Complexity

# Scalable Distributed Decision-Making and Coordination in Large and Complex Systems: Methods, Techniques, and Models

Lead Guest Editor: Marin Lujak
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*Editorial*

# Scalable Distributed Decision-Making and Coordination in Large and Complex Systems: Methods, Techniques, and Models

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Human society, global economy, and Internet are becoming ever more decentralized, while millions of computers connected to the Internet facilitate the engineering of systems whose scale goes beyond spatial and computational boundaries of individual organizations.

The decision-making authority in this context is distributed throughout a system, and the decisions are made locally based on the interactions of an individual with the rest of the system and with its environment (see, e.g., [1]). A desired global behavior following the identifiable interest of the whole system is the result of system intelligence that emerges from the system's beliefs and system's collective actions and, as such, is a shift away from the hierarchical system paradigm. Distributed Decision-Making (DDM) models are usually used to support group decision-making in such large and complex systems, where each agent holds only limited information, and the cooperation between agents is crucial for the system's performance (see, e.g., [2]).

We are pleased to see the publication of this special issue focusing on the design and implementation of new methods, techniques, and models (see, e.g., [3]) that adapt or hybridize findings from Distributed Optimization, Multiagent Systems, Network Science, and Distributed Computing and facilitating distributed/parallel/multiagent decision-making and coordination for solving complex computational and real-life problems in large systems. The applications of DDM vary from coordination problems in groups and crowds (e.g., [4]), Internet (e.g., [5]), emergency logistics (e.g., [6]), multirobot systems (e.g., [7]) to transport (see, e.g., [8]), and beyond.

The main objective of this special issue is to provide an opportunity to study different aspects of intelligent and distributed decision-making and coordination in large and complex systems, including their formal analysis, with an intention to balance between theoretical research ideas and their practicability. Overall, this special issue collects six research articles and one review article on the state-of-the-art in DDM.

J. Li and S. Gong address the topic of coordination of closed-loop supply chain (CLSC) with dual-source supply and low-carbon concern. They construct a CLSC model with two competitive dominant upstream suppliers and one following a downstream (re-)manufacturer and then coordinate supply chain through cost-sharing contract. Based on the industrial case in the area of power battery, they analyze the optimal strategies under competition, cooperation, and coordination structures separately and then investigate the influences of emission reduction effort and collection efficiency on supply chain performance. The results reveal that collection of used products can positively affect the profit of the (re-)manufacturers, but has opposite impact on the new component supplier. Besides, recycling is beneficial to both low-carbon consumers' utility and social welfare, but hurts the total profit of CLSC because of the high investment cost of collection. Therefore, the paper designs a cost-sharing contract, which is applicable and efficient for both economic and environmental development. Furthermore, it can also increase the profit of CLSC up to cooperation case and improve each member's profit, eliminating double marginal effect and achieving supply chain coordination.

I. G. Pérez Vergara et al. study the design of collaborative processes among stakeholders involved in inventory decision making and requiring effective communication and agreements between the leaders of the logistics processes. Traditionally, decision making in inventory management was based on approaches conditioned only by cost or sales volume. These approaches must be overcome by others that consider multiple criteria, involving several areas of the companies and taking into account the opinions of the stakeholders involved in these decisions. Inventory management becomes part of a complex system that involves stakeholders from different areas of the company, where each agent has limited information and where the cooperation between such agents is a key for the system's performance. In this paper, a distributed inventory control approach is used with the decisions allowing communication between the stakeholders and with a multicriteria group decision-making perspective. This work proposes a methodology that combines the analysis of the value chain and the Analytic Hierarchy Process (AHP) technique, in order to improve communication and performance of the areas related to inventory management decision making. This methodology uses the areas of the value chain as a theoretical framework to identify the criteria necessary for the application of the AHP multicriteria group decision-making technique. These criteria are defined as indicators that measure the performance of the areas of the value chain related to inventory management and are used to classify ABC inventory of the products according to these selected criteria. Therefore, the methodology allows us to solve inventory management DDM based on multicriteria ABC classification and was validated in a Colombian company belonging to the graphic arts sector.

M. Simão Filho et al. propose a multicriteria approach to support task allocation in distributed software development (DSD) projects. A typical decision-making problem in the distributed scenario consists of deciding which team should be allocated to each task. That decision takes into account a relative degree of subjectivity. The setting is suitable for applying Verbal Decision Analysis (VDA). This paper introduces an approach to support the allocation of tasks to distributed units in DSD projects, structured on the hybridisation of methods of Verbal Decision Analysis for classification and rank ordering applied to influencing factors and executing units. Firstly, a review of the literature is conducted aiming to identify the approaches to support the allocation of tasks in DSD contexts. Then, an approach is developed by applying VDA-based methods for classification and ordering. Bibliographic research and the application of surveys with professionals allow identifying and characterising the main elements that influence task assignment in DSD projects. Afterwards, experiences are carried out in five real-world companies. Finally, the proposed approach is evaluated by the professionals of the participating companies and by some project management experts. Results of the experiences and evaluations present evidence that the proposed approach is flexible, adaptable, and easy to understand and to use. Moreover, it helps to reduce decision subjectivity and to think of new aspects, supporting the task allocation process in DSD.

X. Pu and C. Xiong study the weighted couple-group consensus of continuous-time heterogeneous multiagent systems with input and communication time delay. They design a novel weighted couple-group consensus protocol based on cooperation and competition interaction, which can relax the in-degree balance condition. They obtain the time delay upper limit that the system may allow by using graph theory, the general Nyquist criterion, and the Gershgorin disc theorem. The conclusion indicates that there is no relationship between weighted couple-group consensus and communication time delay. When the agents input time delay, the coupling weight between the agents, and the systems control parameters are satisfied, and the multiagent system can converge to any given weighted coupling group consistent state. The results of the experimental simulation support the conclusion.

Open challenges of coordination in distributed decision-making systems (DDMS) include finding the relation between the complexity of the decision problem, the problem's predictability and its dynamics, and the applicable coordination mechanisms. These challenges apply to DDMS resided by human decision-makers like firms as well as to systems of software agents in the domain of multiagent systems (MAS).

F. Wall studies the adaptation and emergence of coordination in the course of growing decision-making organizations. For this, an agent-based simulation model based on the framework of NK fitness landscapes is employed. NK landscapes are stochastically generated pseudo-Boolean functions with $N$ bits (genes) and $K$ interactions between genes. The study controls for different levels of complexity of the overall decision problem, different strategies of search for new solutions, and different levels of cost of effort to implement new solutions. The results suggest that, with respect to the emerging coordination mode, complexity subtly interferes with the search strategy employed and cost of effort. In particular, results support the conjecture that increasing complexity leads to more hierarchical coordination. However, the search strategy shapes the predominance of hierarchy in favor of granting more autonomy to decentralized decision-makers. Moreover, the study reveals that the cost of effort for implementing new solutions in conjunction with the search strategy may remarkably affect the emerging form of coordination. This could explain differences in prevailing coordination modes across different branches or technologies or could explain the emergence of contextually inferior modes of coordination.

The work by G. Xiao et al. deals with the problem of autonomous separation of traffic flows in smart reversible lanes. Spacer bars in the smart reversible lanes periodically broadcast messages to share their local observed traffic information with each other. This aims to help other spacer bars acquire the global traffic information and make consistent movement when separating the flows. However, radio interference and vehicles in the traffic may degrade the qualities of wireless communication links and cause frequent message losses in the broadcast. Existing solutions tend to use data forwarding to enhance the message dissemination, which may cause imbalanced load in the spacer bars. The unbalanced distribution of network load has a high risk of blocking the wireless communication links and yield inconsistent movement in the reversible lanes. They propose a Cooperative Bargain (CoB) scheme where each spacer bar carries some received messages to help other spacer bars recover their lost messages. Since the spacer bars can only acquire the local information, they formulate a cooperative bargain game to negotiate how to allocate the task of message recovery with a balanced network load until a consensus is achieved. CoB is evaluated with the real-world Wi-Fi communication traces in isti/rural. Simulation results show that CoB can recover an average of 98.6% messages within 100 milliseconds in a 50-node network. CoB does not require the global network information, but it can still achieve a comparable performance to other broadcast schemes.

Finally, the guest editors of this special issue present a review article on the topic of decentralizing coordination in open vehicle fleets for scalable and dynamic task allocation. One of the major challenges in the coordination of large and open collaborative and commercial vehicle fleets is dynamic task allocation. Self-concerned individually rational vehicle drivers have both local and global objectives, which require coordination using some fair and efficient task allocation method. They review the literature on scalable and dynamic task allocation focusing on deterministic and dynamic two-dimensional linear assignment problems. They focus on a multiagent system representation of open vehicle fleets where dynamically appearing vehicles are represented by software agents that should be allocated to a set of dynamically appearing tasks. They give a comparison and critical analysis of recent research results focusing on centralized, distributed, and decentralized solution approaches. Moreover, they propose mathematical models for dynamic versions of the following assignment problems well-known in combinatorial optimization—the linear assignment problem, bottleneck assignment problem, fair matching problem, dynamic minimum deviation assignment problem, $\sum_k$ – assignment problem, the semiassignment problem, the assignment problem with side constraints, and the assignment problem while recognizing agent qualification—all while considering the main aspect of open vehicle fleets: random arrival of tasks and vehicles (agents) that may become available after assisting previous tasks or by participating in the fleet at times based on individual interest.

## Conflicts of Interest

The editors declare that they have no conflicts of interest regarding the publication of this special issue.

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*Review Article*

# Decentralizing Coordination in Open Vehicle Fleets for Scalable and Dynamic Task Allocation

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One of the major challenges in the coordination of large, open, collaborative, and commercial vehicle fleets is dynamic task allocation. Self-concerned individually rational vehicle drivers have both local and global objectives, which require coordination using some fair and efficient task allocation method. In this paper, we review the literature on scalable and dynamic task allocation focusing on deterministic and dynamic two-dimensional linear assignment problems. We focus on multiagent system representation of open vehicle fleets where dynamically appearing vehicles are represented by software agents that should be allocated to a set of dynamically appearing tasks. We give a comparison and critical analysis of recent research results focusing on centralized, distributed, and decentralized solution approaches. Moreover, we propose mathematical models for dynamic versions of the following assignment problems well known in combinatorial optimization: the assignment problem, bottleneck assignment problem, fair matching problem, dynamic minimum deviation assignment problem, $\Sigma_k$-assignment problem, the semiassignment problem, the assignment problem with side constraints, and the assignment problem while recognizing agent qualification; all while considering the main aspect of open vehicle fleets: random arrival of tasks and vehicles (agents) that may become available after assisting previous tasks or by participating in the fleet at times based on individual interest.

## 1. Introduction

Open collaborative vehicle fleets composed of autonomous self-interested system participants are ever more widespread. However, even though the drivers are autonomous and self-interested, the authority and the ownership of these systems today remain centralized in terms of management, control, and access. The trend seems to be an ever-increasing access to mobility and last-mile services for the average person at the cost of relying on just a few (centralized) worldwide enterprises. The state-of-the-art algorithms for the allocation of tasks to vehicle fleets solve customer requests in very large fleets in almost near real time, but they seem to be limited to centralized systems. Centralization here can be a source of failure (a single bottleneck of the system), obsolete information due to significant computation delay while processing ever-increasing quantity of data, privacy evasion, and mistrust if the interests of the enterprise mismatch the users' interest.

Distributed decision-making (DDM) obviously resolves the drawbacks of centralized systems. The multitude of the connected smart devices of the vehicles' drivers and customers makes it possible to combine their potential and to coordinate fleets at a scale exceeding spatial and computational boundaries. This potential can be exploited for the benefit of the fleet system as a whole as well as for the interest of individual vehicle drivers and customers.

The decision-making authority in the DDM is distributed throughout a system, and the decisions are taken locally based on the local and shared global information and the interactions of an individual with the rest of the system and with the environment. Here, each fleet participant is modelled as an autonomous collaborative individually rational software agent installed on a user's smart device. The agent has only a local vision of the fleet and it needs to cooperate with other agents in order to find the allocation of dynamically appearing tasks faced by the whole fleet.

The behaviour of the fleet as a whole is a result of intervehicle coordination. Distributed task allocation strongly contributes to the shift of knowledge and power from the individual (fleet owner) to the collective (vehicles composing the fleet). A desired behaviour of the fleet emerges from the identifiable interest of its participating vehicles, their beliefs, and collective actions and, as such, is a shift away from the hierarchical organizational paradigm (see, e.g., [1]). A major challenge is the identification of a right decision-maker for each part of the problem, timely exchange of relevant and up-to-date information among vehicle agents, and modelling of complex relations in such a multiagent system. A trade-off between the amount of computation and the quality of the solution is often necessary. Moreover, minimizing the overhead of communication required to converge to a desirable global solution is desirable.

*Decentralized* coordination algorithms may be the means to obtain scalability for task allocation in the context of large-scale open fleets. Here, each self-concerned (vehicle, driver, or courier) agent aims at achieving a desired local objective based on a limited local information and by communicating with the rest of the fleet and interacting with the environment. Due to the limited local information, one of the drawbacks of decentralization is lack of control of the emerging fleet behaviour that cannot be predicted with certainty. Moreover, to facilitate cooperation, assuming individually rational agents, we have to consider efficiency and fairness. How to balance decentralization and centralization to improve system performance is much investigated but still not a completely solved question.

*1.1. Contribution.* In this work, we present a survey on multiagent system (MAS) coordination mechanisms for computationally complex dynamic (one-on-one) task allocation problem (DTAP) and its variations for open vehicle fleet applications. These problems may be modelled by a variety of deterministic and dynamic two-dimensional linear assignment problems, i.e., the problems regarding the assignment of two sets that may be referred to as "agents" and "tasks" with at most one task per agent and one agent per task, where the tasks appear dynamically and the task assignment is fully determined by the (cost, profit, or revenue) parameter values and the initial conditions. We extend mathematical models of the variations of the static task assignment problem to their dynamic counterparts in open vehicle fleet scenarios considering, among others, self-interested and individually rational vehicle drivers, time restrictions, fairness, agent qualification, and personal rank.

We identify some of the main scalable solution methods, i.e., coordination mechanisms, that can be put at work to solve these problems. We investigate the theoretical scalability of these approaches and introduce a taxonomy to classify them in terms of the level of interdependence in decision-making available to individual vehicles and customers during the coordination process (centralized, distributed, and decentralized coordination). Our intention here is not to perform an exhaustive search nor to identify the most scalable solution procedure. Contrarily, we identify and mathematically model the variations of the dynamic task assignment problem applicable to the studied fleet task allocation contexts and provide general scalability characteristics of their solution approaches. Our intention is to make it easier for a researcher to solve some variation of the task allocation problem in large-scale open vehicle fleets by describing state-of-the-art solutions and their theoretical scalability results.

Even though some works exist that include reviews of the state of the art in multiagent-task allocation (see, e.g., [2–6]) and in vehicle fleet coordination (see, e.g., [7–9]) or ride-sharing optimization (see, e.g., [10, 11]), none of them addresses one-on-one dynamic task assignment problems in open vehicle fleets. In addition, a few approaches apply methods of multiagent-task allocation to the field of vehicle fleet coordination (see, e.g., [12]) but, to the best of our knowledge, there is no systematic survey combining both fields.

The paper is organized as follows. In Section 2, we discuss some relevant concepts in the context of coordination for dynamic task allocation in open systems with the focus on distribution and decentralization of decision-making. In Section 3, we present mathematical models of various static and dynamic task assignment problems applicable in the open vehicle fleet context. Centralized, distributed, and decentralized state-of-the-art solution methods and mechanisms for the problems presented in Section 3 are discussed in Section 4. We conclude the paper emphasizing open issues and challenges for possible future research directions in Section 5.

## 2. Coordination in Open Vehicle Fleets

In this section, we introduce some key concepts and characteristics of the target domains related to decentralizing coordination for scalable and dynamic task allocation. The coordination problem arises due to the distributed nature of the control exercised by the fleet's vehicles.

Generally, coordination may be defined as "the process of organizing people or groups so that they work together properly and well" (https://www.merriam-webster.com/dictionary/coordination). By the coordination in open vehicle fleets for task allocation, we refer to the organization and management of decision-making within the fleet with the aim to improve given key performance indicators of a fleet's task allocation.

The topics of coordination and task allocation are the object of studies in multiple disciplines, e.g., operations research, economics, and computer science. The corresponding definitions and related concepts may vary based on the specific discipline at hand. In the so-called field of coordination models and languages, for instance, the focus is on the general-purpose abstractions (so-called *coordination* *media*) that can be generally used to model and engineer the patterns of interaction between computational agents—with no specific reference to a particular application scenario or coordination problem. In our survey, and in the following, we focus on the specific issues of dynamic task allocation and distributed/decentralized coordination, with a particular emphasis on open vehicle fleets.

*2.1. Fleet Coordination.* We consider the context with cooperative vehicles in a large vehicle fleet, which functions as an organization that constrains the cooperation schemes within it. The coordination problem here can be tackled from a bottom-up point of view, considering the emergence of global properties from the interfleet direct vehicle-to-vehicle communication and fleet-environment interaction.

For simplicity and without loss of generality, we consider a two-dimensional space in which tasks may appear randomly at any location in space and time while the vehicles circulate through a transportation network within the space to reach them. Each vehicle can have three states: *idle*, in which a vehicle is waiting for the assignment of a task, *assigned* in which a vehicle is assigned to a task but has still not reached the task, and *assisting* in which the vehicle has reached its assigned task and is assisting it. Only idle and assigned vehicles can be assigned or reassigned from one task to another. Once assigned, the vehicles start moving towards their assigned task. A task is considered completed once when it is reached and assisted by a vehicle.

Given a dynamically changing set (fleet) of idle and assigned vehicles, a dynamically changing set of randomly appearing tasks, and a cost function of the assignment of each task to every idle and assigned vehicle (e.g., the distance or time traveled or a given execution cost), the objective is to dynamically assign these vehicles to tasks in a given time horizon reaching a globally minimum cost assignment considering that each task must be performed by exactly one vehicle.

Coordinating the vehicles in this respect requires that they find the globally best allocation in a distributed or decentralized way and resolve conflicts that violate local constraints. An efficient strategy in this context is a dynamic (re-)assignment of the vehicles in the fleet to the tasks as they appear. The vehicles require continuous communication and processing for task allocation. The coordination system must ensure a balanced use of shared resources, e.g., vehicle-to-cloud (V2C) communication bandwidth and vehicle processing capacities.

V2C communication is limited in bandwidth and latency, so is the vehicle processing capacity. Coordination strategies that ignore these communication and computation constraints may fail to find a fleet's action plan in close to real time and thus may be inapt for the application in real-time fleets (see, e.g., [13]). These fleets require both autonomous and collaborative behaviours since vehicles have localized viewpoints, knowledge, and control and lack the overview of the global data integrated from various locations beyond their local capabilities. Such a dynamic context requires for coordination fault detection that indicates if the coordination exists within the fleet (see, e.g., [14]). Once a coordination fault is detected, a coordination recovery process can begin in which cooperation can be rebuilt.

Vehicle fleets that rely on one-on-one vehicle task assignment are, for example, rescue fleets (see, e.g., [15]), ride-hailing and taxi service (see, e.g., [16]), ambulance assistance of urgent out-of-hospital patients (see, e.g., [17]), and home-delivered restaurant hot meal services (see, e.g., [18]). Ride-hailing and restaurant hot meal delivery services are examples of open vehicle fleets that use online on-demand service platforms (see, e.g., [19]) to allocate in real-time customers and independent private vehicle owners, drivers, or couriers, using their personal vehicles. These platforms usually exploit sensor and GPS data to track the delivery process in real time [20].

Our focus is on the dynamic scenario with nonrecurring prearranged and spontaneously requested single-rider (customer), single-driver trips with at most one pickup and delivery for each rider and driver. Dynamically appearing riders (customers) should be allocated to drivers in a one-on-one manner. Before the allocation, in ride-hailing, a customer chooses the driver based on the time of arrival and the price of the ride. In case of hot meal delivery, the system gives an estimated delivery time to the customer and assigns a courier that meets such an estimate.

*Coordination* here is the key issue, including the stages of communication, resource allocation, and agreement. The allocation of the dynamically appearing customers over time needs to be performed in real time and it fails if not completed within a specified deadline relative to an arrival of a customer; deadlines must always be met, regardless of the system load. Conventionally, the matching is based just on the rider's personal preferences and the nearby drivers' availabilities. Reallocation of already matched drivers to riders that are awaiting the service is not possible even if a more efficient matching exists. At the end of each trip, every driver is available for a new rider allocation.

Speedy meal delivery services are constrained in geographic availability and timing. Usually, restaurants, riders, and customers have access to the system through an app. A customer detects his/her location and displays restaurants that participate in the platform in the region of interest and are open at the time. Couriers participate in this open fleet context by delivering whenever they choose and they may get paid on the individual delivery basis. Once a customer requests a meal from a restaurant via his/her app, the corresponding delivery is assigned to a courier available nearby. The courier picks up the delivery from the restaurant and delivers it to the customer. After the delivery, a courier is available for new deliveries.

The allocation of a courier to the customer is conventionally done based on the shortest arrival time to the restaurant (first-come-first-served strategy) and the availability of the courier; reallocation is not possible once the courier is allocated. The challenge here is to assign couriers to dynamically appearing pickups and deliveries in order to maximize customer satisfaction (which can be measured in different ways, as explored in [20]) without violating delivery times agreed at the time of the customer's hot meal request.

Task allocation problem in open vehicle fleets considers both providers of transportation services (vehicle drivers) and their customers and thus both of them may be considered active participants in the transportation process. In the ride-hailing scenario, drivers are usually modelled as agents and riders as tasks, while in the hot meal delivery scenario, couriers are agents while meal deliveries are tasks.

Even though the ownership of most of the open fleet systems today is centralized, not only customers but also drivers with vehicles may appear dynamically and spontaneously in time and space influenced by a variety of factors unknown in advance such that it is reasonable to assume that they appear randomly. In this *dynamic task allocation* context, available vehicles are assigned to pending customers as they appear. Each agent and task is assumed to be characterized by a set of attributes that influences the cost or profit resulting from an agent-task allocation. In this way, the *task allocation* problem that assigns tasks to agents in time is simplified to *task assignment* problem focusing on the one agent-one task allocation at the time (see, e.g., [17, 21]). Optimized and dynamic task (re-)assignment may considerably improve the performance of the fleet while considering individual fairness and efficiency (see, e.g., [21]). If dynamic courier (rider) reallocation is allowed, a substantial increase in efficiency may be observed, as in the case of ambulance allocation to out-of-hospital patients (see, e.g., [8, 21, 22]).

*2.2. Coordination Models for Open Vehicle Fleets.* Based on the ownership of the fleet, its structure, and the level of decentralizing coordination that we want to achieve in the fleet task allocation, we can design the following models:

A *centralized coordination model*, where the task allocation problem is solved in a single block by only one decision-maker (e.g., a single enterprise) having total control over and complete information about the vehicle fleet.

A *distributed coordination model*, where the global task allocation problem is decomposed such that each customer is represented by an autonomous decision-maker (agent) that may solve its own subproblem only with its own local decision variables and parameters. The allocation of a limited number of vehicles (global constraints) is done through the interaction between competing customer agents and a vehicle fleet owner (a single autonomous agent) having available all the fleet information. Customer agents that compete for the resources are not willing to disclose their complete information but will share a part of it if it facilitates achieving their local objectives. The vehicle fleet owner agent is responsible for achieving globally efficient resource allocation by interacting with customer agents usually through an auction. The problem decomposition here is done to gain computational efficiency since customer agents can compute their bids in parallel. However, the resource allocation decisions are still made by a single decision-maker (vehicle fleet owner) with the requirement on synchronous bidding of customer agents (see, e.g. [23–25]).

A *decentralized coordination model*, which further decentralizes the distributed model by allowing for multiple resource owner (vehicle) agents, multiple competing customer agents requesting the transportation service, and asynchrony in decision-making. Customer agents compete for fleet's vehicles held by multiple resource owners while each customer and resource owner agent has access only to its local information with no global information available. Therefore, they must negotiate resource allocation by running localized algorithms while exchanging relevant (possibly obsolete) information. Localized algorithms make the achievement of a desired global objective easier through simple local interactions of agents with their environment and other agents, with no need for a central decision-maker. The decisions specifying these interactions emerge from local information. Fairness in resource allocation here plays a major role. The same as in the distributed model, an agent is not willing to disclose its complete information but will share a part of it if it facilitates achieving its local objective. Resource allocation here is achieved by the means of a decentralized protocol.

Generally speaking, coordination is distributed when complex behaviour within a system does not emerge due to the control of the system owner, but through interactions and communication of individual agents operating on local information, while sharing globally relevant knowledge. This form of control is typically known as *distributed control*, that is, control where each agent is equally responsible for contributing to the global, complex behaviour by acting properly on local information. Agents are implicitly aware of the interaction rules through mechanisms that are based on the agent's interaction with other agents and the environment. The system behaviour is then an emergent property of distributed coordination mechanisms (algorithms) that act upon agents, rather than the result of a control mechanism of a centralized system owner. In decentralized algorithms, no global clock is assumed, no agent has complete information about the systems' state, every agent takes decisions based only on local information, and failure of one agent does not prevent the system to continue running. An example is Bitcoin: Instead of one central server owned and operated by a single entity, Bitcoin's ledger is distributed across the globe making it impossible to shut down, break in, or hack as there is no single central bottleneck of the system.

Let us notice the main difference between distributed and decentralized coordination models. Distributed coordination relies on local and shared (global) parameters and variables. Local parameters and variables are private, whereas shared and global parameters and variables need to be shared among two or more agents—even among all the agents of the system. If we assume self-concerned agents, resource owner can manipulate these parameters and variables or deceive agents in communicating their values to influence the individual decision-making of each one of them and thus obtain the behaviour of the system the resource owner wants. This can be prevented by ensuring individual agent access to nonobsolete and truthful information—using, e.g., blockchain technology. Reaching a globally optimal solution with quality of solution guarantees is then possible, contrary to the decentralized coordination case. In the latter case, due to the lack of the global nonobsolete and truthful information, quality of solution guarantees generally do not exist. In general, solution approaches for decentralized coordination concentrate on finding a feasible (admissible) solution without quality of solution guarantees. Contrary to the distributed case most often studied in the operations research field where the emphasis is on the method's optimality gap, decentralized coordination methods are mostly approximate heuristics-based methods without quality of solution guarantees but with proven completeness, soundness, and termination.

Open vehicle fleets are intrinsically distributed systems since they comprise a multitude of geographically distributed and mutually communicating customers' and vehicle drivers' apps. Traditionally, distributed systems refer to systems consisting of sequential processes (each one with an independent thread of control, possibly located on geographically distributed processors) that coordinate their actions by exchanging messages to meet a common goal (see, e.g., [26, 27]). The common goal in this context is an efficient and cost-effective transportation service of the vehicle fleet while considering individual rationality, preferences, and constraints whether it is of drivers, riders, or hot meal delivery customers. Quality of solution guarantees play a crucial role of sustainable competitive advantage in any transportation network company.

Distributed open vehicle fleets exhibit some clear strong points over their centralized counterparts. First of all, they are more robust than their centralized counterparts because they can rely on their intrinsic built-in redundancy. They can operate at a larger scale and assist more customers at once since they are aggregating vehicle capacity and customer throughput across all their individual vehicle drivers. However, distributed open vehicle fleets also have to deal with intervehicle communication and coordination overhead that can sometimes make them slower or more difficult to control than their centralized counterparts. Applying trustless distributed systems that are meant to operate in an adversarial environment, such as Bitcoin, in open fleets entails an additional overhead.

## 3. Task Assignment Models for Open Vehicle Fleets

Assignment problems (APs) are among the earliest optimization problems studied in the operations research field. They involve optimally matching the elements of two or more sets, where the dimension of the problem refers to the number of sets to be matched [28]. For example, in two-dimensional assignment problems, given is a set of agents $A$ and a set of tasks $T$ and we have to match (assign) tasks to agents. Tasks are assumed atomic, i.e., each task cannot be decomposed into subtasks and it can be completed by a single vehicle. In general, two-dimensional assignment problems can be solved in polynomial time, while $d$-dimensional assignment problems, with $d > 2$, in general are NP-hard (see, e.g., [29]).

We distinguish between the static and dynamic assignment problems (see, e.g., [30]). The former refers to the assignment of a set of tasks to a set of agents in a given static environment in which the problem data does not change during the planning horizon, while in the dynamic task assignment problems, both agents and tasks may appear and disappear dynamically over time. In the open vehicle fleet setting, agents can be in one of the following three states: *idle*, *assigned* without still having reached the customer, or *assisting a customer*, and only idle and assigned agents that have still not reached their customers can be (re)assigned to unassisted tasks. In general, agents are assumed renewable, i.e., after completing a task, an agent's state changes from *assisting a customer* to *idle* and it becomes assignable again to customers (tasks) that have not been assisted yet. This is a special case of a more general computationally complex dynamic vehicle routing problem (DVRP) in which, for each (vehicle) agent, we find a minimum cost route that visits a dynamically changing set of tasks (customers) [31]. Due to the high computational complexity, myopic algorithms are the most usual solution approaches for DVRP. For simplicity, we can assume that agents are nonrenewable, i.e., an agent can be assigned only to one task; if, after completing a task, it is still available for new task assignment, it appears as a new agent.

The static and deterministic AP is a computationally easy problem, which allows us (in theory) to find an optimal solution in close to real time (in the nonrenewable agent case). Dynamic AP can be solved by (suboptimal) myopic approaches that consider only the information available at the present time with no consideration for future events and possibly reassign tasks among idle and already assigned agents to improve the system's efficiency (see, e.g., [8, 17, 21, 22]). However, in the case where tasks are not randomly appearing, this approach can be significantly improved by considering future developments.

*3.1. Static Task Assignment.* Based on the categorization of the AP models presented in [28], in this section, we consider the classic assignment problem and its variations relevant in the open fleet vehicle task assignment considering self-interested and individually rational vehicle users whose tasks can be performed simultaneously: the classic linear assignment problem (LAP), assignment problem recognizing agent qualification (APRAQ), the bottleneck assignment problem (BAP), the fair matching problem (FMP), the minimum deviation assignment problem (MDAP), the $\Sigma_k$-assignment problem ($\Sigma_k$-AP), the semiassignment problem (SAP), and the assignment problem with side constraints (APSC). In Figure 1, we give a framework for easier understanding of the characteristics of both the static and dynamic version of these problems.

For self-completeness of this article, we bring in the following the descriptions of these problems. Considering that the number of publications concerning assignment problems is

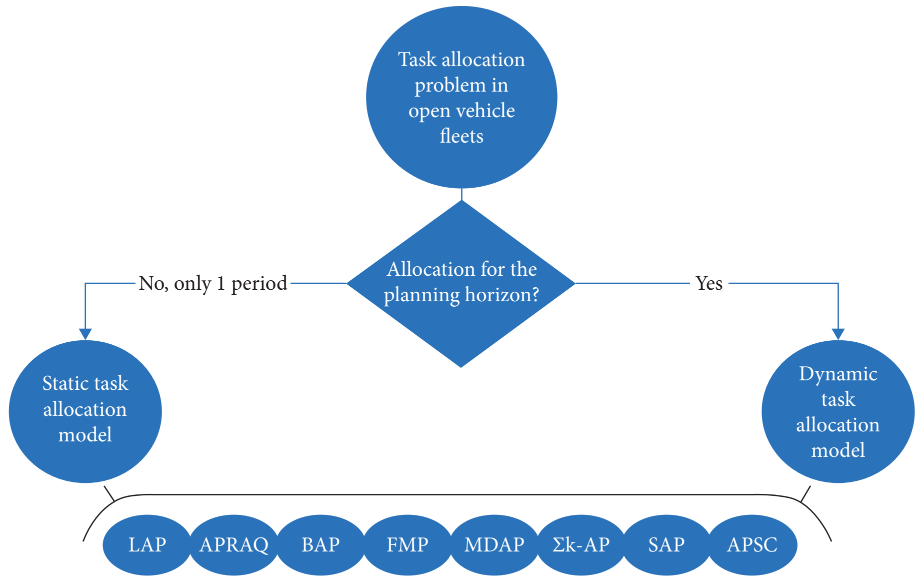


Figure 1: Static and dynamic task assignment problems in open vehicle fleets.

enormous, the references in this section constitute only a very limited part of them. For the details and other assignment problem variations, the reader is referred to [28].

*3.1.1. Classic (Linear) Assignment Problem (LAP).* The static classic linear assignment problem involves two sets of the same size and consists of finding, in a weighted complete bipartite graph, a perfect matching in which the sum of weights of the matched edges is as low as possible, i.e., a *minimum-weight perfect matching*. Perfect weighted matching implies that each node must be matched to some other node by minimizing the total cost of the arcs in the (perfect) matching.

The classic linear assignment problem (LAP) can be defined as follows: given a weighted complete bipartite graph $G = (A \cup T, E)$ with two vertex sets $A$ and $T$, with $n = |A| = |T|$, and an edge set $E = A \times T$, with edge weights $c_{ij}$ on edge $(i, j) \in E$, find a minimum-weight perfect matching of $G$, i.e., a perfect matching among vertices in $A$ and vertices in $T$ such that the sum of the costs of the matched edges is minimum. An edge $(i, j) \in E$ is matched if two extreme vertices $i$ and $j$ are mutually matched, and a matching is perfect if every vertex $i$ of $A$ is matched (assigned) exactly to one vertex $j$ of $T$, and vice versa. The LAP is equivalent to the weighted bipartite matching, since we may assume that the bipartite graph is always complete by letting the weights of the edges that are missing being sufficiently large. If $|A| \neq |T|$, we can add a number of dummy nodes to the set with lower cardinality and connect them by dummy arcs of zero cost to the other set. The number of dummy nodes should be sufficient to balance the cardinalities of the two sets.

The LAP is equivalent to the maximum weighted bipartite matching (with edge weights $w_{ij} \geq 0$), since we may assume that the bipartite graph is always complete by letting the weights of the edges that are missing being sufficiently large. Furthermore, also in this case, we can assume that the two vertex sets of the bipartite graph have the same size. At this point, we can reformulate the problem as a minimization problem by considering costs $c_{ij} = W - w_{ij}$, where $W$ is larger than the maximum of the $w_{ij}$, and hence, this problem corresponds to the LAP.

The LAP is a special case of the transportation problem assuming an equal number of supplier agents and customer agents and each one with their unitary supply and unitary demand, respectively. The transportation problem is one of the special cases of the minimum cost flow problem together with, e.g., the shortest path problem and the max flow problem. While it is possible to solve this problem using the simplex algorithm, specialized algorithms take advantage of its special network structure and are thus more efficient.

From the multiagent systems' point of view, in the assignment problem, a number of agents need to be assigned to a number of tasks based on the given cost of agent-task assignment. In general, each agent can be assigned to any task. In case an agent is not capable of performing a task, a given agent-task assignment cost is modelled as a very large number. All tasks should be performed with the objective to minimize the total cost of the assignment such that exactly one agent is assigned to each task and exactly one task to each agent. The mathematical formulation of the problem is as follows:

$$\min \sum_{i,j} c_{ij} x_{ij}, \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall j \in T, \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \forall i \in A, \tag{3}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in A, j \in T. \tag{4}$$

Constraints (2) ensure that every task is assigned to only one agent and constraints (3) ensure that every agent is assigned to only one task.

The structure of the problem, i.e., the total unimodularity of the constraint matrix, makes the binary requirements on the variables unnecessary. In fact, in this case, it can be proven that the linear relaxation has always an optimal binary solution (see, e.g., [32, 33]) and, therefore, the LAP is a linear programming (LP) problem.

*3.1.2. The Classic Assignment Problem Recognizing Agent Qualification (APRAQ).* Caron et al. in [34] propose a mathematical model in which not every agent is qualified to do every task, and the objective is utility maximization:

$$\max \sum_{i,j} p_{ij} x_{ij}, \tag{5}$$

subject to

$$\sum_{i \in A} q_{ij} x_{ij} \leq 1, \quad \forall j \in T, \tag{6}$$

$$\sum_{j \in T} q_{ij} x_{ij} \leq 1, \quad \forall i \in A, \tag{7}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in A, j \in T, \tag{8}$$

where parameter $q_{ij} = 1$ if agent $i$ is qualified to perform task $j$, 0 otherwise, parameter $p_{ij}$ is the utility of assigning agent $i$ to task $j$ (with $p_{ij} = 0$ if $q_{ij} = 0$), and variable $x_{ij} = 1$ if agent $i$ is assigned to task $j$, 0 otherwise. Constraints (6) ensure that no more than one qualified agent is assigned to any task, while constraints (7) guarantee that each agent is assigned to not more than one task.

The classic assignment problem does not consider fairness. The solution of classic AP (1)–(4) maximizes utilitarian social welfare (see, e.g., [35]), but it may be unfair and unsatisfactory since there may be one or more agents with a much higher task cost than the rest. This is why it is best applied to centralized open vehicle fleets with a single owner of the fleet's vehicles that is interested in the minimization of the overall cost of the fleet's operation costs but not in how they are distributed among the vehicles.

*3.1.3. Bottleneck Assignment Problem (BAP).* To resolve the issues with fairness and workload distribution, we may minimize maximum cost among the individual agent-task assignments and thus maximize the system's egalitarian social welfare (see, e.g., [36]). The mathematical program for the BAP is as follows: minimize $\max_{i,j}\{c_{ij}x_{ij}\}$ or minimize $\max_{i,j}\{c_{ij} \mid x_{ij} = 1\}$ subject to constraints (2)–(4) and definitions of the LAP.

Note that here the integrality requirements cannot be relaxed. Contrary to the classic AP model, the BAP model pursues the objective of fairness among agents. It is based on the optimization of the worst-off performance and provides a good solution when the minimum requirements of all agents should be satisfied. However, only the most costly agent-task assignment influences the objective function, while the contribution of the rest of the agents is ignored. For this reason, this approach deteriorates the system efficiency and thus the system's utilitarian social welfare.

*3.1.4. The Fair Matching Problem (FMP).* The fair matching problem minimizes the difference between the maximum and minimum assignment values [37]: minimize $\max_{i,j}\{c_{ij} \mid x_{ij} = 1\} - \min_{i,j}\{c_{ij} \mid x_{ij} = 1\}$ subject to the same constraints and definitions as in the classic AP.

This formulation of fairness is not unique. Sun and Yang in [38] study the concept of fair and optimal allocations. They define an allocation to be fair and optimal if it is envy-free and the sum of compensations is maximized, subject to the compensation assigned to each object is less than or equal to the maximum compensation limit. They prove that fair and optimal allocations exist and demonstrate that the fair and optimal allocation mechanism achieves efficiency, fairness, and strategy-proofness simultaneously. Andersson [39] demonstrates that it is also coalitionally strategy-proof, i.e., it is not possible for any agent or any coalition of agents to successfully manipulate the allocation rule.

*3.1.5. The Minimum Deviation Assignment Problem (MDAP).* The objective here is to minimize the difference between the maximum and average assignment costs:

$$\text{minimize} \min\{n,m\} \times \max_{p,q}\{c_{pq}x_{pq}\} - \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}, \tag{9}$$

or to minimize the difference between the average and minimum assignment profit:

$$\text{minimize} \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}x_{ij} - \min\{n,m\} \times \min_{s,t}\{p_{st}x_{st}\}, \tag{10}$$

subject to constraints (2)–(4). Here, $n$ is the cardinality of agent set $A$, and $m$ of task set $T$, and other definitions are the same as in the LAP [40, 41].

*3.1.6. The $\Sigma_k$-Assignment Problem ($\Sigma_k$-AP).* Since there may be generally multiple different sets of assignments with the same minimum value for $\max\{c_{ij}x_{ij}\}$, the objective here is to find a set of assignments for which the sum of the $k$ largest values is minimized. The BAP and LAP can be viewed as special cases of $\Sigma_k$-AP with $k = 1$ and $k = n$, respectively.

A recent study on generic mixed integer problem with $\Sigma_k$ optimization is done by Filippi et al. [42].

*3.1.7. The Semiassignment Problem (SAP).* This is the version of the assignment problem where every agent or task may not be unique. This results in a constraint matrix containing a number of rows or columns with equal coefficients. Kennington and Wang in [43] show examples of such a problem in workforce and project planning and scheduling as use case examples. Here, constraints (2) from the classic LAP are substituted by

$$\sum_{i=1}^{m} x_{ij} = d_j, \quad \forall j, \tag{11}$$

everything else being the same as in the classic LAP for the situation in which there are $n$ agents and $m$ task categories. Here, $m \leq n$, and $d_j$ is the number of tasks in task group $j$ with $\sum_j d_j = n$.

Note that if also the agents are not unique and are clustered into agent groups, with $q_i$ agents in each group $i$, where $\sum_j d_j = \sum_i q_i$, the problem is equivalent to the transportation problem.

*3.1.8. The Assignment Problem with Side Constraints (APSC).* Classic assignment problem can be solved by multiple centralized and efficient polynomial algorithms. However, by introducing side constraints, generally, this problem becomes NP-hard. Side constraints may include budgetary limitations, degree of technical training of personnel, the rank of personnel, or time restrictions that limit the assignment of agents to tasks.

Aggarval [44] introduces to the classical LAP problem an additional knapsack-type constraint:

$$\sum_{i,j} r_{ij} x_{ij} \leq b, \tag{12}$$

where $r_{ij}$ is the amount of resource used if agent $i$ is assigned to task $j$ and $b$ is the amount of a resource available. Adding constraint (12) to LAP results in a resource-constrained assignment problem (RCAP), which is a knapsack problem under perfect matching over a bipartite network. Constraint (12) deranges the unimodularity of the LAP set of constraints so that the optimal solution of the linear relaxation of the problem is no more always within the values $\{0, 1\}$ and, hence, integrality constraints cannot be relaxed. The resulting problem belongs to the class of NP-complete problems for which no polynomially bounded algorithm is likely to exist (see, e.g., [44]).

Mazzola and Neebe [45] present a general model for the assignment problem with side constraints that generalizes the general assignment problem (GAP) (see, e.g., [46]) and adds the following constraints to either the classic LAP model or the classic LAP recognizing agent qualifications:

$$\sum_{i,j} r_{ijk} x_{ij} \leq b_k, \quad \forall k, \tag{13}$$

where $r_{ijk}$ is the amount of resource $k$ used if agent $i$ is assigned to task $j$ and $b_k$ is the amount of resource $k$ available.

By side constraints, we can model drivers that belong to different seniority classes and customers that have different priority levels. Seniority constraints impose for the solution to be such that no unassigned agent can be assigned to a task unless an assigned agent with the same or higher seniority becomes unassigned, while priority constraints specify that the solution must be such that no unassigned task can become assigned without a task with the same or higher priority becoming unassigned [34].

*3.2. Dynamic Task Assignment.* In this section, we propose extensions of the static assignment problem models presented previously to the dynamic versions in which new agents and tasks may enter the system in each time period and the costs or profits of agent-task assignment are updated in (close to) real time. This problem is similar to the online bipartite matching problem, in which tasks that appear in sequence should be assigned to the agents immediately as they appear. Relating to the previously presented terminology of the static AP, a set of available (idle and assigned) agents $A$ (that are not assisting any customer) is known in the given weighted bipartite graph $G = (A \cup T, E)$. Tasks in $T$ (along with their incident edges) arrive online. Upon the arrival of a task $j \in T$, we must assign it to one of agents $i \in A$ with an existing edge $(i, j) \in E$. At all times, the set of matched edges must form a (feasible) matching, i.e., each agent should be matched with at most one task and vice versa. In case of different cardinalities of the two sets, to balance the two, dummy elements are added to the set with lower cardinality.

We assume random arrivals of customer demands (tasks) over time. In open fleets, we also assume that agents (drivers and couriers) either become available randomly after assisting previous tasks (customers) or by entering and leaving the fleet based on personal interest, available time, and/or other individual constraints and preferences. Given are attribute parameters both for agents and tasks that define their main characteristics in terms of the assignment.

We consider deterministic on-demand task allocation where the (re-)assignment of vehicles (agents) to tasks is performed as soon as a new vehicle or task enters the system. Close to real-time reassignment is beneficial here since the parameters and variables of the assignment problem are perfectly known.

Spivez and Powell [30] propose a Markov decision process model for the dynamic assignment problem. In this paper, inspired by their work, we propose mathematical programming models for the variations of the static task assignment described in the previous section while respecting agent-task taxonomy used previously in this paper.

The decisional variables in the dynamic AP receive a third index such that

$$x_{ij\tau} = \begin{cases} 1, & \text{if task } j \in T \text{ is assigned to agent } i \in A \text{ at period } \tau \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Moreover, we introduce two additional binary variables $\alpha_{\tau i}$ and $\beta_{\tau j}$, for all $i \in A$, $j \in T$ defined as follows:

$$\alpha_{i\tau} = \begin{cases} 1, & \text{if agent } i \in A \text{ is known and available for assignment in period } \tau, \\ 0, & \text{otherwise,} \end{cases}$$
$$\beta_{j\tau} = \begin{cases} 1, & \text{if task } j \in T \text{ is known and available for assignment in period } \tau, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Let $\mathcal{T}$ be a set of consecutive time periods of the planning time horizon. The mathematical formulation of the deterministic and dynamic LAP problem considering utility maximization is then given by

$$Z = \max \sum_{\tau\in\mathcal{T}} \sum_{i\in A} \sum_{j\in T} p_{ij\tau} x_{ij\tau}, \tag{16}$$

subject to

$$\sum_{j\in T} x_{ij\tau} \leq \alpha_{i\tau}, \quad \forall i, \tau \tag{17}$$

$$\sum_{i\in A} x_{ij\tau} \leq \beta_{j\tau}, \quad \forall j, \tau, \tag{18}$$

$$\alpha_{i,\tau+1} = \alpha_{i\tau} - \sum_{j\in T} x_{ij\tau} + \widehat{A}_{i,\tau+1}, \quad \forall i, \forall \tau \in \{1, \ldots, |\mathcal{T}| - 1\}, \tag{19}$$

$$\beta_{j,\tau+1} = \beta_{j\tau} - \sum_{i\in A} x_{ij\tau} + \widehat{T}_{j,\tau+1}, \quad \forall j, \forall \tau \in \{1, \ldots, |\mathcal{T}| - 1\}, \tag{20}$$

$$\alpha_{i,1} = \widehat{A}_{i,1}, \quad \forall i, \tag{21}$$

$$\beta_{j,1} = \widehat{T}_{j,1}, \quad \forall j, \tag{22}$$

$$x_{ij\tau} \in \{0, 1\}, \quad \forall i \in A, j \in T, \tau \in \mathcal{T}, \tag{23}$$

$$\alpha_{i\tau} \in \{0, 1\}, \quad \forall i \in A, \tau \in \mathcal{T}, \tag{24}$$

$$\beta_{j\tau} \in \{0, 1\}, \quad \forall j \in T, \tau \in \mathcal{T}, \tag{25}$$

where $p_{ij\tau}$ is the utility of assigning agent $i$ to task $j$ at period $\tau$ (note that it may vary through time) and $\widehat{A}$ and $\widehat{T}$ are given parameters such that

$$\widehat{A}_{i\tau} = \begin{cases} 1, & \text{if agent } i \in A \text{ enters into set } A \text{ (the fleet) in period } \tau, \\ 0, & \text{otherwise.} \end{cases}$$
$$\widehat{T}_{j\tau} = \begin{cases} 1, & \text{if task } j \in T \text{ becomes known in period } \tau, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

Moreover, based on the assumption of nonrenewable agents and tasks, we assume that $\sum_{\tau\in\mathcal{T}} \widehat{A}_{i\tau} \leq 1$ and $\sum_{\tau\in\mathcal{T}} \widehat{T}_{j\tau} \leq 1$, i.e., every agent and task are unique and enter into the fleet and thus become available for assignment only once.

The aim is maximizing the total utilitarian social welfare over the planning time horizon, which is achieved by maximizing the assignment utility (16) over all agent-task assignments in all periods of the planning time horizon. Constraints (17) guarantee that each available agent at time period $\tau$ is assigned to at most one task while unavailable agents cannot be assigned to any task. Constraints (18) ensure that at most one agent is assigned to any available task while no agent can be assigned to any unavailable task.

Constraints (19) and (20) represent the dynamics of dependent variables $\alpha_{\tau i}$ and $\beta_{\tau j}$, assuming that both agents and tasks disappear from the system at the end of the period when they are assigned. Furthermore, constraints (21) and (22) represent the initial conditions of the problem, while the variable ranges are given by (23)–(25).

We can also consider cost minimization problem where we substitute (16) with the following objective function:

$$Z = \min \sum_{\tau\in\mathcal{T}} \sum_{i\in A} \sum_{j\in T} c_{ij\tau} x_{ij\tau}, \tag{27}$$

subject to

$$\sum_{i\in A} \sum_{j\in T} \sum_{\tau\in\mathcal{T}} x_{ij\tau} = n, \tag{28}$$

and (17)–(25). Constraint (28) guarantees the assignment of all the tasks and/or agents in the planning time horizon, depending on the relative size of these two sets.

*3.2.1. The Dynamic Classic Assignment Problem Recognizing Agent Qualification.* Here, the objective function is again the utility maximization (16), while constraints (17) and (18) are substituted by the following ones, everything else remaining the same as in the dynamic LAP:

$$\sum_{j\in T} q_{ij\tau} x_{ij\tau} \leq \alpha_{\tau i}, \quad \forall i, \tau, \tag{29}$$

$$\sum_{i\in A} q_{ij\tau} x_{ij\tau} \leq \beta_{\tau j}, \quad \forall j, \tau, \tag{30}$$

where parameter $q_{ij\tau} = 1$ if agent $i$ is qualified to perform task $j$ at period $\tau$, 0 otherwise, parameter $p_{ij\tau}$ is the utility of assigning agent $i$ to task $j$ at period $\tau$ (with $p_{ij\tau} = 0$ if $q_{ij\tau} = 0$), and variable $x_{ij\tau} = 1$ if agent $i$ is assigned to task $j$ at period $\tau$, 0 otherwise. Constraints (29) guarantee that no more than one qualified agent is assigned to any task, while constraints (30) ensure that each agent is assigned to not more than one task. Instead of the profit maximization, here, we can introduce cost minimization by substituting (16) with (27) and introducing (28) into the constraint set.

*3.2.2. The Dynamic Bottleneck Assignment Problem (DBAP).* The objective function of the DBAP problem can be formulated as follows: at each period $\tau \in \mathcal{T}$, maximize

$Z = \min_{i,j}\{p_{ij\tau}x_{ij\tau}\}$ or maximize $Z = \min_{i,j}\{p_{ij\tau} \mid x_{ij\tau} = 1\}$. This maxmin problem can be expressed by maximizing an additional variable $L$ that is a lower bound for each of the individual values $\{p_{ij\tau} \mid x_{ij\tau} = 1\}$ as follows: max $L$ subject to constraints $L \le \sum_{j\in T} p_{ij\tau}x_{ij\tau}$ for all $i \in A_\tau$, $\tau \in \mathcal{T}$, and (17)–(25) and definitions of the dynamic LAP.

*3.2.3. The Dynamic Fair Matching Problem (DFMP).* Here, at each period $\tau \in \mathcal{T}$, we minimize the objective function $\max_{i,j}\{c_{ij\tau} \mid x_{ij\tau} = 1\} - \min_{i,j}\{c_{ij\tau} \mid x_{ij\tau} = 1\}$ and subject to constraints (17)–(25). Similarly, we can minimize the difference between the maximum and minimum profit obtained among agents, i.e., minimize ($\max_{i,j}\{p_{ij\tau} \mid x_{ij\tau} = 1\} - \min_{i,j}\{p_{ij\tau} \mid x_{ij\tau} = 1\}$ and subject to constraints (17)–(25).

*3.2.4. The Dynamic Minimum Deviation Assignment Problem (DMDAP).* At each period $\tau \in \mathcal{T}$, the objective function is as follows:

$$\text{minimize} \quad \min\{n, m\} \times \max_{p,q}\{c_{pq\tau}x_{pq}\} - \sum_{i\in A}\sum_{j\in T} c_{ij\tau}x_{ij\tau}, \tag{31}$$

or

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij}x_{ij} - \min\{n, m\} \times \min_{s,t}\{p_{st}x_{st}\}, \tag{32}$$

subject to constraints (17)–(25) and definitions of the minimum deviation assignment problem.

*3.2.5. The Dynamic $\Sigma_k$-Assignment Problem (D$\Sigma_k$-AP).* Given parameter $k$, objective function (16) is modified to

$$Z = \max \sum_{\tau\in\mathcal{T}}\sum_{i=1}^{k}\sum_{j\in T} p_{ij\tau}x_{ij\tau}, \tag{33}$$

subject to constraints (17)–(25) and definitions of the dynamic LAP.

*3.2.6. The Semiassignment Problem.* Here, constraints (18) from the dynamic LAP are substituted by

$$\sum_{i=1}^{m} x_{ij\tau} = d_j\beta_{j\tau}, \quad \forall j, \tau, \tag{34}$$

everything else being the same as in the dynamic LAP for the situation in which there are $n$ agents and $m$ task categories, where $m \le n$.

*3.2.7. The Assignment Problem with Side Constraints.* Side constraints (13) here include also the time index:

$$\sum_{j\in T} r_{ij\tau}x_{ij\tau} \le b_{\tau i}, \quad \forall k, \tau, \tag{35}$$

where $r_{ijk\tau}$ is the amount of resource $k$ used if agent $i$ is assigned to task $j$ at period $\tau$ and $b_{k\tau}$ is the amount of resource $k$ available at period $\tau \in \mathcal{T}$. Constraints (35) are simply added to the formulation of the dynamic LAP.

*3.3. Bottom Line.* To sum up, in Table 1, we give the overview of the characteristics of the treated (static and dynamic) task assignment problems related to (i) the kind of the social welfare they optimize (utilitarian, egalitarian, elitist, or a difference between them), (ii) whether agents are qualified to perform only certain tasks or not, (iii) including fairness or not, (iv) whether the agents are considered homogeneous or not, (v) time restrictions, (vi) personal ranking, and (vii) technical training.

Note that once we introduce additional constraints to the classic assignment problem, the resulting model is, generally, no more resolvable in polynomial time and is highly computationally expensive. Additionally, *we consider tasks and agents that may be known both at some future time period and at the first period of the planning time horizon*. Therefore, we can use this model to coordinate task allocation for planned tasks and agents that schedule their appearance in advance for some future time period, but also for the tasks and agents that need to be allocated on short notice or immediately as they get known and enter the system. To this aim, we must use highly computationally efficient close to real-time solution approaches and, generally, exact methods do not suffice for this purpose. Therefore, we are obliged to use heuristic-based approximations.

## 4. Coordination Approaches in Task Allocation to Fleet's Vehicles

In this section, we recall the main (coordination) solution methods for the task allocation problem in open vehicle fleets in general and the treated assignment problems in particular, categorizing them in centralized, distributed, and decentralized (Figure 2), with special attention to those with the best time complexity. Recall that the static classic assignment problem consists in finding the minimum cost perfect matching of a complete bipartite graph $G = (A \cup T, E)$, with $E = A \times T$ and $n = |A| = |T|$.

*4.1. Centralized Coordination Approaches.* There are a huge number of algorithms for the linear assignment problem (LAP). They can be subdivided into *primal*, *dual*, and *primal-dual* algorithms. The worst-case time complexity of the best algorithms is $O(n^3)$.

We preliminary recall the mathematical formulation of the dual problem of the linear formulation of the LAP:

$$\max \sum_{i=1}^{n} u_i + \sum_{j=1}^{n} v_j, \tag{36}$$

subject to

$$u_i + v_j \le c_{ij}, \forall i, j \in \{1, \ldots, n\}, \tag{37}$$

Table 1: Characteristics of the discussed task assignment models.

| Model | Soc. welfare | Agent qualif. | Fairness | Unique ag./tasks | Time restr. | Pers. rank | Tech. train. |
|---|---|---|---|---|---|---|---|
| LAP | Util. | No | No | Yes | No | No | No |
| APRAQ | Util. | Yes | No | Yes | No | No | No |
| BAP | Egal. | No | No | Yes | No | No | No |
| FMP | El. – Eg. | No | No | Yes | No | No | No |
| MDAP | El. – ut. | No | No | Yes | No | No | No |
| | Ut. – el. | No | No | Yes | No | No | No |
| $\Sigma_k$-AP | Egal. | No | Yes | No | No | No | No |
| SAP | Util. | No | No | Yes | No | No | No |
| APSC | Util. | No | No | No | Yes | Yes | Yes |

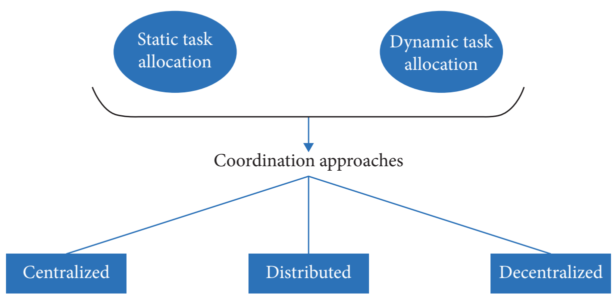


Figure 2: Coordination approach framework for task allocation.

where $u_i$ and $v_j$ are the (dual) variables.

*4.1.1. Primal Algorithms.* Primal algorithms are in general special implementations of the network simplex algorithm: one of the best primal algorithms is proposed in [47] and runs in $O(n^3)$ time.

*4.1.2. Dual Algorithms.* Dual algorithms are iterative algorithms which at each iteration maintain a feasible dual solution, and only at the final iteration, they come up with a primal solution (i.e., a feasible assignment). In this regard, also the primal-dual algorithms can be viewed as special dual algorithms. Typical dual algorithms are those based on successive shortest paths, signature, pseudoflow, interior point, and auction methods. In the following, we concentrate on the auction methods because from the latter, one can easily derive distributed versions of the same. For additional details, the reader is referred to [29, 36].

For a short survey on the above solution algorithms for the LAP, the reader is referred to a not so recent but detailed experimental comparison of some of the algorithms in [48]. Another survey on the state-of-the-art algorithms for the LAP is provided in [36].

*4.1.3. Auction Algorithms.* The first auction algorithm for the LAP was given by Bertsekas [49] and successively improved by Bertsekas and Eckstein [50] through a scaling technique providing an algorithm that runs in $O(n^3 \log(nC))$, where $C = \max\{|c_{ij}|\}$. A survey of iterative combinatorial auction algorithms for task allocation in multiagent systems can be found in, e.g., [4, 51–53].

The auction algorithm proposed by Bertsekas in [49] is an iterative algorithm that at each iteration maintains a triple $(x, (u, v))$ of primal and dual solutions that satisfy the complementary slackness conditions such that the dual solution is feasible. The algorithm terminates when also the corresponding primal solution is feasible. At each iteration, the dual solution is updated and the corresponding primal solution (with respect to complementary slackness conditions) is found.

In particular, given a dual vector $v$, the optimal (feasible) dual vector $u$ can be obtained by considering $u_i = \min_j\{c_{ij} - v_j\}$, and, hence, the dual problem can be rewritten as

$$\max q(v) = \sum_{i=1}^{n} \min_j\{c_{ij} - v_j\} + \sum_{j=1}^{n} v_j. \quad (38)$$

Denoting with $j_i = \arg - \min_j\{c_{ij} - v_j\}$, the primal solution $x$, with $x_{i,j_i} = 1$ and 0 for $j \neq j_i$, with $i = 1, \dots, n$, satisfies the complementary slackness conditions.

