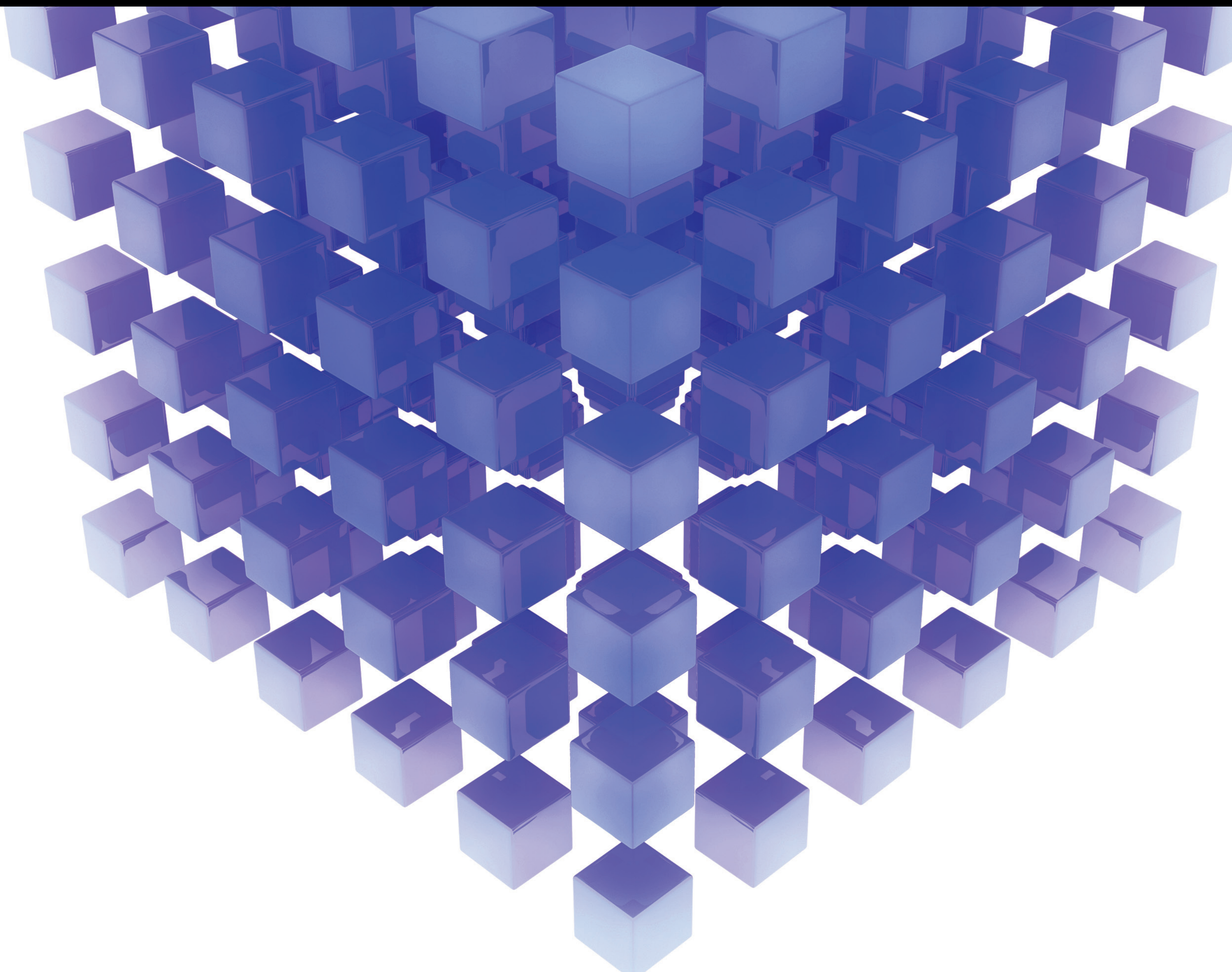
Mathematical Problems in Engineering

Mathematical Problems in Emerging Manufacturing Systems Management

Guest Editors: Taho Yang, Mu-Chen Chen, Felix T.S. Chan, Chiwoon Cho, and Vikas Kumar



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Editorial

Mathematical Problems in Emerging Manufacturing Systems Management

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This special issue aims to address the *mathematical problems* associated with the management of innovative emerging manufacturing systems. The scope of innovative manufacturing systems management in this special issue addresses the emerging issues from production and operations management, manufacturing strategy, lean/agile manufacturing, supply chain and logistics management, healthcare systems management, and so forth. The contributions gathered in this special issue offer a snapshot of different interesting researches, problems, and solutions. In the following, we briefly highlight these topics and synthesize the content of each paper.

The paper "Impacts of Transportation Cost on Distribution-Free Newsboy Problems," by M.-H. Shu et al., addresses a distribution-free newsboy problem (DFNP) for a vendor to decide a product's stock quantity in a single-period inventory system to sustain its least maximum-expected profits. The transportation cost is formulated as a function of shipping quantity and is modeled as a nonlinear regression form. An optimal solution of the order quantity is computed on the basis of Newton's approach to ameliorate its complexity of computation. The empirical results are quite competitive with the results from the existing literature.

The paper "The Integration of Group Technology and Simulation Optimization to Solve the Flow Shop with Highly Variable Cycle Time Process: A Surgery Scheduling Case Study," by T. K. Wang et al., introduces a case of healthcare system application. It proposes an algorithm that allows the estimation of the mean effective process time and the coefficient of variation. It also develops a group technology based takt time. A simulation model is combined with the case study, and the capacity buffers are optimized against the remaining variability for each group. The empirical results from a practical application are quite promising.

The paper "Undesirable Outputs' Presence in Centralized Resource Allocation Model," by G. Tohidi et al., extends the existing Data Envelopment Analysis (DEA) literature and proposes a new Centralized Resource Allocation (CRA) model to assess the overall efficiency of system consisting of Decision Making Units (DMUs) by using directional distance function when DMUs produce desirable and undesirable outputs.

The paper "A Multiple Attribute Group Decision Making Approach for Solving Problems with the Assessment of Preference Relations," by T. Yang et al., proposes to use a fuzzy preference relations matrix which satisfies additive consistency in solving a multiple attribute group decision making (MAGDM) problem. It takes a heterogeneous group of experts into consideration. A numerical example is used to test the proposed approach; and the results illustrate that the method is simple, effective, and practical.

The paper "Integrated Supply Chain Cooperative Inventory Model with Payment Period Being Dependent on Purchasing Price under Defective Rate Condition," by M.-F. Yang et al., aims at finding the maximum of the joint expected total profit and at coming up with a suitable inventory policy. It solves the trade-off between increased postponed payment deadline and the decreased profit for a buyer and vice versa for a vendor. Its numerical illustrations provide useful managerial insights.

The paper "Clustering Ensemble for Identifying Defective Wafer Bin Map in Semiconductor Manufacturing," by C.-Y. Hsu, proposes a clustering ensemble approach to facilitate wafer bin map defect detection problem from semiconductor manufacturing. It adopts a series of algorithms to solve the proposed problem such as mountain function, k-means, particle swarm optimization, and neural network model. The numerical results are promising.

The paper "Joint Optimization Approach of Maintenance and Production Planning for a Multiple-Product Manufacturing System," by L. Mifdal et al. deals with the problem of maintenance and production planning for randomly failing multiple-product manufacturing system. It establishes sequentially an economical production plan and an optimal maintenance strategy, taking into account the influence of the production rate on the system's degradation. Analytical models are developed in order to minimize sequentially the total production/inventory cost and then the total maintenance cost. Finally, a numerical example is presented to illustrate the usefulness of the proposed approach.

The paper "The Dynamics of Bertrand Model with Technological Innovation," by F. Wang et al., studied the dynamics of a Bertrand duopoly game with technology innovation, which contains bounded rational and naive players. The stability of the equilibrium point, the bifurcation, and chaotic behavior of the dynamic system have been analyzed. It concludes that technology innovation can enlarge the stability region of the speed and control the chaos of the dynamic system effectively.

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Research Article

Clustering Ensemble for Identifying Defective Wafer Bin Map in Semiconductor Manufacturing

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Wafer bin map (WBM) represents specific defect pattern that provides information for diagnosing root causes of low yield in semiconductor manufacturing. In practice, most semiconductor engineers use subjective and time-consuming eyeball analysis to assess WBM patterns. Given shrinking feature sizes and increasing wafer sizes, various types of WBMs occur; thus, relying on human vision to judge defect patterns is complex, inconsistent, and unreliable. In this study, a clustering ensemble approach is proposed to bridge the gap, facilitating WBM pattern extraction and assisting engineer to recognize systematic defect patterns efficiently. The clustering ensemble approach not only generates diverse clusters in data space, but also integrates them in label space. First, the mountain function is used to transform data by using pattern density. Subsequently, k-means and particle swarm optimization (PSO) clustering algorithms are used to generate diversity partitions and various label results. Finally, the adaptive response theory (ART) neural network is used to attain consensus partitions and integration. An experiment was conducted to evaluate the effectiveness of proposed WBMs clustering ensemble approach. Several criterions in terms of sum of squared error, precision, recall, and F-measure were used for evaluating clustering results. The numerical results showed that the proposed approach outperforms the other individual clustering algorithm.

1. Introduction

To maintain their profitability and growth despite continual technology migration, semiconductor manufacturing companies provide wafer manufacturing services generating value for their customers through yield enhancement, cost reduction, on-time delivery, and cycle time reduction [1, 2]. The consumer market requires that semiconductor products exhibiting increasing complexity be rapidly developed and delivered to market. Technology continues to advance and required functionalities are increasing; thus, engineers have a drastically decreased amount of time to ensure yield enhancement and diagnose defects [3].

The lengthy process of semiconductor manufacturing involves hundreds of steps, in which big data including the wafer lot history, recipe, inline metrology measurement, equipment sensor value, defect inspection, and electrical test data are automatically generated and recorded. Semiconductor companies experience challenges integrating big data from various sources into a platform or data warehouse and lack intelligent analytics solutions to extract useful manufacturing intelligence and support decision making regarding production planning, process control, equipment monitoring, and yield enhancement. Scant intelligent solutions have been developed based on data mining, soft computing, and evolutionary algorithms to enhance the operational effectiveness of semiconductor manufacturing [4–7].

Circuit probe (CP) testing is used to evaluate each die on the wafer after the wafer fabrication processes. Wafer bin maps (WBMs) represent the results of a CP test and provide crucial information regarding process abnormalities, facilitating the diagnosis of low-yield problems in semiconductor manufacturing. In WBM failure patterns the spatial dependences across wafers express systematic and random effects. Various failure patterns are required; these pattern types facilitate rapidly identifying the associate root causes of low yield [8]. Based on the defect size, shape, and location on the wafer, the WBM can be expressed as specific patterns such as rings, circles, edges, and curves. Defective dies caused by random particles are difficult to completely remove and typically exhibit nonspecific patterns. Most WBM patterns consisted of a systematic pattern and a random defect [8–10].

In practice, thousands of WBMs are generated for inspection and engineers must spend substantial time on pattern judgment rather than determining the assignable causes of low yield. Grouping similar WBMs into the same cluster can enable engineers to effectively diagnose defects. The complicated processes and diverse products fabricated in semiconductor manufacturing can yield various WBM types, making it difficult to detect systematic patterns by using only eyeball analysis.

Clustering analysis is used to partition data into several groups in which the observations are homogeneous within a group and heterogeneous between groups. Clustering analysis has been widely applied in applications such as grouping [11] and pattern extraction [12]. However, most conventional clustering algorithms influence the result based on the data type, algorithm parameter settings, and prior information. For example, the k-means algorithm is used to analyze substantial amount of data that exhibit time complexity [13]. However, the results of the k-means algorithm depend on the initially selected centroid and predefined number of clusters. To address the disadvantages of the k-means algorithm evolutionary methods have been developed to conduct data clustering such as the genetic algorithm (GA) and particle swarm optimization (PSO) [14]. PSO is particularly advantageous because it requires less parameter adjustment compared with the GA [15].

Combining results by applying distinct algorithms to the same data set or algorithm by using various parameter settings yields high-quality clusters. Based on the criteria of the clustering objectives, no individual clustering algorithm is suitable for whole problem and data type. Compared with individual clustering algorithms, clustering ensembles that combine multiple clustering results yield superior clustering effectiveness regarding robustness and stability, incorporating conflicting results across partitions [16]. Instead of searching for an optimal partition, clustering ensembles capture a consensus partition by integrating diverse partitions from various clustering algorithms. Clustering ensembles have been developed to improve the accuracy, robustness, and stability of clustering; such ensembles typically involve two steps. The first step involves generating a basic set of partitions that can be similar to or distinct from those of various parameters and cluster algorithms [17]. The second step involves combining the basic set of partitions by using a consensus function [18]. However, with the shrinking integrated circuit feature size and complicated manufacturing process, the WBM patterns become more complex because of various defect density, die size, and wafer rotation. It is difficult to extract defect pattern by single specific clustering approach and needs to incorporate different clustering aspects for various complicated WBM patterns.

To bridge the need in real setting, this study proposes a WBM clustering ensemble approach to facilitate WBM defect pattern extraction. First, the target bin value is categorized into binary value and the wafer maps are transformed from two-dimensional to one-dimensional data. Second, k-means and PSO clustering algorithms are used to generate various diversity partitions. Subsequently, the clustering results are regarded as label representations to facilitate aggregating the diversity partition by using an adaptive response theory (ART) neural network. To evaluate the validity of the proposed method, an experimental analysis was conducted using six typical patterns found in the fabrication of semiconductor wafers. Using various parameter settings, the proposed cluster ensembles that combine diverse partitions instead of using the original features outperform individual clustering methods such as k-means and PSO.

The remainder of this study is organized as follows. Section 2 introduces a fundamental WBM. Section 3 presents the proposed approach to the WBM clustering problem. Section 4 provides experimental comparisons, applying the proposed approach to analyze the WBM clustering problem. Section 5 offers a conclusion and the findings and future research directions are discussed.

2. Related Work

A WBM is a two-dimensional failure pattern. Based on various defects types, random, systematic, and mixed failure patterns are primary types of WBMs generated during semiconductor fabrication [19, 20]. Random failure patterns are typically caused by random particles or noises in the manufacturing environment. In practice, completely eliminating these random defects is difficult. Systematic failure patterns show the spatial correlation across wafers such as rings, crescent moon, edge, and circles. Figure 1 shows typical WBM patterns which are transformed into binary values for visualization and analysis. The dies that pass the functional test are denoted as 0 and the defective dies are denoted as 1. Based on the systematic patterns, domain engineers can rapidly determine the assignable causes of defects [8]. Mixed failure patterns comprise the random and systematic defects on a wafer. The mixed pattern can be identified if the degree of the random defect is slight.

Defect diagnosis of facilitating yield enhancement is critical in the rapid development of semiconductor manufacturing technology. An effective method of ensuring that the causes of process variation are assignable is analyzing the spatial defect patterns on wafers. WBMs provide crucial guidance, enabling engineers to rapidly determine the potential root causes of defects by identifying patterns. Most studies have used neural network and model-based approaches to extract common WBM patterns. Hsu and Chien [8] integrated spatial statistical analysis and an ART neural network to conduct WBM clustering and associated the patterns with manufacturing defects to facilitate defect diagnosis. In addition to ART neural network, Liu and Chien [10] applied moment invariant for shape clustering of WBMs. Model-based clustering algorithms are used to construct a model for each cluster and compare the likelihood values between clusters to identify defect patterns. Wang et al. [21] used model-based clustering, applying a Gaussian expectation maximization algorithm to estimate defect patterns. Hwang and Kuo [22] modeled global defects

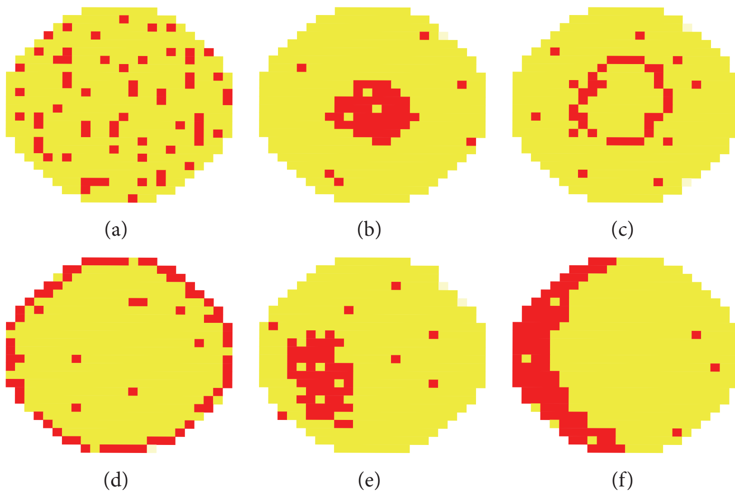


Figure 1: Typical WBM patterns.

and local defects in clusters exhibiting ellipsoidal patterns and local defects in clusters exhibiting linear or curvilinear patterns. Yuan and Kuo [23] used Bayesian inference to identify the patterns of spatial defects in WBMs. Driven by continuous migration of semiconductor manufacturing technology, the more complicated types of WBM patterns have been occurred due to the increase of wafer size and shrinkage of critical dimensions on specific aspect of complex WBM pattern and little research has evaluated using the clustering ensemble approach to analyze WBMs and extract failure patterns.

3. Proposed Approach

The terminologies and notations used in this study are as follows:

N_g: number of gross dies;

N_w: number of wafers;

N_p: number of particles;

N_c: number of clusters;

N_b: number of bad dies;

i: wafer index, $i = 1, 2, \ldots, N_w$;

j: dimension index, $j = 1, 2, \ldots, N_g$;

k: cluster index, $k = 1, 2, \ldots, N_c$;

l: particle index, $l = 1, 2, \ldots, N_p$;

q: clustering result index, $q = 1, 2, \ldots, M$;

r: bad die index, $r = 1, 2, \ldots, N_b$;

s: clustering subobjective in PSO clustering, $s = 1, 2, 3$;

U: uniform random number in the interval $[0, 1]$;

ω_v: inertia weight of velocity update;

ω_s: weight of clustering subobjective;

c_p: personal best position acceleration constants;

c_g: global best position acceleration constants;

β: a normalization factor;

m: a constant for approximate density shape in mountain function;

y_r: the rth bad die on a wafer;

n_k: the number of WBMs in the kth cluster;

n_{lk}: the number of WBMs in the kth cluster of lth particle;

C_k: subset of WBMs in the kth cluster;

$x^{\max}$: maximum value in the WBM data;

$\mathbf{m}_k$: vector of the kth cluster centroid, $\mathbf{m}_k = [m_{k1}, m_{k2}, \ldots, m_{kN_g}]$;

$\mathbf{m}_{lk}$: vector centroid of the kth cluster of lth particle;

$\mathbf{p}_l$: vector centroids of the lth particle, $\mathbf{p}_l = [m_{l1}, m_{l2}, \ldots, m_{lk}]$;

θ_{lj}: position of the lth particle at the jth dimension;

V_{lj}: velocity of the lth particle at the jth dimension;

ψ_{lj}: personal best position ($p$best) of the lth particle at jth dimension;

ψ_{gj}: global best position ($g$best) at the jth dimension;

$\mathbf{x}_i$: vector of the ith WBM, $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{iN_g}]$;

$\mathbf{\Theta}_l$: vector position of the lth particle, $\mathbf{\Theta}_l = [\theta_{l1}, \theta_{l2}, \ldots, \theta_{lN_g}]$;

$\mathbf{V}_l$: vector velocity of the lth particle, $\mathbf{V}_l = [V_{l1}, V_{l2}, \ldots, V_{lN_g}]$;

$\boldsymbol{\psi}_l$: vector personal best of the lth particle, $\boldsymbol{\psi}_l = [\psi_{l1}, \psi_{l2}, \ldots, \psi_{lN_g}]$;

$\boldsymbol{\psi}_g$: vector global best position, $\boldsymbol{\psi}_g = [\psi_{g1}, \psi_{g2}, \ldots, \psi_{gN_g}]$.

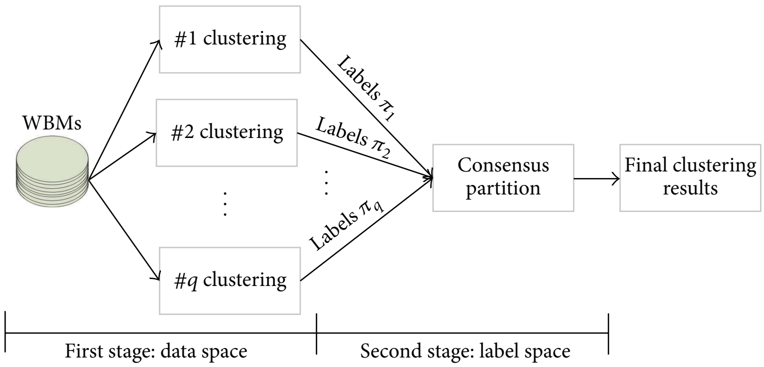


Figure 2: A framework for WBMs clustering ensemble.

3.1. Problem Definition of WBM Clustering Ensemble. Clustering ensembles can be regarded as two-stage partitions, in which various clustering algorithms are used to assess the data space at the first stage and consensus function is used to assess the label space at the second stage. Figure 2 shows the two-stage clustering perspective. Consensus function is used to develop a clustering combination based on the diversity of the cluster labels derived at the first stage.

Let $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N_w}\}$ denote a set of N_w WBMs and $\Pi = \{\pi_1, \pi_2, \ldots, \pi_M\}$ denote a set of partitions based on M clustering results. The various partitions of $\pi_q(x_i)$ represent a label assigned to x_i by the qth algorithm. Each label vector π_q is used to construct a representation Π, in which the partitions of $\mathbf{X}$ comprise a set of labels for each wafer $\mathbf{x}_i$, $i = 1, \ldots, N_w$. Therefore, the difficulty of constructing a clustering ensemble is locating a new partition Π that provides a consensus partition satisfying the label information derived from each individual clustering result of the original WBM. For each label π_q, a binary membership indicator matrix $H^{(q)}$ is constructed, containing a column for each cluster. All values of a row in the $H^{(q)}$ are denoted as 1 if the row corresponds to an object. Furthermore, the space of a consensus partition changes from the original N_g features into N_w features. For example, Table 1 shows eight WBMs grouped using three clustering algorithms (π_1, π_2, π_3); the three clustering results are transformed into clustering labels that are transformed into binary representations (Table 2). Regarding consensus partitions, the binary membership indicator matrix $H^{(q)}$ is used to determine a final clustering result, using a consensus model based on the eight features $(v_1, v_2, \ldots, v_8)$.

Table 1: Original label vectors.

	π_1	π_2	π_3
$\mathbf{x}_1$	1	1	1
$\mathbf{x}_2$	1	1	1
$\mathbf{x}_3$	1	1	1
$\mathbf{x}_4$	2	2	1
$\mathbf{x}_5$	2	2	2
$\mathbf{x}_6$	3	1	2
$\mathbf{x}_7$	3	1	2
$\mathbf{x}_8$	3	1	2

Table 2: Binary representation of clustering ensembles.

Clustering results		v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$H^{(1)}$	h_{11}	1	1	1	0	0	0	0	0
	h_{12}	0	0	0	1	1	0	0	0
	h_{13}	0	0	0	0	0	1	1	1
$H^{(2)}$	h_{21}	1	1	1	0	0	1	1	1
	h_{22}	0	0	0	1	1	0	0	0
$H^{(3)}$	h_{31}	1	1	1	1	0	0	0	0
	h_{32}	0	0	0	0	1	1	1	1

3.2. Data Transformation. The binary representation of good and bad dies is shown in Figure 3(a). Although this binary representation is useful for visualisation, displaying the spatial relation of each bad die across a wafer is difficult.

To quantify the spatial relations and increase the density of a specific feature, the mountain function is used to transform the binary value into a continuous value. The mountain method is used to determine the approximate cluster center by estimating the probability density function of a feature [24]. Instead of using a grid node, a modified mountain function can employ data points by using a correlation self-comparison [25]. The modified mountain function for a bad die r on a wafer $M(y_r)$ is defined as follows:

$$M(y_r) = \sum_{r=1}^{N_b} e^{-m\beta d(y_r, y_s)}, \quad r = 1, 2, 3, \ldots, N_b, \tag{1}$$

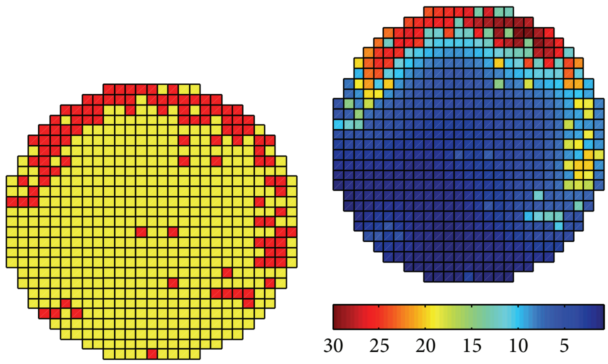
where

$$\beta = \left(\frac{d(y_r - y_{\mathrm{wc}})}{N_b}\right)^{-1} \tag{2}$$

and $d(y_r, y_s)$ is the distance between dices r and s. Parameter β is the normalization factor for the distance between bad die r and the wafer centroid y_{wc}. Parameter m is a constant. Parameter $m\beta$ determines the approximate density shape of the wafer. Figure 3(b) shows an example of WBM transformation. Two types of data are used to generate a basic set of partitions. Moreover, each WBM must sequentially transform

(1) Randomly select k data as the centroid of cluster
(2) Repeat
  For each data vector, assign each data into the group with respect to the closest centroid by minimum Euclidean distance.
  recalculate the new centroid based on all data within the group.
  end for
(3) Steps 1 and 2 are iterated until there is no data change.

Procedure 1: k-means algorithm.



(a) Binary value (b) Continuous value

Figure 3: Representation of wafer bin map by binary value and continuous value.

from a two-dimensional map into a one-dimensional data vector [8]. Such vectors are used to conduct further clustering analysis.

3.3. Diverse Partitions Generation by k-Means and PSO Clustering. Both k-means and PSO clustering algorithms are used to generate basic partitions. To consider the spatial relations across a wafer, both the binary and continuous values are used to determine distinct clustering results by using k-means and PSO clustering. Subsequently, various numbers of clusters are used for comparison.

K-means is an unsupervised method of clustering analysis [13] used to group data into several predefined numbers of clusters by employing a similarity measure such as the Euclidean distance. The objective function of the k-means algorithm is to minimize the within-cluster difference, that is, the sum of the square error (SSE) which is determined using (3). The k-means algorithm consists of the following steps as shown in Procedure 1:

$$\mathrm{SSE} = \sum_{k=1}^{N_c} \sum_{\mathbf{x}_i \in C_k} (\mathbf{x}_i - \mathbf{m}_k)^2 . \quad (3)$$

Data clustering is regarded as an optimisation problem. PSO is an evolutionary algorithm [14] which is used to search for optimal solutions based on the interactions amongst particles; it requires adjusting fewer parameters compared with using other evolutionary algorithms. van der Merwe and Engelbrecht [26] proposed a hybrid algorithm for clustering data, in which the initial swarm is determined using the k-means result and PSO is used to refine the cluster results.

A single particle $\mathbf{p}_l$ represents the k cluster centroid vectors: $\mathbf{p}_l = [m_{l1}, m_{l2}, \ldots, m_{lk}]$. A swarm defines a number of candidate clusters. To consider the maximal homogeneity within a cluster and heterogeneity between clusters, a fitness function is used to maximize the intercluster separation and minimize the intracluster distance and quantisation error

$$f(\mathbf{p}_i, \mathbf{Z}_l) = \omega_1 \times J_e + \omega_2 \times \bar{d}_{\max}(\mathbf{p}_l, \mathbf{Z}_l) + \omega_3 \times (X_{\max} - d_{\min}(\mathbf{p}_l)), \quad (4)$$

where $\mathbf{Z}_l$ is a matrix representing the assignment of the WBMs to the clusters of the lth particle. The following quantization error equation is used to evaluate the level of clustering performance:

$$J_e = \frac{\sum_{k=1}^{N_c} \left[\sum_{\forall \mathbf{x}_i \in C_k} d(\mathbf{x}_i, m_k) / n_k \right]}{K}. \quad (5)$$

In addition,

$$\bar{d}_{\max}(\mathbf{p}_i, \mathbf{Z}_l) = \max_{k=1,2,\ldots,N_c} \left[\sum_{\forall \mathbf{x}_i \in C_{lk}} \frac{d(\mathbf{x}_i, \mathbf{m}_{lk})}{n_{lk}} \right] \quad (6)$$

is the maximum average Euclidean distance of particle to the assigned clusters and

$$d_{\min}(\mathbf{p}_l) = \min_{\forall u,v,u \neq v} [d(\mathbf{m}_{lu}, \mathbf{m}_{lv})] \quad (7)$$

is the minimum Euclidean distance between any pair of clusters. Procedure 2 shows the steps involved in the PSO clustering algorithm.

(1) Initialize each particle with k cluster centroids.
(2) For iteration $t = 1$ to $t = \max$ do
For each particle l do
For each data pattern $\mathbf{x}_i$
calculate the Euclidean distance to all cluster centroids and assign pattern $\mathbf{x}_i$ to cluster c_k which has the minimum distance
end for
calculate the fitness function $f(\mathbf{p}_i, \mathbf{Z}_l)$
end for
find the personal best and global best positions of each particle.
update the cluster centroids by the update velocity equation (i) and update coordinate equation (ii).

$$\mathbf{V}_i(t+1) = \omega_v \mathbf{V}_i(t) + c_p u(\boldsymbol{\psi}_l(t) - \boldsymbol{\Theta}_l(t)) + c_g u(\boldsymbol{\psi}_g(t) - \boldsymbol{\Theta}_l(t)) \quad \text{(i)}$$

$$\boldsymbol{\Theta}_l(t+1) = \boldsymbol{\Theta}_l(t) + \mathbf{V}_l(t+1) \quad \text{(ii)}$$

end for
(3) Step 2 is iterated until these is no data change

Procedure 2: PSO clustering algorithm.

3.4. Consensus Partition by Adaptive Response Theory. ART has been used in numerous areas such as pattern recognition and spatial analysis [27]. Regarding the unstable learning conditions caused by new data, ART can be used to address stability and plasticity because it addresses the balance between stability and plasticity, match and reset, and search and direct access [8]. Because the input labels are binary, the ART1 neural network [27] algorithm is used to attain a consensus partition of WBMs.

The consensus partition approach is as follows.

Step 1. Apply k-means and PSO clustering algorithms and use various parameters (e.g., various numbers of clusters and types of input data) to generate diverse clusters.

Step 2. Transform the original clustering label into binary representation matrix H as an input for ART1 neural network.

Step 3. Apply ART1 neural network to aggregate the diverse partitions.

4. Numerical Experiments

In this section, this study conducts a numerical study to demonstrate the effectiveness of the proposed clustering ensemble approach. Six typical WBM patterns from semiconductor fabrication were used such as moon, edge, and sector. In the experiments, the percentage of defective dies in six patterns is designed based on real cases. Without losing generality of WBM patterns, the data have been systematically transformed for proprietary information protection of the case company. Total 650 chips were exposed on a wafer. Based on various degrees of noise, each pattern type was used to generate 10 WBMs for estimating the validity of proposed clustering ensemble approach. The noise in WBM could be caused from random particles across a wafer and test bias in CP test, which result in generating bad die randomly on a wafer and generating good die within a group of bad dies. It means that some bad dices are shown as good dice and the density of bad die could be sparse. For example, the value of degree of noise is 0.02 which represents total 2% good die and bad dies are inverse.

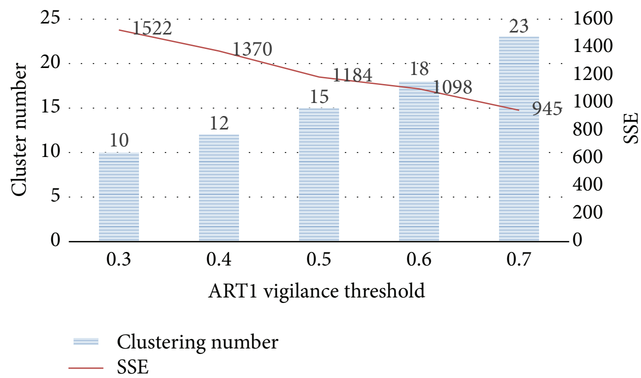


Figure 4: Comparison of various ART1 vigilance threshold.

The proposed WBM clustering ensemble approach was compared with k-means, PSO clustering method, and the algorithm proposed by Hsu and Chien [8]. Six numbers of clusters were used for single k-means methods and single PSO clustering algorithms. Table 3 showed the parameter settings for PSO clustering. The number of clusters extracted by ART1 neural network is sensitive to the vigilance threshold value. The high vigilance threshold is used to produce more clusters and the similarity within a cluster is high. In contrast, the low vigilance threshold results in fewer numbers of clusters. However, the similarity within a cluster could be low. To compare the parameter setting of ART1 vigilance threshold, various values were used as shown in Figure 4. Each clustering performance was evaluated in terms of the SSE and number of clusters. The SSE is used to compare the cohesion amongst various clustering results, and a small SSE indicates that the WBM within a cluster is highly similar. The number of clusters represents the effectiveness of the WBM grouping. According to the objective of clustering is to group

Table 3: Parameter settings for PSO clustering.

Parameter	Value	Parameter	Value
m	20	ω	1
X^{max}	1	a_1	0.4
c_p	2	a_2	0.3
c_g	2	a_3	0.3
Iteration	500		

Table 4: Results of clustering methods by SSE.

Methods		Noise degree				
		0.02	0.04	0.06	0.08	0.10
Hsu and Chien [8]		1184	1192	1203	1248	1322
Individual clustering	KB	2889	3092	3003	4083	3570
	KC	3331	2490	2603	3169	2603
	PB	5893	3601	6566	5839	6308
	PC	4627	4873	3330	3787	6112
Clustering ensemble	KB and PB	1827	1280	1324	1801	2142
	KC and PC	2272	2363	2400	1509	1718
	KB and PC	1368	1459	2400	1509	2597
	KC and PB	2100	2048	1421	1928	2043
	KB and PB and KC and PC	1586	1550	1541	1571	1860

the WBM into few clusters in which the similarities among the WBMs within a cluster are high as possible. Therefore, the setting of ART1 vigilance threshold value is used as 0.50 in the numerical experiments.

WBM clustering is to identify the similar type of WBM into the same cluster. To consider only six types of WBMs that were used in the experiments, the actual number of clusters should be six. Based on the various degree of noise in WBM generation as shown in Table 4, several individual clustering methods including ART1 [8], k-means clustering, and PSO clustering were used for evaluating clustering performance. Table 4 shows that the ART1 neural network yielded a lower SSE compared with the other methods. However, the ART1 neural network separates the WBM into 15 clusters as shown in Figure 5. The ART1 neural network yields unnecessary partitions for the similar type of WBM pattern. In order to generate diverse clustering partitions for clustering ensemble method, four combinations with various data scale and clustering algorithms including k-means by binary value (KB), k-means by continuous value (KC), PSO by binary value (PB), and PSO by continuous value (PC) are used. Regardless of the individual clustering results based on six numbers of clusters, using K-means clustering and PSO clustering individually yielded larger SSE values than using ART1 only.

Table 4 also shows the clustering ensembles that use various types of input data. For example, the clustering ensemble method KB&PB integrates the six results including the k-means algorithm by three kinds of clusters (i.e., $k = 5, 6, 7$) and PSO clustering by three kinds of clusters (i.e., $k = 5, 6, 7$), respectively, to form the WBM clustering via

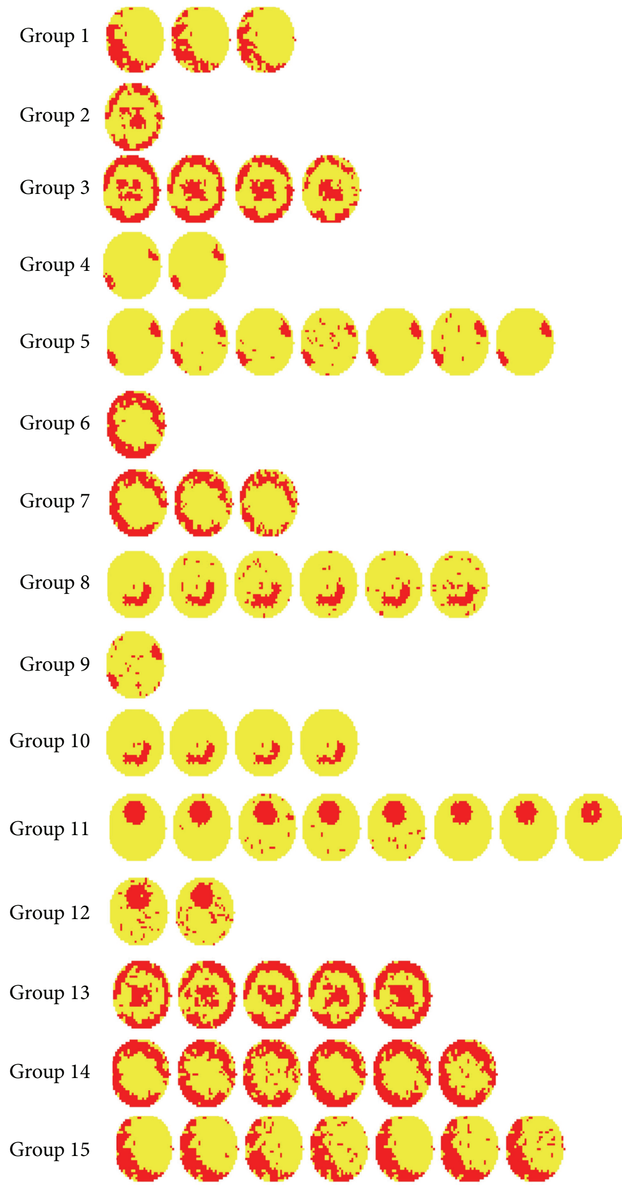


Figure 5: Clustering result by ART1 (15 clusters).

label space. In general, the clustering ensembles demonstrate smaller SSE values than do individual clustering algorithms such as the k-means or PSO clustering algorithms.

In addition to compare the similarity within the cluster, an index called specificity was used to evaluate the efficiency of the evolved cluster over representing the true clusters [28]. The specificity is defined as follows:

$$\text{specificity} = \frac{t_c}{T_e}, \quad (8)$$

where t_c is the number of true WBM patterns covered by the number of evolved WBM patterns and T_e is the total number of evolved WBM patterns. As shown in the ART1 neural network clustering results, the total number of evolved WBM clusters is 15 and number of true WBM clusters is 6. Then, the specificity is 0.4. Table 5 shows the results of specificity

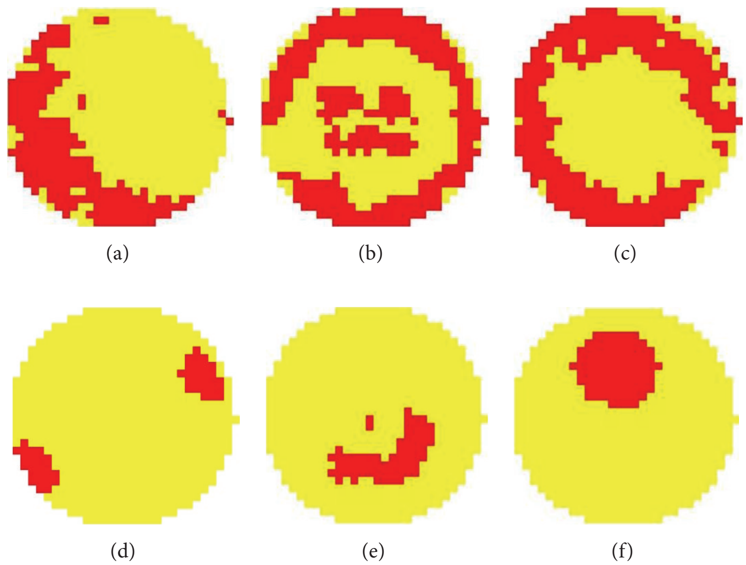


Figure 6: Six types of WBM patterns.

Table 5: Results of clustering methods by specificity.

Methods		Noise degree				
		0.02	0.04	0.06	0.08	0.10
Hsu and Chien [8]		0.4	0.4	0.4	0.4	0.4
Individual clustering	KB	1.0	1.0	1.0	1.0	1.0
	KC	1.0	1.0	1.0	1.0	1.0
	PB	1.0	1.0	1.0	1.0	1.0
	PC	1.0	1.0	1.0	1.0	1.0
Clustering ensemble	KB and PB	0.7	0.5	0.5	0.5	0.8
	KC and PC	0.5	0.8	0.9	0.8	0.6
	KB and PC	0.5	0.7	0.9	0.8	0.7
	KC and PB	0.9	0.5	0.5	0.6	0.7
	KB and PB and KC and PC	1.0	0.9	0.9	0.9	1.0

among clustering methods. The ART1 neural network has the lowest specificity due to the large number of clusters. The specificity of individual clustering is 1 because the number of evolved WBM patterns is fixed as 6. Furthermore, compared with individual clustering algorithms, combining various clustering ensembles yields not only smaller SSE values, but also smaller numbers of clusters. Thus, the homogeneity within a cluster can be improved using proposed approach. The threshold of ART1 neural network yields maximal cluster numbers. Therefore, the proposed clustering ensemble approach considering diversity partitions has better results regarding the SSE and number of clusters than individual clustering methods.

To evaluate the results among various clustering ensembles and to assess cluster validity, WBM class labels are employed based on six pattern types as shown in Figure 6. Thus, the indices including precision and recall are two classification-oriented measures [29] defined as follows:

$$\text{precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}, \qquad \text{recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}, \tag{9}$$

where TP (true positive) is the number of WBMs correctly classified into WBM patterns, FP (false positive) is the number of WBMs incorrectly classified, and FN (false negative) is the number of WBMs that need to be classified, but not to be determined incorrectly. The precision measure is used to assess how many WBMs classified as Pattern (a) are actually Pattern (a). The recall measure is used to assess how many samples of Pattern (a) are correctly classified.

However, a trade-off exists between precision and recall; therefore, when one of these measures increases, the other decreases. The F-measure is a harmonic mean of the precision and recall which is defined as follows:

$$F = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} = \frac{2\text{TP}}{\text{FP} + \text{FN} + 2\text{TP}}. \tag{10}$$

Specifically, the F-measure represents the interaction between the actual and classification results (i.e., TP). If the classification result is close to the actual value, the F-measure is high.

Tables 6, 7, and 8 show a summary of various metrics among six types of WBM in precision, recall, and F-measure, respectively. As shown in Figure 6, Patterns (b) and (c) are similar in the wafer edge, demonstrating smaller average precision and recall values compared with the other patterns. The clustering ensembles which generate partitions by using k-means make it difficult to identify in both Patterns (b) and (c). Using a mountain function transformation enables

Table 6: Clustering result on the index of precision.

		Hsu and Chien [8]	Clustering ensemble				
			KB and PB	KC and PC	KB and PC	KC and PB	KB and PB and KC and PC
Precision	A	0.70	0.84	0.92	0.92	0.92	0.98
	B	0.50	0.66	0.96	0.92	0.62	0.96
	C	0.60	0.64	1.00	1.00	0.60	1.00
	D	0.70	0.98	0.92	0.92	0.98	1.00
	E	0.60	0.94	0.82	0.82	0.98	0.98
	F	0.80	0.98	0.76	0.76	0.98	0.98
	Avg.	**0.65**	**0.84**	**0.90**	**0.89**	**0.85**	**0.98**

Table 7: Clustering result on the index of recall.

