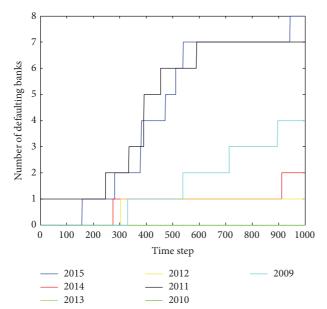
FIGURE 3: Time variations in number of defaulting banks in Kenyan banking system with the network estimated by maximum entropy estimation method.



FIGURE 4: Time variations in number of defaulting banks in Kenyan banking system with the network estimated by maximum entropy estimation method and the minimum density approach in the year 2015 with parameter $c = 10$.

24.3% and 19.2%, respectively, from 2008 to 2015. Therefore, we can examine which banks borrow (lend) more than they lend (borrow) in the Kenyan network in terms of percentage. Following is a breakdown of the percentage figures that these banks borrow in order of magnitude and they include Kenya Commercial Bank (24.3%) and Equity Bank (19.2%). The list of banks that lends includes NIC Bank (55.3%), Diamond Trust Bank (53.3%), HFCK Bank (49.5%), National Bank of Kenya (46.2%), Cooperative Bank (38.9%), and Barclays Bank (35.8%).

*4.2. Bank Default Analysis.* We estimate the theoretical number of defaulting banks during the estimation period of 2009–2015, which is presented in Figure 3. Figure 3 indicates the time variations of the number of defaulting banks in the Kenyan banking industry with the complete network that is estimated by the Maximum Entropy Method. It basically shows that in 2015 the Kenyan banking system is more unstable than other years. The years in 2013 and 2010 are more stable than other years, because no banks defaulted.

There has also been a banking crisis in Kenya since 2015 mostly due to weak supervision and outright fraud by bank directors. An example of this is the nonlisted Chase Bank which was put under receivership in the same year. Several other banks including National bank also showed signs of collapsing due to the same. Analysts have been warning banks since 2012 to stop understating loan provisions and to increase their capitalization. A lot of work is still needed especially with the regulators including the CMA (Capital Market Authority) and Kenyan banking sector poses a challenge of lack of trust in the banking industry as most clients move to other rudimentary means of saving their money.

Since the Kenyan banking system in the year 2015 is most unstable, we compare the effect of network structure on the time variations in the number of defaulting banks in 2015, which is presented in Figure 4. From Figure 4, we can see that the network topology estimation methods cause not much effect on the time variations in the number of defaulting banks. Next, we compare the evolution of the network topology, which is showed in Figures 5 and 6, respectively. In Figures 5 and 6, the time step is showed in each bank defaults; for example, in Figure 5, when the time step is 158, then HFCK defaults. After HFCK is removed from the bank system, the topology of the network system is changed after 158 time steps. Figures 5 and 6 show that although the estimation methods are not similar, the results are similar, perhaps due to the low number of banks in the network. However, we note in Figure 4 that the number of defaults increases earlier in the case of the minimum density approach. Considering that the maximum entropy estimation produces a complete network, which lowers systemic risk measures, and that the minimum density approach produces a network that increases systemic risk measures, we can claim that the time variations in the number of defaulting banks in the Kenyan banking system, for the real (nonobserved) network, is around the range provided by the two methods.
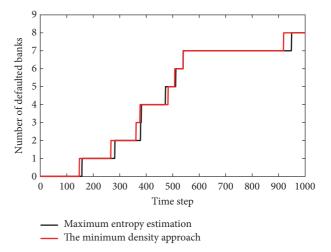
*4.3. Stress Testing.* Since the effect of the topology of the bank network system estimated by two methods is not relevant, we conduct a stress test to confirm the strength of the Kenyan banking system with the minimum density approach in 2009, 2011, and 2015, which would provide higher systemic risk measures (more conservative). Our test is somewhat different from typical macro stress tests, which first remove a bank from the Kenyan banking system and then find how many banks defaults the removed bank can cause, namely, contagious defaults. The results of stress test are listed in Tables 2, 3, and 4 as follows. Table 2 shows that in the Kenyan banking system the KCB bank defaulting can result in four defaults banks (namely, contagious default), because the KCB
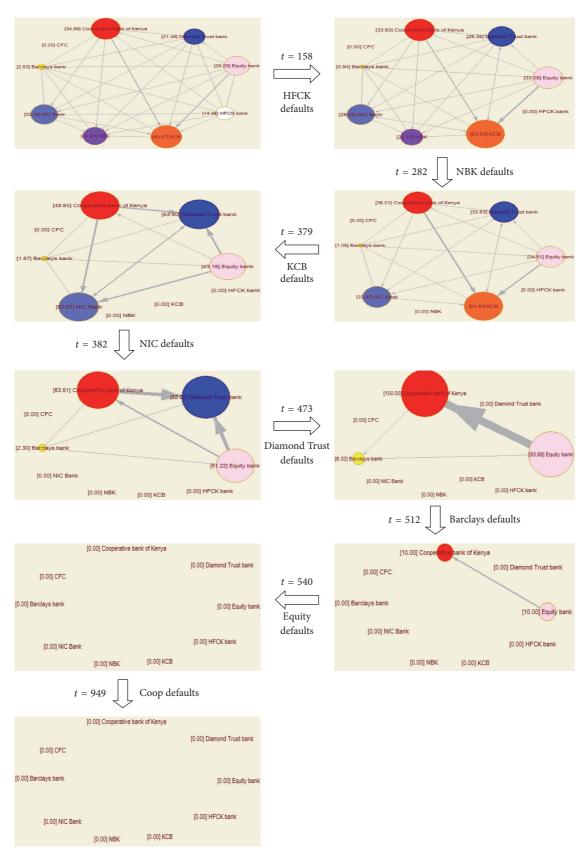
FIGURE 5: The evolution of the network topology in 2015 estimated by maximum entropy; the size of each node represents the total percentage of in-strength and out-strength of each bank and the number of size is marked beside each node.
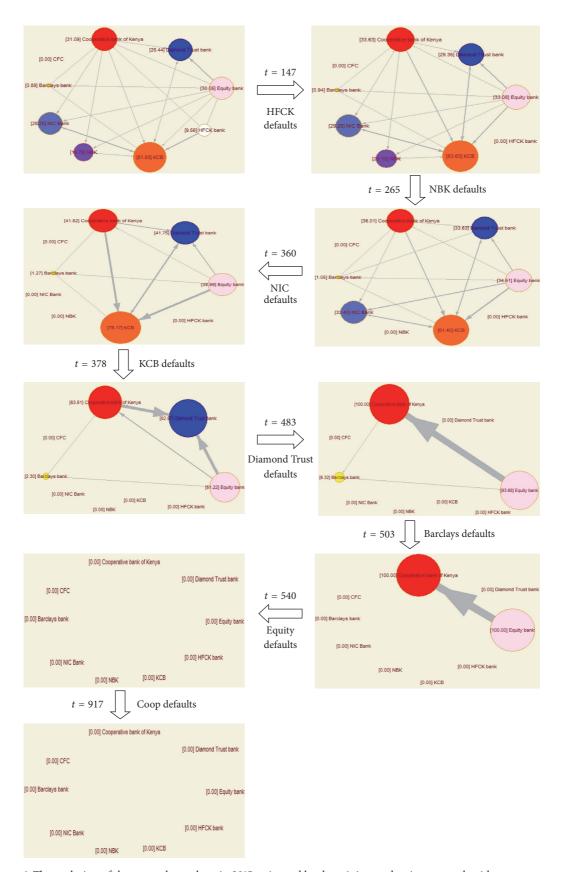
FIGURE 6: The evolution of the network topology in 2015 estimated by the minimum density approach with parameter $c = 10$.

TABLE 2: Results of stress test in 2009; CDs number: number of contagious defaults; IC: interconnectedness: the total strength, namely, sum of in-strength and out-strength; TA: total assets; IA: interbank assets (loans and advances to banks). Total assets are measured at market value, whereas interbank assets are measured at book value. We use the letters s1, s2, s3, s4, s5, s6, s7, and s8 to stand for Barclays Bank, Coop, Diamond Trust, Equity Bank, HFCK, KCB, NBK, and NIC, respectively. The unit of currency is Shs.

| Number | Bank name | Collapsed banks | CDs number | IC | TA | IA |
|---|---|---|---|---|---|---|
| S1 | Barclays Bank | S1, S6, S7, S8 | 3 | 0.0518 | 155,151,000 | 1061000 |
| S2 | Coop | S1, S2, S6, S7, S8 | 4 | 0.2198 | 110531373 | 4642338 |
| S3 | Diamond Trust | S1, S3, S6, S7, S8 | 4 | 0.2713 | 47146767 | 5638340 |
| S4 | Equity Bank | S1, S4, S6, S7, S8 | 4 | 0.3821 | 96512000 | 2022000 |
| S5 | HFCK | S1, S5, S6, S7, S8 | 4 | 0.1624 | 18280761 | 2106419 |
| S6 | KCB | S1, S2, S3, S6, S7, S8 | 5 | 0.6858 | 172384128 | 5936128 |
| S7 | NBK | S1, S6, S7, S8 | 3 | 0.0445 | 51404408 | 1154271 |
| S8 | NIC | S1, S6, S7, S8 | 3 | 0.1824 | 47558241 | 4936616 |

TABLE 3: Results of stress test in 2011.

| Number | Bank name | Collapsed banks | CDs Number | IC | TA | IA |
|---|---|---|---|---|---|---|
| S1 | Barclays Bank | S1, S2, S5, S7 | 3 | 0.0227 | 166269000 | 913000 |
| S2 | Coop | S2, S5, S7 | 2 | 0.2137 | 167772390 | 7437716 |
| S3 | Diamond Trust | S2, S3, S5, S7 | 3 | 0.2473 | 77453024 | 9452751 |
| S4 | Equity Bank | S2, S3, S4, S5, S7 | 4 | 0.4791 | 176911000 | 1094000 |
| S5 | HFCK | S2, S5, S7 | 2 | 0.1242 | 31972113 | 4724183 |
| S6 | KCB | S2, S3, S5, S6, S7, S8 | 5 | 0.7407 | 282493553 | 17648880 |
| S7 | NBK | S2, S5, S7 | 2 | 0.0741 | 5564998 | 3388191 |
| S8 | NIC | S2, S5, S7, S8 | 3 | 0.0981 | 73581321 | 4486475 |

TABLE 4: Results of stress test in 2015.

| Number | Bank name | Collapsed banks | CDs number | IC | TA | IA |
|---|---|---|---|---|---|---|
| S1 | Barclays Bank | S1, S3, S5, S6, S7, S8 | 5 | 0.009 | 241152698 | 252867 |
| S2 | Coop | S2, S3, S5, S6, S7, S8 | 5 | 0.3109 | 339549808 | 13869273 |
| S3 | Diamond Trust | S3, S5, S6, S7, S8 | 4 | 0.2843 | 190947903 | 4973737 |
| S4 | Equity Bank | S3, S4, S5, S6, S7, S8 | 5 | 0.3004 | 341329318 | 16554308 |
| S5 | HFCK | S3, S5, S6, S7, S8 | 4 | 0.0956 | 68808654 | 5517670 |
| S6 | KCB | S3, S4, S5, S6, S7, S8 | 5 | 0.5193 | 467741173 | 9254721 |
| S7 | NBK | S3, S5, S6, S7, S8 | 4 | 0.1976 | 117789712 | 1850368 |
| S8 | NIC | S1, S3, S5, S6, S7, S8 | 5 | 0.2828 | 156762225 | 5464120 |

bank is the highest interconnectedness and its total assets and interbank assets are the largest.

In 2011, Table 3 shows the relationship between the number of contagious defaults and interconnectedness, the total assets, and the interbank assets. With high interconnectedness, the total assets, and the interbank assets, the default banks cause more contagious defaults banks, for example, Equity Bank and KCB bank, which cause the number of contagious defaults to be 4 and 5, respectively. In 2015, the Kenyan banking system is more unstable seen from Table 4, because any bank defaults can cause more than 4 banks to have contagious defaults.

The stress tests results indicate the number of contagious defaults caused by a SIB's (systematically important bank) default. In general, the banks that trigger over four contagious

defaults have significantly more interbank exposures as well as greater interconnectedness measured in terms of strength than the other banks do. In contrast, the banks that trigger less contagious defaults do not necessarily have more interbank exposures or greater interconnectedness compared to SIB's banks. As per the above results, we do not have any bank that has triggered five contagious results, only KCB. Therefore, we found the KCB is the systematically important bank in the Kenyan banking system.

## 5. Conclusion

The present paper proposed a theoretical framework to find the time evolution of the systemic risk by calculating the number of defaults of banks using sequences of daily financial

data. The framework combines the asset value estimation algorithm [26, 27], maximum entropy estimation method [14], the minimum density approach [28], and obligation clearing algorithm [25], effortlessly to deal with the dynamic problem—the time evolution of the systemic risk. The asset value estimation algorithm is used to approximate the asset values of the banks at each day which are required to calculate the time evolution of systemic risk. The obligation clearing algorithm is used to calculate the systemic risk given the tuples of data on a daily basis.

In the present paper, we evaluated the systemic risk of the Kenyan banking system using the theoretical framework proposed. The Kenyan interbank market involves various domestic contracts and transactions. First, we clarified the network structure of the Kenyan interbank market and theoretically analyzed its network structure using the estimated bilateral exposures matrix. We also analyzed the interconnectedness of each bank in the Kenyan interbank market using the in- and out-strength measure. Significantly, we found that the banks designated as systematically important banks (SIBs) play a central role in the Kenyan interbank market and these are Kenya Commercial Bank (KCB) and Equity Bank.

We modeled contagious defaults in the Kenyan interbank network using real aggregate banking data from the portal of African Markets and theoretically analyzed the mechanism of contagious defaults conditional on a basic default during a seven-year period (2009–2015).

Further analysis theoretically showed the occurrence of some contagious defaults in 2009, 2011, and 2015, and these years are very unstable than other years. We also conducted a stress test and analyzed the likelihood of contagious defaults conditional on a bank's basic default at an evaluation time point in the future. Some banks designated as SIBs were confirmed to have the potential to trigger the contagious defaults of other banks. In general, the banks that trigger over more contagious defaults have significantly more interbank exposures as well as greater interconnectedness measured in terms of strength than the other banks do. We found that the KCB is the most systematically important bank in the Kenyan banking system.

To finalize, we are convinced that, in order to uphold the stability of the Kenyan banking system, there is a need to apply systemic risk assessment practices. These could also be useful in the execution of bank-internal systemic stress tests of default contagion.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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*Research Article*

# The Application of Macroprudential Capital Requirements in Managing Systemic Risk

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When setting banks regulatory capital requirement based on their contribution to the overall risk of the banking system we need to consider that the risk of the banking system as well as each banks risk contribution changes once bank equity capital gets redistributed. Therefore the present paper provides a theoretical framework to manage the systemic risk of the banking system in Nigeria based on macroprudential capital requirements, which requires banks to hold capital that is proportional to their contribution to systemic risk. Using a sample of 10 Nigerian banks, we reallocate capital in the system based on two scenarios; firstly in the situation where the system shocks do not exist in the system, we find that almost all banks appear to hold more capital; secondly, we also consider the situation where the system shocks exist in the system; we find that almost all banks tend to hold little capital on four risk allocation mechanisms. We further find that despite the heterogeneity in macroprudential capital requirements, all risk allocation mechanisms bring a substantial decrease in the systemic risk. The risk allocation mechanism based on ΔCoVaR decreases the average default probability the most. Our results suggest that financial stability can be substantially improved by implementing macroprudential regulations for the banking system.

## 1. Introduction

The downfall of Lehman Brothers in mid-2008 unveils that the modern financial system was extremely fragile. The financial system deteriorated due to the distress and in some cases failure of important institutions, leading to further distress and the spread of shocks to the real economy [1]. The crisis emphasizes the need of identifying the underlying factors that destabilize the financial institutions, which could result in systemic risk.

Bisias et al. [2] have defined systemic risk as the risk of disruption to financial services that is caused by an impairment of all or parts of the financial system and has the potential to have serious negative consequences on the real economy. Systemic risk is created endogenously within the banking system due to banks' common exposures to macroeconomic factors and propagated through interbank connections (contagion); thus systemic risk encompasses two aspects which are basic and contagious defaults.

To deal with the systemic risk better, the financial stability board has pinpointed the need for a macroprudential approach to financial system analysis. Researchers like Galati and Moessner [3], Cerutti et al. [4], Ebrahimi Kahou and Lehar [5], Lehar (2005) [6], and Hanson et al. [7] argue that macroprudential policy is aimed to mitigate the systemic risk and reduce its aggregate cost for the real economy; thus the bank regulation should be designed on macroprudential perspective so as to downsize the amount of systemic risk. On the other hand, Basel III requires a capital conservation buffer in normal times consisting of a further amount of core Tier 1 equity capital equal to 2.5% of risk weighted assets. This provision is designed to ensure that banks build up capital during normal times so that it cannot be affected when losses are incurred during periods of financial distress. Therefore it is much easier for banks to raise capital during normal times than during periods of stressed market conditions. In a situation where the capital conservation buffer has been wholly or partially used up, banks are required to constrain their dividends until the capital has been replenished. However, the bank regulators in some countries require banks to hold more capital than the minimum specified by the Basel Committee and some banks themselves have a target for the

capital they will hold that is higher than that specified by their bank supervisors.

In a banking system both the overall risk and each bank's contribution are endogenous and hinge on the banks' equity capital. This means that as banks hold more capital the probability of default through either direct losses or contagion is less; therefore redistributing bank capital changes the banks' default probabilities, overall risk of the banking system, and each bank's risk contribution. In this work we investigate one regulatory mechanism that is macroprudential capital requirements that require each bank to hold a buffer of equity capital that corresponds to the banks contribution to the overall risk of the system. We use the complex network theory to construct the network model of banking system, where each bank is regarded as a node and connected with others through the interbank bilateral exposures. We investigate the systemic risk of banking system by measuring bank's basic default caused by the insolvency of bank itself and contagious default due to the interbank bilateral exposures.

We derive macroprudential capital requirements as a fixed point using four risk allocation mechanisms: Component and incremental value at risk [8], ΔCoVaR [9], and a risk allocation mechanism based on Shapley value [10]. Among the mentioned approaches, CoVaR has the ability to detect the risk on the system by individually systemically important financial institutions, which are so interconnected and large that they can cause negative risk spillover effects on others and in addition to the smaller institutions that are systemic when acting as part of a crowd. Moreover this measure does not rely on contemporaneous price movements and therefore can be used to anticipate systemic risk as it captures institutional externalities such as "too big to fail," "too interconnected to fail," and crowded trade positions. We show that reallocation of bank capital in the system through the new capital requirements can change the probability of the bank to default and change total systemic risk. We supplement the model by applying the interbank clearing network algorithm designed by Eisenberg and Noe [11]. Finally, we compare the effects of macroprudential regulations for banking system under four risk allocation mechanisms to measure the stability of the banking system.

Our study considers Africa that is mostly excluded by researchers in the study of systemic risk because its stock market capitalization is low in the global banking assets. We use the data of 10 major banks in Nigeria in terms of assets value and test its validity even though the country was not affected heavily by the financial crisis. Moreover this study is intended to show if banks keep enough capital for the risks they take and therefore we estimate macroprudential capital requirements as a fixed point problem. Thus, supported by both theoretical and empirical evidence in Nigerian banking system, our studies analyze how macroprudential capital requirements can reduce the level of systemic risk.

## 2. Literature Review

There has been a wide heterogeneous discussion of measuring and evaluating contagion and systemic risk around the globe; however there is a scant of literature reviews on the macroprudential policy particularly regulatory capital requirements on managing of systemic risk. The main impediment for a true implementation of macroprudential capital requirements is that each bank's capital requirement would in part be driven by the actions of other banks, and therefore a bank cannot exercise full control over its own capital requirements.

Two types of the risk of contagion have been studied in the literature. One is the network of banks investing in similar types of assets, in which one bank failure can lead to a fall in the price of its assets and then affect the solvency of other banks that hold the same assets [12, 13]. The other is the risk of contagion in the interbank market, which concerns the liquidity risk of contagion at a form of interlocking exposure; such exposure is very short term, mainly overnight. In this study we focus on the interbank network as a contagion channel of systemic risk, which is when some banks are not able to honor their promises in the interbank market they might push other banks into insolvency which might again lead to defaults of other banks.

The literature on contagion starts with the work of Allen and Gale [14] who give a model of risk propagation through interbank exposure network. They consider that the possibility for contagion depends on the precise structure of the interbank market and show that a complete structure of claims in which every bank has symmetric exposures to all other banks is much more stable than an incomplete structure; thus for same shocks some structures would result in contagion while others would not. Freixas et al. [15] consider that for contagion to happen in a system with money-centre banks where the institutions on the periphery are connected to banks at the centre but not to each other depends on the precise values of the models parameters. Researchers regard financial networks as robust-yet-fragile which means that they can absorb smaller shocks to the system but may show contagion and cascade effects when exposed to a large enough shock [16–18]. Allen and Gale [14] show that if there is no liquidity shock, all banks can survive; however in liquidity shock case, number of defaulting banks change depending on network completeness. Upper [19] makes a summary of contagion studies in the literature and shows that as the shock some papers are considering individual bank failures and some are using failure of group of banks. Lubloy [20] grouped banks according to their FX exposures and let all banks in a given category fail jointly. Elsinger et al. [21] use loan registry data to model common shocks to loan books and banks foreign exchange and stock market exposures to model shocks from financial markets. Their approach included bankruptcy costs in the simulation of the Austrian banking system and show that the system is able to absorb shocks well for small bankruptcy costs while large dead weight losses can crash the banking system. Rogers and Veraart [22] model clearing in the interbank networks with bankruptcy costs and provide an analysis of those situations in which banks have incentives to bail out distressed bank. Moreover, Frisell et al. [23] use detailed Swedish data to model common shocks; they use asset prices correlations to get a covariance matrix for the shock process. They show

that contagion is much more common and reflected in the Swedish banking system. Caccioli et al. [24] model common shocks emanating from overlapping securities portfolios. Their study reveals that, upon bank default, when the threat of contagion looms, other banks might be in weak position as well making them more vulnerable to contagion. Gauthier et al. [25] model loan losses of banks using detailed information on banks' loan books and common industry exposures and follow an iterative procedure in which they compute each bank's risk contribution, adjust the bank's capital level to that risk contribution, and then recompute both systemic risk and each bank's risk contribution. Furfine [26] considers the effect of liquidity in the contagion process in the case where the largest lender in the federal funds market is unable to lend, therefore forcing its counterparties of that institution to look elsewhere for funds or reduce their own lending.

Several studies have also examined the empirical features of interbank networks in various countries such as Upper and Worms [27] in Germany, van Lelyveld and Liedorp [28] in the Netherlands, Wells [29] in the UK, and Mistrulli [30] in Italy. These studies have revealed the heterogeneity of interbank network and are systematically surveyed in [19].

A number of studies have examined the dynamic model of network structure such as Georg [31] who developed a dynamic model of a banking system that can be used to analyze the impact of the interbank network structure on financial stability in way that when depositors decide on deposit investment via a random walk process, banks pay their maturing loans depending on their liquidity position. Lux [32] examined the dynamic model of interbank credit relationships. He assumed that banks initially choose potential trading partners randomly but form preferential relationships via an elementary reinforcement learning algorithm. The dynamic evolution of the system showed a formation of a core-periphery structure with mainly the largest banks assuming the roles of money centre banks mediating between the liquidity needs of many smaller banks, On the other hand Xu et al. [33] developed a dynamic network model based on agent behavior to explain the formation mechanism of interbank market and found that the interbank network structure keeps dynamic stability in the network evolution process, while Bluhm et al. [34] built a dynamic banking system with banks optimizing decisions and market adjustment; they found that higher liquidity requirements result in more concentrated network and lower systemic risk.

In our study we believe that deriving macroprudential capital requirements should take into account the dynamic evolution of the banking system when measuring the systemic risk of the banking system. Therefore, we extend the Eisenberg-Noe framework [11] to a dynamic multiperiod. Besides, the total asset value and equity value of the bank change dynamically, which can be estimated from real-world data instead of theoretical assumption. Furthermore, for shock scenarios, we add a system shock artificially in the system to observe the time evolution of the banking system; hence measure the systemic risk and then adjust macroprudential capital requirements according to each bank contributing to the whole system risk. We take into account two scenarios, namely, where the system shocks exist and do not exist in the banking system. For shock scenarios, we measure the systemic risk by checking whether a bank system can withstand certain strength of system shocks. Some banks may be bankrupt at a certain time point due to this artificially added system shock; therefore both the structure and the state of the bank change dynamically. We measure the systemic risk by recording the number of banks which undergo bankruptcy during the time. If the strength of the system shock is fixed, then a bank system with more banks which undergo bankrupt during the whole time course of its evolution is believed to suffer more systemic risk. Moreover we highlight that changing capital requirements change the risk in the banking system and that macroprudential capital requirements have to be seen as a fixed point problem.

The rest of the paper is organized as follows. Section 3 provides the network model of the banking system which encompasses methodology for estimating the matrix of bilateral exposures, the process of estimating the time evolution of balance sheet in the banking system, the methods of estimating the time evolution of bank's default, and the methodology of calculating bank's macroprudential capital. Section 4 provides the macroprudential capital requirements under different risk allocation mechanisms, Section 5 provides data used, Section 6 presents the results, and lastly Section 7 provides the concluding remark.

## 3. The Network Model of Banking System

We use the complex network theory to construct the network model of banking system, where each bank is regarded as a node and connected with others through the interbank bilateral exposures. We investigate the systemic risk of banking system by measuring bank's basic default caused by the insolvency of bank itself and contagious default due to the interbank bilateral exposures. We show explicitly that reallocation of bank capital in the system through the new capital requirements can change the probability of the bank to default and change total systemic risk. Thus we set the macroprudential capital requirements as a fixed problem and use four risk allocation mechanisms to compute bank's macroprudential capital requirements. We also compare the effects of macroprudential regulations for banking system under four risk allocation mechanisms which help us to measure the stability of the banking system. Figure 1 illustrates the underlying theoretical frameworks in this study.

*3.1. Estimation of Bilateral Exposure Matrix.* In this section, we introduce the methodology used in [19] for estimating the bilateral exposure matrix of banking system. The interbank exposures cannot be fully observed due to the fact that some bank's information is not transparent; therefore, we estimate the interbank bilateral exposure matrix by minimizing the uncertainty of bank's lending information based on the initial total interbank assets data $a_i$ and total interbank liabilities data $b_i$ in the balance sheet. The lending relationship in
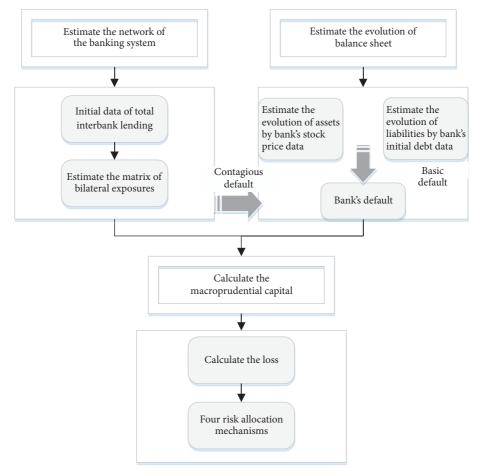
FIGURE 1: The theoretical framework.

the interbank market is represented by $(N \times N)$ nominal interbank matrix $X$:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N1} & & x_{Nj} & & x_{NN} \end{bmatrix} \begin{matrix} \sum_j \\ b_1 \\ \vdots \\ b_i \\ \vdots \\ b_N \end{matrix} \quad (1)$$

$$\begin{matrix} \sum_i & a_1 & \cdots & a_j & \cdots & a_N \end{matrix}$$

where $x_{ij}$ represents the outstanding loans of bank $i$ to bank $j$, the sum of each row $b_i$ represents the total interbank liability of bank $i$, and the sum of each column $a_j$ represents the total interbank assets of bank $j$. It is donated as

$$b_i = \sum_j x_{ij},$$
$$a_j = \sum_i x_{ij}. \quad (2)$$

We minimize the uncertainty of bank's lending information by standardizing $\sum_j a_j = \sum_i b_i = 1$, to get the results $x_{ij} = a_j * b_i$, which represents the standardized lending relationship of bank $i$ to bank $j$. We know that the diagonal elements of $X$ have to be zero; thus we make new definitions for the elements $x_{ij}$ in the interbank matrix $X$ as follows:

$$x_{ij}^0 = \begin{cases} 0, & i = j, \\ a_j b_i, & \text{otherwise.} \end{cases} \quad (3)$$

However, $X^0 = (x_{ij}^0)$ violates the summing constraints expressed in (2). The standard way in the literature to handle this problem is to optimize the elements in the interbank matrix according to (4):

$$\min \quad \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \ln \left( \frac{x_{ij}}{x_{ij}^0} \right)$$

$$\sum_{j=1}^{N} x_{ij} = b_i, \quad \sum_{i=1}^{N} x_{ij} = a_j, \quad x_{ij} \geq 0. \quad (4)$$

### 3.1.1. The Algorithm of Estimating the Bilateral Exposures Matrix

**Step 1.** Start the iteration for the elements in the interbank matrix $X$, $x_{ij}^0 = a_j b_i$ if $i \neq j$; otherwise $x_{ij}^0 = 0$.

**Step 2.** Take the rows constraint and set

$$x_{ij}^1 = \frac{x_{ij}^0 b_i}{\sum_{i=1}^N \sum_{j=1}^N (b_i * a_j)}, \quad i \in 1, 2, \ldots, N. \tag{5}$$

**Step 3.** Take the columns constraint and set

$$x_{ij}^2 = \frac{x_{ij}^1 a_j}{\sum_{i=1}^N \sum_{j=1}^N (b_i * a_j)}, \quad j \in 1, 2, \ldots, N. \tag{6}$$

The $K$ iteration runs across the rows and columns constraints show that

$$x_{ij}^K = \frac{x_{ij}^{K-1} b_i}{\sum_{i=1}^N \sum_{j=1}^N (b_i * a_j)}, \quad i \in 1, 2, \ldots, N,$$

$$x_{ij}^{K+1} = \frac{x_{ij}^K a_j}{\sum_{i=1}^N \sum_{j=1}^N (b_i * a_j)}, \quad j \in 1, 2, \ldots, N. \tag{7}$$

The iteration is stopped when $x_{ij}^{K+1} - x_{ij}^{K-1} < \varepsilon$, where $\varepsilon > 0$.

### 3.2. Estimation of the Time Evolution of $V_i$.

The asset value of bank is not daily observable, but we can obtain the data at the end of year from the balance sheet. The daily data of equity prices can be collected from the stock market on daily basis. Following Merton [35] we interpret equity as a call option on bank assets; thus we can estimate the time evolution of asset value by the stochastic model and maximum likelihood function based on the time series data of bank's equity prices. We assume that the asset value $V_i$ of bank $i$ follows a geometric Brownian motion with drift $u_i$ and volatility $\sigma_i$:

$$dV_i = u_i V_i dt + \sigma_i V_i d_Z. \tag{8}$$

The equity value of bank $i$ $S_t(t)$ is given by the Black-Scholes model as follows:

$$S_t(t) = V_i(t) \phi(d_t) - D_i(t) \phi(d_t - \sigma_i \sqrt{T}). \tag{9}$$

Then equity $S_t(t)$ can be seen as a call option on the assets of bank $i$ with a strike price equal to the future notional value of bank $i$'s debt $D_i(t)$, which is assumed to have a maturity of $T$. $\phi(\bullet)$ is the standard normal distribution function, $T = 365$ days, $t$ represents the evolution of days, and

$$d_t = \frac{\ln(V_i(t)/D_i(t)) + ((1/2)\sigma_i^2)T}{\sigma_i \sqrt{T}}. \tag{10}$$

By using the time series of equity prices $\{S_i(0), S_i(1), \ldots, S_i(T)\}$ from the stock market, the face value of total liabilities $\{D_i(0), D_i(1), \ldots, D_i(T)\}$, where $D_i(t) = D_i(0)e^{rt}$ from the balance sheet, the arbitrary initial value of $\mu_i(0)$, $\sigma_i(0)$, and the risk-free rate $r$ that is obtained from the Central bank of Nigeria from 2008 to 2014, we can estimate the time series of assets value $\{\widehat{V}_i(1), \widehat{V}_i(2), \ldots, \widehat{V}_i(T)\}$ according to (9). Then we use the following maximization likelihood function to estimate the parameters $\widehat{u}_i$ and $\widehat{\sigma}_i$:

$$L\left(\widehat{u}_i, \widehat{\sigma}_i; \widehat{V}_i(1), \widehat{V}_i(2), \ldots, \widehat{V}_i(T)\right)$$

$$= -\frac{T}{2}\ln(2\pi\sigma_i^2 h) - \frac{T}{2}\sum_{k=1}^T \frac{\left(R_i(k) - (u_i - \sigma_i^2/2)h\right)^2}{\sigma_i^2 h} \tag{11}$$

$$- \sum_{k=1}^T \ln \widehat{V}_k,$$

where $R_i(k) = \ln(\widehat{V}_i(t)/\widehat{V}_i(t-1))$, $h = 1/365$. In this case $h$ represents business days instead of calendar days. According to the stock market data source, $h$ may equal about 250, which will be different at each year.

The estimated parameters are denoted as $\widehat{u}_i$, $\widehat{\sigma}_i$ respectively. We compare the estimated parameters $\widehat{u}_i$, $\widehat{\sigma}_i$ with the initial value $u_i(0)$, $\sigma_i(0)$; if $\widehat{u}_i$, $\widehat{\sigma}_i$ do not equal to $u_i(0)$, $\sigma_i(0)$; we replace the initial value $u_i(0)$, $\sigma_i(0)$, $V_i(0)$ with $\widehat{u}_i$, $\widehat{\sigma}_i$, $\widehat{V}_i(0)$ and then repeat the estimation method from (9), (10), and (11) once again until the estimated parameters $\widehat{u}_i$, $\widehat{\sigma}_i$ equal to $u_i(0)$, $\sigma_i(0)$. Accordingly we get the estimated parameters $u_i$ and $\sigma_i$. Then according to (12), we get the evolution of $V_i(t)$.

$$V_i(t) = V_i(0) e^{u_i - (\sigma_i^2/2)th + \sigma_i \sqrt{th * z_i(t)}}, \tag{12}$$

where $z_i(t)$ obeys normal distribution ($N(0, 1)$).

The above estimation method of $V_i(t)$ is the same as [6].

To test the stability of the bank network system, we add a system shock to the bank system. If the banks withstand a strong shock, then we say that the system is stable. If the bank system collapses when a weak shock applies, then we say that the bank system is unstable. Thus, we apply a medium shock to count the number of banks which undergoes bankruptcy. If the probabilities of the banks which undergo bankruptcy are large for a medium shock, then the bank system is unstable. The system shock is added to the banking system by replacing $z_i(t)$ in (12) with $(1 - \xi)z_i(t) + \xi * \omega(t)$ where $\xi$ represents the strength of the system shock and $\omega(t)$ is the system shock which is the same for all banks. $\omega(t)$ follows the normal distribution ($N(0, 1)$), where $\xi = 0.1$. Thus the evolution of $V_i(t)$ can be estimated as follows:

$$V_i(t) = V_i(0) e^{u_i - (\sigma_i^2/2)th + \sigma_i \sqrt{th * [(1-\xi)z_i(t) + \xi * \omega(t)]}}. \tag{13}$$

### 3.3. The Measures of Banks Defaults.

In the present paper, we consider the risk of bank defaults that are basic defaults and contagious defaults. When bank $i$ is insolvent, we define it as the basic default which satisfied

$$V_i(t) - D_i(t) < 0, \tag{14}$$

where $D_i(t)$ can be described as follows:

$$D_i(t) = \left(V_i(0) - C_i(0)\right)e^{rt}. \tag{15}$$

In (15), $r$ is risk-free rate. We set $C_i(0) = 7\% * V_i(0)$, namely, $(V_i(0) - D_i(0))/V_i(0) = 7\%$, where $D_i(0) = V_i(0) - C_i(0)$. The 7% is a capital adequacy ratio according to Basel III; therefore $C_i(0)$ is the initial capital of bank $i$.

If bank $i$ satisfies (14), bank $i$ occurs as basic default. The losses come from $V_i(t) - D_i(t)$. Due to the variable $V_i(t)$ following random walk, it will fluctuate around the drift $u_i$; however, $D_i(t)$ is a variable that is increasing with time. Therefore, with the time step increasing, there must be some banks default. Then through the interbank market, bank $i$ may cause the other banks default, which is the contagion default. Here, we extend the clearing payment mechanism [11] to suit the calculation of the time evolution of the contagion default. We define a new matrix $\Pi \in [0,1]^{N\times N}$ to standardize the total interbank liabilities:

$$\Pi_{ij}(t) = \begin{cases} \dfrac{x_{ij}(t)}{b_i(t)} & b_i(t) > 0 \\ 0 & \text{otherwise}, \end{cases} \tag{16}$$

where $b_i(t) = \sum_j x_{ij}(t)$; this shows the total interbank liabilities of bank $i$ at time step $t$. We also define a clearing payment vector $p^*(t)$ that respects the limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism defined as

$$p_i^*(t) = \begin{cases} b_i(t) & \displaystyle\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) \geq b_i(t), \\[2ex] \displaystyle\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) & 0 \leq \displaystyle\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) < b_i(t), \\[2ex] 0 & \displaystyle\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) < 0, \end{cases} \tag{17}$$

where $e_i(t) = V_i(t) - D_i(t)$. Here, we adopt the default algorithm [11] to find a clearing payment vector. If bank $i$ cannot default according to (14), it may default when other banks are not able to keep their promises; that is, contagious default of bank $i$ occurs if

$$\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) - b_i(t) < 0. \tag{18}$$

*3.3.1. The Evolution of the Bilateral Exposures $X(t+1)$, $a_j(t+1)$, and $b_i(t+1)$.* After calculating a clearing payment vector at time step $t$, we can calculate the new matrix of $X$ at the time step $t+1$. We should note that when bank $i$ defaults, bank $i$ can pay only a part of its liabilities to other banks. The ratio is defined as follows:

$$\chi_i = \frac{\sum_{j=1}^{N}\Pi'_{ij}(t)\,p_j^*(t) + e_i(t)}{b_i(t)}. \tag{19}$$

The total assets and liabilities of bank $j$ from time step $t+1$ to $T$ will be updated as follows:

$$V_j(t+1:T) = V_j(t) - x_{ji}$$
$$D_j(t+1:T) = D_j(t) - x_{ji} \tag{20}$$
$$V_j(t+1:T) = V_j(t) - (1-\chi_i)\,x_{ij}.$$

When bank $i$ defaults, we set $x_{i,j}(t) = 0$, $x_{j,i}(t) = 0$ and clear out bank $i$ from the network bank system. Then, we need to recalculate the bilateral exposures matrix $X_{t+1}$ according to the algorithm in Section 3.1.1. Thus, the evolution of $a_j(t+1)$ and $b_i(t+1)$ is described as

$$a_j(t+1) = \sum_{i=1}^{N} x_{i,j}(t+1)$$

$$b_i(t+1) = \sum_{j=1}^{N} x_{i,j}(t+1). \tag{21}$$

Thus, we measure the stability of the banking system by calculating the probability of bank's default. The probability of basic default of bank $i$ is calculated as the ratio of the times of basic default of bank $i$ occurring and the total times of the simulation. Similarly, the probability of the contagious default of bank $i$ is calculated as the ratio of the times of contagious default of bank $i$ occurring and the total times of the simulation. The probability of the total default of banking system is the sum of the basic defaults probability and contagious defaults probability.

*3.4. The Measure of Bank's Macroprudential Capital.* The core algorithm of calculating bank's macroprudential capital lies in measuring bank's losses, which is defined as

$$l_i(t) = \min\left(\sum_{j=1}^{N}\Pi_{ji}(t)\,p_j^*(t) + e_i(t) - b_i(t), 0\right). \tag{22}$$

The dynamic evolution (time step $t$) of banking system is iterated for test of times (test $= 1,2,3,\ldots,M$); $l_{i,\text{test}}(t)$ represents the loss of bank $i$ at every time of simulation.
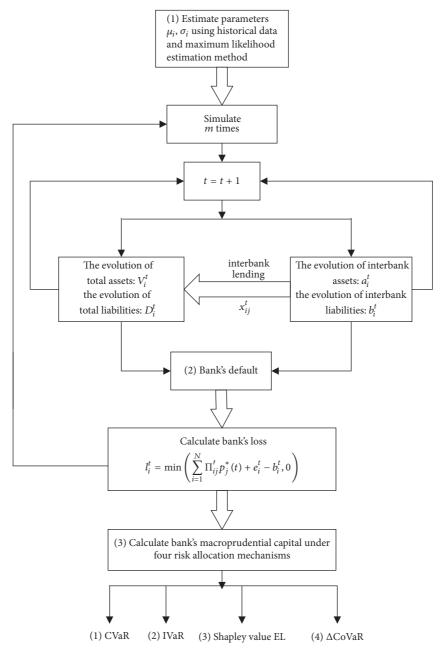
FIGURE 2: The algorithm flowchart of calculating macroprudential capital requirements.

Therefore, we get a $N \times M$ loss matrix of $N$ banks in $M$ times of simulation. The loss matrix under the given capital $C_i = (C_1, C_2, \ldots, C_N)$ can be expressed as $l(C)$; then, we use the risk allocation mechanism $f(\bullet)$ to allocate systemic risk to every bank, which is described as $f_i(l(C))$. Thus, the macroprudential capital of bank $i$ is computed as

$$C_i^* = f_i\left(l\left(C_i^*\right)\right) \times \left(\sum_{i=1}^{N} C_i^0\right)$$

$$\sum_{i=1}^{N} f_i\left(l\left(C\right)\right) = 1, \quad f_i\left(l\left(C\right)\right) \geq 0,$$

(23)

where $C_i^0$ is the initial capital of bank $i$; $C_i^*$ is the redistributed capital of bank $i$. The algorithm flowchart of calculating bank's macroprudential capital is shown in Figure 2.