The dual problem has a nice economical interpretation. Assume that $p_j = -v_j$ represents the price that any agent will pay for being assigned to task $j$ and $u_i$ is the utility for agent $i$ for being assigned to a task. The dual assignment problem consists in determining $u_i$ and $p_j$ (i.e., $-v_j$) maximizing the agents' total net utility, such that agents' net utilities cannot be greater than the costs $c_{ij}$ they face. LP duality theory states that the maximum agents' total net utility equals the total assignment cost. At optimum, each task is assigned exactly to one agent, and the LP duality theory and complementary slackness conditions in particular assure that each agent $i$ is assigned to the most profitable task $j_i$, which guarantees that agent net utility $u_i - p_{j_i}$ is exactly equal to the assignment cost $c_{i,j_i}$.

From the LP duality theory applied to the AP, we can derive the following auction algorithm [51]. Assume that agents are assigned to tasks through a market mechanism, with agent $i$ acting according to its own best interest. Assume that task prices $p_j = -v_j$ are given. The total agent utility $(\sum_j u_j)$ is maximized if we set each $u_j$ to its largest value allowed by the dual constraints, that is, $u_i = \min_j\{c_{ij} + p_j\}$. From the complementary slackness conditions, it follows that each agent $i$ will bid for the most profitable task $j_i$, i.e., with $c_{ij_i} + p_{j_i} = u_i$ in order to be assigned to it. If no task is bid by more than one agent, we reach an equilibrium and the assignment is optimal; otherwise, we may change (increase) task prices $p_j$ in order to discourage agents to bid for the same task. This mechanism may be regarded as a naive auction algorithm that proceeds in rounds and halts if we get an equilibrium. We call it naive because it contains a flaw (as we will show next), but it motivates a more sophisticated and correct algorithm.

At each round of the naive auction algorithm, we start with a partial assignment and a given set of task prices and repeat the following two steps until all agents are assigned to their desired task (when we are at the equilibrium):

(1) Bidding step: given task prices $p_j$ and a partial assignment of agents to tasks, (i) each unassigned agent $i$ bids for its most profitable task $j_i = \arg - \min_j \{c_{ij} + p_j\}$ with an offer equal to $p_{j_i} + \gamma_i$, with $\gamma_i = \beta_i - \alpha_i$, where $\alpha_i = \min_j \{c_{ij} + p_j\}$ and $\beta_i = \min_{j \neq j_i} \{c_{ij} + p_j\}$, while (ii) each already assigned agent still submits the previous winning bid (without changing their bid offers).

(2) Pricing step: each task $j$ is assigned to the highest offering bidder (agent) for that target. The price $p_j$ of each task $j$ receiving a new (greater) bid is increased to the highest received offer, i.e., the new price value will be equal to $p_j + \gamma_i$.

Unfortunately, this naive auction mechanism does not always work. It gets trapped in a cycle when (a) there is at least one unassigned agent and (b) each new winner bidder $i$ submitted an offer for its preferred task $j_i$ at its given target price $p_{j_i}$, i.e., $\gamma_i = 0$, meaning that its first and second best choices have the same cost.

In order to avoid this to happen, we need to keep rising the prices of tasks receiving new bids by at least a small amount $\epsilon > 0$. Therefore, we assume that agent $i$ will bid for its preferred task $j_i$ by offering $p_{j_i} + \gamma_i + \epsilon$.

This means that agent $i$ desires to be assigned to task $j_i$ if $c_{ij_i} + p_{j_i} \leq \min_j \{c_{ij} + p_j\} + \epsilon = \alpha_i + \epsilon$, which therefore is not necessarily its best choice. The above condition is known as $\epsilon$-complementary slackness (see, e.g., [51]).

With this correction, the auction algorithm works ending in a finite number of rounds (depending on $\epsilon$), with each task receiving a bid. At the end, we are almost at an equilibrium with agent $i$ assigned to its almost desired task $j_i$. In general, this corresponds to an almost optimal solution for the assignment problem, since complementary conditions are only almost satisfied, while primal and dual complementary solutions are both feasible. It can be proved that if the cost $c_{ij}$ are integers and $0 < \epsilon < 1/n$, then the (corrected) auction algorithm ends with an optimal solution for the assignment problem (see, e.g., [51]).

Without loss of generality, let us assume that $c_{ij} \geq 0$, and let $C = \max_{ij} \{c_{ij}\}$. In this case, it can be proved that the auction algorithm runs in $O(n^3 (C/\epsilon))$ time (see, e.g., [51]). Then, choosing $0 < \epsilon < 1/n$, the algorithm returns an optimal solution in $O(n^4 C)$ time. By using the scaling technique, Bertsekas and Eckstein in [50] proposed a modified version of the above-described auction algorithm that runs in $O(n^3 \log(nC))$ time. In real-world vehicle networks, the quality of solution in localized algorithms for task assignment is related to the communication network quality and range of communication. In [54], the influence of the communication range and different strategies of movement on the task assignment value in the auction algorithm was evaluated in simulations in mobile (robot) agent-task allocation scenarios.

*4.1.4. Primal-Dual Algorithms.* Primal-dual algorithms start from a dual feasible solution $(u, v)$. From this solution, a restricted primal problem is defined and solved, consisting in finding the maximum cardinality matching on the bipartite subgraph $G' = (A \cup T, E')$, where $E' = \{(i, j) \in E \mid c_{ij} - u_i - v_j = 0\}$. If the optimal matching has a size equal to $n$, we are done; otherwise, the dual solution is improved (the dual objective function is increased), while assuring that also the size of $E'$ is increased, and the procedure is repeated.

Note that also the auction algorithms for LAP consider simultaneously primal and dual solutions but, differently from primal-dual algorithms, they can improve as well as worsen both the primal and the dual cost through the intermediate iterations, although at the end, the optimal assignment is found (see, e.g., [51]).

*4.1.5. Hungarian Algorithm.* In particular, the Hungarian algorithm proposed by Munkres [55] is a primal-dual algorithm. The original version of the algorithm runs in $O(n^4)$ time and was improved to $O(n^3)$ by Lawler in 1976 (see, e.g., [32]) by using successive shortest path technique when finding a new maximum cardinality matching after having updated the dual variables.

In the following, we give some insights of the Hungarian algorithm that will be also useful for describing a decentralized version of the same. The Hungarian algorithm proceeds as follows:

Start with any feasible dual solution $(u, v)$ and any matching $M \subseteq E' = \{(i, j) \in E \mid c_{ij} - u_i - v_j = 0\}$. For the starting dual solution, we can consider $v_j = \min_j \{c_{ij}\}$, with $j = 1, \ldots, n$, and $u_i = \min_i \{c_{ij} - v_j\}$, with $i = 1, \ldots, n$.

While $M$ is not perfect, repeat the following:

(1) Given $M$ and $G' = (A \cup T, E')$, find an alternating augmenting path $P$ (i.e., a sequence of an odd number of edges that alternate edges of $E' \setminus M$ and edges of $M$, starting and ending with nonmatched edges); augment the matching by considering the new matching $M\prime = M \setminus P \cup P \setminus M$. Note that $|M\prime| = |M| + 1$. Update the matching $M$ (with $M\prime$) and repeat until no new alternating augmenting path exists. $M$ is the maximum cardinality matching of $G'$.

(2) If $M$ is not perfect, update the dual solution such that at least a new edge is added to the set of (admissible) edges $E' = \{(i, j) \in E \mid c_{ij} - u_i - v_j = 0\}$, and continue with a new iteration. In particular, we can achieve this result by updating the values of $u_i$ with $u_i + \delta$ and the values of $v_j$ with $v_j - \delta$, where

$\delta = \min\left\{c_{ij} - u_i - v_j \mid i \in A\prime, j \in T\prime\right\}$ with $A\prime$ and $T\prime$ being the subsets of the vertices incident to the edges of the matching.

Searching for the alternating augmenting path can be done by a graph visiting algorithm that identifies a forest of alternating trees of $G'$. Note that in each step of the loop, we will either be increasing the size of $M$ or the size of $E'$ so this process must terminate. Furthermore, when the process terminates, $M$ will be a perfect matching of $G' = (A \cup T, E')$, whose edge set $E'$ is defined according to a feasible dual solution $(u, v)$. Since the matching is perfect also for the complete bipartite graph $G$, the former represents a feasible primal solution for the assignment problem, respecting complementary constraints (by construction of $E'$); therefore, the primal and dual solutions are optimal.

*4.1.6. Parallel Primal-Dual Algorithms.* A certain number of parallel algorithms for the linear assignment problem have been proposed. They are parallelized versions of primal-dual algorithms based on shortest path computations, of the auction algorithm, and of primal simplex-based methods. Among the most efficient parallel algorithms for the LAP is the one proposed by Orlin and Stein [56] that adopting the cost scaling technique solves the problem using $\Omega(n^4)$ processors in $O(\log^3 n \cdot \log(\max\{cij\}))$ time. For a review, the reader is referred to [36, 51, 57].

*4.1.7. Algorithms for the Bottleneck Assignment Problem.* The bottleneck assignment problem can be solved in polynomial time, for example, by the so-called *threshold algorithm* that alternates two phases (see, e.g., [36, 58]). In the first one, a threshold value $\bar{c}_{ij}$ is chosen, and in the second phase, it is checked if the bipartite graph $G' = (A \cup T, E')$ admits a perfect matching or not, where $E' = \left\{(i, j) \in E \mid c_{ij} \le \bar{c}_{ij}\right\}$.

One possible way to implement the first phase is applying a binary search. This leads to a threshold algorithm that runs in $O(T(n)\log n)$ time, where $O(T(n))$ is the time complexity for perfect matching checking. One of the best time complexity algorithms is by Punnen and Zhang (see, e.g., [59, 60]) that runs in $O(m\sqrt{n\log n})$, where $m$ is the number of finite entries of the cost matrix $\left\{c_{ij}\right\}$.

*4.1.8. Algorithms for the Fair Matching Problem.* The balanced assignment problem can be solved in polynomial time, for example, by means of an iterative algorithm based on a feasibility subroutine that runs in $O(kT(n))$ (see, e.g., [37]), where $k \le n^2$ is the number of distinct values of $c_{ij}$ and $O(T(n))$ is the time required to test if there is a feasible assignment on a subset $\overline{E} \subseteq E$ of the edges of the complete bipartite graph $G = (A \cup T, \overline{E})$. Testing if there is a feasible assignment on $\overline{E}$ corresponds to check if the bipartite graph $\overline{G} = (A \cup T, \overline{E})$ admits a perfect matching that can be done by solving the maximum cardinality matching of $\overline{G}$, e.g., in $O(n^{2.5})$ time [61]. Hence, since $k \le n^2$, the overall algorithm runs in $O(n^{4.5})$ time. Martello et al. in [37] improved the algorithm time complexity to $O(n^4)$ with a special refinement of the same.

*4.1.9. Algorithms for an Online Bipartite Matching.* Karp et al. in [62] evaluate an online algorithm for bipartite matching by comparing its performance by the worst-case ratio of its profit to that of the optimal offline algorithm. They propose an optimal online $1 - 1/e$ competitive simple randomized online algorithm to maximize the size of the matching in an unweighted bipartite graph. The best approximation algorithm for this problem is presented in [63] that applies the power of two choices paradigm, i.e., compute two offline matchings and use them to guide the adaptive online solutions.

Haeupler et al. in [64] study the unrestricted weighted problem in the stochastic arrival model and present the first approximation algorithms for it. They improve $1 - 1/e$-approximation for the online stochastic weighted matching problem to a 0.667-approximation. Moreover, they apply a discounted LP technique to give an improved competitive algorithm for the online stochastic matching problem and use the dual of the tightened LP to obtain a new upper bound on the optimal solution with a competitive ratio of 0.684. Via pseudomatching, they obtain an algorithm with a competitive ratio of 0.7036. They also present simple adaptive online algorithms to solve the online (weighted) stochastic matching problem optimally for the union of two matchings.

In [65], at each time step, a task is sampled independently from the given distribution and it needs to be matched upon its arrival to an agent. The goal is to maximize the number of allocations. An online algorithm is presented for this problem with a competitive ratio of 0.702. A key idea of the algorithm is to collect statistics about the decisions of the optimum offline solution using Monte Carlo sampling and use these statistics to guide the decisions of the online algorithm. The algorithm achieves a competitive ratio of 0.705 when the rates are integral.

*4.1.10. Summary.* While it is possible to solve most of these problems using the simplex algorithm, each AP variation has specialized more efficient algorithms designed to take advantage of its special structure.

Many centralized algorithms have been developed for solving the assignment problem in polynomial time (see, e.g., [36]). One of the first such algorithms was the Hungarian algorithm [55]. Other solution approaches include augmenting path methods (see, e.g., [66, 67]), adaptations of the primal simplex method (see, e.g., [68]), relaxation methods and auction algorithms (see, e.g., [51]), and signature methods (see, e.g., [69]).

The complexity of the Hungarian method by using Fibonacci heaps is $O(mn + n^2\log n)$ [70]. Duan and Su's approach in [71] give an algorithm whose running time for integer weights is $O(m\sqrt{n}\log N)$, where $m$ and $n$ are the number of edges and vertices and $N$ is the largest weight magnitude. Sankowski in [72] gave an $\tilde{O}(Wn^{\omega})$ ($\tilde{O}$ denotes the so-called "soft O" notation) time, where $\omega$ is the matrix multiplication exponent and $W$ is the highest edge weight in the graph.

Duan and Pettie in [73] find an $O(m\epsilon^{-1}\log\epsilon^{-1})$ running time algorithm that computes $(1-\epsilon)$-approximate maximum weight matching for any fixed $\epsilon$.

Dell'Amico and Toth in [48] consider the classic linear assignment problem with a min-sum objective function, and the most efficient and easily available sequential codes for its solution that include shortest path algorithms APC, CTCS, and LAPm; shortest augmenting path algorithm with reduction transfer procedure JV, naive auction and sequential shortest path algorithm NAUCTION SP, two different implementations of the auction method, AFLP and AFR, and pseudoflow cost scaling algorithm CSA. Based on the results of the computational experiments obtained on dense instances containing both randomly generated and benchmark problems, it is not possible to obtain a precise ranking of the eight algorithms. However, APC is the fastest code for the two cost class and has a behaviour, on average, similar to that of CTCS for the other classes. Algorithm LAPm is the winner for the uniform random and the geometric classes and for the instances from the OR library. No dominance with respect to NAUCTION SP, CTCS, and APC exists for the remaining classes. Code JV has a good and stable average performance for all the classes, and it is the best algorithm for the uniform random (together with LAPm) and for the single-depot class. CSA performance strongly depends on the class, and it wins for no-wait flow-shop classes.

*4.2. Distributed Coordination Approaches.* By distributed, we consider the algorithms that combine the concepts of centralized and decentralized coordination, and principally *market-based approaches*, where solutions are built based on a bidding-auctioning procedure between the bidders (agents) and coordinators that play the role of auctioneers for allocating tasks to agents. There may be one or more coordinator agents as intermediaries in the task assignment process. The most known such algorithm is the auction algorithm that is presented in the following.

In this section, we recall two distributed solution approaches, respectively, based on auction algorithm and on primal-dual Hungarian method.

The Bertsekas auction algorithm (see, e.g., [51]) can be naturally implemented in a decentralized fashion. Zavlanos et al. [23] provide a distributed version of the auction algorithm proposed by Bertsekas for the considered networked systems with the lack of global information due to the limited communication capabilities of the agents. Updated prices necessary for accurate bidding can be obtained in a multihop fashion only by local exchange of information between adjacent agents. No shared memory is available, and the agents are required to store locally all the pricing information. This approach calculates the optimal solution in $O(\Delta n^3 C)$ time, with $\Delta \leq n-1$ being the maximum network diameter of the communication network.

Another market-based algorithm has been proposed more recently by Liu and Shell in [74] that instead of auctioning via a series of selfish bids from customers (agents) adopts a mechanism from the perspective of a merchant. The algorithm is capable of producing a solution (equilibrium) that satisfies both merchant and customers and is globally optimal; its running time is $O(n^3\log n)$.

Otte et al. in [75] study various auction algorithms for task assignment in the multirobot context and study how lossy communication between the auctioneer and bidders affects solution quality. They demonstrate both analytically and experimentally that even though many auction algorithms have similar performance when communication is perfect, they degrade in different ways as communication quality decreases from perfect to nonexistent. They compare six auction algorithms including standard implementations of the sequential auction, parallel auction, combinatorial auction; a generalization of the prim allocation auction called G-Prim; and two multiround variants of a repeated parallel auction. Variants of these auctions are also considered in which award information from previous rounds is rebroadcast by the auctioneer during the later round. They conclude that the best performing auction changes based on the reliability of the communication between the bidders and the auctioneer.

Giordani et al. in [24, 25] propose a distributed version of the Hungarian method for solving the LAP, based on the concept of alternating augmenting paths that are searched by maintaining a forest of alternating trees that is updated during the execution of the algorithm. In particular, given the current bipartite subgraph $G' = (A \cup T, E')$, where $E' = \{(i,j) \in E \mid c_{ij} - u_i - v_j = 0\}$, and $A$ and $T$ are agent and task vertices, respectively, the algorithm maintains forest $F_1$ of all the alternating trees rooted at free task vertices. Moreover, it maintains forest $F_2$ of the alternating trees of $G'$ rooted at agent vertices containing all the agent/task vertices not contained in $F_1$. Clearly, the alternating trees in $F_2$ are not connected with vertices in $F_1$.

The algorithm involves root agents that initiate message exchange with other agents in the network via a depth-first search and synchronize the decision rounds (iterations, each containing multiple communication hops) across all agents. Through autonomous calculations and the communication with the (agent) neighbors, with respect to the position of the vertex representing the agent in the spanning alternating forests, agents get and share the information about the position of each task vertex (whether in $F_1$ or $F_2$), the values of dual variables related to tasks, the value of $\delta$ for the dual variables' update, the new admissible edge entering in a set of admissible edges of $G'$ due to the dual variables' update, and the root agents $r(F_1)$ and $r(F_2)$ of forests $F_1$ and $F_2$, respectively. All these data are locally stored by each agent. In this way, there is no common coordinator or a shared memory of the agent's system. The agents, depending on the positions of the related vertices in the forests, change their roles and accordingly execute some of the steps of the distributed Hungarian algorithm. The total computational time is $O(n^3)$ as well as the total number of messages exchanged by the robots; nonetheless, the computational time required to perform the local calculation by each robot is $O(n^2)$. Regarding the robustness of the proposed method, if the agent during the execution of the algorithm stops responding, it is considered erroneous and is eliminated from the further calculations. In the case where the agent was unmatched in forest $F_2$, the calculation continues without any modifications, ignoring the agent in question. Otherwise, the algorithm starts from the beginning excluding the same.

Chopra et al. in [76] propose a novel distributed version of the Hungarian method for solving the LAP that does not use any coordinator or shared memory. Specifically, each agent runs a local routine to execute ad hoc substeps of the centralized Hungarian method and exchanges estimates of the solution with neighboring robots. The authors show that with their approach, all agents converge to a common optimal assignment in a finite number ($O(n^3)$) of communication rounds if agents act synchronously. The overall performance of their approaches in terms of running time is only evaluated experimentally.

Eiselt and Marianov in [77] propose a model for the task assignment to employees with heterogeneous capabilities and multiple goals. Employees and tasks are mapped into the skill space where, after finding feasible matchings, they are assigned to each other by minimizing employee-task distance to minimize assignment cost, boredom, and unfairness between employees' workloads.

Peters and Zelewski in [78] develop two goal programming models for the employee assignment to workplaces according to both their competencies and preferences and the workplace requirements and attributes to ensure effective and efficient task performance. A review and classification of the literature regarding workforce planning problems incorporating skills can be found in [79].

The bottleneck assignment problem can be solved in polynomial time, for example, by the so-called *threshold algorithm* that alternates two phases (see, e.g., [36, 58]). In the first one, a threshold value $\overline{c}_{ij}$ is chosen, and in the second phase, it is checked if the bipartite graph $G' = (A \cup T, E')$ admits a perfect matching or not, where $E' = \{(i, j) \in E \mid c_{ij} \leq \overline{c}_{ij}\}$.

One possible way to implement the first phase is applying a binary search. This leads to a threshold algorithm that runs in $O(T(n) \log n)$ time, where $O(T(n))$ is the time complexity for perfect matching checking. One of the best time complexity algorithms by Punnen and Zhang (see, e.g., [59, 60]) that runs in $O(m\sqrt{n \log n})$, where $m$ is the number of finite entries of the cost matrix $\{c_{ij}\}$. Efrat et al. in [80] propose algorithms that, assuming planar objects, run in roughly $O(n^{1.5} \log n)$ time. Pothen and Fan in [81] propose a parallel algorithm with $O(nm)$ time complexity, which is currently among the best practical serial algorithms for maximum matching. However, its performance is sensitive to the order in which the vertices are processed for matching.

In [82], Azad et al. study the performance improvement of augmentation-based parallel matching algorithms for bipartite cardinality matching on multithreaded machines over serial algorithms and report extensive results and insights on efficient multithreaded implementations of three classes of algorithms based on their manner of searching for augmenting paths: breadth-first search, depth-first search, and a combination of both.

In [80], algorithms for the balanced assignment problem and minimum deviation assignment are presented that run in roughly $O(n^{10/3})$ and, as such, are more efficient than the algorithms in [37, 41] that run in $O(n^4)$ time on general bipartite graphs. Kennington and Wang in [43] present a shortest augmenting path algorithm for solving the semi-assignment problem in which each iteration during the final phase of the procedure (also known as the endgame) obtains an additional assignment.

*4.3. Decentralized Coordination Approaches.* In contrast to centralized and distributed coordination approaches to task allocation where full knowledge of global information is assumed available to every relevant decision-maker (central decision-maker or fleet coordinator (fleet owner) and (vehicle) bidder agents), in the decentralized task assignment approaches, there is no coordinator and each vehicle agent disposes only of its local (possibly incomplete and imperfect) information and finds its local assignment based exclusively on this information and the communication with the rest of the agents and interaction with its environment.

In general, decentralized approaches have several advantages, i.e., real-time property, robustness, and scalability. These characteristics are in general absent in centralized and distributed approaches that outperform decentralized approaches in terms of efficiency especially for large-scale instances. The decentralized decision-making does not include any intermediary. In case of imperfect communication, conflicts may occur. This is why the related literature in decentralized multivehicle cooperative control is related to consensus, i.e., the agreement of all vehicles on some common features by negotiating with their local neighbors. General consensus issues are related to, e.g., positions, velocities, and attitudes. In the following, we analyze localized, scalable, and decentralized heuristic algorithms for coordination of deterministic and dynamic task assignment in open vehicle fleets. We concentrate on the approaches resulting in both task assignment feasibility and efficiency even though these approaches usually have no quality of solution guarantees.

Decentralized task assignment approaches have been mostly developed in the multirobot and unmanned aerial vehicle (UAV) coordination domain. The most known ones are sequential auction-based or consensus and negotiation-based algorithms (e.g., [83–85]).

One of the most known approaches for the decentralized task assignment in the coordination of a fleet of unmanned vehicles when all-to-all intervehicle communication is not possible is the consensus-based auction algorithm (CBAA) and its more general version that allows for the assignment of bundles of tasks to each agent called the consensus-based bundle algorithm (CBBA) [84].

The CBAA is a polynomial time market-based decentralized task selection agreement protocol running in two phases: in the first phase, each vehicle places a bid on a task asynchronously with the rest of the fleet, and in the second, consensus phase, conflicting assignments are identified and resolved through local communication between neighboring agents within certain predefined rules to avoid task conflicts. The agents use a consensus strategy to converge on the list of winning bids and use that list to determine the winner and associated winning scores. The list accounts for inconsistent information among agents guaranteeing a conflict-free assignment for all. This allows conflict resolution over all tasks that are robust to inconsistencies in the situational awareness across the fleet and the changes in the communication network topology. If the resulting scoring scheme satisfies a diminishing marginal gain property (i.e., the value of a task does not increase as other tasks are assigned to the same agent before it), a feasible, conflict-free solution is guaranteed.

Provided that the scoring function abides by the principle of diminishing marginal gains, the CBBA has convergence guarantees. In a synchronized conflict resolution phase over a static communication network, it produces the same solution as the sequential greedy algorithm sharing across the fleet the corresponding winning bid values and winning agent information. Moreover, the convergence time is bounded from above and it does not depend on the inconsistency in the situational awareness over the agent set.

In [84], it is analytically shown that CBAA produces the same solution as some centralized sequential greedy procedures, and this solution guarantees 50% optimality. Segui-Gasco et al. [86] propose a decentralized algorithm for multirobot task allocation with a constant factor approximation of 63% for positive-valued monotone submodular utility functions and of 37% for general positive-valued submodular utility functions. Therefore, the authors improve the approximation guarantee of Choi et al. [84] for monotone positive-valued submodular utility functions from 50% to 37%.

The CBBA has also been extended to consider coupled constraints [87, 88]. Choi et al. in [87] extended CBBA for heterogeneous task allocation to UAV agents with different qualifications and various cooperation constraints. The CBBA was extended with task decomposition and a scoring modification to allow for soft constraints related to cooperation preferences and a decentralized task elimination protocol that ensures the satisfaction of the hard constraints related to cooperation requirements. The performance of the algorithms was analyzed in Monte Carlo simulations in some randomly generated experiments.

The CBBA was also extended in [88] to consider the assignment of tasks with assignment constraints and also with different types of coupled and temporal constraints, where it was assumed that assigned tasks are executed in the order defined by their temporal precedence.

The temporal sequential single-item (TeSSI) auction algorithm [83] allocates tasks with time windows to cooperative robot agents using a variant of the sequential single-item auction algorithm. Contrary to the CBBA algorithm that does not let the change of the start time of the tasks once they are allocated and thus reduces the number of tasks that the algorithm allocates, the TeSSI algorithm overcomes this limitation by allowing tasks' start times to change, which results in higher allocation rates.

The main features of the TeSSI algorithm are a fast and systematic processing of temporal constraints and two bidding methods that optimize either completion time or a combination of completion time and distance. The main objective function used in the TeSSI algorithm is the makespan (the time the last task is finished) even though it is also combined with the total distance traveled. Each robot maintains the temporal consistency of its allocated tasks using a simple temporal network. The authors show that TeSSI outperforms a baseline greedy algorithm and the CBBA through random experiments and related work datasets.

Ponda et al. in [89] further extend the CBBA to tasks with time windows and address replanning in dynamic environments and consider agents with different capabilities. Agents obtain new plans based on the changes in the environment considering new tasks while pruning older or irrelevant ones.

One of the drawbacks of the CBBA algorithm is that it relies on global synchronization mechanisms which are hard to enforce in decentralized environments. Johnson et al. [85] proposed the asynchronous CBBA (ACBBA) for agents that communicate asynchronously. To allow for asynchrony in communication, the ACBBA contains a set of local deconfliction rules that do not require access to the global information. In ACBBA, agents locally replan their actions that, possibly, affect only a limited number of agents.

Johnson et al. [90] propose a situational awareness algorithm for task assignment when agents predict the bids of their neighbors, in order to obtain more informed decisions in a cooperative way.

To respond to the problem with local information consistency assumption that reduces optimization capabilities compared to global information assumption approaches, Johnson et al. [91] proposed a bid warped consensus-based bundle algorithm that converges for all deterministic objective functions and has nontrivial performance guarantees for submodular and some non-submodular objective functions. They analyze the convergence and performance of the algorithm and show its efficiency compared with some other relevant local and global information approaches.

Another extension to the CBBA is provided by Binetti et al. [92] that consider the decentralized surveillance problem by a team of robots. Tasks are assigned to each robot with the additional constraint that a subset of the tasks called critical tasks must be assigned. The authors use the CBBA incorporating hard constraints in order to ensure that the critical tasks are not left unassigned.

In [93], Garcia and Casbeer present a robust task assignment algorithm that reduces communication between vehicles in uncertain environments. Piece-wise optimal decentralized allocation of tasks is considered for a group of unmanned aerial vehicles. They present a framework for multiagent cooperative decision-making under communication constraints. Each vehicle estimates the position of all other vehicles in order to assign tasks based on these estimates, and it also implements event-based broadcasting strategies that allow the multiagent system representing the vehicle fleet to use communication resources more efficiently. The agents implement a simple decentralized auction scheme in order to resolve possible conflicts.

Cui et al. in [94] investigate game theory-based negotiation for task allocation in the multirobot task assignment context. Tasks are initially allocated using an approach based on contract net (see [95]), after which a negotiation approach employing the utility functions to select the negotiation robot agents and construct the negotiation set is proposed. Then, a game theory-based negotiation strategy achieves the Pareto-optimal solution for the task reallocation. Extensive simulation results demonstrate the efficiency of such a task assignment approach.

Yet another extension of the consensus-based bundle algorithm (CBBA) allowing for the fast allocation of new tasks without a full reallocation of existing tasks is CBBA with partial replanning (CBBA-PR) [96]. The algorithm enables the multiagent system to trade-off between convergence time and increased coordination by resetting a portion of their previous allocation at every round of bidding on tasks. By resetting the last tasks allocated by each agent, the convergence of the MAS to a conflict-free solution is assured. CBBA-PR can be further improved by reducing the team size involved in the replanning, further reducing the communication burden of the team and runtime of CBBA-PR.

In [97], Sayyaadi and Moarref investigate a proportional task assignment problem in which it is desired for (robot) agents to have an equal duty to capability ratios, i.e., the agents with more capability should perform more tasks. They address this problem as a combination of deployment and consensus problems in which agents should reach consensus over the value of their duty to capability ratios. They propose a distributed, asynchronous and scalable algorithm for this problem in the continuous time domain.

Duran et al. in [98] study the problem of finding the list of solutions with strictly increasing cost for the semi-assignment problem. Four different algorithms are described and compared. The results show that they find the exact list of solutions and considerably reduce the computation times in comparison with the other exact approaches.

Spivey et al. in [99] propose a distributed, flexible, and scalable control scheme that evenly allocates tasks. Dynamic load balancing exploits feedback information about the status of tasks and vehicles with the objective to keep a balanced task load and, thus, force cooperation in the solution of the randomized bottleneck task assignment problem.

In summary, most of the state-of-the-art decentralized and deterministic coordination approaches for task allocation are heuristic algorithms developed for multirobot or UAV task allocation scenarios that often include both operational and tactical constraints of a vehicle fleet and its environment. Even though their adaptation for the use in open vehicle fleets does not seem difficult, it remains an open challenge, especially if we consider task allocation efficiency, the key performance indicator of commercial open fleets.

## 5. Challenges in Open Vehicle Fleet Coordination

In this paper, we proposed new mathematical programming models of dynamic versions of the following assignment problems well known in combinatorial optimization and applicable in open vehicle fleets: the assignment problem, bottleneck assignment problem, fair matching problem, dynamic minimum deviation assignment problem, $\sum_k$–assignment problem, the semiassignment problem, the assignment problem with side constraints, and the assignment problem while recognizing agent qualification. The goal of the studied problems is finding an optimal (minimum cost or maximum profit) assignment to the (vehicle) agents of the tasks that are known at the time of decision-making. These approaches do not take into account unknown tasks that may appear once when the current tasks are completed.

With the long-term objective of decentralizing and democratizing shared mobility, we categorized solution approaches for static and dynamic task assignment problems applicable in open vehicle fleets into centralized, distributed, and decentralized and discussed their main characteristics. The presented distributed and decentralized task assignment methods are applicable in distributed and decentralized open vehicle fleets, respectively. In case of decentralized fleets, the issues related to privacy, trust, and control intrinsic to centralized systems are gone.

We focused on homogeneous vehicle agents and tasks, i.e., each vehicle agent is able to complete each task with equal efficiency but varying cost or profit. In the real world, that might not be the case since in open vehicle fleets, the vehicles tend to be heterogeneous. The proposed mathematical programs can easily be adapted to this case by varying the agent-task assignment cost/profit depending on the performance efficiency of an agent; in case of an agent inapt to perform a task, its agent-task assignment cost is assigned a very large value.

With fully decentralized scalable coordination of task allocation, there is no need to put limits to the size of the system. However, even though scalable task allocation and related coordination mechanisms are essential for efficiently managing large-scale open vehicle fleet systems, it should be noticed that, for real-world applications, they need to be complemented with scalable and efficient solution approaches to other combinatorial optimization problems depending on the context, e.g., dial-a-ride problem and traveling salesperson problem, etc.

We dealt with the deterministic and dynamic assignment problem where real-time reassignment is beneficial since both the variables and parameters of the optimization problem are perfectly known at each period. However, when dealing with real-world stochastic environments with increased sensor noise, a too high frequency of task reassignment may result in a churning effect in the assignment and may lead to increased human errors. Thus, a chosen coordination method must consider churning in this context to obtain good overall task allocation performance (see, e.g., [100]).

A truly open vehicle fleet system should work also based on heterogeneous software agents produced by multiple producers. The agent software could be an open source and/or there may be multiple proprietary software companies working on a common open fleet coordination standard. The Agreement Technologies (AT) paradigm [101] identifies and relates various such technologies. It provides a sandbox of mechanisms to support coordination among (heterogeneous) autonomous software agents, which focuses on the concept of agreement between them. To this respect, AT-based systems not only support the interactions for reaching an agreement in a coordinated manner (e.g., as part of a distributed or decentralized algorithm) but are also endowed with means to specify and govern the "space" of agreements that can be reached, as well as monitoring agreement execution. In particular, in truly open vehicle fleet systems where there may be a multitude of (possibly heterogeneous) software providers, semantic mismatches among vehicle agents need to be dealt with through the alignment of ontologies, so that vehicle agents can reach a common understanding on the elements of agreements.

Furthermore, (weak) constraints on agreement and agreement processes (often also called *norms*) need to be defined and represented in a declarative manner, so autonomous agents can decide as to whether they will adopt them, determine as to how far they are applicable in a certain situation, dynamically generate priorities among conflicting norms depending on the context, etc. In addition, trust and reputation models are necessary for keeping track of whether the agreements reached, and their executions, respect the requirements put forward by norms and organizational constraints. So, norms and trust can be conceived as a priori and a posteriori approaches, respectively, to support the security in relation to the coordination process. How to find seamless and effective means of integrating the different distributed and decentralized algorithms outlined in this paper in such a framework is still an open issue that we will treat in our future work.

The presented distributed and decentralized coordination methods for dynamic task assignment may be applied to semiautonomous and autonomous vehicles and are a necessary part of reaching full vehicle fleet autonomy. They may not fix the mobility concerns, but they will definitely improve them as they are directly related to giving a higher control both to an individual driver (or to an autonomous vehicle) and to a customer (rider). Intrinsically, these methods aid in changing the hierarchical tree structure of the transportation networks to a more horizontal one. Indirect benefits of such coordination methods, among others, include higher efficiency, smaller carbon footprints, and less traffic jams. In the long run, they will facilitate more decentralized, autonomous, and transparent open vehicle fleets, but above all, they will further the task allocation efficiency and fair rewards and benefits of vehicles, drivers, customers, and riders proportional to their participation in large and open fleets.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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*Research Article*

# Coordination of Closed-Loop Supply Chain with Dual-Source Supply and Low-Carbon Concern

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Considering the impact of dual-source supply and low-carbon manufacturing on a closed-loop supply chain (CLSC) system, this article constructs a CLSC model with two competitive dominant upstream suppliers and one following a downstream (re-) manufacturer, then coordinates supply chain through cost-sharing contract. Based on the industrial case in the area of power battery, we analyze the optimal strategies under competition, cooperation, and coordination structures separately and then investigate the influences of emission reduction effort and collection efficiency on supply chain performance. The results reveal that collection of used products can positively affect the (re-)manufacturer's profit but has opposite impact on the new component supplier. Besides, recycling is beneficial to both low-carbon consumers' utility and social welfare, but hurts the total profit of CLSC because of the high investment cost of collection. Therefore, the paper designs a cost-sharing contract, which is of applicability and efficiency for both economic and environmental development. Furthermore, it can also increase the profit of CLSC up to cooperation case and improve each member's profit, eliminating double marginal effect and achieving supply chain coordination.

## 1. Introduction

With the shortage of raw material resources and the increase of environmental concerns, such as carbon emissions and global warming, resources conservation and environmental protection have become the most concerned topics all over the world [1, 2]. Manufacturing has the positive influences on the economic development but negative effects on environmental protection [3, 4]. The integration of the reverse supply chain system with multiforward channels has drawn public attention. Manufacturing activities, although can be beneficial to economic development, will prevent the environment and resource conservation [5]. Hence, it is necessary and important to create a closed-loop supply chain system for improving the efficiency of collecting and remanufacturing [6]. Moreover, unlike the traditional supply chain, in which competition only exists in its forward supply channel, the suppliers start to focus on the recycling activities in CLSC, which can create a huge profit margin. Meanwhile, firm's optimal decisions in the reverse channel can affect the performance in the forward channel, forcing manufacturers to take pricing strategy and emission reducing strategy into consideration simultaneously. Nowadays, consumers have growing low-carbon awareness and are willing to purchase ecofriendly products even though have to pay a higher price, which facilitates manufacturers to reduce carbon emissions in producing process, then decreases negative impact on environment, improves social welfare, and stimulates low-carbon demand. For instance, Fuji Xerox, a Japanese firm which produces printers and duplicators, started its recycling business (such as collects printer consumables and remanufactures printers) from 2008, and had obtained over 200 million dollars cost saving in five years. At the same time, its carbon emissions incurred by producing had been effectively reduced. As to Hewlett-Packard, it began recycling business in 1990s and its collection efficiency was relatively higher than other enterprises. Early in 2012, HP collected over 160,000 tons of consumables and more than 80% were reused, which dramatically increased HP's competition in PC market. Likewise, in China, many electric vehicle manufacturers and power battery producers, such as FAW Group and CATL, have realized the importance of recycling and emission reduction. They began to develop circular economy and build a conservation-oriented manufacturing system by collecting used batteries and remanufacturing new electric vehicles [7]. Therefore, this paper investigates the CLSC with competitive suppliers and (re-)manufacturer and then analyzes the optimal strategy of each member, which can increase the utilization of used products and decrease the carbon emissions in the entire society [8, 9].

According to a survey conducted by Chinese National Bureau of Statistics, "2018 Statistical Bulletin of National Economic and Social Development," the GDP of the automobile manufacturing industry increased by 15.5% while the recycling and remanufacturing development was stagnant due to the inefficiency in combining pricing and collecting strategies at the same time. In 2017, the number of private electric vehicles in China reached more than 1.7 million, the year-on-year growth was 61.7% and kept growing. In general, a power battery's lifetime is around five years, which will definitely cause a sharp increase of scrap rate in the near future. Nevertheless, the recycling number of used vehicles only accounts for 1.5% of the total private cars in 2016. Apparently, if the collecting and remanufacturing activities cannot be implemented efficiently, there will be huge waste of resources and environmental pollution. Therefore, the Chinese government offers subsidy to electric vehicle enterprises to support them in manufacturing as well as recycling, which is beneficial not only to social carbon emission reduction but also to resource conservation. Current studies should pay emphasis on several aspects: collecting used products, remanufacturing with emission reduction concern, and selling products to low-carbon consumers. Furthermore, it will be of great significance for researchers to focus on designing an effective recycling supply chain system and its coordination mechanism.

Up to now, there were a great number of research studies which investigated CLSC, but most of them focused on downstream firms and consumers' strategy, analyzing the competition or cooperation between manufacturers, retailers, and collectors. Few research studies discussed the performance of competitive suppliers in forward channels when taking reverse flow and coordination contracts into consideration. Xie et al. [10] built a dual-channel model and discussed pricing competition between two retailers, neither concerning suppliers' competition nor carbon emission reduction. Although Giri et al. [11] considered the competitive forward channel in CLSC, they did not investigate the impacts of emission reduction on supply chain performance. In CLSC research area, most studies focus on the optimal decision analysis [12, 13], coordination contract design [14, 15], recycling and remanufacturing strategy [16], differential pricing strategies of new and remanufactured products [17], and so on. Furthermore, concentrating on multireverse channels, many research studies discussed two types of collection ways: recycling by oneself [18, 19] or by third-party collector [20, 21]. However, these reviewed papers did not take emission reduction into consideration.

On the other hand, as to recyclable products, such as automobiles and electronic appliances, only the core components are worth to be collected and remanufactured. When the traditional suppliers started to collect used products, a dual-source supply system would immediately form and competition occurred between the new component suppliers and recycled component suppliers. Thus, the stronger supplier can gain more benefit by increasing the wholesale price, while the weaker one would inevitably lose profits [22]. Generally, when the recycling suppliers enter the market, they not only compete with the original suppliers but also count on them for new components. When this supply chain mode generates, it brings more conflicts and incoordination. Therefore, it is very urgent to study the optimal decision-making process and coordination mechanism of CLSC which is composed of new and recycled component suppliers as well as a (re-)manufacturer [23]. In existing research studies, many works designed coordination mechanism in the forward channel. Zheng et al. [24] analyzed a coordination model for CLSC with cooperative and noncooperative games. Xie et al. [25] further built a coordination model with two competitive sale channels and one reverse channel. However, these articles all neglected the impact of carbon emission reduction on supply chain and consumer's strategies.

With the increasing emphasis paid by the government on resource and environment issues, the recycling of used products has received more attention, and then a series of policies and regulations were implemented. Recycling suppliers gained more power to compete with traditional suppliers [26]. As to dual-source supply, Li et al. [27] studied the procurement and pricing strategies in the dual-suppliers case and designed a coordination mechanism to effectively increase the total profit of CLSC. Moreover, Zhang and Chen [28] discussed the impact of different contract coordination mechanisms under dual-source supply and found that the impact of wholesale price contract and revenue-sharing contract in the dual-source supply chain is very different from the single-supply system. Xiong et al. [29] indicated that manufacturers are more willing to collect and remanufacture the used products. Furthermore, Cui et al. [30] focused on the utility of RFID technology in the dual-source supply chain and identified the optimal order quantity and profit. Nevertheless, these papers all ignored the low-carbon consumers' influence on manufacturing and recycling.

As discussed above, the environmental concerns and consumers' environmental awareness stimulates enterprises to make more ecofriendly efforts. In addition, emission reduction is an effective method to enhance the environmental features of products and attract more low-carbon consumers [31]. Recently, studies began to focus on this factor in supply chain management. Xu et al. [32] further investigated the optimal reduction degree in a CLSC, and Du et al. [33] found that proper supply chain contracts could positively affect the decision-making process of supply chain members. However, few studies consider emission reduction under competitive situation. Some studies have not proposed coordination contracts, and others have not taken emission reduction into CLSC. Hence, this paper for the first time proposes a new method to simultaneously coordinate emission reduction and competitive dual-source supply decisions in a CLSC.

*1.1. Research Gaps and Contributions.* From the literature review, the following research gaps are identified:

(1) Most previous research studies have not paid enough attention to the competition between new and recycled component suppliers when the latter one also acts as a collector, not even researched on the coordination of this dual-source supply system. (2) Few research studies have considered supply competition under coordination contracts when reverse logistics is provided by one of the suppliers. (3) Although some studies have discussed the competition in the supply channel, few of them introduce the ecofriendly factor, such as emission reduction, into the supply chain, which will dramatically influence the consumer's willingness to pay and manufacturer's profit.

This paper investigates a two-echelon CLSC comprising of one manufacturer, two competitive suppliers in forward logistics, and one of suppliers also acts as a monopolistic collector in reverse logistics. This model fits in many practical cases, such as home appliance, electronic equipment, production facility, and electric vehicle, while the (re-) manufacturer faces three challenges. (1) How to increase the recycling rate of recycled products? (2) How to enhance the consumers' willingness to pay? (3) How to satisfy the stricter carbon emission requirements? The model has been investigated under various decision-making modes: centralized, decentralized, and coordinated, which can be regarded as the competition, cooperation, and coordination structures in business operation. First, firms make pricing, recycling, and emission reducing strategies under the aim of maximizing their respective profits in the competition model. Second, the cooperation model is an optimal, benchmark case, in which firms make joint decisions for maximizing the total profit of CLSC. Ultimately, due to the double marginal effect caused by decentralized decision in the competition model, we design and implement a cost-sharing contract to coordinate CLSC. Under the proposed contract, firms in CLSC are simultaneously coordinated in which all members' profit are improved and total profit reach up to the level of the cooperation model.

## 2. Model Description and Assumption

In this paper, a dual-channel CLSC with competitive forward supplying and monopolistic reverse recycling is taken into account. The article builds a CLSC model consisting of one (re-)manufacturer and two competing suppliers in which one of suppliers also play the role of recycling recycled products. In the forward channel, (re-)manufacturers produce or remanufacture products from both new and recycled component suppliers. In this case, the new supplier competes on selling components to the (re-)manufacturer with the recycling supplier. The suppliers who take charge of the core technology can dominate the whole supply chain firstly decide their wholesale price and collection rate individually. Then, the (re-)manufacturer decides the pricing and emission reducing strategies. On the other hand, in the reverse channel, the recycled component supplier is responsible for collecting recycled products from the consumers to the (re-) manufacturer. Therefore, the collection rate is a key factor, which significantly affects profit, which needs to be optimized. For instance, in the case of HP printers, Hewlett-Packard commissioned the recycled plastics supplier, Lavergne Group, to collect the used ink cartridges and then further reprocess them. At the same time, HP also purchases new ink cartridges from other cartridge suppliers, manufactures all cartridges from different supply channels, and then sells to consumers. Recently, HP has recognized the importance of environmental products and considered emission reduction in producing process, not only to attract low-carbon consumers but also to satisfy government emission regulation.

The proposed model is analyzed under three decision-making structures: competition, cooperation, and coordination models. In the two-echelon decentralized model, firms make their own strategies only considering their own profit, forming a Stackelberg game model in which the equilibriums can be obtained by the Nash backward induction method [34]. The dominant suppliers make decisions first, and then the (re-)manufacturer acts according to the optimal strategies of suppliers. However, the double marginal effect occurs in this decision model, decreasing the supply chain efficiency. Therefore, a coordination model needs to be introduced. In order to design a proper contract, we investigate a cooperation model as a benchmark, in which all supply chain members make joint decisions under the aim of optimizing the entire CLSC profit. Finally, a coordination model is proposed to facilitate the cooperation between firms and achieve channel coordination. The structure of CLSC is shown in Figure 1.

*2.1. Notations.* The notations used in this study are summarized in Table 1.

*2.2. Assumptions.* In order to simplify the calculation, we make necessary assumptions without loss of generality:

(1) In the low-carbon market, demand is affected by the consumers' preference of environmental protection. In addition to the price, consumers also take the emission reduction level of products into consideration. Therefore, the higher the environmental performance of products (in terms of carbon emission reduction degree), the higher the demand. In this paper, linear demand function is used as follows, which is also used in studies of Chen [35], and Kouvelis and Zhao [36]. Market demand function is $D(p, L) = d - \alpha p + \beta L$, where $d$ denotes the market size, $\alpha$ denotes the price elasticity of demand, $\beta$ denotes the consumer's preference intensity for low-carbon products, and $\alpha > \beta > 0$.

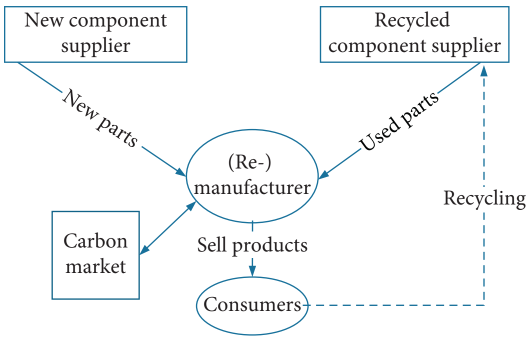


FIGURE 1: Framework of the competitive supply chain.

(2) The collection rate is depended on the investment in collecting recycled products. According to the research of Huang et al. [37], the collection rate increases with the growth of investment, $\theta = \sqrt{2I_\theta/b}$. In order to increase the collection rate, the recycling supplier should invest more, such as promoting and advertising events. In general, the investment increases faster in the higher range of the collection rate. $b$ is a scalar parameter which should be large enough to ensure the range of the collection rate in $[0, 1]$.

(3) The downstream (re-)manufacturer can produce from new components with cost $c_n$ and recycled components with cost $c_o$, $c_o < c_n$. Manufactured and remanufactured products have the same quality level and can be sold as the same price. Same assumption was considered in the paper of Xie et al. [10].

(4) In reality, the environmental enterprises will make effort to reduce carbon emissions during producing process, through improving technology or equipment. According to Giri et al. [38], emission reduction degree is positively correlated with reduction efforts. The cost will increase more rapidly with the growth of emission reduction. In other words, the cost of emission reduction increases concavely with respect to reduction degree. Therefore, (re-)manufacturer's investment function can be assumed as a quadratic function, $c(L) = kL^2/2$. The higher the $k$, the higher the technical resistance and the investment in reduction. In order to ensure the existence of solutions, $k$ should be assumed as large as possible.

## 3. Model Formulation

*3.1. Competition Structure.* In the decentralized CLSC, that is, the competition structure, (re-)manufacturer, and new and recycling suppliers decide their strategies individually in order to maximize their own profit. In this two-echelon Stackelberg game model, the dominant suppliers first decide the wholesale price and collection rate simultaneously, and then the following (re-)manufacturer decides its sales price and emission reduction degree. Two competitive suppliers ignore the effect of their strategies on others. According to Nash equilibrium, the optimal decision of each member in CLSC can be solved. The following sections analyze the problems in detail.

*3.1.1. (Re-)manufacturer's Problem.* The (re-)manufacturer profit consists of four terms: sales revenue, cost of products by new components, cost of products by recycled components, and cost of emission reduction. The profit function is as follows:

$$\max_{p,L} \pi_M = pD - (w + c_n)(1-\theta)D - (w + c_o)\theta D - \frac{kL^2}{2}. \tag{1}$$

**Proposition 1.** *In this CLSC, the (re-)manufacturer makes pricing and emission reduction policies after suppliers. The optimal decisions exist due to the concavity of profit function w.r.t p and L. The optimal profit is dramatically influenced by the suppliers' wholesale price and collection rate.*

*Proof.* Take the second order derivative of the (re-)manufacturer's profit function with respect to carbon emission reduction effort and sales price separately, obtaining $\partial^2\pi_M/\partial p^2 = -2\alpha < 0$, $\partial^2\pi_M/\partial L^2 = -k < 0$, and Hessian matrix $\begin{vmatrix} \partial^2\pi_M/\partial p^2 & \partial\pi_M/\partial L\,\partial p \\ \partial\pi_M/\partial p\,\partial L & \partial^2\pi_M/\partial L^2 \end{vmatrix} = 2k\alpha - \beta^2 > 0$. The profit function of the (re-)manufacturer under the competition structure is concave with respect to $p$ and $L$. Then, according to the backward induction method, the optimal decisions are solved by FOC; substituting in $\partial\pi_M/\partial L = 0$ and $\partial\pi_M/\partial p = 0$ gives

$$L = \frac{d\beta - \alpha\beta[w + (1-\theta)c_n + \theta c_o]}{2k\alpha - \beta^2}, \qquad p = \frac{kd + (k\alpha - \beta^2)[w + (1-\theta)c_n + \theta c_o]}{2k\alpha - \beta^2}, \tag{2}$$

which when substituted into the objective function yields

$$\pi_M = \frac{k[d - w\alpha - \alpha c_n(1-\theta) - \alpha\theta c_o]^2}{4k\alpha - 2\beta^2}. \tag{3}$$

□

*3.1.2. Suppliers' Problem.* The new component supplier's profit consists of the sales revenue minus supply cost, and its supply quantity is equal to the market demand minus the quantity of recycled products. The profit function is as follows:

$$\max_{w_n} \pi_N = (w - sc_n)(1-\theta)D. \tag{4}$$

Likewise, the profit function of the recycling supplier can be expressed as follows:

$$\max_{\theta} \pi_O = (w - a_o)\theta D - I_\theta. \tag{5}$$

In the first stage of decision-making process, two competitive suppliers simultaneously and individually make

Table 1: Decision variables and parameters.

| | |
|---|---|
| *Parameters* | |
| $c_n$ | Unit manufacture cost from a new component |
| $c_o$ | Unit manufacture cost from a recycled component, $c_o < c_n$ |
| $sc_n$ | Unit supply cost of new component supplier |
| $a_o$ | Unit acquisition price of recycled products paid to consumers, $a_o < sc_n$ |
| $k$ | Cost coefficient of emission reduction effort |
| $b$ | Cost coefficient of investment in collecting used products |
| $I_\theta$ | Investment of recycling supplier in collecting used products from consumers |
| $L_0$ | Initial unit carbon emission of (re-)manufacturer before reducing emissions |
| $L_G$ | Unit carbon quota set by government, $0 < L_G < L_0$ |
| $p_c$ | Unit carbon emission right trading price in carbon market |
| $D$ | Market demand function |
| $\pi_N$ | New component supplier's profit |
| $\pi_M$ | (Re-)manufacturer's profit |
| $\pi_O$ | Recycled component supplier's profit |
| $\pi_S$ | Supply chain total profit |
| *Decision variables* | |
| $L$ | Unit emission reduction degree decided by (re-)manufacturer |
| $p$ | Unit sales price decided by (re-)manufacturer |
| $\theta$ | Collection rate of used products decided by recycling supplier, $0 < \theta < 1$ |
| $w$ | Wholesale price decided by new component supplier |

their optimal decisions, wholesale price and collection rate. The equilibrium can be calculated by the backward induction method.

**Proposition 2.** *In this CLSC, two competitive suppliers make pricing and collection investment policies firstly. The optimal decisions exist due to the concavity of profit functions w.r.t $w$ and $\theta$.*

*Proof.* Substitute equation (2) into profit functions (4)–(5). Take the second order derivative of the suppliers' profit functions with respect to the wholesale price and collection rate separately, obtaining $\partial^2\pi_N/\partial w^2 = -(2k\alpha^2(1-\theta)^2/2k\alpha - \beta^2) < 0$ and $\partial^2\pi_O/\partial\theta^2 = -b - (2k\alpha^2(w-a_o)(c_o-c_n)/2k\alpha - \beta^2) < 0$. The profit functions of suppliers under the competition structure are both concave with respect to their own decision variables. Then, the optimal decisions are solved by FOC; substituting in $\partial\pi_N/\partial w = 0$ and $\partial\pi_O/\partial\theta = 0$ gives

$$w^{*d} = \frac{d - \alpha[(1-\theta^*)c_n + \theta^* c_o - sc_n)]}{2\alpha},$$
$$\theta^{*d} = \frac{k\alpha[d - \alpha(c_n + w^*)](w^* - a_o)}{b(2k\alpha - \beta^2) - 2k\alpha^2(w^* - a_o)(c_n - c_o)}. \quad (6)$$

Substituting (6) into (2) yields

$$L^{*d}(\theta^*) = \frac{\beta[d - \alpha(1-\theta^*)c_n - \alpha(\theta^* c_o + sc_n)]}{4k\alpha - 2\beta^2},$$
$$p^{*d}(\theta^*) = \frac{2\alpha dk + (k\alpha - \beta^2)[d + \alpha((1-\theta^*)c_n + \theta^* c_o + sc_n)]}{2\alpha(2k\alpha - \beta^2)}. \quad (7)$$

Finally, the optimal profits of enterprises in CLSC obtain

$$\pi_M{}^{*d} = \frac{k[d - \alpha(1-\theta)c_n - \alpha(\theta c_o + sc_n)]^2}{16k\alpha - 8\beta^2},$$
$$\pi_N{}^{*d} = \frac{k(1-\theta)[d - \alpha(1-\theta)c_n - \alpha(\theta c_o + sc_n)]^2}{8k\alpha - 4\beta^2}, \quad (8)$$
$$\pi_O{}^{*d} = \frac{k\theta[d - \alpha(2a_o + (1-\theta)c_n + \theta c_o - sc_n)][d - \alpha(1-\theta)c_n - \alpha(\theta c_o + sc_n)]}{8k\alpha - 4\beta^2} - \frac{b\theta^2}{2}.$$

□

**Lemma 1.** *In this CLSC under the competition structure, when the collection investment of the recycled component supplier gradually increases, that is, the collection rate grows, the wholesale price paid by the (re-)manufacturer will increase.*

**Lemma 2.** *In this CLSC, the sales price decreases with the growth of the collection rate, but the emission reduction degree increases. Recycling is beneficial to both consumers and social welfare.*

*Proof.* Take the first derivative order of the optimal wholesale price w.r.t. collection investment, $\partial w^{*d}/\partial\theta = c_n - c_o/2$. Because the cost of producing from recycled components is less than the new components, $c_n > c_o$; therefore, $\partial w^{*d}/\partial\theta > 0$. As to the (re-)manufacturer's sales price and emission reduction policies, obtain $\partial p^{*d}/\partial\theta = -((k\alpha-\beta^2)(c_n-c_o)/4k\alpha-2\beta^2) < 0$ and $\partial L^{*d}/\partial\theta = (\alpha\beta(c_n-c_o)/4k\alpha-2\beta^2) > 0$.

According to Lemma 1 and 2, in the CLSC, when the competitive suppliers dominate the whole supply chain and the (re-)manufacturer makes decisions accordingly, the collection investment of the recycled component supplier has a positive effect on the wholesale price paid by the (re-)manufacturer. This is because the order quantity of new components reduces when the quantity of recycled components rises. In order to maintain profit, the new component supplier can increase the wholesale price due to the leadership power. On the other hand, when the collection rate increases, the (re-)manufacturer's producing cost decreases, leading the dominant suppliers gain more advantages from the (re-) manufacturer.

As to the products' sales price and emission reduction degree, their changing trends are different with the growth of the collection rate: sales price decreases and emission reduction degree increases. This is because recycling from used products can reduce the producing cost of the (re-)manufacturer and further push him to make more effort on attracting low-carbon consumers, which is beneficial to both himself and his upstream firms-suppliers. □

**Lemma 3.** *In this CLSC, as the collection investment of the recycling supplier increases, the profit of the new supplier decreases, but the (re-)manufacturer's profit rises. Furthermore, as to the cost-saving efficiency of recycling, $\Delta = c_n - c_o$, it can facilitate the influence of the collection rate on profits.*

*Proof.* According to the assumptions, $\partial\pi_M^{*d}/\partial\theta = (k\alpha(c_n - c_o)[d-\alpha(1-\theta)c_n-\alpha(\theta c_o+sc_n)]/8k\alpha-4\beta^2) > 0$ and $\partial\pi_N^{*d}/\partial\theta = -(k(d-3\alpha(1-\theta)c_n+\alpha(2-3\theta)c_o-\alpha sc_n)[d-\alpha(1-\theta)c_n-\alpha(\theta c_o+sc_n)]/8k\alpha-4\beta^2) < 0$. The increasing slope of $\partial\pi_M^{*d}/\partial\theta$ and decreasing slope of $\partial\pi_N^{*d}/\partial\theta$ will become sharper when $\Delta$ increases.