		Hsu and Chien [8]	Clustering ensemble				
			KB and PB	KC and PC	KB and PC	KC and PB	KB and PB and KC and PC
Recall	A	1.00	1.00	1.00	0.93	1.00	1.00
	B	1.00	0.97	0.7	0.78	0.83	1.00
	C	1.00	0.94	0.67	0.84	0.67	0.97
	D	1.00	0.81	1.00	1.00	1.00	1.00
	E	1.00	0.79	1.00	1.00	1.00	1.00
	F	1.00	1.00	1.00	1.00	1.00	1.00
	Avg.	**1.00**	**0.92**	**0.90**	**0.93**	**0.92**	**1.00**

Table 8: Clustering result on the index of F-measure.

		Hsu and Chien [8]	Clustering ensemble				
			KB and PB	KC and PC	KB and PC	KC and PB	KB and PB and KC and PC
F-measure	A	0.82	0.91	0.96	0.92	0.96	0.99
	B	0.67	0.79	0.81	0.84	0.71	0.98
	C	0.75	0.76	0.8	0.91	0.63	0.98
	D	0.82	0.89	0.96	0.96	0.99	1.00
	E	0.75	0.86	0.90	0.90	0.99	0.99
	F	0.89	0.99	0.86	0.86	0.99	0.99
	Avg.	**0.78**	**0.87**	**0.88**	**0.90**	**0.88**	**0.99**

considering the defect density of the spatial relations between the good and bad dies across a wafer. Based on the F-measure, the clustering ensembles obtained using all generated partitions exhibit larger precision and recall values and superior levels of performance regarding each pattern compared with the other methods. Thus, the partitions generated by using k-means and PSO clustering in various data types must be considered.

The practical viability of the proposed approach was examined. The results show that the ART1 neural network performing into data space directly leads to worse clustering performance in terms of precision. However, the true types of WBM can be identified through transforming original data space into label space and performing consensus partition by ART1 neural network. The proposed cluster ensemble approach can get better performance with fewer numbers of clusters than other conventional clustering approaches including k-means, PSO clustering, and ART1 neural network.

5. Conclusion

WBMs provide important information for engineers to rapidly find the potential root cause by identifying patterns correctly. As the driven force for semiconductor manufacturing technology, WBM identification to the correct pattern becomes more difficult because the same type of patterns is influenced by various factors such as die size, pattern density, and noise degree. Relying on only engineers' experiences of visual inspections and personal judgments in the map patterns is not only subjective, and inconsistent, but also very time-consuming and inefficient. Therefore, grouping similar WBM quickly helps engineer to use more time to diagnose the root cause of low yield.

Considering the requirements of clustering WBMs in practice, a cluster ensemble approach was proposed to facilitate extracting the common defect pattern of WBMs, enhancing failure diagnosis and yield enhancement. The advantage of the proposed method is to yield high-quality clusters by applying distinct algorithms to the same data set and by using various parameter settings. The robustness of clustering ensemble is higher than individual clustering method because the clustering from various aspects including algorithms and parameter setting is integrated into a consensus result.

The proposed clustering ensemble has two stages. At the first stage, diversity partitions are generated using two types of input data: various cluster numbers and distinct clustering algorithms. At the second stage, a consensus partition is attained using these diverse partitions. The numerical analysis demonstrated that the clustering ensemble is superior to using individual k-means or PSO clustering algorithms. The results demonstrate that the proposed approach can effectively group the WBMs into several clusters based on their similarity in label space. Thus, engineers can have more time to focus the assignable cause of low yield instead of extracting defect patterns.

Clustering is an exploratory approach. In this study, we assume that the number of clusters is known. Evaluating the clustering ensemble approach, prior information is required regarding the cluster numbers. Further research can be conducted regarding self-tuning the cluster number in clustering ensembles.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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Research Article

A Multiple Attribute Group Decision Making Approach for Solving Problems with the Assessment of Preference Relations

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A number of theoretical approaches to preference relations are used for multiple attribute decision making (MADM) problems, and fuzzy preference relations is one of them. When more than one person is interested in the same MADM problem, it then becomes a multiple attribute group decision making (MAGDM) problem. For both MADM and MAGDM problems, consistency among the preference relations is very important to the result of the final decision. The research reported in this paper is based on a procedure that uses a fuzzy preference relations matrix which satisfies additive consistency. This matrix is used to solve multiple attribute group decision making problems. In group decision problems, the assessment provided by different experts may diverge considerably. Therefore, the proposed procedure also takes a heterogeneous group of experts into consideration. Moreover, the methods used to construct the decision matrix and determine the attribution of weight are both introduced. Finally a numerical example is used to test the proposed approach; and the results illustrate that the method is simple, effective, and practical.

1. Introduction

There are many situations in daily life and in the workplace which pose a decision problem. Some of them involve picking the optimum solution from among multiple available alternatives. Therefore in many domain problems multiple attribute decision making methods, such as simple additive weighting (SAW), the technique for order preference by similarity to ideal solution (TOPSIS), analytical hierarchy process (AHP), data envelopment analysis (DEA), or grey relational analysis (GRA) [1–5], are usually adopted, for example, layout design [6–8], supply chain design [9], push/pull junction point selection [10], pacemaker location determination [11], work in process level determination [12], and so on.

If more than one person is involved in the decision, the decision problem becomes a group decision problem. Many organizations have moved from a single decision maker or expert to a group of experts (e.g., Delphi) to accomplish this task successfully [13, 14]. Note that an "expert" represents an authorized person or an expert who should be involved in this decision making process. However, no single alternative works best for all performance attributes, and the assessment of each alternative given by different decision makers may diverge considerably. As a consequence, multiple attribute group decision making (MAGDM) is more difficult than cases where a single decision maker decides using a multiple attribute decision making method.

MAGDM is one of the most common activities in modern society, which involves selecting the optimal one from a finite set of alternatives with respect to a collection of the predefined criteria by a group of experts with a high collective knowledge level on these particular criteria [15]. When a group of experts wants to choose a solution from among several alternatives, preference relations is one type of assessment that experts could provide. Preference relations are comparisons between two alternatives for a particular attribute. A higher preference relation means that there is a higher degree of preference for one alternative over another.

However, different experts may use different assessment types to express the preference relation. It is possible that in group decision making different experts express their preference in different formats [16–21].

In addition, after experts have provided their assessment of the preference relation, the appropriateness of the comparison from each expert must be tested. Consistency is one of the important properties for verifying the appropriateness of choices [22]. If the comparison from an expert is not logically consistent for a specific attribute, it means that at least one preference relation provided by the expert is defective. Therefore, the lack of consistency in decision making can lead to inconsistent conclusions.

Quite apart from the type of assessment, there can be considerable variation between experts as to their evaluation of the degree of the preference relation. In general, it would be possible to aggregate the preferences of experts by taking the weight assigned by every expert into consideration. However, heterogeneity among experts should also be considered [23]. For example, if the expert who assigns the greatest weight to a preference relation also makes choices that are not appropriate and quite different from the evaluations of the other experts who assign lower weights, then the group decision procedure can be distorted and imperfect.

Moreover, the determination of attribute weight is also an important issue [24]. In some decision cases, some attributes are considered to be more important in the experts' professional judgment. However, for these important attributes, the preference relation provided by experts may be quite similar for all alternatives. Even for the attribute with the highest weight, the degree of influence on the final decision would be very small in this case. In this way this kind of attribute can become unimportant to the final decision [25].

Therefore, during the multiple attribute group decision process, 5 aspects should be noted:

(i) considering different assessment types simultaneously;

(ii) insuring the preference relations provided by experts are consistent;

(iii) taking heterogeneous experts into consideration;

(iv) deciding the weight of each attribute;

(v) ranking all alternatives.

Group decision making has been addressed in the literature. In recent years, Ölçer and Odabaşi [23] proposed a fuzzy multiple attribute decision making method to deal with the problem of ranking and selecting alternatives. Experts provide their opinion in the form of a trapezoidal fuzzy number. These trapezoidal fuzzy numbers are then aggregated and defuzzified into a MADM. Finally, TOPSIS is used to rank and select alternatives. In the method, experts can provide their opinion only by trapezoidal fuzzy number.

Boran et al. [26] proposed a TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision making environment. Intuitionistic fuzzy weighted averaging (IFWA) operator is utilized to aggregate individual opinions of decision makers for rating the importance of criteria and alternatives. Cabrerizo et al. [27] presented a consensus model for group decision making problems with unbalanced fuzzy linguistic information. This consensus model is based on both a fuzzy linguistic methodology to deal with unbalanced linguistic term sets and two consensus criteria—consensus degrees and proximity measures. Chuu [28] builds a group decision making model using fuzzy multiple attributes analysis to evaluate the suitability of manufacturing technology. The proposed approach involved developing a fusion method of fuzzy information, which was assessed using both linguistic and numerical scales.

Lu et al. [29] developed a software tool for supporting multicriteria group decision making. When using the software, after inputting all criteria and their corresponding weights, and the weighting for all the experts, all the experts can assess every alternative against each attribute. Then the ranking of all alternatives can be generated. In the software only one assessment type is allowed and there is no function that can be used to ensure that the preference relations provided by experts are consistent. Zhang and Chu [30] proposed a group decision making approach incorporating two optimization models to aggregate these multiformat and multigranularity linguistic judgments. Fuzzy set theory is utilized to address the uncertainty in the decision making process.

Cabrerizo et al. [14] proposed a consensus model to deal with group decision making problems, in which experts use incomplete unbalanced fuzzy linguistic preference relations to provide their preference. However, the model requires that preference relations should be assessed in the same way, and no allowance is made for heterogeneous experts. Cebi and Kahraman [31] proposed a methodology for group decision support. The methodology consists of eight steps which are (1) definition of potential decision criteria, possible alternatives, and experts, (2) determining the weighting of experts, (3) identifying the importance of criteria, (4) assigning alternatives, (5) aggregating experts' preferences, (6) identifying functional requirements, (7) calculating information contents, and (8) calculating weighted total information contents and selecting the best alternative. The methodology does not include a check on the consistency of preference relations provided by the experts.

The novelty of the present study is that it proposes a multiple attribute group decision making methodology in which all of the five issues mentioned above are addressed. A review of the literature related to this research suggests that no previous research has addressed all of the issues simultaneously. For managers who are not experts in fuzzy theory, group decision making, MADM, and so on, this research can provide a complete guideline for solving their multiple attribute group decision making problem.

The remainder of this paper is organized as follows. In Section 2 all the issues set out above are discussed and appropriate methodologies for dealing with them are proposed. Then an overall approach is proposed in Section 3. The proposed model is tested and examined with a numerical example in Section 4. Finally Section 5 contains the discussion and conclusions.

2. Multiple Attribute Group Decision Making Methodology

2.1. Assessment and Transformation of Preference Relations. There are two types of preference relations that are widely used. One is fuzzy preference relations, in which r_{ij} denotes the preference degree or intensity of the alternative i over j [32–35]. If $r_{ij} = 0.5$, it means that alternatives i and j are indifferent; if $r_{ij} = 1$, it means that alternative i is absolutely preferred to j, and if $r_{ij} > 0.5$, it means that alternative i is preferred to j. r_{ij} is reciprocally additive; that is, $r_{ij} + r_{ji} = 1$ and $r_{ii} = 0.5$ [35, 36].

The other widely used type of preference relations is multiplicative preference relations, in which a_{ij} indicates a ratio of preference intensity for alternative i to that of alternative j; that is, it is interpreted as meaning that alternative i is a_{ij} times as good as alternative j [17]. Saaty [3] suggested measuring a_{ij} on an integer scale ranging from 1 to 9. If $a_{ij} = 1$, it means that alternatives i and j are indifferent; if $a_{ij} = 9$, it means that alternative i is absolutely preferred to j, and if $8 \geq r_{ij} \geq 2$, it means that alternative i is preferred to j. In addition, $a_{ij} \times a_{ji} = 1$, and $a_{ij} = a_{ik} \times a_{kj}$.

For these two preference types, Chiclana et al. [17] proposed an equation to transform the multiplicative preference relation into the fuzzy preference relation, as shown by

$$r_{ij} = 0.5\left(1 + \log_9 a_{ij}\right). \tag{1}$$

However, for both preference types, it is possible that some experts would not wish to provide their preference relation in the form of a precise value. In the fuzzy preference relations, experts can use the following classifications:

(i) a precise value, for example, "0.7";

(ii) a range, for example, (0.3, 0.7); the value is likely to fall between 0.3 and 0.7;

(iii) a fuzzy number with triangular membership function, for example, (0.4, 0.6, 0.8); the value is between 0.4 and 0.8 and is most probably 0.6;

(iv) a fuzzy number with trapezoidal membership function, for example, (0.3, 0.5, 0.6, 0.8); the value is between 0.3 and 0.8, most probably between 0.5 and 0.6.

In this paper, the four classifications set out above are unified by transferring them into trapezoidal membership functions. Thus, 0.7 becomes (0.7, 0.7, 0.7, 0.7), (0.3, 0.7) becomes (0.3, 0.3, 0.7, 0.7), and (0.4, 0.6, 0.8) then becomes (0.4, 0.6, 0.6, 0.8). If experts provide their assessment in the format of multiplicative preference relations, it will be transformed into a trapezoidal membership function first, and then using (1) it will be further transformed into the format of fuzzy preference relations. For example, (3, 4, 5, 6) can be transferred into (0.75, 0.82, 0.87, 0.91) by using (1). Therefore, this paper will mention only fuzzy preference relations in what follows.

2.2. The Generation of Consistent Preference Relations. The property of consistency has been widely used to establish a verification procedure for preference relations, and it is very important for designing good decision making models [22]. In the analytical hierarchy process, for example, in order to avoid potential comparative inconsistency between pairs of categories, a consistency ratio (CR), an index for consistency, has been calculated to assure the appropriateness of the comparisons [3]. If the CR is small enough, there is no evidence of inconsistency. However, if the CR is too high, then the experts should adjust their assessments again and again until the CR decreases to a reasonable value. For fuzzy preference relations, Herrera-Viedma et al. [22] designed a method for constructing consistent preference relations which satisfy additive consistency. Using this method, all experts need only to provide preference relations between alternatives i and $i+1$, $r_{i(i+1)}$, and the remaining preference relations can be calculated using (2) if $i > j$ and (3) if $i < j$:

$$r_{ij} = \frac{i-j+1}{2} - r_{j(j+1)} - r_{(j+1)(j+2)} - \cdots - r_{(i-1)i}, \quad \forall i > j, \tag{2}$$

$$r_{ij} = 1 - r_{ji}, \quad \forall i < j. \tag{3}$$

To illustrate the generation of preferential relations, we provide an empirical example of four alternatives as follows. First, the expert provides the three preference relations as $r_{12} = 0.3$, $r_{23} = 0.6$, and $r_{34} = 0.8$.

According to (2),

$$\begin{aligned} r_{21} &= 1 - 0.3 = 0.7, \\ r_{31} &= 1.5 - 0.3 - 0.6 = 0.6, \\ r_{41} &= 2 - 0.3 - 0.6 - 0.8 = 0.3, \\ r_{32} &= 1 - 0.6 = 0.4, \\ r_{42} &= 1.5 - 0.6 - 0.8 = 0.1, \\ r_{43} &= 1 - 0.8 = 0.2. \end{aligned} \tag{4}$$

According to (3),

$$\begin{aligned} r_{13} &= 1 - 0.6 = 0.4, \\ r_{14} &= 1 - 0.3 = 0.7, \\ r_{24} &= 1 - 0.1 = 0.9. \end{aligned} \tag{5}$$

Therefore, the preference relations matrix, PR, is

$$\mathrm{PR} = \begin{bmatrix} 0.5 & 0.3 & 0.4 & 0.7 \\ 0.7 & 0.5 & 0.6 & 0.9 \\ 0.6 & 0.4 & 0.5 & 0.8 \\ 0.3 & 0.1 & 0.2 & 0.5 \end{bmatrix}. \tag{6}$$

In general, experts are asked to evaluate all pairs of alternatives and then construct a preference matrix with full information. However, it is difficult to obtain a consistent preference matrix in practice, especially when measuring preferences on a set with a large number of alternatives [22].

2.3. Assessment Aggregation for a Heterogeneous Group of Experts. For each comparison between a pair of alternatives, the preference relations provided by different experts would vary. Hsu and Chen [37] proposed an approach to aggregate fuzzy opinions for a heterogeneous group of experts. Then, Chen [38] modified the approach and Ölçer and Odabaşi [23] present it as the following six-step procedure.

(1) Calculate the Degree of Agreement between Each Pair of Experts. For a comparison between two alternatives, let there be E experts in the decision group, (a_1, a_2, a_3, a_4) and (b_1, b_2, b_3, b_4) are the preference relations provided by experts a and b, $1 \leq a \leq E$, $1 \leq b \leq E$, and $a \neq b$. The similarity between these two trapezoidal fuzzy numbers, S_{ab}, can be measured by

$$S_{ab} = 1 - \frac{|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4|}{4}. \quad (7)$$

(2) Construct the Agreement Matrix. After all the agreement degrees between experts are measured, the agreement matrix (AM) can be constructed as follows:

$$\mathrm{AM} = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1E} \\ S_{21} & 1 & \cdots & S_{2E} \\ \vdots & \vdots & S_{ab} & \vdots \\ S_{E1} & S_{E2} & \cdots & 1 \end{bmatrix}, \quad (8)$$

in which $S_{ab} = S_{ba}$, and if $a = b$, then $S_{ab} = 1$.

(3) Calculate the Average Degree of Agreement for Each Expert. The average degree of agreement for expert a (AA_a) can be calculated by

$$\mathrm{AA}_a = \frac{1}{E-1} \sum_{b=1, a \neq b}^{E} S_{ab}, \quad \forall a. \quad (9)$$

(4) Calculate the Relative Degree of Agreement for Each Expert. After calculating the average degree of agreement for all experts, the relative degree of agreement for expert a (RA_a) can be calculated by

$$\mathrm{RA}_a = \frac{\mathrm{AA}_a}{\sum_{a=1}^{E} \mathrm{AA}_a}, \quad \forall a. \quad (10)$$

(5) Calculate the Coefficient for the Degree of Consensus for Each Expert. Let ew_a be the weight of expert a, and $\sum_{a=1}^{E} \mathrm{ew}_a = 1$. The coefficient of the degree of consensus for expert a (CC_a) can be calculated by

$$\mathrm{CC}_a = \beta \cdot \mathrm{ew}_a + (1 - \beta) \cdot \mathrm{RA}_a, \quad \forall a, \quad (11)$$

in which β is a relaxation factor of the proposed method and $0 \leq \beta \leq 1$. It represents the importance of ew_a over RA_a. When $\beta = 0$, it means that the group of experts is considered to be homogeneous.

(6) Calculate the Aggregation Result. Finally, the aggregation result of the comparison between two alternatives i and j is $\tilde{r}_{ij}$, where

$$\begin{aligned} \tilde{r}_{ij} = {} & \mathrm{CC}_1 \otimes \tilde{r}_{ij}(1) \oplus \mathrm{CC}_2 \otimes \tilde{r}_{ij}(2) \oplus \cdots \oplus \mathrm{CC}_a \\ & \otimes \tilde{r}_{ij}(a) \oplus \cdots \oplus \mathrm{CC}_E \otimes \tilde{r}_{ij}(E). \end{aligned} \quad (12)$$

In (12), $\tilde{r}_{ij}(a)$ is the preference relation between alternatives i and j provided by expert a, and $\tilde{r}_{ij} = (r_{ij}^1, r_{ij}^2, r_{ij}^3, r_{ij}^4)$. Moreover $\otimes$ and $\oplus$ are the fuzzy multiplication operator and the fuzzy addition operator, respectively.

Let there be N alternatives. Since each expert only provides preference relations between alternatives i and $i+1$, the aggregation process for a heterogeneous group of experts must be executed $N-1$ times in order to generate $N-1$ aggregated trapezoidal fuzzy numbers. These $N-1$ trapezoidal fuzzy numbers can then be converted into a precise value by the use of

$$r_{ij} = \frac{r_{ij}^1 + 2\left(r_{ij}^2 + r_{ij}^3\right) + r_{ij}^4}{6}. \quad (13)$$

After the aggregation procedure, using (2) and (3), an aggregated preference relations matrix for attribute k is constructed as follows:

$$\mathrm{PR}_k = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1N} \\ r_{12} & 1 & \cdots & r_{2N} \\ \vdots & \vdots & 1 & \vdots \\ r_{N1} & r_{N2} & \cdots & 1 \end{bmatrix}. \quad (14)$$

2.4. Attribute Weight Determination. In a preference relations matrix of attribute k, r_{ij} indicates the degree of preference of alternative i over j when attribute k was considered. Therefore, $\sum_{j=1, j\neq i}^{N} r_{ij}$ indicates total degree of preference of alternative i over the other $N-1$ alternatives. In the same way, $\sum_{j=1, j\neq i}^{N} r_{ji}$ indicates the total degree of preference of the other $N-1$ alternatives over alternative i. Fodor and Roubens [39] proposed (15) to define δ_{ik}, the net degree of preference of alternative i over the other $N-1$ alternatives by attribute k, and the bigger δ_{ik} is, the better alternative i by attribute k is:

$$\delta_{ik} = \sum_{j=1, j\neq i}^{N} r_{ij} - \sum_{j=1, j\neq i}^{N} r_{ji}, \quad \forall i, k. \quad (15)$$

Thus, the problem is reduced to a multiple attribute decision making problem:

$$\mathrm{DM} = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1M} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{N1} & \delta_{N2} & \cdots & \delta_{NM} \end{bmatrix}. \quad (16)$$

For the decision matrix constructed in Section 2.4, Wang and Fan [25] proposed two approaches, absolute deviation maximization (ADM) and standard deviation maximization (SDM), to determine the weight of all attributes. For a certain attribute, if the difference of the net degree of preference among all alternatives shows a wide variation, this means this attribute is quite important. ADM and SDM used absolute deviation (AD) and standard deviation (SD) to measure the degree of variation. An attribute with a bigger value of AD and SD will be a more important attribute.

When ADM was adopted, the weight of attribute k, aw_k, was calculated by using (17), while if SDM was adopted, (18) was used for calculating the weight of attribute k:

$$\mathrm{aw}_k = \frac{\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\left|\delta_{ik}-\delta_{jk}\right|\right)^{1/(p-1)}}{\sum_{l=1}^{M}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\left|\delta_{il}-\delta_{jl}\right|\right)^{1/(p-1)}}, \quad \forall k;\ p>1, \tag{17}$$

$$\mathrm{aw}_k = \frac{\left(\sum_{i=1}^{N}\delta_{ik}^{2}\right)^{1/2(p-1)}}{\sum_{l=1}^{M}\left(\sum_{i=1}^{N}\delta_{il}^{2}\right)^{1/2(p-1)}}, \quad \forall k;\ p>1, \tag{18}$$

where p is the parameter of these two functions for calculating weights. Setting the variable to different values will lead to different weights and when $p = \infty$, all weights will be equal. Therefore, in order to reflect the differences among the attribute weights, Wang and Fang [25] suggested preferring a small value for parameter p. Further details of the demonstration of the use of ADM and SDM can be found in the paper by Wang and Fan [25].

2.5. Alternative Ranking. Once the weights of all attributes are determined by (17) or (18), the multiple attribute decision making problem constructed by (16) can be solved by the application of a multiple attribute decision making method, such as SAW, TOPSIS, ELECTRE, or GRA [1, 2, 5]. According to Kuo et al. [40], different MADM methods would lead to different results, but similar ranking of alternatives. In this research, SAW was selected for the MADM problem. Since the weight calculated by (17) and (18) has been normalized, and $\sum_{k=1}^{M}\mathrm{aw}_k = 1$, the score of alternatives i, C_i, can be calculated directly by

$$C_i = \sum_{k=1}^{M}\mathrm{aw}_k\delta_{ik}, \quad i = 1, 2, \ldots, N. \tag{19}$$

The bigger the C_i is, the better the alternative i is. After the scores of all alternatives have been calculated, the alternatives can be ranked by C_i.

3. The Proposed Approach

Following from the consideration of issues which were set out in the Introduction and further developed in Section 2, this research proposes a 5-step procedure for multiple attribute group decision making problems as shown in Figure 1.

In Step 1, experts provide their preference relations for all attributes using their preferred format of expression. In order to ensure the additive consistency of these preference relations, only the preference relations between alternatives i and $i+1$ are assessed. Then these preference relations provided by the experts are transformed into trapezoidal membership functions. If the preference relations are multiplicative preference relations, (1) is used to transform them into fuzzy preference relations.

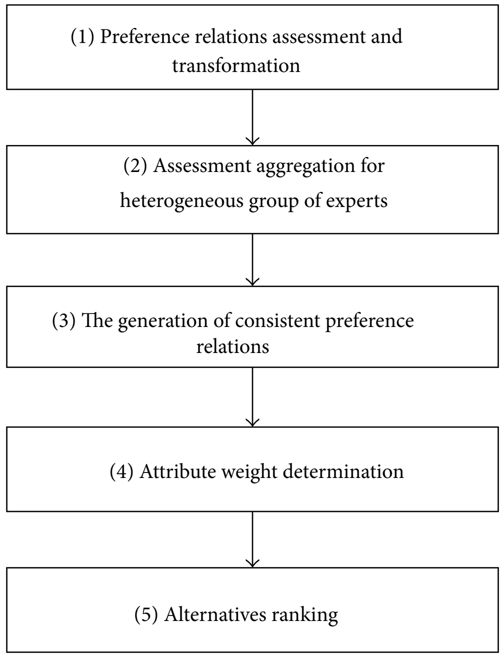

Figure 1: The proposed MAGDM procedure.

In Step 2, in order to take the heterogeneity of the experts into consideration, the trapezoidal membership function of fuzzy preference relations for all experts is aggregated by a six-step procedure given by Ölçer and Odabaşi [23]. Then (2) and (3) are used to calculate the remaining preference relations which had not been provided by the experts, and these are then used to construct preference relation matrixes which are additively consistent in Step 3.

In Step 4, these preference relation matrixes are transformed into a traditional multiple attribute decision matrix and used to determine the weight of all attributes using (17) and (18). Finally, all the scores of alternatives can be calculated using (19) and the alternatives can be ranked in Step 5.

4. Numerical Example

The proposed MAGDM methodology allows two types of preference relations, fuzzy reference relations and multiplicative preference relations, which are explained in Section 2.1. The former ones are transformed to numerical number through fuzzy membership functions and the latter ones directly use numerical numbers. They are then aggregated through the proposed aggregation and ranking procedure as discussed in Sections 2.2 to 2.5. Due to both the transformation and aggregation procedures, the resulting numbers are real numbers.

In this section, we provide a numerical example to illustrate the implementation of the proposed methodology. Consider four alternatives, three experts, and two attribute MAGDM problems as follows.

Step 1 (preference relations assessment and transformation). The preference relations assessments of Attribute 1 provided by these three experts were given as follows. in which R_{ak} is the assessment of attribute k provided by expert a:

$$R_{11} = \begin{bmatrix} - & \text{Low} & - & - \\ - & - & \text{Low} & - \\ - & - & - & \text{Medium} \\ - & - & - & - \end{bmatrix},$$

$$R_{21} = \begin{bmatrix} - & \text{More low} & - & - \\ - & - & \text{Medium} & - \\ - & - & - & \text{Medium} \\ - & - & - & - \end{bmatrix},$$

$$R_{31} = \begin{bmatrix} - & \frac{1}{3} & - & - \\ - & - & \frac{1}{4} & - \\ - & - & - & 1 \\ - & - & - & - \end{bmatrix}. \tag{20}$$

In this example, Experts 1 and 2 preferred to provide assessment by fuzzy preference relations, and Expert 3 preferred to provide assessment by multiplicative preference relations. However, Expert 1 used the membership function as shown in **Figure 2**, Expert 2 used the membership function as shown in **Figure 3**, and Expert 3 used precise values for providing his/her preference relations. All assessments are then transformed into the type of trapezoidal membership function as shown below:

$$R_{11} = \begin{bmatrix} - & 0.125,\ 0.225,\ 0.325\ 0.425 & - & - \\ - & - & 0.125,\ 0.225,\ 0.325,\ 0.425 & - \\ - & - & - & 0.350,\ 0.450,\ 0.550,\ 0.650 \\ - & - & - & - \end{bmatrix},$$
$$R_{21} = \begin{bmatrix} - & 0.200,\ 0.300,\ 0.400,\ 0.500 & - & - \\ - & - & 0.350,\ 0.450,\ 0.550,\ 0.650 & - \\ - & - & - & 0.350,\ 0.450,\ 0.550,\ 0.650 \\ - & - & - & - \end{bmatrix}, \tag{21}$$
$$R_{31} = \begin{bmatrix} - & \frac{1}{3},\ \frac{1}{3},\ \frac{1}{3},\ \frac{1}{3} & - & - \\ - & - & \frac{1}{4},\ \frac{1}{4},\ \frac{1}{4},\ \frac{1}{4} & - \\ - & - & - & 1,\ 1,\ 1,\ 1, \\ - & - & - & - \end{bmatrix}.$$

The preference relations' assessments of Attribute 2 which have been transformed into the type of trapezoidal membership function were given as follows:

$$R_{12} = \begin{bmatrix} - & 0.125,\ 0.225,\ 0.325\ 0.425 & - & - \\ - & - & 0.350,\ 0.450,\ 0.550,\ 0.650 & - \\ - & - & - & 0.125,\ 0.225,\ 0.325,\ 0.425 \\ - & - & - & - \end{bmatrix},$$

$$R_{22} = \begin{bmatrix} - & 0.050,\ 0.150,\ 0.250,\ 0.350 & - & - \\ - & - & 0.500,\ 0.600,\ 0.700,\ 0.800 & - \\ - & - & - & 0.200,\ 0.300,\ 0.400,\ 0.500 \\ - & - & - & - \end{bmatrix},$$

$$R_{32} = \begin{bmatrix} - & \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} & - & - \\ - & - & 1, 1, 1, 1 & - \\ - & - & - & \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \\ - & - & - & - \end{bmatrix}. \quad (22)$$

Using (1) the multiplicative preference relations in R_{31} and R_{32} can be transformed into fuzzy preference relations and then become R'_{31} and R'_{32} as follows. R_{31} and R_{32} were then replaced by R'_{31} and R'_{32} for the rest of the analysis:

$$R'_{31} = \begin{bmatrix} - & 0.250, 0.250, 0.250, 0.250 & - & - \\ - & - & 0.185, 0.185, 0.185, 0.185 & - \\ - & - & - & 0.500, 0.500, 0.500, 0.500 \\ - & - & - & - \end{bmatrix},$$

$$R'_{32} = \begin{bmatrix} - & 0.185, 0.185, 0.185, 0.185 & - & - \\ - & - & 0.500, 0.500, 0.500, 0.500 & - \\ - & - & - & 0.250, 0.250, 0.250, 0.250 \\ - & - & - & - \end{bmatrix}. \quad (23)$$

Step 2 (assessment aggregation for heterogeneous group of experts). In this example, the weights of Experts 1, 2, and 3 are 0.3, 0.3, and 0.4, respectively. Following the method set out in Section 2.3, the six steps can be used to aggregate the assessments provided by the heterogeneous group of experts. Let the relaxation factor $\beta = 0.5$. The results are then summarized in Table 1.

Therefore, the aggregated preference relations matrixes PR_1 and PR_2 are as shown in the following:

$$\mathrm{PR}_1 = \begin{bmatrix} - & 0.290 & - & - \\ - & - & 0.311 & - \\ - & - & - & 0.500 \\ - & - & - & - \end{bmatrix},$$

$$\mathrm{PR}_2 = \begin{bmatrix} - & 0.218 & - & - \\ - & - & 0.547 & - \\ - & - & - & 0.290 \\ - & - & - & - \end{bmatrix}. \quad (24)$$

Step 3 (the generation of consistent preference relations). In Step 3, the results in PR_1 and PR_2 are incomplete. Equations (2) and (3) are then used to calculate the remaining preference relations and to construct additively consistent preference relation matrixes. The complete preference relation matrixes PR'_1 and PR'_2 are

$$\mathrm{PR}'_1 = \begin{bmatrix} 0.500 & 0.290 & 0.100 & 0.100 \\ 0.710 & 0.500 & 0.311 & 0.311 \\ 0.900 & 0.689 & 0.500 & 0.500 \\ 0.900 & 0.689 & 0.500 & 0.500 \end{bmatrix},$$

$$\mathrm{PR}'_2 = \begin{bmatrix} 0.500 & 0.218 & 0.265 & 0.055 \\ 0.782 & 0.500 & 0.547 & 0.337 \\ 0.735 & 0.453 & 0.500 & 0.290 \\ 0.945 & 0.663 & 0.710 & 0.500 \end{bmatrix}. \quad (25)$$

According to the proposition and proof from Herrera-Viedma et al. [22], a fuzzy preference relation $\mathrm{PR} = (r_{ij})$ is consistent if and only if $r_{ij} + r_{jk} + r_{ki} = 3/2, \forall i \le j \le k$. It can be found that above PR'_1 and PR'_2 are consistent.

Step 4 (attribute weight determination). Using (15) to calculate all δ_{ik}, the decision matrix DM can be constructed as follows:

$$\mathrm{DM} = \begin{bmatrix} -2.019 & -1.923 \\ -0.336 & 0.331 \\ 1.178 & -0.045 \\ 1.178 & 1.637 \end{bmatrix}. \quad (26)$$

According to the constructed decision matrix, when ADM and SDM were adopted, the weight of Attributes 1 and 2 can be calculated by (17) and (18), respectively. A value of $p = 2$ has been adopted arbitrarily for the sake of this demonstration. If ADM is adopted, the weights of Attributes 1 and 2 are 0.501 and 0.499, respectively. If SDM is adopted the weights of Attributes 1 and 2 are 0.509 and 0.491,respectively.

Table 1: Aggregation of heterogeneous group of experts for Attribute 1.

	$\tilde{r}_{12}$	$\tilde{r}_{23}$	$\tilde{r}_{34}$
Expert 1	(0.125, 0.225, 0.325, 0.425)	(0.125, 0.225, 0.325, 0.425)	(0.350, 0.450, 0.550, 0.650)
Expert 2	(0.200, 0.300, 0.400, 0.500)	(0.350, 0.450, 0.550, 0.650)	(0.350, 0.450, 0.550, 0.650)
Expert 3	(0.250, 0.250, 0.250, 0.250)	(0.185, 0.185, 0.185, 0.185)	(0.500, 0.500, 0.500, 0.500)
Degree of agreement (S_{ab})			
S_{12}	0.925	0.775	1.000
S_{13}	0.900	0.880	0.900
S_{23}	0.875	0.685	0.900
Average degree of agreement of expert a (AA_a)			
AA_1	0.913	0.828	0.950
AA_2	0.900	0.730	0.950
AA_3	0.888	0.783	0.900
Relative degree of agreement of expert a (RA_a)			
RA_1	0.338	0.354	0.339
RA_2	0.333	0.312	0.339
RA_3	0.329	0.334	0.321
Consensus degree coefficient of expert a (CC_a) for $\beta = 0.5$			
CC_1	0.319	0.327	0.320
CC_2	0.317	0.306	0.320
CC_3	0.364	0.367	0.361
Aggregated results	$\tilde{r}_{12} = (0.19, 0.26, 0.32, 0.38)$	$\tilde{r}_{23} = (0.22, 0.28, 0.34, 0.41)$	$\tilde{r}_{34} = (0.40, 0.47, 0.53, 0.60)$
Converted results	$r_{12} = 0.290$	$r_{23} = 0.311$	$r_{34} = 0.500$

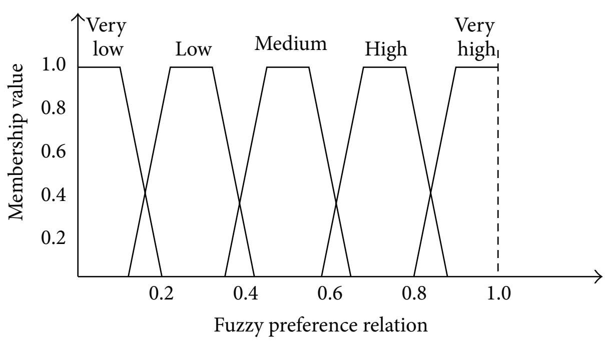

Figure 2: Membership functions adopted by Expert 1.

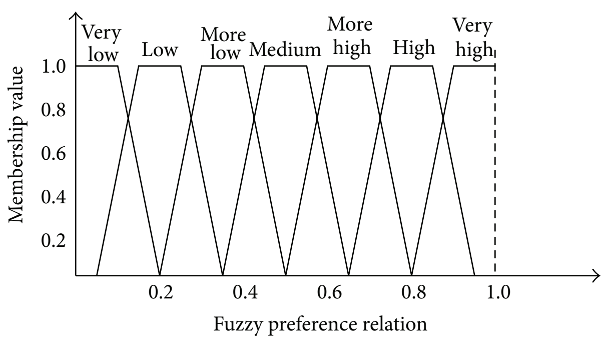

Figure 3: Membership functions adopted by Expert 2.

Table 2: The scoring results by weight determination methods ADM and SDM.

Alternative i	δ_{ik}		C_i (ADM)	C_i (SDM)	Ranking results
1	−2.019	−1.923	−1.971	−1.972	4
2	−0.336	0.331	−0.003	−0.009	3
3	1.178	−0.045	0.567	0.577	2
4	1.178	1.637	1.407	1.403	1
aw_k by ADM	0.501	0.499			
aw_k by SDM	0.509	0.491			

Step 5 (ranking alternatives). After generating the weights of Attributes 1 and 2, using SAW, the score of all alternatives C_i can be calculated by (9). The scoring results are as shown in Table 2. In Table 2, C_i (ADM) and C_i (SDM) indicate the scores of all alternatives using attribute weight determining approaches ADM and SDM, respectively. The bigger values of C_i indicate that the alternative i is better. In the case of the values of C_i (ADM), for example, because C_4 (ADM) > C_3 (ADM) > C_2 (ADM) > C_1 (ADM), the group decision selected Alternative 4 as the first priority. Moreover, according to the values of C_i (SDM), the results also show Alternative 4 as the first priority.

Although the theoretical development involves complicated technical details, the implementation is relatively straightforward in light of the numerical implementation.

Therefore, the proposed methodology is applicable for a practical application. Its contribution can be justified accordingly.

5. Conclusion

This paper proposes a procedure for solving multiple attribute group decision making problems. In the proposed procedure, the transformation of assessment type, the property of consistency, the heterogeneity of a group of experts, the determination of weight, and scoring of alternatives are all considered. It would be a useful tool for decision makers in different industries. A review of the literature related to this research suggests that no previous research has addressed all of the issues simultaneously. The proposed procedure has several important properties as follows.

(i) Experts can provide their preference relations in various formats, which can then be transformed into a standard type.

(ii) Because all preference relation types are transformed into fuzzy preferences, and experts only provide preference relations between alternatives i and $i+1$, it is possible to construct preference relations matrixes that satisfy the property of additive consistency.

(iii) Experts who are highly divergent from the group mean will have their weights reduced.

(iv) The weights of each attribute depend on the degree of variation; the higher the variation of the attribute, the higher its weight.

(v) Decision makers can select suitable MADM methods, such as SAW, GRA, or TOPSIS, for the final ranking step.

In the proposed procedure all the steps are adopted in response to observations made in the related literature and are understood by managers who are not experts in fuzzy theory, group decision making, MADM, or similar issues. A numerical example was described to illustrate the proposed procedure. It was demonstrated that the proposed procedure is simple and effective and can be easily applied to other similar practical problems.

The proposed procedure has some weaknesses in several of its properties. The weight of each expert depends on the divergence of his (or her) assessment from the opinions of other experts. Sometimes the real expert provides the most accurate assessment but is highly divergent from the mean of group. This characteristic would reduce the quality of the group decision. Moreover, the proposed procedure assumes that an attribute is quite important if the difference of the net degree of preference among all alternatives shows a wide variation. However, if an attribute is very important and has a relatively high weight, any small divergence in the assessment of the attribute can influence the ranking produced by the group decision. These weaknesses can provide the opportunity for future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Integrated Supply Chain Cooperative Inventory Model with Payment Period Being Dependent on Purchasing Price under Defective Rate Condition

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In most commercial transactions, the buyer and vendor may usually agree to postpone payment deadline. During such delayed period, the buyer is entitled to keep the products without having to pay the sale price. However, the vendor usually hopes to receive full payment as soon as possible, especially when the transaction involves valuable items; yet, the buyer would offer a higher purchasing price in exchange of a longer postponement. Therefore, we assumed such permissible delayed period is dependent on the purchasing price. As for the manufacturing side, defective products are inevitable from time to time, and not all of those defective products can be repaired. Hence, we would like to add defective production and repair rate to our proposed model and discuss how these factors may affect profits. In addition, holding cost, ordering cost, and transportation cost will also be considered as we develop the integrated inventory model with price-dependent payment period under the possible condition of defective products. We would like to find the maximum of the joint expected total profit for our model and come up with a suitable inventory policy accordingly. In the end, we have also provided a numerical example to clearly illustrate possible solutions.

1. Introduction

Inventory occurs in every stage of the supply chain; therefore, managing inventory in an effective and efficient way becomes a significant task for managers in the course of supply chain management (SCM). Fogarty [1] pointed out that the purpose of inventory is to retrieve demand and supply in an uncertain environment. Frankel [2] considered supply chain to be closely related to controlling and preserving stocks. A good inventory policy should contain a right venue to order, to manufacture, and to distribute accurate supply quantities at the right moment which will then store inventory at the right place to minimize total cost. Another reason for the need to collaborate with other members in the supply chain is to remain competitive. Better collaboration with customers and suppliers will not only provide better service but also reduce costs [3]. Beheshti [4] considered inventory policy as the key to affect conditions during the supply chain, and applying inappropriate inventory policy would result in great loss. Therefore, it is crucial for SCM practice to generate suitable inventory policy. Since the EOQ model proposed by Harris [5] and researchers as well as practitioners have shown interest in optimal inventory policy, Harris [5] focused on inventory decisions of individual firms; yet from the SCM perspective, collaborating closely with members of the supply chain is certainly necessary. Goyal [6] is the first researcher to point out the importance of performance when integrating a supplier and a customer's inventory policies. The single-supplier single-customer model showed the total relevant cost reduction compared with traditional independent inventory strategy. Jammernegg and Reiner [7] pointed out that effective inventory management can enhance the value of the full supply chain. Olson and Xie [8] proposed purchasers and sellers should have a common inventory system when they cooperate with each other. Since supply chain is formed with multiple firms, focusing on a vendor and a buyer's inventory problem is not sufficient. In other words, multiechelon inventory problem is one of the leading issues in SCM. Huang et al. [9] developed an inventory model as three-level dynamic noncooperative game by using the Nash equilibrium. Giannoccaro and Pontrandolfo [10] developed an inventory forecast for three-echelon supply chain to minimize the joint total cost. Cárdenas-Barrón et al. [11] made complements to some shortcomings in the model proposed by Sana [12] and then introduced alternative algorithm to obtain shorter CPU time and fewer total cost [3]. Sana [12] coordinated production and inventory decisions across the supplier, the manufacture, and the customer to maximize the total expected profits. Chung et al. [13] combined deteriorating items with two levels of trade credit under three-layer condition in the supply chain system. A new economic production quantity (EPQ) inventory is then proposed to minimize the total cost. Yang and Tseng [14] assumed that defective products occurred in the supplier and the manufacturer stage, and then backorder is allowed to develop a three-echelon inventory model. Permissible delay in payments and controllable lead time are also considered in the model.