## 4. Macroprudential Capital Measures Underlying Four Risk Allocation Mechanisms

Following Caccioli et al. [24] and Liao et al. [36], we use four risk allocation mechanisms to calculate each bank's macroprudential capital requirements, namely, Component VaR, Incremental VaR, Shapley value, and ΔCoVaR. The methods

for calculating bank's macroprudential capital requirements are summarized as follows.

## 4.1. Component VaR.

The core of this allocation mechanism is to reallocate the capital based on the contribution $\beta$ of each bank's loss $l_i$ to the total loss of banking system $\sum_{i=1}^{n} l_i(l_p)$, and $\beta_i = \text{cov}(l_i, l_p)/\sigma^2(l_p)$. The macroprudential capital of bank $i$ under the Component VaR mechanism is given by

$$C_i^{\text{CVaR}} = \beta_i \sum_{i=1}^{n} C_i^0. \tag{24}$$

The change in capital under the Component VaR mechanism is $(C_i^{\text{CVaR}} - C_i^0)/C_i^0$.

## 4.2. Incremental VaR.

The theme of this allocation procedure is to reallocate the capital according to the change in the overall risk due to the exclusion of a bank in the system.

Individual bank losses are simulated over ten thousand scenarios and for each scenario we compute the 5% VaR of the total losses $l_p$ in the system, denoted by $\text{VaR}_P$. Then, we calculate the 5% VaR of total losses excluding bank $i$ denoted by $\text{VaR}^{-i}$. Thus, the increment VaR of bank $i$ is calculated as

$$\text{IVaR}_i = \text{VaR}_P - \text{VaR}^{-i}. \tag{25}$$

The macroprudential capital of bank $i$ under the increment VaR mechanism is given by

$$C_i^{\text{IVaR}} = \frac{\text{IVaR}_i}{\sum_{i=1}^{n} \text{IVaR}_i} \sum_{i=1}^{n} C_i^0. \tag{26}$$

The change in capital under the increment VaR mechanism is $(C_i^{\text{IVaR}} - C_i^0)/C_i^0$.

## 4.3. Shapley Value EL.

The Shapley value EL method is the arithmetic average of $n$ times of simulation based on the increment VaR mechanism. Thus, a new $\text{IVaR}_i$ is represented as $\phi_i$ by the calculation of arithmetic average based on $\text{VaR}_P$ and $\text{VaR}^{-i}$.

Thus we compute the macroprudential capital requirements of bank $i$ under the Shapley value EL mechanism by

$$C_i^{\text{Shapley EL}} = \frac{\phi_i}{\sum_{i=1}^{n} \phi_i} \sum_{i=1}^{n} C_i^0. \tag{27}$$

The change in capital under the Shapley value EL mechanism is $(C_i^{\text{Shapley EL}} - C_i^0)/C_i^0$.

## 4.4. ΔCoVaR.

We define CoVaR of bank $i$ as the total loss of banking system conditional on bank $i$ realizing a loss corresponding to its VaR. $\text{CoVaR}_i$ is described as

$$\Pr\left(l_p < \text{CoVaR}_i \mid l_i \in \left[(1 - \ell)\,\text{VaR}_i, (1 + \ell)\,\text{VaR}_i\right]\right)$$
$$= 0.5\%, \quad \ell = 0.1. \tag{28}$$

Here $\Delta\text{CoVaR}_i$ is defined as the difference of $\text{CoVaR}_i$ and the VaR of the total losses in the banking system conditional on bank $i$ making a loss at its median:

$$\Delta\text{CoVaR}_i = \text{CoVaR}_i - \left(\text{VaR}_p \mid l_i = \text{median}\,(l_i)\right). \tag{29}$$

The macroprudential capital of bank $i$ under the $\Delta$CoVaR mechanism is given by

$$C_i^{\Delta\text{CoVaR}} = \frac{\Delta\text{CoVaR}_i}{\sum_{i=1}^{n} \Delta\text{CoVaR}_i} \sum_{i=1}^{n} C_i^0. \tag{30}$$

The change in capital under the $\Delta$CoVaR mechanism is $(C_i^{\Delta\text{CoVaR}} - C_i^0)/C_i^0$.

# 5. Data

In this paper, we use financial data collected from the African markets website. The study involves 10 major public traded commercial banks in Nigeria from 2008 to 2014. The sample of 10 banks is selected based on banks with large assets value, namely, Zenith International bank (₦ 3.3 trillion), FBN Holdings (₦ 3.2 trillion), Guaranty bank (₦ 3.15 trillion), United bank of Africa (₦ 2 trillion), Diamond bank (₦ 1.93 trillion), Access bank (₦ 1.835 trillion), Fidelity (₦ 1.19 trillion), Union bank of Nigeria (₦ 1.049 trillion), Skye bank ₦ (1 trillion), and Sterling bank (₦ 841 billion) as of 2015 (https://www.relbanks.com).

# 6. Results

## 6.1. Changes in Capital Requirements.

In this section we present the changes in capital requirements to reach the fixed point of the four capital allocation mechanism presented in Section 4 in percent of actual observed capital requirements. We use numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 to portray Access bank, Diamond bank, FBNH, Fidelity bank, Guaranty bank, Skye bank, Sterling bank, Union bank of Nigeria, United bank of Africa, and Zenith International bank, respectively. We use different risk allocation mechanisms to reallocate the current capital in the system as shown in Section 4. Reallocation of capital in the system through new capital requirements changes a banks probability of default and systemic risk. In the first scenario, we assume the case where the system shocks do not exist in the system for the period of 2008-2009.

Table 1 presents the increase in capital as a share of these banks' risk weighted assets from the current observed capital level $C_i^0$ to macroprudential capital requirement $C_i^*$. The change is presented as a proportion of current observed capital level $C_i^0$; for example, $(C_i^* - C_i^0)/C_i^0$.

The methods of four risk allocation mechanisms are quite different from each other. Since Shapley value is based on incremental VaR, in Table 1, these two results are similar. The method of the Component VaR calculates the correlation between each bank's losses and the total loss of banking system. In Table 1, it seems the losses of bank 10 are not very related to other 9 banks. The method of $\Delta$CoVaR is related to the total loss of banking system conditional on bank $i$

TABLE 1: Increase in capital as a share of banks' risk weighted assets for macroprudential capital allocation mechanism in 2008.

| Bank | Component VaR | Incremental VaR | Shapley value EL | ΔCOVAR |
|---|---|---|---|---|
| 1 | −0.224490152 | 0.075807121 | 0.079291353 | −0.53067966 |
| 2 | −0.224490152 | −0.098024516 | −0.108952535 | 1.278613545 |
| 3 | −0.224490152 | −0.093480509 | −0.097516656 | −0.413673209 |
| 4 | −0.224490152 | −0.162273193 | −0.164247199 | 0.181725936 |
| 5 | −0.224490152 | −0.144122078 | −0.144675929 | 1.185156475 |
| 6 | −0.224490152 | 0.129450668 | 0.130866402 | 1.171263759 |
| 7 | −0.224490152 | −0.052641093 | −0.050383221 | 1.862680122 |
| 8 | −0.224490152 | 0.739933973 | 0.74625635 | −0.329806275 |
| 9 | 1.205658115 | −0.144056852 | −0.144084074 | −0.507137053 |
| 10 | −0.224490152 | −0.151642289 | −0.151117088 | −0.631962395 |

TABLE 2: Increase in capital as a share of banks' risk weighted assets for macroprudential capital allocation mechanisms in 2009.

| Bank | Component VaR | Incremental VaR | Shapley value EL | ΔCOVAR |
|---|---|---|---|---|
| 1 | −0.838983488 | −0.321911 | −0.277249974 | 0.339348356 |
| 2 | −0.838983488 | −0.324542322 | −0.291759584 | 0.492986513 |
| 3 | −0.838983488 | 0.037474798 | 0.026574758 | −0.49003279 |
| 4 | −0.838983488 | −0.392470575 | −0.377802565 | 0.792001406 |
| 5 | −0.838983488 | −0.318295374 | −0.313198791 | −0.10427448 |
| 6 | −0.838983488 | −0.324506098 | −0.3050429 | 0.453956728 |
| 7 | −0.838983488 | −0.367389192 | −0.345570211 | 3.392491614 |
| 8 | −0.838983488 | 0.735790757 | 0.738495391 | −0.18476422 |
| 9 | −0.838983488 | 0.565740871 | 0.550344543 | −0.0511749 |
| 10 | 4.219043349 | −0.292376022 | −0.31859484 | −0.41648809 |

realizing a loss corresponding to its VaR, and the method is more related to the median of each bank's losses; therefore, it may eliminate the effect of extreme value. Therefore, the results of ΔCoVaR for ten banks in Table 1 seem more uniform than the method of Component VaR.

All risk allocation rules suggest that banks 3 and 10 hold more capital than their contribution to the overall risk the system would require. The results are mixed for both banks, under three out of four capital allocations mechanisms suggesting that banks 2, 4, 5, 7, and 9 hold more capital for unexpected losses while bank 6 holds little capital. For bank 1 and 8 the results shows that two out of four capital allocation mechanisms suggest that the bank holds little; the remaining holds more capital.

In Table 2, all risk allocation rules suggest that bank 5 holds more capital than its contribution to the overall risk the system would require. Three out of four capital allocations mechanisms suggest that banks 1, 2, 4, 6, 7, and 10 hold more capital. For banks 3, 8, and 9 the results show that two out of four capital allocation mechanisms suggest that the bank holds little, the remaining holds more capital.

We also examine the changes in the capital requirements when the system shock exists in the system for the period 2008–2014. We use the same risk allocation mechanism to compute the current capital in the system. Our results are presented in Figure 3. Figure 3 portrays the changes of capital expressed in percentage according to macroprudential capital requirements in situation where the system shock exists

in Nigerian banking system. We note that almost all risk allocation rules suggest that almost all banks appear to be undercapitalized because nearly all the changes are positive.

6.2. Probability of Bank Defaults and Macroprudential Capital Requirements. Macroprudential capital requirements can serve as buffers against the risk created in the banking system. To supplement our findings which are the core elements of our study, we thus examine the extent in which macroprudential capital requirements reduce the average bank default probability compared to the probability under Basel equal. We use different risk allocation mechanisms to show the individual bank default probabilities. Table 3 depicts the differences in macroprudential capital requirements under different risk mechanisms; almost all these allocations reduce default probabilities compared to the benchmarks "Basel equal approach."

Table 3 depicts the individual bank default probabilities under the Basel equal and macroprudential capital requirements computed with four systemic risk allocation mechanisms expressed in percent. Average shows the average default probability. The table depicts that Skye bank, Guaranty bank, and Diamond bank are bit weaker banks in the system as they have the highest total probabilities of basic and contagious default as shown in Table 5; overall total default probabilities range between 0.0081% and 0.1422%.

In addition, the findings show that capital requirements under ΔCOVAR lead to the lowest default probability. The
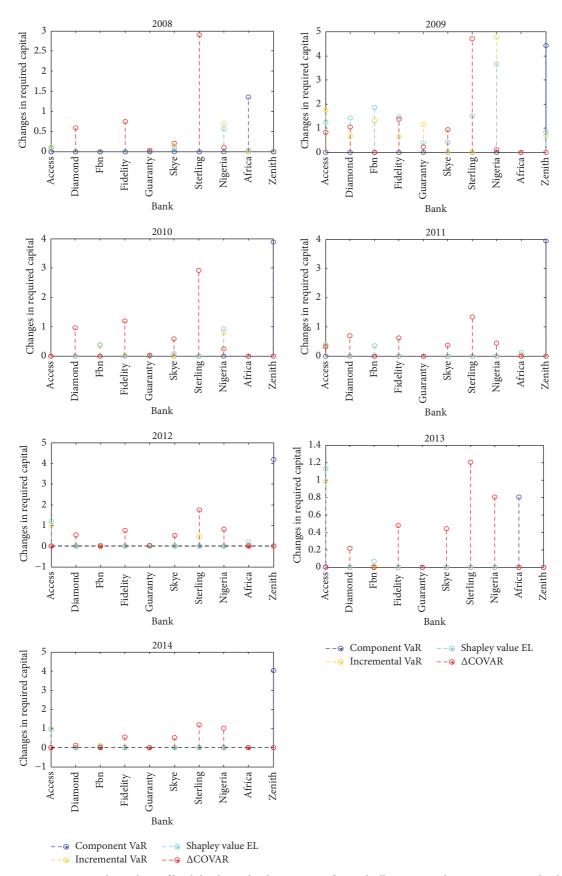
FIGURE 3: Increase in capital as a share of banks' risk weighted assets using four risk allocation mechanisms assuming shocks exist.

TABLE 3: Individual bank default probabilities under the macroprudential capital requirements in 2008.

| Bank | Basel equal | Component VaR | Incremental VaR | Shapley value EL | ΔCOVAR |
|---|---|---|---|---|---|
| 1 | 0.0552 | 0.0552 | 0.0332 | 0.0405 | 0.0552 |
| 2 | 0.1344 | 0.1344 | 0.1344 | 0.1344 | 0.0238 |
| 3 | 0.0952 | 0.0952 | 0.0952 | 0.0952 | 0.0952 |
| 4 | 0.0787 | 0.0787 | 0.0787 | 0.0787 | 0.0033 |
| 5 | 0.1356 | 0.1356 | 0.1356 | 0.1356 | 0.1268 |
| 6 | 0.1422 | 0.1422 | 0.0991 | 0.1201 | 0.0845 |
| 7 | 0.1247 | 0.1247 | 0.1247 | 0.1247 | 0 |
| 8 | 0.0081 | 0.0081 | 0 | 0.0001 | 0.0032 |
| 9 | 0.1065 | 0.0002 | 0.1065 | 0.0986 | 0.1065 |
| 10 | 0.0511 | 0.0511 | 0.0511 | 0.0511 | 0.0511 |
| Average | 0.09317 | 0.08254 | 0.08585 | 0.0879 | 0.05496 |

TABLE 4: Individual bank default probabilities under the macroprudential capital requirements in 2009.

| Bank | Basel equal | Component VaR | Incremental VaR | Shapley value EL | ΔCOVAR |
|---|---|---|---|---|---|
| 1 | 0.1177 | 0.1177 | 0 | 0.0006 | 0.0065 |
| 2 | 0.1311 | 0.1311 | 0.0111 | 0.0001 | 0.0018 |
| 3 | 0.1 | 0.1 | 0 | 0 | 0.1 |
| 4 | 0.1508 | 0.1508 | 0.0197 | 0.0005 | 0.0005 |
| 5 | 0.157 | 0.157 | 0.0023 | 0.0558 | 0.0752 |
| 6 | 0.1643 | 0.1643 | 0.1643 | 0.0543 | 0.0063 |
| 7 | 0.18 | 0.18 | 0.18 | 0.0002 | 0 |
| 8 | 0.0722 | 0.0722 | 0 | 0 | 0.0263 |
| 9 | 0.9982 | 0.9982 | 0.9982 | 0.9982 | 0.9982 |
| 10 | 0.1222 | 0.0886 | 0.0094 | 0.0037 | 0.1222 |
| Average | 0.21935 | 0.21599 | 0.1385 | 0.11134 | 0.1337 |

average probability of bank defaults based on ΔCOVAR decreases from 0.09317% to 0.05496%, a 0.4101% reduction in risk. This proves that increased capital levels downsize default risk and thus macroprudential capital requirements serve as a buffer against internally created risk in the banking system. More importantly, other measures like Component VaR, Shapley value, and incremental value at risk also decrease individual bank default probabilities.

Table 4 portrays that United bank of Africa is a weaker bank in the system; overall total probabilities range between 0.1% and 0.9982%. Moreover, the findings show that capital requirements under Shapley value lead to the lowest default probability. The average probability of bank defaults based on Shapley value decreases from 0.21935% to 0.11134%, a 0.4924% reduction in risk; other measures like Component VaR, incremental value at risk and ΔCOVAR also decrease individual bank default probabilities.

Figure 4 portrays individual bank total default probabilities under macroprudential capital requirements for the entire sample of 2008–2014.

From Figure 4, we see that in 2010 Access Bank seems to be the weaker bank in the system. We find that the overall total default probabilities range between 0.0247% and 0.9501%, with ΔCOVAR leading to the lowest default probability. Moreover the average probability of bank defaults

based on ΔCOVAR decreases from 0.13716% to 0.110084%, that is, a 0.1974% reduction in risk. Other measures also appear to decrease individual bank defaults.

For year 2011, Sterling and United bank of Africa are bit weaker banks in the system. The overall total probabilities in the system range between 0.0091% and 0.977%. Our results reveal that capital requirements under ΔCOVAR lead to the lowest default probability with the average probability of bank defaults based on this measure decreasing from 0.05885% to 0.022254%, which is approximately a 0.62185% reduction in risk; other measures appear to decrease the individual bank default probabilities. Meanwhile for 2012 we note that Access bank is a weaker bank in the system; overall total probabilities bank defaults range between 0.0122% and 0.8829%. The findings also unveil that capital requirement under ΔCOVAR leads to the lowest default probability with the average probability of bank defaults decreasing from 0.1271% to 0.10012%, a 0.21227% reduction in risk; other measures do the same.

For year 2013 it can be seen that the overall total probabilities of bank defaults range between 0.001% and 0.0473%; at the same time the findings reveal that ΔCOVAR lead to lowest default probability with the average probability of bank defaults decreasing from 0.02212% to 0.00703%, which implies a reduction in risk by 0.6822%. Sterling and

TABLE 5: Probabilities of Basic and contagious default in 2008.

| Bank | Basel equal | Component VaR | Incremental VaR | Shapley value EL | ΔCOVAR |
|---|---|---|---|---|---|
| *Basic default probabilities* | | | | | |
| Access bank | 0.0546 | 0.0546 | 0.0329 | 0.0403 | 0.0546 |
| Diamond bank | 0.1342 | 0.1342 | 0.1342 | 0.1342 | 0.0234 |
| Fbn holdings | 0.0949 | 0.0949 | 0.0949 | 0.0949 | 0.0949 |
| Fidelity bank | 0.0783 | 0.0783 | 0.0783 | 0.0783 | 0.0033 |
| Guaranty bank | 0.1349 | 0.1349 | 0.1349 | 0.1349 | 0.1256 |
| Skye bank | 0.1418 | 0.1418 | 0.0986 | 0.1198 | 0.0837 |
| Sterling bank | 0.1247 | 0.1247 | 0.1247 | 0.1247 | 0 |
| Union bank of Nigeria | 0.008 | 0.008 | 0 | 0.0001 | 0.0031 |
| United bank of Africa | 0.1057 | 0.0002 | 0.1057 | 0.0982 | 0.1057 |
| Zenith international bank | 0.0504 | 0.0504 | 0.0504 | 0.0504 | 0.0504 |
| Average | 0.09275 | 0.0822 | 0.08546 | 0.08758 | 0.05447 |
| *Contagion default probabilities* | | | | | |
| Access bank | 0.0006 | 0.0006 | 0.0003 | 0.0002 | 0.0006 |
| Diamond bank | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0004 |
| Fbn holdings | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| Fidelity bank | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0 |
| Guaranty bank | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0012 |
| Skye bank | 0.0004 | 0.0004 | 0.0005 | 0.0003 | 0.0008 |
| Sterling bank | 0 | 0 | 0 | 0 | 0 |
| Union bank of Nigeria | 0.0001 | 0.0001 | 0 | 0 | 0.0001 |
| United bank of Africa | 0.0008 | 0 | 0.0008 | 0.0004 | 0.0008 |
| Zenith international bank | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 |
| Average | 0.00042 | 0.00034 | 0.00039 | 0.00032 | 0.00049 |

Union bank of Nigeria appear to be weaker banks in the system in this year. For 2014 our findings unfold that United bank of Africa is a weaker bank in the system; overall total probabilities of bank defaults range between 0.006% and 0.0747%; moreover the capital requirements under ΔCOVAR lead to the lowest default probability. The average probability of bank defaults based on ΔCOVAR decreases from 0.02731% to 0.01631%, a 0.4028% reduction in risk. More importantly, other measures like Component VaR, incremental value at risk, and Shapley value also decrease individual bank default probabilities.

Generally we note that despite the heterogeneity in macroprudential capital requirements across risk allocation mechanisms, all risk allocation rules bring a substantial improvement in bank stability relative to the existing regulatory framework. Compared to the bench mark, all capital allocations mechanisms reduce the probability of banks defaults, the lowest probability being shown by ΔCOVAR measures. Our findings prove that increased capital levels reduce default risk, in line with macroprudential capital requirements serving as insurance against the risk created within the financial system.

## 7. Conclusion

One of the major objectives of macroprudential policy is to internalize the risk within the banking system so as to enhance financial stability. We used the complex network theory to construct the network model of banking system, whereby each bank is regarded as a node and connected with others through the interbank bilateral exposures. We used different risk allocation mechanisms to reallocate the current capital in the system. We realized that the financial system risk, individual bank risk contribution, and bank default probability change upon the reallocation of capital in the system.

Thus we used a method to compute a fixed point for which capital redistribution is accordant with the contributions of each bank to the total risk of the banking system under proposed capital allocation mechanisms. Upon reallocation of capital in the system, we based on two scenarios; firstly in the situation where the system shocks do not exist in the system; in this scenario, we found that almost all banks appear to hold more capital. This ensures that banks build up capital during normal times so that it cannot be affected when losses are incurred during periods of financial difficulties. Secondly, we consider the situation where the system shock exists in the system; we found that almost all banks tend to hold little capital on both risk allocation mechanisms.

Our results show that United bank of Africa is the weakest in the system with high default probability in 2009, 2011, and 2014 followed by Access bank in 2010 and 2012; regulators should apply stringent supervision to these banks. We reveal that, under diverse risk allocation mechanisms,
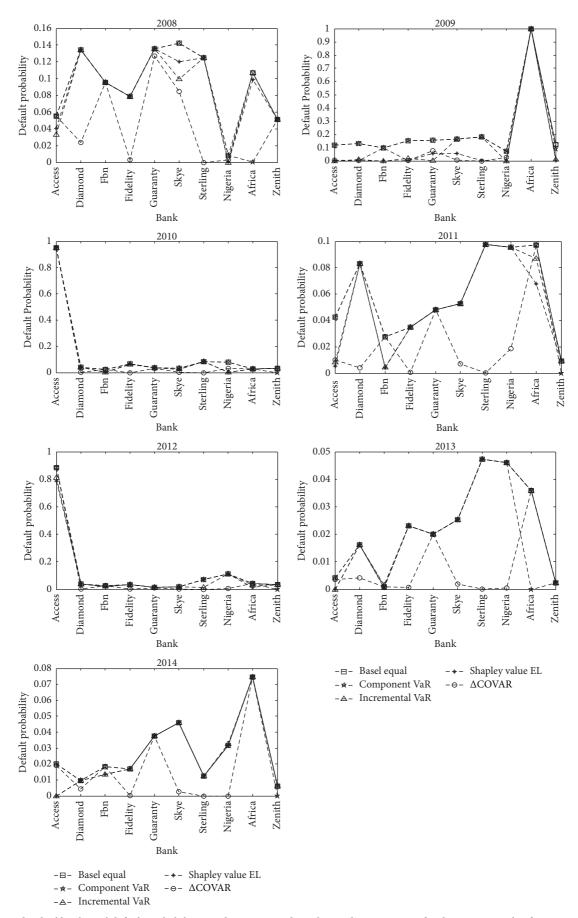
FIGURE 4: Individual bank total default probabilities under macroprudential capital requirements for the entire sample of 2008–2014. Within the banking system, ΔCOVAR downsizes the average probability of default the most.

both the average default probability of individual institutions and multiple banks defaults can be substantially reduced, with ΔCoVaR decreasing the average default probability the most. The findings suggest that risk allocation mechanisms can improve the stability of the system and regulate the financial system from a macroprudential dimension. The macroprudential capital allocation mechanisms work as an instrument of prudent bank regulation and importantly can reduce the risk of banks as well as the risk of the banking system.

The approach of estimating bilateral exposure matrix in the present paper assumes that the topology of the interbank network is complete network and does not reproduce incomplete interbank market. Depending on the actual network structure this may negatively or positively bias the results. In the future work, we can adopt the maximum entropy estimation method [19] and minimum density approach [37] to estimate the bilateral exposure matrix, and then we can give an interval value of macroprudential capital allocation, which will be more practical.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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*Research Article*

# Incorporating Contagion in Portfolio Credit Risk Models Using Network Theory

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Portfolio credit risk models estimate the range of potential losses due to defaults or deteriorations in credit quality. Most of these models perceive default correlation as fully captured by the dependence on a set of common underlying risk factors. In light of empirical evidence, the ability of such a conditional independence framework to accommodate for the occasional default clustering has been questioned repeatedly. Thus, financial institutions have relied on stressed correlations or alternative copulas with more extreme tail dependence. In this paper, we propose a different remedy—augmenting systematic risk factors with a contagious default mechanism which affects the entire universe of credits. We construct credit stress propagation networks and calibrate contagion parameters for infectious defaults. The resulting framework is implemented on synthetic test portfolios wherein the contagion effect is shown to have a significant impact on the tails of the loss distributions.

## 1. Introduction

One of the main challenges in measuring the risk of a bank's portfolio is modelling the dependence between default events. Joint defaults of many issuers over a fixed period of time may lead to extreme losses; therefore, understanding the structure and the impact of default dependence is essential. To address this problem, one has to take into consideration the existence of two distinct sources of default dependence. On the one hand, performance of different issuers depends on certain common underlying factors, such as interest rates or economic growth. These factors drive the evolution of a company's financial success, which is measured in terms of its rating class or the probability of default. On the other hand, default of an issuer may, too, have a direct impact on the probability of default of a second dependent issuer, a phenomenon known as contagion. Through contagion, economic distress initially affecting only one issuer can spread to a significant part of the portfolio or even the entire system. A good example of such a transmission of pressure is the Russian crisis of 1998-1999 which saw the defaults of corporate and

subsovereign issuers heavily clustered following the sovereign default [1].

Most portfolio credit risk models used by financial institutions neglect contagion and rely on the conditional independence assumption according to which, conditional on a set of common underlying factors, defaults occur independently. Examples of this approach include the Asymptotic Single Risk Factor (ASRF) model [2], industry extensions of the model presented by Merton [3] such as the KMV [4, 5] and CreditMetrics [6] models, and the two-factor model proposed recently by Basel Committee on Banking Supervision for the calculation of Default Risk Charge (DRC) to capture the default risk of trading book exposures [7]. A considerable amount of literature has been published on the conditional independence framework in standard portfolio models; see, for example, [8, 9].

Although conditional independence is a statistically and computationally convenient property, its empirical validity has been questioned on a number of occasions, where researchers investigated whether dependence on common factors can sufficiently explain the default clustering which

occurs from time to time. Schönbucher and Schubert [10] suggest that the default correlations that can be achieved with this approach are typically too low in comparison with empirical default correlations, although this problem becomes less severe when dealing with large diversified portfolios. Das et al. [11] use data on US corporations from 1979 to 2004 and reject the hypothesis that factor correlations can sufficiently explain the empirically observed default correlations in the presence of contagion. Since a realistic credit risk model is required to put the appropriate weight on scenarios where many joint defaults occur, one may choose to use alternative copulas with tail dependence which have the tendency to generate large losses simultaneously [12]. In that case, however, the probability distribution of large losses is specified a priori by the chosen copula, which seems rather unintuitive [13].

One of the first models to consider contagion in credit portfolios was developed by Davis and Lo [14]. They suggest a way of modelling default dependence through infection in a static framework. The main idea is that any defaulting issuer may infect any other issuer in the portfolio. Giesecke and Weber [15] propose a reduced-form model for contagion phenomena, assuming that they are due to the local interaction of companies in a business partner network. The authors provide an explicit Gaussian approximation of the distribution of portfolio losses and find that, typically, contagion processes have a second-order effect on portfolio losses. Lando and Nielsen [16] use a dynamic model in continuous time based on the notion of mutually exciting point processes. Apart from reduced-form models for contagion, which aim to capture the influence of infectious defaults to the default intensities of other issuers, structural models were developed as well. Jarrow and Yu [17] generalize existing models to include issuer-specific counterparty risks and illustrate their effect on the pricing of defaultable bonds and credit derivatives. Egloff et al. [18] use network-like connections between issuers that allow for a variety of infections between firms. However, their structural approach requires a detailed microeconomic knowledge of debt structure, making the application of this model in practice more difficult than that of Davis and Lo's simple model. In general, since the interdependencies between borrowers and lenders are complicated, structural analysis has mostly been applied to a small number of individual risks only.

Network theory can provide us with tools and insights that enable us to make sense of the complex interconnected nature of financial systems. Hence, following the 2008 crisis, network-based models have been frequently used to measure systemic risk in finance. Among the first papers to study contagion using network models was [19], where Allen and Gale show that a fully connected and homogeneous financial network results in an increased system stability. Contagion effects using network models have also been investigated in a number of related articles; see, for example, [20–24]. The issue of too-central-to-fail was shown to be possibly more important than too-big-to-fail by Battiston et al. in [25], where DebtRank, a metric for the systemic impact of financial institutions, was introduced. DebtRank was further extended in a series of articles; see, for example, [26–28].

The need for development of complexity-based tools in order to complement existing financial modelling approaches was emphasized by Battiston et al. [29], who called for a more integrated approach among academics from multiple disciplines, regulators, and practitioners.

Despite substantial literature on portfolio credit risk models and contagion in finance, specifying models, which take into account both common factors and contagion while distinguishing between the two effects clearly, still proves challenging. Moreover, most of the studies on contagion using network models focus on systemic risk and the resilience of the financial system to shocks. The qualitative nature of this line of research can hardly provide quantitative risk metrics that can be applied to models for measuring the risk of individual portfolios. The aforementioned drawback is perceived as an opportunity for expanding the current body of research by contributing a model that would account for common factors and contagion in networks alike. Given the wide use of factor models for calculating regulatory and economic capital, as well as for rating and analyzing structured credit products, an extended model that can also accommodate for infectious default events seems crucial.

Our paper takes up this challenge by introducing a portfolio credit risk model that can account for two channels of default dependence: common underlying factors and financial distress propagated from sovereigns to corporates and subsovereigns. We augment systematic factors with a contagion mechanism affecting the entire universe of credits, where the default probabilities of issuers in the portfolio are immediately affected by the default of the country where they are registered and operating. Our model allows for extreme scenarios with realistic numbers of joint defaults, while ensuring that the portfolio risk characteristics and the average loss remain unchanged. To estimate the contagion effect, we construct a network using credit default swaps (CDS) time series. We then use CountryRank, a network-based metric, introduced in [30] to quantify the impact of a sovereign default event on the credit quality of corporate issuers in the portfolio. In order to investigate the impact of our model on credit losses, we use synthetic test portfolios for which we generate loss distributions and study the effect of contagion on the associated risk measures. Finally, we analyze the sensitivity of the contagion impact to rating levels and CountryRank. Our analysis shows that credit losses increase significantly in the presence of contagion. Our contributions in this paper are thus threefold: First, we introduce a portfolio credit risk model which incorporates both common factors and contagion. Second, we use a credit stress propagation network constructed from real data to quantify the impact of deterioration of credit quality of the sovereigns on corporates. Third, we present the impact of accounting for contagion which can be useful for banks and regulators to quantify credit, model, or concentration risk in their portfolios.

The rest of the paper is organized as follows. Section 2 provides an overview of the general modelling framework. Section 3 presents the portfolio model with default contagion and illustrates the network model for the estimation of contagion effects. In Section 4 we present empirical analysis of two

synthetic portfolios. Finally, in Section 5, we summarize our findings and draw conclusions.

# 2. Merton-Type Models for Portfolio Credit Risk

Most financial institutions use models that are based on some form of the conditional independence assumption, according to which issuers depend on a set of common underlying factors. Factor models based on the Merton model are particularly popular for portfolio credit risk. Our model extends the multifactor Merton model to allow for credit contagion. In this section, we present the basic portfolio modelling setup, outline the model of Merton, and explain how it can be specified as a factor model. A more detailed presentation of the multivariate Merton model is provided by [9].

*2.1. Basic Setup and Notations.* This subsection introduces the basic notation and terminology that will be used throughout this paper. In addition, we define the main risk characteristics for portfolio credit risk.

The uncertainty of whether an issuer will fail to meet its financial obligations or not is measured by its *probability of default*. For comparison reasons, this is usually specified with respect to a fixed time interval, most commonly one year. The probability of default then describes the probability of a default occurring in the particular time interval. The *exposure at default* is a measure of the extent to which one is exposed to an issuer in the event of, and at the time of, that issuer's default. The default of an issuer does not necessarily imply that the creditor receives nothing from the issuer. The percentage of loss incurred over the overall exposure in the event of default is given by the *loss given default*. Typical values lie between 45% and 80%.

Consider a portfolio of $m$ issuers, indexed by $i = 1, \ldots, m$, and a fixed time horizon of $T = 1$ year. Denote by $e_i$ the exposure at default of issuer $i$ and by $p_i$ its probability of default. Let $q_i$ be the loss given default of issuer $i$. Denote by $Y_i$ the default indicator, in the time period $[0, T]$. All issuers are assumed to be in a nondefault state at time $t = 0$. The default indicator $Y_i$ is then a random variable defined by

$$Y_i = \begin{cases} 1 & \text{if issuer } i \text{ defaults} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

which clearly satisfies $\mathbb{P}(Y_i = 1) = p_i$. The overall portfolio loss is defined as the random variable

$$L := \sum_{i=1}^{m} q_i e_i Y_i. \tag{2}$$

With credit risk in mind, it is useful to distinguish potential losses in *expected losses*, which are relatively predictable and thus can easily be managed, and *unexpected losses*, which are more complicated to measure. Risk managers are more concerned with unexpected losses and focus on risk measures relating to the tail of the distribution of $L$.

*2.2. The Model of Merton.* Credit risk models are typically distinguished in structural and reduced-form models, according to their methodology. Structural models try to explain the mechanism by which default takes place, using variables such as asset and debt values. The model presented by Merton in [3] serves as the foundation for all these models. Consider an issuer whose asset value follows a stochastic process $(V_t)_{t \geq 0}$. The issuer finances itself with equity and debt. No dividends are paid and no new debt can be issued. In Merton's model the issuer's debt consists of a single zero-coupon bond with face value $B$ and maturity $T$. The values at time $t$ of equity and debt are denoted by $S_t$ and $B_t$ and the issuer's asset value is simply the sum of these; that is,

$$V_t = S_t + B_t, \quad t \in [0, T]. \tag{3}$$

Default occurs if the issuer misses a payment to its debtholders, which can happen only at the bond's maturity $T$. At time $T$, there are only two possible scenarios:

(i) $V_T > B$: the value of the issuer's assets is higher than its debt. In this scenario the debtholders receive $B_T = B$, the shareholders receive the remainder $S_T = V_T - B$, and there is no default.

(ii) $V_T \leq B$: the value of the issuer's assets is less than its debt. Hence, the issuer cannot meet its financial obligations and defaults. In that case, shareholders hand over control to the bondholders, who liquidate the assets and receive the liquidation value in lieu of the debt. Shareholders pay nothing and receive nothing; therefore we obtain $B_T = V_T$, $S_T = 0$.

For these simple observations, we obtain the below relations:

$$S_T = \max(V_T - B, 0) = (V_T - B)^+, \tag{4}$$

$$B_T = \min(V_T, B) = B - (B - V_T)^+. \tag{5}$$

Equation (4) implies that the issuer's equity at maturity $T$ can be determined as the price of a European call option on the asset value $V_t$ with strike price $B$ and maturity $T$, while (5) implies that the value of debt at $T$ is the sum of a default-free bond that guarantees payment of $B$ plus a short European put option on the issuer's assets with strike price $B$.

It is assumed that under the physical probability measure $\mathbb{P}$ the process $(V_t)_{t \geq 0}$ follows a geometric Brownian motion of the form

$$dV_t = \mu_V V_t dt + \sigma_V V_t dW_t, \quad t \in [0, T], \tag{6}$$

where $\mu_V \in \mathbb{R}$ is the mean rate of return on the assets, $\sigma_V > 0$ is the asset volatility, and $(W_t)_{t \geq 0}$ is a Wiener process. The unique solution at time $T$ of the stochastic differential equation (6) with initial value $V_0$ is given by

$$V_T = V_0 \exp\left(\left(\mu_V - \frac{\sigma_V^2}{2}\right) T + \sigma_V W_T\right) \tag{7}$$

which implies that

$$\ln V_T \sim \mathcal{N}\left(\ln V_0 + \left(\mu_V - \frac{\sigma_V^2}{2}\right) T, \sigma_V^2 T\right). \tag{8}$$

Hence, the real-world probability of default at time $T$, measured at time $t = 0$, is given by

$$\mathbb{P}\left(V_T \le B\right) = \mathbb{P}\left(\ln V_T \le \ln B\right)$$

$$= \Phi\left(\frac{\ln\left(B/V_0\right) - \left(\mu_V - \sigma_V^2/2\right)T}{\sigma_V \sqrt{T}}\right). \quad (9)$$

A core assumption of Merton's model is that asset returns are lognormally distributed, as can be seen in (8). It is widely acknowledged, however, that empirical distributions of asset returns tend to have heavier tails; thus, (9) may not be an accurate description of empirically observed default rates.

### 2.3. The Multivariate Merton Model.

The model presented in Section 2.2 is concerned with the default of a single issuer. In order to estimate credit risk at a portfolio level, a multivariate version of the model is necessary. A multivariate geometric Brownian motion with drift vector $\boldsymbol{\mu}_V = (\mu_1, \ldots, \mu_m)'$, vector of volatilities $\boldsymbol{\sigma}_V = (\sigma_1, \ldots, \sigma_m)$, and correlation matrix $\Sigma$, is assumed for the dynamics of the multivariate asset value process $(\mathbf{V}_t)_{t \ge 0}$ with $\mathbf{V}_t = (V_{t,1}, \ldots, V_{t,m})'$, so that for all $i$
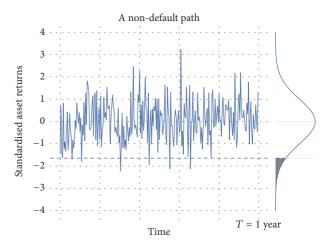
$$V_{T,i} = V_{0,i} \exp\left(\left(\mu_i - \frac{1}{2}\sigma_i^2\right)T + \sigma_i W_{T,i}\right), \quad (10)$$

where the multivariate random vector $\mathbf{W}_T$ with $\mathbf{W}_T = (W_{T,1}, \ldots, W_{T,m})'$ is satisfying $\mathbf{W}_T \sim N_m(\mathbf{0}, T\Sigma)$. Default takes place if $V_{T,i} \le B_i$, where $B_i$ is the debt of company $i$. It is clear that the default probability in the model remains unchanged under simultaneous strictly increasing transformations of $V_{T,i}$ and $B_i$. Thus, one may define

$$X_i := \frac{\ln V_{T,i} - \ln V_{0,i} - \left(\mu_i - (1/2)\sigma_i^2\right)T}{\sigma_i \sqrt{T}},$$

$$d_i := \frac{\ln B_i - \ln V_{0,i} - \left(\mu_i - (1/2)\sigma_i^2\right)T}{\sigma_i \sqrt{T}} \quad (11)$$

and then default equivalently occurs if and only if $X_i \le d_i$. Notice that $X_i$ is the standardized asset value log-return $\ln V_{T,i} - \ln V_{0,i}$. It can be easily shown that the transformed variables satisfy $(X_1, \ldots, X_m)' \sim N_m(\mathbf{0}, \Sigma)$ and their copula is the Gaussian copula. Thus, the probability of default for issuer $i$ is satisfying $p_i = \Phi(d_i)$, where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. A graphical representation of Merton's model is shown in Figure 1. In most practical implementations of the model, portfolio losses are modelled by directly considering an $m$-dimensional random vector $\mathbf{X} = (X_1, \ldots, X_m)'$ with $\mathbf{X} \sim N_m(\mathbf{0}, \Sigma)$ containing the standardized asset returns and a deterministic vector $\mathbf{d} = (d_1, \ldots, d_m)$ containing the critical thresholds with $d_i = \Phi^{-1}(p_i)$ for given default probabilities $p_i$, $i = 1, \ldots, m$. The default probabilities are usually estimated by historical default experience using external ratings by agencies or model-based approaches.



FIGURE 1: In Merton's model, default of issuer $i$ occurs if at time $T$ asset value $V_{T,i}$ falls below debt value $B_i$, or equivalently if $X_i := (\ln V_{T,i} - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T)/\sigma_i \sqrt{T}$ falls below the critical threshold $d_i := (\ln B_{T,i} - \ln V_{0,i} - (\mu_i - (1/2)\sigma_i^2)T)/\sigma_i \sqrt{T}$. Since $X_i \sim \mathcal{N}(0,1)$, $i$'s default probability, represented by the shaded area in the distribution plot, is satisfying $p_i = \Phi(d_i)$. Note that default can only take place at time $T$ does not depend on the path of the asset value process.