From the perspective of Lemma 3, when the recycling supplier invests more in collecting used products, such as advertising and door-to-door service, the downstream enterprise in CLSC–(re-)manufacturer can gain more profit from the recycling supplier's action. However, the new component supplier's profit decreases when the competitor craves up more market. This result is consistent with the reality. This is because the producing cost of the remanufactured component is less than the new component so that when the wholesale prices are equal and the recycling capacity increases, the cost of the (re-)manufacturer decreases while the market demand does not, leading to a growth in the manufacturer's profit. At the same time, the dominant, the competitive supplier gets less profit instead because of losing demand.

Generally, with the continuous development of recycling and remanufacturing technology, the recycling cost will gradually decrease. Therefore, the products' sales price and reduction degree will change more dramatically with the collection rate. Likewise, the profits of supply chain members also change sharply. This shows that the advancement of producing technology and recycling methods and management can be beneficial to not only the low-carbon consumers but also the social welfare. Eventually, recycling of used products hurts the new component suppliers, forcing them to prevent the entry and development of the recycling supplier at the initial stage. Thus, it is necessary and important to eliminate the double marginal effect caused by decentralized decision-making and keep the high profit margins of CLSC, which are the main objectives of the following sections. □

*3.2. Cooperation Structure.* In the centralized decision-making structure, that is, the cooperation structure, all CLSC members collectively make policy from the whole supply chain view, maximizing the entire CLSC profit. Therefore, the total profit function is a sum of the profits of the (re-) manufacturer and suppliers as follows:

$$\max_{p,L,\theta} \pi_S = pD - D(1-\theta)(c_n + sc_n) - \theta D(a_o + c_o) - \frac{kL^2}{2} - \frac{b\theta^2}{2}. \tag{9}$$

**Proposition 3.** *In the cooperation model, the optimal decisions exist due to the concavity of profit function w.r.t p, L, and θ.*

*Proof.* As discussed above, the concavity of the CLSC can be verified by $\partial^2\pi_S/\partial p^2 = -2\alpha < 0$, $\partial^2\pi_S/\partial L^2 = -k < 0$, and $\partial^2\pi_S/\partial\theta^2 = -b < 0$. Hessian matrix $\begin{vmatrix} \partial^2\pi_M/\partial p^2 & \partial\pi_M/\partial L\,\partial p \\ \partial\pi_M/\partial p\,\partial L & \partial^2\pi_M/\partial L^2 \end{vmatrix} = 2k\alpha - \beta^2 > 0$ and $\begin{vmatrix} \partial^2\pi_S/\partial p^2 & \partial\pi_M/\partial p\,\partial L & \partial\pi_M/\partial p\,\partial\theta \\ \partial\pi_M/\partial L\,\partial p & \partial^2\pi_S/\partial L^2 & \partial\pi_M/\partial L\,\partial\theta \\ \partial\pi_M/\partial\theta\,\partial p & \partial\pi_M/\partial\theta\,\partial L & \partial^2\pi_S/\partial\theta^2 \end{vmatrix} = -2k\alpha b + b\beta^2 + k\alpha^2(c_n - c_o + sc_n - a_o)^2 < 0$. According to the FOC and backward induction method, the optimal sales price, emission reduction degree, and collection rate are solved by $\partial\pi_S/\partial p = 0$, $\partial\pi_S/\partial L = 0$, and $\partial\pi_S/\partial\theta = 0$ as follows:

$$p^{*c} = \frac{b(c_n + sc_n)(k\alpha - \beta^2) + dkb - dk\alpha(c_n - c_o + sc_n - a_o)^2}{b(2k\alpha - \beta^2) - k\alpha^2(c_n - c_o + sc_n - a_o)^2},$$

$$L^{*c} = \frac{b\beta[d - \alpha(c_n + sc_n)]}{b(2k\alpha - \beta^2) - k\alpha^2(c_n - c_o + sc_n - a_o)^2},$$

$$\theta^{*c} = \frac{k\alpha(c_n - c_o + sc_n - a_o)[d - \alpha(c_n + sc_n)]}{b(2k\alpha - \beta^2) - k\alpha^2(c_n - c_o + sc_n - a_o)^2}, \tag{10}$$

which when substituted into the objective function yields

$$\pi_S^{*c} = \frac{bk[d - \alpha(c_n + sc_n)]^2}{b(4k\alpha - 2\beta^2) - 2k\alpha^2(c_n - c_o + sc_n - a_o)^2}. \tag{11}$$

□

**Lemma 4.** *The double marginal effect can be eliminated under the cooperation structure. Meanwhile, the sales price is relatively lower but the emission reduction degree is higher than the competition case. Similarly, the changing trends of the sales price and emission reduction are the same with the competition case.*

*Proof.* Substituting the optimal collection rate function into sales price, emission reduction and profit function obtains $\pi_S^{*c} - \pi_S^{*d} = (k[d - \alpha(c_n + \mathrm{sc}_n) + \alpha\theta(2\mathrm{sc}_n - 2a_o + c_n - c_o)]^2/16k\alpha - 8\beta^2) > 0$, $p^{*c} - p^{*d} = -((k\alpha - \beta^2)[d - \alpha(c_n - \theta(2sc_n - 2a_o + c_n - c_o) + sc_n)]/2\alpha(2k\alpha - \beta^2)) < 0$, and $\pi_S^{*c} - \pi_S^{*d} = (k[d - \alpha(c_n + \mathrm{sc}_n) + \alpha\theta(2\mathrm{sc}_n - 2a_o + c_n - c_o)]^2/16k\alpha - 8\beta^2) > 0$. Then, investigate the changing trend with respect to the collection rate: $\partial L^{*c}/\partial\theta = \alpha\beta(c_n - c_o + sc_n - a_o)/\ 2k\alpha - \beta^2 > 0$,

$$\frac{\partial p^{*c}}{\partial\theta} = -\frac{(k\alpha - \beta^2)(c_n - c_o + sc_n - a_o)}{2k\alpha - \beta^2} < 0. \tag{12}$$

It can be learnt from Lemma 4 that, since all CLSC members can be treated as one and have the same pursuit in centralized decision-making process, double marginal effect disappears and the total profit is distributed according to their bargaining power. A higher profit will motivate the (re-)manufacturer to pursue more market demand by increasing investment in emission reduction and reduce sales price. In fact, the dominant suppliers will also force the (re-)manufacturer to do so for more ordering quantity of components. This shows that the cooperation structure is more efficient for both consumers and enterprises in CLSC. The cooperation structure as a benchmark achieves the optimal performance. However, in reality, although the upstream and downstream enterprises cooperate in one supply chain, they hardly completely reach an agreement and share their information without distinction. Therefore, a proper coordination mechanism, which can be accepted by all firms, needs to be proposed.

Due to the complexity of calculation, the comparison and sensitivity analysis are shown by numerical examples in the next section. □

## 4. Numerical and Sensitivity Analysis

This section analyzes the performance of various models through the numerical examples, investigates variations of the optimal decisions and profits, and discusses the impacts of firms' decisions on consumers utility and social welfare. Furthermore, to investigate the performance of the proposed models, sensitivity analyses on the critical parameters are provided.

According to our model, a real case of a Chinese power battery (EV Cell) manufacturing enterprise, CATL, can be used. In practice, CATL can produce electric vehicle batteries from both new and recycled cells because cells' material can be extracted and purified repeatedly without reducing the battery's efficiency. Generally, CATL has two types of suppliers: one provides new cells and the other collects used batteries and provides recycled cells. Furthermore, in order to promote environmental sustainability in electric vehicles and increase low-carbon consumers' willingness to pay, it is important for CATL to consider carbon emission reduction and invest in manufacturing technology. Due to the complexity of valuing parameters, we set some number based on the assumptions and CATL case.

In the forward channel, two suppliers compete in providing different types of components. The cost of providing a new component is $sc_n = 6$, which is higher than that of the recycled component, $a_o = 4$. The producing cost with the new component, $c_n = 2$, is also higher than the recycled one, $c_o = 1$, because producing by the new component cause the extra cost of fittings. Furthermore, in order to satisfy consumers' low-carbon preference, the (re-)manufacturer invests in emission reduction. The reduction cost coefficient $k = 10^4$ and its impact on demand with coefficient $\beta$. The potential market demand is $d = 10^4$ and the price coefficient is $\alpha = 100$. The collection cost is associated with the recycling supplier's investment which has cost coefficient $b = 4 \times 10^5$, which should be extremely high to ensure the rational range of the collection rate.

Results of the numerical studies with various decision-making structures are indicated in the following figures. Firstly, in the presence of competition structure, the decision variables and profits of supply chain members are all significantly influenced by consumers' low-carbon preference, as shown in Figure 2. $\mathbb{Z}$ is a constant which can make figures more intuitive without affecting the conclusions.

As can be seen from Figure 2, the wholesale price decided by the new component supplier, the collection rate set by the recycling supplier as well as the emission reduction degree and sales price determined by the (re-)manufacturer all increase with the growth of the consumers' low-carbon preference, and the stronger the preference, the steeper the slope scale. When the CLSC faces the low-carbon market, the consumers' willingness to pay is greatly influenced by the products reduction degree. The higher the reduction level, the more the demand. Therefore, the manufacturer will firstly increase the investment in reduction, which causes a growth in producing cost and eventually raises the sales price. Because suppliers dominate the whole supply chain, they can immediately increase the wholesale price in order to

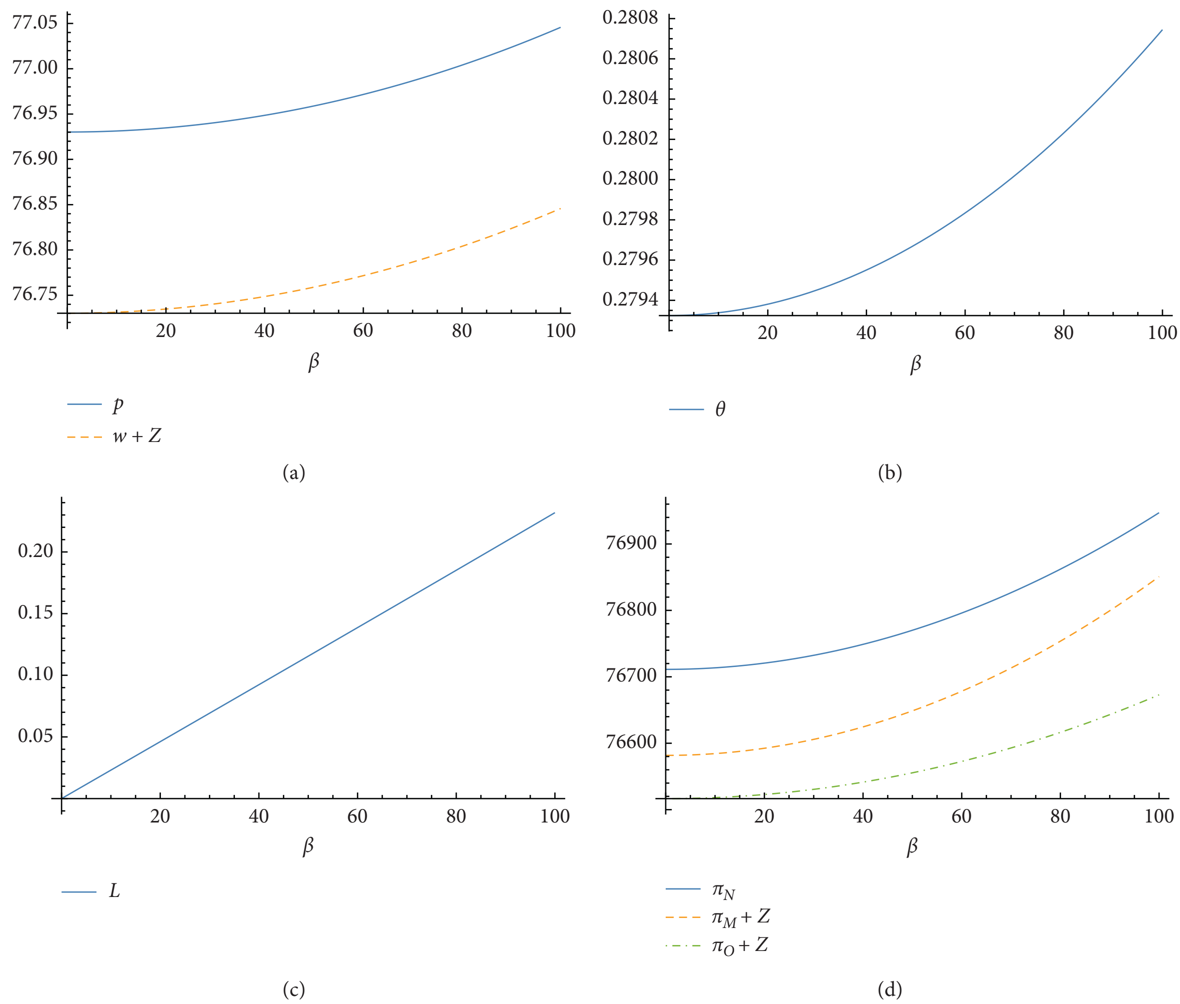


Figure 2: Optimal decisions and profits over low-carbon preference.

carve up more revenue from the following manufacturer. Although sales price slightly negatively affects consumer's utility, the low-carbon factor will catch more attention of ecofriendly consumers which eventually can bring in sales. On the other hand, the low-carbon preference also can boost recycling. That is, because the growth in demand is beneficial to the whole supply chain, leading the dominant recycling supplier gain more profit thus promoting it to invest more in collecting used products. In order to develop society sustainably and harmoniously, government should pay more emphasis on improving customers' environmental awareness because it is not only the most effective way to promote waste recycling and emission reduction simultaneously but also beneficial to all the supply chain members' profits. Similarly, the coefficient of collection investment cost also can affect firms' profits, as shown in Figure 3.

As can be seen from Figure 3, the coefficient of the collection cost has a positive correlation with the profit of the new component supplier, but affects the recycling supplier in an exact opposite way. When the recycling supplier costs more in increasing the collection rate, the profit of itself will definitely decrease. However, the profit of its competitor, new component supplier, will increase. That is, because these two suppliers dominate the whole supply chain and monopolize the market, when one of them face higher cost, the other one will obtain the price advantage and attract more order quantity from the (re-)manufacturer. Therefore, it is extremely necessary and urgent for the recycling supplier to develop the collection technology, service, and advertising campaign, in order to not be squeezed out of the market by the original supplier. Furthermore, when the collection rate increases, that is, when the manufacturer uses more recycled components to produce, the advantage of low-cost manufacturing appears. Both the (re-)manufacturer and recycling supplier can gain more profit. In general, recycling of used products has benefits to the overall supply chain. It can not only bring economic benefits to enterprises but also improve social welfare, especially meeting low-carbon consumers' environmental protection requirements. Thus, the government should also pay emphasis on promoting enterprises to collect used products and further remanufacture them.

In order to compare the efficiency of different decision structures: cooperation and competition structures, the

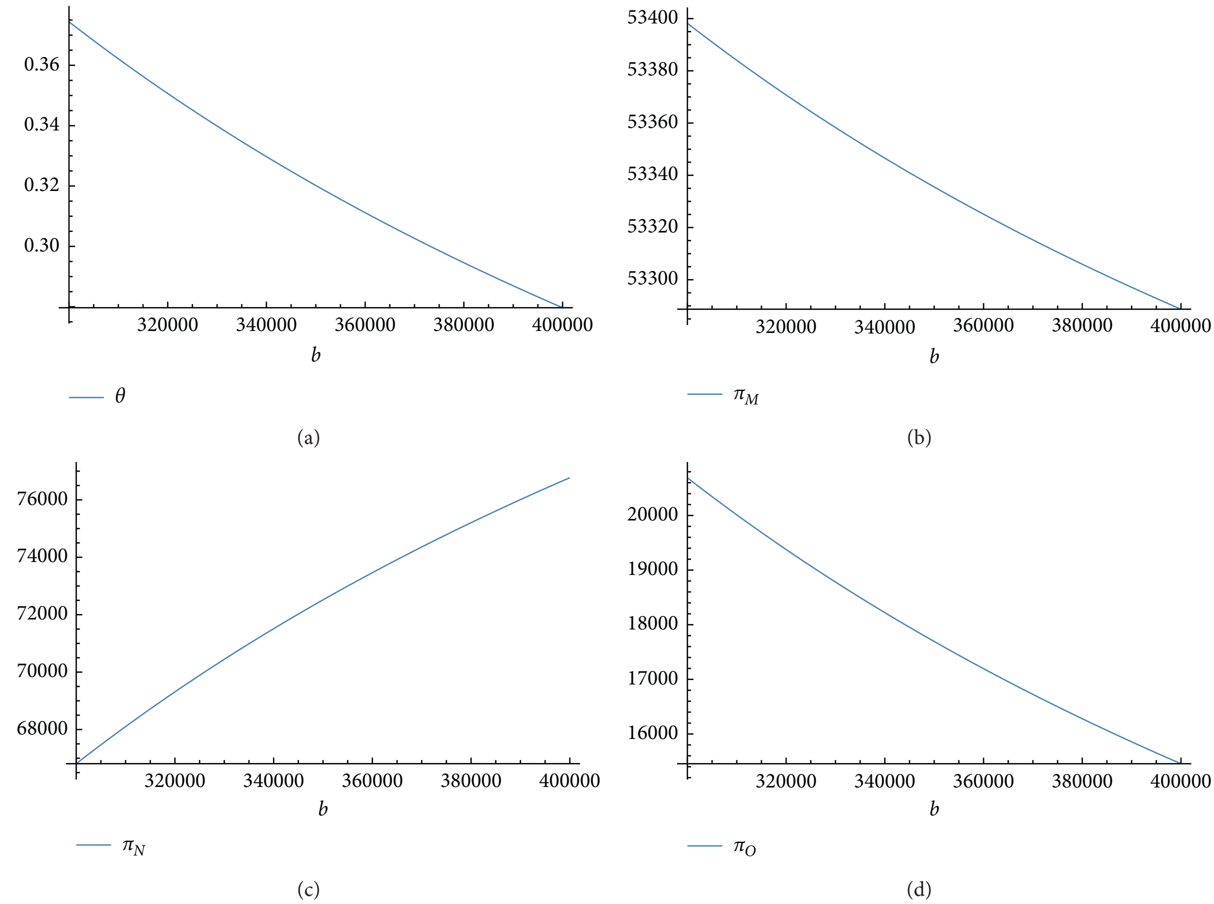


Figure 3: Optimal suppliers' profits over collection cost coefficient.

optimal decision variables and profits of each supply chain member in different decision models can be obtained, as shown in Figure 4.

According to Figure 4, the sales price and the collection rate in cooperation model are both lower than that of the competition model. Meanwhile, the emission reduction degree is higher in the cooperation structure. Apparently, the total profit of the supply chain is much higher in the cooperation model. This is consistent with the reality; centralized decision-making is the most efficient way for firms to cooperate, for consumers to get high utility, and for the government to achieve social emission reduction. However, as to the collection rate, it is, surprisingly, lower in the cooperation mode than the competition mode. Although they both decrease with the collection cost coefficient, the decreasing slope scale in the competition model is larger than the cooperation model. This is because the recycling supplier needs to invest more in collecting for improving its competitiveness, grabbing more order quantity from the (re-)manufacturer. When all enterprises cooperate as one decision-maker, the recycling supplier loses motivation to invest more because there is no competition in CLSC. Nevertheless, the carbon emission reduction degree is significantly improved in the cooperation model, bringing benefits to low-carbon consumers and environmental government.

Overall, the cooperation model is much more profitable and efficient for both firms and society, but in practice, it is extremely hard to achieve a full-cooperation among supply chain members due to the information barriers and conflicts of interest. Therefore, a coordination structure is introduced in the following section.

## 5. Coordination Structure

According to the above analysis, it can be concluded that the cooperation structure, as a benchmark, is more efficient than competition one. However, in most practical cases of CLSC operation, enterprises usually prefer not to fully cooperate, which allows them to reserve own right to make independent choices. In this section, we design a coordination mechanism to improve the total profit of CLSC and ensure each firm can increase their profits. First of all, in order to make sure all the supply chain members are willing to participate in coordination contract, the necessary conditions include: (1) overall profit of the coordinated supply chain is greater than the competition model and (2) all members' profits in the coordination structure are greater than the competition structure.

Since this CLSC is dominated by upstream suppliers, so they can take priority to set a higher wholesale price and take advantages from the downstream (re-)manufacturer.

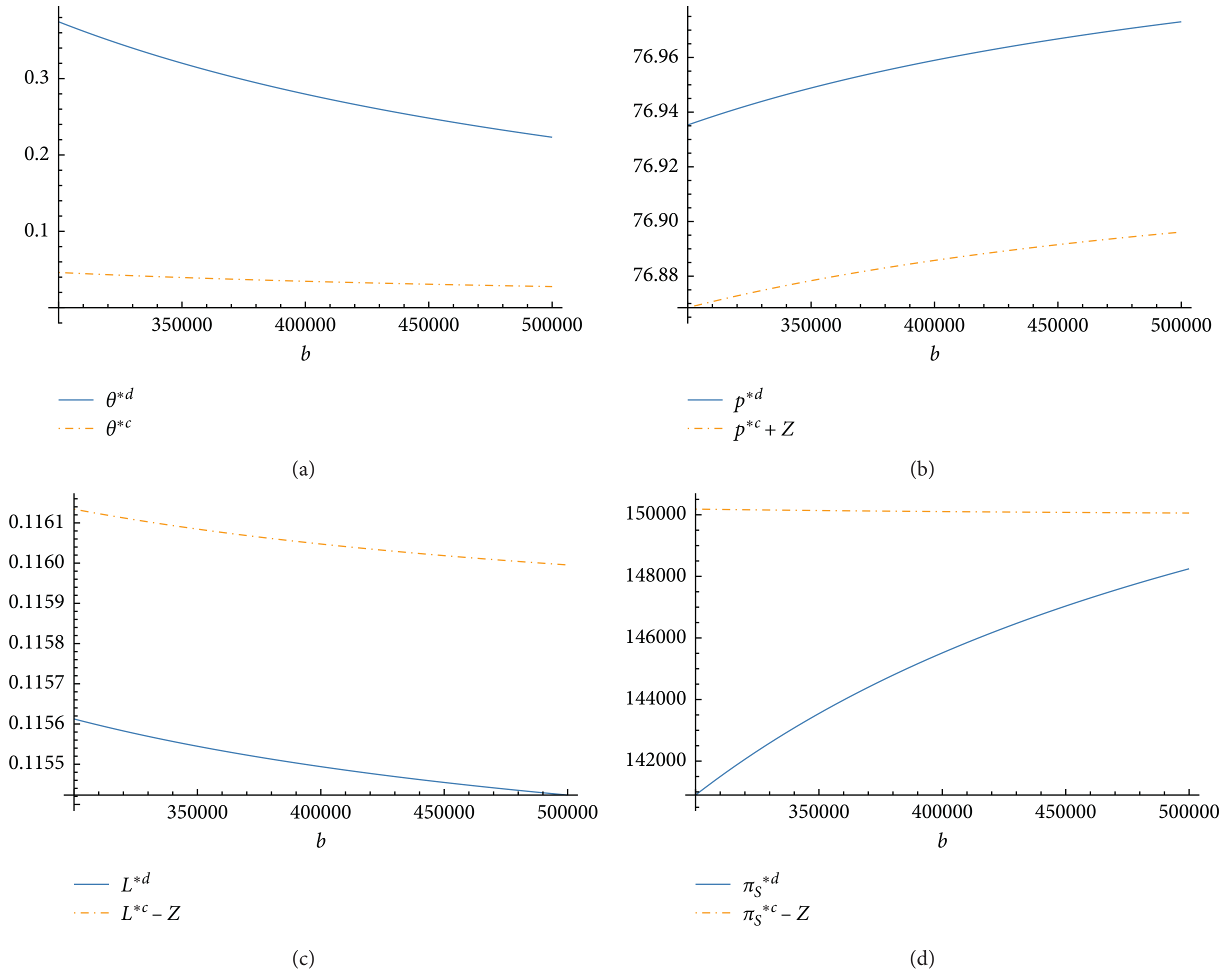


Figure 4: Optimal decisions and profits compared in different structures.

Therefore, the following enterprise has motivation to share the component producing cost in order to get a lower wholesale price in return. The lower wholesale price can further stimulate the (re-)manufacturer to make more effort for emission reduction and then boost the market demand. Thus, a cost-sharing contract is introduced in this coordination structure, in which the (re-)manufacturer shares the producing cost of the new component supplier and obtains a new wholesale price, which can be divided into two parts: a sharing part $Tsc_n$ and a wholesale price $w^{co}$. The profit functions in the coordination structure are derived as follows:

$$\begin{aligned} &U_M = pD - (w^{co} + c_n + Tsc_n)(1-\theta)D - (w + c_o)\theta D - \frac{kL^2}{2}, \\ &U_N = [w^{co} - (1-T)sc_n](1-\theta)D, \\ &U_O = (w^{co} - a_o)\theta D - I_\theta, \\ \text{s.t.}\quad &U_M \geq \pi_M, U_N \geq \pi_N, U_O \geq \pi_O. \end{aligned} \tag{13}$$

**Proposition 4.** *In the cost-sharing coordination mechanism, if the pricing strategy* $(w^{co}, T)$ *satisfies the conditions:* $w^{co} = d - \alpha(1-\theta)c_n - \alpha\theta c_o + \alpha s c_n[1 - T(2-\theta)]/2\alpha$ *and* $T = d - \alpha[c_n - \theta(c_n - c_o + 2sc_n - 2a_o) + sc_n]/\alpha\theta s c_n$*, the coordination mechanism can achieve the Pareto improvement.*

*Proof.* As discussed above, the concavity can be verified by the second order derivative conditions. According to the FOC and backward induction method, the optimal sales price, emission reduction degree, and collection rate are solved as $p^{*co}$, $L^{*co}$, and $\theta^{*co}$. In order to achieve the coordination, the equations $p^{*co} = p^{*c}$, $L^{*co} = L^{*c}$, and $\theta^{*co} = \theta^{*c}$ must be satisfied.

According to the results, this contract can achieve supply chain coordination through a cost-sharing contract, motivating the (re-)manufacturer to sell products and reduce carbon emissions at the level of the centralized decision-making case. Therefore, the double marginal effect in the competition structure can be eliminated and the total profit reaches the level of cooperation case. Ultimately, supply chain members share the total profit according to their bargaining power. Without loss of generality, participants are assumed to have bargaining powers as follows: $bp_N = 0.5$, $bp_M = 0.35$, and $bp_O = 0.15$. When the supply chain is coordinated, it meets $U_M = bp_M \pi^{*c} \geq {\pi_M}^{*d}$, $U_N = bp_N \pi^{*c} \geq {\pi_N}^{*d}$, and $U_0 = bp_O \pi^{*c} \geq {\pi_O}^{*d}$.

Table 2: Comparison of different decision-making structures in Test 1 ($\theta = 0.45$).

| | Competition structure | Cooperation structure | Coordination structure, $T = 34.91$ |
|---|---|---|---|
| $p$ | 76.916 | 53.383 | 53.383 |
| $L$ | 0.116 | 0.234 | 0.234 |
| $w$ | 52.225 | — | $w^{co} + \mathrm{Tsc}_n = 99.35$ |
| $\pi_M$ | 53485.62 | — | 62169.88 |
| $\pi_O$ | 9719.79 | — | 26644.23 |
| $\pi_N$ | 58834.18 | — | 88814.11 |
| $\pi_S$ | 122039.60 | 177628.22 | 177628.22 |

Table 3: Comparison of different decision-making structures in Test 2 ($\theta = 0.25$).

| | Competition structure | Cooperation structure | Coordination structure, $T = 62.17$ |
|---|---|---|---|
| $p$ | 76.967 | 53.683 | 53.683 |
| $L$ | 0.115 | 0.232 | 0.232 |
| $w$ | 52.125 | — | $w^{co} + Tsc_n = 98.75$ |
| $\pi_M$ | 53254.46 | — | 70991.63 |
| $\pi_O$ | 15281.80 | — | 30424.98 |
| $\pi_N$ | 79881.69 | — | 101416.62 |
| $\pi_S$ | 148417.94 | 202833.23 | 202833.23 |

Table 4: Comparison of different decision-making structures in Test 3 ($\theta = 0.05$).

| | Competition structure | Cooperation structure | Coordination structure, $T = 307.5$ |
|---|---|---|---|
| $p$ | 77.02 | 53.983 | 53.983 |
| $L$ | 0.115 | 0.231 | 0.231 |
| $w$ | 52.025 | — | $w^{co} + Tsc_n = 98.15$ |
| $\pi_M$ | 53023.79 | — | 74219.69 |
| $\pi_O$ | 5032.79 | — | 31808.44 |
| $\pi_N$ | 100745.21 | — | 106028.13 |
| $\pi_S$ | 158801.80 | 212056.26 | 212056.26 |

According to Proposition 4, when the enterprises in CLSC make decentralized decision-making in the competition model, the profit decline caused by the double marginal effect can be solved by designing a coordination mechanism, and the (re-)manufacturer can further cooperate with the new and recycled component suppliers. Due to the complexity of calculation, we further discuss the effectiveness of the coordination contract in the next section. □

*5.1. Numerical Analysis of Coordination Effectiveness.* The effectiveness of coordination mechanism is verified through numerical analysis. Similarly, each parameter is valued as in the previous section. The results of the case study under competition, cooperation, and coordination structures are indicated in Tables 2–4. As can be seen from Tables 2–4, the cooperation structure is optimal and the coordination structure can improve all members' profits compared with the competition structure. Furthermore, in order to indicate the robustness of the proposed models under various cases, test problems 1–3 are designed. In Test 1, the collection rate is relatively higher than Test 2 and 3, and collection rate in Test 3 is the lowest.

From Table 2, the proposed cost-sharing contract improves the profit of overall CLSC and its members compared to the competition model. Moreover, under the contract, the emission reduction degree increases in comparison with the competition structure. In addition, under the cooperation model, the sales price decreases which is consistent with the result of previous research studies. Accordingly, the proposed cost-sharing contract as a coordination scheme is able to improve both the economic and environmental aspects.

Results of comparison of competition, cooperation, and coordination structures in Test 2 are shown in Table 3. The proposed cost-sharing contract also can improve the profit of overall CLSC and its members compared to the competition structure, when the collection rate decreases. Moreover, under the coordination contract, the sales price decreases which is consistent with the previous results of other scholars, and the emission reduction degree improved compared to those of the competition model. When the collection rate of used products goes further lower, similar results are shown in Table 4.

As can be seen from Tables 2–4, the cost-sharing contract coordination mechanism can achieve the optimal supply chain performance and improve each firm's profit. Meanwhile, through Test 1–3, the robustness and effectiveness of coordination structures are proved.

*Conclusion 1.* The cost-sharing contract can achieve the Pareto improvement of the CLSC system and ensure all enterprises in CLSC are willing to participate in the contract, by keeping their own profit increasing. In addition, the coordination mechanism is beneficial to both economic and environmental aspects.

Under these three types of decision-making structures, the sales price decreases and emission reduction effort increases with the growth of the collection rate. This indicates that recycling is beneficial to both consumers' utility and social welfare. However, the total profits cannot increase with the collection rate. This is because collecting used products is not only the cost of the supplier but also the source of its income. Thus, the recycling supplier needs to investigate a proper collection rate, in order to preserve a higher profit. Furthermore, cost of investment in collecting is too high for CLSC to reach an increasing in total profit over the collection rate. It is necessary for each firm in CLSC to pay emphasis on technical innovation and improvement.

## 6. Conclusion

Most previous research studies have not paid enough attention to the competition between new and recycled component suppliers when the latter one also acts as a collector, not even researched on the coordination of this dual-source supply system. Although some studies have discussed the competition in the supply channel, few of them introduce the ecofriendly factor, such as emission reduction, into the supply chain, which will dramatically influence the consumer's willingness to pay and manufacturer's profit. Therefore, this article focuses on a closed-loop supply chain system consisting of a (re-)manufacturer, a new component supplier, and a recycled component supplier, in which the (re-)manufacturer considers carbon emission reduction and the recycling supplier considers used product collection. Decisions on the sales price, emission reduction degree, wholesale price, and collection rate are analyzed and further compared under three decision-making models: competition, cooperation, and coordination structures on the basis of Stackelberg game and Nash equilibrium. In order to eliminate the negative effects of the competition structure, this paper designs a cost-sharing contract to coordinate the whole supply chain. The results indicate that

(1) In the CLSC system in which the upstream suppliers dominate and the (re-)manufacturer focuses on emission reduction, the wholesale price increases with the growth of collection investment, but the sales price of products decreases. Meanwhile, the emission reduction degree rises with the collection investment decided by the recycling supplier. Moreover, recycling is beneficial to both consumers' utility and social welfare.

(2) Under competition structure, the collection investment in used component recycling has positive effect on the profit of the (re-)manufacturer, but has negative impact on the new component supplier. Moreover, cost-saving efficiency of recycling can facilitate these influences.

(3) The double marginal effect can be eliminated under the cooperation structure compared to the competition structure. Meanwhile, the sales price is relatively lower but emission reduction degree is higher in the cooperation case, which is beneficial to both consumers and enterprises.

(4) Due to the difficulty of collection and high cost of investment, recycling negatively affects the profit and efficiency of CLSC. Although recycling is beneficial to social benefits and environmental protection requirements, it hurts enterprises' profits to a certain extent.

(5) Although centralized decision-making can optimize the CLSC system, it is extremely hard to achieve in reality and practice. Therefore, a coordination mechanism is introduced, cost-sharing contract. It can fully coordinate CLSC and achieve Pareto improvement, making the total profit reach the level of the cooperation structure as well as bringing benefits to environmental protection.

For future research, this study can be extended in several directions. First, the paper investigates a symmetric game model of two competitive suppliers and one recycling channel; it can be extended by several collecting channels, such as the third-part collector and second-hand market, and then it considers information barriers and market demand uncertainty. Second, this model can be extended by taking other coordination mechanisms into consideration, including two-part tariff, quantity discount, and buy back contract. Moreover, this paper assumes that the costs of collecting, manufacturing, and producing components are constant. In fact, due to the uncertain quality of used products and stochastic producing output, these costs should be assumed as uncertainty. Finally, this model only studies on a single-period game model and there is no competition between manufacturers. Thus, in future research, this model can be extended into multiperiod and multicompetition among collectors and manufacturers, in addition to suppliers.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# Improving Distributed Decision Making in Inventory Management: A Combined ABC-AHP Approach Supported by Teamwork

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The need of organizations to ensure service levels that impact on customer satisfaction has required the design of collaborative processes among stakeholders involved in inventory decision making. The increase of quantity and variety of items, on the one hand, and demand and customer expectations, on the other hand, are transformed into a greater complexity in inventory management, requiring effective communication and agreements between the leaders of the logistics processes. Traditionally, decision making in inventory management was based on approaches conditioned only by cost or sales volume. These approaches must be overcome by others that consider multiple criteria, involving several areas of the companies and taking into account the opinions of the stakeholders involved in these decisions. Inventory management becomes part of a complex system that involves stakeholders from different areas of the company, where each agent has limited information and where the cooperation between such agents is key for the system's performance. In this paper, a distributed inventory control approach was used with the decisions allowing communication between the stakeholders and with a multicriteria group decision-making perspective. This work proposes a methodology that combines the analysis of the value chain and the AHP technique, in order to improve communication and the performance of the areas related to inventory management decision making. This methodology uses the areas of the value chain as a theoretical framework to identify the criteria necessary for the application of the AHP multicriteria group decision-making technique. These criteria were defined as indicators that measure the performance of the areas of the value chain related to inventory management and were used to classify ABC inventory of the products according to these selected criteria. Therefore, the methodology allows us to solve inventory management DDM based on multicriteria ABC classification and was validated in a Colombian company belonging to the graphic arts sector.

## 1. Introduction

Nowadays, the level of business competitiveness has to be high when facing the opening of markets as an intrinsic factor of globalization. It is necessary for companies to be competitive to respond to the requirements of increasingly demanding customers in terms of cost, quality, and product delivery time. Likewise, small and medium-sized enterprises (SMEs) must compete with multinationals belonging to the same sector, which have a greater infrastructure, in terms of processes and finance strength, which means that SMEs need to increase productivity levels through decision models.

Being able to develop competitive advantages with customer service orientation allows companies to excel in local and foreign markets. Therefore, it is important to carry out an internal analysis of the company's processes, which should be focused on achieving the satisfaction of internal and external customers, as well as guaranteeing the best performance in the operation.

The decisions related to inventory management are especially relevant in a customer service orientation approach. These decisions are part of a complex system and involve stakeholders from different areas of the company, where each agent has only limited information and where the cooperation among such agents is key for the system's performance. The most used tool for the identification of these stakeholders, and to model this complex system, is the analysis of the value chain. In addition, the value chain provides a framework for identifying the criteria to be considered [1] in decision-making inventory management through all areas of the company. On the other hand, even though, from a centralized point of view, these inventory management decisions are interesting topics to investigate, we are going to approach these issues within the distributed decision-making (DDM) framework that considers all areas of the value chain.

Several DDM structures are possible within this scenario. In this work, a single-level distributed inventory control approach was used with the decisions made by the stakeholders involved at the management team level, allowing communication and coordination among the decision makers with a multicriteria group decision-making perspective.

It is important to highlight that the objective of this research work is to propose a methodology that integrates value chain analysis and a multicriteria decision-making method. This methodology tends to identify and improve the relationship among the stakeholders involved in one of the most important logistics processes for companies, such as inventory management.

This research was validated in a Colombian company belonging to the graphic arts sector. This sector is shown to have a big influence in the national economy due to its contribution of 3.7% of GDP in Colombia. Therefore, any effort that tends to improve the competitiveness of this sector will reflect directly on the economy of the region and later on the economic and commercial position of Colombia.

Due in large part to the expansionist trend that this sector has undergone in recent years, especially in the cities of Bogotá, Medellín, and Cali, companies have focused efforts on the acquisition of specialized software for production planning, acquisition of equipment for manufacturing processes, and human resources expansion. However, the efforts associated with the development and analysis of the value chain considering multicriteria decisions for inventory management have not been appreciated. This generates a problematic environment for the decisions made on how to control the inventory by the stakeholders.

One of the main pieces of evidence of this problem, in the SMEs of graphic arts, is the noncompliance with the delivery dates agreed with the clients. There is also a high level of obsolescence of stocks. This situation is largely caused by the lack of inventory policies that should allow the identification of when and how many product units to order from suppliers. Additionally, as mentioned in [2], the process of decision making in inventory management is complex, which is why different perspectives are needed from the department managers of each area of the company to control stocks in a more efficient way. In this regard, several research works have been undertaken focused on methodologies based on multiple criteria for inventory planning and control taking into account, i.e., cost, quality, and delivery [3, 4]. This will be addressed in the following sections of this article.

As mentioned at the beginning of the section, the purpose of this paper is to propose a methodology for value chain analysis that considers multicriteria decisions for inventory management. This methodology allows us, in the first instance, to consider an internal analysis of the company's value chain that recognizes the relevance of decision-making processes in inventory management. At the same time, it allows us to identify the best criteria to classify and control the inventory based on the opinions of the stakeholders involved in the process. Finally, the methodology establishes the guidelines of an ABC classification based on the multicriteria technique analytic hierarchy process (AHP) [5] in order to categorize items correctly.

In Section 2 of this article, the theoretical foundations of the proposal are presented. In Section 3, the methodology proposed by the authors will be explained, and each of its components will be described. In Section 4, the process of planning and inventory control in the company is characterized. In the same section, the validation of the methodology, being applied to a Colombian graphic arts company, was shown. In the two last sections, we present the conclusions of the research, limitations, and recommendations for future research work on the subject.

## 2. Literature Review

*2.1. Value Chain.* Organizations grant a set of physical and intangible features and benefits to customers. These are the result of a logical and progressive process that, when carried out efficiently, achieves one of the main objectives of the entire company: customer satisfaction, also known as value approach [6].

The value chain is considered as a technique for the analysis of manufacturing and service companies that determines how an organization can develop and deliver value to its stakeholders, customers (both internal and external), through the analysis and identification of sources of value for the optimization of adjacent processes. According to some previous works [6–9], the synthesis of business activity is divided into two types of activities: the primary and secondary. The primary activities are associated with the manufacture, transfer, and sale of the products to the buyer. On the other hand, the secondary activities are related and serve as support for the previous ones, such as procurement, information systems, and communications among others. Figure 1 shows the generic scheme by types of activities that make up the value chain.

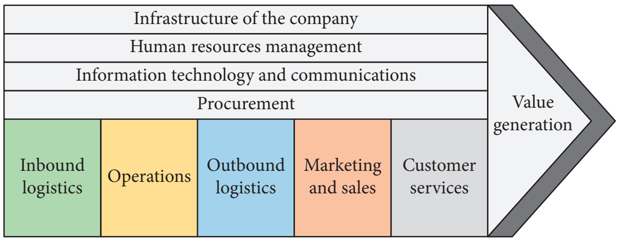


Figure 1: Value chain structure [6].

The final part of the chain represents the margin or profit, which is seen as the intangible increase in the value appreciated by internal customers (company areas) or external customers (end users).

The value chain of the companies, which covers all its functions, consists of suppliers, purchases, operations, marketing, sales, customers, human resources, and finances depending on the products and services provided [10]. The analysis of the components of the value chain allows us to define the factors that most affect company competitiveness [11]. In this sense, business analysis is based on the analysis of the company's value chain.

The value chain can be defined as a conceptual structure, and its components can be used to lay the foundations for the company's performance in inventory management, identifying the main criteria for this purpose. Value chain analysis helps to diagnose the sources of information and communication in inventory management. Therefore, the value chain will be used as a conceptual framework to identify the areas related to inventory management, involving stakeholders in decision-making processes, increasing communication and coordination with each other, and improving the performance of these areas in inventory management.

*2.2. Multicriteria ABC Classification.* Traditionally, companies often use the well-known ABC classification technique to identify the most representative inventory items and, at the same time, have an efficient control over them. Companies handle a large number of inventory items, which makes the management and control process more complex given the limited amount of resources. To have an efficient inventory management, the most appropriate action is to group the stock and to focus on the most important items [12].

The ABC classification, which is based on the Pareto principle, has three classes or families of products: class A, including the most important articles, class B that are of moderate importance, and class C that have a low importance. Consequently, once the classes are established among the articles, control policies can be defined as those presented by Silver et al. [13].

Traditional methods only consider one criterion, commonly the unit cost of acquisition. This is largely due to the fact that companies have been focused on analysing the products that generate the greatest sensitivity to cash flows and to the profitability of the operation. However, in the literature, inventory classification approaches are portrayed under multiple criteria, known as multicriteria inventory classification (MCIC). Some previous works [14–17] illustrate the implementation of these methodologies.

According to the literature review carried out by Van Kampen et al. [18], in many bibliographic studies, multicriteria techniques have been proven to be a good alternative to help control and classify inventory items. The use of the AHP is especially addressed in these studies [14, 19–22], since its structure deals with the subjectivity of the experts about the pronouncement of judgments [17].

On the other hand, several authors follow two main trends to determine the thresholds for the classification of products in each category A, B, or C, (i) those that set them on a percentage basis of the quantity of products, most of them based on ranking methods [19, 23–26], and (ii) those who propose them based on advanced methods, such as sorting methods or artificial intelligence (AI) methods [27–33]. The choice of the path forward will depend on the skill and knowledge of the experts, since, in practice, accurate data and criteria affecting ABC classification are not always available. Therefore, many times, managers prefer linguistic values than numerical values for measuring the criteria in practical applications of ABC classification. These linguistic values are closer to the knowledge and experience of the experts. Hence, in the literature, there are many applications of type (i). In these applications, the authors handle various percentage values for classification and there is no prevalence of specific classification percentages for categories A, B, and C. One of the most frequently used classifications is 20, 30, and 50% for products A, B, and C, respectively [26, 34].

*2.3. Analytic Hierarchy Process (AHP) in Inventory Management Distributed Decision Making.* Distributed decision making (DDM) addresses an important and rapidly developing field in general decision theory. It comprises several areas among other group decision making [35].

Classical decision theory and decision analysis are centralized, i.e., there is only one decision maker with a utility function and a set of subjective probabilities about the state of the world. The presence of multiple decision makers makes the problem more complicated because the decision makers may have different utility functions and/or different assumptions about the underlying uncertainty. Even when all the decision makers have the same point of view and are going to make their decisions cooperatively, there is still the problem of defining optimality for multiple utility functions. One approach is given by multiattribute utility theory where an organizational utility function is constructed from the individual utilities. Another approach is given by team decision theory, which considers decision making by multiple decision makers with a single common objective but different information about the underlying uncertainty. Physically, one may imagine the decision makers to be connected by a communication network that is imperfect.

Following Schneeweiss [36] and his classification proposal, decision problems with various decision-making units (DMUs) and *team-based DDM systems* are like one-party systems and are denoted as conflict-free DDM problems. In these cases, the team has the same utility function and just one coordinating decision must occur. This requires a DDM system of partners to be symmetrically informed.

Saaty [5] developed the analytic hierarchy process (AHP), a mathematical technique based on matrix concepts, which allows solving complex problems in convergence terms of human judgments. AHP is a multicriteria decision-making technique that can be included in the multiattribute utility theory [37] and whose application, widely extended in various fields, is also used in group decision making [38].

In the process of decision making in inventory management, it can be considered that all the stakeholders, although with different information and perspectives depending on the area of the company to which each one belongs, form a single DMU (management team). This is because they share the same objective in terms of inventory management. In this way, this issue can be treated as a case of multicriteria group decision making.

According to Saaty [39], there are two important issues in group decision making: how to add individual judgments in a group in a single representative judgment for the whole group and how to build a group choice based on individual choices. The reciprocal property plays an important role in combining the judgments of several individuals to obtain a unique judgment for the group. The judgments must be combined so that the reciprocal of the synthesized judgments is equal to the synthesis of the reciprocals of these judgments. It has been shown that the geometric mean, not the arithmetic mean used frequently, is the only way to do it. If individuals are experts, they may not want to combine their judgments, but only the results obtained by each from their own preferences. In that case, the geometric mean of the final results is taken.

*2.3.1. Theoretical Background of the Analytic Hierarchy Process (AHP).* The AHP technique formalizes and makes a systematic decision-making process, which is largely subjective and, therefore, facilitates "precise" judgments. As a result of the method, the decision makers receive information of the implicit weights of the evaluation criteria. AHP allows better communication, leading to a clearer understanding and consensus among the members of the decision-making groups and therefore a greater commitment to the chosen alternative. This method has the following steps:

Identification of the problem: before starting any numerical calculation, it should be checked that the problem in question can be displayed as a structured model, where the criteria and the alternatives of the process are identified. At the top level of the structure, the objective or *goal of the decision problem* to be solved must be identified.

Selection of criteria: in this stage, the criteria associated with the multicriteria decision-making process are selected, which will be assessed and weighted in subsequent stages. Some criteria for the classification of inventory items addressed in the literature are presented in Table 1. It is important that accurate data of the criteria are available, since a satisfactory result of the process depends on it.

Pairwise comparison of criteria: each criterion $i$ is compared to criterion $j$ through the relative scale of priority presented in Table 2: Saaty's 1–9 scale [46]. Such comparisons are located in a square matrix of order $n$ and reciprocal. This process is repeated with each of the experts and stakeholders involved in the assessment process.

Priority calculation: the weights of each one of the criteria are calculated through

$$A\omega = \lambda_{\max}\omega, \tag{1}$$

where $A$ is a $n$ dimensional of the comparison matrix, $\lambda_{\max}$ is the largest eigenvalue of $A$, and $\omega$ is the eigenvector corresponding to $\lambda_{\max}$.

Consistency index: equation (2) is used to calculate the consistency of the decision-making process.

$$\text{C.I} = \frac{((\lambda_{\max} - n)/(n-1))}{\text{R.I}}, \tag{2}$$

where $n$ is the number of the criteria and R.I is a random index corresponding to the $n$ criteria. Table 3 shows the variation of the R.I according to $n$.

Coefficient of consistency: this coefficient measures the degree of homogeneity between the judgments issued by the experts or stakeholders of the process. $A$ value less than 0.1 is considered admissible. However, when more than 5 criteria are handled within a multicriteria decision-making process, this threshold can increase to 0.15 or 0.18. To calculate it, use

$$\text{C.C} = \frac{\text{C.I}}{\text{R.I}}. \tag{3}$$

*2.3.2. The Use of Multicriteria Analysis in Value Chain and Inventory Classification.* In order to achieve a level of efficiency in the logistics activities of the value chain, aligned with the competitive tendencies of the industrial market, these activities must be supported by multicriteria methodologies for decision making. For example, for the selection and evaluation of suppliers, there are diverse applications such as the following [47, 48].

It is important to note that efforts have also been focused on applying multicriteria techniques for the distribution of products, such is the case of the research work presented by Bravo et al. [49]. This work shows a distribution prioritization methodology that considers several criteria when making shipments to customers. For this purpose, the AHP method was used to weigh criteria and to determine which are the most important when are making operational distribution decisions.

In the research work carried out by Guarnieri et al. [50], a reference framework is presented for the hiring of a 3PL

Table 1: Criteria for the classification of inventory items (literature review).

| References | Method | Lead time | Criticality | Durability | Substitutability | Obsolescence | Availability | Unit cost | Annual use of the dollar | Rotation | Number of requests/ year | Shortage | Reparability | Order size requirement | Storage capacity | Demand distribution | Exhaustion penalty cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [12] | Matrix of criteria | | | | | | | X | X | | | | | | | | |
| [19] | Weighted sum | X | | X | | | | X | X | | | | | | | | |
| [14] | AHP | | | | | | | | | | | | | | | | |
| [17] | Linear optimization | X | X | | X | X | | X | | | X | X | X | X | X | X | X |
| [40] | AHP fuzzy | X | | | | | | X | | | | | | | | X | |
| [31] | AHP and K-means | X | X | | | | | | X | | | | | | | | |
| [4] | AHP | X | X | X | X | X | X | X | X | X | | X | X | X | X | X | X |
| [26] | Optimization | X | | | | | | X | X | | | | | | | | |
| [31] | AHP | X | X | | | | | X | X | X | | | | | | X | |
| [15] | AHP | | | | | | | X | X | X | X | | | | | X | |
| [41] | Cluster analysis | X | | | | | | X | X | | | | | | | | |
| [42] | Chebyshev theorem | X | | | | | | X | X | | | | | | | | |
| [43] | Machine learning | X | | | | | | X | | | | | | X | | X | |
| [44] | Machine learning | X | | | | | | X | | | | | | X | | X | |
| [45] | Neuronal networks, multihoming algorithm | | | | | | | X | | | | | | | | X | |

Table 2: Scale intensity of relative importance [5].

| Value $a_{ij}$ | Description |
|---|---|
| 1 | Criterion $i$ and criterion $j$ are considered to be equally important |
| 3 | Criterion $i$ is considered to be slightly more important than criterion $j$ |
| 5 | Criterion $i$ is considered to be significantly more important than criterion $j$ |
| 7 | Criterion $i$ is considered to be far more important than criterion $j$ |
| 9 | Criterion $i$ is considered to be absolutely more important than criterion $j$ |
| 2, 4, 6, 8 | Intermediate values |

Table 3: Scale of variation of the random index [5].

| Number of criteria ($n$) | Random index |
|---|---|
| 2 | — |
| 3 | 0.58 |
| 4 | 0.90 |
| 5 | 1.12 |
| 6 | 1.24 |
| 7 | 2.32 |
| 8 | 1.41 |
| 9 | 1.45 |

supplier of reverse logistics, 3PRPL—third-party reverse logistics supplier, taking into account environmental regulations. Several multicriteria techniques are proposed for the selection of suppliers, among which the use of AHP stands out.

Decision making in inventory management from a multicriteria analysis has been widely studied in the literature. The first contribution on MCIC was provided in [19] that applied AHP to classify inventory items. Later, AHP was adopted by some authors such as [14, 22], and others have used modified versions of AHP applied in MCIC, for instance, AHP Fuzzy [40], or a new hybrid method based on AHP and the $K$-means [31]. Several authors have used AHP for spare parts classification [20, 21, 32].

MCIC is a specific issue that can be faced with the application of AHP, where the alternatives correspond to the inventory items [19, 31], since AHP can solve problems with qualitative and quantitative evaluations. These evaluations are entered into a pairwise comparison matrix. The importance of the criteria and ranking of the alternatives are then derived with the eigenvalue [5, 51]. As the value of the items on each criterion in MCIC is often precisely measureable [19], these values are normalised in order to be combinable and rankable in a weighted global score.

Nowadays, only a few applications on machine learning classification algorithms to MCIC have been developed [44, 52], and the only one which has been extended to the inventory system is the study carried out by Lolli et al. [43]. These applications can reduce classification cost and human errors when sets of thousands of inventory items must be managed.

By carrying out a thorough bibliographic analysis, the most commonly used criteria in the literature for the classification of inventory items were identified, as shown in Table 1.

## 3. Proposed Methodology

*3.1. Basic Foundations of the Methodology.* The main objective of this work is to propose a methodology that combines the analysis of the value chain and the AHP multicriteria decision-making technique, in order to improve the communication and the performance of the areas related to the inventory management decision-making process. The criteria used in the decision-making process will be those indicators that allow us to analyse the performance of the areas of the value chain related to inventory management. On the other hand, with the application of AHP, it is possible to solve inventory management DDM based on multicriteria ABC classification.

Before performing an analysis of the value chain, it becomes relevant to know the level of maturity of the company to identify the processes that are not aligned with the objectives and interests of the organization. In this way, a qualitative analysis of the level of communication and coordination among the areas that make up the value chain is carried out. Once this level has been identified, it is necessary to define the performance in the processes of each area related to inventory management. For this purpose, indicators that analyse this performance are established.

Therefore, to achieve this objective, the methodology is based on three key aspects:

Maturity level of the company: according to Alonso-Manzanedo et al. [53], the maturity level of the company can be set in 5 levels as shown in Table 4.

Definition and evaluation of performance measurement: according to Augusto et al. [54], companies should possess a model that measures the characteristics and parameters of multifaceted performance through a number of specific indicators approved by experts. Performance indicators allow us to identify how close or far is the proposed goal. However, there are two common problems when measuring the performance of a process: the first is that a goal is difficult to obtain due to the lack of information or communication of the stakeholders. To this end, a level 3 of maturity of the company, which is considered necessary to establish coordinated decisions in inventory management, must be reached. This level is an achievable one for any organization that considers common objectives in decision making. The second problem is the poor mathematical construction of the indicators. This is frequent when the stakeholders have little expertise in the work context. It is important that the members of each area of the organization have knowledge of the inputs, processing, and outputs required by the inventory management process and know how to connect with the other areas.

Value chain framework: once the two previous steps have been completed, it is proposed to carry out the

Table 4: Maturity level of a company [53].

| Level | Description |
|---|---|
| 1 | The processes are unstructured and address the interests of each area |
| 2 | The processes are defined and documented and the relationships among areas are based only on the transfer of information |
| 3 | There is feedback from each area leader and the objectives are shared |
| 4 | The members of the organization collaborate in other processes and not only information is shared but also resources |
| 5 | There is reciprocal trust and mutual dependence among the members of an organization to achieve common objectives |

analysis of the value chain together with the multicriteria decision-making technique as the central basis of the methodology. The proposed methodology will use the areas of the value chain as a theoretical framework to identify the criteria necessary for the application of the AHP multicriteria group decision-making technique. To this end, all the areas of the value chain are analysed in order to identify the stakeholders involved in inventory management and the performance indicators, which will be the criteria in the AHP model. These criteria were defined taking into account the literature review shown in Table 1 and the opinion of all the stakeholders involved in inventory management as experts in these decision-making processes.

As a main goal when defining performance indicators in inventory management, it was established *to maximize compliance with the service level of the inventory items* (the goal of the decision-making problem in the AHP model). On the other hand, to help the mathematical construction of the combination of indicators that measure the performance of these areas, it is proposed to complement the methodology with the use of AHP (see Section 2.2), which establishes how these criteria should be weighted when classifying inventories. In Figure 2, this methodology is related.

*3.2. Flowchart of the Methodology.* As shown in Figure 2, the methodology begins with the identification of the level of maturity of the organization. It is important that problems that exist in the company are identified in terms of lack of communication among the areas, failure to meet common objectives for the company, and loss of trust among the members of each area.

Next, consensus meetings should be scheduled in which each of the stakeholders reviews their capacities, limitations, and opinions about the process. The consensus meetings should be held periodically and should measure the progress and commitments proposed by each area. All of the stakeholders, as DDM system partners, have to share information and have to be symmetrically informed. In this type of meetings, work must be done to improve communication and coordination until reaching the level of maturity necessary for the organization (level 3).

*3.2.1. Value Chain Analysis.* In this step, performance indicators that have an impact on inventory management for each of the areas of the value chain are established so that their measurement considers relevant elements for the areas with which they are related. For example, the customer service area must handle a cumulative service level indicator that not only considers the dates and quantities agreed with the customer but also considers the level of service provided by the raw material inventory (logistics area) and tracks when stock breakage occurs.

When these requirements are achieved, a thorough analysis of the value chain is carried out in order to identify experts from all the necessary areas. In this sense, all processes that add value to the company should be considered. For the specific case of inventory management, all the internal and external processes shown in Figure 1 are taken into account: the procurement section must issue orders to suppliers, inbound logistics must receive, store, and control all the items of inventory, and outbound logistics must distribute the product according to the location of customers.

Finally, when the company obtains the results of the analysis of the value chain, different improvement actions are proposed that intend to add greater value to the company.

*3.2.2. AHP Method.* Following Flores et al. [19], the methodology applies an AHP-based approach (see Saaty [5]) that synthesises several weighted criteria into a single priority score for each item. The values of the items on each criterion in MCIC are normalised in order to be commensurable and combinable in a weighted global score.

Following the steps of the AHP method, the opinions of each of the experts are considered and consensus criteria are selected by these stakeholders as those indicators that are capable of measuring performance in all areas of the value chain. These criteria will be extracted from the set of criteria proposed by the literature, shown in Table 1, in order to classify the inventory items. Next, the iterations corresponding to the AHP are performed, raising the pairwise comparison matrix to limiting powers, and the weights of each criterion are obtained through the aggregation of the experts' judgments by means of the geometric mean. Mathematical foundations and the steps of the AHP technique can be found in [5].

In the final step of the methodology, the classification of the inventory items in the classes defined as A, B, and C is carried out by means of weighted sum, obtaining a single priority score for each item [19].

As mentioned in Section 2.2, one of the most frequently used classifications is 20, 30, and 50% for products A, B, and C, respectively. In this proposal, the decision making in inventory classification that best suits the conditions of the problem to be solved was left to the experts.

