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Editorial Board

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Editorial

Black Holes: Insights and Enigmas

Izzet Sakalli,[1] **Eduardo Guendelman**,[2] **Douglas Singleton**,[3] **and S. Habib Mazharimousavi**[1]

[1]*Physics Department, Arts and Sciences Faculty, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey*
[2]*Physics Department, Ben-Gurion University of the Negev, 84105 Beer-Sheva, Israel*
[3]*Physics Department, California State University, Fresno, Fresno, CA 93740, USA*

Correspondence should be addressed to Izzet Sakalli; izzet.sakalli@emu.edu.tr

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The study of black holes has been a central feature of general relativity since its birth in 1915. Shortly after Einstein wrote down the field equations of general relativity, Karl Schwarzschild discovered the first and simplest solutions which represented the gravitational field of a massive body with spherical symmetry, but without any other features like angular momentum or charge. It was noticed that this Schwarzschild solution had a place where the metric apparently became singular, the Schwarzschild radius, which for a spherical body of mass M occurred at the distance $2\,\mathrm{Gm/c^2}$. It was initially hoped that "realistic" objects could never be compressed to the extent that the material of the body would be less than this Schwarzschild radius. In the late 1930s starting with the work of Oppenheimer and Snyder, it became clear that it was possible for a material object to be compressed past its Schwarzschild radius and become a black hole. But what of the apparent singularity at the Schwarzschild radius? In the early 1960s, Kruskal and Szekeres were able to find a coordinate system which showed that the Schwarzschild singularity at $r = 2\,\mathrm{Gm/c^2}$ was an artifact of the coordinates that had been used by Schwarzschild and others. The only real singularity in the Schwarzschild solution was the one at the origin, namely, $r = 0$. Thus the study of black holes has been a prompt for pushing forward the understanding of the physical meaning of general relativity as well as better understanding what the complex math was saying physically.

In the mid-1970s, research led by Bekenstein, Hawking, and others showed that there was a deep connection between black hole physics and quantum mechanics and thermodynamics. Bekenstein was the first to argue that a black hole should have an entropy associated with it and that this entropy should be proportional to the surface area of the event horizon as defined by the Schwarzschild radius. Following on this work, Hawking showed, by applying quantum field theory in the background of a black hole, that black holes emitted thermal radiation and had a temperature now known as the Hawking temperature. These seminal works have led to researchers viewing black holes as a theoretical laboratory to giving hints as to the proper path toward a formulation of the as yet undiscovered theory of quantum gravity. This has led to the concepts like black hole information paradox, the holographic principle and the firewall puzzle, and a host of other interesting conjectures and puzzles.

Finally, with the advent of more powerful telescopes and observing equipment, both on the ground and in space, researchers have begun to gather observational evidence for the existence of black holes, including finding that in the heart of most galaxies there are enormous black holes of million plus solar masses. In our galaxy this "center of the galaxy" black hole is called Sagittarius A*. More recently, with the detection of gravitational waves by the LIGO scientific collaboration, we have very strong evidence of binary black hole coalescence, which resulted in the awarding of the 2017 Physics Nobel Prize.

This special issue is devoted to works which carry on the tradition of studying various aspects of black holes to understand the workings of gravity as well as other branches of physics.

In the paper by D.-Q. Sun et al. entitled "Hawking Radiation-Quasinormal Modes Correspondence for Large AdS Black Holes" the authors investigate the not exactly thermal nature of the Hawking radiation emitted by a black hole. They find a connection between Hawking radiation and the quasi-normal modes of black holes. They apply their analysis to Schwarzschild, Kerr, and nonextremal Reissner-Nordstrom black holes. They pay particular attention to these black holes in anti-de Sitter spacetime.

In the paper "P-V Criticality of a Specific Black Hole in $f(R)$ Gravity Coupled with Yang-Mills Field" by A. Övgün, a study is made of specific charge anti-de Sitter black holes in $f(R)$ gravity with a Yang-Mills field. Using thermodynamics analogies, it is found that this complex gravitational system behaves like the thermodynamic system of a van der Waals gas at critical points of the system. It is also found that the phase transition between small and large specific charge AdS black holes is a first-order phase transition.

In the paper by S. Chakraborty, entitled "Field Equations for Lovelock Gravity: An Alternative Route," the author studies an alternative derivation of the gravitational field equations for Lovelock gravity starting from Newton's law, which is closer in spirit to the thermodynamic description of gravity. Projecting the Riemann curvature tensor appropriately and taking a cue from Poisson's equation, the generalized Einstein's equations in the Lanczos-Lovelock theories of gravity are derived.

In the paper by W. Zhang and X.-M. Kuang, entitled "The Quantum Effect on Friedmann Equation in FRW Universe," the quantum mechanically modified Friedmann equation for the Friedmann-Robertson-Walker universe is derived. The authors also analyze the modified Friedmann equations using the conjecture by Padmanabhan of a modified entropy-area relation.

In the paper by M. He et al., entitled "Discussion of a Possible Corrected Black Hole Entropy" by using T. Padmanabhan's local formalism, which avoids addressing the asymptotically flat structure of spacetimes, the authors derive the analogue of the first law of thermodynamics for the AdS black holes in Eddington-inspired Born-Infeld (EiBI) gravity with and without matter fields. The same formalism has also been used to express Einstein's field equations in the form of the first law of thermodynamics, for a static spherically symmetric spacetime near any horizon of a definite radius. In accordance with their results, the authors conclude that since Einstein gravity and EiBI gravity give the same results, from thermodynamics' point of view, both of these theories of gravity could be equivalent on the event horizon.

The present volume collects together works which use black holes as a theoretical laboratory for understanding how gravity works and how gravity might fit in with the other theories of modern physics and in particular quantum mechanics.

Izzet Sakalli
Eduardo Guendelman
Douglas Singleton
S. Habib Mazharimousavi

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Research Article

P-v Criticality of a Specific Black Hole in $f(R)$ Gravity Coupled with Yang-Mills Field

Ali Övgün [1,2,3]

[1] *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla Postal 4950, Valparaíso, Chile*
[2] *Physics Department, Arts and Sciences Faculty, Eastern Mediterranean University, Famagusta, Northern Cyprus, Mersin 10, Turkey*
[3] *TH Division, Physics Department, CERN, 1211 Geneva 23, Switzerland*

Correspondence should be addressed to Ali Övgün; ali.ovgun@pucv.cl

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We study the P-v criticality of a specific charged AdS type black hole (SBH) in $f(R)$ gravity coupled with Yang-Mills field. In the extended phase space, we treat the cosmological constant as a thermodynamic pressure. After we study the various thermodynamical quantities, we show that the thermodynamic properties of the SBH behave as a Van der Waals liquid-gas system at the critical points and there is a first-order phase transition between small-large SBH.

1. Introduction

Important contribution on black holes' thermodynamics in anti-de-Sitter (AdS) spacetime is made by Hawking and Page [1], where a first-order phase transition is discovered between the Schwarzschild-anti-de-Sitter (SAdS) black holes that is known as the Hawking-Page transition. Then Chamblin et al. and Cvetic et al. show that the first-order phase transition among Reissner Nordstrom (RN) AdS black holes and the similarities between charged AdS black holes and liquid-gas systems in grand canonical ensemble [2–4]. Moreover, in the seminal papers of Kubiznak and Mann [5], the cosmological constant Λ is used as dynamical pressure [6]

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi l^2} \tag{1}$$

for the RN-AdS black holes in the extended phase space, instead of treating the Λ as a fixed parameter (in standard thermodynamic), and its conjugate variable has dimension of volume

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q}. \tag{2}$$

Calculating the critical components and finding the phase transition of the RN-AdS black holes, it is shown that RN-AdS black holes behave similar to the Van der Waals fluid in the extended phase space where a first-order small/large black hole's phase transition occurs at a critical temperature below [7–11]. The Van der Waals equation,

$$\left(P + \frac{a}{v^2}\right)(v - b) = kT, \tag{3}$$

where its pressure is P, its temperature is T, its specific volume is $v = V/N$, the Boltzmann constant is k, and the positive constants are a and b, takes into account the attractive and repulsive forces between molecules and gives an improved model for ideal gas behaviour to describe the basic properties of the liquid-gas phase transition with the ratio of $P_c V_c / T_c = 3/8$ at critical points [12–17]. Afterwards, applications of the thermodynamical law's to the black hole's physics have gain attention. Different researches are done by using the variation of the first law of thermodynamics of black holes and also application of the P-v criticality on black holes [18–88]. Furthermore, AdS-CFT correspondence is the other reason for studying the AdS black hole.

In this paper we use a black hole's solution in the Yang-Mills field which is the one of the most interesting nonabelian gauge theories. By using the string theory models they find the Yang-Mills fields equations in low energy limit and then

Yasskin found the first black hole solution in the theory of Yang-Mills coupled to Einstein theory [89].

Our main aim is to check P-V criticality of a specific charged AdS type black hole (SBH) in $f(R)$ gravity coupled with Yang-Mills field (YMF) [90] by comparing its result with the Van der Waals system. The Yang-Mills field is acted inside the nuclei with short range and $f(R)$ gravity which is an extension of Einstein's General Relativity with the arbitrary function of Ricci scalar $f(R)$ [91–94]. It would be of interest to study the P-v criticality of SBH in $f(R)$ gravity coupled with YMF in the extended phase space treating the cosmological constant as a thermodynamic pressure. In this paper, we first study the thermodynamics in the extended phase space and then we obtain its critical exponents to show the existence of the Van der Waals like small-large black hole phase transitions.

The paper is organized as follows: in Section 2 we will briefly review the SBH in $f(R)$ gravity coupled with YMF. In Section 3 P-v criticality of the SBH in $f(R)$ gravity coupled with YMF will be studied in the extended phase space by calculating its critical exponents. In Section 4 we conclude with final remarks.

2. SBH in $f(R)$ Gravity Coupled with YMF

In this section, we briefly present a solution of SBH in $f(R)$ gravity coupled with YMF with a cosmological constant in d-dimensions [90]. Then we discuss its temperature, entropy, and other thermodynamic quantities. The action of the $f(R)$ gravity minimally coupled with YMF ($c = G = 1$) is [90]

$$S = \int d^d x \sqrt{-g} \left[\frac{f(R)}{16\pi} + \mathscr{L}(F) \right], \tag{4}$$

where $f(R)$ is a function of the Ricci scalar R and $\mathscr{L}(F)$ stands for the Lagrangian of the nonlinear YMF with $F = (1/4)\mathrm{tr}(F^{(a)}_{\mu\nu} F^{(a)\mu\nu})$, where the 2-form components of the YMF are $\mathbf{F}^{(a)} = (1/2)F^{(a)}_{\mu\nu} dx^\mu \wedge dx^\nu$. Here is the internal index (a) for the degrees of freedom of the nonabelian YMF. It is noted that this nonlinear YMF can reduce to linear YM field ($\mathscr{L}(F) = -(1/4\pi)F^s$) for $s = 1$ and $f_R = df(R)/dR = \eta r$ which η is an integration constant. Solving Einstein field equations for the $f(R)$ gravity coupled with YMF gives to the spherically symmetric black hole metrics (see equation (36) in [90])

$$\begin{aligned} ds^2 &= -f(r)\, dt^2 + \frac{dr^2}{f(r)} \\ &\quad + r^2 \left(d\theta_1^2 + \Sigma_{i=2}^{d-2} \Pi_{j=1}^{i-1} \sin^2\theta_j d\theta_i^2 \right) \end{aligned} \tag{5}$$

with $0 \le \theta_{d-2} \le 2\pi,\ 0 \le \theta_i \le \pi,\ 1 \le i \le d-3$, in which the metric function $f(r)$ is

$$\begin{aligned} f(r) &= \frac{d-3}{d-2} - \Lambda r^2 - \frac{m}{r^{d-2}} \\ &\quad - \frac{(d-1)(d-2)^{(d-1)/2}(d-3)^{(d-1)/4}}{2^{(d-5)/2}\eta d} \frac{Q^{(d-1)/2} \ln r}{r^{d-2}}. \end{aligned} \tag{6}$$

Note that $\Lambda = -1/l^2$, M is the mass of the black hole, and η is a constant. Furthermore for the limit of $Q^{7/2} \to 0$, it becomes well-known solutions in $f(R)$ gravity. The Bekenstein-Hawking temperature [95–98] of the black hole is calculated by $T = (1/4\pi)(\partial f(r)/dr)|_{r=r_h}$

$$T = \frac{(-1 + (d-2)\ln(r)) Q^{d/2-1/2} (d-1)(d-2)^{d/2-1/2} (d-3)^{d/4-1/4} + d 2^{d/2-5/2} \left(-2r^2 \Lambda r^{d-2} + m(d-2) \right) \eta}{4\pi r^{d-2} \eta d 2^{d/2-5/2} r}, \tag{7}$$

where r_h is the horizon of the black hole, and solving the equation $f(r_h) = 0$, the total mass of the black hole is obtained as

$$m = -\frac{4Q^{d/2-1/2} (d/2-1)^{d/2} \ln(r) \sqrt{2} (d-1)(d-2)(d-3)^{d/4-1/4} + \left(\left((f-1)d - 2f + 3 \right) r^{d-2} + r^d \Lambda (d-2) \right) d \sqrt{d-2}\eta}{\eta d (d-2)^{3/2}}. \tag{8}$$

The entropy of the black hole can be derived as

$$S = \frac{A_h}{4} \eta r_h, \tag{9}$$

where $A_h = ((d-1)/\Gamma((d+1)/2))\pi^{(d-1)/2} r_h^{d-2}$ is the area of the black hole's event horizon. Then, in the extended phase space, we calculate the pressure in terms of cosmological constant

$$P = -\frac{\Lambda}{8\pi} \tag{10}$$

and its thermodynamic volume is

$$V = \frac{\Omega_{d-2} r_h^{d-1} \eta}{n-1}, \tag{11}$$

where Ω_{d-2} is the volume of the unit sphere. Now the mass can be also written in terms of P as follows:

$$m = -\frac{4Q^{d/2-1/2}(d/2-1)^{d/2}\ln(r)\sqrt{2}(d-1)(d-2)(d-3)^{d/4-1/4} + \left(\left(\left(f-1\right)d-2f+3\right)r^{d-2} - 8r^{d}P\pi(d-2)\right)d\sqrt{d-2}\eta}{(d-2)^{3/2}\eta d}. \quad (12)$$

The first law of the black hole thermodynamics in the extended phase space is

$$dm = TdS + \Phi dQ + VdP, \quad (13)$$

where the thermodynamic variables can be obtained as $T = (\partial m/\partial S)_{Q,P}$, $\Phi = (\partial m/\partial Q)_{S,P}$, and $V = (\partial m/\partial P)_{S,Q}$.