### 2.4. Merton Model as a Factor Model.

The number of parameters contained in the correlation matrix $\Sigma$ grows polynomially in $m$, and thus, for large portfolios it is essential to have a more parsimonious parametrization which is accomplished using a factor model. Additionally, factor models are particularly attractive due to the fact that they offer an intuitive interpretation of credit risk in relation to the performance of industry, region, global economy, or any other relevant indexes that may affect issuers in a systematic way. In the following we show how Merton's model can be understood as a factor model. In the factor model approach, asset returns are linearly dependent on a vector $\mathbf{F}$ of $p < m$ common underlying factors satisfying $\mathbf{F} \sim N_p(0, \Omega)$. Issuer $i$'s standardized asset return is assumed to be driven by an issuer-specific combination $\widetilde{F}_i = \boldsymbol{\alpha}_i' \mathbf{F}$ of the systematic factors

$$X_i = \sqrt{\beta_i}\widetilde{F}_i + \sqrt{1 - \beta_i}\epsilon_i, \quad (12)$$

where $\widetilde{F}_i$ and $\epsilon_1, \ldots, \epsilon_m$ are independent standard normal variables and $\epsilon_i$ represents the idiosyncratic risk. Consequently, $\beta_i$ can be seen as a measure of sensitivity of $X_i$ to systematic risk, as it represents the proportion of the $X_i$ variation that is explained by the systematic factors. The correlations between asset returns are given by

$$\rho\left(X_i, X_j\right) = \operatorname{cov}\left(X_i, X_j\right) = \sqrt{\beta_i \beta_j}\operatorname{cov}\left(\widetilde{F}_i, \widetilde{F}_j\right)$$

$$= \sqrt{\beta_i \beta_j}\boldsymbol{\alpha}_i' \Omega \boldsymbol{\alpha}_j \quad (13)$$

since $\widetilde{F}_i$ and $\epsilon_1, \ldots, \epsilon_m$ are independent and standard normal and $\operatorname{var}(X_i) = 1$.
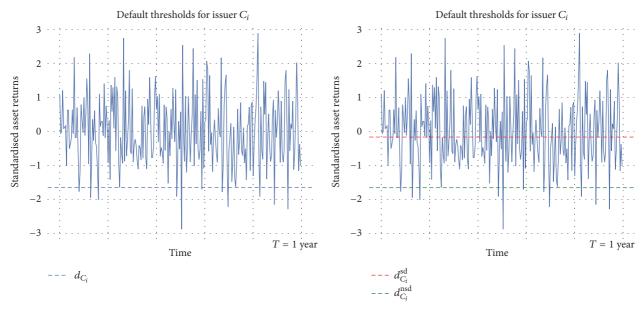
FIGURE 2: Under the standard Merton model, the default threshold $d_{C_i}$ for corporate issuer $C_i$ is set to be equal to $\Phi^{-1}(p_{C_i})$. Under the proposed model, the threshold increases in the event of sovereign default, making $C_i$'s default more likely as the contagion effect suggests.

## 3. A Model for Credit Contagion

In the multifactor Merton model specified in Section 2.4, the standardized asset returns $X_i$, $i = 1, \ldots, m$, are assumed to be driven by a set of common underlying systematic factors, and the critical thresholds $d_i$, $i = 1, \ldots, m$, are satisfying $d_i = \Phi^{-1}(p_i)$ for all $i$. The only source of default dependence in such a framework is the dependence on the systematic factors. In the model we propose, we assume that, in the event of a sovereign default, contagion will spread to the corporate issuers in the portfolio that are registered and operating in that country, causing default probability to be equal to their CountryRank. In Section 3.1, we demonstrate how to calibrate the critical thresholds so that each corporate's probability of default conditional on the default of the corresponding sovereign equals its CountryRank, while its unconditional default probability remains unchanged. In Section 3.2, we show how to construct a credit stress propagation network and estimate the CountryRank parameter.

*3.1. Incorporating Contagion in Factor Models.* Consider a corporate issuer $C_i$ and its country of operation $S$. Denote by $p_{C_i}$ the probability of default of $C_i$. Under the standard Merton model, default occurs if $C_i$'s standardized asset return $X_{C_i}$ falls below its default threshold $d_{C_i}$. The critical threshold $d_{C_i}$ is assumed to be equal to $\Phi^{-1}(p_{C_i})$ and is independent of the state of the country of operation $S$. In the proposed model, a corporate is subject to shocks from its country of operation; its corresponding state is described by a binary state variable. The state is considered to be stressed in the event of sovereign default. In this case, the issuer's default threshold increases, causing it more likely to default, as the contagion effect suggests. In case the corresponding

sovereign does not default, the corporates liquidity state is considered stable. We replace the default threshold $d_{C_i}$ with $d_{C_i}^*$, where

$$
d_{C_i}^*
= \begin{cases} d_{C_i}^{\mathrm{sd}} & \text{if the corresponding sovereign defaults} \\ d_{C_i}^{\mathrm{nsd}} & \text{otherwise} \end{cases}
\tag{14}
$$

or equivalently

$$
d_{C_i}^* = \mathbb{1}_{\{Y_S=1\}} d_{C_i}^{\mathrm{sd}} + \mathbb{1}_{\{Y_S=0\}} d_{C_i}^{\mathrm{nsd}}.
\tag{15}
$$

We denote by $p_S$ the probability of default of the country of operation and by $\gamma_{C_i}$ the CountryRank parameter which indicates the increased probability of default of $C_i$ given the default of $S$. An example of the new default thresholds is shown in Figure 2. Our objective is to calibrate $d_{C_i}^{\mathrm{sd}}$ and $d_{C_i}^{\mathrm{nsd}}$ in such way that the overall default rate remains unchanged and $\mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1) = \gamma_{C_i}$. Denote by

$$
\phi_2(x, y; \rho) := \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 + y^2 - 2\rho xy}{2(1-\rho^2)}\right),
\tag{16}
$$

$$
\Phi_2(h, k; \rho) := \int_{-\infty}^{h} \int_{-\infty}^{k} \phi_2(x, y; \rho)\, dy\, dx
$$

the density and distribution function of the bivariate standard normal distribution with correlation parameter $\rho \in (-1, 1)$.

Note that $d_{C_i}^*(\omega) = d_{C_i}^{sd}$ for $\omega \in \{Y_{C_i} = 1, \; Y_S = 1\} \subset \{Y_S = 1\}$, and $d_{C_i}^*(\omega) = d_{C_i}^{nsd}$ for $\omega \in \{Y_{C_i} = 1, \; Y_S = 0\} \subset \{Y_S = 0\}$. We rewrite $\mathbb{P}(Y_{C_i} = 1 \mid Y_S = 1)$ in the following way:

$$
\begin{aligned}
& \mathbb{P}\left(Y_{C_i} = 1 \mid Y_S = 1\right) \\
&= \frac{1}{\mathbb{P}\left(Y_S = 1\right)} \mathbb{P}\left(Y_{C_i} = 1, \; Y_S = 1\right) \\
&= \frac{1}{p_S} \mathbb{P}\left[X_{C_i} < d_{C_i}^{sd}, \; X_S < d_S\right] \\
&= \frac{1}{p_S} \Phi_2\left(d_{C_i}^{sd}, d_S; \rho_{SC_i}\right).
\end{aligned}
\tag{17}
$$

Using the above representation and given $d_S = \Phi^{-1}(p_S)$ and $\rho_{SC_i}$, one can solve the equation

$$
\mathbb{P}\left(Y_{C_i} = 1 \mid Y_S = 1\right) = \gamma_{C_i}
\tag{18}
$$

over $d_{C_i}^{sd}$. We proceed to the derivation of $d_{C_i}^{nsd}$ in such way that the overall default probability remains equal to $p_{C_i}$. This constraint is important, since contagion is assumed to have no impact on the average loss. Clearly,

$$
\begin{aligned}
p_{C_i} &= \mathbb{P}\left(Y_{C_i} = 1\right) \\
&= \mathbb{P}\left(Y_{C_i} = 1, \; Y_S = 1\right) + \mathbb{P}\left(Y_{C_i} = 1, \; Y_S = 0\right) \\
&= \mathbb{P}\left(Y_{C_i} = 1 \mid Y_S = 1\right)\mathbb{P}\left(Y_S = 1\right) \\
&\quad + \mathbb{P}\left(Y_{C_i} = 1, \; Y_S = 0\right)
\end{aligned}
\tag{19}
$$

and thus

$$
\mathbb{P}\left(Y_{C_i} = 1, \; Y_S = 0\right) = p_{C_i} - \gamma_{C_i} \cdot p_S.
\tag{20}
$$

The left-hand side of the above equation can be represented as follows:

$$
\begin{aligned}
\mathbb{P}\left(\text{corp.def} \cap \text{nosov.def}\right) &= \mathbb{P}\left[X_{C_i} < d_{C_i}^{nsd}, \; X_S > d_S\right] \\
&= \mathbb{P}\left[X_{C_i} < d_{C_i}^{nsd}\right] - \mathbb{P}\left[X_{C_i} < d_{C_i}^{nsd}, \; X_S < d_S\right] \\
&= \Phi\left(d_{C_i}^{nsd}\right) - \Phi_2\left(d_{C_i}^{nsd}, d_S; \rho_{SC_i}\right).
\end{aligned}
\tag{21}
$$

By use of the above and given $d_S = \Phi^{-1}(p_S)$ and $\rho_{SC_i}$, one can solve the previous equation over $d_{C_i}^{nsd}$.

### 3.2. Estimation of CountryRank.
In this section, we elaborate on the estimation of the CountryRank parameter [30], which serves as the probability of default of the corporate conditional on the default of the sovereign. In addition, we provide details on the construction of the credit stress propagation network.

### 3.2.1. CountryRank.
In order to estimate contagion effects in a network of issuers, an algorithm such as DebtRank

[31] is necessary. In the DebtRank calculation process, stress propagates even in the absence of defaults and each node can propagate stress only once before becoming inactive. The level of distress for a previously undistressed node is given by the sum of incoming stress from its neighbors with a maximum value of 1. Summing up the incoming stress from neighboring nodes seems reasonable when trying to estimate the impact of one node or a set of nodes to a network of interconnected balance sheets where links represent lending relationships. However, when trying to quantify the probability of default of a corporate node given the infectious default of a sovereign node, one has to consider that there is significant overlap in terms of common stress, and thus, by summing we may be accounting for the same effect more than once. This effect is amplified in dense networks constructed from CDS data. Therefore, we introduce CountryRank as an alternative measure which is suited for our contagion model.

We assume that we have a hypothetical credit stress propagation network, where the nodes correspond to the issuers, including the sovereign, and the edges correspond to the impact of credit quality of one issuer on the other. The details of the network construction will be presented in Section 3.2.2. Given such a network, the CountryRank of the nodes can be defined recursively as follows:

(i) First, we stress the sovereign node and as a result its CountryRank is 1.

(ii) Let $\gamma_S$ be the CountryRank of the sovereign and let $e_{(j,k)}$ denote the edge weight between nodes $j$ and $k$. Given a node $C_i$, let $p = SC_1C_2 \cdots C_{i-1}C_i$ be a path without cycles from the sovereign node $S$, to the node $C_i$. The *weight* of the path $p$ is defined as

$$
w\left(p\right) = \gamma_S e_{(1,2)} \cdots e_{(i-1,i)},
\tag{22}
$$

where $e_{(j,k)}$ are the respective edge weights between nodes $j$ and $k$ for $j \in \{1, \ldots, i-1\}$ and $k \in \{2, \ldots, i\}$. Let $p_1, \ldots, p_m$ be the set of all acyclic paths from the sovereign node to the corporate node $C_i$ and let $w(p_1), \ldots, w(p_m)$ be the corresponding weights. Then the CountryRank of node $C_i$ is defined as

$$
\gamma_{C_i} = \max_{1 \leq j \leq m} w\left(p_j\right).
\tag{23}
$$

In order to compute the conditional probability of default of a corporate given the sovereign default analytically, we would need the joint distribution of probabilities of default of the nodes, which has an exponential computational complexity, and it is therefore intractable. Thus, we approximate the conditional probability by choosing the path with the maximum weight in the above definition for CountryRank.

The example in Figure 3 illustrates calculation of CountryRank for a hypothetical network. The network consists of a sovereign node $S$ and corporate nodes $C_1, C_2, C_3, C_4$. The edge labels indicate weights in network between two nodes. We initially stress the sovereign node which results in a CountryRank of 1 for node $S$. In the next step, the stress propagates to node $C_1$ and as a result its CountryRank is 0.9.

(a) Sovereign node in stress

(b) Stress propagates to node $C_1$

(c) Stress propagates to node $C_2$

(d) Stress propagates to node $C_3$

(e) Stress propagates to node $C_4$
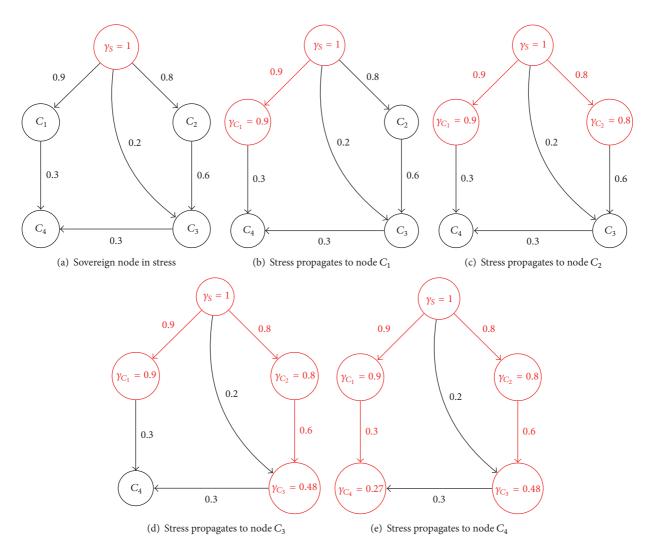
FIGURE 3: Illustration of the CountryRank parameter using a hypothetical network. The subfigures (a)–(e) show the propagation of stress in the network starting from the sovereign node to corporate nodes. At each step, the stress spreads to a node using the path with the maximum weight from the sovereign node.

Then, node $C_2$ gets stressed giving it a CountryRank value 0.8. For node $C_3$, there are two paths from node $S$, so we pick the path through node $C_2$ having a higher weight of 0.48. Finally, there are three paths from node $S$ to node $C_4$, and the path with maximum weight is 0.27.

*3.2.2. Network Construction.* Credit default swap spreads are market-implied indicators of probability of default of an entity. A credit default swap is a financial contract in which a protection seller A insures a protection buyer B against the default of a third party C. More precisely, regular coupon payments with respect to a contractual notional $N$ and a fixed rate $s$, the CDS spread, are swapped with a payment of $N(1 - RR)$ in the case of the default of C, where RR, the so-called recovery rate, is a contract parameter which represents the fraction of investment which is assumed be recovered in the case of default of C.

*Modified $\epsilon$-Draw-Up.* We would like to measure to what extent changes in CDS spreads of different issuers occur simultaneously. For this, we use the notion of a *modified $\epsilon$-draw-up* to quantify the impact of deterioration of credit quality of one issuer on the other. Modified $\epsilon$-draw-up is an alteration of the $\epsilon$-draw-ups notion which is introduced in [32]. In that article, the authors use the notion of $\epsilon$-draw-ups to construct a network which models the conditional probabilities of spike-like comovements among pairs of CDS spreads. A modified $\epsilon$-draw-up is defined as an upward movement in the time series in which the amplitude of the movement, that is, the difference between the subsequent local maxima and current local minima, is greater than a threshold $\epsilon$. We record such local minima as the modified $\epsilon$-draw-ups. The $\epsilon$ parameter for a local minima at time $t$ is set to be the standard deviation in the time series between days $t - n$ and $t$, where $n$ is chosen to be 10 days. Figure 5 shows

the time series of Russian Federation CDS with the calibrated modified $\epsilon$-draw-ups using a history of 10 days for calibration.

*Filtering Market Impact.* Since we would like to measure the comovement of the time series $i$ and $j$, we exclude the effect of the external market on these nodes as follows. We calibrate the $\epsilon$-draw-ups for the CDS time series of an index that does not represent the region in question; for instance, for Russian issuers we choose the iTraxx index which is the composite CDS index of 125 CDS referencing European investment grade credit. Then, we filter out those $\epsilon$-draw-ups of node $i$ which are the same as the $\epsilon$-draw-ups of the iTraxx index including a time lag $\tau$. That is, if iTraxx has a modified $\epsilon$-draw-up on day $t$, then we remove the modified $\epsilon$-draw-ups of node $i$ on days $t, t + 1, \ldots, t + \tau$. We choose a time lag of 3 days for our calibration based on the input data which is consistent with the choice in [32].

*Edges.* After identifying the $\epsilon$-draw-ups for all the issuers and filtering out the market impact, the edges in our network are constructed as follows. The weight of an edge in the credit stress propagation network from node $i$ to node $j$ is the conditional probability that if node $i$ has an epsilon draw-up on day $t$, then node $j$ also has an epsilon draw-up on days $t, t + 1, \ldots, t + \tau$, where $\tau$ is the time lag. More precisely, let $N_i$ be the number of $\epsilon$-draw-ups of node $i$ after filtering using iTraxx index and $N_{ij}$ epsilon draw-ups of node $i$ which are also epsilon draw-ups for node $j$ with the time lag $\tau$. Then, the edge weight $w_{ij}$ between nodes $i$ and $j$ is defined as $w_{ij} = N_i/N_{ij}$. Figure 6 shows the minimum spanning tree of the credit stress propagation network constructed using the CDS spread time series data of Russian issuers.

*Uncertainty in CountryRank.* We test the robustness of our CountryRank calibration by varying the number of days used for $\epsilon$-parameter. The figure in Appendix B shows that the $\epsilon$-parameter for Russian Federation CDS time series remains stable when we vary the number of days. We initially obtain time series of $\epsilon$-parameters by calculating standard deviation in the last $n = 10$, 15, and 20 days on all local minima indices of Russian Federation CDS. Subsequently, we calculate the mean of the absolute differences between the epsilon time series calculated and express this in units of the mean of Russian Federation CDS time series. The percentage difference is 1.38% between the 10-day $\epsilon$-parameter and 15-day $\epsilon$-parameter and 2.22% between the 10-day and 20-day $\epsilon$-parameters.

Further, we quantify the uncertainty in CountryRank parameter as follows. For an corporate node, we calculate the absolute difference in CountryRank calculated using $n = 15$ and 20 days with CountryRank using $n = 10$ days for the $\epsilon$-parameter. We then calculate this difference as a percentage of the CountryRank calculated using 10 days for $\epsilon$-parameter for all corporates and then compute their mean. The mean difference between CountryRank calibrated using $n = 15$ days and $n = 10$ days is 6.84% and $n = 20$ days and $n = 10$ days is 9.73% for the Russian CDS data set.

TABLE 1: Systematic factor: index mapping.

| Factor | Index |
|---|---|
| Europe | MSCI EUROPE |
| Asia | MSCI AC ASIA |
| North America | MSCI NORTH AMERICA |
| Latin America | MSCI EM LATIN AMERICA |
| Middle East and Africa | MSCI FM AFRICA |
| Pacific | MSCI PACIFIC |
| Materials | MSCI WRLD/MATERIALS |
| Consumer products | MSCI WRLD/CONSUMER DISCR |
| Services | MSCI WRLD/CONSUMER SVC |
| Financial | MSCI WRLD/FINANCIALS |
| Industrial | MSCI WRLD/INDUSTRIALS |
| Government | ITRAXX SOVX GLOBAL LIQUID INVESTMENT GRADE |

## 4. Numerical Experiments

We implement the framework presented in Section 3 to synthetic test portfolios and discuss the corresponding risk metrics. Further, we perform a set of sensitivity studies and explore the results.

*4.1. Factor Model.* We first set up a multifactor Merton model, as it was described in Section 2. We define a set of systematic factors that will represent region and sector effects. We choose 6 region and 6 sector factors, for which we select appropriate indexes, as shown in Table 1. We then use 10 years of index time series to derive the region and sector returns $F_{R(j)}$, $j = 1, \ldots, 6$ and $F_{S(k)}$, $k = 1, \ldots, 6$, respectively, and obtain an estimate of the correlation matrix $\Omega$, shown in Figure 7. Subsequently, we map all issuers to one region and one sector factor, $F_{R(i)}$ and $F_{S(i)}$, respectively. For instance, a Dutch bank will be associated with Europe and financial factors. As a proxy of individual asset returns, we use 10 years of equity or CDS time series, depending on the data availability for each issuer. Finally, we standardize the individual returns time series $(X_{i,t})$ and perform the following Ordinary Least Squares regression against the systematic factor returns

$$X_{i,t} = \alpha_{R(i)} F_{R(i),t} + \alpha_{S(k)} F_{S(k),t} + \epsilon_{i,t} \qquad (24)$$

to obtain $\widehat{\alpha}_{R(i)}$, $\widehat{\alpha}_{S(i)}$, and $\widehat{\beta}_i = R^2$, where $R^2$ is the coefficient of determination, and it is higher for issuers whose returns are largely affected by the performance of the systematic factors.

*4.2. Synthetic Test Portfolios.* To investigate the properties of the contagion model, we set up 2 test portfolios. For these portfolios, the resulting risk measures are compared to those of the standard latent variable model with no contagion. Portfolio A consists of 1 Russian government bond and 17 bonds issued by corporations registered and operating in the Russian Federation. As it is illustrated in Table 2, the issuers are of medium and low credit quality. Portfolio B represents a similar but more diversified setup with 4 sovereign bonds

TABLE 2: Rating classification for the test portfolios.

| Rating | Portfolio A | | Portfolio B | |
|---|---|---|---|---|
| | Issuers | % | Issuers | % |
| AAA | - | 0.00% | 3 | 3.75% |
| AA | - | 0.00% | 3 | 3.75% |
| A | - | 0.00% | 22 | 27.50% |
| BBB | 1 | 5.56% | 39 | 48.75% |
| BB | 15 | 83.33% | 9 | 11.25% |
| B | 2 | 11.11% | 3 | 3.75% |
| CCC/C | - | 0.00% | 1 | 1.25% |
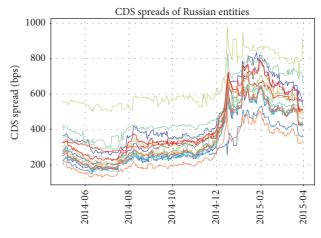
TABLE 3: Sector classification for the test portfolios.

| Sector | Portfolio A | | Portfolio B | |
|---|---|---|---|---|
| | Issuers | % | Issuers | % |
| Materials | 5 | 27.78% | 12 | 15.00% |
| Consumer products | - | 0.00% | 12 | 15.00% |
| Services | 3 | 16.67% | 19 | 23.75% |
| Financial | 7 | 38.89% | 25 | 31.25% |
| Industrial | - | 0.00% | 6 | 7.50% |
| Government | 3 | 16.67% | 6 | 7.50% |

issued by Germany, Italy, Netherlands, and Spain and 76 corporate bonds by issuers from the aforementioned countries. The sectors represented in Portfolios A and B are shown in Table 3. Both portfolios are assumed to be equally weighted with a total notional of €10 million.

*4.3. Credit Stress Propagation Network.* We use credit default swap data to construct the stress propagation network. The CDS raw data set consists of daily CDS liquid spreads for different maturities from 1 May 2014 to 31 March 2015 for Portfolio A and 1 July 2014 to 31 December 2015 for Portfolio B. These are averaged quotes from contributors rather than exercisable quotes. In addition, the data set also provides information on the names of the underlying reference entities, recovery rates, number of quote contributors, region, sector, average of the ratings from Standard & Poor's, Moody's, and Fitch Group of each entity, and currency of the quote. We use the normalized CDS spreads of entities for the 5-year tenor for our analysis. The CDS spreads time series of Russian issuers are illustrated in Figure 4.

*4.4. Simulation Study.* In order to generate portfolio loss distributions and derive the associated risk measures, we perform Monte Carlo simulations. This process entails generating joint realizations of the systematic and idiosyncratic risk factors and comparing the resulting critical variables with the corresponding default thresholds. By this comparison, we obtain the default indicator $Y_i$ for each issuer and this enables us to calculate the overall portfolio loss for this trial. The only difference between the standard and the contagion model is that in the contagion model we first obtain the default indicators for the sovereigns, and their values determine which default thresholds are going to be



FIGURE 4: Time series of CDS spreads of Russian issuers.

TABLE 4: Portfolio losses for the test portfolios and additional risk due to contagion.

(a) Panel 1: Portfolio A

| Quantile | Loss standard model | Loss contagion model | Contagion impact | |
|---|---|---|---|---|
| 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| 99.50% | 1,443,579 | 3,003,949 | 1,560,370 | 108% |
| 99.90% | 2,258,857 | 4,968,393 | 2,709,536 | 120% |
| 99.99% | 3,543,441 | 5,713,486 | 2,170,045 | 61% |
| Average loss | 71,807 | 71,691 | | |

(b) Panel 2: Portfolio B

| Quantile | Loss standard model | Loss contagion model | Contagion impact | |
|---|---|---|---|---|
| 99% | 373,013 | 379,929 | 6,915 | 2% |
| 99.50% | 471,497 | 520,467 | 48,971 | 10% |
| 99.90% | 775,773 | 1,009,426 | 233,653 | 30% |
| 99.99% | 1,350,279 | 1,847,795 | 497,516 | 37% |
| Average loss | 44,850 | 44,872 | | |
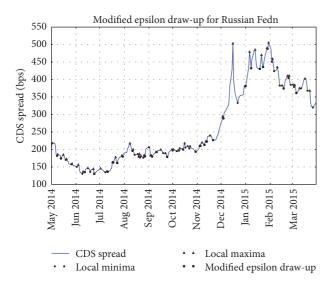


FIGURE 5: Time series for Russian Federation with local minima, local maxima, and modified $\epsilon$-draw-ups.

used for the corporate issuers. The quantiles of the generated loss distributions as well as the percentage increase due to contagion are illustrated in Table 4. A liquidity horizon of 1 year is assumed throughout and the figures are based on a simulation with $10^6$ samples.

For Portfolio A, the 99.90% quantile of the loss distribution under the standard factor model is €2,258,857, which corresponds to approximately 23% of the total notional. This figure jumps to €4,968,393 (almost 50% of the notional) under the model with contagion. As shown in Panel 1, contagion has a minimal effect on the 99% quantile, while at 99.5%, 99.90%, and 99.99% it results in an increase of 108%, 120%, and 61%, respectively. This is to be expected

as the probability of default for Russian Federation is less than 1% and thus, in more than 99.9% of our trials, default will not take place and contagion will not be triggered. For Portfolio B, the 99.90% quantile is considerably lower under both the standard and the contagion model, at €775,773 or 8% of the total notional and €1,009,426 or 10% of the total notional, respectively, reflecting lower default risk. One can observe that the model with contagion yields low additional losses at 99% and 99.5% quantiles, with a more significant impact at 99.90% and 99.99% (30% and 37%, respectively). An illustration of the additional losses due to contagion is given by Figure 8.

*4.5. Sensitivity Analysis.* In the following, we present a series of sensitivity studies and discuss the results. To achieve a candid comparison, we choose to perform this analysis on the single-sovereign Portfolio A. We vary the ratings of sovereign and corporates, as well as the CountryRank parameter, to draw conclusions about their impact on the loss distribution and verify the model properties.

*4.5.1. Sovereign Rating.* We start by exploring the impact of the credit quality of the sovereign. Table 5 shows the quantiles of the generated loss distributions under the standard latent variable model and the contagion model when the rating of the Russian Federation is 1 and 2 notches higher than the original rating (BB). It can be seen that the contagion effect appears less strong when the sovereign rating is higher. At the 99.9% quantile, the contagion impact drops from 120% to 62% for an upgraded sovereign rating of BBB. The drop is even higher, when upgrading the sovereign rating to A, with only 11% additional losses due to contagion. Apart from having a less significant impact at the 99.9% quantile, it is clear that, with a sovereign rating of A, the contagion impact is zero at the 99% and 99.5% levels, where the results of the
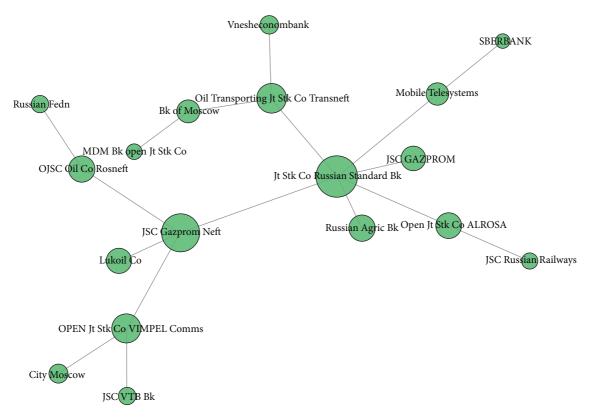
Figure 6: Minimum spanning tree for Russian issuers.
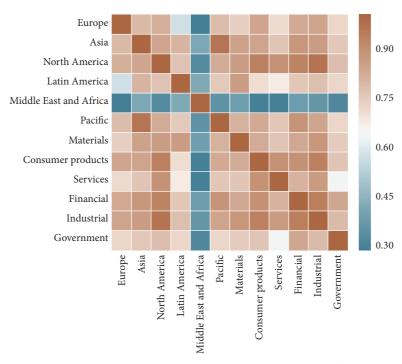


Figure 7: Estimated systematic factor correlation matrix $\widehat{\Omega}$.
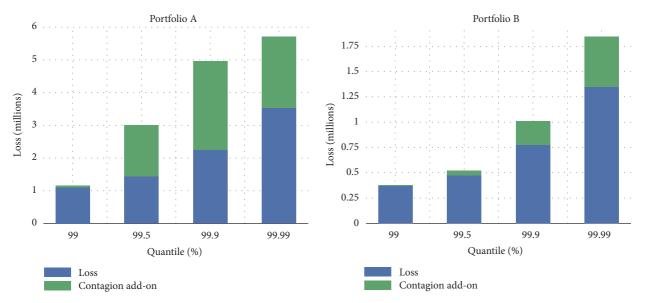
FIGURE 8: Additional losses due to contagion for Portfolios A and B.

TABLE 5: Varying the sovereign rating.

| Sovereign rating | Quantile | Loss standard model | Loss contagion model | Contagion impact | |
|---|---|---|---|---|---|
| BB | 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| | 99.50% | 1,443,579 | 3,003,949 | 1,560,370 | 108% |
| | 99.90% | 2,258,857 | 4,968,393 | 2,709,536 | 120% |
| | 99.99% | 3,543,441 | 5,713,486 | 2,170,045 | 61% |
| BBB | 99% | 1,115,153 | 1,115,153 | - | 0% |
| | 99.50% | 1,443,009 | 1,490,755 | 47,746 | 3% |
| | 99.90% | 2,229,742 | 3,613,625 | 1,383,883 | 62% |
| | 99.99% | 3,496,264 | 5,432,236 | 1,935,972 | 55% |
| A | 99% | 1,114,583 | 1,114,583 | - | 0% |
| | 99.50% | 1,443,009 | 1,443,009 | - | 0% |
| | 99.90% | 2,229,737 | 2,469,922 | 240,185 | 11% |
| | 99.99% | 3,455,199 | 5,056,639 | 1,601,439 | 46% |

contagion model match those of the standard model. This is to be expected since a rating of A corresponds to a probability of default less than 0.01%, and as explained in Section 4.4, when sovereign default occurs seldom, the contagion effect can hardly be observed.

*4.5.2. Corporate Default Probabilities.* In the next test, the impact of corporate credit quality is investigated. As Table 6 illustrates, contagion has smaller impact when the corporate default probabilities are increased by 5%, which is in line with intuition since the autonomous (not sovereign induced) default probabilities are quite high, meaning that they are likely to default whether the corresponding sovereign defaults or not. For the same reason, the impact is even less significant when the corporate default probabilities are stressed by 10%.

*4.5.3. CountryRank.* In the last test, the sensitivity of the contagion impact to changes in the CountryRank is investigated. In Table 7, we test the contagion impact when CountryRank is stressed by 15% and 10%, respectively. The results are in line with intuition, with a milder contagion effect for lower CountryRank values and a stronger effect in case the parameter is increased.

# 5. Conclusions

In this paper, we present an extended factor model for portfolio credit risk which offers a breadth of possible applications to regulatory and economic capital calculations, as well as to the analysis of structured credit products. In the proposed framework, systematic risk factors are augmented

Table 6: Varying corporate default probabilities.

| Corporate default probabilities | Quantile | Loss standard model | Loss contagion model | Contagion impact | |
|---|---|---|---|---|---|
| Unstressed | 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| | 99.50% | 1,443,579 | 3,003,949 | 1,560,370 | 108% |
| | 99.90% | 2,258,857 | 4,968,393 | 2,709,536 | 120% |
| | 99.99% | 3,543,441 | 5,713,486 | 2,170,045 | 61% |
| Stressed by 5% | 99% | 1,115,153 | 1,250,570 | 135,417 | 12% |
| | 99.50% | 1,443,579 | 3,003,949 | 1,560,370 | 108% |
| | 99.90% | 2,333,935 | 4,968,393 | 2,634,458 | 113% |
| | 99.99% | 3,584,506 | 5,713,486 | 2,128,979 | 59% |
| Stressed by 10% | 99% | 1,162,329 | 1,260,422 | 98,093 | 8% |
| | 99.50% | 1,503,348 | 3,003,949 | 1,500,602 | 100% |
| | 99.90% | 2,375,570 | 4,968,393 | 2,592,823 | 109% |
| | 99.99% | 3,642,099 | 5,713,486 | 2,071,387 | 57% |

Table 7: Varying CountryRank.

| CountryRank | Quantile | Loss standard model | Loss contagion model | Contagion impact | |
|---|---|---|---|---|---|
| Unstressed | 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| | 99.50% | 1,443,579 | 3,003,949 | 1,560,370 | 108% |
| | 99.90% | 2,258,857 | 4,968,393 | 2,709,536 | 120% |
| | 99.99% | 3,543,441 | 5,713,486 | 2,170,045 | 61% |
| Stressed by 5% | 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| | 99.50% | 1,443,579 | 3,196,958 | 1,753,379 | 121% |
| | 99.90% | 2,258,857 | 5,056,634 | 2,797,777 | 124% |
| | 99.99% | 3,543,441 | 5,713,495 | 2,170,054 | 61% |
| Stressed by 10% | 99% | 1,115,153 | 1,162,329 | 47,176 | 4% |
| | 99.50% | 1,443,579 | 3,389,398 | 1,945,818 | 135% |
| | 99.90% | 2,258,857 | 5,296,249 | 3,037,392 | 134% |
| | 99.99% | 3,543,441 | 5,801,727 | 2,258,286 | 64% |

with an infectious default mechanism which affects the entire portfolio. Unlike models based on copulas with more extreme tail behavior, where the dependence structure of defaults is specified in advance, our model provides an intuitive approach, by first specifying the way sovereign defaults may affect the default probabilities of corporate issuers and then deriving the joint default distribution. The impact of sovereign defaults is quantified using a credit stress propagation network constructed from real data. Under this framework, we generate loss distributions for synthetic test portfolios and show that the contagion effect may have a profound impact on the upper tails.

Our model provides a first step towards incorporating network effects in portfolio credit risk models. The model can be extended in a number of ways such as accounting for stress propagation from a sovereign to corporates even without sovereign default or taking into consideration contagion between sovereigns. Another interesting topic for future research is characterizing the joint default distribution of issuers in credit stress propagation networks using Bayesian network methodologies, which may facilitate an improved approximation of the conditional default probabilities in comparison to the maximum weight path in the current definition of CountryRank. Finally, a conjecture worthy of further investigation is that a more connected structure for the credit stress propagation network leads to increased values for the CountryRank parameter, and, as a result, to higher additional losses due to contagion.

| Allianz SE | Robert Bosch GmBH |
| EnBW Energie Baden | Rheinmetall AG |
| Wuerttemberg AG | RWE AG |
| BASF SE | Siemens AG |
| Bertelsmann SE Co KGaA | UniCredit Bk AG |
| Bay Motoren Werke AG | Volkswagen AG |
| Bayer AG | Aegon N.V. |
| Bay Landbk Giroz | Koninklijke Ahold N V |
| Commerzbank AG | Akzo Nobel NV |
| Contl AG | Eneco Hldg N V |
| Daimler AG | Essent N V |
| Deutsche Bk AG | ING Groep NV |
| Deutsche Bahn AG | ING Bk N V |
| Fed Rep Germany | Koninklijke DSM NV |
| Deutsche Post AG | Koninklijke Philips NV |
| Deutsche Telekom AG | Koninklijke KPN N V |
| EON SE | UPC Hldg BV |
| Fresenius SE & Co KGaA | Kdom Neth |
| Grohe Hldg Gmbh | Nielsen Co |
| Hannover Rueck SE | NN Group NV |
| HeidelbergCement AG | PostNL NV |
| Henkel AG & Co KGaA | Rabobank Nederland |
| Linde AG | Royal Bk of Scotland NV |
| Deutsche Lufthansa AG | SNS Bk NV |
| METRO AG | Stmicroelectronics N V |
| Munich Re | Unilever N V |
| TUI AG | Wolters Kluwer N V |

Figure 9: Time series of CDS spreads of Dutch and German entities.



| ALTADIS SA |
| Bco de Sabadell S A |
| Bco Bilbao Vizcaya Argentaria S A |
| Bco Pop Espanol |
| Endesa S A |
| Gas Nat SDG SA |
| Iberdrola SA |
| Instituto de Cred Oficial |
| REPSOL SA |
| Bco SANTANDER SA |
| Kdom Spain |
| Telefonica S A |
| Assicurazioni Generali S p A |
| ATLANTIA SPA |
| Mediobanca SpA |
| Bco Pop S C |
| CIR SpA CIE Industriali Riunite |
| ENEL S p A |
| ENI S.p.A. |
| Edison S p A |
| Finmeccanica S p A |
| Rep Italy |
| Bca Naz del Lavoro S p A |
| Bca Pop di Milano Soc Coop a r l |
| Bca Monte dei Paschi di Siena S p A |
| Intesa Sanpaolo SpA |
| Telecom Italia SpA |
| UniCredit SpA |

Figure 10: Time series of CDS spreads of Spanish and Italian entities.

## Appendix

## A. CDS Spread Data

The data used to calibrate the credit stress propagation network for European issuers is the CDS spread data of Dutch, German, Italian, and Spanish issuers as shown in Figures 9 and 10.

## B. Stability of $\epsilon$-Parameter

The plot in Figure 11 shows the time series of the epsilon parameter for different number days used for $\epsilon$-draw-up calibration.



| 10 days |
| 15 days |
| 20 days |

Figure 11: Stability of $\epsilon$ parameter for Russian Federation CDS.

## Disclosure

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*
# A Network-Based Dynamic Analysis in an Equity Stock Market

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We study how changes in the structure of a brokers' transaction network affect the probability with which the returns and volume of the traded financial assets change significantly. We analyze how the dynamics of the brokers' transaction network are associated with the returns and volume observed in the Chilean stock market. To do this, we construct and validate an index that synthesizes the daily changes of the brokers' transaction network structure of equity market transactions. We find that the changes of this structure are significantly correlated with variables that describe the local and international economic-financial environments. In addition, changes in the brokers' transaction network structure are associated with a greater probability of positive shocks of more than two standard deviations in the stock exchange index return and total traded stock volume. These results suggest that the structure of the brokers' trading relations plays a role in determining the returns and volume of transactions in the Chilean stock market.

## 1. Introduction

Trading stocks, bonds, or currencies create a transaction network among brokers which stems from the architecture of the financial market, which in turn affects the formation of capital asset prices and returns. This network of brokers' transactions, in a broad sense, reflects both the investor-lender choices based on the information they both have and the brokers' information. The asset pricing literature usually assumes that the markets operate free of transaction costs and without any friction. The latter assumption is challenged both theoretically and empirically. First, from the market design perspective, a frictionless market implies ideal trading conditions [1]. Thus, changes in the network of transactions between brokers should not affect the returns and the volumes traded. However, from the financial market microstructure perspective, asset prices and return formation is a process governed by the rules that regulate and create the interactive mechanisms between investors (buyers), lender (sellers), and brokers in the market [2]. These rules also determine the existence and participation of brokers in the market. This occurs because these rules impose participation costs, funding constraints, and search costs that not only create imperfect competition but also may affect prices and returns through broker's trading behavior [3]. Second, the empirical

evidence shows that there exist frictions in financial markets. Thus, changes in the brokers' transaction network can reflect frictions stemming from qualitatively significant changes in the heterogeneous information available to market agents for a given set of market rules and therefore affect capital assets pricing and returns. For example, during the financial crisis of 2008, the valuation of financial assets was affected by two distinct behaviors: liquidity hoarding, and its implied reduction on transactions, and *flight to quality*, that is, the flight from toxic assets toward risk-free assets [4]. Both behaviors not only affected the observed prices and returns, but also added further tension to the financial system and highlighted the difficulty of modeling how the information flows can change the structure and implications on the returns of the investor-lender financial network and the brokers' transaction network, henceforth *the transaction network*. This difficulty centers mainly on the fact that networks are multidimensional mathematical objects, whose structure cannot be completely described by a single dimension. That is, changes in broker's information and/or her financial conditions may lead to changes in each broker's decisions on how much to trade and who to trade with, thereby varying the direct and indirect trading relations between brokers or structure of the transaction network. Moreover, the multiple dimensions that characterize a network's topology may be interrelated, which,

in the case of a financial network, can have nonlinear effects on the prices and returns of the traded assets as the literature has reported [5, 6].

The latter argument partially illustrates the complexity of the issue of modeling the effect of a transaction network structure on the risk and returns of capital assets. Additionally, when considering that the network structure varies over time, it is necessary to study how the changes of the network structure across time affect the equilibrium of financial markets. Thus, we study how changes in the brokers' transaction network structure affect the probability with which the return and volume of financial assets change significantly. Our approach is dynamic and emphasizes the analysis of the structural changes in the brokers' transaction network and their effects on the return and volume of a financial market. Given that variations in financial assets' returns affect the financial position of financial intermediaries, our research question is related to what the literature has called systemic risk. However, the study of systemic risk is not our focus; rather our aim is to shed some light on the effect of changes in the transaction network's structure on market dynamics and its implied returns and volume in the context of an equity market.

The literature that examines networks in financial markets has focused on the implications of three static properties of the networks and their relationship with the stability and fragility of financial systems. First, Kiyotaki and Moore [7], Allen and Gale [8], and Freixas and others [9] study the effect of financial system structure on systemic risk, developing the first formal contagion models in banking/financial networks. They find that, in the context of a banking system, more interconnected network architecture or topology increases the system's resilience to the insolvency of one particular bank, because the losses generated by an insolvent bank are absorbed by a larger number of actors in the network. Thus, a larger number of interconnections or links reduce the effect of negative shocks on the rest of the system.