## 4. Case Study

This methodology was validated in a Colombian SME belonging to the graphic arts sector. This organization manufactures and distributes products such as labels, stickers,

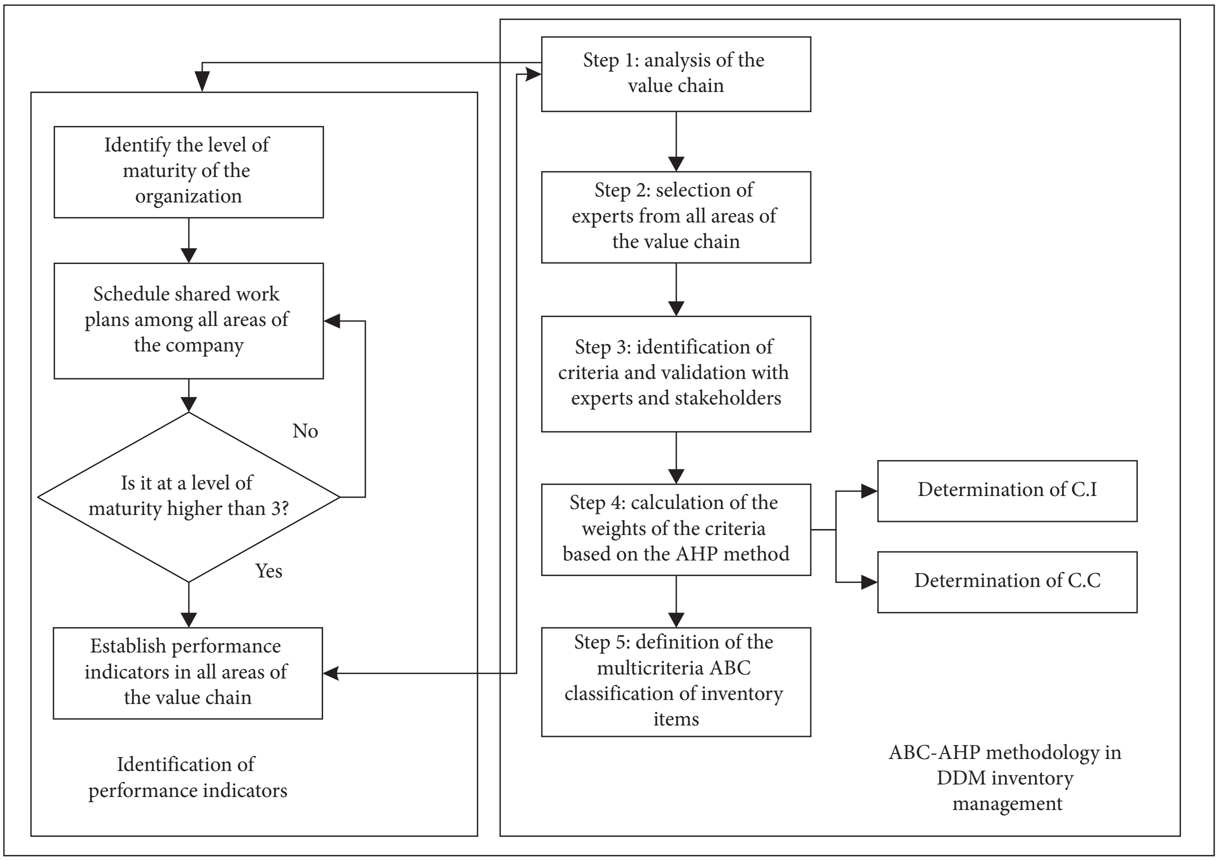


Figure 2: Proposed methodology.

leaflets, and different advertising materials. According to the information provided by the company, there were problems of shortages and supplies of raw materials. Moreover, the information among the areas related to the inventory management did not handle the same data about the level of stocks, costs, and dates of product delivery.

The following describes the implementation of the methodology in the case study.

*4.1. Steps 1 and 2: Value Chain Analysis and Selection of Experts from All Areas of the Value Chain.* An analysis of the value chain was conducted, following step 1 of the methodology as shown in Table 5, so interviews based on checklists about the process of inventory management of the company were made to the managers of planning, logistics, manufacturing, procurement, finance, and commercial areas (that make up the management team of the DDM). This management team was selected to act as the experts involved in inventory decision-making processes.

Table 6 presents the most representative results of the value chain analysis.

Table 7 shows the experts selected in step 2.

As can be seen in Table 6, poor communication and lack of synergy among the areas related to the inventory process lead to problems, such as an imbalance in the plant, continuous breaches in the delivery of customers' orders, and deterioration of the image of the company. All these problems position the company below level 3 of maturity. In order for the company to reach level 3, several meetings were necessary to address the communication problem, to share all the information related to inventory management, to unify points of view of the five areas involved, and to define the common objective of achieving a coordinated decision making. This common objective is *to maximize compliance with the service level of the inventory items.*

One of the aspects that stood out in the analysis of the value chain is that only one criterion is used to classify the inventory, leaving aside the perspective of other criteria that must be taken into account, such as lead time, criticality of the item, degree of substitutability, and distribution of demand, among others. It should be noted that prior to the application of this methodology, inventory items were classified only by the criterion *unit cost.* This is because there were items imported from Chile, which had a strong impact on the cash flow of the company, and therefore the general manager and the finance manager carried out the negotiation process with the suppliers.

The use of AHP as a multicriteria decision-making tool is highly relevant because it makes it possible to deal with the subjectivity and the pronouncement of expert judgments.

*4.2. Steps 3 and 4: Determination of Criteria and Calculation of Their Weights Based on AHP Method.* For the development of step 3, there were five stakeholders involved in the

Table 5: Application of step 1 of the methodology (analysis of the value chain).

| | |
|---|---|
| Analysis of the value chain (step 1) | The value chain of the company that covers all the functions of the company consists of suppliers, procurement, operations, marketing, sales, customers, human resources, and finance<br>The components of the value chain were used to lay the foundations for the performance indicators in inventory management |

Table 6: Value chain analysis results.

| Areas | Value chain analysis results |
|---|---|
| Inbound logistics | There is no classification system by importance of inventory items<br>The logistics area classifies the products according to the rotation of the article, while the finance area is based on the cost of acquisition<br>There are no inventory policies<br>The logistics area does not communicate in time to the procurement area the quantity of orders that must be launched |
| Operations | There is a high rate of downtime in the plant due to the fact that there is no raw material to start the daily production program<br>The sales area does not take into account the capacity of the plant when it comes to confirming orders with customers, so they must pay overtime or subcontract units |
| Outbound logistics | Decreases in the indicator of correct deliveries due to noncompliance with customer delivery dates<br>There is continuous rescheduling of routes due to orders not being shipped on time |
| Customers services | There are decreases in the indicator of fulfilment of order delivery, since the finished product is released late by the production area<br>The commercial area makes estimates using simple averages and does not take into account the variability of demand |
| Procurement | There is no collaborative relationship with suppliers<br>Performance evaluation is not carried out for each provider |
| Information technology and communications | An MRP system is being implemented to make more reliable production plans |
| Human resources management | Training is being programmed to improve communication among the members of the organization; however, it is a process that takes a long time to show improvements |
| Infrastructure | The company is acquiring new state-of-the-art equipment for printing processes |

Table 7: Application of step 2 of the methodology (selection of the experts).

| | |
|---|---|
| Selection of the experts (step 2) | The information comes from the opinion of experts with long experience and knowledge of the company: Planning manager, manufacturing manager, finance manager, logistics manager, and procurement manager. |

inventory management process, selected in step 2 of the methodology. After the analysis of the value chain, the group of experts determined which factors were more important in inventory management of the company and defined the performance indicators. These performance indicators were defined in two participatory workshops through discussion and subsequent agreement. By consensus, the experts selected the performance indicators used to classify the inventory items from the list of criteria shown in Table 1.

With the collaboration of the experts, during two face-to-face participatory workshops, 3 indicators were finally selected. The first session, in which the experts were shown the value chain and its key areas, lasted two hours. The session focused on the discussion about the key areas, particularly planning, manufacturing, finance, logistics and procurement, and the indicators associated with them. From this first session, 5 indicators were chosen: *Unit cost, Lead time, Rotation, Criticality*, and *Substitutability*. In the second session, the experts consensually expressed their interest in reducing the number of indicators to 3, as can be seen in Table 8.

In step 4, we proceeded to apply the AHP multicriteria method that resulted in the global priority vector of the criteria. This was calculated using the geometric mean to aggregate the priority vectors of the criteria of each of the 5 experts. The consistency of the decision-making process was also calculated in this step. These results are shown in Table 9.

At this stage, and through criteria comparison questionnaires answered by the experts, the degree of the importance among the criteria was obtained using Saaty's 1–9 scale [46].

An example of the questionnaire designed to allow the comparison analysis is shown in Figure 3.

The weights of the 3 criteria were obtained based on the geometric mean value of the priorities expressed by each expert.

According to the results presented in Table 9, we can observe that there is convergence among the results issued by the stakeholders, who prefer *Unit cost* as a predominant criterion for the classification of the items. However, the three most relevant criteria were considered to carry out the

Table 8: Application of step 3 of the methodology (identification of criteria).

| | |
|---|---|
| Criteria (step 3) | *Unit cost:* it is the acquisition cost of the inventory item measured in \$/unit.<br>*Lead time:* it is the time that elapses from when a purchase order is issued to the supplier until it is available to be delivered to the production area. It is measured in days.<br>*Rotation:* it is the number of times that an item of the inventory has been renewed in a period of time and is measured in number of times per year. |

Table 9: Application of step 4 of the methodology (AHP method results).

| | |
|---|---|
| Calculation of criteria weights and consistency (step 4) | Unit cost: 38.31%<br>Lead time: 33.80%<br>Rotation: 27.89%<br>C.I: 0.0042<br>R.I: 0.58<br>C.C: 0.007 |

With respect to the goal "*to maximize compliance with the service level of the inventory items*" for each pair of Inventory Management criterion, please indicate which of the two you consider to be most important and to what extent.

The inventory management criterion must be compared pairwise, by asking to what degree criterion $C_i$ has a greater importance compared with criterion $C_j$, using the following scale (Saaty's scale):

$C_{ij}$ = 1: criterion *i* and criterion *j* are considered to be equally important

$C_{ij}$ = 3: criterion *i* is considered to be slightly more important than criterion *j*

$C_{ij}$ = 5: criterion *i* is considered to be significantly more important than criterion *j*

$C_{ij}$ = 7: criterion *i* is considered to be far more important than criterion *j*

$C_{ij}$ = 9: criterion *i* is considered to be absolutely more important than criterion *j*

---

C1: Unit cost

C2: Lead time

| | | | | | |
|---|---|---|---|---|---|
| Which Inventory Management Criterion do you consider more important? | C1 | C2 | | | |
| To what extent? | 1 | 3 | 5 | 7 | 9 |

Figure 3: Sample of questionnaire used for comparison of criteria.

ABC classification, since they were performance measurement indicators common to all areas of the value chain and accepted by consensus among all the stakeholders in those areas.

As can be seen in Table 9, there is a convergence between the judgments issued by the experts, given that the consistency coefficient (C.C) is less than 0.1. Additionally, the most important criterion is *Unit cost* with 38.31%. This occurs because there is a strong preference of the finance and procurement managers towards this criterion, which prioritizes the purchase to the lowest cost suppliers. *Lead time* criterion with 33.80% occupies the second position and has a strong preference of the logistics and planning managers, due in large part to the fact that low levels of inventory lead to unreliability of production plans.

*4.3. Step 5: Multicriteria ABC Classification.* In this step, the items were ordered based on the level of compliance of each item for the 3 criteria, thus obtaining the results of Table 10.

Table 10: Multicriteria ABC Classification results.

| Class | Items | Percentage | Weighted score |
|---|---|---|---|
| A | 32 | 19.75 | >0.14 |
| B | 49 | 30.24 | >0.083 |
| C | 81 | 50.00 | >0.05 |
| Total | 162 | 100 | |

The results of the prioritization of the items according to inventory management criteria lead to categorizing the items in ABC classes. This prioritization was calculated by means of the weighted sum of the values of each item for each criterion, as explained in Table 11.

It is important to note that the company has 453 items, of which 162 are classified as critical within the manufacturing process. The development of this proposal was applied to these 162 items of the total handled by the company.

Table 10 shows the representative results of this step.

Table 11: Application of step 5 of the methodology (multicriteria ABC classification).

| | |
|---|---|
| Multicriteria ABC Classification (step 5) | To classify the inventory items, the values of each item for each criterion must be normalised and multiplied by their corresponding weights resulting from the AHP method. Later, the items were ordered in descending order of the weighted score and groups A, B, and C were established by the experts (see table in the supplementary material (STEP5_ABC CLASSIFICATION)). |

## 5. Conclusions

The customer service orientation approach requires cooperation at the operational level in the company and even the entire supply chain to improve delivery times, as well as flexibility in some processes and efficiency in the performance of the operation. A better synchronization means lower costs throughout the chain, a high level of quality, and an improvement in the image of the organization. However, the synchronization of all members of an organization is not an easy task because there must be shared information, as well as communication and coordination among all the areas of the company. The value chain analysis can be used as a theoretical framework to identify the main areas that need coordination to solve a problem within the organization.

This research work describes a new methodology, based on analytic hierarchy process (AHP), to solve problems in inventory management distributed decision making (DDM) involving all areas of the company. This approach evaluates the performance in inventory management of companies in a trustworthy and efficient way. It covers an indicator selection process adapted to the company using value chain analysis.

The approach combines the use of an AHP multicriteria group decision-making technique with value chain analysis to identify performance indicators for all areas of the company that can be used as criteria for ranking items in ABC inventory classification. The methodology includes all the stakeholders involved in the decision-making process, considering the different information and perspectives they have. These different perspectives depend on the area to which each one belongs. AHP is used due to its ability to obtain quantitative values from the qualitative opinions of the experts and also because it allows the aggregation of the priorities of the selected experts. The experts were selected according to their experience and knowledge in the different areas of the company's value chain (management team). They are treated as a single decision-making unit (DMU) since they share the same objective in terms of inventory management.

The analysis of the value chain of companies helps to identify the first performance indicators of the areas of the company related to inventory management. These performance indicators will then be weighted with the implementation of AHP in the proposed methodology. For its validation, the methodology was applied to a company in the graphic arts sector of Colombia.

The weighting of the criteria (indicators) provides some important insights into the general philosophy and the underlying conception of the experts on inventory management. The data resulting from the indicators show that the most important criterion in ABC inventory classification is *Unit cost* because there is a strong preference of the finance and procurement managers for this criterion. The second one is *Lead time* which is strongly preferred by managers of logistics and planning.

The results obtained from the analysis of the company's value chain diagnosed the lack of communication and coordination among the different areas and proposed improvement actions aligned with the objective of the decision problem. These improvement actions allow the company to connect inventory management processes with other areas and are necessary for any organization that shares common objectives in a coordinated decision making. A single-level distributed inventory control approach was used with the decisions made by all the stakeholders involved at the management team level, allowing communication among the decision makers.

The experts agreed on the selection of most of the indicators, but not on the weights assigned to the indicators. However, all five experts agreed with the final result obtained through the aggregation of their priorities and with the procedure followed in the methodology. The case study showed that the geometric mean proposed by AHP to add priorities helps to balance the extreme positions among the decision makers and is useful in the cases of *team-based DDM systems* (which are like one-party systems with a single DMU).

Based on the findings of this study, we can conclude that it is not so important for an organization to measure the performance in the inventory management of all areas of the company's value chain. On the contrary, it is relevant for any organization to have clear objectives, the criteria of prioritization in the classification of items and their corresponding weights, since this contributes directly to reaching the objective. The AHP method contributes efficiently to solve a multicriteria decision-making process with several stakeholders from the different areas involved in the company, and the results obtained in this work allow us to conclude that AHP is an adequate tool for ABC inventory classification.

Even though the new proposal has been applied specifically to the graphic arts sector, this tool can be adapted to any industrial sector, provided that the criteria are correctly identified. This tool constitutes a very promising line of future research in the field of distributed decision making in inventory management.

## 6. Limitations and Future Work

Some limitations of this work must be pointed out. On the one hand, a limitation of this approach is that classifying thousands of items with respect to several criteria is a complex and time-consuming task. Machine learning approaches like support vector machines and deep neural networks [43] can help to overcome this issue.

On the other hand, another limitation could be treating ABC inventory classification as a ranking problem, not as a sorting problem. In the case of this paper, the experts decided the final sorting step, including each item in a critical class (A, B, or C). The following situation could have occurred: two items with exactly the same score could have been in two different clusters. This method is conditioned [31] by the subjective opinion of the experts on the criticality of the item.

Accordingly Ishizaka and Nemery [55], inventory item classification requires a sorting method. This last issue could constitute a subject for further research. We propose the use of AHPSort [27] in a future work, applying this method to the same set of items. In this way, we can compare the results obtained and present these new results to the managers in order to achieve their approval. Although, a priori, a larger number of meetings with the experts are necessary to agree on the limiting profiles of the classes, the level of satisfaction with the results is likely to be higher due to the overall time savings. However, they can see that with the limiting profiles established in advance, the classification decision achieves greater objectivity.

## Data Availability

The multicriteria ABC classification data used to support the findings of this study are included within the supplementary information file.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Supplementary Materials

This file consists of an excel file with two calculation sheets: (i) STEPS 3 AND 4_AHP METHOD: with the data of the pairwise comparison matrices used by the experts for the calculation of the criteria weights and (ii) STEP 5_ABC CLASSIFICATION: with the data of the items' values for each criterion used for the ABC inventory classification. (*Supplementary Materials*)

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*Research Article*

# Emergence of Coordination in Growing Decision-Making Organizations: The Role of Complexity, Search Strategy, and Cost of Effort

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Coordination among decision-makers of an organization, each responsible for a certain partition of an overall decision-problem, is of crucial relevance with respect to the overall performance obtained. Among the challenges of coordination in distributed decision-making systems (DDMS) is to understand how environmental conditions like, for example, the complexity of the decision-problem to be solved, the problem's predictability and its dynamics shape the adaptation of coordination mechanisms. These challenges apply to DDMS resided by human decision-makers like firms as well as to systems of artificial agents as studied in the domain of multiagent systems (MAS). It is well known that coordination for increasing decision-problems and, accordingly, growing organizations is in a particular tension between shaping the search for new solutions and setting appropriate constraints to deal with increasing size and intraorganizational complexity. Against this background, the paper studies the adaptation of coordination in the course of growing decision-making organizations. For this, an agent-based simulation model based on the framework of NK fitness landscapes is employed. The study controls for different levels of complexity of the overall decision-problem, different strategies of search for new solutions, and different levels of cost of effort to implement new solutions. The results suggest that, with respect to the emerging coordination mode, complexity subtly interferes with the search strategy employed and cost of effort. In particular, results support the conjecture that increasing complexity leads to more hierarchical coordination. However, the search strategy shapes the predominance of hierarchy in favor of granting more autonomy to decentralized decision-makers. Moreover, the study reveals that the cost of effort for implementing new solutions in conjunction with the search strategy may remarkably affect the emerging form of coordination. This could explain differences in prevailing coordination modes across different branches or technologies or could explain the emergence of contextually inferior modes of coordination.

## 1. Introduction

The coordination of decision-making among a set of agents, with each being responsible for a certain partition of an overall decision-problem, is a fundamental issue in the design of distributed decision-making systems (DDMS)—may they be, for example, organizations like firms or teams of robots (e.g., [1–6]). Coordination within DDMS is studied in various domains of organizational thinking. For example, a predominant issue in organization theory is mechanisms to manage interdependencies between activities within an organization [7–9]; in the domain of management control, the so-called Management Control Systems (MCS) are intended to ensure that decision-making is consistent with objectives and strategies of a firm by employing a multitude of mechanisms and techniques to coordinate managers' choices [10–12]; in the domain of multiagent systems, for example, "plan merging" in terms of integrating partial plans into an overall plan is one of the issues discussed in the context of coordination (e.g., [5, 13]).

A common, though mostly implicit, idea in these schools of organizational thinking is, however, that there is a well-informed designer knowing the "true" nature of the overall decision-problem and, with this, the coordination need as affected by the problem's complexity; hence, the designer can organize the DDMS accordingly. However, this assumption does not necessarily universally apply, since the system may be newly set up or it may operate in an environment which has undergone an external shock. In a similar vein, the underlying decision-problem does not need to keep its structure over time; it may, for example, grow in size: A firm may produce and sell additional products or an increased geographical area may be provided with services by a (growing) fleet of unmanned aerial vehicles.

In this line of thought, for coordination theory in multiagent systems (MAS), Lesser and Corkill [14] recently argue that among the issues so far underrepresented in MAS research is to explicitly take into account environmental conditions like, for example, the predictability of task characteristics and the dynamic adaptation of coordination when the environment or the problem to solve is evolving dynamically.

Against this background, the research objective of this paper is to study *which types of coordination emerge in DDMS when the underlying decision-problem is not known in advance and is subject to growth*. The paper seeks to provide some answers to this research question while adopting the perspective of contingency theory, i.e., assuming that the performance of a system is shaped by the fit between its situational context and its internal arrangements, taking the particular interrelation among complexity theory and contingency theory ([15], pp. 411) into account. Based on this understanding, the research endeavor presented takes three contingent factors into account.

*1.1. Complexity of the Decision-Problem.* Corresponding to the seminal paper of Simon [16], the complexity of an overall decision-problem and, hence, the need for coordination is shaped by the interactions among its components: In case the overall decision-problem is (nearly) decomposable, it can be separated into (nearly) disjoint sub-problems such that intra-sub-problem linkages are stronger than the inter-sub-problem interactions. In consequence, sub-problems can be solved independent from each other without taking positive or negative interactions with respect to the overall problem's solution into account. In contrast, if an overall decision-problem is nondecomposable, no decomposition into sub-problems can be found that (nearly) diminishes inter-sub-problem interactions (e.g., [17, 18]). Hence, if complexity in terms of interactions is high, also the need for coordination across the sub-problems is high when superior solutions to the overall decision-problem are pursued.

*1.2. Search Strategy.* The strategies for finding new solutions for decision-problems have been studied in various domains. For DDMS "resided" by human decision-makers a cornerstone was set by Simon [19] who argues that in "situations of any complexity" (pp. 104) decision-makers are unable to survey the *entire* search space and, hence, cannot identify the optimal solution of their decision-problem "at once"; rather, they search stepwise for superior solutions following a satisfying approach. In the domains of organizational search and innovation, the idea of stepwise search often is characterized by the extent of changes in terms of the distance of the newly found options compared to a status quo, i.e., explorative, exploitative, or ambidextrous strategies of search (e.g., [20, 21]). Constraints regarding the allowed or enforced extent of changes induced by alternative solutions are part of the boundary system in MCS: The boundary system sets behavioral constraints to decision-makers and is regarded as a prerequisite for the delegation of decision-making [22–24]. In growing organizations, the boundary system is in a particular tension between shaping the search for new opportunities and innovation on the one hand and on the other, setting behavioral constraints to deal with increasing size and intraorganizational complexity [25, 26].

*1.3. Cost of Effort.* The search strategy shapes the potential extent of changes made compared to a status quo and changes may come along with notable extra costs often termed "switching cost" or "cost of effort," e.g., for cognitive effort or additional consumption of time or resources. For example, organizational changes may cause costs for reorganization and for handling resistance of certain stakeholders [27], choosing another supplier could result in costs for technical conversions [28, 29], changing the direction of a robot's movement in a multirobot system could cost some extra time [30–32], and altering the task assignment in the course of job scheduling may rise some extra costs of, for example, learning—in case of humans as well as "artificial" agents [33, 34]. The level of cost of effort, therefore, affects the propensity to implement an alternative to the status quo as identified according to the search strategy.

The paper employs an agent-based simulation model *to "grow" organizations from scratch and to observe which modes of coordination predominantly emerge for different settings of the aforementioned contingent factors*. Simulation appears an appropriate method since it allows to capture long-term and processual phenomena that—depending on the subject—would require rather challenging longitudinal studies in empirical research (e.g., [35–37]). Simulation further helps to analyze "borderline" cases which could be hardly studied in a sufficient number empirically in order to systematically explore the effects of contingency the effects of contingency factors in interaction with each other [15]. To capture and be able to control the complexity of a DDMS's overall decision problem, the study relies on the framework of NK fitness landscapes which was originally introduced in the domain of evolutionary biology [38, 39] and, since then, broadly employed in managerial science (for overviews, see [21, 37]). In particular, an agent-based simulation is employed in which growing organizations—with an increasing number of decentralized decision-makers—search for superior solutions to their task (specified according to the NK framework) and where the organizations may employ different search styles. From time to time, the organizations adapt their coordination mode based on reinforcement learning.

The remainder of this paper is organized as follows: Section 2 relates this paper to streams of prior research. Section 3 introduces a theoretical model of growing DDMS before in Section 4 the simulation experiments including the parameter settings are described. Section 5 reports and discusses the results obtained from the simulations, and Section 6 provides concluding remarks.

## 2. Related Work

The emergence of coordination is a key question in various domains and, accordingly, the related approaches in research are rather manifold. For example, complex systems science studies the emergence of "collective intelligence" (for an overview, see [40]) as apparent in linguistic conventions [41]; in economics, it is examined how individuals learn to coordinate in respect to collective objectives like in public good games (e.g., [42, 43]); prominent issues of multiagent systems are the self-adaptation of consensus mechanisms in variable networks or the convergence of consensus algorithms (for an overview, see [44]).

While these streams of research clearly are of general relevance for this research endeavor, with respect to the research question posed in the Introduction, the paper particularly focuses on the emergence of coordination in DDMS which face decision-problems of increasing size by means of an agent-based simulation.

Herewith, the paper also relates to the two domains of "agent-based computing" as Niazi and Hussain [45] put it: The research question is predominantly directed to the *understanding* of emergent phenomena in the context of complex decision-problems and, thus, it is related to the stream of research employing agent-based simulation as a research method—often termed as "*social simulation*" [46]; however, the research question also relates to the domain of multiagent systems (MAS) with its primary *design* focus [45, 47].

With this, the paper particularly builds on three lines of research which are outlined subsequently: (1) coordination and, in particular, constraint-setting mechanisms via MCS in growing organizations; (2) agent-based simulation in the domain of organizations and management control; and (3) coordination of decision-making in MAS.

*2.1. Management Control Systems in Growing Organizations.* In this line of research, the empirical study of Davila [48] on the emergence of management control systems (MCS) in small growing organizations provides a cornerstone according to which organizational size has a positive impact on the overall level of use of MCS. A key argument is that when organizations grow, informal controls become too costly and/or ineffective and, hence, more formal controls for purposes of motivation and monitoring are employed as they are captured in MCS. As mentioned in the Introduction, the research endeavor of this paper addresses *complexity* in terms of interactions among sub-problems in growing decision-problems. However, in this respect, research findings on MCS are somewhat ambiguous. While low levels of interactions have been found to be linked to budgets, operating procedures, and statistical reports, the latter combined with informal coordination were found to be employed when complexity is high [49]. Some findings indicate that with higher levels of complexity, more emphasis is put on communication between subordinate and superior decision-makers and that aggregated and integrative information becomes more important [50]. Moreover, it was argued that with higher levels of inter-sub-problem interactions the uncertainty derived from a lack of control over the supplying decision-makers increases and this, in turn, may result in more formal controls, while, at the same time, increasing the need for flexibility resulting in an emphasis on informal controls [51]. In a recent study [24], a high level of interdependencies was found to, in tendency, be related to the establishment of vertical information flows for solving coordination problems. With respect to the search strategy (see Introduction), first of all, it is worth mentioning that MCS are intended to provide focus for the search for novel solutions, i.e., aligning them with the organization-wide strategic orientation [25]. Constraining the decision-making scope via boundary systems (as subsystems of MCS) was found to be positively associated with performance of exploitative innovations with their typically tightly coupled activities—particularly, because risk is reduced that subordinate decision-makers pursue activities which are not in line with established processes [26, 52]. In contrast, in the long run, boundary systems are argued to reduce the propensity of exploration in terms of experimentation [53].

*2.2. Agent-Based Simulation in the Domain of Organizations and Management Control Systems.* Several studies employ agent-based modeling—following the tradition of "social simulation"—in the domain of organizations [54], and, as in this paper, employ the NK-framework—with its particular capabilities of capturing the level of complexity of the decision-problem and of studying the effects of different search strategies for superior solutions at the level of the entire organization (for overviews, see [21, 37, 55, 56]) For example, the performance of different coordination mechanisms in turbulent and complex task environments is analyzed by Siggelkow and Rivkin [57]: the authors identify—for different levels of task complexity—organizational configurations which appear appropriate to cope with environments undergoing some external shock; e.g., when complexity and turbulence are at high levels, ample coordinative power and strategies fostering broad search processes turn out to perform best; however, the results also suggest that subtle interferences with distribution of decision-making authority in the organizations may affect overall performance. These results relate to prior research which indicates on the subtle interaction between complexity and the appropriate organizational design, namely, hierarchical coordination (e.g., [55, 58, 59]). Regarding the *search strategy* (see Introduction), not only experiential search (i.e., based on processes of local search) has been studied but also compared to, for example, forward-looking search (capturing decision-makers' potentially imprecise beliefs about the actions-outcome-relations) [60] or "search" via imitation [61]: imitation turns out to be effective for the imitator particularly at intermediate levels of complexity while with high complexity of the decision-problem small errors in imitation can lead to severe deviations from the intended solutions and performance losses [62]. With respect to *cost of effort* for implementing new solutions (see Introduction), prior research suggests on subtle interactions with problem complexity: On the one hand, higher cost of effort makes changes less attractive and, hence, increase the peril of inertia, which is a particular relevant aspect in complex decision-problems; on the other hand, they may stabilize search and prevent frequent mutual ("hyperactive") adjustments within an organization [63, 64].

*2.3. Coordination of Decision-Making in Multiagent Systems.* Coordination among agents is a key issue of multiagent systems (MAS) and, accordingly, can build on a vast body of prior research (for reviews, see [3, 6, 65–67]). With respect to the issues focused in this paper, the unpredictability of the environment, imperfect information about the environment as well as the fellow agents behavior, and imperfect communication are of particular interest: For multiagent systems, a general theoretical specification of this kind of problem is given by the framework of decentralized partially observable Markov decision processes (dec-POMDPs) [67, 68] and overviews of the multitude of techniques for coordinating distributed (or local) plans are given by [5, 13, 69]. According to the recent survey of Torreño et al. [69], the multitude of multiagent planning can be classified according to six key features: (1) agent distribution (involvement of multiple agents in formation and/or execution of plans), (2) computational processes (centralized or split among several processing units), (3) plan synthesis schemes (i.e., how and when the coordination takes place), (4) communication mechanisms (which is highly related to aspects 2 and 3), (5) heuristic search processes, and (6) privacy preservation (i.e., coordination of plans without that sensitive information becomes publicly available).

Within this framework, the research question studied in this paper is mainly related to aspect 3, plan synthesis ("plan merging"): The *formation* of plans is, at least, partially distributed across agents, and once each agent has formed a plan regarding its sub-problem, the emerging modes of whether, and, if so, how the local plans are synthesized into an overall plan [2, 13] are studied for different levels of the planning problem's complexity and search strategies (aspect 5 as mentioned above).

In MAS, two principle ways for plan synthesis are distinguished: *unthreaded* planning and coordination in terms of sequential activities [69] vs. *interleaved* coordination which implies an immediate integration of planning and coordination [70]. This paper focuses on unthreaded planning. A general model of coordination of plans according to unthreaded planning was introduced by Martial [2] including negotiations among agents for resolution of conflicts. In this paper, synthesizing schemes like concatenation of distributed plans according to certain rules [71] or iterative response planning (i.e., sequential information and adjustment) [72] are employed (for details, see Section 3.4).

For coordination theory in MAS, recently Lesser and Corkill [14] raised four challenges which they argue based on empirical observations, so far, are reflected in research to a lesser extent. Two of these challenges reflect key aspects of the research endeavor of this paper (see Introduction). In particular, according to Lesser and Corkill [14] (in a similar vein [69])

(1) environmental conditions like, for example, predictability of task characteristics shape which coordination strategies are appropriate
(2) the dynamic adaptation of coordination seems crucial when the environment or the problem to solve is evolving dynamically

Both aspects correspond to key issues of this research effort (see Introduction): the first since this paper adopts a contingency perspective on coordination taking different levels of complexity, different types of search behavior, and different levels of cost of effort into account; the second since the research question posed in this paper boils down to which type of coordination emerges for evolving, in terms of growing decision-problems.

## 3. A Theoretical Model of Distributed Decision-Making in Growing Organizations

This part introduces a model of distributed decision-making in organizations facing a decision-problem which grows over time reflected in organizational growth accordingly. The organizations learn about appropriate modes of coordination in terms of plan synthesis (Section 2.3) in the course of growth. The description of the model is structured into the following steps: first, the model of growing decision-problems is described (Section 3.1); next, Section 3.2 introduces the two types of agents "residing" in the organizations before Section 3.3, in particular, goes into details of the distributed (or local) decision-making agents including their respective sub-problems, their search strategies, and their formation of preferences. The different schemes of plan synthesis for coordination among distributed decision-makers captured in the model are described in Section 3.4 before the organizations' learning about the appropriate mode of coordination is modeled (Section 3.5).

*3.1. Decision-Problem Based on Growing NK Fitness Landscapes.* The decision-problems to be solved by the artificial organizations are modeled according to the framework of NK-fitness landscapes. A major feature of NK fitness landscapes is that they allow to easily control for the complexity of an $N$-dimensional decision-problem captured by parameter $K$; from a "technical" perspective, NK landscapes are stochastically generated pseudo-Boolean functions with $N$ bits, i.e., $F : \{0,1\}^N \longrightarrow \mathbb{R}^+$ [73, 74], for a brief description of NK landscapes as used in managerial science (see [75]). In line with the NK-framework, at time step $t$ the organizations face an $N$-dimensional binary decision problem, i.e., $\vec{d_t} = (d_{1t}, \ldots, d_{Nt})$ with $d_{it} \in \{0,1\}$, $i = 1, \ldots, N$, out of $2^N$ different binary vectors possible.

Each of the two states $d_{it} \in \{0,1\}$ provides a contribution $C_{it}$ to the overall performance $V(\overrightarrow{d_t})$, where the $C_{it}$ are randomly drawn from a uniform distribution with $0 \le C_{it} \le 1$. The parameter $K$ (with $0 \le K \le N-1$) reflects the number of those choices $d_{jt}$, $j \ne i$ which also affect the performance contribution $C_{it}$ of choice $d_{it}$ and, thus, captures the complexity of the decision problem to be solved in terms of the interactions among the single decisions. Hence, contribution $C_{it}$ may not only depend on the single choice $d_{it}$ but also on $K$ other choices:

$$C_{it} = f_i\left(d_{it}; d_{i_1 t}, \ldots, d_{i_K t}\right), \quad (1)$$

with $\{i_1, \ldots, i_K\} \subset \{1, \ldots, i-1, i+1, \ldots, N\}$. In case of no interactions among choices, $K$ equals 0, and $K$ is $N-1$ for the maximum level of complexity where each single choice $i$ affects the performance contribution of each other binary choice $j \ne i$. The overall performance $V_t$ achieved in period $t$ results as normalized sum of contributions $C_{it}$ from

$$V_t = V\left(\overrightarrow{d_t}\right) = \frac{1}{N}\sum_{i=1}^{N} C_{it}. \quad (2)$$

While this, so far, captures the key elements of the "standard" NK framework, a distinguishing feature of the model presented in this study, is that—due to growth—the number of single choices to be made by an organization may increase over time, i.e., $N(t)$, and, with this, also the level of complexity may rise, i.e., $K(t)$. Figure 1(b) gives an example reflecting, in the first growth stage, a rather small decision-problem with $N_1 = 6$ and $K_1 = 3$, in the second stage, a medium-sized decision-problem with $N_2 = 9$ and $K_2 = 4$ growing up to a size $N_3 = 12$ and $K_3 = 5$ in the third growth stage. More formally, we have

$$N(t) = \begin{cases} N_1, & \text{for } 0 \le t \le t_1, \\ \vdots & \\ N_s, & \text{for } t_{s-1} < t \le t_s, \\ \vdots & \\ N_S, & \text{for } t_{S-1} < t \le T, \end{cases} \quad (3)$$

where $s = (1, \ldots, S)$ captures the growth stage and $T$ denotes the entire observation period. Correspondingly, $K_s$ gives the level of complexity in the respective periods of time. The overall performance (see equation (2)), herewith, modifies to

$$V_t = V\left(\overrightarrow{d_t}\right) = \frac{1}{N_s}\sum_{i=1}^{N_s} C_{it}. \quad (4)$$

With this, the overall performance is "dynamically" normalized to the problem size Ns; moreover, in the analysis of the simulation results, the final performance VT 750 is given in relation to the global maxima of the respective performance landscapes: otherwise the results could not be compared over time and across different performance landscapes.

*3.2. Agents and Their Capabilities.* Two types of agents reside in the organizations:

(1) The distributed (or local) "decision-making agents" (type 1) which are in primary responsibility of decision-making where each of these agents is in charge for a distinct sub-problem of the overall (growing) decision-problem. These agents could, for example, reflect managers each being the head of a department in an organization.

(2) Each organization has one central agent (type 2)—capturing, for example, the headquarter of a firm—being responsible for

   (a) learning-based selection of a mode of coordination in every $T^L$-th period
   (b) eventually—i.e., subject to the coordination mode selected—actively intervening in plan synthesis
   (c) registering the performance achieved by each distributed decision-making agent (see above) and eventually rewarding it accordingly

As familiar in agent-based computational economics (e.g., [76, 77]), the agents captured in the model show some form of bounded rationality [19]. In particular, both types of agents, i.e., distributed decision-making agents and headquarter, are assumed to decide on basis of bounded information in terms of not knowing the entire search space and having limited computational power [78–80]. Hence, although distributed decision-makers are self-interested pursuing their particular goals and, in a similar vein, the central agent seeks to maximize overall performance (equation (4)), the agents are not optimizers but conduct stepwise search processes in terms of neighborhood search. Moreover, the agents are not able to perfectly evaluate newly found options to their respective decision-problems.

The properties of the distributed decision-making agents (type 1) are described more precisely in Section 3.3 while the central agent (type 2) is depicted more in detail in Sections 3.4 and 3.5.

### *3.3. Distributed Decision-Making Agents' Search and Formation of Preferences*

*3.3.1. Decomposition and Delegation.* The $N_s$-dimensional decision problem is partitioned into $M_s$ disjoint partial problems of, for the sake of simplicity, equal size $N_s^r$. Each of these sub-problems is delegated to one distributed decision-maker $r$ (i.e., one agent of type 1)—with particular competencies of decision-maker $r$ related to the respective sub-problem being subject to the mode of coordination implemented at that time. However, the distributed decision-makers are, at least, *preparing* choices regarding their partition of the overall $N_s$-dimensional decision-problem in each period $t$ (i.e., depicting "unthreaded planning" from MAS; see Section 2.3).

The growth processes in the problem space are reflected in the number of distributed decision-makers (type 1), accordingly. More formally, we have the number of distributed decision-makers being a function of time, i.e., $M_s(t)$ and, at each given growth stage $s$, the scope of competencies of the

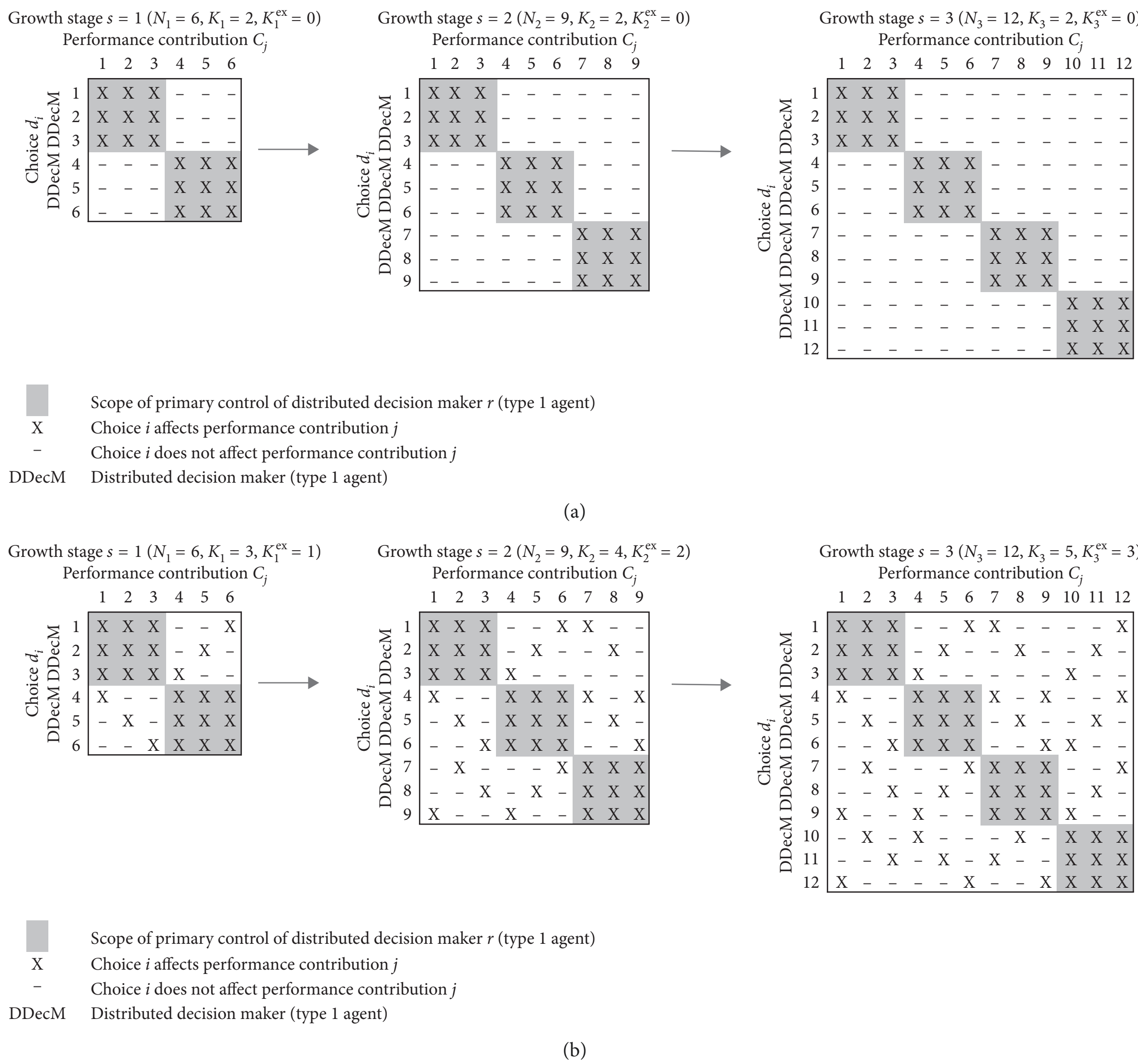


Figure 1: Interaction structures and growth stages in the simulation experiments: (a) decomposable and (b) nondecomposable.

distributed decision-makers are of equal size in terms of number of single choices $N_s^r$ assigned to them (see, for example, the shaded areas in Figure 1).

*3.3.2. Search Strategies.* In the model, as mentioned above, distributed decision-makers are not able to survey the entire search space of their decision-problem and, hence, they are not able to "locate" the optimal solution of their partial decision problem "at once"; rather, they search stepwise for superior solutions. In each time step $t$ distributed decision-maker $r$ considers two alternative solutions $\overrightarrow{d_t^{r,a1}}$ and $\overrightarrow{d_t^{r,a2}}$ for its partial decision-problem compared to the status quo $\overrightarrow{d_{t-1}^{r*}}$. With this, in each of the search strategies modeled and in every time step, decision-maker $r$ has three options to choose from—including staying with the status quo.

However, the model captures that—as a part of the boundary system in the MCS—some boundaries related to the *distances* of the two alternatives $\overrightarrow{d_t^{r,a1}}$ and $\overrightarrow{d_t^{r,a2}}$ compared to the status quo are set and enforced by the central unit (type 2 agent). In particular, the boundary system enforces search strategies being of an exploitative, explorative, or ambidextrous nature [20, 58, 81] and, accordingly, the model comprises the following three search strategies:

(i) "*exploitation only*": both alternatives $\overrightarrow{d_t^{r,a1}}$ and $\overrightarrow{d_t^{r,a2}}$ discovered by distributed decision-maker $r$ differ from the status quo $\overrightarrow{d_{t-1}^{r*}}$ by one digit, respectively. Hence, the Hamming distance $h(\overrightarrow{d^{r,a1}}) = \sum_{i=1}^{N^r} |\overrightarrow{d_{t-1}^{r*}} - \overrightarrow{d_t^{r,a1}}|$ of the first alternative to the status quo equals 1 as well as is the case for the second alternative (i.e., $h(\overrightarrow{d^{r,a2}}) = 1$).

(ii) "*exploitation and exploration*": the local decision-makers are allowed to alternatively consider an option with one bit flipped and another where two digits are altered, i.e., the Hamming distances are $h(\overrightarrow{d^{r,a1}}) = 1$ and $h(\overrightarrow{d^{r,a2}}) = 2$.

(iii) "*exploration only*": in each of both alternatives $\overrightarrow{d_t^{r,a1}}$ and $\overrightarrow{d_t^{r,a2}}$, two bits are flipped compared to the status quo $\overrightarrow{d_{t-1}^{r*}}$, i.e., we have $h(\overrightarrow{d^{r,a1}}) = 2$ and $h(\overrightarrow{d^{r,a2}}) = 2$.

*3.3.3. Distributed Decision-Makers' Objectives and Cost of Effort.* Each local decision-maker $r$, $r = 1, \ldots, M_s$ pursues its "own" objective (i.e., related to its respective sub-problem). In particular, each distributed decision-maker seeks to identify that option $\overrightarrow{d_t^r}$ out of $\overrightarrow{d_{t-1}^{r*}}$, $\overrightarrow{d_t^{r,a1}}$, and $\overrightarrow{d_t^{r,a2}}$ which promises the highest net performance $A^r$, i.e., the highest difference between partial performance $P_t^r$—resulting from the contributions $C_{it}$ of those particular single choices assigned to decision-maker $r$—and the related cost of efforts for implementing $\overrightarrow{d_t^r}$:

$$A_t^r\left(\overrightarrow{d_t^r}\right) = P_t^r\left(\overrightarrow{d_t^r}\right) - Z_t^r\left(\overrightarrow{d_t^r}\right), \tag{5}$$

where the partial performance $P_t^r(\overrightarrow{d_t^r})$ of decision-maker $r$'s contribution to overall performance $V_t$ (see equation (4)) is given by

$$P_t^r\left(\overrightarrow{d_t^r}\right) = \frac{1}{N}\sum_{i=1+w}^{N_s^r} C_{it}, \tag{6}$$

with $w = \sum_{p=1}^{r-1} N_s^p$ for $r > 1$ and $w = 0$ for $r = 1$.

In case of interactions across the sub-problems, captured by $K_s^{ex} > 0$, choices of decision-maker $r$ might affect the contribution of the other decision-makers' $q \neq p$ choices on $q$'s parochial performance and vice versa.

In the model, the number of single choices altered in terms of the Hamming distance $h(\overrightarrow{d_{t-1}^{r*}}, \overrightarrow{d_t^r})$ to the status quo (i.e., the number of bits flipped) is regarded as the *effort* to be taken by decision-maker $r$ in order to implement an option. With this, the *possible range of effort* is shaped by the search strategy as introduced in Section 3.3.2: the lower bound for the effort in all three search strategies equals 0 for the case that the status quo $\overrightarrow{d_{t-1}^{r*}}$ is chosen to be kept; the upper bound is 1 in the "exploitation only" and equals 2 in the "exploration only" as well as in the ambidextrous strategy.

For modeling the cost of effort, as customary in economics (e.g., [82, 83]), it is assumed that higher levels of effort are increasingly costly. Hence, for distributed decision-maker $r$'s cost of effort $Z^r$ we have $Z^r(h)' > 0$ and $Z^r(h)'' > 0$. In particular, the cost of effort $Z^r(\overrightarrow{d_t^r})$ of decision-maker $r$ is modeled to be quadratically increasing with the Hamming distance $h$ to the status quo, i.e.,

$$Z_t^r\left(\overrightarrow{d_t^r}\right) = z^r \cdot \left(h\left(\overrightarrow{d_{t-1}^{r*}}, \overrightarrow{d_t^r}\right)\right)^2, \tag{7}$$

where $z^r$ is a cost coefficient. For the sake of simplicity, in the simulation experiments the cost coefficient is the same for every decision-makers $r$ and does not change over time. For this reason, subsequently, the index $r$ indicating on a decision-maker $r$ is skipped when addressing the cost coefficient (considering decision-makers which are heterogeneous in respect to their cost functions (as it could be captured by different cost coefficients $z^r$) would be a natural extension of the research effort presented here).

*3.3.4. Evaluation of Options.* When evaluating their options, the decentralized decision-makers suffer from two types of limited information: First, without eventual further communication prescribed by the coordination mechanism (see Section 3.4), decision-maker $r$ is not able to anticipate the other decision-makers' $q \neq r$ choices and, thus, assumes that they will stay with the status quo, i.e., that they will opt for $\overrightarrow{d_{t-1}^{q*}}$ (for this see also [84]). This is particularly relevant in case of inter-sub-problem interdependencies when other local decision-makers' actions may affect performance of decision-maker $r$.

Second, decentralized decision-makers are not able to perfectly ex-ante evaluate their newly discovered options' $\overrightarrow{d_t^{r,a1}}$ and $\overrightarrow{d_t^{r,a2}}$ effects on their partial performance $P_t^r(\overrightarrow{d_t^r})$ (see equation (6)). Rather the ex-ante evaluation is afflicted with some noise. In particular, for the sake of simplicity, a decision-maker $r$'s *perceived* partial performance ($\widetilde{P}^r$) is distorted by a relative error imputed to the true performance. With this, our local decision-makers search on noisy partial performance landscapes (for further types of errors, see [85]). The error terms follow a Gaussian distribution $N(0; \sigma)$ with expected value 0 and standard deviations $\sigma^r$ for every decision-maker $r$; errors are assumed to be independent from each other. Hence, the *perceived* performance $\widetilde{P}_t^r(\overrightarrow{d_t})$ of distributed decision-maker $r$ is given by

$$\widetilde{P}_t^r\left(\overrightarrow{d_t^r}\right) = P_t^r\left(\overrightarrow{d_t^r}\right) + e^r\left(\overrightarrow{d_t^r}\right), \tag{8}$$

and the objective function in equation (5) modifies to

$$\widetilde{A_t^r}\left(\overrightarrow{d_t^r}\right) = \widetilde{P}_t^r\left(\overrightarrow{d_t^r}\right) - Z_t^r\left(\overrightarrow{d_t^r}\right). \tag{9}$$

With equation (8), each distributed decision-maker $r$ has a distinct partial and distorted "view" of the true fitness landscape: each decision-maker $r$ is exclusively in charge of an "own" part of the entire $N$-dimensional decision-problem as it is shaped in the current growth stage and imperfectly estimates the performance contributions of newly discovered options. Thus, at a given growth stage, the model captures $M_s + 1$ heterogeneous (distorted) views of the true landscape, i.e., one view per local decision-maker $r = 1, \ldots, M_s$ plus a headquarters' perspective (for the latter, see Section 3.4.3).

However, for the status quo option, we assume that each decision-maker $r$ remembers the performance obtained from the last period and, with this, knows the actual performance $P_{t-1}^r$ of the status quo should it be implemented in period $t$ again.

Based on the evaluation of options, each decision-maker $r$ compiles a list $L_t^r = \left\{\overrightarrow{d_t^{r,p1}}, \overrightarrow{d_t^{r,p2}}, \overrightarrow{d_t^{r,p3}}\right\}$ of preferences where $\overrightarrow{d_t^{r,p1}}$ indicates the most preferred option out of $\overrightarrow{d_{t-1}^{r*}}$, $\overrightarrow{d_t^{r,a1}}$, and $\overrightarrow{d_t^{r,a2}}$. Correspondingly, $\overrightarrow{d_t^{r,p2}}$ denotes the second-most and $\overrightarrow{d_t^{r,p3}}$ the third-most preferred option.

*3.4. Coordination Modes for Synthesizing Plans.* As a result of the search and (imperfect) evaluation of options, so far each distributed decision-maker has an ordered list $L_t^r$ of preferences which, in terms of multiagent systems, reflects the individual [2] or local plan [69]. The next step within each period $t$ is to come to a decision on the organization's overall problem $\vec{d_t}$, i.e., to merge the decentralized decision-makers' preferences on $\vec{d_t^r}$ captured in the $M_s$ lists $L_t^r$ into the overall configuration $\vec{d_t}$. This requires to employ a mode of coordination for synthesizing the local plans—and the very core of this paper is to study which mode emerges under which conditions of search strategy and cost of effort.

As it was outlined in Section 2, a multitude of modes for synthesizing local plans has been proposed in various domains. Out of the numerous feasible mechanisms, the model comprises three modes which could represent particular pronounced (for not to say "extreme") forms of coordination regarding

(i) (de-)centralization in terms of (locus of) authority
(ii) direction of alignment, i.e., lateral vs. vertical
(iii) parallelization vs. sequencing of final decision regarding the local plans

In particular, the three types of coordination captured in the model (see Table 1) differ in the communication channels (the model assumes that communication in the course of coordination works perfectly; for an investigation of the effects of unintended communications errors, see [86]), the information employed in decision-making, the locus of final decision-making, and, in the end, the tightness of coordination provided (for an overview and further modes, see, e.g., [2, 57, 69, 87, 88]).

*3.4.1. Decentralized Mode.* The highest level of autonomy is granted to the $M_s$ distributed decision-makers if each of them is allowed to choose its most preferred option. Then, the overall organizational configuration $\vec{d_t}$ results from

$$\vec{d_t} = \left(\overrightarrow{d_t^{1,p1}}, \ldots, \overrightarrow{d_t^{r,p1}}, \ldots, \overrightarrow{d_t^{M_s,p1}}\right). \quad (10)$$

Hence, for decision-maker $r$'s partial decision problem $N_s^r$ the option according to

$$\overrightarrow{d_t^{r*}} = \overrightarrow{d_t^{r,p1}} \, \forall r \in \{1, \ldots M_s\}, \quad (11)$$

is implemented. Obviously, this type of coordination does not require any form of communication between the decentralized decision-makers or with the headquarter (central agent). The headquarter does not intervene in decision-making directly and its role is limited to registering the achieved performances $V_t(\vec{d_t})$ and $P_t^r(\vec{d_t^r})$ in the end of each period $t$ and, eventually, to reward the decentralized decision-makers accordingly.

*3.4.2. Sequential Mode.* This mode captures the idea of "sequential planning": The distributed decision-makers make their final choices sequentially with, for the sake of simplicity, the sequence being given by the index $r$ of the decision-makers. In particular, in time step $t$ decision-maker $r-1$ informs decision-maker $r$ with $1 < r \leq M_s$ about the choices made so far, i.e., made by the "preceding" decision-makers $< r$. Decision-maker $r > 1$ re-evaluates its "own" options $\overrightarrow{d_{t-1}^{r*}}$, $\overrightarrow{d_t^{r,a1}}$, and $\overrightarrow{d_t^{r,a2}}$ taking these "prior" choices into account—potentially resulting in an adjusted list of preferences $L_t^r = \left\{\overrightarrow{d_t^{r,p1}}; \overrightarrow{d_t^{r,p2}}; \overrightarrow{d_t^{r,p3}}\right\}$ and chooses $\overrightarrow{d_t^{r,p1}}$ which here depends on $\overrightarrow{d_t^{r-1,p1}}$, i.e., is a function of the choices of the preceding decision-makers. Hence, only decision-maker 1 does not have to consider previous choices and the choice of decision-maker $r = (1, \ldots, M_s)$ is made according to

$$\overrightarrow{d_t^{r*}} = \begin{cases} \overrightarrow{d_t^{r,p1}}, & \text{if } r = 1, \\ \overrightarrow{d_t^{r,p1}}\left(\overrightarrow{d_t^{r-1,p1}}\right), & \text{if } 1 < r \leq \mathrm{M}, \end{cases} \quad (12)$$

and with equation (12) the overall configuration is given by $\vec{d_t} = \overrightarrow{d_t^{M*}}$. The headquarter (type 2 agent) does not intervene in decision-making and—similar to the decentralized mode—is confined to observing the performances achieved and, eventually, rewarding the decentralized decision-makers accordingly.

*3.4.3. Proposal Mode.* Each local decision-maker transfers its list $L_t^r$ of preferences to the headquarter which compiles the first preferences to a composite vector $\overrightarrow{d^C} = (\overrightarrow{d_t^{1,p1}}, \ldots, \overrightarrow{d_t^{r,p1}}, \ldots, \overrightarrow{d_t^{M_s,p1}})$, and then evaluates the overall performance $V(\overrightarrow{d^C})$ (see equation (4)) that this solution promises.

However, as with decentralized decision-makers, in the model it is assumed that the headquarter is not capable to perfectly ex-ante evaluate new options, i.e., other solutions than the status quo (see Section 3.2). Rather, similar to the local decision-makers, the headquarter suffers from some relative noise following a Gaussian distribution with expected value 0 and standard deviations $\sigma^{\text{cent}}$ resulting in a *perceived* overall performance $\widetilde{V}(\overrightarrow{d^C})$. The headquarter

Table 1: Overview of the modes of coordination in the model.

| | Type of coordination mode | | |
|---|---|---|---|
| | Decentralized | Sequential | Proposal |
| Lateral communication | No | Yes | No |
| Vertical communication | No | No | Yes |
| Headquarter intervening | No | No | Yes |
| Re-evaluation of local plans | No | Yes | Yes |
| Information employed for final decisions | Expected parochial performance | Expected parochial performance taking predecessor's choices into account | Expected organization-wide aggregated performance |
| Locus of final decisions on local plans | Decentralized | Decentralized | Centralized |
| Temporal order of final decisions on local plans | In parallel | Asynchronously (in sequence) | Synchronously ("at once") |
| Final configurations for distributed decision-makers' sub-problems | First preferences $\overrightarrow{d_t^{r,p1}} \forall r \in \{1,\ldots,M_s\}$ | First preferences subject to preceding decision-makers' choices, i.e., $\overrightarrow{d_t^{r,p1}}\,(\overrightarrow{d_t^{r-1,p1}})$ for $r>1$; $\overrightarrow{d_t^{r,p1}}$ for $r=1$ | First (or second) preferences $\overrightarrow{d_t^{r,p1}}$ [or $\overrightarrow{d_t^{r,p2}}$] if $\widetilde{V}(\overrightarrow{d^C}) \geq V(\overrightarrow{d_{t-1}})$ |

decides in favor of the composite vector, i.e., $\overrightarrow{d_t} = \overrightarrow{d^C}$, if $\overrightarrow{d^C}$ promises the same or a higher performance as the status quo $\overrightarrow{d_{t-1}}$, i.e., if

$$\widetilde{V}\left(\overrightarrow{d^C}\right) \geq V\left(\overrightarrow{d_{t-1}^*}\right). \tag{13}$$

If the composite vector $\overrightarrow{d^C}$ assembled from the local decision-makers' first preferences does not meet the condition in equation (13), the headquarter evaluates a vector composed from the decentralized decision-makers' second preferences according to equation (13). If this also does not, at least, promise the performance of the status quo, then the organization stays with the status quo, i.e., then $\overrightarrow{d_t} = \overrightarrow{d_{t-1}}$.

*3.5. Learning-Based Adaptation of Coordination.* The very core of this research endeavor is to study which modes of coordination emerge within growth processes of the artificial organizations. In order to capture some kind of self-adaptation of coordination, the model employs a simple mode of reinforcement learning (for overviews, see [89, 90]) based on statistical learning, i.e., a generalized form of the Bush–Mosteller model [91, 92].