Then we write the generalized Smarr relation for the black hole, which can be derived also using the dimensional scaling, as

$$m = 2TS + \Phi Q - 2VP. \quad (14)$$

We introduce the cosmological constant as thermodynamic pressure in the extended phase space in (10), and it is seen that the first law of the black hole's thermodynamics and the Smarr relations is matched well.

3. P-v Criticality

In this section, we investigate the critical behaviour of the SBH in the extended phase space. The critical point can be defined as

$$\frac{\partial P}{\partial v} = \frac{\partial^2 P}{\partial v^2} = 0. \quad (15)$$

Now we consider the case of four dimensions ($d = 4$), where the metric function becomes

$$f = \frac{1}{2} - \frac{m}{r^2} - r^2\Lambda - \frac{3Q^{3/2}\ln(r)}{r^2\eta} \quad (16)$$

and corresponding mass of the black hole is calculated as

$$m = \frac{r^2}{2} - r^2\Lambda - \frac{3Q^{3/2}\ln(r)}{\eta}. \quad (17)$$

The temperature of the four dimensional SBH is

$$T = -\frac{3Q^{3/2} - r^2\eta + 4r^4\eta\Lambda}{4\pi r^3\eta}. \quad (18)$$

Then we write the temperature in terms of P ($P = -\Lambda/8\pi$) as follows:

$$T = 8rP - \frac{3Q^{3/2}}{4r^3\eta\pi} + \frac{1}{4r\pi}. \quad (19)$$

Afterwards one can easily obtain the pressure P in terms of the temperature T:

$$P = \frac{T}{8r} + \frac{3Q^{3/2}}{32r^4\eta\pi} - \frac{1}{32r^2\pi}. \quad (20)$$

To consider the P-v criticality using the extended phase space, we write the black hole radius in terms of the specific volume v as $r_h = (d-2)v/4$. Using the condition of (15), we derive the critical Bekenstein-Hawking temperature T_c, critical pressure P_c, and critical specific volume v_c as follows:

$$T_c = -24\frac{Q^{3/2}}{v^3\eta\pi} + \frac{1}{v\pi},$$
$$v_c = 24\frac{\sqrt{2}\sqrt{Q^{3/2}}}{\sqrt{\eta}}, \quad (21)$$
$$P_c = \frac{\eta}{1152Q^{3/2}\pi}.$$

One can also find this relation which is same with a Van der Waals fluid

$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}. \quad (22)$$

It is noted that Figures 1 and 2 show that P-r diagram is the same with the diagram of the Van der Waals liquid-gas system.

Let us now analyze the Gibbs free energy of the system. We first use the mass as enthalpy instead of internal energy and the Gibbs free energy in the extended phase space for the SBH in $f(R)$ gravity coupled with Yang-Mills field is calculated as

$$G = m - TS = \frac{6Q^{3/2} + r^2\eta - 16P\pi r^4\eta - 18Q^{3/2}\ln(r)}{6\eta}. \quad (23)$$

We plot the change of the free energy G with T in Figure 3. There is a small-large black hole phase transition as seen in Figure 3.

4. Conclusion

In this paper, we first treat the cosmological constant Λ as a thermodynamical pressure P and the thermodynamics

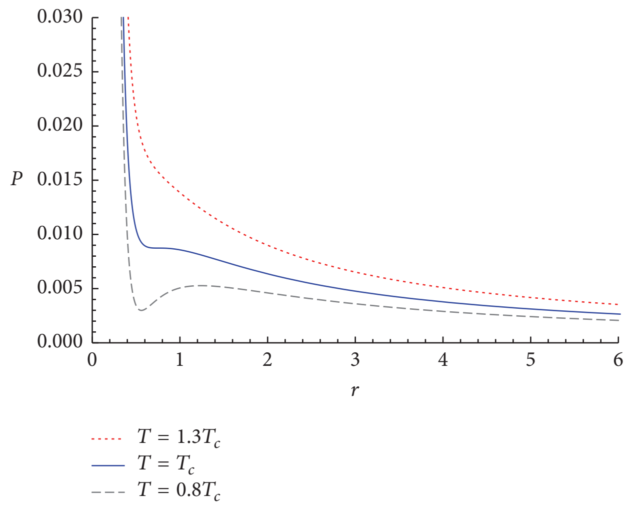

FIGURE 1: P-r diagram of a SBH in a $f(R)$ gravity coupled with YMF for $Q = 0.1$ and $\eta = 1$.

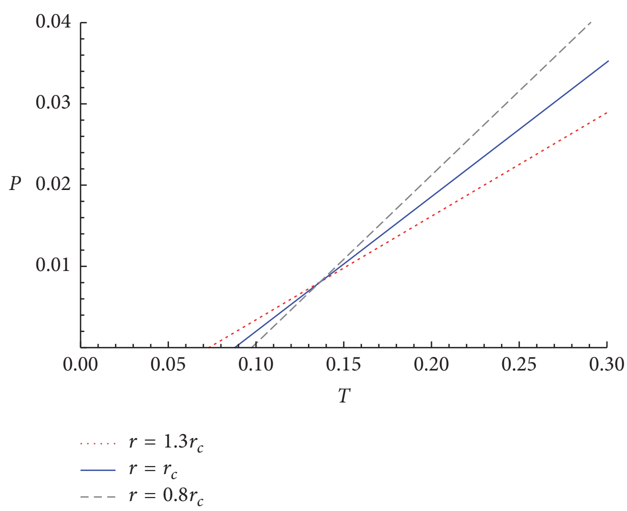

FIGURE 2: P-T diagram of a SBH in a $f(R)$ gravity coupled with YMF for $Q = 0.1$ and $\eta = 1$.

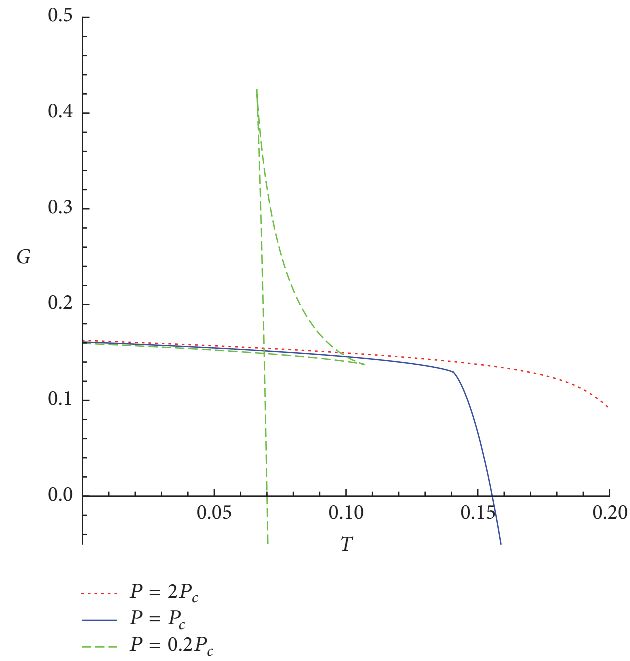

FIGURE 3: G-T diagram of a SBH in a $f(R)$ gravity coupled with YMF for different values of P ($P < P_c$, $P = P_c$, and $P > P_c$) with $Q = 0.1$ and $\eta = 1$.

and P-v criticality of the SBH in $f(R)$ gravity coupled with YMF is studied in the extended phase space. It is shown that there is a phase transition between small-large black holes. Furthermore, after we obtain the critical exponents, the critical behaviour of SBH in $f(R)$ gravity coupled with YMF in the extended space behaves also similarly as Van der Waals liquid-gas systems with the ratio of $P_c v_c / T_c = 3/8$ at critical points. Hence it would be of great importance to obtain the P-V criticality of SBH in $f(R)$ gravity coupled with YMF. Hence the critical ratio $P_c v_c / T_c = 3/8$ is universal and independent from the modified gravities. The YMF has a parameter of η but has no effect on the universal ratio of 3/8.

It is also interesting to study the holographic duality of SBH in $f(R)$ gravity coupled with YMF. It is noted that without thermal fluctuations black hole is holographic dual with Van der Waals fluid given by $(P + a/V^2)(V - b) = T$, where k is the Boltzmann constant [84, 85], $b > 0$ is nonzero constant which is the size of the molecules of fluid, and the constant $a > 0$ is a value of the interaction measurement between molecules. We leave this problem for the future projects.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Field Equations for Lovelock Gravity: An Alternative Route

Sumanta Chakraborty

Department of Theoretical Physics, Indian Association for the Cultivation of Science, Kolkata 700032, India

Correspondence should be addressed to Sumanta Chakraborty; sumantac.physics@gmail.com

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We present an alternative derivation of the gravitational field equations for Lovelock gravity starting from Newton's law, which is closer in spirit to the thermodynamic description of gravity. As a warm up exercise, we have explicitly demonstrated that, projecting the Riemann curvature tensor appropriately and taking a cue from Poisson's equation, Einstein's equations immediately follow. The above derivation naturally generalizes to Lovelock gravity theories where an appropriate curvature tensor satisfying the symmetries as well as the Bianchi derivative properties of the Riemann tensor has to be used. Interestingly, in the above derivation, the thermodynamic route to gravitational field equations, suited for null hypersurfaces, emerges quiet naturally.

1. Introduction

Equivalence principle acts as the guiding lighthouse to understand how matter fields behave in a curved spacetime background. Unfortunately, there exists no such principle which helps to answer the opposite, namely, how matter fields curve the spacetime [1–7]. Lack of such principle resulted in the vast landscape of various alternative gravity theories among which general relativity remains the most useful one. Despite this large variety of alternative gravitational theories one can do better by imposing some physical requirements on the systems of interest. In particular, the restriction to the class of theories having utmost second-order derivatives of the metric is a judicious one, since this helps to overcome the well known Ostrogradski instability by evading the existence of any ghost modes in the theory [8–10]. Surprisingly enough the above criteria turn out to be very interesting ones, as they single out a very specific class of unique gravitational theories, known in the literature as the Lanczos-Lovelock models of gravity [11–14]. In particular, all these models satisfy the criterion that Bianchi derivative of the field tensor identically vanishes [15, 16]. One can add to this list a large number of interesting properties; a few among them are as follows: (a) the action functional for Lanczos-Lovelock gravity can be broken into a bulk part and a surface contribution, which is closely related to the Wald entropy associated with the black hole horizon [17]; (b) the field equations in Lanczos-Lovelock gravity can be expressed as a thermodynamic identity on an arbitrary null surface [18–26]; (c) the difference between a suitably defined surface and bulk degrees of freedom in the context of Lanczos-Lovelock gravity can be interpreted as the evolution of the spacetime [27]; (d) the surface and bulk terms in the Lanczos-Lovelock gravity are related by a holographic relation between them [28, 29].

In the standard textbook treatment, one first introduces the gravitational Lagrangian L, which is $\sqrt{-g}R$ for general relativity, while the Lagrangian is a polynomial in the Riemann tensor for general Lanczos-Lovelock gravity. Variation of the action functional due to arbitrary variation of the metric, with appropriate boundary conditions [30, 31], leads to the corresponding field equations for gravity, namely, the rule by which matter tells the spacetime how to curve (see also [32]). However, as emphasized earlier there is no physical principle which guides us to the precise mathematical structure of the gravitational action; what is more the field equations obtained equate matter which is intrinsically *quantum* with spacetime geometry describing gravity *classically*. Despite this conceptual discomfort, there exist two additional representations of the gravitational dynamics, which are intrinsically observer dependent. The first one uses timelike observers with four-velocity u^a and equates two quantities where this observer measures in the geometric as well as in the matter sector, leading to a scalar equation

$$G_{ab}u^a u^b = \kappa T_{ab}u^a u^b. \quad (1)$$

Here κ stands for the gravitational constant in four dimensions and enforcing this equation for all observers with four-velocity u_i leading to Einstein's equations $G_{ab} = \kappa T_{ab}$. The other approach uses null vectors ℓ_a (i.e., $\ell^2 = 0$) and leads to the following scalar equation:

$$R_{ab}\ell^a \ell^b = \kappa T_{ab}\ell^a \ell^b. \quad (2)$$

In this situation as well validity of the above expression for all null vectors ℓ_a, along with two times contracted Bianchi identity and covariant conservation of matter energy momentum tensor, amounts to furnishing the ten components of Einstein's equations. Following [33], in this article, we would like to derive (1) and (2) from a geometrical point of view for Lanczos-Lovelock theories of gravity. We would also argue why the route taken in this work is a natural one compared to the standard derivation of Einstein's equations from the gravitational action. We have organized the paper as follows: In Section 2 we briefly review the derivation of Einstein's equations, which will be generalized in Section 3 to Lanczos-Lovelock theories of gravity. Finally we provide the concluding remarks before presenting a derivation regarding the curvature tensor associated with the Lanczos-Lovelock theory of gravity in Appendix.