Yield rate is an important factor in manufacturing industry. Production can be imperfect, which may have resulted from insufficient process control, wrongly planned maintenance, inadequate work instructions, or damages during handling (Rad et al., [15]). High defective rate will increase not only production costs but also inspecting costs and repair costs, which may likely cause shortage during the process. In early researches, defective production was rarely considered in economic ordering quantity (EOQ) model; however, defective production is a common condition in real practice. Schwaller [16] added fixed defective rate and inspecting costs to the traditional EOQ model. Paknejad et al. [17] developed an imperfect inventory model under random demands and fixed lead time. Liu and Yang [18] developed an imperfect inventory model which included good products, repairable products, and scrap to maximize the joint total profits. Salameh and Jaber [19] indicated that all products should be divided into good products and defective products; they found that EOQ will increase as defective products increase. Eroglu and Ozdemir [20] extended Salameh and Jaber's [19] model, who indicated how defective rate affects economic production quantity (EPQ) with defective products and permissible shortage. All defective products can be inspected and sold separately from good products. Pal et al. [21] developed a three-layer integrated production-inventory model considering out-of-control quality may occur in the supplier and manufacturer stage. The defective products are reworked at a cost after the regular production time. Using Stakelberg's approach, we can see that the integrated expected average profit was being compared with the total expected average profits. Sarkar et al. [22] extended such work and developed three inventory models considering that the proportion of products could follow different probability distribution: uniform, triangular, and beta. The models allowed planned backorders and the defective products to be reworked [23]. The comparison table was made to show that the minimum cost is obtained in the case of triangular distribution. Soni and Patel [24] assumed that an arrival order lot may contain defective items, and the number of defective items is a random variable which follows beta distribution in a numerical example. The demand is sensitive to retail price, and the production rate will react to demand.

Recently, permissible delay in payments has become a common commercial strategy between the vendor and the buyer. It will bring additional interests or opportunity costs to each other as permissible delayed period varies; hence, delayed period is a critical issue that researchers should consider when developing inventory models. In traditional EOQ assumptions, the buyer has to pay upon product delivery; however, in actual business transactions, the vendor usually gives a fixed delayed period to reduce the stress of capital. During such period, the buyer can make use of the products without having to pay to the vendor; both parties can earn extra interests from sales. Goyal [25] developed an EOQ model with delays in payments. Two situations were discussed in the research: (1) time interval between successive orders was longer than or equal to permissible delay in settling accounts; (2) time interval between successive orders was shorter than permissible delay in settling accounts. Aggarwal and Jaggi [26] quoted Goyal's [25] assumptions to develop a deteriorating inventory model under fixed deteriorating rate. Jamal et al. [27] extended Aggarwal and Jaggi's [26] model and added shortage condition. Teng [28] also amended Goyal's [25] EOQ model and acquired two conclusions. (1) The EOQ decreases and the order cycle period shortens. It is different from Goyal's [25] conclusion. (2) If the supplier wants to decrease the stocks, the supplier has to set higher interest rate to the retailer unpaid payments after the payment periods are overdue, but this will cause the EOQ to be higher than traditional EOQ model. Huang et al. [29] developed a vendor-buyer inventory model with order processing cost reduction and permissible delay in payments. They considered applying information technologies to reduce order processing cost as long as the vendor and the buyer are willing to pay additional investment costs. They also showed that Ha and Kim's [30] model is actually a special case. Lou and Wang [31] extended Huang's [32] integrated inventory model which discussed the relationship between the vendor and the buyer in trade credit financing. They relaxed the assumption that the buyer's interest earned is always less than or equal to the interests charged. They also established a discrimination term to determine whether the buyer's replenishment cycle time is less than the permissible delay period. Li et al. [33] extended the model of Meca et al. [34] by adding permissible payment delays into the corresponding inventory game. They also showed that the core of the inventory game is nonempty and the grand coalition is stable in a myopic perspective; therefore, largest consistent set (LCS) is applied to improve the grand coalition. While most of EOQ models are considered with infinite replenishment rate, Sarkar et al. [35] developed EOQ model for various types of time-dependent demand when delay in payment and price discount are permitted by suppliers in order to obtain the optimal cycle time with finite replenishment rate.

The main purpose of this paper is to maximize the expected joint total profits. Based on Yang and Tseng's [14] model, we also considered the fact that some defective products can be repaired. Furthermore, we proposed functions between purchasing costs and permissible delayed payment period to balance the opportunity costs and interests income when we promote cooperation. We first defined the parameters and assumptions in Section 2, and then we started to develop the integrated inventory model in Section 3. In Section 4, we tried to solve the model to get the optimal solution. A series of numerical examples would be discussed to observe the variations of decision variables by changing parameters in Section 5. In the end, we summarized the variation and present conclusions.

2. Notations and Assumptions

We first develop a three-echelon inventory model with repairable rate and include permissible delay in payments dependent on sale price. The expected joint total annual profits of the model can be divided into three parts: the annual profit of the supplier, the manufacturer, and the retailer. We then observe how purchasing cost may affect permissible delayed period, EOQ, the number of delivery per production run, and the expected joint total annual profits under different manufacturer's production rate and defective rate.

2.1. Notations. To establish the mathematical model, the following notations and assumptions are used. The notations are shown as follows.

The Parameters and the Decision Variable

Q_i: Economic delivery quantity of the ith model, $i = 1, 2, 3, 4$, a decision variable

n_i: The number of lots delivered in a production cycle from the manufacturer to the retailer of ith model, $i = 1, 2, 3, 4$, a positive integer and a decision variable.

(i) Supplier Side

C_s: Supplier's purchasing cost per unit

A_s: Supplier's ordering cost per order

h_s: Supplier's annual holding cost per unit

I_{sp}: Supplier's opportunity cost per dollar per year

I_{se}: Supplier's interest earned per dollar per year.

(ii) Manufacturer Side

P: Manufacturer's production rate

X: Manufacturer's permissible delayed period

C_m: Manufacturer's purchasing cost per unit

A_m: Manufacturer's ordering cost per order

Z: The probability of defective products from manufacturer

R: The probability of defective products can be repaired

W: Manufacturer's inspecting cost per unit

C_{rm}: Manufacturer's repair cost per unit

G: Manufacturer's scrap cost per unit

t_s: The time for repairing all defective products at manufacturer

F_m: Manufacturer's transportation cost per shipment

h_m: Manufacturer's annual holding cost per unit

L_m: The length of lead time of manufacturer

I_{mp}: Manufacturer's opportunity cost per dollar per year

I_{me}: Manufacturer's interest earned per dollar per year.

(iii) Retailer Side

D: Average annual demand per unit time

Y: Retailer's permissible delayed period

P_r: Retailer's selling price per unit

C_r: Retailer's purchasing cost per unit

A_r: Retailer's ordering cost per order

F_r: Retailer's transportation cost per shipment

h_r: Retailer's annual holding cost per unit

L_r: The length of lead time of retailer

I_{rp}: Retailer's opportunity cost per dollar per year

I_{re}: Retailer's interest earned per dollar per year

TP_s: Supplier's total annual profit

TP_m: Manufacturer's total annual profit

TP_r: Retailer's total annual profit

EJTP_i: The expected joint total annual profit, $i = 1, 2, 3, 4$.

Note. "i" represents four different cases due to the relationship of lead time and permissible payment period of manufacturer and the relationship of lead time and permissible payment period of retailer. We will have more detailed discussions in Section 3.

2.2. Assumptions

(1) This supply chain system consists of a single supplier, a single manufacturer, and a single retailer for a single product.

(2) Economic delivery quantity multiplied by the number of deliveries per production run is economic order quantity (EOQ).

(3) Shortages are not allowed.

(4) The sale price must not be less than the purchasing cost at any echelon, $P_r \geq C_r \geq C_m \geq C_s$.

(5) Defective products only happened in the manufacturer and can be inspected and separated into repairable products and scrap immediately.

(6) Scrap cannot be recycled, so the manufacturer has to pay to throw away.

(7) The seller provides a permissible delayed period (X and Y). During the period, the purchaser keeps selling the products and earning the interest by selling revenue. The purchaser pays to the seller at the end of the time period. If the purchaser still has stocks, it will bring capital cost.

(8) The lead time of manufacturer is equal to the cycle time ($L_m = nQ/D$). The lead time of supplier is equal to the cycle time ($L_r = Q/D$).

(9) The purchasing cost is in inverse to the permissible delayed period. It means that the cheaper the purchasing cost, the longer the permissible delayed period.

(10) The time horizon is infinite.

3. Model Formulation

In this section, we have discussed the model of supplier, manufacture, and retailer, and we combined them all into an integrated inventory model. We extended Yang and Tseng's [14] research to compute opportunity costs and interests income. Finally, we used the function between purchasing costs and the permissible delayed payment period to discuss and observe the variation of the expected joint total annual profits.

3.1. The Supplier's Total Annual Profit. In each production run, the supplier's revenue includes sales revenue and interest income; the supplier's includes ordering cost, holding cost, and opportunity cost. Under the condition of permissible delay in payments, if the payment time of the manufacturer (X) is longer than the lead time of the manufacturer (L_m), it will bring additional interests income based on its interest rate (I_{me}) to the manufacturer. On the other hand, it causes the supplier to pay additional opportunity cost based on its interest rate (I_{sp}). If the payment time of the manufacturer (X) is shorter than the lead time of the manufacturer (L_m), it will bring not only additional interests income but also the opportunity costs based on its interest rate (I_{me} and I_{sp}) separately to the manufacturer because of the rest of stocks; however, it causes the supplier to pay additional opportunity costs but gains additional interests income based on its interest rate (I_{sp} and I_{se}) separately.

Before we start to establish the inventory model, we have to discuss how defective rate (Z) and repair rate (R) can affect yield rate. In each production run, the manufacturer outputs defective products because of the imperfect production line. In other words, yield rate is $(1-Z)$. There is fixed proportion to repair these defective products, which means that the proportion of repaired products is (ZR). Since the repaired

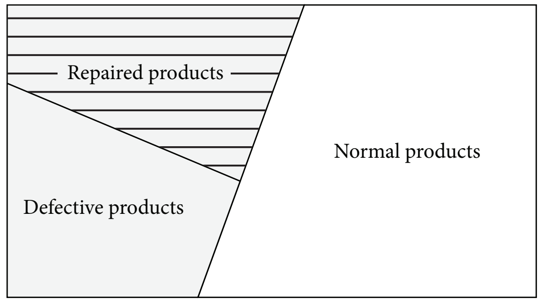


Figure 1: Three kinds of products in the production run.

products are counted in the yield products, we have to revise yield rate by adding the proportion of repaired products. Figure 1 showed the relationship of defective rate, repair rate, and yield rate. So revised yield rate is $(1-Z(1-R))$. In order to satisfy the demand in each production run, the manufacturer will request the supplier to deliver $(nQ)/[1-Z(1-R)]$.

Figure 2 showed the supplier, manufacturer, and retailer's inventory level. As mentioned before, the retailer needs (nQ) to satisfy the demand, while the manufacturer produces $(nQ)/[1-Z(1-R)]$ due to defective rate and repair rate, and the supplier would need to prepare $(nQ)/[1-Z(1-R)]$ to prevent storage.

Case 1 ($L_m < X$). If $L_m < X$, the manufacturer will earn interests income, but the manufacturer's interests income will be transferred into opportunity costs for the supplier (see Figure 3). Consider the following.

(i) Sales revenue $= D(C_m - C_s)/(1-Z(1-R))$.

(ii) Ordering cost $= A_s D/n_i Q_i$.

(iii) Holding cost $= h_s D n_i Q_i/2P[1-Z(1-R)]^2$.

(iv) Transfer opportunity cost $= C_s I_{\text{sp}}(2DX - n_i Q_i)/2[1-Z(1-R)]$.

Thus, TP_{s1} is given by

$$\begin{aligned}\text{TP}_{s1} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\ &\quad - \text{transfer opportunity cost} \\ &= \frac{D(C_m - C_s)}{1-Z(1-R)} - \frac{A_s D}{n_i Q_i} - \frac{h_s D n_i Q_i}{2P[1-Z(1-R)]^2} \\ &\quad - \frac{C_s I_{\text{sp}}(2DX - n_i Q_i)}{2[1-Z(1-R)]}.\end{aligned} \tag{1}$$

Case 2 ($L_m \geq X$). If $L_m \geq X$, the manufacturer will not only earn interests income but also pay the opportunity costs due to the rest of stocks. The manufacturer's interests income and opportunity costs will be transferred into opportunity costs and interests income for the supplier (see Figure 4). Consider the following.

(i) Transfer opportunity cost $= C_s I_{\text{sp}}(2DX - n_i Q_i)/2[1-Z(1-R)]$.

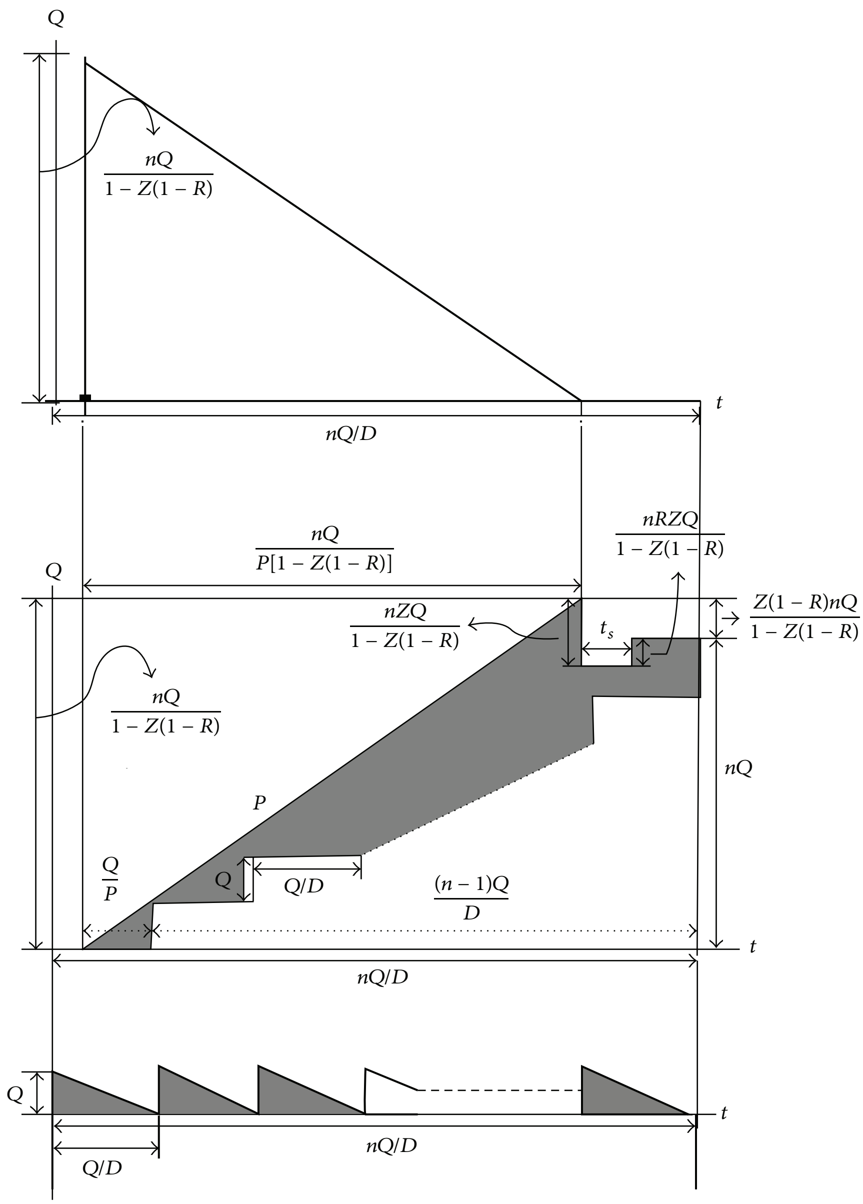


FIGURE 2: The inventory pattern for the three firms.

(ii) Transfer interest income = $C_m I_{se}(n_i Q_i - DX)^2/2n[1 - Z(1-R)]Q_i$.

Thus, TP_{s2} is given by

$$\begin{aligned} TP_{s2} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\ &\quad - \text{transfer opportunity cost} + \text{interest income} \\ &= \frac{D(C_m - C_s)}{1 - Z(1-R)} - \frac{A_s D}{n_i Q_i} - \frac{h_s D n_i Q_i}{2P[1 - Z(1-R)]^2} \\ &\quad - \frac{C_s I_{sp}(2DX - n_i Q_i)}{2[1 - Z(1-R)]} + \frac{C_m I_{se}(n_i Q_i - DX)^2}{2n[1 - Z(1-R)]Q_i}. \end{aligned} \tag{2}$$

3.2. The Manufacturer's Total Annual Profit. In each production run, the manufacturer's revenue includes sales revenue and interests income; the manufacturer's cost includes ordering costs, holding costs, transportation costs, inspecting costs, repair costs, scrap costs, and opportunity costs. We have discussed the relationship between the lead time of the manufacturer (L_m) and the payment time of the manufacturer (X). This relationship can be also used to discuss the retailer's lead time (L_r) and the payment time (Y); therefore, the manufacturer's total annual profit has four different cases. In the middle of Figure 2 is the manufacturer's inventory level which has been the effect of defective rate and repair rate.

Case 1 ($L_m < X$, $L_r < Y$). If $L_m < X$ and $L_r < Y$, both the manufacturer and the retailer will earn interests income, but the retailer's interests income will be transferred

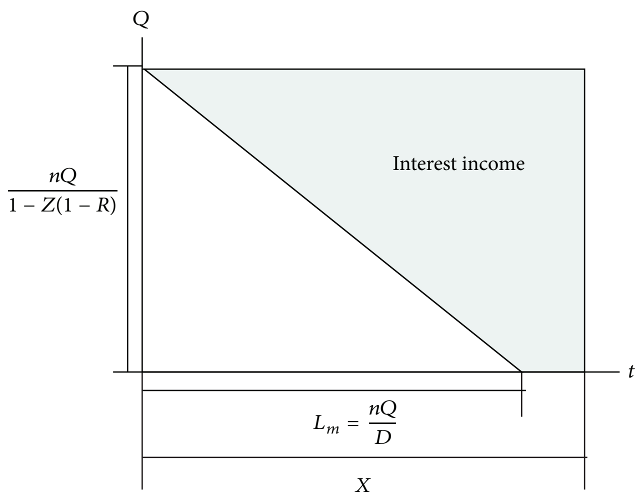


Figure 3: $L_m < X$.

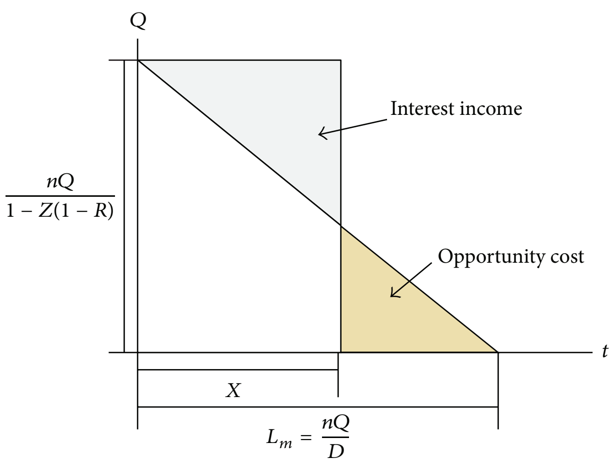


Figure 4: $L_m \geq X$.

into opportunity costs for the manufacturer. Consider the following.

(i) Sales revenue = $D[C_r - C_m/(1 - Z(1 - R))]$.

(ii) Ordering cost = $A_m D/n_i Q_i$.

(iii) Holding cost = $h_m D\{Q_i[(n_i - 1)/2D + \{1 - 2[1 - Z(1 - R)]\}n_i/2P[1 - Z(1 - R)]^2 + 1/P] - t_s Z R n_i/(1 - Z(1 - R))\}$.

(iv) Transportation cost = $F_m D/n_i Q_i$.

(v) Inspecting cost = $WD/(1 - Z(1 - R))$.

(vi) Repair cost = $WD/(1 - Z(1 - R))$.

(vii) Scrap cost = $GZ(1 - R)D/(1 - Z(1 - R))$.

(viii) Interest income = $C_r I_{\mathrm{me}}(2DX - n_i Q_i)/2[1 - Z(1 - R)]$.

(ix) Transfer opportunity cost = $C_m I_{\mathrm{mp}}(DY - Q_i/2)$.

Thus, TP_{m1} is given by

$$\begin{aligned}
\mathrm{TP}_{m1} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} - \text{inspecting cost} - \text{repair cost} \\
&\quad - \text{scrap cost} + \text{interest income} \\
&\quad - \text{transfer opportunity cost} \\
&= D\left[C_r - \frac{C_m}{1 - Z(1 - R)}\right] - \frac{A_m D}{n_i Q_i} \\
&\quad - h_m D\left\{Q_i\left[\frac{n_i - 1}{2D} + \frac{\{1 - 2[1 - Z(1 - R)]\}n_i}{2P[1 - Z(1 - R)]^2} + \frac{1}{P}\right] - \frac{t_s Z R n_i}{1 - Z(1 - R)}\right\} \\
&\quad - \frac{F_m D}{n_i Q_i} - \frac{D[W + C_{\mathrm{rm}} Z R + G Z(1 - R)]}{1 - Z(1 - R)} \\
&\quad + \frac{C_r I_{\mathrm{me}}(2DX - n_i Q_i)}{2[1 - Z(1 - R)]} - C_m I_{\mathrm{mp}}\left(DY - \frac{Q_i}{2}\right).
\end{aligned} \tag{3}$$

Case 2 ($L_m < X$, $L_r < Y$). If $L_m < X$ and $L_r \geq Y$, the manufacturer will earn interests income while the retailer will not due to the rest of stocks, but the retailer's interests income and opportunity costs will be transferred into opportunity costs and interests income for the manufacturer:

$$\text{Interest income} = \frac{C_r I_{\mathrm{me}}(2DX - n_i Q_i)}{2[1 - Z(1 - R)]}. \tag{4}$$

Consider the following.

(i) Transfer opportunity cost = $C_m I_{\mathrm{mp}}(DY)^2/2Q_i$.

(ii) Transfer interest income = $C_r I_{\mathrm{me}}(Q_i - DY)^2/2Q_i$.

Thus, TP_{m2} is given by

$$\begin{aligned}
\mathrm{TP}_{m2} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} - \text{inspecting cost} - \text{repair cost} \\
&\quad - \text{scrap cost} + \text{interest income} \\
&\quad - \text{transfer opportunity cost} + \text{transfer interest income} \\
&= D\left[C_r - \frac{C_m}{1 - Z(1 - R)}\right] - \frac{A_m D}{n_i Q_i} \\
&\quad - h_m D\left\{Q_i\left[\frac{n_i - 1}{2D} + \frac{\{1 - 2[1 - Z(1 - R)]\}n_i}{2P[1 - Z(1 - R)]^2} + \frac{1}{P}\right] - \frac{t_s Z R n_i}{1 - Z(1 - R)}\right\}
\end{aligned}$$

$$-\frac{F_m D}{n_i Q_i}-\frac{D\left[W+C_{\mathrm{rm}} Z R+G Z(1-R)\right]}{1-Z(1-R)}+\frac{C_r I_{\mathrm{me}}\left(2 D X-n_i Q_i\right)}{2[1-Z(1-R)]}-\frac{C_m I_{\mathrm{mp}}(D Y)^2}{2 Q_i}+\frac{C_r I_{\mathrm{me}}\left(Q_i-D Y\right)^2}{2 Q_i}. \tag{5}$$

Case 3 ($L_m \geq X$, $L_r < Y$). If $L_m \geq X$ and $L_r < Y$, the manufacturer will not earn interests income but also pay opportunity costs, and the retailer will earn interests income but such income will be transferred into opportunity costs for the manufacturer. Consider the following.

(i) Opportunity cost $= C_m I_{\mathrm{mp}}(n_i Q_i - DX)^2/2[1-Z(1-R)]n_i Q_i$.

(ii) Interest income $= C_r I_{\mathrm{me}}(DX)^2/2[1-Z(1-R)]n_i Q_i$.

(iii) Transfer opportunity cost $= C_m I_{\mathrm{mp}}(DY - Q_i/2)$.

Thus, TP_{m3} is given by

$$\begin{aligned}
\mathrm{TP}_{m3} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} - \text{inspecting cost} - \text{repair cost} \\
&\quad - \text{scrap cost} - \text{opportunity cost} + \text{interest income} \\
&\quad - \text{transfer opportunity cost} \\
&= D\left[C_r-\frac{C_m}{1-Z(1-R)}\right]-\frac{A_m D}{n_i Q_i} \\
&\quad -h_m D\left\{Q_i\left[\frac{n_i-1}{2 D}+\frac{\{1-2[1-Z(1-R)]\} n_i}{2 P[1-Z(1-R)]^2}+\frac{1}{P}\right]-\frac{t_s Z R n_i}{1-Z(1-R)}\right\} \\
&\quad -\frac{F_m D}{n_i Q_i}-\frac{D\left[W+C_{\mathrm{rm}} Z R+G Z(1-R)\right]}{1-Z(1-R)} \\
&\quad -\frac{C_m I_{\mathrm{mp}}\left(n_i Q_i-D X\right)^2}{2[1-Z(1-R)] n_i Q_i}+\frac{C_r I_{\mathrm{me}}(D X)^2}{2[1-Z(1-R)] n_i Q_i} \\
&\quad -C_m I_{\mathrm{mp}}\left(D Y-\frac{Q_i}{2}\right).
\end{aligned} \tag{6}$$

Case 4 ($L_m \geq X$, $L_r \geq Y$). If $L_m \geq X$ and $L_r \geq Y$, both the manufacturer and the retailer will not earn interests income but need to pay opportunity costs, and the retailer's interests income and opportunity costs will be transferred into opportunity costs for the manufacturer. Consider the following.

(i) Opportunity cost $= C_m I_{\mathrm{mp}}(nQ - DX)^2/2[1-Z(1-R)]n_i Q_i$.

(ii) Interest income $= C_r I_{\mathrm{me}}(DX)^2/2[1-Z(1-R)]n_i Q_i$.

(iii) Transfer opportunity cost $= C_m I_{\mathrm{mp}}(DY)^2/2Q_i$.

(iv) Transfer interest income $= C_r I_{\mathrm{me}}(Q_i - DY)^2/2Q_i$.

Thus, TP_{m4} is given by

$$\begin{aligned}
\mathrm{TP}_{m4} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} - \text{inspecting cost} - \text{repair cost} \\
&\quad - \text{scrap cost} - \text{opportunity cost} + \text{interest income} \\
&\quad - \text{transfer opportunity cost} + \text{transfer interest income} \\
&= D\left[C_r-\frac{C_m}{1-Z(1-R)}\right]-\frac{A_m D}{n_i Q_i} \\
&\quad -h_m D\left\{Q_i\left[\frac{n_i-1}{2 D}+\frac{\{1-2[1-Z(1-R)]\} n_i}{2 P[1-Z(1-R)]^2}+\frac{1}{P}\right]-\frac{t_s Z R n_i}{1-Z(1-R)}\right\} \\
&\quad -\frac{F_m D}{n_i Q_i}-\frac{D\left[W+C_{\mathrm{rm}} Z R+G Z(1-R)\right]}{1-Z(1-R)} \\
&\quad -\frac{C_m I_{\mathrm{mp}}(n Q-D X)^2}{2[1-Z(1-R)] n_i Q_i}+\frac{C_r I_{\mathrm{me}}(D X)^2}{2[1-Z(1-R)] n_i Q_i} \\
&\quad -\frac{C_m I_{\mathrm{mp}}(D Y)^2}{2 Q_i}+\frac{C_r I_{\mathrm{me}}\left(Q_i-D Y\right)^2}{2 Q_i}.
\end{aligned} \tag{7}$$

3.3. The Retailer's Total Annual Profit

In each production run, the retailer's revenue includes sales revenue and interests income; the retailer's costs include ordering costs, holding costs, transportation costs, and opportunity costs. The relationship between the retailer's lead time (L_r) and payment time (Y) has been discussed before. The retailer may gain additional interests income or pay opportunity costs according to two different cases shown as follows.

Case 1 ($L_r < Y$). If $L_r < Y$, the retailer will earn interest income. Consider the following.

(i) Sales revenue $= D(P_r - C_r)$.

(ii) Ordering cost $= A_r D/n_i Q_i$.

(iii) Holding cost $= h_r Q_i/2$.

(iv) Transportation cost $= F_r D/Q_i$.

(v) Interest income $= P_r I_{\mathrm{re}}(DY - Q_i/2)$.

Thus, TP_{r1} is given by

$$\begin{aligned}
\mathrm{TP}_{r1} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} + \text{interest income} \\
&= D\left(P_r - C_r\right) - \frac{A_r D}{n_i Q_i} - \frac{h_r Q_i}{2} - \frac{F_r D}{Q_i} + P_r I_{\mathrm{re}}\left(DY - \frac{Q_i}{2}\right).
\end{aligned} \tag{8}$$

Case 2 ($L_r \geq Y$). If $L_r \geq Y$, the retailer will not only earn interests income but also pay opportunity costs due to the rest of stocks. Consider the following.

(i) Opportunity cost $= C_r I_{\mathrm{rp}}(Q_i - DY)^2/2Q_i$.

(ii) Interest income $= P_r I_{\mathrm{re}}(DY)^2/2Q_i$.

Thus, TP_{r2} is given by

$$\begin{aligned}
\mathrm{TP}_{r2} &= \text{sales revenue} - \text{ordering cost} - \text{holding cost} \\
&\quad - \text{transportation cost} - \text{opportunity cost} \\
&\quad + \text{interest income} \\
&= D\left(P_r - C_r\right) - \frac{A_r D}{n_i Q_i} - \frac{h_r Q_i}{2} - \frac{F_r D}{Q_i} - \frac{C_r I_{\mathrm{rp}}\left(Q_i - DY\right)^2}{2Q_i} \\
&\quad + \frac{P_r I_{\mathrm{re}}\left(DY\right)^2}{2Q_i}.
\end{aligned} \tag{9}$$

3.4. The Expected Joint Total Annual Profit. According to different conditions, the expected joint total annual profit function, $\mathrm{EJTP}(Q_i, n_i)$, can be expressed as

$$\mathrm{EJTP}_i\left(Q_i, n_i\right) = \begin{cases}
\mathrm{EJTP}_1\left(Q_1, n_1\right) = \mathrm{TP}_{s1} + \mathrm{TP}_{m1} + \mathrm{TP}_{r1} & \text{if } L_m < X,\ L_r < Y \\
\mathrm{EJTP}_2\left(Q_2, n_2\right) = \mathrm{TP}_{s1} + \mathrm{TP}_{m2} + \mathrm{TP}_{r2} & \text{if } L_m < X,\ L_r \geq Y \\
\mathrm{EJTP}_3\left(Q_3, n_3\right) = \mathrm{TP}_{s2} + \mathrm{TP}_{m3} + \mathrm{TP}_{r1} & \text{if } L_m \geq X,\ L_r < Y \\
\mathrm{EJTP}_4\left(Q_4, n_4\right) = \mathrm{TP}_{s2} + \mathrm{TP}_{m4} + \mathrm{TP}_{r2} & \text{if } L_m \geq X,\ L_r \geq Y,
\end{cases} \tag{10}$$

where

$$\begin{aligned}
&\mathrm{EJTP}_1\left(Q_1, n_1\right) \\
&= D\left[P_r - \frac{C_s + W + C_{\mathrm{rm}} ZR + GZ\left(1-R\right)}{1 - Z\left(1-R\right)}\right] \\
&\quad - h_m D\left\{Q_1\left[\frac{n_1 - 1}{2D} + \frac{\left\{1 - 2\left[1 - Z\left(1-R\right)\right]\right\} n_1}{2P\left[1 - Z\left(1-R\right)\right]^2} + \frac{1}{P}\right]\right. \\
&\qquad \left. - \frac{t_s Z R n_1}{1 - Z\left(1-R\right)}\right\} - \frac{h_r Q_1}{2} - \frac{h_s D n_1 Q_1}{2P\left[1 - Z\left(1-R\right)\right]^2} \\
&\quad - \frac{D\left(A_s + A_m + F_m + A_r + F_r n_1\right)}{n_1 Q_1} \\
&\quad + \frac{\left(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}\right)\left(2DX - n_1 Q_1\right)}{2\left[1 - Z\left(1-R\right)\right]} \\
&\quad + \left(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}}\right)\left(DY - \frac{Q_1}{2}\right),
\end{aligned}$$

$$\begin{aligned}
&\mathrm{EJTP}_2\left(Q_2, n_2\right) \\
&= D\left[P_r - \frac{C_s + W + C_{\mathrm{rm}} ZR + GZ\left(1-R\right)}{1 - Z\left(1-R\right)}\right] \\
&\quad - h_m D\left\{Q_2\left[\frac{n_2 - 1}{2D} + \frac{\left\{1 - 2\left[1 - Z\left(1-R\right)\right]\right\} n_2}{2P\left[1 - Z\left(1-R\right)\right]^2} + \frac{1}{P}\right]\right. \\
&\qquad \left. - \frac{t_s Z R n_2}{1 - Z\left(1-R\right)}\right\} - \frac{h_r Q_2}{2} - \frac{h_s D n_2 Q_2}{2P\left[1 - Z\left(1-R\right)\right]^2} \\
&\quad - \frac{D\left(A_s + A_m + F_m + A_r + F_r n_2\right)}{n_2 Q_2} \\
&\quad + \frac{\left(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}\right)\left(2DX - n_2 Q_2\right)}{2\left[1 - Z\left(1-R\right)\right]} \\
&\quad + \frac{\left(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}}\right)\left(Q_2 - DY\right)^2}{2Q_2} + \frac{\left(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}}\right)\left(DY\right)^2}{2Q_2},
\end{aligned}$$

$$\begin{aligned}
&\mathrm{EJTP}_3\left(Q_3, n_3\right) \\
&= D\left[P_r - \frac{C_s + W + C_{\mathrm{rm}} ZR + GZ\left(1-R\right)}{1 - Z\left(1-R\right)}\right] \\
&\quad - h_m D\left\{Q_3\left[\frac{n_3 - 1}{2D} + \frac{\left\{1 - 2\left[1 - Z\left(1-R\right)\right]\right\} n_3}{2P\left[1 - Z\left(1-R\right)\right]^2} + \frac{1}{P}\right]\right. \\
&\qquad \left. - \frac{t_s Z R n_3}{1 - Z\left(1-R\right)}\right\} - \frac{h_r Q_3}{2} - \frac{h_s D n_3 Q_3}{2P\left[1 - Z\left(1-R\right)\right]^2} \\
&\quad - \frac{D\left(A_s + A_m + F_m + A_r + F_r n_3\right)}{n_3 Q_3} \\
&\quad + \frac{\left(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}\right)\left(DX\right)^2}{2\left[1 - Z\left(1-R\right)\right] n_3 Q_3} \\
&\quad + \frac{\left(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}}\right)\left(n_3 Q_3 - DX\right)^2}{2\left[1 - Z\left(1-R\right)\right] n_3 Q_3} \\
&\quad + \frac{\left(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}}\right)\left(n_3 Q_3 - DX\right)^2}{2\left[1 - Z\left(1-R\right)\right] n_3 Q_3} \\
&\quad + \left(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}}\right)\left(DY - \frac{Q_3}{2}\right),
\end{aligned}$$

$$\begin{aligned}
&\mathrm{EJTP}_4\left(Q_4, n_4\right) \\
&= D\left[P_r - \frac{C_s + W + C_{\mathrm{rm}} ZR + GZ\left(1-R\right)}{1 - Z\left(1-R\right)}\right]
\end{aligned}$$

$$
\begin{aligned}
&- h_m D \left\{ Q_4 \left[\frac{n_4 - 1}{2D} + \frac{\{1 - 2[1 - Z(1-R)]\} n_4}{2P[1 - Z(1-R)]^2} + \frac{1}{P} \right] \right. \\
&\qquad \left. - \frac{t_s Z R n_4}{1 - Z(1-R)} \right\} - \frac{h_r Q_4}{2} - \frac{h_s D n_4 Q_4}{2P[1 - Z(1-R)]^2} \\
&- \frac{D(A_s + A_m + F_m + A_r + F_r n_4)}{n_4 Q_4} \\
&+ \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}})(DX)^2}{2[1 - Z(1-R)] n_4 Q_4} \\
&+ \frac{(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}})(n_4 Q_4 - DX)^2}{2[1 - Z(1-R)] n_4 Q_4} \\
&+ \frac{(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}})(Q_4 - DY)^2}{2Q_4} + \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})(DY)^2}{2Q_4}.
\end{aligned} \tag{11}
$$

4. Solution Procedure

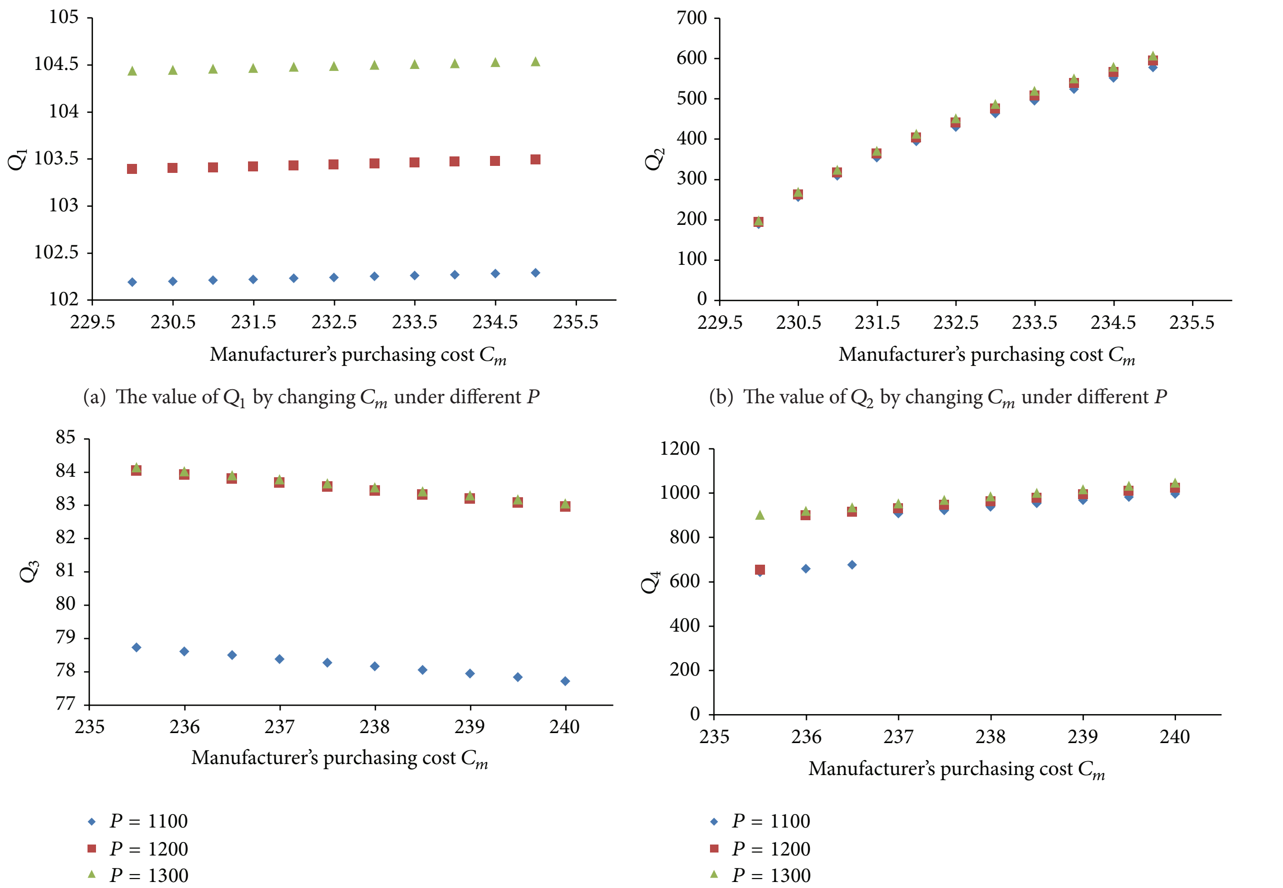
4.1. Determination of the Optimal Delivery Quantity Q_i for Any Given n_i. We would like to find the maximum value of the expected total profit EJTP(Q_i, n_i). For any n_i, we will take the first and second partial derivations of EJTP(Q_i, n_i) with respect to Q_i. We have

$$
\begin{aligned}
&\frac{\partial \mathrm{EJTP}_1(Q_1, n_1)}{\partial Q_1} \\
&\quad = \frac{D(A_s + A_m + F_m + A_r + F_r n_1)}{n_1 Q_1^2} \\
&\qquad - h_m D \left\{ \frac{n_1 - 1}{2D} + \frac{\{1 - 2[1 - Z(1-R)]\} n_1}{2P[1 - Z(1-R)]^2} + \frac{1}{P} \right\} \\
&\qquad - \frac{h_r}{2} - \frac{h_s D n_1}{2P[1 - Z(1-R)]^2} - \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}) n_1}{2[1 - Z(1-R)]} \\
&\qquad - \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})}{2},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&\frac{\partial \mathrm{EJTP}_2(Q_2, n_2)}{\partial Q_2} \\
&\quad = \frac{D(A_s + A_m + F_m + A_r + F_r n_2)}{n_2 Q_2^2} \\
&\qquad - h_m D \left\{ \frac{n_2 - 1}{2D} + \frac{\{1 - 2[1 - Z(1-R)]\} n_2}{2P[1 - Z(1-R)]^2} + \frac{1}{P} \right\} \\
&\qquad - \frac{h_r}{2} - \frac{h_s D n_2}{2P[1 - Z(1-R)]^2} - \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}) n_2}{2[1 - Z(1-R)]} \\
&\qquad - \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})(DY)^2}{2Q_2^2} \\
&\qquad + \frac{(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}})[Q_2^2 - (DY)^2]}{2Q_2^2},
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&\frac{\partial \mathrm{EJTP}_3(Q_3, n_3)}{\partial Q_3} \\
&\quad = \frac{D(A_s + A_m + F_m + A_r + F_r n_3)}{n_3 Q_3^2} \\
&\qquad - h_m D \left\{ \frac{n_3 - 1}{2D} + \frac{\{1 - 2[1 - Z(1-R)]\} n_3}{2P[1 - Z(1-R)]^2} + \frac{1}{P} \right\} \\
&\qquad - \frac{h_r}{2} - \frac{h_s D n_3}{2P[1 - Z(1-R)]^2} \\
&\qquad - \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}})(DX)^2}{2[1 - Z(1-R)] n_3 Q_3^2} \\
&\qquad + \frac{(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}})[(n_3 Q_3)^2 - (DX)^2]}{2[1 - Z(1-R)] n_3 Q_3^2} \\
&\qquad - \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})}{2},
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&\frac{\partial \mathrm{EJTP}_4(Q_4, n_4)}{\partial Q_4} \\
&\quad = \frac{D(A_s + A_m + F_m + A_r + F_r n_4)}{n_4 Q_4^2} \\
&\qquad - h_m D \left\{ \frac{n_4 - 1}{2D} + \frac{\{1 - 2[1 - Z(1-R)]\} n_4}{2P[1 - Z(1-R)]^2} + \frac{1}{P} \right\} \\
&\qquad - \frac{h_r}{2} - \frac{h_s D n_4}{2P[1 - Z(1-R)]^2} - \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}})(DX)^2}{2[1 - Z(1-R)] n_4 Q_4^2} \\
&\qquad - \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})(DY)^2}{2Q_4^2} \\
&\qquad + \frac{(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}})[(n_4 Q_4)^2 - (DX)^2]}{2[1 - Z(1-R)] n_4 Q_4^2} \\
&\qquad + \frac{(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}})[Q_4^2 - (DY)^2]}{2Q_4^2},
\end{aligned} \tag{15}
$$



(a) The value of Q_1 by changing C_m under different P

(b) The value of Q_2 by changing C_m under different P

(c) The value of Q_3 by changing C_m under different P

(d) The value of Q_4 by changing C_m under different P

Figure 5: The value of delivery quantity by changing C_m in Q_i, for $i = 1, 2, 3, 4$.

