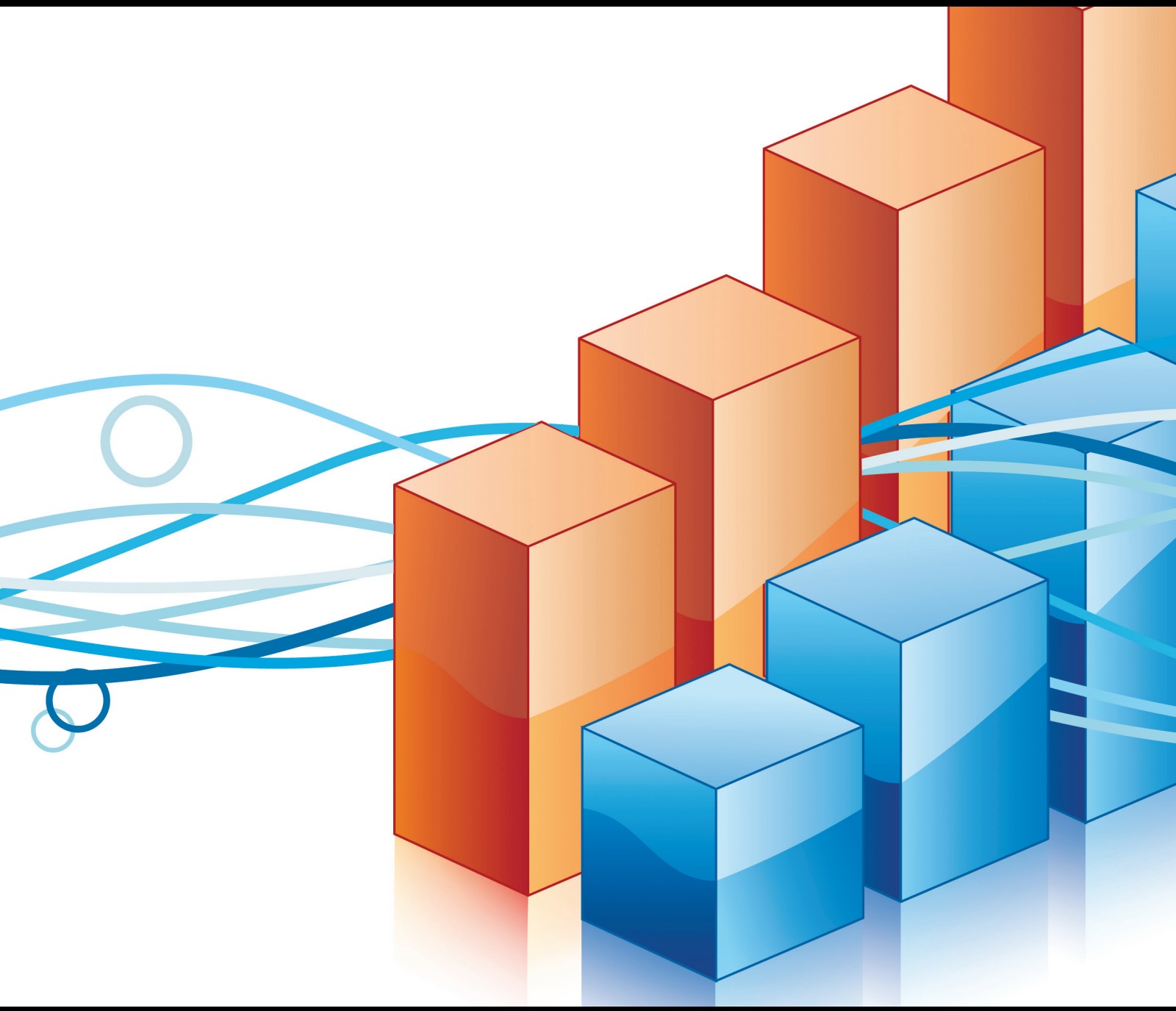
Advances in Operations Research

Decision-Making for Urban Planning and Regional Development

Lead Guest Editor: Marta Bottero
Guest Editors: Alessandra Oppio and Chiara D'Alpaos



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Editorial Board

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Editorial

Decision-Making for Urban Planning and Regional Development

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Urban and regional development can be considered as multidimensional concepts which involve socioeconomic, ecological, cultural, technical, and ethical perspectives. Decision problems in the domain of urban and regional development processes represent "weak" or unstructured problems as they are characterized by multiple actors, many and often conflicting values and views, a wealth of possible outcomes, and high uncertainty.

Under these circumstances, evaluation of alternative projects is therefore a complex decision problem, where different aspects need to be considered simultaneously, and both technical elements, based on empirical observations, and non-technical elements, based on social visions, preferences, and feelings, need to be taken into account. This complexity requires multidimensional approaches and specific qualitative/quantitative methods to analyse and synthesize the full variety of aspects involved in transformation processes, that range from the environmental impacts of urban renewal to its impacts on energy consumption/production patterns and mobility; from the social and economic impacts of a specific urban transformation strategy to its effects on landscape and cultural heritage.

This special issue addresses recent advances on the role of evaluation in supporting decision-makers in urban planning and regional development. 6 papers are published in this special issue; each paper was reviewed by at least two reviewers and revised according to review comments. The accepted papers show the role of evaluation procedures to support decisions in the context of urban management and territorial transformations.

The paper "A New Robust Dynamic Data Envelopment Analysis Approach for Sustainable Supplier Evaluation" by Nikfarjam et al. presents a new dynamic Data Envelopment Analysis (DEA) approach for suppliers selection which takes into account social, environmental and economic criteria and considers differently from previous literature, contiguous time periods. In detail efficient Decision Making Units (DMUs) are identified in each time period and as well as an ideal DMU by implementing a robust scenario-based optimization approach.

The paper "Multicriteria Evaluation of Urban Regeneration Processes: An Application of PROMETHEE Method in Northern Italy" by M. Bottero et al. proposes an original multimethodological evaluation procedure, which combines SWOT Analysis, Stakeholders Analysis, and PROMETHEE method, to evaluate alternative renewal strategies in an urban area in Northern Italy and provide decision-makers with useful tools in making welfare-maximizing urban planning decisions.

The paper "Measuring Conflicts Using Cardinal Ranking: An Application to Decision Analytic Conflict Evaluations" by T. Fasth et al. provides: (a) an application of the cardinal ranking method for preference elicitation to inform decision-makers with respect to controversies; (b) and two indexes to measure potential conflicts within a group of stakeholders or between two groups of stakeholders.

The paper "Minimizing Cost Travel in Multimodal Transport Using Advanced Relation Transitive Closure" by R. Oucheikh et al. proposes a new method for travel cost optimization, which can be applied either on path optimization

for graphs or on binary constraint reduction in Constraint Satisfaction Problem (CSP). In addition, it introduces the mathematical background for the transitive closure of binary relations.

The paper "Multiobjective Optimization for Multimode Transportation Problems" by L. Lemarchand et al. presents a model to solve service facilities localization problems in a multimode transportation context, by implementing an adapted ε-constraint multiobjective method and exploring the implementation of heuristic methods based on evolutionary multiobjective frameworks.

The paper "Integration between Transport Models and Cost-Benefit Analysis to Support Decision-Making Practices: Two Applications in Northern Italy" by P. Beria et al. contributes to the assessment of sustainable mobility transport plans and infrastructure projects, and presents an operative application of Cost Benefit Analysis to the evaluation of alternative scenarios, complemented by the implementation of transportation models and GIS.

The papers in this special issue represent a scientifically based support to address the complexity of decisions making in urban planning and regional development, improve the effectiveness and soundness of choices, and increase transparency in collective decision-making, by enhancing shared learning processes. We hope that this special issue will attract attention for further research into complex urban/territorial transformation processes, and will prove to be a valuable resource in the improvement of knowledge that the development of future cities and society requires.

Conflicts of Interest

This is to confirm that as guest editors of the special issue titled "Decision-Making for Urban Planning and Regional Development" we have not any possible conflicts of interest or private agreements with companies.

Marta Bottero
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Research Article

A New Robust Dynamic Data Envlopment Analysis Approach for Sustainable Supplier Evaluation

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Supplier selection is one of the intricate decisions of managers in modern business era. There are different methods and techniques for supplier selection. Data envelopment analysis (DEA) is a popular decision-making method that can be used for this purpose. In this paper, a new dynamic DEA approach is proposed which is capable of evaluating the suppliers in consecutive periods based on their inputs, outputs, and the relationships between the periods classified as desirable relationships, undesirable relationships, and free relationships with positive and negative natures. To this aim various social, economic, and environmental criteria are taken into account. A new method for constructing an ideal decision-making unit (DMU) is proposed in this paper which differs from the existing ones in the literature according to its capability of considering periods with unit efficiencies which do not necessarily belong to a unique DMU. Furthermore, the new ideal DMU has the required ability to rank the suppliers with the same efficiency ratio. In the concerned problem, the supplier that has unit efficiency in each period is selected to construct an ideal supplier. Since it is possible to have more than one supplier with unit efficiency in each period, the ideal supplier can be made with different scenarios with a given probability. To deal with such uncertain condition, a new robust dynamic DEA model is elaborated based on a scenario-based robust optimization approach. Computational results indicate that the proposed robust optimization approach can evaluate and rank the suppliers with unit efficiencies which could not be ranked previously. Furthermore, the proposed ideal DMU can be appropriately used as a benchmark for other DMUs to adjust the probable improvement plans.

1. Introduction

Supplier selection is an important strategic decision of managers for the economic and industry. In recent decades scholars and practitioners have paid special attention to this issue. To name a few relevant samples we can refer to Khan et al. [1] who analyzed the suppliers regarding their ability in transferring the technology. However, they merely considered economic criteria in evaluation of four levels of technology transfer among suppliers of auto industries in Pakistan. Nowadays, the decision-maker duty (in supplier selection) has become more and more intricate. This means that they must care specifically about sustainability criteria while supplier selection. Sustainable supplier evaluation and selection concept is resulted from incorporating environmental and social responsibility factors into economic factors when making decisions regarding supply chain management (SCM) [2]. In recent years, sustainability factors have played pivotal role in supplier evaluation and selection process [3]. Ratan et al. [4] discussed that sustainability principles force companies to select the suppliers which develop products and services, preserve environmental resources and look after manpower and communities. Beamon [5] introduced ethical and social responsibilities criteria as fundamental requirements of sustainable SCM for future decades.

A wide range of multiple criteria models and approaches such as fuzzy, AHP, ANP, TOPSIS and DEA have been proposed to deal with supplier selection issue over the last two decades. Some of the following researchers applied fuzzy, AHP/ANP-based methods to deal with multi-criteria supplier selection problems (e.g. [6–12]). Some other researchers used TOPSIS based methods to evaluate the suppliers (e.g. [9, 13, 14]). For instance, using fuzzy inference system, Amindoust et al. [15] proposed a ranking model based on

fuzzy inference system for sustainable supplier selection. Wen et al. [3] introduced a model for sustainable supplier evaluation using intuitionistic fuzzy sets' group decision-making models.

Another applied method is DEA which is capable to evaluate the suppliers based on weighted ratio of outputs to input. According to Kumar et al. [16], DEA is an applicable and effective tool for supplier selection problem. In the DEA approach, the importance or weights of inputs and outputs are determined through model itself in a pairwise comparison manner without any human's interference [17]. Different models of DEA have been developed in the literature to evaluate the potential suppliers. For this purpose, Weber et al. (2000) proposed a hybrid multi-objective programming (MOP)-DEA model. Farzipoor Saen [18] proposed a DEA model for ranking suppliers in the presence of imprecise data, weight restriction, and nondiscretionary factors. Also, Farzipoor Saen [19] suggested a DEA model for supplier selection in the presence of undesirable outputs and imprecise data. Noorizadeh et al. [20] introduced a model for supplier selection in the presence of dual-role factors, nondiscretionary inputs, and weight restrictions. To help managers for ranking and selecting the best suppliers in the presence of undesirable outputs and stochastic data, Azadi and Farzipoor Saen [21] developed a new slack- based measure model. Azadi et al. [22] developed a chance-constrained DEA (CCDEA) model for supplier selection in the presence of stochastic data and nondiscretionary factors. Kumar et al. [16] proposed a unified green DEA (GDEA) model for selecting the best suppliers using a comprehensive environment friendly approach.

2. Literature Review

Reviewing the relevant literature reveals that traditional models of DEA evaluate efficiency of DMUs merely in one specific past period. Hence, Dynamic DEA (D-DEA) was an appropriate approach which was initially developed by Sengupta [23] and, nowadays, it is used for evaluating DMUs in different periods [24]. In the literature related to dynamic DEA, Färe and Grosspkof [25] proposed a dynamic production frontier using an intermediate output which relates annual production processes. Tone and Tsutsui [26] introduced a new dynamic slack-based measure (DSBM) model to assess DMUs in different periods, using carry-over variables (links). They introduced four types of carry-overs (links) as desirable, undesirable, discretionary, and non-discretionary (fixed) links. Nevertheless, one of the deficiencies of the existing dynamic DEA models is their inability in introducing a strictly efficient DMU with efficiency score of unity in all periods. In fact, strictly efficient DMUs are defined as those that have unit efficiency in all periods. If efficiency score of a DMU in one of the periods has less than unity, it won't be considered as a strictly efficient unit. This deficiency can be seen in the works by Yousefi et al. [27] and Cook et al. [28]. To overcome this problem, we propose a new method for constructing the ideal DMU (IDMU) where in each period a different DMU with unit efficiency is selected. In fact, the ideal DMU is constructed by a combination of different strictly efficient DMUs. In other words, in this paper, we extend the dynamic DEA model to evaluate suppliers in different periods based on their inputs, outputs and the relationships between the periods classified as desirable relationships, undesirable relationships and free relationships with positive and negative natures. The proposed model is used to evaluate suppliers based on sustainable supplier criteria such as social, economic and environmental criteria.

In a methodological point of view, the recent hybrid-approach studies by Tavana et al., 2017; Shabanpour et al., 2017a; Shabanpour et al., 2017b; Yousefi et al., 2016 and Yousefi et al., 2015 have played fundamental roles in creating the main idea and the contributions of this study. Tavana et al., [29] developed a hybrid DEA framework for Sustainable Supplier Evaluation. They combined the goal programming and dynamic DEA model to assess efficiency of suppliers over several periods. Their approach enables the decision-maker to provide improved solutions for inefficient suppliers based on the extent to which the suppliers achieve future goals (benchmarks). Merging dynamic DEA with ANN models, Shabanpour et al., [30] created a novel framework for assessing and forecasting prospective efficiency of green suppliers. Likewise, another relevant survey was conducted by Shabanpour et al., [31]. They applied robust values to define managerial goals (improvement solutions) for evaluating suppliers. Given the fact that the management goals are inherently uncertain as well as based on human interference, they created a new robust double-frontier DEA model to assess and rank sustainable suppliers. Yousefi et al., [32] developed a scenario-based robust DEA technique to deal with sustainable suppliers' evaluation. Yousefi et al., [33], furthermore, combined goal programming and network DEA and proposed a novel DEA framework to evaluate supply chains. Their approach has the potential of predicting the DMUs' efficiencies in prospective periods. Accordingly, the decision-maker can not only evaluate suppliers/supply chains but also rank them based on their efficiency trend in several time periods. Tanskanen et al. [34] consider relationship strategies for levels of suppliers with respect to sustainable criteria. Varoutsa and Scapens [35] evaluate the supply chain's agents inside the organization. Patala et al. [36] consider sustainable criteria such as economic, environmental and social criteria in supplier evaluation.

The ideal DMU, as another assessment method, has been used in different performance evaluation problems over the past decade. Wang et al. [37] created an interval DEA model in which efficiency was calculated within the range of an interval. The upper bound of the interval was set to one and the lower bound was established by introducing a virtual IDMU, whose performance was superior to any DMU. Jahanshahloo et al. [38] developed two ranking methods using positive IDMU. They ranked 20 Iranian bank branches by two ranking methods. Hatami-Marbini et al. [39] provided a four-phase fuzzy DEA framework based upon the theory of displaced ideal. They made two hypothetical DMUs namely the ideal and nadir DMUs as reference points to rank the DMUs. Jahanshahloo et al. [40] proposed an interval DEA model to attain an efficiency interval, including evaluations from both the optimistic and the pessimistic perspectives. In

their method, the lower bounds of the DMUs are increased to obtain the maximum value. The derived points from this method were called ideal points. Then, the ideal points are employed to rank DMUs. Wang et al. [41] developed new DEA models for cross-efficiency evaluation by introducing a virtual IDMU and a virtual anti-ideal DMU (ADMU). The purpose of their study was to measure the cross-efficiencies in a neutral and more logical way.

This paper aims to evaluate the suppliers of a home appliance company based on sustainable suppliers' criteria using a new robust dynamic DEA model which is capable to evaluate and rank the suppliers with unit efficiencies which could not be ranked in the previously developed approaches. Furthermore, a new method is developed to construct an ideal DMU. The proposed ideal DMU is made up of a combination of DMUs with unit efficiency in each period. It is possible to have more than one DMU with unit efficiency in each period thus resulting in different combinations or scenarios. Therefore, a two-step scenario-based robust approach is employed to deal with these scenarios all with unit efficiencies. The proposed ideal DMU can be appropriately used as a benchmark for other DMUs to adjust the probable improvement plans.

The main contributions of this paper are summarized as follows:

(i) Presenting a new method for constructing an ideal DMU in the dynamic DEA model (since it is unlikely to find a DMU with unit efficiency in all periods, in each period a different DMU with unit efficiency can be considered)

(ii) Presenting a two-step robust method to deal with different scenarios for ideal DMU (it is possible to have more than one DMU with unit efficiency in each period and therefore different combinations of DMUs result in different scenarios)

(iii) Presenting robust ranks and improvement plans for all efficient and inefficient units

Correspondingly, the following research questions are expected to be addressed in this paper:

(i) What is the efficiency of suppliers in different periods with respect to sustainable criteria?

(ii) What is the rank of suppliers when more than one supplier with unit efficiency exists?

(iii) How can an ideal DMU be constructed to present better improvement methods when no DMU exist with unit efficiency in all periods?

(iv) What if when multiple DMUs have unit efficiency in a period?

The rest of the paper is organized as follows. In Section 2 the proposed dynamic DEA model is presented; first the deterministic model (Model D) is introduced and then the standard form of the model is written (Model S). Section 3 introduces the proposed two-step robust method which are proposed for the dynamic DEA; For step "a," the proposed robust input/output-oriented dynamic DEA model is defined (Model RI_a/RO_a) along with the corresponding linear form (Model LRI_a/LRO_a). Afterwards for step "b," the proposed robust input/output-oriented dynamic DEA model is defined (Model RI_b/RO_b) along with the corresponding linear form (Model LRI_b/LRO_b). In Section 4 the proposed robust dynamic DEA models are investigated on a case study for supplier selection. Finally, in Section 5 conclusions are brought along with future research directions.

3. Problem Statement and Formulation

The purpose of this paper is to evaluate suppliers of a company in consecutive periods based on sustainable criteria within three categories: (1) social, (2) economic, and (3) environmental. In each period, there are a number of inputs and outputs for each supplier. Also some materials may be transferred from one period to the next period(s) such as backorders and uncashed checks. These make relationships between periods which some of them are desirable and some are undesirable. In the context of DEA, each supplier is considered as a DMU. Since the suppliers are evaluated in multiple periods, dynamic version of DEA is appropriate. Dynamic DEA aims at evaluating m DMUs during P periods with respect to relationships between periods. For every DMU, in each period, n inputs and s outputs are considered. Also, three types of relationships such as desirable, undesirable and free are considered which ensure the link between periods. Relationships with desirable and undesirable nature need to be maximized and minimized, respectively. Free relationships are those that lack any essence and their nature cannot be recognized some of them have positive nature and some have negative nature. Figure 1 graphically illustrates the proposed dynamic DEA.

In the dynamic DEA, usually a strictly efficient unit called ideal DMU is specified to be considered as a benchmark so that inefficient DMUs try to reach it. In fact, the ideal DMU is the one that in all periods has unit efficiency and it is a strictly efficient unit. In most cases such a DMU which is efficient in all periods does not exist and according to the existing definition no ideal DMU can be recognized. To overcome this problem, we introduce a new method for constructing the ideal DMU where in each period a different DMU with unit efficiency in that period is selected. Obviously, the proposed ideal DMU is virtual and does not exist in reality. To clarify, consider 3 DMUs in 5 periods (Figure 2). According to the existing method for constructing the ideal DMU, no DMU is selected as an ideal DMU whereas according to the proposed method in this paper, the ideal DMU is composed of DMU 1, DMU 2, DMU 3, DMU 2, and DMU 1, respectively.

In the dynamic DEA, the efficient frontier is constructed in a pairwise comparison between units where units with maximum ratio of outputs to inputs are selected to construct the efficient frontier as presented by dotted line in Figure 3. The improvement method is presented for inefficient units to push them towards this efficient frontier. This issue can be mentioned as another shortcoming of the existing dynamic DEA models where the benchmark(s) as well as improvement plans are merely introduced for inefficient units. Actually, those models do not present improvement methods for

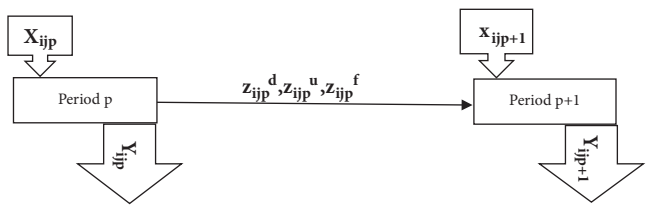

Figure 1: Representation of the proposed dynamic DEA.

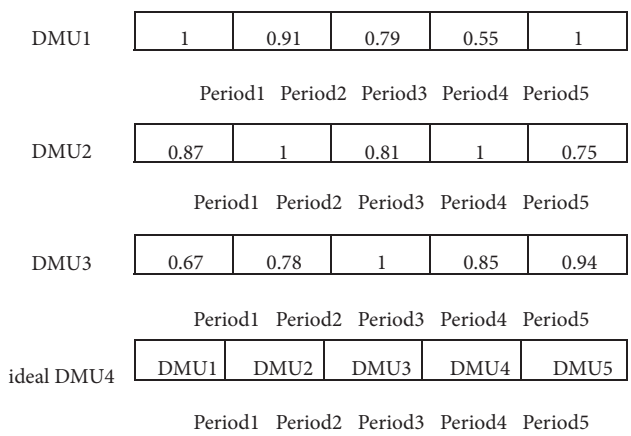

Figure 2: Demonstrating the proposed ideal DMU.

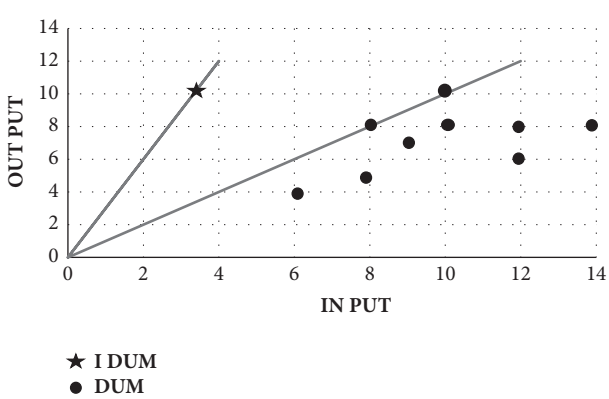

Figure 3: Efficient frontiers by the ideal DMU and efficient units.

benchmarks themselves. The proposed ideal DMU, shown by an asterisk in Figure 3, can be introduced as a benchmark for both inefficient units and efficient units. The boundary which is presented by the solid line in Figure 3 is the frontier which has been constructed by the ideal DMU. As mentioned earlier, the proposed ideal DMU consists of periods with unit efficiencies which do not necessarily belong to a DMU. It is possible to have more than one DMU with unit efficiency in a specific period. Therefore, different scenarios may exist for the ideal DMU. To deal with these scenarios, we employ a scenario-based robust optimization approach and propose a robust dynamic DEA model to present improvement plans for all DMUs. Even when all DMUs obtain similar efficiency, the proposed model can rank those units by considering an absolutely efficient unit called ideal DMU. Actually, the ideal DMU can be used as a unique benchmark based on which improvement methods can be presented for other DMUs. As a matter of fact, initially DMUs are evaluated based on a dynamic DEA approach and then different scenarios are constructed for the ideal DMU (ideal supplier) through a combination of efficient DMUs in each period.

Altogether, the proposed ideal DMU addresses the following concerns about the existing DEA models:

(i) Presenting the improvement methods for efficient units (in existing models the improvement methods can only be presented for inefficient units)

(ii) Considering requirements and opinions of experts in presenting the improvement methods (these opinions are considered in inputs and outputs values of the ideal DMU).

(iii) Modifying the benchmarks in case these units are not acceptable from DM's perspective.

Since in this paper different models are presented, to prevent misunderstanding, we number the models using the following acronyms.

Acronyms for Models

D: Deterministic model
S: Standard model
RI_a: Robust Input-oriented model step a
RI_b: Robust Input-oriented model step b
RO_a: Robust Output-oriented model step a
RO_b: Robust Output-oriented model step b
LRI_a: Linear Robust Input-oriented model step a
LRI_b: Linear Robust Input-oriented model step b
LRO_a: Linear Robust Output-oriented model step a
LRO_b: Linear Robust Output-oriented model step b

Before describing the models, the used notations can be described as follows.

Notations

m: Index of DMUs
n: Index of inputs
s: Index of outputs
p: Index of time periods

n_d, n_u, n_f: Total number of desirable, undesirable, and free relationships, respectively.

x_{ijp}: ith input of the jth DMU in period p (i=1, 2, 3, ..., n)

y_{ijp}: ith output of the jth DMU in period p (i=1, 2, 3, ..., s)

z^d_{ijp}: Desired relationship for the jth DMU in period p, (i=1,..,n_d), (j=1,. . ., m), (p=1,. . ., P).

z^u_{ijp}: Undesirable relationship for the jth DMU in period p. (i=1,..,n_u), (j=1,. . ., m), (p=1,. . ., P).

z^f_{ijp}: Free relationship for the jth DMU in period p. (i=1,..,n_f), (j=1,. . ., m), (p=1,. . ., P).

λ^p_j: Benchmark for the jth inefficient DMU in period p.

o: Index for the under investigation DUM

w^-_i: Weight of the ith input

w^+_i: Weight of the ith output

w^p: Weight of the pth period

ϕ^*_{op}: Efficiency of the under investigated DMU in period p (input-oriented model)

Φ^*_0: Total efficiency of the under investigated DMU (input-oriented model)

ψ^*_{op}: Efficiency of the under investigated DMU in period p (output-oriented model)

Ψ^*_o: Total efficiency of the under investigated DMU (output-oriented model)

3.1. The Proposed Mathematical Model for the Deterministic Dynamic DEA (Model D)

$$\max \Phi^*_0 = \sum_{p=1}^{P} \frac{\phi^*_{op}}{P} \tag{D-1-IN}$$

$$\max \Psi^*_o = \sum_{p=1}^{P} \frac{\psi^*_{op}}{P} \tag{D-1-OUT}$$

$$x_{iop} \geq \sum_{j=1}^{m} x_{ijp}\lambda^p_j \qquad (i = 1, \ldots, n;\ p = 1, \ldots, P) \tag{D-2}$$

$$y_{iop} \leq \sum_{j=1}^{m} y_{ijp}\lambda^p_j \qquad (i = 1, \ldots, s;\ p = 1, \ldots, P) \tag{D-3}$$

$$z^d_{iop} \leq \sum_{j=1}^{m} z^d_{ijp}\lambda^p_j \qquad (i = 1, \ldots, n_d;\ p = 1, \ldots, P) \tag{D-4}$$

$$z^u_{iop} \geq \sum_{j=1}^{m} z^u_{ijp}\lambda^p_j \qquad (i = 1, \ldots, n_u;\ p = 1, \ldots, P) \tag{D-5}$$

$$z^f_{iop}: \text{free of sign} \qquad (i = 1, \ldots, n_f;\ p = 1, \ldots, P) \tag{D-6}$$

$$\lambda^p_j \geq 0 \quad (j = 1, \ldots, m : p = 1, \ldots, P) \tag{D-7}$$

Objective function (D-1-IN) and (D-1-OUT) maximizes the efficiency of the under investigation DMU. (D-1-IN) is the objective function of the input-oriented model and (D-1-OUT) is the objective function of the output-oriented model. At each time either objective function (D-1-IN) or (D-1-OUT) is considered. Constraint set (D-2) ensures that the ith input of the under investigation DMU in period p be greater than or equal to weighted sum of input i in period p for all DMUs. Constraint set (D-3) ensures that the ith output of the under investigation DMU in period p be less than or equal to weighted sum of output i in period p for all DMUs. Constraint set (D-4) indicates that the value of the ith desirable relationship of the under investigation DMU in period p be less than or equal to weighted sum of the desirable relationship i in period p for all DMUs. Constraint set (D-5) indicates that the value of the ith undesirable relationship of the under investigation DMU in period p be greater than or equal to weighted sum of the undesirable relationship i in period p for all DMUs. Constraint set (D-6) defines the free relationship variables which are free of sign. Constraint set (D-7) defines the weight variables of improvement plans.

Note that the right hand sides of the above constraints, i.e., x_{ijp}, y_{ijp}, and z^d_{ijp} z^u_{ijp}, are positive values. The left hand side of the mentioned constraints, i.e., x_{iop}, y_{iop}, z^d_{iop}, z^u_{iop}, and z^f_{iop}, is connected together through λ^p_j. The continuity of the flow representing the relationship between pth and (p+1)th periods is ensured through (1), where $\propto$ is a general index which can be d, u, or f representing desirable, undesirable, and free relationships. These constraints are important in the proposed dynamic DEA model, since they connect period p to period p+1 and ensure having a series of time periods.

$$\sum_{j=1}^{m} x^{\propto}_{ijp}\lambda^p_j = \sum_{j=1}^{m} z^{\propto}_{ijp}\lambda^{p+1}_j \qquad (i = 1, \ldots, n\ \propto;\ p = 1, \ldots, P) \tag{1}$$

3.2. The Standard Mathematical Model for the Deterministic Dynamic DEA (Model S). After writing the standard form for the constraints of model (D), the final standard model (S) whose constraints have equal sign is obtained as follows:

$$\min \Phi^*_0 = \frac{1}{P}\sum_{p=1}^{P} w^p \left[1 - \frac{1}{\boldsymbol{m} + \boldsymbol{n_u} + \boldsymbol{n_f}}\left(\sum_{i=1}^{n}\frac{w^-_i s^-_{ip}}{x_{iop}} + \sum_{i=1}^{n_u}\frac{s^u_{ip}}{z^u_{iop}} + \sum_{i=1}^{n_f}\frac{s^{f-}_{ip}}{z^f_{iop}}\right)\right] \tag{S-1-IN}$$

$$\max \frac{1}{\Psi_o^*} = \frac{1}{P}\sum_{p=1}^{P} w^p \left[1 - \frac{1}{\boldsymbol{s}+\boldsymbol{n_d}+\boldsymbol{n_f}}\left(\sum_{i=1}^{s}\frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d}\frac{s_{ip}^d}{z_{iop}^d} + \sum_{i=1}^{n_f}\frac{s_{ip}^{f+}}{z_{iop}^f}\right)\right] \tag{S-1-OUT}$$

$$x_{iop} = \sum_{j=1}^{m} x_{ijp}\lambda_j^p + s_{ip}^- \quad (i = 1,\ldots,n;\ p = 1,\ldots,P) \tag{S-2}$$

$$y_{iop} = \sum_{j=1}^{m} y_{ijp}\lambda_j^p - s_{ip}^+ \quad (i = 1,\ldots,s;\ p = 1,\ldots,P) \tag{S-3}$$

$$z_{iop}^d = \sum_{j=1}^{m} z_{ijp}^d\lambda_j^p - s_{ip}^d \quad (i = 1,\ldots,n_d;\ p = 1,\ldots,P) \tag{S-4}$$

$$z_{iop}^u = \sum_{j=1}^{m} z_{ijp}^u\lambda_j^p + s_{ip}^u \quad (i = 1,\ldots,n_u;\ p = 1,\ldots,P) \tag{S-5}$$

$$z_{iop}^f = \sum_{j=1}^{m} z_{ijp}^f\lambda_j^p + s_{ip}^f \quad (i = 1,\ldots,n_f;\ p = 1,\ldots,P) \tag{S-6}$$

$$\begin{aligned} &s_{ip}^f : \text{free of sign} \quad (\forall i, p),\\ &s_{ip}^u \geq 0,\\ &s_{ip}^p \geq 0,\\ &s_{ip}^+ \geq 0,\\ &s_{ip}^- \geq 0,\\ &\lambda_j^p \geq 0 \end{aligned} \tag{S-7}$$

In model (S) like model (D), two alternative cases are considered to calculate the efficiency of the under investigated DMU. One is input-oriented (S-1-IN) and the other is output-oriented (S-1-OUT) which are described in the followings. As a matter of fact, the choice of these objective functions depends on the DEA approach, i.e., whether it is input-oriented or output-oriented, which is used for presenting improvement plans.

Objective function (S-1-IN) represents the total efficiency of the input-oriented model. This objective function is based on the nonredial input-oriented model which considers undesirable relationships, i.e., s_{ip}^u and s_{ip}^{f-}, along with surplus of inputs, i.e., s_{ip}^-, which should be simultaneously minimized. If all these variables become zero, the efficiency of the considered DMU in period p is one. Obviously, a DMU with overall efficiency equal to one is the one which has unit efficiency in all periods. This objective function calculates the weighted mean of the efficiencies in all periods whose value is between 0 and 1, i.e., $(0 \leq \Phi_o^* \leq 1)$, $(0 \leq \phi_{op}^* \leq 1)$. The optimal value for the efficiency of period p in input-oriented model is according to

$$\phi_{op}^* = 1 - \frac{1}{\boldsymbol{m}+\boldsymbol{n_u}+\boldsymbol{n_f}}\left(\sum_{i=1}^{n}\frac{w_i^- s_{ip}^-}{x_{iop}} + \sum_{i=1}^{n_u}\frac{s_{ip}^u}{z_{iop}^u} + \sum_{i=1}^{n_f}\frac{s_{ip}^{f-}}{z_{iop}^f}\right), \quad (p = 1,\ldots,P) \tag{2}$$

Objective function (S-1-OUT) represents the total efficiency of the output-oriented model. The optimal value for the efficiency of period p in output-oriented model is according to

$$\psi_{op}^* = \frac{1}{1 - \left(1/\left(\boldsymbol{s}+\boldsymbol{n_d}+\boldsymbol{n_f}\right)\right)\left(\sum_{i=1}^{s}\left(w_i^+ s_{ip}^+/y_{iop}\right) + \sum_{i=1}^{n_d}\left(s_{ip}^d/z_{iop}^d\right) + \sum_{i=1}^{n_f}\left(s_{ip}^{f+}/z_{iop}^f\right)\right)}, \quad (p = 1,\ldots,P) \tag{3}$$

The denominator of the objective function deals with the slack of outputs, s_{ip}^{+}, free relationships with positive nature, s_{ip}^{f+}, and desirable relationships, s_{ip}^{d}. If these values become zero, the denominator becomes one and therefore the efficiency of the considered DMU in period p is one. If these slacks get values more than one, the denominator becomes more than one. Therefore, the efficiency of the considered DMU in period p is less than one. Consequently, the total efficiency in the objective function gets a value between zero and one, i.e., $(0 \le \Psi_o^* \le 1)$, $(0 \le \psi_{op}^* \le 1)$.

In Constraint (S-2), s_{ip}^{-} represents the slack for the ith input in period p. The left hand side of this constraint is the inputs of the underinvestigated DMU in period p. If the value of s_{ip}^{-} be zero, it means that the supplier does not have excess consumption for that input in period p. In constraint (S-3), s_{ip}^{+} represents the surplus for the ith output in period p. In the rest of the constraints, $s_{ip}^{d}, s_{ip}^{u}, s_{ip}^{f}$, respectively, represent the slack of the desirable relationship, surplus of the undesirable relationship, and the deviation of the free relationship. Note that the auxiliary variables used to standardize constraints have negative natures. For inputs it means excess consumption, for outputs it means shortage in production, for desirable relationships it means shortage in this relationship, for undesirable relationships it means excess in this relationship, and for free relationships it means deviation in this relationship. Constraint (S-6) contains a free of sign variable. The deviation of the free relationship can either be stated as slack or surplus. Therefore, to deal with the free of sign variable s_{ip}^{f}, two positive variables, s_{ip}^{f-} and s_{ip}^{f+}, are defined and the following constraints are considered:

$$\begin{gathered} s_{ip}^{f} = s_{ip}^{f-} - s_{ip}^{f+} \\ s_{ip}^{f+} * s_{ip}^{f-} = 0, \\ s_{ip}^{f-} \ge 0,\ s_{ip}^{f+} \ge 0 \end{gathered} \tag{4}$$

Consequently, the following constraints are substituted for Constraint (S-6):

$$z_{iop}^{f} = \sum_{j=1}^{m} z_{ijp}^{f} \lambda_j^{p} + s_{ip}^{f-} \qquad \left(i = 1, \ldots, n_f;\ p = 1, \ldots, P\right) \tag{S-6-1}$$

$$z_{iop}^{f} = \sum_{j=1}^{m} z_{ijp}^{f} \lambda_j^{p} - s_{ip}^{f+} \qquad \left(i = 1, \ldots, n_f;\ p = 1, \ldots, P\right) \tag{S-6-2}$$

4. Robust Dynamic DEA Model

As mentioned previously, one of the deficiencies of the existing dynamic DEA is the lack of a strictly efficient unit as a unit that can be introduced as a benchmark. To overcome this deficiency, we introduce a new method for constructing the ideal DMU which is constructed by making use of the results obtained from the dynamic DEA. The proposed ideal DMU is made up of different periods each of which contains DMUs with unit efficiency. As a matter of fact, in each period, DMUs are evaluated and DMU(s) with unit efficiency are selected to construct the ideal DMU. Since more than one DMU with unit efficiency may exist in each period, different combinations of DMUs may be generated for the ideal DMU. Each of these combinations is called a "scenario". More than one DMU with unit efficiency in each period leads to different combinations or scenarios for the ideal DMU whose probabilities of occurrence are considered the same in this paper. Different scenarios for the ideal DMU result in different improvement plans. By taking the advantages of the scenario-based robust optimization method and applying it for the studying dynamic DEA, we evaluate and rank the suppliers with respect to these scenarios for the ideal DMU. This process is done in two steps. The first step (step a) formulates the robust optimization model where one of the scenarios is under investigation. The second step (step b) formulates the robust optimization model where other DMUs along with the selected scenario unit are under investigation.