2. Einstein's Equations following Newton's Path

To set the stage for Lanczos-Lovelock theories of gravity we would like to briefly review the corresponding situation in general relativity, which will mainly follow from [33]. In Newtonian theory one describes gravity using the gravitational potential $\phi(\mathbf{x}, t)$. Given the matter density $\rho(\mathbf{x}, t)$, the dynamics of the gravitational field is being determined through Poisson's equation $\nabla^2\phi = (\kappa/2)\rho$. When one invokes principle of equivalence, the potential ϕ is readily identified with components of the metric, in particular, $g_{00} = -(1+2\phi)$. One then interprets Poisson's equation in metric language as $-\nabla^2 g_{00} = \kappa T_{00}$, where matter energy density is interpreted as the time-time component of the divergence free, symmetric, matter energy momentum tensor T_{ab}. In order to have a relativistic generalization one naively makes the following replacement $\nabla^2 \to \Box$. This immediately suggests looking for second derivatives of the metric which is a second rank tensor as well as divergence free, paving the way to the Einstein tensor. On the other hand, for the right hand side the natural choice being the matter energy momentum tensor T_{ab}, one ends up equating Einstein's tensor with the matter energy momentum tensor resulting in Einstein's equations (for a more detailed discussion, see [33]).

However, we could have just followed Newton's path and look for relativistic generalization of Poisson's equation itself, possibly leading to a scalar equation describing gravity in general relativity. As a first step one must realize that the right hand side of Poisson's equation is intrinsically observer dependent, as it is the energy density of some matter field measured by an observer with some four-velocity u_i. Given the symmetric and conserved matter energy momentum tensor T_{ab}, the matter energy density ρ as measured by an observer with four-velocity u_a is $T_{ab}u^a u^b$. Hence the same observer dependence must continue to exist in general relativity and the right hand side of the desired general relativistic equation for gravity should be $T_{ab}u^a u^b$ [33].

To derive the left hand side, that is, analogue of $\nabla^2\phi$, we note that this requires deriving a scalar object which involves two spatial derivatives of the metric (since this is what is responsible for the $\nabla^2\phi$ term in nonrelativistic limit) and necessarily depends on the four-velocity u^i. The only tensor depending on two derivatives of the metric that can be constructed corresponds to the curvature tensor R_{abcd}. In order to obtain double spatial derivatives acting on the metric one has to project all the components of the curvature tensor on the plane orthogonal to u_a using $h^a_b = \delta^a_b + u^a u_b$ and hence construct a scalar thereof. To keep generality, instead of looking for a timelike vector u_i, we will concentrate on a particular vector ℓ^a, with norm $\ell_a \ell^a \equiv \ell^2$. Then one can immediately introduce the following tensor:

$$\begin{aligned}
\mathbb{P}^a_b &= \delta^a_b - \frac{1}{\ell^2}\ell^a \ell_b; \\
\mathbb{P}^a_b \mathbb{P}^b_c &= \left(\delta^a_b - \frac{1}{\ell^2}\ell^a \ell_b\right)\left(\delta^b_c - \frac{1}{\ell^2}\ell^b \ell_c\right) = \delta^a_c - \frac{1}{\ell^2}\ell^a \ell_c \\
&= \mathbb{P}^a_c
\end{aligned} \quad (3)$$

for which the final property ensures that it is a projector. Then one can define a projected Riemann curvature with respect to the vector ℓ^a by projecting all the indices of the Riemann curvature tensor, leading to (by the very definition, the Riemann tensor R_{abcd} has a generic form $\partial_b\partial_c g_{ad}$ in local inertial frame; thus the above projection ensures that g_{ab} has only double spatial derivatives when ℓ_a is a timelike vector; all time derivatives appearing in R_{abcd} are single in nature and hence vanish in the local inertial frame all together; this is the prime motivation of introduction of this projected Riemann tensor; the same can be ascertained from the Gauss-Codazzi equation as well; see [1, 5])

$$\mathsf{R}_{mnrs} = \mathbb{P}^a_m \mathbb{P}^b_n \mathbb{P}^c_r \mathbb{P}^d_s R_{abcd}. \quad (4)$$

The scalar constructed out of this projected Riemann tensor becomes

$$\mathsf{R} = \mathbb{P}_{mr}\mathbb{P}_{ns}R^{mnrs} = R - \frac{2}{\ell^2}R_{ab}\ell^a \ell^b. \quad (5)$$

Note that we have thrown away another term involving $R_{abcd}\ell^a \ell^b \ell^c \ell^d$, since due to antisymmetry properties of the Riemann tensor the above quantity identically vanishes. Furthermore, the above scalar *by* construction has only two derivatives of the metric and is completely spatial in the instantaneous rest frame of the timelike observer. Hence R is the relativistic generalization of $\nabla^2\phi$ and thus must be equal to the corresponding matter energy density, which in this case corresponds to $-(2\kappa/\ell^2)T_{ab}\ell^a\ell^b$, where T_{ab} is the matter energy momentum tensor. Thus finally one obtains the following equation:

$$R_{ab}\ell^a\ell^b - \frac{1}{2}R\ell^2 = \kappa T_{ab}\ell^a\ell^b. \tag{6}$$

Surprisingly, through the above exercise we have achieved two results in one go. Firstly, for $\ell^2 = -1$, that is, for unit normalized timelike vectors we get back the equation $G_{ab}u^a u^b = \kappa T_{ab}u^a u^b$. Then requiring these equations to hold for all timelike observers will lead to $G_{ab} = \kappa T_{ab}$. At this stage it will be worthwhile to mention that a similar approach as the above one was taken in [34] to derive Einstein's equations. However the key difference between the approach in [34] and in this work is the use of the projection tensor $\mathbb{P}^a_b$ judiciously. For our approach the use of projection tensor is of utmost importance in contrast to [34].

Finally the limit $\ell^2 \to 0$ would lead to the null version of the above equation, which reads $R_{ab}\ell^a\ell^b = \kappa T_{ab}\ell^a\ell^b$. In this case besides demanding the validity of the above equation for all null vectors, it is important to use contracted Bianchi identity as well as covariant conservation of energy momentum tensor, leading to $G_{ab} + \Lambda g_{ab} = \kappa T_{ab}$. Note that in the second situation a cosmological constant has automatically come into existence.

3. Newton Leads the Way to Lovelock

In view of the above result, one immediately asks for the corresponding situation in Lovelock gravity: can field equations of Lovelock gravity be derived following the above procedure? This is what we will explore in this section and shall show that one can indeed derive the field equations for Lovelock gravity following an equivalent procedure. Before jumping into the details let us mention some structural aspects of Lovelock gravity. Briefly speaking, Lovelock gravity corresponds to a class of gravitational Lagrangians which are polynomial in the Riemann curvature tensor yielding field equations which are of second order in the metric. The analytical form of such a polynomial (also called a pure Lovelock term) of order m involves m Riemann curvature tensors contracted appropriately, such that

$$L^{(m)} = \frac{1}{2^m}\delta^{a_1b_1\cdots a_mb_m}_{c_1d_1\cdots c_md_m}R^{c_1d_1}_{a_1b_1}\cdots R^{c_md_m}_{a_mb_m} \equiv \frac{1}{m}P^{ab(m)}_{cd}R^{cd}_{ab}. \tag{7}$$

The above relation defines the tensor $P^{ab(m)}_{cd}$ associated with the mth order Lanczos-Lovelock gravity, having all the symmetries of the Riemann tensor with the following algebraic structure:

$$P^{ab(m)}_{cd} = \frac{m}{2^m}\delta^{aba_2b_2\cdots a_mb_m}_{cdc_2d_2\cdots c_md_m}R^{c_2d_2}_{a_2b_2}\cdots R^{c_md_m}_{a_mb_m}. \tag{8}$$

The tensor $P^{ab(m)}_{cd}$ satisfies an additional criterion $\nabla_a P^{ab(m)}_{cd} = 0$, which ensures that the field equations derived from this Lagrangian are of second order. In what follows we will exclusively concentrate on the mth order Lanczos-Lovelock gravity and hence shall remove the symbol "(m)" from the superscripts of various geometrical expressions.

One can now start from Poisson's equation and fix the right hand side to be $-T_{ab}\ell^a\ell^b/\ell^2$, which is the matter energy density associated with the vector field ℓ^a. In order to get the left hand side we must construct an appropriate curvature tensor suited for the Lovelock gravity and project it using the projector $\mathbb{P}^a_b$ introduced in (3) before constructing a scalar out of it. There are two possible choices for such a curvature tensor among which we will discuss the simpler one in the following, while the complicated one is deferred to Appendix 4. The curvature tensor described here was first introduced in [15, 16] and is defined as follows:

$$\begin{aligned}\mathrm{R}^{ab}_{cd} &= \frac{1}{2}\left(P^{ab}_{mn}R^{mn}_{cd} + P^{mn}_{cd}R^{ab}_{mn}\right) \\ &\quad - \frac{(m-1)}{(d-1)(d-2)}\left(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c\right)L,\end{aligned} \tag{9}$$

where L is the mth order Lovelock polynomial and $P^{ab}_{cd} = \partial L/\partial R^{cd}_{ab}$. Since the mth order Lanczos-Lovelock Lagrangian depends on m powers of Riemann, the above definition for P^{ab}_{cd} ensures that it depends on $(m-1)$ powers of Riemann tensor and hence exactly coincides with (8). Furthermore, the tensor P^{ab}_{cd} can be easily generalized to the full Lovelock polynomial by just adding over different m values, but we will concentrate on a single term in the full Lanczos-Lovelock Lagrangian. Also note that the above defined "Lovelock" Riemann tensor has all the symmetries of the original "Einstein" Riemann tensor R^{ab}_{cd}. Further it satisfies the contracted Bianchi identity, which will be sufficient for our purpose. Given the above mth order "Lovelock" Riemann, one can project it in the plane orthogonal to ℓ_a and obtain the following scalar:

$$\mathbb{R} = \mathrm{R}^{ab}_{cd}\mathbb{P}^c_a\mathbb{P}^d_b. \tag{10}$$

Explicit evaluation of the above scalar can be performed keeping in mind that the tensor R^{ab}_{cd} is antisymmetric under exchange of the indices (a, b) and (c, d), respectively, leading to

$$\begin{aligned}\mathbb{R} &= \mathrm{R}^{ab}_{cd}\delta^c_a\delta^d_b - \frac{2}{\ell^2}\mathrm{R}^{ab}_{cd}\ell^c\ell_a\delta^d_b = P^{ab}_{cd}R^{cd}_{ab} \\ &\quad - \frac{(m-1)}{(d-1)(d-2)}\left(d^2 - d\right)L \\ &\quad - \frac{2}{\ell^2}\left\{P^{ab}_{mn}R^{mn}_{cb} - \frac{(m-1)}{(d-1)(d-2)}\left(d\delta^a_c - \delta^a_c\right)L\right\} \\ &\quad \cdot \ell_a\ell^c = mL - \frac{d(m-1)}{d-2}L - \frac{2}{\ell^2}P^{ab}_{mn}R^{mn}_{cb}\ell_a\ell^c + 2 \\ &\quad \cdot \frac{m-1}{d-2}L = -\frac{2}{\ell^2}P^{ab}_{mn}R^{mn}_{cb}\ell_a\ell^c + L.\end{aligned} \tag{11}$$

Thus equating (11) to the matter energy density through $-(2\kappa/\ell^2)T_{ab}\ell^a\ell^b$ we finally obtain

$$P^a_{bmn}R^{bmn}_c\ell_a\ell^c - \frac{1}{2}L\ell^2 = \kappa T_{ab}\ell^a\ell^b. \tag{12}$$

For unit normalized timelike vectors, $\ell_a = u_a$ and $\ell^2 = -1$, leading to $E_{ab}u^a u^b = \kappa T_{ab}u^a u^b$, where $E_{ab} \equiv P_a{}^{pqr}R_{bpqr} - (1/2)Lg_{ab}$ is the analogue of Einstein tensor in Lovelock gravity. On the other hand, for the null vectors we arrive at $\mathcal{R}_{ab}\ell^a\ell^b = \kappa T_{ab}\ell^a\ell^b$, where the tensor $\mathcal{R}_{ab} \equiv P_a{}^{pqr}R_{bpqr}$ is the analogue of Ricci tensor in Lovelock gravity. Hence even in the case of Lovelock gravity, if one assumes that $E_{ab}u^a u^b = \kappa T_{ab}u^a u^b$ holds for all timelike observers, the field equations for Lovelock gravity $E_{ab} = \kappa T_{ab}$ follow. While in the null case, besides demanding the validity of $E_{ab}\ell^a\ell^b = \kappa T_{ab}\ell^a\ell^b$ for all null vectors, one has to use the Bianchi identity associated with Lovelock theories as well as covariant conservation of matter energy momentum tensor to arrive at the Lovelock field equations $E_{ab} + \Lambda g_{ab} = \kappa T_{ab}$, which inherit the cosmological constant as well. Note that the above result has been derived in the context of mth order Lanczos-Lovelock gravity, which can be generalized to the general Lanczos-Lovelock Lagrangian in a straightforward manner. This results in $\{\sum_m E_{ab}^{(m)}\} + \{\sum_m \Lambda^{(m)}\}g_{ab} = \kappa T_{ab}$. Interestingly, the cosmological constant besides being generated as an integration constant of the field equations also gets contribution from the $m = 0$ term in the Lanczos-Lovelock gravity. Therefore one may choose this term in the Lagrangian appropriately to arrive at the present small value of the cosmological constant. Therefore we conclude that the derivation of field equations for gravity can always be achieved starting from Poisson's equation and subsequently projecting a suitable curvature tensor, whether it is Einstein gravity or Lovelock.

4. Concluding Remarks

By equivalence principle gravity manifests itself by curving the spacetime, which the material particles follow. In particular one can invoke special relativity in locally freely falling frame and hence write down the laws of motion in curvilinear coordinates, thus describing motion in curved spacetime. The notion of locally freely falling observer brings in intrinsic observer dependence in the theory and introduces observers for whom a local spacetime region is causally inaccessible, known as a local Rindler observer. Remarkably the local vacuum state of a test quantum field (as fit for local inertial observers) will appear as thermal to the local Rindler observer [35, 36]. If any matter field (characterized by matter energy momentum tensor T^a_b) crosses the local Rindler horizon, it will appear to be thermalized by the Rindler observer (since the matter will take infinite time to reach the horizon) and the corresponding heat density is being given by $T^b_a\ell^a\ell_b$ (for a perfect fluid the above quantity is given by $\rho + p$, which by Gibbs-Duhem relation is the matter heat density), where ℓ_a is the null normal to the horizon. Note that the above heat density for matter is invariant under the transformation $T^a_b \to T^a_b + (\text{constant})\delta^a_b$.