This mode of learning is chosen for the following reasons: In the vast multitude of forms of learning studied in various domains (e.g., psychology, economics, and computer science), reinforcement learning is regarded to be among the most basic forms of learning [92]. It represents a fundamental possible form of humans' behavior but also provides some basis for learning of artificial agents [90]—both regarded as DDMS in this paper. Moreover, in this model, as a form of experiential learning the mid-termed reinforcement learning on the coordination mode corresponds to the experiential type of short-termed adaptation [60] when the decision-making systems, in every single time-step, search for a superior solution compared to the status quo. However, it would be a natural extension of the model to let the DDMS employ other and, in particular, more advanced forms of learning about which mode of coordination is appropriate within the stage of growth and for the search strategy employed and the cost of effort.

In the model, reinforcement learning on coordination for synthesizing local plans is represented in the following way: In the end of each time step *t*, the central agent (type 2 agent; see Section 3.2) receives information about the overall performance $V_t$ according to equation (4). This allows the central agent, in every $T^L$th period, to compute the performance enhancements achieved within the last $T^L$ periods. Moreover, in every $T^L$th period, the organizations can alter the type of coordination mode as introduced in Section 3.4. Hence, our DDMS face the mid-termed decision-problem which type of coordination mode $a^c(t) \in A^c$ (with $|A^c| = 3$; see Section 3.4) to implement in the next $T^L$ periods. Let $p(a^c, t)$ denote the probability of an alternative $a^c(t)$ to be chosen at time $t$ (with $0 \leq p(a^c, t) \leq 1$ and $\sum_{a^{ac} \in A^c}(p(a^c, t)) = 1$).

The key idea of reinforcement learning is that the probabilities of options are updated according to the positive or negative stimuli resulting from these options. In our context, whether the performance enhancement $\Delta V_t$ obtained under the regime of a certain mode of coordination $a^c(t)$ in the previous $T^L$ periods is regarded positive or negative, depends on whether, or not, it at least equals an aspiration level $v$. $\Delta V_t$ of type $a^c(t)$ of coordination is defined as the relative performance enhancement achieved within the last $T^L$ periods of the adaptive walk, i.e.,

$$\Delta V_t = \frac{V_t - V_{t-T^L}}{V_{t-T^L}}. \tag{14}$$

Hence, the stimulus $\tau(t)$ is

$$\tau(t) = \begin{cases} 1, & \text{if } \Delta V_t \geq v, \\ -1, & \text{if } \Delta V_t < v. \end{cases} \tag{15}$$

The probabilities of options $a^c \in A^c$ are updated according to the following rule, with $\lambda$ (where $0 \leq \lambda \leq 1$) giving the reinforcement strength [92]:

$$p\left(a^{c},t+1\right)=p\left(a^{c},t\right)+\lambda\cdot\begin{cases}\left(1-p\left(a^{c},t\right)\right), & \text{if } a^{c}=a^{c}(t)\wedge\tau(t)=1,\\ -p\left(a^{c},t\right), & \text{if } a^{c}=a^{c}(t)\wedge\tau(t)=-1,\\ -p\left(a^{c},t\right), & \text{if } a^{c}\neq a^{c}(t)\wedge\tau(t)=1,\\ \dfrac{p\left(a^{c},t\right)\cdot p\left(a^{c}(t),t\right)}{1-p\left(a^{c}(t),t\right)}, & \text{if } a^{c}\neq a^{c}(t)\wedge\tau(t)=-1.\end{cases} \quad (16)$$

After the probabilities are updated as given in equation (16) the "next" mode of coordination to be employed from $t+1$ to $t+T^L$ is determined at random—according to the updated probabilities.

## 4. Simulation Experiments: Processual Structure, Parameters, and Analysis

This section is intended to introduce the principle structure of the simulation experiments based on the theoretical model as presented in the previous Section 3. For this, first, Section 4.1 gives an overview of the principle processual structure of the simulations before the parameters settings are motivated (Section 4.2) and the metrics employed for analysis of experiments are introduced (Section 4.3).

*4.1. Process Overview of the Simulation Model.* Figure 2 depicts the principle processual structure of the simulation model based on the theoretical model as introduced in the previous Section 3. In particular, the simulation in its core is characterized by three loops capturing three temporal horizons.

In the short term, in each time step $t$, the artificial organizations search for superior solutions of their $N_s$-dimensional decision-problem where the overall problem is segmented into $M_s$ sub-problems with each delegated to $M_s$ local decision-making agents accordingly (type 1 agents in Section 3.2). In the mid-term, i.e., in each $T^L$th time step, the central unit (type 2 agent) evaluates the current mode of coordination, learns from this evaluation via reinforcement, and, eventually, chooses another mode of coordination for the next $T^L$ periods. In the long term, in every $T^G$th time step, the decision-problem grows by a fixed number $N^r_s$ of additional single choices to be made and the number $M_s$ of distributed decision-makers increases by 1 (for details, see Section 3.1).

*4.2. Parameter Settings*

*4.2.1. Parameters Fixed for all Experiments.* In the simulation experiments—parameter settings are listed in Table 2—organizations are observed over $T=750$ periods where in every $T^L=10$th period, eventually the mode of coordination is switched (the observation period $T$ and the learning interval $T^L=10$ were fixed based on pretests which indicate that the results do not principally change when the organizations are observed for a longer time; the similar holds for an extension of the learning interval (e.g., to $T^L=20$); however, shortening the learning period notably below 10 periods does not leave the different coordination modes "enough time" to unfold their particular potential with respect to the aspiration level).

In every $T^G=250$th period, the organizations undergo a growth in *problem space* as well as in the *number of distributed decision-makers*:

(i) In their first growth stage, the organizations face an $N_{s=1}=6$-dimensional overall decision-problem decomposed into two sub-problems of equal size (i.e., $N^r_{s=1}=3\forall r\in\{1,2\}$) assigned to two decision-makers ($M_{s=1}=2$) accordingly

(ii) In the second growth stage (i.e., from $t=251$ to 500), three additional binary choices are to be made by the organizations for which one additional local decision-maker is responsible—hence, the organizations comprise 3 decentralized decision-makers, i.e., $M_{s=2}=3$

(iii) In the third stage, the decision-problem grows by three further binary choices and; hence, finally the organizations deal with an $N_{s=3}=12$-dimensional problem and a fourth decentralized decision-maker comes into play ($M_{s=3}=4$)

The simulations are run for a moderate level of noise captured by parameters $\sigma^r$ and $\sigma^{\text{cent}}$ relevant for the ex-ante evaluations of options by the local decision-makers and, in case of the proposal mode, the central agent, respectively (see Sections 3.2 and 3.4.3). It is assumed that the information of the local decision-makers—only related to their respective $N^r$-dimensional sub-problems—are more precise than the information of the central unit which is related to the entire $N_s$-dimensional decision problem (i.e., $\sigma^r<\sigma^{\text{cent}}$). This is intended to capture differentiation and specialization [8, 93] due to division of labor. Some empirical evidence suggests that noise of about 10% of the true value in the domain of management control is at a reasonable range [94].

The modes of coordination which the organizations choose of are motivated and introduced in Section 3.4. The organizations employ the same initial mode of coordination, namely, the "decentralized" mode. This "setup" procedure is chosen to make sure that the adaptive processes start from a "defined" initial configuration and without having the learning processes overlaid by the strong performance enhancements that are typically made in the very first periods of the adaptive walks (notwithstanding, further simulations

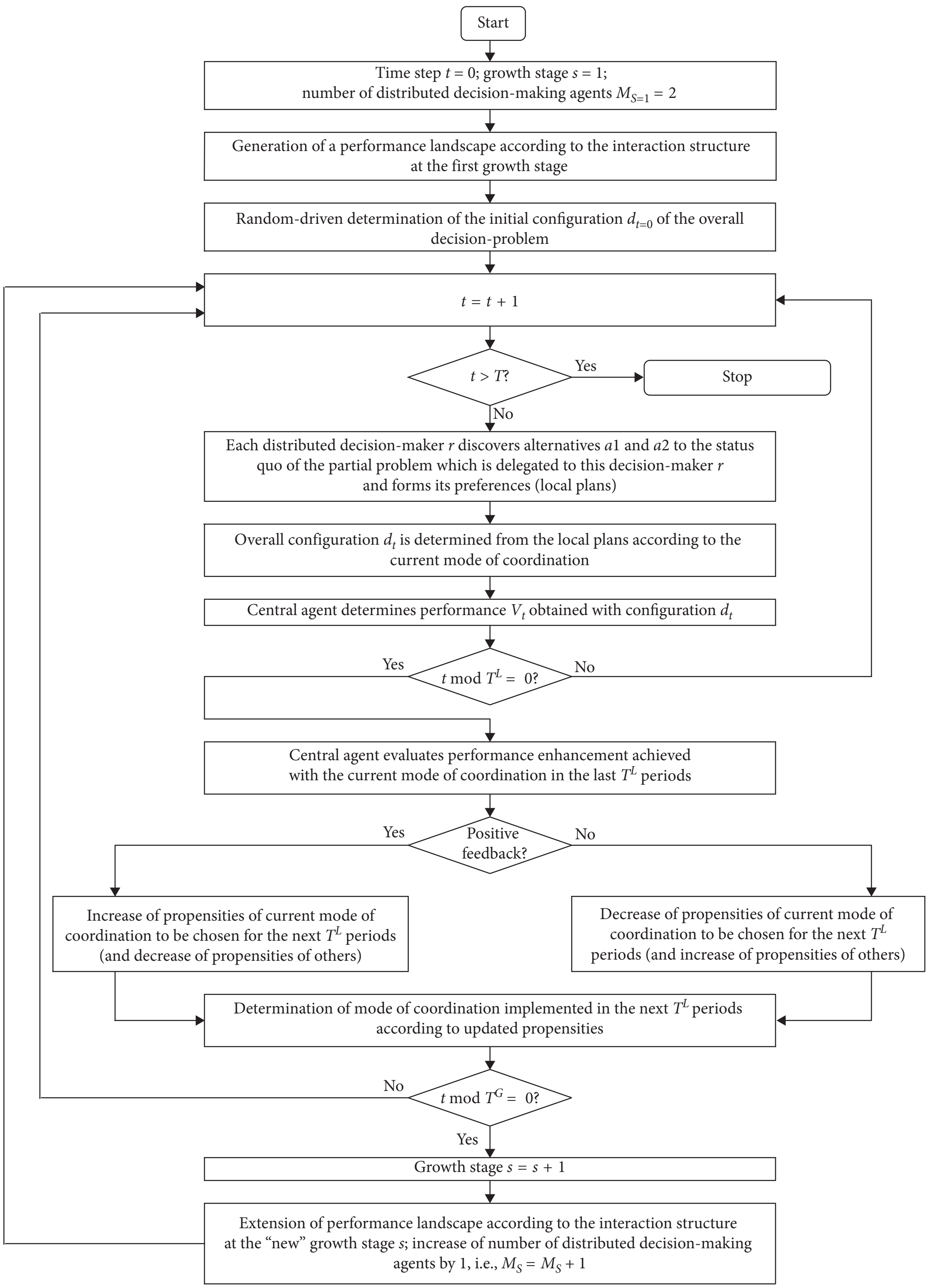


Figure 2: Principle processual structure of the simulation model.

have shown that the overlaying effects appear to be rather negligible). However, an intuitive "story" behind this setup procedure is that the organizations start, in fact, without any particular mode of coordination, i.e., the decentralized mode (for example, because they were newly founded), and keep this until time $t = 10$; in this period, they "discover" the two

Table 2: Parameter settings.

| Parameter | Values/types |
|---|---|
| *Parameters fixed for all experiments (see Section 4.2.1)* | |
| Observation period | $T = 750$ |
| Interval of learning | $T^L = 10$ |
| Interval of growth stages | $T^G = 250$ |
| Growth stages | $s = \{1, 2, 3\}$ |
| Number of choices | In growth stage $s = 1$: $N_{s=1} = 6$<br>In growth stage $s = 2$: $N_{s=2} = 9$<br>In growth stage $s = 3$: $N_{s=3} = 12$ |
| Number $M_s$ of distributed decision-makers | In growth stage $s = 1$: $M_1 = 2$ distributed decision-makers with sub-problems $\overrightarrow{d^1} = (d_1, d_2, d_3)$ and $\overrightarrow{d^2} = (d_4, d_5, d_6)$<br>In growth stage $s = 2$: $M_2 = 3$, i.e., one additional distributed decision-maker 3 with sub-problem $\overrightarrow{d^3} = (d_7, d_8, d_9)$<br>In growth stage $s = 3$: $M_3 = 4$, i.e., one further additional decision-maker 4 with sub-problem $\overrightarrow{d^4} = (d_{10}, d_{11}, d_{12})$ |
| Precision of ex-ante evaluation | Distributed decision-makers: $\sigma^r = 0.05 \forall r$<br>Central agent (headquarter): $\sigma^{\text{cent}} = 0.1$ |
| Modes of coordination | $a^c \in A^c = \{$"decentralized," "sequential," "proposal"$\}$ |
| Aspiration level | $v = 0$ |
| Learning strength | $\lambda = 0.5$ |
| *Parameters subject to variation across experiments (see Section 4.2.2)* | |
| Complexity of decision problem | "Decomposable"; "nondecomposable",<br>for details of interaction structures, see Figure 1 |
| Search strategy | "Exploitation only": $h(\overrightarrow{d^{r,a1}}) = 1$ and $h(\overrightarrow{d^{r,a2}}) = 1$<br>"Exploitation and exploration": $h(\overrightarrow{d^{r,a1}}) = 1$ and $h(\overrightarrow{d^{r,a2}}) = 2$<br>"Exploration only": $h(\overrightarrow{d^{r,a1}}) = 2$ and $h(\overrightarrow{d^{r,a2}}) = 2$ |
| Cost of effort (cost coefficient) | *Baseline scenarios:* $z = 0.001$<br>*Sensitivity analysis:* $z \in \{0, 0.003, 0.005, 0.01, 0.02\}$ |

alternative modes and choose randomly one option $a^c \in A^c$ out of the three options (i.e., $|A^c| = 3$) where each mode has the same initial probability $1/|A^c|$. Then, in every $T^L = 10$th period, probabilities are adjusted according to the performance enhancements. The parameters for learning correspond to a moderate strength of learning [95]. On basis of the updated probabilities, eventually, a new coordination mode is established.

*4.2.2. Contingency Factors: Parameters Subject to Variation across Experiments.* As argued in the Introduction, this research effort takes a contingency perspective and, in particular, intends to study the interacting effects of (1) complexity of decision-problem, (2) search strategy, and (3) cost of effort. Accordingly, these three contingent factors are subject to variation across simulation experiments (see also the lower part in Table 2).

*(1) Complexity of Decision-Problem.* In line with the idea of factorial design of simulation experiments [96], the experiments distinguish between two different levels of complexity of the decision problem to be solved by the organizations which capture two rather pronounced cases.

Based on the seminal work of Simon [16] on the architecture of complexity (near) decomposability as compared to nondecomposability is the key aspect in the understanding of complexity (see also Introduction). Against this background, the simulations are run for a growing decomposable and a growing nondecomposable interaction structure, where the principle "type" of complexity is kept over the growth stages. Hence, the growth processes simulated may capture some kind of organic growth [97–99]. Figure 1 provides a graphical representation of interaction matrices of the growing decision-problems which could capture the following situations (in the simulation experiments, the number of decision-makers mirrors the growth of the decision-problem for avoiding interference with effects of varying size of decision-makers' scope of competency (for further references, see [55, 100, 101])):

(i) "*decomposable*" (Figure 1(a)): This type of organization and growth relates to the idea of an organization consisting of self-contained "sub-problems" or "units" [7, 8] which have intense intra-unit interactions, but no cross-unit interactions. In the beginning, the organization's overall decision-problem is decomposable into 2 distinct sub-problems [17], which, for example, might be related to different products without any interrelations between them. For the 2 products, 2 business units are responsible. In the course of the growth stages, the organization "adds" further products and business units, correspondingly, without interrelations among the "old" and "new" products and, accordingly, the units.

(ii) "*nondecomposable*" (example given in Figure 1(b)): This case of interactions may capture what—according to the prominent classification of Thompson [7]—is called reciprocal interdependencies. In particular, this structure could represent an organization with functional specialization showing the typical high level of interrelations between sub-problems—and departments accordingly. In the course of growth, it may be that the vertical integration is increased (e.g., establishing an inhouse production of certain intermediate products or sales logistics).

*(2) Search Strategy*. As described in Section 3.3.2, the decentralized decision-makers employ one of three fundamental search strategies which are intended to capture exploitation, exploration, and an ambidextrous strategy [20] as enforced by the boundary system—each characterized by the number of options and the Hamming distances allowed. For example, if an exploration strategy is pursued, with $M_3 = 4$ local decision-makers and $N_3 = 12$ in the final growth stage, at maximum, 8 digits of the overall decision-problem could be flipped, i.e., two-thirds of the configuration $\vec{d_t}$ could be altered in one time step.

*(3) Cost of Effort*. While the search strategy shapes the maximum of binary choices that could be flipped, the actual alterations are affected by the local decision-makers' preferences together with the coordination mode employed. According to equations (7) and (9) in Section 3, the distributed decision-makers' preferences are affected by the cost of effort and, in particular, by cost coefficient $z$.

However, in order to be clear and concise, the simulation experiments are conducted in two steps. First, to gain a basic understanding of the emergence of coordination, in the baseline scenarios the cost of effort is at a moderate level $z = 0.001$ for all local decision-makers. Second, a sensitivity analysis is conducted in whose course the cost of effort is varied from costless effort (i.e., $z = 0$) to higher levels of cost of effort (i.e., $z > 0.001$) in order to analyze the effects of cost of effort on the emergence of coordination.

*4.3. Metrics Employed for Analysis of Simulations.* In the baseline scenarios, with 2 interaction structures and 3 search strategies under investigation at a given level of cost of effort, 6 different scenarios of parameters are simulated. For the sensitivity analysis, 5 additional levels of cost of effort are simulated which results in 30 further scenarios. For each of, in sum, the 36 scenarios, 2,500 simulations are run with 10 runs on 250 performance landscapes.

The research question of this paper boils down to the question which coordination modes emerge predominantly for which contingencies in terms of complexity, search strategy, and cost of effort. Hence, for answering this question, the key metric is the *relative frequencies of the coordination modes in the end of the observation period* $t = 750$.

Moreover, the relative frequencies of coordination modes over observation time are depicted. This allows to gain an understanding of how their relative shares evolve in the course of growth and learning.

As described in Section 3.5 the propensities to opt for the one or the other mode of coordination is shaped by the performance gains achieved under the current mode in the last $T^L$ periods. Hence, the level of performance according to equations (2) and (4), respectively, in Section 3.1, i.e., the final performance achieved at the end of the observation period for the different coordination modes is of interest. For this, the 2,500 simulations for each scenario were grouped according to that mode of coordination which was "active" (i.e., has emerged) in the last period $t = 750$ of the observation time.

These subgroups were analyzed individually and, in particular, the *mean of the final performance* $V_{t=750}$ *achieved for each coordination mode selected at* $t = 750$ is computed as well as the respective *confidence interval* at a 99.9% level of confidence.

Moreover, another metric informing about the effectiveness of the search processes is the *relative frequency of the global maximum found in* $t = 750$ of the respective performance landscape which is computed for each subgroup (based on the coordination mode in $t = 750$) separately.

In order to gain some deeper understanding of the search processes conducted by the DDMS, two further metrics were observed and analyzed for each subgroup: the *ratio of periods in which the status quo is altered* and the *ratio of periods with false-positive alterations*, i.e., alterations in favor of a false-positive option (i.e., reducing $V_t$) to the $T = 750$ periods of observation. These metrics put some focus on the efficiency of search.

## 5. Results and Discussion

The results are presented in two steps. First, for gaining a basic understanding of the effects of intraorganizational complexity and search strategy on the emergence of coordination, the baseline scenarios are presented and analyzed (Section 5.1). While in the baseline scenarios the distributed decision-makers operate at a rather moderate, though nonzero level of cost of effort, the sensitivity analysis (Section 5.2) illustrates the effect of cost of effort at the local decision-makers' side on the coordination mode emerging at the system's level.

*5.1. Complexity and Search Strategy: Baseline Scenarios*

*5.1.1. Overview.* For the baseline scenarios, Table 3 displays condensed results obtained from the simulation experiments according to the metrics introduced in Section 4.3. In addition, Table 4 reports on the significances of mean

Table 3: Condensed results of the baseline scenarios (moderate cost coefficient of effort $z = 0.001$).

| Search strategy | Frequency of coordination mode $a^c$ in $t = 750$ (a) | | | Final performance $V_{t=750}$ (conf. interval*) (b) | | | Frequency of global maximum found in $t = 750$ (c) | | | Frequency of alterations (false positives) of $\overrightarrow{d_t}$ (d) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. |
| *Decomposable structure* | | | | | | | | | | | | |
| Exploitation only | 33.9% | 35.0% | 31.1% | 0.953 ± 0.0052 | 0.950 ± 0.0055 | 0.952 ± 0.0055 | 18.8% | 17.5% | 15.8% | 4.3% (2.3%) | 4.7% (2.0%) | (2.1%) |
| Exploitation and exploration | 41.4% | 38.8% | 19.8% | 0.992 ± 0.0017 | 0.992 ± 0.0018 | 0.988 ± 0.0038 | 52.5% | 57.2% | 47.7% | 4.3% (1.9%) | 4.5% (2.0%) | (2.2%) |
| Exploration only | 35.2% | 36.3% | 28.4% | 0.913 ± 0.0070 | 0.915 ± 0.0066 | 0.915 ± 0.0075 | 5.7% | 5.3% | 5.0% | 3.0% (1.4%) | 3.1% (1.4%) | 3.0% (1.3%) |
| *Nondecomposable structure* | | | | | | | | | | | | |
| Exploitation only | 27.9% | 35.5% | 36.6% | 0.896 ± 0.0083 | 0.894 ± 0.0070 | 0.911 ± 0.0064 | 7.6% | 6.4% | 7.5% | 4.3% (2.1%) | 4.3% (2.0%) | 4.4% (2.1%) |
| Exploitation and exploration | 22.1% | 33.0% | 45.0% | 0.891 ± 0.0142 | 0.910 ± 0.0092 | 0.933 ± 0.0064 | 10.3% | 9.3% | 18.3% | 12.4% (6.2%) | 12.4% (6.1%) | 14.2% (7.0%) |
| Exploration only | 17.5% | 27.1% | 55.4% | 0.847 ± 0.0133 | 0.848 ± 0.0100 | 0.872 ± 0.0057 | 3.0% | 2.1% | 2.6% | 8.0% (4.0%) | 9.0% (4.5%) | 9.7% (4.8%) |

*Confidence intervals for $V_{t=750}$ are given at a level of 99.9%. Each row represents averaged/aggregated results of 2,500 simulation runs: 250 landscapes with 10 runs on each.

Table 4: Mean differences of final performances $V_{t=750}$ and half-lengths of individual 99.9% confidence intervals for pairwise comparisons in the baseline scenarios (moderate cost coefficient of effort $z = 0.001$).

| Search strategy | | Mode of coordination: Decentralized | *Sequential* |
|---|---|---|---|
| *Decomposable structure* | | | |
| Exploitation only | Proposal | 0.0015 ± 0.0059 | −0.002 ± 0.0061 |
| | Decentralized | | −0.0034 ± 0.0059 |
| Exploitation and exploration | Proposal | 0.004 ± 0.0033* | 0.004 ± 0.0033* |
| | Decentralized | | 0.0000 ± 0.0019 |
| Exploration only | Proposal | −0.0017 ± 0.0079 | −0.0001 ± 0.0078 |
| | Decentralized | | 0.0017 ± 0.0074 |
| *Nondecomposable structure* | | | |
| Exploitation only | Proposal | −0.015 ± 0.0082* | −0.0169 ± 0.0074* |
| | Decentralized | | −0.0019 ± 0.0085 |
| Exploitation and exploration | Proposal | −0.0418 ± 0.0122* | −0.0234 ± 0.0088* |
| | Decentralized | | 0.0184 ± 0.0133* |
| Exploration only | Proposal | −0.0256 ± 0.0113* | −0.0239 ± 0.009* |
| | Decentralized | | 0.0017 ± 0.013 |

*Significant difference. For parameter settings, see Table 2.

differences of the final performances $V_{t=750}$ achieved on average for the simulation runs grouped by the modes of coordination active in the last observation period employing Welch's method [102, 103]. The plots in Figure 3 display—for the two interaction structures and the three search strategies under investigation in the baseline scenarios—the relative frequencies of the three modes of coordination within the observation time.

According to the results, the coordination modes emerging in the course of growth and adaptation differ remarkably across the two interaction structures and for the three search strategies.

Broadly speaking, for the growing decomposable structure, the results suggest that with the exploitative and the explorative search strategy no particular coordination mode emerges predominantly; in contrast, when the organizations allow more flexibility in terms of an ambidextrous search strategy, coordination modes prevail that leave the decision-making authority at the side of the decentralized decision-makers.

For growing nondecomposable structures, hierarchical coordination (proposal mode) increasingly predominates in the course of growth. Moreover, the level of predominance varies with the search strategy employed. A purely explorative strategy is most likely to emphasize hierarchical elements for coordination of decentralized decision-making.

These results are discussed for the two types of interaction structures more into detail subsequently.

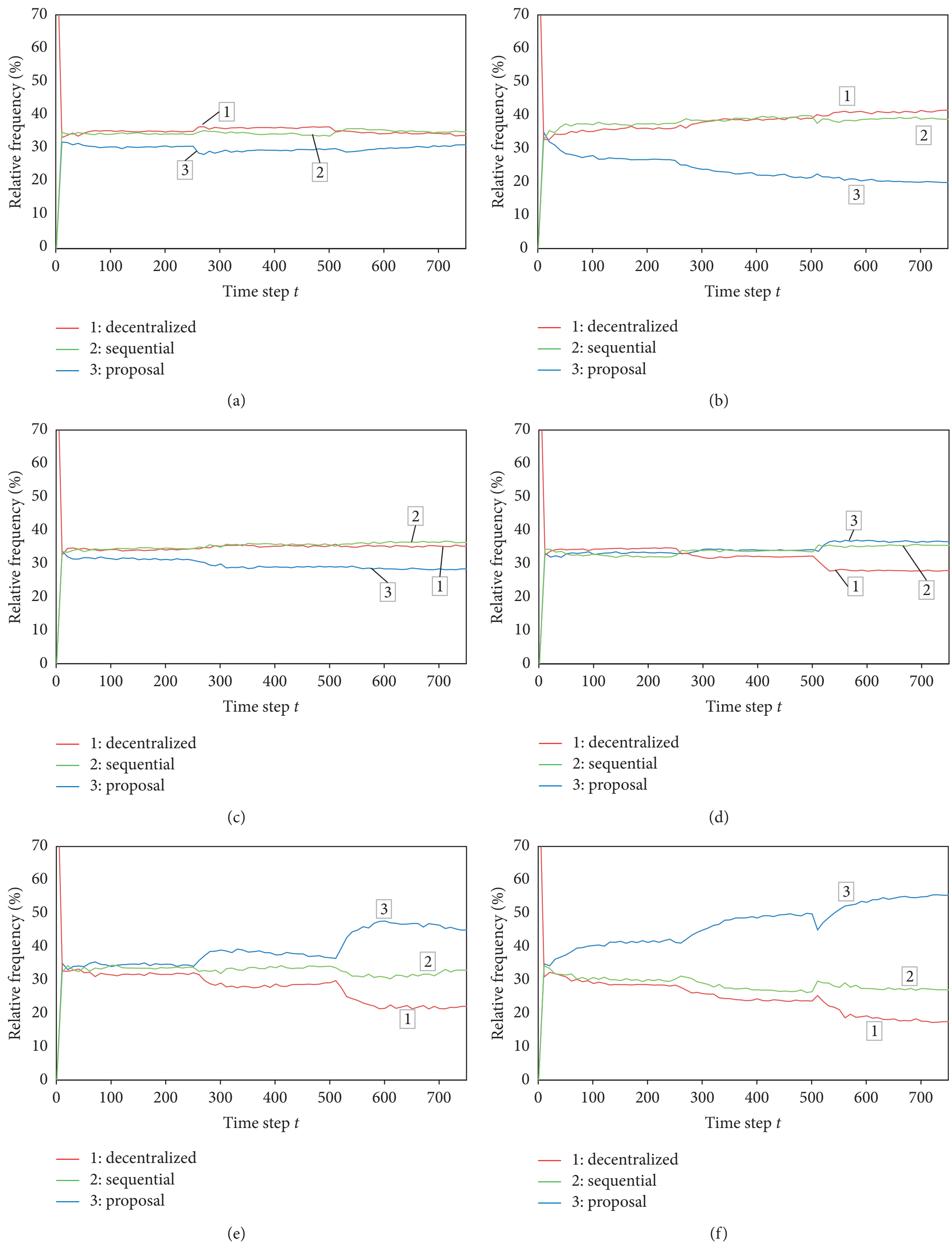


FIGURE 3: Relative frequencies of coordination modes in the course of growth in the baseline scenarios (i.e., moderate cost coefficient of effort $z = 0.001$). Each plot represents results of 2,500 simulation runs: 250 landscapes with 10 runs on each. For parameter settings, see Table 2. (a) Decomposable-exploitation only, (b) decomposable-exploitation and exploration, (c) decomposable-exploration only, (d) nondecomposable-exploitation only, (e) nondecomposable-exploitation and exploration, and (f) nondecomposable-exploration only.

*5.1.2. Decomposable Interaction Structure.* For a closer analysis, a starting point is the coordination need, and in the decomposable structure there is, in fact, no need for coordination across the sub-problems which could result from the nature of the "growing" task: The sub-problems do not show any interactions among each other (see Figure 1(a)). With this, intuition suggests that—for a given search strategy and as far as no cost of coordination are taken into account like in this study—the three mechanisms of coordination under investigation should not show remarkable differences in terms of performance $V_t$ provided and, in consequence, the learning-based frequencies of occurrence.

For the "exploitation only" and the "exploration only" search strategies this conjecture is broadly supported by the results: frequencies of occurrences throughout the observation period (Figures 3(a) and 3(c)) and final performances (columns (a) and (b) in Table 3) are at a similar level for the three modes of coordination. However, for the ambidextrous search strategy, results suggest that the proposal mode (i.e., employing hierarchy), in the course of growth, is increasingly predominated by the other modes and the final performances achieved with the decentralized and the sequential mode go beyond the level obtained when the proposal mode is "active" at $t = 750$ (employing Welch's method [102, 103], the performance differences against the proposal mode are significant at a confidence level of 99.9% as can be seen in Table 4.). Moreover, it is worth mentioning that in the ambidextrous strategy the final performances exceed the levels obtained with the "exploitation only" strategy by around 3.5 and with the "exploration only" strategy by around 8 points of percentage.

Hence, from these observations, two interrelated questions arise: (1) What may cause the performance excess of the ambidextrous strategy compared to the other strategies and (2) what drives the imbalance in the coordination modes in the ambidextrous strategy?

For suggesting answers to these questions, it appears helpful to consider the sources of coordination need which are, broadly speaking,

(1) interactions across sub-problems and distributed decision-makers accordingly which result from decomposition into sub-problems (e.g., [16, 93])

(2) distributed decision-makers pursuing not the DDMS's overall, but their parochial objectives—as extensively elaborated in contract theory with its applications in management control (e.g., [83, 104])

(3) decision-makers having different and imperfect information of the problem to be solved—may it be in the tradition of information economics as elaborated in the seminal paper of Sah and Stiglitz [105], in contract theory (e.g., [83]) or following the tradition of bounded rationality according to Simon [19]

Since in the decomposable structure, there are no interactions across sub-problems and distributed decision-makers, no inter-sub-problem coordination need (1) exists, and for this reason even the merely parochial objectives (2) of the local decision-makers should not affect the overall performance achieved which here results—across all growth stages—as a sum of the decentralized decision-makers' performances without complementarities or substitutes to be taken into account. However, apparently aspect (3) is of relevance: in the model, none of the decision-makers—neither the distributed nor the central type (see Section 3.2)—disposes of perfect information and, in the proposal mode, the rather imprecise information of the central agent enters in decision-making without that its broad perspective makes a relevant contribution since there is no need for coordination resulting from aspects (1) or (2).

Apparently, this is particularly crucial in the ambidextrous strategy: This strategy gives the distributed decision-makers the highest flexibility in terms of shaping the novelty of solutions to their partial problems, and, combined with their rather precise information, allows them to adjust rather fast to the local maxima of their particular sub-problems. These beneficial effects of flexibility of search captured in the ambidextrous strategy show up in the frequency of the global maximum found (column (c) in Table 3) which is at a considerably higher level than in the other strategies. However, involving the central agent—with its imprecise information whose broadness does not contribute in case of decomposable structures—reduces the effectiveness of search and, hence, the proposal mode is less often selected.

More broadly speaking, the results, so far, suggest that the search strategy subtly interferes with the coordination mode emerging even if coordination need is at a low level.

*5.1.3. Nondecomposable Interaction Structure.* In contrast to the decomposable structure, for noncomposable decision-problems, the emergence of coordination modes differs remarkably across search strategies. For a start, it is worth emphasizing that now—due to cross-problem interactions—superior configurations or even the global optimum in the performance landscape cannot be found by just locating superior (or optimal) solutions to the sub-problems. Moreover, stepwise search processes, particularly, when conducted in a decentralized manner as in our model, are likely to end up in local maxima causing inertia of the search (with further references [21, 37, 73]).

Each of the aforementioned three sources of coordination need is relevant now: (1) cross-departmental interactions with the complexity increasing in the course of growth, (2) distributed decision-makers focusing on parochial performance which in the nondecomposable structure is not necessarily in line with overall performance, and (3) decision-makers operating with imperfect information in various senses.

As can be seen in Table 3, in the "exploitation only" strategy, the three modes of coordination emerge with rather similar relative frequencies. In contrast, in the ambidextrous strategy, the proposal mode's share reaches a level of 45%, and in the "exploitation only" strategy, it is established in more than 55% of the runs in the end of the observation period. Figures 3(d)–3(f) illustrate these differences in the emergence over time.

As reported in Table 3, the final performances achieved with the different settings of search strategy and coordination mode differ remarkably. With "exploitation only", in all coordination modes, a medium level of around 90% of the maximal performance of 1 is achieved—though the proposal mode significantly provides the best results (according to the Welch's test (see Table 4), the performance excess is significant at a confidence level of 99.9%.). In contrast, with "pure" exploration, final performance, at maximum, is around 87%, and in the decentralized and the sequential mode around 2.5 points of percentage less. In the ambidextrous strategy, the final performance achieved in the proposal mode is significantly higher than that obtained with the decentralized (+4.18 points of percentage) and the sequential mode (+2.34 points of percentage), and with 93.3%, it is even the highest obtained for this structure across all search strategies. The frequency of the global maximum found directs in a similar direction.

Apparently, in growing task environments with high complexity throughout growth, the central agent is increasingly involved in coordination in terms of using upward communication and employing aggregate, organization-wide information for decision-making. This corresponds to results of empirical studies examining the effect of intraorganizational interdependencies as a contingent factor on management controls as reported in Section 2 indicating that higher levels of interdependencies are associated by more vertical information flows and use of aggregated information [24, 49, 50]. In a similar vein, though not explicitly controlling for internal complexity, Davila [48] argues that the use of formal controls increasing in organizational size may be driven by increasing complexity which is supported by a positive relation between size and emphasis on action controls (corresponding to the boundary system) in his empirical study.

While the aforementioned effect shows up for all search strategies, the different search strategies appear to be differently sensitive to the coordination mode emerging: As mentioned above, the purely explorative strategy performs worst in this interaction structure, with hierarchical coordination ("proposal mode") leading to the relatively best results and emerging, by far, most often. This indicates on the tension incorporated in the boundary system in general and as also captured in the model: As argued by Simons et al. [53, 106], the boundary system could facilitate renewal enforcing decision-makers to search for largely new ways—which our model seeks to capture in the "exploration only" strategy; at the same time, boundaries as established in the "proposal mode" help to align parochial choices to the overall objectives of the organization. However, rigid limits on the scope of search as established in the "exploitation only" strategy reduce the diversity of search and, in this sense, more rigid coordination (as with the proposal mode) may be beneficial (as is supported by the empirical study of Bedford [26]) though less than that in the "exploration only" strategy. This is broadly reflected in the simulation results since in the "exploitation only" strategy the final performances obtained in the three modes of coordination reach a similar level and the predominance of the proposal mode is not as clear as for the other search strategies.

The ambidextrous search strategy appears to be particularly sensitive to the coordination mode in respect of the (spread of) performance levels obtained. As mentioned before, this strategy grants the highest flexibility to the distributed decision-makers in terms of allowing for varying levels of novelty of their solutions; apparently, the combination of flexibility on the side of decentralized decision-makers and centralized final choices provides the best results when complexity is (increasingly) high. This relates to the tension captured in ambidextrous strategies which, in face of empirical results, lets Bedford [26] argue that in organizations pursuing this search strategy setting boundaries may be an alternative or substitute to other components of the MCS rather than being necessarily balanced with the other components.

### *5.2. Sensitivity to Cost of Effort*

*5.2.1. Overview.* So far, the emergence of the coordination mode was studied for organizations with the distributed decision-makers operating at a moderate level of cost of effort (i.e., cost coefficient $z = 0.001$). In the next step of this simulation study, the effect of cost of effort is analyzed. In particular, for the two interaction structures and the three search strategies under investigation, simulations with different levels of cost of effort are run. Apart from simulations for costless effort (i.e., $z = 0$), also experiments for cost coefficients $z \in \{0.003, 0.005, 0.01, 0.02\}$ were conducted. Figures 4 and 5 present results obtained for a medium cost level $z = 0.005$ and a high cost level of $z = 0.02$, respectively.

In order to be clear and concise, only the key findings and selected aspects of the sensitivity analysis are addressed explicitly. For this reason, the case of zero cost of effort is not discussed more in detail, since the experiments suggest that results obtained in the baseline scenarios (i.e., moderate cost level) do not change substantially when the decentralized decision-makers operate with *costless* effort. However, with increasing cost of effort the emergence of coordination modes (given by their relative frequencies) notably changes compared to the baseline scenarios. The most interesting results show up for the nondecomposable structure which is why this will be discussed more extensively—particularly in conjunction with the "exploitation and exploration" strategy.

With respect to the *decomposable structure*, comparing results obtained for the moderate cost level (Figure 3) against those for medium and high cost (Figures 4 and 5) reveals the most obvious difference that—in each search strategy—hierarchical coordination (proposal mode) is the less often implemented, the higher the cost level. This effect is more pronounced for the ambidextrous and the explorative strategies. Analyzing the results more in detail reveals an interesting dichotomy between the relative frequency of emergence and the final performance $V_{t=750}$ of the coordination modes; this effect is even stronger in the nondecomposable structures and, therefore, will be discussed in the context below.

In contrast, in the *nondecomposable structure*, the emerging coordination mode appears to be remarkably sensitive to an increase in the cost of effort. In particular, for the ambidextrous search strategy and the purely explorative

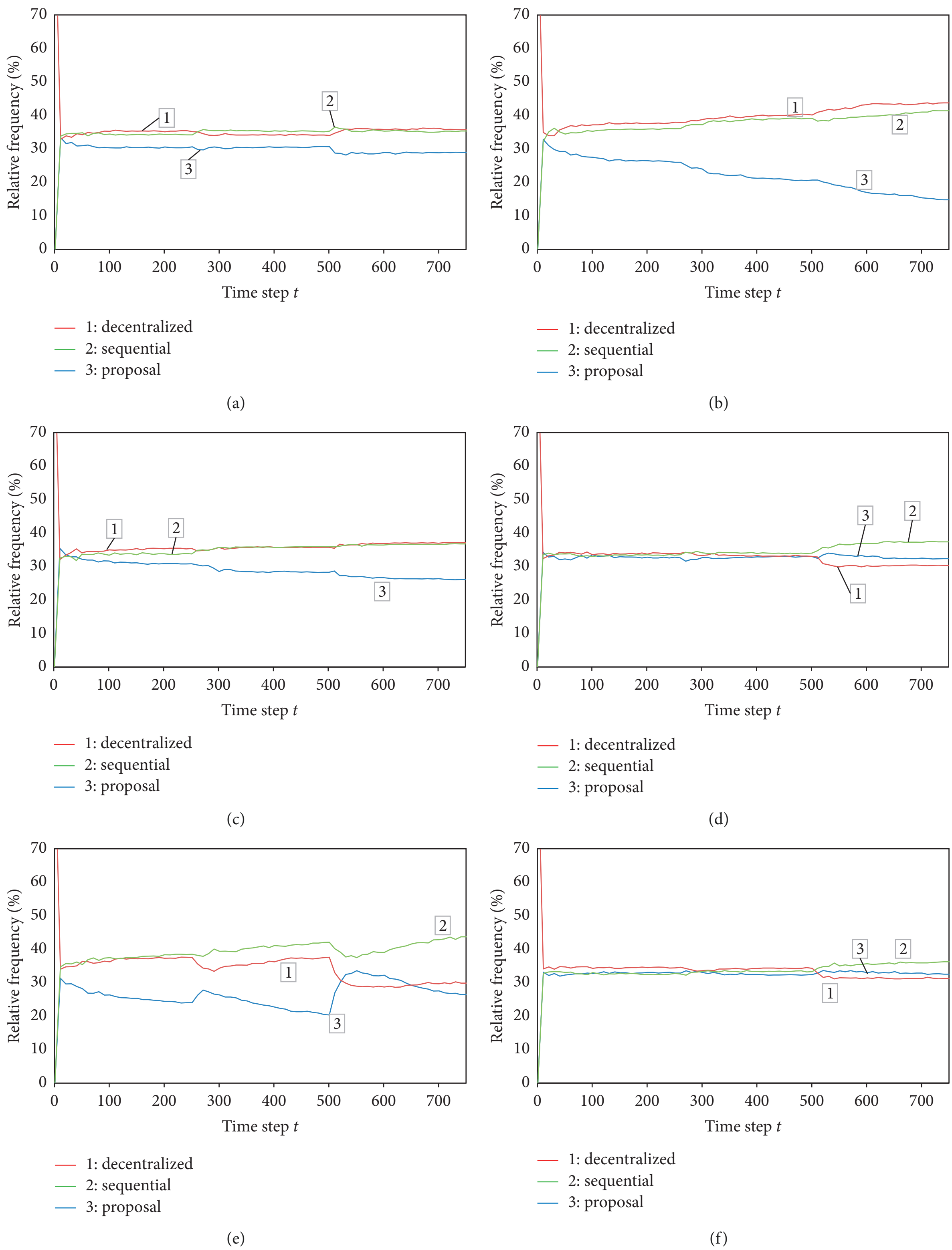


Figure 4: Relative frequencies of coordination modes in the course of growth with medium cost of effort (cost coefficient $z = 0.005$). Each plot represents results of 2,500 simulation runs: 250 landscapes with 10 runs on each. For parameter settings, see Table 2. (a) Decomposable-exploitation only, (b) decomposable-exploitation and exploration, (c) decomposable-exploration only, (d) nondecomposable-exploitation only, (e) nondecomposable-exploitation and exploration, and (f) nondecomposable-exploration only.

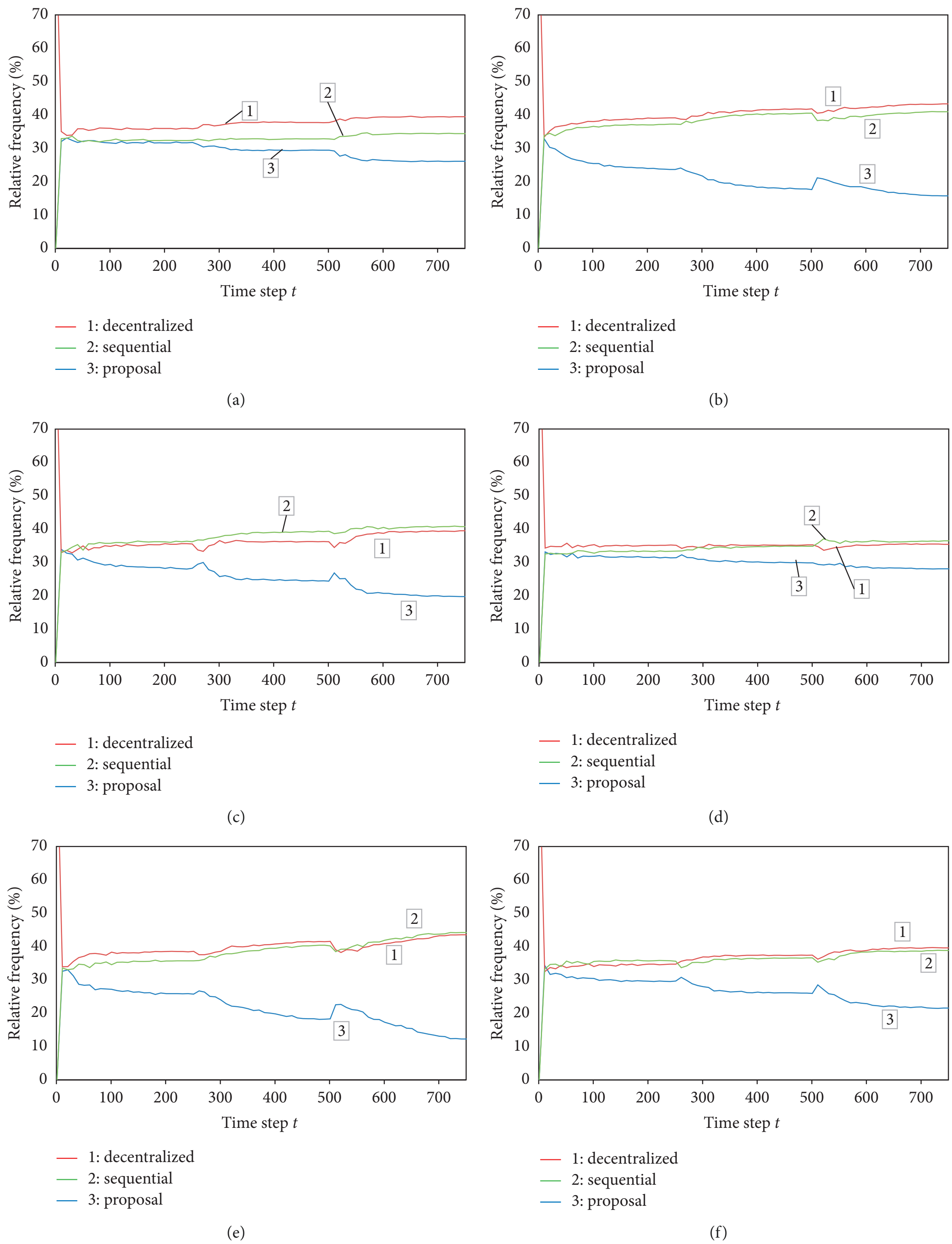


FIGURE 5: Relative frequencies of coordination modes in the course of growth with high cost of effort (cost coefficient $z = 0.02$). Each plot represents results of 2,500 simulation runs: 250 landscapes with 10 runs on each. For parameter settings, see Table 2. (a) Decomposable-exploitation only, (b) decomposable-exploitation and exploration, (c) decomposable-exploration only, (d) nondecomposable-exploitation only, (e) nondecomposable-exploitation and exploration, and (f) nondecomposable-exploration only.

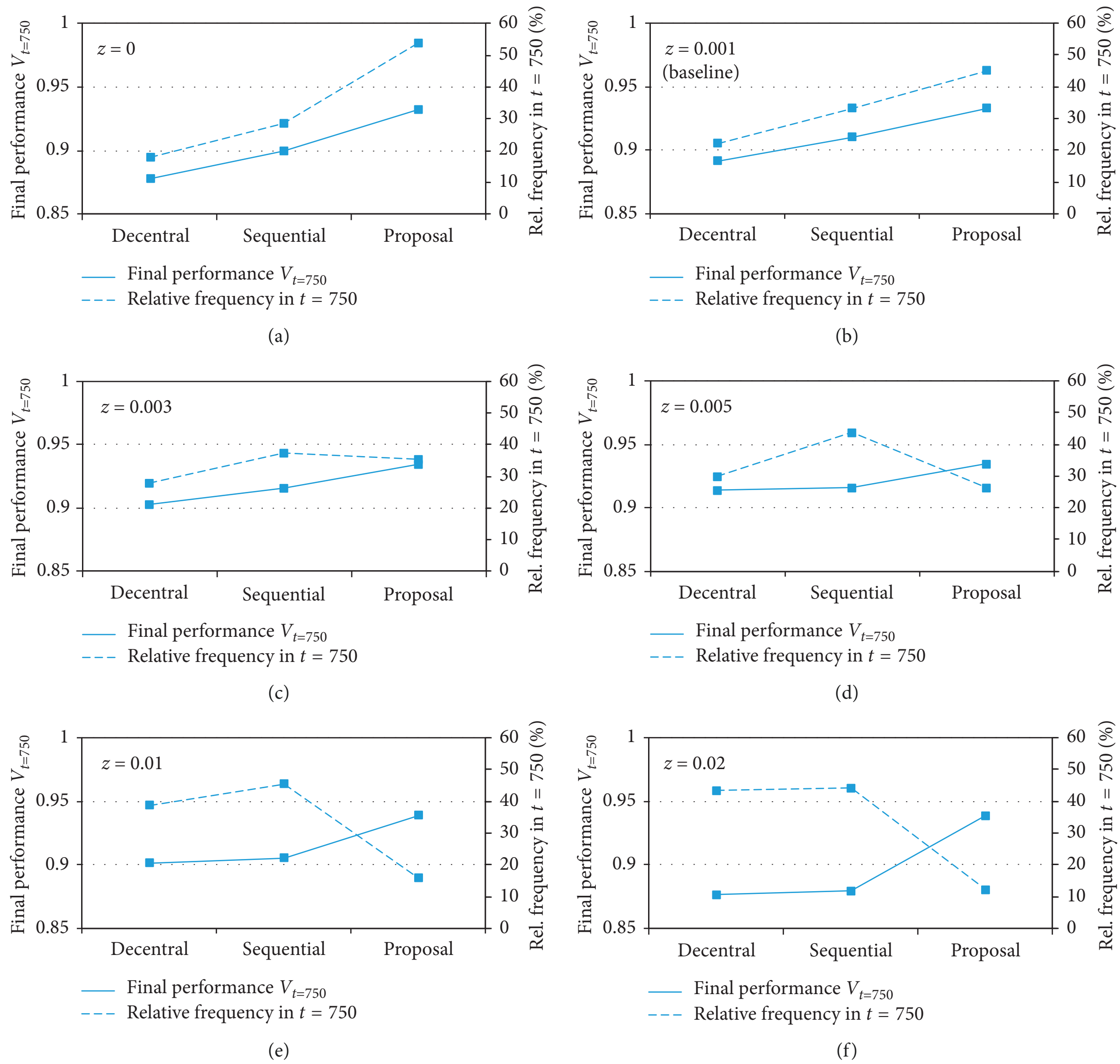


Figure 6: Sensitivity of final performance and relative frequencies of coordination modes to the cost of effort with the "exploitation and exploration" search strategy for the nondecomposable interaction structure. Each plot represents results of 2,500 simulation runs: 250 landscapes with 10 runs on each. For parameter settings, see Table 2. (a) $z = 0$, (b) $z = 0.001$ (baseline), (c) $z = 0.003$, (d) $z = 0.005$, (e) $z = 0.01$, and (f) $z = 0.02$.

strategy the "order" of coordination mechanisms changes compared to the baseline scenarios: recall that at the moderate cost level (baseline), the proposal mode clearly predominates (with a share up to more than 55%); now, with higher cost, this is the least often emerging coordination mode (partially with only about 12%).

Moreover, this low frequency of occurrence of hierarchical coordination goes along with the *highest* final performances compared to the other coordination modes. This clearly runs against intuition since one would expect that the better performing coordination mode is selected more often as it is the case for lower cost levels as shown in the baseline scenarios (see Section 5.1).

To illustrate this effect, Figure 6 plots—for the different cost levels simulated—the final performances $V_{t=750}$ and the relative frequencies in the end of the observation time for the case of the ambidextrous search strategy in the nondecomposable structure, while Table 5 reports on the respective details. As it becomes apparent, with a cost coefficient level of $z = 0.003$ and higher, final performance and frequency of the hierarchical coordination run apart—or put the other way round: the worse performing coordination modes emerge more often (a similar effect shows up in the "explorative only" search strategy, and even for the "explorative only," it is worth mentioning that the coordination modes show similar frequencies of occurrence while the final performance obtained with hierarchical coordination ("proposal") goes remarkably beyond the level obtained with the other coordination modes).

*5.2.2. On the Dichotomy of Coordination Modes in Performance and Frequency.* Hence, an interesting question is what may cause this dichotomy in final performance and

Table 5: Condensed results obtained for the nondecomposable interaction structure in the "exploitation and exploration" search strategy for different levels of cost of effort.

| Cost of effort (cost coefficient) | Frequency of coordination mode $a^c$ in $t = 750$ (a) | | | Final performance $V_{t=750}$ (conf. interval*) (b) | | | Frequency of global maximum found in $t = 750$ (c) | | | Frequency of alterations (false positives) of $\overrightarrow{d_t}$ (d) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. | Decent. | Sequ. | Prop. |
| $z = 0$ | 17.9% | 28.4% | 53.8% | 0.878 ± 0.0185 | 0.899 ± 0.0120 | 0.932 ± 0.0058 | 13.0% | 12.4% | 18.5% | 14.4% (7.1%) | 15.4% (7.6%) | 16.3% (8.0%) |
| $z = 0.001$ | 22.1% | 33.0% | 45.0% | 0.891 ± 0.0142 | 0.910 ± 0.0092 | 0.933 ± 0.0064 | 10.3% | 9.3% | 18.3% | 12.4% (6.2%) | 12.4% (6.1%) | 14.2% (7.0%) |
| $z = 0.003$ | 27.6% | 37.2% | 35.2% | 0.903 ± 0.0102 | 0.915 ± 0.0079 | 0.934 ± 0.0068 | 11.1% | 14.1% | 19.2% | 8.5% (4.2%) | 8.3% (4.1%) | 11.3% (5.5%) |
| $z = 0.005$ | 29.8% | 43.7% | 26.4% | 0.914 ± 0.0080 | 0.916 ± 0.0063 | 0.935 ± 0.0080 | 11.7% | 11.9% | 17.7% | 5.8% (2.8%) | 6.0% (2.9%) | 9.0% (4.4%) |
| $z = 0.01$ | 38.8% | 45.6% | 15.6% | 0.901 ± 0.0071 | 0.905 ± 0.0062 | 0.940 ± 0.0100 | 7.0% | 9.1% | 20.7% | 3.2% (1.5%) | 3.1% (1.4%) | 5.0% (2.4%) |
| $z = 0.02$ | 43.5% | 44.2% | 12.3% | 0.877 ± 0.0070 | 0.880 ± 0.0070 | 0.940 ± 0.0100 | 3.0% | 4.2% | 18.9% | 1.9% (0.8%) | 1.8% (0.8%) | 3.0% (1.3%) |

*Confidence intervals for $V_{t=750}$ are given at a level of 99.9%. Each row represents averaged/aggregated results of 2,500 simulation runs: 250 landscapes with 10 runs on each.

frequency of emergence for higher cost of effort in the "exploitation and exploration" strategy as shown in Figure 6 and Table 5.

A starting point for an explanation is that higher cost of effort at the distributed decision-makers' side increase the propensity that they prefer to stay with the status quo. This, in turn, is best overcome by hierarchical coordination though with the risk of false positive alterations.

In particular, with an increase in cost of effort, from a local decision-maker's perspective, leaving the status quo in favor of an alternative configuration becomes less rewarding, or in other words, the higher the cost of effort the more promising an alternative to the status quo—inducing effort—has to be for being selected, i.e., only alterations that promise more than the cost of effort are preferable from a decentralized decision-maker's perspective. The search strategies enforce different levels of effort to be taken for leaving the status quo; for example, the purely explorative strategy requires making long jumps (i.e., switching two single choices) which—according to the quadratic cost function in equation (7)—induces rather high cost of effort.

Hence, with an increasing cost coefficient $z$, in tendency, in all three modes of coordination, keeping the status quo becomes more attractive for the local decision-makers and this effect is the more pronounced the higher the effort enforced by the search strategy. However, the coordination modes are differently prone to this kind of "inertia": In the decentralized mode the local decision-makers' first preferences are implemented without any further revision. In the sequential mode, distributed decision-makers' first preferences may be revised, though from a parochial perspective.

In contrast, in the proposal mode it is more likely that the decentralized decision-makers' preferences are overrided by the central agent with respect to the overall performance and, hence, alterations, though causing cost on the decentralized side, may be induced. This, in turn, makes it more likely that the status quo is abandoned and a new configuration $\overrightarrow{d_t}$ is implemented. However, since the central agent disposes of rather noisy information ($\sigma^{\text{cent}}$), these alterations could also be in favor of a false positive alteration. This explanation is broadly confirmed by the ratios of periods with alterations and with false positive alterations (for the metrics, see Section 4.3): These ratios are particularly higher in case of hierarchical coordination (proposal mode) as reported in Table 5 for the example of the ambidextrous search strategy (with increasing levels $z$ of the cost coefficient the alterations decrease for all coordination modes under investigation but the relative differences among the modes increase; the confidence intervals of the final performance achieved shows in a similar direction: with levels of cost coefficient $z = 0.005$ and higher the confidence intervals for the proposal mode are higher than those of the other coordination modes).

With respect to the emergence of coordination modes, the stability in terms of, at least, keeping a once achieved performance level provided by a particular mode of coordination drives the propensity of being implemented in the future. Hence, since the decentralized and the sequential mode in combination with higher cost coefficients of effort and a search strategy enforcing exploration induce more inertia and, on average, lower performance than hierarchical coordination, according to the simulation results, these coordination mechanisms predominate.

In this sense, the somewhat counter-intuitive results for higher levels of cost of effort may provide an explanation for the ambiguous results obtained in empirical studies on the relation between tightness of coordination on the one hand and organizational size as well as intraorganizational complexity on the other hand as reported in Section 2.

For a further interpretation of results, it appears worth mentioning that the level of cost of effort in the model (captured by a simple cost coefficient $z$) may represent rather different aspects and—depending on its particular context—the results of this study show in quite different directions. For example, the cost of effort could capture "switching costs" for altering a status quo in favor of a new solution which may be specific to a certain branch or industry due to technological aspects [107, 108]. In this sense, the results may be regarded as an indication that, for example, in different branches different coordination modes might be predominant.

In another interpretation, cost of effort could capture the costs for dealing with resistance of certain stakeholders or (emotional) costs for loosing old and building new intra- or extraorganizational relations when new solutions are to be implemented [109–111]. Regarded in this sense, the results suggest that organizations where, for example, high resistance or high relational costs for implementing the "new" are to be expected, in tendency, might stick to decentralized coordination modes—though hierarchical coordination could lead to higher levels of performance.

## 6. Conclusion

This paper presents a computational study on the emergence of the mode of coordination in the course of an increasing decision-problem and, in consequence, a growing number of decentralized decision-makers in DDMS. In the theoretical model and the simulation model accordingly, the mode of coordination employed is subject to learning-based emergence, while the search strategy—being exploitative, explorative or of an ambidextrous nature—is regarded as given (e.g., by a central authority). The same holds for the level of cost of effort that the distributed decision-makers face for implementing new solutions.