The procedure for the proposed supplier evaluation and rank model is summarized in the following procedure.

Procedure: Supplier Evaluation and Rank through the Proposed Robust Dynamic DEA

Begin

(1) Determine inputs, outputs, desirable, and undesirable relationships for suppliers in each period.

(2) Consider each supplier as a DMU and employ the dynamic (input/ouput-oriented) DEA model to determine the efficiency values of each supplier in each period.

(3) If there is a DMU with unit efficiency values in all periods consider it as a strictly efficient unit.

(4) Otherwise, build a virtual ideal DMU whose periods belong to DMUs with unit efficiency thus leading to different scenarios for the ideal DMU.

(5) In step "a," evaluate and rank scenarios (with equal probability and based on the punishment and encouragement values considered for each scenario) using the proposed linear (input/output-oriented) robust dynamic DEA model (LRI_a/LRO_a). The best scenario is considered as a unique benchmark for presenting improvement plans.

(6) In step "b," consider the selected scenario from step "a" along with other suppliers (resulting in $m+1$ number of DMUs) and evaluate the suppliers through models (LRI_b/LRO_b).

End.

Notations Used in the Proposed Robust Method

Π_s: The probability of occurrence for scenario s

k_i^s: The unit cost for ith input of sth scenario in period p.

g_i^s: The unit cost for ith undesirable relationship of sth scenario in period p.

v_i^s: The unit cost for ith free relationship of sth scenario in period p.

h_i^s: The unit revenue for ith output of sth scenario in period p.

b_i^s: The unit revenue for ith desirable relationship of sth scenario in period p.

e_i^s: The unit revenue for ith free relationship with positive nature of sth scenario in period p.

4.1. Step a: A Scenario Unit Is under Investigation. In this section the efficiency of scenarios are investigated through models RI_a or RO_a, depending on the decision-maker's approach which could be input-oriented or output-oriented. At each time one of the scenarios is under investigation and the efficiencies of scenarios for the ideal DMU are calculated and they are ranked. The high ranked scenario is selected based on which the improvement plan is presented. As a matter of fact, the best scenario has more distance from other DMUs (see Figure 3) and the improvement plan for other DMUs is presented in the worst case. Therefore, the proposed improvement plan is robust against different scenarios which could be considered for the ideal DMU.

4.1.1. Robust Optimization for Input Oriented DEA Model Step a ($\mathbf{RI_a}$)

$$\min \beta \times \text{Average} + (1-\beta) \times \sum_{s=1}^{S} \Pi_s$$

$$\times \left| \frac{1}{P} \sum_{p=1}^{P} w^p \left[\frac{1}{1 - \left(1/\left(m + n_u + n_f\right)\right)\left(\sum_{i=1}^{n}\left(w_i^- s_{ip}^-/x_{iop}\right) + \sum_{i=1}^{n_u}\left(s_{ip}^u/z_{iop}^u\right) + \sum_{i=1}^{n_f}\left(s_{ip}^{f-}/z_{iop}^f\right)\right)} \right] - \text{Average} \right| \quad (\mathbf{RI_a}\text{-1})$$

$$+ \sum_{s=1}^{S} \Pi_s \left(\sum_{i=1}^{n} k_i^s x s_{isp} \lambda S_s^p + \sum_{i=1}^{n_u} g_i^s z s_{isp}^u \lambda s_s^p + \sum_{i=1}^{n_f} v_i^s z s_{isp}^{f-} \lambda s_s^p - \sum_{i=1}^{s} h_i^s y s_{isp} \lambda s_s^p - \sum_{i=1}^{n_d} b_i^s z s_{isp}^d \lambda s_s^p - \sum_{i=1}^{n_f} e_i^s z s_{isp}^{f+} \lambda s_s^p \right)$$

$$\textbf{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{m + n_u + n_f} \left(\sum_{i=1}^{n} \frac{w_i^- s_{isp}^-}{x_{iop}} + \sum_{i=1}^{n_u} \frac{s_{isp}^u}{z_{iop}^u} + \sum_{i=1}^{n_f} \frac{s_{isp}^{f-}}{z_{iop}^f} \right) \right] \quad (\mathbf{RI_a}\text{-2})$$

$$x_{isp} = \sum_{j=1}^{m} x_{ijp} \lambda_j^p + x s_{isp} \lambda S_s^p + s_{isp}^- \quad (i = 1, \ldots, n;\ p = 1, \ldots, P;\ s = 1, \ldots, S) \quad (\mathbf{RI_a}\text{-3})$$

$$y s_{isp} = \sum_{j=1}^{m} y_{ijp} \lambda_j^p + y s_{isp} \lambda s_s^p - s_{isp}^+ \quad (i = 1, \ldots, s;\ p = 1, \ldots, P;\ s = 1, \ldots, S) \quad (\mathbf{RI_a}\text{-4})$$

$$z s_{iop}^f = \sum_{j=1}^{m} z_{ijp}^f \lambda_j^p + z s_{isp}^f \lambda s_s^p + s_{isp}^{f-} \quad (i = 1, \ldots, n_f;\ p = 1, \ldots, P) \quad (\mathbf{RI_a}\text{-5})$$

$$z s_{iop}^f = \sum_{j=1}^{m} z_{ijp}^f \lambda_j^p + z s_{isp}^f \lambda s_s^p - s_{isp}^{f+} \quad (i = 1, \ldots, n_f;\ p = 1, \ldots, P) \quad (\mathbf{RI_a}\text{-6})$$

$$z s_{iop}^d = \sum_{j=1}^{m} z_{ijp}^d \lambda_j^p + z s_{isp}^d \lambda s_s^p - s_{isp}^d \quad (i = 1, \ldots, n_d;\ p = 1, \ldots, P) \quad (\mathbf{RI_a}\text{-7})$$

$$z s_{isp}^u = \sum_{j=1}^{m} z_{ijp}^u \lambda_j^p + z s_{isp}^u \lambda s_s^p + s_{isp}^u \quad (i = 1, \ldots, n_u;\ p = 1, \ldots, P;\ s = 1, \ldots, S) \quad (\mathbf{RI_a}\text{-8})$$

$$s_{isp}^u \geq 0,$$

$$s_{ips}^+ \geq 0,$$

$$s_{isp}^- \geq 0,$$

$$\lambda_j^p \geq 0,$$

$$s_s^p \geq 0,$$

$$\mathbf{s}_{\mathbf{isp}}^{f+} \geq \mathbf{0},$$
$$\mathbf{s}_{\mathbf{isp}}^{f-} \geq \mathbf{0},$$
$$s_{isp}^{d} \geq \mathbf{0} \tag{$\mathbf{RI_a}$-9}$$

The objective function ($\mathbf{RI_a}$-1) consists of three terms. The first term calculates the average efficiency of scenarios. The second term calculates the deviation of efficiency of scenarios from the average value. The third term calculates the total profit or loss resulted from scenarios. In fact, in each scenario, outputs, desirable relationships and free relationships with positive natures, yield return. Whilst inputs, undesirable relationships, and free relationships with negative natures result in cost. Note that in this term, the values of returns are subtracted from the values of costs. Therefore, if this term is negative it means that the considering scenario is profitable; otherwise it makes losses.

It is worth noting that since each scenario is a combination of DMUs with unit efficiency selected from different periods, the efficiency of each scenario is one. Therefore, the average efficiency of all scenarios is one and consequently the standard deviation is zero.

*(1) Linear Robust Input-Oriented Model Step a (*L$\mathbf{RI_a}$*).* To deal with the absolute function and make the model linear, two positive variables Q_s^+ and Q_s^- are defined.

$$\begin{aligned}
&\mathbf{min}\ \boldsymbol{\Phi}_0^* \\
&= \boldsymbol{\beta} \times \mathbf{Average} + (\mathbf{1} - \boldsymbol{\beta}) \times \sum_{s=1}^{S} \Pi_s \times (\mathbf{Q}_s^+ + \mathbf{Q}_s^-) \\
&+ \sum_{s=1}^{S} \Pi_s \left(\sum_{i=1}^{n} k_i^s \mathrm{xs}_{\mathrm{isp}} \lambda \mathrm{S}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_u} g_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{u}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_f} v_i^s zs_{isp}^{f-} \lambda s_s^p - \sum_{i=1}^{s} h_i^s ys_{isp} \lambda s_s^p - \sum_{i=1}^{n_d} b_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{d}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} - \sum_{i=1}^{n_f} e_i^s zs_{isp}^{f+} \lambda s_s^p \right)
\end{aligned} \tag{L$\mathbf{RI_a}$-1}$$

$$\frac{\mathbf{1}}{\boldsymbol{P}} \sum_{\mathrm{p}=1}^{\mathrm{P}} \mathrm{w}^p \left[\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{m} + \boldsymbol{n_u} + \boldsymbol{n_f}} \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{w}_{\mathrm{i}}^- \mathrm{s}_{\mathrm{isp}}^-}{\mathrm{x}_{\mathrm{iop}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_u} \frac{\mathrm{s}_{\mathrm{isp}}^{\mathrm{u}}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{u}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{isp}}^{f-}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] - \mathbf{Average} = \mathbf{Q}_s^+ - \mathbf{Q}_s^- \tag{L$\mathbf{RI_a}$-2}$$

Other constraints of model ($\mathbf{RI_a}$) hold.

4.1.2. Robust Optimization for Output-Oriented DEA Model Step a ($\mathbf{RO_a}$). The robust optimization for the output-oriented model differs with the input oriented model in the objective function and also in the average and the linearized constraints. Other constraints are the same for both.

$$\begin{aligned}
&\min \boldsymbol{\beta} \times \mathbf{Average} + (\mathbf{1} - \boldsymbol{\beta}) \times \sum_{s=1}^{S} \Pi_s \\
&\times \left| \frac{\mathbf{1}}{\boldsymbol{P}} \sum_{p=1}^{P} w^p \left[\mathbf{1} - \frac{\mathbf{1}}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{ip}}^{f+}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] - \mathbf{Average} \right| \\
&+ \sum_{s=1}^{S} \Pi_s \left(\sum_{i=1}^{n} k_i^s \mathrm{xs}_{\mathrm{isp}} \lambda \mathrm{S}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_u} g_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{u}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_f} v_i^s zs_{isp}^{f-} \lambda s_s^p - \sum_{i=1}^{s} h_i^s ys_{isp} \lambda s_s^p - \sum_{i=1}^{n_d} b_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{d}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} - \sum_{i=1}^{n_f} e_i^s zs_{isp}^{f+} \lambda s_s^p \right)
\end{aligned} \tag{$\mathbf{RO_a}$-1}$$

$$\mathbf{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{\mathbf{1}}{\boldsymbol{P}} \sum_{p=1}^{P} w^p \left[\mathbf{1} - \frac{\mathbf{1}}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{ip}}^{f+}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] \tag{$\mathbf{RO_a}$-2}$$

Other constraints of model ($\mathbf{RI_a}$) hold.

Like the input-oriented model, the objective function ($\mathbf{RO_a}$-1) consists of three terms. The first term minimizes the average efficiency of scenarios with weight importance of β. The second term minimizes the standard deviation of efficiencies with weight importance of 1- β. Finally, the third term calculates the total profit or loss resulted from scenarios.

(1) Linear Robust Output-Oriented Model Step a ($\mathbf{LRO_a}$*).* By considering two positive variables Q_s^+ and Q_s^- and substituting it with the absolute function, the model is linearized.

$$\min \boldsymbol{\beta} \times \mathbf{Average} + (\mathbf{1} - \boldsymbol{\beta}) \times \sum_{s=1}^{S} \Pi_s \times (\mathbf{Q}_s^+ + \mathbf{Q}_s^-)$$
$$+ \sum_{s=1}^{S} \Pi_s \left(\sum_{i=1}^{n} k_i^s \mathrm{xs}_{\mathrm{isp}} \lambda \mathrm{S}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_u} g_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{u}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} + \sum_{i=1}^{n_f} v_i^s zs_{isp}^{f-} \lambda s_s^p - \sum_{i=1}^{s} h_i^s ys_{isp} \lambda s_s^p - \sum_{i=1}^{n_d} b_i^s \mathrm{zs}_{\mathrm{isp}}^{\mathrm{d}} \lambda \mathrm{s}_{\mathrm{s}}^{\mathrm{p}} - \sum_{i=1}^{n_f} e_i^s zs_{isp}^{f+} \lambda s_s^p \right) \quad (\mathbf{LRO_a}\text{-1})$$

$$\left[\frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{ip}}^{f+}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] - \mathbf{Average} \right] = \mathbf{Q}_s^+ - \mathbf{Q}_s^- \quad (\mathbf{LRO_a}\text{-2})$$

Other constraints of model ($\mathrm{RI_a}$) hold.

4.2. Step b: A DMU from Other DMUs Is under Investigation. In the previous section the under investigation DMU was one of the scenarios and we evaluated scenarios and selected the suitable one. In fact, model ($\mathrm{RI_a}/\mathrm{RO_a}$) calculates benefit or loss resulted from each scenario and we can select the best one accordingly. In this section, other DMUs are evaluated along with the selected scenario and therefore the number of DMUs increases by one (i.e., m+1). Actually the best scenario is considered in model ($\mathrm{RI_b}/\mathrm{RO_b}$).

4.2.1. Robust Optimization for Input-Oriented DEA Model Step b ($\mathrm{RI_b}$*).* The following model ($\mathrm{RI_b}$) evaluates other DMUs along with the strictly efficient DMU (i.e., the selected scenario as an ideal DMU) as a benchmark. Then the improvement methods can be presented for other DMUs based on their distance from the efficient frontier and image inefficient DMUs to the efficient frontier. The main difference of model ($\mathrm{RI_b}$) with model ($\mathrm{RI_a}$) is that in model ($\mathrm{RI_b}$) the number of DMUs is m+1. Another difference is that the objective function consists of two terms, i.e., the average efficiency and the standard deviation of efficiencies of the considered DMU in different periods respectively with importance weights β and $1 - \beta$. Note that the objective function of model ($\mathrm{RI_b}$) does not consider the cost since by taking the ideal DMU into consideration; we can rank DMUs and present improvement methods.

$$\min \quad \Phi_0^*$$
$$= \beta \times \mathrm{Average} + (1 - \beta) \times \sum_{s=1}^{S} \Pi_s \times \left(\frac{1}{P} \sum_{\mathrm{p}=1}^{\mathrm{P}} \mathrm{w}^p \left[1 - \frac{1}{m + n_u + n_f} \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{w}_\mathrm{i}^- \mathrm{s}_{\mathrm{ip}}^-}{\mathrm{x}_{\mathrm{iop}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_u} \frac{\mathrm{s}_{\mathrm{ip}}^{\mathrm{u}}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{u}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{ip}}^{f-}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] - \mathrm{Average} \right) \quad (\mathrm{RI_b}\text{-1})$$

$$\text{s.t.} \quad \mathrm{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{1}{P} \sum_{\mathrm{p}=1}^{\mathrm{P}} \mathrm{w}^p \left[1 - \frac{1}{m + n_u + n_f} \left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{w}_\mathrm{i}^- \mathrm{s}_{\mathrm{ip}}^-}{\mathrm{x}_{\mathrm{iop}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_u} \frac{\mathrm{s}_{\mathrm{ip}}^{\mathrm{u}}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{u}}} + \sum_{\mathrm{i}=1}^{\mathrm{n}_f} \frac{\mathrm{s}_{\mathrm{ip}}^{f-}}{\mathrm{z}_{\mathrm{iop}}^{\mathrm{f}}} \right) \right] \quad (\mathrm{RI_b}\text{-2})$$

$$\mathrm{x}_{\mathrm{iop}} = \sum_{\mathrm{j}=1}^{\mathrm{m}+1} \mathrm{x}_{\mathrm{ijp}} \lambda_{\mathrm{j}}^{\mathrm{p}} + \mathrm{s}_{\mathrm{ip}}^- \quad (i = 1, \ldots, n;\ p = 1, \ldots, P;\ j = 1, \ldots, m + 1) \quad (\mathrm{RI_b}\text{-3})$$

$$y_{iop} = \sum_{j=1}^{m+1} y_{ijp} \lambda_j^p - s_{ip}^+ \quad (i = 1, \ldots, s;\ p = 1, \ldots, P;\ j = 1, \ldots, m + 1) \quad (\mathrm{RI_b}\text{-4})$$

$$\mathrm{z}_{\mathrm{iop}}^{\mathrm{u}} = \sum_{\mathrm{j}=1}^{\mathrm{m}+1} \mathrm{z}_{\mathrm{ijp}}^{\mathrm{u}} \lambda_{\mathrm{j}}^{\mathrm{p}} + \mathrm{s}_{\mathrm{ip}}^{\mathrm{u}} \quad (i = 1, \ldots, n_u;\ p = 1, \ldots, P;\ j = 1, \ldots, m + 1) \quad (\mathrm{RI_b}\text{-5})$$

$$\mathrm{zs}_{\mathrm{iop}}^{\mathrm{f}} = \sum_{\mathrm{j}=1}^{\mathrm{m}+1} \mathrm{z}_{\mathrm{ijp}}^{\mathrm{f}} \lambda_{\mathrm{j}}^{\mathrm{p}} + \mathrm{s}_{\mathrm{ip}}^{f-} \quad (i = 1, \ldots, n_f;\ p = 1, \ldots, P;\ j = 1, \ldots, m + 1) \quad (\mathrm{RI_b}\text{-6})$$

$$zs_{iop}^{f} = \sum_{j=1}^{m} z_{ijp}^{f} \lambda_j^p - s_{ip}^{f+} \quad (i = 1, \ldots, nf;\ p = 1, \ldots, P;\ j = 1, \ldots, m+1) \tag{RI$_b$-7}$$

$$zs_{iop}^{d} = \sum_{j=1}^{m} z_{ijp}^{d} \lambda_j^p - s_{ip}^{d} \quad (i = 1, \ldots, n_d;\ p = 1, \ldots, P;\ j = 1, \ldots, m+1) \tag{RI$_b$-8}$$

$$\begin{aligned} & s_{ip}^{u} \geq 0, \\ & s_{ip}^{+} \geq 0, \\ & s_{ip}^{-} \geq 0, \\ & \lambda_j^p \geq 0, \\ & s_{ip}^{f+} \geq 0, \\ & s_{ip}^{f-} \geq 0, \\ & s_{ip}^{d} \geq 0 \end{aligned} \tag{RI$_b$-9}$$

(1) Linear Robust Input-Oriented Model Step b (LRI$_b$). By considering two positive variables Q_s^+ and Q_s^- and substituting it with the absolute function, the model is linearized.

$$\min \quad \Phi_0^* = \beta \times \text{Average} + (1-\beta) \times \sum_{s=1}^{S} \Pi_s \times (Q_s^+ + Q_s^-) \tag{LRI$_b$-1}$$

$$\text{s.t.} \quad \text{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{\boldsymbol{m} + \boldsymbol{n_u} + \boldsymbol{n_f}} \left(\sum_{i=1}^{n} \frac{w_i^- s_{ip}^-}{x_{iop}} + \sum_{i=1}^{n_u} \frac{s_{ip}^u}{z_{iop}^u} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f-}}{z_{iop}^f} \right) \right] \tag{LRI$_b$-2}$$

$$\frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{\boldsymbol{m} + \boldsymbol{n_u} + \boldsymbol{n_f}} \left(\sum_{i=1}^{n} \frac{w_i^- s_{ip}^-}{x_{iop}} + \sum_{i=1}^{n_u} \frac{s_{ip}^u}{z_{iop}^u} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f-}}{z_{iop}^f} \right) \right] - \text{Average} = Q_s^+ - Q_s^- \tag{LRI$_b$-3}$$

Other constraints of model (RI$_b$) hold.

4.2.2. Robust Optimization for Output-Oriented DEA Model Step b (RO$_b$)

$$\begin{aligned} \min \quad & \Phi_0^* = \beta \times \text{Average} + (1-\beta) \\ & \times \sum_{s=1}^{S} \Pi_s \times \left(\frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f+}}{z_{iop}^f} \right) \right] - \text{Average} \right) \end{aligned} \tag{RO$_b$-1}$$

$$\text{s.t.} \quad \textbf{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f+}}{z_{iop}^f} \right) \right] \tag{RO$_b$-2}$$

Other constraints of model (RI$_b$) hold.

(1) Linear Robust Output-Oriented Model Step b (LRO$_b$). By considering two positive variables Q_s^+ and Q_s^- and substitut-

ing them with the absolute function, the model is linearized.

$$\min \quad \Phi_0^* = \beta \times \text{Average} + (1-\beta) \times \sum_{s=1}^{S} \Pi_s \times (Q_s^+ + Q_s^-) \tag{LRO$_b$-1}$$

$$\text{s.t.} \quad \textbf{Average} = \sum_{s=1}^{S} \Pi_s \times \frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f+}}{z_{iop}^f} \right) \right] \tag{LRO$_b$-2}$$

$$\frac{1}{P} \sum_{p=1}^{P} w^p \left[1 - \frac{1}{s + n_d + n_f} \left(\sum_{i=1}^{s} \frac{w_i^+ s_{ip}^+}{y_{iop}} + \sum_{i=1}^{n_d} \frac{s_{ip}^d}{z_{iop}^d} + \sum_{i=1}^{n_f} \frac{s_{ip}^{f+}}{z_{iop}^f} \right) \right] - \text{Average} = Q_s^+ - Q_s^- \tag{LRO$_b$-3}$$

Other constraints of model (RI_b) hold.

5. Case Study Implementation and Performance Evaluation

In this section, the performance of the proposed robust dynamic DEA approach is investigated via a case study taken from NANIWA (http://www.naniwa.ir) appliances production plant. The mentioned firm aims at evaluating its 35 suppliers in 4 time periods based on environmental, social, and economic criteria. For the purpose of evaluation, for each supplier as a DMU, 4 inputs, 3 desirable and undesirable relationships, and 4 outputs are considered.

Inputs

(1) Price offered by suppliers (as an economic criterion): It is the money (in $1000) that is paid to suppliers for each unit of products.

(2) Cost of recoverable packages (as an environmental criterion): This input is the cost (in $100) that is imposed to the company by the supplier for using recoverable packages. It is worth noting that it is mandatory for Naniwa Company to ship their products in suitable pallets with recoverable packages to prevent damaging the environment.

(3) Final transportation cost (as an economic criterion): This is the final cost (in $100) which is imposed by the supplier to the firm for transportation of shipped pallets. The farthest the supplier is and the less accessible the paths are, the more cost imposed. This cost is the main concern of the decision-makers in the company.

(4) Work safety and labor health (as a social criterion): This is the cost that each supplier pays for dangers that exist and accidents which happen in the workplace. The less damage and casualties are, the less cost is paid by each supplier.

Relationships

(1) Used technology in the production line of suppliers (as a desirable relationship and an economic criterion): The used technology is scored by expert's opinion using 9-point Likert spectrum according to Table 1. Likert scale is used to convert qualitative factors into quantitative values [42]. There are variety of scales which can be rated as 1 to 5, 1 to 7, and 1 to 9. Valuation of factors in this scale is performed according to concept of each factor [43].

(2) Green research and development (as a free relationship and an environmental criterion): The corresponding budget per year (in $100). Green research and development is a dual-role factor which plays the role of both undesirable and desirable factors. Green research and development can be considered as an undesirable criterion since it is cost of performing green researches. On the other hand, green research and development is a desirable criterion, because it implies innovations in manufacturing green products and services and environmental efficiency enhancement.

(3) Shortages (as an undesirable relationship and an economic criterion): The amount of shipments (in terms of pallets) that have not delivered in the past period and should be met in the next period by the supplier.

Outputs

(1) Obtaining ISO certificates and observance of standards: this kind of output is scored by expert's opinion using 9-point spectrum with respect to qualitative and environmental criteria and standards and also workplace standards. Table 2 shows the 9-point spectrum.

(2) Quality (as an economic criterion): Quality of products is evaluated by Likert scale. In Table 3, using a 9-point Likert scale, valuation of quality of parts supplied by suppliers is presented.

(3) Supply capacity (g_2): maximum amount of materials that supplier can send to Naniwa Co.

(4) Efficiency of energy consumption (as an environmental criterion): Efficiency of the energy consumption is the third output which is an environmental criterion. To determine an appropriate scale for evaluating

Table 1: 9-point Likert spectrum for Technologic power.

Value	9	7	5	3	1	2-4-6-8
Technology	High Technology	Good Technology	Medium Technology	Weak Technology	Very weak Technology	Intermediate values for Technology

Table 2: 9-point Likert spectrum for Standards.

Value	9	7	5	3	1	2-4-6-8
ISO standards	High Standards	Good Standards	Medium Standards	Weak Standards	Very Weak Standards	Intermediate values for Standards

efficiency of energy consumption we apply a scoring method which is shown by A+++ to G scale. The letter A+++ shows the lowest energy consumption. The letter G indicates the highest energy consumption. Using 100-point scale. Table 4 shows the energy consumption of the suppliers.

Figure 4 illustrates the proposed dynamic DEA approach on the case study to evaluate suppliers in periods 2011 to 2014. The inputs, outputs, and relationships between periods is illustrated in this figure.

The inputs, relationships, and outputs data for 35 suppliers in 4 time periods are presented in Table 5 in the Appendix.

In Table 6, the DMUs (i.e., 35 suppliers) are evaluated by making use of the existing input-oriented dynamic DEA model on data shown in the Table 5 presented in the Appendix. The DEA model is selected according to the DM's approach for which he/she want to present their proposed improvement plans. Table 6 shows the efficiency values for 35 suppliers in 4 periods from 2011 to 2014.

The results of Table 6 show that none of the suppliers have unit efficiency values in all 4 periods. Therefore, none of them are strictly efficient. To construct a strictly efficient unit as a benchmark for other units, in each period the DMU with unit efficiency is selected as a candidate. As a result, 8 scenarios are generated for ideal DMU which are different combinations of DMUs (suppliers) with unit efficiency in each period. These resulting scenarios are presented in Table 7 with probability of 0.125 for each one. If these scenarios are taken into consideration and evaluated along with other 35 suppliers, the efficiency of scenarios becomes one because they consist of DMUs with unit efficiencies in all periods. Therefore, we can claim that the existing dynamic DEA model cannot evaluate and rank the scenarios. In the proposed model (RI_a), since all scenarios have unit efficiency, the average efficiency is also one and the deviation of efficiencies is zero. Therefore, we set $\beta = 1, 1 - \beta = 0$. To deal with this difficulty, a punishment (Cost) and encouragement (Income) value is considered for the inputs, outputs, and relationships of each scenario.

The costs and incomes associated with inputs, outputs, and relationships are presented in Table 8 with the following notations:

$\boldsymbol{k_j^{ALL}}$: Unit cost for input j for all scenarios (j: price, packaging, transportation cost, and workforce health cost): this is a penalty that is imposed to a scenario by the decision-maker for each unit of inputs.

$\boldsymbol{h_j^{ALL}}$: Unit income for output j for all scenarios (j: quality, production capacity, the number of acquired ISO and standards, and efficiency of the energy consumption): this is the encouragement that is considered for a scenario for each unit of outputs.

$\boldsymbol{g_1^{ALL}}$: Unit cost of shortages in shipped pallets for all scenarios (undesired relationships): this is the penalty that is decided by the decision-maker for each unit of this undesired relationship. In this paper for backlogs or goods shipped with a delay a penalty is considered for the supplier in that period.

$\boldsymbol{v_1^{ALL}}$: Unit cost of green research and development for all scenarios (free relationships): if the free relationship be an undesired relationship a penalty value will be assigned for each unit of this relationship. In our case study, if the green R&D is considered as an undesired relationship for the under investigation DMU, a penalty is considered for that supplier for each unit of this relationship.

$\boldsymbol{e_1^{ALL}}$: Unit income of green research and development for all scenarios (free relationships): if the free relationship is a desired relationship an encouragement value will be assigned for each unit of this relationship. In our case study, if the green R&D is considered as a desired relationship for the under investigation DMU, an encouragement value is considered for that supplier for each unit of this relationship.

$\boldsymbol{b_1^{ALL}}$: Unit income of technological power for all scenarios (desirable relationships).

As mentioned earlier, the existing dynamic DEA model cannot rank different scenarios for the ideal supplier. In this paper, first we evaluate and rank these scenarios through the proposed input-oriented robust dynamic DEA model (RI_a). In Table 9, the punishment and encouragement for each scenario resulting from model (RI_a) is presented. As presented in Table 9, the second scenario is the best scenario since it has the maximum value of benefit. Generally, if the objective function of model (RI_a) is positive, the scenario will generate loss while it is negative the scenario will make benefits. If all scenarios generate loss, the scenario with minimum loss will be selected. The values of landa obtained from model (RI_a) admit that the scenarios can be considered

Table 3: 9-point Likert spectrum for Quality.

Value	9	7	5	3	1	2-4-6-8
Quality	High Quality	Good Quality	Medium Quality	Weak Quality	Very Weak Quality	Intermediate values for Quality

Table 4: Spectrum for efficiency of the energy consumption.

Score	100	90	80	70	60	50	40	30	20	10
Efficiency of the energy consumption	A+++	A++	A+	A	B	C	D	E	F	G

as a benchmark. In the next step, the second scenario which is the best scenario is considered as the ideal DMU and it is evaluated along with other DMUs (suppliers) through model (RI_b). The ideal DMU which is actually the ideal scenario is considered as a unique benchmark which owns both the property of a real supplier in that it consists of some periods and the property of a virtual supplier in that it is strictly efficient and consists of periods with unit efficiency. Therefore, the proposed ideal DMU can present improvement plans for all suppliers and can rank the suppliers with same efficiency. The results obtained from model (RI_b) are reported in Table 10. As presented in Table 10, the ideal DMU gives the ability of ranking the suppliers that had the same rank in Table 6. From Table 6 we can see that suppliers 2 and 13, 19 and 3, 8 and 31, 6 and 4, two by two have the same rank because the total efficiency of these DMUs are equal. Using the proposed model (RI_b) for evaluating the suppliers and constructing an ideal DMU help us to rank the suppliers. This ranking considers the standard deviation of efficiencies in calculating the efficiency of each supplier. Furthermore, the proposed ideal DMU gives us the opportunity of presenting improvement methods for all DMUs. As a matter of fact, the improvement methods are presented based on their distance from the ideal DMU (supplier). The high ranked scenario resulting from model (RI_a), which is the second scenario, has maximum distance from other suppliers and it is the worst case that can be happen for an ideal scenario based on which improvement plans can be presented. Through model (RI_b) other suppliers can be ranked based on their distance from this worst case ideal scenario. Therefore, we could claim that the resulting ranks and improvement plans cannot get worse when each of the other scenarios, which could be happen with a given probability, is considered thus resulting in a robust solution.

6. Conclusions and Future Research Directions

This paper proposed a new DEA approach for evaluating suppliers based on sustainable supplier criteria such as social, economic, and environmental criteria. The proposed model considers suppliers in different periods thus leading to a dynamic DEA model. Existing dynamic DEA models just present improvement plans and do not have the ability of ranking DMUs. The proposed model apart from having the ability of ranking DMUs can present improvement plans. In most DEA models, when different time periods are considered in evaluating DMUs, no DMUs can be introduced as a strictly efficient unit which is efficient in all periods. Our contribution is introducing a new method for constructing the ideal DMU such that apart from the previous definition for ideal DMU which considers one of the DMUs which has unit efficiency in all periods as an ideal DMU, a virtual DMU is considered as an ideal DMU. The new proposed ideal DMU has the ability of presenting an improvement method for all suppliers and also ranking the suppliers with the same efficiency. The proposed ideal DMU introduces a strictly efficient unit by building a combination of DMUs with unit efficiency in each period. It is possible that multiple DMUs have unit efficiency in a period. Therefore, different combinations of these units lead to different scenarios for the ideal DMU each of which can happen with a specific probability. The existing dynamic DEA model cannot evaluate and rank the scenarios which all have unit efficiency. To deal with these scenarios, a scenario-based robust optimization model for the dynamic DEA is developed which is capable of ranking the scenarios based on a punishment and encouragement value assigned to each scenario.

The proposed robust method is implemented in two steps and it is is able to obtain a solution which is immunized against different scenarios exist in constructing the ideal DMU. In the first step, at each time one of the scenarios is under investigation. The high ranked scenario which has more distance from other DMUs is selected as a unique benchmark based on which the improvement plan is presented in the worst case. In fact, in the second step other DMUs along with the worst case scenario are under investigation and improvement plans are proposed based on their distance from the ideal DMU. Therefore, the proposed method can rank the suppliers with the same efficiency and propose an improvement plan which is robust against different scenarios which could be considered for the ideal DMU.

For future study on this work, apart from the uncertainty in different scenarios for ideal DMU, one can consider the input, output, and relationship values as uncertain parameters which can vary in a convex uncertainty set. Then a hybrid robust optimization approach, i.e., a hybridization of the scenario-based and robust counterpart methods, can be developed to deal with these uncertainties.

Appendix

See Table 5.

Table 5: The inputs, relationships and outputs data for 35 suppliers in 4 time periods.

	DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
	IN1	26	31	29	30	26	34	32	16	29	30	33	28	24	31	29	26	25	29	28	31	35	15	33	21	26	24	28	23	30	27	26	31	25	29	33
	IN2	15	18	23	22	20	19	14	9	17	18	19	25	22	20	18	17	16	23	19	22	19	10	19	13	15	16	17	19	20	18	12	16	16	18	21
	IN3	13	12	11	10	9	6	7	2	8	10	6	9	9	11	14	5	8	6	9	5	7	3	9	8	12	5	6	7	8	9	10	5	8	12	9
	IN4	10	9	7	8	9	6	4	8	10	9	8	11	8	9	9	10	6	9	7	8	10	2	9	9	8	6	6	7	8	9	10	6	8	8	9
	OUT1	4	4	6	5	8	4	9	6	5	7	6	5	4	4	5	6	7	8	6	3	4	9	5	8	7	4	5	7	6	5	4	6	5	9	8
2011	OUT2	8	6	5	9	5	6	9	7	8	7	5	7	8	6	7	9	6	8	7	7	9	9	7	7	6	8	7	8	7	9	9	7	6	8	7
	OUT3	78	53	59	64	69	71	88	68	63	59	55	49	69	72	66	67	81	71	63	65	73	95	62	56	84	79	83	78	66	69	59	74	95	76	85
	OUT4	60	90	80	80	90	90	60	100	80	90	60	70	90	80	80	60	70	50	60	50	40	100	70	70	80	60	60	50	70	60	40	70	60	50	80
	LINK1	8	7	8	6	7	9	6	9	5	5	7	9	7	8	6	8	9	5	5	6	7	9	9	8	6	5	8	8	6	7	8	6	6	7	8
	LINK2	28	31	30	32	29	28	35	18	26	26	27	29	30	33	27	25	29	30	27	29	27	15	25	26	37	32	34	26	29	29	28	26	33	34	25
	LINK3	20	16	17	18	13	15	15	2	15	18	16	18	15	13	6	8	9	7	6	8	11	3	12	19	15	13	14	6	16	9	11	12	14	15	10
	IN1	19	33	32	27	29	29	28	30	32	33	36	29	28	32	31	29	28	32	33	28	30	15	29	33	28	30	29	29	27	32	30	33	29	25	38
	IN2	12	16	25	24	22	20	16	18	16	17	20	26	19	23	22	19	18	20	16	18	20	10	16	21	19	14	20	16	15	20	16	20	18	16	18
	IN3	3	9	13	12	10	5	6	7	7	9	5	7	8	14	19	7	9	5	7	9	10	5	7	9	14	9	5	7	9	7	9	7	10	9	10
	IN4	4	11	8	9	8	9	10	6	9	7	9	10	9	13	12	9	8	10	8	6	9	5	11	10	9	8	7	5	6	7	9	8	6	7	10
	OUT1	9	6	7	8	6	6	5	7	6	8	6	7	8	6	5	8	6	7	5	8	6	9	7	8	5	6	6	8	7	7	6	5	8	6	7
2012	OUT2	9	6	7	8	6	5	5	8	9	9	6	9	8	7	7	6	8	6	9	8	7	9	6	6	7	8	5	8	6	7	8	9	7	8	8
	OUT3	95	59	60	70	77	66	68	52	70	59	63	52	73	78	72	69	73	69	74	72	69	89	73	66	78	66	91	65	73	72	60	85	80	66	80
	OUT4	100	80	80	80	90	80	50	60	80	70	50	70	70	80	80	70	70	50	60	60	40	100	70	70	70	60	60	60	80	60	50	70	70	50	80
	LINK1	9	6	7	8	8	8	7	7	6	6	7	8	6	7	7	7	8	6	6	7	8	9	8	7	7	7	7	7	7	6	8	7	7	8	7
	LINK2	12	30	28	31	33	29	29	25	19	28	34	36	33	28	34	29	25	32	33	30	26	20	30	31	28	26	29	30	32	25	27	29	32	26	28
	LINK3	5	12	10	11	17	16	15	13	17	16	14	11	10	11	15	12	12	17	14	12	16	5	10	10	17	10	12	11	18	14	13	15	18	12	10

Table 5: Continued.

	DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
	IN1	31	32	34	26	30	26	29	32	35	32	30	26	30	30	29	22	29	31	30	25	32	26	28	32	29	31	30	28	26	33	28	30	26	24	15
	IN2	18	18	27	25	24	22	19	19	19	18	22	28	17	25	25	17	19	24	19	19	22	19	18	23	17	18	21	17	17	22	18	21	19	15	9
	IN3	14	10	14	13	11	8	9	8	8	10	8	9	9	11	15	8	10	7	8	8	12	10	8	10	12	8	8	9	10	6	7	8	8	4	2
	IN4	15	14	12	11	10	10	12	8	7	9	10	12	10	15	14	11	10	14	9	9	10	10	15	14	10	11	12	10	5	8	8	11	9	10	4
	OUT1	7	9	7	9	8	9	9	8	7	7	9	8	9	6	9	7	8	8	9	9	8	8	6	9	8	7	6	9	9	7	8	8	6	7	9
2013	OUT2	6	8	9	8	6	9	7	8	6	8	7	9	9	8	6	7	9	9	8	7	9	8	6	7	8	9	9	8	8	9	8	7	9	8	9
	OUT3	63	70	72	64	86	70	64	65	65	60	70	60	65	85	80	75	80	70	65	83	82	70	65	80	81	75	76	55	65	80	50	80	77	65	95
	OUT4	70	70	80	60	90	70	50	60	70	50	40	70	60	80	70	70	70	70	60	60	50	60	70	70	70	80	60	50	80	60	60	70	70	60	100
	LINK1	8	7	7	8	9	8	7	9	6	5	7	9	6	7	8	5	8	5	8	7	8	6	8	7	8	6	8	7	8	6	8	8	7	8	9
	LINK2	33	32	27	29	30	34	31	39	28	30	30	36	33	29	31	30	26	27	29	30	31	36	34	33	29	30	32	29	37	29	34	33	26	28	10
	LINK3	15	19	21	20	19	18	17	16	14	15	19	18	16	17	15	12	18	17	13	15	16	14	10	12	14	11	13	15	16	14	12	15	16	12	2
	IN1	20	34	37	29	34	33	32	36	37	34	34	35	34	32	33	28	32	34	33	31	30	30	32	34	30	33	32	30	30	35	33	32	28	28	15
	IN2	14	22	30	28	27	26	25	20	23	24	20	26	20	27	29	20	22	26	20	21	25	21	19	26	19	21	23	21	20	24	21	22	20	17	12
	IN3	4	14	15	15	13	12	12	10	10	12	10	10	11	14	17	16	12	18	10	11	15	12	9	14	15	9	7	10	12	14	10	16	9	10	5
	IN4	8	15	14	13	12	12	9	10	12	10	10	11	10	18	13	12	11	13	10	10	13	12	14	11	12	11	14	12	9	11	10	14	11	9	7
	OUT1	9	7	7	9	9	8	8	8	9	7	7	8	9	9	7	8	6	9	7	9	8	8	9	8	7	7	8	9	9	8	8	7	9	8	9
2014	OUT2	9	9	9	9	7	9	7	9	7	9	8	9	9	8	8	7	9	9	8	8	9	8	8	8	9	9	9	9	9	9	8	9	9	8	9
	OUT3	89	59	63	72	73	72	68	68	60	68	67	73	72	66	64	69	65	43	70	72	71	74	73	70	72	66	67	59	68	75	63	78	69	59	95
	OUT4	100	90	80	80	90	70	60	60	90	70	50	60	60	90	70	80	70	80	70	60	50	50	60	80	70	80	60	70	100	60	60	80	70	70	100
	LINK1	9	7	7	8	9	9	8	9	7	7	8	9	7	8	8	6	8	6	9	7	9	7	8	6	9	7	8	8	8	8	8	8	9	8	9
	LINK2	20	30	29	30	32	32	33	35	29	32	32	35	34	30	33	32	28	29	30	29	30	35	36	34	31	32	34	32	30	32	36	30	25	31	22
	LINK3	5	9	11	10	14	16	13	11	12	10	14	16	16	18	5	17	16	18	15	10	10	13	12	14	16	7	10	21	18	14	11	13	15	10	3

Table 6: Efficiency values for 35 suppliers in 4 periods (Input-oriented dynamic DEA model).

DMU	ϕ_{2011}	ϕ_{2012}	ϕ_{2013}	ϕ_{2014}	Φ_j	RANK
1	0.856	1	0.986	1	0.9605	1
2	0.763	0.943	0.915	0.966	0.8967	4
3	0.88	0.849	0.965	0.824	0.8795	11
4	0.867	0.825	0.814	0.894	0.85	26
5	0.728	0.831	0.824	0.872	0.8137	33
6	0.809	0.941	0.769	0.881	0.85	26
7	0.911	0.817	0.964	0.764	0.864	19
8	1	0.866	0.813	0.821	0.875	13
9	0.766	0.835	0.849	0.938	0.847	29
10	0.849	0.961	0.827	0.933	0.846	31
11	0.867	0.985	0.863	0.74	0.8637	20
12	0.857	0.938	0.917	0.768	0.87	16
13	0.892	0.846	0.965	0.884	0.8967	4
14	0.893	0.851	0.824	0.825	0.8482	28
15	0.846	0.837	0.764	0.864	0.8277	32
16	0.937	0.84	0.819	0.897	0.8732	15
17	0.967	0.869	0.893	0.831	0.89	7
18	0.955	0.815	0.84	0.843	0.8632	21
19	0.934	0.834	0.933	0.817	0.8795	11
20	0.871	0.769	0.968	0.934	0.8855	9
21	0.768	0.941	0.824	0.92	0.8632	22
22	1	1	0.866	0.819	0.9212	3
23	0.869	0.911	0.814	0.8349	0.8572	24
24	0.942	0.771	0.867	0.822	0.8505	25
25	0.852	0.764	0.789	0.796	0.8002	34
26	0.789	0.859	0.984	0.753	0.8462	30
27	0.844	0.915	0.943	0.749	0.8627	23
28	0.809	0.864	0.972	0.921	0.8915	6
29	0.78	0.91	0.928	0.966	0.8842	10
30	0.922	0.752	0.846	0.946	0.8665	18
31	0.818	0.92	0.837	0.925	0.875	13
32	0.846	0.881	0.819	0.933	0.8697	17
33	0.856	0.825	0.935	0.928	0.886	8
34	0.736	0.714	0.966	0.784	0.8	35
35	0.855	0.891	1	1	0.9365	2

Table 7: Different scenarios for Ideal DMU.

Ideal DMU	Period 2011	Period 2012	Period 2013	Period 2014
Scenario 1	Supplier 22	Supplier 22	Supplier 35	Supplier 1
Scenario 2	Supplier 22	Supplier 22	Supplier 35	Supplier 35
Scenario 3	Supplier 22	Supplier 1	Supplier 35	Supplier 1
Scenario 4	Supplier 22	Supplier 1	Supplier 35	Supplier 8
Scenario 5	Supplier 8	Supplier 22	Supplier 35	Supplier 1
Scenario 6	Supplier 8	Supplier 22	Supplier 35	Supplier 35
Scenario 7	Supplier 8	Supplier 1	Supplier 35	Supplier 1
Scenario 8	Supplier 8	Supplier 1	Supplier 35	Supplier 35

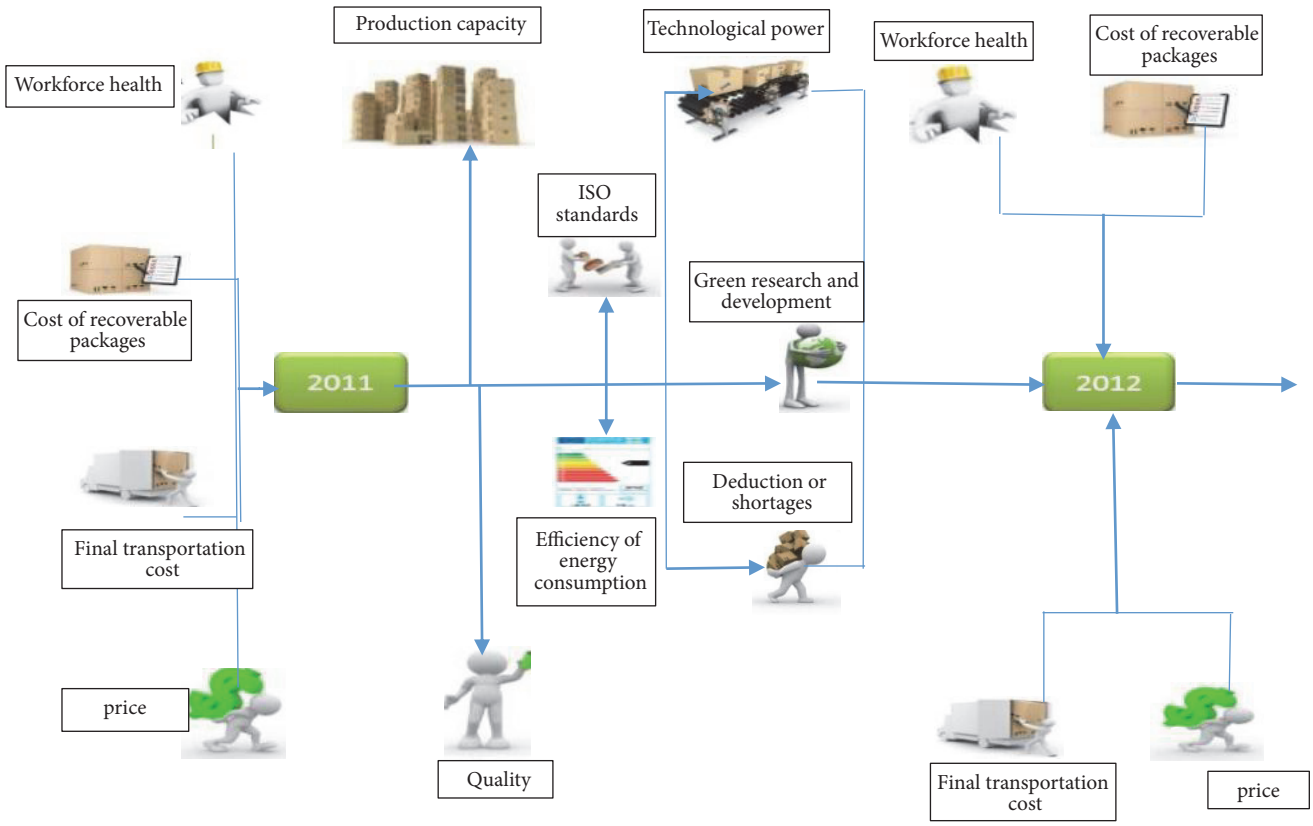

Figure 4: Demonstration of the proposed dynamic DEA on the considered case study.

Table 8: Costs and incomes associated with inputs, outputs and relationships.

Parameters	k_1^{ALL}	k_2^{ALL}	k_3^{ALL}	k_4^{ALL}	g_1^{ALL}	v_1^{ALL}
Costs	*10*	*5*	*7*	*3*	*11*	*9*
Parameters	h_1^{ALL}	h_2^{ALL}	h_3^{ALL}	h_4^{ALL}	e_1^{ALL}	b_1^{ALL}
Incomes	*20*	*15*	*18*	*12*	*15*	*18*

Table 9: Evaluating different scenarios for the Ideal DMU (Model RI_a).

Scenario	1	2	3	4	5	6	7	8	Worst case
Efficiency	1	1	1	1	1	1	1	1	1
Benefit/loss	10027	10988	10103	9655	10320	10768	9435	9883	10147.375

Table 10: Evaluating suppliers along with the ideal supplier (Model RI_b).

DMU	1	2	3	4	5	6	7	8	9
Efficiency	0.489	0.478	0.445	0.354	0.317	0.359	0.412	0.437	0.341
RANK	2	5	13	28	34	27	20	15	30
DMU	10	11	12	13	14	15	16	17	18
Efficiency	0.325	0.406	0.428	0.475	0.348	0.321	0.432	0.469	0.401
RANK	32	21	17	6	29	33	16	8	22
DMU	19	20	21	22	23	24	25	26	27
Efficiency	0.449	0.462	0.392	0.481	0.377	0.362	0.309	0.332	0.384
RANK	12	10	23	4	25	26	35	31	24
DMU	28	29	30	31	32	33	34	35	Ideal DMU
Efficiency	0.472	0.458	0.416	0.439	0.422	0.466	0.305	0.484	0.5
RANK	7	11	19	14	18	9	36	3	1

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Multicriteria Evaluation of Urban Regeneration Processes: An Application of PROMETHEE Method in Northern Italy

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The paper illustrates the development of an evaluation model for supporting the decision-making process related to an urban regeneration intervention. In particular, the study proposes an original multi-methodological approach, which combines SWOT Analysis, Stakeholders Analysis and PROMETHEE method for the evaluation of alternative renewal strategies of an urban area in Northern Italy. The article also describes the work carried out within an experts' panel that has been organized for validating the structuring of the decision problem and for evaluating the criteria of the model.

1. Introduction

In recent years many European cities have implemented relevant renewal programmes for enhancing physical, environmental, social, and economic long-term development of old industrial sites or areas under decline. Integrated regeneration processes represent the main concern in many experiences. Physical transformations are embedded within social, environmental, and economic as well as institutional aspects [1]. How to achieve a balance among interrelated and often conflictual goals in order to improve the quality of urban systems is still an open challenge. On one side the need of replacing top-down strategies with collaborative models, based on needs, expectations, and values shared by all the parties involved, is widely acknowledged as one of the driver of success [2–4]. On the other one, local oppositions often arise against both public and private works, thus causing interruptions and delays to development processes [5].

Territorial and urban regeneration programmes specifically point out the need of developing new combinations between analytical tools and participatory approaches, in order to strengthen the choices' legitimacy and to address the wealth of contradictory visions, and preferences of the different actors to a shared vision according to a multilevel governance perspective. A critical review of the notion of reuse over time has revealed an emerging attention to the quality issue that does not only depend on development and design tools focused on environmental targets, but also on the managerial approach of local authorities in structuring multiform partnerships [6].

Under these circumstances, evaluation plays a crucial role since it allows to codefine and rank alternative projects with respect to both technical elements, which are based on empirical observations, and non-technical elements, which are based on social visions, preferences, and feelings [7].

In this context, a very useful support is provided by Multiple Criteria Decision Analysis (MCDA) techniques, which are used to make a comparative assessment of alternative projects or heterogeneous measures [8, 9]. These methods allow several criteria to be taken into account simultaneously in a complex situation and they are designed to help decision-makers (DMs) to integrate the different options, which reflect the opinions of the involved actors, in a prospective or retrospective framework. Participation of decision-makers in the process is a central part of the approach.

Aware of the advantages and disadvantages of the many available MCDA techniques, this paper aims at testing the PROMETHEE (Preference Ranking Organisation Method

for Enrichment Evaluations), as an outranking method [10] to support decisions in urban planning and regeneration processes. Given the lack of robust assumptions on the decision maker preferences, the PROMETHEE can be effectively integrated with participatory methods in order to get enough information to understand whether one alternative is at least as good as another.

In particular, the paper refers to the assessment of different urban regeneration scenarios for the city of Collegno (Italy). Differently from Bottero et al. [11], who modeled urban resilience dynamics in Collegno by using Fuzzy Cognitive Maps, and complementing Bottero et al. [12] who combined Stakeholder Analysis and Stated Preference Methods to assess the social value of urban regeneration scenarios in Collegno and their related willingness to pay, we combine the PROMETHEE approach with SWOT Analysis and Stakeholder Analysis, to rank six urban regeneration alternatives and identify the solution that outranks the others, thus providing decision-makers with useful tools in making welfare-maximizing urban planning decisions. We thus aim to contribute framing a multimethodological evaluation process which can be transferred, once validated, in other decision contexts [13].

The remainder of the paper is organized as follows. Section 2 provides a methodological background and a brief literature review; Section 3 illustrates the application of PROMETHEE in the evaluation of urban renewal projects in the city of Collegno (Italy); in Section 4 results are discussed and conclusions are drawn.

2. Methodological Background

The PROMETHEE method is one of the most recent Multicriteria Decision Analysis (MCDA) methods which was firstly proposed by Brans in the early Eighties [10] and subsequently extended by Brans and Vincke [14], Brans et al. [15], Brans and Mareschal [16], and Brans and Mareschal [17]. Usually a multicriteria problem is an ill-posed mathematical problem as it does not find a solution which optimizes all of the criteria simultaneously. As other multicriteria methods, the PROMETHEE requires additional information to overcome the poorness of dominance relation on Preference (P) and Indifference (I), thus enriching the dominance graph [18]. The PROMETHEE is an outranking method for ranking a finite set of alternative actions when multiple criteria, which are often conflicting, and multiple decision-makers are involved [8]. PROMETHEE uses partial aggregation and by a pairwise comparison of alternative actions, it allows to verify whether under specific conditions one action outranks or not the others. The PROMETHEE methods are a family of outranking methods [19]: PROMETHEE I (partial ranking); PROMETHEE II (complete ranking); PROMETHEE III (ranking based on intervals); PROMETHEE IV (continuous case); PROMETHEE V (including segmentation constraints); and PROMETHEE VI (evaluating the degree of hardness of a multicriteria decision problem with respect to the weights given to the criteria, i.e., for human brain representation). In addition, in 2004 Figueira et al. [20] proposed two extensions of the PROMETHEE, namely PROMETHEE TRI to solve sorting problems, and PROMETHEE CLUSTER for nominal classification.

In this paper we implement PROMETHEE II in order to rank alternatives according to different criteria which have to be maximized or minimized. Once the decision group was constituted, we proceeded according to the following subsequent steps.

Step 1 (construction of an evaluation matrix). A double entry table for the selected criteria and alternatives has been compiled by using cardinal (quantitative) and ordinal (qualitative) data. This matrix accounts for deviations of evaluations on pairwise comparisons of two alternatives, a and b, on each criterion.

Step 2 (identification of the preference function $P_j(a, b)$ for each criterion j). The preference function is used to determine how much alternative a is preferred to alternative b and it translates the difference in evaluations of the two alternatives into a preference degree. These preferences are represented in a numerical scale ranging between 0 and 1. The value "1" represents a strong preference of alternative a over b, whereas "0" represents the indifferent preference value between the two alternatives. Six types of preference functions have been proposed by the developers of the PROMETHEE methodology: Usual criterion, Quasi criterion (U-shape), Criterion with linear preference (V-shape), Level criterion, Linear criterion, and Gaussian criterion [15, 21].

Step 3 (calculation of the overall preference index $\Pi(a, b)$). The overall preference index $\Pi(a, b)$ represents the intensity of preference of a over b and it is calculated as follows (1):

$$\Pi(a, b) = \sum_{j=1}^{k} w_j P_j(a, b) \tag{1}$$

where $\Pi(a, b)$ is the overall preference intensity of a over b with respect to all of the K criteria, w_j is the weight of criterion j, and $P_j(a, b)$ is the preference function of a over b with respect to criterion j. Clearly $\Pi(a,b) \sim 0$ implies a weak global preference of a over b, whereas $\Pi(a,b) \sim 1$ implies a strong global preference of a over b.

Step 4 (calculation of the outranking flows, i.e., positive flow $\Phi^+(a)$ and negative flow $\Phi^-(a)$). In PROMETHEE method two flow measures can be determined for each alternative. There are a positive flow (it expresses how alternative a is outranking all the others)

$$\Phi^+(a) = \frac{1}{n-1} \sum_{b \in A} \Pi(a, b) \tag{2}$$

and negative flow (it expresses how alternative a is outranked by all the others)

$$\Phi^-(a) = \frac{1}{n-1} \sum_{b \in A} \Pi(b, a) \tag{3}$$

Step 5 (comparison of the outranking flows to define the alternatives complete ranking). In detail, PROMETHEE II, here

implemented, provides a complete ranking of the alternatives by calculating the net flow (4):

$$\Phi(a) = \Phi^{+}(a) - \Phi^{-}(a). \quad (4)$$

The higher the net flow, the better the alternative. When PROMETHEE II is considered, no incomparability remains, as all the alternatives are comparable on all the criteria. It is worth noting that the net flow provides a complete ranking and thus can be compared with a utility function.

In the past decade, a growing interest arose in identifying solutions which reflect reality as much as possible by modeling it in a clear and understandable way by both analysts and decision-makers. Conceptually, PROMETHEE is a rather simple ranking method compared with other methods for multicriteria analysis [15] and the number of its applications to real world decision problems increased significantly [22]. The applications of PROMETHEE methods are varied and cover as major fields environmental management, water management, business and financial management, logistics and transportation, and energy management [22]. There are several applications as well in social sciences starting from seminal works by D'Avignon and Mareschal [23] and Urli and Beaudry [24] on hospital services and allocation of funds to development programs, respectively. Nonetheless, PROMETHEE applications in urban and territorial planning are quite recent. Mavrotas et al. [25] adopted PROMETHEE for comparatively evaluating control strategies to reduce air pollution in Tessaloniki and base their procedure on active involvement of local and central authorities; Anton et al. [26] applied PROMETHEE for the management and disposal of solid wastes in an Andine area; Juan et al. [27] used the PROMETHEE method combined with fuzzy set theory to determine the priority of 13 urban renewal projects in Taipei City, whereas Roozbahani et al. [28] combined PROMETHEE with Precedence Order in the Criteria (POC) to urban water supply management in Melbourne to assess operation rules in single or group decision-making contexts. More recently Cilona and Granata [29] implemented the PROMETHEE approach to support prioritization of subprojects in complex renewal projects at neighborhood scale; Esmaelian et al. [30] implemented PROMETHEE IV and GIS to identify most vulnerable urban areas to earthquakes and they prove its efficacy in electing the most suitable locations for the construction of emergency service stations; Polat et al. [31] proposed an integrated approach which combines the Analytic Hierarchy Process (AHP) and the PROMETHEE method to support construction companies to select urban renewal projects to invest in; Bottero et al. [32] used PROMETHEE methods to analyze different urban regeneration scenarios in Gran Canaria island; Cerreta and Daldanise [33] proposed PROMETHEE to support urban regeneration by a learning and negotiation process; Dirutigliano et al. [34] applied PROMETHEE as a support tool for promoting energy retrofitting of urban districts in Torino; Mendonça Silva et al. [35] used PROMETHEE method to solve an urban planning conflict in Recife; Wagner [36] adopted PROMETHEE to assist the decision-making process in spatial urban planning, whereas Tscheikner-Gratl et al. [37] compared PROMETHEE to other four multicriteria decision aiding/making methods (i.e., ELECTRE, AHP, WSM, and TOPSIS) in rehabilitation planning of urban water networks.

3. Application

The case study considered in the present paper is related to the urban regeneration program of the city of Collegno, located in the metropolitan area of Torino (Northern Italy). The program, promoted by the Municipal Administration, aims at finding answers to the economic and social needs of the city and to provide a coherent development strategy to a territory afflicted by an unregulated development and by the presence of many abandoned areas.

The objectives of the program are mainly related to the regeneration of the city as "Collegno Social Town". The creation of a nice and livable place and the elimination of physical and environmental limits are the key elements of the development strategy. The area of the Fermi metro station, including the site of Campo Volo, represents a crucial portion of the territory under investigation.

3.1. Structuring of the Decision Problem. The first step for the evaluation refers to the structuring of the decision problem, i.e. identifying the possible alternative strategies for the urban regeneration program and defining the criteria to be included in the model. For this purpose, an integrated framework has been proposed in the present application that aims at setting the problem and highlighting its key elements. More precisely, two different analyses have been performed, namely, the SWOT Analysis and the Stakeholders Analysis.

In detail,

(i) the SWOT (Strengths, Weaknesses, Opportunities, and Threats) Analysis is a technique used to define strategies, in those context which are characterized by complexity and uncertainty, such as urban regeneration. The analysis was used for a critical interpretation of the case under investigation and for supporting the definition of the goal of the transformation and the construction of the alternative projects;

(ii) the Stakeholders Analysis allows to define who are the actors of the process under investigation. As stated by Yang (2013), in the context of urban transformation real-world problems, only if stakeholders' interests are identified, it is possible to sufficiently empower them in the decision-making process. Moreover, the analysis permits to define which resources and objectives the actors are able to bring into play, showing possible conflicts. Finally, by means of Stakeholders Analysis the complexity of the decisional process can be represented, suggesting the evaluation criteria to be considered for the comparison of the alternative strategies (Figure 1).

3.2. Alternative Transformation Projects. In this experimentation, we have implemented an integrated approach to evaluate six different alternatives related to the development of the urban regeneration program of the city of Collegno.

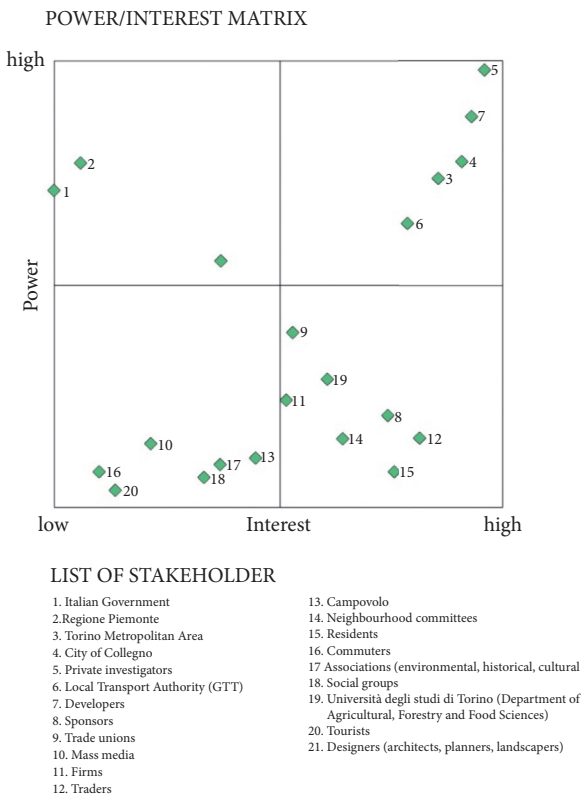

Figure 1: Stakeholders mapping for the case under investigation.

In detail, starting from the alternatives analyzed by Bottero et al. [11, 12], we have selected six alternative projects, which we consider the most relevant according to the SWOT and Stakeholders Analyses. These alternatives can be described as follows (Figure 2):

(1) Cultural district: this strategy is based on the creation of new cultural services for the area, including a new public library and residences for university students.

(2) Smart City: the goal of this strategy consists in providing a new identity to the area based on the concept of smart city.

(3) Start up: this project focuses on the creation of innovative business activities in the area.

(4) City and craft: this strategy is based on the valorization of the small economic activities in the area and on the creation of a new urban park in the Northern part of the area.

(5) Sharing city: the objective of this project is mainly related to the valorization of the public spaces in the area, with special attention to innovative shared solution for living and working.

(6) Green infrastructure: the main intent of this strategy is to improve the livability of the territory, with particular attention to the creation of new green infrastructures, such as pedestrian and bicycle paths.

3.3. Definition and Evaluation of the Criteria. In accordance with the results of the two aforementioned analyses, we identified the most important drivers of the transformation that can be summarized in Table 1. In particular, SWOT and Stakeholders Analysis allowed breaking down the complexity of the problem and identifying general aspects that characterize the transformation to be defined, namely, environmental, economic, social, regeneration, mobility, and services factors. These aspects have been then further investigated in order to obtain a set of measurable attributes for the evaluation of the alternatives.

The subsequent step consists in assessing the performance of the alternatives from the point of view of the evaluation criteria and in assigning a preference function with related thresholds of the criteria (q, p) (Table 2).

3.4. Weights Determination. For the development of the PROMETHEE II method, different decision scenarios have been taken into account. The different scenarios reflect the point of view of different actors who can face the problem under investigation. For this purpose, in the application of the methodology personal interviews with experts in different fields and local decision-makers were developed. In particular, 5 experts have been considered for the evaluation, whose expertise was in urban design, economic evaluation, history of architecture, landscape architecture, and sociology. According to the revised Simos procedure [38], the interviews were carried out through the set of cards methodology that allows for setting the criteria weights and determining their priority, according to actors' preferences. The weight values obtained by different experts are shown on the axes of radar charts displayed in Figure 3. As it is possible to see, all the actors agreed in considering the regeneration aspects as the most important ones. On the contrary, the criteria related to parking spaces and new commercial developments are not important according to all the actors involved in the evaluation.

3.5. Results. The ranking of alternative options was derived by implementing the decision support software Visual PROMETHEE 1.4 [39].

Figure 4 shows the final ranking of the alternative strategies with reference to the sets of weights resulting from the interviews to different actors involved. By direct inspection of Figure 4, it emerges that the ranking is preserved in all the cases and for all the strategies. The "Sharing city" alternative is confirmed as the best performing strategy for the successful implementation of the urban transformation/regeneration process. According to our results, the "Green infrastructures" alternative is worth of consideration too, as it is placed as second in the actors' ranking.

To complement the discussion of our results, we consider worth of mentioning the novelty of our approach to the evaluation of complex urban transformation processes and their long-term effects.

Decision problems in urban planning, and specifically those which are concerned with the design and implementation of urban transformation/regeneration process, are often ill-structured problems, as they involve multiple actors

Table 1: Evaluation criteria for the PROMETHEE model [Table 1 is reproduced from Bottero et al. [11]].

Criteria	Description
C_1 Public/private spaces	Ratio between public and private surfaces
C_2 Co-working spaces	Surface of the structures for workshop, meeting, training courses
C_3 Co-housing inhabitants	Number of residents in new co-housing buildings
C_4 Permeable surface/territorial surface	Ratio between permeable areas and overall territorial surface of the program
C_5 Urban gardens	Total area used for community and private urban gardens
C_6 Waste production	Amount of waste produced in a year by the activities of the program
C_7 Residential areas	Surface for residential functions
C_8 Retail areas	Surfaces for commercial functions
C_9 Sport and leisure areas	Surfaces for sport and cultural activities
C_{10} Mixité index	Index that describes the functional mix of the area
C_{11} Slow mobility	Surface of the pedestrian tracks and bicycle lanes
C_{12} New public parking	Number of new public parking lots
C_{13} Car sharing/bike sharing	Number of car and bike sharing points
C_{14} Total Economic Value	Estimate of the social benefits delivered by the program
C_{15} Investment cost	Total cost of the program
C_{16} New jobs	Number of new jobs created
C_{17} Regeneration	Regenerated surface
C_{18} Via De Amicis regeneration	Qualitative index showing the level of the regeneration of Via De Amicis
C_{19} Territorial index	Ratio between the maximum buildable volume and the territorial surface

Figure 2: Alternative strategies considered in the evaluation model [Figure 2 is reproduced from Bottero et al. [11]].

Table 2: Input matrix for the PROMETHEE evaluation.

(a)

	SOCIAL		ENVIRONMENT				SERVICES			
	Ratio between public and private services	*Surface of the structures for workshop, meeting, training*	*No. of residents in new co-housing buildings*	*Ratio between permeable areas and overall territorial surface of the program*	*Total area used for community and private urban gardens*	*Amount of waste produced in a year by the activities of the program*	*Surface for residential function (SLP in m^2)*	*Surface for commercial functions (SLP in m^2)*	*Surface for sport and cultural activities (SLP in m_2)*	*Mixité index*
CULTURAL DISTRICT	4,31	20425	398	0,69	8.527	1.350.845	70.880	28.031	48.150	0,71
SMART CITY	3,25	24260	150	0,39	2.130	2.332.234	117.736	59.169	81.796	0,46
START UP	1,33	49880	255	0,58	25.569	2.692.663	82.330	95.000	26.960	1
CITY AND CRAFTS	8,35	11328	421	0,52	66.894	1.817.205	164.925	84.248	21.458	0,30
SHARING CITY	2,76	5108	2513	0,53	23.118	3.014.301	538.018	40.192	114.725	0,30
GREEN INFRASTRUCTURE	4,20	3300	1036	0,71	12.888	1.631.941	75.252	25.515	37.920	0,64

(b)

	MOBILITY		ECONOMICS				REGENERATION		
	Surface of the pedestrian tracks and bicycle lanes (m^2)	*No. of new parking lots*	*No. of car and bike sharing points*	*Estimate the social benefits delivered by the program (VET in €)*	*Total cost of the program (€)*	*No. of new jobs created*	*Regenerated surface (Regenerated SLP/Total SLP)*	*Qualitative Index for the regeneration of Via De Amicis*	*Ratio between the maximum buildable volume and the territorial surface (m^2)*
CULTURAL DISTRICT	68.326	1.385	7	2.550.746	233.336.184	1.010	0,2	3	0,38
SMART CITY	171.609	2.567	12	537.692	279.468.,021	1.545	0,12	5	0,16
START UP	16.000	2.100	2	3.500.000	100.000.000	300	0,51	4	0,23
CITY AND CRAFTS	132.541	1.137	3	7.471.328	183.948.594	736	0,36	4	0,52
SHARING CITY	624.933	1.689	14	7.707.778	494.055.026	3.229	0,06	5	0,40
GREEN INFRASTRUCTURE	251.831	1.394	19	531.155	231.527.860	768	0,20	5	0,13

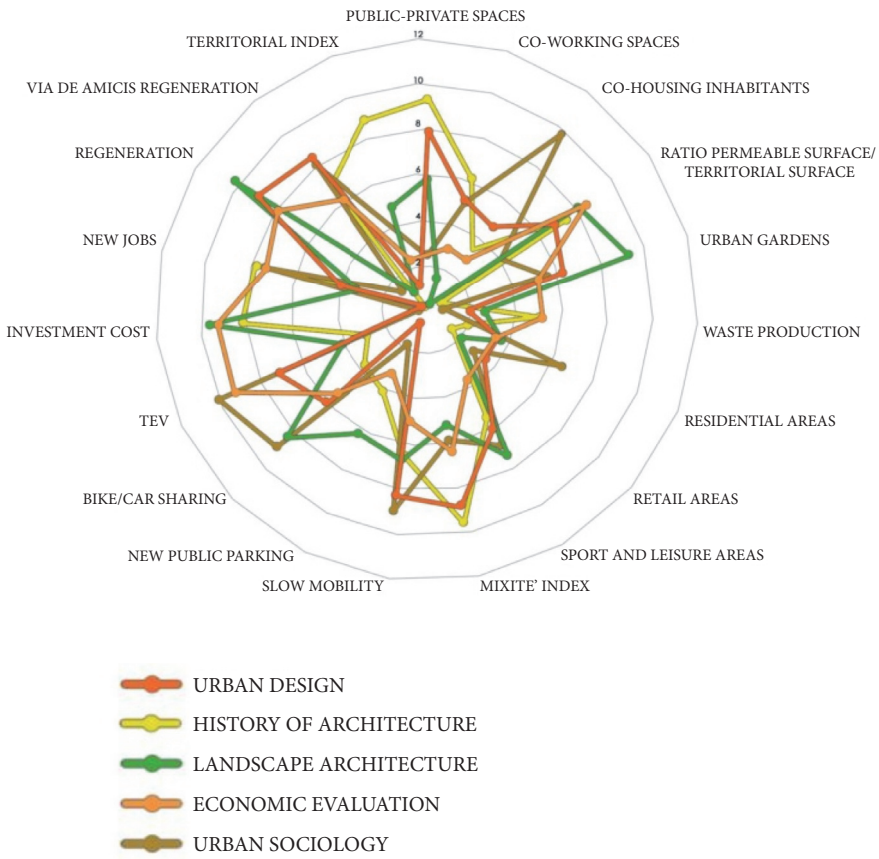

Figure 3: Sets of weights resulting from the different actors.

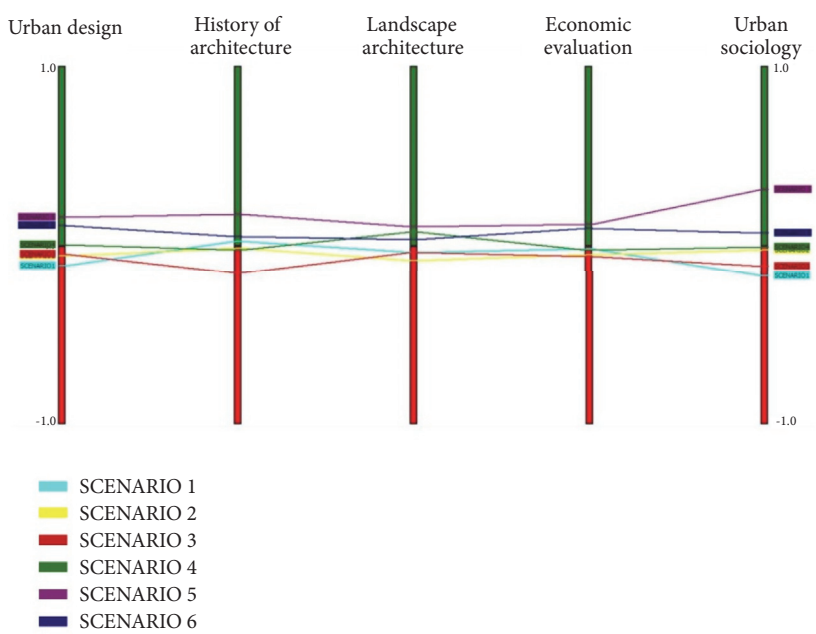

Figure 4: Ranking comparison for the different actors.

and stakeholders, often conflicting objectives and views and are characterized by significant uncertainty over potential outcomes of alternative design options and planning actions. In this context the valuation of alternative scenarios is a complex process, where various aspects need to be accounted for simultaneously. These aspects comprise both technical and non-technical issues and characteristics. The formers build on empirical observations, whereas the latters are usually based on social visions, preferences and feelings [13].

In this paper we adopted a mixed-method research approach to address the issue of urban planning and projects evaluation. In detail, in accordance with Creswell et al. [40], we developed a multiphase mixed-method that allows for considering the subsequent phases of projects formulation and implementation, and thus considering as inputs for the next analysis the results/outputs of the previous one. We combined different methods for the design and selection of alternative urban regeneration projects and strategies, and structured a multiphase decision aiding process meant to support strategic planning. To structure the decision problem we implemented a SWOT Analysis and a Stakeholder Analysis. Problem structuring is in fact a fundamental phase in any decision problem, which involves multiple actors and perspectives, and conflicting stakes to be conciliated, but it becomes of greater importance when alternatives are not a priori designed in detail as in this case [41–45]. We firstly carried out a SWOT Analysis, which provided an in-depth knowledge of the problem and context under investigation, and of the correlation between endogenous and exogenous factors. In this phase, data and information were collected, the objectives were identified and potential alternative scenarios were defined at a preliminary stage. We then performed a Stakeholder Analysis, informed by the SWOT Analysis, through which we identified the actors involved in the problem, and their values and objectives. Stakeholder Analysis allowed to identify conflicting interests at an early stage of the process and develop a strategic view of the human and institutional framework, the relationships among different actors and their concerns. In fact it plays a key role in strategic planning and urban regeneration processes. The above-mentioned analyses informed the last phase of the mixedmethod approach (e.g., criteria express actors' objectives and needs), in which PROMETHEE method was implemented to assess the alternative scenarios under investigation, obtain a list of priorities, and identify the best performing urban regeneration strategy. Table 3 provides an insight in our multiphase decision aiding process, synthetizes strengths and limitations of SWOT Analysis, Stakeholder Analysis and PROMETHEE method respectively, and illustrates main results obtained from their implementation in the city of Collegno case study.