However, and in contrast to the previous conclusion, Vivier-Lirimont [10] and Blume and others [11, 12] indicate that a network with a high interconnection density can act as a destabilizing force on the system, increasing the probability of systemic defaults while increasing the number of counterparts in a banking network. These apparently contradictory views illustrate the complex relation between the network structure and systemic risk. Acemoglu and others [6] and Haldane [4] show that the denser the interconnections are, the more stable the network will be in the event of small negative shocks. However, in the event of a larger shock on a *key bank in the network or hub*, the same interconnections propagate the shocks, increasing the fragility of the system due to the possibility of contagion. In other words, one characteristic of the network may generate more resilience under certain condition or *threshold* or may act as a source of systematic risk and instability[1]. A second aspect explored in the literature is how the distribution of interconnections affects the systemic reaction to shocks. When links are formed randomly, a Gaussian distribution of links with *"fat middle and thin tail"* is obtained [13, 14]. However, a financial network characterized by a distribution of links with *thin middle and long fat tail* is more robust to random shocks but is weaker against specific shocks in the *hub* nodes, which can collapse the functioning of the entire network [15]. Finally, the third characteristic of studied financial networks is related to the degree of node separation—the *"small world" property*. For networks where there are certain agents playing a key role in the connectivity of the network or *key nodes*, local disturbances are more likely to have global effects on the network. Thus, one particular property of the network structure implies that a local problem may lead to a global one, increasing the systemic risk of the financial system [16].

These three static characteristics mentioned are studied by Haldane [4] in a sample of 18 developed and emerging countries for the period of 1985–2005. Haldane finds high levels of interconnection, *long-tailed degree* distributions, and small degrees of separation in *the global financial network* of this set of nations. Then, from the point of view of financial system stability, the global financial network of this sample may be understood as a *robust-yet-fragile* system. In this, systems under certain conditions, such as loss of confidence in certain key nodes, would favor a rapid transmission of global financial shocks.

Another implication derived from this *robust-yet-fragile* characteristic of financial networks is related to how other network structure characteristics, such as their centrality measures, may affect the systemic risk. However, no consensus has been reached regarding the measurement and estimation of the systemic risk of a financial network, and the traditional robustness measurements do not capture its fragility 100% [4][2]. In an attempt to gain more accuracy in the estimation of systemic risk in banks, Guerra and others [17] propose novel systemic risk indicators that can measure the effect of one bankruptcy over the entire system.

An alternative approach is to study the parallelism between the analysis of the systemic risk in financial networks and electrical, ecological, or ecosystem networks. This makes it possible to apply analytical models and tools from these last fields to the analysis of financial markets. May and others [18] find that the impact of an external shock on the contagion rate in a network depends not only on the network topology but also on the *feedback* between the agents that comprise it. That is to say, a system's reaction to a shock depends not only on the current network structure but also on its evolution over time, and therefore it emphasizes the importance of studying the dynamic properties of the network topology. An advance in this direction is the work of Sensoy and Tabak [19] who apply a dynamic approach to analyze the time-varying dependence structure of a stock market.

It is worth noting that the financial agents' trading decisions are not solely determined by current information, but they follow a dynamic process of purchase and sale decisions that continuously give feedback on internal and external network information. For example, the investment and divestment decisions of money market intermediaries during the crisis of 2008 affected liquidity, interbank interest rates, and therefore fixed income, equity, and derivative instrument prices, among others. This behavior and its subsequent chain reaction are not captured by traditional risk analysis [20]. This occurs because static risk measures focus on estimating the

risk of specific nodes not giving an overall risk assessment of the network and its feedback processes.

In summary, the evidence indicates that various structural dimensions of a network affect the characteristics of the equilibrium in financial markets. Moreover, these relations tend to be nonlinear and involve the interaction among several network structure characteristics. Additionally, the static network analysis must be complemented with dynamic measures that capture the evolution and complexity of the relationships between financial system actors or a subgroup of these. In this line of research, Haldane [4] proposes a complementarity between the development of dynamic measures of systemic risk and measures of idiosyncratic risk of the nodes. Aligned with Haldane's proposal and considering the empirical evidence that indicates that transaction costs and frictions in financial markets affect asset's values, returns, and volatility [21–23], our analysis takes the dynamic perspective to study how changes in the structure of a brokers' transaction network in an equity market affect the probability of qualitatively significant changes in the stock returns and in their level of activity. In other words, we present a dynamic and global analysis of the brokers' (intermediaries') transaction network, which departs from the traditional static and idiosyncratic investor or lender-agent centered approach in the literature. Specifically, we study the following hypotheses:

(1) The transaction network's structure changes with variations in the financial information set available for the market agents.

(2) Changes in the transaction network structure affect the probability of qualitatively significant variations in the returns of the traded assets.

(3) Changes in the transaction network structure affect the probability of qualitatively significant variations in the volume of the traded assets.

In order to test these hypotheses, we construct and validate a measure of how much a network structure changes over time. Such measure must consider the change in several possible dimensions to capture the complexity of a network's change. Thus, we propose a measure, summarized in an index, which synthesizes the complexity of the daily changes in the transaction network structure, using data from the Santiago Stock Exchange (SSE)[3] for the period of 2006–2015. The results show that this index is sensitive to the variation of a series of local and international financial environment variables, which provides evidence to support the first hypothesis. In addition, we show that changes in the transaction network structure, captured through the index, are strongly and significantly associated with a greater probability of positive shocks on the stock return at the aggregate level. We also show that there is no evidence of a significant correlation between a change in the network structure and negative shocks on returns. These findings support the second hypothesis. Finally, we find that larger changes of the transaction network captured by the index increase the probability of a rise in traded volumes, which supports the third hypothesis.

Our results complement the financial market microstructure evidence that frictions and imperfections at the brokers'

level affect asset prices and returns. This literature finds that the valuation effect of frictions at brokerage level (such as trading fees, cost of processing orders, or simply the search for counterparties in a negotiation of purchase and sale of shares) is relevant. These brokers' level frictions reduce assets' values and lead to lower securities' liquidity levels and higher short term price volatility[4]. However, if transaction network's changes affect returns, then the transaction network's structural change may capture sources of brokers' level frictions previously unidentified by the financial market microstructure literature[5].

The rest of the paper is structured as follows: in Section 2, we describe our data[6] and explain our focus on the Chilean market. Then, we explain how we characterize the network, which allows us to study how the definition of when an interaction between agents is generated also plays a role in the analysis of the network structure. Finally, we present the index that we use to summarize the structural network change. In Section 3, we present the methodology and the results of our analysis. First, we specify the econometric models used to test our hypotheses. Then, we present the main results. In Section 4, we present an analysis of robustness, and finally in the fifth section we provide our conclusions.

## 2. The Data
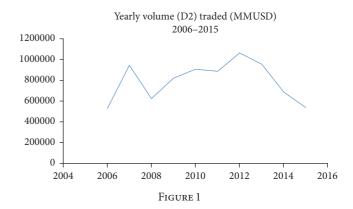
*2.1. Main Data Sources.* In order to empirically analyze the changes in the transaction network of local stockbrokers, we constructed a database with the universe of the daily stock transactions conducted by the stockbrokers at the SSE for the period of 2006–2015. This study relies on the level of detail and disaggregation of the data. The dataset includes precise information of day of transaction, instrument traded, type of operation, units exchanged, purchase price, and identification of the buying and selling brokers involved. Therefore, we are able to construct the daily transaction network for the Chilean equity market for the period of 2006–2015. We construct an index that synthesizes and measures the structural change of the transaction network by comparing the networks at dates $t$ and $t − 1$. The details of the index are presented in Section 2.4.

Then, we use this measure of the structural change of a network (index) to econometrically study whether the changes of the network's structure are correlated with changes (evolution) of local and international financial environment variables and indicators over time[7]. We consider variables that are standard in the literature, and our aim is to capture all possible factors that affect stock prices. Fratzscher and others [24], Estrada and others [25], Lavigne and others [26], and Chen and others [27] describe four channels by which *spillovers* are transmitted from developed to emerging markets: (a) portfolio balance: portfolio rebalancing that lowers risk premiums, boosts assets prices, and lowers yields; (b) signaling or confidence: carry trade and capital flows generated by larger differentials in interests rates; (c) exchange rate: flows that generate appreciation and depreciation of exchanges rates; and (d) trade flows: changes in the demands for emerging countries' exports. According to this and the fact that Chile is an emerging country, the main local

variables that should be affected by a shock are the IPSA blue-chip index (IPSA), which groups the 40 most liquid stocks with the greatest market capitalization on the Chilean market; the exchange rate between the Chilean peso and the dollar (CLP); the credit default swaps at 5 years of the Chilean sovereign debt (CDS5), and, as a measure of the aggregate activity of the local stockbroker network, the total traded stock volume (volume). Among the international variables, we considered the S&P 500 Index (S&P 500) and the Chicago Board Options Exchange Market Volatility Index (VIX) [28]. Finally, since Chile is an emerging market and is a country exposed to the evolution of commodities prices, we also include the MSCI emerging market index (MSCI) and the copper (CU) and oil (Pe) prices, as they are Chile's main export and import products, respectively [29].

*2.2. Chilean Stock Market.* Studying the transaction network in the SSE is interesting for several reasons. First, having exact data of the daily transactions made by brokers allows us to explore, in a dynamic setting, a part of the financial network mostly ignored by the literature[8]. Second, the Chilean equity market has less liquidity than the OECD average or other developing markets, but it is larger in terms of market capitalization to GDP. Third, global factors such as monetary policy in developed countries and the appetite for global risk are key cyclical factors in the liquidity of the equity market and its level of activity [30]. Fourth, the main stock market actors are pension funds managers, insurance companies, and mutual funds [31]. These entities channel their stock transactions via stockbrokers and not through the *over-the-counter* (OTC) market; consequently, the stock purchase and sale decisions by the main professional investors in the Chilean market are captured through the SSE broker network. Finally, Chile is a developing country with a medium level of financial development and occupies the 29th place out of a total of 62 countries[9]; therefore, the lessons learned from the study of the stock market transactions and the formation of the broker network are useful for other developing countries on the road to greater financial development.

*2.3. Characterization of the Brokers' Transaction Network.* In order to construct the daily equity brokers' transaction network for the period of 2006–2015[10], we calculate the daily total amount sold from one broker to another, excluding OTC operations. The latter allows us to distinguish between the selling behavior and the buying behavior of the SSE brokers. This approach implies that, for each pair of brokers A and B, there are two possible links: A sells to B and B sells to A. Given the wide spread of amounts traded daily within a calendar year, defining when two agents are interconnected is not an easy task. We decided to consider different alternatives for this definition. First, we define a traded amount threshold above which the interconnection between the agents is granted. We construct alternative networks varying the threshold of daily traded total amounts. In particular, we report results based on thresholds defined by the second, fourth, sixth, and eighth deciles and the average of the daily transactions made within a calendar year between any



Yearly volume (D2) traded (MMUSD)
2006–2015

FIGURE 1
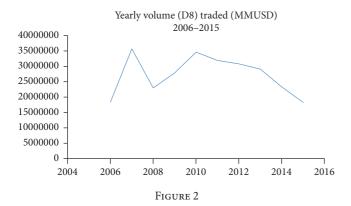
two SSE brokers. Additionally, we generated analogous results for every decile and the mode[11]. For each day, a link in the network indicates that the total (gross) amount of equity sold by a seller to a buyer in the day is above the threshold, as opposed to indicating that net amount sold from one broker to another exceeds the threshold. We decided to use gross sales to build a directed brokers' transaction network to minimize the information loss regarding the brokers' behavior (transaction decisions) due to two motives. First, netting transactions (within a day) generates a loss of information due to the decrease in the number of links. To illustrate the latter argument, consider the following example. Assume that the threshold considered to acknowledge a relation between two brokers is US$40. Now, suppose that, on Monday, broker A sells US$100 worth equity in the morning to broker B and then broker B sells US$100 worth equity to broker A in the afternoon. If transactions were netted, the link between A and B would not be formed, and then the transaction network would not capture the actual behavior that brokers A and B exhibit by trading in the morning and in the afternoon. Second, we use a directed network to represent the transaction network to avoid information loss due to confounding the selling and buying behavior of the brokers. Considering "A selling to B" identical to "B selling to A" implicitly decreases the possible informational frictions in financial markets, which may affect returns[12].

Formally, $N$ is the set of SSE brokers in the period of 2006–2015 and $G_t$ is the set of total transactions between brokers in $N$ in day $t$. Then, for a given $t$, the link $ij \in G_t$ with $i, j \in N$ indicates that the total amount sold by broker $i$ to broker $j$ in stocks, denoted by $\tau_{ij}^t$, is such that $\tau_{ij}^t \geq c$, where $c$ is the threshold for the amount traded[13]. Thus, $g^t = (N, G_t)$ is the directed transaction network between the brokers from $N$ in day $t$[14].

Figures 1, 2, and 3 show the evolution of three threshold criteria, second decile, eighth decile, and average, in current MMUSD. As can be seen, in the three figures, there is a considerable variability in the amounts traded.

Figure 4 shows how the three threshold criteria (second decile, eighth decile, and average on 3 January 2012) determine the structure of the transaction network on a given day in the sample. The structure of the transaction network defined by the "second decile traded amount" is

## Yearly volume (D8) traded (MMUSD) 2006–2015



FIGURE 2

## Yearly volume (D8) traded (MMUSD) 2005–2016



FIGURE 3

qualitatively different compared to the network defined by the "eighth decile traded amount" or "average traded amount." This qualitative difference suggests that the threshold used to define the transaction network affects the measurement of the transaction network's structural change.

In order to describe the change in $g_t$ over time, first we characterize the network structure using a wide *set of structural* characteristics. In Section 2.4, we delve deeper into why we consider several characteristics of the network structure instead of a single measure. Specifically, we calculate the ratio of brokers that trade to the total number of registered brokers in a calendar year, the ratio of sale-purchase relations to the total number of possible directed links between registered brokers, the density of $g_t$ (ratio of brokers to purchase-sale relations), the percentage of nodes with the same number of in-and-out degrees, the percentage of nodes with an in-degree greater than 0 (percentage of brokers that sell), the percentage of nodes with an out-degree greater than 0 (percentage of brokers that buy), the assortativity coefficient based on the number of links (indicating similarity between connected brokers based on number of links), the reciprocity coefficient, and the clustering coefficient for the network (network cohesion indicator). In addition, we determine whether the network was connected, if it had Hamiltonian cycles, if it was bipartite, if it was acyclic, if it was planar, and if it had loops. We normalized the number of nodes and directed links as explained above in order to give an equal weight to the change of any dimension used to characterize the network at $t$ and to avoid the situation where any specific characteristic of the network could skew the change in $g_t$ over time[15]. Each of these dimensions is a component of a vector of
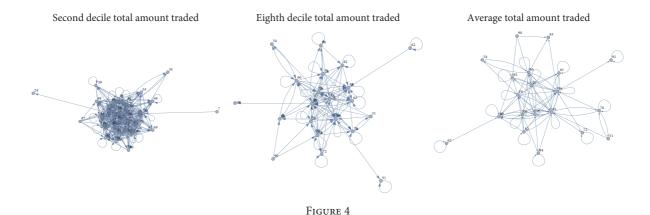
network characteristics at $t$ called $v_t = (v_{t,1}, \ldots, v_{t,D})$, where $D$ is the number of network characteristics that we include in our analysis.

*2.4. Proposal of an Index of Network Change.* Networks are objects that have a multiplicity of dimensions that characterize them, each one changing over time. For this reason, it is not surprising that different studies, focused on the static relation of a single characteristic of a financial network (or a part of it) with some measure of financial performance, have obtained apparently contradictory results, as previously mentioned. These apparently contradictory conclusions can be explained in at least two ways. First, there may be conditions external to the network which vary over time and explain a financial resilience that changes over time for the same network topology. This is the approach of Acemoglu and others [6] who show that the magnitude and number of negative shocks on a financial system determine if greater interconnectivity within the network increases or reduces the robustness of a banking system. Second, a network is a complex object for which each of its diverse characteristics can change over time [17]. Therefore, to assume that financial performance depends exclusively on only one specific characteristic of the network over a measurement of financial performance omits the role that other characteristics of the network, their interrelations, and the dynamics of the network structure can have on the mechanisms which determine both the functioning of the financial system and its returns and volumes traded. Then discerning between the potential causes that affect the resilience of financial systems is in essence an empirical problem. We circumvent this problem assuming that the networks are complex objects evolving over time. Moreover, the investor's broker choice and the broker's decisions on who to trade with reveal an additional aspect of the financial network, which may affect its resilience. If there are no frictions in the stock market, then the investor's broker choice should be random, and returns and volume traded should not be affected by the selection of a broker. This occurs because there is no gain from choosing one broker over another. However, if there are frictions in the stock market, then investors may decide to trade through a specific broker purposely to increase (decrease) his benefits (costs) from trade, which should affect the structure of the transaction network and its change over time. Therefore, the change of the transaction network's structure may provide information about how some financial frictions affect the stocks' returns and volumes traded. In the latter case, a structural change of the transaction network should affect the IPSA return (hypothesis 2) and volume traded (hypothesis 3).

Our goal is to produce a measure of the daily *structural change* on the transaction network. Considering the multidimensionality of the object to study, we proposed two alternative indices that capture this change:

$$\kappa_t^1 = \left| \left[ \arg\min_{\lambda \in \mathbb{R}} \sum_{d=1}^{D} \left( \lambda \cdot v_{t,d}\left(g_t\right) - v_{t-1,d}\left(g_{t-1}\right)\right)^2 \right] - 1 \right|,$$

$$\kappa_t^2 = \left[ \arg\min_{\lambda \in \mathbb{R}} \sum_{d=1}^{D} \left( \lambda \cdot v_{t,d}\left(g_t\right) - v_{t-1,d}\left(g_{t-1}\right)\right)^2 \right],$$

(1)

Second decile total amount traded        Eighth decile total amount traded        Average total amount traded



FIGURE 4

where $g_t$ is the transaction network at date $t$ in the SSE and $v_{t,d}$ corresponds to the $d$th characteristic of the network $g_t$. That is, in both $\kappa_t^1$ and $\kappa_t^2$, $\lambda \in \mathbb{R}$ scales up or down the vector of network characteristics at $t$ (i.e., $v_t$) in order to make it as similar as possible to $v_{t-1}$. Thus, a smaller $\lambda$ indicates that the *structural change* in the transaction network in $t - 1$ and $t$ is smaller. Therefore, $g^t$ and $g^{t-1}$ are more alike. Given these definitions, $\kappa_t^1, \kappa_t^2 \in \mathbb{R}_+$. Both indices, $\kappa_t^1$ and $\kappa_t^2$, measure the change in the transaction network. However, $\kappa_t^1$ and $\kappa_t^2$ differ in that $\kappa_t^1$ solely indicates the magnitude of a change from $g_t$ to $g_{t-1}$, whereas $\kappa_t^2$ also indicates whether $g_t$ looks more or less alike a complete network than $g_{t-1}$ [16]. That is, the difference between $\kappa_t^1$ and $\kappa_t^2$ is that $\kappa_t^1$ measures the change in the transaction network at two points in time without assigning any interpretation regarding the type of structure that the network is changing to. On one hand, $\kappa_t^1$ is best suited to study, for instance, if greater structural changes of the transaction network are positively or negatively associated with a better (or worse) performance of the financial system measured by positive (negative) jumps of the return of the market's price index [17]. On the other hand, $\kappa_t^2$ is best suited to study if structural changes toward a complete network have different effect than structural changes away from a complete network. The absence of change in the network at $t$ and at $t - 1$ is reflected in $\kappa_t^1 = 0$ (and in $\kappa_t^2 = 1$). A greater daily transaction network's structural change between $t$ and $t - 1$ implies a larger $\kappa_t^1$ but not necessarily a larger $\kappa_t^2$, because the latter index increases as the network changes toward the complete network. Because we are giving equal weights to every component in $v_t$, $\kappa_t^1$ can be interpreted as the magnitude (absolute value) of first difference of a summary statistic of the change between $v_t$ and $v_{t-1}$. Analogously, $\kappa_t^2$ can be interpreted as the first difference of a summary statistic of the change between $v_t$ and $v_{t-1}$.

Considering that the literature is still debating if a complete network helps or is detrimental for the financial system's resilience, we decided to focus on the role of the structural change of the network independently of the topology that the network moves to. Therefore, our preferred measure of structural change is the index $\kappa_t^1$. We present only the basic results for $\kappa_t^2$.

## 3. Econometric Analysis

*3.1. Specification of the Model.* As was discussed before, the first step consists in studying whether changes in the transaction network measured by the indices are correlated with the local and international financial environment variables that we described in Section 2.1. First, we empirically assess which of the two possible measures of structural change ($\kappa_t^1$ or $\kappa_t^2$) better captures changes in the financial system (CLP, CDS5, VIX, MSCI, CU, and Pe). Table 1 shows the correlation between the returns or growth rate of the financial environment variables described in Section 2.1 and the two possible definitions of the index. We estimate an ordinary least squares regression where the dependent variable is the index to obtain the correlations. Adding controls for month and year allows us to isolate the correlations of seasonal variations or a specific year relationship. Table 1 shows regressions for all finance variables described above. Going forward, we focus on the relationship between the index and the financial variables that present a statistically significant correlation with the network structure.

It should be noted that, given the construction of the measures of network change $\kappa_t^1$ and $\kappa_t^2$, it is not possible to determine a priori how the financial variables will be related to the transaction network's structural change. Furthermore, the definition of when agents are linked (thresholds) and, therefore, how the transaction network is structured provides an additional degree of freedom to consider when assessing the correlation between the financial environment variables and the change in the transaction network structure.

One difference established in Table 1 between $\kappa_t^1$ and $\kappa_t^2$, which is in line with the contradictory results described in the aforementioned works, is that the measure that is neutral with respect to the interpretation of movements in the network ($\kappa_t^1$) correlates with more financial variables than $\kappa_t^2$ and confirms our theoretical choice that $\kappa_t^1$ is the best suited index for our study. Table 1 also provides support for our first hypothesis. Specifically, the network structure moves together with the financial environment variables. Moreover, Table 1 shows that this relationship does not follow an easily discernible simple pattern. Going forward, we focus on $\kappa_t^1$, because the $\kappa_t^1$ index correlates significantly with more of the

TABLE 1: Basic model. This table summarizes the results of the following basic model: $y_t = \alpha + \beta * X_t + m_t + N_t + \varepsilon_t$. The dependent variable is the proposed index of network change, calculated using the average transaction as a threshold for the existence of a link. $X_t$ corresponds to the returns or growth rate of variables such as IPSA, VIX, Pe, CU, CLP, S&P 500, volume, CDS5, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Columns (1) and (2) show the results for the indexes $\kappa_t^1$ and $\kappa_t^2$, and column (3) displays our preferred specification.

| Variables | (1) Average ($\kappa^1$) | (2) Average ($\kappa^2$) | (3) Average ($\kappa^1$) |
|---|---|---|---|
| VIX | 0.112*** | −0.187*** | 0.109*** |
| | [0.0412] | [0.0629] | [0.0410] |
| IPSA | −0.605** | 0.327 | −0.554** |
| | [0.280] | [0.377] | [0.277] |
| MSCI | 0.481** | −0.487 | 0.556** |
| | [0.243] | [0.305] | [0.242] |
| Pe | −0.372*** | 0.193 | −0.316*** |
| | [0.126] | [0.145] | [0.115] |
| CU | 0.218 | −0.235 | |
| | [0.153] | [0.187] | |
| S&P 500 | 0.698*** | −0.948** | 0.679** |
| | [0.270] | [0.386] | [0.269] |
| CLP | 0.876** | −0.451 | 0.719* |
| | [0.394] | [0.541] | [0.376] |
| CDS5 | −0.0230 | 0.0690 | |
| | [0.0651] | [0.0832] | |
| Volume | −0.00145 | −0.0158*** | |
| | [0.00375] | [0.00448] | |
| Constant | 0.133*** | 0.930*** | 0.135*** |
| | [0.0122] | [0.0143] | [0.0118] |
| Observations | 2,340 | 2,340 | 2,352 |
| R-squared | 0.036 | 0.028 | 0.035 |

Robust standard errors are given in brackets. ***$p < 0.01$, **$p < 0.05$, and *$p < 0.1$.

financial system variables, and it is our theoretical index of choice[18].

To deepen our understanding of how the financial system is related to the transaction network's structure, we analyze the correlations presented in Table 1 for different thresholds that define the links that form the network. For instance, using the second decile as a threshold implies that most of the transactions are able to establish links among the agents, and therefore the transaction network's structure changes "little" and constantly due to the daily financial fluctuations. On the other hand, if we use the eighth decile, the only links generated are associated with transactions that are relatively extraordinary. Therefore, the transaction network that is formed in this case is associated with high volume transactions that occur only seldom within a year. Thus, any such network can be expected to change "more dramatically" than networks defined by smaller thresholds.

Another interesting result stemming from Table 1 is the lack of correlation between traded volume and the index. One possible concern regarding the informational content of the index is that changes in financial activity may endogenously affect the transaction network's structure through the creation and destruction of links and nodes.

However, by construction, $\kappa_t^1$ and $\kappa_t^2$ capture many topological characteristics of $g_t$ such as cyclicality and planarity and thus not only reflect changes in the number of links and nodes. This defining characteristic of the indices alleviates the endogeneity concerns. Moreover, the results in Table 1 indicate that these latter network topological characteristics are playing an important role in the index, and consequently traded volume is, indeed, not correlated with the index.

In Table 2, the results of the estimation presented in Table 1 are compared, but using deciles 2, 4, 6, and 8 as well as the average as the threshold definition, we can see that, for the transaction network defined with the average, changes in the VIX, IPSA, MSCI, Pe, S&P 500, and CLP correlate with the network showing more structural changes[19]. It is worth noting that as we move up on the deciles, only the VIX and the S&P 500 show correlation with the index when the threshold is set to the eighth decile. This result suggests that changes of external factors correlate with changes of the transaction network defined by low volume transactions threshold (lower deciles) or by less common high volume transactions threshold (higher deciles).

Now, to better understand how changes in the transaction threshold define how the network is constructed,

TABLE 2: Basic model using different thresholds. This table summarizes the results of the following basic model: $y_t = \alpha + \beta * X_t + m_t + N_t + \varepsilon_t$. The dependent variable is the proposed index of network change for different thresholds that define a link. $X_t$ corresponds to the returns or growth rate of variables such as IPSA, VIX, Pe, S&P 500, CLP, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Column (1) uses the average transaction as a threshold. Columns (2) to (5) use the second, fourth, sixth, and eighth deciles as thresholds.

| Variables | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile |
|---|---|---|---|---|---|
| VIX | **0.109**\*\*\* | **0.0735**\*\* | −0.00858 | 0.0443 | **0.119**\*\*\* |
|  | [0.0410] | [0.0372] | [0.0381] | [0.0334] | [0.0372] |
| IPSA | **−0.554**\*\* | 0.0845 | −0.248 | −0.237 | −0.0450 |
|  | [0.277] | [0.183] | [0.248] | [0.194] | [0.233] |
| MSCI | **0.556**\*\* | 0.202 | 0.129 | 0.122 | 0.113 |
|  | [0.242] | [0.135] | [0.161] | [0.171] | [0.165] |
| Pe | **−0.316**\*\*\* | −0.0818 | −0.0138 | −0.0175 | −0.0519 |
|  | [0.115] | [0.0795] | [0.0711] | [0.0712] | [0.0796] |
| S&P 500 | **0.679**\*\* | **0.381**\* | 0.0800 | **0.374**\*\* | **0.821**\*\*\* |
|  | [0.269] | [0.218] | [0.195] | [0.173] | [0.220] |
| CLP | **0.719**\* | **0.606**\*\* | 0.184 | 0.478 | 0.219 |
|  | [0.376] | [0.276] | [0.331] | [0.323] | [0.269] |
| Constant | **0.135**\*\*\* | **0.0699**\*\*\* | **0.0636**\*\*\* | **0.0623**\*\*\* | **0.102**\*\*\* |
|  | [0.0118] | [0.0104] | [0.00879] | [0.00638] | [0.00933] |
| Observations | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 |
| R-squared | 0.035 | 0.032 | 0.028 | 0.016 | 0.025 |

Robust standard errors are given in brackets. \*\*\*$p < 0.01$, \*\*$p < 0.05$, and \*$p < 0.1$.

TABLE 3: Descriptive statistics of the network structure. This table summarizes the descriptive statistics of the network structure in terms of the number of links and nodes for the second, fourth, sixth, and eighth deciles.

|  | Number of links | | | | Number of nodes | | | |
|---|---|---|---|---|---|---|---|---|
|  | Second decile | Fourth decile | Sixth decile | Eighth decile | Second decile | Fourth decile | Sixth decile | Eighth decile |
| Average | 412.66 | 309.79 | 206.72 | 103.46 | 30.93 | 28.49 | 25.48 | 20.85 |
| Median | 413.00 | 310.00 | 204.00 | 101.00 | 31.00 | 28.00 | 26.00 | 21.00 |
| Mode | 430.00 | 275.00 | 206.00 | 101.00 | 31.00 | 28.00 | 26.00 | 20.00 |
| Std. dev. | 56.15 | 49.99 | 42.00 | 29.31 | 2.29 | 2.11 | 1.98 | 2.24 |

Table 3 shows descriptive statistics about the structure of the networks formed with different thresholds (second, fourth, sixth, and eighth decile and the average for the daily amount traded). The table shows that when we increase the decile that defines the threshold, the average number of links in the network is reduced substantially. In particular, it drops from 412 links on average when using the second decile to only 103 links on average for the eighth decile. This decrease is also noted in the number of brokers involved, which lowers from 30 to 20. Nevertheless, the fall in the number of links is more dramatic, since it is a drop close to 75% versus a decrease of 33% in the number of active brokers when rising from the second to the eighth decile.

Another interesting aspect is that in relative terms the network formed with thresholds associated with higher deciles is more volatile in the number of links than the networks made up of less restrictive thresholds.
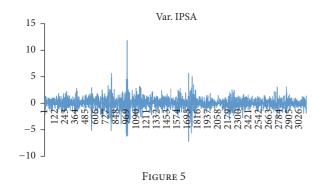
One important concern when studying the correlation between the index and the returns of the financial variables is the possibility of autocorrelation in the index. That is, we want

to determine if the index is correlated with contemporaneous returns of financial variables or if it is the lagged index that is correlated with contemporaneous returns of financial variables. Table 4 shows estimations of the model presented in Table 1, adding lags of 1 and 2 days of the index. Other autocorrelation structures were estimated[20], but the time lags beyond two days are not statistically significant. Table 4 shows that the index constructed using the average amount daily traded shows no autocorrelation. For the second and fourth deciles, the index tends to reduce its change after one day[21]. For the eighth decile, if there were structural changes in the network yesterday, it is very possible that changes are occurring today. These autocorrelations show that the changes in the network structure are relatively localized over time and are not long-term trends.
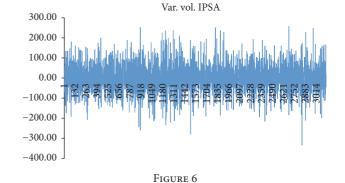
In order to analyze econometrically if the index $\kappa_t^1$ is able to capture a relationship between changes of the transaction network and changes in the local and international financial environment variables, we define a shock to variable $x$ as the change of two standard deviations on the value of $x$. We

TABLE 4: Basic model controlling for autocorrelation. This table summarizes the results of the following model: $y_t = \alpha + \beta * X_t + \gamma_1 * y_{t-1} + \gamma_2 * y_{t-2} + m_t + N_t + \varepsilon_t$. The dependent variable is the proposed index of network change for different thresholds that define a link. We add to the basic model the index lagged in one and two days to study possible autocorrelation. $X_t$ corresponds to the returns or growth rate of variables such as IPSA, VIX, Pe, CLP, S&P 500, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Column (1) uses the average transaction as a threshold. Columns (2) to (5) use the second, fourth, sixth, and eighth deciles as thresholds.

| Variables | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile |
|---|---|---|---|---|---|
| VIX | 0.107*** | 0.0758** | −0.00762 | 0.0424 | 0.110*** |
| | [0.0410] | [0.0373] | [0.0380] | [0.0334] | [0.0374] |
| IPSA | −0.562** | 0.0818 | −0.241 | −0.238 | −0.0739 |
| | [0.277] | [0.184] | [0.248] | [0.195] | [0.230] |
| MSCI | 0.550** | 0.221 | 0.124 | 0.120 | 0.0482 |
| | [0.242] | [0.135] | [0.162] | [0.171] | [0.163] |
| Pe | −0.319*** | −0.0814 | −0.0104 | −0.0198 | −0.0352 |
| | [0.115] | [0.0795] | [0.0715] | [0.0714] | [0.0794] |
| S&P 500 | 0.679** | 0.387* | 0.0829 | 0.369** | 0.827*** |
| | [0.268] | [0.218] | [0.195] | [0.173] | [0.223] |
| CLP | 0.716* | 0.633** | 0.182 | 0.477 | 0.225 |
| | [0.377] | [0.276] | [0.332] | [0.323] | [0.267] |
| Index lagged 1 day | 0.0182 | −0.0553*** | −0.0314** | 0.00869 | 0.0921*** |
| | [0.0193] | [0.0126] | [0.0150] | [0.0172] | [0.0236] |
| Index lagged 2 days | −0.00677 | −0.00574 | 0.0149 | −0.0189 | −0.0286* |
| | [0.0199] | [0.0225] | [0.0244] | [0.0192] | [0.0173] |
| Constant | 0.132*** | 0.0744*** | 0.0647*** | 0.0628*** | 0.0914*** |
| | [0.0128] | [0.0108] | [0.00889] | [0.00654] | [0.00860] |
| Observations | 2,351 | 2,351 | 2,351 | 2,351 | 2,351 |
| R-squared | 0.036 | 0.035 | 0.029 | 0.016 | 0.034 |

Robust standard errors are given in brackets. *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.



FIGURE 5



FIGURE 6
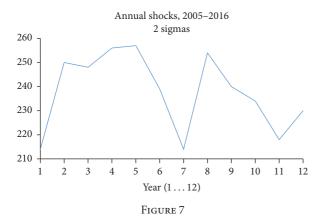
differentiate positive and negative changes, and we focus on shocks to stock returns (IPSA returns) and volume traded. We construct a daily dummy variable *upward (downward) shock_{ity}* that takes the value of 1 if the daily variation of variable $i$ on day $t$ of year $y$ is greater than two standard deviations above the average daily variation of $i$ in year $y$ (upward). Thus, taking the IPSA equity market return as an example, out of a total of 3,130 daily observations (see Figure 5), we obtained 311 upward and 85 downward 2-sigma shocks. In the case of the volume traded on the market (see Figure 6), we obtained 347 upward and 80 downward 2-sigma shocks.

Figure 7 illustrates all the 2-sigma shocks generated in the period of 2006–2015 for all the variables described in Section 2.1.

As Figure 7 shows, there are a significant number of financial shocks in the market. In terms of the variability of these events, it is observed that over time there is no uniformity in these shocks. This could suggest that if these shocks are related to changes in the financial network, the network possibly could affect the behavior of the market as a whole. In the next section, we study if changes in the

Figure 7

transaction network are correlated with these episodes of positive and negative shocks to returns and volume.

### 3.2. Main Results

*3.2.1. Types of Transaction and Movements in the Equity Market.* The previous results indicate a correlation between changes in the transaction network and movements in the financial variables. However, we are interested in learning if the change in the transaction network may have significant effects on both returns and volume.

In order to avoid the situation in which the threshold defining the network is relevant for our main results, the estimations are shown for different thresholds. These different networks allow us to observe econometrically both the performance of the index and what type of transaction network performs statistically better.

First, we estimate a probit model, where the dependent variable $I_t$ is 0 when there is no shock and takes the value of 1 when a positive shock of two standard deviations affects the IPSA. In other words, we estimate the following equation:

$$\Pr\left(I_t = 1\right) = \Phi\left(\alpha + \lambda * y_t + \beta * X_t + m_t + N_t\right), \quad (2)$$

where $y_t$ is the index at time $t$ to measure how much the transaction network structure has changed from $t-1$ to $t$. $X_t$ is a set of finance variables, expressed in returns or growth rates, such as IPSA, VIX, Pe, CLP, S&P 500, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year.

Panel A in Table 5 shows the results of this model for networks constructed with the aforementioned thresholds. Surprisingly, mainly no contemporaneous change in financial variables statistically increases or decreases the probability of a positive shock. Note, however, that a contemporaneous change in the network structure, captured through variations of the index, increases the probability that there will be a positive shock. This implication holds true for all possible thresholds. In other words, structural changes in the transaction network defined by low volume transaction threshold or high volume transaction threshold increase the probability of a substantial positive shock on stock returns.

Yet, it is possible that the index is related to the probability of a positive shock with a certain time lag. Panel B in Table 5

shows estimations for the probit, when a time lag is included for the index ($\kappa_t^1$), for the same networks constructed with different thresholds. In light of the results presented in Table 4, we only use one lag for the autocorrelation structure. It is clear from panel B that only the contemporaneous structural change of the network is correlated with the probability of a positive shock.

Table 5 shows that changes in the network structure, captured through the index, are not only correlated with the financial variables but are also capturing relevant information at brokers level or at market level, which are not seized by other aggregate financial variables about positive shocks in the financial system, such as good corporate results, better growth prospects, and political changes. Additionally, this result is robust even when we change the threshold that defines the network. Then, the index makes it possible to systematically analyze how the structural changes in the highly complex and volatile networks associated with transactions in the financial market are related to the performance of the financial returns. These results indicate that the brokers' network matters for the market behavior and performance, in line with Easley and O'Hara [2] and Vayanos and Wang [3].

However, it is possible that positive shocks and negative shocks are different in nature. One possible difference may be related to how information is transmitted through the transaction network. It is plausible that negative shocks are mostly unexpected, but positive shocks go through an information-diffusion process. In the latter process, more informed agents or brokers could act first, thereby transmitting private information to other less informed agents. One possible interpretation of Table 5 is that the change in the network structure is correlated to positive shocks on returns due to the information-diffusion process through transaction among informed brokers. Therefore, one interesting test is to study how the changes in the network correlate with unexpected shocks.

Table 6 shows the estimation of a probit model to study the correlation between the structural change of the network and large negative shocks to the IPSA return (greater than two standard deviations). We present the results controlling for possible autocorrelation, but they are similar for the specification without including the lagged index. Table 6 shows that the contemporaneous change of the transaction network is not correlated with the probability of a negative shock. This result is consistent with the notion of an unexpected shock that does not affect the network structure. Moreover, other contemporaneous financial variables display correlation with the negative shock. For instance, an increase in the percentage change of oil price (Pe) augments the probability of a negative shock in the stocks return. Similarly, a drop in the S&P 500 (return) leads to an increase in the probability of a negative shock in the stocks return. These results indicate that our estimations are able to capture contemporaneous changes that affect the probability of substantial changes in the IPSA returns.

## 4. Additional Analysis

We present additional econometric analyses to provide further insights and to test the robustness of the main result.

TABLE 5: Probit model, IPSA positive shocks. This table summarizes the results of the following probit model: $\Pr(I_t = 1) = \Phi(\alpha + \lambda_1 * y_t + \lambda_2 * y_{t-1} + \beta * X_t + m_t + N_t)$. The probit is defined over $I_t$, which is 1 when the daily return on the IPSA surpasses two standard deviations, and it is 0 otherwise. The independent variables are the index $y_t$ and a set of variables $X_t$, that include the returns or growth rate of variables such as IPSA, VIX, Pe, CLP, S&P 500, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Column (1) uses the average transaction as a threshold. Columns (2) to (5) use the second, fourth, sixth, and eighth deciles as thresholds. Panel A is the basic probit estimation, and panel B includes control for autocorrelation for the respective index.

| Variables | Panel A | | | | | Panel B | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile |
| Index | 0.989** | 0.719* | 1.378*** | 0.855*** | 1.185** | 0.979** | 0.746** | 1.448*** | 0.844* | 1.039** |
| | [0.391] | [0.374] | [0.371] | [0.435] | [0.487] | [0.393] | [0.377] | [0.378] | [0.437] | [0.487] |
| VIX | 0.477 | 0.517 | 0.550 | 0.550 | 0.440 | 0.454 | 0.466 | 0.525 | 0.487 | 0.394 |
| | [1.121] | [1.125] | [1.103] | [1.118] | [1.118] | [1.118] | [1.132] | [1.104] | [1.111] | [1.118] |
| IPSA | 44.74*** | 43.72*** | 44.01*** | 43.84*** | 44.29*** | 44.88*** | 43.77*** | 44.01*** | 44.05*** | 44.48*** |
| | [8.606] | [8.542] | [8.463] | [8.501] | [8.587] | [8.640] | [8.560] | [8.456] | [8.485] | [8.636] |
| MSCI | -1.978 | -1.639 | -1.692 | -1.683 | -1.890 | -1.985 | -1.842 | -1.578 | -1.607 | -2.383 |
| | [5.826] | [5.848] | [5.742] | [5.803] | [5.866] | [5.842] | [5.850] | [5.780] | [5.805] | [5.917] |
| Pe | -0.0191 | -0.331 | -0.434 | -0.364 | -0.245 | -0.129 | -0.334 | -0.492 | -0.510 | 0.0493 |
| | [2.259] | [2.254] | [2.270] | [2.255] | [2.273] | [2.261] | [2.255] | [2.261] | [2.240] | [2.298] |
| S&P 500 | 6.183 | 6.499 | 7.018 | 6.706 | 5.974 | 6.005 | 6.365 | 6.903 | 6.254 | 6.127 |
| | [7.084] | [7.057] | [7.036] | [7.046] | [7.071] | [7.085] | [7.082] | [7.073] | [7.000] | [7.155] |
| CLP | 5.404 | 5.505 | 5.644 | 5.702 | 6.122 | 5.206 | 4.993 | 5.419 | 5.757 | 6.262 |
| | [8.426] | [8.368] | [8.340] | [8.296] | [8.383] | [8.398] | [8.387] | [8.346] | [8.271] | [8.351] |
| Index lag. 1 day | | | | | | 0.310 | 0.490 | 1.218*** | 0.603 | 1.143** |
| | | | | | | [0.417] | [0.382] | [0.344] | [0.392] | [0.500] |
| Constant | -1.831*** | -1.745*** | -1.779*** | -1.746*** | -1.830*** | -1.872*** | -1.791*** | -1.888*** | -1.790*** | -1.936*** |
| | [0.219] | [0.210] | [0.208] | [0.207] | [0.215] | [0.225] | [0.212] | [0.209] | [0.206] | [0.225] |
| Observations | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 |
| Pseudo R-squared | 0.148 | 0.145 | 0.152 | 0.145 | 0.148 | 0.149 | 0.146 | 0.159 | 0.147 | 0.153 |

Robust standard errors are given in brackets. *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

TABLE 6: Probit model, IPSA negative shocks. This table summarizes the results of the following probit model: $\Pr(I_t = 1) = \Phi(\alpha + \lambda_1 * y_t + \lambda_2 * y_{t-1} + \beta * X_t + m_t + N_t)$. The probit is defined over $I_t$ which is 1 when the daily losses on the IPSA surpass two standard deviations, and it is 0 otherwise. The independent variables are the index $y_t$ and a set of variables $X_t$ that include the returns or growth rate of variables such as IPSA, VIX, Pe, CLP, S&P 500, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Column (1) uses the average transaction as a threshold. Columns (2) to (5) use the second, fourth, sixth, and eighth deciles as thresholds.

| Variables | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile |
|---|---|---|---|---|---|
| Index | −0.0989 | −0.319 | −0.448 | 0.956 | −1.210 |
| | [0.736] | [0.674] | [0.708] | [0.670] | [1.056] |
| VIX | −2.001 | −1.799 | −1.972 | −2.130 | −2.099 |
| | [1.555] | [1.547] | [1.560] | [1.537] | [1.541] |
| IPSA | **−144.2***** | **−146.1***** | **−144.9***** | **−145.8***** | **−148.1***** |
| | [23.26] | [23.13] | [23.05] | [23.09] | [23.39] |
| MSCI | 2.009 | 3.782 | 2.454 | 2.940 | 2.748 |
| | [10.25] | [10.49] | [10.50] | [10.60] | [10.81] |
| Pe | **11.33***** | **11.13***** | **11.02***** | **11.01***** | **11.43***** |
| | [3.384] | [3.327] | [3.345] | [3.297] | [3.410] |
| S&P 500 | **−25.06**** | **−24.41**** | **−24.60**** | **−26.45**** | **−26.58**** |
| | [10.32] | [10.13] | [10.23] | [10.53] | [10.53] |
| CLP | −7.024 | −7.076 | −6.786 | −8.895 | −7.219 |
| | [13.54] | [13.47] | [13.43] | [13.45] | [13.19] |
| Index lag. 1 day | −0.259 | **−1.962**** | −0.506 | −0.652 | **−2.891**** |
| | [0.828] | [0.992] | [0.748] | [0.978] | [1.451] |
| Constant | **−3.104***** | **−3.064***** | **−3.100***** | **−3.168***** | **−2.804***** |
| | [0.395] | [0.415] | [0.412] | [0.412] | [0.422] |
| Observations | 2,352 | 2,352 | 2,352 | 2,352 | 2,352 |
| Pseudo $R$-squared | 0.593 | 0.597 | 0.594 | 0.595 | 0.603 |

Robust standard errors are given in brackets. *** $p < 0.01$ and ** $p < 0.05$.