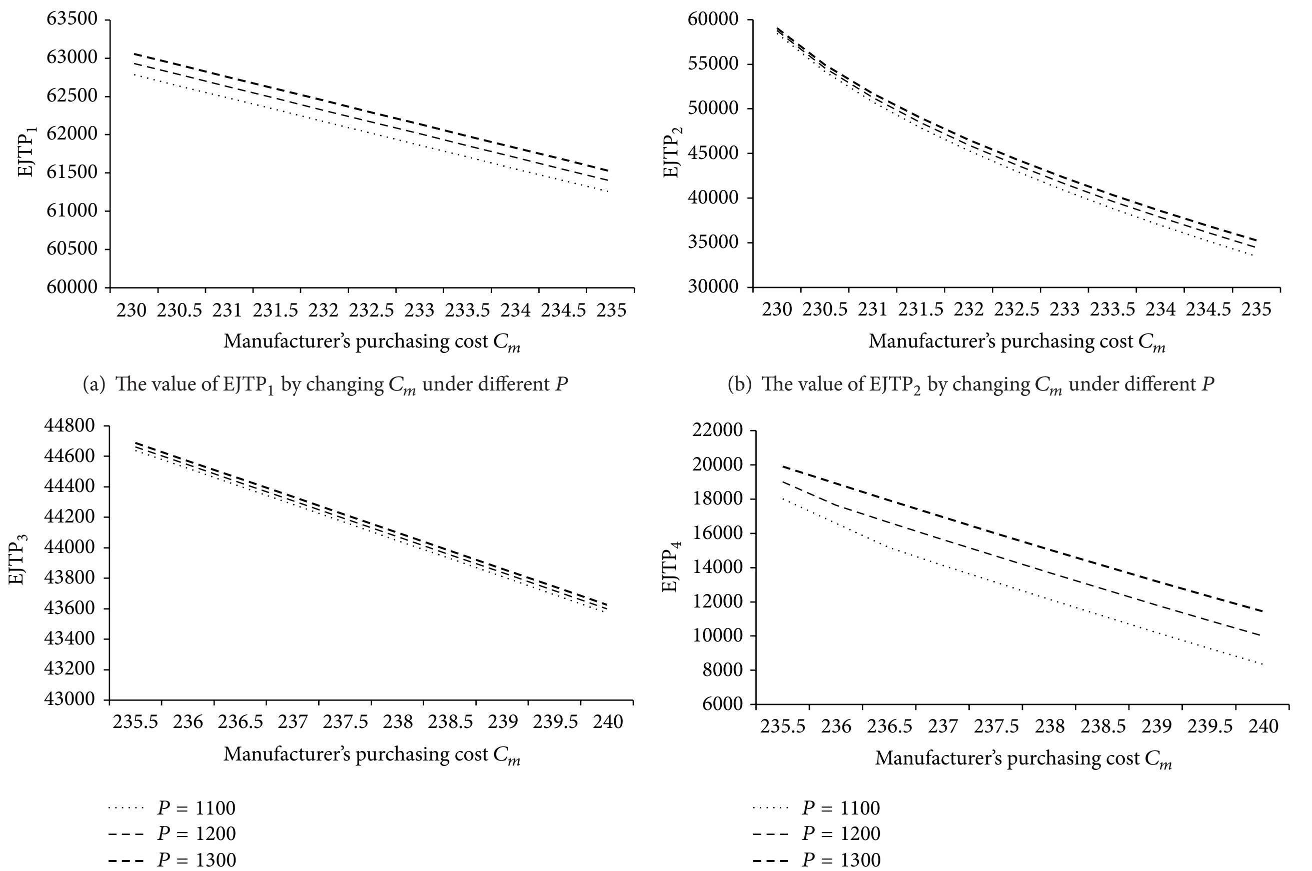
$$\frac{\partial^2 \mathrm{EJTP}_1(Q_1, n_1)}{\partial Q_1^2} = -\frac{2D(A_s + A_m + F_m + A_r + F_r n_1)}{n_1 Q_1^3} < 0, \tag{16}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{EJTP}_2(Q_2, n_2)}{\partial Q_2^2} &= -\frac{2D(A_s + A_m + F_m + A_r + F_r n_2)}{n_2 Q_2^3} \\ &\quad + \frac{(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}})(DY)^2}{Q_2^3} + \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})(DY)^2}{Q_2^3} \\ &< 0,\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{EJTP}_3(Q_3, n_3)}{\partial Q_3^2} &= -\frac{2D(A_s + A_m + F_m + A_r + F_r n_3)}{n_3 Q_3^3} \\ &\quad + \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}})(DX)^2}{[1 - Z(1-R)]\, n_3 Q_3^3} + \frac{(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}})(DX)^2}{[1 - Z(1-R)]\, n_3 Q_3^3} \\ &< 0,\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{\partial^2 \mathrm{EJTP}_4(Q_4, n_4)}{\partial Q_4^2} &= -\frac{2D(A_s + A_m + F_m + A_r + F_r n_4)}{n_4 Q_4^3} \\ &\quad + \frac{(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}})(DX)^2}{[1 - Z(1-R)]\, n_4 Q_4^3} + \frac{(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}})(DX)^2}{[1 - Z(1-R)]\, n_4 Q_4^3} \\ &\quad + \frac{(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}})(DY)^2}{Q_4^3} + \frac{(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}})(DY)^2}{Q_4^3} \\ &< 0.\end{aligned} \tag{19}$$

Because (16), (17), (18), and (19) < 0, therefore $\mathrm{EJTP}(Q_i, n_i)$ is concave function in Q_i for fixed n_i. We can find a unique value of Q_i that maximize $\mathrm{EJTP}(Q_i, n_i)$. Let



(a) The value of EJTP$_1$ by changing C_m under different P

(b) The value of EJTP$_2$ by changing C_m under different P

(c) The value of EJTP$_3$ by changing C_m under different P

(d) The value of EJTP$_4$ by changing C_m under different P

FIGURE 6: The value of profit by changing C_m in EJTP$_i$, for $i = 1, 2, 3, 4$.

$\partial \mathrm{EJTP}_i(Q_i, n_i)/\partial Q_i = 0$ in (16), (17), (18), and (19), so we can get that Q_i are as follows.

The original equations are too long, so in order to shorten them, we let $[1 - Z(1 - R)] = U$; $(C_r I_{\mathrm{me}} - C_s I_{\mathrm{sp}}) = M$; $(P_r I_{\mathrm{re}} - C_m I_{\mathrm{mp}}) = W$; $(C_r I_{\mathrm{me}} - C_r I_{\mathrm{rp}}) = B$; $(C_m I_{\mathrm{se}} - C_m I_{\mathrm{mp}}) = E$. Then we substitute them into the original equations

$$Q_1^* = \Big(\big(2DPU^2\left(A_s + A_m + F_m + A_r + F_r n_1\right)\big) \times \Big(n_2\left\{PU\left[U\left(h_m\left(n_1 - 1\right) + h_r + W\right) + Mn_1\right] + D\left[n_1\left(h_s + h_m\left(1 - 2U\right)\right) + 2h_m U^2\right]\right\}\Big)^{-1}\Big)^{1/2}, \tag{20}$$

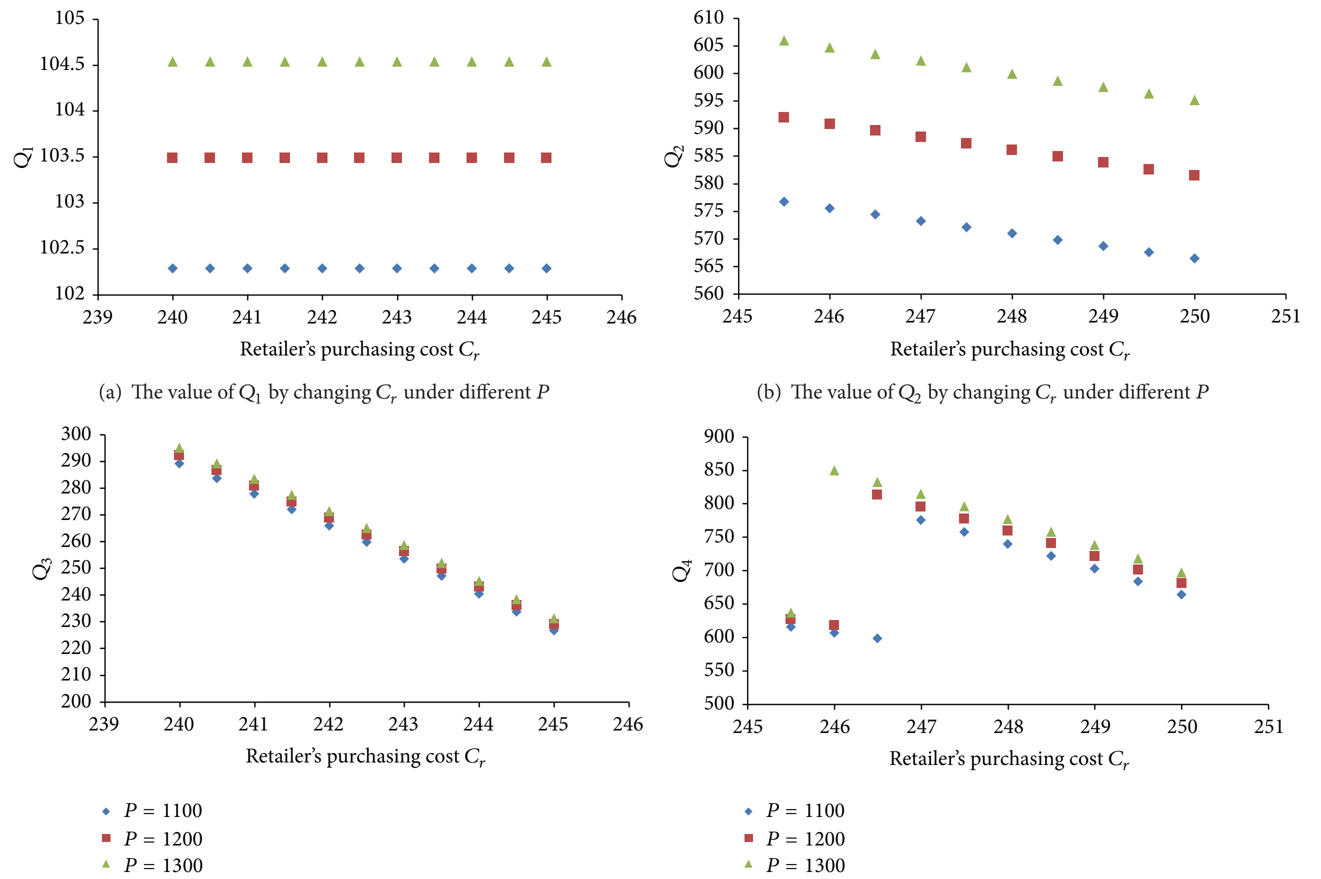
$$Q_2^* = \Big(\big(PU^2\left[2D\left(A_s + A_m + F_m + A_r + F_r n_2\right) - n_2\left(B + W\right)\left(DY\right)^2\right]\big) \times \Big(n_2\left\{PU\left[U\left(h_m\left(n_2 - 1\right) + h_r - B\right) + Mn_2\right] + D\left[n_2\left(h_s + h_m\left(1 - 2U\right)\right) + 2h_m U^2\right]\right\}\Big)^{-1}\Big)^{1/2}, \tag{21}$$

$$Q_3^* = \Big(\big(PU\left[2DU\left(A_s + A_m + F_m + A_r + F_r n_3\right) - \left(M + E\right)\left(DX\right)^2\right]\big) \times \Big(n_3\left\{PU\left[U\left(h_m\left(n_3 - 1\right) + h_r + W\right) - E\right] + D\left[n_3\left(h_s + h_m\left(1 - 2U\right)\right) + 2h_m U^2\right]\right\}\Big)^{-1}\Big)^{1/2}, \tag{22}$$

$$Q_4^* = \Big(\big(PU\left[2DU\left(A_s + A_m + F_m + A_r + F_r n_4\right) - \left(M + E\right)\left(DX\right)^2 - Un_4\left(B + W\right)\left(DY\right)^2\right]\big) \times \Big(n_4\left\{PU\left[U\left(h_m\left(n_4 - 1\right) + h_r - B\right) - E\right] + D\left[n_4\left(h_s + h_m\left(1 - 2U\right)\right) + 2h_m U^2\right]\right\}\Big)^{-1}\Big)^{1/2}. \tag{23}$$

Algorithm. To summarize the above arguments, we established the algorithm to obtain the optimal values of $\mathrm{EJTP}(n_i, Q_i)$.

Equation (10) shows the situations of each case; obviously, each case is mutual exclusive. In other words, before we start



(a) The value of Q_1 by changing C_r under different P

(b) The value of Q_2 by changing C_r under different P

(c) The value of Q_3 by changing C_r under different P

(d) The value of Q_4 by changing C_r under different P

Figure 7: The value of delivery quantity by changing C_r in Q_i, for $i = 1, 2, 3, 4$.

to find the optimal solutions, we have to recognize which equations should be used first.

Step 1. Examine the relationship of L_m, X and L_r, Y to use corresponding equations.

Step 2. Let $n_i = 1$ and substitute into (20), (21), (22), or (23) to find Q_1, Q_2, Q_3, or Q_4.

Step 3. Find EJTP_i by substituting n_i, Q_i, and different production rate (P).

Step 4. Let $n = n_i + 1$ and repeat Step 2 to Step 3 until $\text{EJTP}_{i(n_i)} > \text{EJTP}_{i(n_i+1)}$.

5. Numerical Example

In Section 5, we will observe the variation of Q_i, n_i, and EJTP_i by changing C_m and C_r separately under different production rate or defective rate. We consider an inventory system with the following data.

Consider $D = 1000$ unit/year, $C_s = 200$ per unit, $A_s = 80$ per order, $h_s = 20$ per unit, $I_{\text{sp}} = 0.025$ per year, $I_{\text{se}} = 0.0254$ per year, $C_m = 235$ per unit, $A_m = 100$ per order, $h_m = 23$ per unit, $W = 5$ per unit, $C_{\text{rm}} = 10$ per unit, $G = 10$ per unit, $F_m = 100$ per time, $Z = 0.1$, $R = 0.9$, $t_s = 0.0055$ year, $I_{\text{mp}} = 0.0256$ per year, $I_{\text{me}} = 0.02$ per year, $C_r = 245$ per unit, $A_r = 120$ per order, $h_r = 25$ per unit, $F_r = 150$ per time, $P_r = 280$ per unit, $I_{\text{rp}} = 0.02$ per year, and $I_{\text{re}} = 0.021$ per year.

5.1. The Variation under Different P. In Section 5.1, we supposed that the maximum of the production rate is 1300. The manufacturer can change the production rate under any condition; furthermore, the extra payment by changing the rate is ignored. Let us observe the value of delivery quantity and profit with $P = 1100$, $P = 1200$, and $P = 1300$ by changing the manufacturer's purchasing costs, and we set the function of L_m and X is $X = 3000/C_m$, or changing the retailer's purchasing costs, and we set the function of L_r and Y is $Y = 3000/C_r$.

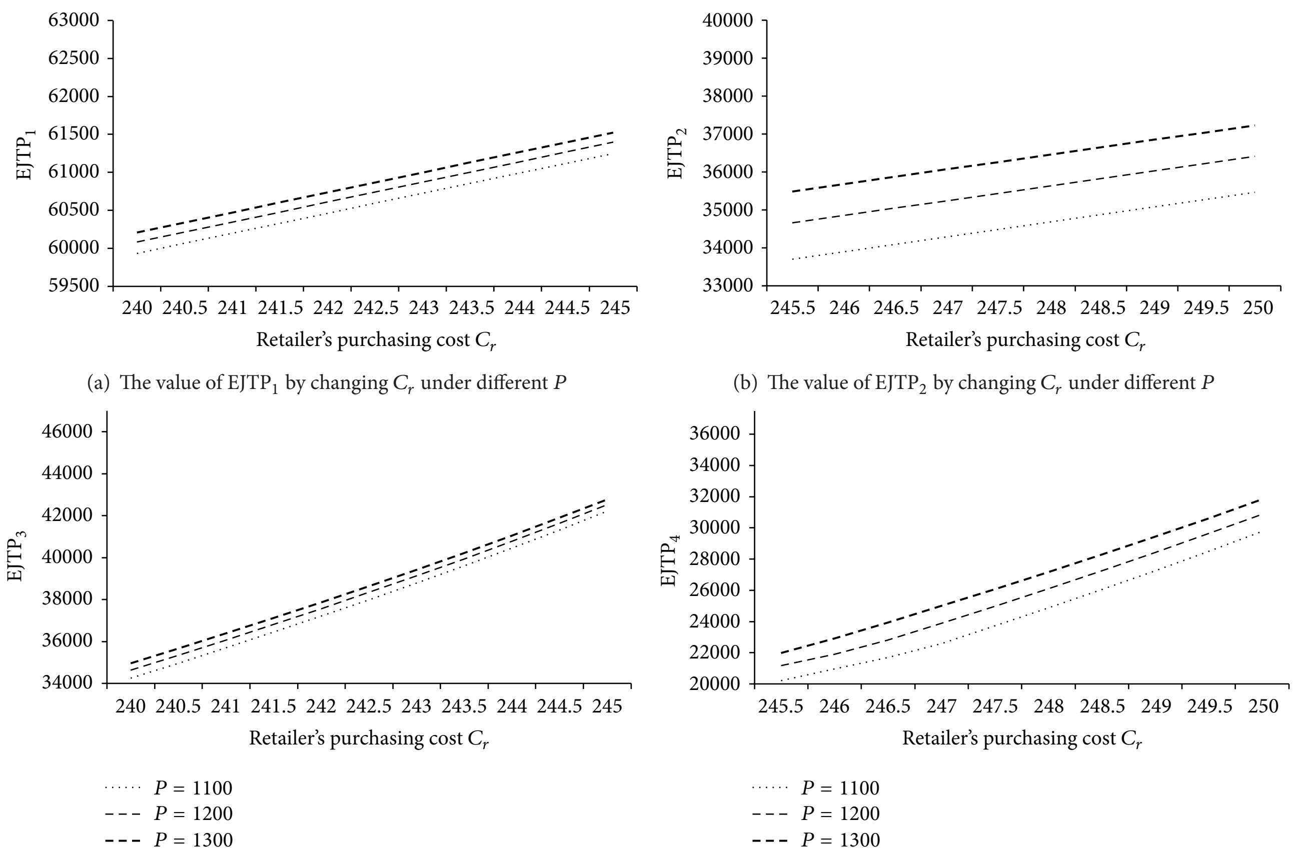
5.1.1. The Permissible Period X and EJTP. We have changed C_m by 0.5 per unit. In order to find out which condition is more beneficial to the proposed inventory model, we formed the details shown in Table 1 and the solution results are illustrated in Figures 5 and 6.

We have discussed that if the payment time is longer than the lead time, it will bring additional interests income

Table 1: The value of profit in different condition by changing C_m.

	$P = 1100$	$P = 1200$	$P = 1300$
C_m	230.0~235.0	230.0~235.0	230.0~235.0
n_1	2	2	2
Q_1	102.19~102.29	103.39~103.49	104.44~104.54
$EJTP_1$	62782.89~61249.25	62930.18~61396.67	*63056.73~61523.33
C_m	230.0~235.0	230.0~235.0	230.0~235.0
n_2	1	1	1
Q_2	189.02~577.86	194.04~593.21	198.62~607.21
$EJTP_2$	58465.23~33503.15	58779.07~34462.59	59051.28~35294.77
C_m	235.5~240	235.5~240	235.5~240
n_3	14	13	13
Q_3	78.73~77.72	84.04~82.95	84.15~83.06
$EJTP_3$	44637.85~43572.97	44660.66~43599.22	44686.91~43625.13
C_m	235.5~240	235.5~240	235.5~240
n_4	2 (C_m = 235.5~236.5) 1 (C_m = 235.5~236.5)	2 (C_m = 235.5) 1 (C_m = 236.0~236.5)	1
Q_4	641.72~675.19 (C_m = 235.5~236.5) 906.62~996.36 (C_m = 237.0~240.0)	653.70 (C_m = 235.5) 897.88~1022.77 (C_m = 235.5~236.5)	901.78~1046.84
$EJTP_4$	18007.04~8353.20	19000.21~10007.45	19908.57~11442.18

*Optimal solution of $EJTP_i$.



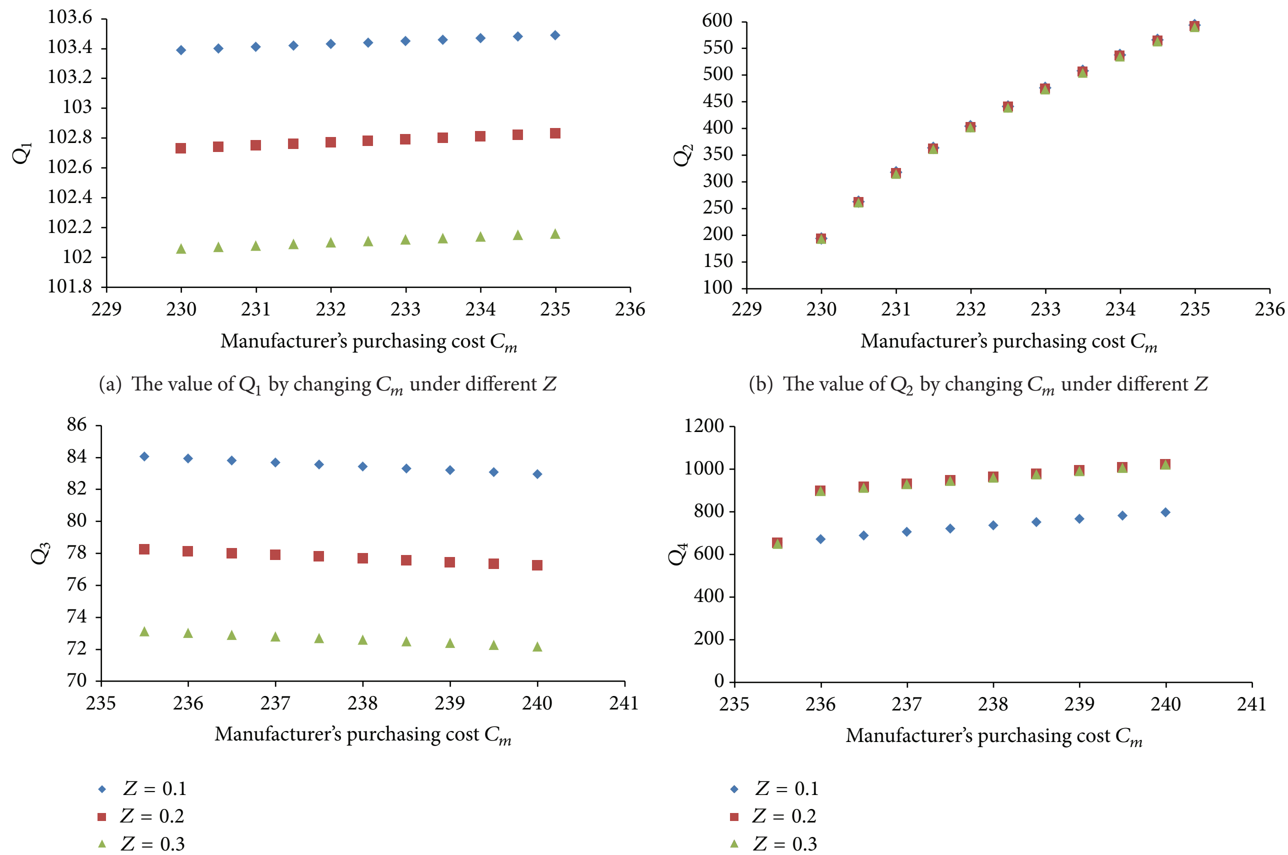
(a) The value of $EJTP_1$ by changing C_r under different P

(b) The value of $EJTP_2$ by changing C_r under different P

(c) The value of $EJTP_3$ by changing C_r under different P

(d) The value of $EJTP_4$ by changing C_r under different P

Figure 8: The value of profit by changing C_r in $EJTP_i$, for $i = 1, 2, 3, 4$.



(a) The value of Q_1 by changing C_m under different Z

(b) The value of Q_2 by changing C_m under different Z

(c) The value of Q_3 by changing C_m under different Z

(d) The value of Q_4 by changing C_m under different Z

FIGURE 9: The value of delivery quantity by changing C_m in Q_i, for $i = 1, 2, 3, 4$.

to the buyer. However, if the payment time is shorter than the lead time, it will bring additional interests income and opportunity costs to the buyer due to the rest of stocks. After computing and comparing the results in Table 1, we have found that the optimal profits will occur in $\mathrm{EJTP}_1(Q_1, n_1)$ under the manufacturer's production rate being 1300 units per year. Also, the worst profit will occur in $\mathrm{EJTP}_4(Q_4, n_4)$ under the manufacturer's production rate being 1100 units per year.

5.1.2. The Permissible Time X and EJTP. In Section 5.1.2, we changed the retailer's purchasing cost to observe the value of profit; the solution results are illustrated in Figures 7 and 8, and the detailed result is shown in Table 2.

From Table 2, we have found that the optimal profits will occur in $\mathrm{EJTP}_1(Q_1, n_1)$ under the manufacturer's production rate being 1300 units per year which is the same as in Section 5.1.1. Also, the worst profit will occur in $\mathrm{EJTP}_4(Q_4, n_4)$ under the manufacturer's production rate being 1100 units per year.

5.2. The Variation under Different Z. In Section 5.2, we supposed that the maximum of defective rate is 0.3. The manufacturer can change the production rate under any condition; also, the extra payment by changing the rate is ignored.

5.2.1. The Permissible Period X and EJTP. We have changed manufacturer's purchasing cost C_m by 0.5 per unit. In order to compare which condition is more beneficial, we formed detailed results in Table 3. The solution results are illustrated in Figures 9 and 10.

From Table 3, we have found that the optimal profits will occur in $\mathrm{EJTP}_1(Q_1, n_1)$ under the manufacturer's defective rate being 0.1. Also, the worst profits will occur in $\mathrm{EJTP}_4(Q_4, n_4)$ under the manufacturer's defective rate being 0.3.

5.2.2. The Permissible Period Y and EJTP. We have changed retailer's purchasing costs C_r by 0.5 per unit. In order to know which condition is more beneficial, we formed detailed results in Table 4. The solution results are illustrated in Figures 11 and 12.

From Table 4, we have found the optimal profits will occur in $\mathrm{EJTP}_1(Q_1, n_1)$ under the manufacturer's defective rate being 0.1. Also, the worst profits will occur in $\mathrm{EJTP}_4(Q_4, n_4)$ under the manufacturer's defective rate being 0.3.

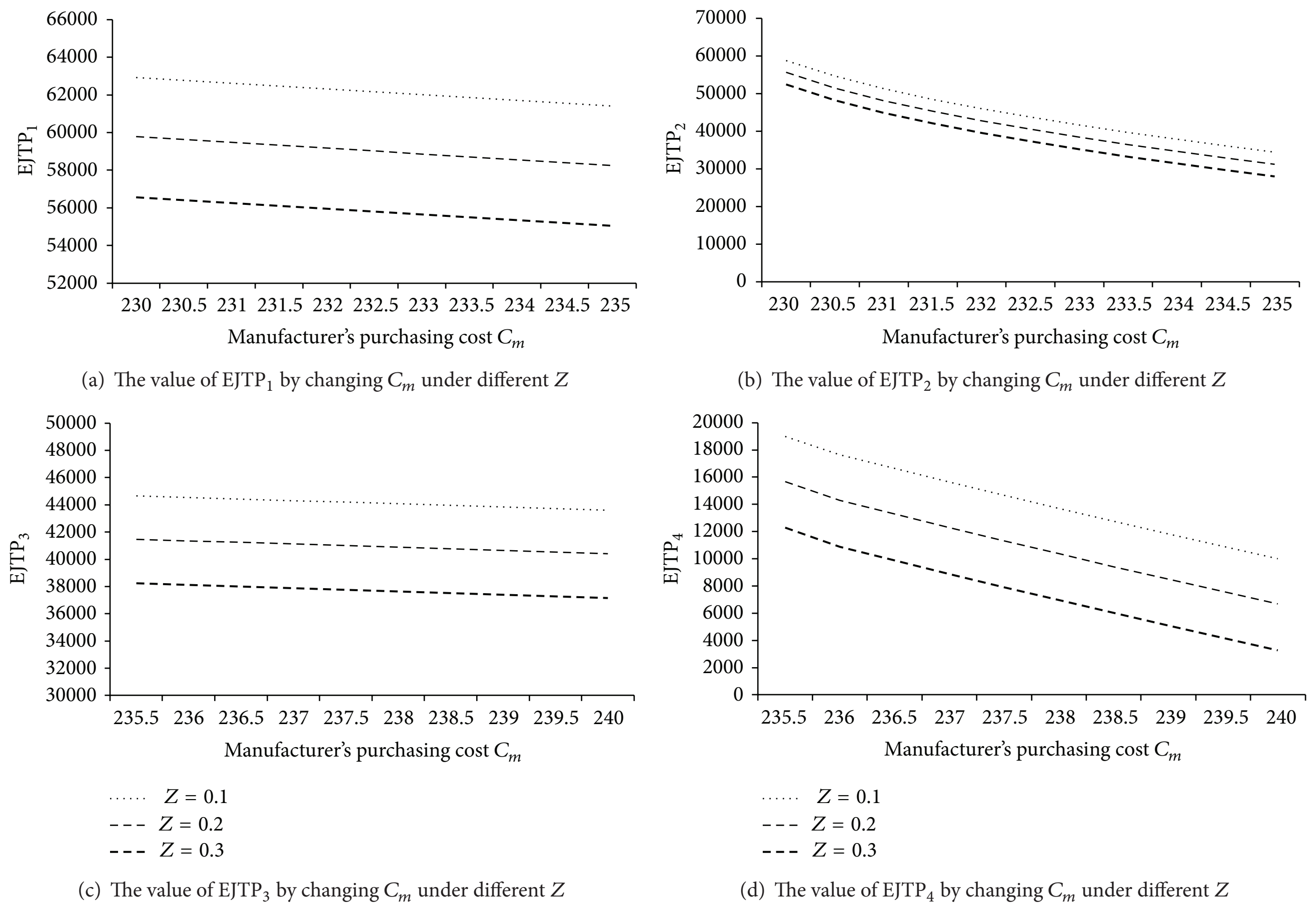
5.3. Observation (See Figures 5–12 and Tables 1–4). In Section 5.1, we observed the variation of quantity per delivery, numbers of delivery, and EJTP by changing manufacturer's

Table 2: The value of profit in different condition by changing C_r.

	$P = 1100$	$P = 1200$	$P = 1300$
C_r	240.0~245.0	240.0~245.0	240.0~245.0
n_1	2	2	2
Q_1	**102.29	***103.49	****104.54
EJTP_1	59935.40~61249.25	60082.94~61396.66	*60209.70~61523.33
C_r	245.5~250.0	245.5~250.0	245.5~250.0
n_2	1	1	1
Q_2	576.70~566.45	592.02~581.48	605.98~595.17
EJTP_2	33700.94~35468.36	34659.34~36408.84	35488.95~37224.54
C_r	240.0~245.0	240.0~245.0	240.0~245.0
n_3	4	4	4
Q_3	289.19~226.66	292.34~229.13	295.07~231.27
EJTP_3	34255.30~42211.53	34641.54~42514.20	34971.60~42772.87
C_r	245.5~250.0	245.5~250.0	245.5~250.0
n_4	2 (C_r = 245.5~246.5) 1 (C_r = 247.0~250.0)	2 (C_r = 245.5~246.0) 1 (C_r = 235.5~250.0)	2 (C_r = 245.5) 1 (C_r = 236.0~250.0)
Q_4	615.74~598.22 (C_r = 245.5~246.5) 775.30~663.75 (C_r = 247.0~250.0)	627.23~618.37 (C_r = 245.5~246.0) 813.38~681.35 (C_r = 246.5~250)	637.48 (C_r = 245.5) 850.10~697.38 (C_r = 246~250)
EJTP_4	20215.40~29779.79	21168.25~30881.82	21988.73~31837.60

*Optimal solution of EJTP_i.

,*,***We cannot observe the variation because of low increasing rate; in fact, Q_1 will decrease slightly when C_r increases.



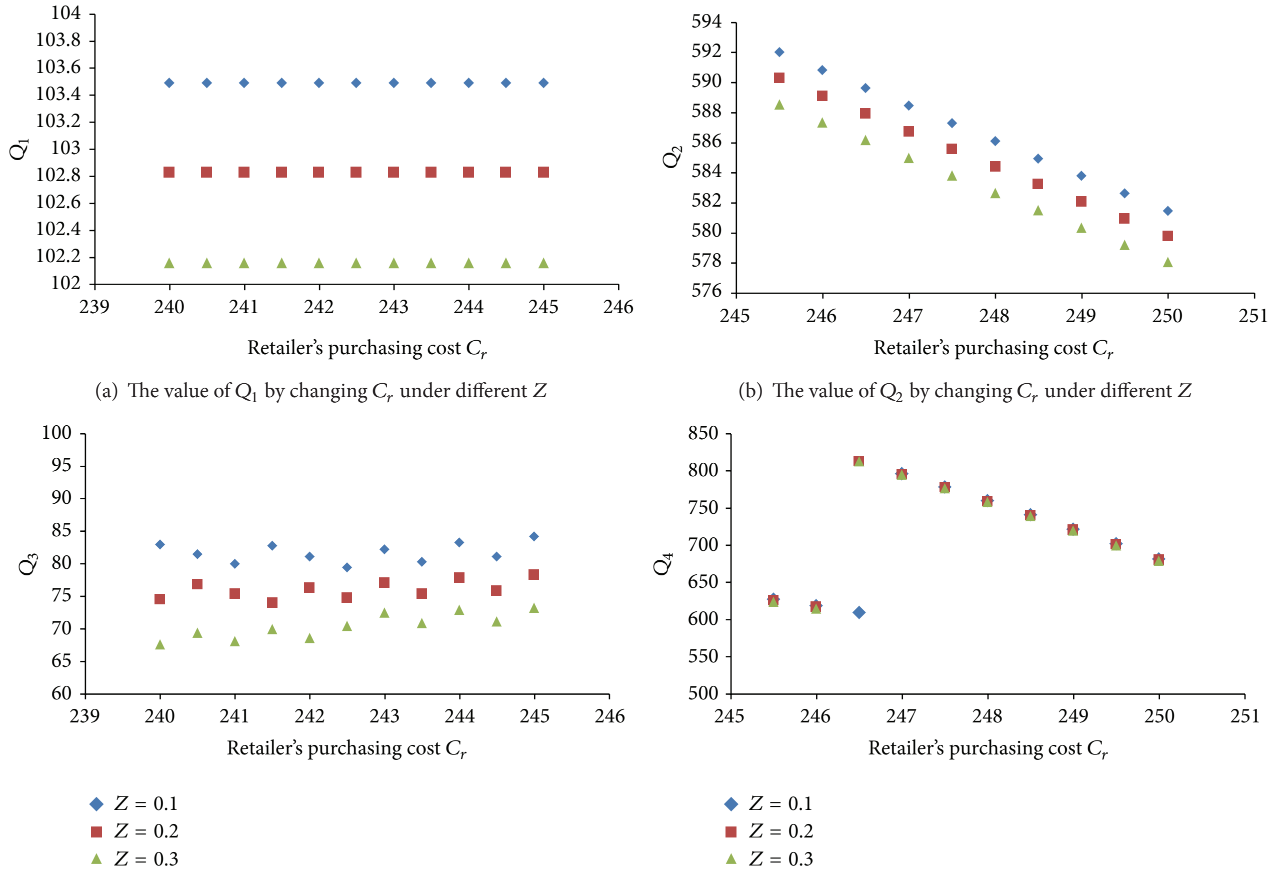
(a) The value of EJTP_1 by changing C_m under different Z

(b) The value of EJTP_2 by changing C_m under different Z

(c) The value of EJTP_3 by changing C_m under different Z

(d) The value of EJTP_4 by changing C_m under different Z

Figure 10: The value of profit by changing C_m in EJTP_i, for $i = 1, 2, 3, 4$.



(a) The value of Q_1 by changing C_r under different Z

(b) The value of Q_2 by changing C_r under different Z

(c) The value of Q_3 by changing C_r under different Z

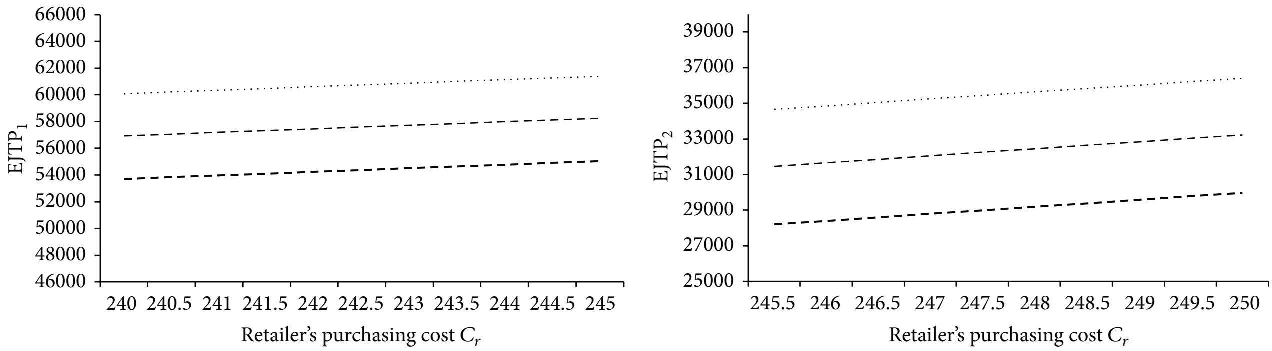
(d) The value of Q_4 by changing C_r under different Z

FIGURE 11: The value of delivery quantity by changing C_r in Q_i, for $i = 1, 2, 3, 4$.

TABLE 3: The value of profit in different condition by changing C_m.

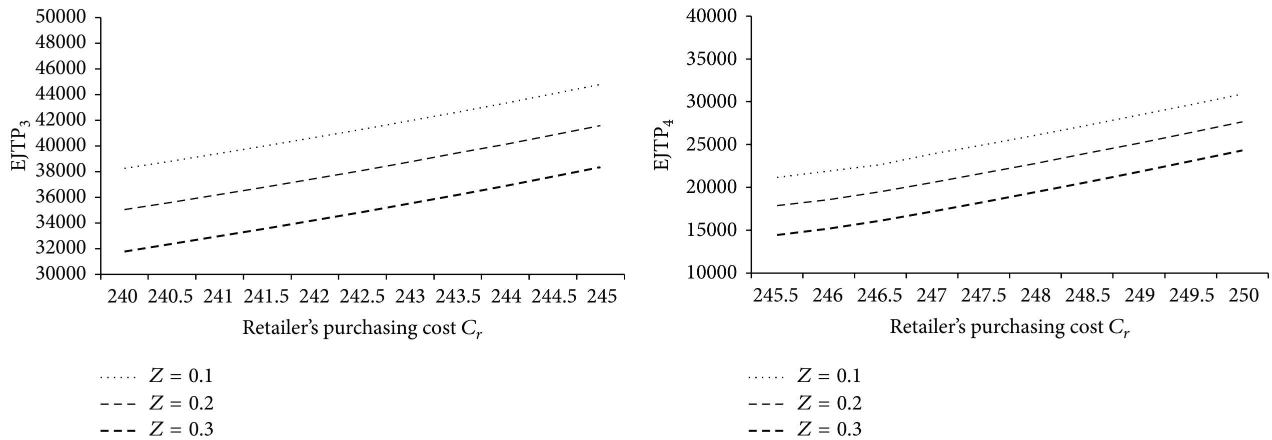
	$Z = 0.1$	$Z = 0.2$	$Z = 0.3$
C_m	230.0~235.0	230.0~235.0	230.0~235.0
n_1	2	2	2
Q_1	103.39~103.49	102.73~102.83	102.06~102.16
$EJTP_1$	*62930.18~61396.67	59783.53~58250.30	56571.47~55038.52
C_m	230.0~235.0	230.0~235.0	230.0~235.0
n_2	1	1	1
Q_2	194.04~593.21	193.48~591.50	192.90~589.73
$EJTP_2$	58779.07~34462.59	55616.48~31264.91	52388.27~28000.05
C_m	235.5~240	235.5~240	235.5~240
n_3	13	14	15
Q_3	84.04~82.95	78.23~77.23	73.12~72.29
$EJTP_3$	44660.66~43599.22	41473.30~40411.53	38226.15~37164.46
C_m	235.5~240	235.5~240	235.5~240
n_4	2 ($C_m = 235.5$) 1 ($C_m = 236$~240)	2 ($C_m = 235.5$) 1 ($C_m = 236$~240)	2 ($C_m = 235.5$) 1 ($C_m = 236$~240)
Q_4	653.70 ($C_m = 235.5$) 897.88~1022.77 ($C_m = 236$~240)	652.03 ($C_m = 235.5$) 897.51~1021.71 ($C_m = 236$~240)	650.29 ($C_m = 235.5$) 897.08~1020.58 ($C_m = 236$~240)
$EJTP_4$	19000.21~10007.45	15675.17~6673.83	12278.57~3629.20

*Optimal solution of $EJTP_i$.



(a) The value of EJTP$_1$ by changing C_r under different Z

(b) The value of EJTP$_2$ by changing C_r under different Z



(c) The value of EJTP$_3$ by changing C_r under different Z

(d) The value of EJTP$_4$ by changing C_r under different Z

FIGURE 12: The value of profit by changing C_r in EJTP$_i$, for $i = 1, 2, 3, 4$.

purchasing costs C_m or retailer's purchasing costs C_r under different production rate. Obviously, higher production rate will yield higher profits. All EJTP of each case decreases when C_m increases. In Section 5.1.1, the optimal profits occur in EJTP$_1(Q_1, n_1)$ under $P = 1300$; in Section 5.1.2, the optimal profits also occur in EJTP$_1(Q_1, n_1)$ under $P = 1300$.

In Section 5.2, the observations are shown under different defective rate consideration. Surely, higher defective rate leads manufacturer to pay more costs to rework defective items and deal with scrap. As C_m increases, all EJTP of each case decreases; nevertheless, increasing C_r brings decreasing EJTP contrarily. In Section 5.2.1, the optimal profits occur in EJTP$_1(Q_1, n_1)$ under $Z = 0.1$; in Section 5.2.2, the optimal profits also occur in EJTP$_1(Q_1, n_1)$ under $Z = 0.1$.

Because of the relationship between the price and payment period, the decision-makers can get different payment period by varying the price. When the supply chain is successfully integrated, this variation can lead to unnecessary costs reduction or enhance the performance.

6. Conclusions and Future Works

The main purpose of every firm is to maximize profits. There are two ways to enhance profits: one is to raise the products' selling price, and the other is to lower the relevant costs in supply chain. To raise the products' selling price, firms have to enhance products' quality and show uniqueness to convince customers. Alternatively, firms can provide proper strategies to reduce relevant costs such as purchasing costs, production costs, holding costs, and transportation costs.

Permissible delay in payments is a common commercial strategy in real business transactions, since the purpose of business strategies is to enhance the flexibility of capital. In other words, firms can obtain additional interests income from sales revenue during the payment period; yet, upstream firms simply grant loans to downstream firms without any interests. Thus, it is of great importance to decide the length of payment period in an SCM setting. There are many ways to balance the costs or revenue of each firm. From the reward perspective, providing discounts is a direct way to attract downstream firms in accepting shorter payment period. On the other hand, which is from the punishment perspective, downstream firms must pay extra costs if they wish to enjoy a longer payment period. Whether it is from the rewards or the punishments perspective, the purpose is always about shortening the payment period. In this paper, we have used different ways to determine the payment period. We set the relationship of purchasing costs and payment period as inverse proportion; that is, payment period is floating and higher purchasing costs will bring shorter payment period. From the results in Section 5, decision-makers should negotiate with their upstream or downstream firms to enhance supply chain performance. From the supplier and manufacturer's

Table 4: The value of profit in different condition by changing C_r.