At this stage, one can ask a natural question, "what about heat density of gravity?". Surprisingly, one can answer the same in the above setting. Considering a general null surface it turns out that one can interpret $R^a_b\ell^b\ell_a$ as the heat density of the spacetime. This originates from the fact that one can have a one to one correspondence between $R_{ab}\ell^a\ell^b$ and the viscous dissipation term $2\eta\sigma_{ab}\sigma^{ab}+\zeta\theta^2$, where σ_{ab} is the shear of the null congruence ℓ_a and θ is its expansion, with η and ζ being shear and bulk viscous coefficients. Thus the term $R_{ab}\ell^a\ell^b$ is related to heating of the spacetime [37]. Given this thermodynamic backdrop, it is clear that the Einstein's equations when written as (2) not only yield the geometrical input of the gravitational theory but are also physically well-motivated since the equality of (2) can be thought of as an equilibrium situation, where the heat produced by gravity is being compensated by that of matter.

Thus the equation $2G_{ab}\ell^a\ell^b = T_{ab}\ell^a\ell^b$ arises more naturally from the relativistic generalization of Newton's law and the usefulness of writing Einstein's equations in this manner stems from the fact that one might interpret both sides of these equations independently and the equations themselves follow due to a balancing act performed by spacetime itself [33, 38, 39]. We would like to reiterate that the field equations derived in this context are purely geometrical and follow Newton's path. One first realizes that energy density associated with any material body is intrinsically observer dependent and surprisingly one can construct a tensor (again dependent on observer) which contains spatial derivatives of the metric alone. Keeping this as a curved spacetime generalization of Newton's law one uniquely arrives at Einstein's equations when one makes use of all observers (or all the null surfaces). This shows that the most natural generalization of Newton's law to curved spacetime is $R_{ab}\ell^a\ell^b = 8\pi T_{ab}\ell^a\ell^b$ (of course, leading to Einstein's equations, but at a secondary level) bolstering the claim that gravity is intrinsically a thermodynamic phenomenon.

Appendix

An Alternative Riemann Tensor for Lovelock Gravity

In Lovelock gravity it is possible to define two tensors having the symmetry properties of Riemann and satisfying Bianchi identity. The first one corresponds to defining a $(2m \times 2m)$ tensor for mth order Lovelock polynomial by multiplying m such curvature tensors and then an alternating tensor of rank $(2m \times 2m)$ as [40]

$$\mathcal{R}^{b_1 b_2\cdots b_{2m}}_{a_1 a_2\cdots a_{2m}} = \delta^{b_1 b_2\cdots b_{2m}}_{c_1 c_2\cdots c_{2m}}\delta^{d_1 d_2\cdots d_{2m}}_{a_1 a_2\cdots a_{2m}} R^{c_1 c_2}_{d_1 d_2}\cdots R^{c_{2m-1}c_{2m}}_{d_{2m-1}d_{2m}}. \quad (A.1)$$

Note that the above tensor is completely antisymmetric in both upper and lower indices.

Let us now project all the indices on a lower dimensional spacelike hypersurface using the projection tensor $\mathbb{P}^a_b = \delta^a_b - (1/\ell^2)\ell^a\ell_b$, such that one obtains another $(2m \times 2m)$ tensor, but whose inner product with the normal ℓ_a identically vanishes. Hence one arrives at

$$m!\mathbb{R}^{p_1 p_2\cdots p_{2m}}_{q_1 q_2\cdots q_{2m}} = \mathbb{P}^{p_1}_{b_1}\cdots\mathbb{P}^{p_{2m}}_{b_{2m}}\mathbb{P}^{a_1}_{q_1}\cdots\mathbb{P}^{a_{2m}}_{q_{2m}}\mathcal{R}^{b_1 b_2\cdots b_{2m}}_{a_1 a_2\cdots a_{2m}}. \quad (A.2)$$

One can immediately construct a scalar out of the projected $(2m \times 2m)$ tensor, leading to

$$m!\mathbb{R} = \mathbb{P}^{a_1}_{b_1} \cdots \mathbb{P}^{a_{2m}}_{b_{2m}} \mathscr{R}^{b_1 b_2 \cdots b_{2m}}_{a_1 a_2 \cdots a_{2m}}. \tag{A.3}$$

There would be two terms contributing to the above expression, one when all the projectors are related by Kronecker deltas and when one of the projector $\mathbb{P}^a_b$ is replaced by $\ell^a \ell_b$ while all the others are replaced by Kronecker deltas. In any other case, for example if two projectors are replaced by the normal, then it would identically vanish, thanks to the completely antisymmetric nature of $\mathscr{R}^{b_1 b_2 \cdots b_{2m}}_{a_1 a_2 \cdots a_{2m}}$. Thus finally we obtain

$$\begin{aligned} m!\mathbb{R} &= \delta^{a_1}_{b_1} \cdots \delta^{a_{2m}}_{b_{2m}} \mathscr{R}^{b_1 b_2 \cdots b_{2m}}_{a_1 a_2 \cdots a_{2m}} \\ &\quad - \frac{2m}{\ell^2} \ell^{a_1} \ell_{b_1} \delta^{a_2}_{b_2} \cdots \delta^{a_{2m}}_{b_{2m}} \mathscr{R}^{b_1 b_2 \cdots b_{2m}}_{a_1 a_2 \cdots a_{2m}} \\ &= \delta^{b_1 b_2 \cdots b_{2m}}_{c_1 c_2 \cdots c_{2m}} \delta^{d_1 d_2 \cdots d_{2m}}_{b_1 b_2 \cdots b_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}} \\ &\quad - \frac{2m}{\ell^2} \ell^{a_1} \ell_{b_1} \delta^{b_1 b_2 \cdots b_{2m}}_{c_1 c_2 \cdots c_{2m}} \delta^{d_1 d_2 \cdots d_{2m}}_{a_1 b_2 \cdots b_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}}. \end{aligned} \tag{A.4}$$

One can now use the following identities:

$$\delta^{b_1 b_2 \cdots b_{2m}}_{d_1 d_2 \cdots d_{2m}} \delta^{c_1 c_2 \cdots c_{2m}}_{b_1 b_2 \cdots b_{2m}} = \frac{m!}{2^m} \delta^{c_1 c_2 \cdots c_{2m}}_{d_1 d_2 \cdots d_{2m}} \tag{A.5}$$

as well as

$$\begin{aligned} &\delta^{b_1 b_2 \cdots b_{2m}}_{c_1 c_2 \cdots c_{2m}} \delta^{d_1 d_2 \cdots d_{2m}}_{a_1 b_2 \cdots b_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}} \\ &\quad = \frac{m!}{2^m} \delta^{b_1}_{c_1} \delta^{d_1 d_2 \cdots d_{2m}}_{a_1 c_2 \cdots c_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}} \end{aligned} \tag{A.6}$$

such that one arrives at

$$\begin{aligned} \mathbb{R} &= \frac{1}{2^m} \delta^{d_1 d_2 \cdots d_{2m}}_{c_1 c_2 \cdots c_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}} \\ &\quad - \frac{2m}{\ell^2} \frac{1}{2^m} \ell^{a_1} \ell_{b_1} \delta^{b_1}_{c_1} \delta^{d_1 d_2 \cdots d_{2m}}_{a_1 c_2 \cdots c_{2m}} R^{c_1 c_2}_{d_1 d_2} \cdots R^{c_{2m-1} c_{2m}}_{d_{2m-1} d_{2m}} \\ &= L - \frac{2}{\ell^2} \ell_c \ell^d R^{cp}_{qr} P^{qr}_{dp} = L - \frac{2}{\ell^2} \mathscr{R}_{ab} \ell^a \ell^b, \end{aligned} \tag{A.7}$$

where $\mathscr{R}_{ab}$ stands for the analogue of Ricci tensor associated with the Lovelock gravitational action. Hence this particular Lovelock Riemann tensor reproduces the Lovelock field equations for gravity as well if the procedure outlined above is being followed. The above exercise explicitly demonstrates the robustness of the idea presented here.

Conflicts of Interest

The author states that there are no conflicts of interest.

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Research Article

Discussion of a Possible Corrected Black Hole Entropy

Miao He, Ziliang Wang, Chao Fang, Daoquan Sun, and Jianbo Deng

Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China

Correspondence should be addressed to Jianbo Deng; dengjb@lzu.edu.cn

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Einstein's equation could be interpreted as the first law of thermodynamics near the spherically symmetric horizon. Through recalling the Einstein gravity with a more general static spherical symmetric metric, we find that the entropy would have a correction in Einstein gravity. By using this method, we investigate the Eddington-inspired Born-Infeld (EiBI) gravity. Without matter field, we can also derive the first law in EiBI gravity. With an electromagnetic field, as the field equations have a more general spherically symmetric solution in EiBI gravity, we find that correction of the entropy could be generalized to EiBI gravity. Furthermore, we point out that the Einstein gravity and EiBI gravity might be equivalent on the event horizon. At last, under EiBI gravity with the electromagnetic field, a specific corrected entropy of black hole is given.

1. Introduction

Black hole thermodynamics has been proposed for many years since the entropy and temperature were found by Bekenstein and Hawking [1, 2], even getting many interesting results, like the four laws of black hole thermodynamics. It established the connection between the gravity and thermodynamics.

The entropy is assumed to be proportional to its horizon area [1], and it is well-known that the so-called area formula of black hole entropy holds only in Einstein gravity. However, when some higher order curvature terms appear in some gravity theory, the area formula has to be modified [3]. A logarithmic term often occurs in the correction like the black hole entropy in loop quantum gravity (quantum geometry) [4–6] and thermal equilibrium fluctuation [7, 8]. The correction of entropy has been studied in Gauss-Bonnet gravity [9], Lovelock gravity [10], and $f(R)$ gravity [11]. In the apparent horizon of FRW universe, the entropy also has a correction [12].

Einstein's equation can be derived from the thermodynamics [13]; on the other side, the thermodynamic route to the gravity field equation, which could get the first law of thermodynamics in Einstein gravity, was proposed by Padmanabhan [14–16]. It indicated a generic connection between thermodynamics of horizons and gravity, although it is not yet understood at a deeper level [17]. This technique has been used in Gauss-Bonnet gravity and Lanczos-Lovelock gravity [15]; the corrected entropy is the same in [18, 19], respectively.

The EiBI gravity was inspired by Bañados and Ferreira [20]. It is completely equivalent to the Einstein gravity in vacuum, but in the presence of matter it would show many interesting results, like an alternative theory of Big Bang singularity in early universe [21] and the mass inflation in EiBI black holes [22, 23]. However, there are few investigations for the thermodynamic properties of black hole in EiBI gravity, as it has a complicated spherically symmetric solution when the electromagnetic field is considered [24].

In this paper, inspired by Padmanabhan [14–16], we derive the first law of black hole thermodynamics with the commonly accepted thermodynamics quantities from Einstein's equation. We also use this technique in EiBI gravity and get the known first law; the results show a more general formula of entropy, which also holds for AdS Schwarzschild black hole and AdS R-N black hole. Motived by this, supposing a more general static spherically symmetric metric, we get the same result in Einstein gravity.

This paper is organized as follows: In Section 2, there is a derivation of the thermodynamic identity from Einstein gravity; we also get a more general formula of entropy. In Section 3, we investigate the EiBI gravity by using the thermodynamic route to the field equation and get the formula of entropy in EiBI gravity. Conclusions and discussion are given in Section 4.

2. Black Hole Thermodynamic Identity from Einstein's Equation

Einstein's equation can be derived from the thermodynamics [13]. On the other side, it is possible to interpret Einstein's equation near the spherical symmetric event horizon as the first law of thermodynamics which was proposed by Padmanabhan [14–16]. However, the thermodynamic quantities might not be consistent with the normal ones, especially the pressure and internal energy. Fortunately, through restudying the field equation, we can also derive the first law of thermodynamics with the commonly accepted thermodynamic quantities [25, 26].

Considering a static spherically symmetric space-time

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad (1)$$

and the event horizon $r = r_H$ satisfying $f(r_H) = 0$, then one can get its thermodynamic quantities

$$\begin{aligned} T &= \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi}, \\ S &= \pi r_H^2, \\ V &= \frac{4\pi}{3} r_H^3. \end{aligned} \quad (2)$$

If we consider AdS space-time with a negative cosmological constant Λ, there would be a pressure $P = -\Lambda/8\pi$ [25]. The mass of black hole was treated as enthalpy [26] and the first law of black hole thermodynamics is

$$dH = dM = TdS + VdP. \quad (3)$$

Through the Legendre transformation, one can get the internal energy

$$dU = TdS - PdV. \quad (4)$$

Particularly, the internal energies for AdS Schwarzschild black hole and AdS R-N black hole are

$$\begin{aligned} U_{\text{Schwarzschild}} &= \frac{r_H}{2}, \\ U_{\text{R-N}} &= \frac{r_H}{2} + \frac{Q^2}{2r_H}. \end{aligned} \quad (5)$$

Einstein's equation with a cosmological constant is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (6)$$

If the metric has the form of (1), in the case of $T_{\mu\nu} = 0$, one can obtain the $\theta\theta$ component of the equation

$$-1 + f(r) + rf'(r) = -\Lambda r^2. \quad (7)$$

Setting $r = r_H$ and then multiplying the above equation by dr_H, we can rewrite (7) as

$$\frac{f'(r_H)}{4\pi} d\left(\pi r_H^2\right) - d\left(\frac{r_H}{2}\right) = -\frac{\Lambda}{8\pi} d\left(\frac{4\pi r_H^3}{3}\right). \quad (8)$$

Noticing (2), the above equation can be regarded as the first law of black hole thermodynamics, since $U = r_H/2$ for the Schwarzschild solution.