The financial literature indicates that price changes tend to be accompanied by changes in the transaction volume. For example, Huddart and others [32] show that the transaction volume significantly increases in the case of stock price changes higher or lower than their past trading range. For this reason, we study whether structural changes of the transaction network are also correlated with an increase in the probability that substantial changes in the traded volumes are observed. Table 7 shows estimations for positive and negative shocks to the traded volume[22]. Not surprisingly, our results suggest that the structural change of the network has a positive and significant effect on the probability of a positive shock on the traded volume. However, when studying the probability of a negative shock, the index (constructed with networks defined by different daily amount traded thresholds) shows no correlation with the negative shock. However, both the index and the lagged index show a negative correlation with a negative shock in the volume.

This result reinforces the idea that contemporaneous structural changes of the transaction network have an effect on the financial market behavior. As with the case of returns, this result is consistent across thresholds, and, therefore, we conclude that structural change of a transaction network is pertinent (significantly correlated) for the positive shocks on returns and traded volume regardless of the threshold considered to construct the network.

Finally, we calculate all the marginal effects for the average amount daily traded for each variable for the probit regressions displayed above[23]. The latter calculations show, consistently, that a change of the transaction network's structure has a positive and statistically significant effect on the probability of a positive shock in returns or volume.

## 5. Conclusions

The financial transactions may depend on the information withheld by the investors. However, the brokers' transaction network reflects the decisions that brokers make based on their own available information (public or private). We show that these latter decisions affect the returns and volume of the assets traded in the Santiago Stock Exchange (SSE) equity market. In this study, we generate a measure of the change of the brokers' transaction network based on several characteristics of the transaction network's structure in the SSE. This measure, which we call the $\kappa_t^1$ index, is able to capture the complexity of the dynamic evolution of this structure over time. Using an econometric analysis, we show that this index is sensitive to the variation of a series of local and international financial variables, which suggests that the transaction network structure is sensitive to the change in the economic/financial information available to the

TABLE 7: Probit model, IPSA volume shocks. This table summarizes the results of the following probit model: $\Pr(I_t = 1) = \Phi(\alpha + \lambda * y_t + \beta * X_t + m_t + N_t)$. The probit is defined over $I_t$, which is 1 when the daily volume on the IPSA surpasses two standard deviations, and it is 0 otherwise. The independent variables are the index $y_t$ and a set of variables $X_t$, that include the returns or growth rate of variables such as IPSA, VIX, Pe, CLP, S&P 500, volume, and MSCI. $m_t$ and $N_t$ are month and year fixed effects to control for seasonality and variation associated with any specific year. Column (1) uses the average transaction as a threshold. Columns (2) to (5) use the second, fourth, sixth, and eighth deciles as thresholds. Panel A is the basic probit estimation, and panel B includes control for autocorrelation for the respective index.

| Variables | Panel A: positive shock | | | | | Panel B: negative shock | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile | (1) Average | (2) Second decile | (3) Fourth decile | (4) Sixth decile | (5) Eighth decile |
| Index | 0.990*** | 1.053*** | 1.422*** | 1.306*** | 1.064** | -1.806** | -1.412 | 0.560 | -0.388 | 0.186 |
| | [0.378] | [0.389] | [0.401] | [0.445] | [0.476] | [0.749] | [0.901] | [1.183] | [0.882] | [0.922] |
| VIX | 1.464 | 1.463 | 1.559 | 1.585 | 1.430 | 3.948 | 3.878 | 3.989 | 4.565* | 3.977 |
| | [0.994] | [1.011] | [0.974] | [0.991] | [0.992] | [2.898] | [2.639] | [2.753] | [2.609] | [2.609] |
| IPSA | 2.779 | 1.603 | 1.431 | 2.080 | 1.797 | -43.66*** | -36.58*** | -33.21** | -32.28** | -43.63*** |
| | [5.577] | [5.610] | [5.385] | [5.549] | [5.483] | [14.95] | [12.61] | [13.16] | [12.87] | [14.46] |
| MSCI | 5.752 | 5.889 | 6.295 | 6.027 | 5.804 | 15.81* | 15.10* | 12.89 | 12.12 | 13.89 |
| | [4.542] | [4.607] | [4.516] | [4.578] | [4.563] | [8.761] | [8.125] | [8.202] | [8.535] | [8.683] |
| Pe | 0.0824 | -0.0650 | -0.0736 | -0.177 | 0.0852 | 9.485* | 6.114 | 5.050 | 2.996 | 6.291 |
| | [2.234] | [2.245] | [2.222] | [2.244] | [2.268] | [5.609] | [5.022] | [5.026] | [4.225] | [5.264] |
| S&P 500 | 4.099 | 4.346 | 4.667 | 4.619 | 4.242 | 30.18 | 26.68 | 26.72 | 33.60** | 29.96 |
| | [5.426] | [5.432] | [5.367] | [5.363] | [5.383] | [18.92] | [18.83] | [18.77] | [15.92] | [18.52] |
| CLP | 2.598 | 2.627 | 3.142 | 3.230 | 3.696 | -3.954 | -2.881 | -7.122 | -7.266 | -8.233 |
| | [7.998] | [8.020] | [7.879] | [7.894] | [7.939] | [17.30] | [15.63] | [17.38] | [16.75] | [16.02] |
| Index lag. 1 day | 0.255 | 0.547 | 0.872** | 0.118 | 0.922** | -3.548*** | -0.174 | 0.829 | 2.727** | -4.650*** |
| | [0.413] | [0.405] | [0.369] | [0.415] | [0.468] | [0.886] | [0.849] | [0.935] | [1.325] | [1.352] |
| Volume | 0.574*** | 0.576*** | 0.574*** | 0.576*** | 0.568*** | -2.890*** | -2.704*** | -2.720*** | -2.807*** | -2.800*** |
| | [0.0801] | [0.0805] | [0.0802] | [0.0805] | [0.0805] | [0.440] | [0.431] | [0.422] | [0.470] | [0.456] |
| Constant | -1.801*** | -1.757*** | -1.813*** | -1.736*** | -1.844*** | -3.236*** | -3.719*** | -3.936*** | -4.109*** | -3.616*** |
| | [0.215] | [0.206] | [0.210] | [0.204] | [0.220] | [0.672] | [0.679] | [0.573] | [0.774] | [0.737] |
| Observations | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 | 2,340 |
| Pseudo-R-squared | 0.141 | 0.141 | 0.148 | 0.142 | 0.143 | 0.689 | 0.670 | 0.669 | 0.681 | 0.683 |

Robust standard errors are given in brackets. *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$.

participants of that market. Using a study of cases with significant price movements, we show that changes in the index and thus in the network structure are significantly associated with a greater probability of positive shocks on the stocks' returns and traded volume. Nevertheless, there is no consistent significant correlation between the change in the network and negative shocks. This asymmetry in the relation of the transaction network's structure and shocks on the stocks' returns and volume suggests that contemporaneous changes of the transaction network to positive shocks on the IPSA returns respond to information or signals spread from brokers who trade large volumes to brokers who trade smaller volumes. On the other hand, the absence of a robust and significant relation between changes of the transaction network and negative shocks to the IPSA returns suggests that negative shocks are not accompanied by a flow of information in the transaction network. Therefore, the agents in the market cannot rationally internalize their consequences or anticipate them either. Furthermore, we show that our index is correlated positively and significantly with important changes in the transaction volume observed. In terms of possible mechanisms that can explain our results, the literature in market microstructure provides a natural relationship between volume traded and returns of an asset. Our more aggregate study of the relationship between the transaction network and the probability of positive changes on volume and returns may be capturing this natural relationship between volume and returns. We conjecture that further research in how changes in the structure of the network affect returns and volume would complement the microstructure literature in terms of both new frictions at the brokers' level and how volume can affect returns.

Therefore, our study provides evidence that suggests that the structural changes of the transaction network correlate with variations in the set of financial information available for the agents in the market (hypothesis 1), that structural changes of the transaction network affect the occurrence probability of qualitatively significant returns variations of the traded assets (hypothesis 2), and that structural changes of the transaction network affect the occurrence probability of qualitatively significant variations in the volume of traded assets (hypothesis 3). Finally, our results propose an alternative to study dynamically several characteristics of a transaction network simultaneously. This approach serves as a basis for future studies about how the structural changes of the transaction network of the equity market affect systemic risk. We believe that a promising avenue would be to establish the link between the change in the probability of a positive shock on the stock returns and the change in the financial position of the brokers.

## Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

## Acknowledgments

## Endnotes

1. This property, known as *"robust-yet-fragile"* in financial networks, exhibits nonlinear dynamics. In other words, small shocks can cause large chain reactions.

2. In order to measure the relative importance of a node within a network, numerous measures such as volume of transactions, links, connectivity, and reciprocity have been used to identify the nodes of systematically important financial institutions (SIFIs). Kuzubaş and others [33] and Martínez-Jaramillo and others [34] show that centrality measures applied to risk analysis are a useful tool in the ex post analysis of a crisis, because they provide a good understanding for a subsequent financial regulation as they identify those institutions that are *"too interconnected too fail"*. However, the existence of nonlinearities in the network interactions means that the centrality measures are not always the best tool to estimate the systemic importance and contribution to the systemic risk of a financial institution. Acemoglu and others [6] show that, in the presence of large shocks, the commonly applied centrality measures in empirical works fail to identify the SIFIs.

3. Bolsa de Comercio de Santiago de Chile (BCS) in Spanish.

4. See Stoll [1] for a survey.

5. The financial market microstructure literature identifies two sources of frictions. Some frictions are the result of the asset-associated characteristics (opacity, corporate governance, etc.). The second source of frictions is associated with the brokers, such as size or financial position.

6. The key relational data in this study were provided by the SSE in anonymized form, with no association between the brokers' names and their transactions.

7. All the data used to construct the change of the local and international financial variables were obtained from the BLOOMBERG and SEBRA terminals at the SSE (http://www.sebra.cl).

8. Namely, the intermediaries'/brokers' transaction network.

9. World Economic Forum. The Financial Development Report 2012. (https://www.weforum.org).

10. It should be noted that SSE assigns a code to each stockbroker; therefore, we did not identify them by name.

11. The interested reader can require the results for the transaction networks based on nonreported deciles and the mode from the authors.

12. In addition, it is common for investors to use multiple brokers to trade in the same day, so netting transactions could generate a loss of real stock market activity.

13. The threshold is defined for each calendar year. To not overburden the notation, we avoid indexing the threshold by year.

14. A directed network considers that $ij \neq ji$ for every $ij$, $ji \in G_t$.

15. Every network characteristic (dimension) takes a value between $-1$ and $1$ (i.e., $v_{t,d} \in [-1, 1]$ for all $t$ and $d \in [1, 2, \ldots, D]$).

16. A complete network is a network where every possible link between each pair of nodes in the network exists. Because we use directed networks, the complete network is a network where every broker has a directed link to and from every other broker in the network.

17. To the extent that in some general equilibrium models price changes have a normative interpretation, one could extend the positive interpretation of such a correlation to a normative correlation. However, this is not our intent in this paper and, thus, the latter could be the subject of further research.

18. All explanatory variables are percentage changes of the financial variables.

19. Remember that the measure $k1$ implies that when it increases in value, the network has changed more.

20. Autocorrelograms are available upon request.

21. This result should not be interpreted as returning to the original network. It only shows that the network has a tendency to reduce its structural change after two days of considerable structural movement.

22. Note that such shocks coincide in 60% of the cases with the shocks to IPSA.

23. They are available upon request.

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*Research Article*
# Partially Overlapping Ownership and Contagion in Financial Networks

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Using historical banking data for the United States from the years 2000 to 2015 we characterize the probability and extent of a financial contagion using a calibrated network model of heterogeneous interbank exposures. Both the probability and the average extent of a contagion begin to rise in 2007 prior to the US financial crisis. Including a common asset in the model increases both the probability and extent of contagion, especially during the years of the financial crisis. Based on rising institutional ownership in the banking industry, we introduce a partially overlapping ownership asset that devalues endogenously. The addition of this asset increases the extent of a financial contagion. Our results show that trends in capital buffers and the distribution and type of assets have a significant effect on the predictions of financial network contagion models and that the rising trend in ownership of banks by banks amplifies shocks to the financial system.

## 1. Introduction

Following the 2008-2009 US and global financial crisis, there has been a growing interest in the role that the network structure of banks and the types and distribution of their assets have in determining the probability and extent of a potential financial contagion. Chinazzi and Fagiolo [1] provide a concise survey of recent literature in this area. Two limitations common in much of this research relate to the complexity of the network and asset structure and the availability of data. First, the assets and network structures observed in real world financial systems tend to be more complicated than assumed in typical financial network models. Second, detailed data on the structure of real world financial networks is often extremely limited, especially for a major nation like the United States. Empirical papers tend to focus on nations other than the US and often employ data from a single or small number of years. In this paper we improve on these two limitations.

First, in addition to the standard assumption of direct exposures (interbank loans) and independent external assets, we add an external common asset (similar to [2]) and we introduce the concept of a partially overlapping ownership asset, or an asset held by some banks that has value endogenously determined by failures within the banking industry (i.e., such as an investment in an indexed portfolio of stock for the banking industry). In addition to these assets, we consider a core-periphery network structure, which is increasingly becoming the preferred representation of the banking industry. We contrast our results with those from a scale-free network, which is another common network structure. Finally, we use historical financial information for depository institutions in the United States, for the years prior to and following the financial crisis, to calibrate key financial network characteristics such as the size of the network, total assets, and capital buffers for individual banks as well as to characterize and distribute the partially overlapping ownership asset. These steps allow us to investigate how the predictions of a financial contagion model change based on observed trends in the banking industry.

Equity ownership of banks by banks is becoming increasingly common in the United States. Between 2000 and 2015 the number of banks with ownership in other banks doubled in the United States. Over the same time, the total value of

this ownership across the banking system has been growing, especially following the financial crisis. Between 2011 and 2015 the total value of ownership of banks by banks increased by 211% (over this same time period the value of the S&P 500 rose by 55%). This growth in bank ownership by banks has the potential to add a new and significant feedback channel that amplifies shocks. For example, a negative shock to the banking industry reduces the value of ownership in banks, which, in turn, further negatively affects balance sheets of banks, further reducing the value of bank ownership and adding additional stress to the system. To investigate this trend in institutional ownership, we introduce a partially overlapping ownership asset, to capture the rise in equity ownership of banks by banks, and demonstrate how the addition of this asset amplifies the effects of financial contagions.

Our approach is related to Sensoy's paper [3], in which the author investigates the effects of firm size and ownership structure on commonality in liquidity using unique ownership data for Turkey. The author analyzes both institutional and individual ownership and finds that, in addition to the amount of ownership and firm size, the characteristics of investors, and the microstructure of the market are important in determining systematic liquidity. While related, in our case we focus on the role of institutional ownership in banking networks in order to study financial contagion.

Our results show that for the US banking system between 2000 and 2015 the average capital buffers held by banks increased overall and the probability of contagion declined, with some significant exceptions in 2007 and 2008. Based on a network of direct exposures only, in 2007 and 2008, prior to the US financial crisis, the probability of contagion resulting from the failure of a core bank rose from 19.7% to 24.6% (a 24% increase) and the average extent of a potential contagion more than tripled. The addition of a common asset greatly increases the probability of a contagion in all years, bringing it close to 100% from a random core bank failure between 2008 and 2010 and close to 100% in 2009 from a random periphery bank failure. A common asset also more than doubles the average extent of contagion from a core bank failure. While adding a partially overlapping ownership asset increases the probability of contagion from a core bank failure by only about 5%, it doubles the extent of a contagion on average across all years. These results are similar for both core-periphery and scale-free network structures. Before focusing on the data and model we begin with a brief review of literature related to this topic.

Caccioli et al. [4] study the Australian interbank network and show that the interplay of multiple channels of exposures is a major contributor to systemic risk and contagion. They conduct stress tests to analyze contagion through direct exposures, overlapping portfolios, and the combination of these two channels. They conclude that contagion due to counterparty risk can be strongly amplified by the addition of a common portfolio. In another paper, Caccioli et al. [2] extend the analysis of contagion caused by overlapping portfolios to a scenario with multiple assets. They characterize how the average level of diversification in bank portfolios, the ratio of the number of banks to the number of assets, and the leverage attained by banks all affect system stability

with respect to an initial shock on a single asset or bank. By conducting analytical simulations on a stylized network (a random network with Poisson degree distributions for both banks and assets), they estimate the region of parameter space where global cascades occur. Poledna et al. [5] analyze a four-layered interbank network and show that a traditional measure of systemic risk based only on a single layer of deposits and loans, which is common in most studies, dramatically underestimates (by as much as 90%) the risk inherent in a financial system.

While earlier literature provides empirical evidence of real world financial networks following a scale-free network structure [6–8], more recent papers argue that evidence from interbank markets suggest that a core-periphery network structure better represents interbank exposures for various counties, such as for the interbank systems of Netherlands [9], Italy [10], Germany [11], UK [12], Brazil [13], and Mexico [14]. A core-periphery network classifies nodes into two different types: core nodes and periphery nodes. Core nodes are well connected to each other and to periphery nodes, while periphery nodes have connections only with core nodes. Several papers have investigated the core-periphery structure in the context of financial networks, both from a theoretical and dynamic perspective. Lux [15] develops a simple dynamic model of an interbank market where banks initially choose trading partners randomly due to idiosyncratic liquidity shocks. He shows that with heterogeneity in balance sheets and a simple reinforcement learning scheme governing potential trading counterparts, the system quickly converges to a core-periphery network structure. In the paper of van der Leij et al. [16], they propose a simple model of the overnight interbank lending market in which banks compete for intermediation benefits. They find that a complete core-periphery network is not stable while an incomplete core-periphery network may be stable with heterogeneity between banks and inequality in payoffs corresponding to inequality in sizes. In their paper, the banks are ex antehomogeneous and they show that heterogeneity plays a key role in forming a stable core-periphery network. Silva et al. [13] bridges the empirical and theoretical literature by developing a method of measuring how close a financial network is to a perfect core-periphery structure and then applying this measure to the Brazilian interbank market.

Galeotti et al. [17] study how financial linkages in networks affect individual payoffs and risk to the system by constructing an ownership matrix and exploring the effects of changes to this network. They find that the effects of integration (strengthening of current links) and diversification (spreading of links to more neighbors) depend crucially on the topology of the network. Specifically, they show that, in a core-periphery network, core banks take more risk than periphery banks, which is consistent with our data, results, and other literatures [9]. They also show that when the network is homogenous, individuals take on too little risk relative to the socially optimal portfolios, while when the network is homogenous, they take on too much risk.

In addition to the empirical literature, which focuses primarily on non-US financial networks, and the theoretical literature, there have been several studies on the structure of
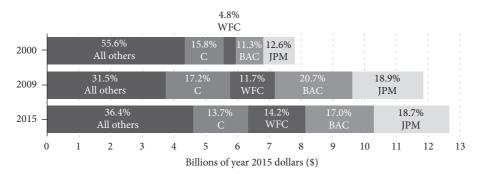
Figure 1: Combined total assets and distribution for years 2000, 2009, and 2015 among the largest four banks: JPMorgan & Chase Co. (JPM), Bank of America Corporation (BAC), Wells Fargo & Company (WFC), and Citigroup (C).

the US financial market following the financial crisis. McCord and Prescott [18] identify significant changes to the structure of the US banking system after the financial crisis. They find a sharp decline in the number of banks, mostly among smaller banks. They provide evidence that this decrease is not due to more banks exiting the market, but rather to a decline in the number of new entries. They also find trends in asset concentration which are consistent with our data. While we focus on banks with $1 billion or more in total assets, Kowalik et al. [19] studied mergers of US banks with $1 billion or less in total assets following the financial crisis. They also find a decline in the number of banks and argue that since the end of the recession, voluntary mergers have been the main reason for this decline. They find that larger banks tend to acquire smaller banks in order to more quickly expand loan operations and gain access to cash and deposits to support future loan growth.

The rest of this paper is organized in the following way. Section 2 presents data from 2000 to 2015 on the US banking system and discusses key trends in the number of banks, total assets, capital buffers, and equity ownership. Section 3 explains the underlying model, network algorithm, and calibration to the observed data. Section 4 presents the results based on different combinations of the three assets types: direct exposures, a common asset and a partially overlapping ownership asset. Section 5 provides a brief summary of our results and some concluding remarks.

## 2. Data

Our data are based on balance sheet and ownership information from the FactSet Fundamentals database for the years 2000 to 2015 or approximately seven years before and after the 2008/2009 US financial crisis. We restrict our attention to firms (henceforth "banks") classified as depository credit intermediation institutions (NAICS code 5221) and head-quartered in the United States and with total assets exceeding $1 billion in year 2015 dollars (adjusted for inflation using the Bureau of Labor Statistics' Consumer Price Index for all urban consumers, US city average series for all items). Between 2000 and 2015 there are a total of 696 banks that satisfy this criteria with an average of 361 active banks in a given year. These data represent an unbalanced panel in which institutions may enter and exit the sample based on

mergers, acquisitions, and bankruptcies. A large proportion of institutions, 145 out of the average 361 or 40% in a given year, are active and filed financial statements across all 16 years. While a complete analysis of the structure of the US banking system is beyond the scope of this paper (for a more detailed characterization of some of changes to the US banking system over this time see [18, 19]), we focus on major trends in the number of banks, size and distribution of total assets, changes in capital buffers, and trends in ownership of other banks.

Table 1 presents the summary statistics for number of banks, total assets adjusted for inflation and capital buffers. The total number of banks in the data peaks in 2003 at 390 declines to a low of 333 in 2012 consistent with [18, 19] and then returns by 2015 to roughly the same level as in 2000.

Between 2000 and 2015 mean total assets rose consistently, growing 63.3% (3.3% on average annually). Over the same time period, median total assets generally declined until 2009 before rising again for a total growth between 2000 and 2015 of 17.1% (1.1% on average annually). The large difference between mean and median is driven primarily by three outliers. Between 2000 and 2015 the assets of Wells Fargo & Company grew by $1.43 trillion (+385%), JPMorgan Chase & Co. by $1.39 trillion (+142%), and Bank of America Corporation by $1.27 trillion (+144%). The growth of these banks is primarily the result of consolidation in the years leading up to and during the 2007/2008 financial crisis (among the most significant were the 2004 mergers of Bank One Corporation with JPMorgan & Chase Co. and FleetBoston Financial Corporation with Bank of America Corporation and then in 2008 the purchase of Washington Mutual out of seizure by JPMorgan & Chase Co. and the acquisition of Wachovia Corp. by Wells Fargo). This can also be seen in Figure 1, which shows the combined total assets in the data, highlighting the largest four banks, for the years of 2000, 2009, and 2015. These data show extreme consolidation and growth among the four largest banks up to and during the US financial crisis. In 2000 the four largest banks owned 44.4% (or $3.46 billion) of all assets in the system, in 2009 this grew to 68.5% (or $8.12 billion), and in year 2015 they owned 63.6% (or $8.06 billion).

*2.1. Trends in Capital Buffers.* Table 1 also shows summary statistics by year for capital buffers. Consistent with Gai

TABLE 1: Summary statistics (observations, total assets, and capital buffers).

| | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Obs. | 349 | 348 | 368 | 390 | 375 | 389 | 385 | 369 | 378 | 370 | 353 | 338 | 333 | 337 | 345 | 347 |
| Total assets (billions of year 2015 dollars) | | | | | | | | | | | | | | | | |
| *Mean* | 22.4 | 23.9 | 23.4 | 23.8 | 26.5 | 26.4 | 26 | 32.1 | 32.1 | 32.1 | 33.8 | 34.7 | 36.5 | 36.1 | 36.8 | 36.5 |
| *Median* | 2.69 | 2.74 | 2.76 | 2.72 | 2.69 | 2.53 | 2.58 | 2.63 | 2.56 | 2.44 | 2.48 | 2.49 | 2.83 | 2.78 | 3.02 | 3.14 |
| *Top 5%* | 62.1 | 73.1 | 68.2 | 65.4 | 64.7 | 63.5 | 55.5 | 65.1 | 46.4 | 51.9 | 55.3 | 48.8 | 57.7 | 57.8 | 56.1 | 59.7 |
| *Top 25%* | 7.9 | 7.8 | 7.7 | 6.9 | 7.5 | 7.1 | 6.8 | 6.8 | 6.2 | 6 | 6.6 | 6.8 | 7 | 7.2 | 7.4 | 8.3 |
| *Bottom 25%* | 1.55 | 1.56 | 1.48 | 1.42 | 1.5 | 1.42 | 1.47 | 1.44 | 1.44 | 1.45 | 1.46 | 1.4 | 1.48 | 1.46 | 1.52 | 1.59 |
| *Bottom 5%* | 1.1 | 1.07 | 1.08 | 1.06 | 1.07 | 1.04 | 1.06 | 1.05 | 1.06 | 1.07 | 1.07 | 1.08 | 1.08 | 1.08 | 1.1 | 1.07 |
| Capital buffer (%, net-worth/total assets) | | | | | | | | | | | | | | | | |
| *Mean* | 8.9 | 9 | 9.3 | 9.1 | 9.5 | 9.5 | 10 | 10.3 | 9.7 | 9.6 | 10.3 | 10.6 | 10.9 | 11 | 11.3 | 11.1 |
| *Median* | 8.4 | 8.6 | 8.8 | 8.6 | 8.8 | 8.8 | 9.2 | 9.2 | 9.1 | 9.3 | 9.8 | 10.2 | 10.5 | 10.7 | 10.8 | 10.6 |
| *Top 5%* | 13.9 | 12.8 | 13.4 | 13.8 | 14.8 | 15.3 | 17.7 | 18.3 | 16.1 | 16.4 | 16.1 | 15.8 | 15.8 | 15.7 | 16 | 15.4 |
| *Top 25%* | 9.7 | 9.8 | 10.1 | 9.8 | 10.2 | 10.5 | 11 | 11 | 10.8 | 11.2 | 11.9 | 12.2 | 12.4 | 12.3 | 12.5 | 12.4 |
| *Bottom 25%* | 7.3 | 7.7 | 7.7 | 7.3 | 7.5 | 7.5 | 7.7 | 7.8 | 7.5 | 7.5 | 8.2 | 8.8 | 9.2 | 9.2 | 9.4 | 9.3 |
| *Bottom 5%* | 5.9 | 5.9 | 6 | 6 | 6.1 | 6 | 6.2 | 6 | 5.1 | 4.1 | 5.4 | 6.2 | 7.4 | 7.7 | 7.8 | 7.8 |
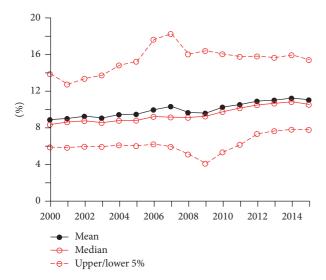
Figure 2: Capital buffer levels by year (as a percent of total assets).

Table 2: Number of banks with ownership and average ownership (as a percent of total assets) by bank size in 2015.

| Total Assets Range | Banks (#) | Owner-banks (#) | Mean share of assets |
|---|---|---|---|
| $100 billion+ | 14 | 5 | 0.47% |
| $10–100 billion | 54 | 12 | 0.30% |
| $5–10 billion | 60 | 10 | 1.16% |
| Under $5 billion | 219 | 17 | 2.24% |



Figure 3: Trends in bank ownership in other banks. Total value of ownership in billions of year 2015 dollars (left axis) and number of owner-banks (right axis) by year.

and Kapadia's specification in [20] we define an institution's capital buffer as their net-worth (total assets minus total liabilities) divided by total assets. Capital buffers show a trend similar to mean total assets and are generally increasing, from a mean of 8.9% in 2000 to 11.1% in 2015. The overall rise in capital buffers is likely in part due to the tightening of the Basel Accords during this time (see Appendix A for discussion on the role of the Basel Accords over this time period). One significant exception to the trend occurs in 2007 and 2008 just before the US financial crisis and can be seen in Figure 2. Mean capital buffers fell from 10.3% in 2007 to 9.7% in 2008 and for the highest 5% percentile, capital buffers declined from 27.2% to 22.8%. For banks in the lowest 5% percentile of capital buffers these buffers declined from 6.0% in 2007 to 5.1% in 2008 to 4.1% in 2009. The sharp decline during 2007 and 2008 for these lowest 5% of banks will play an important in our results, as the definition of a financial contagion is when at least 5% of banks in the system fail. Based on this definition, in 2009 an exogenous loss of just over 4% of total assets at these banks (ignoring any subsequent losses due to direct exposures and other common and overlapping assets) would alone be sufficient to trigger a contagion. Following 2009, mean and median capital buffers begin to climb again. Figure 2 shows this trend in capital buffers over time. There is a weak correlation between total assets and capital buffers that is negative from 2000 to 2006 and positive from 2007 to 2015 this correlation is statistically significant at the 5% level and negative in years 2000 and 2001 and positive in years 2011, 2012, and 2015, based on an OLS regression of capital buffer = $\alpha + \beta \log(\text{Total Assets})$. In year 2000 a 100% increase in total assets results in a 0.25-percentage point decrease in capital buffer while in year 2015 a 100% increase in total assets results in a 0.27-percentage point increase in capital buffer. This suggests that prior to the US financial crisis larger banks tended to carry relatively smaller capital buffers and after the crises relatively larger buffers.
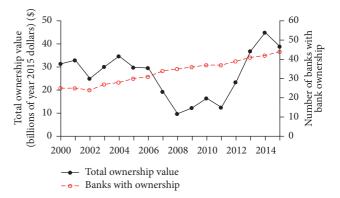
*2.2. Trends in Ownership and Owner-Banks.* In addition to balance sheet information, we collect equity ownership data for each bank from the FactSet Fundamentals database. We then match this ownership data to the banks in our data to construct a matrix of bank equity ownership in other banks. To simplify discussion, banks with ownership in other banks will be referred to as "owner-banks." Between 2000 and 2015 there was a marked increase in both the number of owner-banks and the total value of that ownership. In the year 2000, there were 25 banks with ownership in other banks that totaled $31.3 million in value. By year 2015 there were 44 banks with ownership that totaled $38.9 million.

The increase in the total value of ownership is especially pronounced since the end of the financial crisis. Between 2011 and 2015 the total system value of bank ownership by owner-banks increased from $12.5 billion to $38.9 billion (a 211% increase). Figure 3 shows the total value of ownership within the system and the number of owner-banks. While some of this increase can be attributed to the rise in the stock market (the S&P 500 rose 55.1% over the same period), this only explains a small portion of the increase in ownership value and does not explain the sustained increase in the number of owner-banks (between 2008 and 2015 the total value of bank ownership increased by 299% while the S&P 500 rose only 44.7%).

While ownership in other banks is more likely among larger banks, it is not exclusive to them. Table 2 shows for year 2015 the number of owner-banks and the proportion of owner-bank total assets in ownership, broken down by various sizes of total assets. Of the 14 largest banks (those

with over \$100 billion in total assets) five were owner-banks. For these five, the mean value of this ownership was equal to 0.47% of their total assets. For smaller banks, those with total assets between \$5 and \$10 billion and those with assets below \$5 billion, the value of their ownership, as a proportion of total assets, was much higher (1.16% and 2.24%, resp.). Thus, investing in ownership of other banks is not limited to large banks, and smaller banks leverage a larger portion of their assets in bank ownership.

For most owner-banks this ownership is relatively diversified. For example, in 2015 owner-banks had, on average, ownership in over 27 other banks (median 12.5), with larger banks generally having ownership in a larger number of other banks. Owner-banks also tend to limit their exposure to ownership in a single bank, with the average owner-bank in 2015 investing at most 40% of their ownership assets in a single bank. In 2015, the average ownership amount per bank was 12% of their total ownership value. There are similarities in the banks that owner-banks choose to invest in. In 2015, more than half of total system ownership value was in just two banks, (33% of the total system ownership value was in Bank of America and 17.7% in Wells Fargo). In addition, out of the 44 owner-banks, 42 have a share of ownership in Wells Fargo, 40 in JPMorgan, and 33 in US Bank. As a result, the failure of any of these highly owned banks would have a significant effect on the value of ownership assets across the system.

Overall, ownership of banks by banks still represents a relatively small portion of total assets. In 2015, for the ten banks with the largest ownership positions (as a percent of total assets) represented on average only 3.6% of their total assets and across all owner-banks in 2015 the average value of ownership was only 1.26% of total assets. While investment in bank ownership is a relatively small asset class, it is growing. Furthermore, there may be significant ownership through private equity and debt that we cannot easily observe. The growth of bank ownership by banks is potentially concerning given that it has the potential to feedback into and amplify any shocks to the banking industry.

## 3. Model and Methods

The model we use in our analysis is based on Gai and Kapadia's model from [20], with the addition of heterogeneous link-weights as in [21] and a common asset as in [4]. We also introduce a new type of asset, a partially overlapping ownership asset, which is held by only some banks and has value affected endogenously by bank failures. We begin by going over the details of the model and assets before discussing the generation of the financial network, calibration to US banking data, and the method behind the financial contagion simulations.

*3.1. Banks and the Financial Environment.* Assume that there are $N$ financial institutions (banks) in a network and each bank is represented by a node in the network. Let $A_i^{\text{TOT}}$ and $L_i^{\text{TOT}}$ be the total assets and liabilities for bank $i$. Each bank holds interbank assets, $A_i^I$, as well as some combination of a common asset, $A_i^C$, a partially overlapping ownership asset,

$A_i^O$, and other external assets, $A_i^M$. Each bank has two types of liabilities, customer deposits, $D_i$, and interbank liabilities, $L_i^I$. If a bank becomes insolvent at any point (i.e., $A_i^{\text{TOT}} < L_i^{\text{TOT}}$) it immediately fails and defaults on its interbank liabilities. The value of any interbank assets at a failed bank becomes zero.

The financial network is defined by a network of interbank lending and borrowing. Let $A \subset \mathscr{R}_+^2$ be $N$ by $N$ matrix that represents the network of interbank exposures. Each element $a_{ij}$ represents the amount of assets bank $i$ loans to bank $j$. By convention, $a_{ii} = 0$. Therefore $A_i^I = \sum_{j=1}^N a_{ij}$ is the total interbank lending by bank $i$ and $L_i^I = \sum_{j=1}^N a_{ji}$ is the total interbank borrowing of bank $i$. Matrix $A$ represents a weighted directed network with heterogeneous link-weights. Incoming links of a node reflect the interbank assets of the node. Outgoing links of a node represent the interbank liabilities of that node. Let $w_{ij} = a_{ij}/A_i^I$ be the proportion of interbank assets belonging to bank $i$ held by bank $j$.

In addition to interbank assets, each bank invests its remaining assets across a combination of a common asset, a partially overlapping ownership asset and external assets. The risks and exposures associated with each of these assets are different. The common asset reflects an asset held by all banks with a common value determined exogenously (e.g., mortgage-backed securities). The common asset decreases in value only due to an exogenous shock, however, any decrease in value will affect the balance sheets of all banks.

The partially overlapping ownership asset reflects equity ownership in a portfolio of banks within the system and may decline in value following the failure of any bank included in the portfolio. Unlike the common asset, neither do all banks hold the partially overlapping ownership asset, nor all banks hold necessarily included in the portfolio. The partially overlapping ownership asset can be interpreted as a single investment fund indexed to select banking stocks. The network of ownership generated by this asset can be thought of as a second, completely distinct, structure from the network of interbank exposures. In a given year, we define the share of this portfolio invested in bank $j$ as $P_j$. If bank $j$ fails, then the value of its stock falls to zero and the partially overlapping ownership asset will devalue to reflect this or suffer a $1 - P_j$ loss. This loss, in turn, is reflected in the value of the partially overlapping ownership asset held by some banks. All banks that choose to invest in the partially overlapping ownership asset purchase the same portfolio.

Finally, any remaining assets not invested in interbank assets, the common asset or the partially overlapping ownership asset, are invested in a risk-free external asset. This external asset represents other assets outside the scope of the model.

Our main interest is in determining the probability and extent of a financial contagion stemming from a single shock to the financial system. All scenarios we study begin by imposing the failure of a random bank on the financial network. In scenarios in which the common asset is present, we simultaneously impose an exogenous shock of size $\phi$ to this common asset in addition to the initial failure of a random bank. Following this initial failure (and possibly

the decline in value of the common asset), the shock may then be transmitted across the financial network through two channels. First, the failure and default of the initial bank will eliminate any interbank assets of other banks held at this bank. Second, if the failed bank is contained in the portfolio of the partially overlapping ownership asset then the value of this asset will decline for banks holding this asset. These two effects combine to eliminate or reduce the value of assets for some banks across the network, which in turn may cause additional banks to fail.

To formalize the condition under which bank $i$ fails following the initial failure of another bank (and possible simultaneous devaluing of the common asset) let $K_i = A_i^{\text{TOT}} - L_i^{\text{TOT}} = (A_i^I + A_i^M + A_i^C + A_i^O) - (L_i + D_i)$ represent the capital buffer of bank $i$. Following an initial failure of bank $j$, the solvency condition for bank $i$ is

$$L_i + D_i < A_i^M + \left(1 - w_{ij}\right) A_i^I + (1 - \phi) A_i^C \\ + \left(1 - P_j\right) A_i^O \tag{1}$$

and bank $i$ will fail if

$$w_{ij} > \frac{K - \phi A_i^C - P_j A_i^O}{A_i^I}. \tag{2}$$

Should bank $i$ fail, they then default on their interbank liabilities and the partially overlapping ownership asset may devalue. This process continues (with the solvency condition updated to reflect prior failures) until no further banks fail.

Consistent with [20], if over 5% of all banks in the system fail then we define this as a financial contagion. The extent of such a financial contagion is defined as the proportion of banks that fail or the expected proportion of banks that fail conditioned on at least 5% of banks failing.

*3.2. Calibration and Simulation Methods.* For each year we begin by calibrating the number of banks, as well as the total assets and total liabilities for individual banks to the observed data. The private nature of interbank loans in the United States means that data on the structure of the interbank asset network is limited. As a result, for each year we simulate 100 core-periphery network structures to represent the network of interbank assets, using an algorithm calibrated to match the typical characteristics of the interbank market of other nations (while 100 simulations may seem low, because assets and liabilities of individual banks are drawn directly from the data and because the network generation algorithm is based in part on these data, it is sufficient to allow our results to converge. Increasing simulations to 1000 per year does not have a significant effect on our results). See Section 3.3 for additional details on the algorithm and parameters used to generate the network.

While the network of interbank exposures is based on underlying balance sheet data, the partially overlapping asset is generated from separate data on the equity ownership of banks by banks that is not directly related to interbank exposures (for a brief discussion of when exposures from ownership may be equivalent to exposures from interbank lending and when it is appropriate to treat these as two distinct assets see Section 5). To generate the partially overlapping ownership asset, in each year we construct a single asset that reflects a representative portfolio of all observed ownership in banks. The share of this portfolio invested in bank $j$ is equal to the total value of equity ownership by all banks in bank $j$ as a fraction of the total value of equity ownership by all banks in all other banks, which can be expressed as $P_j = \sum_{i=1}^{N}(\widehat{P}_{ij}/\sum_{k=1}^{N} \widehat{P}_{ik})$, where $\widehat{P}_{ij}$ is the observed dollar value of ownership held by bank $i$ in bank $j$. After constructing the ownership asset portfolio $P = \{P_1, P_2, \ldots, P_N\}$, we then distribute the amount of assets; we observe each bank investing in equity ownership to this portfolio.