4. Discussion and Conclusions

Multicriteria Analysis is nowadays widely implemented in decision and valuation processes, and specifically in urban planning. Urban planning and urban regeneration processes are multidimensional concepts and involve socioeconomic, environmental, technical, and ethical perspectives, which are strongly interconnected and cannot be addressed by referring exclusively to economic issues: urban renewal projects are often faced by many challenges, such as destruction of existing social networks, expulsion of vulnerable groups, and adverse impacts on the living environment.

Therefore, in urban planning, due to intrinsic complexities and to the high number of stakeholders and actors involved in the decision process, multicriteria techniques and methodologies can be efficiently implemented to identify efficient solutions, which accounts for decision-makers and actors preferences, as well as for public choice policy objectives [46]. To some extent, urban planning is meant to respond to challenges, improve communication between government or public administrations and stakeholders, allocate budgets according to a list of priorities, and favor long and mid-term investments. In addition, to be effective and successful, urban planning requires a commitment by the government to achieve strategic goals, a common understanding on prioritization of actions, and the involvement of the society and the private sector that collaborate to develop and implement strategic plans.

This paper shows how the PROMETHEE II method can be usefully implemented in decision problems related to urban planning and development projects; namely, in this paper the PROMETHEE method is used to determine the projects' priority. In detail, we evaluated different regeneration scenarios for the city of Collegno according to a set of qualitative and quantitative criteria, which account for social, environmental, mobility and economic key factors. As the dominance relation is poor on preference and indifference, incomparability holds for most of pairwise comparisons and additional information is needed to make a decision. By outranking relations, the PROMETHEE method provides realistic enrichments of the dominance relation despite incomparability relations are not completely eliminated. In this respect, the integration of SWOT Analysis and Stakeholder Analysis increased the information useful for ranking the scenarios, thus confirming the importance of supporting cross-sector approaches in sustainable regeneration projects.

The scenarios under investigation were evaluated according to experts judgments, local stakeholders and decision-makers' preferences, values and objectives.

According to the results of PROMETHEE II, scenario 5 the "Sharing city project" is the most desirable and comprising alternative to implement, whereas scenario 6 the "Green Infrastructure" is ranked as second, except for the judgments expressed by the expert in landscape architecture. Our results show that the other alternatives cannot be listed in the same descending order of their net flows for each expert. As multiactor analysis shows, the "Sharing City" alternative encompasses the preferences of the entire group of five experts involved in the decision process. The results obtained from the Visual PROMETHEE software highlight the usefulness of multicriteria outranking methods in spatial decision-making problems. Multiactors analysis was indeed useful in clarifying the most appropriate project, by taking into account the point of views of different actors.

The comprehensive and integrated approach proposed in this paper accounts for key factors in urban renewal, provides

Table 3: Strengths and limitations of the proposed evaluation methods and relative results from the city of Collegno case study.

Evaluation method	Strengths and Limitations	Results from the city of Collegno case study
SWOT Analysis	+ Improvement of overall understanding of the decision problem general framework + Provision of a systematic approach to analyse and decompose complex problems + Identification of correlation between internal factors, strengths and weaknesses and external factors, opportunities and threats + Ease of use and understanding of results – Open nature and unstructured method – Preliminary level of analysis – Tendency to overemphasize opportunities – Lack of prioritisation of factors (no requirement for their classification and evaluation) – Risk of oversimplification – Risk of over-subjectivity in the generation of factors	The SWOT Analysis allowed the definition of the guidelines for the design and implementation of the general masterplan as well as the identification of the transformation process layout. It played a key role in supporting experts and planners in the identification of alternative scenarios for urban transformation/regeneration.
Stakeholders Analysis	+ Improvement in stakeholders management and mobilization of their support in achieving a goal + Identification of purpose and time-dimension of interest + Identification of time-frame and resource availability + Provision of comprehensive analysis meant to produce new knowledge about policy-making processes + Prediction or encouragement of stakeholder alliances – Need for great reliance on quantitative approaches to data collection – Need for iterative processes in data collection and analysis – Inappropriateness of feedback of results when stakeholders may influence or control analysis results – Uncertainty over validity and reliability of results – Potential biases generated by analysts who become implicitly stakeholders who bring to the analysis their own values, perspectives and problem understanding	The Stakeholders Analysis provided the identification of relevant actors in the transformation and relative values and perspectives. These actors are mostly private investors and developers, who have financial resources available for undertaking investments and carry out the urban regeneration process. The Municipality of Collegno and the social groups involved in the process proved to be relevant actors as well.

Table 3: Continued.

Evaluation method	Strengths and Limitations	Results from the city of Collegno case study
PROMETHEE Approach	+ Ease of use + Provision of a complete ranking + Accuracy of results + Adoption of the concordance non-discordance principle in the definition of the overall preference index + Limited total compensation between pros and cons + No assumption on the requirement of criteria to be proportionate +Avoidance of the commensurability problem – Non triviality in preference structuring in detail – Assignment of weights that does not build on a clear method – No information on the cost-effectiveness or profitability of alternatives (are they welfare-maximizing?) – Assignment of values that does not build on a clear method – Difficulties in selecting the generalized criterion functions and the associated thresholds for each criterion – Computational limitations with respect to the number of decision alternatives	The analysis performed by implementing the PROMETHEE method allowed the comparisons of urban transformation alternative options and the identification of the best performing solution for the regeneration process. The results show that the considered sets of weights converge in ranking the "Sharing city" alternative as the most preferred option, and the "Green infrastructure" alternative as the second best option.

a useful tool to assess renewal projects from the standpoint of urban competitiveness and sustainability, and may have interesting policy implications by providing policy makers with useful guidelines for investments to be undertaken. Successful implementation of urban renewal is de facto a crucial driver in promoting sustainable urban development and improving urban competitiveness and attractiveness. In this respect the PROMETHEE method can be useful in assisting decision-makers in selecting urban renewal programs and projects in a more objective and realistic way.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Research Article

Measuring Conflicts Using Cardinal Ranking: An Application to Decision Analytic Conflict Evaluations

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One of the core complexities involved in evaluating decision alternatives in the area of public decision-making is to deal with conflicts. The stakeholders affected by and involved in the decision often have conflicting preferences regarding the actions under consideration. For an executive authority, these differences of opinion can be problematic, during both implementation and communication, even though the decision is rational with respect to an attribute set perceived to represent social welfare. It is therefore important to involve the stakeholders in the process and to get an understanding of their preferences. Otherwise, the stakeholder disagreement can lead to costly conflicts. One way of approaching this problem is to provide means for comprehensive, yet effective stakeholder preference elicitation methods, where the stakeholders can state their preferences with respect to actions part of the current agenda of a government. In this paper we contribute two supporting methods: (i) an application of the cardinal ranking (CAR) method for preference elicitation for conflict evaluations and (ii) two conflict indices for measuring stakeholder conflicts. The application of the CAR method utilizes a *do nothing* alternative to differentiate between positive and negative actions. The elicited preferences can then be used as input to the two conflict indices indicating the level of conflict within a stakeholder group or between two stakeholder groups. The contributed methods are demonstrated in a real-life example carried out in the municipality of Upplands Väsby, Sweden. We show how a questionnaire can be used to elicit preferences with CAR and how the indices can be used to semantically describe the level of consensus and conflict regarding a certain attribute. As such, we show how the methods can provide decision aid in the clarification of controversies.

1. Introduction

One of the core complexities involved in evaluating decision alternatives in the area of public decision-making is to deal with conflicts. The stakeholders affected by and involved in the decision often have conflicting preferences regarding the actions under consideration. For an executive authority, these differences of opinion can be problematic during both implementation and communication, even though the decision is rational with respect to an attribute set perceived to represent social welfare; see, e.g., [1–3].

Of particular interest for the decision-making forum are preferential conflicts between stakeholders, since such conflicts may cause delays in the decision process due to obstructions, hassles, and/or locked negotiations [4–6]. For instance, Hansson et al. [6] describe that the development plans of Husby, a suburb to Stockholm, had been on hold for several years due to conflicts. Another example is given in Danielson et al. [5], where three infrastructure decisions in Nacka, a municipality in the Stockholm region, were delayed for several years due to conflicts between stakeholders. Therefore, to avoid stakeholder conflicts it is important for the executive authority to involve the stakeholders in the process and to get an understanding of their preferences [7, 8].

This calls for a desire to become better informed with regard to potential controversies, and it has been discussed in contemporary decision analysis literature that interaction with stakeholders using web-based techniques is one feasible approach to obtain stakeholder preferences and

then inform decision-makers by utilizing decision analysis techniques. For instance, French et al. [1] suggest that web-based approaches can be used to support and structure the democratic process. Hansson et al. [6] suggest that a solution to the above-mentioned problem of conflicts could be to actively interact with stakeholders early in the process by the use of social media.

In this approach, an important first step is to elicit the opinions or attitudes of the citizens regarding a set of "actions". These are potential actions that in the future may be redefined into more well-defined projects. It has been recognized that the selected methods must be easy to use for less experienced stakeholders and at the same time be sufficiently powerful to enable them to provide meaningful feedback [9]. To facilitate scalable elicitation, many approaches (including ours) promote the use of web-based surveys or questionnaires distributed to stakeholders early in the planning phase [3].

In this paper we contribute with an application of the cardinal ranking (CAR) method [10], extended with a feature for conflict evaluations, and a method for measuring conflict within a stakeholder group and between two stakeholder groups. The contributions support the decision process in a public decision-making setting.

Cardinal Ranking for Conflict Evaluations. To the best of our knowledge, no method exists that utilizes preferences elicited by a cardinal ranking approach in conflict evaluations. In this application of CAR [10], the respondents, in addition to cardinally ranking the elements, state the performance of the elements relative to a *do nothing* alternative. For example, supporting statements are like *"action 1 is better than action 2, and both actions are negative relative the do nothing alternative* and *"action 2 is much better than action 3, action 2 is positive whereas Action 3 is negative relative the do nothing alternative"*.

Conflict Indices. In a group decision analysis setting, it is of interest to investigate whether a certain alternative is conflict-prone with respect to one or more attributes. The preferences elicited by the application of CAR can be used for assessing the level of conflict within or between stakeholder groups. We describe two such indices, the Within-Group Conflict Index and the Between-Group Conflict Index, both utilizing a sum of squares, used in Ward's clustering method [11, p. 466] and extended with a positive stakeholder scaling constant. The underlying group formation is similar to the collective attractiveness and unattractiveness indices presented by [12]. The approaches differ in how the value functions are created and utilized. Bana e Costa [12] creates one value function per criterion by applying MACBETH and its semantic categories. In our approach the respondents use CAR to create their individual value functions per criterion. In turn, this enables individual value estimates of the *do nothing* alternative which is not necessarily viewed as a "neutral" alternative.

With this we contribute to the first important step in a participatory group decision process by elucidating both conflict-prone actions and nonconflicting actions. An action is conflict-prone when there are strong opposing preferences regarding the performance of the action, either within a stakeholder group or between two stakeholder groups. An action is a nonconflicting action when the stakeholders have similar preferences.

1.1. Case Setting. In this paper, we describe a case conducted in Upplands Väsby municipality slightly north of Stockholm City. In this case, we analyzed the citizens preferences regarding a set of actions that could be implemented in the future. A previous reporting of the case can be found in Chapter 7 of Ekenberg et al. [13], in which a simplified conflict measure was utilized. In the reported method, the stakeholders were divided into two groups, the con- and the pro-group. The conflict index was then measured as the difference between the arithmetic means of the groups part-worth values. The methods presented in this paper are rather based on Ward's method [11].

Typically, the actions are described quite briefly, for example, in short sentences such as "Build residential area near the lake" or "Build apartments in the town centre". In the paper, we use the term "action" to distinguish from the more traditional term "alternatives" employed within the field of decision analysis. In our setting, an action is a tentative project proposal without an associated cost but rather a line of direction for a future project. This is somewhat similar to the tentative proposals defined as "topics" by [14].

In order to understand attitudes, the questionnaire must allow for the participating stakeholders to state positive or negative preferences regarding an action's performance relative to a *do nothing* action. In order to measure conflict, the statements need to be represented in a manner allowing for such a measure to be meaningful. Thus, the attitudes must be represented using a measurable value function where the questionnaire is used as a tool for eliciting preferences. Conventional preference elicitation methods for approximating such value functions are typically considered to be cognitively demanding [15]. The nature of the actions being so loosely defined renders the use of more elaborate preference elicitation techniques, since they rely on well-defined alternatives. To resolve this situation, the questionnaire we propose is inspired by the attitude surveys often employing different versions of the Likert scale.

An alternative approach to reducing the cognitive burden of the decision-maker is to use preference disaggregation techniques. The techniques do this by utilizing global preferences, e.g., a ranking of a subset of alternatives, or by a pairwise comparison of alternatives, to infer value functions; see, e.g., [16–19] for details.

The questionnaire, which was previously reported in [13, Ch. 7], enables the capturing of negative and positive attitudes of the actions relative to a *do nothing* action. Then, methods for cardinal ranking are used to interpret the responses in terms of surrogate values and attribute weights. Lastly, the questionnaire contains a section where the respondent enters demographic information. This information can then be used to analyze the preferences of the citizens, e.g., to find stakeholder groups with conflicting interests.

2. Modeling of Preferences

In the decision analysis field, we distinguish between two types of preference representation functions, the value function and the utility function. A value function represents preferences over certain outcomes, as opposed to the utility function which represents preferences over lotteries with uncertain outcomes, and the two representations are based upon different axiom systems. The focus in this paper is on preference representations under certainty, and in the absence of uncertain consequences (or risk), preferences over objects x are typically represented by means of a value function $v(x) \longmapsto [0,1]$, such that the object x is preferred to object y (commonly denoted by $x \succ y$) by the decision-maker if and only if $v(x) > v(y)$. If $v(x) = v(y)$ then the decision-maker is indifferent between the two objects. The value function is measurable and is defined over an interval scale if $v(w) - v(x) > v(y) - v(z)$ and $v(w) > v(x), v(y) > v(z)$ entails that exchanging x for w is preferred compared to exchanging z for y; see [20] for a comprehensive treatment.

In the case of multiple decision-makers or stakeholders $S_1, S_2, \ldots, S_n$ having different individual measurable value functions $v^1(), v^2(), \ldots, v^n()$, in group decision analysis, it is of concern to aggregate the set of individual value functions using some proper preference aggregation rule providing a group value function sharing the properties of the individual value functions. For this purpose, given that all individual value functions of stakeholder k are of the form v^k and that domain and range are shared among the stakeholders, it has been shown that an additive aggregation of individual measurable value functions provides a measurable group value function $W(A_j)$ for an alternative A_j, such that $W(A_j) = \sum \lambda_k v^k(A_j)$, where $v^k(A_j)$ is the value of alternative A_j for stakeholder S_k, and λ_k are stakeholder scaling constants subject to $\lambda_i \geq 0$ and $\sum \lambda_i = 1$ whenever exchange independence is satisfied (if all stakeholders are indifferent between two exchanges then the whole group must also be indifferent) [21].

In the multiattribute setting, considering a set of evaluation attributes $\boldsymbol{G} = \{G_1, G_2, \ldots, G_m\}$, the value of an alternative A_j is obtained by aggregating the corresponding set of value functions $v_1(x_1), v_2(x_2), \ldots, v_m(x_m)$. The most common way of aggregation is the additive approach such that the value $V(A_j)$ of A_j is given by $V(A_j) = \sum_i w_i v_{ij}$, where v_{ij} is the value of alternative A_j under attribute G_i, and w_i is the weight of attribute G_i under the condition that $0 \leq w_i \leq 1$ and $\sum_i w_i = 1$. This additive aggregation provides a measurable multiattribute value function given that the conditions of mutually preferential independence and difference independence hold; see, e.g., [22]. The weights w_i then act as attribute scaling constants, scaling the value contribution to an alternative from an attribute. The scaled value $q_{ij} = w_i \cdot v_{ij}$ is called the "part-worth" value of attribute G_i to alternative A_j. In a group setting, the group's value of alternative A_j is then given by

$$W\left(A_j\right) = \sum_k \lambda_k \sum_i q_{ij}^k \tag{1}$$

where q_{ij}^k is the part-worth value of attribute G_i to alternative A_j for stakeholder S_k; i.e., $q_{ij}^k = w_i^k \cdot v_{ij}^k$. Henceforth in this paper, q_{ij}^k notation for a stakeholders part-worth value will be used, and we will denote the stakeholder S_k's value of alternative A_j under attribute G_i with v_{ij}^k. In order to obtain these values, different approaches have been suggested to elicit them from decision-makers. For an overview of elicitation methods, see, e.g., [22, 23].

3. Rank-Based Elicitation

The procedure in rank-ordering methods is to elicit the preferences as ranks, which are then converted into cardinal surrogate weights. Two such methods, rank-sum (RS) and rank-reciprocal (RR), are described in [24], and a third method the Rank Order Centroid (ROC) is described in [25, 26].

Rank-ordering methods have desirable advantages over more precise elicitation techniques, such as (i) being easier to elicit the vague preferences (less cognitively demanding) and (ii) having increased likelihood for a group to come to agreement [27, 28]. But rank ordered weight elicitation may be problematic. For example, Jia et al. [29] point out that, in a real-life setting, uncertainty may exist regarding both the magnitudes and ordering of weights, and even though information regarding the difference in importance may exist, the information is not considered in the transformation from rank orders into weights.

A method taking the difference in importance into consideration is the cardinal ranking (CAR) method [10, 30]. In CAR, a prerequisite is an ordinal ranking, to which cardinal information is added; see Figure 1 for a visualization of the difference between ordinal and cardinal ranking. The cardinal information is used to denote the strength of preference between pairs of elements in the ranking. This strength of preference is interpreted as the number of steps between each pair of elements on an underlying importance scale. The notation $\succ_i$ is used for describing this, where i is the number of steps. The cardinality can be described by semantic expressions, e.g., obtained from a linguistic analysis,

$\sim_0$ equally important, 0 steps

$\succ_1$ slightly more important, 1 step

$\succ_2$ more important, 2 steps

$\succ_3$ much more important, 3 steps

This enables the decision-maker to make statements such as the following:

(i) Attribute A is equally important ($\sim_0$) as attribute B

(ii) Attribute B is slightly more important ($\succ_1$) than attribute C

(iii) Attribute C is more important ($\succ_2$) than attribute D

(iv) Attribute D is much more important ($\succ_3$) than attribute E

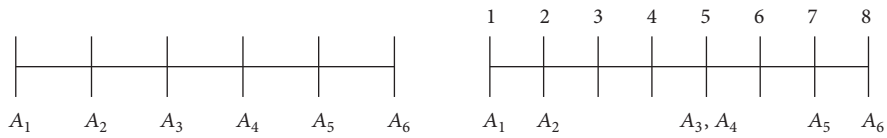

Figure 1: The left picture visualizes an ordinal ranking of six elements, while the right picture visualizes a cardinal ranking of the elements.

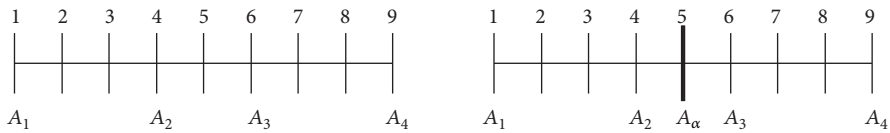

Figure 2: The left picture visualizes a cardinal ranking of the four alternatives. The right picture visualizes a cardinal ranking of the alternatives, with the *do nothing* alternative inserted between alternatives A_2 and A_3.

Note that similar linguistic translations are commonly used in other MCDA methods such as AHP [31] and MACBETH [32, 33].

The CAR method has been demonstrated using both linear inequalities to represent cardinal ranking statements [34] and closed formulas for obtaining surrogate weights [10, 30]. More recently, rank-based methods have been suggested for probability elicitation as well, with a particular aim for use in time scarce environments [35].

4. Cardinal Ranking for Conflict Evaluations

In this section we introduce an application of CAR which captures negative or positive preferences with regard to an alternative's performance relative to a *do nothing* alternative over a set of attributes. The method enables respondents to express the strength of preference between ordered pairs of alternatives using steps of preference intensities and at the same time express whether the alternatives are negative or positive relative to a *do nothing* alternative.

Assume that we have a set of alternatives $\boldsymbol{A} = \{A_1, A_2, \ldots, A_n\}$ which are evaluated against a set of attributes $\boldsymbol{G} = \{G_1, G_2, \ldots, G_m\}$. To capture the negative or positive preferences we need to introduce a *do nothing* alternative A_α to the set of alternatives. The *do nothing* alternative represents the current state; i.e., it should be considered whether the actions under consideration are better/worse than this alternative. See, e.g., Lahdelma et al. [36] for arguments supporting this technique.

We conform to the CAR method's procedure for eliciting the alternatives' values [10, 30] but extend it with a third step, the step where the *do nothing* alternative is inserted in the ranking:

(1) An ordinal number is assigned to each position on the underlying measurement scale.

(2) The underlying scale consists of Q positions in decreasing order of importance. Alternative A_i has a position $p(i)$ on the scale, such that $1 \leq p(i) \leq Q$, where $Q \geq m$. The strength, or cardinality, of preference s_i between two adjacent alternatives $A_i \succ_{s_i} A_j$, is then $s_i = |p(i) - p(j)|$.

(3) A do nothing alternative A_α is inserted into the ranking by providing it with a position p.

(4) The cardinal ranking is normalized to a proportional $[0, 1]$-value scale according to the following equation:

$$v_i^{\mathrm{CAR}} = \frac{Q - p(i)}{Q - 1} \tag{2}$$

4.1. Example of CAR for Conflict Evaluations. Assume that a decision-maker evaluates the performance of four alternatives, $\{A_1, \ldots, A_4\}$, with regard to attribute C_1.

(1) The decision-maker orders the alternatives as $A_1 \succ A_2 \succ A_3 \succ A_4$.

(2) He/she adds cardinal information to the ordinal ranking by introducing cardinality (strength of preference) steps between pairs of alternatives:

(i) A_1 is much better ($\succ_3$) than A_2,
(ii) A_2 is better ($\succ_2$) than A_3
(iii) A_3 is much better ($\succ_3$) than A_4,

which gives the following cardinal rank $A_1 \succ_3 A_2 \succ_2 A_3 \succ_3 A_4$; see Figure 2.

(3) He/she states that alternatives A_1 and A_2 are considered to be positive and A_3 and A_4 negative relative to the do nothing alternative. The do nothing alternative A_α is therefore inserted at position 0, between A_2 and A_3, giving the following cardinal ranking, $A_1 \succ_3 A_2 \succ_1 A_\alpha \succ_1 A_3 \succ_3 A_4$; see Figure 2.

(4) The cardinal ranking results in the following positions on the underlying scale:

(i) A_1 at position $p(4)$,
(ii) A_2 at position $p(1)$,
(iii) A_α at position $p(0)$,
(iv) A_3 at position $p(-1)$,
(v) A_4 at position $p(-4)$.

These positions are then mapped onto a proportional $[0, 1]$-value scale, giving the alternatives the following

values, where v_{ij} denotes alternative A_j:s value under attribute G_i. As [37] points out, the do nothing alternative is not always assigned a score of 0. Note that in our approach the do nothing alternative can be assigned a value between 0 and 1 depending on its position in the ordinal ranking. In this example, $Q = 9$ and $p(4) = 1$, $p(1) = 4$, $p(0) = 5$, $p(-1) = 6$ and $p(-4) = 9$.

(i) $v_4 = 1.000$,
(ii) $v_3 = 0.625$,
(iii) $v_\alpha = 0.500$,
(iv) $v_2 = 0.375$,
(v) $v_1 = 0.000$.

4.2. Conflicts. In a group decision analysis setting, it may be of interest to investigate whether a certain alternative is conflict-prone with respect to one or more attributes. The preferences elicited in the application of CAR for conflict evaluations can be used for assessing the level of conflict within or between stakeholder groups. In the following section, we will propose two such indices, the within-group conflict index and the between-group conflict index. Based upon these indices we can define consensus properties that textually describe the level of conflict associated with an attribute with respect to a specific alternative. A similar approach was presented by Bana e Costa [12].

Before presenting the indices, we stipulate the following conditions for the concept of conflict.

Definition 1 (conflict). Given a set of stakeholders $\boldsymbol{S}$ and a set of alternatives $\boldsymbol{A}$, conflict exists in $\boldsymbol{S}$ if there are two or more stakeholders in $\boldsymbol{S}$ with positive scaling constants and these have differing preferences towards at least one alternative $A_i \in \boldsymbol{A}$. Formally, there must exist $S_k, S_l \in \boldsymbol{S}$ with $\lambda_k, \lambda_l > 0$ such that $v_j^k < v_\alpha^k$ and $v_j^l \geq v_\alpha^l$.

Definition 1 explicitly says that if all stakeholders share preferences towards an alternative, there is no conflict. In this sense it can be argued that this definition is a very strong definition of conflict. In the case of multiple attributes, Definition 1 can be extended to attribute conflict. Needless to say, given an attribute G_i with a positive weight $w_i > 0$ we have that $v_{ij}^k < v_{i\alpha}^k$ implies $q_{ij}^k < q_{i\alpha}^k$ and $v_{ij}^k \geq v_{i\alpha}^k$ implies $q_{ij}^k \geq q_{i\alpha}^k$ since $q_{ij}^k = w_i v_{ij}^k$. We can now define the meaning behind attribute conflict and measurable conflict.

Definition 2 (attribute conflict). Given a set of stakeholders $\boldsymbol{S}$, a set of alternatives $\boldsymbol{A}$, and a set of evaluation attributes $\boldsymbol{G}$, conflict exists for $G_i \in \boldsymbol{G}$ if there exist $S_k, S_l \in \boldsymbol{S}$ with $\lambda_k, \lambda_l > 0$ such that $q_{ij}^k < q_{i\alpha}^k$ and $q_{ij}^l \geq q_{i\alpha}^l$.

4.2.1. Measurable Conflict. The definitions above do not consider different opinions on how "good" (or "bad") two alternatives are given that they are both considered as productive (or counter-productive) to count for conflict. In other words, the definitions do not account for value differences, which lead us to measurable conflict.

In a two-stakeholder setting, given that attribute conflict exists for attribute G_i such that $q_{ij}^k < q_{i\alpha}^k$ and $q_{ij}^l \geq q_{i\alpha}^l$, we will argue that it is reasonable to base a measure of conflict upon the value differences:

$$d_{ij}^k = \left| q_{i\alpha}^k - q_{ij}^k \right| \tag{3}$$

The intuition behind (3) is that stakeholder S_k considers A_j to be $|q_{i\alpha}^k - q_{ij}^k|$ worse than the *do nothing* alternative while another stakeholder S_l considers A_j to be $|q_{i\alpha}^l - q_{ij}^l|$ better than the *do nothing* alternative. Given two stakeholders S_k, S_l and two alternatives A_1, A_2 such that

$$q_{i1}^k \leq q_{i2}^k \leq q_{i\alpha}^k \tag{4}$$

$$q_{i1}^l \geq q_{i2}^l \geq q_{i\alpha}^l, \tag{5}$$

then conflict cannot be smaller for A_2 compared to A_1. Further, conflict is larger for A_1 compared to A_2 if S_k would rather exchange A_1 for A_α than exchange A_2 for A_α, or if S_l would rather exchange A_α for A_1 than exchange A_α for A_2.

However, a generalized measure of conflict should consider more than two stakeholders. In addition to measuring how far from the *do nothing* alternative two stakeholders in conflict are, we need to consider disparities between sets of stakeholders. For this reason we adopt a cluster distance approach to conflict measurement by partitioning the stakeholders into two sets based upon their preferences towards the alternative.

Let $\boldsymbol{S} = \{S_1, S_2, \ldots, S_n\}$ be a set of stakeholders. For each attribute G_i and alternative A_j, we partition the stakeholder set $\boldsymbol{S}$ into two partitions, the con-group $\boldsymbol{S}_{ij}^-$ and the pro-group $\boldsymbol{S}_{ij}^+$. The members of the $\boldsymbol{S}_{ij}^-$ evaluated v_{ij}^k to be less than the value of the *do nothing* alternative $v_{i\alpha}^k$, and the stakeholders in $\boldsymbol{S}_{ij}^+$ evaluated v_{ij}^k to be greater than or equal to $v_{i\alpha}^k$; see the following equations:

$$\boldsymbol{S}_{ij}^- = \left\{ S_k \in \boldsymbol{S} : v_{ij}^k < v_{i\alpha}^k \right\}_{k=1}^n \tag{6}$$

$$\boldsymbol{S}_{ij}^+ = \left\{ S_k \in \boldsymbol{S} : v_{ij}^k \geq v_{i\alpha}^k \right\}_{k=1}^n \tag{7}$$

Hence, the *do nothing* alternative A_α is used to separate the stakeholders into either $\boldsymbol{S}_{ij}^-$ or $\boldsymbol{S}_{ij}^+$. The intuition behind the conflict index is to measure the distance between these two groups such that if two stakeholders $S_k \in \boldsymbol{S}_{ij}^-, S_l \in \boldsymbol{S}_{ij}^+$ in general disagree to a large extent, i.e., they both have a large differences $|q_{i\alpha}^k - q_{ij}^k|$ and $|q_{i\alpha}^l - q_{ij}^l|$, then the conflict is greater than if they have smaller differences. Further, it can be argued that if the power balance between $\boldsymbol{S}_{ij}^-$ and $\boldsymbol{S}_{ij}^+$ is more equal, then the conflict is stronger since it has been demonstrated that less powerful stakeholders are more willing to accept an alternative even though they deem it counterproductive [38]. In our case, this is represented as follows: if the difference

$$\sum_{S_k \in \boldsymbol{S}_{ij}^-} \lambda_k - \sum_{S_l \in \boldsymbol{S}_{ij}^+} \lambda_l \tag{8}$$

Table 1: The positions on the underlying measurement scale produced by the cardinal rankings.

Stakeholder	Position					
S_k	$p(A_1)$	$p(A_2)$	$p(A_3)$	$p(A_4)$	$p(A_5)$	$p(A_\alpha)$
S_1	8	6	8	1	1	8
S_2	8	6	8	1	1	8
S_3	8	7	8	2	1	8
S_4	8	7	8	3	1	8
S_5	8	7	8	3	1	8
S_6	1	2	1	5	8	1
S_7	1	2	1	5	8	1
S_8	1	2	1	6	8	1
S_9	1	3	1	7	8	1
S_{10}	1	3	2	7	8	1

is small, then the power balance is more equal entailing larger conflict.

To measure these properties reflecting the distance between the groups, we utilize three sums of squares similarly to how Ward's clustering method measures distances between clusters [11, p. 466]. We obtain the sum of squared differences between each d_{ij}^k and the group's mean distance, i.e., the differences

$$d_{ij}^k - \frac{\sum_{S_k \in \mathbf{S}_{ij}} d_{ij}^k}{\left|\mathbf{S}_{ij}\right|} \tag{9}$$

for the con-group (12), the pro-group (13), and the combined group (14), respectively.

Definition 3 (within-group conflict index). A within-group conflict index $d_{ij}^{\mathbf{S}}$ for stakeholder set $\mathbf{S}$ with two or more stakeholders, under attribute G_i and alternative A_j, is given by

$$d_{ij}^{\mathbf{S}} = \sqrt{\beta\left(T_{ij}^{\mathbf{S}} - \left(C_{ij}^{\mathbf{S}} + P_{ij}^{\mathbf{S}}\right)\right)} \tag{10}$$

where

$$\beta = \frac{1}{\sum_{S_k \in S} \lambda_k{}^2} \tag{11}$$

$$C_{ij}^{\mathbf{S}} = \sum_{S_k \in \mathbf{S}_{ij}^-} \lambda_k{}^2 \left(d_{ij}^k - \frac{\sum_{S_k \in \mathbf{S}_{ij}^-} d_{ij}^k}{\left|\mathbf{S}_{ij}^-\right|} \right)^2 \tag{12}$$

$$P_{ij}^{\mathbf{S}} = \sum_{S_k \in \mathbf{S}_{ij}^+} \lambda_k{}^2 \left(d_{ij}^k - \frac{\sum_{S_k \in \mathbf{S}_{ij}^+} d_{ij}^k}{\left|\mathbf{S}_{ij}^+\right|} \right)^2 \tag{13}$$

$$T_{ij}^{\mathbf{S}} = \sum_{S_k \in \mathbf{S}_{ij}} \lambda_k{}^2 \left(d_{ij}^k - \frac{\sum_{S_k \in \mathbf{S}_{ij}} d_{ij}^k}{\left|\mathbf{S}_{ij}\right|} \right)^2 \tag{14}$$

In (10), β is used to normalize the value onto $[0, 1]$. It can be noted that $0 \leq d_{ij}^{\mathbf{S}} \leq 1$, where $d_{ij}^{\mathbf{S}} = 1$ occurs when all stakeholders in the con-group rank such that $q_{i\alpha} = 1$ and $q_{ij} = 0$, and all stakeholders in the pro-group rank such that $q_{i\alpha} = 0$ and $q_{ij} = 1$ and the power balance is equal. Further, $d_{ij}^{\mathbf{S}} = 0$ occurs if either the con-group or the pro-group is empty.

4.3. Consensus Properties. The semantic meaning of the conflict indices can then be explained by dividing the conflict range $[0, 1]$ into a number of subintervals each describing a certain level of consensus. We define the points a and b as the interval's lower and upper bound and stipulate n interval points $\{x_1, x_2, \ldots, x_{n-1}, x_n\}$. These points are to divide the scale into the n intervals such as $[a, x_1], [x_1, x_2], \ldots, [x_{n-1}, b]$. Each interval is then associated with a consensus property label describing the level of consensus, e.g., *consensus aligned attribute*, *potentially controversial attribute,* and *controversial attribute*, with regard to a specific action.

4.4. Example. Assume that ten stakeholders $\mathbf{S} = \{S_1, \ldots, S_{10}\}$ evaluate the performance of five actions $\{A_1, \ldots, A_5\}$, with regard to attribute G_1 using CAR for conflict evaluations. The actions' positions on the underlying measurement are presented in Table 1 and the normalized values in Table 2. Attribute G_1 is for illustrative purposes given the weight of 1.

Properties of G_1 for each action can now be defined. First, we stipulate classes of controversy based upon the magnitude of conflict. Assume four such classes which range from "consensus aligned", i.e., low conflict, to "very controversial" where the conflict is high; see Table 3. Note that the subranges and the semantical labels are defined by the authors for illustrative purposes. In a real setting the ranges and labels should be defined by the decision-maker.

To analyze conflict within the stakeholder group, for each action we form a con-group (6) and a pro-group (7) and subsequentially calculate the squared distance within the con-group (12) and the pro-group (13), the squared distance between the con- and pro-group (14), and the within-group conflict index $d_{ij}^{\mathbf{S}}$ (10).

The results are shown in Table 4. Action A_1 has $d_{1,1}^{\mathbf{S}} = 0$; i.e., there exists no conflict between the stakeholders since all stakeholders are members of the pro-group and no stakeholders are members of the con-group. As seen in Table 2,

Table 2: The stakeholders' cardinal rankings normalized to a proportional [0, 1]-value scale.

Stakeholder	Value						Weight
S_k	$v_{1,1}^k$	$v_{1,2}^k$	$v_{1,3}^k$	$v_{1,4}^k$	$v_{1,5}^k$	$v_{1,\alpha}^k$	w_1
S_1	0.00	0.29	0.00	1.00	1.00	0.00	1.00
S_2	0.00	0.29	0.00	1.00	1.00	0.00	1.00
S_3	0.00	0.14	0.00	0.86	1.00	0.00	1.00
S_4	0.00	0.14	0.00	0.71	1.00	0.00	1.00
S_5	0.00	0.14	0.00	0.71	1.00	0.00	1.00
S_6	1.00	0.86	1.00	0.43	0.00	1.00	1.00
S_7	1.00	0.86	1.00	0.43	0.00	1.00	1.00
S_8	1.00	0.86	1.00	0.29	0.00	1.00	1.00
S_9	1.00	0.71	1.00	0.14	0.00	1.00	1.00
S_{10}	1.00	0.71	0.86	0.14	0.00	1.00	1.00

Table 3: The table shows four conflict index intervals, each associated with a consensus property label which semantically describes the level of consensus.

Conflict index interval	Consensus property label
[0.00, 0.10)	Consensus aligned
[0.10, 0.20)	Potentially controversial
[0.20, 0.50)	Controversial
[0.50, 1.00]	Very controversial

all stakeholders placed A_1 on the same position as A_α in the cardinal ranking. However, stakeholders $S_1, \ldots, S_5$ stated that all other actions were better than A_1, and stakeholders $S_6, \ldots, S_{10}$ stated that all other actions were worse than action A_1. Thus, when the ranking was normalized to proportional [0, 1]-value scale, the value of action A_1 for stakeholders $S_1, \ldots, S_5$ was $v_{1,1}^k$=0, and for stakeholders $S_6, \ldots, S_{10}$ the value was $v_{1,1}^k$=1. Action A_2 has $\mathrm{d}_{1,2}^{\boldsymbol{S}} = 0.2$, since both the con-group and the pro-group estimated the value of action to lie close to A_α. Action A_3 has an even lower conflict index, $d_{1,3}^{\boldsymbol{S}} = 0.0429$, since all stakeholders except one are members of the pro-group. The pro-group members estimated the value of A_3 to be equal to the value of A_α, and the member of the con-group estimated the value of A_3 to be slightly less than the value of A_α. Action A_4 has $d_{1,4}^{\boldsymbol{S}} = 0.7857$ since, as seen in Table 2, the stakeholders of both groups estimated it to lie closer towards the most preferred or least preferred alternative for both groups, action A_5. Lastly, action A_5 has $d_{1,5}^{\boldsymbol{S}} = 1$, since all members of the con- and pro-group estimate it to be the worst and, respectively, best action. See the classification of the actions in Table 5.

Note that the conflict index measures the disagreement between the con-group and the pro-group. If either the con-group or the pro-group is empty, C_{ji}^S (or P_{ij}^S) cancels T_{ij}^S out.