For a charged AdS black hole, the metric also takes the form of (1). The energy-momentum tensor of electromagnetic field is

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\sigma}F^{\sigma}_{\nu} - \frac{1}{4}g_{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}\right). \quad (9)$$

Its nonzero components are

$$\begin{aligned} T_{tt} &= \frac{fE_0^2}{8\pi}, \\ T_{rr} &= -\frac{f^{-1}E_0^2}{8\pi}, \\ T_{\theta\theta} &= \frac{r^2E_0^2}{8\pi}, \\ T_{\phi\phi} &= \frac{r^2\sin^2\theta E_0^2}{8\pi}, \end{aligned} \quad (10)$$

where $E_0 = Q/r^2$, and Q represents the charge of black hole. According to Einstein's equations, one can also get

$$-1 + f(r) + rf'(r) = -\Lambda r^2 - r^2 E_0^2. \quad (11)$$

By the same technique, and treating Q as a constant, we get

$$\frac{f'(r_H)}{4\pi} d\left(\pi r_H^2\right) - d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) = Pd\left(\frac{4\pi r_H^3}{3}\right), \quad (12)$$

which can be also treated as the first law with the thermodynamic quantities in (2), and one can verify that $U = r_H/2 + Q^2/2r_H$ for the AdS R-N black hole.

Here we keep Q as a constant, which means a chargeless particle falls into the AdS R-N black hole; (12) is consistent with the first law. While a charged particle falls into the AdS R-N black hole, the event horizon r_H would arise due to changes of dM and dQ; then (12) could be rewritten as [15]

$$\begin{aligned} &\frac{f'(r_H)}{4\pi} d\left(\pi r_H^2\right) - d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) + \frac{Q}{r_H}dQ \\ &\quad = Pd\left(\frac{4\pi r_H^3}{3}\right). \end{aligned} \quad (13)$$

Then it would adopt to the first law with the formulation

$$dU = TdS - PdV + \Phi dQ. \quad (14)$$

We should point out that $T_{\mu\nu}$ contributes to the internal energy U.

Thus Einstein's equation can be interpreted as the first law of thermodynamic near the event horizon. The technique was first proposed by Padmanabhan, and some relevant comments about the meaning of thermodynamic quantities for this result were given [14–17]. Since we used the different thermodynamics quantities in our derivation compared with Padmanabhan's work, there are some comments we would like to add. Firstly, this method can be applied to single horizon and multiple horizons space-time, but this is just a local description of horizon thermodynamics which means the temperature, entropy, and energy are local quantities just for one horizon. In addition, this method still has some problem to solve, when it was applied to cosmic horizon or de Sitter horizon, because the definition of the temperatures for cosmic horizon is quite different. Secondly, it seems that there is a manifest arbitrariness or freedom in the derivations. One could multiply the entire equation by an arbitrary function; then the expressions for entropy, internal energy, and volume would be another one. In fact, if we consider the initial value, that is, $S = A/4$, $V = 4\pi r_H^3/3$, for Schwarzschild space-time, this problem would disappear. Finally, the perturbation of the static space-time can be interpreted in terms of the physical process of black hole evaporation or hawking radiation. One can also interpret the relations by, say, dropping test particles into the black hole.

Note that the structure of the equation itself allows us to read off the expression for entropy. This technique has been used for Gauss-Bonnet gravity and Lovelock gravity [15], in which their entropy formulas are the same as [18, 19], respectively. We would like to consider a more general static spherical symmetric metric

$$ds^2 = -\psi(r)^2 f(r)\,dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{15}$$

One can calculate its nonzero components of Ricci tensor:

$$\begin{aligned}
R_{tt} &= \frac{\left(\psi^2 f\right)'' f}{2} - \frac{\left(\psi^2 f\right)' f}{4}\left(-\frac{f'}{f} + \frac{\left(\psi^2 f\right)'}{\psi^2 f}\right) + \frac{\left(\psi^2 f\right)' f}{r}, \\
R_{rr} &= -\frac{\left(\psi^2 f\right)''}{2\psi^2 f} + \frac{\left(\psi^2 f\right)'}{4\psi^2 f}\left(-\frac{f'}{f} + \frac{\left(\psi^2 f\right)'}{\psi^2 f}\right) - \frac{f'}{rf}, \\
R_{\theta\theta} &= 1 - \frac{rf}{2}\left(\frac{f'}{f} + \frac{\left(\psi^2 f\right)'}{\psi^2 f}\right) - f, \\
R_{\phi\phi} &= \sin^2\theta R_{\theta\theta}.
\end{aligned} \tag{16}$$

And Einstein's equation with a cosmological constant can be written as

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right). \tag{17}$$

For the Schwarzschild vacuum $T_{\mu\nu} = 0$, one can get $\psi = C$, a constant. We can always have $\psi = 1$ by choosing the coordinate time $d\hat{t} = C dt$ without changing the killing vector; then the metric would return back to the spherically symmetric metric (1). For the charged black hole, we have got the Reissner-Nordstrom metric, which also implies $\psi = 1$; thus the first law could be obtained. The reason of this result might be that the matter field leads to $\psi = 1$.

To get a general case, we just consider the $\theta\theta$ component of (17), which can be expressed as

$$1 - \frac{rf(r)}{2}\left(\frac{f'(r)}{f(r)} + \frac{\left(\psi^2 f(r)\right)'}{\psi^2 f(r)}\right) - f(r) = \Lambda r^2 + 8\pi\left(T_{\theta\theta} - \frac{1}{2}Tr^2\right). \tag{18}$$

We assume the metric satisfies $f(r_H) = 0$ and $\psi(r_H) \neq 0$ on the event horizon. By setting $r = r_H$ and considering the matter field contributes to the internal energy U and then multiplying dr_H one can get

$$dU - \frac{r_H}{2}f'(r_H)\,dr_H = -Pd\left(\frac{4\pi r_H^3}{3}\right), \tag{19}$$

where

$$dU = \left[\frac{1}{2} + 4\pi\left(T_{\theta\theta} - \frac{1}{2}Tr_H^2\right)\right]dr_H. \tag{20}$$

One can verify that for the AdS Schwarzschild black hole, it reduces to $U = r_H/2$ and for the AdS R-N black hole it gives $U = r_H/2 + Q^2/2r_H$. So the internal energy expression is just a generalization.

The Hawking temperature on the event horizon becomes

$$T = \frac{\kappa}{2\pi} = \frac{\psi(r_H)\,f'(r_H)}{4\pi}. \tag{21}$$

Now rewrite (19) as

$$dU - T\left(\frac{2\pi r_H}{\psi(r_H)}dr_H\right) = -PdV. \tag{22}$$

One would find the entropy has to satisfy

$$dS = \frac{2\pi r_H}{\psi(r_H)}dr_H, \tag{23}$$

or

$$S = \int \frac{2\pi r_H}{\psi(r_H)}dr_H. \tag{24}$$

Thus, we generalize the corrected entropy formula to Einstein gravity for static spherically symmetric metric equation (15). Once $\psi = 1$, it is obvious that $S = \pi r_H^2 = A/4$, which is the well-known Bekenstein-Hawking black hole entropy. For the Schwarzschild black hole and R-N black hole, which all have $\psi = 1$, $S = A/4$.

In fact, Matt Visser has proposed that the entropy of "dirty" black holes might not equal quarter of area of event horizon [27, 28]. Generically, a "dirty" black hole is a black hole with various classical matter fields, higher curvature terms in the gravity Lagrangian, or some other versions of quantum hair. For the Einstein gravity, a more general metric (15) could be caused by some special matter fields like electromagnetism with Dilaton fields [29]. So the entropy also should be corrected in Einstein gravity. In the next section, we would like to consider the EiBI gravity [20]; its Lagrangian includes the higher curvature terms in (25), which could be interpreted as the self-gravity; we will show our discussion of this method in the Eddington-inspired Born-Infeld gravity and get its thermodynamic quantities.

3. The Entropy in Eddington-Inspired Born-Infeld Gravity

The Eddington-inspired Born-Infeld theory of gravity is based on the Palatini formulation which treats the metric and connection as independent fields [20]. Its action can be written as

$$S = \frac{1}{8\pi\kappa}\int d^4x\left[\sqrt{\left|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)\right|}-\lambda\sqrt{g}\right] + S_M(g,\Gamma,\Psi), \tag{25}$$

where $g_{\mu\nu}$ is the metric of space-time and its determinant is g, $R_{\mu\nu}$ is the symmetric Ricci tensor related to Γ, the dimensionless parameter $\lambda = 1+\kappa\Lambda$, and the parameter κ has the inverse dimension of cosmological constant Λ.

By varying the action with respect to $g_{\mu\nu}$ and Γ, one obtains the equation of motion:

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \tag{26}$$

$$\sqrt{|q|}q^{\mu\nu} = \lambda\sqrt{|g|}g^{\mu\nu} - 8\pi\kappa\sqrt{|g|}T^{\mu\nu}, \tag{27}$$

where $q_{\mu\nu}$ is the auxiliary metric compatible with the connection to

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}q^{\lambda\sigma}\left(q_{\mu\sigma,\nu}+q_{\nu\sigma,\mu}-q_{\mu\nu,\sigma}\right). \tag{28}$$

By combining (26) and (27) and then expanding the field equations to 2nd order of κ [20]

$$R_{\mu\nu} \simeq \Lambda g_{\mu\nu} + 8\pi\left(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu}\right) + 8\pi\kappa\left[S_{\mu\nu}-\frac{1}{4}Sg_{\mu\nu}\right], \tag{29}$$

where $S_{\mu\nu} = T^{\alpha}_{\mu}T_{\alpha\nu} - (1/2)TT_{\mu\nu}$, one can find that the equation is the 1st-order correction to Einstein's equation. On the other hand, EiBI gravity can be interpreted as a correction of the matter term compared with Einstein gravity. Even the EiBI gravity is fully equivalent to the Einstein gravity in vacuum.

Let us consider the thermodynamics from the field equation in this gravity model. Generally, a static spherically symmetric metric $g_{\mu\nu}$ could be

$$ds_g^2 = -\psi^2(r)f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \tag{30}$$

and the auxiliary metric $q_{\mu\nu}$ is assumed as [24]

$$ds_q^2 = -G^2(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + H^2(r)\left(d\theta^2+\sin^2\theta d\phi^2\right). \tag{31}$$

The Ricci tensor was calculated as follows:

$$\begin{aligned} R_{tt} &= 2\frac{GG'H'F^2}{H} + \frac{G^2FF'H'}{H} + \frac{3}{2}GG'FF' + GG''F^2 + \frac{1}{2}G^2FF'', \\ R_{rr} &= -2\frac{H''}{H} - \frac{F'H'}{FH} - \frac{3}{2}\frac{G'F'}{GF} - \frac{G''}{G} - \frac{F''}{2F}, \\ R_{\theta\theta} &= 1 - HH'F' - \frac{G'}{G}HH'F - H'^2F - HH''F, \\ R_{\phi\phi} &= \sin^2\theta R_{\theta\theta}, \end{aligned} \tag{32}$$

where $X' = \partial X/\partial r$, which we will use in the rest of this paper.

Without the matter fields, (27) reduces to

$$\begin{aligned} \frac{H^2}{GF} &= \frac{\lambda r^2}{\psi f}, \\ GH^2F &= \lambda r^2\psi f, \\ G &= \lambda\psi, \end{aligned} \tag{33}$$

and thus one can obtain

$$\begin{aligned} G &= \lambda\psi, \\ F &= \lambda^{-1}f, \\ H^2 &= \lambda r^2. \end{aligned} \tag{34}$$

Plugging these into (26), then the $\theta\theta$ component is

$$1 - rf'(r) - \left(\frac{r\psi'}{\psi}+1\right)f(r) = \frac{1}{\kappa}(\lambda-1)r^2. \tag{35}$$

Near the event horizon $r = r_H$ ($f(r_H) = 0$), we have

$$d\left(\frac{r_H}{2}\right) - \frac{f'(r_H)}{4\pi}d\left(\pi r_H^2\right) = -Pd\left(\frac{4\pi r_H^3}{3}\right). \tag{36}$$

As the EiBI gravity is fully equivalent to the Einstein gravity in vacuum [20], the above equation could imply the first law. In fact, the black hole solution to EiBI gravity with no source is the same as Schwarzschild-de Sitter metric, which illustrates $\psi = 1$ and $f = 1 - 2m/r + \Lambda r^2/3$ [20]. The thermodynamic quantities in (36) also hold in EiBI gravity. Thus the first law can also be got from the EiBI gravity. Next, we would consider the EiBI gravity with electromagnetic field.