For owner-banks or those banks which we observe having ownership assets in the data, we multiply the amount of the observed ownership, or $A_i^O = \sum_{i=1}^{N} \widehat{P}_{ij}/A_i^{\text{TOT}}$, by a factor of 15, capped at 40% of their total assets. The factor of 15 was chosen to increase the average ownership by owner-banks to be equal to approximately 20% of total assets or half of the proportion we assign to the common asset (the observed ownership by owner-banks as a percent of their total assets in 2015 was equal to 1.26% on average. See Section 2.2 for more details). Banks that we observe having no assets in equity ownership invest zero in the ownership asset. We use this form of a representative portfolio of ownership, rather than the observed ownership network structure, and amplify the value of this ownership, to reflect the possibility of other unobserved sources of ownership, such as private equity and debt ownership, as well as to more clearly capture the effect of this asset class on financial contagions.

In the results presented in Section 4 we consider four scenarios. In all four scenarios each bank holds 20% of their total assets in interbank assets ($A_i^I = 0.2A_i^{\text{TOT}}$) and the differences are in how the remaining 80% of total assets are invested. The four scenarios are as follows:

(1) *Direct exposures only*: in Section 4.1 we first consider the simplest case with only direct exposures from interbank assets ($A_i^C = A_i^O = 0$).

(2) *Common asset*: next, in Section 4.2, in addition to direct exposures, we add a common asset equal to 40% of total assets to all banks which depreciates by 10% simultaneously with the initial shock ($A_i^C = 0.4A_i^{\text{TOT}}, A_i^O = 0$).

(3) *Partially overlapping ownership asset*: in Section 4.3 instead of the common asset, we add to direct exposures the partially overlapping asset, which is held by some banks and may reflect up to 40% of their total assets ($A_i^C = 0, 0.4A_i^{\text{TOT}} \geq A_i^O \geq 0$).

(4) *Common and partially overlapping ownership assets*: finally, in Section 4.4 we consider the combined effects of these assets by combining the partially overlapping asset with the common asset ($A_i^C = 0.4A_i^{\text{TOT}}, 0.4A_i^{\text{TOT}} \geq A_i^O \geq 0$).

In each scenario, any assets not invested in direct exposures (interbank assets), the common asset or the partially

TABLE 3: Comparing the CP model for the Dutch interbank market to Germany (Craig and Von Peter, 2014 [11]), Italy (Fricke and Lux, 2014 [10]), UK (Langfield et al., 2014 [12]), and Mexico (Solis-Montes, 2013 [14]).

| Country | Netherlands | Germany | Italy | UK | Mexico |
|---|---|---|---|---|---|
| The number of banks | 100 | 1800 | 120 | 176 | 46 |
| Average number of core banks | ±15 | ±45 | ±30 | 16 | ±16 |
| Average core size | ±15% | ±2.5% | ±25% | 9.1% | ±35% |
| Error frequency, as % of links | 29% | 12% | 42% | 47% | 25% |

overlapping ownership asset, are invested in other external assets ($A_i^M$).

*3.3. Network Structure.* To generate the network of interbank exposures we follow the definition of a core-periphery network in in't Veld and van Lelyveld [9] and van der Leij et al. [16].

*Definition 1.* A network is a perfect core-periphery network if there exists a set of core nodes $K \subset N$ and periphery nodes $P = N \setminus C$ such that

$$
\begin{aligned}
g_{ij} &= 1, \\
g_{ji} &= 1, \\
&\quad \forall i, j \in K, \\
g_{ij} &= 0, \\
g_{ji} &= 0, \\
&\quad \forall i, j \in P, \\
g_{ij} &= 1, \quad \forall i \in K, \ \exists j \in P, \\
g_{hi} &= 1, \quad \forall i \in K, \ \exists h \in P,
\end{aligned}
\tag{3}
$$

where $g_{ij}$ represents a directed link between nodes $i$ and $j$. This defines a perfect directed core-periphery network.

If we form a network of interbank assets as a perfect core-periphery structure, using the matrix $A$ to represent the links between different banks, we will get

$$
A = \begin{bmatrix} CC & CP \\ PC & PP \end{bmatrix} = \begin{bmatrix} 1 & CP \\ PC & 0 \end{bmatrix}.
\tag{4}
$$

A number of recent papers [9–14] find that real financial networks exhibit a similar structure to a perfect core-periphery networks, only with the addition of link "errors." These "errors" are missing links between core nodes as well as extra links between periphery nodes. Table 3 represents data from in't Veld and van Lelyveld [9] characterizing the number of core banks and error frequency of banking systems in different countries.

To simulate a network of interbank exposures to reflect the US banking system we first we assume that core banks are those banks with the highest total assets in the system. We fix the number of core banks at 25, which is the average number of core banks across the countries in Table 3. After

constructing the perfect core-periphery bank network, we calibrate error links. The error frequency of our network is set at 31%, which is the average of error frequencies of different countries in Table 3.

In addition to simulating the network of interbank exposures as a core-periphery network structure, we also consider a scale-free structure as in [21] and find results very similar to that of the core-periphery structure. The algorithm to generate the scale-free network as well as the results can be found in Appendix B.

## 4. Results

The results in this section are organized into subsections in which we sequentially introduce each asset type. Section 4.1 begins with only direct exposures through interbank lending. Section 4.2 adds a common asset owned by all banks and Section 4.3 adds a partially overlapping ownership asset to direct exposures. Finally, Section 4.4 adds the combination of a common asset and a partially overlapping ownership asset to direct exposures. When a common asset is present we present contagion results from both the initial failure of a random core node and the initial failure of a random periphery node. In the absence of a common asset, contagions do not result from the initial failure of any periphery nodes and only the results from an initial core node failure are given (in the cases where the initial failure of periphery nodes does not result in a contagion the overall contagion probability (resulting from the initial failure of any node) can be determined by multiplying the probability of contagion from a core node failure by the proportion of nodes that are core nodes, or $25/N$, which is equal, on average, to 0.069). In addition, we restrict our focus to the core-periphery network structure, as it is more representative of the US banking system. The results for the scale-free network structure are extremely similar and for the complete results for the scale-free network structure see Appendix B.

*4.1. Direct Exposures Only.* As a baseline case we begin by considering the possibility of a financial contagion arising only through direct exposures to counterparties through interbank lending. The approach was first developed in [20] and then expanded in [21, 22]. Network characteristics such as the number of nodes, the total assets of each node, and the capital buffer of each node are calibrated from data on the US financial system and interbank exposures are generated with a core-periphery network structure (see Sections 3.2 and 3.3 for further details). Our main results are that, with the calibrated data for the US financial system and only
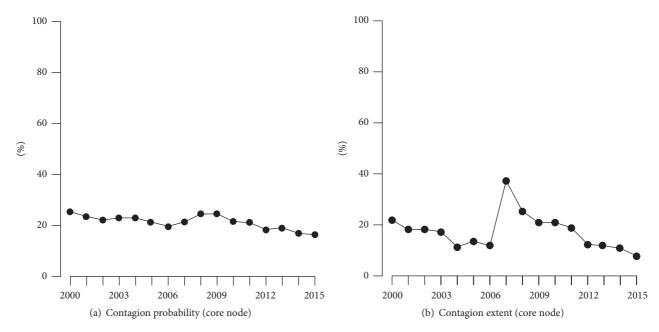
(a) Contagion probability (core node)



(b) Contagion extent (core node)

Figure 4: *Contagions with Direct Exposures Only*. Probability (a) and extent (b) of a financial contagion for a core-periphery network with direct exposures only, conditioned on a random core bank failing.

direct exposures, financial contagions originate only from the failure of core banks and that there was a significant increase in the probability and extent of a financial contagion beginning in 2007, prior to the US financial crisis.

With only direct exposures, financial contagions begin only with the initial failure of a core node or the failure of one of the largest 25 banks. Due to the concentration of lending at core nodes and their greater connectedness, the failure of a single core node will have greater and wider-reaching implications than the failure of a periphery node. Figure 4 shows the simulated probability of contagion (a) and average extent of contagion (b) based on the initial failure of a random core node.

Overall between 2000 and 2015 the probability of a financial contagion resulting from the failure of a random core node generally declined (from 25.4% to 16.4%). A significant exception was in 2007, when the probability rose from 19.7% to 21.4%, and 2008, when it rose to 24.6%, prior to the beginning of the US financial crisis (the US financial crises are often marked as beginning with the bankruptcy of Lehman Brothers in September 2008. The effect in 2007 is also present and more pronounced in the scale-free model, where the probability of contagion rose from 17.6% to 19.5% in 2007). This rise in the probability of contagion is accompanied by a more than tripling in the average contagion extent in 2007, from 11.8% to 37.3%. Viewed ex post, these results can be interpreted as a significant warning sign in 2007 of the coming financial crisis. The results from the scale-free network structure (see Appendix B) are similar.

While the rise in probability and extent of a financial contagion in 2007 and 2008 are consistent with the financial crisis, the result that an initial failure of a periphery node never leads to a financial contagion suggests that direct exposures through interbank lending alone may not provide

a rich enough set of exposures to explain more than a small class of extreme financial contagions. To address this, in the next sections we consider additional types of assets and exposures.

*4.2. Common Asset.* In this section we add a common asset, in a manner consistent with [4], to the existing direct exposures. This common asset provides an additional exposure common to every bank, which increases the chance of a financial contagion. Consistent with the results of [4], we find that the addition of a common asset to direct exposures significantly increases both the probability and extent of financial contagions. Unlike the previous case in Section 4.1 with direct exposures only, during some years the probability of contagion from the initial failure of periphery node is positive.

Figure 5 shows the simulated probability of contagion from the failure of a random core node (a) and a random periphery node (b) as well as the average extent of contagion for each type of initial failure ((c) and (d), resp.). Included in each figure in red for comparison are the results from Section 4.1 for direct exposures only.

With the addition of a common asset, the probability that the failure of a core node triggers a contagion is both extremely high and highly volatile (Figure 5(a)), especially beginning in year 2008 and through the years of the US financial crisis. The probability of contagion drops from 82% in 2000 to 47% in 2006 before jumping to 99% in 2008 and then eventually dropping to 39% in 2015. While this probability may appear extremely high, recall that it is conditioned on the failure of a core node or one of the 25 largest banks. The overall probability of contagion resulting from the initial failure of any random node (core or periphery) for year 2000 is just 5.9%.

(a) Contagion probability (core node)

(b) Contagion probability (periphery node)

(c) Contagion extent (core node)

(d) Contagion extent (periphery node)
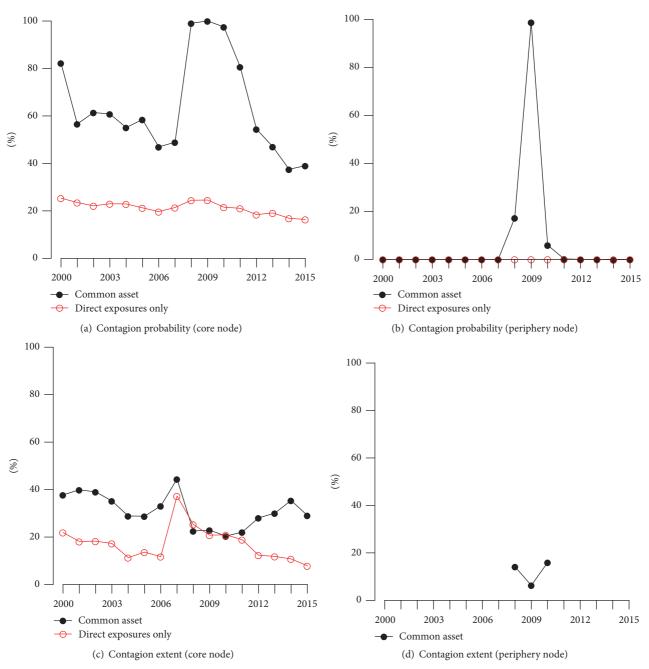
Figure 5: *Contagions with a Common Asset*. Probability of a contagion ((a) and (b)) and average extent of contagion ((c) and (d)) for a core-periphery network with a common asset and direct exposures (black solid circles) and direct exposures only (red empty circles) conditioned on a random core bank failing ((a) and (c)) and conditioned on a random periphery bank failing ((b) and (d)). Contagion extent shown only when average contagion frequency is greater than 1%.

The addition of a common asset also greatly increases the average extent of a financial contagion stemming from the failure of a core node for most years (Figure 5(c)). Between years 2000 to 2006 and 2012 to 2015 the contagion extent from a core node failure is 15 to 25 percentage points higher with the addition of a common asset. During 2007 to 2011, the years leading up to and during the financial crisis, the average extent is the same with and without a common asset, due to a large increase in the number of smaller contagions, which is discussed further below.

In addition to significantly increasing the probability and extent of a contagion resulting from the failure of a core node, the addition of a common asset introduces the possibility of a contagion from a periphery node, although only during the immediate years of the financial crisis. Prior to 2008 and after 2010 the probability of a contagion from a periphery node is zero (Figure 5(b)). In 2008 this probability rises to 17.2%, then to 98.7% in 2009 before falling to 5.9% in 2010. Combined with the results for the core node failures, in 2009 the probability of a contagion resulting from the failure of any

single node (core or periphery) was 98.9%, or a near certainty. However, this near certainty of a contagion is offset by an extremely low contagion extent for periphery node failures (Figure 5(d)). In 2009, the average extent of a contagion resulting from the failure periphery node was only 6.3%.

The extremely high probability of contagion in 2009 for both core and periphery nodes is being primarily driven by the decline in capital buffers we observe in the data among banks with the lowest capital buffers. In 2009 the 5% of banks with the lowest capital buffers (see Figure 2 and Table 1) had capital buffers at or below 4.1%. Since the exogenous depreciation of the common asset can be interpreted as a four-percentage point loss in total assets this depreciation alone results in the failure of a significant number of nodes. During most years the initial depreciation of the common asset results in between 0 and 3 failures directly (i.e., before the failure of a random node and the spread of the shock through direct exposures). However, in 2009 due to low capital buffers this depreciation of the common asset directly results in the failure of 17 nodes (4.59% of the network). Combined with our imposed failure of one node, only one additional node needs to fail through interbank exposures in this year to meet the definition of a contagion. As a result of the decline in capital buffers a financial contagion is almost guaranteed with a common asset in 2009; however, the extent of such a contagion may be relatively small. This result also explains the convergence of contagion extent we see for core node failures in 2008, 2009, and 2010.

While for most years the average contagion extent from a core node failure with a common asset is 15 to 25 percentage points higher than with direct exposures only, between 2008 and 2010 this extent drops and converges to that of the direct exposures (see Figure 5(c)). This convergence of contagion extent is an artifact of the introduction of a large number of small contagions during these years (see Figure 5(a)). As in [21], the separation of contagions into "mild contagions" (when 5–30% of nodes fail) and "moderate contagions" (when over 30% of nodes fail) would continue to show significantly higher contagion extents from core node failures with the common asset compared to direct exposures only.

The addition of a common asset results in a significantly larger contagion probability and extent for all years. However, one potential criticism of including a common asset is that it has an extremely homogenizing effect on exposures. Because the asset is truly "common" to all banks, any depreciation in the asset will necessarily weaken the balance sheet of all banks. In addition, without a separate market and pricing structure for this common asset any depreciation must be external and arbitrary. In the next section we introduce a partially overlapping asset or a common asset which only some banks chose to hold (i.e., it is not common to all banks). In addition, we interpret this asset as a fixed ownership portfolio of other banks, which allows us to endogenously devalue this asset following the failure of a bank based on its weight in the portfolio of ownership.

*4.3. Partially Overlapping Ownership Asset.* In this section we return to the case of direct exposures (i.e., without a common asset) and then add a partially overlapping ownership asset.

Unlike the common asset in Section 4.2, this asset will be held by only some banks (hence "partially overlapping"). We also interpret this asset as reflecting a fixed portfolio of ownership in other banks. For a discussion of this ownership observed in the data see Section 2.2 and of how we implement this ownership in the model see Section 3.2. One of the advantages of interpreting this asset as ownership is that it provides a natural way to devalue the asset following the failure of a bank. With the failure of a bank, the value of ownership in that bank falls to zero and the ownership asset devalues according to share of that bank held in the portfolio.

With the addition of this partially overlapping ownership asset, our main results are a small increase in the probability of a contagion and a large increase in the extent of a contagion stemming from the failure of a core node. As in the case of direct exposures only, contagions do not arise from the failure of a periphery node and figures for an initial periphery failure are omitted. Figure 6 shows the simulated probability of contagion (a) and average extent of contagion (b) based on the failure of a random core node. Included in each figure in red for comparison are the results from Section 4.1 for direct exposures only.

The addition of the partially overlapping ownership asset has a small but positive effect on the probability of a contagion stemming from the failure of a core node (Figure 6(a)). Unlike the case of the common asset in Section 4.2, the partial ownership asset neither consistently depreciates for all failures nor uniformly transmits the this shock to all other banks. For core nodes that are weighted little or not at all in the portfolio, the effect of their failure will be minimal. For example, in 2012, while 91% of the ownership portfolio was in core banks, three-quarters of this ownership was concentrated in just four of these banks: Bank of America (33%), Wells Fargo (18%), SunTrust Banks (10%), and Citibank (8%). The effect of a failure of any core bank outside of these four banks would be similar to the case with direct exposures only (i.e., without the partially overlapping ownership asset). Because periphery banks are minimally represented in the ownership asset, the probability of a contagion from the failure of a periphery bank remains at zero, as in Section 4.1.

While the effect of a partially overlapping ownership asset on the probability of a financial contagion may be small, the effect on the average extent of a contagion is relatively large. This is because the ownership portfolio is heavily concentrated in a small number of core node banks. For example, in 2012 the largest shares of ownership by the portfolio are in Bank of America (33%), Wells Fargo (18%), SunTrust Banks (10%), and Citibank (8%). In the event that one of these banks fails, a contagion due solely to direct exposures is already highly likely and the partially overlapping ownership asset serves as a channel to transmit the contagion to owner-banks who may not have otherwise been affected solely through direct (and indirect) exposures. Ultimately, the partially overlapping ownership asset has the effect of increasing the importance of a select few core banks and worsening the extent of a contagion should one of these banks fail. In the next section we combine this partially overlapping ownership asset with the common asset.

(a) Contagion probability (core node)

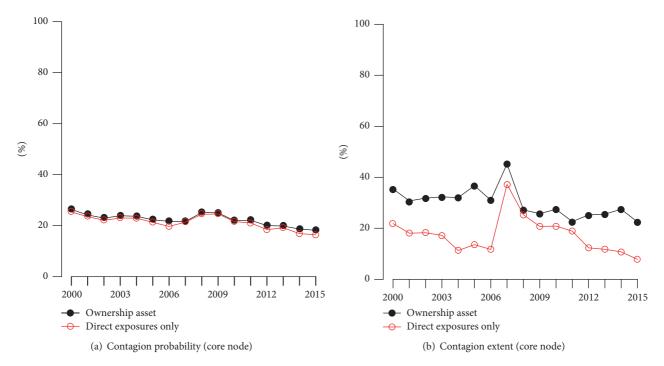(b) Contagion extent (core node)

FIGURE 6: *Contagions with a Partially Overlapping Ownership Asset.* Probability of a contagion (a) and average extent of contagion (b) for a core-periphery network with a partially overlapping ownership asset and direct exposures (black solid circles) and direct exposures only (red empty circles) conditioned on a random core bank failing.

*4.4. Common and Partially Overlapping Ownership Assets.* In this section we add both the partially overlapping ownership and common asset to direct exposures. Figure 7 shows the simulated probability of contagion (a) and average extent of contagion (c) based on the failure of a random core node as well as the probability of contagion (b) and average extent of contagion (d) based on the failure of a random periphery node. Included in each figure in red for comparison are the results from Section 4.2 for direct exposures with a common asset.

The addition of the partially overlapping ownership asset with the common asset and direct exposures has similar effects to those outlined in Section 4.3. There is a small increase in the probability of contagion from core nodes in the years prior to and after the financial crisis (Figure 7(a)) and there is a substantial increase in the extent of contagion (Figure 7(b)). There is no significant effect on contagions resulting from periphery node failures. The lack of a significant interaction between the common asset and partially overlapping ownership asset is not surprising given that the first disproportionately affects contagions from periphery node failures while the second disproportionately affects contagions from core node failures. This is primarily the case due to the observed makeup of the ownership asset portfolio which heavily reflects core node banks (e.g., in 2015 over 92% of the ownership portfolio was in core banks). While the partially overlapping ownership asset is held by many periphery banks, failures from periphery banks have a limited, if any, effect on the value of this asset. If, hypothetically, the ownership asset was constructed primarily of periphery banks the results would be quite different.

## 5. Conclusion

Using historical data on the US banking industry from 2000 to 2015 we calibrate a core-periphery financial network model and characterize the probability and average extent of a financial contagion over time from the random failure of a core or periphery node. The financial network is composed of a network of heterogeneous direct exposures, calibrated to data where possible. In addition to direct exposures we consider two other types of assets. The first is a common asset as in [2] and for the second we introduce a partially overlapping ownership asset to capture the growing equity ownership of banks by banks that we observe in the data. This partially overlapping ownership asset is held by a subset of banks and devalues endogenously based on the failure of banks within the system.

The results show that with direct exposures only, in 2007 and 2008, prior to the US financial crisis, the probability of contagion resulting from the failure of a core bank rose from 19.7% to 24.6% (a 24% increase) and the average extent of a potential contagion more than tripled. The addition of a common asset greatly increases the probability of a contagion in all years, bringing it close to 100% for core banks between 2008 and 2010 and close to 100% in 2009 for periphery banks, primarily due to the decline in capital buffers among certain banks in 2009. Adding a common asset also more than doubles the average extent of contagion from a core bank failure. The addition of a partially overlapping ownership asset only slightly increases the probability of contagion from a core bank failure (by about 5%) but doubles the extent of a contagion on average across all years. The combination of

(a) Contagion probability (core node)

(b) Contagion probability (periphery node)

(c) Contagion extent (core node)

(d) Contagion extent (periphery node)
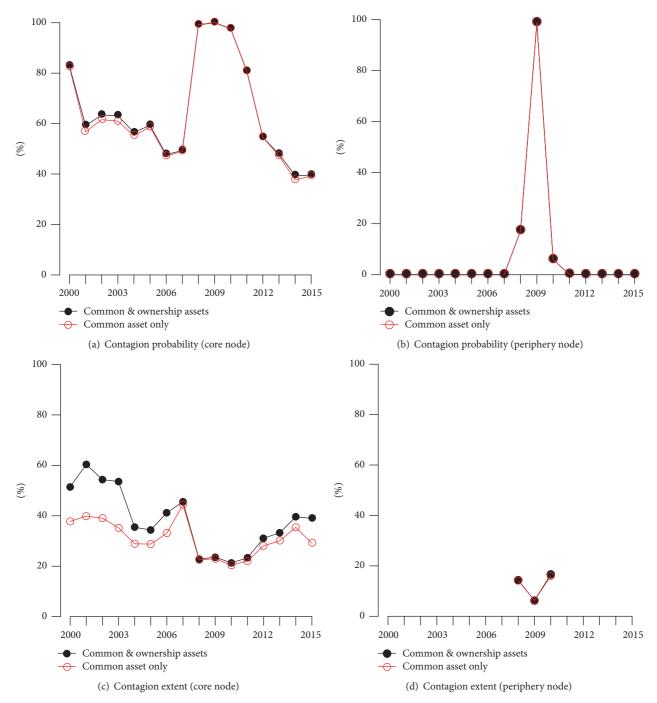
FIGURE 7: *Contagions with a Partially Overlapping Ownership and Common Asset*. Probability of a contagion ((a) and (b)) and average extent of contagion ((c) and (d)) for a core-periphery network with common and partially overlapping ownership assets and direct exposures (black solid circles) and common asset and direct exposures only (red empty circles) conditioned on a random core bank failing ((a) and (c)) and conditioned on a periphery bank failing ((b) and (d)). Contagion extent shown only when average contagion frequency is greater than 1%.

both a common asset and a partially overlapping ownership asset increases both the probability and average extent of a contagion, but there does not appear to be any significant interaction or amplification.

Our data show that many key financial network characteristics, such as number of nodes, total assets, and capital buffers, change significantly over time and these changes have a large effect on the probability and average

extent of a potential financial contagion. As a result, researchers may want to consider data from more than a single or small number of years when characterizing a financial network for contagion analysis. A comparison between results from core-periphery networks and scale-free networks (see Appendix B) shows that the differences in probability and average extent of a financial contagion across the two network structures are smaller than the
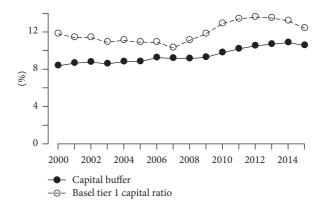
FIGURE 8: Comparison of median capital buffers and median Basel tier 1 capital ratios.

differences across years due to changes in financial network characteristics.

While we interpret equity ownership as investment in a single portfolio asset, such as an index fund of banking stock, a complete network of ownership exposures may also be interpreted as observationally equivalent to a second network, similar to that of interbank lending. In such a case, ownership can be captured by appropriately amplifying the network of interbank exposures. However, there are scenarios in which equity ownership is more appropriately treated as a separate asset from debt. For example, ownership may include the ownership of other firms outside the network of banks (such as insurance or other financial firms) or may reflect more complex types of ownership. In other situations, the order of liquidation in a bankruptcy may be relevant, with interbank debt claims being satisfied prior to, or more wholly than, interbank equity claims. Finally, by its nature, debt typically requires mutual consent to the transaction, while equity does not. There may be a scenario in which one bank deliberately chooses not to borrow from another bank but is unable to prevent this other bank from buying their equity. The distinction between debt and equity become more relevant to the addition of strategic behavior.

In the future we would like to expand this research to look at the effect of partially overlapping ownership assets with other characteristics. For example, if the portfolio was more heavily weighted toward periphery banks then this may increase the probability of contagion more significantly and amplify shocks further once a contagion begins. We would also like to expand the ownership portfolio to include other sources of ownership, such as debt ownership, as well as the ownership of other firms in related financial areas, such as nondepository banks and insurance firms.

Another way to expand this research is to add a dynamic component to the model. For example, if banks in the financial network respond to a first round shock strategically, their response may affect the contagion results. Reference [2] discusses a scenario where banks try to rebalance their portfolio to reach their target leverage level and they conclude that this rebalancing of portfolio destabilizes the system. In contrast, [21] shows that if banks rebalance their portfolio

by reducing investment in potentially weakened banks, this rebalancing stabilizes the system to future shocks.

## Appendix

## A. Basel Accords

The definition of capital buffers used in this paper and common in the literature, beginning with [20] and earlier, is of net-worth (total liabilities minus total assets) divided by total assets. This definition is the most literal definition of solvency but does not take into consideration the riskiness of different asset classes. Another interpretation of capital buffers that does consider the riskiness of assets comes from the Basel Accords or the international recommendations on banking regulations. Basel I was developed in 1988 by the Basel Committee on Banking Supervision (BCBS) and was adopted in law by the Unites States and other G-10 countries in 1992. Since then the Basel Accord has gone through revisions (Basel II in 2004 and Basel III in 2010). In the United States Basel II became effective on April 1, 2008, but with some rules initially delayed or waived due the 2007/2008 financial crises. The Basel III recommendations were approved on July 9, 2013, in the United States.

Under the Basel Accords the regulatory capital buffer is measured in part by the tier 1 capital ratio or the ratio of a bank's core equity capital to its total risk-weighted assets. While the tier 1 capital ratio more accurately measures solvency riskiness than actual solvency, it may be useful to compare this measure with the more traditional definition of capital buffers. Figure 8 compares the median capital buffer (as defined in this paper) with the tier 1 capital ratio under the Basel Accord. In all years the tier 1 capital ratio is significantly higher than the capital buffer, reflecting that it accounts for the riskiness of relatively safe or risk-free assets on the balance sheet. Overall the trend we see in the data for tier 1 capital ratios is similar to the trend in capital buffers, other than a sharper decline during the years leading up to the US financial crisis. Another reason we do not use the tier 1 capital ratio in our model is that participation in the Basel Accord was voluntary prior to 2008 (in 2008 97.1% of our sample reported tier 1 capital buffers under the Basel Accords,

(a) Contagion probability (core node)

(b) Contagion probability (periphery node)

(c) Contagion extent (core node)

(d) Contagion extent (periphery node)
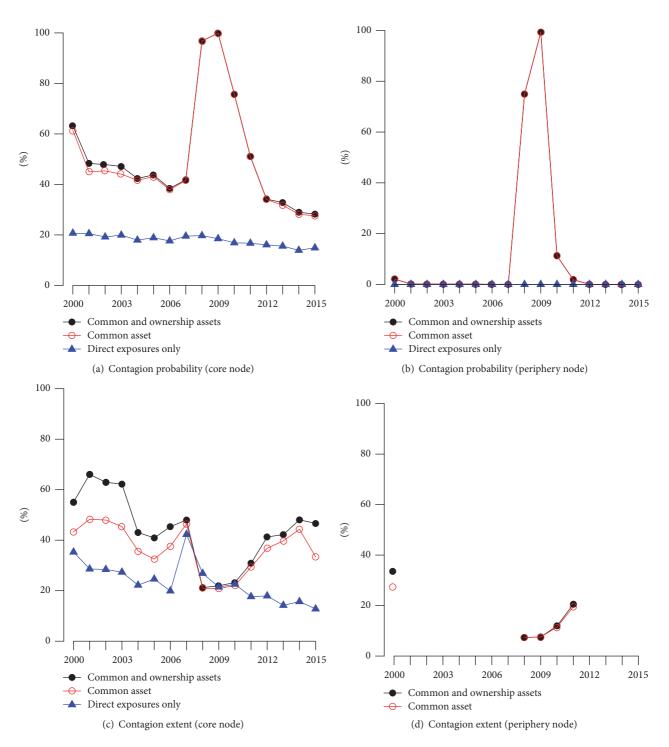
FIGURE 9: *Results for the Scale-Free Network Structure.* Probability of a contagion ((a) and (b)) and average extent of contagion ((c) and (d)) for a scale-free network with common and partially overlapping ownership assets and direct exposures (black solid circles), common asset and direct exposures only (red empty circles), and direct exposures only (blue solid triangles), conditioned on a random core bank failing ((a) and (c)) and conditioned on a periphery bank failing ((b) and (d)). Contagion extent shown only when average contagion frequency is greater than 1%.

compared to only 62.3% in 2003 and 6.6% in 2000), and the rules were further changed in 2010.

## B. Results from Scale-Free Network Structure

For the purpose of comparing our results for the core-periphery network structure with the commonly used scale-free structure we use Barabási-Albert model to construct a scale-free network, which reflects the preferential attachment characteristic of scale-free networks. The algorithm we use here is a directed version of Barabási-Albert model from Bollobás et al. [23].

Assume that $\alpha$, $\beta$, $\gamma$, $\delta_{in}$, and $\delta_{out}$ are nonnegative real numbers such that $\alpha + \beta + \gamma = 1$. Starting with an initial graph $G0 = G(t_0)$, we form $G(t + 1)$ from $G(t)$ according to the following steps:

(1) With probability $\alpha$, add a new vertex $v$, together with an edge from $v$ to an existing vertex $w$, where $w$ is chosen according to $d_{in} + \delta_{in}(P(w = w_i) = (d_{in}(w_i) + \delta_{in})/(t + \delta_{in}n(t)))$, where $d_{in}(w_i)$ represents the incoming degree for node $i$ and $n(t)$ represents the number of vertices in the graph at time $t$.

(2) With probability $\beta$, add an edge from an existing vertex $v$ to an existing vertex $w$, where $v$ and $w$ are chosen independently, $v$ is chosen according to $d_{out} + \delta_{out}$, and $w$ is chosen according to $d_{in} + \delta_{in}$.

(3) With probability $\gamma$, add a new vertex $w$, together with an edge from an existing vertex $v$ to $w$, where $v$ is chosen according to $d_{out} + \delta_{out}$.

To simplify comparisons with the core-periphery network structure, we use the terminology "core node" to refer to the 25 largest nodes (by total assets) in the scale-free model and "periphery node" to refer to other nodes. Figure 9 presents for the scale-free network that are comparable to those of the core-periphery structure presented in Sections 4.1, 4.2, and 4.4. Both network structures yield extremely similar results.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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*Research Article*

# Network Entropy and Systemic Risk in Dynamic Banking Systems

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We investigate network entropy of dynamic banking systems, where interbank networks analyzed include random networks, small-world networks, and scale-free networks. We find that network entropy is positively correlated with the effect of systemic risk in the three kinds of interbank networks and that network entropy in the small-world network is the largest, followed by those in the random network and the scale-free network.

## 1. Introduction

There exist financial connections in the interbank market, which make it possible for the interbank market to be represented as a network. It is important to study the financial connections in the interbank market from the network perspective. The reason for this is that the financial connections can become a channel for propagation and amplification of shocks, which is directly linked to the stability of economic/financial systems [1]. In fact, many empirical studies have shown that interbank lending relationships reflect some typical network structures (e.g., [2–6]), such as random structures, small-world structures, and scale-free structures. And there is a rapidly growing literature on bank network models and systemic risk (e.g., [7–19]).

According to the above literature, we can know that banking systems can be modeled as the complex networks, which are useful to investigate systemic risk. In the realm of complex networks, the entropy has been adopted as a measure to characterize properties of the network topology [20–22]. However, there is limited research to adopt entropy to investigate interbank networks and systemic risk. And this paper aims to add to the current literature on understanding systemic risk in banking systems from the perspective of network entropy. The rare instance is the work of Lee [23] who

applies the measure of network entropy to BIS global financial network database in order to study highly connected global banking networks. The study of Lee [23] mainly investigates how network structure of global banking networks among core countries has evolved during the global financial crisis of 2007–2009 in terms of diversification and probes into financial linkages between core countries and periphery countries. Besides, some studies show that the notion of entropy can be used to build an early warning indicator for systemic risk [24, 25].

Based on the above analysis, it can be seen that entropy measures have been rarely adopted to analyze interbank networks and banking systemic risk. And the single study only adopts network entropy to measure diversity of highly connected global banking networks. Besides, there are a lot of researches on adopting the entropy to investigate complex networks (e.g., [20–22, 26]). This paper aims to contribute to investigating characteristics of network entropy of dynamic banking systems and study whether network entropy can be used as a measure of robustness for banking systems from the perspective of systemic risk.

Therefore, in this paper, we apply the measure of network entropy to the dynamic banking systems, where interbank networks analyzed include random, small-world, and scale-free networks. In the context of the analysis of interbank

networks, we transform adjacency matrices into stochastic matrices and then apply the concept of entropy. In this paper, we find that network entropy is positively correlated with the effect of systemic risk in the three kinds of interbank networks and that network entropy in the small-world network is the largest, followed by those in the random network and the scale-free network.

The rest of the paper is organized as follows. Section 2 describes the methodology. Section 3 presents the results of numerical simulations. And the conclusion is drawn in Section 4.

## 2. Methodology

*2.1. Modeling of Dynamic Banking Systems.* The modeling of dynamic banking systems in this paper is based on the study of Lux [27]. However, we are different in the formation of interbank lending relationships and liquidity shocks. Besides, we take bank defaults into consideration, while Lux does not. To develop a dynamic model of a banking system with $N$ banks, we start from the description of its stylized balance sheets. We assume that the assets of bank $k$ at time $t$ include investments, interbank loans, and liquid assets, denoted by $I_k(t)$, $L_k(t)$, and $M_k(t)$, respectively, and that its liabilities are composed of deposits, interbank borrowing, and net worth, denoted by $D_k(t)$, $B_k(t)$, and $W_k(t)$, respectively. At the initial time, we assume that the interbank market does not yet exist and set the structure of the balance sheet of bank $k$ at time $t = 0$ as $I_k(0) = \alpha TA_k(0)$, $M_k(0) = (1 - \alpha)TA_k(0)$, $W_k(0) = \beta TI_k(0)$, and $D_k(0) = (1-\beta)TI_k(0)$, where $TA_k(0)$ and $TI_k(0)$ denote the total assets and the total liabilities of bank $k$ at the initial time, respectively. In the simulation, we adopt the following algorithm to determine how the banking system evolves from one state to another.

The first phase is the update of liquid assets and net worth. At the beginning of time $t$, liquid assets and net worth of bank $k$ are updated as follows:

$$
\begin{aligned}
M_k(t) = M_k(t-1) &+ [D_k(t) - D_k(t-1)] \\
&- \sum_{i \in \Psi_k(t-1)} B_{ki}(t-1) [1 + r_{ki}(t-1)] \\
&+ \sum_{j \in \Phi_k(t-1)} b_{jk}(t-1) [1 + r_{jk}(t-1)], \\
W_k(t) = W_k(t-1) &- \sum_{i \in \Psi_k(t-1)} b_{ki}(t-1) r_{ki}(t-1) \\
&+ \sum_{j \in \Phi_k(t-1)} b_{jk}(t-1) r_{jk}(t-1),
\end{aligned}
\tag{1}
$$

where $\Phi_k(t)$ and $\Psi_k(t)$ denote the set of debtors and the set of creditors of bank $k$ at time $t$, respectively. $B_{ij}(t)$ is the actual amount borrowed by bank $i$ from bank $j$, and its interest rate is $r_{ij}(t)$. In this paper, we assume that the duration of all debts is one and that banks' investments remain constant over time and disregard the return from them for simplicity. Following

the study of Gatti et al. [28], we assume that the lender $j$ sets the interest rate $r_{ij}(t)$ on loans to the borrower $i$.

$$
r_{ij}(t) = \eta \left(W_j(t)\right)^{-\eta} + \eta \left(\delta_i(t)\right)^{\eta},
\tag{2}
$$

where $\eta > 0$ and $\delta_i(t) = B_i(t)/W_i(t)$. According to the study of Georg [29], we introduce the following shocks to deposits, which is the source of the formation of interbank credit lending relationships:

$$
D_k(t) = \left(1 - \gamma_k + 2\gamma_k X\right) D_k(t-1),
\tag{3}
$$

where $\gamma_k$ is a scaling parameter for the level of deposit fluctuations and $X$ is a random variable ($X \in [0, 1]$).

The second phase is default settlement. After the first phase, bank $k$ defaults due to insolvency if $W_k(t) \leq 0$. If bank $k$ defaults, it will result in the loss of its creditors. We assume that the loss caused by the default of bank $k$ is shared proportionally by its creditor banks with their respective lending sizes. After the adjustment of liquid assets and net worth by subtracting the loss, a creditor bank of bank $k$ also defaults if its net worth is less than or equal to zero, which is caused by the default contagion. This procedure keeps circulating until no bank defaults.

The third phase is the credit lending. We assume that there is a threshold ($\overline{M}_k(t)$) of liquid assets for bank $k$ at time $t$, which guarantees the continuance of regular business operations, and $\overline{M}_k(t) = \theta TA_k(t)$. For a bank with positive net worth, it is a potential borrower if its liquid assets are less than the threshold; otherwise it is a potential lender. The demand or supply of liquidity for a bank is equal to $|M_k(t) - \overline{M}_k(t)|$. From time $t = 1$, in every period, we assume that there exists a potential interbank network; namely, potential borrowers can only borrow money from potential lenders who have links with them. If the potential borrower does not obtain enough liquidity from the first randomly selected potential lender, it contacts other potential banks for the remaining funds until its total demand for liquidity is satisfied or there is no more liquidity to be allocated. For the potential lender, if the total amount of demand for liquidity received from potential borrowers is less than its loanable liquidity, all its potential borrowers' demands for liquidity can be satisfied. Otherwise, the potential lender satisfies its potential borrowers in a sequence according to the rank of their net worth from high to low until all its loanable liquidity is completely allocated.

In this paper, we assume that the funds do not transfer from lenders to borrowers until borrowers' demand for liquidity is satisfied. Banks' balance sheets will be updated according to the actual borrowing or lending. If potential borrowers' demand for liquidity is not satisfied, they default. For the sake of simplicity, the total number of banks in the banking system is constant over time. Therefore, in this paper, we assume a simple mechanism of entry-exit: a default bank is replaced by a new one. The balance sheet structure of the new bank is the same as the initial balance sheet structure of the default bank. This can be interpreted as the entry of new banks into the interbank market. In fact, this mechanism is present in the existing literature, such as the study of Gatti et al. [30].

TABLE 1: Benchmark parameters of the model.

| Parameter | Description | Benchmark value | Range of variation |
|---|---|---|---|
| $N$ | Number of banks | 100 | Positive integer |
| $[A_{\min}, A_{\max}]$ | Range of values of initial assets | $[5, 200]$ | $0 < A_{\min} < A_{\max}$ |
| $\tau$ | Pareto distribution parameter | 1.2 | Positive number |
| $\alpha$ | Initial proportion of investments | 0.9 | $(0, 1)$ |
| $\beta$ | Initial proportion of net worth | 0.08 | $(0, 1)$ |
| $\theta$ | Proportion of $\overline{M}_k(t)$ to $TA_k(t)$ | 0.08 | $(0, 1)$ |
| $\eta$ | Interest rate parameter | 0.01 | Positive number |
| $\gamma$ | Scaling parameter for the level of deposit fluctuations | 0.1 | Positive number |
| $P_1$ | Probability of connection between any two nodes in random networks | 0.3 | $[0, 1]$ |
| $K$ | Number of nearest neighbors of a node in small-world networks | 30 | Positive integer |
| $P_2$ | Probability of randomly rewiring each edge of the lattice for small-world networks | 0.01 | $[0, 1]$ |
| $\xi$ | Initial number of nodes in scale-free networks | 25 | Positive integer |
| $\zeta$ | Number of edges of a new node in scale-free networks | 15 | Positive integer |

*2.2. Network Entropy.* According to the above modeling of dynamic banking systems, we can obtain dynamic interbank networks, which can be represented by adjacency matrix $A(t) = (a_{ij}(t))_{1 \leq i,j \leq N}$. $a_{ij}(t)$ denotes the liability of bank $j$ towards bank $i$ at time $t$. In order to apply the concept of entropy, we need to transform adjacency matrices into stochastic matrices. Demetrius and Manke [31] propose the analysis of the stochastic matrix in the context of network robustness. In this paper, we adopt their method to define network entropy for dynamic banking systems. According to the adjacency matrix $A(t)$, we can obtain a stochastic matrix $(p_{ij}(t))$ from the following formula [23]:

$$p_{ij}(t) = \frac{a_{ij}(t)}{\sum_{j=1}^{N} a_{ij}(t)}. \tag{4}$$

Given a stochastic matrix $(p_{ij}(t))$, we apply the Shannon entropy formula [32] to the transition probability distribution which corresponds to the $i$th row of the stochastic matrix, and we can obtain Shannon entropy $(E_i(t))$ of node $i$ at time $t$, which is given as follows:

$$E_i(t) = -\sum_{j=1}^{N} p_{ij}(t) \log p_{ij}(t). \tag{5}$$

Moreover, we can obtain Shannon entropy $(E(t))$ of the interbank network at time $t$, which is defined as the weighted sum of entropies of nodes and is given as

$$E(t) = \sum_{i=1}^{N} \pi_i(t) E_i(t), \tag{6}$$

where $\pi_i(t)$ is the $i$th component of the unique invariant distribution of the corresponding stochastic matrix $(p_{ij}(t))$.