For moderate levels of cost of effort, the results suggest that DDMS facing a growing nondecomposable decision-problem would increasingly employ vertical information flows, broad information, and decision-making via hierarchy rather than granting high autonomy to decentralized decision-makers. In contrast, when growth means that additional self-contained decision-problems are to be solved, according to the simulation results, with the exploitative and the explorative search strategy, no predominance in the coordination mode appears; in an ambidextrous search strategy, coordination modes tend to prevail that leave the decision-making authority at the side of decentralized decision-makers. For the levels of complexity studied, results suggest that the ambidextrous strategy bears the highest potential of superior performance while enforcing high levels of novelty by a "purely" explorative strategy leads to inferior performance.

However, the results suggest that the higher the cost of effort on the side of decentralized decision-makers are, the more likely it may become that those coordination mechanisms emerge which provide high autonomy to decentralized decision-makers though employing hierarchical coordination could lead to higher performance obtained by the DDMS.

These results could, for example, be regarded as an explanation for some ambiguous empirical findings since different industries may differ in the "switching" costs in favor of new solutions due to technological aspects. This study may also provide an explanation for organizational inertia when resistance against change is of relevance or change comes along with high relational costs to decision-makers.

Moreover, a particular contribution of this study may also lie in its method: It appears worth mentioning that with the computational study introduced here some empirical findings related to (the emergence of) the coordination mode as a part of the boundary system obtained in prior empirical research [26, 48–51, 106] were "replicated." In particular, via growing DDMS "from scratch," the coordination mode at the system's level emerged from some rather simple (for not to say simplistic) components and behavioral rules at the individual agents' level. In this sense, computational studies directed to growth processes could contribute to the research in the various domains of organizational thinking since they allow to capture growth processes without having to deal, for example, with methodological challenges of longitudinal empirical research as well as hardly controllable contingent factors [35].

This study calls for further research activities: First, it is worth mentioning that this study, by far, does not represent the width of modes of coordination. Hence, a natural extension of the research effort presented here would be to integrate further coordination modes, like, for example, negotiations among distributed decision-makers.

Second, the learning mechanism employed in this study, i.e., reinforcement learning, is rather simple, and more sophisticated learning modes should be studied. For example, in contexts representing organizations resided by human decision-makers, belief-based learning [92] could be a reasonable alternative.

Third, it is to be mentioned that the model presented here builds on rather simple cost functions: As such, the model does not capture any cost for coordination which, of course, vary across coordination modes (e.g., [6]). Furthermore, the cost of effort could be modeled in a more sophisticated way by, for example, distinguishing between industry-specific technological "switching cost," cost of effort related to the capabilities of agents, as well as costs due to intraorganizational resistance or costs due to specific investments to name but a few.

Fourth, a natural extension would be to study the emergence of coordination in the context of further types of controls: For example, in the context of DDMS with human decision-makers, according to the "Levers of Control" framework [22, 106], the emergence of coordination in the context of commonly shared values as part of the beliefs system or of incentive systems being part of the diagnostic control system is of interest. These ideas relate to the growing body of research emphasizing the internal fit and the balance between the various types of control mechanisms. Hence, further computational studies may build on more comprehensive models of mechanisms to control decision-making and examine the interrelations among these mechanisms for different contingent factors like, for example, the complexity of the decision-problem.

## Data Availability

No empirical data were used to support this study.

## Conflicts of Interest

The author declares that there are no conflicts of interest.

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*Research Article*

# Cooperative Bargain for the Autonomous Separation of Traffic Flows in Smart Reversible Lanes

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Spacer bars in the smart reversible lanes make periodically broadcast of messages to share their local observed traffic information with each other. This aims to help other spacer bars acquire the global traffic information and make consistent movement when separating the flows. However, radio interference and vehicles in the traffic may degrade the qualities of wireless communication links and cause frequent message losses in the broadcast. Existing solutions tend to use data forwarding to enhance the message dissemination, which may cause imbalanced load in the spacer bars. For instance, the nodes close to the sink have to forward more messages, whereas the ones far away from the sink have fewer messages for forwarding. The unbalanced distribution of network load has a high risk of blocking the wireless communication links and yield inconsistent movement in the reversible lanes. In this paper, we propose a Cooperative Bargain (CoB) scheme where each spacer bar carries some received messages to help other spacer bars recover their lost messages. Since the spacer bars can only acquire the local information, we formulate a cooperative bargain game to negotiate how to allocate the task of message recovery with a balanced network load until a consensus is achieved. CoB is evaluated with the real-world Wi-Fi communication traces in Isti/rural. Simulation results show that CoB can recover an average of 98.6% messages within 100 milliseconds in a 50-node network. CoB does not require the global network information but it can still achieve a comparable performance to other broadcast schemes.

## 1. Introduction

The rapidly growing demand on congestion-free driving makes the smart traffic control a critical functionality in intelligent transportation systems (ITSs) [1, 2]. To make full utilization of road resources, the smart reversible lanes are deployed on roads to reduce directional traffic congestion in rush hours. The smart reversible lanes monitor traffic environment and work as movable centre dividers to separate traffic flows from different directions, aiming to increase the traffic capacity in the peak direction by borrowing some unused lanes from off-peak direction. As shown in Figure 1, a smart reversible lane is composed of several spacer bars that are connected with longitudinal barriers. There are two categories of spacer bars in a smart reversible lane, i.e., active spacer bars and passive spacer bars. The active spacer bar is equipped with a radio transceiver, integrated sensors (e.g., video camera and ultrasonic unit), and a step motor. The radio transceiver is used for wireless communications, and the integrated sensors are used for monitoring the traffic environment. The step motor is installed on the base and powered by batteries to drive the spacer bar forward or backward. In contrast, the passive spacer bar has no radio transceiver, integrated sensors, or step motor. It is mechanically connected with the adjacent spacer bars and moves passively when dragged by others.

Considering the hardware investment and maintenance cost, the number of active spacer bars are limited and they are sparely distributed in the smart reversible lane. These active spacer bars form a backbone and drive the smart reversible lane forward or backward when separating the traffic flows. Each active spacer bar monitors the local traffic conditions, e.g., the volume of traffic flow and average vehicle speed, and then decides to move forward, backward, or stand still. Since the active spacer bars are distributed in a decentralized environment, they can only observe the local traffic information. This may yield dispute in the moving decisions and cause inconsistent movement of spacer bars as well as mechanical damages to the reversible lanes [3]. To eliminate the dispute in the moving decisions, the active spacer bars broadcast some messages to exchange their local observed traffic information with others and negotiate how to move the reversible lane consistently. However, the empirical studies in [4] have shown that, even in rural areas, the radio jamming and multiple-path interference can decrease the packet delivery ratio (PDR) by 50%. Put it in another way, not all the active spacer bars can receive the messages. Since each message contains the local observed traffic information, the frequent message losses will cause inconsistent movement of the spacer bars and mechanical damages to the reversible lanes.

A straightforward approach to address this issue is *data forwarding*. Take a line network for example. The node forwards its received messages to the next node until they are successfully delivered to the sink. However, it will cause unbalanced load in the network, e.g., the nodes that are close to the sink have more messages to forward, whereas the nodes that are far away from the sink have few messages to forward [5]. In [6], a flooding-based scheme is proposed where each node forwards all received messages to help other nodes recover their lost ones. However, the fast expense of message duplications will increase the communication overhead and chock the communication channels with a high risk of broadcast storms. Another frequently used method is *cooperative broadcasting*, where the task of message delivery is offloaded to each node regarding its local resources, e.g., the quality of communication links and the remaining energy in the battery. Based on this key idea, a number of cooperative broadcasting schemes are proposed with different metrics for the task offloading, e.g., the geolocation-based schemes [7], the energy-based schemes [8, 9], and the neighborhood-based schemes [10, 11]. Note that most of these schemes require the global information for central administration and task offloading, which is challenging in wireless networks due to the unstable qualities of communication links.

In this paper, we develop a Cooperative Bargain (CoB) scheme to improve the message delivery ratio (MDR) in the broadcast. CoB only uses the local information at the active spacer bars but can reduce most disputes in the moving decisions and avoid inconsistent movements in the smart reversible lanes. Each active spacer bar carries some received messages to help others recover their lost messages. Since the messages are carried in each active spacer bar's broadcast, no extra retransmission is needed, yielding high MDRs with small communication overhead and short time delay. Besides, a decentralized cooperative bargain game is formulated to offload the task of message recovery regarding each active spacer bar's local resource [12]. The active spacer bars keep on negotiating with each other on which messages to carry until a consensus is achieved, so that they can make consistent movement in the reversible lanes.

The rest of this paper is organized as below. Section 2 reviews some related works on the reliable broadcast in wireless networks. Section 3 describes the system model and formulates the procedure of cooperative broadcast as an integral optimization problem. In Section 4, a cooperative bargain game is formulated to offload the task of message recovery. Section 5 proves that the achieved consensus in the cooperative bargain game is an optimal solution based on the theory of Nash equilibrium. Section 6 presents the simulation results where CoB is compared with some other broadcast schemes, and Section 7 concludes the whole paper.

## 2. Related Work

There have been active research studies on the reliable broadcast in wireless networks, but few can be applied to the wireless communications in smart reversible lanes. The reasons are twofold: (1) the signal jams and multiple-path interference in the traffic may degrade the qualities of wireless communication links between the spacer bars, causing frequent message losses in the broadcast of messages [13, 14] and (2) the active spacer bar can only obtain incomplete network topology and local traffic information, which may yield inconsistent moving actions and mechanical damages to reversible lanes [15]. Existing solutions can be divided into two categories, i.e., cooperative broadcast and noncooperative broadcast, depending on whether the nodes cooperate with others during the message recovery.

Flooding is a widely used noncooperative broadcast solution, where each node forwards all received messages through all outgoing channels to improve the broadcast reliability [16]. To reduce the communication overhead resulting from excessive duplications in the broadcast, some variants of flooding are proposed. As shown in [17], Dash et al. proposed a hop-controlled flooding to restrict the number of hops when forwarding the received messages. It describes a novel hop-count update procedure using a history database called Info-Base. Current hop-count update procedure reduces the hop count at each routing node. However, the inherent redundant duplication in the flooding-based schemes remains unchanged and the nodes still suffer from the high risk of broadcast storms. In [18], Byeon et al. proposed an opportunistic flooding-based scheme where the duplication of messages is strictly limited. Depending on the contribution level for the entire network, the proposed technique enhances transmission efficiency through priority adjustment and the removal of needless relay nodes. However, it is prone to blind spots or redundant overlaps if the parameter $p$ is not well designated, as it highly depends on the broadcast environment and application scenarios.

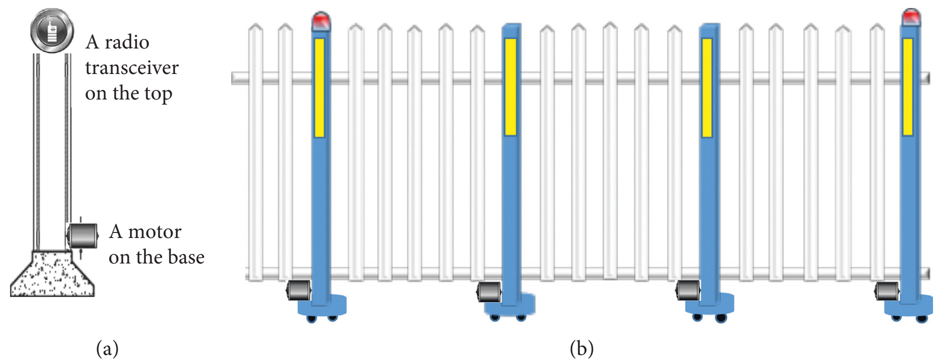


Figure 1: (a) The mechanical structure of an active spacer bar. (b) A smart reversible lane where the spacer bars are connected with longitudinal barriers.

The greedy-based scheme is a typical noncooperative solution for reliable dissemination of messages, where each node tries to recover the lost messages individually with little or seldom cooperation with others [19]. Xiao et al. [20] proposed a greedy-based piggybacking scheme to recover Cooperative Awareness Messages (CAMs) in the vehicular networks. Each vehicle broadcasts a request list to inform others which CAMs it has lost, and the CAM with maximum request-times will be selected for piggybacking first. However, the vehicles act too much greedy and little cooperation can be observed in the recovery of lost messages, which may cause redundant piggybacking and a low MDR with long time delay and large communication overhead. In [21], Xu et al. formed a noncooperative game for distributed wireless sensor networks to control the topology for energy saving and network load balancing. A price function is developed to calculate the reward for forwarding messages and the penalty for declining messages in the broadcast. Since all the nodes tend to maximize the reward, both the MDR and the network lifetime can be enhanced when a Nash equilibrium is achieved. However, Nash equilibrium is not sufficient to guarantee a global optimal solution. A simple example is the prisoner's dilemma in the game theory.

Recently, much attention has been paid to the cooperative broadcast, where the message recovery is offloaded to the nodes in a cooperative manner [22, 23]. For instance, a cooperative beacon broadcast scheme is proposed in [24] to provide the vehicles with more traffic information when driving on roads. Each vehicle selects $w$ neighbors and piggybacks their awareness messages in the periodical beacon broadcast. It aims to improve the driving safety via the exchange of traffic information as it can provide sufficient traffic awareness when driving on roads. The drawback is that each vehicle has to dynamically maintain the geographic information of all its neighboring vehicles, which will consume a lot of network resources and introduce large communication overhead in the networks. A cooperative volunteer-based broadcast is studied in [25], where each vehicle exchanges its routing table with others to explore hidden neighbors in the non-line of sight (NLOS) area. The vehicles that have reliable links to the hidden neighbors will be selected as forwarders to retransmit the messages. Similarly, this scheme assumes that all nodes can obtain the global network information when offloading the task of message recovery, which is quite challenging for the active spacer bars in smart reversible lanes [26, 27].

This paper provides a novel cooperative broadcast scheme called CoB to improve MDR in the broadcast of messages, aiming to eliminate the bias and making consistent movements in reversible lanes. Besides, a cooperative bargain game is formulated to negotiate the recovery strategies and moving decisions in all nodes until a consensus is achieved. Therefore, the task of message recovery can be offloaded to the nodes regarding their local resources, which is proved to be a global optimal solution to the cooperative recovery problem. Since the messages can be delivered reliably and efficiently, the spacer bars will acquire the same traffic information and thus move consistently to separate the traffic flows.

## 3. System Model and Problem Statement

Consider a smart reversible lane with $|N|$ active spacer bars. Each active spacer bar is equipped with a radio transceiver for transmitting and receiving messages, and the radio transceivers have the same transmission range of $r$ meters. A stepping motor is installed on the base of each active spacer bar to move it forward or backward for separating the traffic flows. We model the smart reversible lane as a line network $\mathbb{G} = (N, L)$. The vertices in $N = \{n_i \mid i \leq |N|, i \in \mathbb{Z}^+\}$ represent the active spacer bars, which is referred to as *node*(*s*) in the rest of this paper. The edges in $L = \{l_{ij} \mid i, j \leq |N|, i, j \in \mathbb{Z}^+, i \neq j\}$ denote the wireless communication links between the active spacer bars.

Time is synchronized within a consensus-based synchronization method. Each node uses the carrier sense multiple access/collision detection (CSMA/CD) scheme to access the channels for broadcasting, which can achieve a high utilization of channel resources which do not require the central administration to schedule the nodes for broadcasting [28]. Once the local traffic conditions have changed (e.g., the vehicle density has changed by 0.1 vehicle/m and the shockware speed has changed by 2 vehicle/s), the node will broadcast a message to inform other nodes of the up-to-date local traffic information [29]. Thereby, the nodes will acquire more traffic information to make consistent movement in the reversible lanes when separating the traffic flows. The messages received by node $n_i$ are cached in its receiving buffer $F_i$. Each message is associated with a time-to-live (TTL) of $\delta$ ms and will be discarded when the TTL expires.

However, not all the nodes can receive the messages due to the fragile communication links, which may cause inconsistent movements in the reversible lanes when separating the traffic flows. To address this issue, each node carries some received messages when it is scheduled for broadcasting and helps other nodes recover their lost messages. Let $s_i^t$ be node $n_i$'s task in the message recovery, and the combination of each node's task in the message recovery, $\mathcal{S}_i^t = \left\{s_1^t, s_2^t, \ldots, s_{|N|}^t\right\}$ is defined as the joint recovery strategy in the broadcast. Suppose $M_j$ is the set of messages generated by node $n_j$ in the broadcast, and $|N_i(M_j, \mathcal{S}^t)|$ is the number of messages in $M_j$ that are received by node $n_i$ under the joint recovery strategy $\mathcal{S}^t$. Then, node $n_i$'s message reception ratio (MRR) regarding node $n_j$'s messages, denoted by $r_{ij}(\mathcal{S}^t) = |N_i(M_j, \mathcal{S}^t)|/|M_j|$, is defined as the ratio between the number of messages received by node $n_i$ and the total number of messages in $M_j$. Thereby, we can convert the process of cooperative broadcasting in the reversible lane into an integral optimization problem as shown in the following equation:

$$\max \sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}(\mathcal{S}^t)}{|N|^2}. \tag{1}$$

Equation (1) indicates that the joint recovery strategy $\mathcal{S}^t$ should be capable of maximizing the average MRR for all the nodes in the broadcast. It aims to find an optimal joint recovery strategy $\mathbb{S}^{t*}$ that can maximize the MRR in the broadcast, so that they can acquire the same messages and make consistent movement when separating the traffic flows.

## 4. Cooperative Bargaining

In this section, we formulate a cooperative bargain game to allocate the task of cooperative recovery regarding each node's local resources, and then propose a Cooperative Bargain scheme to maximize the objection function in 1.

*4.1. Game Formulation.* To achieve a consensus in the recovery of lost messages, each node $n_i$ generates a bargain proposal $B_i^t$ and exchanges it with other nodes for negotiation. The bargain proposal contains a recommending strategy $\mathcal{R}_i^t$ and a bidding strategy $\mathcal{P}_i^t$. The recommending strategy $\mathcal{R}_i^t = \left\{r_{i1}^t, r_{i2}^t, \ldots, r_{i|N|}^t\right\}$ is a set of recommendations generated by node $n_i$, and it indicates which message node $n_i$ expects other nodes to carry. The bidding strategy $\mathcal{P}_i^t = \left\{p_{i1}^t, p_{i2}^t, \ldots, p_{i|N|}^t\right\}$ is a set of bidding prices offered by node $n_i$ corresponding to each recommendation in $\mathcal{R}_i^t$. The higher bidding price the node $n_i$ offers, the higher priority the recommended message will be piggybacked first.

After generating a bargain proposal, each node will receive a reward, depending on how much its bargain proposal deviates from other nodes' bargain proposals. The bargain proposal with less deviation will yield more rewards and vice versa. To maximize the rewards, the nodes will negotiate with each other to update their bargain proposal until a consensus is achieved. The deviations of bargain proposals are referred to as *bargain bias* in the rest of this paper, and the definition is presented as below:

*Definition 1.* Consider two bargain proposals generated by node $n_i$ and $n_j$ at time $t$, respectively, i.e., $B_i^t = \left\{\mathcal{R}_i^t, \mathcal{P}_i^t\right\}$ and $B_j^t = \left\{\mathcal{R}_j^t, \mathcal{P}_j^t\right\}$. The bargain bias of $B_i^t$ against $B_j^t$, denoted by $\theta_{ij}^t$ as shown in equation (3), indicates how much node $n_i$'s bargain proposal deviates from node $n_j$'s bargain proposal:

$$\theta_{ij}^t = \left(\mathcal{P}_j^t - \mathcal{P}_i^t\right) \cdot (1)^T + \left(\mathcal{R}_j^t - \mathcal{R}_i^t\right) \cdot \operatorname{diag}\left(\mathcal{P}_i^t\right) \cdot \left(\mathcal{R}_j^t - \mathcal{R}_i^t\right)^T. \tag{2}$$

According to Definition 1, if nodes $n_i$ and $n_j$ have generated the same bargain proposal, e.g., they have made the same recommending strategy and offered the same bidding prices, the bargain bias is calculated as 0; otherwise, the bargain bias is nonzero. To make consistent movement in the reversible lanes, the nodes should eliminate the bargain bias and enforce all the bargain proposals converge to a consensus. The challenge is that the nodes can only acquire the local information, and it is difficult to compare the bargain proposal with all the other nodes in the decentralized environment. To address this issue, we connect all the nodes into a virtual ring and divide them into several overlapped groups. Each group is composed of three nodes, e.g., a heading node, a central node, and an ending node, and the group overlaps with a preceding group and a succeeding group, respectively. Put it in another way, the heading node in the current group is the ending node of its preceding group, whereas the ending node is the heading node of its succeeding group. For example, consider a set of $|N|$ nodes in Figure 2. If $|N|$ is an odd number, the nodes can be divided as, $\{(n_1, n_3, n_5), (n_3, n_5, n_7), \ldots, (n_{|N|-4}, n_{|N|-2}, n_{|N|}), (n_{|N|-2}, n_{|N|}, n_{|N|-1}), (n_{|N|}, n_{|N|-1}, n_{|N|-3}), \ldots, (n_4, n_2, n_1), (n_2, n_1, n_3)\}$. If the number of nodes $|N|$ is even, the nodes can be divided as $\{(n_1, n_3, n_5), (n_3, n_5, n_7), \ldots, (n_{m-5}, n_{m-3}, n_{m-1}), (n_{m-3}, n_{m-1}, n_m), (n_{m-1}, n_m, n_{m-2}), \ldots, (n_4, n_2, n_1), (n_2, n_1, n_3)\}$. Since each group overlaps with its preceding group and the succeeding group, the central node only has to compare its bargain proposal inside of the group because the preceding node and the successive node will carry on comparing the bargain proposal with other nodes in the overlapped groups. Thereby, the bargain proposals will be compared among all the nodes group by group along the virtual ring until a consensus is achieved.

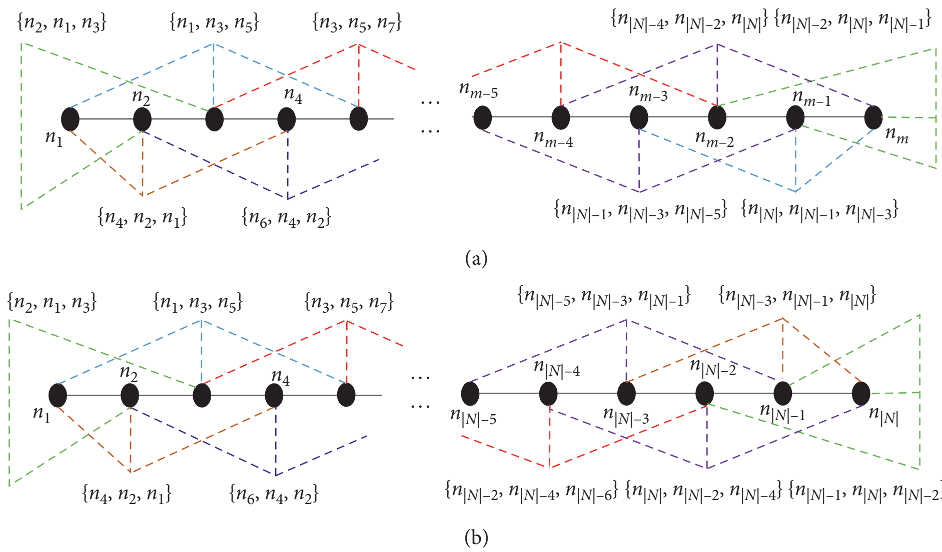


Figure 2: Dividing all nodes into groups for the comparison of bargain proposals. (a) The division of groups when the number of nodes $|N|$ is odd. (b) The division of groups when the number of nodes $|N|$ is even.

*Definition 2.* Consider a group with three nodes ($n_i, n_j$, and $n_k$) in the cooperative bargain game. $B_j^t$ is the bargain proposal generated by the central node $n_j$. It will receive a reward $w(B_i^t)$, which is defined as the difference between the bargain bias $\theta_{ij}^t$ and $\theta_{jk}^t$ in the following equation:

$$\begin{aligned} w\left(B_i^t\right) = \theta_{ij}^t - \theta_{jk}^t = \left(\mathscr{P}_i^t - \mathscr{P}_j^t\right) \cdot (1)^T + \left(\mathscr{R}_i^t - \mathscr{R}_j^t\right) \cdot \operatorname{diag}\left(\mathscr{P}_i^t\right) \\ \cdot \left(\mathscr{R}_i^t - \mathscr{R}_j^t\right)^T - \left(\mathscr{P}_j^t - \mathscr{P}_k^t\right) \cdot (1)^T - \left(\mathscr{R}_j^t - \mathscr{R}_k^t\right) \\ \cdot \operatorname{diag}\left(\mathscr{P}_j^t\right) \cdot \left(\mathscr{R}_j^t - \mathscr{R}_k^t\right)^T. \end{aligned} \tag{3}$$

As shown in equation (3), if the bargain bias between nodes $n_i$ and $n_j$ is less than the bargain bias between nodes $n_j$ and $n_k$, node $n_i$ will receive a positive reward $w(B_i^t) > 0$ as a bonus for its bargain proposal; otherwise, node $n_i$ will receive a negative reward $w(B_i^t) < 0$ as a penalty for its bargain proposal. Since each node aims to maximize the rewards, the bargain bias will be eliminated ultimately. However, eliminating the bargain bias does not mean the messages have been recovered successfully, e.g., the nodes may achieve a consensus that no one piggybacks any message. The nodes will not receive any penalty but they fail to recover the lost messages in the broadcast. To solve this problem, we combine the reward with MRR and form a utility function as shown in equation (4).

*Definition 3.* For each node $n_i$ in the cooperative bargain game, the utility function, denoted by $u(B_i^t = \{\mathscr{R}_i^t, \mathscr{P}_i^t\})$, is defined as the sum of MRR following its recommending strategy $\mathscr{R}_i^t$ and the reward for its bargain proposal $w(B_i^t)$:

$$u\left(B_i^t\right) = \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^t\right)}{|N|^2} + w\left(B_i^t\right). \tag{4}$$

For each node in the cooperative bargain, it keeps on updating the bargain proposal to eliminate the bargain bias and maximize the utility function. According to the definition of Nash equilibrium in [30], all the bargain proposals will converge to a consensus, which is referred to as *balance point* in the Nash equilibrium. When the balance point is achieved, none of the nodes is willing to change its bargain proposal as the utility function has been maximized according to the node's local information.

*4.2. Cooperative Bargain.* In this subsection, a Cooperative Bargain (CoB) scheme is proposed to maximize the utility function in equation (4). The key ideas of CoB can be described as follows:

(1) For each message in the broadcast, node $n_i$ generates a set of recommendations to indicate which node it expects to carry the message. Considering all the messages in the broadcast, the combination of node $n_i$'s recommendations to each other node, i.e., $\mathscr{R}_i^t = \{r_{i1}^t, r_{i2}^t, \ldots, r_{i|N|}^t\}$, is defined as its recommending strategy.

(2) For each recommendation in $\mathscr{R}_i^t$, node $n_i$ offers a bidding price. The higher bidding price it offers, the higher priority the recommended message is expected to be carried first. The set of bidding prices offered by node $n_i$, denoted by $\mathscr{P}_i^t = \{p_{i1}^t, p_{i2}^t, \ldots, p_{i|N|}^t\}$, is defined as its bidding strategy.

(3) The combination of piggybacking strategy and bidding strategy, i.e., $B_i^t = \{\mathscr{R}_i^t, \mathscr{P}_i^t\}$, is defined as node $n_i$'s bargain proposal. Node $n_i$ compares the bargain proposal with the heading node and ending

node in the group, and it calculates the reward $w(B_i^t)$ regarding the bargain bias between them.

(4) The node $n_i$ keeps on adjusting its bargain proposal to maximize the utility function until a consensus is achieved.

A key issue in CoB is how to generate and update the bargain proposals. If the nodes know the qualities of wireless communication links, they will recommend the one with a good link to piggyback the messages. Meanwhile, the bidding price can be offered regarding how many nodes have lost the message. The more nodes that have lost the message, the higher bidding price will be offered for recommending other nodes to carry it for recovery. To address this issue, we develop the following two principles for generating and updating the bargain proposals.

*Principle 1.* The nodes with fewer stable communication links should be recommended for recovering the messages first.

The nodes with more stable communication links tend to have more options when carrying the messages for recovery. If these nodes are recommended for recovery first, they may choose to carry some messages that can also be recovered by other nodes, but fail to carry the messages that can only be recovered by themselves.

*Principle 2.* If the bargain proposal is different from that of others, the node should

(1) Choose the mainstreamed one as the new bargain proposal if it can recover the message with fewer forwarders, shorter time delays, or higher success probability

(2) Remain the bargain proposal unchanged if the mainstreamed one fails to recover one or more messages

Since the nodes can only acquire the local information, they may generate different bargain proposals for the message recovery. To eliminate the bargain bias, each node should update its bargain proposal to achieve a consensus. Note that some nodes have already achieved a local consensus (e.g., the mainstreamed bargain proposal can be regarded as a local consensus), and they have a higher probability to make an optimal bargain proposal as they have acquired more information via exchanging the bargain proposals. Therefore, if a node has generated a bargain proposal that is different from the mainstream one, it should update its bargain proposal to coincide with the mainstreamed one. However, it might be possible that the node has detected a lost message which fails to be detected by the nodes with the mainstreamed bargain proposals. Therefore, the node should check whether the mainstreamed bargain proposal can recover all the messages. If the mainstreamed bargain proposal fails to recover one or more messages, the node should remain its bargain proposal unchanged on recovering these messages, and thus the other nodes will be informed that the mainstreamed bargain proposal failed to recover all the messages.

The pseudocode of CoB is given in Algorithm 1, where each node $n_i$ exchanges its bargain proposal with other nodes for comparison and updating until a consensus is achieved. Upon broadcasting a packet, each node $n_i$ will infer the lost messages and qualities of wireless communication links in the broadcast. Then, it will generate a bargain proposal based on the inferred information to maximize the utility function. Upon receiving a packet from another node $n_j$, node $n_i$ will check each piggybacked message in the packet and cache it in the receiving buffer $F_i$. The node $n_j$'s bargain proposal in the received packet will be used to update node $n_i$'s bargain proposal if it is different from the mainstreamed one.

## 5. Theoretical Analysis

In CoB, each node generates bargain proposals based on its local information and then exchanges them with other nodes for negotiation and updating so that the bargain bias can be eliminated. In this section, we prove that the achieved consensus in CoB is Nash equilibrium (NE), and the bargain proposal at the NE is an optimal solution for the cooperative recovery of lost messages in the broadcast.

**Theorem 1.** *If a consensus is achieved in the cooperative bargain game, the combination of each node's bargain proposal is a Nash equilibrium to the objection function in equation (1).*

*Proof.* Suppose $\mathbb{R}^t = \{\mathcal{R}_1^t, \mathcal{R}_2^t, \ldots, \mathcal{R}_{|N|}^t\}$ is the set of recommending strategies generated by all the nodes at time $t$ and $\mathbb{P}^t = \{\mathcal{P}_1^t, \mathcal{P}_2^t, \ldots, \mathcal{P}_{|N|}^t\}$ is the set of bidding prices corresponding to each recommending strategy in $\mathbb{R}^t$. Denote the achieved consensus by $\mathbb{BP}^{t*} = \{B_i^{t*} = (\mathcal{R}_i^{t*}, \mathcal{P}_i^{t*}) \mid n_i \in N\}$. Define $\mathbb{R}^{t*} = \{\mathcal{R}_1^{t*}, \mathcal{R}_2^{t*}, \ldots, \mathcal{R}_{|N|}^{t*}\}$ and $\mathbb{P}^{t*} = \{\mathcal{P}_1^{t*}, \mathcal{P}_2^{t*}, \ldots, \mathcal{P}_{|N|}^{t*}\}$ as the joint recommended strategies and bidding prices at the consensus, respectively. According to the definition of Nash equilibrium, the achieved consensus $\mathbb{BP}^{t*}$ is a balance point where no node tends to change its bargain proposal or bidding price unilaterally, where each node's utility function has been locally maximized. Considering the group division in Figure 2 and the definition of rewards in equation (3), the sum of rewards at each node is zero as shown in the following equation:

$$\sum_{n_i \in N} w\left(B_i^{t*}\right) = \left(\theta_{13}^t - \theta_{35}^t\right) + \left(\theta_{35}^t - \theta_{57}^t\right) + \cdots + \left(\theta_{42}^t - \theta_{21}^t\right) + \left(\theta_{21}^t - \theta_{13}^t\right) = 0. \tag{5}$$

By substituting equation (5) into the utility function in equation (4), we can convert the objection function into the following equation:

//Upon broadcasting a packet:
  Infer the qualities of wireless communication links and the lost messages in the broadcast;;
  **for** *each lost message in the broadcast* **do**
      Recommend the node with the fewest stable communication links to piggyback the message
    **if** *the recommendation is different from the mainstreamed one* **then**
        Rank all the received recommendations on piggybacking this message, and choose the top-ranked one as the new recommendation;
        Broadcast a packet with the bargain proposal to inform other nodes of node $n_i$'s recommendations
//Upon receiving a packet from node $n_j$:
  **for** *each piggybacked message in the received packet* **do**
    **if** *there is no such a message in the receiving buffer $F_i$* **then**
      Cache it in the receiving buffer $F_i$.
    **else**
      Keep the one that has the latest time stamp.
  Cache node $n_j$'s bargain proposal for comparison;

Algorithm 1: CoB at each node $n_i$.

$$\sum_{n_i\in N}\frac{\sum_{n_j\in N} r_{ij}\left(\mathcal{R}_i^{t*}\right)}{|N|^2} = \sum_{n_i\in N}\left(u\left(B_i^{t*}\right)-w\left(B_i^{t*}\right)\right) = \sum_{n_i\in N} u\left(B_i^{t*}\right) - \sum_{n_i\in N} w\left(B_i^{t*}\right) = \sum_{n_i\in N} u\left(B_i^{t*}\right). \tag{6}$$

Since each node's utility function achieves a Nash equilibrium at $\mathbb{BP}^{t*} = \{(\mathcal{R}_i^{t*}, \mathcal{P}_i^{t*}) \mid n_i \in N\}$, the object function in equation (1) will also achieve a Nash equilibrium at $\mathbb{BP}^{t*} = \{(\mathcal{R}_i^{t*}, \mathcal{P}_i^{t*}) \mid n_i \in N\}$ based on the equivalence relationship as shown in equation (6).

Theorem 1 proves that the achieved consensus is a Nash equilibrium in the cooperative bargain game. However, it does not mean that the achieved Nash equilibrium is the optimal bargain proposal. In other words, maximizing $u(B_i^{t*})$ locally at each node is not equivalent to maximizing $\sum_{n_i\in N} u(B_i^{t*})$ globally. The following Theorem 2 proves that the Nash equilibrium achieved in the cooperative bargain game is an optimal solution to the objective function in equation (1). □

**Theorem 2.** *The combination of each node's bargain proposal at the achieved consensus in the cooperative bargain game is an optimal solution to the objective function in equation (1).*

*Proof.* Consider a bargain proposal $\mathbb{BP}^t = \{B_i^t = (\mathcal{R}_i^{t*}, \mathcal{P}_i^t) \mid n_i \in N\}$ in the cooperative bargain game. It cannot yield a higher value in the object function than the one at the Nash equilibrium, i.e., $\mathbb{BP}^{t*} = \{B_i^{t*} = (\mathcal{R}_i^{t*}, \mathcal{P}_i^{t*}) \mid n_i \in N\}$. Therefore, we can have the following inequality:

$$\begin{aligned}
&\sum_{n_i\in N} u\left(B_i^t\right) \le \sum_{n_i\in N} u\left(B_i^{t*}\right),\\
&B_i^t = \left\{\left(\mathcal{R}_i^{t*}, \mathcal{P}_i^t\right)\right\},\\
&B_i^{t*} = \left\{\left(\mathcal{R}_i^{t*}, \mathcal{P}_i^{t*}\right)\right\},\\
&\mathbb{BP}^t = \left\{B_1^{t*}, B_2^{t*}, \ldots, B_i^t, \ldots, B_{|N|-1}^{t*}, B_{|N|}^{t*}\right\},\\
&\mathbb{BP}^{t*} = \left\{B_1^{t*}, B_2^{t*}, \ldots, B_i^{t*}, \ldots, B_{|N|-1}^{t*}, B_{|N|}^{t*}\right\}.
\end{aligned} \tag{7}$$

Since the utility function $u(B_i^t)$ is strictly monotone decreasing of $w(B_t^i)$, we can have the following inequality:

$$\forall n_i \in N, \quad w\left(B_i^t\right) \le w\left(B_i^{t*}\right). \tag{8}$$

Suppose node $n_i$ is the heading nodes in the group $(n_i, n_j, \text{and}\, n_k)$, and then we can rewrite Inequality (8) into Inequality (9) based on the definition of $w(B_i^t)$:

$$\left(\mathcal{P}_i^t - \mathcal{P}_i^{t*}\right) + \left(\mathcal{R}_i^t - \mathcal{R}_j^{t*}\right)^T \operatorname{diag}\left(\mathcal{P}_i^t - \mathcal{P}_i^{t*}\right)\left(\mathcal{R}_i^t - \mathcal{R}_j^{t*}\right) \le 0. \tag{9}$$

Since Inequality (9) stands up for all bidding prices, we can have Inequality (10) by substituting $\mathcal{P}_i^t = 2\mathcal{P}_i^{t*}$ and $\mathcal{P}_i^t = 0$ into Inequality (9), respectively:

$$\mathcal{P}_i^{t*} + \left(\mathcal{R}_i^t - \mathcal{R}_j^{t*}\right)^T \operatorname{diag}\left(\mathcal{P}_i^{t*}\right)\left(\mathcal{R}_i^t - \mathcal{R}_j^{t*}\right) = 0. \tag{10}$$

Substitute equation (10) into equation (3), and the reward of node $n_i$ at the Nash equilibrium is calculated as follows:

$$w\left(B_i^{t*}\right) = \left(\mathcal{P}_k^{t*} - \mathcal{P}_j^{t*}\right)(1)^T. \tag{11}$$

Consider the definition of the Nash equilibrium again, and we can have another inequality (12) as below:

$$
\begin{aligned}
&\sum_{n_i \in N} u\left(B_i^{t'}\right) \leq \sum_{n_i \in N} u\left(B_i^{t*}\right), \\
&B_i^{t'} = \left\{\left(\mathscr{R}_i^{t'}, \mathscr{P}_i^{t'}\right)\right\}, \\
&B_i^{t*} = \left\{\left(\mathscr{R}_i^{t*}, \mathscr{P}_i^{t*}\right)\right\}, \\
&\mathbb{BP}^t = \left\{B_1^{t*}, B_2^{t*}, \ldots, B_i^{t'}, \ldots, B_{|N|-1}^{t*}, B_{|N|}^{t*}\right\}, \\
&\mathbb{BP}^{t*} = \left\{B_1^{t*}, B_2^{t*}, \ldots, B_i^{t*}, \ldots, B_{|N|-1}^{t*}, B_{|N|}^{t*}\right\}.
\end{aligned}
\tag{12}
$$

By substituting Inequality (11) into Inequality (12), we can have the following inequality:

$$
\begin{aligned}
&\sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^{t'}\right)}{|N|^2} + \left[\left(\mathscr{P}_i^{t'} - \mathscr{P}_j^{t*}\right) - \left(\mathscr{P}_j^{t*} - \mathscr{P}_k^{t*}\right)\right] \cdot (1)^T \\
&\quad + \left(\mathscr{R}_i^{t'} - \mathscr{R}_j^{t*}\right) \cdot \operatorname{diag}\left(\mathscr{P}_i^{t'}\right) \cdot \left(\mathscr{R}_i^{t'} - \mathscr{R}_j^{t*}\right)^T, \\
&\quad - \left(\mathscr{R}_j^{t*} - \mathscr{R}_k^{t*}\right) \cdot \operatorname{diag}\left(\mathscr{P}_j^{t*}\right) \cdot \left(\mathscr{R}_j^{t*} - \mathscr{R}_k^{t*}\right)^T \\
&\leq \sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^{t*}\right)}{|N|^2} + \left(\mathscr{P}_k^{t*} - \mathscr{P}_j^{t*}\right)(1)^T.
\end{aligned}
\tag{13}
$$

Since Inequality (13) holds for all values of $\mathscr{P}_i^{t'}$, we can obtain Inequality 14 by setting $\mathscr{P}_i^{t'} = 0$ in Inequality (13):

$$
\sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^{t'}\right)}{|N|^2} \leq \sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^{t*}\right)}{|N|^2}. \tag{14}
$$

Putting it in another way, Inequality (14) can be transformed into equation (15) as follows:

$$
\begin{aligned}
&\mathscr{R}_i^{t*} = \operatorname{argmax} \sum_{n_i \in N} \frac{\sum_{n_j \in N} r_{ij}\left(\mathscr{R}_i^{t}\right)}{|N|^2}, \\
&\text{s.t.} \quad B_i^{t*} = \left\{\left(\mathscr{R}_i^{t*}, \mathscr{P}_i^{t*}\right)\right\}.
\end{aligned}
\tag{15}
$$

Therefore, we prove that $B_i^{t*}$ is the optimal solution in the feasible solution space of equation (1), where the objective function reaches the maximum value at the achieved consensus. Putting it in another way, the Nash equilibrium $B_i^{t*} = \{(\mathscr{R}_i^{t*}, \mathscr{P}_i^{t*})\}$ is the optimal solution to the cooperative piggybacking problem in equation (1). □

## 6. Simulations

*6.1. Simulation Setup.* In this section, we evaluate CoB with some real-world Wi-Fi communication data traces in the Isti/rural dataset. It is compared with the following broadcast schemes in terms of MDR, time delay, and communication overhead:

(1) *Probabilistic Flooding* (*PF*). Each node forwards its received messages with a predefined probability (say $p = 50\%$ in our simulation) to improve MDR in the broadcast.

(2) *Retransmission-Based Recovery* (*RR*). The node carries a request list in its message to inform others which one it has lost. The source node, i.e., the one that generated this message, will make the retransmission until it is received by all the nodes.

(3) *Greedy Recovery* (*GR*). Each node carries a request list in its packets. It also carries some messages to help other nodes recover their lost ones. The more the nodes request for recovering a message, the higher priority it will be carried for recovery first.

(4) *Centralized Recovery* (*CR*). Assume there exists a central node that can obtain the global information in the network. The central node develops the recovery strategy by using the global information and then disseminates it to the other nodes. These nodes recover the lost messages following the strategy developed by the central node.

Isti/rural contains a set of real-world communication data traces which records the link qualities within a Wi-Fi network. Each node in the Wi-Fi network is equipped with a CNet CNWLC-811 IEEE 802.11b PCMCIA wireless card and a standard driver in the ad hoc mode. Fragmentation, RTS/CTS, retransmissions, and dynamic rate switching are disabled and each message is only broadcast once. This helps to sample the link qualities quickly and accurately compared to the setup where retransmission is enabled after each message loss. For each data trace in the Isti/rural, it records the status of message delivery (which can be regarded as the link qualities) in the broadcast, where the distance between the sending node and the receiving node is set as a fixed value. If a message can be received successfully, the link quality is marked as "1"; otherwise, it is marked as "0". Isti/rural is composed of a number of such data traces by varying the distance between the sending node and the receiving node with a difference of every 20 meters, starting from 40 to 300 meters. In our simulations, we set the gap between two adjacent spacer bars as 20 meters. For each wireless communication link in the broadcast, we allocate it with a unique data trace in the Isti/rural, where the gap between the two spacer bars along the wireless communication link is equal to the distance between the sending node and the receiving node in the data trace. If the wireless communication link is reliable, i.e., the status of message delivery in the allocated data trace is marked as "1," the simulator will deliver the message to the receiving node; otherwise, the simulator will discard the message. The TTL of messages is set as 100 milliseconds, and the nodes can carry at most two messages in each transmission.

*6.2. Broadcast Reliability.* As shown in equation (1), we calculate the MDR for each node in the broadcast, and define the average value of each message's MDR as the broadcast

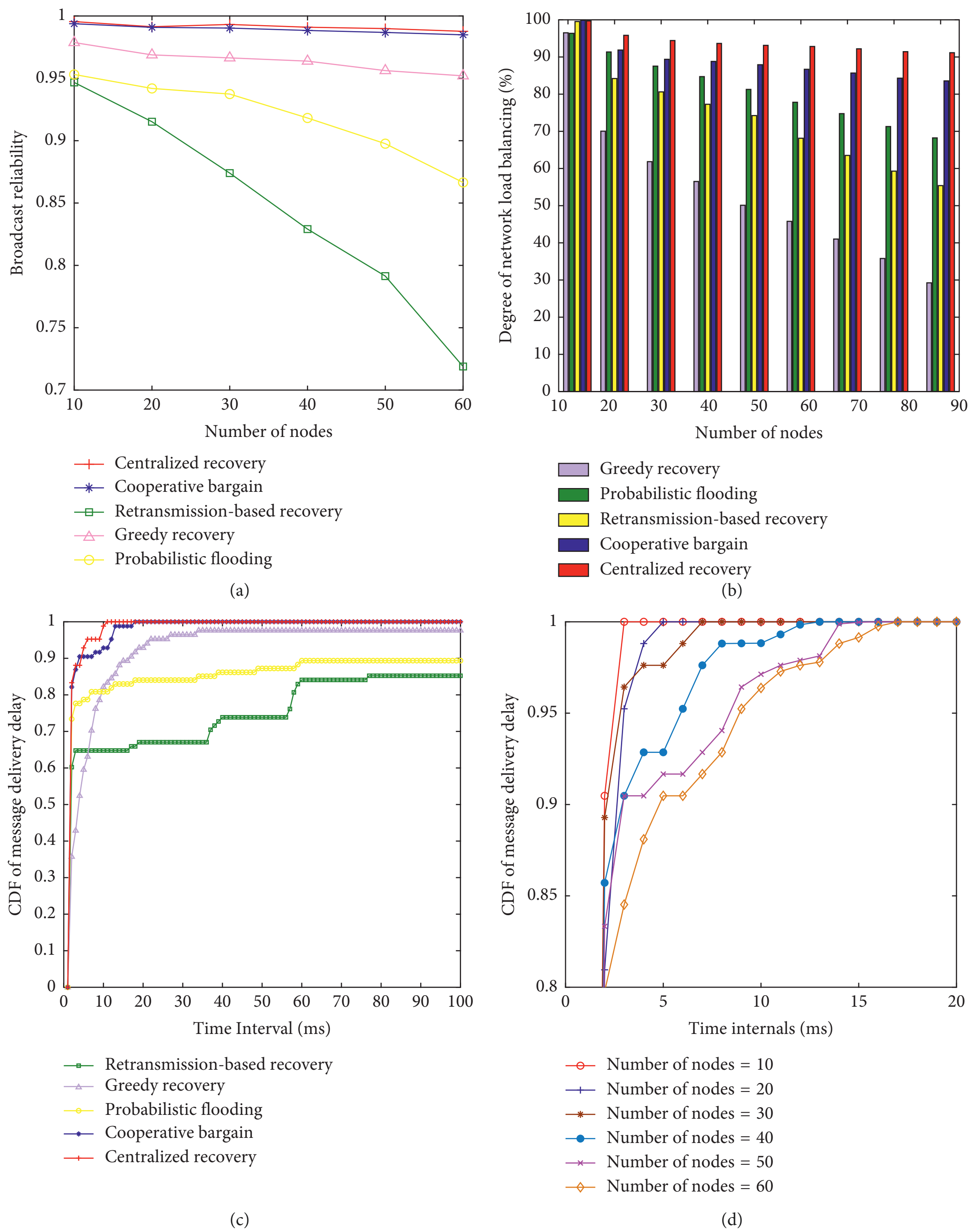


Figure 3: The performance evaluations on cooperative bargain. (a) Broadcast reliability. (b) Network load balancing. (c) Message delivery delay. (d) Recovery accuracy.

reliability. Figure 3(a) illustrates the MDR of different broadcast schemes when the number of nodes varies from 5 to 30. Since CP assumes all the nodes can acquire the global information, it has the highest MDR compared with other broadcast schemes. Note that the proposed CoB can achieve a comparable MDR to CP, only using each node's local information for the recovery of lost messages. Moreover, when the number of nodes increases, the MDRs of CoB and CP are basically unchanged. It indicates that both schemes are stable and robust, which are little impacted by the number of nodes. GP follows behind CoB with an average MDR of 96%, and it decreases slightly when the number of nodes increases. This mainly results from the greedy recovery of lost messages in GP, as the messages that are lost by

Table 1: The average number of transmitted packets for each successfully received message.

| | Number of nodes | | | | | |
|---|---|---|---|---|---|---|
| | 4 | 6 | 8 | 10 | 12 | 14 |
| CBP | 63 | 72 | 87 | 98 | 106 | 125 |
| CP | 87 | 111 | 125 | 169 | 234 | 271 |
| GP | 91 | 113 | 134 | 150 | 150 | 150 |
| SP | 181 | 285 | 337 | 377 | 436 | 475 |
| PF | 237 | 359 | 525 | 692 | 949 | 1157 |

more nodes will be carried with higher priority, which can benefit more nodes in the recovery of lost messages. PF has a lower MDR (around 92%) when the number of nodes is small, and it drops to 85% when the number of nodes increases to 30, as the redundant forwarding of messages may cause frequent data collisions and yield a high risk of overloading the communication channels in the broadcast. DR has the lowest MDR which drops significantly when the number of nodes increases from 5 to 30.

*6.3. Network Load Balancing.* Let $|N_i|$ be the number of messages transmitted by vehicle $v_i$ in the broadcast, and $|\overline{N}|$ is the average number of transmitted messages at each node in the broadcast. Then, we define the degree of network load balancing (DNLB) as $(\sum_{i=1}^{|N|} P_{\Psi_i})^2 / |N| \cdot \sum_{i=1}^{|N|} (P_{\Psi_i})^2$. If the nodes have transmitted the same number of messages in the broadcast, DNLB is maximized as 1. If one node has transmitted all the messages, whereas the other nodes have transmitted no messages, DNLB is minimized as $1/n$. As shown in Figure 3(b), CoB has a high DNLB which decreases slightly when the number of nodes increases. This results from the negotiation for eliminating the bias among the nodes where the task of message recovery is allocated regarding each node's local information. CP has a similar high DNLB as it assumes the central node can acquire the global information and thus make optimal recovery strategy with balanced network load. GP has the lowest DNLB as the nodes always tend to carry the same message for recovery, i.e., the one that is requested by most nodes, which will cause imbalanced network load in the nodes.

*6.4. Delay of Message Delivery.* We define the delay of message delivery (DMD) as the time interval between the instant when a message is generated and the instant when it is delivered or recovered at all the receiving nodes. Figure 3(c) shows the cumulative distribution function (CDF) of DMDs when the number of node is set as 30. CoB and CP have the shortest DMD in the recovery of lost messages, which are always smaller than 15 milliseconds. GP fails to recover all lost messages, and the CDF climbs to 0.98 when the TTL expires. PF only recovers 85% lost messages at the end of TTL, as all the nodes forward their received packets which will cause severe data collisions in the broadcast. DR has the worst performance which only recovers 80% lost messages. This is because the success probability of message recovery highly depends on the qualities of wireless communication links, and retransmission helps little in the recovery of lost messages if the qualities of wireless communication links remain unstable.

Another factor that may impact the DMD is the network size, i.e., the number of nodes in the network. When the number of nodes is small, the nodes tend to have a low probability of data collisions when accessing the communication channel for broadcasting messages. In contrast, when the number of nodes increases, the nodes suffer from a high probability of data collisions when accessing the communication channel for broadcasting. This will also deteriorate the success probability in the recovery of lost messages and yield a long time delay in DMDs. As shown in Figure 3(d), the DMDs for recovering lost messages within a 5-node network are averagely shorter than 20 ms, whereas the DMDs for recovering lost messages within a 30-node network rise to 75 ms. The DMDs increase when the number of nodes increases, but CoB can still recover most messages before the TTLs expire.

*6.5. Recovery Accuracy.* We define the recovery accuracy as the average number of transmitted messages for each received message, that is, the ratio between the total number of transmitted messages and the number of successfully received messages. As shown in Table 1, fewer messages are transmitted in CoB compared to that of CP. This is because the nodes in CoB work in a decentralized environment and they only use the local information to make cooperative piggybacking. In contrast, the central node in CP has to acquire the global information via exchanging messages with all the other nodes. After developing the optimal recovery strategy, the central nodes have to communicate with the other nodes again to disseminate the developed optimal recovery strategy, which will yield a large communication overhead in the broadcast. GP has transmitted more messages in the broadcast, and the number of transmitted packets increases when the network size becomes large. SP and PF have the worst performance in terms of network load, where 475 and 1157 packets are transmitted in each broadcast period, respectively, which may cause a high risk of data collisions in the broadcast.

## 7. Conclusion

This paper proposes a decentralized Cooperative Bargain scheme, namely CoB, to improve the broadcast reliability when disseminating messages in the reversible lanes. It aims to drive the spacer bars make consistent movement for the autonomous separation of traffic flows. To achieve this target, each active spacer bar is allowed to carry some received messages to help others recover the lost ones. Besides, a cooperative bargain game is formulated to allocate the task of message recovery regarding each spacer bar's local information. Each active spacer bar keeps on negotiating with others and updating its recovery strategy until a consensus is achieved, which is proved to be an optimal solution to the recovery of lost messages. Our future work is to improve the convergence speed, that is, minimizing the process of negotiation in the recovery of lost messages, and extend the proposed scheme to other ad hoc networks.

## Data Availability

The real-world Wi-Fi communication data trace (Isti/rural) used to support the findings of this study were supplied by RAWDAD (a community resource for archiving wireless data at Dartmouth, http://www.crawdad.org/isti/rural) under license and so cannot be made freely available. Requests for access to these data should be made to the web administrator via http://crawdad@crawdad.org. Isti/rural contains a group of dataset of transmission distance vs. packet loss measurement on a Wi-Fi network in rural areas. A series of measurements for relating transmission distance and packet loss on a Wi-Fi network in rural areas are conducted to propose a model that relates distance with packet loss probability. The data/time of the measurement was released on 2007-12-19, and the date/time of measurement started on 2005-03-25 and ended on 2006-04-23.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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*Research Article*

# A Multicriteria Approach to Support Task Allocation in Projects of Distributed Software Development

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Organizations are increasingly investing in Distributed Software Development (DSD) over the years. A typical decision-making problem in the distributed scenario consists of deciding which team should be allocated each task. That decision takes into account a relative degree of subjectivity. That setting is suitable for applying Verbal Decision Analysis (VDA). This paper introduces an approach to support the allocation of tasks to distributed units in DSD projects, structured on the hybridisation of methods of Verbal Decision Analysis for classification and rank ordering applied to influencing factors and executing units. Firstly, a review of the literature was conducted aiming to identify the approaches to support the allocation of tasks in DSD contexts. Then, an approach was developed by applying VDA-based methods for classification and ordering. Bibliographic research and the application of surveys with professionals allowed identifying and characterising the main elements that influence task assignment in DSD projects. Afterwards, experiences were carried out in five real-world companies. In the end, the proposed approach has been submitted to the evaluation by the professionals of the participating companies and by some project management experts. The proposed approach comprises a workflow containing responsible actors and descriptions of the activities. Automated tools are also employed in automating the implementation of the approach. After applying the approach in five companies, task assignment recommendations are presented in groups for each company, according to the task type, i.e., requirements, architecture, coding, and testing, ranging from the most to the least preferable office. Results of the experiences and evaluations held during this work present evidence that the proposed approach is flexible, adaptable, and easy to understand and to use. Moreover, it helps to reduce decision subjectivity and to think of new aspects, supporting the task allocation process in DSD.

## 1. Introduction

Software development can be structured in several activities, such as requirements, design, architecture, and programming. Companies that have a mature and organized development process can direct activities to be developed in different locations, taking advantage of the skills of local teams.

A typical decision-making problem in the distributed scenario consists of deciding which team should be allocated each task. The specific literature mentions some methodologies and models that address the decision problem on the division of tasks in distributed projects using diverse strategies. Some approaches consist of simple tools whereas others consist of complex systems. Some solutions define simple procedures while others comprise whole processes. The granularity of the object to be allocated also varied widely, from single tasks to full stages (sets of tasks).

However, few models use multicriteria approaches and, when they do, they usually focus on quantitative or numerical aspects, often requiring a heavy mathematical framework. Furthermore, only a small set of criteria was usually used to decide on the allocation of tasks, especially the job costs. Team skills and experience, and cultural issues, among other relevant factors, have been often neglected [1]. In practice, what happens is that, most times, the professional responsible for the project takes into account only their experience and some knowledge about the project and the teams to decide how the tasks will be distributed, that is, in a subjective and restricted way.

Situations like these bring some degree of subjectivity to the decision-making procedure. This scenario is suitable for Verbal Decision Analysis, which is a methodology that employs qualitative analysis to solve multicriteria problems [2]; that is, the methods that integrate the VDA framework address the subjectivity of the criteria. In VDA methods most decision-making problems can be represented in a qualitative way. In addition, the methods use the verbal representation of the problems to carry out the decision process [3].

Considering the given scenario, in this paper we introduce DiVA, an approach to support the assignment of tasks to distributed teams Distributed Software Development Projects, structured on the hybridisation of methods of Verbal Decision Analysis for classification and rank ordering, applied to influencing factors and executing units. As an executing unit, we understand individuals, teams, departments, offices, or any other entity that develops the work itself.

The remainder of the work is organized as described next. In Section 2, some theoretical reference necessary to support the understanding of the research is provided. Section 3 presents the main works found in the literature related to the subject of this paper. Section 4 introduces and describes the proposed task allocation approach to DSD projects. In Section 5, we report the experiences of using the proposed approach performed in real companies, as well as providing the results, evaluations and limitations of the work. Section 6 contains the conclusions and some suggestions for further works.

## 2. Theoretical Reference

*2.1. Task Allocation in Distributed Software Development.* Organizing tasks is one of the main assignments of a project manager. He or she is responsible for determining which tasks are essential to the completion of each step of the process and for allocating those tasks to the team members of different backgrounds and experience. Workload, expertise, maturity, and profile of each team member are criteria commonly taken into account in the task allocation process.

In this research, we adopt the PMBOK (Body of Knowledge in Project Management [4]) definition of "task", which is a responsibility that can be allocated within the project management plan in such a way that the designated resource is subject to the obligation to perform the task requirements. PMBOK also defines task assignment in software development as an activity that settles the way in which the tasks will be carried out and how and which professionals will be assigned the tasks, considering the imposed restrictions [4]. These tasks set out what should be done to achieve not only the project goals but also the organization's objectives [5].

When we identify tasks in distributed projects, we usually think of the constraints, whether technical, social, or professional, related to assigning tasks to distributed groups. In other words, the task allocation planning should consider people's characteristics (such as personal skills, for example) as well as the characteristics of relationships among distributed teams (such as cultural differences and communication issues). In addition, the characteristics of the project or the product to be developed, such as complexity, dependencies, and abilities necessary for its development, cannot be ignored. [1].