The energy-momentum tensor of electromagnetic field could be

$$\begin{aligned} T^{tt} &= \frac{\left(\psi^2 f\right)^{-1} E_0^2}{8\pi}, \\ T^{rr} &= -\frac{f E_0^2}{8\pi}, \\ T^{\theta\theta} &= \frac{r^{-2} E_0^2}{8\pi}, \\ T^{\phi\phi} &= \frac{r^{-2} \sin^{-2}\theta E_0^2}{8\pi}, \end{aligned} \tag{37}$$

where $E_0 = Q/r^2$ and Q represents the charge of black hole. Then (27) becomes

$$\begin{aligned} \frac{H^2}{GF} &= \left(\lambda + \kappa E_0^2\right) \frac{r^2}{\psi f}, \\ GH^2 F &= \left(\lambda + \kappa E_0^2\right) r^2 \psi f, \\ G &= \left(\lambda - \kappa E_0^2\right) \psi, \end{aligned} \tag{38}$$

and one can get

$$\begin{aligned} G &= \left(\lambda - \kappa E_0^2\right) \psi, \\ F &= \left(\lambda - \kappa E_0^2\right)^{-1} f, \\ H^2 &= \left(\lambda + \kappa E_0^2\right) r^2. \end{aligned} \tag{39}$$

The $\theta\theta$ component of (26) is written as

$$\begin{aligned} 1 - \frac{\lambda + \kappa E_0^2 + \kappa r E_0 E_0'}{\lambda - \kappa E_0^2} \cdot r f' - \frac{Y}{\lambda - \kappa E_0^2} f \\ = \frac{1}{\kappa} (\lambda - 1) r^2 + E_0^2 r^2, \end{aligned} \tag{40}$$

where

$$Y = 2\kappa r E_0 E_0' + \frac{G'}{G} H H' - H'^2. \tag{41}$$

If we assume that the event horizon satisfies $f(r_H) = 0$ and $\psi(r_H) \neq 0$, then set $r = r_H$ in (40) and multiply it by dr_H; noting $E_0' = -2E_0/r$, it gives

$$d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) - \frac{r_H}{2} f'(r_H)\, dr_H = -P d\left(\frac{4\pi r_H^3}{3}\right). \tag{42}$$

This equation should be the first law since it can go back to (36) when $Q = 0$. And one can also confirm this by noticing that $dU = d(r_H/2 + Q^2/2r_H)$ and $dV = d(4\pi r_H^3/3)$. Moreover, it gives the same result in Einstein gravity. So we should identify that

$$T dS = \frac{r_H}{2} f'(r_H)\, dr_H. \tag{43}$$

However, as the metric takes the form of (30), the surface gravity could be [29]

$$\kappa = \lim_{r \to r_H} \frac{1}{2} \frac{\partial_r g_{tt}}{\sqrt{g_{tt} g_{rr}}} = \frac{\psi(r_H) f'(r_H)}{2}. \tag{44}$$

One can obtain the temperature on the event horizon

$$T = \frac{\psi(r_H) f'(r_H)}{4\pi}. \tag{45}$$

Then (43) would imply that

$$dS = \left(\frac{2\pi r_H}{\psi(r_H)}\right) dr_H, \tag{46}$$

or

$$S = \int \frac{2\pi r_H}{\psi(r_H)} dr_H. \tag{47}$$

Obviously, when $\psi = 1$, one can get $S = \pi r_H^2$.

Thus, we get the entropy formula in EiBI gravity. Surprisingly, we find it also holds in Einstein gravity once metric takes the form of (30). Therefore, we can also get the first law for a more general static spherically symmetric metric in EiBI gravity. Moreover, one can find that (12) is the same as (42). It implies that the Einstein gravity and EiBI gravity might be equivalent on the event horizon from the view of black hole thermodynamics.

In fact, the black hole solution with electromagnetic field in EiBI gravity has been found, while $f(r_H) = 0$ and $\psi(r_H) \neq 0$ [20, 24]. It is given as follows:

$$\psi(r) = \frac{\sqrt{\lambda} r^2}{\sqrt{\lambda r^4 + \kappa Q^2}}. \tag{48}$$

With this result we can get the corrected entropy

$$\begin{aligned} S &= \int \frac{2\pi r_H}{\psi(r_H)} dr_H = \pi \int \frac{1}{r_H^2} \sqrt{r_H^4 + \frac{\kappa}{\lambda} Q^2}\, dr_H^2 \\ &= \pi \sqrt{r_H^4 + \frac{\kappa}{\lambda} Q^2} - \pi \sqrt{\frac{\kappa}{\lambda}} |Q| \\ &\quad \cdot \ln\left(\sqrt{\frac{\kappa}{\lambda}} \frac{|Q|}{r_H^2} + \sqrt{1 + \frac{\kappa}{\lambda} \frac{Q^2}{r_H^4}}\right). \end{aligned} \tag{49}$$

A logarithmic term occurs in this formula as a corrected entropy. When $Q = 0$, one gets $S = \pi r_H^2$, which is consistent with the vacuum case, and so it is the same as Einstein gravity. When $\kappa \to 0$, EiBI gravity would reduce to the Einstein gravity, and entropy becomes the Bekenstein-Hawking one.

4. Conclusion and Discussion

In this paper, we restudied Padmanabhan's work that it is possible to write Einstein's equation for spherically symmetric space-time in the form of the first law of thermodynamics [14–17], but the thermodynamic quantities might not be consistent with the normal ones, especially the pressure and internal energy. By using this technique, we reproduced the first law with the commonly accepted thermodynamic quantities in the AdS space-time, and this technique provides an effective approach to read off the thermodynamic quantities. Next, we investigated a more general static spherically symmetric metric taking form (15) in Einstein gravity. It is surprising to find that the entropy might have a correction in Einstein gravity.

Since it provided a convenient approach to study the black hole thermodynamics just from the field equation, we investigated the black hole thermodynamic in EiBI gravity. We found that there is nothing different from Einstein gravity in vacuum, but entropy could be different from the R-N black hole when the electromagnetic field was considered. The corrected entropy from EiBI gravity should be (47), which can reduce to the Bekenstein-Hawking entropy when $\psi = 1$, and it is also the same as Einstein gravity when $Q = 0$ without the matter field [20]. Thus, the corrected entropy in Einstein gravity could be generalized to EiBI gravity.

Moreover, as the Einstein gravity and EiBI gravity hold the same result, we remarked that these two theories of gravity could be equivalent on the event horizon from the view of thermodynamics.

At last, as an example, a specific corrected entropy of the charged black hole in EiBI gravity was given. The entropy form would lead to something different for the black hole thermodynamics in EiBI gravity, like the phase transition and critical phenomenon. These would be left for our further research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

The Quantum Effect on Friedmann Equation in FRW Universe

Wei Zhang[1] **and Xiao-Mei Kuang**[2]

[1] *Department of General Studies, Nanchang Institute of Science & Technology, Nanchang 330108, China*
[2] *Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, Yangzhou 225009, China*

Correspondence should be addressed to Xiao-Mei Kuang; xmeikuang@gmail.com

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We study the modified Friedmann equation in the Friedmann-Robertson-Walker universe with quantum effect. Our modified results mainly stem from the new entropy-area relation and the novel idea of Padmanabhan, who considers the cosmic space to be emerging as the cosmic time progresses, so that the expansion rate of the universe is determined by the difference of degrees of freedom between the holographic surface and the bulk inside. We also discuss the possibility of having bounce cosmological solution from the modified Friedmann equation in spatially flat geometry.

1. Introduction

In the 1970s, the thermodynamic property of black holes has been proposed [1–3], and it reveals that the gravitational dynamics is entwined with thermodynamics. Inspired by Bekenstein's entropy-area theorem [1], Bardeen et al. put forward the four thermodynamical laws of black hole systems [2]. In 1995, Jacobson considered Einstein's field equation as an equation of state. Afterwards, he reproduced Einstein's field equation by demanding that the fundamental relation $\delta Q = TdS$ holds for all local Rindler causal horizons through each space-time point, and δQ and T are treated as the energy flux and Unruh temperature, respectively, felt by an accelerated observer inside the horizon [4]. In 2010, Verlinde defined gravity as an entropic force due to the changes of the information related to the positions of the materials, and the space is emergent based on the holographic principle in his discussions [5]. Moreover, Verlinde's proposal has been applied to reproduce the Friedmann equation into brane cosmology [6] and Friedmann-Robertson-Walker (FRW) universe [7], respectively.

On the other hand, it was addressed in [8–12] that the Friedmann equation can be modified by a bounce solution of the universe as

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{1}$$

in loop quantum cosmology (LQC). Then the authors of [13] attempted to derive the Friedmann equation by borrowing the Clausius relation, that is, $\delta Q = TdS$, and an entropy-area relation with quantum correction; however, they failed to reproduce the same modified Friedmann equation as that in LQC with bounce solution. Luckily, the difficulty was overcome by the authors of [14], where they proposed a modified dispersion relation at quantum phenomenological level and then obtained the modified Friedmann equation for a bounce solution to the flat FRW universe in LQC. It was found that the role of their modified dispersion relation is to explicitly modify the Clausius relation. This proposal has also been extended into the spatially curved cases and the corresponding modified entropy-area relations have been derived [15].

However, the above studies are only involved in the gravity as an emergent phenomenon rather than the space-time itself as an emergent structure. This situation has been improved by Padmanabhan. In detail, he proposed in [16] that the accelerated expansion of the universe is related to the difference between the surface degrees of freedom (N_{sur}) and the bulk degrees of freedom (N_{bulk}) in a region of space; that is, $\Delta V = \Delta t(N_{\text{sur}} - N_{\text{bulk}})$, where V is the Hubble volume and t is the cosmic time in Planck units. Moreover, the standard evolution of the universe was also reproduced directly from the proposed relation. This proposal inspired plenty of related studies and remarkable progress [17–33]. However, in the framework of Padmanabhan's conjecture, the study of quantum effect is missing. So, it is interesting to introduce the quantum effect to Padmanabhan's conjecture and study the related cosmology.

Thus, in this paper, we will introduce the modified dispersion relation in the framework of Padmanabhan's conjecture and derive the modified Friedmann equation. Then we will analyze whether the quantum effect in Padmanabhan's conjecture will bring in modified Friedmann equation (1) with bounce solution. It is notable that starting from the Clausius relation to the apparent horizon along with the modified dispersion relation, one can easily get the modified Friedmann equation with bounce solution to the FRW universe [14, 15], but the answer is not direct in Padmanabhan's conjecture in the emergent universe. Our study will give an insight into the answer.

Our paper is organized as follows. In next section we briefly review Padmanabhan's idea that the cosmic space is emergent as cosmic time progresses and give the standard Friedmann equation governing the dynamical evolution of the FRW universe. Then, in Section 3, we will analyze the modified Friedmann equation from Padmanabhan's conjecture based on modified entropy-area relation. Finally, we will give our summary and discussions in Section 4. In this paper, we use the natural units with $c = \hbar = k_B = 1$.

2. The Emergence of Cosmic Space of the FRW Universe

In this section, we will give a brief review on the process of obtaining standard Friedmann equation in the emergent universe, which was addressed by Padmanabhan in [16]. The main idea is that the expansion of the universe (or the emergence of space) tends to fulfill the holographic equipartition condition, which stated that the number of degrees of freedom (N_{bulk}) inside the Hubble volume is equal to the number of degrees of freedom (N_{sur}) on the spherical surface of Hubble radius; that is, $N_{\text{bulk}} = N_{\text{sur}}$. So in our asymptotic de Sitter universe, the natural law governing the emergence of space in an infinitesimal interval dt is

$$\frac{dV}{dt} = G\left(N_{\text{sur}} - N_{\text{bulk}}\right), \tag{2}$$

where $V = 4\pi/3H^3$ is the Hubble volume and t is the cosmic time.

For a spatially flat FRW universe with Hubble constant H and apparent horizon $r_A = 1/H$, we have

$$N_{\text{sur}} = 4S = \frac{4\pi}{GH^2}, \tag{3}$$

where $S = A/4G = \pi/GH^2$ is the entropy of the apparent horizon, and

$$N_{\text{bulk}} = \frac{2|E|}{T} = -\frac{2(\rho + 3p)V}{T}, \tag{4}$$

where in the second equality we recalled the horizon temperature $T = H/2\pi$ and Komar energy $|E| = -(\rho + 3p)V$ for accelerating part with dark energy having $\rho + 3p < 0$ (it is notable that in [16] the author discussed the contributions of the matter with $|E| = (\rho+3p)V$ in the bulk degrees of freedom and the derivation of Friedmann equation was unaffected). Subsequently, one can reduce (2) into

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{5}$$

which is the standard dynamical Friedmann equation of flat FRW universe in general relativity. Furthermore, recalling the continuity equation,

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{6}$$

and integrating (5) gives us the standard Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3}. \tag{7}$$

Note that in [13] the integration result is $H^2 + k/a^2 = 8\pi G\rho/3$ with general geometry, where the authors interpreted the integration constant k as the spatial curvature of the FRW universe.

3. The Quantum Effect on Friedmann Equation in Cosmic Space of the FRW Universe

In this section, we will apply the proposal described in last section to study the quantum effect on the Friedmann equation. We only consider the quantum effect at the phenomenological level and borrow the modified dispersion relation (MDR) [14]:

$$\frac{\sin\left(\eta l_p E\right)}{\eta l_p} = \sqrt{p^2 + m^2}. \tag{8}$$

Here p and E are the momentum and energy of a particle with mass m, respectively. The Planck length is $l_p = \sqrt{8\pi G} = 1/M_p$, where M_p is the Planck mass. η is a dimensionless parameter and $\eta \to 0$ goes to the standard dispersion relation $E^2 = p^2 + m^2$.

With the use of thermodynamical description on the apparent horizon, the authors of [14] derived the modified Friedmann equation of a spatially flat universe from MDR (8). Later, the extended study in general FRW universe with $k = 0, \pm 1$ was presented in [15].

Here, we will derive the modified Friedmann equation by following the steps of emergent cosmic space shown in last section. According to the study in [15], MDR (8) modified the entropy for the first energy branch as

$$\begin{aligned} S_M = {} & \frac{A}{4G}\sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} \\ & + \frac{\pi\eta^2 l_p^2}{G}\ln\left[\sqrt{\frac{A}{4\pi\eta^2 l_p^2}} + \sqrt{\frac{A}{4\pi\eta^2 l_p^2} - 1}\right], \end{aligned} \tag{9}$$

where $A = 4\pi r_A^2 = 4\pi/(H^2 + k/a^2)$ is the area of the apparent horizon at the classical level. To proceed, we define an effective apparent horizon area with the quantum effect

$$\begin{aligned}\widetilde{A} &= 4GS_M \\ &= A\sqrt{1-\frac{4\pi\eta^2 l_p^2}{A}} \\ &\quad + 4\pi\eta^2 l_p^2 \ln\left[\sqrt{\frac{A}{4\pi\eta^2 l_p^2}} + \sqrt{\frac{A}{4\pi\eta^2 l_p^2}-1}\right] \\ &= 4\pi r_A^2\sqrt{1-\frac{\eta^2 l_p^2}{r_A^2}} \\ &\quad + 4\pi\eta^2 l_p^2 \ln\left[\frac{r_A}{\eta l_p} + \sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}\right].\end{aligned} \tag{10}$$

Note that when $\eta \to 0$, $\widetilde{A}$ is equal to A and recovers the usual result.