## 3. Simulation Results

According to the studies [2–4], we analyze network entropies in three kinds of potential interbank networks, namely,

random networks, small-world networks, and scale-free networks. According to the algorithms provided by Erdös and Rényi [33], Watts and Strogatz [34], and Barabási and Albert [35], we construct the three kinds of potential interbank networks. Following the study of Lux [27], we assume that the initial total assets of all banks are drawn from a truncated Pareto distribution over the range $[A_{\min}, A_{\max}]$ with the power-law parameter $\tau$. Referring to the studies [27–29], we set the parameter values in Table 1. If not stated otherwise, our numerical simulations are performed with the parameters given in Table 1. For each set of parameters, we repeat every simulation 100 times to average out stochastic effects.

*3.1. Network Entropy and Systemic Risk.* Demetrius and Manke [31] study the process of network disintegration under random node removal for the three types of networks with different topological entropies and find that network entropy is positively correlated with robustness, where networks analyzed include scale-free networks, random networks, and regular networks. In the study of Demetrius and Manke [31], robustness pertains to the insensitivity of measurable parameters of the system to changes in its internal organization and includes dynamical robustness and topological robustness. Now we investigate whether network entropy can be used as a measure of robustness for banking systems from the perspective of systemic risk. In this paper, we adopt the number of default banks ($S$) to measure the effect of systemic risk. Figure 1 shows the results of network entropies and the effects of systemic risk, where (a), (b), and (c) correspond, respectively, to the results of random, small-world, and scale-free networks. From Figure 1, it can be seen that the change trend of network entropies is similar to that of systemic risk.

Moreover, we adopt Pearson's correlation to investigate the correlation between network entropies and the effects of systemic risk. Table 2 shows Pearson's correlation coefficients between network entropies and the effects of systemic risk under different parameter values. We can observe that network entropy is positively correlated with the effect of systemic risk in the three kinds of interbank networks. Therefore, we provide computational and analytical support
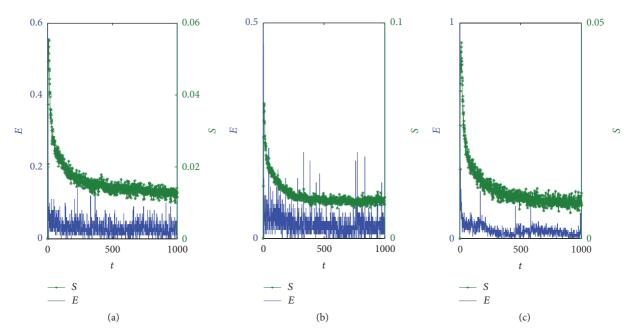
Figure 1: Network entropies and effects of systemic risk. (a), (b), and (c) correspond, respectively, to the results of random, small-world, and scale-free networks.

Table 2: Pearson's correlation coefficients between network entropies and systemic risk.

| Parameter | Random network | Small-world network | Scale-free network |
| --- | --- | --- | --- |
| $\gamma_k = 0.1$ | 0.3077 | 0.2729 | 0.5523 |
| $\gamma_k = 0.2$ | 0.3108 | 0.3236 | 0.5508 |
| $\gamma_k = 0.3$ | 0.1992 | 0.2845 | 0.5119 |
| $\overline{M}_k = 0.06$ | 0.2556 | 0.2934 | 0.4834 |
| $\overline{M}_k = 0.08$ | 0.3077 | 0.2729 | 0.5523 |
| $\overline{M}_k = 0.1$ | 0.2922 | 0.3176 | 0.5516 |
| $\tau = 1.2$ | 0.3077 | 0.2729 | 0.5523 |
| $\tau = 2$ | 0.3098 | 0.2106 | 0.4747 |
| $\tau = 3$ | 0.2644 | 0.1734 | 0.4688 |

for that network entropy which can reflect the robustness of banking systems against systemic risk to a certain extent. Moreover, the network entropy can predict the direction of changes of systemic risk and characterize the stable states of dynamic banking systems.

*3.2. Network Entropy and Network Structure.* We now investigate the difference of network entropies in the three kinds of interbank networks. Figure 2 shows the network entropies in the random network, the small-world network, and the scale-free network. We can see that the values of network entropies present a trend of decrease after the increase, and they tend to be stable with time evolution. In fact, from Figure 1, we know that the effect of systemic risk has the same change trend as network entropies. Moreover, bank defaults change interbank network structures and then result in the above evolution characteristics of network entropies. Besides, we can observe that the value of network entropy in the small-world network is the largest among the three kinds of interbank networks, followed by those in random and scale-free networks.
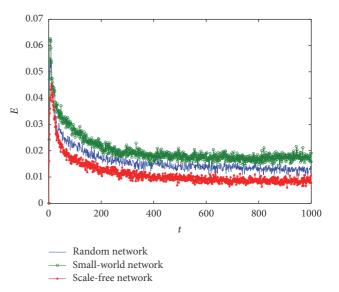


Figure 2: Network entropies in the three kinds of interbank networks.
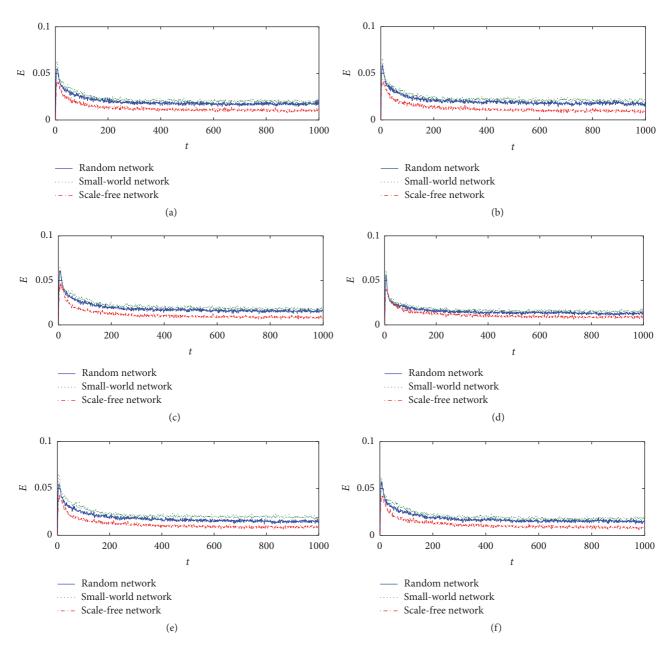
FIGURE 3: Network entropies in the three kinds of interbank networks under different parameter values. (a)–(f) are the results when $\gamma_k = 0.2$, $\gamma_k = 0.3$, $\overline{M}_k = 0.06$, $\overline{M}_k = 0.1$, $\tau = 2$, and $\tau = 3$, respectively.

According to the above model, we can see that $\gamma_k$, $\overline{M}_k$, and $\tau$ are the key parameters in the establishment of interbank networks. Therefore, we investigate the effect of these parameters on the above results, which is shown in Figure 3. We can observe that the above results do not change, despite the fact that the values of the parameters are different. From Figure 3, it can be seen that the network entropy trajectories are very similar among completely different network structures, and this similarity holds if we change some parameters. The probable reason for this result is that the effects of systemic risk are similar among different network structures.

## 4. Conclusions

In this paper, we first construct artificial banking systems and then investigate network entropy of dynamic banking systems, where the three kinds of potential interbank networks are analyzed, namely, random networks, small-world networks, and scale-free networks. First, simulation analysis shows that the change trend of network entropies is similar to that of systemic risk and that network entropy is positively correlated with the effect of systemic risk in the three kinds of interbank networks. Besides, we find that the value of network entropy in the small-world network is the largest

among the three kinds of interbank networks, followed by those in random and scale-free networks.

In this paper, we analyze the network entropy in known network topologies. However, several works in the systemic risk and banking literatures show that financial networks are organized in a core-periphery structure. Therefore, we believe that more research needs to be done in order to understand how network entropy behaves in financial networks. For example, how does network entropy behave in core-periphery structures? Moreover, is the entropy dependent on the network core size? Or does it show the same pattern regardless of the periphery and core sizes? Similar to most of the literature in this field, we define systemic risk as the number of defaulting banks. In the future, we would consider the total loss of capitalization of the banking system as a robustness indicator.

## Conflicts of Interest

## Acknowledgments

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*Research Article*

# Connectivity, Information Jumps, and Market Stability: An Agent-Based Approach

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We propose a metamodel to assess simulated market stability by introducing information connectivity in an agent-based network. The market is occupied by heterogeneous agents with different behaviors, strategies, and information connectivity. A jump-diffusion process simulating events that may occur in the market is introduced. Agents information awareness varies along with agents propensity to respond to the information jump and jump size. A jump reshuffles market positions based on agents risk preferences determined by behavior and strategy. We examine the effect of information awareness on the volatility index of the simulated market in a scale-free market network. The analysis is performed by developing five experiments wherein the first one corresponds to systemic information ignorance state. Three experiments examine the role of hubs, normal agents, and hermits in the network when intermediate combinations of agent types have information awareness. The fifth experiment corresponds to the systemic information awareness with all agents being informed. The results show that the simulated market is driven to instability in a similar manner to patterns observed in a crisis where all agents become homogeneous in information awareness of events. Hubs contribute to increased connectivity and act as amplifiers of good, bad, or inaccurate information or sentiment.

## 1. Introduction

How should a *financial economic environment simulator* (FEES) be designed and what should it inform? Our agent-based approach ultimately provides such a FEES just as a flight simulator offers the pilot a close-to-reality flying experience. However, the stage of reaching a truly complex financial environment simulator is not yet mature enough and this paper is one more step in that direction as it introduces information jumps, heterogeneous agent behavioral responses, information connectivity attributes, and a regulatory model in one financial network metamodel; the term metamodel in this paper is used in the sense that the ABM approach for a financial market network simulation constitutes a *model of models*. For the desired level of complexity, the financial economic dynamics cannot be, thus far, expressed analytically through a set of neatly justified equations. Thus, the advantage of agent-based metamodels lies in their ability

to create sufficiently rich scenarios once the appropriate components of a financial system are included and endowed with their rules of interaction and transaction along with an initial state.

In this paper, we build on the previous work of Khashanah and Alsulaiman [1] where bounded rational heterogeneous market metamodel of network of networks was developed to explore the effect of the interaction of heterogeneous agents on the market volatility and capital. This paper develops the first metamodel for incorporating information jumps as shocks in an agent-based approach and to equip agents with heterogeneous response to information jump stimuli. Shocks are essential for meaningful financial systemic risk simulations. It is our philosophy to emphasize the quantitative-simulative-empirical approach, which aims to quantify the quantitative aspects of the problem, simulate the nonquantifiable aspects of the problem, and calibrate the metamodel parameters with empirical data. We do not agree with the

excessive emphasis of some economic literature on analyzing systemic risk from the perspective of banking systems only. Markets, banks, intermediaries, media, and many other types of agents collectively interact as nonlinear dynamical systems in the financial ecosystem. Markets act as forward looking indicators of collective intelligence in all sectors including the banking system.

The novelty of our approach in this paper is to equip agents with new features representing their response to information flows including jumps. The idea that prices are information-driven was explained by many researchers starting with the work of Merton [2], yet those models assume that information has been factored into the price and may cause a jump in the price. Therefore, we obtain a jump-diffusion model depicting postulated price dynamics. However, market metamodels are interested in the mechanics by which the network supports the conversion of news into price impact. So far, market models in ABM have not included information sources as drivers of emerging patterns in market metamodels. As a first attempt to addressing this shortcoming, we observe that news are mappings of events into information from variable agent perspectives. Information packets flow in the network wherever there is a channel that supports that flow. Therefore, agents connectivity in our network refers to the ability to conduct information between two nodes. Thus we call it information connectivity. Since events map into information and events come in bursts of variable sizes, it makes sense to model information flow in a financial network as a stochastic jump-diffusion process that accounts for market events. Agents become receivers of information flows. To equip agents with this feature, we introduce the idea of agent *information awareness* in the network. Agents awareness variability of events and their variability of mapping events into actionable information both impact financial systemic patterns appearing in the financial system. In general, agents have heterogeneous information awareness; otherwise, agents are said to be homogeneous in information awareness.

In ABM we calibrate the model to either theoretical finance or empirical observations and then explore alternative scenarios that describe the relationship between macrobehaviors and micromotivations via the change of market microstructure. This relationship may be explored to produce the systemic patterns of markets under various scenarios. In this study, we take the maximum volatility index as a proxy measure of the systemic stability of the simulated market.

For the paper to be manageable, the analysis is carried out by developing five scenarios for agents information awareness of market events. The first and last scenarios are extreme cases where it is assumed that none of the agents is informed and, in the other, all agents are uniformly informed of the events. The second scenario assumes that the hub agents in the network are aware of events and they pass this information to the agents that are directly connected to them. Similarly, the third and fourth scenarios assume that agents with normal connections and hermits are aware of information jumps in the market, respectively.

The outline of this paper is as follows: Section 2 offers a literature review. Section 3 presents definitions of the market metamodel, models, and rules of interactions. Section 4 extends the connectivity model and Section 5 shows the simulation results and analysis of the experiments. We conclude with Section 6.

## 2. Literature Review

Researchers attempted to discover the causes of the anomalies in the stock market even before the development of the ABM. For over two decades researchers used ABM to explore complexities of a financial market. This includes the pioneering work of Kim and Markowitz [3] in 1989, the zero-intelligence model of Gode and Sunder [4] in 1993, the heterogeneous belief system of Brock and Hommes [5] in 1998, the Santa Fe artificial stock market [6] in 1999, and the Ising Model by Iori [7] in 2002.

Rules of interactions among market agents often find its roots outside the ABM literature. For example, Campbell and Kyle [8] derived an equilibrium model to examine the dynamics of stock prices. The model was constructed in a way that noise traders interact with smart money investors "fundamental traders." They conclude that "overreaction" to the news about the fundamental value creates an important type of noise to stock movements; the term "noise" has a different connotation than the one used in signal processing.

Existing models for analyzing financial systems are based on top-down approaches, as explained in Bookstaber [9], and focus on a comprehensive view while endeavoring to break down the system into components, which may result in losing a portion of variability such as behavior and agent interactions. Additionally, Bookstaber categorizes the applications of ABM in economics and finance into banks and assets management; real-estate markets; mortgage payments; payment systems and credit risk; financial market microstructure; and macroeconomics. In [10] Bookstaber et al. suggest that the new version of the stress test should require more emphasis on institutional interactions and feedback effects. Therefore, Bookstaber et al. recommend using more adaptable models, such as agent-based model, that are capable of capturing more elements of systemic complexity and can evolve freely from the history in a characteristic design similar to the real systems.

Sornette [11] called attention to utilizing models of complexity theory, such as network analysis and agent-based models (ABM) to assess systemic risk. Helbing [12] expressed that the catastrophe engendered by human factors can not be clarified by analytical approaches only but rather requires a collective comprehension of the social dynamics.

Thurner et al. [13, 14], as an implementation of ABM in banking and asset management, studied the effect of leverage on market behavior by building an ABM that allows borrowing long assets with a margin. The results of such actions cause fluctuations in prices, fat tail returns, and volatility clustering. As a mechanism for contagion, a bank may liquidate some of its assets to raise sufficient capital in response to a margin call. Asset liquidation to cover margin calls becomes a systemic risk when fire sale liquidations cause a downward spiral price destruction and eventual crashes in extreme cases.

Poledna et al. [15] developed an ABM for simulating asset credit regulation policy. The assessments were performed under three different scenarios where in the first scenario the regulator imposes limits on maximum leverage only. In the second and third scenarios, Poledna et al. assessed the Basel II policy and alternative hypothetical ones by which the banks utilize options to hedge the risk completely. Kuzubaş et al. [16] simulate a financial network to examine the leverage effect in the banking system. The results indicate that the difference in leverage significantly affects the measure of systemic risk. The paper studied the impact of Basel III regulations for charging higher capital requirements for banks with higher leverage. The results confirmed that such a rule would increase the resilience of the system. In real-estate market application, Gilbert et al. [17] built up an ABM to research the housing market of the UK. The model accounts for interactions between the buyers, seller, and realtors given a set of endogenous and exogenous parameters representing agents attributes such as wealth and regulator control factors like interest rate. The results of the model show that house prices are sensitive to interactions of interest rate and loan-to-value (LTV) ratio. In addition, agents tend to cluster with high/low priced houses according to wealth. Also, altering the tax rate has an insignificant effect on house prices. Furthermore, a shock in the market causes fluctuations in house prices. Erlingsson et al. [18] generated an ABM of credit network and performed a set of simulation experiments to inspect the requirements for bank mortgages. The results show that lowering the mortgage standards increases house prices in the short term but leads to an unstable economic system in the long run and may lead to a recession. However, limiting home loans prerequisites helps market stability despite the possibility of slowing down economic growth. Other studies of systemic risk due to real-estate markets include the work of Carstensen [19], Bjarnason [20], and Ge [21].

Also, researchers addressed the role of network theory and agents direct interactions of model such as the models of Panchenko et al. [22], Yeh and Yang [23], Delellis et al. [24, 25], and Khashanah and Alsulaiman [1, 26]. For more details of the ABM in market microstructure, readers may refer to [27–30].

In postmodern finance with superconnectivity, agents in markets trade information dynamics. This paper is the first to introduce an information-based connectivity to model a financial network of market agent interactions, including information jumps, with heterogeneous agent response system to information flows using the ABM methodology. This paper investigates the impact of information connectivity on systemic volatility, which is proposed as an elementary proxy for the simulated market stability.

## 3. The Metamodel and Models

In this section, based on the metamodel of Khashanah and Alsulaiman [1, 26], new definitions are introduced and the new features are explained. In the previous work the authors showed how the market volatility changes under heterogeneous agents behavior. In this paper we simulate

financial market realizations to assess what may cause market instability as a function of information flow with jumps with heterogeneous agent information awareness and variable connectivity. For this purpose, we introduce the following definitions

*Definition 1.* The set $\mathscr{B} = \{B_1, B_2, \ldots, B_m\}$ denotes agent behavioral types; the set $\mathscr{S} = \{S_1, S_2, \ldots, S_k\}$ denotes the set of $k$ strategies; and $\mathscr{D} = \{0, 1, \ldots, n-1\}$ denotes a possible degree of an agent with $n$ the number of agents in the metamodel.

*Definition 2.* An agent is defined in terms of the attribute vector $\mathbf{a}^{(i)} = (b_j^{(i)}, s_k^{(i)}, d_l^{(i)})$, $1 \le i \le n$, $b_j^{(i)} \in \mathscr{B}$, $s^{(i)} \in \mathscr{S}$, and $d_l^{(i)} \in \mathscr{D}$.

*Definition 3.* The agent lattice $\mathscr{A}$ is the set $\mathscr{A} = \mathscr{B} \times \mathscr{S} \times \mathscr{D}$.

*Definition 4.* A market $\mathscr{M}$ consists of an agent lattice $\mathscr{A}$, a set of assets $\mathscr{T}$, a set of regulatory frameworks $\mathscr{R}$, and a set of connectivity characteristics $\mathscr{C}$ summarized as the 4-tuple $\mathscr{M} = (\mathscr{A}, \mathscr{T}, \mathscr{R}, \mathscr{C})$.

*Definition 5.* A market $\mathscr{M}$ is said to be homogeneous in *behavior* if agents exhibit one behavior so that $\mathscr{B}$ is a singleton; otherwise the market is said to be heterogeneous in behavior. A market $\mathscr{M}$ is said to be homogeneous in *strategy* if agents have one strategy so that $\mathscr{S}$ is a singleton; otherwise, it is heterogeneous in strategy. A market $\mathscr{M}$ is said to be homogeneous in connectivity if agents are equally likely to be connected forming a random network; otherwise, $\mathscr{M}$ is said to be heterogeneous in connectivity.

*Definition 6.* A metamodel is a model of models. A market $\mathscr{M}$ is a metamodel consisting of its component models and their interactions expressed symbolically as the 4-tuple $\mathscr{M} = (\mathscr{A}, \mathscr{T}, \mathscr{R}, \mathscr{C})$.

Current practices of scientific methodology assume linear interactions of models to produce a metamodel whose accuracy is as good as the minimum accuracy of its component models; this topic deserves attention as it has impact on advancing both complexity science and science complexity. A new definition of approximation and intermodel propagation of error is being developed in the context of quantitative-simulative-empirical methodology.

The metamodel is designed so that agents may interact directly in the trading environment where the network topology follows a scale-free network type that is structured based on the preferential attachment algorithm [31] with the initial number of hubs $H$. In the experiments of this paper, with Definitions 1 through 6, agents $\mathbf{a}^{(i)}$, $i = 1, 2, \ldots, 211$, with attribute vector $\mathbf{a}^{(i)} \equiv (b_j^{(i)}, s_k^{(i)}, d_l^{(i)})$ with $k = 1, \ldots, 4$, $j = 1, 2, \ldots, 6$, and $l = 0, 1, \ldots, 210$ referring to an agent degree. The behavioral types set is identified with $\mathscr{B} = \{R, RO, RC, L, LO, LC\}$ where $R, RO, RC$ stand for the risk averse, risk averse with overconfidence and risk averse with conservative types. In addition, $L, LO, LC$ stand for the loss averse, loss averse with overconfidence and loss averse with conservative types.

The strategies set, $\mathcal{S}$, consists of four strategies: the first strategy accounts for arbitrary traders that make investment decisions randomly, called zero-intelligence agents, and denoted by $Z$. The second investment strategy accounts for fundamental traders, denoted by $F$, who concentrate on the fundamental value of the asset, not on historical prices. The third type is referred to as momentum traders or technical traders, denoted by $T$, and the fourth type is called adaptive traders, denoted by $N$, with artificial intelligence capabilities using neural networks (NN); thus, $\mathcal{S} = \{Z, F, T, N\}$. The last two types distinguish themselves from the first two types in their practice that historical prices contain relevant information to future decisions.

Using Definition 5, the market $\mathcal{M}$ under consideration is heterogeneous in behavior, strategy and connectivity. The agent lattice $\mathcal{A}$ in this experiment is the collection of agents possible states. Thus the market $\mathcal{M}$ contains $|\mathcal{B} \times \mathcal{S}| = 24$ possible types of agents as a result of combinations of behaviors and strategies. Allowing for degrees to change, we obtain the number of points in the agent lattice to be $|\mathcal{A}| = 24 \times 210 = 5040$, however, only 211 points in the agent lattice can be occupied at a given state of the market. Putting a practical upper limit on the agent degree reduces the agent lattice cardinality to reduce computational complexity.

The set of assets $\mathcal{T}$ in this experiment contains two assets: the risk-free rate asset and one risky asset. Furthermore, the regulatory set $\mathcal{R} = \{r_f, c, B, S\}$ with the risk-free rate $r_f$, taxation $c$, and buying and selling power $B$ and $S$ as control parameters. The regulatory environment can be designed to approximately map its real constructs in various financial economic systems.

The set of *information connectivity* characteristics depends on the network topology of the market. In this paper, our metamodel is heterogeneous in information connectivity in the sense of Definition 5 since the underlying network is scale-free. As part of connectivity characteristics, we introduce the new feature of heterogeneous ability to acquire information and be aware of information flows and call it *information awareness* in the market network. We consider this feature to be a function of agent connectivity and the coordinates of the agent in the agent lattice. More precisely, it is postulated that this property is a function of betweenness centrality of an agent. In our metamodel, heterogeneous agent awareness and the variability of mapping events into actionable information depending on agent awareness impact systemic patterns. The relationship between the interconnectivity and systemic risk depends on the definition of connectivity. In most studies on connectivity and systemic risk, connectivity is implied by the level of cash flow and risk flow, which are two important dimensions. In our context, information connectivity in terms of information flow is emphasized and, with third-party settlement, there is no direct transactional connectivity in this model. There is a reason for this type distinction between connectivity types: information dynamics, including market sentiments, lead to decisions on positions (such as buy, sell, or hold), which result in some transactions. In general, an information connectivity network is not identical to the corresponding transactional or cash flow connectivity network of the same

market but rather the networks are dynamically linked with a variable lag. In an upcoming work, multidimensional connectivity types are considered including agent-bank, transactional, cash flow, and risk connectivity.

Agents belong to the market $\mathcal{M}$ to achieve their objectives. Agents myopically aim to maximize their utility function given the wealth constraint [32]. The solution of the optimization problem is given by

$$x^*_{i,j,t} = \frac{E_{i,j,t}\left(p_{t+1} + d_{t+1}\right) - \left(1 + r_f\right) p_t \pm c p_t}{\lambda_{i,j} \nu_{i,j} \beta_{i,j} \sigma^2_{i,t,p_{t+1}+d_{t+1}}}, \quad (1)$$

where $j$ denotes the behavior of the agent, which is identified with the set of behavioral types $\mathcal{B}$. With keeping the order, $\{R, RO, RC, L, LO, LC\} = \{1, 2, 3, 4, 5, 6\}$ so that, for example, $j = 1$ corresponds to risk averse type R and so on. The expectation $E_{i,j,t}(p_{t+1} + d_{t+1})$ is the expected price and dividend for the next time step, which is crucial for the determination of optimal holding. The expectations of heterogeneous agents are by necessity diverse and they are determined based on investment strategies explained in the next section. Here $\sigma^2_{i,t,p_{t+1}+d_{t+1}}$ is the conditional standard deviation of price and dividend at time $t + 1$. For simplicity, $\sigma^2_{i,t,p_{t+1}+d_{t+1}}$ is assumed to be constant of unit value. The change in the sign in the above equation opposite the state of $E_{i,j,t}(p_{t+1} + d_{t+1}) - (1 + r_f)p_t$ makes $x^*_{i,j,t} = 0$. By the change in the sign, we mean that the negative sign follows the positive state of $E_{i,j,t}(p_{t+1} + d_{t+1}) - (1 + r_f)p_t$ and the positive sign follows the negative state of $E_{i,j,t}(p_{t+1} + d_{t+1}) - (1 + r_f)p_t$.

Agent behaviors are quantified by assigning values to coefficients. Hence, the risk aversion coefficient $\lambda = 4$, loss aversion coefficient $\beta = 2.5$ as $j = 4$, 5 *or*, 6, and $\Delta W_{i,j,t} < 0$. Consider overconfidence/conservative $\nu$ where $\nu = \nu_c = 3$ as $j = 3$ or 6, $\nu = 1$ if $j = 1$ or 4, and $\nu = \nu_o = 0.75$ as $j = 2$ *or* 5. The intuition behind the agent behaviors quantification is that the overconfidence/conservative agents tend to hold, sell higher, or lower portion of stock over the neutral traders. Also, the loss averse traders tend to reduce their positions as confronted with a loss at a given reference point $\Delta W_{i,j,t}$ as it has been indicated by Kahneman and Tversky [33]—individuals are impacted by losses more than profits.

The expectation $E_{i,j,t}(p_{t+1} + d_{t+1})$ in (1) is a critical variable in the model. It is estimated individually given the agent trading strategy. The zero-intelligence agents quantitatively and randomly expect future prices and dividends as

$$\widehat{E}_{i,j,t} = U\left(p_0 - 2a, p_0 + 2a\right) + U\left(d_0 - 2b, d_0 + 2b\right), \quad (2)$$

where $a$ and $b$ are fixed at $a = 270$ and $b = 8$ in this study to reduce computational complexity and $U = U(a, b)$ is the uniform distribution on $[a, b]$ whose parameters can be calibrated.

The fundamental agents keep an eye on the fundamental process of the asset price where it is assumed that the dividends follow the process:

$$d_t = d_{t-1} e^{(\mu_d - (1/2)\sigma^2_{t-1})\Delta t + \sigma_{t-1}\sqrt{\Delta t}W_t}, \quad (3)$$

where $\mu_d = 0.05$ is the growth rate of the dividend, $\sigma_t$ is the dividend volatility, and $W_t$ is a Wiener process with normal distributions $\sim N(0, \sigma_{W_t})$. The variance $\sigma_t^2$ may be estimated using GARCH (1,1):

$$\sigma_t^2 = \alpha_0^g + \alpha_1^g r_{t-1}^2 + \beta^g \sigma_{-1}^2, \tag{4}$$

where $\alpha_0^g = 0.0097$, $\alpha_1^g r_{t-1}^2 = 0.168$, and $\beta^g \sigma_{-1}^2 = 0.766$. The fundamental prices of the asset then follow Williams [34]:

$$p_t^f = \frac{d_t}{r_f}, \tag{5}$$

where $r_f$ is the risk-free rate. This fundamental value method works well as long as $0 < r_0 \leq r_f$.

The expected dividends and prices for agent $i$ are

$$\widehat{E}_{i,j,t}\left[p_{t+1}\right] = \frac{\widehat{E}_{i,j,t}\left[d_{t+1}\right]}{r_f}, \tag{6}$$

$$\widehat{E}_{i,j,t}\left[p_{t+1} + d_{t+1}\right] = \widehat{E}_{i,j,t}\left[p_{t+1}\right] + \widehat{E}_{i,j,t}\left[d_{t+1}\right].$$

The momentum agents verify the saying "the trend is your friend," and they buy/sell if the previous returns are positive/negative:

$$\widehat{E}_{i,j,t}\left[d_{t+1}\right] = \begin{cases} d_t + d_t\left(\varphi_{i,j,t}\right) & \text{if } r_{d,t} > 0 \\ d_t - d_t\left(\varphi_{i,j,t}\right) & \text{if } r_{d,t} < 0 \\ d_t & \text{if } r_{d,t} = 0, \end{cases}$$

$$\tag{7}$$

$$\widehat{E}_{i,j,t}\left[p_{t+1}\right] = \begin{cases} p_t + p_t\left(\varphi_{i,j,t}\right) & \text{if } r_{d,t} > 0 \\ p_t - p_t\left(\varphi_{i,j,t}\right) & \text{if } r_{d,t} < 0 \\ p_t & \text{if } r_{d,t} = 0, \end{cases}$$

where $\varphi_{i,j,t} \sim U(0.8, 1.2)$.

The last trading strategy in $\mathcal{S} = \{Z, F, T, N\}$ in the strategy model is an adaptive trading strategy $N$ where the neural network is utilized for prediction execution. The designed neural network is a feed-forward network where the input to nodes $IN_{i,j}$ represents past returns of the stock that are uniformly distributed among the agents with the minimum of one previous return and maximum of ten past returns. Also, the neural network consists of one hidden layer that may be composed of one to ten nodes $HN_{i,j}$ with equal probability for agent $i$. The output of the neural network is the expected return at time $t + 1$ that will be mapped to $\widehat{E}_{i,j,t}[p_{t+1}]$ and comparative approach applies to $\widehat{E}_{i,j,t}[d_{t+1}]$. Agents will learn by updating the weights inside the neural network from the training data $L$ where the training follows uniform distribution from 10 to 100. The neural network is invoked customarily according to parameter $K = 0.1$. The stopping criteria are subject to maximum iteration $iter$ or base error $e$.

Agents may change their initial decisions of stock holding $x_i^*$ as a consequence of the interaction with other agents. Whenever agents have a direct interaction with each other

with a chance to share their market sentiment, agents may be influenced to change their outlook on the market as a result of the interaction. Notice that market sentiment sharing is a form of information connectivity. The final holding decision $X_i^*$ is then constructed as the weighted average of the agent initial decision and initial decisions of the connected agents:

$$X_{i,j,t}^* = \begin{cases} \alpha_{i,j} x_{i,j,t}^* + \dfrac{\left(1 + \alpha_{i,j}\right)}{\sum_{k=1, k \neq i}^N I_{ik}} x_{i,j,t}^* & \text{if connections} > 0 \\ x_{i,j,t}^* & \text{otherwise,} \end{cases} \tag{8}$$

where $X_{i,j,t}^*$ is the final decision for agent $i$ and $\alpha$ is a given weight for the initial decision of holding shares of stock $x_{i,j,t}^*$ for agent $i$, $N$ is the total number of agents, and $I_{ik} = 1$ if $k$ is connected to $i$ and zero otherwise.

The price formation will follow the price adjustment method [35–38]. The price adjustment process achieves local price equilibrium in the market based on the aggregate bids and offers

$$p_t = p_{t-1}\left[1 + \eta\left(B_t - O_t\right)\right], \tag{9}$$
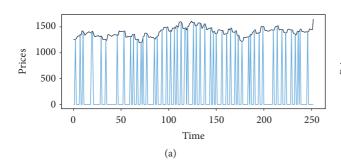
where $p_t$ is the market price at time $t$, $\eta$ is the price adjustment speed relative to the spread, that is, a simplified form of market efficiency. Further, $B_t$ represents the total number of bids among all agents and $O_t$ is the total number of offers.

## 4. Exogenous Factors and Market Reactions

In this study, information flow including jumps is introduced in the ABM metamodel. The market $\mathcal{M}$ "consumes" information and converts it through agent responses into new realizations of prices and positions. The market $\mathcal{M}$ may be influenced by nonsystematic events such as political, economic, and natural disasters, wars, or, in this day and age, tweets and fake news. For asset price dynamics, Merton's jump-diffusion model superposes jumps on a diffusion process [2] with jumps following a compound Poisson process. Merton's model is a special case of Lévy processes. Most literature in finance on jumps has been in conjunction with option pricing as the underlying asset undergoes a jump. A good reference on the subject of jumps is the book by Tankov [39]. For the purpose of this paper, and as a first approximation, we postulate that the information flow $I_t$ process follows a jump-diffusion process. The process of jumps follows the notation and construct in [40], with the usual definition of a probability space $(\Omega, \mathcal{F}, P)$ as a probability space with information filtration $\{\mathcal{F}_t\}_{t \geq 0}$ and the information process $I_t$ defined on the probability space with dynamics expressed by the stochastic differential equation (SDE)

$$\frac{dI_t}{I_t} = \mu dt + \sigma dW_t + J(Q)\, dN_t, \tag{10}$$

where $\mu$ is the drift coefficients and $\sigma$ is the volatility—both are assumed to be constant. $W_t$ is a continuous Brownian

FIGURE 1: Simulated process $\lambda = 0.2$ (a) and empirical jumps of 2008 S&P 500 prices (b).

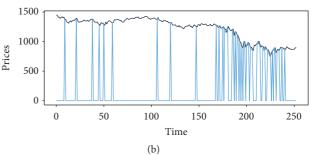TABLE 1: Population size of agents types.

|  | $Z$ | $F$ | $M$ | $N$ |
|---|---|---|---|---|
| Population size |  |  |  |  |
| R | 7 | 8 | 10 | 8 |
| RO | 10 | 7 | 10 | 10 |
| RC | 7 | 8 | 10 | 9 |
| L | 8 | 9 | 10 | 9 |
| LO | 7 | 7 | 7 | 10 |
| LC | 11 | 14 | 7 | 8 |

motion process and $N_t$ is Poisson process adapted to the filtration with constant rate $\lambda$ with $P(N_t = n) = ((\lambda t)^n/n)e^{-\lambda t}$. As in [40], the term $J(Q)$ is a jump amplitude process. The three processes, $W_t$, $N_t$, and $J(Q)$, are assumed to be independent and the jump process, as in Merton model, is taken to be a compound Poisson process in the sense that we can sum jump amplitudes of Poisson jumps occurring over a given time interval. Using Itô calculus the solution of (10) takes the form

$$I_t = I_{t-1} e^{(\mu - (1/2)\sigma^2)\Delta t + \sigma \sqrt{\Delta t} W_t + Q \Delta N_t}. \tag{11}$$

The price process is assumed to follow the same dynamics. Our implementation calibrates $\lambda$ to empirical jumps of price time series with a predefined jump threshold. For example, for the S&P500 price process, for the year of 2008, with a threshold of ±10%, we collect return jumps in the set $J_{2008}\{r_{t_k} : |r_{t_k}| > 0.1\}$, which is a time subseries of the return time series.

The jump size distribution, $Q$, can be assumed to be normally distributed which sufficiently achieves our implementation for this paper

$$Q_t = \mu + \sigma \epsilon_t \tag{12}$$

with constant $\hat{\mu}$ being the jump mean and $\hat{\sigma}$ being the jump standard deviation. Kou and Wang assume double exponential distribution as in "option pricing under a double exponential jump-diffusion model," Management Science, 50 (2004), pp. 1178–1192. For example, in the 2008 sample we find

$\hat{\mu} = -0.052$ for negative jumps and $\hat{\mu} = 0.045$ for positive jumps. The $\hat{\sigma} = 0.022$ is the jump standard deviation and $\epsilon_t \sim N(0, 1)$.

Figure 1 shows the empirical jumps of the S&P500 in 2008 (a) and the simulated process for the corresponding period with $\lambda = 0.2$. The empirical jumps exhibit the phenomenon of jump clustering, which invites, in an upcoming work, the use of more sophisticated jump processes such as Hawkes self-exciting processes.

In this metamodel, the jump is restricted to being in response to an exogenous shock of the nonsystematic type as Merton [2] explained "the Poisson distributed 'event' is an arrival of an important piece of information about the stock." However, the news arrival is converted by agents into actions of buy, sell, or hold, which collectively reflect in the jump price. In other words, the price jump is in fact an aggregate response to the stimuli of a jump in the information space. If the market is homogeneous in behavior, strategy, and connectivity, agents' response will be homogeneous.

Heterogeneous awareness of news produces variable time responses, so, as a first attempt, we relate the instances of jumps to agents' connectivity. There are a total of 211 agents in the trading space by which they remain distributed according to the behavior and strategy as shown in Table 1.

The number of direct relations is varied among agents. The network degrees follow a power-law distribution with $H = 5$. We divided agents in terms of connectivity into three classes: hubs, normal, and hermits. The links for each agent are demonstrated in Table 2.

TABLE 2: Distribution of connections among the agents (∗∗ for hubs, ∗∗∗ for normal, and ∗ for hermits).

| Agent type | Agent index | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| RZ | 6* | 5* | 5* | 5* | 5* | 6* | 5* | — | — | — | — | — | — | — |
| RF | 33** | 42** | 32** | 36** | 23** | 49** | 32** | 35** | — | 29** | — | — | — | — |
| RM | 42** | 31** | 45** | 48** | 52** | 31** | 25** | 35** | 45** | — | — | — | — | — |
| RN | 24** | 23** | 14** | 15** | 13** | 18** | 21** | 24** | — | 9*** | — | — | — | — |
| ROZ | 15** | 17** | 11** | 11** | 18** | 16** | 20** | 11** | 17** | — | — | — | — | — |
| ROF | 19** | 19** | 19** | 13** | 11** | 15** | 7*** | — | — | — | — | — | — | — |
| ROM | 16** | 8*** | 9*** | 10** | 13** | 8*** | 9*** | 17** | 13** | 11** | — | — | — | — |
| RON | 21** | 12** | 8*** | 9*** | 10** | 10** | 6* | 9*** | 16** | 12** | — | — | — | — |
| RCZ | 10** | 13** | 9*** | 10** | 10** | 5* | 8*** | — | — | — | — | — | — | — |
| RCF | 6* | 7** | 12** | 6* | 9*** | 11** | 7*** | 11** | — | — | — | — | — | — |
| RCM | 13** | 8*** | 8*** | 6* | 11** | 8*** | 10** | 10** | 8*** | 6* | — | — | — | — |
| RCN | 6* | 8*** | 6* | 8*** | 8*** | 9*** | 7*** | 9*** | 8*** | — | — | — | — | — |
| LZ | 5* | 7*** | 6* | 7*** | 6* | 6* | 7*** | 6* | — | — | — | — | — | — |
| LF | 15** | 8*** | 8*** | 8*** | 11** | 6* | 9*** | 8*** | 6* | — | — | — | — | — |
| LM | 6* | 6* | 6* | 5* | 6* | 7*** | 6* | 6* | 8*** | 6* | — | — | — | — |
| LN | 6* | 7*** | 8*** | 7*** | 5* | 5* | 6* | 5* | 6* | — | — | — | — | — |
| LOZ | 9*** | 9*** | 6* | 9*** | 8*** | 8*** | 6* | — | — | — | — | — | — | — |
| LOF | 4 | 8*** | 7*** | 6* | 7*** | 9*** | 10** | — | — | — | — | — | — | — |
| LOM | 6* | 7*** | 7*** | 7*** | 6* | 6* | 7*** | 8*** | — | — | — | — | — | — |
| LON | 5* | 5* | 6* | 5* | 5* | 5* | 6* | 8*** | 5* | 6* | — | — | — | — |
| LCZ | 10** | 6* | 7*** | 6* | 6* | 7*** | 6* | 5* | 5* | 5* | 7*** | — | — | — |
| LCF | 6* | 6* | 5* | 6* | 5* | 6* | 7*** | — | 6* | 6* | 6* | 11** | 6* | 6* |
| LCM | 7*** | 5* | 5* | 5* | 6* | 8*** | 5* | 5* | — | — | — | — | — | — |
| LCN | 5* | 5* | 6* | 6* | 5* | 5* | 7*** | 5* | — | — | — | — | — | — |

TABLE 3: Possible states of information flow awareness.

| Systemic ignorance | Only hubs | Only normal | Only hermits | Normal and hermits | Hubs and hermits | Hubs and normal | Systemic awareness |
|---|---|---|---|---|---|---|---|
| 000 | 100 | 010 | 001 | 011 | 101 | 110 | 111 |

The market's environment contains 69 individuals that have more than 9 connections. Also, it accommodates 60 agents with degrees between 7 and 8 and 82 agents with less than or equal to 6 edges. In addition, hubs connect with 1189 links, the normal class include 661 links, and hermits needed 464 links.