4.5. Between-Group Conflict Index. Of interest for the application case was first to measure the conflict within one group of stakeholders, i.e., the within-group conflict index. Second, it was of interest to measure the conflict between two stakeholder groups, e.g., people living in different parts of the town or belonging to different generations.

The between-group conflict index measures the conflict between two stakeholder groups. The intuition behind the between-group conflict index is that if the two groups disagree to the same extent on the same matters, there is low conflict between the groups. The between-group conflict should thus reveal to what extent two groups disagree differently from each other. Let D and E be two subsets of $\boldsymbol{S}$. For each alternative A_j and attribute G_i, we partition D and E into two partitions: the con-group $\boldsymbol{S}_{ij}^{D-}$ and the pro-group $\boldsymbol{S}_{ij}^{D+}$ for D, and the con-group $\boldsymbol{S}_{ij}^{E-}$ and the pro-group $\boldsymbol{S}_{ij}^{E+}$ for E. The members of $\boldsymbol{S}_{ij}^{D-}$ and $\boldsymbol{S}_{ij}^{E-}$ estimated the part-worth value q_{ij}^k to be less than the part-worth value of the *do nothing* alternative $q_{i\alpha}^k$, and the members of $\boldsymbol{S}_{ij}^{D+}$ and $\boldsymbol{S}_{ij}^{E+}$ estimated it to be greater than or equal to $q_{i\alpha}^j$; see the following equation:

$$\begin{aligned}
\boldsymbol{S}_{ij}^{D-} &= \left\{S_k \in D : q_{ij}^k < q_{i\alpha}^k\right\}_{j=1}^n \\
\boldsymbol{S}_{ij}^{D+} &= \left\{S_k \in D : q_{ij}^k \geq q_{i\alpha}^k\right\}_{j=1}^n \\
\boldsymbol{S}_{ij}^{E-} &= \left\{S_k \in E : q_{ij}^k < q_{i\alpha}^k\right\}_{j=1}^n \\
\boldsymbol{S}_{ij}^{E+} &= \left\{S_k \in E : q_{ij}^k \geq q_{i\alpha}^k\right\}_{j=1}^n
\end{aligned} \tag{15}$$

The squared distances within the con-group $C_{ij}^D, C_{ij}^E, C_{ij}^{D,E}$ (12), and the pro-group $P_{ij}^D, P_{ij}^E, P_{ij}^{D,E}$ (13), and the squared distance between the con- and pro-group $T_{ij}^D, T_{ij}^E, T_{ij}^{D,E}$ (14) for groups D and E are then obtained according to the given equations. The conflict with respect to alternative A_j under attribute G_i between the two groups D and E can now be represented by the between-group conflict index $d_{ij}^{D,E}$.

Definition 4 (between-group conflict index).

$$d_{ij}^{D,E} = \sqrt{\beta\left|\left(T_{ij}^{D,E} - \left(T_{ij}^D + T_{ij}^E\right)\right) - \left(\left(C^{D,E} - \left(C_{ij}^D + C_{ij}^E\right)\right) + \left(P_{ij}^{D,E} - \left(P_{ij}^D + P_{ij}^E\right)\right)\right|} \quad \text{where } \beta = \frac{1}{\sum_{S_k \in S} \lambda_k^2} \tag{16}$$

Table 4: The distance within the con- and pro-group, the distance between the con- and pro-group, and the conflict index.

	A_1	A_2	A_3	A_4	A_5
$C_{1j}^{\boldsymbol{S}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$P_{1j}^{\boldsymbol{S}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$T_{1j}^{\boldsymbol{S}}$	0.0000	0.0045	0.0002	0.0634	0.1000
$\boldsymbol{d_{1j}^{S}}$	**0.0000**	**0.2000**	**0.0429**	**0.7857**	**1.0000**

Table 5: The classification of the actions consensus properties.

Consensus Property	A_1	A_2	A_3	A_4	A_5
Consensus aligned	×		×		
Potentially controversial		×			
Controversial					
Very controversial				×	×

Table 6: The con- and pro-indices for both groups and the conflict between the groups.

	A_1	A_2	A_3	A_4	A_5
$C_{i1}^{\boldsymbol{D}}$	0.0000	0.0000	0.0000	0.0000	0.0000
$P_{i1}^{\boldsymbol{D}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$T_{i1}^{\boldsymbol{D}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$C_{i1}^{\boldsymbol{E}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$P_{i1}^{\boldsymbol{E}}$	0.0000	0.0000	0.0000	0.0000	0.0000
$T_{i1}^{\boldsymbol{E}}$	0.0000	0.0002	0.0002	0.0008	0.0000
$C_{i1}^{\boldsymbol{D,E}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$P_{i1}^{\boldsymbol{D,E}}$	0.0000	0.0002	0.0000	0.0008	0.0000
$T_{i1}^{\boldsymbol{D,E}}$	0.0000	0.0045	0.0002	0.0634	0.1000
$d_{i1}^{\boldsymbol{D,E}}$	**0.0000**	**0.2000**	**0.0143**	**0.7857**	**1.0000**

The intuition behind the between-group conflict index is that conflict increases when two stakeholder groups' con- and pro-groups have greater value differences between them. This can be noted by observing that $0 \leq d_{ij}^{D,E} \leq 1$, where $d_{ij}^{D,E} = 1$ when the two stakeholders groups have opposing preferences; for example, $C_{ij}^{D} = 0$, $C_{ij}^{E} = 0$, $C_{ij}^{D,E} = 0$, $P_{ij}^{D} = 0$, $P_{ij}^{E} = 0$, $P_{ij}^{D,E} = 0$, $T_{ij}^{D} = 0$, $T_{ij}^{E} = 0$, $T_{ij}^{D,E} = 1$; i.e., $(1-(0+0)) - ((0-(0+0)) + (0-(0+0))) = 1$.

4.6. Example. Using the preference data in Table 2, we divide the group of stakeholders into two sets, $D = \{S_1, S_2, S_3, S_4, S_5\}$ and $E = \{S_6, S_7, S_8, S_9, S_{10}\}$, based on some demographic variable, e.g., age or sex. These sets are further divided into the con-group sets $\boldsymbol{S}_{ij}^{D-}$ and $\boldsymbol{S}_{ij}^{E-}$ and the pro-group sets $\boldsymbol{S}_{ij}^{D+}$ and $\boldsymbol{S}_{ik}^{E+}$. The conflict between the stakeholder groups is then given by the conflict index $d_{ij}^{D,E}$. The results of the calculations are presented in Table 6.

As in the within-group conflict index, the between-group conflict index range is $[0, 1]$. In this example we use the consensus properties defined in the within-group example in Section 4.4; see the conflict index intervals and the associated consensus properties in Table 3.

As seen in Table 6, the result for action A_1 is $d_{1,1}^{\boldsymbol{D,E}} = 0$, since there is no conflict between the groups. Table 2 shows that all stakeholders of both subsets $\boldsymbol{D}$ $(S_1, \ldots, S_5)$ and $\boldsymbol{E}$ $(S_6, \ldots, S_{10})$ are members of the pro-group and that they all placed A_1 on the same position as A_α in the cardinal ranking; i.e., no con-groups exist. The stakeholders in $\boldsymbol{D}$ expressed that all other actions were better than A_1, and stakeholders in $\boldsymbol{E}$ expressed that all other actions were worse than action A_1. Thus, when the ranking was normalized to proportional $[0, 1]$-value scale, the value for A_1 was $v_{1,1}^{k}$=0 for $S_1, \ldots, S_5$, and $v_{1,1}^{k}$=1 for $S_6, \ldots, S_{10}$. Action A_2 has $d_{1,2}^{\boldsymbol{D,E}} = 0.2$, since the two stakeholder groups have similar preferences. Both groups estimated the value of the action to lie close to A_α. Action A_3 has $d_{1,3}^{\boldsymbol{D,E}} = 0.0143$, note that this is because only stakeholder S_{10} estimates that the action is negative, and all other stakeholders estimate it to be equal to A_α. Action A_4 has $d_{1,4}^{\boldsymbol{D,E}} = 0.7857$, since the two stakeholder groups have opposing preferences, as seen in Table 2. Action A_5 has $d_{1,5}^{\boldsymbol{D,E}} = 1$, since all members of $\boldsymbol{D}$ estimate that the action is the best of all actions, and all members of $\boldsymbol{E}$ estimate that the action is the worst of all actions. The results of the classification of the actions are presented in Table 7.

5. Application of CAR for Conflict Evaluations at Upplands Väsby Municipality

In a current research project, researchers from Stockholm University cooperate with the Royal Institute of Technology

Table 7: The classification of the actions consensus properties.

Consensus Property	A_1	A_2	A_3	A_4	A_5
Consensus aligned	×		×		
Potentially controversial		×			
Controversial					
Very Controversial				×	×

Table 8: Analysis I: the number of members of the con- and pro-groups, the con- and pro-indices, and the conflict in the total population.

	A_1	A_2	A_3	A_4	A_5
$\lvert S_{8j}^{-}\rvert$	88	11	24	133	120
$\lvert S_{8j}^{+}\rvert$	851	928	915	806	819
$C_{8j}^{\boldsymbol{S}}$	$1.04 \cdot 10^{-7}$	$2.74 \cdot 10^{-8}$	$7.32 \cdot 10^{-8}$	$1.64 \cdot 10^{-7}$	$1.52 \cdot 10^{-7}$
$P_{8j}^{\boldsymbol{S}}$	$4.91 \cdot 10^{-6}$	$5.57 \cdot 10^{-6}$	$5.54 \cdot 10^{-6}$	$4.87 \cdot 10^{-6}$	$4.68 \cdot 10^{-6}$
$T_{8j}^{\boldsymbol{S}}$	$7.12 \cdot 10^{-6}$	$6.01 \cdot 10^{-6}$	$6.25 \cdot 10^{-6}$	$7.09 \cdot 10^{-6}$	$6.64 \cdot 10^{-6}$
$\boldsymbol{d_{8j}^{S}}$	**0.0444**	**0.0197**	**0.0244**	**0.0440**	**0.0412**

in developing models for public planning and decision processes in Swedish municipalities. We investigate how to increase expressiveness of the media used for communication between the parties in these processes. The focus here lies on developing tools that enrich this kind of communication. As a part of the project we conduct three case studies, of which one is in cooperation with civil servants and politicians at Upplands Väsby (UV), a municipality in the Stockholm region.

5.1. A Case Study. As reported in [13, Ch. 7], the politicians and civil servants wanted to get in-depth information regarding the citizens preferences regarding actions, which possibly could be implemented in the future. It was especially of interest to investigate if there were any actions that potentially could lead to citizen conflicts. To facilitate this, it was decided to use a web-based questionnaire (the Appendix) as a front-end for preference elicitation. The content of the questionnaire was developed together with civil servants at UV. The questionnaire consisted of four parts:

Part I consisted of ten questions regarding different "focus areas" (or criteria in an MCDA setting). Under each focus area, the respondents used an implementation of CAR for conflict evaluations to rank five actions; see Figure 3.

Part II consisted of one question where the ten focus areas are given weights using an implementation of CAR; see Figure 4.

Part III consisted of three questions where the respondents stated their preferences regarding two contradicting actions.

Part IV consisted of questions regarding demographic information.

An invitation letter containing a URL to the questionnaire was sent by mail to 10,000 citizens. The sample was chosen by conducting simple random sampling on a sampling frame consisting of 31,408 citizens. In the section below we present an analysis of the results from the eighth focus area, *School*. In total we received 939 answers; of these 465 were male, 456 female, 15 did not want to disclose their gender, and 3 selected other/agender. The analysis consists of two parts, the analysis of (i) the difference in preferences in the total population and (ii) the difference in preference between females and males.

5.1.1. Results. Five actions $A_1, \ldots, A_5$ are evaluated under the focus area/attribute G_8 *School*:

(i) A_1 reduces preschool child groups.

(ii) A_2 raises the quality of teaching.

(iii) A_3 increases professional development for schools and teachers.

(iv) A_4 increases modern information technology (IT) in education.

(v) A_5 involves caretakers more in school.

The respondents assess the affect associated with each action with regard to a *do nothing* alternative (current state). Note that this assessment may not be related to the cost of implementing the action, rather the feeling/affect related to the implementation. In the questionnaire implementation of the CAR for conflict evaluations (Figure 3) the *do nothing* alternative is represented by the tick mark located in the middle of the scale.

Analysis I: Conflict in the Population. In the first analysis we investigate the difference in preference in the total population. We used four consensus property labels to semantically describe the level of conflict. The within-group conflict index range $[0, 1]$ was divided into four subranges each representing one level of consensus; see Table 3. Note that the subranges and the consensus properties are defined by the authors for illustrative purposes.

The results of this analysis are presented in Table 8. The results show that all actions have a very low conflict index, indicating that the respondents have similar preferences. Actions A_1 ($d^{\boldsymbol{S}_{8,1}} = 0.0444$), A_4 ($d^{\boldsymbol{S}_{8,1}} = 0.0440$), and A_5 ($d^{\boldsymbol{S}_{8,5}} = 0.0412$) have the largest conflict indices. Actions A_3

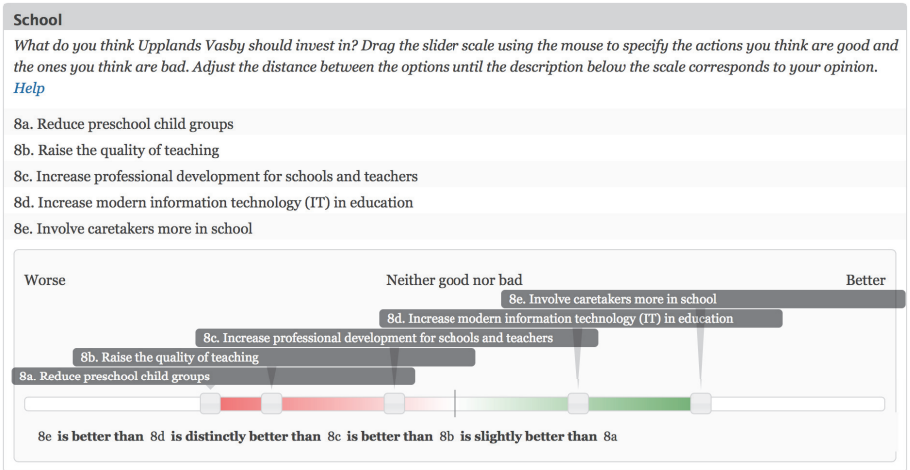

Figure 3: Question 1 from Part 1 of the questionnaire translated to English. A handle on the slider represents one action. Actions to the left of "Neither good nor bad" are regarded to be worse, and actions to the right are considered to be better than the *do nothing* action. The preference intensity is represented by a colored gradient on the slider (red to the left of Neither good nor bad, and green to the right). A textual description of the strength of preference between pairs of actions is found underneath the slider. The questionnaire text was originally in Swedish.

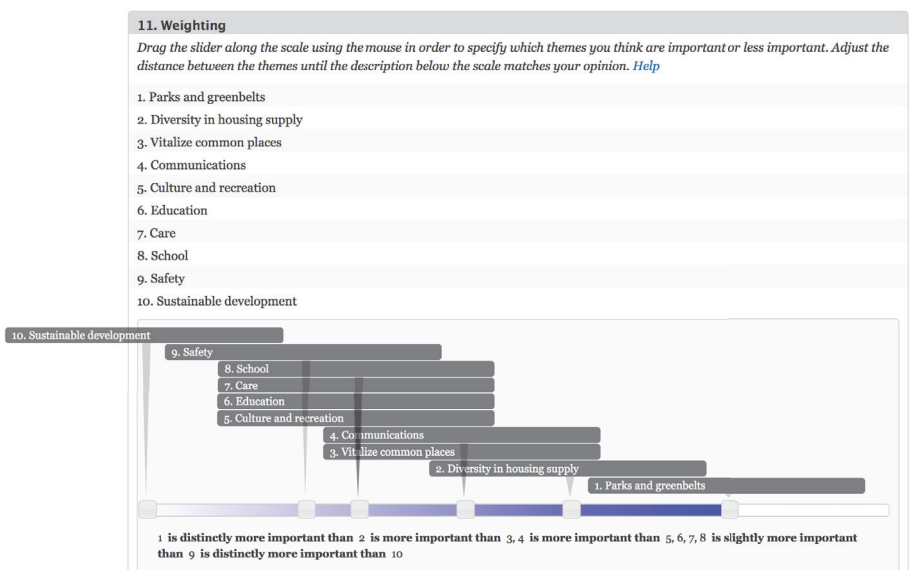

Figure 4: The weighting of the focus areas translated to English. A handle on the slider represents one focus area. The importance of a focus area is represented by the blue colored gradient on the slider, and the rightmost focus area is the most important one. A textual description of the strength of preference between pairs of actions is found underneath the slider. The questionnaire text was originally in Swedish.

and A_2 have the lowest conflict indices of $d^{S_{8,3}} = 0.0244$ and $d^{S_{8,2}} = 0.0197$, respectively. The attribute properties are presented in Table 9, showing that all actions are consensus aligned.

Analysis II: The Difference in Preference between Females and Males. In the second analysis we investigate the difference in preference between females and males. We use the same consensus property labels as in the first analysis to semantically describe the level of conflict. The between-group conflict index range $[0, 1]$ was divided into four subranges each representing one level of consensus; see Table 3. Note that the consensus properties are defined by the authors for illustrative purposes.

The results of the calculations are presented in Table 10. All actions have very low conflict indices, which also is reflected in the attribute properties (Table 11), where all actions are *consensus aligned*. The action with the highest conflict index is A_1 ($d^{S_{8,1}} = 0.0107$), followed by A_4 ($d^{S_{8,4}} = 0.0052$), A_3 ($d^{S_{8,3}} = 0.0027$), A_5 ($d^{S_{8,5}} = 0.0019$), and A_2 ($d^{S_{8,2}} = 0.0005$). The difference in value between the actions' conflict indices is small since both groups have very similar preferences.

Table 9: Analysis I: the classification of the actions consensus properties.

Consensus Property	A_1	A_2	A_3	A_4	A_5
Consensus aligned	×	×	×	×	×
Potentially controversial					
Controversial					
Very Controversial					

Table 10: Analysis II: the number of members of the con- and pro-groups, the con- and pro-indices, and the conflict between females and males.

	A_1	A_2	A_3	A_4	A_5
$\lvert S_{8j}^{D-}\rvert$	25	6	10	60	57
$\lvert S_{8j}^{D+}\rvert$	431	450	446	396	399
$\lvert S_{8j}^{E-}\rvert$	60	5	13	69	61
$\lvert S_{8j}^{E+}\rvert$	405	460	452	396	404
$C_{8j}^{\boldsymbol{D}}$	$3.60 \cdot 10^{-8}$	$1.29 \cdot 10^{-8}$	$6.00 \cdot 10^{-8}$	$6.45 \cdot 10^{-8}$	$6.87 \cdot 10^{-8}$
$P_{8j}^{\boldsymbol{D}}$	$2.39 \cdot 10^{-6}$	$2.70 \cdot 10^{-6}$	$2.68 \cdot 10^{-6}$	$2.07 \cdot 10^{-6}$	$2.31 \cdot 10^{-6}$
$T_{8j}^{\boldsymbol{D}}$	$3.20 \cdot 10^{-6}$	$2.94 \cdot 10^{-6}$	$3.07 \cdot 10^{-6}$	$2.95 \cdot 10^{-6}$	$3.20 \cdot 10^{-6}$
$C_{8j}^{\boldsymbol{E}}$	$6.82 \cdot 10^{-8}$	$1.56 \cdot 10^{-8}$	$6.20 \cdot 10^{-9}$	$9.61 \cdot 10^{-8}$	$8.70 \cdot 10^{-8}$
$P_{8j}^{\boldsymbol{E}}$	$2.57 \cdot 10^{-6}$	$3.02 \cdot 10^{-6}$	$3.03 \cdot 10^{-6}$	$2.85 \cdot 10^{-6}$	$2.52 \cdot 10^{-6}$
$T_{8j}^{\boldsymbol{E}}$	$3.86 \cdot 10^{-6}$	$3.24 \cdot 10^{-6}$	$3.33 \cdot 10^{-6}$	$4.23 \cdot 10^{-6}$	$3.62 \cdot 10^{-6}$
$C_{8j}^{\boldsymbol{D,E}}$	$1.05 \cdot 10^{-7}$	$2.85 \cdot 10^{-8}$	$7.39 \cdot 10^{-8}$	$1.61 \cdot 10^{-7}$	$1.57 \cdot 10^{-7}$
$P_{8j}^{\boldsymbol{D,E}}$	$5.05 \cdot 10^{-6}$	$5.72 \cdot 10^{-6}$	$5.71 \cdot 10^{-6}$	$4.99 \cdot 10^{-6}$	$4.83 \cdot 10^{-6}$
$T_{8j}^{\boldsymbol{D,E}}$	$7.27 \cdot 10^{-6}$	$6.18 \cdot 10^{-6}$	$6.41 \cdot 10^{-6}$	$7.22 \cdot 10^{-6}$	$6.82 \cdot 10^{-6}$
$\boldsymbol{d_{8j}^{D,E}}$	**0.0107**	**0.0005**	**0.0027**	**0.0052**	**0.0019**

Table 11: Analysis II: the classification of the actions consensus properties.

Consensus Property	A_1	A_2	A_3	A_4	A_5
Consensus aligned	×	×	×	×	×
Potentially controversial					
Controversial					
Very Controversial					

6. Concluding Remarks

Public decision problems can be complex. They involve multiple actions and multiple stakeholders which may have conflicting preferences with regard to the actions. A problem in such situations is that these opposing preferences might lead to conflicts between stakeholders, which may lead to delays in the decision process. An approach that can be used to increase the understanding of the stakeholders' opinions is to allow them to state their opinions, e.g., by using a web-based questionnaire.

In this paper, we showed how the CAR method can be applied, enabling respondents to state negative or positive preferences with regard to an alternative's performance relative to a *do nothing* alternative. We applied CAR for conflict evaluations in a case study conducted in cooperation with Upplands Väsby municipality. The citizens' preferences were elicited by a web-based questionnaire that used an implementation of the method. We showed how the method can be used to highlight conflict between and within different stakeholders groups and how the conflict can be conceptualized into semantic attribute properties. The results of such an analysis can aid decision-makers in the process of making well-informed decisions by clarifying the actions that can be conflict-prone.

Appendix

The Questionnaire

Part I. What Should Upplands Väsby Focus on in the Future?

(1) Parks and greenbelts

(1a) Preserve existing larger greenbelts
(1b) Build parks in existing urban districts
(1c) Build homes near greenbelts
(1d) Renovate existing parks
(1e) Improve accessibility to major greenbelts

(2) Diversity in housing supply

(2a) Offer more housing types
(2b) Offer more apartment sizes
(2c) Offer small-scale land ownership
(2d) Preserve the conceptual foundations of the buildings from the 1970s
(2e) Offer more housing near the water

(3) Vitalize common places

(3a) Mix different types of traffic
(3b) Place parking along the streets
(3c) Turn entrances to streets
(3d) Place public locales in transparent ground floors
(3e) Secure parking solutions under houses

(4) Communications

(4a) Pair the new streets with existing ones to strengthen the connection to the adjacent neighborhoods and reduce the barriers that the main roads pose.
(4b) Improve communications at night between various parts of the municipality
(4c) Improve communications to and from Uppsala
(4d) Improve the north-south and east-west routes through a fine-mesh and well-integrated metropolitan area networks.
(4e) Improve communications to and from downtown Stockholm

(5) Culture and recreation

(5a) Expand the range of cultural sports and recreational activities
(5b) Create better opportunities for festivals and concerts
(5c) Create more opportunities for outdoor sports
(5d) Create outdoors marketplaces
(5e) Provide municipal grants for cultural and recreational projects

(6) Education

(6a) Renovate older schools
(6b) Build new schools
(6c) Improve the physical environment of schoolyards
(6d) Improve the quality of primary education
(6e) Improve the quality of secondary education

(7) Care

(7a) Increase cultural and recreational activities for the elderly
(7b) Increase cultural and recreational activities for children and young people
(7c) Improve care for the elderly in the municipality
(7d) Increase youth centres and field assistants
(7e) Reduce preschool child groups

(8) School

(8a) Reduce preschool child groups
(8b) Raise the quality of teaching
(8c) Increase professional development for schools and teachers
(8d) Increase modern information technology (IT) in education
(8e) Involve caretakers more in school

(9) Safety

(9a) Increase safety around the station area
(9b) Increase police presence in central Väsby
(9c) Improve the lighting in the centre of Väsby
(9d) Narrow opening hours for alcohol outlets in central Väsby
(9e) Extend the opening hours of shops in the city center

(10) Sustainable development

(10a) Reduce energy consumption
(10b) Reduce transport and sound pollution
(10c) Increase climate change adaptation and recycling
(10d) Prioritize environmentally friendly transport modes (walking, cycling, public transport)
(10e) Reducing environmental toxins and hazardous chemicals in nature

Part II. How Important Is Each Focus Area?

(11) Weighting

(1) Parks and greenbelts
(2) Diversity in housing supply
(3) Vitalize common places
(4) Communication
(5) Culture and recreation
(6) Education
(7) Care
(8) School
(9) Safety
(10) Sustainable development

Part III. Contradictions

(12) Water or housing

(12a) Build homes near water
(12b) Preserving nature and shorelines intact

(13) Services or green areas

(13a) Densify the city centre and increase the range of services
(13b) Preserve green areas

(14) Regional centre or smaller urban areas

(14a) Develop central Upplands Väsby
(14b) Develop smaller urban areas

Part IV. Your Background

(i) Where do you live? (This question consisted of 41 residential areas to choose among)

(ii) What is your highest level of education?

(a) Not finished elementary school or equivalent compulsory school
(b) High school or equivalent compulsory school
(c) High school, folk high-school or equivalent
(d) College/university
(e) Other post-secondary education
(f) Postgraduate

(iii) What is your main occupation?

(a) Employed
(b) Self-employed
(c) Student
(d) Pensioner/retired
(e) Sickness or activity compensation
(f) Long term sick leave (more than 3 months)
(g) Leave of absence or parental leave
(h) Job seeker or in labour market activity
(i) Homemaker
(j) Other

(iv) How long have you lived in Upplands Väsby?

(a) 0-4 years
(b) 5-9 years
(c) 10 years or more

(v) How old are you?

(a) 0-14
(b) 15-24
(c) 25-34
(d) 35-44
(e) 45-54
(f) 55-64
(g) 65+

(vi) What is your gender?

(a) Female
(b) Male
(c) Other/agender
(d) Do not want to disclose

Data Availability

The dataset is available on data.world, https://data.world/samuel-bohman/2015-upplands-vasby-municipality.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Research Article

Minimizing Cost Travel in Multimodal Transport Using Advanced Relation Transitive Closure

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The optimization computation is an essential transversal branch of operations research which is primordial in many technical fields: transport, finance, networks, energy, learning, etc. In fact, it aims to minimize the resource consumption and maximize the generated profits. This work provides a new method for cost optimization which can be applied either on path optimization for graphs or on binary constraint reduction for Constraint Satisfaction Problem (CSP). It is about the computing of the "transitive closure of a given binary relation with respect to a property." Thus, this paper introduces the mathematical background for the transitive closure of binary relations. Then, it gives the algorithms for computing the closure of a binary relation according to another one. The elaborated algorithms are shown to be polynomial. Since this technique is of great interest, we show its applications in some important industrial fields.

1. Introduction

The optimization computation is an essential branch of operations research which aims to minimize the resource consumption and maximize the generated profits. It is primordial in many technical and industrial fields: system design, supply chain management, transport, energy, finance, networks, etc. This type of computation has enjoyed a great growth in the last century, and it presents the goal of many works in operations research and graph theory.

This paper introduces a new method for cost optimization, applied either on path optimization for graphs or on binary constraint reduction for Constraint Satisfaction Problem (CSP). It is a problem-solving technique that is applied to improve decision-making and efficiency. This method aims to fill the gap in the binary constraints optimization, in particular where these constraints are of different structures. In addition, it has a low algorithmic complexity that is shown to be polynomial. Moreover, it can be widely used in many operations research application fields, namely, management, decision-making, network optimization, scheduling, etc.

Before citing the related works and making the comparison with what is done in the literature, we start by describing the problem. First, we begin by defining the binary constraints on which the elaborated method is based. A binary constraint is a relation between two variables. Concretely, a relation can be, for example, an association of employees' names and their salaries, or a pairing of calendar years with automobile production figures. In the case when we have relationships between just two elements, the relation is called a binary relation and it is represented by set of ordered pairs (x, y) where x and y are objects of sets A and B, respectively, and x bears a relation to y.

Many binary relations display identifiable properties. For example, the relation "less than or equal to" has the property that if an object bears a relation to a second which further bears the same relation to a third then the first bears this relation to the third ($x < y$ and $y < z$ and then $x < z$). Such relations are said to be transitive.

Transitivity is an important property used in several domains. However, it is not satisfied by many relations. In fact, the transitive closure computation has been identified as an important and frequent problem in many computer science applications, we mention among others path optimization, redundant synchronization removal [1], reachability analysis of transition graphs in communication networks [2],

construction of parsing automata in compilers [3], evaluation of recursive database queries, and iteration space slicing and code generation [4, 5].

In this paper, we investigate a new extension of the transitive closure concept: "the transitive closure according to a given property." That is, for a given property P, and a relation R, we are interested in computing the smallest transitive relation containing R such that the property P holds. To begin, we enumerate the main contributions of this paper:

(i) The introduction of the concept of transitive closure according to a given property.

(ii) The proposition of a new algorithm to compute this transitive closure.

(iii) The profit of our new concept in many research operations applications, especially those requiring the constraints reduction or path optimization.

Related Works. Many efficient algorithms of transitive closure have been developed in the field of algorithmic graph theory: we mention Roy Warshal algorithm [6] and Warren algorithms [7] based on a matrix representation of the graph. We cite also the Schmitz algorithm [8] which takes benefit of Tarjan's algorithm. Other interesting works in the same line are those of Ioannadis [9]. However, all these previous works consider only the computation of the transitive closure. To the best of our knowledge, the concept of transitive closure according to a given property has not been introduced before and there is no elaborated technique to solve this problem.

In the case of many relations, Wlodzimierz, Verdoolaege, Cohen, and Beletska [10, 11] worked on transitive closure of the union of affine relations on integer tuples. The general form of a parameterized affine integer tuple relation is $\{[x] \longrightarrow [y] \mid \bigvee_{i=1}^{n}(\exists \alpha_i,\ A_i(x, y, \alpha_i) \geq bi)\}$ where x, y, and α_i are integer tuples, A_i is an integer matrix, and b_i is a tuple of integers and symbolic value. They presented iterative algorithms to calculate the exact, the underapproximation, and the overapproximation transitive closure of such relations, within Presburger arithmetics. However, they consider only the union of integer tuple relations that are commutative and have the same dimensions.

To show the insufficiency of their study, let us discuss an example of a closure case that is not included in their work. For this purpose, we begin by giving some definitions.

Definition 1. Let $\mathbb{N}$ be the set of nonnegative integers and k be a positive integer. A set $S \subseteq \mathbb{N}^k$ is a **linear set** if there exist vectors $v_0, v_1, \ldots, v_t$ in $\mathbb{N}^k$ such that $S = \{v \mid v = v_0 + a_1 v_1 + \cdots + a_t v_t, \text{for } a_1, a_2, \ldots, a_t \text{ in } \mathbb{N}\}$. V_0 is constant vector and $v_0, v_1, \ldots, v_t$ are called generators of the linear set S (called also periods).

A subset of $\mathbb{N}^k$ for $k \in \mathbb{N}$ is **semilinear** if it is a finite union of linear sets.

Example. In $\mathbb{N}^2$ the set $A = \{(x, y) \mid x \geq 4\}$ is a linear set, namely, the constant vector is $(4, 0)$, and the generators are $(1, 0)$ and $(0, 1)$.

Let $X = (x_1, \ldots, x_k)$ be a vector of distinct variables and $V(X) = (v(x_1), \ldots, v(x_k))$ be the vector which associates with each variable in the vector X its valuation.

Definition 2. A set $S \subseteq \mathbb{N}^k$ is said to be denoted by $\Phi(X)$ where Φ is Presburger formula with variables in $x_1, \ldots, x_k\}$ if $S = \{V(X) \mid V \vDash \Phi\}$. **S is called a Presburger set.**

Theorem 3. *Guinsburg and Spanier proved in [12] that the family of Presbureger sets of $\mathbb{N}^k$ is identical with the family of semilinear sets of $\mathbb{N}^k$.*

From this theorem, we can deduce that the works above are insufficient since the nonsemilinear sets are not studied. To explain this, we give a concrete example where a set B is defined as $B = \{i^i \mid i \in \mathbb{N}\}$. This latter is not a Presburger set since it is not semilinear.

Within Presburger logic, we consider a relation R over $\mathbb{N}$ defined as $\{[x] \longrightarrow [y] \mid x = y+3 \vee x = y+4\}$ where $(x, y) \in \mathbb{N}$. Let A be the partition of singletons on R^* (R^* designate the transitive closure of R). We define the following mapping: for each x in B we associate $(x^{1/x}, x^{1/x} + 1)$ in $\mathbb{N}^2 \cap A^2$. Take the following property on elements of B: $P(x) = \exists y, x = 2y$.

Assuming this data, up to now, there is no elaborated method which computes the transitive closure of R according to a property P in this case.

Other close techniques are hypergraph theory and linear programming. The hypergraphs do not express more than one relation. In fact, they present a simple generalization of the graphs in which an edge can join any number of vertices. This means that each hypergraph includes just one relation even if it is not binary.

Moreover, linear programming (LP) aims to optimize a linear objective function subject to linear inequality constraints. In our case there is no objective function to optimize. In addition, computation complexity using LP is higher than the complexity of our method.

The rest of this paper is organized as follows: In Section 2, we present the basic definitions, concepts, and notations used in this paper. In Section 3, we show the properties of our closure and we elaborate the computation algorithm. In Section 4, we apply our method in multimodal transport by adapting the algorithm to minimize the cost travel. Finally, we conclude and draw perspectives in Section 5.

2. Background and Basic Definitions

In the following, $\mathbb{R}$ denotes the set of real numbers and $\wedge$ (reps. $\vee$) logical conjunction (resp. disjunction).

Definition 4. Let A and B be two sets. A binary relation R from A (called domain of R) to B (called codomain of R) is defined by a subset G of $A * B$. If $(x, y) \in G$, we say that x is related to y and we write x R y.

When A = B, we say that R is a binary relation over A or defined on A. Some important properties of a binary relation R over A are as follows:

(i) Reflexivity: $\forall x \in A, xRx$.

(ii) Symmetry: $\forall x, y \in A^2, (xRy) \Longrightarrow (yRx)$.

(iii) Transitivity: $\forall x, y, z \in A^3, (xRy) \wedge (yRx) \Longrightarrow (xRz)$.

A relation that is reflexive, symmetric, and transitive is called an equivalence relation. When R is an equivalence relation over A, the equivalence class of an element $x \in A$ is the subset of all elements in A that bear this relation to x. Note that the equivalence classes are disjoint. The quotient set of A by R is the set of the equivalence classes. C_r will denote the cardinal of this quotient set.

Definition 5. The transitive closure of a relation R on a set A is the smallest transitive relation R^* over A containing R.

Directly from this definition, we can deduce the following properties:

(i) If R is transitive, then $R = R^*$

(ii) Any transitive relation R' defined on A, containing R, contains also R^*

In this paper, we will investigate the problem of computing the smallest transitive relation containing R such that a given property P holds. This problem may be viewed as an extension of the usual transitive closure problem: we are not looking only for the smallest transitive relation but a relation which satisfies also a given property. Formally, one has the following.

Definition 6. Let A and B be two sets, R be a relation over A, ϕ be a mapping from B to A^2, and P be a logical formula on elements of B. The transitive closure of R with respect to P is the smallest transitive relation defined on A, noted R_p, containing R, such that, $\forall x \in B, P(x) \Longrightarrow \phi(x) \in R_p$.

P expresses desired properties using a given logic. ϕ maps each element of B to an element of A^2. In this case, R_p is the smallest transitive relation containing R such that if an element of B satisfies the property P then its image must be in R_p.

Computing R_p in general case can be hard and depends on the decidability of the used logic. So, in this paper we will consider only the following case:

(i) $B = A^2$ and $\phi(x) = x \ \forall x \in B$.

(ii) $P(x) = \exists x' \in R_p \wedge x'Sx$, where S is a given relation over B.

In this case, Definition 6 becomes as follows.

Definition 7. Let R (resp. S) be a relation over a set A (resp. A^2). The transitive closure of R according to S is the smallest transitive relation over A, noted R_s, and containing R such that, $\forall (x, y, z, w) \in A^4, (x, y)S(z, w) \wedge xR_s y \Longrightarrow zR_s w$.

Example 8. To illustrate this concept, let us consider the following system of inequalities:

$$\begin{aligned} x_1 - x_2 &\leq 12 \\ x_3 - x_4 &\leq 8 \\ x_3 - x_5 &\leq 4 \\ x_5 - x_4 &\leq 2 \\ \max(x_1 - x_2) &\leq \max(x_3 - x_4) \end{aligned} \tag{1}$$

The system has two types of inequalities:

(i) Variable inequalities of the form $\boldsymbol{x_i} - \boldsymbol{x_j} \leq \boldsymbol{c_{ij}}$ with $(\boldsymbol{x_i}, \boldsymbol{x_j}, \boldsymbol{c_{ij}}) \in \mathbb{R}^3$.

(ii) Inequalities of maximal bound difference of the form $\max(\boldsymbol{x_k} - \boldsymbol{x_l}) \leq \max(\boldsymbol{x_p} - \boldsymbol{x_q}$ with $(\boldsymbol{x_k}, \boldsymbol{x_l}, \boldsymbol{x_p}, \boldsymbol{x_q}) \in \mathbb{R}^4$.