	$Z = 0.1$	$Z = 0.2$	$Z = 0.3$
C_r	240.0~245.0	240.0~245.0	240.0~245.0
n_1	2	2	2
Q_1	*103.49	**102.83	***102.16
$EJTP_1$	*60082.94~61396.67	56923.45~58250.30	53698.28~55038.52
C_r	245.5~250	245.5~250	245.5~250
n_2	1	1	1
Q_2	592.02~581.48	590.31~579.80	588.54~578.07
$EJTP_2$	34658.46~36408.84	31462.16~33225.06	28198.72~29974.40
C_r	240.0~245	240.0~245	240.0~245
n_3	17 (C_r = 240~241) 16 (C_r = 241.5~242.5) 15 (C_r = 243~243.5) 14 (C_r = 244~244.5) 13 (C_r = 245)	19 (C_r = 240) 18 (C_r = 240.5~241.5) 17 (C_r = 242~242.5) 16 (C_r = 243~243.5) 15 (C_r = 244~244.5) 14 (C_r = 245)	21 (C_r = 240) 20 (C_r = 240.5~241) 19 (C_r = 241.5~242) 18 (C_r = 242.5) 17 (C_r = 243~243.5) 16 (C_r = 244~244.5) 15 (C_r = 245)
Q_3	82.95~79.97 (C_r = 240~241) 82.77~81.1 (C_r = 241.5~242.5) 82.21~80.32 (C_r = 243~243.5) 83.25~81.12 (C_r = 244~244.5) 84.17 (C_r = 245)	74.53 (C_r = 240) 76.84~73.99 (C_r = 240.5~241.5) 76.29~74.71 (C_r = 242~242.5) 77.09~75.33 (C_r = 243~243.5) 75.82 (C_r = 244~244.5) 78.34 (C_r = 245)	67.62 (C_r = 240) 69.40~68.14 (C_r = 240.5~241) 69.98~68.60 (C_r = 241.5~242) 70.48 (C_r = 242.5) 72.52~70.88 (C_r = 243~243.5) 72.96~71.13 (C_r = 244~244.5) 73.23 (C_r = 245)
$EJTP_3$	38237.07~44777.73	35037.87~41590.40	31785.46~38343.24
C_r	245.5~250	245.5~250	245.5~250
n_4	2 (C_r = 245.5) 1 (C_r = 246~250)	2 (C_r = 245.5~246) 1 (C_r = 246.5~250)	2 (C_r = 245.5~246) 1 (C_r = 246.5~250)
Q_4	627.23 (C_r = 245.5) 618.37~681.35 (C_r = 246~250)	625.65~616.76 (C_r = 245.5) 246.5~250 (C_r = 246~250)	624.00~615.08 (C_r = 245.5~246) 812.57~679.24 (C_r = 246~250)
$EJTP_4$	21168.36~30881.82	17850.43~27637.25	14461.21~24324.25

*Optimal solution of $EJTP_i$.
**,***,***We cannot observe the variation because of low increasing rate; in fact, Q_1 will decrease slightly when C_r increases.

viewpoint, EJTP moves up when the purchasing costs of manufacturer go down. However, there is a contrary result on the manufacturer and supplier's side. Higher purchasing costs of the supplier will lead to lower profits. Decision-makers should know where their firms are positioned in the supply chain and may thus make appropriate decisions.

Defective rate is also an important factor in the manufacturing process. The higher the probability of defective product occurrence, the higher the cost and more time will be spent by the manufacturer; these may include reordering the materials and reproducing, repairing, and declaring the scrap. Additionally, defective rate is one of the direct factors to affect the amount of storage. If retailers do not have enough stocks to satisfy customers' needs, customers may lose their patience and therefore choose other retailers. Surely, it is important to accurately grasp the situation of production lines.

From what has been discussed above, we developed a three-echelon inventory model to determine optimal joint total profits. Firstly, we have developed four inventory models in Section 3 according to different permissible delay payment period and lead time. Secondly, we computed the decision variables, economical delivery quantity, and the number of deliveries per production run from the manufacturer to the retailer. Finally, we observed and found the optimal profits by varying the manufacturer's purchasing costs or the supplier's purchasing costs.

Compared with Yang and Tseng's [14] article, although they considered the defective products to occur in the three echelons, we only assumed the defective products occur in the manufacturing process. In this paper, we also focused on the relationship between materials/finished product's sale price and the permissible delay period. We assumed that the relationship is inverse proportion and developed the function while Yang and Tseng's [14] simply focused on variable lead time and assumed that the permissible delay period is constant.

In the future, we can add more conditions or assumptions such as ignoring the backorder and variable lead time which were considered by Yang and Tseng's [14]. The assumptions can be added again to develop more practical inventory models. Besides, multiple sellers or multiple purchasers are not unusual situations in commerce. Moreover, the parameters in this paper are fixed while some of them (such as demand or defective rate) may be unfixed in practice by using fuzzy theory. The fuzzy variables can lead to better results. The issue regarding deteriorating products is worthy of deliberation in the inventory model since all products would face deterioration (i.e., rust or decay) sooner or later. We look forward to illustrating real-world numerical exam.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Joint Optimization Approach of Maintenance and Production Planning for a Multiple-Product Manufacturing System

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This paper deals with the problem of maintenance and production planning for randomly failing multiple-product manufacturing system. The latter consists of one machine which produces several types of products in order to satisfy random demands corresponding to every type of product. At any given time, the machine can only produce one type of product and then switches to another one. The purpose of this study is to establish sequentially an economical production plan and an optimal maintenance strategy, taking into account the influence of the production rate on the system's degradation. Analytical models are developed in order to find the production plan and the preventive maintenance strategy which minimizes sequentially the total production/inventory cost and then the total maintenance cost. Finally, a numerical example is presented to illustrate the usefulness of the proposed approach.

1. Introduction

Manufacturing companies must manage several functional capacities successfully, such as production, maintenance, quality, and marketing. One of the keys to success consists in treating all these services simultaneously. On the other hand, the customer satisfaction is one of the first objectives of a company. In fact, the nonsatisfaction of the customer on time is often due to a random demand or a sudden failure of production system. Therefore, it is necessary to develop maintenance policies relating to production, reducing the total production and maintenance cost. One of the first actions of decision-making hierarchy of a company is the development of an economical production plan and an optimal maintenance strategy.

It is necessary to find the best production plan and the best maintenance strategy required by the company to satisfy customers. This is a complex task because there are various uncertainties due to external and internal factors. External factors may be associated with the inability to precisely define the behaviour of the application during periods of production. Internal factors may be associated with the availability of hardware resources of the company. In this context, Filho [1] treated a stochastic scheduling problem in terms of production under the constraints of the inventory.

Establishing an optimal production planning and maintenance strategy has always been the greatest challenge for industrial companies. Moreover, during the last few decades, the integration of production and maintenance policies problem has received much research attention. In this context, Nodem et al. [2] developed a method to find the optimal production, replacement/repair and preventive maintenance policies for a degraded manufacturing system. Gharbi et al. [3] assumed that failure frequencies can be reduced through preventive maintenance, and developed joint production and preventive maintenance policies depending on produced part inventory levels. An analytical model and a numerical procedure which allow determining a joint optimal inventory control and an age based on preventive maintenance policy for a randomly failing production system was presented by Rezg et al. [4].

This work examined a problem of the optimal production planning formulation of a manufacturing system consisting of one machine producing several products in order to meet several random demands. This type of problem was studied by Kenne et al. [5]. They presented an analysis of production control and corrective maintenance problem in a multiple-machine, multiple-product manufacturing system. They obtained a near optimal control policy of the system through numerical techniques by controlling both production and repair rates. Feng et al. [6] developed a multiproduct manufacturing systems problem with sequence dependent setup times and finite buffers under seven scheduling policies. Sloan and Shanthikumar [7] presented a Markov decision process model that simultaneously determines maintenance and production schedules for a multiple-product, single-machine production system, accounting for the fact that equipment condition can affect the yield of different product types differently. Filho [8] developed a stochastic dynamic optimization model to solve a multiproduct, multiperiod production planning problem with constraints on decision variables and finite planning horizon.

Looking at the literature on integrated maintenance policies, we noticed that the influence of the production rate on the degradation system over a finite planning horizon was rarely addressed in depth. Recently, Zied et al. [9–11] took into account the influence of production plan on the equipment degradation, in the case of a system composed of single machine producing one type of product under randomly failing and satisfying a random demand over a finite horizon. In the same context, Kenne and Nkeungoue [12] proposed a model, where the failure rate of a machine depends on its age; hence, the corrective and preventive maintenance policies are machine-age dependent.

Motivated by the work in the Zied et al. [9–11], we treat the production and maintenance problem in another context that we consider a more complex and real industrial system, composed of one machine that produces several products during a finite horizon divided into subperiods. This study displays that it has a novelty and originality relative to this type of problem which considers the influence of several products on the degradation degree of the considered machine and consequently on the average number of failure as well as on the maintenance strategy.

This paper is organized as follows: Section 2 states the problem. Section 3 presents the notations. The production and maintenance models are developed, respectively, in Sections 4 and 5. A numerical example and sensitivity study are presented, respectively, in Sections 6 and 7. Finally, the conclusion is included in Section 8.

2. Statement of the Industrial Problem

This study treated an industrial case. The problem concerns a textile company located in North Africa specialized in clothing manufacturing. The company's production system consists of a conversion of three types of fiber into yarn, then fabric, and textiles. These are then fabricated into clothes or other artefacts. The production machine is called the loom, and it uses a jet of air or water to insert the weft. The loom ensures pattern diversity and faultless fabrics by a flexible and gentle material handling process. Fabrics can be in one plain color with or without a simple pattern or they can have decorative designs.

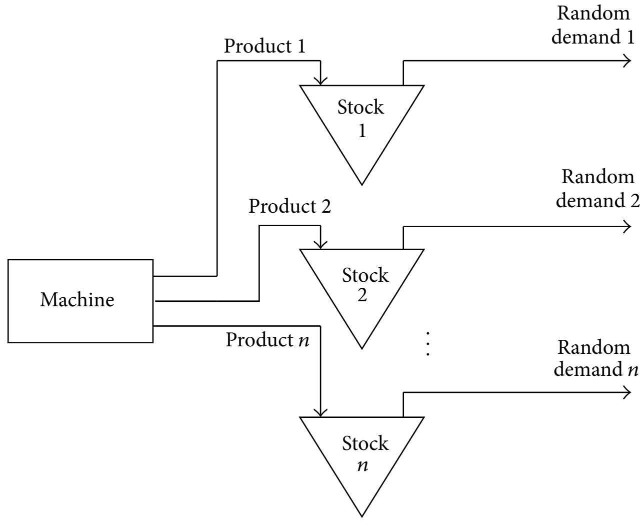

Figure 1: Problem description.

Based on the industrial example described, this study was conducted to deal with the problem of an optimal production and maintenance planning for a manufacturing system. The system is composed of a single machine which produces several products in order to meet corresponding several random demands. The problem is presented in (Figure 1).

The considered equipment is subject to random failures. The degradation of the equipment increases with time and varies according to the production rate. The machine is submitted to a preventive maintenance policy in order to reduce the occurrence of failures. In the literature, the influence of the production rate on the material degradation is rarely studied. In this study, this influence was taken into consideration in order to establish the optimal maintenance strategy.

The model developed in this study is based on the works of Zied et al. [9–11]. These studies seek to determine an economical production plan followed by an optimal maintenance policy but for the case of only one product.

Firstly, for a randomly given demand, an optimal production plan was established to minimize the average total storage and production costs while satisfying a service level. Secondly, using the obtained optimal production plan and considering its influence on the manufacturing system failure rate, an optimal maintenance schedule is established to minimize the total maintenance cost.

3. Notations

In this paper, we shall as far as possible use the notation summarized as follows:

$\mathrm{Cp}(i)$: unit production cost of product i,

$\mathrm{Cs}(i)$: holding cost of one unit of product i during Δt,

$\mathrm{St}(i)$: setup cost of product i,

Mc: corrective maintenance action cost,

Mp: preventive maintenance action cost,

H: total number of periods,

n: total number of products,

p: total number of subperiods during each period,

Δt: production period duration,

$U_{i\,\mathrm{nom}}$: nominal production quantity of product i during Δt,

θ_i: probabilistic index (related to customer satisfaction) of product i,

$d_i(k)$: demand of product i during period k,

$S_{i,(k\times p)-(p-j)}$: inventory level of product i at the end of subperiod j of period k,

$Z(U)$: the total expected cost of production and inventory over the finite horizon,

$\mathrm{Var}(d_i(k))$: the demand variance of product i at period k,

$\varphi(\theta_i)$: cumulative Gaussian distribution function,

$\varphi^{-1}(\theta_i)$: inverse distribution function,

$\Gamma(N)$: the total cost of maintenance,

$\lambda_{(k\times p)-(p-j)}(\cdot)$: failure rate function at subperiod j of the period k,

$\lambda_n(\cdot)$: nominal failure rate,

$\phi(\cdot)$: the average number of failures,

T: intervention period for preventive maintenance actions.

Decision Variables

$U_{i,j,k}$: production quantity of product i during subperiod j of period k,

$\delta_{(k\times p)-(p-j)}$: duration of subperiod j at period k,

$y_{i,j,k}$: a binary variable, which is equal to 1 if product i is produced in subperiod j of the period k, and 0 otherwise,

N: number of preventive maintenance actions during the finite horizon.

4. Production Policy

In this section, we developed an analytical model which minimizes the total cost of production and storage. The decision variables are the production quantities $U_{i,j,k}$, the binary variable $y_{i,j,k}$, and the duration of subperiods $\delta_{(k\times p)-(p-j)}$. Our objective consists in determining an economical production plan U^* ($U^* = U^*_{i,j,k}, y^*_{i,j,k}$ and $\delta^*_{(k\times p)-(p-j)}$ $\forall\{i = 1,\ldots,n\}, \{j = 1,\ldots,p\}, \{k = 1,\ldots,H\}$) for a finite time horizon $H\times\Delta t$. The production plan must satisfy random demands under the requirement of a given level of service, while minimizing the cost of production and storage. The production of each product i will take place at the beginning of subperiods, and delivery to the customer will be at the end of periods.

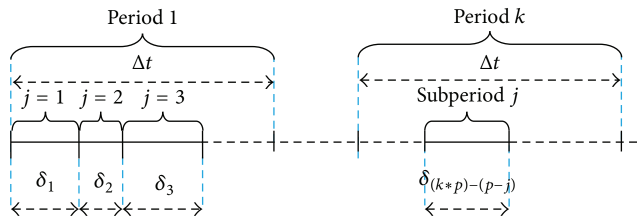

Figure 2: Production plan.

The state of the stock is determined at the end of each subperiod. Figure 2 shows an example of a production plan.

4.1. Stochastic Model of the Problem. To develop this section, the following assumptions are specifically made:

(i) holding and production costs of each product are known and constant;

(ii) only a single product can be produced in each subperiod;

(iii) as described in (Figure 2), we have divided the period k into p equal subperiods, with $p = n$ (the total number of products);

(iv) the standard deviation of demand $\sigma(d_i)$ and the average demand $\hat{d}_i$ for each product and each period k are known and constant.

The model has the following basic structure:

$$\text{To Minimize } [(\text{production cost}) + (\text{Holding cost})] \quad (1)$$

under the constraints below:

(i) the inventory balance equation,

(ii) the service level,

(iii) the admissibility of production plan,

(iv) the maximum production capacity.

Formally

(i) The Cost Functions. Consider

Production cost

$$= \sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n} y_{i,j,k} \times \left(\mathrm{St}(i) + \mathrm{Cp}(i) \times U_{i,j,k}\right).$$

Holding cost

$$= \sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n} \mathrm{Cs}(i) \times \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times S_{i,(k\times p)-(p-j)}. \quad (2)$$

(ii) The Inventory Balance Equation. The available stock at the end of each subperiod j of period k for each product i is

formulated in the form of flow balance constraints (inflow = outflow):

$$S_{i,(k\times p)-(p-j)} = S_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Int}\left[\frac{j}{p}\right] \times d_i(k) \quad (3)$$
$$\forall \{i = 1, \ldots, n\}, \{j = 1, \ldots, p\}, \{k = 1, \ldots, H\},$$

where $S_{i,0}$ is *the initial stock level of product i*.

This equation shows that the stock of product i at the end of each subperiod j of each period k $((k \times p) - (p - j))$ is determined by the state of the stock of product i at the end of the subperiod $(k \times p) - (p - j) - 1$.

(iii) The Admissibility of Production Plan and Service Level Constraints. The service level of product i is determined by the probability constraint on the stock level at the end of each period k:

$$\operatorname{Prob}\left(S_{i,(k\times p)} \geq 0\right) \geq \theta_i \quad \forall \{k = 1, \ldots, H\}, \{i = 1, \ldots, n\}. \quad (4)$$

We can transform the probabilistic constraint of stock level to a deterministic constraint.

Formally, the function becomes

$$\sum_{s=1}^{k} D(i,s) + \text{Stock\_min}(i,k) \leq \sum_{s=1}^{k}\sum_{j=1}^{p}\left(y_{(i,j,s)} \times U_{i,j,s}\right) + \text{stock\_init}(i, s = 0) \quad (5)$$
$$\forall \{i = 1, \ldots, n\},$$

where $D(i,s)$ is the estimated demand of product i during the period s, Stock_ min(i,k) is the minimum stock level of product i required at the end of period k, and stock_init$(i, s = 0)$ is the initial stock level of product i.

(iv) The Maximum Production Capacity. The production quantity of the machine for each product i, $\{i = 1, \ldots, n\}$, is limited and is presented as follows:

$$0 \leq U_{i,j,k} \leq \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times U_{i\,\text{nom}} \quad (6)$$
$$\forall \{i = 1, \ldots, n\}, \{j = 1, \ldots, p\}, \{k = 1, \ldots, H\}.$$

The term $\langle\langle \delta_{(k\times p)-(p-j)}/\Delta t \rangle\rangle$ allows taking into account the influence of duration of subperiods $\delta_{(k\times p)-(p-j)}$ on the maximum quantity of production. If $\delta_{(k\times p)-(p-j)}$ tends to 0, the maximum quantity of production tends also to 0, and if $\delta_{(k\times p)-(p-j)}$ tends to Δt, the maximum quantity of production tends to $U_{i\,\text{nom}}$ (with $U_{i\,\text{nom}}$ Nominal production quantity of product i during Δt).

However, the term $\langle\langle (\delta t_{(k\times p)-(p-j)}/\Delta t) \times U_{i\,\text{nom}} \rangle\rangle$ represents the maximum production quantity of product i during the subperiod j of period k.

4.2. Problem Formulation. We recall that, in this study, we assume that the horizon is divided into H equal periods and each period is divided into p subperiods with different durations. Figure 2 shows the distribution of the production plan for the finite horizon $H \times \Delta t$. Each product i is produced in a single subperiod j in each period k. The demand of each product i is satisfied at the end of each period k.

The mathematical formulation of the proposed problem is based on the extension of the model described by Zied et al. [11] for the one product case study.

Their problem is defined as follows:

$$\operatorname{Min}\left[\operatorname{Cs} \times E\left[S(H)^2\right] + \sum_{k=0}^{H-1}\left(\operatorname{Cs} \times E\left[S(k)^2\right] + \operatorname{Cp} \times E\left[u(k)^2\right]\right)\right], \quad (7)$$

where Cp is unit production cost and Cs is holding cost of a product unit during the period k.

Formally, our stochastic production problem is defined as follows:

$$\operatorname{Min}\left(\mathrm{Z}(U)\right)$$
$$U = U_{i,j,k} \quad \forall \{i = 1, \ldots, n\}, \{j = 1, \ldots, p\}, \{k = 1, \ldots, H\} \quad (8)$$

with

$$Z(U) = \sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}\left[y_{i,j,k} \times \left(\operatorname{St}(i) + \left(\operatorname{Cp}(i) \times E\left[\left(U_{i,j,k}\right)^2\right]\right)\right) + \left(\operatorname{Cs}(i) \times \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]\right)\right], \quad (9)$$

where $E[\cdot]$ is the mathematical expectation.

Under the following constraints:

$$S_{i,(k\times p)-(p-j)} = S_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Int}\left[\frac{j}{p}\right] \times d_i(k) \quad (10)$$
$$\forall \{i = 1, \ldots, n\}, \{j = 1, \ldots, p\}, \{k = 1, \ldots, H\},$$

$$\operatorname{Prob}\left(S_{i,(k\times p)} \geq 0\right) \geq \theta_i \quad \forall \{i = 1, \ldots, n\}, \{k = 1, \ldots, H\}, \quad (11)$$

$$0 \leq U_{i,j,k} \leq \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times U_{i\,\text{nom}} \quad (12)$$
$$\forall \{i = 1, \ldots, n\}, \{j = 1, \ldots, p\}, \{k = 1, \ldots, H\},$$

$$\sum_{j=1}^{p}\delta_{(k\times p)-(p-j)} = \Delta t \quad \forall \{k = 1, \ldots, H\}. \quad (13)$$

The first constraint stands for the inventory balance equation for each product i, $\{i = 1,\dots,n\}$ during each subperiod j, $\{j = 1,\dots,p\}$, of period k, $\{k = 1,\dots,H\}$. Equation (11) refers to the satisfaction level of demand of product i in each period k. Constraint (12) defines the upper production quantity of the machine for each product i. The aim of (13) is to divide each period k into p different subperiods.

The constraints below should also be taken into account:

$$\sum_{i=1}^{n} y_{i,j,k} = 1 \quad \forall\{j = 1,\dots,p\} \text{ for } \{k = 1,\dots,H\},$$
$$\sum_{j=1}^{p} y_{i,j,k} = 1 \quad \forall\{i = 1,\dots,n\} \text{ for } \{k = 1,\dots,H\}, \tag{14}$$

$$y_{i,j,k} \in \{0,1\} \quad \forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\}. \tag{15}$$

Equation (14) indicates that only one type of product will be produced in subperiod j of period k. Constraint (15) states that $y_{i,j,k}$ is a binary variable. We note that $y_{i,j,k}$ is equal to 1 if product i is produced in subperiod j of the period k, and 0 otherwise.

For each subperiod j of period k, the equation of the stock status is determined by the first constraint. This equation remains random because of the uncertainty of fluctuating demand. Therefore, the variables of production and storage are stochastic. Their statistics depend on a probabilistic distribution function of demand. It is, therefore, necessary to use constraint (11) for decision variables. These constraints can help us to analyse the various production scenarios to improve the performance of the production system.

4.3. The Deterministic Production Model. We admit that a function $f_{(i,j,k)}$ $\forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\}$, represents the cost of storage and production which is relative to the proposed plan and $E[\cdot]$ represents the value of the mathematical expectation. The quantity stocked of product i at the end of the subperiod j of period k is stood for by $S_{i,(k\times p)-(p-j)}$. The production quantity required to satisfy the demand of product i at the end of period k is $U_{i,j,k}$, where j represents the subperiod during which the product i is produced.

Thus, the problem formulation can be presented as follows:

$$U^{*} = \text{Min}\left[E\left[f_{(i,j,k)}\left(U_{i,j,k}, S_{i,(k\times p)-(p-j)}\right)\right]\right]. \tag{16}$$

The purpose, then, is to determine the decision variables ($U_{i,j,k}$, $y_{i,j,k}$ and $\delta_{(k\times p)-(p-j)}$) required to satisfy economically the various demands under the constraints seen in the previous subsection.

The resolution of the stochastic problem under these assumptions is generally difficult. Thus, its transformation into a deterministic problem facilitates its resolution.

(i) Inventory Balance Equation. The stochastic inventory balance equation is

$$\begin{aligned} S_{i,(k\times p)-(p-j)} &= S_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) \\ &\quad - \text{Int}\left[\frac{j}{p}\right] \times d_i(k), \\ &\forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\} \end{aligned} \tag{17}$$

with $S_{i,0}$ being the initial stock level of product i.

We suppose that the means and variance of demand are known and constant for each product i in each period k.

Therefore,

$$\begin{gathered} E\left[d_i(k)\right] = \hat{d}_i(k), \qquad \text{Var}\left[d_i(k)\right] = \sigma^2\left(d_i(k)\right) \\ \forall\{i = 1,\dots,n\},\{k = 1,\dots,H\}. \end{gathered} \tag{18}$$

The inventory equation $S_{i,(k\times p)-(p-j)}$ is statistically described by its means:

$$\begin{gathered} E\left[S_{i,(k\times p)-(p-j)}\right] = \hat{S}_{i,(k\times p)-(p-j)} \\ \forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\}. \end{gathered} \tag{19}$$

We note that

$$E\left[U_{i,j,k}\right] = \hat{U}_{i,j,k} = U_{i,j,k} \tag{20}$$

because $U_{i,j,k}$ is constant for each interval $\delta_{(k\times p)-(p-j)}$.

And

$$\begin{gathered} \text{Var}\left(U_{i,j,k}\right) = 0 \\ \forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\}. \end{gathered} \tag{21}$$

Then, the balance equation (10) can be converted into an equivalent inventory balance equation:

$$\begin{aligned} \hat{S}_{i,(k\times p)-(p-j)} &= \hat{S}_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) \\ &\quad - \text{Int}\left[\frac{j}{p}\right] \times \hat{d}_i(k) \\ &\forall\{i = 1,\dots,n\},\{j = 1,\dots,p\},\{k = 1,\dots,H\}, \end{aligned} \tag{22}$$

with $\hat{S}_{i,0}$ being *the average initial stock level of product*.

(ii) Service Level Constraint. The second step is to convert the service level constraint into a deterministic equivalent constraint by specifying certain minimum cumulative production quantities that depend on the service level requirements.

Lemma 1. *Consider the following:*

$$\begin{aligned} &\sum_{j=1}^{p}\left(y_{i,j,k} \times U_{i,j,k}\right) \geq \text{Var}\left(d_i(k)\right) \times \varphi^{-1}\left(\theta_i\right) + \hat{d}_i(k) - S_{i,(k-1)\times p} \\ &\qquad \forall\{i = 1,\dots,n\},\{k = 1,\dots,H\}. \end{aligned} \tag{23}$$

Proof. We know that

$$\operatorname{Prob}\left(S_{i,(k\times p)} \geq 0\right) \geq \theta_i \quad \forall \{i=1,\dots,n\},\{k=1,\dots,H\}, \tag{24}$$

$$\begin{aligned}
S_{i,(k\times p)} &= S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - d_i(k) \\
&\qquad \forall \{i=1,\dots,n\},\{k=1,\dots,H\} \\
\Longrightarrow \operatorname{Prob}&\left(S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - d_i(k) \geq 0\right) \geq \theta_i \\
&\qquad \forall \{i=1,\dots,n\},\{k=1,\dots,H\} \\
\Longrightarrow \operatorname{Prob}&\left(S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) \geq d_i(k)\right) \geq \theta_i \\
&\qquad \forall \{i=1,\dots,n\},\{k=1,\dots,H\} \\
\Longrightarrow \operatorname{Prob}&\left(S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - \widehat{d}_i(k) \geq d_i(k) - \widehat{d}_i(k)\right) \geq \theta_i \\
&\qquad \forall \{i=1,\dots,n\},\{k=1,\dots,H\} \\
\Longrightarrow \operatorname{Prob}&\left(\frac{S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - \widehat{d}_i(k)}{\operatorname{Var}\left(d_i(k)\right)} \geq \frac{d_i(k) - \widehat{d}_i(k)}{\operatorname{Var}\left(d_i(k)\right)}\right) \geq \theta_i \\
&\qquad \forall \{i=1,\dots,n\},\{k=1,\dots,H\}.
\end{aligned} \tag{25}$$

Noting that

$$X = \frac{d_i(k) - \widehat{d}_i(k)}{\operatorname{Var}\left(d_i(k)\right)}, \tag{26}$$

X is a Gaussian random variable for demand $d_i(k)$.

Hence,

$$\operatorname{Prob}\left(\frac{S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - \widehat{d}_i(k)}{\operatorname{Var}\left(d_i(k)\right)} \geq X\right) \geq \theta_i \quad \forall \{i=1,\dots,n\},\{k=1,\dots,H\}. \tag{27}$$

We recall that θ_i represents the probabilistic index (related to customer satisfaction) of product i and $\operatorname{Var}(d_i(k))$ represents the demand variance of product i at period k.

The distribution function is invertible because it is an increasing and differentiable function.

Hence,

$$S_{i,(k-1)\times p} + \sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) - \widehat{d}_i(k) \geq \operatorname{Var}\left(d_i(k)\right)\times\varphi^{-1}\left(\theta_i\right) \quad \forall \{i=1,\dots,n\},\{k=1,\dots,H\}. \tag{28}$$

Therefore,

$$\sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) \geq \operatorname{Var}\left(d_i(k)\right)\times\varphi^{-1}\left(\theta_i\right) + \widehat{d}_i(k) - S_{i,(k-1)\times p} \quad \forall \{i=1,\dots,n\},\{k=1,\dots,H\}. \tag{29}$$

□

(iii) The Expression of the Total Production and Storage Cost. In this step, we proceed to a simplification of the expected cost of production and storage.

The expression of the total cost of production is presented as follows.

Lemma 2. *Consider the following:*

$$\begin{aligned}
Z(U) = \sum_{k=1}^{H}\sum_{j=1}^{P}\sum_{i=1}^{n}\Bigg\{ & y_{i,j,k}\times\left(\operatorname{St}(i) + \left(\operatorname{Cp}(i)\times U^{2}{}_{i,j,k}\right)\right) \\
& + \operatorname{Cs}(i)\times\frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \\
& \times\Bigg[\sigma^{2}\left(S_{i,0}\right) \\
& + \left(\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Int}\left(\frac{l}{p}\right)\times\sigma^{2}\left(d_i(Q)\right)\right) \\
& + \left(\sum_{l=1}^{j}\operatorname{Int}\left(\frac{l}{p}\right)\times\sigma^{2}\left(d_i(k)\right)\right) \\
& + \left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^{2}\Bigg]\Bigg\}.
\end{aligned} \tag{30}$$

Proof. See Appendix A. □

(iv) In Summary. The deterministic optimization problem becomes as follows.

(a) The Objective Function. Consider

$$U^* = \text{Min} \sum_{k=1}^{H}\sum_{j=1}^{P}\sum_{i=1}^{n}\Bigg[y_{i,j,k} \times \left(\text{St}(i) + \left(\text{Cp}(i) \times U^2{}_{i,j,k}\right)\right) + \text{Cs}(i) \times \frac{\delta_{(k\times p)-(p-j)}}{\Delta t} \times \Bigg[\sigma^2(S_{i,0}) + \left(\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\text{Int}\left(\frac{l}{p}\right) \times \sigma^2(d_i(Q))\right) + \left(\sum_{l=1}^{j}\text{Int}\left(\frac{l}{p}\right)\times\sigma^2(d_i(k))\right) + \left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\Bigg]\Bigg]. \quad (31)$$

(b) The Constraints. Consider

$$\widehat{S}_{i,(k\times p)-(p-j)} = \widehat{S}_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k}\times U_{i,j,k}\right) - \text{Int}\left[\frac{j}{p}\right]\times \widehat{d}_i(k)$$
$$\forall\{i=1,\dots,n\},\{j=1,\dots,p\},\{k=1,\dots,H\},$$
$$\sum_{j=1}^{p}\left(y_{i,j,k}\times U_{i,j,k}\right) \geq \text{Var}(d_i(k))\times\varphi^{-1}(\theta_i) + \widehat{d}_i(k) - S_{i,(k-1)\times p}$$
$$\forall\{i=1,\dots,n\},\{k=1,\dots,H\},$$
$$0 \leq U_{i,j,k} \leq \frac{\delta_{(k\times p)-(p-j)}}{\Delta t}\times U_{i\,\text{nom}}$$
$$\forall\{i=1,\dots,n\},\{j=1,\dots,p\},\{k=1,\dots,H\},$$
$$\sum_{j=1}^{p}\delta_{(k\times p)-(p-j)} = \Delta t, \quad \forall\{k=1,\dots,H\}. \quad (32)$$

5. Maintenance Strategy

5.1. Description of the Maintenance Strategy. The maintenance strategy adopted in this study is known as preventive maintenance with minimal repair. The actions of preventive maintenance are practiced in the period $q\times T$ $(q = 1,2,\dots)$. The replacement rule for this policy is to replace the system with another new system (as good as new) at each period $q\times T$. At each failure between preventive maintenance actions, only one minimal repair is implemented. If we note Mc, the cost of corrective maintenance actions, and Mp, the cost of preventive maintenance actions and degradation of the machine is linear, the total cost of maintenance is expressed as follows:

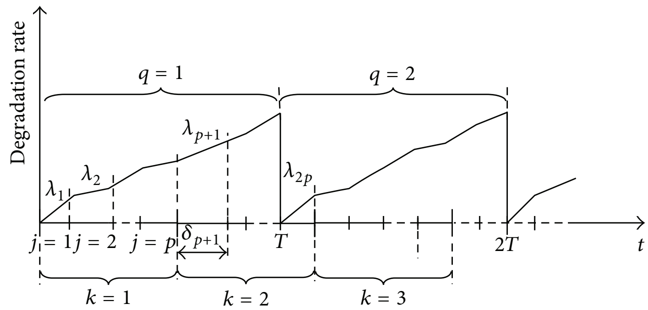

FIGURE 3: Degradation rate.

$$\Gamma(N) = \text{Mc}\times\phi_{(N,U)} + \text{Mp}\times N. \quad (33)$$

To develop the analytical model, it was assumed that

(i) durations of maintenance actions are negligible;

(ii) Mp and Mc costs incurred by the preventive and corrective maintenance actions are known and constant, with Mc $\gg$ Mp;

(iii) preventive maintenance actions are always performed at the end of the subperiods of production.

The aim of this maintenance strategy is to find the optimal number of preventive maintenance actions N^* $(N = 1,2,\dots)$ minimizing the total cost of maintenance over a given horizon $H\times\Delta t$. The existence of an optimal number of partitions N^* and, therefore, the optimal preventive maintenance period T^* is proven in the literature. It has been proven that T^* exists if the failure rate is increasing [13].

Before determining the analytical model minimizing the total cost of maintenance, we need first to develop the expression of the failure rate $\lambda_{(k\times p)-(p-j)}(t)$ and then the average number of failures expression $\phi_{(U,N)}$, during the finite horizon $H\times\Delta t$.

5.2. Expression of Failure Rate. Recall that the key of this study is the influence of the variation of the production rates on the failure rate.

Figure 3 represents the general description of the evolution of the failure rate, which depends on both the production rate and the failure rate of the previous period.

As presented in Figure 3, the failure rate is reset after each $q\times T$, with $q = 1,\dots,N+1$.

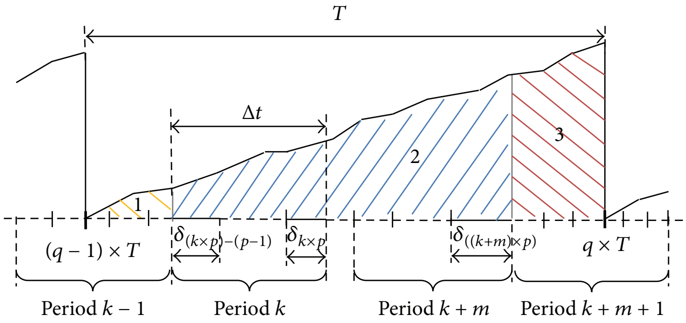

FIGURE 4: The evolution of the failure rate during the interval $[(q-1)\times T, q\times T]$.

Thus, the expression of the failure rate depending on time and production rate can be written as follows:

$$\begin{aligned}\lambda_{(k\times p)-(p-j)}(t) &= \Bigg[\left(\lambda_{(k\times p)-(p-j)-1}\left(\delta_{(k\times p)-(p-j)-1}\right)\right) \\ &\quad \times \left(1-\operatorname{In}\left[\frac{(k\times p)-(p-j+1)}{q\times T}\right]\right) \\ &\quad + \sum_{i=1}^{n}\frac{U_{i,j,k}}{\delta_{(k\times p)-(p-j)}}\times\frac{1}{U_{i\,\mathrm{nom}}/\Delta t}\times\lambda_n(t)\Bigg] \\ &\forall t\in\left[0,\delta_{(k\times p)-(p-j)}\right],\quad \forall\{k=1,\ldots,H\},\{j=1,\ldots,p\}.\end{aligned} \tag{34}$$

The term $\langle\langle U_{i,j,k}/\delta_{(k\times p)-(p-j)}\rangle\rangle$ represents the production rate of product i during subperiod j of period k.

The term $\langle\langle U_{i\,\mathrm{nom}}/\Delta t\rangle\rangle$ represents the nominal production rate of product i during Δt.

Therefore,

$$\begin{aligned}\lambda_{(k\times p)-(p-j)}(t) &= \Bigg[\left(\lambda_{(k\times p)-(p-j)-1}\left(\delta_{(k\times p)-(p-j)-1}\right)\right) \\ &\quad \times \left(1-\operatorname{In}\left[\frac{(k\times p)-(p-j+1)}{q\times T}\right]\right) \\ &\quad + \sum_{i=1}^{n}\frac{U_{i,j,k}\times\Delta t}{U_{i\,\mathrm{nom}}\times\delta_{(k\times p)-(p-j)}}\times\lambda_n(t)\Bigg] \\ &\forall t\in\left[0,\delta_{(k\times p)-(p-j)}\right],\quad \forall\{k=1,\ldots,H\},\{j=1,\ldots,p\}.\end{aligned} \tag{35}$$

The aim of the expression $(1-\operatorname{In}[((k\times p)-(p-j))/(q\times T)])$ is to reset the failure rate after each $q\times T$ with $q=1,\ldots,N+1$.

Note that

$$q=\operatorname{In}\left[\frac{(k\times p)-(p-j+2)}{T}\right]+1, \tag{36}$$

where $\operatorname{In}[x]$ is the integer part of number x.

Lemma 3. *Consider the following:*

$$\begin{aligned}\lambda_{(k\times p)-(p-j)}(t) &= \Bigg[\Bigg(\lambda_0+\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\sum_{i=1}^{n}\frac{U_{i,l,Q}\times\Delta t}{U_{i\,\max}\times\delta_{(Q\times p)-(p-l)}} \\ &\quad \times\lambda_n\left(\delta_{(Q\times p)-(p-l)}\right) \\ &\quad + \sum_{l=1}^{j-1}\sum_{i=1}^{n}\frac{U_{i,l,k}\times\Delta t}{U_{i\,\max}\times\delta_{(k\times p)-(p-l)}} \\ &\quad \times\lambda_n\left(\delta_{(k\times p)-(p-l)}\right)\Bigg) \\ &\quad \times\left(1-\operatorname{In}\left[\frac{(k\times p)-(p-j+1)}{q\times T}\right]\right) \\ &\quad + \sum_{i=1}^{n}\frac{U_{i,j,k}\times\Delta t}{U_{i\,\max}\times\delta_{(k\times p)-(p-j)}}\times\lambda_n(t)\Bigg] \\ &t\in\left[0,\delta_{(k\times p)-(p-j)}\right],\quad \forall\{k=1,\ldots,H\},\{j=1,\ldots,p\}.\end{aligned} \tag{37}$$

Proof. See Appendix B. □

5.3. Expression of the Average Number of Failures. In order to reduce the complexity of the generation of the optimal number of preventive maintenance, we assume that interventions are made at the end of subperiods.

Hence, the function of the period of intervention is presented as follows:

$$T=\operatorname{Round}\left[\frac{H\times p}{N}\right], \tag{38}$$

where $\operatorname{Round}[x]$ is a round number of x.

To determine the average number of failures expression $\phi_{(U,N)}$ during the finite horizon $H\times\Delta t$, we will focus on the calculation of the average number of failures during the interval $[(q-1)\times T, q\times T]$, which we designate $\phi^{T}_{(U,N)}$. Hence, we have to calculate the three surfaces {1}, {2}, and {3} mentioned in Figure 4.

Therefore, the average number of failures expression during the interval $[(q-1)\times T, q\times T]$ is presented as follows:

$$\begin{aligned}\phi^{T}_{(U,N)} = \Bigg[& \sum_{j=((q-1)\times T+1)-(\mathrm{In}[((q-1)\times T)/\Delta t]\times p)}^{p} \int_{0}^{\delta_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \\ & + \sum_{k=\mathrm{In}_{\mathrm{sup}}[((q-1)\times T+1)/\Delta t]+1}^{\mathrm{In}[(q\times T)/\Delta t]} \sum_{j=1}^{p} \int_{0}^{\delta_{(k\times p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)\,dt \\ & + \sum_{j=1}^{q\times T-\mathrm{In}[(q\times T)/\Delta t]\times p} \int_{0}^{\delta_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \Bigg],\end{aligned} \tag{39}$$

where $\mathrm{In}_{\mathrm{sup}}[x]$ is the superior integer part of number x.

Thus, the average number of failures expression $\phi_{(U,N)}$ during the finite horizon $H\times\Delta t$ is defined by

$$\phi_{(U,N)} = \sum_{q=1}^{N+1} \phi^{T}_{(U,N)}. \tag{40}$$

Therefore, we have the following lemma.

Lemma 4. *Consider the following:*

$$\begin{aligned}\phi_{(U,N)} = \sum_{q=1}^{N+1} \Bigg[& \sum_{j=((q-1)\times T+1)-(\mathrm{In}[((q-1)\times T)/\Delta t]\times p)}^{p} \int_{0}^{\delta_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \\ & + \sum_{k=\mathrm{In}_{\mathrm{sup}}[((q-1)\times T+1)/\Delta t]+1}^{\mathrm{In}[(q\times T)/\Delta t]} \sum_{j=1}^{p} \int_{0}^{\delta_{(k\times p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)\,dt \\ & + \sum_{j=1}^{q\times T-\mathrm{In}[(q\times T)/\Delta t]\times p} \int_{0}^{\delta_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \Bigg].\end{aligned} \tag{41}$$

Note that $N = 1, 2, \ldots$.

5.4. Expression of the Total Cost of Maintenance. We recall that the initial expression of the total cost of maintenance presented in (33) is

$$\Gamma(N) = \mathrm{Mc}\times\phi_{(U,N)} + \mathrm{Mp}\times N. \tag{42}$$

Using the average number of failures $\phi_{(U,N)}$ established in Lemma 4, we can deduce that the analytical expression of the total maintenance cost is expressed as follows:

$$\begin{aligned}\Gamma(N) = \Bigg[\mathrm{Mc}\times \sum_{q=1}^{N+1} \Bigg[& \sum_{j=((q-1)\times T+1)-(\mathrm{In}[((q-1)\times T)/\Delta t]\times p)}^{p} \int_{0}^{\delta_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[((q-1)\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \\ & + \sum_{k=\mathrm{In}_{\mathrm{sup}}[((q-1)\times T+1)/\Delta t]+1}^{\mathrm{In}[(q\times T)/\Delta t]} \sum_{j=1}^{p} \int_{0}^{\delta_{(k\times p)-(p-j)}} \lambda_{(k\times p)-(p-j)}(t)\,dt \\ & + \sum_{j=1}^{q\times T-\mathrm{In}[(q\times T)/\Delta t]\times p} \int_{0}^{\delta_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}} \lambda_{(\mathrm{In}[(q\times T)/\Delta t]+1)\times p-(p-j)}(t)\,dt \Bigg] + \mathrm{Mp}\times N\Bigg].\end{aligned} \tag{43}$$

The goal is to find the optimal number of preventive maintenance actions N^* that minimizes the total cost of maintenance $\Gamma(N)$. Using this decision variable, we can deduce the optimal period of intervention T^*, knowing that $T^* = \text{Round}[(H \times p)/N^*]$.