The task allocation is a crucial step for project management, mainly if it is a distributed project. The allocation of tasks to distributed teams should simultaneously match the right task with the right team in the correct order while defining the way to carry out the tasks. Moreover, assuming that every project deals with limited resources, such as people and time, to accurately plan, project managers must take into account (a) the attributes of the teams, (b) the features of the products that will be build, (c) the properties of the tasks and the relationships between them, and, finally, (d) the organization's objectives [5]. Thus, it is critical to map the main aspects that influence the assignment of tasks in distributed projects. We will call such aspects "influencing factors", and we will deal with them later.

*2.2. Verbal Decision Analysis.* Decision-making is an everyday activity in people's and companies' lives. To make an assertive decision, we evaluate a problem by judging a set of characteristics or attributes; i.e., our analysis addresses many factors, which we call criteria. Decisions can have a considerable impact on the business context, as choosing the wrong alternative can lead to loss of time, money, and resources, causing detrimental effects for the company. Such management decisions usually take into account several factors. For these cases, it is advisable to adopt methodologies to support the decision-making process.

The decision-making process involving the analysis of alternatives from various points of view is called multicriteria decision analysis. There are many methodologies that support multicriteria decision analysis [6]. Such methodologies help to increase the confidence of the decision-maker as they promote the generation of knowledge about the decision-making context [7].

Given this perspective, Verbal Decision Analysis (VDA) is an approach that employs qualitative analysis for solving multicriteria problems [2]. In VDA, the problems assume a verbal representation. The VDA methods are applied for classifying and rank ordering the alternatives. Some examples of VDA's classification methods are ORCLASS [3], SAC [8], DIFCLASS [9], and CYCLE [9]. Examples of VDA's ordering methods are PACOM [3], ARACE [10], STEPZAPROS, ZAPROS-LM, ZAPROS III, and ZAPROS III-i [11].

There are many studies describing the application of VDA methods in real-world problems. Some of them are provided next. Machado et al. [12] used VDA to help the selection of specific CMMI practices. Mendes et al. [13] and Tamanini et al. [14] applied VDA in digital TV scenario. Machado [15] developed a hybrid model of VDA for selecting project management approaches and, in [16], Machado et al. employed VDA for helping to choose the best SCRUM practices. Tamanini et al. [17] created a VDA-based model to cashew chestnut industrialization process. Tamanini et al. ([18, 19]) and Castro et al. ([20–22]) carried out studies using VDA to support the diagnosis of the Alzheimer's disease. Gomes et al. [23] applied VDA in the marketing business. Barbosa et al. [24] described the use of VDA to prioritize software requirements. Finally, Simão Filho et al. [25] employed VDA in project portfolio ranking.

ORCLASS (ORdinal CLASSification) is a method of the VDA framework that aims to classify the alternatives in a set. For this, it is necessary that these alternatives be classified by the decision-maker in a small number of decision groups or classes, often two. The first group will contain preferable alternatives, and the second group will include nonpreferable alternatives [3]. In addition, a set of criteria and their respective values should be established to allow the evaluation of alternatives.

Some advantages are associated with the ORCLASS method, which we highlight below [3]. First, the comparison of the values is performed verbally since the preferences elicitation is performed in the native language of the decision-maker. Second, the classification rule is organized according to the presentation of the most significant combinations of criteria for the decision-maker. Such strategy decreases the number of questions that the decision-maker would be required to answer during the preferences elicitation. Third, it is possible to check the preferences consistency of the decision-maker. Finally, it is possible to formulate a verbal explanation for the decisions from the classification rules obtained. Such advantages motivated its choice for this study.

In order to make the application of the ORCLASS method easy, the tool ORCLASSWEB was developed and then described by Machado et al. [26]. It can be accessed through the link http://www2.unifor.br/OrclassWeb. ORCLASSWEB aims to make the process of alternatives comparison automatic, providing a result for the problem to the decision-maker, following the ORCLASS method specification. Another factor that favoured the choice of ORCLASS for this study was the existence of an automated tool to run the model.

The ZAPROS III-i method also belongs to the VDA framework. It integrates the ZAPROS family, and its function is to rank alternatives to problems with a small set of criteria and criteria values and a high number of alternatives. The ZAPROS III-i method adopts the same idea of criteria and criteria values employed by the ORCLASS method to assess the alternatives. The method uses values representing the distances between judgments on the scale of the same criteria or between two criteria to elicit the preferences of the decision-maker. A scale of preferences is then built in such a way that allows the alternatives comparison.

The main advantages of the ZAPROS method are emphasised next [27]. First, during the preferences elicitation stage, the model exposes issues that are apprehensible to the decision-maker, according to the criteria values. This process has psychological validity since it accepts the limitations of the human information processing system. Such property consists of the most significant benefit of the method. Besides, the method can detect contradictory inputs from the decision-maker and request a solution for them. Finally, the ZAPROS provides all qualitative comparison information in a language that is natural to the decision-maker. Such advantages influenced its choice for this study.

To facilitate the application of the ZAPROS III-i method and to allow it to be executed consistently, the ARANAÚ tool was implemented ([28–30]). Although initially ARANAÚ has been built for the ZAPROS III method, an updated version for the ZAPROS III-i method was used in this work. As in the previous method, the existence of an automated tool to run the model contributed to its choice.

## 3. Related Work

Today, more and more companies are conducting projects in a distributed way. Assigning tasks is a crucial activity for projects in general, especially in a distributed environment. Much research on the distribution of tasks in DSD has been conducted recently in order to clarify this issue, its difficulties, and its challenges. In this perspective, some approaches, methodologies, and models for distributing tasks considering remote teams were created.

Aiming to identify the related works to the present research, we have developed a systematic review of the literature, which is described in detail in [31]. Our main objectives in conducting the systematic review were (i) to discover which approaches were used to allocate tasks in projects of DSD; (ii) to know whether the approach was based on qualitative multicriteria decision-making methods; (iii) to find out if the approach was based on VDA methods; and (iv) to identify if the study mentioned some factor or criterion that influenced the assignment of tasks in DSD scenarios.

To guide the literature review, based on the objectives cited above, we formulated a series of questions, for which we seek answers in the studies researched. Table 1 shows the research questions. "PQ" identifies primary questions, and "SQ" identify secondary questions.

As a result of the literature review, some relevant papers were selected, which are briefly described next. Some of the most relevant works are those of Lamersdorf and his colleagues. In [1], Lamersdorf et al. analysed different approaches to the distribution of responsibilities. The analysis included various approaches in the areas of distributed development, distributed generation, and distributed systems. As a conclusion, they pointed to the Bokhari algorithm as a high applicability model in DSD. Lamersdorf and Münch [32] presented TAMRI (Task Allocation based on Multiple cRIteria), an approach structured on the Bokhari algorithm and Bayesian networks that adopts various criteria and influences factors to assist the systematic assignment of tasks in DSD projects.

Ruano-Mayoral et al. [33], in turn, proposed a methodological framework to distribute work packages to professionals in global software development (GSD) projects. The proposed approach was composed of two stages: definition and validation. The paper also provided the results of the application of the proposed approach. In [34], Almeida et al. introduced McDSDS (Multicriteria Model for Planning Distributed Software Development Projects with Scrum). The proposed model involves a multicriteria decision applied to planning and fine-tuning project plans. It was based on cognitive mapping and MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique) [6]. In [35],

Table 1: Codes used in the data consolidation table.

| Question Id. | Description |
|---|---|
| PQ1 | What approach was used or suggested for task assignment in DSD projects? |
| PQ2 | Is the approach based on qualitative multi-criteria methods of decision-making? |
| PQ3 | Is the approach based on any method of Verbal Decision Analysis framework? |
| PQ4 | Does the study mention any factor or criterion that influences the assignment of tasks in distributed software environments? |
| SQ1 | Does the approach describe a process? |
| SQ2 | Does the approach have any automated support tool? |
| SQ3 | Does the approach apply influencing factors extracted from the specific literature? |
| SQ4 | Can the approach be instantiated from an organisation standard? |
| SQ5 | Does the approach reuse knowledge? |
| SQ6 | Does the approach presuppose consensus meeting? |
| SQ7 | Does the approach presuppose feedback of results? |
| SQ8 | Does the approach allow using a subset of the whole process? |
| SQ9 | Does the approach allow modifying the main parameters (allocation object and executing unit)? |
| SQ10 | Has the approach been applied in real-world situations? |
| SQ11 | Has the approachbeen evaluated by users or professionals? |
| SQ12 | Has the approachbeen evaluated by knowledge area experts? |

Almeida et al. reported the application of the McDSDS in the selection of DSD Scrum-based project plans, whose chances of success are higher.

Pedras et al. ([36, 37]) introduced DIMANAGER, a tool integrated to DiSEN environment (Distributed Software Engineering Environment) ([38, 39]) whose goal is to assist the choice of team members according to their skills, knowledge, and availability. DIMANAGER used Fuzzy Logic to quantify the identified criteria.

Another model was developed by Setamanit et al. ([40–42]), who described an approach that identifies dynamic and discrete factors of a DSD domain and produces data regarding the dedicated resources, productivity, coordination, and communication throughout the project. By using the model, it is possible to configure several remote sites and their procedures involving several steps and assignment policies, enabling make the best choice to the assignment systematics.

Prikladnicki et al., in their turn, introduced MuNDDoS ([43, 44]) a Reference Framework for DSD. The authors created a project allotment flow that selects projects to be conducted in each remote office, taking into account the allocation strategy established by the company.

In [45], Jalote and Jain described a 24-hour Development Method that adopts a Directed Acyclic Graph (DAG). In such a DAG, the vertices symbolise the tasks, and the edges denote the current operational dependencies. Besides, the method involves three sets of resources, denoting professionals from each remote office related to each fraction of 8 hours of work. By using the critical path model, the method performs the attribution aiming to shorten the duration of the project.

Mullick et al. [46] introduced the Global Studio Project, a distribution model designed to optimise the work allocation. In such a model, groups of professionals from several universities worldwide execute distributed development methods and techniques. All development work is divided into related work packages along with the background needed to perform them and their temporal dependencies. From this data and the professionals' knowledge, the packs are allocated manually.

In [47], Mak and Kruchten developed NextMove, an approach structured on object-oriented process modelling and project management practices to support the coordination and task assignment for remote professionals in an agile development scenario. NextMove applies the AHP (Analytic Hierarchy Process) approach to indicate the most suitable professional to receive a task according to their abilities.

Barcus and Montibeller [48] presented a real case study wherewith they constructed a multicriteria approach to assist the distributed tasks assignment decision for a global software company. The approach was formulated with the support of several software development project managers, combining decision conferencing and value assessment of several factors. The approach encompasses software engineering factors as well as strategic and non-strategic issues such as professionals' motivation and possibilities for education.

Finally, dos Santos et al. [49] proposed a recommendation framework for assigning development teams in distributed projects of software product lines. The framework consists of an activity flow that, in the end, indicates the best recommendation for the allocation. They reported the application of the framework in real situations.

After analysing the works in detail, we elaborated Table 2, which provides an overview of the papers as well as the answers to the research questions. Cells marked with "√"

Table 2: Mapping of approaches and research questions.

| Approach | PQ1 | PQ2 | PQ3 | PQ4 | SQ1 | SQ2 | SQ3 | SQ4 | SQ5 | SQ6 | SQ7 | SQ8 | SQ9 | SQ10 | SQ11 | SQ12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TAMRI [32, 58] | An approach structured on the Bokhari algorithm and Bayesian networks and adopts various criteria and influence factors to assist the systematic assignment of tasks in DSD projects. | - | - | √ | √ | √ | √ | √ | √ | - | - | √ | √ | √ | √ | √ |
| Ruano-Mayoral's Methodological Framework [33, 50] | A methodological framework to distribute work packages to professionals in GSD projects. | - | - | √ | √ | √ | √ | √ | - | - | - | - | √ | √ | √ | √ |
| McDSDS [34, 35, 71] | A multi-criteria decision approach for planning DSD projects with Scrum. | √ | - | √ | - | √ | √ | - | - | - | - | - | - | √ | - | - |
| DIMANAGER [36–39] | A tool integrated to DiSEN environment to assist the choice of team members. It uses Fuzzy Logic as its basis. | - | - | √ | - | √ | √ | - | - | - | - | - | - | √ | √ | √ |
| GSD Simulation Model [40–42] | An approach that identifies dynamic and discrete factors of a DSD domain and produces data regarding the involved resources, productivity, coordination, and communication throughout the project. | - | - | √ | - | - | √ | √ | √ | - | √ | √ | √ | - | - | - |
| 24 hours Development Model [45] | A method that adopts a DAG Directed Acyclic Graph) and uses the critical path model. It performs the attributions aiming to shorten the duration of the project. | - | - | √ | - | - | √ | √ | - | - | - | - | √ | √ | - | - |
| MuNDDoS [43, 44] | A reference model for DSD, which involves a project allotment flow that selects projects to be conducted in each remote office, taking into account the allocation strategy established by the company. | - | - | √ | √ | - | √ | √ | √ | √ | √ | √ | √ | - | - | - |

Table 2: Continued.

| Approach | PQ1 | PQ2 | PQ3 | PQ4 | SQ1 | SQ2 | SQ3 | SQ4 | SQ5 | SQ6 | SQ7 | SQ8 | SQ9 | SQ10 | SQ11 | SQ12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Global Studio Project [46] | A distribution model designed to optimize the work allocation. All development work is divided into related work packages along with the background needed to perform them and their temporal dependencies. | - | - | √ | - | √ | √ | - | - | - | - | √ | √ | - | - | - |
| NextMove [47] | An approach structured on object-oriented process modelling and project management practices to support the coordination and task assignment for remote professionals in an agile development scenario. It uses AHP model as its basis. | - | - | √ | - | √ | √ | √ | √ | - | √ | - | - | - | - | - |
| Barcus and Montibeller [48] | A multi-criteria approach to assist the distributed tasks assignment decision, which combines decision conferencing and value assessment of several factors. | √ | - | √ | √ | - | √ | √ | √ | - | √ | - | √ | - | - | √ |
| Recommendation Framework for allocation of dev teams in dist. proj. SPL [49] | A framework that gives suggestions for team allocation in distributed Software Product Lines projects. | - | - | √ | √ | - | √ | √ | √ | - | √ | √ | √ | √ | - | - |

indicate that a positive answer to the corresponding question was found in the investigated works. Cells filled with "-" denote that we could not conclude anything about the research question in the works consulted. It is worth emphasising that our evaluation considered the information that was published and accessible throughout this research. Also, the author made his interpretations when he did not find explicit information in the texts read.

From these studies, we have noted some variation in the granularity of the task to be allocated. Some approaches consider individual tasks, such as [38, 45, 47, 48], while others focus on groups or packages of tasks ([33, 48]), or even entire projects, such as [43].

We also detected variations about to the target of the assignment. Some authors have explored the allocation of tasks to individuals [38, 45, 47], others consider teams, offices, or branches ([1, 34, 40, 43, 48]).

Some works have elaborated abstract models, such as reference models and frameworks ([43, 50]), while others implemented software tools from the proposed models ([36]).

It is worth noting that considering the works analysed, only three presented approaches based on multicriteria decision-making methods ([34, 47, 48]).

Nevertheless, only two studies used qualitative analysis as a basis, ([34, 48]), and none considered Verbal Decision Analysis methods. Besides the above-listed studies, other works did not present approaches but referred to some factors or criteria that influence or affect the allocation of tasks in DSD. Among them, we highlight [51–57].

Considering the fifteen objective questions assessed in this study (three primary questions, PQ2 to PQ4, and twelve secondary questions), PQ4 and SP3 questions, which concerned the influencing factors for task assignment in DSD projects, were successful for all the investigated approaches. Meanwhile, only one approach addressed the question SQ6 (consensus meeting), and no approach satisfied the question P3 (structured on some VDA method). Table 3 details the performance of the issues investigated.

From this analysis, we have noted that some works researched have developed multicriteria models, but generally considering the quantitative or mathematical approach. Moreover, in most cases, a small number of elements were applied to guide the attribution of tasks such as labour expenses, mainly. Thus, we could identify gaps in the set of existing approaches for task allocation in DSD that allowed us to drive some research to use multicriteria methods based on Verbal Decision Analysis. Moreover, many influencing factors were identified in the DSD task allocation decision process, which may be useful for future works. Besides, several aspects have been analysed in the existing approaches that have enabled us to think of some desirable features for a new approach in this field, such as the issues addressed in the secondary questions.

*3.1. Influencing Factors for Task Allocation in DSD.* After the analysis described in the previous subsection, it was possible to establish the main factors that impact or influence the allocation of tasks in DSD projects. This work was mainly based on the studies of Lamersdorf et al., such as [1, 58, 59]. They carried out literature studies regarding the distributed scenario and explored the criteria or factors that affect the activity of task allocation in GSD. Other works were also used as a basis for composing the relation of influencing factors in the allocation of tasks used in this approach, such as [5, 51–57]. In the end, we have selected the fifteen most cited factors in these works.

Table 4 lists the fifteen factors selected. These factors will be an important part of the approach presented in this paper.

## 4. DiVA—An Approach to Support the Allocation of Software Development Tasks to Distributed Teams Based on Verbal Decision Analysis

In this section, we introduce DiVA, an approach to support the allocation of software development tasks to distributed teams based on Verbal Decision Analysis. The approach is based on Verbal Decision Analysis methods for classification and ordering. For the reasons discussed previously, the methods adopted are ORCLASS for classification and ZAPROS III-i for ordination. These methods work with some common concepts and have already proven successful in their hybridisation, as shown in previous works such as [16, 60]. Preliminary works on VDA application to the problem of task allocation in DSD context can be found in [61–67].

The approach is divided into four main stages, namely, context characterisation, ranking of the influencing factors, ranking of the executing units, and evaluation and decision. Context characterisation stage is responsible for the first definitions of the basic concepts, such as tasks, criteria, criteria values, and preference groups. In the second stage, ranking of the influencing factors, the VDA methods are employed to classify and order the influencing factors considering the selected criteria and criteria values, taking into account the types of tasks chosen. In the third stage, ranking of the executing units, the preferable influencing factors identified in the previous stage are now converted into criteria for evaluating the possible destination of the tasks, which may be offices, departments, or teams. Finally, in the last stage, evaluation and decision, restrictions, conditions, and limiting factors are identified, and the results are evaluated taking these issues into account. After the evaluation, the decision is made according to the process of each organization. Figure 1 shows a procedural view of DiVA with actors responsible for each activity. The process shows the activities grouped in their respective stages. Note that depending on the type of data the company already has, it may choose to start the process from stage 2 or stage 3, not necessarily from stage 1. Likewise, depending on the outcome of the evaluation of the data and the decisions made, it is possible to feedback the approach, returning to stage 1, 2, or 3. The company can also choose to instantiate the flow once each cycle (semiannual, annual), and then the company can only calibrate the model from the data feedback. A more detailed description of DiVA stages and activities is provided next.

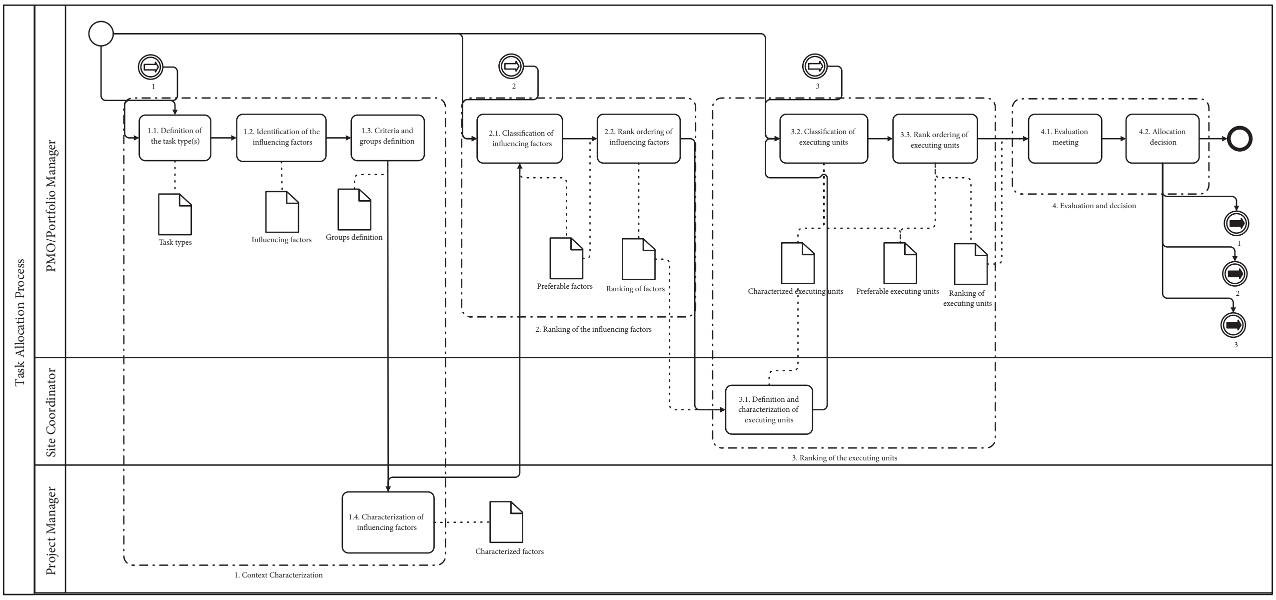



Figure 1: The process of the DiVA approach.

Table 3: Total approaches that scored for each question.

| Q. Id. | Description | Count |
|---|---|---|
| PQ4 | Does the study mention any factor or criterion that influences the assignment of tasks in distributed software environments? | 11 |
| SQ3 | Does the approach apply influencing factors extracted from the specific literature? | 11 |
| SQ4 | Can the approach be instantiated from an organisation standard? | 8 |
| SQ9 | Does the approach allow modifying the main parameters (allocation object and executing unit)? | 8 |
| SQ2 | Does the approach have any automated support tool? | 6 |
| SQ5 | Does the approach reuse knowledge? | 6 |
| SQ10 | Has the approach been applied in real-world situations? | 6 |
| SQ1 | Does the approach describe a process? | 5 |
| SQ7 | Does the approach presuppose feedback of results? | 5 |
| SQ8 | Does the approach allow using a subset of the whole process? | 5 |
| SQ12 | Has the approachbeen evaluated by knowledge area experts? | 4 |
| SQ11 | Has the approachbeen evaluated by users or professionals? | 3 |
| PQ2 | Is the approach based on qualitative multi-criteria methods of decision-making? | 2 |
| SQ6 | Does the approach presuppose consensus meeting? | 1 |
| PQ3 | Is the approach based on any method of Verbal Decision Analysis framework? | 0 |

(1) Context Characterisation: This stage comprises four activities, which are detailed below:

(1.1) Definition of the Task Type(s): Define the work object of the approach, i.e., which tasks, type of tasks, or groups of tasks that will be analysed for allocation.

(1.2) Identification of the Influencing Factors: Identify which factors influence the distribution of tasks in the context being analysed.

Table 4: Influencing factors on task allocation in DSD projects.

| ID | Factors | Description |
|---|---|---|
| Factor1 | Technical skills | Knowledge and ability on techniques, programming languages, frameworks, tools, APIs, necessary for the professionals to carry out the task. |
| Factor2 | Knowledge in business | Knowledge of the teams about the area, domain or business of the clients. |
| Factor3 | Project manager maturity | Experience, background, and maturity of project managers in their profession |
| Factor4 | Proximity to the customer | The office whose team will perform the task is positioned geographically close to the client. |
| Factor5 | Low turnover rate | The turnover rate of office employees is low, that is, typically in the branch, there are few changes in teams. |
| Factor6 | Team availability | Teams seek to be free, available to perform tasks. |
| Factor7 | Team maturity | Teams are mature / experienced concerning the task being performed. |
| Factor8 | Team personal trust | Managers and colleagues believe and trust in themselves and each other. |
| Factor9 | Same time zone | The various teams associated with the task execution work in the same time zone. |
| Factor10 | Cultural similarities | The various teams associated with the task execution share the same cultural aspects. |
| Factor11 | Team willingness | Teams are motivated, excited and interested in the work. |
| Factor12 | Low labour cost | The cost of professionals in the office is low/attractive. |
| Factor13 | Maturity in the process | The teams are mature and experienced concerning the software development process adopted. |
| Factor14 | Language fluency | The teams have fluency in the foreign languages commonly used in the office. |
| Factor15 | Good communication infrastructure | The office has a good communication infrastructure (speed, availability, redundancy, among others). |

(1.3) Definition of Criteria and Preference Groups: Determine the criteria and their respective values to be used to assess the influencing factors in task assignment. Moreover, define the preference groups to categorize the influencing factors. The groups usually defined are (i) the preferable and (ii) the nonpreferable factors.

(1.4) Characterisation of Influencing Factors: Characterise the influencing factors according to the criteria and criteria values, by task group or task. This means that the company must evaluate the factors against the criteria chosen.

(2) Ranking of the Influencing Factors: This stage consists of two activities, which are described next:

(2.1) Classification of Influencing Factors: Use the ORCLASS method to classify the influencing factors identified in Activity (1.2) into the preference groups defined in Activity (1.3). The preferable influencing factors, properly characterised (as a result of Activity (1.4)), will be processed by the ORCLASS method in this step. The influence factors classified as nonpreferable should be discarded.

(2.2) Rank Ordering of Influencing Factors: Use the ZAPROS III-i method to rank order the preferable influencing factors (those from the group (i) resulting from the previous Activity (2.1)), using the same factors characterisation generated in the Activity (1.4).

(3) Ranking of the Executing Units (Individuals/Offices/Teams): This stage is composed of three activities, which are detailed next:

(3.1) Definition and characterisation of the executing units: After identifying the main influencing factors in task assignment in DSD projects, the approach now considers the influencing factors as criteria and the executing units (company offices or teams) as the alternatives for the VDA methods. In this activity, relations must be established between the influencing factors and the executing units; that is, the company's professionals should evaluate the offices, teams, or other executing units regarding the influencing factors.

(3.2) Classification of the Executing Units: Use the ORCLASS method to classify the executing units (offices/teams) into preference groups (defined in the Activity (3.1)), usually two: (i) the preferable and (ii) the nonpreferable ones. As in the previous stage, nonpreferable executing units should be discarded.

(3.3) Rank Ordering of the Executing Units: Use the ZAPROS III-i method to rank order the preferable executing units (those from the group (i) resulting from the previous Activity (3.2)), using the same units characterisation generated in the Activity (3.1).

(4) Evaluation and Decision: This stage consists of two activities, which are described below:

(4.1) Evaluation Meeting: The portfolio management team or equivalent meets to discuss the results of the approach, assessing any restrictions that may exist. Results may change as a consequence of this activity.

(4.2) Allocation Decision: Finally, decisions about task allocation are made, and tasks are assigned to the responsible teams according to the organization's process or procedure.

The next section reports on the experiences of use carried out with the objective of evaluating the proposed approach.

## 5. Some Experiences with the Proposed Approach

*5.1. Preparation.* To evaluate DiVA and obtain indications about how it supports organizations in the allocation of tasks in DSD, we conducted some experiences. The application of DiVA was carried out in five real-world companies. After conducting the experiences of use, we chose to evaluate the application of the approach qualitatively, according to the stakeholders' perception, collected through questionnaires. Note that the study scenarios can be classified as in vivo [68], that is, a validation carried out in real development environments with the supervision or participation of researchers and professionals.

The purpose of the experiences of use was, for each company, to establish rankings of the most recommended executing units to assign a particular type of software development task. For this scenario, some assumptions have been made:

(i) We considered the companies' offices as the executing units, that is, the entities that would perform the tasks themselves;

(ii) Four different types of tasks were taken into account, namely: requirements-related tasks, architecture/design-related tasks, implementation-related tasks, and testing-related tasks.

Due to time restrictions of the companies' professionals and to generate a knowledge base to be used in future instantiations of the approach, we chose to structure the application of the approach considering two phases. The first phase, which covered the stages "Characterisation of the Context" and "Ranking of the Influencing Factors", was common to all participating companies. At the end of the first phase, the approach generated four ordered lists of influencing factors, i.e., one list for each task type (requirements, architecture/design, and implementation/coding and testing).

The second phase involved the stage of "Ranking of the executing units" for the participating companies. In this case, the application was individualized per company. The stage "Evolution and Decision" was not carried out as part of these experiences of use due to the obvious difficulties of putting into practice within the companies. Table 5 shows the details of the strategy for the application of the approach.

We sought to select companies with different profiles, with subsidiaries in various cities, and even in several countries. The size of the companies ranged from medium to large. Figure 2 shows the geographic distribution of the companies' offices involved in the research.

### 5.2. Execution

*5.2.1. Context Characterisation.* In order to perform Stages 1 and 2, the influencing factors previously identified and exposed in Table 4 served as alternatives for the classification and ordering VDA methods (ORCLASS and ZAPROS III-i).

For this activity, we followed the PMBOK's concepts [4] to help us choose the criteria because it has great acceptance in the software industry worldwide. Therefore, for our research, we decided to adopt the iron triangle, which is composed of time, cost, and quality, as the criteria for measuring success in project management. The factors were then characterised according to following criteria: time, cost, and quality of the task. These criteria have three possible values (high gain, some gain, and no gain) with the following interpretations (the adopted methods do not accept negative values):

(i) High gain: The existence or presence of the factor causes a very positive influence on the time/cost/quality of the task to be performed.

(ii) Some gain: The existence or presence of the factor provokes some positive influence on the time/cost/quality of the task to be performed.

(iii) No gain: The existence or presence of the factor produces no influence on the time/cost/quality of the task to be performed.

To build the characterisation vectors for the alternatives of our problem, we needed to find out the impact of influencing factors regarding time, cost, and quality in tasks in DSD. For

(a) 1st Company (b) 2nd Company (c) 3rd Company (d) 4th Company (e) 5th Company

FIGURE 2: Geographic distribution of the companies' offices.

TABLE 5: Strategy for application of the approach.

| Stages | Activities | Strategy |
|---|---|---|
| (1) Context Characterisation | (1.1) Definition of tasks types<br>(1.2) Identification of influencing factors<br>(1.3) Criteria and groups definition<br>(1.4) Characterisation of influencing factors | Performed once for all participating companies. The data will be shared between companies. |
| (2) Ranking of the influencing factors | (2.1) Classification of influencing factors into preference groups<br>(2.2) Rank ordering of influencing factors | |
| (3) Ranking of the executing units | (3.1) Definition and characterisation of offices<br>(3.2) Classification of offices into preference groups<br>(3.3) Rank ordering of offices | Individual per company. |

this, we conducted a survey with 111 professionals who played the role of software project manager in several organizations spread throughout Brazil. The survey questions look like this:

*(1) Factor.* Technical skills – knowledge and ability on techniques, programming languages, frameworks, tools, APIs, necessary for the professionals to carry out the task.

*(1.1) Requirements-Related Tasks.*

| *Criteria\Values* | *1. High gain* | *2. Some gain* | *3. No gain* |
|---|---|---|---|
| A. Quality of the task | A1. ( ) | A2. ( ) | A3. ( ) |
| B. Time of the task | B1. ( ) | B2. ( ) | B3. ( ) |
| C. Cost of the task | C1. ( ) | C2. ( ) | C3. ( ) |

After conducting the survey, we were able to characterise the alternatives (in this stage, the influencing factors), which served as inputs for the VDA methods. An example of alternatives characterisation can be seen in Table 6, where the cells in bold represent the most voted criteria values and, therefore, the ones that were considered in the final characterisation vector (note the final vector column). The numbers in the cells represent the total votes each criterion value received for that factor.

*5.2.2. Ranking of the Influencing Factors.* After defining the criteria and criteria values, alternatives (factors) and preference groups, we moved to the Stage of Ranking of the Influencing Factors. As explained, first, the ORCLASS method was applied with the support of the ORCLASSWEB tool, following the steps listed below:

(1) Definition of the criteria and criteria values;

(2) Definition of the alternatives;

(3) Construction of the classification rule; and

(4) Results generation.

We performed this stage once for each of the four task types determined initially. First, we introduced the problem's criteria into the ORCLASSWEB. In this step, we specified the criteria's names and their possible values. Next, we entered the problem's alternatives into the ORCLASSWEB. We provided the alternatives' names and their representations in terms of criteria values, considering the criteria established in the previous step (values in the column "final vector" in Table 6). The ORCLASSWEB then builds the classification rules by asking questions to the decision-maker. At that time, we got the support of five project management experts with more than ten years of experience in software development and that have a master's degree. They all lived in Fortaleza (Brazil) and worked for different organizations. They played the role of

Table 6: Example of alternatives characterisation for requirements tasks.

| Criteria/alternatives | Quality of the task | | | Time of the task | | | Cost of the task | | | Final vector |
|---|---|---|---|---|---|---|---|---|---|---|
| | A1 | A2 | A3 | B1 | B2 | B3 | C1 | C2 | C3 | |
| Factor1 | **61** | 40 | 10 | 43 | **56** | 12 | 39 | **48** | 24 | A1B2C2 |
| Factor2 | **97** | 10 | 4 | **81** | 27 | 3 | **75** | 33 | 3 | A1B1C1 |
| Factor3 | 52 | **55** | 4 | **56** | 49 | 6 | **51** | 50 | 10 | A2B1C1 |
| | | | | | . . . | | | | | |

decision-makers and answered the questions of the method to classify the alternatives. The series of responses build the classification rule, which allows separating the alternatives into groups. Finally, the ORCLASSWEB classified all the alternatives and provided the results. For example, in our case, for requirement tasks, the A1B2C2 alternative fell in the group I while the A2B2C2 alternative was classified in group II.

At the end of ORCLASSWEB application, the factors were distributed between group I (the preferential factors) and group II (the nonpreferable factors) for each type of task (see Table 7). Factors from group I are the most relevant ones that project managers should take into account when assigning tasks in DSD projects, whereas factors from group II are the least relevant ones and should be less considered or even not considered when attributing tasks in DSD environment.

Once the preferable factors are identified by applying the ORCLASS method, we advanced to the step of ordering in the Stage of Ranking of the Influencing Factors. In this step, we employed the ZAPROS III-i method to order the preferable factors so as to establish a ranking of them. As already explained, the least preferable factors should be discarded in order to reduce our workspace. Like the previous step, this one was performed once for each of the four defined types of task.

As explained previously, the ZAPROS III-i method was performed with the support of the ARANAÚ software, following the steps described below:

(1) Definition of the criteria and criteria values;

(2) Elicitation of preferences;

(3) Definition of the alternatives; and

(4) Results generation.

Initially, we inserted the criteria into the ARANAÚ. Next, the same group of five project management experts from the previous step, in the role of decision-makers, decided the preferences in order to formulate the scale of preferences. The procedure happens in two steps: preferences elicitation for quality variation of the same criterion and preferences elicitation between pairs of criteria. After the preference scale has been formulated, the alternatives of the problem must be defined. The alternatives of the problem under study are the factors that constitute the group I, i.e., the factors that should be preferentially regarded in the allocation of tasks in DSD projects.

Once the data were entered and all the questions of the model were answered, ARANAU executed the process according to ZAPROS III-i, generating the outputs at the end, that is, an ordered list of the most relevant factors that the project managers should think about when distributing tasks in DSD projects. In our study, once we performed the model for all task types, ARANAÚ produced four ordered scales. The tool shows the alternatives of the problem, its representations in terms of criteria values and their rankings.

Note that, among the fifteen factors initially considered, for the tasks related to requirements, eleven factors were classified in group 1; that is, they were indicated as preferable, whereas another four factors were classified in group 2, that is, not preferable. The reasoning is the same for all other types of tasks. Table 8 shows the classification summary after the application of the ORCLASS method. The rank column indicates the position of the influencing factor on the preference scale generated by the model. For example, for requirement-related tasks, Factors 2, 4, 5, 6, 7, 11, 13, 14, and 15 were ranked at position 1; after all, they all have the best representation, i.e., A1B1C1.

The ARANAÚ also draw a graph showing the dominance relations among the alternatives. For all task types, we created the corresponding dominance graphs, which are exhibited in Figure 3. The arrow direction indicates dominance. It is important to emphasise that all such knowledge produced at this stage of the experiences of use can be reused in companies that intend to use the approach proposed in this research. Thus, a company may decide not to perform stage 1, Context Characterisation, and adopt the characterisation of the influencing factors generated by this work. Likewise, a company may also choose not to perform Stage 2, Ranking of the Influencing Factors, and thus make use of the ranking of influencing factors generated in this work. We understand that this ranking of influencing factors constitutes one of the significant contributions of this work since it is the result of very comprehensive research, which had the participation of more than a hundred professionals from the project management area in Brazil.

*5.2.3. Ranking of the Executing Units.* Once we completed the stages that are common for all the companies involved and have selected the main influencing factors by task type, we moved to the individualized stage per company (Stage 3). In this step, the companies' offices served as alternatives for the VDA classification (ORCLASS) and ordering (ZAPROS III-i) methods. The set of the preferable influencing factors resulting from Stage 2 served as criteria to evaluate the offices.

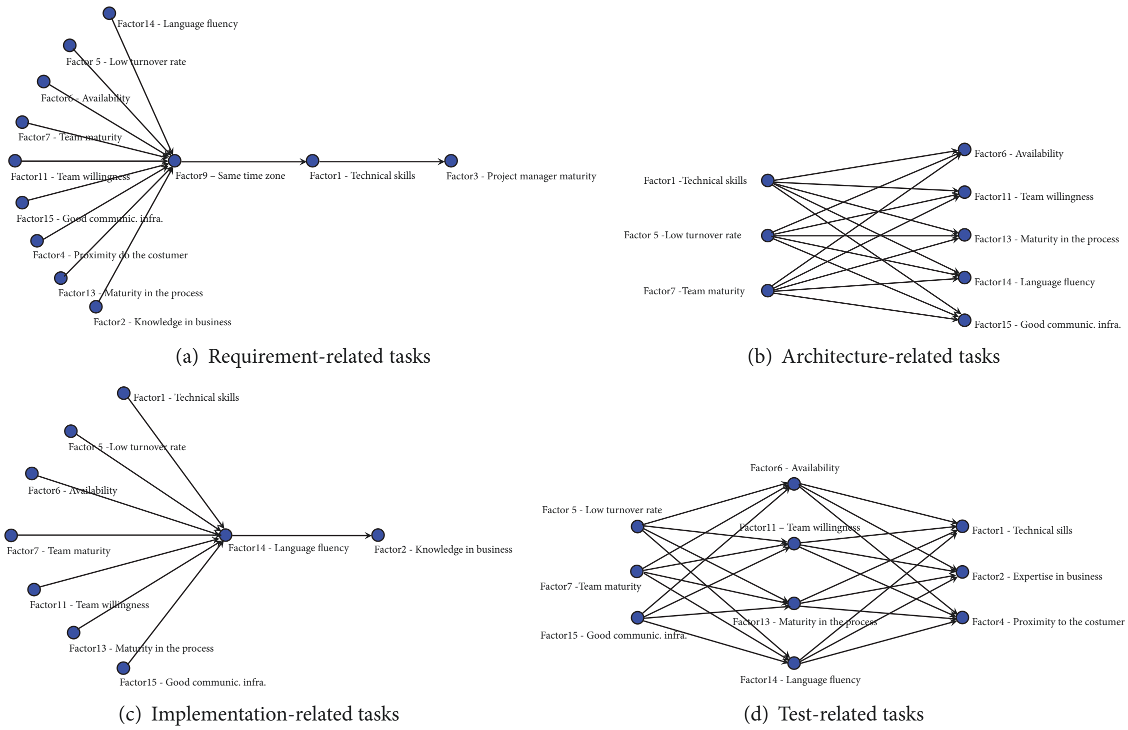
For this study, we defined that each criterion could be evaluated as excellent, good, or regular. Thus, these were the criteria values adopted for this process stage. Once the criteria and their values have been defined, we proceeded to

Table 7: Results of ORCLASS application separated by task types and preference groups.

| | Group I – The Preferable Alternatives | | Group II - The Non-preferable Alternatives | |
|---|---|---|---|---|
| | Criteria Values | Factors | Criteria Values | Factors |
| Requirements | A1B1C1 | Factor2 - Knowledge in business | A2B2C2 | Factor8 – Team personal trust |
| | | Factor4 - Proximity to the customer | | Factor10 - Cultural similarities |
| | | Factor5 - Low turnover rate | A3B3C1 | Factor12 - Low labour cost |
| | | Factor6 - Team availability | | |
| | | Factor7 - Team maturity | | |
| | | Factor11 - Team willingness | | |
| | | Factor13 - Maturity in the process | | |
| | | Factor14 - Language fluency | | |
| | | Factor15 - Good communic. infra. | | |
| | A2B1C1 | Factor3 - Project manager maturity | | |
| | A1B1C2 | Factor9 - Same time zone | | |
| | A1B2C2 | Factor1 - Technical skills | | |
| | Criteria Values | Factors | Criteria Values | Factors |
| Architecture | A1B1C1 | Factor1 - Technical skills | A2B2C2 | Factor2 - Knowledge in business |
| | | Factor5 - Low turnover rate | | Factor3 - Project manager maturity |
| | | Factor7 – Team maturity | | Factor4 - Proximity to the customer |
| | A1B1C2 | Factor6 - Team availability | | Factor8 - Team personal trust |
| | | Factor11 - Team willingness | | Factor9 - Same time zone |
| | | Factor13 - Maturity in the process | A2B2C3 | Factor10 - Cultural similarities |
| | | Factor14 - Language fluency | A3B3C1 | Factor12 - Low labour cost |
| | | Factor15 - Good communic. infra. | | |
| | Criteria Values | Factors | Criteria Values | Factors |
| Implementation | A1B1C1 | Factor1 - Technical expertise | A2B2C2 | Factor3 - Project manager maturity |
| | | Factor5 - Low turnover rate | | Factor4 - Proximity to the customer |
| | | Factor7 - Team maturity | | Factor8 - Team personal trust |
| | | Factor6 - Team availability | | Factor9 - Same time zone |
| | | Factor11 - Team willingness | A2B3C3 | Factor10 - Cultural similarities |
| | | Factor13 - Maturity in the process | A3B3C1 | Factor12 - Low labour cost |
| | | Factor15 - Good communic. infra. | | |
| | A1B1C2 | Factor14 - Language fluency | | |
| | A1B2C2 | Factor2 - Knowledge in business | | |
| | Criteria Values | Factors | Criteria Values | Factors |
| Tests | A1B1C1 | Factor5 - Low turnover rate | A2B2C2 | Factor3 - Project manager maturity |
| | | Factor7 - Maturity of the team | | Factor8 - Team personal trust |
| | | Factor15 - Good communic. infra. | | Factor9 - Same time zone |
| | A1B1C2 | Factor6 - Team availability | A2B2C3 | Factor10 - Cultural similarities |
| | | Factor11 - Team willingness | A3B3C1 | Factor12 - Low labour cost |
| | | Factor13 - Maturity in the process | | |
| | | Factor14 - Language fluency | | |
| | A1B2C2 | Factor1 - Technical skills | | |
| | | Factor2 - Knowledge in business | | |
| | | Factor4 - Proximity to the customer | | |

Table 8: Detailed results of ARANAÚ application for each type of tasks.

| | Rank | Alternative | Representation Quality | Representation Time | Representation Cost |
|---|---|---|---|---|---|
| Requirements | 1 | Factor2 - Knowledge of business | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor4 - Proximity to the customer | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor5 - Low turnover rate | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor6 - Availability | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor7 - Maturity of the team | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor11 - Willingness at site | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor13 - Maturity in the process | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor14 - Language fluency | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor15 - Good communication infrastructure | A1. High gain | B1. High gain | C1. High gain |
| | 2 | Factor9 - Same time zone | A1. High gain | B1. High gain | C2. Some gain |
| | 3 | Factor3 - Maturity of the project manager | A2. Some gain | B1. High gain | C1. High gain |
| | 4 | Factor1 - Technical skills | A1. High gain | B2. Some gain | C2. Some gain |
| | Rank | Alternative | Representation Quality | Representation Time | Representation Cost |
| Architecture | 1 | Factor1 - Technical skills | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor5 - Low turnover rate | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor7 - Maturity of the team | A1. High gain | B1. High gain | C1. High gain |
| | 2 | Factor6 - Availability | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor11 - Willingness at site | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor13 - Maturity in the process | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor14 - Language fluency | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor15 - Good communication infrastructure | A1. High gain | B1. High gain | C2. Some gain |
| | Rank | Alternative | Representation Quality | Representation Time | Representation Cost |
| Implementation | 1 | Factor1 - Technical expertise | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor5 - Low turnover rate | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor6 - Availability | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor7 - Site maturity | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor11 - Willingness at site | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor13 - Maturity in the process | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor15 - Good communication infrastructure | A1. High gain | B1. High gain | C1. High gain |
| | 2 | Factor14 - Language fluency | A1. High gain | B1. High gain | C2. Some gain |
| | 3 | Factor2 - Expertise in business | A1. High gain | B2. Some gain | C2. Some gain |
| | Rank | Alternative | Representation Quality | Representation Time | Representation Cost |
| Tests | 1 | Factor5 - Low turnover rate | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor7 - Maturity of the team | A1. High gain | B1. High gain | C1. High gain |
| | 1 | Factor15 - Good communication infrastructure | A1. High gain | B1. High gain | C1. High gain |
| | 2 | Factor6 - Availability | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor11 - Willingness at site | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor13 - Maturity in the process | A1. High gain | B1. High gain | C2. Some gain |
| | 2 | Factor14 - Language fluency | A1. High gain | B1. High gain | C2. Some gain |
| | 3 | Factor1 - Technical skills | A1. High gain | B2. Some gain | C2. Some gain |
| | 3 | Factor2 - Knowledge of business | A1. High gain | B2. Some gain | C2. Some gain |
| | 3 | Factor4 - Proximity to the customer | A1. High gain | B2. Some gain | C2. Some gain |




(a) Requirement-related tasks (b) Architecture-related tasks

(c) Implementation-related tasks (d) Test-related tasks

Figure 3: Dominance relations among the alternatives by tasks type.

the characterisation of the offices as regards the influencing factors, we mean; considering the influencing factors as criteria and the company offices as alternatives, we wanted to know how each company office was evaluated against the influencing factors. That characterisation will be useful for the next steps. The companies' professionals were then asked to rate all influencing factors for the offices of the company he or she works for, according to the following questions template. We replicated the questions to the other offices and factors.

*(1) Office Located in São Paulo*.

| *Influencing Factor* | *Excellent* | *Good* | *Regular* |
|---|---|---|---|
| Factor 1 – Technical skills: knowledge and ability on techniques, programming languages, frameworks, tools, APIs, necessary for the professionals to carry out the task. | ( ) | ( ) | ( ) |
| Factor 2 – Knowledge in business: Knowledge of the teams about the area, domain or business of the clients. | ( ) | ( ) | ( ) |
| Factor 3 . . . | | | |

The survey continues with the other offices.

After characterising the offices, we ran ORCLASS and ZAPROS III-I methods to get the ordered lists of recommended offices for each task type. Figure 4 shows the ORCLASSWEB screen for assembling the classification rule for the ORCLASS method. In this step, PMO staff, portfolio managers, or both should play the role of decision-maker and answer the questions posed by the model in order to classify the alternatives.

They should respond "Yes" or "No" for all questions. The series of answers contribute to assembling the classification rule. Thus, it is possible to divide the alternatives (influencing factors) into preference groups. A positive response classifies that combination into the preferable group. Otherwise, the combination goes to the nonpreferable group. Thereby, the classification rule will be completed according to the decision-makers selections. At last, the ORCLASSWEB software processes the complete classification of the alternatives.

Figure 5 shows the ARANAÚ interface for the preferences elicitation between pairs of criteria as part of ZAPROS III-i method application. In this scenario, the decision-maker should choose the best alternative, considering that the other values are at their best or worst degree. The choices set the scale of preferences among the criteria.

*5.3. Results.* In the end, after applying DiVA for the five companies, we obtained the results (see Table 9). A detailed description of these experiences of use can be found in [69].

*5.4. Evaluation of the Proposed Approach.* To evaluate the DiVA approach, we performed two types of analysis. The first evaluation sought to analyse the approach from the perspective of the five project management experts. The other analysis aimed to evaluate the approach by the companies'

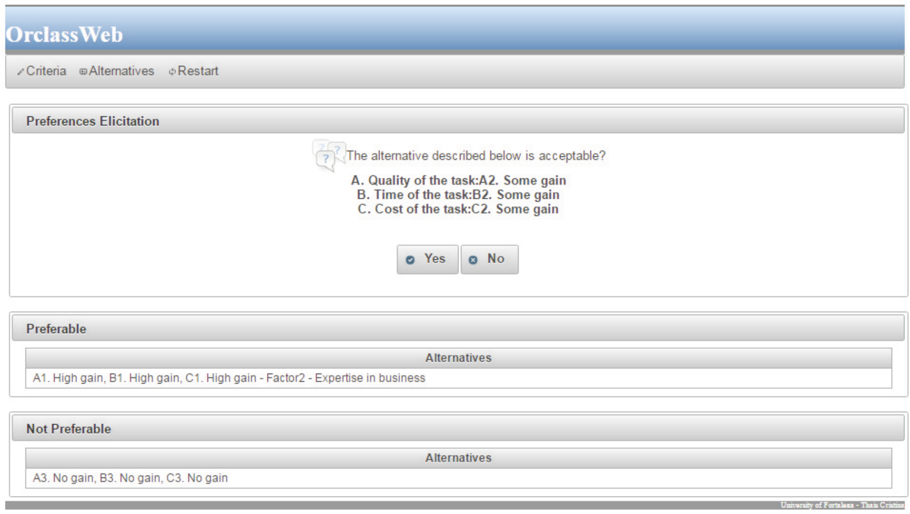



Figure 4: ORCLASSWEB's interface for construction of the classification rule.

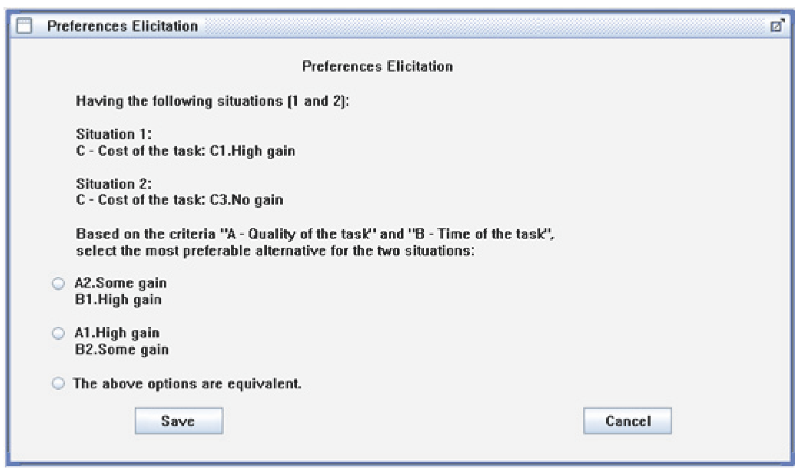



Figure 5: ARANAÚ's interface for structuring the problem scale of preferences between pairs of criteria.

professionals who participated in the experiences of use. Thirteen professionals from the companies participated in the experiences of use. However, three of them abstained from collaborating with the evaluation questionnaire. Table 10 summarizes the responses of both evaluations, which are briefly discussed in Section 5.5.

*5.5. Discussion.* In both evaluations, most respondents agreed that the approach is easy to understand and to use. Furthermore, most of them also agreed that the approach helps reduce subjectivity and helps think of new aspects. Finally, most participants evaluated that it helps in task allocation in DSD. As points of attention, the assessment of the adaptability/extensibility and of the effort to use did not allow drawing precise conclusions.

Analysing the proposed approach based on the experiences of use conducted and the evaluations carried out, we have the following about DiVA:

(i) It describes a process, providing a visual flowchart, which makes easy its understanding;

(ii) It is supported by automated tools (ORCLASSWEB and ARANAÚ), facilitating its application;

(iii) It uses influence factors extracted from the specialized literature, bringing global knowledge to the process;

(iv) It can be instantiated from an organization standard; the experiences of use carried out followed this concept; that is, in the beginning, a common structure was defined, and, then, each company entered with its respective data. In the same way, this can be done within an organization, creating a common base and then customizing for each project.

Table 9: Results after applying DiVA.

| Company | Order | Requirements related tasks | Architecture/Design related tasks | Implementation related tasks | Testing related tasks |
|---|---|---|---|---|---|
| 1 | 1 | Fortaleza (BR) | São Paulo (BR) | São Paulo (BR) | São Paulo (BR) |
| | 2 | São Paulo (BR) | Fortaleza (BR) | Fortaleza (BR) | Fortaleza (BR) |
| | 3 | Quixadá (BR) | Quixadá (BR) | Quixadá (BR) | Quixadá (BR) |
| Company | Order | Requirements related tasks | Architecture/Design related tasks | Implementation related tasks | Testing related tasks |
| 2 | 1 | Recife (BR) | Natal (BR) | Natal (BR) | Recife (BR) |
| | 2 | Fortaleza (BR) | Recife (BR) | Fortaleza (BR) | Natal (BR) |
| | 3 | Natal (BR) | Fortaleza (BR) | Recife (BR) | Fortaleza (BR) |
| Company | Order | Requirements related tasks | Architecture/Design related tasks | Implementation related tasks | Testing related tasks |
| 3 | 1 | São Paulo (BR) | São Paulo (BR) | São Paulo (BR) | São Paulo (BR) |
| | 2 | Rio de Janeiro (BR) | Rio de Janeiro (BR) | Rio de Janeiro (BR) | Rio de Janeiro (BR) |
| | 3 | Porto Alegre (BR) | Porto Alegre (BR) | Porto Alegre (BR) | Porto Alegre (BR) |
| | 4 | Fortaleza (BR) | Fortaleza (BR) | Fortaleza (BR) | Fortaleza (BR) |
| Company | Order | Requirements related tasks | Architecture/Design related tasks | Implementation related tasks | Testing related tasks |
| 4 | 1 | New York (USA) | Buenos Aires (ARG) | New York (USA) | New York (USA) |
| | 2 | Buenos Aires (ARG) | Fortaleza (BR) | Buenos Aires (ARG) | Buenos Aires (ARG) |
| | 3 | Santiago (CHI) | New York (USA) | Santiago (CHI) | São Paulo (BR) |
| | 4 | Fortaleza (BR) | São Paulo (BR) | Fortaleza (BR) | Santiago (CHI) |
| | 5 | São Paulo (BR) | Santiago (CHI) | São Paulo (BR) | Fortaleza (BR) |
| Company | Order | Requirements related tasks | Architecture/Design related tasks | Implementation related tasks | Testing related tasks |
| 5 | 1 | Aracaju (BR) | Aracaju (BR) | Aracaju (BR) | Aracaju (BR) |
| | 2 | Juiz de Fora (BR) | Campina Grande (BR) | Juiz de Fora (BR) | Jaguariúna (BR) |
| | 3 | Curitiba (BR) | Juiz de Fora (BR) | Campina Grande (BR) | Curitiba (BR) |
| | 4 | Brasília (BR) | Rio de Janeiro (BR) | Jaguariúna (BR) | Juiz de Fora (BR) |
| | 5 | Jaguariúna (BR) | Curitiba (BR) | Curitiba (BR) | Campina Grande (BR) |
| | 6 | Campina Grande (BR) | Jaguariúna (BR) | Rio de Janeiro (BR) | Rio de Janeiro (BR) |
| | 7 | Belém (BR) | Porto Alegre (BR) | Belém (BR) | Belém (BR) |

(v) It reuses knowledge because it allows the use of knowledge from other sources (factors, criteria, relations, among others); besides, it is possible to feedback the approach with the generated knowledge itself;

(vi) It presupposes consensus meetings since it provides for meetings between the company's specialists to analyse and evaluate the recommendations suggested by the approach;

(vii) It presupposes feedback of results as its flow provides for feedback of information and results to calibrate the model and improve decisions;

(viii) It proposes to be flexible and adaptable, since it has a modular structure, which makes it possible to use of a subset of the whole process, besides allowing the modification of the main parameters (object of allocation and executing unit).

In addition, DiVA was applied in real-world situations, since the experiences of use were conducted with the collaboration of five software companies with distributed structure, and both knowledge area specialists and business professionals evaluated it.

### *5.6. Threats and Limitations*

*5.6.1. Threats to Validity.* Possible threats were identified during the execution of the experiences of use. The first point that may have influenced the results of the experiences of use

Table 10: Evaluation of the approach by the project management experts (PME) and by the companies' professionals (CP).

| | Strongly disagree | | Disagree | | Neither agree nor disagree | | Agree | | Strongly agree | |
|---|---|---|---|---|---|---|---|---|---|---|
| | PME | CP | PME | CP | PME | CP | PME | CP | PME | CP |
| RQ1. Easy to understand | - | 1 | - | 1 | - | 1 | 5 | 7 | - | - |
| RQ2. Easy to use | - | 1 | - | 1 | 1 | 1 | 2 | 6 | 2 | 1 |
| RQ3. Adaptable/extensible | - | - | - | - | 1 | 5 | 1 | 4 | 3 | 1 |
| RQ4. Low effort to use | - | - | 1 | 2 | 2 | 4 | - | 3 | 2 | 1 |
| RQ5. Helps in reducing subjectivity | - | - | - | 1 | - | - | 3 | 8 | 2 | 1 |
| RQ6. Helps in thinking of new aspects | - | - | - | 1 | - | 1 | 3 | 5 | 2 | 3 |
| RQ7. Helps in task allocation in DSD | - | - | - | - | - | 3 | 1 | 7 | 4 | - |

is the fact that the author of the work was the manager of the process executed in the experiences of use. Prior knowledge of the objectives proposed for DiVA may have had a positive influence on the obtained results. However, it was not possible to adopt an action capable of mitigating this bias, since ideally some professional from each company who had not been involved in this study should have executed DiVA. This was not possible due to staff restrictions and employees' workload in the companies. Thus, the directors requested that the author of the research conducted the experiences of use.

For obvious reasons of time restriction of companies' employees, it was not possible to perform the entire approach within the companies. Thus, we have chosen to structure the application of the approach in two phases. This may have caused some confusion to the companies' professionals for not having participated in the selection of the influencing factors. We tried to mitigate this bias by explaining the whole approach in the questionnaires applied to professionals. The analysis of the results obtained with the application of DiVA by the author of this research can also be considered a threat. To mitigate that thread, the analysis of the results obtained with the execution of the DiVA and the evaluations made by companies' professionals and project management experts were discussed by two other researchers and by two project managers who were not involved with the research. Moreover, the analysis, although subjective, has taken into consideration only the answers regarding the agreement or not with certain statements related to the objectives of the research and can be easily verified from the results presented in the tables throughout the text of the paper.

Finally, another threat of this study concerns the generalization of the results. Once it has been performed in a limited way in a restricted group of companies, it is not allowed to generalize the results. It is only possible to observe indications about how the application of DiVA supports organizations in the task allocation process.

*5.6.2. Limitations.* The user experiences have been developed partly in collaboration with the professionals of the participating companies. Better results could have been obtained if performed entirely by the companies' professionals. It was also not possible to obtain information on the time, cost, and effort of applying DiVA, due to the circumstances reported. Furthermore, another important limitation perceived was the lack of an integrated automated tool. Although there are specific automated tools for the VDA methods adopted, an integrated tool comprising the entire process would likely bring more benefits to the decision-making process as well as better results in approach evaluation.