The set of solutions of this system, when it is not empty, is included in the intersection of the half-planes defined by individual inequalities of type 1. It is therefore a convex set. Now, we have to check if this convex set is not empty and in this case compute its canonical form (the representation in which all inequalities are maximally tight). This problem may be expressed as transitive closure of a relation R according to a relation S as follows:

(i) $R = \{(\boldsymbol{x_i}, \boldsymbol{x_j}) \mid \boldsymbol{x_i} - \boldsymbol{x_j} \leq \boldsymbol{c_{ij}}\}$

(ii) $S = \{(\boldsymbol{x_k}, \boldsymbol{x_l}), (\boldsymbol{x_p}, \boldsymbol{x_q}) \mid \max(\boldsymbol{x_k} - \boldsymbol{x_l}) \leq \max(\boldsymbol{x_p} - \boldsymbol{x_q})\}$.

Firstly, we close transitively R. This gives us $\boldsymbol{x_3} - \boldsymbol{x_4} \leq \boldsymbol{6}$ instead of 8. As we have this implication $(\boldsymbol{x_3}, \boldsymbol{x_4})\boldsymbol{S}(\boldsymbol{x_1}, \boldsymbol{x_2}) \wedge \boldsymbol{x_3}\boldsymbol{R}\boldsymbol{x_4} \Longrightarrow \boldsymbol{x_1}\boldsymbol{R}\boldsymbol{x_2}$ then $\boldsymbol{x_1} - \boldsymbol{x_2} \leq \boldsymbol{6}$. Therefore, computing R_s will give us the canonical form of the previous system.

Since the closure is done in an iterative way, we will introduce the following additional notations: for a relation R (reps. S) over A (resp. A^2).

(i) Completeness: $(R)_s = R \cup \{(z, w) \mid \exists (x, y), (x, y)S(z, w) \wedge xRy\}$

(ii) Iteration: for $i \in \mathbb{N}$,

$$\begin{aligned} R_s^0 &= R \quad if\ i = 0 \\ R_s^i &= \left(\left(R_s^{i-1}\right)^*\right)_s \quad otherwise \end{aligned} \tag{2}$$

Since $R \subset R^*$, it is easy to show that for all $i \in \mathbb{N}$ we have $R_s^i \subset R_s^{i+1}$ and $R_s = \bigcup_{i=0}^{\infty} R_s^i$.

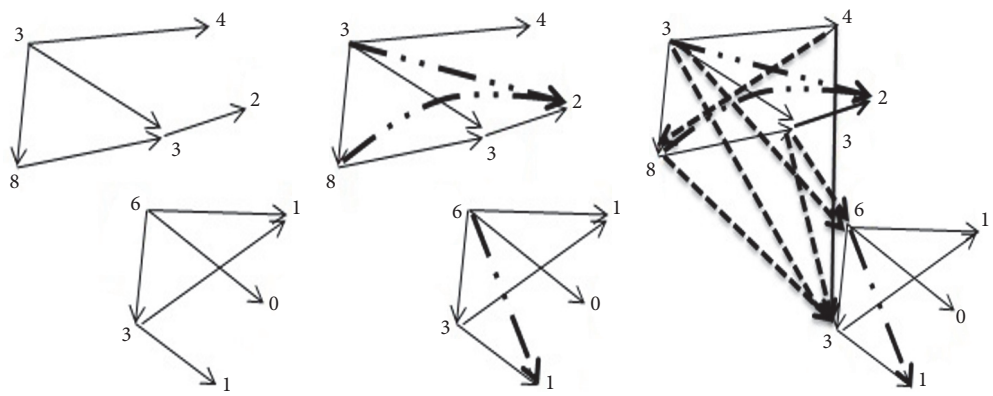
Example 9. This example shows the utility of this closure in a state machine with variables.

Let G be a graph and $A = (v_1, \ldots, v_n)$ be the set of its edges. We assume that $B = \mathbb{N}^2$, the property on elements of B is $P(x, y) : y + x = 0 \ mod\ 3$, and we suppose that the mapping $\phi(x, y)$ associates for each (x, y) in B an element (v_x, v_y). We affirm that for each two elements in B that satisfy property P, their images by the mapping ϕ are linked by the relation R.

Figure 1 shows the closure steps, in the first stage we have the graph presenting the relation R, and then, in the second step, we close transitively this graph. In the third step, we close the obtained graph according to the desired property. These last two steps are repeated iteratively until the graph is stationary.

3. Properties of the Transitive Closure of a Relation according to Another One

Let R (resp. S) be a relation over A (resp. A^2). In this section, we will dress the fundamental properties of R_s.



(a) Graph with just the relation R (b) The transitive closure of R (c) The transitive closure of R according to a property

Figure 1: Illustration of the closure steps.

3.1. Case of Equivalence Relation. The following theorem states the maximal number of iterations to achieve the transitive closure of a relation according to another equivalence relation.

Theorem 10. *If S is an equivalence relation, then the transitive closure of R according to S is*

$$R_s = \bigcup_{i=0}^{N_{max}} R_s^i = R_s^{N_{max}} \tag{3}$$

with $N_{max} = \min(C_s, n^2 - C_s, (n^2 - n)/2)$.

Recall that C_s *is the cardinal of the quotient set of S.*

Intuitively, when S is an equivalence relation, this fundamental theorem gives a way to compute R_s*, by computing* $R_s^{N_{max}}$.

Proof. In each step of the closure, we have at least a couple that has just been connected by S and which will connect all the pairs that belong to its class.

To prove that $N_{max} \leq C_s$ it suffices to show that $\forall j > C_s$ we have $R_s^j = R_s^{C_s}$.

First, we have $R_s^j = (\bigcup_{i=1}^{C_s} R_s^i) \cup (\bigcup_{i=C_s+1}^{j} R_s^i)$.

Since $\bigcup_{i=1}^{C_s} R_s^i = A$, then we will have $R_s^j = A \cup (\bigcup_{i=C_s+1}^{j} R_s^i) = A = R_s^{C_s}$.

However, when each class contains two couples, the maximal bound is a bit different; it is $[n^2/2]$ (we denote by [x] the entire part of x).

In addition, the relations of type (z, t) S (x, x), for $(x, y, z) \in A^3$ (resp. (x, x) S (z, t)), are not useful since they no longer advance the process of the closure (i.e., they do not link any two elements by R). Thus, the classes of type [(x, x), (z, t)] should be eliminated; their minimum number that one can have is equal to n/2. Therefore, to have the final closure we can never exceed $(n^2 - n)/2$ iterations.

If $C_s > [(n^2+1)/2]$ then there are classes that contain only one pair. Each class of them reduces the number of iterations by one, since if the couple is linked in a step we will have no influence in the next steps. Hence, we deduce that if $C_s > [(n^2 + 1)/2]$ then $N_{max} = n^2 - C_s$.

Based on all the above, the following results are deducted: When S is an equivalent relation, we have the maximal number of iterations as follows:

$$\begin{aligned} N_{max} &= \min\left(C_s, \frac{n^2 - n}{2}\right) \quad if\ C_s \leq \left[\frac{n^2 - n}{2}\right] \\ N_{max} &= \min\left(n^2 - C_s, \frac{n^2 - n}{2}\right) \quad otherwise \end{aligned} \tag{4}$$

Hence one has the desired result. □

3.2. General Case of the Relation S. The following theorem determines the maximal number of iterations to achieve the transitive closure of a relation according to any other relation.

Theorem 11. *Let C be the cardinal of* R^**. The transitive closure of R according to S is* $R_s = \bigcup_{i=0}^{N_{max}} R_s^i = R_s^{N_{max}}$ *with*

$$\begin{aligned} N_{max} &= \left[\frac{n^2 - n}{2}\right] \quad if\ C < n \\ N_{max} &= \left[\frac{n^2 - C}{2}\right] \quad otherwise \end{aligned} \tag{5}$$

Proof. In each iteration i, we should have at least one couple (x, y) in A^2 such that $xR_s^{i-1}y$ (the transitive closure should at least bring this relation in the previous iteration) and which is in relation S with at least another couple (z, t): (x, y) S (z, t).

The relations of type (z, t) S (x, x) (resp. (x, x) S (z, t)) do not advance the process of closure. Then, the couples of type (x,x) should be deleted, and their minimum number that we can have is n. Hence the result is

$$N_{max} = \left[\frac{n^2 - n}{2}\right] \tag{6}$$

If we take into consideration the links closed in the first step (by R^*), noted C, we will ensure that if C > n, then $N_{max} = [(n^2 - C)/2]$. □

Input: R, S: Two binary relations, i: integer
Output: R_S: Transitive closure of the relation R according to S
$R_s^0 \longleftarrow R$
Do { $R_s^i \longleftarrow (R_s^{i-1})^*$ // Use of any algorithm which computes the simple transitive closure of R_s^i (i.e. Floyd-Warshall algorithm)
$R_s^i \longleftarrow (R_s^i)_s$ // update R_s^i using the relation S
$i \longleftarrow i + 1$
} ***While*** $(i \leq N_{max})$

Algorithm 1: Algorithm skeleton of transitive closure computation (R_S).

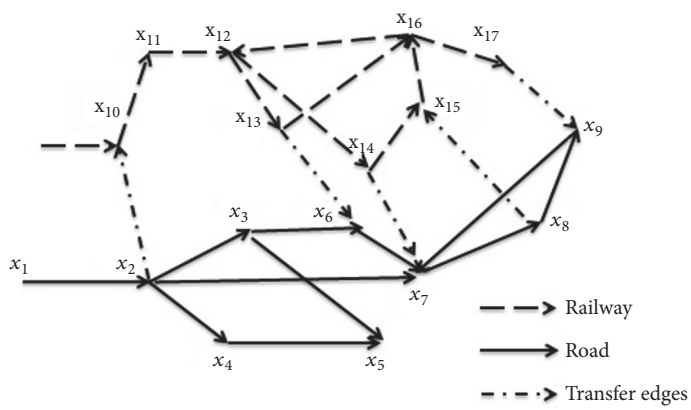


Figure 2: Multimodal transport graph.

3.3. Algorithm Skeleton of Transitive Closure (R_s) Computation. We have shown the maximal number of iterations we need to have the closure of a relation according to another one. We can give the closure computation algorithm as shown in Algorithm 1.

4. Application on Minimizing Cost Travel in Multimodal Transport

Multimodal transport is a logistic problem in which a set of goods have to be transported to different places with the combination of at least two modes of transport, without a change of container for the goods. Thus, it consists of using in the same path or trip several modes of transport (truck, car, train, plane ...). This technique has emerged to deal with problems such as pollution and energy consumption and especially for reason to reduce congestion.

Several issues arise from this idea; we can cite planning of multimodal transportation tasks [13, 14] and modeling the multimodal transportation networks. A lot of algorithms have been elaborated to find the viable shortest path under objectives such as travel time and number of modal transfers [15, 16]. The most important among them remains the calculation of multimodal shortest path.

4.1. Case of Two Modes

4.1.1. Formulation of the Problematic. Assume that we have a transport network as shown in Figure 2. It is presented by a graph $G_M(V, E)$ constituted by two modes. The first mode is the roads (m_1) and the second one presents the railways (m_2). Each mode has its set of nodes $V_{m_i} \subset V$ (its cardinal noted n_i) and its edges $E_{m_i} \subset E$. The graph contains also a set of transfer edges E_t that allows the mode change. These edges are mostly directed; therefore we note the set of their heads H and the set of their tails T.

During the transportation, we are facing constraints on mode changing, such as minimizing the number of transfers, viability, time of conveyance waiting, etc.

For some reasons such as congestion, we can change the mode only if we will reduce the transport cost. For example, we can choose the path $y_1 y_4$ instead of $x_2 x_6$ path only if the following condition is fulfilled: $Cost(y_1 y_4) \leq Cost(x_2 x_6)$. In general, we have $Cost(x_i x_j) \leq Cost(x_k x_l)$, for $(x_i, x_l) \in H^2$ and $(x_j, x_k) \in T^2$.

Given these data and flowing in both modes, the problem is to find the lowest cost between any two points of the graph.

4.1.2. Lowest Cost in Multimodal Transport. We present the transport network by a binary relation noted R. Based on matrix representation, this relation will be represented by a matrix of dimension n (n is the number of nodes): $M^R = \{(M_{ij}^R)_{0 \leq i,j \leq n}\} = \{(c_{ij})_{0 \leq i,j \leq n}\}$, with c_{ij} being the cost needed to achieve the node j from i. The transfer edges are not presented here. The constraints on mode changes are expressed by another binary relation, noted S, represented by a matrix of dimension n^2.$M^S = \{(M_{lk}^S)_{0 \leq l,k \leq n^2} \mid M_{lk}^S = 1 \ if \ Cost(x_i x_j) \leq Cost(x_k x_l)$ with $l = n(i-1) + j$ and $k = n(q-1) + p, 0 \ else\}$.

```
Input: Two matrices representing the two transport mode graphs
Output: Canonical form of the matrix that represents the merge multimodal graph
Do {
                                    // Step 1: Use any algorithm which computes the simple
                                       transitive closure of M^R. Next, we use Floyd-Warshall.
For i: 1 ⟶ n
   { For j: 1 ⟶ n
       { For k: 1 ⟶ n   M^R_ij = min(M^R_ij, M^R_ik + M^R_kj)}}
                                                        // Step 2: Update MR using MS
For i: 1 ⟶ n
   { For j: 1 ⟶ n
      If M^R_ij ≠ ∞ then
      { for k: 1 ⟶ n^2
            If M^S_((i−1)n+j,k) == 1 then
            M^R_([k/n]+1,k−[k/n]∗n) = min(M^R_ij, M^R_([k/n]+1,k−[k/n]∗n))
                                                        // [k/n]: entire part of k/n
            End if }
      End if }}
a ⟵ a + 1} While (a ≤ N_max)
```

Algorithm 2: Lowest cost in multimodal transport graph.

The constraints on mode changes are expressed by another binary relation, noted S.

4.1.3. The Applied Algorithm. Firstly, we accomplish the transitive closure of R. Afterwards, we scan the matrix updated of R^* looking for the coordinates which has non-infinite cost. Then, for every coordinate $M^R_{ij} \neq \infty$ we point out to the line corresponding to (x_i, x_j) in M^S in order to select the coordinates having the cost equal to 1. Assume, for instance, that $Cost((x_i, x_j), (x_p, x_q)) = 1$; this allows us to value the correspondent couples, (x_p, x_q) with $\min(Cost(x_p, x_q), Cost(x_i, x_j))$.

This operation will be repeated until the matrix is stationary, which corresponds to N_{max} iterations as showed in Theorem 10. Then, the algorithm complexity is $\Theta(n^3 N_{max})$ where n is the number of system variables.

4.1.4. General Case. Sometimes, for reason of loading/unloading cost, we can change the mode only if we will earn at least a definite supplementary cost. This can be expressed as $Cost(x_i x_j) - Cost(x_k x_l) \leq c$ for $(x_i, x_l) \in H^2$ and $(x_j, x_k) \in T^2$. This time, the binary relation S is represented by the following matrix: $M^S = \{(M^S_{lk})_{0 \leq l,k \leq n^2} \mid M^S_{lk} = b_{pqij}\ if\ Cost(x_i x_j) - Cost(x_k x_l) \leq b_{pqij}$ with $l = n(i-1) + j$ and $k = n(q-1) + p, 0\ else\}$, where b_{pqij} presents the cost that should be earned in order to change the mode.

It is the same as the previous example with some modifications. After finishing the transitive closure of R, we scan the matrix updated of R^* looking for the coordinates which have noninfinite cost. Then, for every coordinate $M^R_{ij} \neq \infty$, we change the coordinates having the non-infinite costs in the line corresponding to (x_i, x_j) in M^S. Take as an example $Cost((x_i, x_j), (x_p, x_q)) \neq \infty$; this allows us to value the correspondent couples, (x_p, x_q) with $\min(Cost(x_p, x_q), Cost(x_i, x_j) + Cost((x_i, x_j), (x_p, x_q)))$.

This operation will be repeated until the matrix is stationary. The algorithm is then like Algorithm 2.

4.2. Case of More Than Two Modes. The previous algorithm provides the shortest path only if the following proposition is considered: between two crossings of the same mode we pass through an odd number of modes. This implies that the start point and the arrival point belong to the same mode. Consequently, to hold the same computation, the use order of modes in the general case should be as follows: $m_i, m_j, m_k, m_j, m_k, m_l, m_o, m_p, m_o, m_l, m_k, \ldots, m_i$.

In this way, we will have the lowest cost between each two points using the previous algorithm.

5. Conclusion

This paper fits in the frame of the application of operations research in urban planning and transportation. It brought a new optimization method that consists in the concept of the "transitive closure of a given relation with respect to a property." In the literature there is no work that addresses this issue; therefore we have cited the close works done on the transitive closure with many relations. Afterwards, we situated our technique vis-à-vis hypergraph theory and linear programming.

The contributions of this paper are both theoretical and practical. We elaborated methods to compute the transitive closure of a relation according to a property and in particular the case where the property is another relation. After proving the completeness of the closure and the number of necessary iterations to achieve the solution, we design the applied algorithms. What is more interesting is the low algorithmic complexity that is shown to be polynomial. It is in the order

of $\Theta(n^3)$ where n is the number of the constraint system variables, which presents also the number of nodes in the multimodal graph. These algorithms are also useful in many real world problems such as optimal solutions, Constraint Satisfaction Problem, graph optimization, search engines, and system verification.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors hereby declare that there are no conflicts of interest regarding the publication of this paper. Being the authors of the paper, they certify on their honour the accuracy of this information.

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Research Article

Multiobjective Optimization for Multimode Transportation Problems

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We propose modelling for a facilities localization problem in the context of multimode transportation. The applicative goal is to locate service facilities such as schools or hospitals while optimizing the different transportation modes to these facilities. We formalize the School Problem and solve it first exactly using an adapted ϵ-constraint multiobjective method. Because of the size of the instances considered, we have also explored the use of heuristic methods based on evolutionary multiobjective frameworks, namely, NSGA2 and a modified version of PAES. Those methods are mixed with an original local search technique to provide better results. Numerical comparisons of solutions sets quality are made using the hypervolume metric. Based on the results for test-cases that can be solved exactly, efficient implementation for PAES and NSGA2 allows execution times comparison for large instances. Results show good performances for the heuristic approaches as compared to the exact algorithm for small test-cases. Approximate methods present a scalable behavior on largest problem instances. A master/slave parallelization scheme also helps to reduce execution times significantly for the modified PAES approach.

1. Introduction

Localization of facilities such as public services (schools, hospitals, etc.) is an important problem for social planners and policymakers. Most of the time, this problem is formulated as the (single objective) p-median problem which is a central problem in Operations Research (see, for example, [1, 2] for surveys). The p-median was introduced by Hakimi [3] who describes its basic properties. Its basic variant can be defined as follows: given a set of N demand nodes, distance values for each pair of nodes, and a fixed number p of facilities, locating each facility at one of the nodes, while minimizing the sum of distances from each node to its closest facility.

Recent developed algorithms solve the single objective version exactly for instances of thousands of nodes (e.g., 25.000 nodes in [4]). However, for a policymaker, considering additional objectives would be useful when solving p-median problems on real cases, leading to different variants; e.g.,

(i) *dispersion problem*: the p-dispersion problem consists of spanning the p facilities by maximizing the minimal distance between two of them. This objective function is suitable to locate business franchises and also when locating obnoxious facilities [5].

(ii) *p-center problem*: the p-center problem [6] aims at minimizing the maximal distance of demand nodes from their facility, or the average distance of a fraction of those that are the farthest from their closest facility, e.g., 5% farthest of them. This problem formulation can be applied to locate emergency services such as fire stations.

(iii) *multimode transportation location (MTL) problem*: in many real cases, transportation can be done by different means (by foot, bike, car, buses, etc.) depending on criteria like a threshold on the distance to the nearest facility. As an example, pupils are going to school by

foot (category A) or by public transportation (category B), with a threshold on the distance defining the 2 categories, e.g., 2 km. The objective is then to minimize both the mean distance for those of category A and the number of people in category B.

In the following we will focus on the MTL problem with multiple objectives. The practical context is to optimize location of schools. This School Problem is a typical multimode transportation problem since, depending the distance, pupils can go to school by foot or by bus. MTL problems can be seen as multiobjective optimization problems if means of transportation have impact on each other. Optimizing the cost for one of them can degrade the other objective value. When considering a multiple objective optimization problem (MOO), a single solution optimizing all of the objectives simultaneously rarely exists. Let $f : S \to Z$ be an objective function mapping solutions $s \in S$, the search space, to the objective space Z, with $Z = \mathbb{R}^m$. MOO algorithms look for solutions $s \in S$ such that $z = f(s)$ is optimized (in the sequel, we consider minimization). $z \in \mathbb{R}^m$ is a point of the objective space Z, with each of $z_i = f_i(s), i \in [1..m]$ being one of the objective function values to be minimized. Many approaches rely on the *dominance* concept to choose, among a set of solutions S, those ones that represent the best trade-offs of objectives within the search space. We say that a solution $s \in S$ *dominates* another one $s' \in S$ if $\forall i \in [1..m]$, $f_i(s) \leq f_i(s')$ and $\exists i \in [1..m]$, $f_i(s) < f_i(s')$. It is denoted as $s \succ s'$. Namely, s is as good as s' on all objectives and better than s' for at least one of them. Solutions that are not dominated by any member of S are *efficient* solutions and constitute the *Pareto set* $\mathcal{PS}$. The set of points $z \in Z$ corresponding to the efficient solutions is the *Pareto front* $\mathcal{PF} = \{z \in \mathbb{R}^m | \exists s \in S, f(s) = z \text{ and } \nexists s' \in S, s' \succ s\}$. In the sequel, we say for short that a point $z \in Z$ dominates $z' \in Z$ if $z = f(s), z' = f(s'), s \succ s'$.

The goal of MOO algorithms is generally to determine or approximate points of $\mathcal{PF}$ and associated solutions.

Solving multiobjective p-median instances is of course related to p-median problem exact and heuristic resolution approaches but also to general approaches used for solving MOO problems, either exactly or approximately. The former are often based on Integer or Binary Programming (Multiobjective Integer Programming, MOIP), the latter on Evolutionary Multiobjective Algorithms (EMOA). Our main contribution is to develop and evaluate the two kinds of approaches, in order to be able to solve exactly medium size cases of the School Problem and approximately very large scale cases. We exploit some mathematical properties of our targeted problem in order to model it with MOIP and solve it exactly with an ϵ-constraint algorithm. Large test-cases are handled with 2 different general EMOA frameworks, namely, the *Pareto Archived Evolution Strategy* (PAES, [7]) and *Nondominated Sorting Genetic Algorithm 2* (NSGA2, [8]). We have modified the former and mixed it with a local search technique: we have adapted to the multiple objective case an efficient neighborhood evaluation procedure [9] developed for the p-median problem. NSGA2 can also use our local search technique, as a postprocessing step. We show that, in many cases, for an equivalent computational effort, a well-known population-based approach such as NSGA2 is outperformed by the single individual method, PAES [7], thanks to our hybrid approach. Efficient parallelization helps for handling large test-cases. As shown in Section 2, similar approaches exist for MOO p-median, but with different multiple objectives and local search algorithms. Furthermore, to our knowledge, no results have been presented for the parallelization of these approaches.

Section 2 introduces related work for multiple objective optimization problems embedding p-median like formulation. Next, in Section 3, we formalize our bicriteria multimode transportation problem, with its specific objective functions. In Section 4, we present an exact approach for solving the problem, with an ϵ-constraint like technique. The problem can also be solved using popular MOO heuristic approaches like NSGA2 and PAES. We show how to adapt those frameworks to our problem solving, coupling them with an aggregation technique for performing local search. The MOO frameworks are presented in Section 5, and the local search, using limited neighborhood, is detailed in Section 5.2, along with its exploitation by the MOO methods. Evaluation and comparison of the proposed algorithms are realized with the hypervolume standard metric [10], over Beasley's benchmark [11] in Section 6. Last, conclusion and perspectives are detailed in Section 7.

2. Related Work

A few works extend the p-median problem with multiple objectives. The p-median problem is NP-hard [12], and MOO versions are also NP-hard since they embed the single objective version. Thus, if some exact algorithms exist, heuristic approaches are preferred for large test-cases.

The MOO p-median problem with an additional facility cost objective is dealt with in [13]. Each facility is weighted by a building cost, and the goal is to minimize the sum of distances to locations (p-median objective) and the sum of costs of the opened locations. The authors in [13] used two approaches. The first is an ϵ-constraint like formulation that is a mix of two-phases algorithms [14] and classical ϵ-constraint approach [15]. According to the authors, it leads to a close approximation of the full Pareto front. Even if its second objective is different (facility cost instead of number of pupils by public transportation), the technique used is similar to our approach concerning the ϵ-constraint problem formulation. However, since the number of nodes varies in our case for the distance count (for instance, pupils going to school by public transportation are not taken into account for the distance of demands from facilities), the method must be adapted, as it will be shown in Section 4. The second approach used in [13] helps to handle large test-cases and is based on MOGA framework, with a path relinking local search procedure for mixing solutions. Problems with uniform demand at each location (unitary problems) and with up to 400 demands and 20 facilities are processed. As MOGA, NSGA2, which we are testing, also uses a population-based approach.

In [16], the authors formulate a biobjective p-median problem with the following objectives : (1) minimal distance to the closest facility (p-median traditional objective) and

(2) maximization of the minimal distance between facilities (p-dispersion problem objective). They formulate and solve it as a MOIP, with an ϵ-constraint algorithm and also with a customized approach, based on threshold fixing for the minimal distance between facilities. Taking into account the threshold leads to additional constraints in a single objective subproblem. The algorithm iterates on subproblems, in order to provide the exact Pareto front of the biobjective initial problem. Once again, our second objective is different and implies an adaptation of the ϵ-constraint algorithm. Furthermore, the modified MOIP developed in [16] is specific to the dispersion problem, since it iterates on a threshold value of dispersion (minimal closeness of selected locations).

Problems similar to multiobjective p-median have to also be examined in the context of disasters management [17], or tsunami exposure. In [18], the authors solve approximatively a 3-objective optimization problem for school localization. Similarly to our heuristic method, it uses the NSGA2 framework mixed with a local search procedure, but the objective is not exactly the same as with p-median problems, since the number of facilities is not fixed but driven by a cost minimization objective. The second objective is related to tsunami risk exposure, and the third one takes into account both the distance to school and the number of pupils not able to go to school, as it is considered too far away (meaning, to go by foot). As we will see in the next section, this objective is in fact a mix of ours.

The p-median problem and its multiobjectives variants have also some military applications. Localization of repairs, supply, or security facilities can be modelized and addressed as p-median problems (see [19] for a survey). Common objectives are cost of localization, coverage of areas, and rapidity of deployment and also security of the localization that must be resilient. As an example, in [20], a multiobjective model, solved using a genetic annealing heuristic, takes into account cost and response time for Canadian forces prepositioning.

3. The Multimode Transportation Problem Modelling

We want to optimize the location of services, such as schools, in such a way that users go by foot to the closest service if it is possible (according to a threshold α on the distance they can walk) and others take public transports. In the following, we will focus on a particular case of multimode transportation problem, the School Problem. The School Problem is stated as follows: given a set of D demand nodes holding pupils and a set of N candidate nodes for locating schools, a value of α, and distances d_{ij} between each couple (demand node, candidate node) ($1 \le i \le D$, $1 \le j \le N$), define a set of p locations for setting up schools (school nodes) among the N candidate nodes with the following objectives:

(i) The mean distance of demand nodes to the closest school node is minimized for those demand nodes that have a distance to closest school node less than the threshold α.

(ii) The total number of demand nodes with a distance to the closest school greater than α is minimized.

The modelling of the School Problem can be stated as follows:

$$\text{minimize} \quad \frac{\sum_{i=1}^{D}\left(h_i.F_i.\sum_{j=1}^{N} d_{ij}.Y_{ij}\right)}{\sum_{i=1}^{D} h_i.F_i} \tag{1}$$

$$\text{minimize} \quad \sum_{i=1}^{D} h_i.\left(1-F_i\right) \tag{2}$$

$$\text{subject to} \quad \sum_{j=1}^{N} X_j = p \tag{3}$$

$$\begin{aligned} &\forall i, 1 \le i \le D, \\ &\forall j, 1 \le j \le N, \\ &Y_{ij} = 1 \Longrightarrow \\ &X_j = 1 \end{aligned} \tag{4}$$

$$\begin{aligned} &\forall i, 1 \le i \le D, \\ &\sum_{j=1}^{N} Y_{ij} = 1 \end{aligned} \tag{5}$$

$$\begin{aligned} &\forall i, 1 \le i \le D \\ &\left(\sum_{\{j \mid 1 \le j \le N \wedge d_{ij} \le \alpha\}} X_j\right) = 0 \Longleftrightarrow \\ &F_i = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\forall i, 1 \le i \le D, \\ &\forall j, 1 \le j \le N, \\ &F_i \in \{0,1\}, \\ &X_j \in \{0,1\}, \\ &Y_{ij} \in \{0,1\} \end{aligned} \tag{7}$$

where

(i) D is the number of demand nodes;

(ii) N is the number of candidate nodes (number of possible locations for a school);

(iii) p is the number of candidate nodes to be selected as school nodes;

(iv) α is the threshold on walking distance;

(v) h_i is the number of pupils located at demand node i;

(vi) d_{ij} is the distance between demand node i and candidate node j;

(vii) F_i is a decision variable, indicating if the pupils at node i go to school by walk or not (category A nodes).;

(viii) X_j is a decision variable, indicating if the candidate node j is selected or not as a school node;

(ix) Y_{ij} is a decision variable, indicating if the demand node i closest school node is j or not. If $Y_{ij} = 1$, we say that (demand node) i is *covered* by (school node) j.

Equations (1) and (2) reflect the objectives stated above, taking into account the number of pupils located at each demand node. Equation (3) fixes the number of selected school nodes. Equations (4) and (5) ensure that the single selected candidate node for pupils at a demand node is effectively a school node. Equation (6) ensures that pupils at a demand node are accounted to go walking if and only if a candidate node is selected for school location in the neighborhood of the node.

This formulation induces that candidate nodes are restricted to demand nodes, but it could be extended to an arbitrary set of candidate nodes. Also notice that distance data can correspond to Euclidean distances, with known geographical locations for the different nodes, or to shortest path values, computed with an algorithm as Floyd-Warshall if the nodes are embedded within a routing graph, with moving cost values on the edges (e.g., time by walking, distance, and transportation cost). In the latter case, the value and meaning of the threshold are to be adapted. The use of the threshold reflects the public Swedish policy where authorities offer free transportation to some pupils based on the walking distance to school. If $\forall 1 \leq i \leq D, h_i = 1$, we speak about the unitary version of the School Problem.

We show in the next section how to model this problem as a MOIP and how to solve it with an ad-hoc technique.

4. Exact Problem Formulation and Solving

It is possible to solve exactly multiple objective problems using ϵ-constraint approaches. These techniques require a linear formulation of the aimed problem. Since the first objective function of our model is nonlinear (1) and nonconvex, those techniques are not directly applicable. In Section 4.1, we present the problem as a multiobjective mixed integer linear program (MOMILP). Its resolution would allow for computing the optimal value of each objective (by removing other objectives from the formulation and using a MILP solver as CPLEX, [21]). However, even computing only the mean distance (our first objective) with this model required high execution times, as detailed in Section 4.1.

Therefore, we consider a simplified MOIP in Section 4.2 where the computation of the mean distance is replaced by a computation of the cumulative distance. Using this model, we can compute the Pareto front $\mathscr{PF}$ and associated solution set using an adapted ϵ-constraint [22]-like approach presented in Section 4.3. We show that this method provides the exact Pareto set for the problem stated in Section 3.

4.1. MOMILP Modelling. We studied the translation of the School Problem to a MOMILP problem mainly to compute directly the minimum mean distance (1) using a MILP solver. Since this objective function is continuous, its linearization requires using continuous variables. Indeed, starting from the modelling of the School Problem presented in Section 3, we can linearize (1) using a new set of (continuous) variables Z_{ij}, which replaces the set of (Boolean) variables Y_{ij}. The semantics of Z variables is the following: $Z_{ij} \neq 0$ if *demand node i is covered by school node j and i is in category A* (i.e., $F_i Y_{ij} = 1$) and, in this case, $Z_{ij} = 1/\sum_{i=1}^{D} h_i F_i$.

Using this semantics, objective (1) becomes linear, as shown in (1a). Notice that, by removing the variables Y_{ij}, we "forget", for each demand node i in category B (i.e., with $F_i = 0$), which school node covers it. It is possible to keep this information, but it is useless for the resolution of the problem (as long as there is at least one school).

To satisfy the semantics of the variables Z_{ij} and specifically specify $1/\sum_{i=1}^{D} h_i F_i$, we also add a continuous variable G_i for each demand node i and a continuous variable R. The resulting model is as follows:

$$\text{minimize} \quad \sum_{i=1}^{D}\left(h_i.\sum_{j=1}^{N} d_{ij}.Z_{ij}\right) \tag{1a}$$

$$\text{minimize} \quad \sum_{i=1}^{D} h_i.\left(1 - F_i\right) \tag{2}$$

$$\text{subject to} \quad \sum_{j=1}^{N} X_j = p \tag{3}$$

$$Z_{ij} \leq X_j \quad \forall i, 1 \leq i \leq D,\ \forall j, 1 \leq j \leq N \tag{4a}$$

$$\sum_{j=1}^{N} Z_{ij} + \left(1 - F_i\right) \geq R \quad \forall i, 1 \leq i \leq D \tag{5a}$$

$$F_i \geq X_j \quad \forall i, 1 \leq i \leq D,\ \forall j, 1 \leq j \leq N,\ d_{ij} \leq \alpha \tag{6a}$$

$$F_i \leq \left(\sum_{\{j \mid 1 \leq j \leq N \wedge d_{ij} \leq \alpha\}} X_j\right) \quad \forall i, 1 \leq i \leq D \tag{6b}$$

$$G_i \leq R \quad \forall i, 1 \leq i \leq D \tag{R1}$$

$$G_i \leq F_i \quad \forall i, 1 \leq i \leq D \tag{R2}$$

$$\sum_{i=1}^{D} h_i G_i \geq 1 \tag{R3}$$

$$\begin{aligned} &X_j \in \{0, 1\} \quad \forall j, 1 \leq j \leq N \\ &F_i \in \{0, 1\}, \\ &G_i \in [0, 1] \\ &\qquad\qquad \forall i, 1 \leq i \leq D \\ &Z_{ij} \in [0, 1] \\ &\qquad \forall i, 1 \leq i \leq D,\ \forall j, 1 \leq j \leq N \\ &R \in [0, 1] \end{aligned} \tag{8}$$

In this model, (R1), (R2) and (R3), ensure that $R \geq 1/\sum_{i=1}^{D} h_i F_i$: from (R2) and (R3) (and since F_i are binaries), we

have $\sum_{i=1}^{D} h_i F_i G_i \geq 1$. By (R1), we get $\sum_{i=1}^{D} h_i F_i R \geq 1$, which gives the lower bound of R.

From this result, (5a) states that when demand node i is in category A (i.e., $F_i = 1$), $\sum_{j=1}^{N} Z_{ij} \geq R$ (the minimization of (1a) ensures that only one Z_{ij} is equal to R and the other Z_{ij} are zero). Equation (4a) implements Constraint (4), whereas the implementation of Constraint (6) is done by (6a) and (6b) (note that the former is needed for the first objective while the latter is needed for the second one).

We have tried to exploit this formulation for computing the minimal mean distance, with the CPLEX MILP solver [21], using the single objective function (1a). In practice this approach did not work, except on our smallest example (100 nodes and 5 schools). On the other test-cases, CPLEX returned possibly nonoptimal solutions after reaching its memory limit. We think that this failure stems from two issues. The first issue is the variability of R (and, as a result, the values of all nonzero Z_{ij}) which makes node selection a hard problem. The second issue is that when solving relaxed problems, with F_i and X_i partly continuous, (5a) enables putting $\sum_j Z_{ij} = 0$ even when F_i is close to 1, since R is very small. As a result, the algorithm needs to force almost all X_j (and F_i) to integral values before getting a meaningful minimal bound. Since both issues are related to the computation of a mean distance, we considered a MOIP using the cumulative distance instead.

4.2. MOIP Modelling. The MOIP formulation of the School Problem with the cumulative distance can be seen as a simplification of the MOMILP formulation of the previous section. In fact, we just need to state that $R = 1$. As a result, the G_i variables are useless ((R1), (R2), and (R3) disappear). From this result, we propose the MOIP formulation $\mathcal{SP}$ for the School Problem, as follows:

$$\text{minimize} \quad z_1 = \sum_{i=1}^{D} \sum_{j=1}^{N} h_i d_{ij} Z_{ij} \tag{9}$$

$$\text{minimize} \quad z_2 = \sum_{i=1}^{D} h_i \left(1 - F_i\right) \tag{10}$$

$$\text{subject to} \quad \sum_{j=1}^{N} X_j = p \tag{11}$$

$$Z_{ij} - X_j \leq 0 \qquad \forall i, j, 1 \leq i \leq D, 1 \leq j \leq N \tag{12}$$

$$Z_{ij} = 0 \qquad \forall i, j, 1 \leq i \leq D, 1 \leq j \leq N, \ d_{ij} > \alpha \tag{13}$$

$$\sum_{j=1}^{N} Z_{ij} = F_i \quad \forall i, 1 \leq i \leq D \tag{14}$$

$$F_i \geq X_j \qquad \forall i, j, 1 \leq i \leq D, 1 \leq j \leq N, \ d_{ij} \leq \alpha \tag{15}$$

$$X_j \in \{0, 1\} \quad \forall j, 1 \leq j \leq N \tag{16}$$

$$F_i \in \{0, 1\} \quad \forall i, 1 \leq i \leq D \tag{17}$$

$$Z_{ij} \in \{0, 1\} \quad \forall i, j, 1 \leq i \leq D, 1 \leq j \leq N \tag{18}$$

Compared with the previous model, we replace (6b) by (12), (13), and (14), which ensure that demand node i cannot be in A if there is no school node at distance α.

Notice that (13) and (15) both use α, but with different meanings: the former ensures that a pupil cannot walk to a distant node (to minimize the second objective), while the latter prevents an artificial minimization of the cumulative distance by covering a demand by a distant node where a closer one is available.