Mechanically, in order to incorporate the impact of jumps, we assume that agents exposed to the news process will reverse their position to be in the direction of the jump for a period that is proportionate to the jump size. On the other hand, the impact of a jump fades away as time progresses; thus, the design of a jump impact time function $\tau_{t_0}$ can be expressed as

$$\tau_{t_0} = |\omega J(Q)| T \quad \text{if jump occurs at } t_0, \qquad (13)$$

where $\omega \sim N(0.04, 0.01)$ represents an agent propensity to react to a jump, $T$ stands for the simulation termination time, $J(Q)$ stands for the magnitude of the jump, and $\tau_{t_0}$ expresses the period of time an agent is impacted by the jump occurring at $t_0$. The impact is realized through the agent reversal of a position from long (short) to short (long) on a negative (positive) jump with reversal impact time lasting $\tau_{t_0}$. An agent does not change position if the direction of the jump is aligned with his/her position so that, in that case, $\tau_{t_0} = 0$. In the design of this metamodel, the market clock is shared by all agents so that trades occur at uniform discrete times. Agents who reverse positions continue doing so in subsequent trades until the impact fades away, that is, for a period of time $t \in [t_0, \tau_{t_0}]$ or unless a new jump arrives, whichever comes first. When a new jump arrives, the clock $\tau_{t_0}$ is reset to the new jump occurrence. Therefore, the function $\tau_{t_0}$ can be viewed as the time horizon of the impact of a jump of size $J(Q)$ occurring at time $t_0$ on an agent with parameter $\omega$ over a time of simulation $T$. In further work we extend the time horizon impact function to be a function of behavior and strategy.

Therefore, an agent reallocates positions conditional on the agent awareness of the jump according to

$$\left(x_{i,j,t_0}^* \mid a_i \text{ is aware of the jump}\right)$$

$$= \begin{cases} -x_{i,j,t_0}^* & \text{if } x_{i,j,t_0}^* \text{ position is opposite to jump direction} \\ x_{i,j,t_0}^* & \text{otherwise.} \end{cases} \qquad (14)$$

## 5. Information Awareness and the Volatility Index

We examine the market $\mathcal{M}$ under different systemic exposures $g$ to the information flow. The index $g$ represents the state of agents awareness in the market and these states can

be large or infinite in general. For the sake of this paper, we present a discretization of such states by assigning binary states to each type of agent as defined with attribute vectors $\mathbf{a}^{(i)} = (b_j^{(i)}, s_k^{(i)}, d_l^{(i)}), i, 1 \leq i \leq n$. In this experiment, with the agent degree $d_l^{(i)} \in \{4, \ldots, 49\}$, there are three types of agents: hubs, normal, and hermits whose states of awareness are designated as $g_1$, $g_2$, and $g_3$, respectively. In general the index of type awareness can be partially realized but to simplify we assume that the type is either aware or not aware of market information; thus the range of type awareness takes values in $g_i \in \{0, 1\}, i = 1, 2, 3$. This simplification yields eight possible systemic awareness states ranging between *systemic ignorance* corresponding to state 000 and *systemic awareness* in state 111; see Table 3.

The objective is to test the effect of agents information awareness of events on the volatility and standard deviation of returns. Formally we state the following hypothesis:

$$H_0 : g_1 = g_2 = g_3 = 0,$$
$$H_1 : \text{at least one } g_i \neq 0. \qquad (15)$$

To test the hypothesis, a one-factor factorial design is implemented where the factor is type awareness of events. The null hypothesis $H_0$ is systemic ignorance while the alternate $H_1$ represents that at least one agent type has information awareness. The test aims to specify if there is a significant difference between the model outputs of maximum volatility of returns. The hypothesis can be rewritten in terms of the maximum volatility $V^{\max}$ and standard deviation $\sigma_r$ of returns.

A Monte Carlo simulation is implemented in order to collect the necessary statistical outputs for the ANOVA. The experiment replicates the Monte Carlo runs thirty times for each factor for a time horizon of 1000 days. This brings the total collected daily prices to 120,000. From theses collected time series, we have calculated the daily returns and the volatility index. Subsequently, the ANOVA procedure tests whether there is a significant difference between maximum volatility and the standard deviations of the returns in the predescribed cases. Figures 7–11 show the results of simulation experiments. Figure 2 shows the box plot of the obtained maximum volatility index and Figure 3 shows return standard deviations.

ANOVA shows that there is a significant difference between the experiments where the $p$ values were close to zero. Table 4 summarizes the results of the analysis. The Tukey range test clusters the experiments into three groups in terms of the standard deviations of returns. The first group contains the case of systemic ignorance of jump events with the mean of annual volatility of returns over thirty runs of

TABLE 4: ANOVA table.

| | SS | DOF | MS | F-value | p value |
|---|---|---|---|---|---|
| | | | Volatility index | | |
| Intercept | 340546.49 | 1 | 340546.49 | 5618.04398 | 0 |
| Agents exposure | 36287.1146 | 4 | 9071.77865 | 149.658425 | 0 |
| Replication | 1467.84234 | 29 | 50.6152532 | 0.835007046 | 0.70593738 |
| Error | 7031.52075 | 116 | 60.6165582 | | |
| | SS | DOF | MS | F | p |
| | | | Returns standard deviations | | |
| Intercept | 0.081547 | 1 | 0.081547 | 7853.254 | 0 |
| Agents exposure | 0.011833 | 4 | 0.002958 | 284.894 | 0 |
| Replication | 0.000186 | 29 | 0.000006 | 0.616 | 0.933521 |
| Error | 0.001205 | 116 | 0.00001 | | |
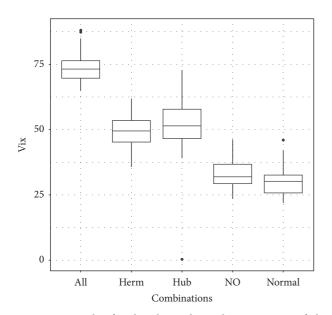


FIGURE 2: Box plot for the obtained simulation outputs of the maximum level of volatility index.
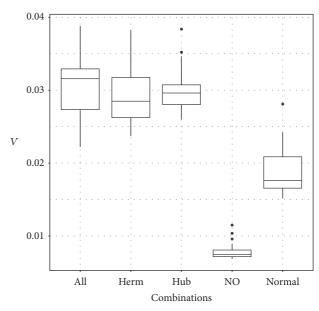


FIGURE 3: Box plot for the obtained simulation outputs of the mean returns standard deviation.

12.5%. The second group includes the group corresponding to the awareness state of 010, that is, when only agents with normal connections are aware of events wherein the mean of the annual volatility of the thirty simulation runs was 29.8%. The third group corresponds to state 111 of systemic awareness of jumps, which demonstrates that the annual volatility increases dramatically in a statistically significant manner compared to the other two states.

In terms of the volatility index, the experiments were classified into three groups as well. However, the results demonstrate that the maximum volatility index is significantly different from the other states when the agents are not aware of jumps or when only agents with normal range connection are exposed to the jumps. On the other hand,

while no significant difference is observed on volatility of awareness between hubs (state 100) and hermits (state 001) agents to jump events in the market, both cases drive the simulated market out of stability. The average of maximum volatility index over the thirty runs realizes 51.1% and 49.2% for states 100 and 001, respectively. Hubs have large connectivity and act as amplifiers of buy or a sell signal as they influence other agents by their decisions. Hubs increase the likelihood of information awareness convergence and in the extreme can lead to herding behavior. On the other hand, the population of hermits agents is quite large offsetting the perceived low number of connections (power of the masses). This property does not allow hermits to be influenced by the sentiment of other agents in the network

and thereby their decisions cumulatively raise fear in the market.

The expected behavior of the market under modern finance axioms of no-arbitrage in a normal regime is similar to the state of systemic ignorance 000. The maximum observed volatility index was 31 for the year 2007 just before the financial crisis when the market shows a normal regime. However, in a rally regime such as that observed in the period between 2010 and 2014, the maximum volatility index was 48. This behavior is similar to that when the hubs and hermits are aware of information. Furthermore, the overall behavior ranges between these cases when the normal agents are aware of the information.

Finally, in systemic awareness case 111, the fear level attains its volatility index maximum of 74.1 given all statistical tests. Empirically, in the presence of successive jumps and systemic awareness, as it was the case during the financial crisis of 2008, this result agrees simulatively with the empirical observation of VIX realizing its maximum of 80.86 on November 20, 2008.

Furthermore, we calculated the degree centralization of the network using Freeman's general formula [41]:

$$C_D = \frac{\sum C_D(n^*) - C_D(i)}{[(N-1)(N-2)]},$$ (16)

where $C_D$ represents the centralization of the network, $C_D(n^*)$ represents the maximum number of edges in the network, $C_D(i)$ represents the edges of node $i$, and $N$ represents the total number of nodes in the network.

The degree centrality of the calibrated network was 0.2. Assuming that the volatility index can serve as a proxy measure of finical stability, we run Monte Carlo simulation of a decentralized network ($C_D = 0$). In the decentralized network, all agents are equally connected. Figures 4 and 5 show the calibrated and decentralized networks, respectively.

Under the assumption that 30% of the population has information awareness, the Monte Carlo simulation for thirty times is performed. The mean of the maximum volatility index records 23.5, which is close to the case of the normal agents awareness and systemic ignorance in the previous experiments. In addition, the mean of the return standard deviations of the runs is 0.014, which is comparable to state 010. Figure 6 shows the market dynamics over the Monte Carlo runs.

In future research, we examine the market dynamics under various levels of network centralization. Also, we consider an assortative network structure wherein agents tend to share information with agents of similar attribute and compare it to a disassortative network structure where opposite-in-attributes may share information awareness.

The results can be partially compared to the model of Panchenko et al. [22] where they investigated the market under various network topology and concluded that the latency in information transmission (such as in lattice and small world network) may cause the market instability. However, the case here is quite different where our analysis is predicated on the market network following a power-law distribution and thereby allowing some nodes to be hubs

with high degree and many hermits nodes with few edges. The main conclusion is that, in this simulated market, the aggressive reaction to the news from a large size of population or a population of highly connected individuals may drive the market out of stability.

## 6. Conclusion

The main contribution of this research is to advance a more realistic metamodel of markets that leads to a financial economic environment simulator. To this end, a formal mathematical definition of a market is introduced along with equipping agents with new attributes such as information awareness of jumps in the context of a scale-free network. This paper focuses on constructing a metamodel for stylized stability of simulated financial markets with emphasis on agents information connectivity, awareness, and propensity to respond to information exposure with a transaction. This paper offers a metamodel of a heterogeneous market where the agent transaction decisions are placed based on a certain set of attributes. Agents decisions change with behaviors, trading strategies, and the interactions with other agents. Trading strategies include arbitrary zero-intelligence, fundamental, trend following, and adaptive strategies that utilize neural network as a learning mechanism. In addition, behaviors such as risk and loss aversion, conservativeness, and overconfidence are included in the model. Also, agents interact directly by sharing market sentiment through a network following the power-law distribution.

A stochastic jump-diffusion process models jump events affecting the market. Agents may or may not be aware of these events depending on their classification. We introduce the concept of agent information awareness as an agent feature and make it a function of connectivity. If the event is known to the agent, the agent adjusts positions depending on the jump direction. The process will continue for a time-frame proportionate to the jump size, and thus we introduced the idea of a time horizon impact of a jump on an agent. Persistence of impact should be proportionate to the size of the jump and to agent propensity to react when a jump occurs.

Five experiments were designed to investigate the effect of agents' awareness of market dynamics and jump events. The analysis of the experiments was implemented using ANOVA. The results show that there is a significant dependence of outputs of various scenarios on the market information awareness states. The mean volatility and volatility index are lowest in the state of systemic ignorance. On the other hand, the volatility and fear index increase as hubs or hermits do observe jumps. The reason is that a large number of connections in the network belong to hubs, which increases the spread of news and impacts market sentiment collectively. The population size of hermits is quite large but relatively isolated from other agents and thus hermits are not influenced by market sentiment and this property can drive the simulated market out of stability. Normal agents information awareness has significant effects on market stability but does not reach the level of hubs and hermits
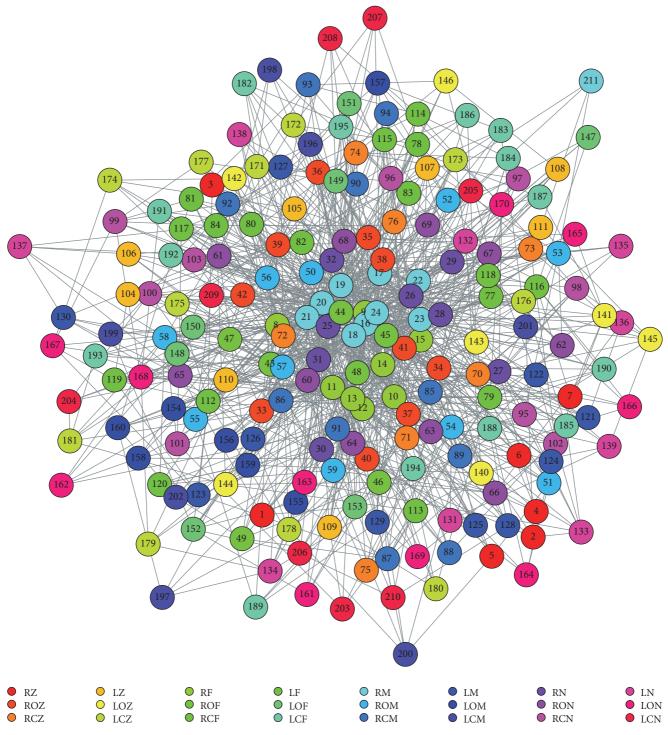
Figure 4: Calibrated network.

effects. In the case where agents information awareness of negative events increases, the fear index reaches its high limit.

In future research and in the context of financial economic environment simulator, we are interested in distinguishing between the state of persistent positive events that

may lead to euphoric responses that ride on hopes of ever higher returns, on the one side, and negative large jumps causing excessive fear in agents and leading to the price destruction. It is postulated that both cases of excessive hope or fear exhibit information awareness convergence—one on the positive, optimistic side (false as it may be) and the other
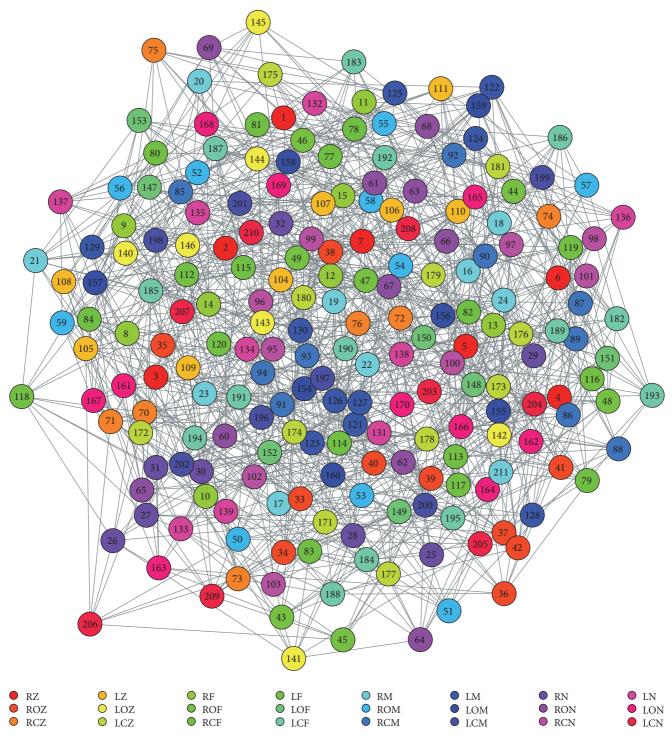
FIGURE 5: Decentralized network.

on the negative pessimistic side. In other words, a future financial economic environment simulator should be able to capture the state prior to the 2008 crisis wherein agents were in the state best described as "irrational exuberance" (Alan Greenspan) of unfounded hope of sustainable systemic

overperformance and be able to model the opposite state of settling fear during a crisis where agents, individuals, institutional traders, and brokers converged to a negative homogeneous information awareness state, summarized by the market becoming to many as "uninvestable".
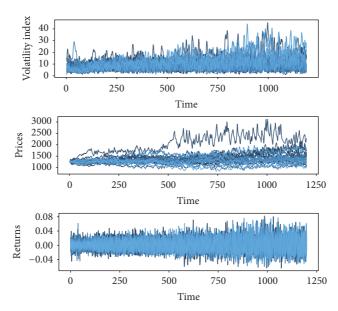
FIGURE 6: The market prices, returns, and volatility index under decentralized network.



(a)

(b)

(c)

FIGURE 7: Time series of volatility index (a), prices (b), and returns (c), when $g = 1 = g = 2 = g = 3 = 0$.



(a)

(b)

(c)

FIGURE 8: Time series of volatility index (a), prices (b), and returns (c), when $g = 1 = 0$ and $g = 2 = g = 3 = 0$.
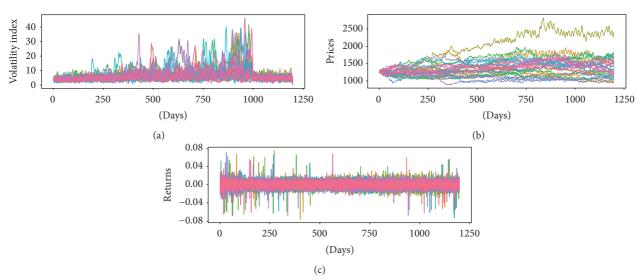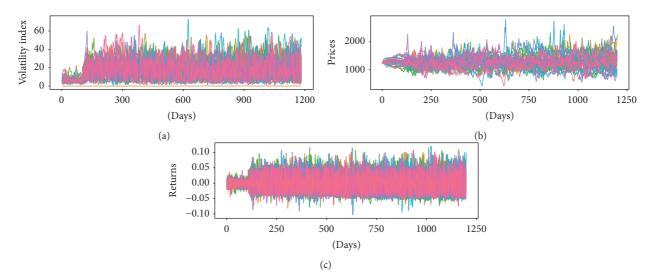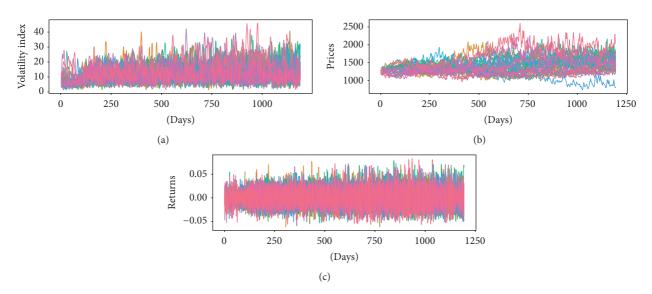
FIGURE 9: Time series of volatility index (a), prices (b), and returns (c) when $g = 2 = 0$ and $g = 1 = g = 3 = 0$.
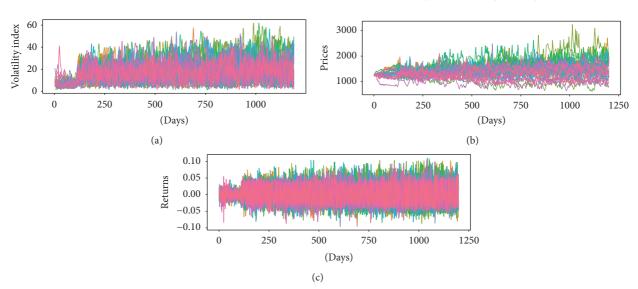


FIGURE 10: Time series of volatility index (a), prices (b), and returns (c) when $g = 3 = 0$ and $g = 1 = g = 2 = 0$.
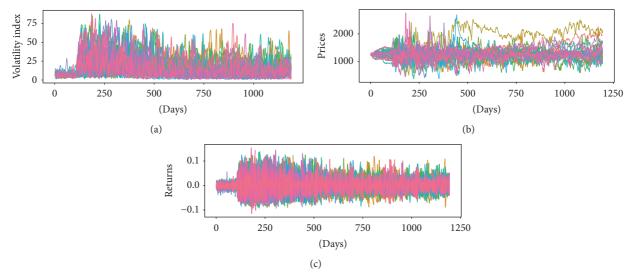


FIGURE 11: Time series of volatility index (a), prices (b), and returns (c), when $g = 1 = g = 2 = g = 3 = 1$.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Acknowledgments

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*Research Article*

# Multiplex Networks of the Guarantee Market: Evidence from China

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We investigate a multiplex network of the guarantee market with three layers corresponding to different types of guarantee relationships in China. We find that three single-layer networks all have the scale-free property and are of disassortative nature. A single-layer network is not quite representative of another single-layer network. The result of the betweenness centrality shows that central companies in one layer are not necessarily central in another layer. And the eigenvector centrality has the same result of the betweenness centrality except the top central company.

## 1. Introduction

The global financial crisis of 2007–2009 has shown how interconnected the global financial system is. Financial institutions' interconnections can serve as a channel for systemic risk and have been directly linked to the stability of financial systems. Network analysis has contributed to characterizing, understanding, and modeling financial institutions' interconnections, which is gaining popularity across academics, regulators, and policymakers.

There are a lot of studies on financial networks and systemic risk (see, e.g., Allen and Gale [1]; Nier et al. [2]; May and Arinaminpathy [3]; Gai et al. [4]; Li [5]; Li and He [6]; Georg [7]; Sachs [8]; Sensoy and Tabak [9]; Aymanns and Georg [10]; Acemoglu et al. [11]; Chen et al. [12]; Betz et al. [13]; Li et al. [14]; Li and Sui [15]; González-Avella et al. [16]; Sensoy et al. [17]; Christiano Silva et al. [18–21]; Silva et al. [22].), while financial institutions interact in just one way in most of these studies. In fact, financial institutions interact in many ways. Such a situation is best modeled with a multiplex network, where a multiplex network is made up with several layers, each of them composed of all relations of the same type and modeled with a simple network [23]. Multiplex networks can explicitly incorporate multiple channels of connectivity and constitute the natural environment to describe systems interconnected through different categories of connections [24].

Mapping out the structure of complex systems as a monoplex network could lead to missing relevant information [25]. For example, Poledna et al. [26] find that modeling contagion using each layer independently can lead to an underestimation of systemic risk. Therefore, the multiplex network method is useful in improving our understanding of complex systems by taking such multilayer features into account.

The theory of multiplex networks is in its early stages and has been introduced within the last three years to analyze the structure of financial systems [27]. The study of financial multiplex networks has only appeared recently. Empirical analyses of the financial multiplex networks of Colombia, UK, Mexico, Italy, Europe, and USA are provided by León et al. [28], Langfield et al. [29], Molina-Borboa et al. [30], Bargigli et al. [31], Aldasoro and Alves [32], and Musmeci et al. [33], respectively.

León et al. [28] investigate a multiplex network of Colombian sovereign securities settlements corresponding to the three sovereign securities' trading and registering environments and find that the multiplex network has some features, such as sparse, inhomogeneous, scale-free, ultra-small-world, and clustered. Langfield et al. [29] construct the multiplex network of the UK interbank market and find that the network of interbank exposures exhibits a core-periphery structure and the funding network has less of a core-periphery structure. Molina-Borboa et al. [30] analyze the persistence

and overlap of relationships between banks in a multiplex network of the Mexican banking system, where the multiplex network includes five layers, namely, collateralized loans between banks, new deposits and loans, securities, outstanding deposits, and loans and derivatives. Bargigli et al. [31] analyze the Italian interbank multiplex network by transaction type and by maturity and find that layers have different topological properties and persistence over time. Aldasoro and Alves [32] adopt data on interbank exposures broken down by both maturity and instrument type to investigate structures of the multiplex network of large European banks and find that the network presents positive correlated multiplexity and a high similarity between layers. Musmeci et al. [33] analyze structural properties of the multiplex network of US stock markets, which includes four layers corresponding, respectively, to linear, nonlinear, tail, and partial correlations among a set of financial time series. They find that some features are unique to the multiplex structure and would not be visible otherwise by the separate analysis of the single-layer networks.

In recent years, China's listed companies widely adopt different forms of guarantee to form a complex guarantee circle, which reduces the difficulty of corporate finance to a certain extent. But this also provides risk contagion channels among listed companies. Generally, listed companies who require guarantees tend to have relatively poor performance and relatively large financial risk. Once a listed company has bankruptcy risk, it may result in risk contagion among listed companies through guarantee connections and even lead to systemic risk. Since 2012, the frequency of China's guarantee circle crisis is increasing, such as, the guarantee circle crisis in Zhejiang, Shanghai, Shandong, and other places.

In this paper, we investigate structures of the multiplex network of the guarantee market in China, which would be conducive to preventing risk contagion in the guarantee market. The study on financial multiplex networks is still preliminary. Existing empirical study on financial multiplex networks only analyzes sovereign securities markets, interbank markets, and stock markets. Therefore, this paper contributes to the literature on financial multiplex networks. Second, we study different segments of the guarantee market by guarantee types, and this would help to understand the economics of guarantee market infrastructures as the collective function of several layers of interaction between financial institutions.

The rest of the paper is organized as follows. Section 2 presents the guarantee multiplex network and the data set. Section 3 presents the results of single-layer networks. Section 4 presents the results of multiplex networks. And the conclusion is drawn in Section 5.

## 2. Guarantee Multiplex Network and Data Description

*2.1. Guarantee Multiplex Network.* We consider a guarantee market consisting of $N$ listed companies and $m$ different types of guarantee relationships. We can adopt the multiplex network to describe the structure of the guarantee market. A network consisting of a type of relationship can be described by a adjacency matrix. Moreover, the structure of

the guarantee market can fully be described by the set of adjacency matrices, which is given as follows:

$$A = \left\{ A^{[1]}, A^{[2]}, \ldots, A^{[m]} \right\}, \tag{1}$$

where $A^{[\theta]} = \{a_{ij}^{\theta}\}$, with $a_{ij}^{\theta} = 1$ if there is the $\theta$ type guarantee relationship between listed company $i$ and listed company $j$; otherwise $a_{ij}^{\theta} = 0$.

If we consider the amount of the guarantee, we can fully describe the structure of the guarantee market by the set of weighted adjacency matrices,

$$W = \left\{ W^{[1]}, W^{[2]}, \ldots, W^{[m]} \right\}, \tag{2}$$

where $W^{[\theta]} = \{W_{ij}^{\theta}\}$ and $W_{ij}^{\theta}$ denotes the amount of the $\theta$ type guarantee. In this paper, we only analyze unweighted multiplex networks of the guarantee market in China.

*2.2. Data Description.* In this paper, we investigate guarantee relationships of listed companies in China, where the data used stem from the Wind database and the time interval is from 2005 to 2015. The Wind database is a leading integrated service provider of financial data in China. In order to simplify data processing, we regard guarantee relationships of subsidiaries as their parent companies'. Besides, if there are more than one guarantee contract between two companies, we assumed that there is only one edge between these companies.

There are 8 types of guarantee relationships among listed companies, namely, joint liability guarantee (JLG), guaranteed guarantee (GUG), credit guarantee (CRG), counter guarantee (COG), general guarantee (GEG), mutual guarantee (MUG), mortgage guarantee (MOG), and pledge guarantee (PLG). Table 1 shows numbers of companies and guarantee relationships from 2005 to 2015. Although various guarantee relationships all play an important role in corporate finance, there are some differences among them. In fact, from Table 1, we can obtain the numbers of the 8 types of guarantee relationships from 2005 to 2015, where the numbers of JLG, GUG, CRG, COG, GEG, MUG, MOG, and PLG are 26126, 1705, 710, 106, 3402, 204, 1121, and 551, respectively. It can be seen that the numbers of JLG, GUG, and GEG are the largest three. Therefore, in this paper, we investigate a multiplex network with three layers corresponding, respectively, to JLG, GUG, and GEG. Due to the different time period of guarantees, in this paper we construct networks based on all data from 2005 to 2015 without special instructions.

## 3. Structure Analysis of Single-Layer Networks

We first investigate topological structures of the three single-layer networks. Figure 1 shows the distributions of degrees of nodes of the JLG network, the GUG network, and the GEG network. It can be seen that degree distributions of these three networks follow a power law, and corresponding exponents are 3.2200, 2.9000, and 2.8900, respectively. Therefore, three single-layer networks all have the scale-free property. This result means that there is a very strong heterogeneity of the

TABLE 1: Numbers of companies and guarantee relationships from 2005 to 2015.

| Year | Number of companies | JLG | GUG | CRG | COG | GEG | MUG | MOG | PLG |
|------|---------------------|------|-----|-----|-----|-----|-----|-----|-----|
| 2005 | 1869 | 1522 | 135 | 48 | 8 | 154 | 37 | 32 | 20 |
| 2006 | 1772 | 1424 | 104 | 42 | 6 | 199 | 40 | 30 | 6 |
| 2007 | 1619 | 1207 | 134 | 66 | 8 | 181 | 43 | 35 | 11 |
| 2008 | 1508 | 1184 | 123 | 36 | 12 | 120 | 29 | 39 | 17 |
| 2009 | 1291 | 967 | 120 | 34 | 14 | 119 | 23 | 46 | 15 |
| 2010 | 2808 | 1834 | 395 | 271 | 25 | 170 | 13 | 88 | 108 |
| 2011 | 2643 | 1937 | 379 | 176 | 29 | 96 | 19 | 86 | 24 |
| 2012 | 3661 | 3019 | 83 | 6 | 4 | 494 | 0 | 155 | 51 |
| 2013 | 4354 | 3618 | 78 | 15 | 0 | 545 | 0 | 199 | 88 |
| 2014 | 4900 | 4102 | 99 | 13 | 0 | 613 | 0 | 184 | 100 |
| 2015 | 6150 | 5312 | 55 | 3 | 0 | 711 | 0 | 227 | 111 |
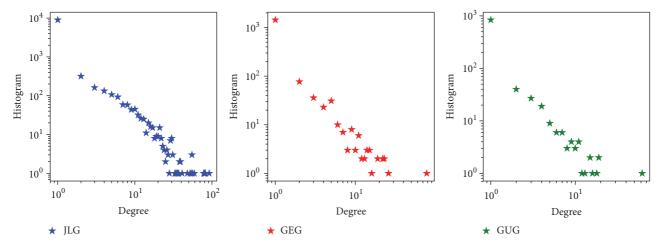


FIGURE 1: Distribution of degrees of nodes of single-layer networks.

single-layer networks, with few important hubs and many nodes with low degree. This means that few listed companies are at the heart of the guarantee market in China. If these listed companies go bankrupt, it is likely to lead to the break of the guarantee chain, which may lead to regional financial crisis.

Next we analyze degree correlations of the three single-layer networks, which can be measured by the following formula [34]:

$$r = \frac{M^{-1} \sum_i j_i k_i - \left[ M^{-1} \sum_i (1/2) (j_i + k_i) \right]^2}{M^{-1} \sum_i (1/2) (j_i^2 + k_i^2) - \left[ M^{-1} \sum_i (1/2) (j_i + k_i) \right]^2}, \quad (3)$$

where $j_i, k_i$ are the degrees of the nodes at the ends of the $i$th edge, with $i = 1, 2, \ldots, M$. This formula measures whether a node of high degree at one end of a link prefers a node of high degree (assortative mixing, $r > 0$) or low degree (disassortative mixing $r < 0$) at the other end. According to this formula, we can obtain that $r$ values of JLG, GUG, and GEG networks are −0.3000, −0.1780, and −0.1670, respectively. This indicates that the three single-layer networks are of disassortative nature; that is, companies of high degrees tend

to be connected to companies with low degrees. The possible reason for this is that companies tend to find companies with high credit for guarantee, where companies with high credit usually have high degrees. Therefore, it is necessary to identify the so-called systemically important listed companies, which would adversely affect large parts of the guarantee network in case of their bankruptcies.

## 4. Structure Analysis of Multiplex Networks

*4.1. Similarity Analysis.* In this section, we focus on analyzing the similarity between different layers, where the similarity analysis assesses to what extent a layer is representative of the other. We adopt the Jaccard similarity $J$ to analyze network similarity, where the Jaccard similarity is the probability of observing a link in a network conditional on the observation of the same link in the other network [31]. The Jaccard similarity $J$ is defined as follows [31]:

$$J(P, Q) = \frac{|P \wedge Q|}{|P \vee Q|}, \quad (4)$$

where $\wedge (\vee)$ is the entry-wise maximum (minimum) of $P$ and $Q$.

TABLE 2: Jaccard similarity of the JLG network over time.

|      | 2006   | 2007   | 2008   | 2009   | 2010   | 2011   | 2012   | 2013   | 2014   | 2015   |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 2005 | 0.4430 | 0.3060 | 0.2280 | 0.1710 | 0.0900 | 0.0700 | 0.0500 | 0.0410 | 0.0350 | 0.0270 |
| 2006 |        | 0.4590 | 0.3000 | 0.2160 | 0.1140 | 0.0860 | 0.0650 | 0.0470 | 0.0390 | 0.0290 |
| 2007 |        |        | 0.4660 | 0.3150 | 0.1490 | 0.1090 | 0.0760 | 0.0550 | 0.0440 | 0.0340 |
| 2008 |        |        |        | 0.4560 | 0.2090 | 0.1470 | 0.0950 | 0.0650 | 0.0530 | 0.0390 |
| 2009 |        |        |        |        | 0.2730 | 0.1890 | 0.1150 | 0.0780 | 0.0600 | 0.0430 |
| 2010 |        |        |        |        |        | 0.4120 | 0.2580 | 0.1770 | 0.1410 | 0.1050 |
| 2011 |        |        |        |        |        |        | 0.3390 | 0.2230 | 0.1730 | 0.1260 |
| 2012 |        |        |        |        |        |        |        | 0.4640 | 0.3200 | 0.2260 |
| 2013 |        |        |        |        |        |        |        |        | 0.5030 | 0.3280 |
| 2014 |        |        |        |        |        |        |        |        |        | 0.4840 |

TABLE 3: Jaccard similarity of the GUG network over time.

|      | 2006   | 2007   | 2008   | 2009   | 2010   | 2011   | 2012   | 2013   | 2014   | 2015   |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 2005 | 0.3580 | 0.2510 | 0.1620 | 0.1230 | 0.0470 | 0.0300 | 0.0090 | 0.0090 | 0.0000 | 0.0000 |
| 2006 |        | 0.4000 | 0.2470 | 0.1850 | 0.0460 | 0.0320 | 0.0330 | 0.0170 | 0.0050 | 0.0060 |
| 2007 |        |        | 0.3110 | 0.2450 | 0.0690 | 0.0410 | 0.0380 | 0.0190 | 0.0040 | 0.0050 |
| 2008 |        |        |        | 0.4820 | 0.1090 | 0.0640 | 0.0400 | 0.0260 | 0.0090 | 0.0110 |
| 2009 |        |        |        |        | 0.1490 | 0.0780 | 0.0410 | 0.0260 | 0.0090 | 0.0120 |
| 2010 |        |        |        |        |        | 0.3460 | 0.0350 | 0.0170 | 0.0100 | 0.0110 |
| 2011 |        |        |        |        |        |        | 0.0290 | 0.0130 | 0.0080 | 0.0090 |
| 2012 |        |        |        |        |        |        |        | 0.1260 | 0.0580 | 0.0950 |
| 2013 |        |        |        |        |        |        |        |        | 0.4630 | 0.1570 |
| 2014 |        |        |        |        |        |        |        |        |        | 0.2830 |

According to (2), we can obtain that the Jaccard similarity $J$ between the JLG network and the GUG network is 0.0340, and that between the JLG network and the GEG network is 0.0460, and that between the GUG network and the GEG network is 0.0390. Therefore, the Jaccard similarity between different layers is relatively low, and thus a single-layer network is not quite representative of another single-layer network. This means that the structure varies greatly from one single-layer network to another, due to the difference of guarantee types.

Moreover, we can adopt the Jaccard similarity to analyze the similarity of individual layers over time, where the results are shown in Tables 2–4. From them, we can see that $J$ values of the JLG market are roughly around 0.4500 for two consecutive years, and that of the GUG market and the GEG market have obvious fluctuation. This result shows that guarantee relationships in the JLG market are much more stable than those in the GUG market and the GEG market. Besides, the Jaccard similarity decreases with the increase of the lag phase in the three types of guarantee markets.

Similarity analysis is a relevant tool in assessing the stability of individual layers over time and the similarity between different layers. The above results show that the joint liability guarantee relationship is relatively stable in China's guarantee market and suggest significant complementarity between different segments of the guarantee market.

### 4.2. Centrality Analysis.
Centrality is an important concept of network theory. In the paper we analyze mostly betweenness and eigenvector centrality. The betweenness centrality quantifies how frequently a node acts as a bridge along the shortest path between two other nodes and is computed as follows [23]:

$$C_B(v) = \frac{1}{(n-1)(n-2)} \sum_{i,j \neq v} \frac{N_{ij}(v)}{N_{ij}}, \qquad (5)$$

where $C_B(v)$ denotes the betweenness of node $v$, $n$ is the number of nodes in the network, and $N_{ij}$ represents the number of shortest paths between nodes $i$ and $j$. $N_{ij}(v)$ is the number of shortest paths between nodes $i$ and $j$ that contain node $v$. Figure 2 reports the betweenness centrality in the JLG market either versus the betweenness in the GUG market (Figure 2(a)) or versus the betweenness in the GEG market (Figure 2(b)). From Figure 2, we can know that, in some cases, the betweenness centrality of a company can be markedly different in different layers. Therefore central companies in one layer are not necessarily central in another layer.

Eigenvector centrality is defined as the principal eigenvector of the adjacency matrix defining the network [35, 36]. If the adjacency matrix of a network is denoted by $A$, then the equation of an eigenvector is as follows:

$$\lambda v = A v, \qquad (6)$$

TABLE 4: Jaccard similarity of the GEG network over time.

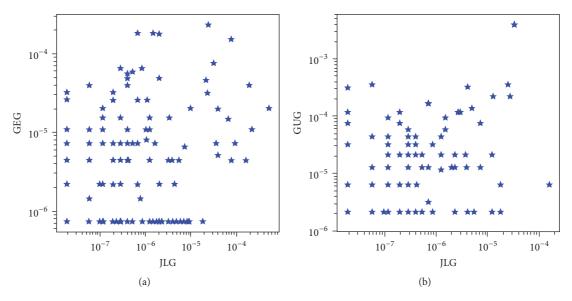| | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2005 | 0.2700 | 0.1240 | 0.1000 | 0.0920 | 0.0550 | 0.0250 | 0.0170 | 0.0090 | 0.0120 | 0.0060 |
| 2006 | | 0.2030 | 0.1560 | 0.1040 | 0.0540 | 0.0310 | 0.0270 | 0.0120 | 0.0160 | 0.0070 |
| 2007 | | | 0.2920 | 0.1860 | 0.0800 | 0.0340 | 0.0270 | 0.0150 | 0.0180 | 0.0110 |
| 2008 | | | | 0.3740 | 0.1240 | 0.0430 | 0.0230 | 0.0150 | 0.0170 | 0.0120 |
| 2009 | | | | | 0.2040 | 0.1080 | 0.0460 | 0.0260 | 0.0240 | 0.0200 |
| 2010 | | | | | | 0.1320 | 0.0680 | 0.0360 | 0.0340 | 0.0230 |
| 2011 | | | | | | | 0.1030 | 0.0420 | 0.0370 | 0.0240 |
| 2012 | | | | | | | | 0.3550 | 0.2270 | 0.1680 |
| 2013 | | | | | | | | | 0.3820 | 0.2780 |
| 2014 | | | | | | | | | | 0.4490 |



(a)



(b)

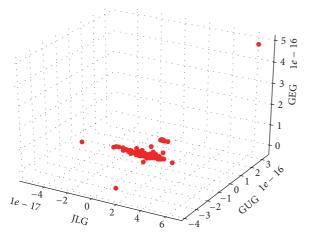FIGURE 2: Betweenness centrality of the multiplex network.



FIGURE 3: Eigenvector centrality of the multiplex network.

where $\lambda$ is a constant and $\nu$ is the eigenvector. Figure 3 shows the three-dimensional plot of the eigenvector centrality of the multiplex network. From it, we can see that the top central node in one layer is also the top central node in another layer. For nodes except the top central node, we can also obtain that the eigenvector centrality of a company can be markedly different in different layers. This result is the same as that of the betweenness centrality.

According to the above analysis, we can see that the centrality measures are important tools, because they can give insight on the degree of specialization of some listed companies as guarantors for some type of guarantee. Therefore, in order to maintain the stability of the guarantee market in China, the centrality measure provides an important tool.

## 5. Conclusion

Financial markets are complex systems, which can be understood better based on network theory. Usually, there are more than one type of relationships between financial institutions. Therefore it is very important to understand financial markets from the perspective of multiplex networks rather than single-layer networks. In this paper, we examine how listed companies relate to each other in different types of guarantee relationships. We investigate a multiplex network with three

layers corresponding, respectively, to joint liability guarantee, guaranteed guarantee, and general guarantee in China.

First, we find that three single-layer networks all have the scale-free property and are of disassortative nature; that is, companies of high degrees tend to be connected to companies with low degrees. Second, according to similarity analysis, we can know that the Jaccard similarity between different layers is relatively low, and thus a single-layer network is not quite representative of another single-layer network. For individual-layer networks, we find that guarantee relationships in the joint liability guarantee market are much more stable than those in the guaranteed guarantee market and the general guarantee market. And the Jaccard similarity decreases with the increase of the lag phase in the three types of guarantee markets.

Finally, we also investigate the betweenness and eigenvector centrality of a company in the network. The result of the betweenness centrality shows that central companies in one layer are not necessarily central in another layer. And the eigenvector centrality has the same result of the betweenness centrality except the top central company.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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