Moreover, the volume (V) and the area (A) of the apparent horizon of an n-sphere with radius r_A satisfy [34]

$$\frac{dV}{dA} = \frac{r_A}{n-1}. \tag{11}$$

Then one can think that the change of the effective volume mainly stems from the change of the effective area, so that we have the time evolution of the effective volume of the FRW universe [34]

$$\frac{d\widetilde{V}}{dt} = \frac{r_A}{2}\frac{d\widetilde{A}}{dt} = \frac{4\pi r_A^3 \dot{r}_A}{\eta l_p\sqrt{r_A^2/\eta^2 l_p^2 - 1}}, \tag{12}$$

from which we can obtain the effective volume

$$\widetilde{V} = \frac{4\pi\eta^3 l_p^3}{3}\left(2+\frac{r_A^2}{\eta^2 l_p^2}\right)\sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}. \tag{13}$$

Also, when $\eta \to 0$, $\widetilde{V} = 4\pi r_A^3/3$ is the usual Hubble volume.

We move on to calculate N_{bulk} in the bulk and N_{sur} in the boundary. Considering the Hawking temperature (similar to (12), we ignore the direct correction to the radius in the Hawking temperature, and the changes of numbers of degrees of freedom directly stem from the corrections of the area of the apparent horizon) $T = 1/2\pi r_A$ and $E = -(\rho + 3p)\widetilde{V}$ with dark energy in the bulk, we obtain

$$\begin{aligned}N_{\text{bulk}} &= \frac{2E}{T} \\ &= -\frac{16\pi^2(\rho+3p)\eta^3 l_p^3 r_A}{3}\left(2+\frac{r_A^2}{\eta^2 l_p^2}\right)\sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}.\end{aligned} \tag{14}$$

The statistical physics has shown that N_{sur} can be calculated from the entropy [18]

$$\begin{aligned}N_{\text{sur}} &= 4S_M \\ &= \frac{4\pi r_A^2}{G}\sqrt{1-\frac{\eta^2 l_p^2}{r_A^2}} \\ &\quad + \frac{4\pi\eta^2 l_p^2}{G}\ln\left[\frac{r_A}{\eta l_p} + \sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}\right].\end{aligned} \tag{15}$$

Substituting (12), (14), and (15) into (2), we get

$$\begin{aligned}&\frac{4\pi r_A^3 \dot{r}_A}{\eta l_p\sqrt{r_A^2/\eta^2 l_p^2 - 1}} \\ &= 4\pi r_A^2\sqrt{1-\frac{\eta^2 l_p^2}{r_A^2}} + 4\pi\eta^2 l_p^2\ln\left[\frac{r_A}{\eta l_p} + \sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}\right] \\ &\quad + \frac{16\pi^2 G(\rho+3p)\eta^3 l_p^3 r_A}{3}\left(2+\frac{r_A^2}{\eta^2 l_p^2}\right)\sqrt{\frac{r_A^2}{\eta^2 l_p^2}-1}.\end{aligned} \tag{16}$$

The expression above looks very complicated; however, with $k = 0$, we have $\dot{r}_A = 1 - (\ddot{a}/a)r_A^2$, so that (16) can be reduced into (with $k = \pm 1$, we have $\dot{r}_A = 1 - ((H\dot{H} - k\dot{a}/a^3 + \sqrt{(H^2 + k/a^2)^3})/\sqrt{(H^2 + k/a^2)})r_A^2$ which makes it difficult to simplify (16); we hope to solve this problem in near future)

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi G(\rho+3p)}{3}\left(1+\frac{\eta^2 l_p^2}{r_A^2} - \frac{2\eta^4 l_p^4}{r_A^4}\right) \\ &\quad + \frac{\eta^2 l_p^2\left[2+\ln\left(\eta^2 l_p^2/4r_A^2\right)\right]}{2r_A^4} \\ &\simeq -\frac{4\pi G(\rho+3p)}{3}\left(1+\frac{\eta^2 l_p^2}{r_A^2}\right) + \frac{\eta^2 l_p^2}{r_A^4},\end{aligned} \tag{17}$$

where, in the third line, we have approximately expanded the expression to the order $\eta^2 l_p^2/r_A^2$ because it is a small quantity. Form (17) is the modified dynamical Friedmann equation for the flat FRW universe, which reduces to the standard dynamical Friedmann equation when $\eta \to 0$. Further combining continuity equation (6) with (17), we obtain the other modified Friedmann equation

$$\begin{aligned}\frac{\dot{a}^2}{a^2} &= \frac{8\pi G\rho}{3} + \frac{8\pi G\eta^2 l_p^2}{3a^2}\int\left(\dot{a}^2\dot{\rho} + \frac{2\dot{a}^3\rho}{a}\right)dt \\ &\quad + \frac{2\eta^2 l_p^2}{a^2}\int\frac{\dot{a}^5}{a^3}dt.\end{aligned} \tag{18}$$

Here we also set the integration constant to be vanished. Again, $\eta \to 0$ in (18) reproduces the standard result of flat FRW universe.

In order to analyze whether (18) admits a bounce solution, we define

$$\rho_c = -\frac{\rho^2}{\left(\eta^2 l_p^2/a^2\right)\int\left(\dot{a}^2\dot{\rho} + 2\dot{a}^3\rho/a\right)dt + \left(3\eta^2 l_p^2/4\pi G\rho a^2\right)\int\left(\dot{a}^5/a^3\right)dt}, \quad (19)$$

so that (18) can be rewritten as (1) for bounce solution. The unsolved integral in ρ_c makes it difficult to give a reliable conclusion; however, we can at least give some discussions. First, without the quantum correction, that is, $\eta \to 0$, ρ_c goes to infinity, so bounce case (1) recovers standard case (7) without bounce. Then, when ρ_c in (19) is positive, (17) and (18) fulfill the bouncing conditions, that is, $a > 0$, $\dot{a} = 0$, and $\ddot{a} > 0$; then (18) admits a bounce solution. Finally, when ρ_c is nonpositive, we can not have any bounce solution.

We note that, for the second energy branch whose entropy is $-S_M$ [15], the procedures above are straightforward and the modified Friedmann equation is the same as (17) and (18). However, for this energy branch, the effective volume $(\widetilde{V})$ and the area $(\widetilde{A})$ of the apparent horizon are negative, which are not physical.

4. Summary and Discussions

In this paper, we studied the quantum effect on the Friedmann equation for the flat FRW universe with the use of Padmanabhan's conjecture in the emergent universe. We obtained modified Friedmann equations (17) and (18) with the quantum correction on dispersion relation (8). For the closed ($k = 1$) and open ($k = -1$) universes, we found it is difficult to simplify the dynamical equation in our process, but we still see the quantum effect on (16) which is supposed to be the modified Friedmann equation. We also argued the condition under which modified Friedmann equation (18) admits a bounce solution in the flat universe.

It is worth pointing out that, in our paper, the modified Friedmann equation was only obtained in the flat FRW universe with $k = 0$, which may imply that key equation (2) is not the basic equation of the emergent universe and it may have to be corrected at the quantum level. This is an interesting point we will study in the near future. On the other hand, the experimental testing of quantum effect in bouncing cosmology is another interesting aspect. There are many literatures discussed this topic, for example, [35, 36] comparing the quantum theory with experimental data and [37, 38] probing the quantum gravity with modified dispersion relation by cold atoms.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Hawking Radiation-Quasinormal Modes Correspondence for Large AdS Black Holes

Dao-Quan Sun, Zi-Liang Wang, Miao He, Xian-Ru Hu, and Jian-Bo Deng

Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China

Correspondence should be addressed to Jian-Bo Deng; dengjb@lzu.edu.cn

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It is well-known that the nonstrictly thermal character of the Hawking radiation spectrum generates a natural correspondence between Hawking radiation and black hole quasinormal modes. This main issue has been analyzed in the framework of Schwarzschild black holes, Kerr black holes, and nonextremal Reissner-Nordstrom black holes. In this paper, by introducing the effective temperature, we reanalyze the nonstrictly thermal character of large AdS black holes. The results show that the effective mass corresponding to the effective temperature is approximatively the average one in any dimension. And the other effective quantities can also be obtained. Based on the known forms of frequency in quasinormal modes, we reanalyze the asymptotic frequencies of the large AdS black hole in three and five dimensions. Then we get the formulas of the Bekenstein-Hawking entropy and the horizon's area quantization with functions of the quantum "overtone" number n.

1. Introduction

Ever since Hawking's astounding discovery [1] that black holes radiate thermally, many studies have been carried out [2–11]. One of the works done by Parikh and Wilczek, considering the conservation of energy, shows that an isolated radiation black hole cannot have an emission spectrum that is precisely thermal [2, 3]. Nonstrictly thermal character of Hawking radiation spectrum implies that the spectrum is not strictly continuous and thus generates a natural correspondence between Hawking radiation and black hole quasinormal modes [12–16]. It is very important for the physical interpretation of black holes quasinormal modes. Hod's intriguing idea [17, 18] suggested that black hole quasinormal modes carry important information about black hole's area quantization. Afterwards, Hod's influential conjecture was refined and clarified by Maggiore [19]. In addition, it is also believed that quasinormal modes would be connected with the microstructure of space-time [20]. In [12–16], it was shown that quasinormal modes can be naturally interpreted in terms of quantum levels, where the emission or absorption of a particle is interpreted as a transition between two distinct levels on the discrete energy spectrum. These results are important to realize the unitary quantum gravity theory as black holes are considered as theoretical laboratories for testing models of quantum gravity.

Based on the nonstrictly thermal character, the effective framework of black holes was introduced by [12–16]. For Schwarzschild black holes [12, 14], the physical solutions for the absolute values of the frequencies can be used to analyze important properties and quantities, like the horizon area quantization, the area quanta number, the Bekenstein-Hawking entropy, and the number of microstates [21], that is, quantities which are considered fundamental to realize the underlying unitary quantum gravity theory.

Research showed that asymptotical AdS solutions are stable [22]. The quantum mechanical and thermodynamic properties of black holes in AdS space-time have been considered [22, 23]. Some researches also show that the quasinormal frequencies of AdS black holes could be interpreted in terms of dual conformal field transformation [24–26]. In this paper, we would like to focus on the quasinormal frequencies of large AdS black holes. We apply the effective framework of nonstrictly thermal character to large AdS black holes in different dimensions. We introduce the effective temperature and show that the effective mass is approximatively the average one in any dimension. As examples, we will give three- and five-dimensional cases. Based on the known forms of frequency in quasinormal modes, we reanalyze the asymptotic frequencies of the large AdS black hole in three and five dimensions and calculate the Bekenstein-Hawking entropy, the horizon area quantization, the area quanta number, and the number of microstates.

The paper is organized as follows: in Section 2, we will study the effective temperature for large AdS black holes and the corresponding quasinormal modes. Section 3 is reserved for conclusions and discussions.

2. Effective Application of Quasinormal Modes to the Large AdS Black Hole

In this section, we will apply the nonstrictly thermal black hole effective framework of [12–16] to large AdS black holes. Working with $G = c = k_B = \hbar = 1/4\pi\varepsilon_0 = 1$ (Planck units) is adopted in the following. The metric of a d-dimensional AdS black hole can be written as

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-2}^2, \qquad f(r) = \frac{r^2}{R^2} + 1 - \frac{\varpi_{d-1}M}{r^{d-3}}, \tag{1}$$

where R is the AdS radius and M is the mass of the black hole. For a large black hole, the metric function $f(r)$ is simplified to

$$f(r) = \frac{r^2}{R^2} - \frac{\varpi_{d-1}M}{r^{d-3}}, \tag{2}$$

where $\varpi_{d-1} = 16\pi/(d-2)A_{d-2}$, $A_{d-2} = 2\pi^{(d-1)/2}/\Gamma((d-1)/2)$ is the volume of a unit $(d-2)$-sphere.

In general, the rate of particle emission from the horizon is as follows [2]:

$$\Gamma \sim e^{-\Delta S_{BH}}. \tag{3}$$

There are two different tunneling methods about calculating the Hawking temperature. Both formulas come from a semiclassical approximation with a scalar field on a curved background to calculate the tunneling amplitude but they differ by a factor of 2 in the resulting temperature. The one method which is called canonically invariant tunneling uses a particular ansatz for the action and then solves the Hamilton-Jacobi equations to find the imaginary part; for details, see [5–11]. In this paper, we will focus on the method proposed by Parikh and Wilczek [2] to calculate the relevant physical quantities. And the form of the probability of the emission for the particle in a d-dimensional AdS black hole is given by the following [27]:

$$\operatorname{Im} S = \frac{1}{8}A_{d-2}\left[r_h^{d-2}(M) - r_h^{d-2}(M-\omega)\right]. \tag{4}$$

For a large AdS black hole, the radius of the horizon is $r_h = [R^2\varpi_{d-1}M]^{1/(d-1)}$. Then, the probability will be

$$\Gamma = \exp\left[-\frac{4\pi}{d-2}(\varpi_{d-1})^{-1/(d-1)} \cdot R^{(2d-4)/(d-1)}\left(M^{(d-2)/(d-1)} - (M-\omega)^{(d-2)/(d-1)}\right)\right]. \tag{5}$$

By making the second-order Taylor expansion of ΔS_{BH} about ω,

$$\Gamma = \exp\left[-4\pi(\varpi_{d-1})^{-1/(d-1)} \cdot R^{(2d-4)/(d-1)}\left(\frac{1}{d-1}\frac{\omega}{M^{1/(d-1)}} + \frac{1}{2(d-1)^2}\frac{\omega^2}{M^{d/(d-1)}}\right)\right]. \tag{6}$$

For a large AdS black hole, the Hawking temperature is $T_H = ((d-1)/4\pi)(r_h/R^2)$. From (6), we transform Γ to get

$$\Gamma = \exp\left[-\frac{\omega}{T_H}\left(1 + \frac{1}{2(d-1)}\frac{\omega}{M}\right)\right], \tag{7}$$

and the leading term gives the thermal Boltzmann factors for the emanating radiation. The second term represents corrections from the response of the background geometry to the emission of a quantum.