As we have seen before, our School Problem is formalized only *approximately* by $\mathcal{SP}$, since (9) uses the *cumulative* distance instead of the *mean* distance for pupils in category A. While the minimization of the cumulative distance can be compared to the p-median problem (indeed, we can get the p-median problem by adding $F_i = 1$ for all i and removing (13)), its exact resolution using MOIP solvers can be more difficult, as the relaxed problem (with continuous variables) has more solutions. To explain this issue, let us consider only two demand nodes (which are also candidate nodes) 1 and 2 such that $d_{12} \leq \alpha$ and one location ($p = 1$). With the relaxed p-median problem ($F_1 = F_2 = 1$), the solver gives directly the optimal cumulative distance d_{12}, whatever the values of X_1 and X_2. Using the relaxation of $\mathcal{SP}$, the optimal solution sets all variable to 1/2 except Z_{12} and Z_{21} which are set to 0 and the cumulative distance is 0.

Using this observation and the fact that the cumulative distance is still not the mean distance lead us to consider an adapted ϵ-constraint method where the first objective is (10).

4.3. Exact Algorithm. ϵ-constraint [22] methods proceed as follows: a series of single objective (*i.e.,* Integer Program, IP) problems are solved, transforming all but one of the objectives of the MOIP considered into IP constraints. The constraint set is updated at each iteration to enforce the exploration of the whole objective space. In their paper [23], Ozlen and Azizoğlu introduce a recursive algorithm to generate a Pareto set for a MOIP problem. They use the set of already solved subproblems and their solutions to avoid solving a large number of IPs. In [14], a two phases algorithm is applied for the biobjective assignment problem. In the 2 objectives case, it first determines the extreme points in the objective space by discarding one objective at a time and solving the resulting single objective problem. Then, in a second phase, it partitions the objective space according to the range of values found at the first step and explores it by slices. It also combines the approach with heuristics specific to the assignment problem for enhancing the execution times.

An adapted ϵ-constraint algorithm which computes all nondominated points sorted by z_2 values (i.e., second objective) is presented in Algorithm 1 for solving $\mathcal{SP}$. This approach uses the fact that given a nondominated point $z = (z_1, z_2)$, other nondominated point $z' = (z'_1, z'_2)$ such that $z'_2 \geq z_2$ must satisfy both $z'_1 < z_1$ and $z'_2 > z_2$. Since the

Data: $\mathcal{SP}$ problem with objectives $z_1 = F_1$ (Eq. (9)) and
$z_2 = F_2$ (Eq. (10)) and constraints set $C = \{$Eq. (11)-(18)$\}$
Result: the Pareto front $\mathcal{PF}$, stored in PF
begin
$maxz_1 := +\infty$
$minz_2 := -\infty$
$PF := \emptyset$
while *true* **do**
add constraint $\{F_1 \leq maxz_1\}$ to C
add constraint $\{F_2 \geq minz_2\}$ to C
solve problem P: $\min z_2 = F_2$ subject to C
if *P not feasible* **then** End
solve problem P:
$\min z_1 = F_1$ subject to $C \cup \{F_2 = z_2\}$
if *P not feasible* **then** End
add point (z_1, z_2) to PF
$C := C \cup \{Eq.(21)\}$
$maxz_1 := z_1$-1
$minz_2 := z_2$+1
end
end

Algorithm 1: An ϵ-constraint based method for solving $\mathcal{SP}$ (with cumulative distance).

minimal cumulative distance is hard to compute (and not really useful as it may not be the minimal mean distance), we do not compute the extreme point associated with it.

In $\mathcal{SP}$, we consider the minimization of the cumulative distance instead of the mean distance. Hence both objective function parameters are of integer values (once all distances are converted into integers), which ensures that our ϵ-constraint method cannot miss any efficient solution for the cumulative distance problem. Furthermore, one can check that, for any feasible solution s of value $z = (z_1, z_2)$, the mean distance for people in category A is

$$\frac{z_1}{\sum_{i=1}^{D} h_i.Z_{ij}} = \frac{z_1}{M - z_2} \tag{19}$$

where $M = \sum_{i=1}^{D} h_i$. Especially $M = D$ if $h_i = 1$ for all i (Unitary Problem). Hence, any efficient solution for the mean distance problem is also an efficient solution of the cumulative distance problem.

Theorem 1. *Let s be a feasible solution for $\mathcal{SP}$(associated with point $z = (z_1, z_2)$ in objective space), such that it is efficient for mean distance optimization, i.e., for each feasible solution s' (point z'):*

$$\frac{z_1}{M - z_2} \leq \frac{z_1'}{M - z_2'} \quad \text{or } z_2 \leq z_2' \tag{20}$$

Then s is efficient for cumulative distance optimization.

Proof. If $z_2 \leq z_2'$, the result is straightforward. Otherwise, $z_1/(M - z_2) \leq z_1'/(M - z_2')$ and $M - z_2 < M - z_2'$, which implies $z_1 < z_1'$. □

Thus all efficient solutions for the School Problem are found by solving $\mathcal{SP}$. But the converse is not true, and nondominated points can also appear in the Pareto front for $\mathcal{SP}$ that do not belong to the Pareto front for the School Problem. However, since the solutions generated are sorted by the value of z_2, it is easy to make the algorithm generate only nondominated points for the School Problem: for each generated nondominated point z, we add the following constraint to the problem:

$$\sum_{i=1}^{D}\sum_{j=1}^{N} \left((M - z_2).d_{ij} - z_1\right) h_i.Z_{ij} \leq -1 \tag{21}$$

Constraint (21) (which has only integer coefficients) ensures that any subsequent solution has a mean distance strictly less than the previous one. Including this constraint, Algorithm 1 computes the Pareto front for the School Problem.

5. Heuristic Problem Solving

MOO problems search spaces are often intractably large [15]. Many heuristics have been developed for searching over huge spaces, particularly using Evolutionary Multiobjective Algorithms (EMOA). Genetic algorithms have been shown to be interesting for solving very large instances of the p-median problem in [24]. We investigate the extension of this work to the MOO case, focusing on chromosome-based EMOA. We investigate the use of two algorithms, NSGA2 and PAES.

NSGA2 is a reference algorithm in the EMOA field, and it compares positively to PAES for standard benchmark problems [8]. But as stated below, NSGA2 (and many other EMOAs) iterates onto a population of individuals (each of them represents a solution), producing eventually a large

offspring at each generation. Even if this is an advantage when exploring the search space for building an interesting front, it can be very costly when employing such an algorithm for solving a problem with a costly evaluation function. The evaluation function can be intrinsically complicated (e.g., physics simulation) or time costly because it runs a local search within the neighborhood of the solution to be evaluated. We have already applied PAES successfully for the former reason in the field of Real-Time Scheduling [25]. We have also used a local search technique for the single objective p-median problem in [24]. Thus our goal is to compare PAES coupled with a local search technique to the reference algorithm NSGA2 for the MOO School Problem.

We first present and compare shortly NSGA2 and PAES and then we detail our declination of both frameworks with a local search technique for solving the School Problem.

5.1. PAES and NSGA2. In many evolutionary algorithms, a solution is represented by a set of parameters called *chromosome* (or *genotype*). The *encoding* of solutions (i.e., the data structure of a chromosome) is specific for each problem. Several researchers have proposed genetic algorithms (GA) for solving the p-median problem [26, 27]. Most of them use a classical string representation; i.e., each chromosome is represented by a single array of integers of length p embedding the index of the selected candidate nodes. As stated in [24], we will use the same encoding, i.e., the chromosome represents the list of school nodes in our case. We add the constraint that in all chromosomes no school is duplicated. The initial population (or first solution) of our algorithms is generated by picking up distinct random candidate nodes, leading to feasible solutions. All the candidate nodes have the same probability to be chosen.

Both EMOA's goals are to produce a front that preserves diversity of solution objective values, with points that are numerous and spread well along $\mathcal{PF}$ and the accuracy of this front, with points that are close to $\mathcal{PF}$. Within a population set of solutions, both PAES and NSGA2 use a crowding metric for handling diversity: PAES by defining an adaptive grid over the objective space and counting points within each portion of the grid and NSGA2 by measuring the distance between points and their closest neighbor in each direction for each objective. To ensure accuracy, both algorithms embed an elitism mechanism: the number of solutions that are kept across generations (elitism) is given either by the population size (NSGA2) or by an external archive (PAES). This size also controls the number aspect of diversity. In order to increase accuracy of solutions and convergence to $\mathcal{PF}$, PAES uses dominance locally, comparing current solution to a single offspring at each generation, while NSGA2 sorts its population into dominance classes. PAES eventually updates its archive with the offspring, while NSGA2 renews its population by generating a series of offsprings at each generation, with a dominance class based selection. With both algorithms, when comparing solutions, if they are nondominated against each other, the crowding metric is used to tie them. Concerning the operators used for generating offspring, we have implemented a specific mutation operator that preserves feasibility within PAES and NSGA2 applies standard binary mutation and crossover onto a binary representation of chromosomes it generates internally. We also provide a penalty metric that helps NSGA2 discarding unfeasible solutions (penalizing duplicated schools).

We will see in the two next sections how to adapt those strategies for mixing the various approaches with a fast local search technique inspired from p-median single objective literature.

5.2. Local Search Technique for MOO. Many heuristic methods have been developed for the (single objective) p-median problem (see surveys [1, 2]). Variable neighborhood search [9] is very popular, coupled with fast evaluation techniques [1]. Those techniques, based on precomputing of the 2 closest schools for each demand node, allow updating the solution cost faster when modifying only partially the solution itself during the local search process: for each demand node, for a given set of schools (solution), the distance to closest school (s_1, the one that covers the demand node) and the second closest school (s_2) and its distance to the node are stored. We use a neighborhood operator called *hypermutation*, inspired from [24]. The neighborhood size is controlled by the number of modified school nodes (seeds): the neighborhood of a solution, for a fixed number of seeds τ, is defined by replacing one by one each seed node by all of the possible other candidate nodes; i.e., those that are not yet selected as school nodes within the solution. The solution cost can be updated faster, using the precomputed tables mentioned above: when a school s is replaced by s', demand nodes covered by it ($s = s_1$) are now covered by their second closest school (s_2), except if s' is closer to them than s_2. Objective functions are updated according to the case. We fix the number of seeds to $\tau = min(p, 5)$ for a School Problem, to obtain a reasonable neighborhood size. This size is $\tau.(n - p)$, with τ the number of seeds, n the number of candidate nodes, and p the number of schools.

The neighborhood and associated evaluation techniques are devoted to the single objective p-median problem. They can be kept when running the MOO case. However, evaluating and comparing to current front multiple neighbors resulting from multiple function evaluations could be very time consuming: evaluation itself can be costly if applied to a large number of neighbors, and these numerous solutions must by compared against existing population or archive, with a dominance checking procedure. Thus, we propose an approach where the neighborhood exploration is run with a single objective metric and only the most interesting solutions found during the local search are compared to the current solutions kept in population or archive. This algorithm is outlined in Algorithm 2. We transform, for the local exploration, the 2 objectives metric of the School Problem into a single one by a classical aggregation technique: weights are given to objectives and $f_{single} = \sigma.z_1 + (1 - \sigma).z_2$ is computed. z_1 and z_2 are normalized values (according to the objective vector of the neighborhood search starting point). f_{single} is evaluated for different values of σ in order to explore the objective space into different directions (e.g., $\{0.0, 0.25, 0.50, 0.75, 1.0\}$). Since σ is restricted to a few values, only a subset of the supported solutions can be found and of course unsupported ones

```
Data: a solution sol (set of p school nodes) to be
      evaluated, n, p (number of candidates nodes and
      schools), a set dirs of λ directions, a number of
      seeds τ
Result: a set offsprings of λ solutions
begin
  foreach σ ∈ dirs do
      best[σ] := (−, ∞)
      { (solution, value) couples }
  end
  (z1, z2) := evaluate(sol)
  choose randomly a subset seeds ⊂ sol of size
  |seeds| = λ
  foreach school s ∈ seeds do
      foreach neighbor sol' = sol ∪ {s' ∉ sol} \ {s} do
          (z'1, z'2) := fastEvaluate(sol')
          {based on sol and (z1, z2)}
          foreach σ ∈ dirs do
              v := z'1.σ + z'2.(1 − σ)
              if v < value(best[σ]) then
              best[σ] := (sol', v)
          end
      end
  end
  offsprings := ∅
  foreach σ ∈ dirs do
      offsprings :=
      offsprings ∪ {solution(best[σ])}
  end
end
```

Algorithm 2: A local search method for supported solutions in λ directions.

cannot be detected by the local search [28]. For each direction searched, only the best solution (i.e., with the minimal f_{single} value for the associated σ weight) into the neighborhood is provided as a result at the end of the local exploration process. This approach can be related to the decomposition techniques used in algorithms like MOEA/D [29].

5.2.1. Cost of Evaluation. The grain of the search is tuned by the number of seeds τ, leading to a neighborhood of size $\tau.(n-p)$, and also by the number λ of different values for σ. Let T by the cost of a (*standard*) evaluation of a solution. It mainly consists of finding the closest school for each children and is in $O(\lambda.n^2.p)$. Evaluating a series of neighbors for a neighborhood of $(\tau.(n-p))$ costs $O(\tau.(n-p).n^2.p)$, also stated as $T_n = O(\tau.(n-p).T)$. During the (*fast*) evaluation of a solution as a neighbor of another one, only a single school is modified and only the distances to school for its covered nodes are to be updated, in $O(n/p)$ time. Thus evaluating a whole neighborhood takes $T'_n = O(T + \lambda.\tau.(n-p).n/p)$, with roughly, a $n.p^2$ factor of gain as compared to T_n.

5.3. Mixing the Local Search Technique with EMOA Algorithms. This local exploration procedure can be mixed with the different EMOAs into 2 different ways:

(i) Applying the local search for each individual: the offspring of an individual is generated via the local search procedure, with λ directions of search.

(ii) Using the local search as a refinement step: applying local search to the members of the population or archive obtained as a postprocessing step.

NSGA2 processes natively multiple offsprings, but the original PAES manipulates a single current solution and a single offspring (it is a (1 + 1) procedure) that replaces (or not) the current solution at next generation, depending on dominance checking. In the first way of mixing, a (1 + 1) algorithm such as PAES must be adapted. Knowles proposes such a (1 + λ) version in [7]. We use instead the same selection mechanism as in [30], developed for costly functions coupling with PAES and well adapted in the context of an hybrid method combining EMOA with (time costly) local search. The selection procedure is as follows: the λ offsprings resulting from the local search are compared to the PAES archive using PAES policy. Then, the new current solution is chosen considering the whole archive as follows, based on a randomly selected optimization criteria: a random integer r in the interval [1..3] is generated. If it corresponds to an objective function (i.e., $r \in [1..2]$), these criteria are chosen;

Table 1: Number of candidate nodes n and school nodes p for the 40 test-cases of the Beasley benchmark.

#	n	p	#	n	p	#	n	p	#	n	p
1	100	5	11	300	5	21	500	5	31	700	5
2	100	10	12	300	10	22	500	10	32	700	10
3	100	10	13	300	30	23	500	50	33	700	70
4	100	20	14	300	60	24	500	100	34	700	140
5	100	33	15	300	100	25	500	167	35	800	5
6	200	5	16	400	5	26	600	5	36	800	10
7	200	10	17	400	10	27	600	10	37	800	80
8	200	20	18	400	40	28	600	60	38	900	5
9	200	40	19	400	80	29	600	120	39	900	10
10	200	67	20	400	133	30	600	200	40	900	90

otherwise ($r = 3$) the crowding criteria is selected. The next current individual is chosen randomly within the 10% best ones within the archive for the elected criteria. Concerning crowding, the best individuals are those in the less crowded areas of the grid.

The goal of the approach is to solve large size problems with a high execution time cost induced by evaluation and proportionally a small amount of time devoted to EMOA internal computations. So we consider here that the cost of choosing next current individual by processing the whole archive is not a drawback for execution times according to the benefit in terms of quality of search that we expect.

6. Algorithms Comparison

The multimode transportation problem or School Problem to be solved is specific, and to our knowledge, no comparable algorithm exists for solving it (see Section 2). Thus, this evaluation section mainly compares the heuristic approaches that we have developed to our customized ϵ-constraint method. The following algorithms are compared:

(i) Exact: the ϵ-constraint method presented in Section 4, which provides the exact Pareto Front $\mathcal{PF}$,

(ii) EMOA approaches presented in Section 5, using NSGA2 and PAES algorithms. For both EMOAs, the standard and the hybrid versions tested: local search realized at each generation with PAES and as a postprocessing step for NSGA2.

We first describe the benchmark set used for comparing algorithms results, detail algorithms setup, and then present results for both quality and execution time aspects.

(a) Data Set. The data set used for the evaluation is the Beasley benchmark set, devoted to the p-median problem [11] and widely used for testing p-median solvers [26, 31, 32]. It is a set of 40 test-cases. Candidate and demand nodes are identical, with a unitary demand at each node. n (the number of nodes) varies from 100 to 900 and p value from 5 to 200. Test-cases are ordered by increasing n value and, for a given n, by increasing p value as shown in Table 1.

We compute the α threshold in such a way that 15% of the distances between nodes are lower than α.

The largest graph of this benchmark has 900 candidate nodes and is not particularly large according to the size of problems current p-median solvers can manage (e.g., instances of up to 24,978 nodes solved exactly in [4]). However, multiple objective optimization could lead to much more costly evaluation of the solution space for the same instances as compared to single-objective formulations, and Beasley benchmark contains large enough instances for our purpose, as execution times will show. It is also comparable to other biobjective p-median problem instances used with others approaches (e.g., 402 nodes for the largest instance in [13]).

(b) Algorithms Setup. The exact method has been implemented in C. It is a modified version of the AIRA software, a general purpose MOIP solver [33], and it uses CPLEX solver 12.3 as IP solver software [21]. As heuristic, we use the version of NSGA2 provided by Deb at [34], with the following parameters: a population size of 100, and 500 generations. We also use our own implementation of PAES, based on the C version provided by J. Knowles at [35]. The depth parameter of the algorithm is used to define the grain of the grid used for measuring the crowding of the objective space by the members of the PAES archive. It defines the number of recursive subdivisions of the range of values for each objective. It is set to 4, according to PAES recommendations for some biobjective problems. The archive size for PAES is of 100 and the number of generations of 1000. The parameters for the local search are $\lambda = 5$ (number of directions for search), corresponding to $\sigma \in \{0.00, 0.25, 0.50, 0.75, 1.00\}$ and $\tau = min(p, 5) = 5$ for the number of seeds, since p is always greater than 5 for the Beasley benchmark test-cases (see Table 1). Those parameters have been set in order to equilibrate the computational effort of the different algorithms, as execution times comparison will show. They lead to 1000 (resp. 50.000) standard evaluations and $25.000\times(n-p)$ (resp. $2.500\times(n-p)$) fast evaluations for hybrid PAES (resp. hybrid NSGA2) (see Section 5.2). All of the tests are performed onto a 48 processors (Xeon E5 at 2.2GHz) SMP machine, with 132GB of memory, and running Linux CentOS.

(c) Quality Evaluation Procedure. The sets of solutions provided by the algorithms are to be compared qualitatively. Many metrics exist [28] that must take into account both the quality of solutions obtained (accuracy) and their number and location along the Pareto front range (diversity). The hypervolume (HV) [10] is often used to compare 2 sets of solutions $front_1$ and $front_2$. It computes the area defined by each set; according to a reference point dominated by all of the points in the sets, the highest value is the best one. Figure 1 illustrates the comparison by HV for 2 fronts, with two objective functions f_1 and f_2 to be minimized.

For each test-case, we run each algorithm in competition 30 times. We collect all of the resulting individuals and compute the associated nondominated front. This front's bounds on objectives are then computed and used for normalizing the objective vectors associated with the individuals into the

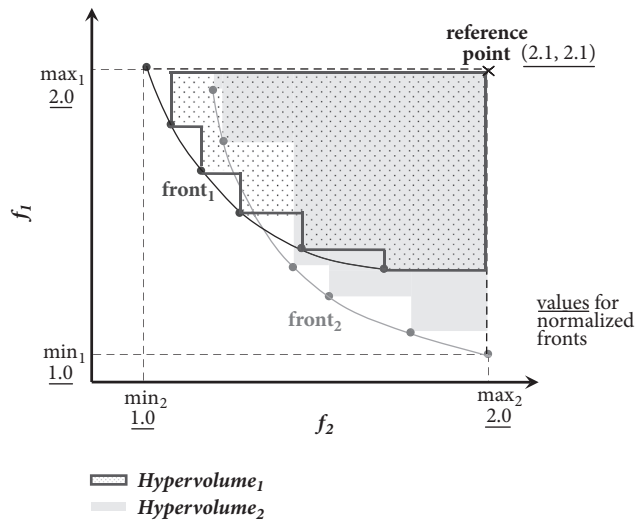

FIGURE 1: Hypervolume (here an area, for 2 objective functions) for 2 fronts (inspired from [36]). Objective functions are to be minimized.

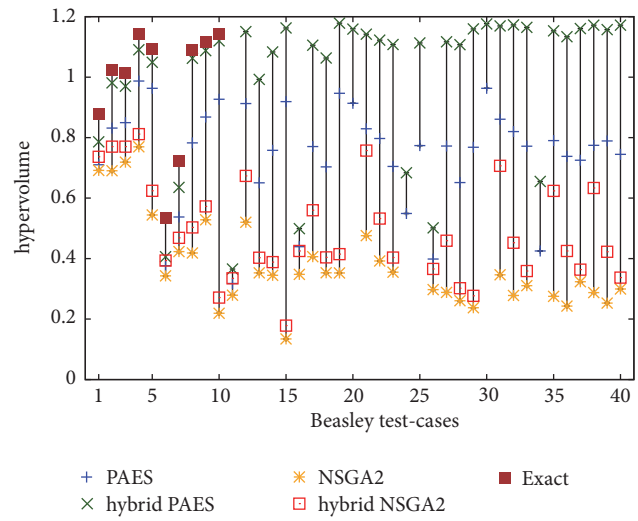

FIGURE 2: Average compared quality of the solution obtained by the different algorithms for 30 runs on the 40 Beasley benchmark test-cases (10 first ones for the Exact method). Quality of a solution is expressed as its hypervolume.

range $[1, 2]$. The point $(2.1, 2.1)$ is thus dominated by all of the vectors and can be used for computing hypervolumes (see values in Figure 1), leading to a maximal possible HV of $(1.1)^2 = 1.21$, if a single objective vector dominates all of the others. We provide results for each algorithm and also make a statistical analysis with the Kruskal-Wallis nonparametric test [37], for comparing the quality value series obtained by running each algorithm 30 times for a given test-case: if and only if a first test for significance of any differences between the samples is passed (H0 hypothesis), at a given confidence value (we use the standard 0.05 value), then the output will be the one-tailored p values for rejecting a null hypothesis of no significant difference between two samples, for each pair-wise combination. If the p value for a test-case is less than 0.05, we considerate that one algorithm beats the other for this test-case with a sufficient confidence. On the contrary, if series are not different enough, or the p value obtained for characterizing the differences is over 0.05, the test-case is not taken into account for comparing the couple of algorithms.

6.1. Hypervolumes Comparison. We compare the hypervolumes obtained for 5 algorithms:

(i) the *Exact* algorithm described in Section 4. The 10 first test-cases of the Beasley benchmark can be solved in less than 1 hour with this method, providing a reference front for comparison with heuristics for this subset of the Beasley benchmark.

(ii) the *PAES* algorithm, with our selection method, but without local search.

(iii) the *hybrid PAES* algorithm, embedding local search at each generation.

(iv) the *NSGA2* algorithm,

(v) and the *hybrid NSGA2* algorithm, using the local search as a postoptimization method

All the heuristics are applied onto the whole benchmark, with parameters as previously described. We have removed from the resulting mean values 5 test-cases (#20, #24, #25, #30, and #34): for those ones, NSGA2 (and thus also hybrid NSGA2) does not provide any feasible solution for at least 5 runs (all runs for #30). This is due to the fact that unfeasibility corresponds to duplicated schools in solutions, and this can arise with a higher probability when p increases. For the discarded test-cases, p is between 100 and 200 (see Table 1). Statistics are thus calculated over the 35 remaining test-cases.

Figure 2 shows the average values of hypervolumes for the different algorithms in competition. On average over the runs for the 40 test-cases, hybrid PAES provides the best results for all test-cases, when considering only EMOAs. It outperforms, respectively, PAES of 24.9%, NSGA2 of 58.7%, and hybrid NSGA2 of 48.0% on average. For the 10 test-cases for which exact solution is known, it provides a front with an HV that is, on average, at 7% of the optimal one. Clearly, the generic NSGA2 framework do not allow to obtain good results, as compared to the specialization of PAES for the School Problem: within NSGA2 version, the only specific component is the objective function, with nonspecialized mutation and crossover operators that can lead to unfeasible solutions. This is managed with NSGA2 by constraints penalties, but, unfortunately, this mechanism does not allow an efficient search of the feasible solution space. As a symptom, the size of the fronts provided by the NSGA2 algorithm is on average 68% less that those found by hybrid PAES (63% less for hybrid NSGA2). It is 49% when comparing PAES and hybrid PAES, showing that local search effectively helps in discovering nondominated solutions. The result is that local search allows improving results: hybrid PAES outperforms PAES by 24.9% and the local search by hybrid NSGA2 allows improving NSGA2 results in terms of HV by 32.7% on average. The latter is very significant, for a single local search phase, but the improvement is realized on the (relatively to hybrid PAES) poor results obtained with NSGA2, which leads room for optimization.

Table 2: Number of test-cases, over the 40 test-cases of the Beasley benchmark, for which the Kruskal-Wallis nonparametric test validates the hypothesis that EMOAs beat each other for hypervolume.

beats ↱	PAES	hybrid PAES	NSGA2	hybrid NSGA2
PAES	-	0	35	32
hybrid PAES	39	-	38	36
NSGA2	0	0	-	0
hybrid NSGA2	7	1	39	-

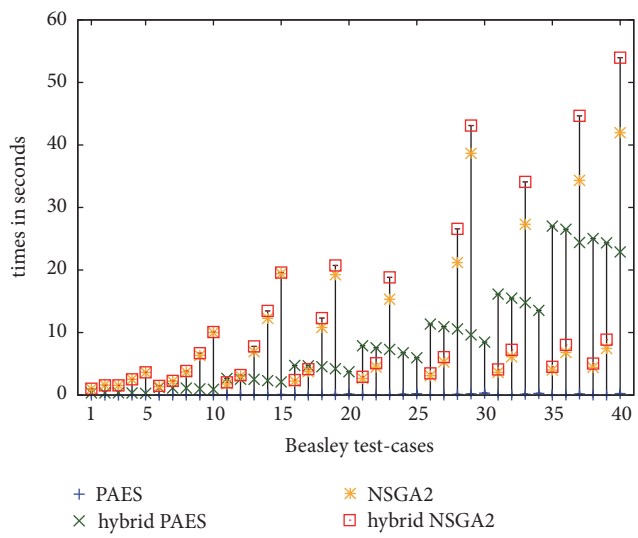

Figure 3: Compared mean execution times (30 runs) for different algorithms on the Beasley benchmark. Exact algorithms execution times, not presented, range from hours.

Table 2 assesses the confidence that can be given when comparing those mean results, by applying the Kruskal-Wallis nonparametric test to the hypervolume series. Even if hybrid PAES always provides mean HV values better than other EMOAs in competition, this is not completely assessed by the statistical test for all of them; e.g., hybrid PAES beats hybrid NSGA2 36 times only with the defined confidence of 0.05. This can be explained by the closeness of the results in some cases: for example, for test-case #6, mean HV is of 0.4061 for hybrid PAES and 0.3937 for hybrid NSGA2, with respective standard deviation of 0.002 and 0.030, meaning that result values of the two algorithms overlap.

6.2. Execution Times. Average execution times of the different heuristic approaches are depicted in Figure 3 for test-cases of Figure 2.

PAES algorithm runs in less than a second for all of the test-cases, thanks to the single evaluation at each generation. Because of the local search executed only as a postprocessing step, hybrid NSGA2 execution time is very close to NSGA2's one, with a local search realized, on average, in 1.62 seconds for the 100 individuals of the population. One can see that the same order of computational effort has been used for the different algorithms, by tuning population size and number of generations (*popsize* = 100 and *generations* = 500 for NSGA2 versions, *generations* = 1000 for PAES and its hybrid version). Those parameters allow for exhibition experimentally that execution times of tested algorithms depend on a combination of p and n (as shown in Section 5.2). Reminding the characteristics of the problems in Table 1, NSGA2 (and the hybrid version) beats hybrid PAES for p values of 5 and 10 if the number of nodes $n \leq 200$, and hybrid PAES is faster in the other cases. Furthermore, NSGA2 execution times are positively correlated to the chromosome size for genetic operations, i.e., to the value of p (e.g., $n = 100$ for instances 1 to 5, with p from 5 to 33). The decreasing steps in curve for the hybrid PAES correspond also to the increasing values of p, for a fixed number of nodes (e.g., $n = 600$ for instances 26 to 30), but with the inverse effect as the one depicted for NSGA2. Concerning hybrid PAES, for the local search, the number of seeds τ is fixed (see Section 5.2), the number of alternatives for each seed is of $(n - p)$, leading to a size of $\tau.(n - p)$ for the neighborhood. Thus, the more the p value, the smaller the neighborhood.

(a) Parallelization. Hybrid PAES is promising in terms of quality as compared to (hybrid) NSGA2 but time consuming as compared to PAES. We have implemented a master-slave scheme for realizing the evaluation and local search in parallel for different individuals, inspired from [30]. Parallel hybrid PAES implementation (in C) uses multiple threads (one per slave plus one for the master process). The OS scheduler allocates each of them to a core if the number of cores is sufficient. Figure 4 shows the speedups

$$\frac{\text{exec. time parallel hybrid PAES with 1 slave}}{\text{exec. time parallel hybrid PAES with X slaves}} \tag{22}$$

obtained when running the algorithm with more than one slave (and thus more than 2 cores) for the evaluation and local search of individuals. The execution times are obtained using a 48 cores SMP system, ensuring that each slave (and the master) is run on a separate core. In order to increase the computational effort, the number of generations is set to 5000 instead of 1000 in previous tests.

Clearly, adding computing resources helps in speeding up the computations and especially when the computational effort is high (last test-cases). The average speedup is 1.89 (resp. 3.76, 7.48, 14.07, and 18.10) for 2 (resp. 4, 8, 16, and 32) slaves. Speedups look linear according to the number of slaves for up to 8 slaves. Classically, the cost of the parallelization is due to the bottleneck in communications induced by the master-slave scheme and also to parallelization itself: for example, for test-case #35, the sequential version of hybrid PAES runs in 29 seconds for 1000 runs (see Figure 3) and the parallel version, with a single slave, runs in 273 seconds for 5000 generations (instead of expected 29 seconds × 5 = 145 seconds). The communication bottleneck degrades the speedup for 16 slaves, but execution times are still improved in all cases as compared to 8 slaves version, except for test-case #5 (average speedup of 14.07). But for 32 slaves, the improvement of speedup is very limited (18.10 on average). As we find again the same steps shape on curves as those observed in Figure 3 for sequential times, with roughly higher speedups for the most costly test-cases, it is possible that the

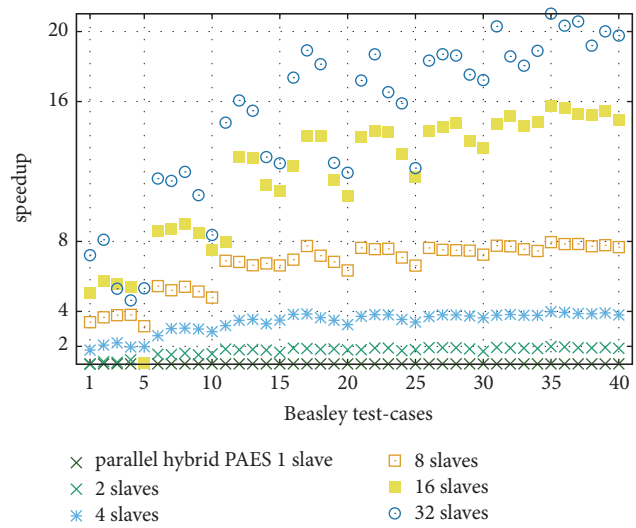

Figure 4: Speedups obtained with up to 32 slaves for hybrid PAES on the Beasley benchmark.

parallelism grain is not sufficient to ensure good acceleration with 32 slaves. Notice that this can be tuned by both σ (number of directions for local search) and τ (number of seeds) parameters of the hybrid PAES (see Section 5.2).

Parallelization helps in reducing the drawback of large execution times for hybrid PAES. For example, the largest execution time (test-case #35) is 273 seconds for the algorithm with a single slave and 5000 generations, and it is decreased to 13 seconds with 32 slaves. Since speedups increase with both the number of slaves and the size of the problem handled, the parallel approach for hybrid PAES looks promising for the processing of real very large data.

7. Conclusion and Future Work

We present in this paper a MOO modelling of a facilities localization problem, applied to a multimode transportation problem, the School Problem. The goal is to optimize the transportation mode for pupils, with two possible modes, namely, by foot and by public transport, with constraints on the walking solution. A modelling for the School Problem is proposed, and an exact resolution method, based onto an Integer Programming formulation, is defined. The IP-based approach solves in a few minutes to one hour each of the 10 smallest instances of the Beasley public benchmark. A heuristic method called hybrid PAES mixes p-median neighborhood search with PAES EMOA. Our approach is able to solve the same 10 instances in less than 10 seconds each, with an average degradation of quality (measured with the hypervolume metric) of 7%. Hybrid PAES also provides solutions for large instances for which the Pareto front is unknown. For those cases, it is competitive with a standard approach as NSGA2, with results that outperform this reference method by 58% and 49% for hybrid NSGA2. The execution times are also improved as compared to NSGA2 when the problem complexity is significant enough ($n > 200$ and $p > 10$). A parallel version of hybrid PAES is also proposed, using a master-slave scheme. The speedup of the algorithm is linear for up to 8 slaves, close to 14 for 16 slaves, but degrades with more slaves (speedup is only 18.2 for 32 slaves). We think that this limitation on the parallelization degree is due to the grain of parallelism induced by the data and the algorithm's setup for the experiments and that it will not hold for larger instances. We plan to apply it to real data, as realized in [24] for single objective p-median problems, dealing with country-scale instances, with nonuniform population distribution. This is important because real problems may exhibit particularities in their data. Another direction of research is to add a third objective function, related to facility implantation costs, with a relaxation of the number of facilities (p becomes a bound instead of a fixed value). This economic cost is useful, for example, in disasters management.

Conflicts of Interest

The authors declare that there is are conflicts of interest regarding the publication of this article.

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Research Article

Integration between Transport Models and Cost-Benefit Analysis to Support Decision-Making Practices: Two Applications in Northern Italy

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Decisions on transport plans and projects involve relevant public investments and may also determine radical changes in users' costs. Unfortunately, it is not rare that—especially at the strategic planning stage—decisions on alternative projects or scenarios are made on a qualitative basis or, at best, by setting some indicators and verifying how much they reach the politically decided targets (e.g., "increasing the use of bicycles by 10%"). In order to reduce subjectivity, a more quantitative and comprehensive approach to the evaluation is needed. A Cost-Benefit Analysis is a tool commonly used to assess public expenditure, but its application to mobility plans introduces further practical and theoretical complexities. In this paper, we will thus try to contribute to the topic of the assessment of both sustainable mobility transport plans and infrastructure projects by presenting the operative application of a CBA methodology that is, at the same time, theoretically coherent and rich in outputs to support the decision-maker. Moreover, we will discuss the possible use of GIS software in order to provide to the decision-makers a clear and immediate "picture" of the effects on the network linked to different scenarios. The structure is as follows. Firstly, we discuss the complexities involved in the evaluation of plans with respect to a single infrastructure. Secondly, we introduce the available approaches for the assessment of consumer surplus, namely, the *Rule of Half* and the *logsum* function method, which allow the perfect integration between CBA and transport models. Thirdly, we present, through some operative case studies, the methodologies applied to the assessment and the network effects visualization of the urban mobility plan and new infrastructures. Finally, we underline how we can make the results more understandable to politicians, policy-makers, stakeholders, and citizens and in general improve the transparency and the awareness of the choices.

1. Introduction: Evaluating Transport Plans and Infrastructural Projects

Modern transport plans try to achieve different objectives and, in general, try to obtain a more sustainable and inclusive transport system, as suggested by the recent European *Sustainable Urban Mobility Plans* (SUMPs) guidelines [1]. This is usually done by foreseeing, simultaneously, the improvement of existing transport services and infrastructure, together with the implementation of sustainable mobility policies. This also allows taking advantage of the benefits of *policy packaging* in terms of higher acceptability, effectiveness, and efficiency [2]. Among the available policies, we can recall the promotion of city walkability, the development of bike transport, road pricing, park pricing, technological investments on networks, vehicles, and communications, smart mobility, vehicle sharing projects, incentives, redefinition of fares structure, mobility credits, and so on. [1, 3, 4].

When performing the evaluation of such complex plans, a mismatch occurs between the available assessment techniques and the contents and expectations of plans. In fact, the economic evaluation of transport investments is usually intended in terms of assessment of transport infrastructure, while plans include a broader range of actions, where expected environmental and social impact can play a dominant role in policy evaluation [5].

Table 1: Policy actions in a mobility plan and type of effect.

Action/policy	Costs	Benefits	Investment costs are dominant versus other costs?
New infrastructure (public transport, roads capacity)	PU, T	pr, C, t	Yes
New/modified services on existing infrastructure	PU, t	pr, d, t	No
Innovative mobility (sharing, pooling, etc.)		pr, d, t	No
Bicycle lanes, bike parking, bike sharing	PU, T	pr, d, t	Yes/No
ITS, traffic management	PU, t	pr, d, t	No
Restrictive policies, car-free zones, traffic calming	pr, d, t	d, t	No
Park and road pricing	pr, d, t	PU, pr, d, t	No
Public transport fares	PU/pr, d, t	Pr/PU, d, t	No

PU: public bodies; pr: private users; C: concentrated in space or limited to groups; d: diffused in space or spread among many users; T: punctual in time, lump sum; t: continuous in time.