5.5. Existence of an Optimal Solution. The following equation determines analytically the optimal solution:

$$\frac{\partial \Gamma(N)}{\partial N} = 0. \quad (44)$$

Since it is difficult to solve analytically the expression of maintenance cost, we use numerical procedure.

We start by proving the existence of a local minimum.

We have the following.

Limits at the terminals of $\Gamma(N)$ are

$$\begin{aligned} \lim_{N\to 1} \Gamma(U,N) &= \lim_{N\to 1} \Bigg(\underbrace{M_c \times \phi(U,N)}_{\to\text{constant}} + \underbrace{M_p \times N}_{\to\text{constant}} \Bigg) \\ &= M_c \times \phi(U,1) + M_p, \\ \lim_{N\to +\infty} \Gamma(U,N) &= \lim_{N\to +\infty} \Bigg(\underbrace{M_c \times \phi(U,N)}_{\to 0} + \underbrace{M_p \times N}_{\to +\infty} \Bigg) \\ &= +\infty. \end{aligned} \quad (45)$$

Note that $\phi(U,N)$ is the average number of failures. Mc and Mp represent, respectively, the corrective and the preventive maintenance costs.

Moreover,

$$\begin{aligned} &\Gamma(U,N+1) - \Gamma(U,N) \geq 0 \\ &\Longrightarrow \left[M_c \times \phi(U,(N+1)) + M_p \times (N+1)\right] \\ &\quad - \left[M_c \times \phi(U,N) + M_p \times N\right] \geq 0 \\ &\Longrightarrow M_c \times (\phi(U,(N+1)) - \phi(U,N)) + M_p \geq 0 \\ &\Longrightarrow \phi(U,(N+1)) - \phi(U,N) \leq \frac{M_p}{M_c}. \end{aligned} \quad (46)$$

In addition,

$$\begin{aligned} &\Gamma(U,N) - \Gamma(U,N-1) \leq 0 \\ &\Longrightarrow \left[M_c \times \phi(U,N) + M_p \times (N)\right] \\ &\quad - \left[M_c \times \phi(U,(N-1)) + M_p \times (N-1)\right] \leq 0 \\ &\Longrightarrow M_c \times (\phi(U,N) - \phi(U,(N-1))) - M_p \leq 0 \\ &\Longrightarrow \phi(U,N) - \phi(U,(N-1)) \geq \frac{M_p}{M_c}. \end{aligned} \quad (47)$$

In summary, there is an optimal number of partition N^*, which is unique and satisfies the previous relations (46) and (47). The following lemma ensures the existence of a local minimum.

Lemma 5. *Consider the following:*

$$\exists N^* si \quad \xi_N \leq \frac{M_p}{M_c} \leq \xi_{N-1}, \quad (48)$$

with

$$\xi_N = \phi(U,N) - \phi(U,(N+1)). \quad (49)$$

Therefore, there exists an optimal number of partition N^*, which satisfies the following expressions:

$$N^* \exists si \begin{cases} \phi(U,(N+1)) - \phi(U,N) \geq 0 \\ \phi(U,N) - \phi(U,(N-1)) \leq 0 \\ \lim_{N\to 1} \Gamma(U,N) = \mathit{Constant} \\ \lim_{N\to +\infty} \Gamma(U,N) = +\infty. \end{cases} \quad (50)$$

The resolution of this maintenance policy, using a numerical procedure, is performed by incrementing the number of maintenance intervals until an N^*, satisfying the two first relations in Lemma 5 and minimizing the total cost of maintenance $\Gamma(N)$ described by (43).

6. Numerical Example

From the industrial example presented in Section 2, we have considered a system producing 3 types of fiber in order to meet three random demands according to every type of product. Using the analytical models developed in previous sections, we start by establishing the optimal production plan and then we determine the optimal maintenance strategy expressed as optimal number of preventive maintenance minimizing the total cost of maintenance over a finite planning horizon: $H = 8$ trimesters (two years). We note that the optimal maintenance strategy is obtained while considering of the influence of the production plan on the system degradation. We supposed that the standard deviation of demand of product i is the same for all periods. The data required to run this model are given in sequence.

6.1. Numerical Example

(i) The Data Relating to Production. The mean demands (in bobbins) as shown in Table 1:

$$\begin{aligned} \widehat{d}_1 &= 200, \quad \sigma(d_1) = 1.5, \\ \widehat{d}_2 &= 110, \quad \sigma(d_2) = 0.9, \\ \widehat{d}_3 &= 320, \quad \sigma(d_3) = 1.2. \end{aligned} \quad (51)$$

The other data are presented as shown in Table 2.

(ii) The Data Relating to System Reliability. System reliability and costs related to maintenance actions are defined by the following data:

(1) the law of failure characterizing the nominal conditions is Weibull. It is defined by

Table 1

	Demands							
	Trim. 1	Trim. 2	Trim. 3	Trim. 4	Trim. 5	Trim. 6	Trim. 7	Trim. 8
Product 1	201	199	198	199	201	202	200	199
Product 2	111	119	108	111	112	110	110	119
Product 3	321	322	323	319	321	317	320	319

Table 2

	Initial stock level $S_{i,0}$ (up)	Nominal production quantities $U_{i\,\text{nom}}$ (up)	Unit production costs Cp(i) (um)	Unit holding costs Cs(i) (um/ut)	Satisfaction rates θ_i (%)
Product 1	110	750	13	3	87
Product 2	85	530	17	5	95
Product 3	145	1150	9	2	90

Table 3: The optimal production plan.

	Trimester 1			Trimester 2			Trimester 3			Trimester 4		
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}
	0.85	0.71	1.44	1.19	1.20	0.61	0.81	1.18	1.01	0.43	0.74	1.83
Product 1	0	169	0	388	0	0	0	321	0	0	151	0
Product 2	150	0	0	0	185	0	134	0	0	0	0	312
Product 3	0	0	507	0	0	230	0	0	387	158	0	0
	Trimester 5			Trimester 6			Trimester 7			Trimester 8		
	δ_{13}	δ_{14}	δ_{15}	δ_{16}	δ_{17}	δ_{18}	δ_{19}	δ_{20}	δ_{21}	δ_{22}	δ_{23}	δ_{24}
	1.82	0.87	0.31	0.56	0.55	1.89	1.36	0.51	1.13	1.05	0.77	1.18
Product 1	0	212	0	0	138	0	272	0	0	130	0	0
Product 2	0	0	52	58	0	0	0	0	92	0	81	0
Product 3	554	0	0	0	0	422	0	202	0	0	0	135

(a) scale parameter (β): 12 months,
(b) shape parameter (α): 2,
(c) position parameter (γ): 0,

(2) the initial failure rate: $\lambda_0 = 0$.

These parameters provide information on the evolution of the failure rate in time.

This failure rate is increasing and linear over time. Thus, the function of the nominal failure rate is expressed by

$$\lambda_n(t) = \frac{\alpha}{\beta} \times \left(\frac{t}{\beta}\right)^{\alpha-1} = \frac{2}{12} \times \left(\frac{t}{12}\right). \tag{52}$$

The preventive and corrective maintenance costs are, respectively, Mp = 800 mu and Mc = 1 500 mu.

6.2. Determination of the Economic Production Plan. The economic production plan obtained is presented in Table 3.

6.3. Determination of the Optimal Maintenance Plan. As described in Figure 5, the optimal maintenance strategy is obtained based on the optimal production plan given in the previous section.

Figure 6 shows the curve of the total cost of maintenance according to N (number of preventive maintenance actions). We conclude that the optimal number of preventive maintenance actions that minimizes the total cost of maintenance during the finite horizon (two years) is $N^* = 2$ times. Hence, the optimal period to intervene for the preventive maintenance is $T^* = 12$ months, and the minimal total cost of maintenance $\Gamma^*(N) = 3316$ mu.

7. The Economical Profit of the Study

We recall that the specificity of this study is that it considered the impact of the production rate variation on the system degradation and consequently on the optimal maintenance strategy adopted in the case of multiple product. In order to show the significance of our study we will consider, in this section, the case of not considering the influence of the production rate variation on the system's degradation. That is to say, we assume that the manufacturing system is exploited at its maximal production rate every time. Analytically, we will consider the nominal failure rate which depends only on time. The results of this study are presented in Table 4.

The optimal number of preventive maintenance obtained in the case when we did not consider the variation of production rate is $N^* = 3$ times and it corresponds to a total cost of maintenance during the finite horizon (two years), $\Gamma^*(N) = 3\,704$ mu. We recall that in our case study when we consider

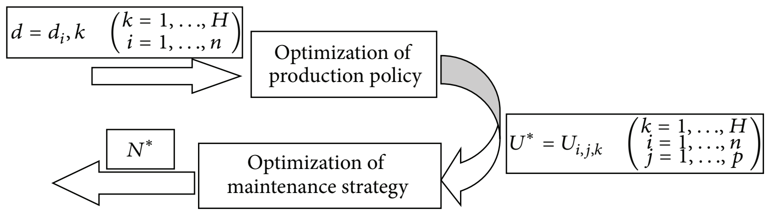

Figure 5: Sequential production and maintenance optimization.

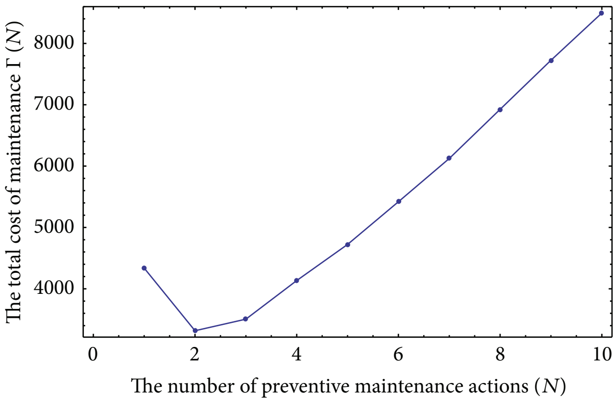

Figure 6: The total cost of maintenance depending to N.

Table 4: The sensitivity study based on the variation of production rate.

	$\Gamma^*(N)$ (um)	N^* (times)
Case 1: considering variation of production rate	3 316	2
Case 2: not considering the variation of production rate	3 704	3

the variation of production rate we have obtained $N^* = 2$ and $\Gamma^*(N) = 3\,316$ mu. We can easily note that we have reduced the optimal number of preventive maintenance with performing an economical gain estimated at 10%.

Several studies have addressed issues related to production and maintenance problem. But, the consideration of the materiel degradation according to the production rate in the case of multiple-product has been rarely studied.

This study was conducted to deal with the problem of an optimal production and maintenance planning for a manufacturing system. The significance of the present study is that we took into account the influence of the production plan on the system degradation in order to establish an optimal maintenance strategy. The considered system is composed of a single machine which produces several products in order to meet corresponding several random demands.

8. Conclusion

In this paper, we have discussed the problem of integrated maintenance to production for a manufacturing system consisting of a single machine which produces several types of products to satisfy several random demands. As the machine is subject to random failures, preventive maintenance actions are considered in order to improve its reliability. At failure, a minimal repair is carried out to restore the system into the operating state without changing its failure rate.

At first we have formulated a stochastic production problem. To solve this problem, we have used a production policy to achieve a level of economic output. This policy is characterized by the transformation of the problem to a deterministic equivalent problem in order to obtain the economic production plan. In the second step, taking into account the economic production plan obtained, we have studied and optimized the maintenance policy. This policy is defined by preventive actions carried out at constant time intervals. The objective of this policy is to determine the optimal number of preventive maintenance and the optimal intervention periods over a finite horizon. This policy is characterized by a failure rate for a linear degradation of the equipment considering the influence of production rate variation on the system degradation and on the optimal maintenance plan in the case of multiple products represents.

The promising results obtained in this thesis can lead to interesting perspectives. A perspective that we are looking for at the short term, is to consider maintenance durations. We recall that, throughout our study, we neglected the durations of actions of preventive and corrective maintenance. It is clear that the consideration of these durations impacts the optimal maintenance plan and the established production plan. In the medium term, it is interesting to concretely consider the impact of logistics service on the study. It is clear that the in-maintenance logistics are absent in most researches. The combination of maintenance logistics and production represents a motivating perspective in this field of study.

Another interesting perspective specifying the manufactured product can be explored.

Appendices

A. Expression of the Total Production and Storage Cost

We have

$$Z(U) = \sum_{k=1}^{H}\sum_{j=1}^{p}\sum_{i=1}^{n}\left[y_{i,j,k}\times\left(\mathrm{St}(i)+\left(\mathrm{Cp}(i)\times E\left[\left(U_{i,j,k}\right)^2\right]\right)\right)\right.$$
$$\left.+\left(\mathrm{Cs}(i)\times\frac{\delta t_{(k\times p)-(p-j)}}{\Delta t}\times E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]\right)\right],$$

$$S_{i,(k\times p)-(p-j)} = S_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Int}\left[\frac{j}{p}\right] \times d_i(k), \quad \forall \{i = 1,\dots,n\}, \{j = 1,\dots,p\}, \{k = 1,\dots,H\}. \tag{A.1}$$

Also,

$$\widehat{S}_{i,(k\times p)-(p-j)} = \widehat{S}_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Ent}\left[\frac{j}{p}\right] \times \widehat{d}_i(k) \quad \forall \{i = 1,\dots,n\}, \{j = 1,\dots,p\}, \{k = 1,\dots,H\}. \tag{A.2}$$

Therefore,

$$\begin{aligned}
&E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\
&\quad = E\left[\left(\left[S_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Ent}\left[\frac{j}{p}\right] \times d_i(k)\right] - \left[\widehat{S}_{i,(k\times p)-(p-j)-1} + \left(y_{i,j,k} \times U_{i,j,k}\right) - \operatorname{Ent}\left[\frac{j}{p}\right] \times \widehat{d}_i(k)\right]\right)^2\right] \\
&\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\
&\quad = E\left[\left(\left[S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right] - \left[\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right]\right)^2\right] \\
&\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)} - \widetilde{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\
&\quad = E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widetilde{S}_{i,(k\times p)-(p-j)-1}\right)^2 - 2 \times \left[\left(S_{i,(k\times p)-(p-j)-1} - \widetilde{S}_{i,(k\times p)-(p-j)-1}\right) \times \left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)\right] + \left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)^2\right] \\
&\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\
&\quad = \left[E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right)^2\right] - 2 \times E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right) \times \left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)\right] + E\left[\left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)^2\right]\right].
\end{aligned} \tag{A.3}$$

$S_{i,(k\times p)-(p-j)-1}$ and $d_i(k)$ are independent random variables, so we can deduce

$$\begin{aligned}
&E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\
&\quad = \left[E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right)^2\right] - 2 \times E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right)\right] \times E\left[\left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)\right] + E\left[\left(\operatorname{Ent}\left[\frac{j}{p}\right] \times \left(d_i(k) - \widehat{d}_i(k)\right)\right)^2\right]\right].
\end{aligned} \tag{A.4}$$

On the other hand, we note that

$$E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right)\right] = E\left[\left(S_{i,(k\times p)-(p-j)-1}\right)\right] - E\left[\left(\widehat{S}_{i,(k\times p)-(p-j)-1}\right)\right] = 0. \tag{A.5}$$

Therefore,

$$E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] = \left[E\left[\left(S_{i,(k\times p)-(p-j)-1} - \widehat{S}_{i,(k\times p)-(p-j)-1}\right)^2\right] + \left(\operatorname{Ent}\left[\frac{j}{p}\right]\right)^2 \times E\left[\left(d_i(k) - \widehat{d}_i(k)\right)^2\right]\right]. \tag{A.6}$$

We know that

$$E\left[\left(x_k - \widehat{x}_k\right)^2\right] = \operatorname{Var}\left(x_k\right), \quad \left(\operatorname{Int}\left[\frac{j}{p}\right]\right)^2 = \operatorname{Int}\left[\frac{j}{p}\right], \quad \text{because } 0 \le \frac{j}{p} \le 1. \tag{A.7}$$

Therefore,

$$E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] = \operatorname{Var}\left(S_{i,(k\times p)-(p-j)-1}\right) + \operatorname{Ent}\left[\frac{j}{p}\right] \times \operatorname{Var}\left(d_i(k)\right). \tag{A.8}$$

Finally,

$$\operatorname{Var}\left(S_{i,(k\times p)-(p-j)}\right) = \operatorname{Var}\left(S_{i,(k\times p)-(p-j)-1}\right) + \operatorname{Ent}\left[\frac{j}{p}\right] \times \operatorname{Var}\left(d_i(k)\right). \tag{A.9}$$

Consequently,

(i) for $k = 1$,

(a) $j = 1$:

$$\operatorname{Var}\left(S_{i,1}\right) = \operatorname{Var}\left(S_{i,0}\right) + \left(\operatorname{Ent}\left[\frac{1}{p}\right]\right) \times \operatorname{Var}\left(d_i(1)\right), \tag{A.10}$$

(b) $j = 2$:

$$\operatorname{Var}\left(S_{i,2}\right) = \operatorname{Var}\left(S_{i,0}\right) + \sum_{l=1}^{2}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(1)\right), \tag{A.11}$$

(c) $j = p$:

$$\operatorname{Var}\left(S_{i,p}\right) = \operatorname{Var}\left(S_{i,0}\right) + \sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(1)\right), \tag{A.12}$$

(ii) for $k = 2$,

(a) $j = 1$:

$$\operatorname{Var}\left(S_{i,p+1}\right) = \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(1)\right) + \operatorname{Ent}\left[\frac{1}{p}\right] \times \operatorname{Var}\left(d_i(2)\right)\right], \tag{A.13}$$

(b) $j = 2$:

$$\operatorname{Var}\left(S_{i,p+2}\right) = \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(1)\right) + \operatorname{Ent}\left[\frac{1}{p}\right] \times \operatorname{Var}\left(d_i(2)\right) + \operatorname{Ent}\left[\frac{2}{p}\right] \times \operatorname{Var}\left(d_i(2)\right)\right], \tag{A.14}$$

(c) $j = p$:

$$\operatorname{Var}\left(S_{i,(2\times p)}\right) = \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(1)\right) + \sum_{l=1}^{P}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(p)\right)\right], \tag{A.15}$$

(iii) for any value of k,

(a) $j = 1$:

$$\operatorname{Var}\left(S_{i,(k\times p)-(p-1)}\right) = \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(Q)\right) + \sum_{l=1}^{1}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(k)\right)\right], \tag{A.16}$$

(b) $j = 2$:

$$\operatorname{Var}\left(S_{i,(k\times p)-(p-2)}\right) = \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(Q)\right) + \sum_{l=1}^{2}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(k)\right)\right], \tag{A.17}$$

(c) for any value of j,

$$\begin{aligned} &\Longrightarrow \operatorname{Var}\left(S_{i,(k\times p)-(p-j)}\right) \\ &= \left[\operatorname{Var}\left(S_{i,0}\right) + \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(Q)\right) + \sum_{l=1}^{j}\operatorname{Ent}\left[\frac{l}{p}\right] \times \operatorname{Var}\left(d_i(k)\right)\right]. \end{aligned} \tag{A.18}$$

On the other hand,

$$\begin{aligned} &E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\ &\quad = E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2 - 2\times S_{i,(k\times p)-(p-j)} \times \widehat{S}_{i,(k\times p)-(p-j)} + \left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\ &\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)} - \widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] \\ &\quad = \left[E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right] - E\left[2\times S_{i,(k\times p)-(p-j)} \times \widehat{S}_{i,(k\times p)-(p-j)}\right] + E\left[\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]\right]. \end{aligned} \tag{A.19}$$

We know that

$$E\left[\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right] = \left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2. \tag{A.20}$$

Hence,

$$\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)}-\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]$$
$$=\left[E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]-2\times\widehat{S}_{i,(k\times p)-(p-j)}\times E\left[S_{i,(k\times p)-(p-j)}\right]+\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right],$$
$$E\left[S_{i,(k\times p)-(p-j)}\right]=\widehat{S}_{i,(k\times p)-(p-j)} \quad \text{(A.21)}$$
$$\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)}-\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]$$
$$=\left[E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]-2\times\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\times E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]+\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right].$$

Consequently,

$$E\left[\left(S_{i,(k\times p)-(p-j)}-\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]=E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]-\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2. \quad \text{(A.22)}$$

Noting that

$$E\left[\left(S_{i,(k\times p)-(p-j)}-\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]=\operatorname{Var}\left(S_{i,(k\times p)-(p-j)}\right)$$
$$\Longrightarrow \operatorname{Var}\left(S_{i,(k\times p)-(p-j)}\right)=E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]-\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2, \quad \text{(A.23)}$$

we deduce from (A.18) and (A.23) that

$$E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]-\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2$$
$$=\left[\operatorname{Var}\left(S_{i,0}\right)+\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right]\times\operatorname{Var}\left(d_i\left(Q\right)\right)+\sum_{l=1}^{j}\operatorname{Ent}\left[\frac{l}{p}\right]\times\operatorname{Var}\left(d_i\left(k\right)\right)\right]$$
$$\Longrightarrow E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]$$
$$=\left[\operatorname{Var}\left(S_{i,0}\right)+\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right]\times\operatorname{Var}\left(d_i\left(Q\right)\right)+\sum_{l=1}^{j}\operatorname{Ent}\left[\frac{l}{p}\right]\times\operatorname{Var}\left(d_i\left(k\right)\right)+\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]. \quad \text{(A.24)}$$

Consequently,

$$E\left[\left(S_{i,(k\times p)-(p-j)}\right)^2\right]$$
$$=\left[\sigma^2\left(S_{i,0}\right)+\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Ent}\left[\frac{l}{p}\right]\times\sigma^2\left(d_i\left(Q\right)\right)+\sum_{l=1}^{j}\operatorname{Ent}\left[\frac{l}{p}\right]\times\sigma^2\left(d_i\left(k\right)\right)+\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]. \quad \text{(A.25)}$$

Substituting (A.25) in the expected cost expression (9),

$$Z\left(U\right)=\sum_{k=1}^{H}\sum_{j=1}^{P}\sum_{i=1}^{n}\left\{y_{i,j,k}\times\left(\operatorname{St}\left(i\right)+\left(\operatorname{Cp}\left(i\right)\times U^2{}_{i,j,k}\right)\right)+\operatorname{Cs}\left(i\right)\times\frac{\delta_{(k\times p)-(p-j)}}{\Delta t}\times\left[\sigma^2\left(S_{i,0}\right)+\left(\sum_{Q=1}^{k-1}\sum_{l=1}^{p}\operatorname{Int}\left(\frac{l}{p}\right)\times\sigma^2\left(d_i\left(Q\right)\right)\right)+\left(\sum_{l=1}^{j}\operatorname{Int}\left(\frac{l}{p}\right)\times\sigma^2\left(d_i\left(k\right)\right)\right)+\left(\widehat{S}_{i,(k\times p)-(p-j)}\right)^2\right]\right\}. \quad \text{(A.26)}$$

B. Expression of Failure Rate

Equation (A.9) is expressed as follows for the different subperiods:

(i) for $k = 1$,

(a) $j = 1$:

$$\lambda_1\left(t\right)=\left(\lambda_0\right)\times\left(1-\operatorname{In}\left[\frac{0}{q\times T}\right]\right)+\sum_{i=1}^{n}\frac{U_{i,1,1}\times\Delta t}{U_{i\,\text{nom}}\times\delta_1}\times\lambda_n\left(t\right), \quad \text{(B.1)}$$

(b) $j = 2$:

$$\lambda_2\left(t\right)=\lambda_1\left(\delta_1\right)\times\left(1-\operatorname{In}\left[\frac{1}{q\times T}\right]\right)+\sum_{i=1}^{n}\frac{U_{i,2,1}\times\Delta t}{U_{i\,\text{nom}}\times\delta_2}\times\lambda_n\left(t\right),$$

$$\lambda_2(t) = \left(\lambda_0 + \sum_{i=1}^{n} \frac{U_{i,1,1} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_1} \times \lambda_n(\delta_{(1)})\right) \times \left(1 - \mathrm{In}\left[\frac{1}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,2,1} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_2} \times \lambda_n(t), \quad \text{(B.2)}$$

(c) $j = p$:

$$\lambda_p(t) = \left(\lambda_{p-1}(\delta_{p-1})\right) \times \left(1 - \mathrm{In}\left[\frac{p-1}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,p,1} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_p} \times \lambda_n(t),$$

$$\lambda_p(t) = \left[\left(\lambda_0 + \sum_{l=1}^{p-1}\sum_{i=1}^{n} \frac{U_{i,l,1} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_l} \times \lambda_n(\delta_{(l)})\right) \times \left(1 - \mathrm{In}\left[\frac{p-1}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,p,1} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_p} \times \lambda_n(t)\right], \quad \text{(B.3)}$$

(ii) for any value of k,

(a) $j = 1$:

$$\lambda_{((k-1)\times p)+1}(t) = \left[\left(\lambda_{(k-1)\times p}(\delta_{(k-1)\times p})\right) \times \left(1 - \mathrm{In}\left[\frac{((k-1)\times p)}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,1,k} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{((k-1)\times p)+1}} \times \lambda_n(t)\right],$$

$$\lambda_{((k-1)\times p)+1}(t) = \left[\left(\lambda_0 + \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\sum_{i=1}^{n} \frac{U_{i,l,Q} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{(Q\times p)-(p-l)}} \times \lambda_n(\delta_{(Q\times p)-(p-l)})\right) \times \left(1 - \mathrm{In}\left[\frac{((k-1)\times p)}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,1,k} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{((k-1)\times p)+1}} \times \lambda_n(t)\right], \quad \text{(B.4)}$$

(b) for any value of j,

$$\lambda_{(k\times p)-(p-j)}(t) = \left[\left(\lambda_0 + \sum_{Q=1}^{k-1}\sum_{l=1}^{p}\sum_{i=1}^{n} \frac{U_{i,l,Q} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{(Q\times p)-(p-l)}} \times \lambda_n(\delta_{(Q\times p)-(p-l)}) + \sum_{l=1}^{j-1}\sum_{i=1}^{n} \frac{U_{i,l,k} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{(k\times p)-(p-l)}} \times \lambda_n(\delta_{(k\times p)-(p-l)})\right) \times \left(1 - \mathrm{In}\left[\frac{(k\times p)-(p-j+1)}{q \times T}\right]\right) + \sum_{i=1}^{n} \frac{U_{i,j,k} \times \Delta t}{U_{i\,\mathrm{nom}} \times \delta_{(k\times p)-(p-j)}} \times \lambda_n(t)\right],$$

$$t \in \left[0, \delta_{(k\times p)-(p-j)}\right] \quad \forall \{k = 1, \ldots, H\}, \{j = 1, \ldots, p\}. \quad \text{(B.5)}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Impacts of Transportation Cost on Distribution-Free Newsboy Problems

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A distribution-free newsboy problem (DFNP) has been launched for a vendor to decide a product's stock quantity in a single-period inventory system to sustain its least maximum-expected profits when combating fierce and diverse market circumstances. Nowadays, impacts of transportation cost on determination of optimal inventory quantity have become attentive, where its influence on the DFNP has not been fully investigated. By borrowing an economic theory from transportation disciplines, in this paper the DFNP is tackled in consideration of the transportation cost formulated as a function of shipping quantity and modeled as a nonlinear regression form from UPS's on-site shipping-rate data. An optimal solution of the order quantity is computed on the basis of Newton's approach to ameliorating its complexity of computation. As a result of comparative studies, lower bounds of the maximal expected profit of our proposed methodologies surpass those of existing work. Finally, we extend the analysis to several practical inventory cases including fixed ordering cost, random yield, and a multiproduct condition.

1. Introduction

A newsboy (newsvendor) problem has been initiated to determine the stock quantity of a product in a single-period inventory system when the product whose demand is stochastic has a single chance of procurement prior to the beginning of selling period. Aiming to maximize expected profit, decisive quantity trades off between the risk of underordering, which fails to gain more profit, and the loss of overordering, which compels release below the unit purchasing cost.

Traditional models for the newsboy problem assume that a single vendor encounters the demand of a product complying with a particular probability distribution function with known parameters, such as a normal, Schmeiser-Deutsch, beta, gamma, or Weibull distribution [1]. With this assumption, several recent studies have to a certain extent succeeded in resolution of certain practical problems. For example, Chen and Ho [2] and Ding [3] analyzed the optimal inventory policy for newsboy problems with fuzzy demand and quantity discounts. Arshavskiy et al. [4] performed experimental studies by implementing the classical newsvendor problem in practice. Ozler et al. [5] studied a multiproduct newsboy problem under value-at-risk constraint; with loss-averse preferences, Wang [6] introduced a problem of multinewsvendors who compete with inventories setting from a risk-neutral supplier. When confronting myriad conditions in markets, however, in many occasions this designated distributional demand failed to best safeguard the vendor's profit.

To cope with the failure, models for the distribution-free newsboy problem (DFNP) have been broadly introduced over the past two decades. Gallego and Moon [7] first outlined a compacted analysis procedure for arranging optimal order quantities to certain inventory models, such as the single product, fixed ordering, random yield, and a multiproduct case. Alfares and Elmorra [8] further employed the procedure for the inventory model which considers shortage penalty cost. Moon and Choi [9] derived an ordering rule for the balking-inventory control model, where probability of per unit sold declines as inventory level falls below balking level.

More recently, Cai et al. [10] provided measurements for deployment of multigenerational product development with the project cost accrued from different phases of a product life cycle such as development, service, and associated risks. Lee and Hsu [11] and Güler [12] developed an optimal ordering rule when an effect of advertising expenditure was reckoned on the inventory model. Kamburowski [13] presented new theoretical foundations for analyzing the best-case and worst-case scenarios. Due to prevalence of purchasing online, Mostard et al. [14] studied a resalable-return model for the distant selling retailers receiving internet orders from customers who have right to return their unfit merchandise in a stipulated period.

Over the past few years, energy prices have risen significantly and become more volatile; transportation of goods has become the highest operational expense as noted by Barry [15]. Many evidences indicate that in the US inbound freight costs for domestically sourced products and imported products typically range from 2 to 4% and from 6 to 12% of gross sales, respectively, and outbound transportation costs typically average 6% to 8% of net sales. In addition, Swenseth and Godfrey [16] reported that depending on the estimates utilized, upwards of 50% of the total annual logistic cost of a product could be attributed to transportation and that these costs were going up. UPS recently announced a 4.9% increase in its net average shipping rate. Ostensibly, the expenditure of the inbound/outbound material transportation has become a critical component of a total annual logistic cost function for determining purchase quantities. Effects of transportation have gained substantial recognition in vendor-buyer coordination problems.

Swenseth and Godfrey [16] unified two freight rate functions into a total annual cost function to understand their brunt on purchasing decisions. For integration of inventory and inbound/outbound transportation decisions, Çetinkaya and Lee [17] enabled an optimal inventory policy and Toptal et al. [18] carried out ideal cargo capacity and minimal costs. Toptal and Çetinkaya [19] further studied a coordination problem between a vendor and a buyer under explicit transportation consideration. More recently, Zhang et al. [20] generalized a standard newsboy model to the freight cost proportional to the number of the containers used. Toptal [21] studied exponentially/uniformly distributed demands and trucking costs. Mutlu and Çetinkaya [22] developed an optimal solution when inventory replenishment and shipment scheduling under common dispatch costs are considered.

Although impacts of the transportation cost on determination of the optimal inventory quantity have become attentive, its influence on the DFNP has not been fully investigated. To bridge the gap, this paper develops analytical and efficient procedures to acquire optimal policies for the DFNP in which the transportation cost function is explicitly joined into the vendor's expected profit structure. We borrowed the idea from the transportation management models [23] that the transportation cost is modeled as a function of delivery quantities; as a result of the computational studies, our proposed optimal-ordering rules increase lower bound of maximized expected profit as much as 4% on average, as opposed to the optimal policies recommended by Gallego and Moon [7]. Moreover, in order to determine and implement the optimal policies in practice, we perform comprehensive sensitivity analyses for the vital parameters, such as the demand mean and variance, unit cost of product, and transportation cost.

Lastly, this paper is organized as follows. Section 2 describes our model formulation for the DFNP in presence of transportation cost whose optimal order quantity Q^* along with lower bound of maximized expected profit $E(Q^*)$ is resolved in Section 3. In Section 4, we study sensitivity analyses and comparative studies. A fixed-ordering cost case is analyzed in Section 5, while a random-yield case is considered in Section 6. In Section 7, we further contemplate a multiproduct case with budget constraint. Conclusions and Implications make up Section 8.

2. Model Formulation for the DFNP with Transportation Cost

For investigating impacts of the DFNP in consideration of the transportation cost, we briefly depict its model assumptions and notations used in this paper. Demand rate from a specific buyer is denoted by D, whose distribution G is unknown with mean μ and variance σ^2. Note that the unknown distribution G is equal to or better off the worst possible distribution ϑ. With a product's unit cost c, a vendor orders size of Q, which arrive before delivering to the buyer. Intuitively, in one replenishment cycle, $\min\{Q, D\}$ units are sold with unit price p and the unsold items $(Q - D)^+$ are salable with unit salvage value s, where $s < p$, where $(Q - D)^+$ defined as the positive part of $Q - D$ are equivalent to $\max\{Q - D, 0\}$. This implies $Q = \min\{Q, D\} + (Q - D)^+$.

Furthermore, we assume transportation cost is a function of the order quantity Q, denoted by $\mathrm{tc}(Q)$. We further assume the transportation cost is in a general form of the tapering (or proportional) function; for example, $\mathrm{tc}(Q) = a + b\ln Q$, for $a, b \geq 0$, where a and b represent fixed and variable transportation cost. Intuitively, high volume corresponds to lower per unit rate of transportation, reflecting that the inequality $[\mathrm{tc}(Q)/Q]' \leq 0$ holds true. That is, $[\mathrm{tc}(Q)/Q]' = (b - a - b\ln Q)/Q^2 \leq 0$ or equivalently $Q \geq \exp(1 - a/b)$, where the regulated minimal quantity level of delivery is $Q^s = \exp(1 - a/b)$ and $Q \geq Q^s$.

The assumption is based on the following observations from the existing works and UPS's on-site data set. First off, economic trade-off for the optimal transportation cost lies between provided service level and shipped quantity [17]. Secondly, in the shipment more weight signifies larger delivery quantity and higher shipment cost [19]. Thirdly, the transportation management models proposed by Swenseth and Godfrey [16] and Toptal et al. [18] indicated that optimal shipping quantity renders minimum of the transportation cost. Finally, we display the on-site shipping data set collected from the UPS worldwide expedited service at zone 7 shown in Figure 1.

Now, we are ready to combat the DFNP in presence of the transportation cost. Our purpose is to decide an optimal stock quantity in a single-period inventory system for a vendor to sustain its least maximum-expected profits when

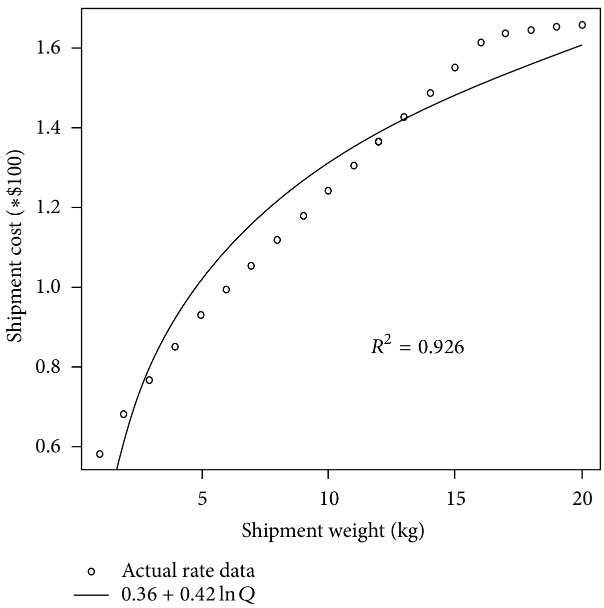

Figure 1: The fitted regression model for the data set of UPS worldwide expedited service at zone 7.

encountering fierce and diverse market circumstances. First, we construct the vendor's expected profit $E(Q)$:

$$\begin{aligned} E(Q) &= pE(\min\{Q,D\}) + sE(Q-D)^{+} - cQ \\ &\quad - \{a + b\ln[E(\min\{Q,D\})]\} \\ &\quad - \{a + b\ln[E(Q-D)^{+}]\} \\ &= pE(\min\{Q,D\}) + sE(Q-D)^{+} - cQ - 2a \\ &\quad - b\ln\{E(\min\{Q,D\})\,E(Q-D)^{+}\}. \end{aligned} \tag{1}$$

Then, according to the relationships of $\min\{Q,D\} = D - (D-Q)^{+}$ and $(Q-D)^{+} = (Q-D) + (D-Q)^{+}$, we further rewrite (1):

$$\begin{aligned} E(Q) &= (p-s)\mu - (p-s)E(D-Q)^{+} \\ &\quad - (c-s)Q - 2a \\ &\quad - b\ln\{[\mu - E(D-Q)^{+}][Q-\mu+E(D-Q)^{+}]\}. \end{aligned} \tag{2}$$

For developing an optimal order quantity for the vendor to sustain its lower bound of maximized expected profit $E(Q)$, we consider G, the distribution of D, to be under the worst possible distribution ϑ. Therefore, based on Gallego and Moon's Lemma 1 in [7], we have the lower bound of expected profit $E(Q)$ for the vendor:

$$\begin{aligned} E(Q) &\geq (p-s)\mu - (p-s) \\ &\quad \times \frac{\left[\sigma^2 + (Q-\mu)^2\right]^{1/2} - (Q-\mu)}{2} \\ &\quad - (c-s)Q - 2a + 2b\ln 2 \\ &\quad - b\ln\left\{-\mu^2 - 1 + Q^2 + 2\mu\left[\sigma^2 + (Q-\mu)^2\right]^{1/2}\right\}. \end{aligned} \tag{3}$$

Lemma 1 (see [7]). *Under the worst possible distribution ϑ, the upper bound of expected value for the positive part of $Q-D$ is*

$$E(D-Q)^{+} \leq \frac{\left[\sigma^2 + (Q-\mu)^2\right]^{1/2} - (Q-\mu)}{2}. \tag{4}$$

Let the right-hand side term of (3) be a continuous function with respect to Q; then, first and second derivatives of $E(Q)$ are elaborately derived as follows:

$$\begin{aligned} \frac{dE(Q)}{dQ} &= \frac{p+s-2c}{2} - \frac{(p-s)(Q-\mu)}{2\left[\sigma^2+(Q-\mu)^2\right]^{1/2}} \\ &\quad - b\frac{2Q + 2\mu(Q-\mu)\left[\sigma^2+(Q-\mu)^2\right]^{-1/2}}{-1-\mu^2+Q^2+4\mu\left[\sigma^2+(Q-\mu)^2\right]^{1/2}}, \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{d^2E(Q)}{dQ^2} &= -\frac{(p-s)\sigma^2}{2\left[\sigma^2+(Q-\mu)^2\right]^{3/2}} \\ &\quad - b\left\{-2 + 2\mu^2 - 2Q^2\right. \\ &\quad + 4\mu\left[\sigma^2+(Q-\mu)^2\right]^{1/2} \\ &\quad + \frac{2\mu\left(-1-\mu^2+4\mu Q-3Q^2\right)}{\left[\sigma^2+(Q-\mu)^2\right]^{1/2}} \\ &\quad - \frac{8\mu^2(Q-\mu)^2}{\sigma^2+(Q-\mu)^2} \\ &\quad \left.+ \frac{(Q-\mu)^2\left(2\mu^3+2\mu-2\mu Q^2\right)}{\left[\sigma^2+(Q-\mu)^2\right]^{3/2}}\right\} \\ &\quad \cdot \left\{-1-\mu^2+Q^2+4\mu\left[\sigma^2+(Q-\mu)^2\right]^{1/2}\right\}^{-2}. \end{aligned} \tag{6}$$

Obviously, $d^2E(Q)/dQ^2$ in (6) is not necessarily being negative. It implies that the generally explicit and analytical close form for the optimal order quantity $\max\{Q^*, Q^s\}$ with the least of maximized expected profits is not available. Therefore, there is a need to develop an efficient search procedure to obtain the optimal order quantity Q^* and its corresponding lower bound of maximized expected profit $E(Q^*)$.

Table 1: The optimal order quantity using Newton's optimization approach.

Iteration i	Q_i	$f'(Q_i)$	$f''(Q_i)$	$f'(Q_i)/f''(Q_i)$	Q_{i+1}
0	9	−0.695	−2.471	0.281	8.719
1	8.719	0.030	−1.729	−0.017	8.736
2	8.736	−0.014	−1.804	0.008	8.728
3	8.728	0.005	−1.772	−0.003	8.731
4	**8.731**	**−0.000**	**−1.785**	**0.000**	**8.731**

3. An Efficient Solution Procedure for Q^* and $E(Q^*)$

Step 1. Start from $i = 0$, let initial order quantity $Q_0 = \mu$, and set the allowable tolerance ε, for example, the acceptable "precision" or "accuracy" selected by the decision maker for the optimal decision policy.

Step 2. Perform Newton's approach (see Hillier and Lieberman [24, pp. 555–557]) to seeking the optimal order quantity of Q.

Let $Q_{i+1} = Q_i - (f'(Q_i)/f''(Q_i))$. According to (5), we set

$$f'(Q_i) = \frac{p+s-2c}{2} - \frac{(p-s)(Q_i-\mu)}{2\left[\sigma^2+(Q_i-\mu)^2\right]^{1/2}} - b\frac{2Q_i+2\mu(Q_i-\mu)\left[\sigma^2+(Q_i-\mu)^2\right]^{-1/2}}{Q_i^2-\mu^2-1+2\mu\left[\sigma^2+(Q_i-\mu)^2\right]^{1/2}}. \tag{7}$$

From (6), we set

$$\begin{aligned} f''(Q_i) = &-\frac{(p-s)\sigma^2}{2\left[\sigma^2+(Q_i-\mu)^2\right]^{3/2}} \\ &- b\left\{-2Q_i^2-2+2\mu^2+4\mu\left[\sigma^2+(Q_i-\mu)^2\right]^{1/2}\right. \\ &+\frac{2\mu\left(4\mu Q_i-3Q_i^2-\mu^2-1\right)}{\left[\sigma^2+(Q_i-\mu)^2\right]^{1/2}} \\ &+\frac{(Q_i-\mu)^2\left(2\mu^3+2\mu-2\mu Q_i^3\right)}{\left[\sigma^2+(Q_i-\mu)^2\right]^{3/2}} \\ &\left.-\frac{8\mu^2(Q_i-\mu)^2}{\sigma^2+(Q_i-\mu)^2}\right\}. \end{aligned} \tag{8}$$

Stop the search when $|Q_{i+1} - Q_i| \le \varepsilon$, so the optimal order quantity Q^* can be found at the value Q_{i+1}.

Step 3. For verifying adequacy of Newton's method, substitute Q^* into (6); if $d^2E(Q^*)/dQ^{*2} < 0$ meaning Newton's method is satisfactory, then the final solution is Q^* whose $E(Q^*)$ is the vendor's lower bound of maximized expected profit; otherwise, go to Step 4 to perform the bisection optimization method.

Step 4. Select l, a quantifiable order quantity. Start $i = 0$ and let $[Q_0^s, Q_0^*]$ be the initial searching interval, where $Q_0^s = \exp(1 - a/b)$ is the regulated minimal quantity level of delivery for the transportation cost $\mathrm{tc}(Q) = a + b\ln Q$ and $Q_0^* = Q^*$.

Step 5. If $|Q_i^* - Q_i^s| < l$, then stop; the optimal order quantity is $Q_i^{**} = (Q_i^s + Q_i^*)/2$ along with the lower bound of maximal expected profit $E(Q_i^{**})$; otherwise, let $Q_i^b = (Q_i^s + Q_i^*)/2$.