## 6. Conclusion and Future Works

The software industry has a high level of competition, which requires IT companies to constantly search for innovative models of management, service supply and software development. On the other hand, the IT companies market, more than in any segment, is global; that is, customers can be spread all over the world. An increasing trend is to create subsidiaries around the world and deliver activities to them to benefit from the best features of each location and thus achieve the gains that the distributed model can provide. However, that solution presents challenges because many issues should be taken into consideration when distributing tasks to remote teams. The context exposed here is a typical decision-making problem that can be structured on multiple criteria. This problem can be tackled by using Verbal Decision Analysis.

In that scenario, this research introduced DiVA, an approach to support the allocation of tasks in DSD projects, based on the hybridisation of methods of Verbal Decision Analysis for classification and ordering applied to influencing factors and executing units. In this way, we intend to aid organizations in task allocation activity, which as presented throughout this work is very critical and involves many difficulties.

Aligned with this objective, a review of the literature was conducted to find out the approaches that support the allocation of tasks in DSD environment. Thus, it was possible to obtain an overview of the existing solutions that proposed to tackle the problem of task assignment in DSD. The literature review also allowed identifying some gaps that were addressed in the proposed approach, named DiVA. For example, the results of the review showed that few solutions proposed by other studies were based on multicriteria decision-making methods and even fewer studies were based on qualitative analysis. Furthermore, no approach based on Verbal Decision Analysis methods was identified.

Thus, for the development of the approach, we applied a hybrid model based on ORCLASS (for classification) and ZAPROS III-i (for ordering) methods, both from Verbal Decision Analysis framework, firstly regarding the factors that influence the task attribution in DSD, and, later, concerning the executing units (teams, offices, or branches). Bibliographic research and application of surveys with professionals allowed identifying and characterising the main factors that influence task assignment in DSD projects.

Experiences were carried out with the collaboration of professionals from five real-world companies that have a distributed structure. For that, tasks were gathered based on their type, that is, requirements, architecture/design, implementation, and testing. In the end, DiVA has been submitted to the evaluation by the professionals of the companies that participated in the experiences of use. In addition, DiVA has also been evaluated by project management experts with extensive experience in the knowledge field being studied.

Several conclusions can be drawn from the use of the approach, among which we can mention: it is supported by automated tools, it uses influence factors extracted from the specialized literature, and it proposes to be flexible and adaptable, since it is possible to use a subset of the whole process, besides allowing the modification of the main parameters (object of allocation and executing unit). Considering the evaluations of the approach, the results provided evidence that the approach is easy to understand and to use and helps to reduce the decision subjectivity and to think of new aspects.

According to [70], in research based on the Design Science Research (DSR), the research method adopted in this work, we seek to find a solution to a particular problem. This solution should not necessarily be optimal but at least satisfactory for the context it proposes. In addition, it is accepted that an actual problem and the artefacts that promote satisfactory solutions to it can share common characteristics that allow its generalization to a determinate category of problems. In this context, based on the results of the evaluations carried out by project management experts from different organizations and by professionals from the companies participating in the experiences of use, it is considered that DiVA approach can support software organizations in allocating tasks in DSD projects.

*6.1. Future Works.* The application of DiVA in real contexts and the evaluation of the results of the approach by professionals of the companies participating in the experiences of use, besides the evaluation of the approach by the specialists, allowed identifying some opportunities for improvement, which we suggest as future works (i) to develop an automated suite aiming the integration of the ORCLASSWEB and ARANAÚ tools in order to make its application easy and minimize the effort to adopt the approach; (ii) to hybridise other classification methods (NORCLASS or SAC) with other ordering methods (PACOM, SNOD, ORCON, or ARACE) to compose variants of DiVA; (iii) to create a knowledge base on factors of influence that would enable companies to select the most convenient factors depending on the context; (iv) to provide a guide or tutorial with information on applying the approach, describing "step by step" the execution of each activity; (v) to apply DiVA considering a reduced set of influencing factors, chosen as more appropriate for each company; (vi) to adapt DiVA to support the allocation of tasks for individual projects; and, finally, (vii) to adapt DiVA to consider dependency relations between tasks.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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*Research Article*

# Weighted Couple-Group Consensus Analysis of Heterogeneous Multiagent Systems with Cooperative-Competitive Interactions and Time Delays

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In this paper, the weighted couple-group consensus of continuous-time heterogeneous multiagent systems with input and communication time delay is investigated. A novel weighted couple-group consensus protocol based on cooperation and competition interaction is designed, which can relax the in-degree balance condition. By using graph theory, general Nyquist criterion and Gerschgorin disc theorem, the time delay upper limit that the system may allow is obtained. The conclusions indicate that there is no relationship between weighted couple-group consensus and communication time delay. When the agents input time delay, the coupling weight between the agents, and the systems control parameters are satisfied, the multiagent system can converge to any given weighted coupling group consistent state. The experimental simulation results verify the correctness of the conclusion.

## 1. Introduction

As an important branch of distributed system, multiagent systems (MASs) have been paid great attention by many scholars due to their wide application in many fields [1–6], such as multirobot system, wireless sensor network, and distributed target tracking. For example, in [5], the distributed formation control problem for multiple nonholonomic wheeled mobile robots would be solved by using a variable transformation, algebraic graph theory, matrix theory, and Lyapunov control approach.

Consensus or synchronization, as one of the most important problems of MASs, is to design an appropriately distributed protocol to make different agents achieve a common state. Group consensus, as an extension of consensus, is very suitable for multitasks and large-scale problems. Up to now, there are many results for consensus or group consensus [7–18]. On the other hand, the controllability problem of multiagent systems has attracted great interests and concern since Tanner proposed it in 2004. In the past decades, many controllability criteria have been given for multiagent systems [19–24]. However, most of these results focused on single time scale. In [25], the group controllability of two-time-scale multiagent networks was firstly proposed and some easy-to-use criteria were proposed for group controllability of two-time-scale multiagent networks compared with the rank criterion. In [26], Long et al. further investigated second-order controllability of two-time-scale multiagent systems, and some more effective second-order controllability conditions would be determined only by the eigenvalues of system matrices. In [27], a new format of time-varying formation shape was proposed, and a new class of distributed adaptive observer-based controllers was designed under the mild assumption that both leaders and followers were introspective. As we know, most existing results have been obtained mainly based on the nodes of the network system. In some other real situations, each agent cannot obtain the neighbors state information in a real networked system. Therefore, in [28], the authors studied the discrete-time nonnegative edge synchronization of networked systems based on neighbors output information, which gives us a novelty and interesting synchronization method.

*1.1. Related Contributions.* It should be noticed that all the aforementioned results are based on the common assumption that the multiagent systems are homogeneous. In this situation, all agents of the whole systems have the same dynamics. However, in real life, almost every agent has its own dynamics because of different external and interaction impacts. Hence, it is natural for us to model heterogeneous multiagent systems. In recent years, some heterogeneous multiagent systems models have been established [29–32]. In [29], dynamical consensus of heterogeneous multiagent systems which consist of the first-order and second-order agent dynamics has been discussed. In [30], a consensus protocol is proposed for high-ordered heterogeneous systems with uncertain communication delays. Furthermore, more and more scholars pay much attention to the group consensus of heterogeneous systems. For example, in [31], a heterogeneous system consisting of first-order and second-order agents has been studied on the basis of fixed and switching topologies. In [32], some sufficient group consensus conditions have been obtained for a kind of heterogeneous system with diverse input time delays based on frequency-domain analysis method and matrix theory. In [33], some sufficient couple-group consensus conditions have been derived for a kind of discrete-time heterogeneous systems consisting of first-order and second- order agents under the influence of communication and input time delays. In [34], Li et al. studied the consensus problem in heterogeneous linear discrete-time MASs. In [35], Cui et al. discussed the consensus problem of heterogeneous chaotic network systems with or without delay. In [36], the consensus problems of linear systems and nonlinear systems were studied separately. In [37], Liu et al. studied the consensus problem of heterogeneous MASs under certain assumptions. In [38], Goldin et al. studied the consensus of heterogeneous networks with undirected topology.

At the same time, some achievements have been made in the research of weighted consensus. For example, in [39], the concept of weighted consensus was proposed, and the multiagent weighted average consensus is studied. In [40], Shi et al. studied the robust consensus control for a class of MASs by PID algorithm and weighted edge dynamics. In addition, MASs based on cooperation-competition interactions are also receiving more and more attention. In [41], Hu et al. studied the second-order consensus problem of heterogeneous MASs. In [42], Hu et al. studied the swarming behavior of multiple Euler-Lagrange systems with cooperation-competition interactions.

*1.2. The Main Motivation.* It is obvious that heterogeneous systems are more complex than homogeneous systems and it is more difficult for us to deal with the relevant crucial topics. Inspired by the recent developments for heterogeneous multiagent systems, this paper will further investigate the weighted group consensus. To the best of our knowledge, most of existing literatures only discuss homogeneous systems, the multiagent systems in which all agent share a common value.

In this paper, we mainly investigate the weighted group consensus for a class of continuous-time heterogeneous multiagent systems with input and communication time delay. In recent years, although group consensus of multiagent systems has derived many significant results. It is worth mentioning that most of the existing results only discussed the situation where all agents possessed a fixed weighted-value, even most of the obtained results mainly focused on the consensus of heterogeneous multiagent systems, and few results were proposed for group consensus of heterogeneous networks with input and communication time delay. Furthermore, all these related conclusions were based only on agents' competitive or cooperative relation. However, in complicated practical situation, the consensus protocol needs to be adjusted with circumstances, cooperative tasks, or other constraint conditions. All these reasons incite us to study the weighted group consensus for heterogeneous multiagent systems with input and communication time delay.

*1.3. Statement of Contributions.* There are three main contributions in this paper. Firstly, the model is different from cooperative or competitive heterogeneous networks, both cooperative and competitive interactions are considered, it extends the scope of the existing research, and a kind of weighted couple-group consensus agreement based on cooperation-competition relationship is introduced, which is quite different from the literature [31, 32, 35, 37, 38]. Relying on the new protocol control, the agents can receive neighbor information more reasonably and speed up the system to achieve group consensus. Secondly, in order to simplify the analysis process, we remove the dynamic virtual speed of the first-order agent, such as in [29, 31, 32, 37]. A novel weighted couple-group consensus protocol is designed, which relaxes the in-degree balance condition and the results are also applicable to directed and undirected graphs. On the other hand, we turn the weighted matrix into a dynamic form, which makes the designed controller more flexible. Thirdly, some sufficient conditions have been obtained for the group consensus of this system by using graph theory, general Nyquist criterion, and Gerschgorin disc theorem. Unlike the [31, 32, 37], we do not require that the system satisfies the condition that the geometric versatility of the zero eigenvalues of the Laplacian matrix is not less than 2, which makes the system's topology more flexible. With the help of these conditions, the time delay upper limit of this system can be computed and the multiagent system can converge to any given state only if the weighted group consensus parameters are satisfied. The simulation results well verify the correctness of the conclusion.

The rest of this article is organized as follows. The second section lists some preliminary knowledge and problem description. The third section presents the main results and proof process of group consensus. The fourth section verifies the correctness and effectiveness of the proposed method through simulation. Finally we come to a conclusion.

*Note*. In this context, $\mathbb{C}$ denotes a complex set and $R$ denotes a real set. $I_N$ represents a unit matrix, where $N$ represents a dimension. $\mathrm{Re}(Z)$ is the real part, and $|Z|$ is the model, where $\forall Z \in \mathbb{C}$. $\lambda_i(A)$ represents the $ith$ eigenvalue of matrix $A$, and $\det(A)$ represents the determinant of the matrix.

## 2. Problem Description and Preliminary Knowledge

In order to facilitate the follow-up work, we need introduce some preliminary knowledge of the graph theory.

*2.1. Graph Theory and Interconnection Topology.* Considering $N$ agents, the topological relationship of the agent is represented by the graph $G = (V, E, A)$, where $V = \{v_1, v_2, \cdots, v_N\}$ represent the set of vertices of the graph. $E \subseteq V \times V$ and $A = (a_{ij})_{N\times N} \in R^{N\times N}$ represent the edge set and the adjacency matrix, respectively. In this article, the case of containing a self-loop is not considered.

Note that the undirected graph can be thought as a special directed graph, and we assume $a_{ij} > 0$ if $e_{ij} \in E$ in this paper. That is, if and only if the node (agent) is able to receive information from the node (agent), $a_{ij} > 0$. At the same time, $N_i = \{j \in V : e_{ij} \in E\}$ represents the set of neighbor nodes, and $D_i = \deg_{in}(i) = \sum_{j=1}^{N} a_{ij}$ represents the set of nodes within the degree, where the in-degree matrix $D$ can also be expressed as $\mathrm{diag}\{d_1, d_2, \cdots d_N\}$. Therefore, $L = D - A$ is defined as a Laplacian matrix. Note. The adjacency matrix $A$ is a symmetric matrix if and only if the graph is an undirected graph.

*2.2. Problem Statement.* Based on the above-mentioned preliminary knowledge of graph theory, in this paper we propose a heterogeneous multiagent system with $N$ agents, which contains second-order and first-order dynamics. In order not to lose generality, it is assumed that the first $n$ agents are second-order dynamics, and the last $m$ agents are first-order dynamics, where $N = m + n$. The specific system model can be designed as follows:

$$\begin{aligned} &\dot{x}_i(t) = v_i(t), \\ &\dot{v}_i(t) = u_i(t), \\ &\qquad i \in o_1 \\ &\dot{x}_i(t) = u_i(t), \quad i \in o_2, \end{aligned} \tag{1}$$

where $o_1 = \{1, 2, \ldots, n\}$, $o_2 = \{n+1, n+2, \ldots, n+m\}$, $o = o_1 \cup o_2$, $x_i(t)$, $v_i(t)$, and $u_i(t) \in R$, where $x_i(t)$ is the location of the agent $i$, $u_i(t)$ is the control rule of the $i$ agent, and $v_i(t)$ is the speed. Obviously, since each agent's neighbors can be first-order or second-order, they are divided into $N_{i,s}$ and $N_{i,f}$. So the neighbor node set $N_i = N_{i,f} \cup N_{i,s}$. Because the dynamics of the agent in the system are heterogeneous, Its adjacency matrix can be expressed as

$$A = \begin{bmatrix} A_s & A_{sf} \\ A_{fs} & A_f \end{bmatrix} \tag{2}$$

where $A_s \in R^{n\times n}$ is an adjacency matrix composed of second-order agents, $A_{sf}$ is composed of coupling weights from second-order agents to first-order agents, $A_{fs}$ is composed of first-order to second-order coupling weights, and $A_f \in R^{m\times m}$ is an adjacency matrix composed of first-order agents. Therefore, we can further write the Laplacian matrix as follows.

$$L = D - A = \begin{bmatrix} L_s + D_{sf} & -A_{sf} \\ -A_{fs} & L_f + D_{fs} \end{bmatrix} \tag{3}$$

The matrix $L$ represents the interaction between only the second-order agents, and the matrix $L_f$ represents the interaction between only the first-order agents. It should be noted that both of the matrices are Laplacian matrices, where $D_{sf} = \mathrm{diag}\{\sum_{j\in N_{i,f}} a_{ij},\ i \in o_1\}$ and $D_{fs} = \mathrm{diag}\{\sum_{j\in N_{i,s}} a_{ij},\ i \in o_2\}$ are the in-degree matrix of the agent $i$, which represents the neighbor information received from different orders.

To facilitate the follow-up work, here are some definitions and lemmas.

*Definition 1.* For the heterogeneous MASs to progressively implement the weighted couple-group consensus, the system should satisfy the following two conditions:

$$\begin{aligned} &\lim_{t\longrightarrow+\infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad \text{if } i, j \in o_k,\ k = 1, 2, \\ &\lim_{t\longrightarrow+\infty} \left\| v_i(t) - v_j(t) \right\| = 0, \quad \text{if } i, j \in o_k,\ k = 1. \end{aligned} \tag{4}$$

*Definition 2.* For the bipartite graph $G = (V, E)$, the vertex set $V$ can be split into two disjoint subsets $V_1$ and $V_2$, where $V_1 \cap V_2 = \emptyset$, and at the same time $\forall e = (w, q) \in E$, where $w \in V_1$ and $q \in V_2$.

**Lemma 3** (see [15]). *For an undirected bipartite graph, $\lambda_i(L) \in R$. At the same time, it should be noted that directed bipartite graphs containing directed spanning trees have the following two properties: (1) rank$(L) = n - 1$, (2) when $\lambda_i(L) \neq 0$,* $\mathrm{Re}(\lambda_i(L)) > 0$, *where n is the number of system agents, matrix* $L = D + A$.

## 3. Main Results

Most existing works are based on the competition or cooperation relationship of agents. At the same time, only a single form of delay is considered. For example, in [38], the grouping of heterogeneous systems with the same input delay is studied. Its system is described as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \sum_{j\in o_1} a_{ij}\left[x_j(t-\tau)-x_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_2} a_{ij}x_j(t-\tau)\\ &\quad+\sum_{j\in o_1} a_{ij}\left[v_j(t-\tau)-v_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_2} a_{ij}v_j(t-\tau),\\ &\qquad i\in o_1.\end{aligned} \tag{5}$$

And

$$\begin{aligned}\dot{x}_i(t) &= v_i(t-\tau)+\sum_{j\in o_2} a_{ij}\left[x_j(t-\tau)-x_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_1} a_{ij}x_j(t-\tau),\\ \dot{v}_i(t) &= \sum_{j\in o_2} a_{ij}\left[x_j(t)-x_i(t)\right]+\sum_{j\in o1} a_{ij}x_j(t),\\ &\qquad i\in o_2.\end{aligned} \tag{6}$$

In (5) and (6), it is not difficult to see that the agents rely on cooperative relationships for information exchange, and there are also speed estimates in the first-order agents. Considering that in practical applications, competitive interactions are inevitable. Therefore, we design a weighted couple-group consensus protocol that utilizes the competition-cooperative interaction of agents. The specific form is as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i(t-\tau)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i(t-\tau)\right]\right]-\beta_i v_i(t-\tau),\\ &\qquad i\in o_1.\end{aligned} \tag{7}$$

And

$$\begin{aligned}\dot{x}_i(t) &= \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i(t-\tau)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i(t-\tau)\right]\right],\quad i\in o_2.\end{aligned} \tag{8}$$

Here $\tau_{ij}$ indicates communication delay between agent $j$ and agent $i$, and $\tau$ represents the identical input delay of the agents. $N_{si}$ denotes a neighbor of the same dynamic as the agent $i$. Similarly, $N_{di}$ denotes a neighbor of a different dynamic from the agent $i$. Meanwhile, $\alpha_i$, $\beta_i$, and $\gamma_i > 0$, where $\alpha_i = \{\alpha_1,\alpha_2,\ldots,\alpha_N\}$, $\beta_i = \{\beta_1,\beta_2,\ldots,\beta_N\}$, $\gamma_i = \{\gamma_1,\gamma_2,\ldots,\gamma_N\}$, $N$ is the number of agents.

*Remark 4.* This paper designs a controller with weighted coefficients. By adjusting the weighted coefficient of the controller, the state of many agents can be globally converged to any given weighted state. Compared with the original controller, the designed controller is more flexible and more adaptable to different states. At the same time, when the agent $j$ and the agent $i$ have the same dynamic, we adopt a cooperative approach. When the agents $j$ and $i$ have different dynamic, we use a competitive approach. By using cooperation-competition relationship, we ensure that heterogeneous MASs can achieve weighted couple-group consensus.

**Theorem 5.** *Based on system (7) and (8), and the undirected bipartite graph is assumed to be the topology of the system. if these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and $\tau \in \{0, \min[1/2\beta_i, 1/2\gamma_i \max\{\widetilde{D}_i\}]\}$, where $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$ and $\widetilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i\in o_2$, then the system can progressively achieve weighted couple-group consensus.*

*Proof.* By performing the Laplace transform on (7) and (8), we can get the following expression:

$$\begin{aligned}sx_i(s) &= v_i(s),\\ sv_i(s) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right]-\beta_i e^{-\tau s}v_i(s),\\ &\qquad i\in o_1.\end{aligned} \tag{9}$$

$$\begin{aligned}sx_i(s) &= \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right],\quad i\in o_2.\end{aligned} \tag{10}$$

Transform $x_i(t)$ and $v_i(t)$ into Laplace forms $x_i(s)$ and $v_i(s)$, respectively. From the (9), we have

$$\begin{aligned}s^2x_i(s) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right]-\beta_i s e^{-\tau s}x_i(s),\\ &\qquad i\in o_1.\end{aligned} \tag{11}$$

After transformation, we can get the following formula:

$$sx_i(s) = \frac{-s^2 x_i(s) + \alpha_i \left[\sum_{j\in N_{si}} a_{ij} \left[e^{-\tau_{ij}s} x_j(s) - e^{-\tau s} x_i(s)\right] - \sum_{j\in N_{di}} a_{ij} \left[e^{-\tau_{ij}s} x_j(s) + e^{-\tau s} x_i(s)\right]\right]}{\beta_i e^{-\tau s}}, \quad i \in o_1. \tag{12}$$

Next, we define $x_s(s) = [x_1(s), x_2(s), \ldots, x_n(s)]^T$, $x_f(s) = [x_{n+1}(s), x_{n+2}(s), \ldots, x_{n+m}(s)]^T$, and

$$\widehat{L} = \left(\widehat{l}_{ij}\right)_{(n+m)\times(n+m)} = \begin{cases} e^{-\tau_{ij}s} a_{ij}, & i \neq j \\ \sum_{j\in N_i} a_{ij} e^{-\tau s}, & i = j. \end{cases} \tag{13}$$

According to (10) and (12), we can get

$$\begin{aligned} sx_s(s) &= \frac{-s^2 C_2 x_s(s) + C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right) x_s(s) - C_2 C_1^{-1} \widehat{A}_{sf} x_f(s)}{e^{-\tau s}}, \\ sx_f(s) &= -C_3^{-1} \widehat{A}_{fs} x_s(s) - C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right) x_f(s). \end{aligned} \tag{14}$$

Here

$$C_1 = \begin{pmatrix} \frac{1}{\alpha_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\alpha_N} \end{pmatrix}, \quad C_2 = \begin{pmatrix} \frac{1}{\beta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\beta_N} \end{pmatrix}, \quad C_3 = \begin{pmatrix} \frac{1}{\gamma_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\gamma_N \end{pmatrix}. \tag{15}$$

Next, we define $y(s) = [x_s^T(s), x_f^T(s)]^T$, and we have

$$sy(s) = \widetilde{\Upsilon}(s)\, y(s). \tag{16}$$

Here

$$\widetilde{\Upsilon}(s) = \begin{bmatrix} \frac{-C_2 s^2 - C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{e^{-\tau s}} & \frac{-C_2 C_1^{-1} \widehat{A}_{sf}}{e^{-\tau s}} \\ -C_3^{-1} \widehat{A}_{fs} & -C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right) \end{bmatrix}. \tag{17}$$

According to (16), we can get $\widetilde{\Theta}(s) = \det(sI - \widetilde{\Upsilon}(s))$. According to the Lyapunov stability criterion, when the $\mathrm{Re}(\lambda_i(\widetilde{\Theta}(s))) < 0$, or $s = 0$, the system achieves group consensus. Next, using general Nyquist criteria, we discuss these two situations.

When $s = 0$, it can be clearly seen that 0 is a characteristic value of the matrix $D + A$, so one root of the formula can be obtained when $s = 0$. At the same time, when $s = 0$, $\widetilde{\Theta}(0) = \det(D + A)(\prod_{i=1}^{n}\alpha_i / \prod_{i=1}^{n}\beta_i)\prod_{i=1}^{m}\gamma_i$.

When $s \neq 0$, set $\widetilde{\Theta}(s) = \det(\Phi(s) + I)$ and

$$\Phi(s) = \begin{bmatrix} \frac{s^2 C_2 + C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{se^{-\tau s}} & \frac{C_2 C_1^{-1} \alpha_i \widehat{A}_{sf}}{se^{-\tau s}} \\ \frac{C_3^{-1} \widehat{A}_{fs}}{s} & \frac{C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right)}{s} \end{bmatrix} \tag{18}$$

where $s = j\omega$. In order for the system to achieve group consensus, the general Nyquist criterion, if and only if the point $(-1, j0)$ is not surrounded by the Nyquist curve, $\widetilde{\Theta}(s)'s$ root is located on the left half of the complex field. Based on the Gerschgorin disk theorem, we can get

$$\lambda(\Phi(j\omega)) \in \{\Phi_i, i \in o_1\} \cup \{\Phi_i, i \in o_2\}. \tag{19}$$

When $i \in o_1$, we have the following.

$$\Phi_i = \left\{ x : x \in \mathbb{C}, \left| x - \frac{\alpha_i}{j\omega\beta_i} \sum_{j\in N_i} a_{ij} - \frac{j\omega}{\beta_i} e^{j\omega\tau} \right| \leq \sum_{j\in N_i} \left| \frac{\alpha_i a_{ij}}{j\omega\beta_i} e^{-j\omega(\tau_{ij}-\tau)} \right| \right\} \tag{20}$$

For the convenience of calculation, we set $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$. At the same time, according to the general criteria, since the point $(-a, j0)$, $a \geq 1$, cannot be encircled in $\Phi_i$, $i \in o_1$, we can further transform the inequality into the following form:

$$\left| -a - \frac{\alpha_i D_i}{j\omega\beta_i} - \frac{j\omega}{\beta_i} e^{j\omega\tau} \right| > \sum_{j\in N_i} \left| \frac{\alpha_i a_{ij}}{j\omega\beta_i} e^{-j\omega(\tau_{ij}-\tau)} \right|. \tag{21}$$

According to the Euler formula and from (21), we can get

$$\left|-a-\frac{\alpha_i D_i}{\omega\beta_i}j-\frac{j\omega}{\beta_i}\left(\cos\omega\tau+j\sin\omega\tau\right)\right| > \left|\frac{\alpha_i D_i}{j\omega\beta_i}\left(\cos\omega\left(\tau_{ij}-\tau\right)-j\sin\omega\left(\tau_{ij}-\tau\right)\right)\right|. \quad (22)$$

After some transformation, we can get the following.

$$a^2-\frac{2a\omega}{\beta_i}\sin\omega\tau+\frac{\omega^2}{\beta_i^2}-\frac{2\alpha_i D_i}{\beta_i^{\,2}}\cos\omega\tau>0 \quad (23)$$

It is easy to see from (23) that when $a \geq 1$, $a^2-(2a\omega/\beta_i)\sin\omega\tau$ is monotonically increasing.

$$1-\frac{2\omega}{\beta_i}\sin\omega\tau+\frac{\omega^2}{\beta_i^2}-\frac{2\alpha_i D_i}{\beta_i^{\,2}}\cos\omega\tau>0 \quad (24)$$

Since $\beta_i$ is a positive number, we can transform (24) into the following form.

$$\beta_i^2-2\omega\beta_i\sin\omega\tau+\omega^2-2\alpha_i D_i\cos\omega\tau>0 \quad (25)$$

According to (24), it is obvious that the following two inequalities are true:

$$2\alpha_i D_i\cos\omega\tau-\beta_i^2<0 \quad (26)$$

and

$$2\omega\beta_i\sin\omega\tau-\omega^2<0. \quad (27)$$

According to (26), we can get $\beta_i^2 > 2\alpha_i D_i$, because $\cos\omega\tau \leq 1$. According to (27), we can change it to the following form.

$$1-2\beta_i\tau\left(\frac{\sin\omega\tau}{\omega\tau}\right)>0 \quad (28)$$

Since $(\sin\omega\tau/\omega\tau) \leq 1$, (27) is established if and only if $\tau \leq (1/2\beta_i)$.

Similarly, when $i \in o_2$, we can get the following inequalities according to the Gerschgorin theorem:

$$\Phi_i=\left\{x:x\in\mathbb{C},\left|x-\frac{\gamma_i}{j\omega}\sum_{j\in N_i}a_{ij}e^{-j\omega\tau}\right| \leq \sum_{j\in N_i}\left|\frac{\gamma_i a_{ij}}{j\omega}e^{-j\omega\tau_{ij}}\right|\right\} \quad (29)$$

so, the point $(-a, j0)$, $a \geq 1$, cannot be encircled in $\Phi_i$, $i \in o_2$, and then the following inequality is obtained.

$$\left|-a-\frac{\gamma_i}{j\omega}\sum_{j\in N_i}a_{ij}e^{-j\omega\tau}\right|>\sum_{j\in N_i}\left|\frac{\gamma_i a_{ij}}{j\omega}e^{-j\omega\tau_{ij}}\right| \quad (30)$$

Next, we define $\widetilde{D}_i=\sum_{j\in N_i}a_{ij}$, $i \in o_2$; then from (30), we have the following.

$$\left|-a+\frac{\gamma_i\widetilde{D}_i}{j\omega}\sum_{j\in N_i}\left(j\cos\omega\tau+\sin\omega\tau\right)\right| > \sum_{j\in N_i}\left|\frac{\gamma_i\widetilde{D}_i}{j\omega}\left(-j\cos\omega\tau_{ij}-\sin\omega\tau_{ij}\right)\right| \quad (31)$$

After some calculations, we can get the following simplified formula.

$$a^2-\frac{2a\gamma_i\widetilde{D}_i}{\omega}\sin\omega\tau>0 \quad (32)$$

From (32), we know that $a^2-(2a\gamma_i\widetilde{D}_i/\omega)\sin\omega\tau$ will gradually increase as $a$ increases. Here we set $a=1$. Obviously, we have the following.

$$1-\frac{2\gamma_i\widetilde{D}_i}{\omega}\sin\omega\tau>0 \quad (33)$$

Since $(\sin\omega\tau/\omega\tau) \leq 1$, (32) is established if and only if $\tau \leq (1/2\gamma_i\widetilde{D}_i)$.

Obviously, we have completed the proof of Theorem 5. □

**Corollary 6.** *Based on system (7) and (8), a bipartite digraph containing a directed spanning tree is assumed to be the topology of the system. If these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and $\tau \in [0, \min\{1/2\beta_i, 1/2\gamma_i \max\{\widetilde{D}_i\}\}]$, where $D_i=\sum_{j\in N_i}a_{ij}$, $i \in o_1$, and $\widetilde{D}_i=\sum_{j\in N_i}a_{ij}$, $i \in o_2$, then the system can progressively achieve weighted couple-group consensus.*

Combined with the previous analysis, it is clear that the theorem is completed.

Using Proof and Lemma 3, it is clear that Corollary 6 is true.

Theorem 5 is proved.

*Remark 7.* From Theorem 5, we can see that the control parameters $\alpha_i$, $\beta_i$, $\gamma_i$ and coupling weight of the system are the key parameters affecting the consensus of the weighted couple-group, and the input time delay is determined by the coupling weight and the control parameters. However, we can see that communication delay has no effect on group consensus.

*Remark 8.* The proposed system (7) and (8) is constructed by using the cooperation-competitive interaction between agents in this paper. Since most of the agents currently working rely on the cooperation or competitive relationship, such as in [17, 18, 29, 31, 32, 34–39], this paper studies the group consensus of heterogeneous complex systems from a new perspective. At the same time, it should be noted that in the proposed protocol, the first-order agent does not contain virtual speed estimation, which can make more rational use of resources and reduce computational cost, for example, in [29, 31, 32, 37].

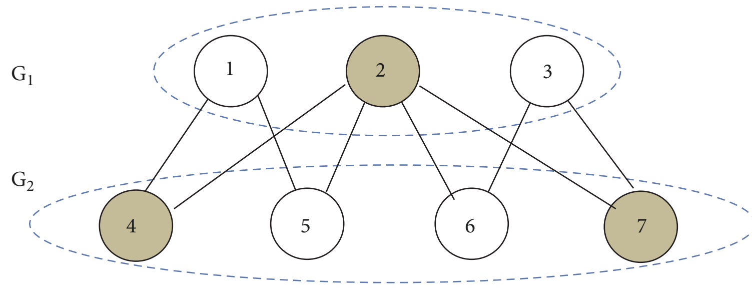



FIGURE 1: The bipartite digraph topology of the heterogeneous MASs.

*Remark 9.* Different from the works in [31, 32, 37], we have relaxed the condition of intra-degree balance, which facilitates communication between agents. In real life, there are many limitations in in-degree balance, because it will result in no actual communication between subsystems [13]. In other words, it will cause the interaction between agents in different subsystems to be offset. At the same time, we do not require that the system satisfies the condition that the geometric versatility of the zero eigenvalues of the Laplacian matrix is not less than 2, which makes the system's topology more flexible.

*Remark 10.* Most of the works are weighted by a fixed value. We use dynamic weighted methods here, namely, $\alpha_i$, $\beta_i$, and $\gamma_i$. The weighted coefficients corresponding to each agent are different, which enables the MASs state to converge globally to any given weighted state. Compared with the original controller, the designed controller is more flexible and more adaptable to different states. In addition, in most of the existing works, the consideration of the delay problem is relatively simple. Only the effects of either input delays [36] or time delays are not considered, such as [31, 32, 36].

To discuss the effect of different input delays and communication delays on the multiagent implementation of group consensus, we rewrite (7) and (8) as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i\left(t-\tau_i\right)\right] - \sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i\left(t-\tau_i\right)\right]\right] - \beta_i v_i\left(t-\tau_i\right), \quad i\in o_1.\end{aligned} \tag{34}$$

$$\dot{x}_i(t) = \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i\left(t-\tau_i\right)\right] - \sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i\left(t-\tau_i\right)\right]\right], \quad i\in o_2. \tag{35}$$

Here $\tau_{ij}$ represents the communication delay between the agents $i$ and $j$, and $\tau_i$ represents the input time delay of the agent $i$.

**Theorem 11.** *Based on Protocol (34) and (35), the undirected bipartite graph is assumed to be the topology of the system. If these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and if $i\in o_1$, $\tau_i\in[0,1/2\beta_i]$ or, otherwise, $\tau_i\in[0,1/2\gamma_i\widetilde{D}_i]$, $i\in o_2$, where $D_i=\sum_{j\in N_i} a_{ij}$, $i\in o_1$, and $\widetilde{D}_i=\sum_{j\in N_i} a_{ij}$, $i\in o_2$, then the system can progressively achieve weighted couple-group consensus.*

**Corollary 12.** *Based on Protocol (34) and (35), a bipartite digraph containing a directed spanning tree is assumed to be the topology of the system. If these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and if $i\in o_1$, $\tau_i\in[0,1/2\beta_i]$ or, otherwise, $\tau_i\in[0,1/2\gamma_i\widetilde{D}_i]$, $i\in o_2$, where $D_i=\sum_{j\in N_i} a_{ij}$, $i\in o_1$, and $\widetilde{D}_i=\sum_{j\in N_i} a_{ij}$, $i\in o_2$, then the system can progressively achieve weighted couple-group consensus.*

The conclusion here is obvious.

*Remark 13.* From Theorem 11, the communication delay of the agent has no effect on the group consensus of the system. At the same time, the upper limit of the input time delay is controlled by the control parameters and coupling weights with the same dynamics, and the delay conditions between different dynamics are different. Communication delay has no effect on the group consensus of the system.

*Remark 14.* Since the system needs some other external conditions when implementing group consensus, our assumed topology is not a specific topology. For example, in [31, 32, 36, 37], the topology of the system is also an undirected graph or a graph containing a directed spanning tree. At the same time, in order to achieve group consensus, some additional assumptions are needed, mentioned in Remarks 7, 8, and 9.

## 4. Simulation

In this section, several simulation results will be used to illustrate the validity of the results obtained. Figure 1 shows a binary topology of a heterogeneous system. The entire system

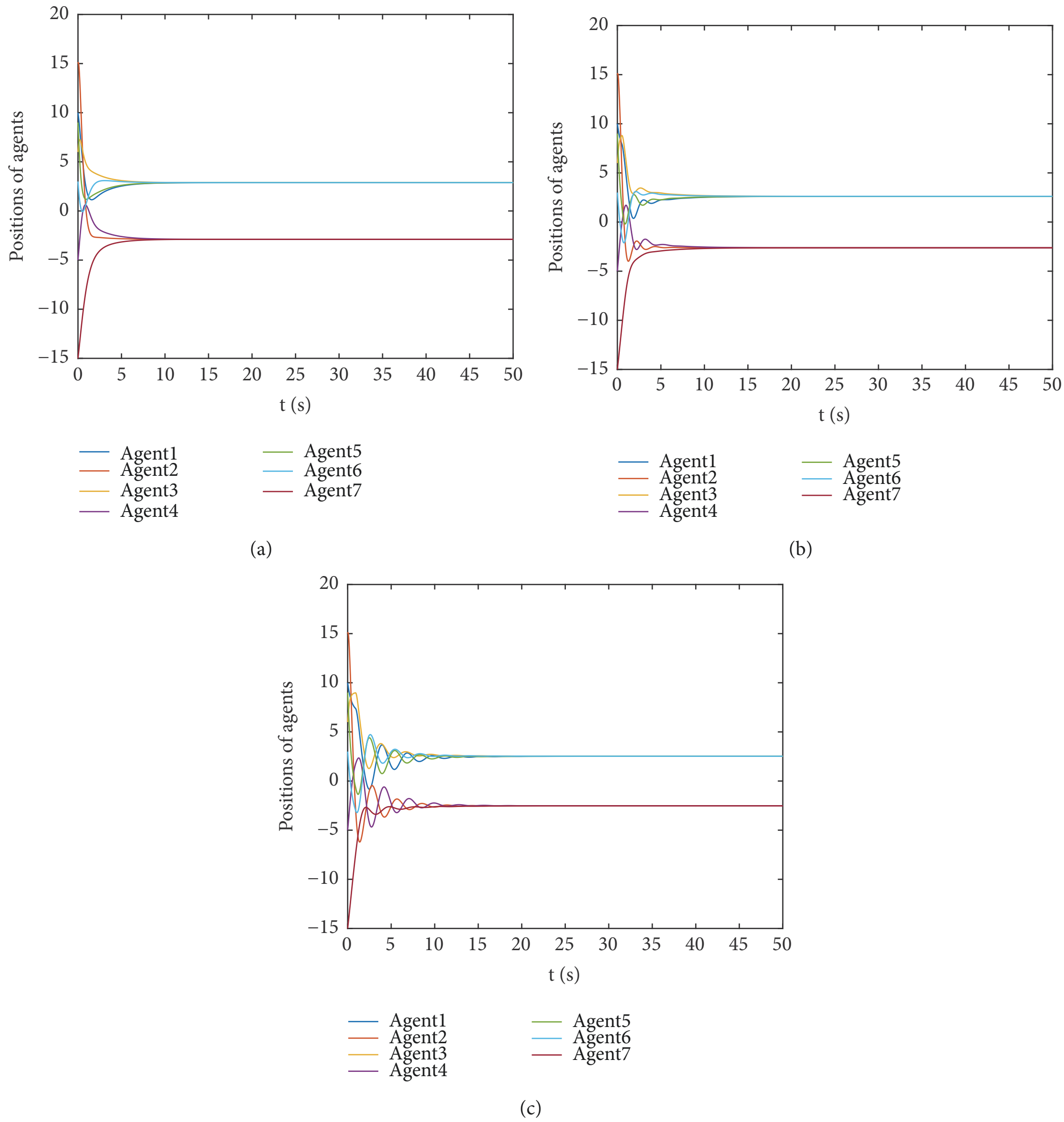



Figure 2: The agents position trajectories, where $\tau = 0.05$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.9$.

is divided into two subgroups, $G_1$ and $G_2$. The system contains agents 1, 2, 3, 4, 5, 6, and 7. In order not to lose generality, we denote 2, 4, and 7 as second-order agents, denoted by $o_1$. The first-order agent includes the remaining agents 1, 3, 5, and 6 and is represented by $o_2$. Obviously, subgroup $G_1$ and subgroup $G_1$ are heterogeneous in Figure 1.

*Remark 15.* From Figure 1, the dynamics of the agents in subgroup $G_1$ and subgroup $G_2$ are heterogeneous. Obviously, we do not require that the dynamics of agents within the same subgroup be homogeneous, such as [32, 37].

*Example 16.* For convenience, we set $a_{ij} = 1$, $i, j \in [1, 7]$, and let $\alpha_i = \text{diag}[1, 1.5, 2, 3, 0.9, 0.8, 0.5]$, $\beta_i = \text{diag}[3, 4, 3, 4, 3, 3, 2]$, $\gamma_i = \text{diag}[1, 1.5, 2, 3, 0.9, 0.8, 0.5]$. Since Figure 1 is an undirected bipartite graph, we can get $d_1 = 2$, $d_2 = 4$, $d_3 = 2$, $d_4 = 2$, $d_5 = 2$, $d_6 = 2$, $d_7 = 2$.

According to the qualification conditions proposed by Theorem 5, we can calculate the range of the input delay as $\tau = \min\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7\}$. In the simulation experiment, we assume $\tau = 0.05$. Obviously, $\tau$ at this time satisfies all the qualifications. To verify the impact of different delays on system group consensus, we assume different input delays and communication delays. In Figure 2, we assume an input delay of $\tau = 0.05$ and then input different communication delays to compare their effects on the system convergence rate. In Figure 3, we fixed the communication delay and then assumed different input delays.

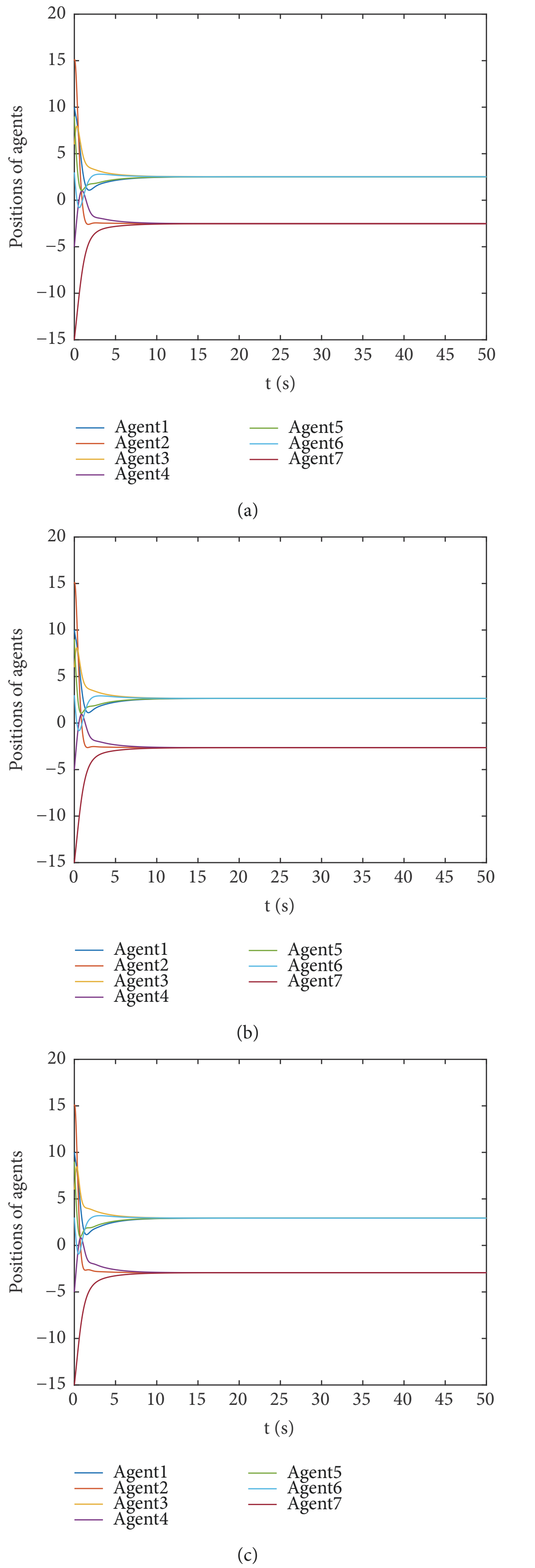



FIGURE 3: The agents position trajectories, where $\tau_{ij} = 0.2$. (a) $\tau = 0$, (b) $\tau = 0.03$, and (c) $\tau = 0.08$.

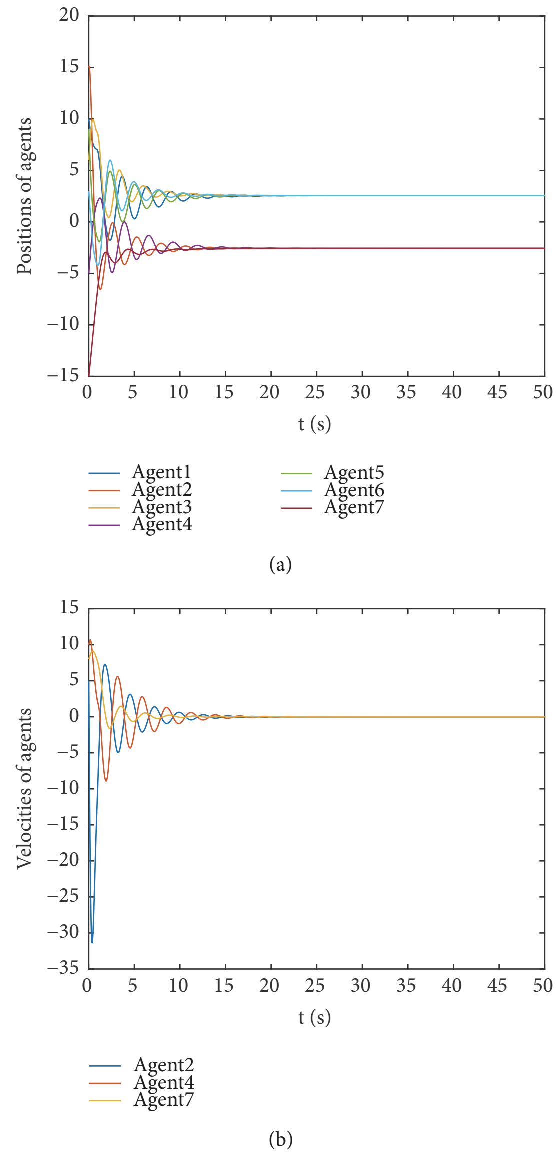
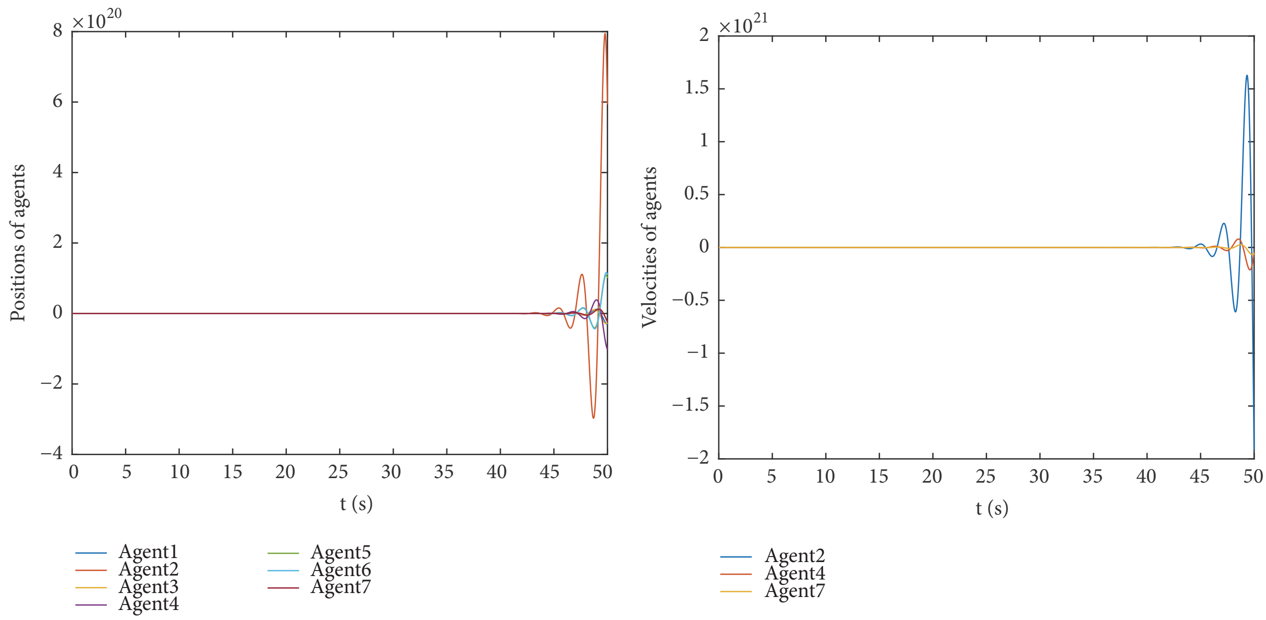



FIGURE 4: The state trajectories of the agents under undirected topology in Figure 1 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

*Remark 17.* It can be seen from Figures 2 and 3 that the input delay and communication delay will affect the convergence trajectory of the agent. When the input delay or the communication delay increases, the convergence speed of the agent decreases, so we can increase the convergence speed by reducing the delay.

From the qualification of Theorem 11, we can calculate the range of input delay for each agent: $\tau_1 = [0, 1/4]$, $\tau_2 = [0, 1/8]$, $\tau_3 = [0, 1/8]$, $\tau_4 = [0, 1/8]$, $\tau_5 = [0, 1/3.6]$, $\tau_6 = [0, 1/3.2]$, $\tau_7 = [0, 1/4]$. Here we take $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$. Obviously all $\tau$ are satisfied with Theorem 11. Figure 4 demonstrates that




(a) (b)

FIGURE 5: The state trajectories of the agents under undirected topology in Figure 1 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.5$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

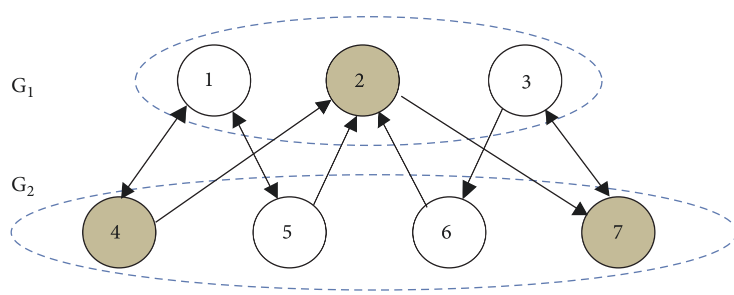



FIGURE 6: The directed graph topology of the heterogeneous MASs.

weighted couple-group consensus is achievable. At the same time, according to the upper bound calculated by Theorem 11, we assume $\tau_2 = 0.5$. As can be seen from Figure 5, the system is divergent at this time.

Next, we will testify Theorem 11 and Corollary 12.

*Example 18.* We assume that the topology of a heterogeneous system contains a directed spanning tree, as shown in Figure 6. Since Figure 6 is an directed bipartite graph, we can get $d_1 = 2$, $d_2 = 3$, $d_3 = 1$, $d_4 = 1$, $d_5 = 1$, $d_6 = 1$, $d_7 = 2$. According to Corollary 6, we can assume that $\tau = 0.05$; obviously, $\tau$ satisfies all the qualifications. In Figure 7, we set the input delay $\tau = 0.05$ to a fixed value and enter different communication delays. According to the topology and Corollary 12 of Figure 6, we set $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.2$, $\tau_4 = 0.1$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, and $\tau_{ij} = 0.9$, as shown in Figure 8. Obviously, from Figures 7 and 8, we can easily find that the system can progressively implement weighted couple-group consensus. When $\tau_4 = 0.5$, the system is divergent, as shown in Figure 9.

## 5. Conclusion

This paper studies the group consensus problem of heterogeneous MASs based on bipartite graph structure. The dynamic weighted couple-group consensus in the case of time delay is considered. A new weighted couple-group consensus protocol is designed by using cooperation and competition interaction between agents. Using graph theory, matrix theory, Gerschgorin disk theorem, and general Nyquist criterion,

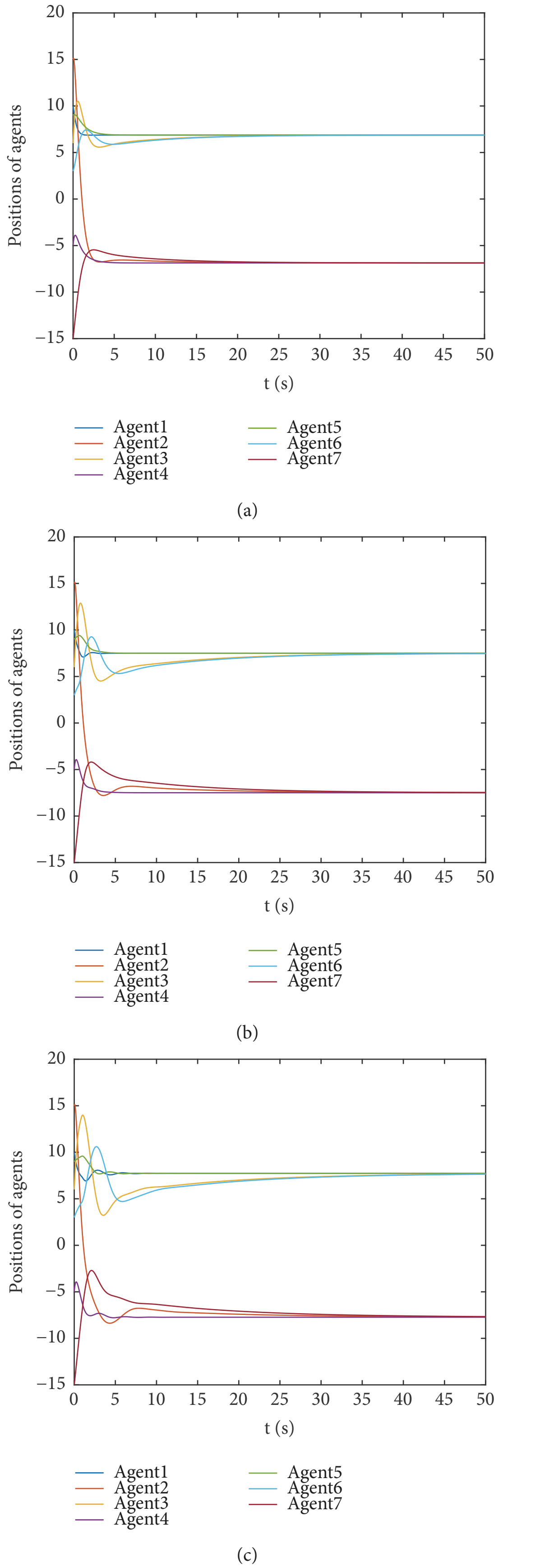



Figure 7: The agents position trajectories, where $\tau = 0.05$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.9$.

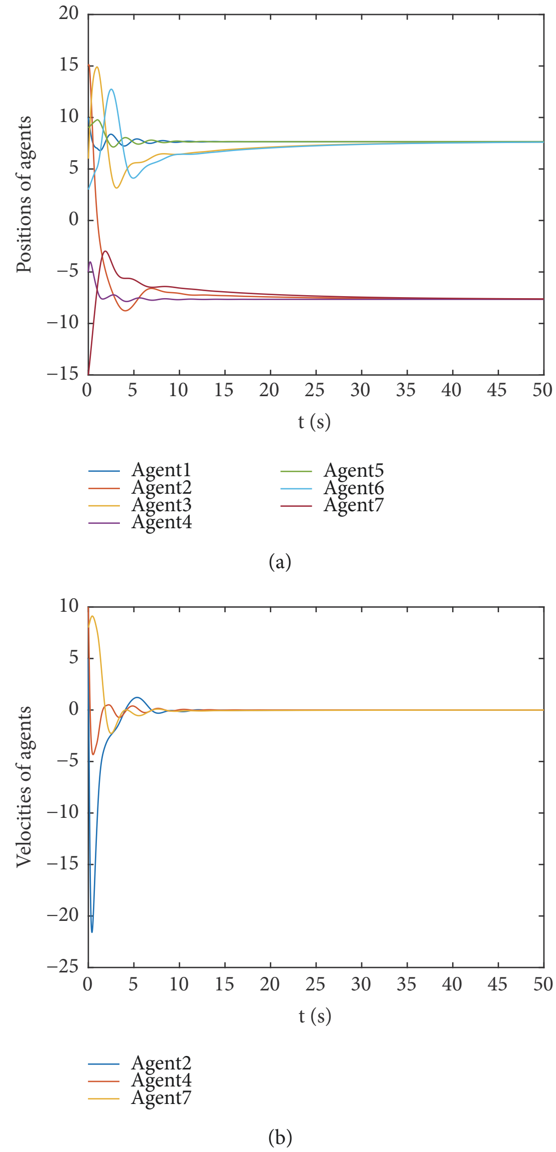



Figure 8: The state trajectories of the agents under directed topology in Figure 6 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

the upper bound of the maximum delay that can be tolerated when the system reaches convergence is obtained. It is not difficult to see from the theoretical results that the weighted couple-group consensus of the heterogeneous MASs is not directly related to the communication delay. The heterogeneous MASs implement weighted couple-group consensus, which is determined by the coupling weight between the agents, the input time delay, and the control parameters. In addition, in order to speed up the convergence of the system, we can reduce the communication delay or input delay, or both of them. The simulation example validated the results. In the future work, we will study the group consensus problems of more complex heterogeneous multiagent systems.

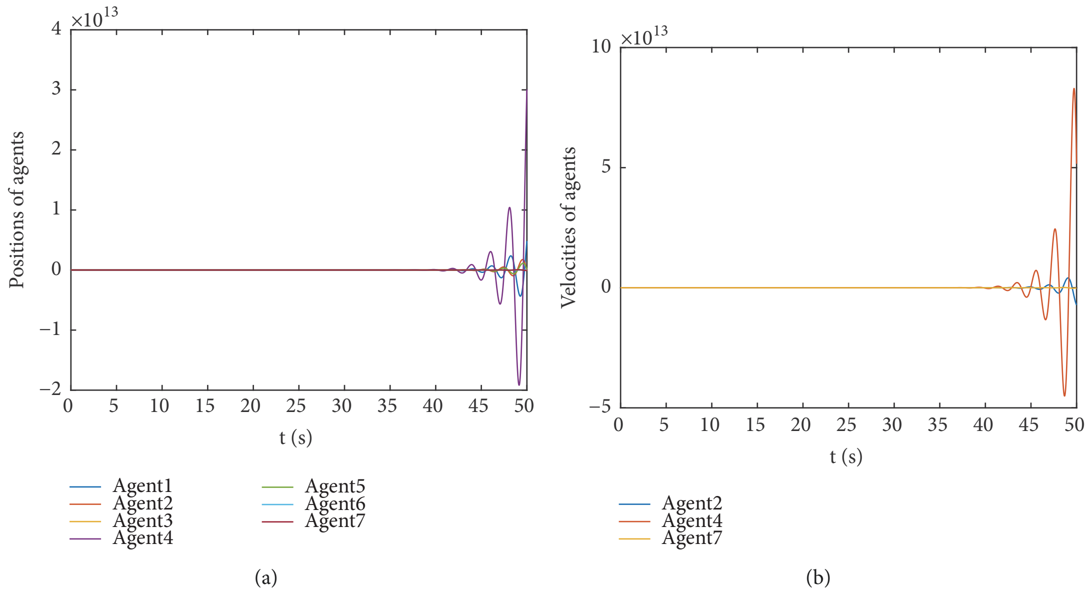



Figure 9: The state trajectories of the agents under directed topology in Figure 6 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.5$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

For example, we will consider switching topology or event driven.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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