Then we introduce the effective temperature:

$$T_E(\omega) = \frac{2(d-1)M}{2(d-1)M+\omega}T_H = \frac{2(d-1)\gamma M^{d/(d-1)}}{2(d-1)M+\omega}, \tag{8}$$

where $\gamma = ((d-1)/4\pi)((R^2\varpi_{d-1})^{1/(d-1)}/R^2)$. Then, (7) can be rewritten in Boltzmann-like form:

$$\Gamma \sim \exp[-\beta_E(\omega)\omega] = \exp\left(-\frac{\omega}{T_E(\omega)}\right), \tag{9}$$

where $\beta_E(\omega) = 1/T_E(\omega)$ and $\exp[-\beta_E(\omega)\omega]$ is the effective Boltzmann factor. From the Hawking temperature $T_H = ((d-1)/4\pi)(r_h/R^2)$ and (8), we can introduce the effective mass $M_E = [2(d-1)]^{d-1}M^d/[2(d-1)M+\omega]^{d-1}$ and, then, we Taylor expand M_E in ω/M. Thus, for d-dimensional large AdS black holes,

$$M_E = M - \frac{\omega}{2}. \tag{10}$$

As one can see, the effective mass is the average quantity between the states before and after the emission. Here, it should be remarked that the original idea is that, during the particle emission, the effective mass is the average quantity, which has only been proved in Schwarzschild black holes, and can be generalized to cases of the large AdS black hole in any dimension. Thus, in any dimension, we may also introduce the other effective quantities, such as the large AdS black hole the effective horizon r_E, effective horizon area A_E, the

Bekenstein-Hawking large AdS black hole effective entropy S_E, and other quantities.

Since the asymptotic forms of quasinormal modes of the 3-dimensional and 5-dimensional large AdS black holes were given by [28], we will reanalyze the asymptotic frequencies of the large AdS black hole in the three and five dimensions by introducing effective temperature.

For the 5-dimensional large AdS black hole, $r_h = (R^2\varpi_4 M)^{1/4}, T_H = (1/\pi)(r_h/R^2) = (R^2\varpi_4 M)^{1/4}/\pi R^2$. From (8), we can easily get

$$T_E(\omega) = \frac{8M}{8M+\omega}T_H = \frac{8\gamma_5 M^{5/4}}{8M+\omega}, \tag{11}$$

where $\gamma_5 = (R^2\varpi_4)^{1/4}/\pi R^2$. The asymptotic form of quasinormal modes of the 5-dimensional large AdS black hole is given as follows [28]:

$$\omega_n = 2\pi T_H n(\pm 1 - i). \tag{12}$$

To consider the nonstrictly thermal spectrum of the large AdS black hole, we will correct the asymptotic form of quasinormal modes of the 5-dimensional large AdS black hole. Namely, we substitute the Hawking temperature T_H in (12) by the effective temperature T_E. Now, (12) yields

$$\omega_n = \pm 2\pi T_E n - i2\pi T_E n = m_n + ip_n, \tag{13}$$

where

$$\begin{aligned} m_n &= \pm\frac{16\pi\gamma_5 M^{5/4}}{8M+|\omega_n|}n, \\ p_n &= -\frac{16\pi\gamma_5 M^{5/4}}{8M+|\omega_n|}n. \end{aligned} \tag{14}$$

Now $|\omega_n|$ is the damped harmonic oscillator's proper frequency that is defined as follows [12, 14, 19]:

$$|\omega_n| = \sqrt{m_n{}^2 + p_n{}^2}. \tag{15}$$

By using the new expression (13) for the frequencies of quasinormal modes, we get

$$\begin{aligned} |\omega_n| &= \sqrt{m_n{}^2 + p_n{}^2} \\ &= \sqrt{\left(\frac{16\pi\gamma_5 M^{5/4}}{8M+|\omega_n|}n\right)^2 + \left(\frac{16\pi\gamma_5 M^{5/4}}{8M+|\omega_n|}n\right)^2} \\ &= \frac{16\sqrt{2}\pi\gamma_5 M^{5/4}}{8M+|\omega_n|}n. \end{aligned} \tag{16}$$

Then, one can get

$$|\omega_n| = -4M \pm 4M\sqrt{1+\sqrt{2}\gamma_5\pi M^{-3/4}n}. \tag{17}$$

Clearly, only $|\omega_n| > 0$ has physical meaning. So, we can get

$$|\omega_n| = 4M\sqrt{1+\sqrt{2}\gamma_5\pi M^{-3/4}n} - 4M. \tag{18}$$

Since the emitted energy is much less than the black hole mass, that is, $|\omega_n| < M$; thus we have

$$4M\sqrt{1+\sqrt{2}\gamma_5\pi M^{-3/4}n} - 4M < M. \tag{19}$$

Then, we give a maximum value for the quantum "overtone" number n:

$$n < n_{\max} = \frac{9R^{3/2}M^{3/4}}{16\sqrt{2}\varpi_4^{1/4}}. \tag{20}$$

The above approach has had some important consequences on the quantum physics of black hole. Then our following discussion will start with the quantization of the black hole horizon area. For the 5-dimensional large AdS black hole the horizon area A is related to the mass through the relation $A = 2\pi^2(R^2\varpi_4 M)^{3/4}$, and a variation ΔM in the mass generates a variation:

$$\Delta A = \frac{3\pi^2 R^{3/2}\varpi_4^{3/4}}{2M^{1/4}}\Delta M. \tag{21}$$

In any case, assuming a transition $n \to n-1$, (13) gives an emitted energy:

$$\Delta M = |\omega_n - \omega_{n-1}| = f_5(M,n), \tag{22}$$

where we have defined

$$\begin{aligned} f_5(M,n) &= 4M\sqrt{1+\sqrt{2}\gamma_5\pi M^{-3/4}n} \\ &\quad - 4M\sqrt{1+\sqrt{2}\gamma_5\pi M^{-3/4}(n-1)}. \end{aligned} \tag{23}$$

Therefore,

$$\Delta A = \frac{3\pi^2 R^{3/2}\varpi_4^{3/4}}{2M^{1/4}} f_5(M,n), \tag{24}$$

due to the black hole horizon area associated with Bekenstein-Hawking entropy $S = A/4$; thus

$$\Delta S = \frac{3\pi^2 R^{3/2}\varpi_4^{3/4}}{8M^{1/4}} f_5(M,n). \tag{25}$$

As can be seen, (24) and (25) give the corrected formula of the horizon's area quantization related to the quantum "overtone" number n by introducing the effective temperature. In the approximation of Taylor expansion,

$$\begin{aligned} f_5(M,n) &\approx \frac{2\sqrt{2}\varpi_4^{1/4}M^{1/4}}{R^{3/2}}, \\ \Delta A &\approx 3\sqrt{2}\pi^2\varpi_4, \\ \Delta S &\approx \frac{3\sqrt{2}}{4}\pi^2\varpi_4. \end{aligned} \tag{26}$$

We can see that under the condition of approximation the area spectrum is equidistant. Our results of (26) are the same as (29) and (31) of [29] which calculated the area and entropy spectra for 5-dimensional large AdS black holes by the Bohr-Sommerfeld quantization condition to the adiabatic invariant quantity [29, 30].

Now, assuming that the horizon area is quantized with quantum $\Delta A = a$, where $a = (3\pi^2 R^{3/2}\varpi_4^{3/4}/2M^{1/4}) f_5(M,n)$, the total horizon area is $A = N\Delta A = Na$, where N is the number of quanta of area. We give

$$N = \frac{A}{\Delta A} = \frac{2\pi^2 \left(R^2\varpi_4 M\right)^{3/4}}{a} = \frac{4M}{3f_5(M,n)}. \tag{27}$$

The above analysis will have important consequences on entropy and microstates. It is usually believed that any candidate for quantum gravity must explain the microscopic origin of the Bekenstein-Hawking entropy.

From (27), we can give

$$S_{\mathrm{BH}} = \frac{A}{4} = \frac{3N\pi^2 R^{3/2}\varpi_4^{3/4}}{8M^{1/4}} f_5(M,n), \tag{28}$$

and the formula of the famous Bekenstein-Hawking [1, 31, 32] entropy becomes function of the quantum "overtone" number n.

According to [33], the number of microstates for this radiation is simply

$$\Omega_{\text{radiation}}(\omega) = \frac{1}{\Gamma(\omega)}, \tag{29}$$

where $\Gamma(\omega)$ is the radiation probability for the an emission ω. Thus, the number of microstates for a large AdS black hole with M is found to be

$$\Omega_5 = \prod_{i=1}^{n} \frac{1}{\Gamma_5(\omega_i)} = e^{(\pi^2/2)(R^2\varpi_4 M)^{3/4}}, \tag{30}$$

where $\Gamma_5(\omega_i) = \exp[-(4\pi/3)R^{3/2}(\varpi_4)^{-1/4}(M^{3/4} - (M-\omega_i)^{3/4})]$. From (27) and (30), we can get

$$\Omega_5 = e^{(3N\pi^2 R^{3/2}\varpi_4^{3/4}/8M^{1/4}) f_5(M,n)}. \tag{31}$$

We can see that the number of microstates for the large AdS black hole becomes a function of the quantum "overtone" number n.

According to discussion of the 5-dimensional AdS large black hole, we will do the same approach to treat the 3-dimensional large AdS black hole.

For the 3-dimensional AdS large black hole $r_h = R(\varpi_2 M)^{1/2}$, $T_{\mathrm{H}} = r_h/2\pi R^2$. From (8) we can easily get

$$T_{\mathrm{E}} = \frac{4\gamma_3 M^{3/2}}{4M + \omega}, \tag{32}$$

where $\gamma_3 = \sqrt{\varpi_2}/2\pi R$. The asymptotic form of quasinormal modes of the 3-dimensional large AdS black hole is given as follows [28]:

$$\omega_n = \pm 4\pi T_{\mathrm{H}} \widehat{p} - i4\pi T_{\mathrm{H}} n, \tag{33}$$

where $\widehat{p}^2 = p/4\pi R T_{\mathrm{H}}$. To consider the nonstrictly thermal spectrum of the large AdS black hole, we will correct the asymptotic form of quasinormal modes of the 3-dimensional large AdS black hole. Namely, we substitute the Hawking temperature T_{H} in (33) by the effective temperature T_{E}. Now, (33) yields

$$\omega_n = \pm 4\pi T_{\mathrm{E}} \widehat{p}_{\mathrm{E}} - i4\pi T_{\mathrm{E}} n = m_n + i p_n, \tag{34}$$

where

$$\begin{aligned} m_n &= \pm\sqrt{\frac{4\pi p}{R}} \frac{2\gamma_3^{1/2} M^{3/4}}{\sqrt{4M + |\omega_n|}}, \\ p_n &= -\frac{16\pi\gamma_3 M^{3/2}}{4M + |\omega_n|} n. \end{aligned} \tag{35}$$

By using the new expression (34) for the frequencies of quasinormal modes, then we get

$$\begin{aligned} |\omega_n| &= \sqrt{{m_n}^2 + {p_n}^2} \\ &= \sqrt{\frac{16\pi p}{R} \frac{\gamma_3 M^{3/2}}{4M + |\omega_n|} + \frac{256\pi^2 \gamma_3^2 M^3}{(4M + |\omega_n|)^2} n^2}, \end{aligned} \tag{36}$$

where the solution of (36) in terms of $|\omega_n|$ will be the answer of $|\omega_n|$. Therefore, given a quantum transition between n and $n-1$, we define

$$\Delta M = |\omega_n - \omega_{n-1}| = f_3(M,n). \tag{37}$$

For the 3-dimensional large AdS black hole, the horizon area $A = 2\pi R\sqrt{\varpi_2 M}$ and a variation ΔM in the mass generates a variation:

$$\Delta A = \pi R \sqrt{\frac{\varpi_2}{M}} \Delta M = \pi R \sqrt{\frac{\varpi_2}{M}} f_3(M,n). \tag{38}$$

Thus,

$$\Delta S = \frac{\pi R}{4} \sqrt{\frac{\varpi_2}{M}} f_3(M,n), \tag{39}$$

$$N = \frac{A}{\Delta A} = \frac{2M}{f_3(M,n)}. \tag{40}$$

From (40), we can give Bekenstein-Hawking entropy:

$$S_{\mathrm{BH}} = \frac{A}{4} = \frac{N}{4} \pi R \sqrt{\frac{\varpi_2}{M}} f_3(M,n), \tag{41}$$

which becomes the function of the quantum "overtone" number n.

Same as the technique with the 5-dimensional large AdS black hole, we can get the number of microstates:

$$\begin{aligned} \Omega_3 &= \prod_{i=1}^{n} \frac{1}{\Gamma_3(\omega_i)} = e^{(1/2)\pi R\sqrt{\varpi_2 M}} \\ &= e^{(N/4)\pi R\sqrt{\varpi_2/M} f_3(M,n)}, \end{aligned} \tag{42}$$

where $\Gamma_3(\omega_i) = \exp[-4\pi R(\varpi_2)^{-1/2}(M^{1/2} - (M-\omega_i)^{1/2})]$.

3. Conclusion

In this paper, by introducing the effective temperature, the analysis on the nonstrictly thermal character of the large AdS black hole is presented. Our results show that the effective mass corresponding to the effective temperature is approximatively the average one in any dimension. And the other effective quantities can also be obtained. Based on the known forms of frequency in quasinormal modes, we reanalyze the asymptotic frequencies of the large AdS black hole in three and five dimensions. Then the formulas of the Bekenstein-Hawking entropy and the horizon's area quantization with functions of the quantum "overtone" number n are got. Moreover, the results we give in the five dimensions have a good consistency with the original one in the approximation of Taylor expansion. We can also see that under the condition of approximation the area spectrum is equidistant.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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