Step 6. If $E(Q_i^b) \ge E(Q_i^*)$, then $Q_{i+1}^* = Q_i^b$ and $Q_{i+1}^s = Q_i^*$; otherwise, $Q_{i+1}^* = Q_i^*$ and $Q_{i+1}^s = Q_i^b$. Go back Step 5 with $i = i + 1$.

To demonstrate the efficient solution procedure for the DFNP incorporating the explicit transportation cost, a numerical example is illustrated.

3.1. Finding Q^* and $E(Q^*)$.

A chosen product has demand mean $\mu = 9$ kg and standard deviation $\sigma = 0.5$. Its unit cost is $c = \$3.5$/kg, unit selling price $p = \$5$/kg, and unit salvage value $s = \$2.5$/kg. Including fuel and handling charges, on-site data of the transportation cost collected from UPS worldwide expedited service at zone 7, from Europe to Taiwan, are 0.58, 0.69, 0.77, 0.85, 0.93, 1.00, 1.06, 1.12, 1.18, 1.24, 1.31, 1.37, 1.43, 1.49, 1.55, 1.61, 1.64, 1.65, 1.66, and 1.66 for shipment weight of $1, 2, \ldots, 20$ kg, respectively. For clarity of description, the costs considered here are all rounded down to a 45-hundred US dollar-scale. By fitting the data through the nonlinear regression model, we have an empirical tampering function $\mathrm{tc}(Q) = 0.36 + 0.42\ln Q$ shown in Figure 1 with R^2 =0.926. We conclude that the fitted function provides high fidelity to represent the actual data.

Then, we follow the proposed search procedure.

Step 1. From $n = 0$ and $i = 0$, set $Q_0 = \mu = 9$ and $\varepsilon = 10^{-3}$.

Step 2. When $n = 1$, we have $Q_1 = Q_0 - (f'(Q_0)/f''(Q_0)) = 9.211$. In this case, $|Q_1 - Q_0| > 0.001$, so continue Newton's search until reaching $|Q_{i+1} - Q_i| \le 0.001$. Then the optimal order quantity $Q^* = Q_{i+1}$. The searching details are listed in Table 1.

Step 3. The optimal order quantity $Q^* = 8.731$ (the condition $d^2E(Q^*)/dQ^{*2} = -1.783 < 0$ holds true). Substituting $Q^* = 8.731$ and known parameters into (5), we obtain lower bound of maximized expected profit $E(Q^*)$, which is \$11.899.

Table 2: The computational results with fixed values of $p = 5$ and $s = 2.5$.

Policy	Parameters setting				Our proposed policy	Gallego and Moon [7]	Profit gain
	μ	σ	c	$\mathrm{tc}(Q)$	$E(Q^*)$	$E(Q^*)$	(%)
1	7	0.4	3	$0.36 + 0.42\ln Q$	12.659(6.824)	12.454(7.300)	1.62
2	11	0.4	3	$0.36 + 0.42\ln Q$	20.470(10.863)	20.262(11.300)	1.02
3	7	0.6	3	$0.36 + 0.42\ln Q$	12.236(6.98)	12.086(7.450)	1.22
4	11	0.6	3	$0.36 + 0.42\ln Q$	20.040(10.700)	19.893(11.450)	0.73
5	7	0.4	4	$0.36 + 0.42\ln Q$	5.939(6.503)	5.533(7.082)	6.84
6	11	0.4	4	$0.36 + 0.42\ln Q$	9.736(10.516)	9.339(11.082)	4.08
7	7	0.6	4	$0.36 + 0.42\ln Q$	5.476(6.509)	5.102(7.122)	6.82
8	11	0.6	4	$0.36 + 0.42\ln Q$	9.271(10.509)	8.907(11.122)	3.93
9	7	0.4	3	$0.31 + 0.56\ln Q$	12.764(6.705)	12.411(7.300)	2.76
10	11	0.4	3	$0.31 + 0.56\ln Q$	20.511(10.754)	20.155(11.300)	1.73
11	7	0.6	3	$0.31 + 0.56\ln Q$	12.250(6.858)	11.988(7.450)	2.13
12	11	0.6	3	$0.31 + 0.56\ln Q$	19.987(10.881)	19.731(11.450)	1.28
13	7	0.4	4	$0.31 + 0.56\ln Q$	6.160(6.444)	5.561(7.082)	9.74
14	11	0.4	4	$0.31 + 0.56\ln Q$	9.889(10.435)	9.303(11.082)	5.93
15	7	0.6	4	$0.31 + 0.56\ln Q$	5.605(6.428)	5.075(7.122)	9.46
16	11	0.6	4	$0.31 + 0.56\ln Q$	9.331(10.414)	8.814(11.122)	5.54
						Average	4.05

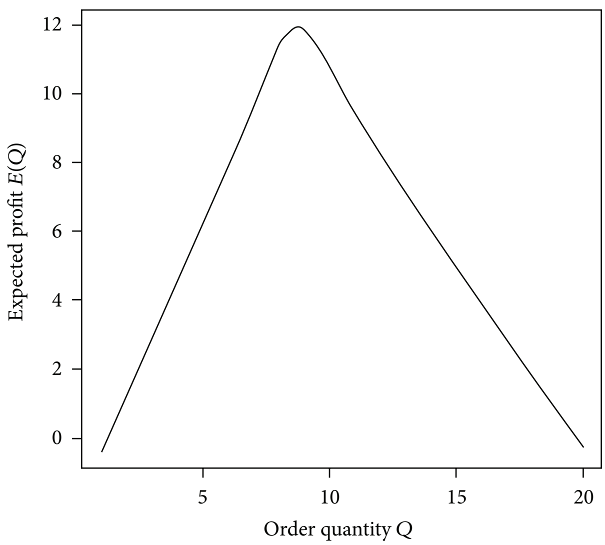

Figure 2: Illustration of the expected profit with respect to order quantity Q.

Figure 2 concavely exhibits $E(Q^*)$ with respect to a wide range of Q^*.

3.2. Models Comparison. For models comparison, we implement the DFNP based on Gallego and Moon [7] whose model does not reckon the transportation cost and perform the similar searching procedure described in Section 3. Their model obtains the optimal order quantity $Q^* = 8.731$ with the lower bound of maximized expected profit $E(Q^*) = \$11.752$. In this case, our proposed model in consideration of the transportation cost has manifested $(11.899 - 11.752)/11.899 = 1.2\%$ of gains in $E(Q^*)$.

4. Sensitivity Analyses and Comparative Studies

Furthermore, we apply a 2^4 factorial design to investigate sensitivity of parameters. They are set as follows. Let the unit selling price be $p = \$5$/kg and the unit salvage value be $s = \$2.5$/kg; two levels are selected for each of the four parameters; that is, mean $\mu \in [7, 11]$, standard deviation $\sigma \in [0.1, 1]$, unit product cost $c \in [3, 4]$, and the transportation cost $\mathrm{tc}(Q) \in [0.36 + 0.42\ln Q, 0.31 + 0.56\ln Q]$ whose selected levels are based on fitting another data set gathered from UPS's transportation cost (worldwide express saver at zone 7, from Europe to Taiwan), US\$ 0.66, 0.78, 0.88, 0.98, 1.08, 1.15, 1.23, 1.31, 1.39, 1.47, 1.55, 1.62, 1.70, 1.79, 1.87, 1.94, 2.01, 2.09, 2.17, and 2.25, respectively, for shipment weight of $1, 2, 3, \ldots, 20$ kg.

Table 2 lists $E(Q^*)$ along with Q^* for our proposed model in the 6th column and Gallego and Moon's model [7] in the 7th column. First, this sensitivity analysis demonstrates significant correlations among the parameters whose simultaneous consideration is imperative for the proposed optimal policy. Moreover, in contrast to Gallego and Moon's model, the percentages of the profit gain obtained from our proposed model are listed in the 8th column. Apparently, our proposed model outperforms Gallego and Moon's model in every policy; especially, in the ordering policies 13 and 15, the profit advance can be more than 9.4%; on average, our proposed policy provides the return gain as much as 4% as opposed to that of the Gallego and Moon's model.

In views of the impact of transportation cost on the DFNP as well as the gains elicited from our proposed policies, we then extend contemplation of the transportation cost into several practical inventory cases such as fixed ordering cost, random yield, and a multiproduct case.

5. The Fixed Ordering Cost Case with Transportation Cost

Let a vendor have an initial inventory I ($I \geq 0$) prior to placing an order $Q > 0$, where ordering cost A is fixed for any size of order. Let r denote the reorder point, known as an inventory level when the order is submitted. Let $S = I + Q$ be end inventory level, an inventory level after receiving the order.

Similarly, $\min\{S, D\}$ units are sold; $S - D$ units are salvaged. For an (r, S) inventory replenishment policy in consideration of the transportation cost, expected profit $E(S)$ is constructed as

$$\begin{aligned} E(S) &= pE(\min\{S, D\}) + sE(S-D)^{+} \\ &\quad - c(S-I) - A1_{[S>I]} - \{a + b\ln[E(\min\{S, D\})]\} \\ &\quad - \{a + b\ln[E(S-D)^{+}]\}, \\ E(S) &= (p-s)\mu - (p-s)E(D-S)^{+} \\ &\quad - (c-s)S + cI - AI_{[S>I]} - 2a \\ &\quad - b\ln\{[\mu - E(D-S)^{+}][S - \mu + E(D-S)^{+}]\}, \end{aligned} \tag{9}$$

where $I_{[S>I]} = \begin{cases} 1 & \text{if } S>I, \\ 0 & \text{otherwise.} \end{cases}$

According to Lemma 1, the expression can be simplified as $\min_{S\geq I}\{AI_{[S>I]} + J(S)\}$, where

$$\begin{aligned} J(S) &= -(p-s)\mu + (p-s)\frac{[\sigma^2 + (S-\mu)^2]^{1/2} - (S-\mu)}{2} \\ &\quad + (c-s)S - cI + 2a - 2b\ln 2 \\ &\quad + b\ln\left\{-\mu^2 - 1 + S^2 + 2\mu[\sigma^2 + (S-\mu)^2]^{1/2}\right\}. \end{aligned} \tag{10}$$

The relationship of $S = I + Q$ implies that acquiring the optimal end inventory level of S for the fixed ordering cost model is equivalent to having optimal order quantity of Q for the single-product model. Clearly, because $I < S$, $J(I) > A + J(S)$. For determining the optimal reorder point of r, $J(r) = A + J(S)$ is set. Then, we have

$$\begin{aligned} &\frac{p-s}{2}\left\{[\sigma^2 + (r-\mu)^2]^{1/2} - r\right\} + (c-s)r \\ &\quad + b\ln\left\{-\mu^2 - 1 + r^2 + 2\mu[\sigma^2 + (r-\mu)^2]^{1/2}\right\} \\ &\quad - A - \frac{p-s}{2}\left\{[\sigma^2 + (S-\mu)^2]^{1/2} - S\right\} - (c-s)S \\ &\quad - b\ln\left\{-\mu^2 - 1 + S^2 + 2\mu[\sigma^2 + (S-\mu)^2]^{1/2}\right\} = 0. \end{aligned} \tag{11}$$

Furthermore, we develop a solution procedure to determine the optimal reorder point.

Step 1. By performing the solution procedure for the optimal order quantity in Section 3, we first obtain Q^*. Then let Q^* be the end inventory level S, where I is set to be 0 for brevity.

Step 2. Start $i = 0$, set the initial reorder point r_0 to be S, and determine the allowable tolerance ε for accuracy of the final result.

Step 3. Perform Newton's search (see Grossman [25, pp.228]) to compute the optimal reorder level of r. That is, $r_{i+1} = r_i - (f(r_i)/f'(r_i))$, where

$$\begin{aligned} f(r_i) &= \frac{p-s}{2}\left\{[\sigma^2 + (r_i-\mu)^2]^{1/2} - r_i\right\} + (c-s)r_i \\ &\quad + b\ln\left\{-\mu^2 - 1 + r_i^2 + 2\mu[\sigma^2 + (r_i-\mu)^2]^{1/2}\right\} \\ &\quad - A - \frac{p-s}{2}\left\{[\sigma^2 + (S-\mu)^2]^{1/2} - S\right\} - (c-s)S \\ &\quad - b\ln\left\{-\mu^2 - 1 + S^2 + 2\mu[\sigma^2 + (S-\mu)^2]^{1/2}\right\}, \\ f'(r_i) &= \frac{2c-p-s}{2} + \frac{(p-s)(r_i-\mu)}{2[\sigma^2 + (r_i-\mu)^2]^{1/2}} \\ &\quad + b\frac{2r_i + 2\mu(r_i-\mu)[\sigma^2 + (r_i-\mu)^2]^{-1/2}}{-\mu^2 - 1 + r_i^2 + 2\mu[\sigma^2 + (r_i-\mu)^2]^{1/2}}. \end{aligned} \tag{12}$$

Stop the search when $|r_{i+1} - r_i| \leq \varepsilon$. Then, the optimal order quantity is r_{i+1}.

Step 4. The optimal policy is to order up to S units if the initial inventory is less than r and not to order otherwise.

5.1. An Example. Continuing the numerical example in Section 3, we assume that the ordering cost is given by $A =$ \$0.3. Using the above solution procedure, we find that the optimal reorder level of r is 8.210 and the end inventory level $S = 8.731$.

6. The Random Yield Case with Transportation Cost

Suppose random variable $G(Q)$ expresses the number of good units produced from ordered quantity Q, where each good unit being ordered or produced has an equal probability of ρ. Thus, $G(Q)$ is a binomial random variable with mean $Q\rho$ and variance $Q\rho q$, where $q = 1-\rho$. Let m be the price markup rate and d the discount rate, so unit selling price $p = (1+m)c/\rho$, and salvage value $s = (1-d)c/\rho$. Thus, the expected profit in (1) can be rewritten as

$$\begin{aligned} E(Q) &= pE(\min\{G(Q), D\}) + sE(G(Q)-D)^{+} - cQ - 2a \\ &\quad - b\ln\{E(\min\{G(Q), D\})\, E(G(Q)-D)^{+}\} \\ &= \frac{c}{\rho}\{(m+d)\mu - (m+d)E[D-G(Q)]^{+} \\ &\qquad -(\rho+d-1)Q\} - 2a \\ &\quad - b\ln\{[\mu - E[D-G(Q)]^{+}] \\ &\qquad \times [Q-\mu+E[D-G(Q)]^{+}]\}. \end{aligned} \tag{13}$$

Applying Lemma 1 to this case, we have

$$E[D-G(Q)]^{+} \le \frac{\left[\sigma^2 + \rho qQ + (\rho Q-\mu)^2\right]^{1/2} - (\rho Q-\mu)}{2}. \tag{14}$$

Substituting the above relationship into (13), we have lower bound of the expected profit in this case. Consider

$$\begin{aligned} E(Q) \ge \frac{c}{\rho}&\Bigg\{(m+d)\mu - (m+d) \\ &\quad \times \frac{\left[\sigma^2 + \rho qQ + (\rho Q-\mu)^2\right]^{1/2} - (\rho Q-\mu)}{2} \\ &\quad -(\rho+d-1)Q\Bigg\} - 2a + 2b\ln 2 \\ &- b\ln\Big\{2(\rho Q+\mu)(1-\rho)Q - \sigma^2 - \rho qQ \\ &\quad + 2(\mu+\rho Q-Q) \\ &\quad \times \left[\sigma^2 + \rho qQ + (\rho Q-\mu)^2\right]^{1/2}\Big\}. \end{aligned} \tag{15}$$

The right-hand side of (15) is a continuous function in terms of Q. Then, first and second derivatives of $E(Q)$ can be derived as

$$\begin{aligned} &\frac{dE(Q)}{dQ} \\ &= -\frac{c(m+d)}{2}\left[\frac{1}{2}X^{-1/2}(q-2\mu+2\rho Q)-1\right] - \frac{c}{\rho}(\rho+d-1) \\ &\quad - b\big(2(1-\rho)(\mu+2\rho Q) - \rho q - 2(1-\rho)X^{1/2} \\ &\qquad + \rho(\mu+\rho Q-Q)(q-2\mu+2\rho Q)X^{-1/2}\big) \\ &\quad \times \big(2(\rho Q+\mu)(1-\rho)Q - \sigma^2 \\ &\qquad - \rho qQ + 2(\mu+\rho Q-Q)X^{1/2}\big)^{-1}, \end{aligned} \tag{16}$$

where $X = \sigma^2 + \rho qQ + (\rho Q-\mu)^2$

$$\begin{aligned} \frac{d^2E(Q)}{dQ^2} &= -\frac{c(m+d)}{2}\left[\frac{-\rho}{4}(q-2\mu+2\rho Q)^2X^{-3/2} + \rho X^{-1/2}\right] \\ &\quad - b\frac{Y'Z - YZ'}{Z^2}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} Y &= 2(1-\rho)(\mu+2\rho Q) - \rho q - 2(1-\rho)X^{1/2} \\ &\quad + \rho(\mu+\rho Q-Q)(q-2\mu+2\rho Q)X^{-1/2}, \\ Z &= 2(\rho Q+\mu)(1-\rho)Q - \sigma^2 - \rho qQ \\ &\quad + 2(\mu+\rho Q-Q)X^{1/2}, \\ Y' &= 4\rho(1-\rho) - 2\rho \\ &\quad \times [(1-\rho)(q-2\mu+2\rho Q) - \rho(\mu+\rho Q-Q)]X^{-1/2} \\ &\quad - \frac{\rho^2}{2}(\mu+\rho Q-Q)(q-2\mu+2\rho Q)^2X^{-3/2}, \\ Z' &= 2(1-\rho)(2\rho Q+\mu) - \rho q \\ &\quad - 2(1-\rho)X^{1/2} + \frac{\rho}{4}(\mu+\rho Q-Q) \\ &\quad \times (q-2\mu+2Q)X^{-1/2}. \end{aligned} \tag{18}$$

Obviously, $d^2E(Q)/dQ^2$ is not necessarily being negative.

Similarly, we develop a solution procedure to find the optimal order quantity in this random yield case.

Step 1. Start $i = 0$ and $Q_0 = \mu$. Set the allowable tolerance ε.

Step 2. Perform Newton's search (see Hillier and Lieberman [24], pp.555–557) to compute the optimal order quantity Q. That is, $Q_{i+1} = Q_i - (f'(Q_i)/f''(Q_i))$, where $f'(Q_i)$ and $f''(Q_i)$ stand for (16) and (17), respectively. Stop the search when $|Q_{i+1} - Q_i| \le \varepsilon$. The optimal order quantity is Q_{i+1}.

Step 3. For verifying adequacy of Newton's method, substitute Q_{i+1} into (19); if $d^2E(Q_{i+1})/dQ_{i+1}^2 < 0$, representing Newton's method, is satisfactory, then the final solution is $Q^* = Q_{i+1}$ whose $E(Q^*)$ is the vendor's lower bound of the maximized expected profit; otherwise, go to Step 4 to perform the bisection optimization method.

Step 4. Select l, a quantifiable order quantity. Start $i = 0$ and let $[Q_0^s, Q_0^*]$ be the initial searching interval, where $Q_0^s = \exp(1 - a/b)$ is the regulated minimal quantity level of delivery for the transportation cost $\mathrm{tc}(Q) = a + b\ln Q$ and $Q_0^* = Q^*$.

Step 5. If $|Q_i^* - Q_i^s| < l$, then stop; the optimal order quantity is $Q_i^{**} = (Q_i^s + Q_i^*)/2$ along with $E(Q_i^{**})$, the lower bound of maximal expected profit; otherwise, let $Q_i^b = (Q_i^s + Q_i^*)/2$.

Step 6. If $E(Q_i^b) \ge E(Q_i^*)$, then $Q_{i+1}^* = Q_i^b$ and $Q_{i+1}^s = Q_i^*$; otherwise, $Q_{i+1}^* = Q_i^*$ and $Q_{i+1}^s = Q_i^b$. Go back Step 5 with $i = i + 1$.

6.1. An Example. We continue Section 3. We assume that, for each unit of Q, the probability of being good is $\rho = 0.9$. We find the optimal order quantity Q^*=10.403, and the lower bound of the maximum expected profit $E(Q^*)$ is 14.573. The condition $d^2E(Q_{i+1})/dQ_{i+1}^2 = -0.916 < 0$ is satisfactory. In contrast, the order quantity placed on the product with perfect quality can be computed as much as 8.731, which is smaller than Q^* = 10.403. Apparently, in therandom yield case the order quantity is increased to provide safeguard against a possible shortage.

7. The Multiproduct Case with Transportation Cost

We now study a multiproduct newsboy problem in the presence of a budget constraint, also known as the stochastic product-mixed problem [26]. Suppose that each product j, for $j = 1,\ldots,N$, has order quantity Q_j received from either purchasing or manufacturing, where a limited budget is allocated due to the limited production capacity in the system. That is, the total purchasing or manufacturing cost for all the N competing products cannot exceed allotted budget B. Denote that each item's unit cost of the jth product is c_j, its unit selling price is p_j, and its unit salvage value is s_j. For the jth product's demand, its mean and variance are denoted by μ_j and σ_j^2, respectively.

In the sequel, under the distribution-free demand jointed with the explicit transportation cost, the vendor is in need of deciding the optimal order quantities for N competing products whose total purchasing or manufacturing cost does not exceed the allocated budge B, where he/she guarantees to possess the least of all possible maximum expected profits.

For solving this problem, we first extend the single product case in (3) to have lower bound of expected profit $E(Q_1,\ldots,Q_N)$ for the vendor, provided that the individual order quantity of $Q_1, Q_2,\ldots$, and Q_N is affected by the budget constraint B. For the vendor to secure the least amount of the maximum expected profit over various situations of market, we maximize (19) with a budget constraint expressed in (20) to determine the optimal order quantities Q_1^*, $Q_2^*,\ldots$, and Q_N^*:

$$\max_{Q_1,\ldots,Q_N} \sum_{j=1}^{N} \Bigg\{ \left(p_j - s_j\right) \frac{\left(Q_j + \mu_j\right) - \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2}}{2} - \left(c_j - s_j\right) Q_j - 2a + 2b \ln 2 - b \ln \Big\{ - \mu_j^2 - 1 + Q_j^2 + 2\mu \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2} \Big\} \Bigg\}, \tag{19}$$

$$\text{Subject to} \quad \sum_{j=1}^{N} c_j Q_j \le B. \tag{20}$$

We further transfer the problem into an unconstrained optimization equation:

$$\begin{aligned} L\left(Q_1,\ldots,Q_N,\lambda\right) &= \sum_{j=1}^{N} \Bigg\{ \left(p_j - s_j\right) \frac{\left(Q_j + \mu_j\right) - \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2}}{2} \\ &\quad - \left(c_j - s_j\right) Q_j - 2a + 2b \ln 2 \\ &\quad - b \ln \Big\{ - \mu_j^2 - 1 + Q_j^2 \\ &\quad + 2\mu_j \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2} \Big\} \Bigg\} \\ &\quad + \lambda \left(\sum_{j=1}^{N} c_j Q_j - B \right), \end{aligned} \tag{21}$$

where λ is the Lagrange multiplier. Hence, we have

$$\begin{aligned} \frac{\partial L\left(Q_1,\ldots,Q_N,\lambda\right)}{\partial Q_j} &= \frac{p_j + s_j - 2c_j}{2} - \frac{\left(p_j - s_j\right)\left(Q_j - \mu_j\right)}{2\left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2}} \\ &\quad - b \frac{2Q_j + 2\mu_j \left(Q_j - \mu_j\right) \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{-1/2}}{-1 - \mu^2 + Q_j^2 + 4\mu_j \left[\sigma_j^2 + \left(Q_j - \mu_j\right)^2\right]^{1/2}} + \lambda c_j. \end{aligned} \tag{22}$$

To find the optimal order quantities $Q_1^*, Q_2^*,\ldots$, and Q_N^* with maximum L, we set $\partial L/\partial Q_j = 0$. In this case, a line search procedure is developed.

Step 1. For multiple products N, let $j = \{1,\ldots,N\}$.

Step 2. Let $\lambda = 0$ and perform the solution procedure proposed in Section 3 to find Q_j^*. If (20) is satisfied, go to Step 6; otherwise, go to Step 3.

Step 3. Substituting each of $Q_1^*, Q_2^*,\ldots$, and Q_N^* into (22), their corresponding λ can be obtained.

Step 4. Start from the smallest nonnegative λ, let its corresponding optimal order quantity be 0 (others are intact), and check the condition of (20).

Step 5. If the condition is satisfactory, then we have the final solution $Q_1^*, Q_2^*,\ldots$, and Q_N^*; otherwise, select the next smallest nonnegative λ to perform the same procedure in Step 4 until (20) is satisfied.

Step 6. Find the least amount of the maximum expected profit $E(Q_1*,\ldots,Q_N*)$.

7.1. An Example. The total budget is \$80 for the four items. The relevant data are as follows: $c = (3.5, 2.5, 2.8, 0.5)$, $p = (5, 4, 3.2, 0.6)$, $s = (2.5, 1.2, 1.5, 0.2)$, $\mu = c(9, 8, 12, 23)$, and $\sigma = c(0.5, 1, 0.7, 1)$. Performing the above procedure, we have the following.

Step 1. Let $j = \{1, 2, 3, 4\}$.

Step 2. Let $\lambda = 0$. We solve the four order quantities by using the solution procedure introduced in Section 3. The optimal order quantities $Q_1^* = 8.731$, $Q_2^* = 7.762$, $Q_3^* = 11.072$, and $Q_4^* = 21.243$. Check $\sum_{j=1}^{4} c_j Q_j^* = \$92 > \$80$, where (20) is not satisfied, so we go to Step 3.

Step 3. Performing a simple line search, we increase the optimal value of the Lagrangian multiplier until $\lambda = 0.147$. In this case, its corresponding Q_3^* is set to 0.

Step 4. Since $\sum_{j=1}^{4} c_j Q_j^* = \$61 < \$80$, (20) is satisfied.

Step 5. The optimal order quantities are 8.731, 7.762, 0, and 21.243 and the lower bound of the maximum expected profit is \$21.667.

8. Conclusions and Implications

Models for the distribution-free newsboy problem have been widely introduced over the past two decades to provide the optimal order quantity for securing the vendor with the least amount of the maximum expected profit when facing a variety of situations in modern business environment.

Over the past few years, energy prices have risen significantly so that the transportation of goods has become a vital component for the vendor's logistic-cost function to determine its required purchase quantities. However, impacts of the transportation cost on previous models for the DFNP were inattentive by either overlooking or deeming it as part of implicit components of ordering cost. In this paper, three main contributions along with their managerial implication have been done.

First, we develop the DFNP incorporating the explicit transportation cost into the expected profit function. In particular, the transportation cost is modeled based on the economic theory from transportation disciplines and fitted a nonlinear regression via actual rate data collected from the shipper. In practice, this way has implied that (1) economic trade-off for the optimal transportation cost lies between provided service level and shipped quantity; (2) in the shipment, more weight signifies larger delivery quantity and higher shipment cost; and (3) optimal shipping quantity renders minimum of the transportation cost.

Secondly, since the expected profit function is neither concave nor convex, the optimization problem underlying this generalization is challenging; therefore, we developed analytical and efficient procedures to acquire the optimal policy. As a result of the computational studies, our proposed optimal ordering rules in comparison with the optimal policy recommended by Gallego and Moon [7] increased the lower bound of the maximal expected profit by as much as 4% on average. This result has demonstrated that the expenditure of the inbound/outbound material transportation has become a critical component of a total annual logistic cost function for determining purchase quantities. Effects of transportation have gained substantial recognition in the DFNP.

Thirdly, according to the results of sensitivity analyses, the parameters, such as demand mean and variance, product's unit cost, and transportation cost, are the key decision variables whose joint reckoning is imperative for the optimal policy proposed. Moreover, we proceed to analyses of several practical inventory cases including fixed ordering cost, random yield, and multiproduct case. These studies further demonstrate the impacts of transportation cost as well as the realized-least profit gains drawn from our recommended policies on the DFNP that explicitly incorporates the transportation cost into consideration. In addition, these numerical findings have implied that joint decision, coordinated operation, or integrated management is crucial in lowering the vendor-and-buyer operating cost as well as balancing a supply-chain operation and structure.

Finally, based on the shipping data sets collected from United Parcel Service (UPS), the transportation cost is modeled using a natural logarithm for a nonlinear regression function in this paper. For future studies, other functional forms may be reckoned to model different transportation costs, such as a step function or a logistic function, to validate a wide variety of applications. Besides, using our proposed model as a basis model in a couple of more advanced studies with certain circumstances, such as the multiproduct newsboy under a value-at-risk and the multiple newsvendors with loss-averse preferences, is intriguing.

Highlights

(i) We extend previous work on the distribution-free newsboy problem where the vendor's expected profit is in presence of transportation cost.

(ii) The transportation cost is formulated as a function of shipping quantity and modeled as a nonlinear regression form based on UPS's on-site shipping-rate data.

(iii) The comparative studies have demonstrated significant positive impacts by using our proposed methodology whose profit gains in comparison with prior research can be as much as 9% and 4% on average.

(iv) The sensitivity analyses jointly reckon the imperative parameters for the optimal policy.

(v) We expand our methodology to several practical inventory cases including fixed ordering cost, random yield, and a multiproduct condition.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Undesirable Outputs' Presence in Centralized Resource Allocation Model

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Data envelopment analysis (DEA) is a common nonparametric technique to measure the relative efficiency scores of the individual homogenous decision making units (DMUs). One aspect of the DEA literature has recently been introduced as a centralized resource allocation (CRA) which aims at optimizing the combined resource consumption by all DMUs in an organization rather than considering the consumption individually through DMUs. Conventional DEA models and CRA model have been basically formulated on desirable inputs and outputs. The objective of this paper is to present new CRA models to assess the overall efficiency of a system consisting of DMUs by using directional distance function when DMUs produce desirable and undesirable outputs. This paper initially reviewed a couple of DEA approaches for measuring the efficiency scores of DMUs when some outputs are undesirable. Then, based upon these theoretical foundations, we develop the CRA model when undesirable outputs are considered in the evaluation. Finally, we apply a short numerical illustration to show how our proposed model can be applied.

1. Introduction

Data envelopment analysis (DEA) was introduced in 1978. DEA includes many models for assessing the efficiency score in the variety of conditions. Many researchers use this technique to evaluate the efficiency and inefficiency scores of decision making units (DMUs). Two of the most common DEA models are CCR (Charnes, Cooper, and Rhodes) and BCC (Banker, Charnes, and Cooper) which were introduced by Charnes et al. [1] and Banker et al. [2], respectively. In addition, there are other important models such as additive (ADD) model which was introduced by Charnes et al. [3] and SMB model (slack-based measure) which was introduced by Tone [4]. Classical DEA models (such as CCR, BCC, ADD, and SMB) rely on the assumption that inputs have to be minimized and outputs have to be maximized. In authentic situations, however, it is possible that the production process consumes undesirable inputs and/or generates undesirable outputs [5, 6]. Consequently, classical DEA models need to be modified in order to deal with the situation because undesirable outputs should not maximize at all.

There frequently exist undesirable inputs and/or outputs in the real applications. Many studies have been done on the undesirable data. Broadly, we can divide these studies into two parts. The first part involves some methods, which use transformation data. For instance, Koopman [6] suggested data transformation. Although the reflection function was used in this method, it caused the positive data to turn into negative data and it was not straightforward to define efficiency score for negative data at that time. Some of the related methods had been suggested by Iqbal Ali and Seiford [7], Pastor [8], Scheel [9], and Seiford and Zhu [10]. However, Golany and Roll [11] and Lovell and Pastor [12] attempted to introduce another form of transformation, which was multiplicative inverse. Being a nonlinear transformation, its behaviors were even more complicated to deal with (Scheel [13]). Therefore, the approaches based on data transformation may unexpectedly produce unfavorable results, such as those discussed by Liu and Sharp [14]. The second part consists of many methods, which can avoid data transformation. As an initial attempt, Liu and Sharp [14] suggested considering undesirable outputs as desirable inputs but undesirable inputs as desirable outputs. This method is currently used as an attractive one in studying operational efficiency because of its simplicity and elegance.

In many authentic situations, there are cases in which all DMUs are under the control of a centralized decision maker (DM) that oversees them and tends to increase the efficiency of all of the system instead of increasing the efficiency of each unit separately. These situations occur when all of the units belong to the same organization (public and/or private) which provides the units with the necessary resources to obtain their outputs, such as bank branches, restaurant chains, hospitals, university departments, and schools. Thus, DM's goal is to optimize the resource utilization of all DMUs across the total entity. Lozano and Villa [15] first introduced the meaning of centralized resource allocation. They presented the envelopment and multiplier form of BCC model with regard to centralized meaning. Mar-Molinero et al. [16] demonstrated that the centralized resource allocation model proposed by Lozano and Villa [15] can be substantially simplified. There are some other similar researches done by Korhonen and Syrjänen [17], Du et al. [18], and Asmild et al. [19]. Multiple-objective model has been used in order to optimize the efficiency of system by Korhonen and Syrjänen [17], and Du et al. [18] proposed another approach for optimization in centralized scenario. Asmild et al. [19] reformulated the centralized model proposed by Lozano and Villa [15] considering adjustments of inefficient units. Hosseinzadeh Lotfi et al. [20] and Yu et al. [21] are other researchers engaged in centralized resource allocation.

In this paper we discuss a DEA model in centralized resource allocation when some of the inputs or outputs are undesirable. This paper is organized as follows. In Section 2, research motivation of this study is given. Section 3 briefly presents some methods for measuring the efficiency scores when some of the outputs are undesirable. Section 4 discusses the centralized resource allocation model and its advantages. We develop the centralized resource allocation model in the undesirable outputs' presence in Section 5. An illustration is given in Section 6 and Section 7 provides the conclusion of the paper.

2. Research Motivation

Traditional DEA models are consecrated to the performance evaluation of DMUs in different situations. Although undesirable outputs treatments have been studied by interested researchers, centralized resource allocation has never dealt with undesirable outputs. Moreover, in many real situations, the production of undesirable outputs is unavoidable; hence, decision makers need scientific methods to deal with the undesirable outputs' production and decrease them when all of DMUs are under their control. Here, we will answer the following question scientifically: how can centralized resource allocation model be modified in order to evaluate the performance of a system involving several DMUs which produce both desirable and undesirable outputs?

3. Undesirable Output Models

Most researchers recently analyze closely the structure of the undesirable data. Undesirable outputs, such as air purification, sewage treatment, and wastewater, can be jointly produced with desirable outputs. When the undesirable outputs are taken into account, the efficiency score's evaluation of DMUs is different. Therefore, traditional DEA models should be modified. Briefly, we review a couple of methods to measure the efficiency scores when some of the data are undesirable and we address some papers for evaluating undesirable data.

Seiford and Zhu [10] showed that the traditional DEA model is used to improve the performance through increasing the desirable outputs and decreasing undesirable outputs by the classification invariance property use. Their model can also be applied to a situation when inputs need to be increased to improve the performance. This model is as follows:

$$\begin{aligned} \max \quad & \phi \\ \text{s.t.} \quad & \lambda X \leq x_o^D \\ & \lambda Y^D \geq \phi y_o^D \\ & \lambda \overline{Y}^U \geq \phi \overline{y_o}^U \\ & e\lambda = 1 \\ & \lambda \geq 0, \end{aligned} \tag{1}$$

in which $\overline{y_o}^U = -Y^U + v > 0$. Hadi Vencheh et al. [22] proposed a model for treating undesirable factors in the framework of DEA as follows:

$$\begin{aligned} \max \quad & \phi \\ \text{s.t.} \quad & \lambda X^D \leq (1-\phi)\, x_o^D \\ & \lambda \overline{X}^U \leq (1-\phi)\, \overline{x_o}^U \\ & \lambda Y^D \geq (1+\phi)\, y_o^D \\ & \lambda \overline{Y}^U \geq (1+\phi)\, \overline{y_o}^U \\ & e\lambda = 1 \\ & \lambda \geq 0, \end{aligned} \tag{2}$$

in which $\overline{y_o}^U = -Y^U + v > 0$ and $\overline{X}^U = -X^U + w > 0$ (Seiford and Zhu [10]). Model (2) evaluates the efficiency level of each DMU by considering desirable and undesirable factors. In fact, model (2) expands desirable outputs and contracts undesirable outputs. A similar discussion holds for the inputs. Jahanshahloo et al. [23] presented an alternative method to deal with desirable and undesirable factors (inputs and outputs) in nonradial DEA models. They demonstrated that their proposed model is feasible, bounded, and unit invariant. The model is given as follows:

$$\begin{aligned}
\min \quad & 1-\left[w_o+\frac{1}{m+s}\left(\sum_{i\in I_D} t_i^{-D}+\sum_{r\in O_D} t_r^{+D}\right)\right] \\
\text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij}^{D}+t_i^{-D}=x_{io}^{D}-w_o \quad i\in I_D \\
& \sum_{j=1}^{n}\lambda_j x_{ij}^{U}+t_i^{-U}=x_{io}^{U}+w_o \quad i\in I_U \\
& \sum_{j=1}^{n}\lambda_j y_{rj}^{D}-t_r^{+D}=y_{ro}^{D}+w_o \quad r\in O_D \\
& \sum_{j=1}^{n}\lambda_j y_{rj}^{U}-t_r^{+U}=y_{ro}^{U}-w_o \quad r\in O_U \\
& \sum_{j=1}^{n}\lambda_j=1,
\end{aligned} \tag{3}$$

in which all variables are restricted to be nonnegative. In model (3), I_D, I_U, O_D, and O_U stand for desirable inputs, undesirable inputs, desirable outputs, and undesirable outputs, respectively. Recently, Wu and Guo [24] suggested a model for measuring the efficiency score which is formulated based on that inputs and undesirable outputs are decreased proportionally. This model is as follows:

$$\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n}\lambda_j x_{ij}\le\theta x_{io} \quad \forall i\in I \\
& \sum_{j=1}^{n}\lambda_j y_{rj}^{D}\ge y_{ro}^{D} \quad \forall r\in O^D \\
& \sum_{j=1}^{n}\lambda_j y_{rj}^{U}\le\theta y_{ro}^{U} \quad \forall r\in O^U \\
& \lambda_j\ge 0 \quad \forall j\in N.
\end{aligned} \tag{4}$$

In model (4), I, O^D, and O^U refer to inputs, desirable outputs, and undesirable outputs sets, respectively. The studies of Scheel [9] and Amirteimoori et al. [25] are another two studies. Indeed, Scheel [9] proposed new efficiency measures which are oriented to desirable and undesirable outputs, respectively. They are based on the assumption that any change of output levels involves both desirable and undesirable outputs. Amirteimoori et al. [25] presented a DEA model which can be used to improve the relative performance via increasing undesirable inputs and decreasing undesirable outputs.

4. Centralized Resource Allocation Model

Measuring the performance plays an important role for a DM providing its weaknesses for the subsequent improvement. Working on the usual DEA framework, assume that there are n DMUs (DMU_j, $j=1,\dots,n$) which consume m inputs (x_i, $i=1,\dots,m$) to produce s outputs (y_r, $r=1,\dots,s$). The first phase of CRA input-oriented model (CRA-I) developed by Lozano and Villa [15] measures the efficiency of system through solving the following linear program:

$$\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij}\le\theta\sum_{j=1}^{n}x_{ij} \quad i=1,\dots,m \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj}\ge\sum_{j=1}^{n}y_{rj} \quad r=1,\dots,s \\
& \sum_{j=1}^{n}\lambda_{jk}=1 \quad k=1,\dots,n \\
& \lambda_{jk}\ge 0 \quad k,j=1,\dots,n.
\end{aligned} \tag{5}$$

In Phase II of CRA model, an additional reduction of any inputs or expansion of any outputs is followed. Phase II is formulated to remove any possible input excesses and any output shortfalls as follows:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{m}s_i^{-}+\sum_{r=1}^{s}t_r^{+} \\
\text{s.t.} \quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij}+s_i^{-}=\theta^{*}\sum_{j=1}^{n}x_{ij} \quad i=1,\dots,m \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj}-t_r^{+}=\sum_{j=1}^{n}y_{rj} \quad r=1,\dots,s \\
& \sum_{j=1}^{n}\lambda_{jk}=1 \quad k=1,\dots,n \\
& s_i^{-}\ge 0, \qquad t_r^{+}\ge 0 \quad i=1,\dots,m,\ r=1,\dots,s \\
& \lambda_{jk}\ge 0 \quad k,j=1,\dots,n.
\end{aligned} \tag{6}$$

Model (5) was formulated based on two important purposes. First, instead of reducing the inputs of each DMU, the aim is to reduce the total amount of input consumption of the DMUs. Second, after solving the problem in Phase II, the projection of all DMUs will be onto the efficient frontier of production possibility set. Indeed, the efficiency score of system is more important than efficiency score of each unit in the centralized scenario. For that reason, decision manager (DM) tries to reallocate resources to have a more efficient system. Toward this end, some of the inputs can be transferred from one DMU to other DMUs. It is not necessary to keep the total value of inputs or outputs in original level because the overall consumption may be decreased and the overall production may be increased.

The improvement activity of DMU_o, which is obtained by the maximum slack solution and is located on the efficiency frontier of production possibility set, is defined as follows:

$$\overline{x_{io}} = \sum_{j=1}^{n} \lambda_j^{o*} x_{ij} = \theta^* x_{io} - s_i^{-*} \quad i = 1, \ldots, m,$$
$$\overline{y_{ro}} = \sum_{j=1}^{n} \lambda_j^{o*} y_{rj} = y_{ro} + t_r^{+*} \quad r = 1, \ldots, s. \tag{7}$$

The difference between the total consumption of improved activity and the original DMUs in each input and output can be found by the following relationship:

$$S_i = \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} \overline{x_{ij}} \geq 0 \quad i = 1, \ldots, m,$$
$$T_r = \sum_{j=1}^{n} \overline{y_{rj}} - \sum_{j=1}^{n} y_{rj} \geq 0 \quad r = 1, \ldots, s. \tag{8}$$

The dual formulation of the envelopment form of the CRA input oriented model to find the common input and output weights, which maximize the relative efficiency score of a virtual DMU with the average inputs and outputs, can be written as follows:

$$\begin{aligned}
\max \quad & \sum_{j=1}^{n} \sum_{r=1}^{s} u_r y_{rj} + \sum_{k=1}^{n} \zeta_k \\
\text{s.t.} \quad & \sum_{j=1}^{n} \sum_{i=1}^{m} v_i x_{ij} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \zeta_k \leq 0 \quad j, k = 1, \ldots, n \\
& u_r \geq 0 \quad r = 1, \ldots, s \\
& v_i \geq 0 \quad i = 1, \ldots, m.
\end{aligned} \tag{9}$$

The above model has $n^2 + 1$ constraints and $m + s + n$ variables. Solving model (9) derives the common set of weights (CSW). It is worth mentioning that we can use this common set of weights to evaluate the absolute efficiency of each efficient DMU in order to rank them. The ranking adopts the CSW generated from model (9), which makes sense because a DM objectively chooses the common weights for the purpose of maximizing the group efficiency. For instance, the government is interested in measuring the performance of DEA efficient banks. The government would determine one common set of weights based upon the group performance of the DEA efficient banks.

5. Proposed Method

Proposing the model in this study, we used the distance directional function to measure the overall efficiency score of each system. Throughout this method, we deal with n DMUs ($j = 1, \ldots, n$) having m inputs ($i = 1, \ldots, m$) and s outputs. The outputs are divided into two sets: one as desirable outputs and one as undesirable outputs. Let the inputs and desirable and undesirable outputs be as follows:

$$X = \{x_{ij}\} \in R_+^{m \times n}, \qquad Y^D = \{y_{rj}^D\} \in R_+^{s^D \times n},$$
$$Y^U = \{y_{tj}^U\} \in R_+^{s^U \times n}, \tag{10}$$

where X, Y^D, and Y^U are input, desirable output, and undesirable output matrices, respectively. In our proposed model, we apply the distance directional function to reformulate the centralized resource allocation model when some outputs are undesirable. In addition, we consider undesirable outputs as inputs in evaluation. The model is as follows:

$$\begin{aligned}
\max \quad & \varphi \\
\text{s.t.} \quad & \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_{jk} x_{ij} \leq \sum_{j=1}^{n} x_{ij} - \varphi R x_i \quad i = 1, \ldots, m \\
& \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_{jk} y_{rj}^D \geq \sum_{j=1}^{n} y_{rj}^D + \varphi R y_r^D \quad r = 1, \ldots, s^D \\
& \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_{jk} y_{tj}^U \leq \sum_{j=1}^{n} y_{rj}^U - \varphi R y_t^U \quad t = 1, \ldots, s^U \\
& \sum_{j=1}^{n} \lambda_{jk} = 1 \quad k = 1, \ldots, n \\
& \lambda_{jk} \geq 0 \quad k, j = 1, \ldots, n,
\end{aligned} \tag{11}$$

where Rx_i, Ry_r^D, and Ry_t^U are parameters; also s^D and s^U stand for the number of desirable outputs and undesirable outputs, respectively. The objective of model (11) is to decrease inputs and undesirable outputs level and increase desirable outputs level with regard to the (Rx_i, Ry_r^D, Ry_t^U) direction. Here, we use the ideal point to assign to the (Rx_i, Ry_r^D, Ry_t^U) vector as follows:

$$Rx_i = \sum_{j=1}^{n} x_{ij} - n\left(\min \{x_{ij}\}_{j=1,\ldots,n}\right) \quad i = 1, \ldots, m,$$
$$Ry_r^D = \sum_{j=1}^{n} y_{rj}^D - n\left(\max \{y_{rj}^D\}_{j=1,\ldots,n}\right) \quad r = 1, \ldots, s^D, \tag{12}$$
$$Ry_t^U = \sum_{j=1}^{n} y_{tj}^U - n\left(\min \{y_{tj}^U\}_{j=1,\ldots,n}\right) \quad t = 1, \ldots, s^U.$$

The optimal objective value of model (11) measures system inefficiency score. It is worth mentioning that another alternative for the directional vector (Rx_i, Ry_r^D, Ry_t^U) can be chosen as follows:

$$\left(Rx_i, Ry_r^D, Ry_t^U\right) = \left(\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} y_{rj}^D, \sum_{j=1}^{n} y_{tj}^U\right). \tag{13}$$

The purposes of model (11) are to reduce the total consumption of inputs, reduce the total production of undesirable

Table 1: Data set with undesirable outputs.

	Inputs		Desirable outputs		Undesirable outputs	
	I1	I2	O1	O2	UO1	UO2
DMU 1	5	8	9	15	4	3
DMU 2	7	5	12	19	9	7
DMU 3	5	4	18	21	4	3
DMU 4	6	8	14	11	10	6
DMU 5	7	7	11	14	8	8
DMU 6	8	3	10	17	4	9
DMU 7	5	5	16	10	6	5
DMU 8	4	9	19	9	5	2
Sum	47	49	109	116	50	43
			Projection points			
DMU 1	5	8	9	15	4	3
DMU 2	7	5	12	19	9	7
DMU 3	5	4	18	21	4	3
DMU 4	6	8	14	11	10	6
DMU 5	7	7	11	14	8	8
DMU 6	8	3	10	17	4	9
DMU 7	5	5	16	10	6	5
DMU 8	4	9	19	9	5	2
Sum	39.2	36	144.8	158.4	32.8	23.2

Table 2: Current and optimized levels of the entire system.

	Inputs		Desirable outputs		Undesirable outputs	
	I1	I2	O1	I1	I2	O1
Current level	47	49	109	116	50	43
Optimal level	39.2	36	144.8	158.4	32.8	23.2
Rate of reduction or increase	16.5%	26.5%	24.7%	26.7%	34.4%	46%

outputs, and increase the overall production of desirable outputs in the direction of (Rx_i, Ry_r^D, Ry_t^U), simultaneously. It should be pointed out that undesirable outputs are considered as inputs in the proposed model.

6. Numerical Example

To illustrate the proposed model (11), consider that a system consists of 8 DMUs and that each DMU consumes two inputs to produce four outputs (two desirable outputs and two undesirable outputs). Table 1 shows the data.

The efficiency score of the entire system can be readily obtained by using model (11), which is 48%. Moreover, the projection points are shown in Table 1. As can be seen from Table 2, we can compare the observed system with the projected system. For instance, model (11) suggests 16.5% and 26.5% saving (reduction) in the first and second inputs, respectively. In addition, by using model (11) to project all of DMUs onto the efficient frontier, DM could have 24.7% and 26.7% increases in producing the desirable output 1 and output 2, respectively.

Increasing the production of desirable output 1 from 109 (current level) to 144.8 (optimum level) and increasing the production of desirable output 2 from 116 (current level) to 158.4 (optimum level) are meaningful. Model (11) also has a significant reduction plan in both undesirable outputs, that is, decreasing the production level of undesirable output 1 from 50 to 32.8 (34.4% reduction) and decreasing the level of production of undesirable output 2 from 43 to 23.2 (46% reduction).

7. Conclusion

The issue of dealing with undesirable data in CRA is an important topic. The existing CRA models have been focused on desirable inputs and outputs. In this paper, we developed an approach proposed by Lozano and Villa [15] for dealing with undesirable outputs by using distance directional function. The CRA model presented here can be used for the analysis of any real situations where a significant number of desirable and undesirable outputs are included.

Moreover, the proposed model is able to suggest a managerial point of view to DM to make decision and come up with a plan for the system. In a similar way, the proposed model can be reformulated to deal with undesirable inputs' treatment in centralized resource allocation scenario. On the basis of the promising findings presented in this paper, work on the remaining issues is continuing and will be presented in future papers. Clearly, in our future research, we intend to concentrate on CRA model with imprecise, interval, and fuzzy data.

Conflict of Interests

The authors have no conflict of interests to disclose.

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Research Article

The Integration of Group Technology and Simulation Optimization to Solve the Flow Shop with Highly Variable Cycle Time Process: A Surgery Scheduling Case Study

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Surgery scheduling must balance capacity utilization and demand so that the arrival rate does not exceed the effective production rate. However, authorized overtime increases because of random patient arrivals and cycle times. This paper proposes an algorithm that allows the estimation of the mean effective process time and the coefficient of variation. The algorithm quantifies patient flow variability. When the parameters are identified, takt time approach gives a solution that minimizes the variability in production rates and workload, as mentioned in the literature. However, this approach has limitations for the problem of a flow shop with an unbalanced, highly variable cycle time process. The main contribution of the paper is to develop a method called takt time, which is based on group technology. A simulation model is combined with the case study, and the capacity buffers are optimized against the remaining variability for each group. The proposed methodology results in a decrease in the waiting time for each operating room from 46 minutes to 5 minutes and a decrease in overtime from 139 minutes to 75 minutes, which represents an improvement of 89% and 46%, respectively.

1. Introduction

Currently, the US healthcare system spends more money to treat a given patient whenever the system fails to provide good quality and efficient care. As a result, healthcare spending in the US will reach 2.5 trillion dollars by 2015, which is nearly 20% of the gross domestic product (GDP). A similar trend is observed by the Organization for Economic Cooperation and Development (OECD), which included Taiwan. The cost of increased healthcare spending will become more important in the coming years. One way to decrease the cost of healthcare is to increase efficiency.

The demand for surgery is increasing at an average rate of 3% per year. To increase access, operating rooms (ORs) must invest in related training for specialized nursing and medical staff. ORs will be a hospital's largest expense at approximately \$10–30/min and will account for more than 40% of hospital revenue [1]. Two types of surgical services are provided by ORs: reaction to unpredictable events in the emergency department (ED) and elective cases, where patients have an appointment for a surgical procedure on a particular day. This paper considers elective cases because an important part of the variance can be controlled by reducing flow variability [2]. The efficiency of ORs not only has an impact on the bed capacity and medical staff requirement but also impacts the ED [3]. Therefore, increasing OR efficiency is the motivation for this study.

Utilization is usually the key performance indicator for OR scheduling. Maximum productivity requires high utilization. However, in combination with high variability, high utilization results in a long cycle time, according to Little's Law [4], as shown in Figure 1. High utilization and low cycle times can be achieved by reducing the flow variability, as shown in Figure 2. Therefore, the identification and reduction of the main sources of variability are keys to optimizing the compromise between throughput and cycle

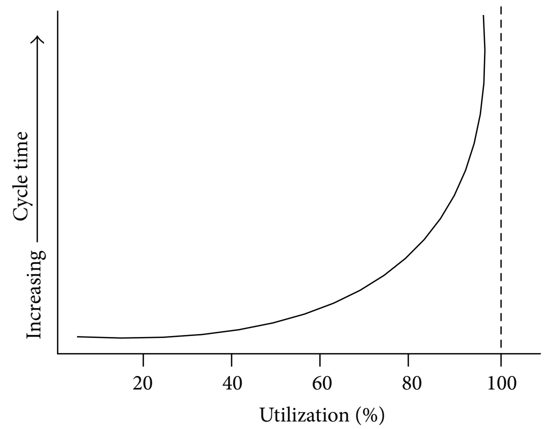

Figure 1: Cycle time versus utilization.

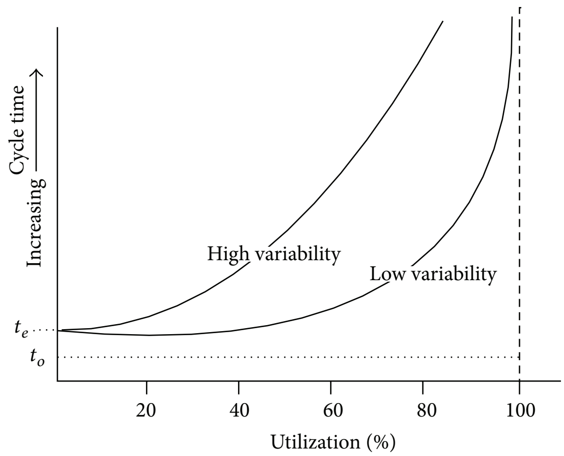

Figure 2: The corrupting influence of variability.

time. Unfortunately, a few measures for flow variability are used in ORs. Such a measure would be highly valuable in reducing variability and would allow more efficient study.

The flow variability determines the average cycle time. There are different sources of variability, such as resource breakdown, setup time, and operator availability. An approach proposed by Hopp and Spearman used the VUT equation to describe the relationship between the waiting time as the cycle time in queue (CT_q), variability (V), utilization (U), and process time (T) for a single process center [5]. The VUT is written in its most general form as (1). This study determines the parameters and the solutions of this equation:

$$\mathrm{CT}_q = VUT. \tag{1}$$

This paper is structured as follows. The analytical VUT equation is applied to a workstation with real surgical scheduling data. The algorithm quantifies the patient flow for the entire OR system and makes the cycle time longer than predicted, due to several parameters. An example then shows the potential of the VUT algorithm for use in cycle time reduction programs. The solution depends on finding the parameters that cause the cycle time variability. A simulation model is used to demonstrate the feasibility of the solution. Finally, the main conclusions and some remarks on future work are given.

2. Literature Review

Timeframe-based classification schemes generally include long, intermediate, and short term processes as follows: (1) capacity planning; (2) process reengineering/redesign; (3) the surgical services portfolio; (4) estimation of the procedural duration; (5) schedule construction; and (6) schedule execution, monitoring, and control [6]. This study focuses on short-term aspects because the shop floor control makes adjustments when the process flow is disrupted by the variability of patients' late arrivals, surgery durations, and resource unavailability in the real world.

The sequencing decision which can be thought of as a list of elements with a particular order and its impact on OR efficiency are addressed in the literature [7, 8]. Most of the studies use a variety of algorithms to improve the utilization under the assumption that the cycle time is deterministic. Studies developed a stochastic optimization model and heuristics to compute OR schedules that reduce the OR team's waiting, idling, and overtime costs [9, 10]. Goldman et al. [11] used a simulation model to evaluate three scheduling policies (i.e., FIFO, longest-case first, and shortest-case first) and concluded that the longest-case first approach is superior to the other two.

Scheduling always struggles to balance capacity utilization and demand in order to let the arrival rate r_a not exceed the effective production rate r_e [12–14]. Then, the utilization at each station is given by the ratio of the throughput to the station capacity ($u = r_a/r_e$). Under the assumption that there is no variability, which includes the assumption that cases are always available at their designated start time, the surgery durations are deterministic and resources never break down. However, it is not possible to predict which patients or staff will arrive late, precisely how long a case will take to perform, or what unexpected problems may delay care [15]. This is why none of a variety of research models has had widespread impact on the actual practice of surgery scheduling over the past 55 years [6]. Therefore, this study will consider these flow variability issues.

Studies show that the management of variability is critical to the efficiency of an OR system. McManus et al. [16] noted that natural variability can be used to optimize the allocation of resources, but no empirical model was included in the study. Managing the variability of patient flow has an effect on nurse staffing, quality of care, and the number of inpatient beds for ED admission and solves the overcrowding problem [17, 18]. However, there is a lack of quantitative analysis to demonstrate which flow variability parameter causes the impact. In summary, this study quantitatively analyzes flow variability, determines which parameters have an impact, and provides relevant solutions for empirical illustration.

Womack et al. [19] stated that high utilization with relatively low cycle time requires a minimum variability.

Although this originates from the Toyota Production System (TPS), its potential applications and in-depth philosophy are not well defined [20]. Different industries apply these principles and develop customized approaches to optimize shop floor processes. The methodology of the study refers to Ohno [21], Monden [22], and Liker [23] for details of development. The five-step process is as follows.

The first step defines the current needs for improvement. Key performance indicators are selected. Performance measures for the OR system fall into two main categories: patient waiting time and staff overtime. Patient waiting is associated with two activities: patients waiting for the preparation of a room and waiting for surgery. There is no waiting time for the recovery process because recovery begins immediately after surgery. Late closure results in overtime costs for nurses and other staff members. A reduction in overtime has a positive effect on the quality of care, decreases surgeons' daily hours, produces annualized cost savings, makes inpatient beds available for ED admission, and positively affects ED overcrowding [17].

The second step incorporates an in-depth analysis of the production line. Before starting detailed time studies, standard movements are observed and mapped. Value stream mapping (VSM) is used to design and analyze an OR's process layer [24]. VSM has a wide perspective and does not examine individual processes. The average cycle time is determined by variability, but VSM does not provide quantifiable evidence and fails to determine how methods can be made more viable. Hopp and Spearman proposed the use of the VUT equation. Equation (2) represents the variability as the sum of the squared coefficients of the variation in the interarrival times, C_a^2, the squared coefficients of the variation in the effective process time, C_e^2, the utilization, U, and the squared coefficients of the variation in departure, C_d^2. The squared coefficient of variation is defined as the quotient of the variance and the mean squared. Therefore, $C_a^2 = \sigma_a^2/t_a^2$ and $C_e^2 = \sigma_e^2/t_e^2$, where t_a and t_e are the mean interarrival time and the mean process time, respectively. The effective process time paradigms, t_e and C_e^2, include the effects of operational time losses due to machine downtime, setup, rework, and other irregularities. Compared with the theoretical process time, t_o, $t_e > t_o$ and $C_e^2 > C_o^2$. C_e^2 is considered low when it is less than 0.5, moderate when it is between 0.5 and 1.75, and high if more than 1.75. Equation (3) shows that, for low utilization, the flow variability of the departing flow equals the variability of the arriving flow, and, for high utilization, the flow variability of the departing flow equals the effective process time variability. The equations give quantifiable evidence of variability:

$$\mathrm{CT}_q = \left(\frac{C_a^2 + C_e^2}{2}\right)\left(\frac{u}{1-u}\right)t_e, \quad (2)$$

$$C_d^2 = u^2 C_e^2 + \left(1-u^2\right)C_a^2. \quad (3)$$

The third step consolidates the current performance data and determines the baseline for efficiency improvement. Because the period of operating time for this study is from 8:00 a.m. to 5:00 p.m., the total overtime after 5:00 p.m. as the baseline per day is 3,336 minutes.

The fourth step defines implementation methods that satisfy the abovementioned subtargets and use the detailed time studies and data analysis from earlier steps. In summary, (2) and (3) clearly show the contribution of variability. The leveling approach minimizes the variability in production rates and work load [25]. However, a leveling approach that only considers a single production level is not applicable to the problem of low volume and high mix production [26]. Only a few papers outline leveling approaches for flow shop environments [27]. The flow shop with an unbalanced, highly variable cycle time process can be solved by takt time grouping [28]. However, this method assumes that the process time for each batch is the same and is not applicable to this study. This study uses a new method of takt time based on group technology to implement the flow environment.

When all of the improvement items are chosen, the fifth step ensures their sustainable implementation. Discrete-event simulation is used to model the behavior of a complex system. By simulating the process, the system behavior is observed and the potential improvements after changes can be evaluated [29]. However, grouping and leveling are still required to achieve the optimal solution for a given problem.

3. Case Description by the Current-State VSM and VUT Equation

3.1. The Current-State VSM. The case studied in this paper is from a Taiwanese medical center that has 21,350 surgical cases per year. The surgical department consists of 24 operating rooms, 15 of which are for specialty procedures. In identifying the overall flow shop procedure using the current-state VSM, which includes the processing time for each process, boxes are used to understand the type of activities that occur in the ORs. VSM allows a visualization of the processes for an entire service rather than just one particular process. This result is plotted in Figure 3. The current value stream mapping shows the cycle time, which includes value-added time and non-value-added time. The non-value-added time is the waiting time, which is 46 minutes.

3.2. The VUT Equation Analysis. To describe the performance of a single workstation, the following parameters are assumed:

t_o: the mean natural process time,

r_a: the arrival rate,

σ_o: the standard deviation for the natural process time,

c_o: the coefficient of variability for the natural process time,

N_s: the average number of cases between setups,

t_s: the mean setup time,

σ_s: the standard deviation for the setup time,

t_e: the mean effective process time,

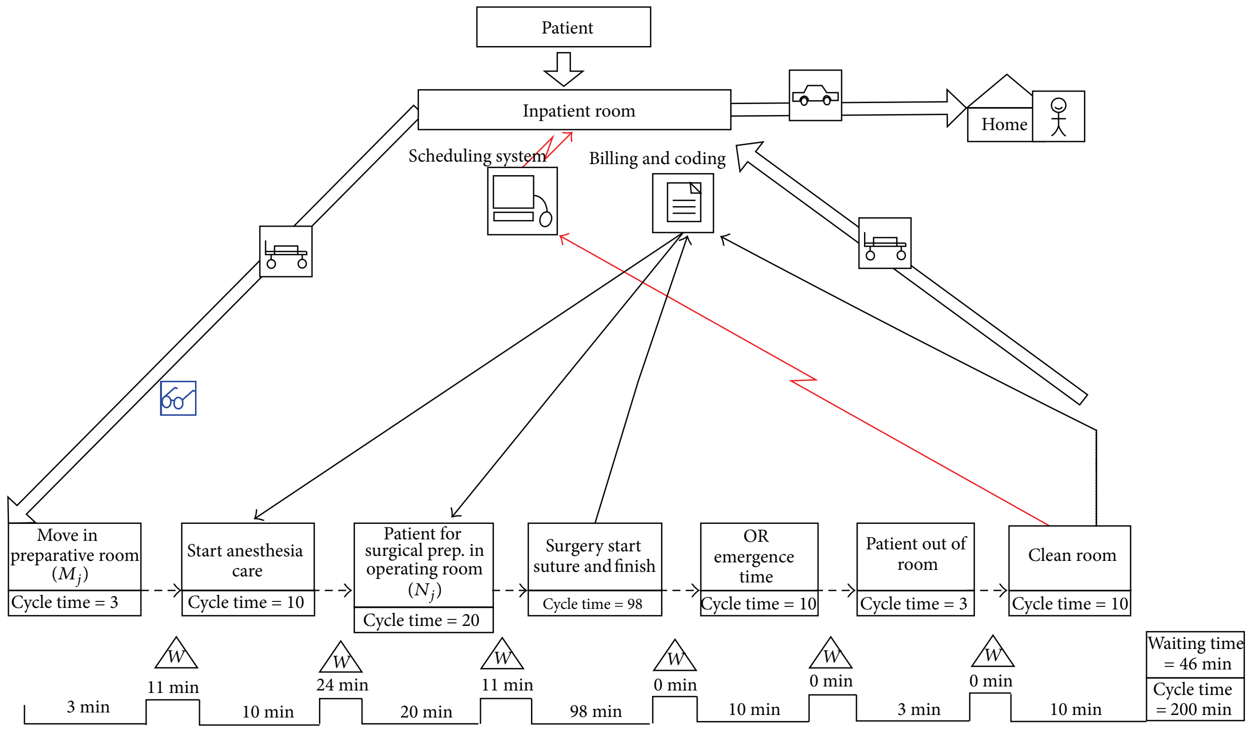

FIGURE 3: The current-state VSM.

σ_e^2: the variance of the effective process time,

c_e^2: the squared coefficient of the variation in the effective process time,

c_a^2: the squared coefficient of the variation in demand arrivals.

The daily surgical scheduling has 80 elective cases on average, according to the effective capacity from 8:00 a.m. to 5:00 p.m. Namely, the arrival rate, r_a, is 8.9 cases/hour. Each patient will go through the two series of stage (W_i), which included the process of preparation (W_1) and operation (W_2). For the worst-case example, at the starting time, patients move into the OR system from wards when the operating room (W_2) is ready. Because the ward and the surgical department are far from each other, the interarrival time is assumed to be exponential ($c_a^2 = 1$). The characterizing flow in the ORs' system passes through the two stages (W_i) shown in Figure 4. The first stage (W_1) checks the patient's documentation, nursing history, and laboratory data. The natural process time mean t_o is 20 minutes, and the natural standard deviation σ_o is 2 minutes. These result in a natural CV of $c_o = \sigma_o/t_o = 0.1$. The capacity of the preparation room (M_j) in the first stage is 12, which is less than the value of 24 for the second stage (N_j), and this is so for all cases. Using a dispatching rule of first-come-first-served (FCFS) in the first stage (W_1), the first stage (W_1) can breakdown under certain conditions (e.g., the patient does not arrive at the start time, when the preparation room (M_j) is ready, or when the number of patients is greater than 12). These situations are called nonpreemptive outages. Specifically, W_1 has a mean time to failure (MTTF), m_f, of 60 minutes and a mean time to repair (MTTR), m_r, of 35 minutes. MTTF is the elapsed time between failures of a system during operation, and MTTR is the average time required to repair a failed operation. The average capacity of W_1 for nonpreemptive outages can be calculated using (4), where the availability $A = 60/(60 + 35) = 0.63$. The effective mean process time, t_e, calculated using (5) is 31.75 minutes. The utilization of the first stage (W_1) is calculated using (6) to be 0.27, and c_e^2 is calculated using (7) as 0.83:

$$A = \frac{m_f}{m_f + m_r}, \tag{4}$$

$$t_e = \frac{t_o}{A}, \tag{5}$$

$$u = \frac{r_a}{r_e} = \frac{r_a t_e}{m}, \tag{6}$$

$$C_e^2 = C_o^2 + 2A(1 - A)\frac{m_r}{t_o}. \tag{7}$$

After the previous patient has left the operating room and following the setup time, the current patient then starts at the second stage (W_2). Both the process time and setup time are stochastic and will be commensurate with the complexity of the disease. The natural mean process time t_o is 120.17 minutes, and the natural standard deviation σ_o is 80.25 minutes. The setup time is regarded as a preemptive outage when they occur due to changes in the following surgery. Trends in the setup time are associated with the type of surgery, and the mean of the setup time t_s is 25.26 minutes and the standard deviation of the setup time σ_s 15.43 minutes.

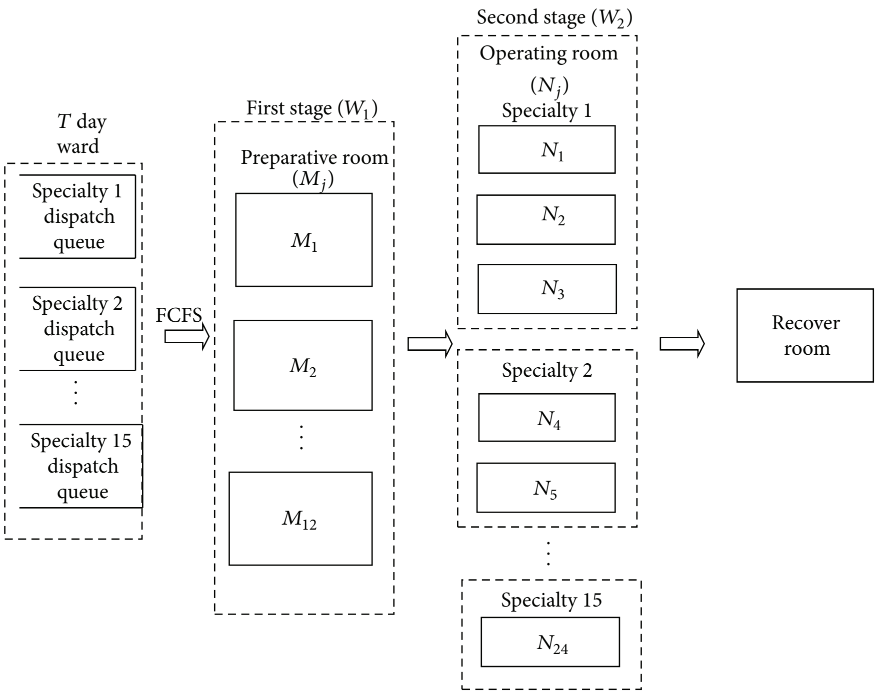

Figure 4: The charactering flow in the ORs' system.

The effective mean process time t_e from (8) is 145.43 minutes. The capacity is 9.9 cases/hour. The utilization of W_2 by (6) is 0.89. Using (9), we can compute $c_e^2 = 7.49$. From the VUT equation, we conclude that this is a stable system in the flow shop with an unbalanced, high variation cycle time process. Consider

$$t_e = t_o + \frac{t_s}{N_s}, \tag{8}$$

$$\sigma_e^2 = \sigma_o^2 + \frac{\sigma_s^2}{N_s} + \frac{N_s - 1}{N_s^2} t_s^2, \qquad c_e^2 = \frac{\sigma_e^2}{t_e^2}. \tag{9}$$

3.3. The Baseline for Efficiency Improvement. The third step consolidates the current performance data and determines the baseline for efficiency improvement. Then, the VUT equation for computing queue time CT_q of W_1 is 10.81 minutes, and c_d^2 is 0.99; however, CT_q of W_2 is 764.74 minutes. After analysis of the VUT (2), we found that the relative differences among the mean of the effective process time t_e and utilization compared to the variability are small. The value of 4.24 comes from two parts: the first is $c_e^2 = 7.49$, which is highly variable based on the process time in the second stage (W_2); the second is $c_a^2 = 0.99$, which is equal to c_d^2 from the first stage (W_1). The departure variability of W_2 depends on the arrival variability of W_1. The $c_e^2 = 0.83$ in the W_1 due to the nonpreemptive outages, which are caused by the interarrival rate from the inpatient ward to the ORs' system. Equations (2) and (3) provide useful models for a deeper understanding of the worst case of natural and flow variability when access to resources is limiting. In practice, balancing the average utilization and the systemic stresses results in a smoother patient flow. Consider

$$\begin{aligned} \mathrm{CT}_q &= \frac{C_a^2 + C_e^2}{2} \frac{u}{1-u} t_e \\ &= \frac{(0.99 + 7.49)}{2} \left(\frac{0.89}{1 - 0.89} \right) 145.43 \\ &= (4.24)(8.09)(145.43). \end{aligned} \tag{10}$$

These are some assumptions in this case study.

(i) The data in analysis of surgical-specific procedure time is the year of 2002.

(ii) Each preparation room (M_j) and operating room (N_j) can process only one case at a time.

(iii) For this study, there should be totally 24 rooms strictly assigned to the different surgical cases. Each case can be carried out in any of the 24 rooms, but each room must be assigned one group at most.

(iv) The period of opening of operating room is from 8:00 a.m. to 5:00 p.m. and the overtime is counted after 5:00 p.m.

(v) Emergency surgeries are not considered. Either patients must have appointments on certain OR days for a medical reason, or any period during which surgeons cannot perform is ignored. In other words, no surgeries are cancelled or added.

(vi) There is no constraint to surgeons or other staff availability. In other words, surgeons are available at any period of the day (i.e., when a case is moved from the morning to the afternoon).

(vii) Each physician can only accept one patient at a time. Once the surgery is started, the operation is not allowed to be interrupted or cancelled. Surgical breakdowns are not considered.

4. Proposed Methodology

The fourth step defines implementation methods that satisfy the abovementioned subtargets and uses the detailed time studies and data analysis from earlier steps. Leveling based on group technology consists of two fundamental steps. In the first step, families are formed for leveling based on similarities. Clustering techniques are used to group families according to their similarities. Using these families, a leveling pattern is created in the second step. Every family and every interval is arranged for a monthly period.

4.1. Group Technology Approach. It has been shown that variability affects the efficiency of the system. Grouping surgeries minimizes the duration variability of surgery [30]. Of these approaches, cluster analysis is the most flexible and therefore the most reasonable method to employ here. K-means is a well-known and widely used clustering method [31]. This method is fast but cannot easily determine the number of groups. If the group is arranged randomly, there will be no obvious difference between each group. Anderberg [32] recommended a two-stage cluster analysis methodology. Ward's minimum variance method is used at first, followed by the K-means method. This is a hierarchical process that forms the initial clusters. Ward's method can minimize the variance through merging the most similar pair of clusters among N elements. Perform those steps until all clusters are merged. The Ward objective is to find out the two clusters whose merger gives the minimum error sum of squares. It determines a number of clusters and then starts the next step. K-means clustering uses the coefficient of variation, which is defined as the ratio of the standard deviation to the mean, as measured by (11). The software SPSS was used for cluster analysis. Consider

$$\text{Coefficient of variation} = \frac{\sigma}{\mu}. \tag{11}$$

4.2. Takt Time Approach. Leveling allocates the volume and variety of surgeries among the ORs' resources to fulfill the patient demand over a defined period of time. The first step in leveling is to calculate the takt time, which is measured by (12). The takt time is a function of time that determines how fast a process must run to meet customer demand [28]. The second step is a pacemaker process selection and leveling of production by both volume and product mix [33]. The pacemaker process must be the only scheduling point in the production system and dictates the production rhythm for the rest of the system, where the pace is based on a supermarket pull system further upstream from this point, as well as First In First Out (FIFO) systems further downstream [34–37]. According to the theory of constraints (TOC), one of the most important points to consider is the bottleneck. Thus, the pacemaker process selection must be located in the second stage (W_2). However, the number of resources for each grouping must still be determined to achieve the optimal solution for a given problem. Consider

$$\text{Takt time} = \frac{\text{Available monthly work time}}{\text{Total monthly volume required}}. \tag{12}$$

4.3. Simulation Modeling and Optimization. The fifth step ensures sustainable implementation. The simulation tool checks the feasibility of integrating the methods into the current system. Simulation is useful in evaluating whether the implementation of the method is justified [38]. Rockwell Arena, a commercial discrete-event simulator, has been used for many studies [39]. To evaluate potential improvements due to the implementation of takt time based on group technology, Rockwell Arena 13.51 was used to build the general simulation model for the OR system. Depending on the nature and the goal of the simulation study, it is classified as either a terminating or a steady-state simulation. This study is a terminating simulation, which signifies that the system has starting and stopping conditions [40].

This study optimizes the capacity buffers against the remaining variability of each surgical group to minimize OR overtime (i.e., work after 5:00 p.m.). Optimization finds the best solution to the problem that can be expressed in the form of an objective function and a set of constraints [41]. Therefore, the difference between the model that represents the system and the procedure that is used to solve the optimization problems is defined within this model. The optimization procedure uses the outputs from the simulation model as an input, and the results of the optimization are fed into the next simulation. This process iterates until the stopping criterion is met. The interaction between the simulation model and the optimization is shown in Figure 5 [42].

5. Empirical Results

5.1. Takt Time Based on a Group Technology Approach Clustering Method. This study focuses on 263 surgical-specific procedures using a Pareto analysis of a total of 1198 types of surgical-specific procedure times in the year 2002. Ward's minimum variance method gives the number of clusters as 5. The following step is segmented into 5 groups, based on Ward's minimum variance method and then K-means clustering, to give the time expression shown in Table 1.

5.2. Takt Time Mechanism. Leveling is used to calculate the takt time for each surgery group. The surgical department organizes the working time according to a monthly time schedule. The monthly time available is 10,800 minutes, as there are 9 hours a day and 5 days in a week in this case. The monthly volume was measured, and the takt time for each group is shown in Table 2.

Table 1: The five groups.

Categories	1	2	3	4	5
Expression	−0.001 + ERLA (28.7, 2)	−0.001 + LOGN (119, 226)	5 + WEIB (91, 0.856)	5 + WEIB (162, 1.2)	5 + GAMM (94.3, 1.51)

Table 2: The monthly volume and takt time of each group.

Group	Monthly time available (minutes)	Monthly volume of surgeries (units)	Takt time (minutes)
1	10800	813	$\frac{10800}{813} \fallingdotseq 13$
2	10800	159	$\frac{10800}{159} \fallingdotseq 68$
3	10800	134	$\frac{10800}{134} \fallingdotseq 81$
4	10800	346	$\frac{10800}{346} \fallingdotseq 31$
5	10800	185	$\frac{10800}{185} \fallingdotseq 58$

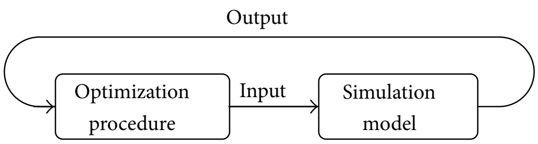

Figure 5: Relationship between simulation model and optimization.

5.3. Simulation Model. Rockwell Arena 13.51 was used to build the simulation model that represents the OR systems. The computer-based module logic design establishes an experimental platform that allows a decision maker to quickly understand the conditions of the system.

When the simulation model is constructed, we wanted to tighten precision cover on the population mean (u); the smaller the confidence interval, the larger the number of required simulation replications. The length of one replication is set as one month. The coefficient of variation (CV), which is defined as the ratio of the sample standard deviation to the sample mean, is used as an indicator of the magnitude of the variance. The value of the CV stabilizes when the number of replications reaches 35, as shown in Figure 6 [43]. We generated the input values from probability distributions in Arena. The simulation model used the time expression with the run length of 1 month and 35 replications. Each replication starts with a both empty and idle system. The individual replication result is independent and identically distributed (IID); we could form a confidence interval for the true expected performance measure μ. In this study, the mean daily cycle time (μ) and the 95% confidence interval are adopted as the system performance measure. We have an initial set of replications 35; we compute a sample average cycle time, 214.28 minutes, and then a confidence interval whose half width is 1.92 minutes. It is noted that the half width of this interval (1.92) is pretty small compared to the value of the center (214.28). The mathematical basis for the above discussion is that, in the 95% of the cases of making 35 simulation replications as we did, the interval formed like this will contain the true expected value of total population.

Table 3: The error between the real system and simulation.

Compare (average)	System	Simulation	Error (%)
Waiting time	46.14	43.10	7

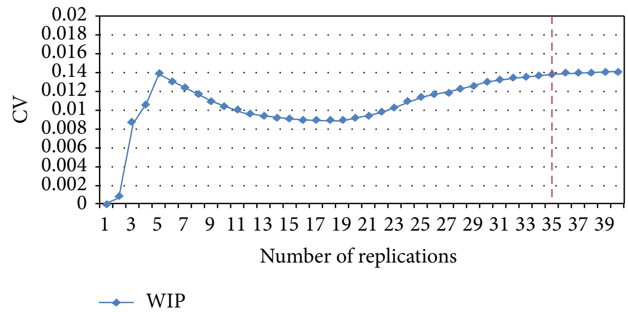

Figure 6: The CV chart.

In this study, simulation models for verification and validation are both used. Verification ensures that the model behaves as intended, and validation ensures that the model behaves like the real system. As shown in Table 3, the error between the simulation and the real system in terms of the daily waiting time in each OR is 7%.

5.4. The Optimal Solution. Identification of the optimal scenario uses one week in July, which in practice is usually 5 days. On each day, each group, i, is available and has an expression time. OptQuest is utilized in conjunction with Arena to provide the optimal solution. The required notations for the formulation are defined as follows.

Parameters:

i = an index for the groups of surgeries, $i \in I$, $I = \{1, 2, 3, 4, 5\}$,

j = an index for the number of operating rooms, $j \in J$, $J = \{1, 2, 3, \ldots, 24\}$.

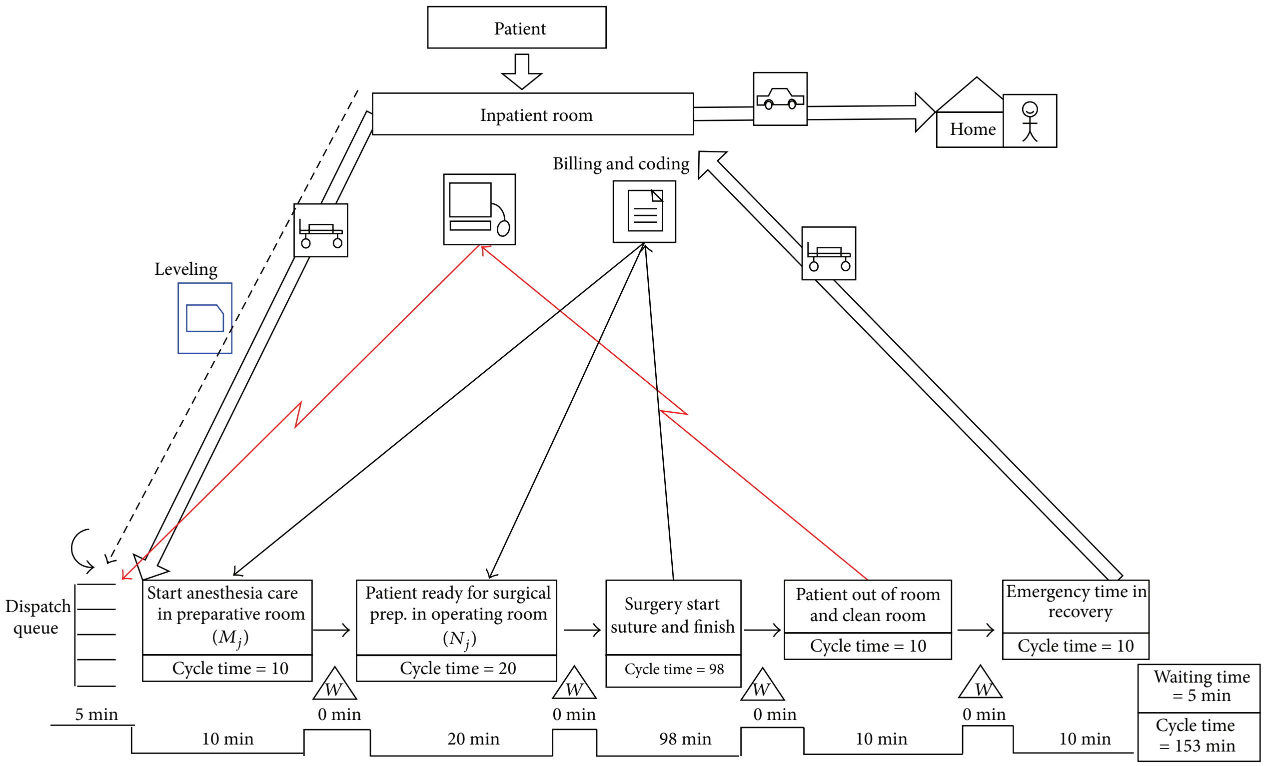

FIGURE 7: The future-state VSM.

Intermediate variables:

O_j = the overtime associated with the ORs.

Decision variables:

A_{ij} = a binary assignment, whether the surgery group, i, is assigned to operating room j (A_{ij} = 1) or not (A_{ij} = 0);

C_i = an index for the number of operating rooms that are allocated to the surgery group, i.

The optimization model solves

$$\text{Minimize } \sum_{j=1}^{24} O_j, \tag{13}$$

subject to the following constraints:

$$\sum_{i=1}^{5} A_{ij} = 1 \quad \forall j, \tag{14}$$

$$C_i \geq 1 \quad \forall i, \tag{15}$$

$$\sum_{i=1}^{5} C_i = 24, \tag{16}$$

$$A_{ij} \in \{0, 1\} \quad \forall ij. \tag{17}$$

The objective function minimizes the total amount of overtime. Constraint (14) specifies that each operating room must be assigned to one group at most. Constraint (15) ensures that each group is allocated at least in one operating room. Constraint (16) sets the limitation of operating rooms for all groups. Constraint (17) as a binary assignment, is whether the surgery group, i, is assigned to operating room j.

5.5. The Result. The results are plotted in Figure 7. The capacity buffers optimized against the remaining variability of each group are $C_1 = 2$, $C_2 = 2$, $C_3 = 8$, $C_4 = 9$, and $C_5 = 3$. In the optimized solution, the computational results show that the waiting time and overtime for each operation room decrease from 46 minutes to 5 minutes and from 139 minutes to 75 minutes, respectively, which is a respective improvement of 89% and 46% as shown in Table 4.

5.6. Conclusions and Further Research. Maximizing the efficiency of the OR system is important because it impacts the profitability of the facility and the medical staff. OR scheduling must balance capacity utilization and demand so that the arrival rate, r_a, does not exceed the effective production rate, r_e. However, authorized overtime is increasing due to the randomness of patient arrivals and cycle times. This paper differs from the existing literature and makes a number of contributions. It focuses on shop floor control and uses a VUT algorithm that quantifies and explains flow variability. When the parameters are identified, the impact on the

Table 4: Optimal results.

	Overtime per operating room (minute)		Waiting time (minute)		Cycle time (minute)	
	Average	Standard deviation	Average	Standard deviation	Average	Standard deviation
Original system	139	26	46	16	200	22
Optimal solution	75	2	5	1	153	2
Improvement (%)	46		89		24	

surgery schedule using leveling based on group technology is illustrated. A more robust model of surgical processes is achieved by explicitly minimizing the flow variability. A simulation model is combined with the case study to optimize the capacity buffers against the remaining variability of each group. The computational result shows that overtime is reduced from 139 minutes to 75 minutes per operating room.

The most significant managerial implications can be summarized as follows.

(i) To achieve a higher return on investment, high utilization and reasonable cycle times, which depend on the level of variability, are necessary. The identification and reduction of the main sources of variability are keys to optimizing the performance instead of utilization.

(ii) This study solves OR scheduling using various heuristic methods and provides the anticipated start times for each case and each operating room. However, most real cases violate the assumptions (e.g., all cases are not ready at the start time, cycle times are stochastic, and resources do not break down, etc.). The schedule cannot be accurately predicted once the assumptions are violated.

(iii) Sequencing patients using takt time based on group technology reduces the flow variability and waiting time by 89%.

(iv) The empirical illustration shows that natural variability is prevented by optimizing the capacity buffers and reducing overtime by 46%.

In practice, there are additional constraints that affect the results, and these require further study.

(i) Although the duration of surgery is analyzed for 263 types of surgical categories and for 340 surgeons, each hospital is different. For example, some hospitals have a higher proportion of complex surgeries and should make comparisons among institutions.

(ii) The tests of model accuracy were performed using the year of 2002; they do account for diurnal variation. However, the year variation should be reflected.

(iii) Additional constraints may arise due to the availability of surgeons or other staff. For example, surgeons may not be available when the case is moved from the morning to the afternoon because they have outpatient clinics or other obligations.

(iv) This study applies to facilities at which the surgeon and patient choose the day and the case is not allowed to be allocated to another day, even if performance may be increased by rescheduling.

(v) Additional constraints may arise due to the availability of the recovery room.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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