The mismatch lies also in the evaluation tools commonly used to support the design of mobility plans and the consequent public decisions, which are substantially referable to

(i) Traditional *Cost-Benefit Analysis* (CBA), required practically everywhere to allocate public money for infrastructure investments, that is, in cases where the public decision is dominated by the alternative allocation of lump sums [6–8];

(ii) *Multicriteria analyses* (MCA) and *Indicators-based assessments,* used to structure and clarify the goals and the (possibly positive) effects of the actions of a plan, such as the foreseen decrease in car ownership or pollutants concentrations [9, 10].

In the assessment of a plan made of both "hard" (infrastructural) investments and "soft" policies, both traditional approaches may fail. In particular, a rigid CBA may not be able to catch all the effects of the plan [7, 8, 11, 12] and, in addition, provides too concise outputs to support the dialogue with public opinion and stakeholders [13]. On the other side, indicators and MCA are, by definition, not able to measure the efficiency of public expenditure, which represents a key element of public decisions [7, 8]. Some EU members state that, in order to include nonmonetized impacts in infrastructure projects or policy evaluation, they opted for MCA in which a CBA is contained [14, 15].

Moreover, one must consider that the "typical" scheme of lump sum public costs versus distributed private benefits is not always the case, especially with pricing or limitation policies. In these cases, the policy gives both advantages to some groups (e.g., public transport users) and disadvantages to others (e.g., car drivers), in some cases entailing no significant public expenditure. A decision on this kind of policy, which is not simply "adding" new transport supply for someone, should obviously take into account not only the benefits, but also the public *and* the private costs necessary to obtain them.

Table 1 provides an example of possible policy actions, with different spatial and time boundaries of their effects, which may be included and assessed in a complex urban mobility plan.

In plans, additive policies, such as new infrastructures whose costs are public and concentrated in time, coexist with restrictive policies, such as traffic calming, pricing, and so on, which changes mobility patterns through raising the private costs. These two extremes must be assessed in a coherent way, but their effects and economic mechanisms are substantially different.

Moreover, the effects act synergistically, so that modal shift is the effect of both the improvement of the destination mode (e.g., public transport) and the worsening of the origin mode (e.g., private road transport). When the existence of such complex cost and benefit structures in the assessment of plans or projects is recognised, theoretical and practical problems may arise. As already mentioned, approaches using only indicators and MCA are not satisfactory for evaluating the trade-offs of public expenditure. At the same time, CBA in the "simple" form, as suggested by numerous guidelines (e.g., [16, 17]), is not sufficiently complex to handle the previously mentioned problems and, in addition, fails in effectively representing the distribution of effects [18], especially for non-*win-win* policies.

Lastly, especially when CBAs are implemented to investigate complex transport scenarios, problems can arise from the imbalance between expert knowledge and various stakeholders' expectations on their "pet-projects" [13]. In order to promote a constructive dialogue in the decision-making process, not only must CBAs be rigorous from a methodological point of view, but also their final and, possibly, intermediate results have to be both intuitive and effective for a general audience.

For such reasons, in this paper, we will contribute to the topic of decision-making for complex plans and projects in a threefold way:

(a) How to manage the consumer surplus from a theoretical viewpoint. While theoretical literature on methods is mature and clear, it does not appear helpful from the point of view of practical applications. On

the other side, guidelines are often too simplistic to handle complex problems, or even misleading.

(b) How to practically calculate the consumer surplus for both a mobility plan and an infrastructural project based on transport model outputs.

(c) How to enrich the outputs of the evaluation in order to make the assessment useful to inform decision-makers' choices.

The paper is structured as follows. Section 2 discusses the two correct ways to calculate consumer surplus, the *Rule of Half* and the *logsum* function method, underlining both their pros and cons and in which situations they are recommended. Section 3 illustrates how these two methods can be calculated, in practice, by means of a transport simulation model, and how to modify the general formula when an active mode is included in simulations. Section 4 presents two evaluation cases: Milan's Sustainable Urban Mobility Plan and the Milan Malpensa airport rail ring closure. Section 5 concludes.

2. Calculating the Consumer Surplus

CBA is a widely used and codified technique. The transformation into monetary values of the majority of variables taken into account in the analysis can be done in an intuitive way (investment costs, running costs, environmental benefits, taxation, etc.; see, e.g., [16, 17]), but we cannot say the same for the consumer surplus. This component, especially when related to complex plans, may be misleading and concurring in the introduction of double counts or internal incoherencies [19, 20].

Many guidelines indicate three possible approaches to estimate the consumer surplus. In practice, their application is seldom discussed and the responsibility to choose among them is left to technicians. For example, in Italy the guideline provided by the Ministry of Transport [21] does not suggest the application of a specific method to calculate user surplus, while the Lombardy Region guidelines define the Rule of Half method as mandatory to evaluate infrastructural projects [22]. Nevertheless, the three methods return results that are not always comparable. In particular, the simplest approach, the generalised costs comparison, has results that are adequate only under very specific and seldom-present conditions [23]. The other two methods ensure a different level of robustness and precision and require different amounts of data but can both be suited to almost any application. In the following, we will illustrate these two latter methods, and we will underline when they are almost perfectly substitutable and when; on the contrary, we can expect considerably different outputs.

2.1. The Rule of Half (RoH) Method. The consumer surplus is defined as the difference between user willingness to pay to make a specific trip and the "price payed" to make it [24]. In transport, the "payed price" by each user to make a specific trip is codified in the so-called *generalised cost* (GC). The GC represents the monetary value that the user associates with his overall trip experience, including out-of-pocket expenses (tolls and fares), operating costs of vehicles (in private transport), and consumed time. Other factors like discomfort, crowding, landscape, and more personal attitudes can also reduce or increase the GC of the same trip for different users (in the paper, we refer to the calculation of user surplus in terms of perceived (or private) costs, which—as we discussed in a former working paper [20]—requires some corrections to balance transfers (e.g., taxes and fares) among different parts of society; another relatively widespread approach, especially when the GC comparison is used, is to calculate the variation in user surplus directly in terms of social costs; the considerations made in this paper remain valid).

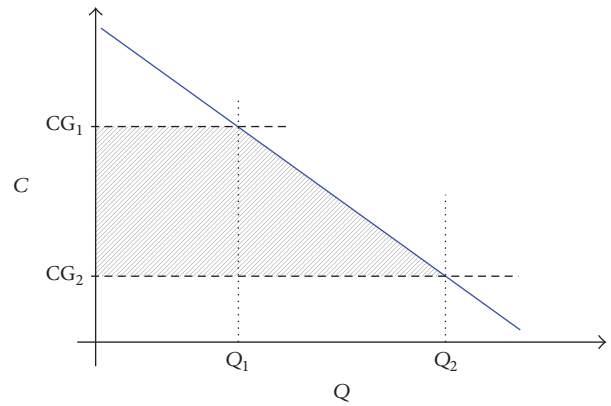


Figure 1: Representation of the Rule of Half (our elaboration).

Thus, intuitively, the variation between the GC before and after a project or policy implementation will return the gain or loss in benefit (surplus) for a specific user even when he shifts between different means of transport.

Unfortunately, information on the single user trip experience is not always available. More realistically, when available, we can use a transport model to obtain the average generalised costs of groups of users (see Section 3), also in a very detailed way. The problem that arises is how to treat shifting users knowing only the GC average value and the total quantity of users in each group.

Traditionally, the most robust and used method to calculate the variations in user surplus when a transport model is not available is to hypothesize the unknown demand curve as a linear function between two points. These points, which are always known, are codified by the intersection between GCs and the total amount of users (Q) before and after the project implementation [16, 19, 20].

This hypothesis allows the application of the so-called *Rule of Half*. The area of the rectangle having base Q_1 (the number of existing users) and height $GC_1 - GC_2$ (the reduction in generalised costs, that is, the unit benefit) represents the surplus variation of existing users. Similarly, the surplus of generated or shifted users is given by the area of the triangle having base $Q_2 - Q_1$ (the number of new users) and the same height $GC_1 - GC_2$ (Figure 1).

The variation in user surplus is thus given by the area of a trapezius as in (1) (hence the name "of half"):

$$\Delta S_{\text{users}} = (Q_1) \times (\text{GC}_1 - \text{GC}_2) + \frac{1}{2}(Q_2 - Q_1) \times (\text{GC}_1 - \text{GC}_2) \quad (1)$$
$$\Delta S_{\text{users}} = \frac{1}{2}(Q_1 + Q_2) \times (\text{GC}_1 - \text{GC}_2).$$

Due to its simplicity and the relatively reduced amount of data needed in the formula, this approach is very widespread. Any other method requires both estimating the GCs' absolute values, a much more difficult task, and knowing the GC of "origin" modes (the estimation of the absolute value of generalised costs is not a simple exercise without a transport model, especially in public transport and in active modes (walking and cycling); the related variation is instead usually made only of easily measurable items, like time savings and/or operating costs).

Moreover, this method, by treating the whole of induced users (i.e., both generated and shifted from other modes, paths, time of the day, etc.) in the same way and not needing the absolute values of the GCs, avoids all the problems linked to the generalised costs comparison approach. These problems are mainly determined by the impossibility of knowing the GC value for each single user [23].

2.2. The Logsum Function Method. The *RoH* assumes a certain distribution of users across average GCs. Depending on the required level of detail (and, of course, on the available data), analysing, by disaggregating as much as possible users in homogenous groups (i.e., in terms of geography, trip purpose, etc.) and applying the *RoH* to each group separately can minimise the error equal to the distance between the single user GC and the average group value.

Despite its methodology consistency, *RoH* returns an approximation of user surplus based on a "strong" preliminary assumption: a linear distribution between demand and GCs.

However, when a calibrated transport model is available, it is possible to use another method to assess the variation in user surplus, which gives a much more detailed representation of the benefits. By measuring, for all alternatives (modes), the variation in the composite utility *(logsum)* returned by the transport model, it is possible to calculate not only the average values of the GCs, but also their implicit distribution among users, thus having a more precise variation in user surplus [25].

Most transport models, in fact, estimate the share of users [p] that will choose a transport mode [m], on the origin-destination pair [od], for the trip purpose [s], using the multinomial logit formula described in (2) (if the model is based on a nested logit, this operation can be done on the first (higher) level of the logit; the GC represents the disutility of the trip) [26, 27]. Parameter [λ_s] is a calibrated value that maximizes the probability that the expected (simulated) modal share is coherent with the observed value (real data used in calibration) for the users travelling for the purpose group [s].

$$p_{\text{od}|s|m} = \frac{e^{\lambda_s \cdot \text{GC}_{\text{od}|s|m}}}{\sum_m e^{\lambda_s \cdot \text{GC}_{\text{od}|s|m}}}. \quad (2)$$

Thus, the unitary surplus variation is equal to the difference in the composite utility (given by the logarithm of the denominator of the logit formula (thus the name, "logsum"), divided by the calibration parameter), before and after projects and/or policies implementation. Once we multiply this unitary value for the number of trips related to each single purpose, we obtain the user surplus variation linked with that trip purpose [ΔS] [25, 28–30] (Bates [31] states that "*with a logit model, there is a closed form solution for the integral under the demand curve,*" which is the surplus).

$$\Delta S_{\text{od}|s} = \text{trips}_s \cdot \frac{1}{\lambda_s} \cdot \left[\left(\ln \sum_m e^{\lambda_s \cdot \text{GC}_{1,\text{od}|s|m}}\right) - \left(\ln \sum_m e^{\lambda_s \cdot \text{GC}_{2,\text{od}|s|m}}\right)\right]. \quad (3)$$

The sum of these variations for all the user groups obviously gives the total variation in consumer surplus associated with the scheme or policy.

If we expect generated demand to appear in the postproject scenario, and we want to keep using the *logsum* approach, Bates [31] suggests applying the *Rule of Half* to the standard *logsum* formula (4):

$$\Delta S_{\text{od}|s} = \frac{1}{2} \cdot \left(\text{trips}_{1,s} + \text{trips}_{2,s}\right) \cdot \frac{1}{\lambda_s} \cdot \left[\left(\ln \sum_m e^{\lambda_s \cdot \text{GC}_{2,\text{od}|s|m}}\right) - \left(\ln \sum_m e^{\lambda_s \cdot \text{GC}_{1,\text{od}|s|m}}\right)\right]. \quad (4)$$

Despite being well-discussed in the literature quoted above, the *logsum* function method has been barely seen from an applicative point of view, when real projects and plans must be practically assessed. The following part of this paper tries to go more deeply in this direction (a more detailed discussion on differences between logsum and Rule of Half methods is presented in Maffii et al. [32], Cascetta [25], Geurs et al. [33], Grimaldi and Beria [20], and Bates [31]).

3. Consistent Integration between Planning, Modelling, and Assessment

3.1. Benefit from a Direct Integration of CBA and Transport Models. The most complex aspect to model in a CBA is the demand curve and its relationship with the replaced assets when generated demand is present. In the transport sector, the demand generated by a certain project is the sum of the new demand induced by the lower transport cost (people who now travel and who did not travel before, given their lower

level of willingness to pay) and the demand attracted from other means of transport.

Generally, transport problems are faced with four-stage multimodal models (guidelines from Lombardy Region [22, p. 18] and the Italian Ministry of Transport [21, p. 30] define the use of multimodal transport models to evaluate infrastructural projects as mandatory). These models allow both the distribution of the existing and generated (induced and/or attracted) demand between OD relations and, considering the relative generalised costs that determined the choices, the calculation of users' paths.

Within the four-stage models, the modal choice can be assimilated to the application of a demand curve. A certain perceived cost (according to the CBA formulation) or generalised cost (according to the modelling formulation), corresponds to a certain quantity consumed. Therefore, the integration between models and CBAs allows greater precision (ensured by the model) in determining the costs for all users on the entire network modelled (instead of considering separately the users on specific segments of the network) and a perfect match between model and evaluation.

In conclusion, while without a model, the CBA must necessarily be based on rather strong assumptions (invariance of the demand or, in the case of demand generated or shifted, assuming a linear trend for the demand curve); with a model, the descriptions of both demand and user surplus variation will be much more detailed. Moreover, parameters such as the value of time can be directly those of the calibrated model.

3.2. Extracting Generalised Costs. When a transport model is available, it is relatively easy to extract the GCs directly from it, guaranteeing a complete consistency between the model and the following Cost-Benefit Analysis. The GCs derived this way can be used both with the *Rule of Half* and with the *logsum* method.

To obtain GCs, it is sufficient to extract the systematic utilities used by the model, which is usually constructed like in (5)(with systematic utility V we mean, according to the definition by Cascetta [25], *"the mean or the expected value of the utility perceived among all the users with the same choice context"*; the perceived utility U is given by the sum of the systematic utility V and the random residual $\mathcal{E}$ (which represents the deviation of the single user with respect to the average value): $U = V + \mathcal{E}$).

$$V_{\mathrm{od}|s|m} = \beta_{s|m}^{\mathrm{Time}} \cdot \mathrm{Time} + \beta_{s|m}^{\mathrm{Cost}} \cdot \mathrm{Cost} + \mathrm{Other\_components}. \tag{5}$$

If we divide the systematic utility by the parameter related to its monetary component, we express it in monetary terms representing the monetary trade-off that users attribute to their trips, which is the GC.

$$\mathrm{GC}_{\mathrm{od}|s|m} = \frac{V_{\mathrm{od}|s|m}}{\beta_{s|m}^{\mathrm{Cost}}}. \tag{6}$$

Some transport modes might not be associated with any direct monetary component (e.g., walking and cycling). There is a way to overcome the issue, though with some loss of consistency (see, e.g., [34] or [29]). We divide the systematic utility by the time parameter (which is always present) instead of by the cost one, thus deriving a *generalised time* instead of the *generalised cost*.

$$\mathrm{GT}_{\mathrm{od}|s|m} = \frac{V_{\mathrm{od}|s|m}}{\beta_{s|m}^{\mathrm{Time}}}. \tag{7}$$

Then we multiply this generalised time by an exogenous value of (in-vehicle) time (VoT), obtaining the generalised cost.

$$\mathrm{GC}_{\mathrm{od}|s|m} = \mathrm{VoT} \cdot \mathrm{GT}_{\mathrm{od}|s|m}. \tag{8}$$

When the GCs are estimated, we can directly apply the formulas of the surplus variation, discussed in Section 2. It must be noticed that this passage is done at the highest level of disaggregation: per origin–destination pair (od), travel purpose (s), and mode (m). This is a computational burden, involving even millions of operations if the study area is divided into hundreds or thousands of zones, like in urban or regional areas. However, this disaggregation allows for enriching the readability and completeness of outputs.

3.3. Presenting the Results. In fact, a nonsecondary aspect deals with the outputs of the analysis. Backing the surplus calculation methods with a calibrated model and working at the origin–destination pair allows the representation of the benefits geographically, and not only in an aggregate way. This kind of representation may be a useful complement to the aggregate CBA indicators (NPV, NBIR, B/C, IRR; [16]) both in the consultation phase and, ultimately, in making better decisions.

Indeed, the network effects visualization, thanks to a GIS software, of a proposed project or policy allows for foreseeing which parts of the territory (and, consequently, which citizen groups) will gain or lose more if that action takes place.

In the next section, we present the methodologies applied for the assessment and network effects visualization of the sustainable urban mobility plan of the Milan municipality (see Section 4.1) and the proposed rail ring of the Malpensa airport (see Section 4.2). Due to their specificities, for the plan evaluation, we utilized the *logsum* method, which assures that nonadditive policies do not bring the risk of double counts, and the *RoH* one for the rail project.

4. Theory Application: Two Case Studies

4.1. Milan's Sustainable Urban Mobility Plan. For the assessment of the new Milan's Sustainable Urban Mobility Plan [35] we interfaced the simulation model developed by the transport authority (AMAT), with the assessment procedure, based on the interaction between a database and spreadsheet software. The transport model has been used extensively in the past years to plan all transport decisions in the city and can rely on solid datasets of observations, used for the calibration. Not only CBA, but also The Strategic Environmental Assessment of Milan's SUMP was based on the same transport model outputs. However, these two evaluations

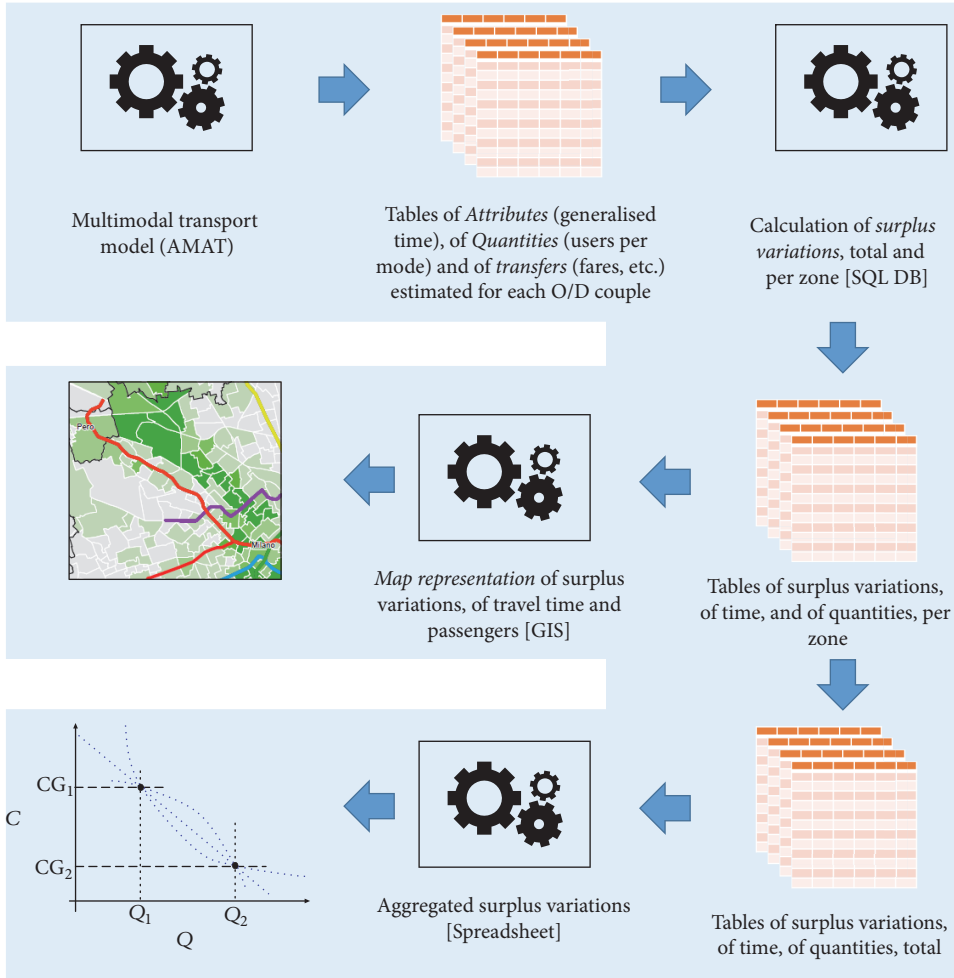


Figure 2: The assessment procedure implemented for the Milan's Sustainable Urban Mobility Plan (SUMP).

differ for some assumptions on input data (e.g., CBA, in the environmental effects calculation, considers, for the year 2024, an equal distribution between the total number of Euro V and VI vehicles, while the latter defines the vehicle's fleet starting from a regression model on gasoline sale trends). However, the overestimation "error" magnitude is contained and does not affect the overall CBA result [36, pag. 112].

A schematic representation of the algorithm developed for SUMP scenario's assessment is depicted in Figure 2.

The transport model provided the generalised cost components of each origin–destination (OD) pair for five different travel purposes (plus the "back-to-home" purpose) and for four modes (car, motorbike, public transport, and active modes). Each segment of the simulated mobility is associated with a set of calibrated β coefficients (5), including the values of time (per trip purpose and per mode). In addition, the model provides the quantities, in terms of passengers/users during the peak hour, for every mode on each OD pair, allowing us to calculate the mode-shifting users. Finally, the model output also includes the data used to correct the monetary transfers in the CBA, in particular the paid fares, the parking tolls, the road pricing tolls, the driven km (useful to calculate the fuel duties paid).

Overall, the output consists of a database file with tables of attributes (generalised time), quantities (amount of users), and transfers (fares, tolls, and taxes) per 390,452 origin–destination pairs and 24 (4×6) mode and travel purpose combinations. By means of SQL procedures, we estimated the surplus variation using the *logsum* method (as described in Section 2.2). It must be noticed that the core calculations are done at the highest disaggregation level, that is, per OD pair, mode, and travel purpose. Later, we proceeded with the aggregation of the results in 829 origin and destination zones, using the same zoning of the model. The same thing has been done for all attributes, quantities, and transfers.

Once the aggregation is done, the datasets become more manageable with other software programs. The procedure is then interfaced with a spreadsheet software for the CBA and with a GIS software to produce cartographical representations of the main variables.

The entire process has been applied for *51 explorative scenarios*. For this reason, we put particular care into automating

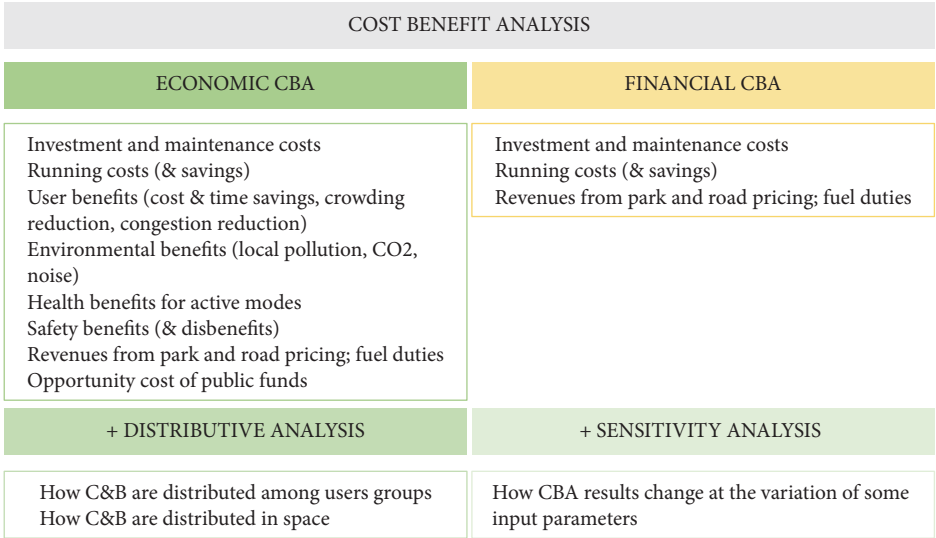


Figure 3: Cost-Benefit Analysis components.

the procedure with scripts. At the same time, we track all the intermediate results throughout every step to facilitate debug procedures. Some controls have been introduced intermediately to manually check the correctness of the results. The process also pointed out some local inconsistencies of the transport model, which have been corrected.

The CBA has been done within a spreadsheet software, in the two variants of Economic CBA and Financial CBA. The elements included are listed in Figure 3, and the indicators calculated are the Net Present Value (NPV), the Net Benefit over Investment Ratio (NBIR), and the Benefit over Cost Ratio (B/C).

In addition, we also developed in the same environment the distributive analysis module (disaggregating the costs and the benefits into six user categories, plus the nonusers, the State, and the Local Administration) and a simple Sensitivity Analysis, testing automatically the effects of user surplus estimation, investment cost, and running costs on results.

The CBA, shortly described here, was directly included in the planning process as suggested in European guidelines [1]. It is worth mentioning because this represents the first case in Italy. In fact, it was only in 2015 that the regional government, within its guidelines, imposed the CBA as mandatory to assess transport projects [22] and, at the national level, this evaluation tool became compulsory one year later [21].

4.1.1. Scenarios. In a preliminary phase, planners, stakeholders, and politicians selected, among all the actions included in previous planning documents and/or indicated by citizens, 51 explorative scenarios to be assessed via CBA. This preselection was carried out by choosing, according to the political goals, the most effective alternative between comparable projects facing a single problem.

The selected policies and projects were heterogeneous in nature, from heavy infrastructural investment (i.e., new metro lines) to punctual traffic calming solutions (i.e., 30 km/h speed limit areas). Table 2 provides an overview of all the scenarios considered in the analysis.

Table 2: SUMP explorative scenarios.

Explorative scenario	Number of subscenarios (alternative options)
Previous land use plan (PGT) infrastructure	3
Metro line 1 extension	3
Metro line 2 extension	5
Metro line 3 extension	3
Metro line 4 extension	4
Metro line 5 extension	3
New Metro line 6	3 (explorative paths)
Tram 7 extension	4
Tram 24, 27, and 178 extensions	2 + 2 + 1
Reorganisation of tram lines in the city centre	1
New urban stations to support rail "circle line" services	7
Change in a suburban rail line path	1
Extension of bike lanes	2
30 km/h speed limit areas	1
Road pricing (Area C) extension	4
Actions to increase commercial speed of surface public transport	2

The aim of this explorative evaluation was to identify those actions that returned positive CBA indicators or, in case of negative performances, under which conditions it was possible to reduce their negative socioeconomic impact. For example, in the case of the 30 km/h speed limit areas, negative indicators were returned when this action was assessed alone.

1: scenario name

2: socio-economic costs & benefits of the project (qualitative & quantitative description, monetised present value)

3: graphical representation of costs and benefits

4: financial analysis – 2024 cash flow

5: CBA Indicators, divided in «base» and «extended»

6: Issues not quantified in the CBA and discussed qualitatively:

- i. Crowding, quality
- ii. Reliability
- iii. Parking space in the city
- iv. Urban landscape
- v. Land value
- vi. Externalities during construction
- vii. Risk of cost overrun

7: distributive analysis – impacts on groups

8: sensitivity analysis

9: summary of the results and short comment

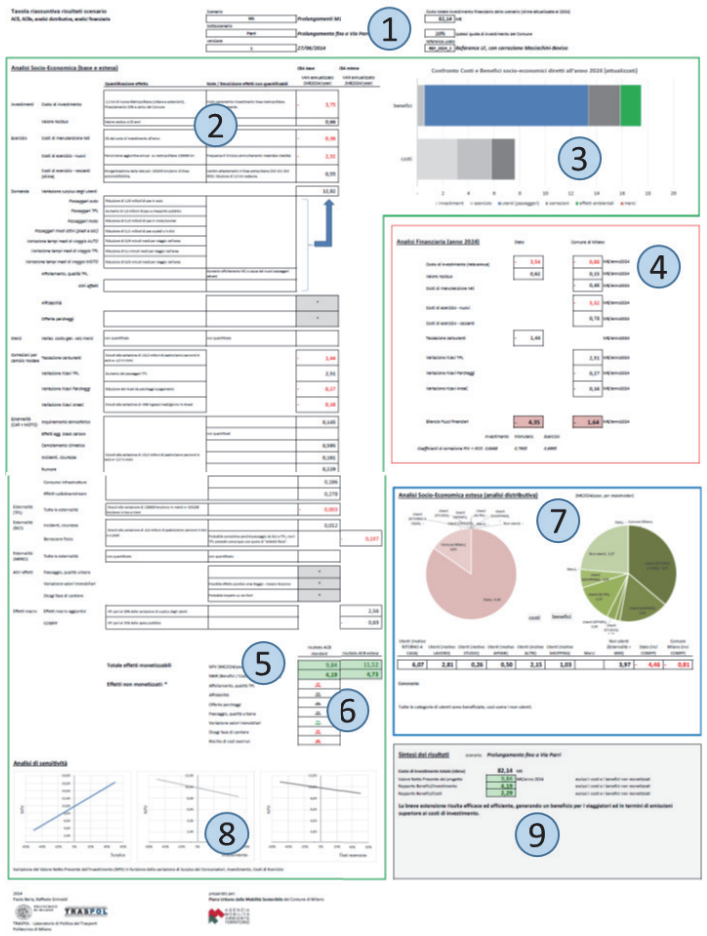


Figure 4: An example of Appraisal Summary Table organisation.

This intermediate result was a consequence of the reduction in road capacity and, consequently, an increase in congestion and travel time for private vehicle users. However, the same policy, evaluated in combination with other actions aimed at promoting a modal shift in favour of public transport, was associated with a positive impact in terms of user surplus.

At the end of this explorative phase, all positively assessed actions have been included in the *final Plan Scenarios* and evaluated together. This last scenario does not represent just a sum of all the positive/negative effects of the explorative ones, but takes into consideration the interdependency between the different actions within them.

4.1.2. Representing the Results. Thanks to the previously mentioned procedure, we were able to draw semiautomatically the outputs for the numerous scenarios. Moreover, final documents and analyses clarify all the main aspects that should be considered for the decisions, not limited to the sole aggregate CBA indicators (NPV, NBIR, and B/C). This result is fundamental in order to simplify the decision process of City Council, allowing different stakeholders to potentially revise their goals on the basis of a transparent and consistent analytical process.

To do that, all the intermediate and final results of the scenarios were presented through the following outputs.

(i) An *Appraisal Summary Table*, directly inspired by the ones used in the UK [37]

(ii) A *Book of Maps*, visualizing on a cartographic basis the effects of the hypothesis on users' behaviours and socioeconomic variables.

The first output included the socioeconomic assessment, the financial assessment, the distributive analysis, and a summary of the results complemented with some general comments (Figure 4).

Due to the different nature of the actions taken into consideration, we provided a double version of the Net Present Value (NPV), the Net Benefit over Investment Ratio (NBIR), and the Benefit Cost Ratio (B/C), calling them, respectively, "base" and "extended." The first set is based on a reliable quantification of all the monetized effects related to the single scenario implementation (investment, user surplus, externalities, etc.). The other set (extended), on the contrary, was an attempt to quantify, from an economic viewpoint, effects related to "soft" mobility policies such as health benefits for active modes, the opportunity cost of

public funds, and an approximation of the possible wider economic effects. In fact, in these cases, these aspects can be considered as the most relevant, and a CBA that does not include them is mainly incomplete. However, due to objective difficulties in quantifying some of these effects, we introduce these indicators mainly to underline a possible positive or negative threshold in comparison with those of the first set. For example, on health benefits related to bicycle users, different sources provide very variable values. From 0.03 €/km [38] up to 0.36 €/km [39] or 0.44 €/km [40]. For our analysis, we considered a weighted average value of 0.365 €/km. Moreover, other nonmonetary elements were returned only in a qualitative way (similar to what Sager [12] describes as a "narrow CBA").

Our aim, in fact, was to provide the decision-makers with not only a synthetic "number" to quantify the goodness of an action, but also a set of tools, giving politicians a certain level of informed discretion on the final decision.

The second output (Book of Maps) included, for each scenario, 24 cartographies showing the variations, both in absolute values and as a percentage, of user surplus, passengers per means of transport, and travel times and distances. Data was aggregated once per zone of origins and subsequently on destinations, allowing a complete visualization of the network effects connected to single scenarios.

This detailed output allows us to depict, for example, that users obtaining benefit from a metro or tram line extension are not only those citizens living or working next to it, but also users of other zones, related to different means of transport, benefiting from road decongestion. Similarly, this output is useful in directly visualizing the modal shift's spatial extension.

This tool is particularly effective in underlining unbalanced situations between different parts of the city (i.e., local benefit or, on the contrary, surplus loss due to punctual actions) and giving hints to policy-makers on how to address specific situations.

4.1.3. How Assessment Changed the Decisions. This approach proved to be a fundamental support for the redefinition of the strategic objectives of the public administration regarding the new type of mobility to sustain the future growth of the city. Through the combination of the aggregate results and benefits distribution maps, we were able to quantitatively effect a change of vision for the decision-maker from a traditional transport plan based on new "heavy" underground infrastructures to a plan made of punctual and cheaper actions homogeneously diffused on the entire Milanese public transport network.

For example, the first scenario considered the transport lines envisaged in the previous Milanese land use plan (PGT). This huge plan hypothesized the creation of 6 new high-performance lines for a total length of 77.3 km (56.1 km of metro lines and 21.2 km of tram lines), providing one run every 3 minutes per direction. In the evaluation, metro lines have been studied once as traditional heavy lines and once as automatic lines, having a higher investment but lower operating costs.

The main results of this new network were

(1) an increase in user surplus of 209 M€/year;

(2) a reduction in bus operating costs of 47.6 M€/year;

(3) an increase in public transport revenues of 25.6 M€/year;

(4) a reduction in negative externalities (accident, environment, and climate change) of 12.1 M€/year.

However, these apparently good outcomes are not capable of counterbalancing the investment, operating, and maintenance costs of the new infrastructure and services. The Benefit/Cost ratio results are 0.80 for the conventional heavy metro version and 0.86 for the automatic light metro. The financial impact for the public budget would exceed 330 M€/year.

In comparison, we tested a scenario based on a generalised speeding up of the entire overground (bus and tram) public transport, mainly through traffic light coordination and intersections redesign. The interventions hypothesized in the simulation consist of

(1) an increase in the commercial speed of some tram line up to 18 km/h in the most congested part of the city;

(2) an increase of 10% in the commercial speed of all other bus and tram lines;

(3) the reintroduction of 3 circular bus and tram lines around the city centre;

(4) the implementation of the new peripheral tram line in the northern part of the Milan municipality.

As for the PGT scenario, the result in terms of user benefit was definitely positive (221 M€/year), but the Benefit/Cost ratio was outstandingly higher (above 15), considering an investment cost of 200 M€.

From the distributive point of view, the traditional metro extension scenario (Figure 6) and the cheaper speed-up scenario (Figure 5) perform similarly. The latter gives homogeneous benefits to almost all zones of the city, while the metro extension presents some more inhomogeneity, with some zones largely benefited and others just marginally.

In conclusion, if the schematic goals of speeding up the public transport network are actually realized, their effect would be enormously positive and equal to that of six new high-frequency mass transit lines. The margin of benefit of this scenario is such that even obtaining only a fraction of the increase in commercial speed simulated can ensure a positive overall outcome.

This type of comparison is useful for testing, and potentially reorienting, decision-makers' vision and strategies, on which plans were founded. Also thanks to these comparative results, City Council decided to renounce a growth based on new "heavy" and costly infrastructure and, nowadays, the Milanese transport authority is working on punctual interventions that are able to ensure higher overground public transport commercial speeds.

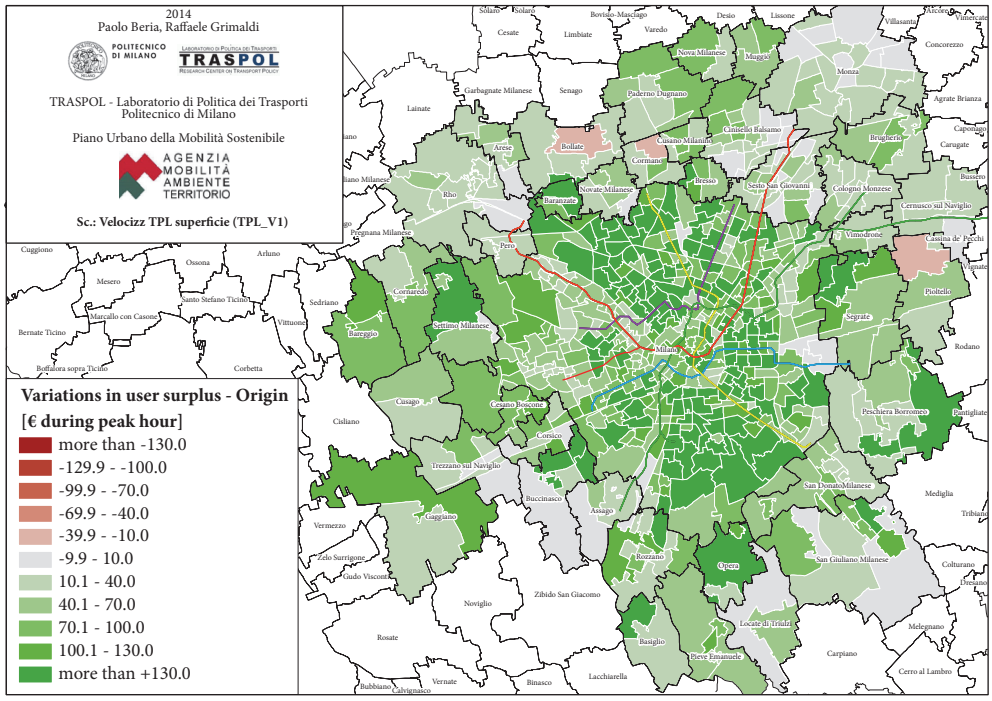


Figure 5: Variation in consumer surplus due to the generalised speeding up of the entire overground public transport (€ in the morning peak hour; our elaboration).

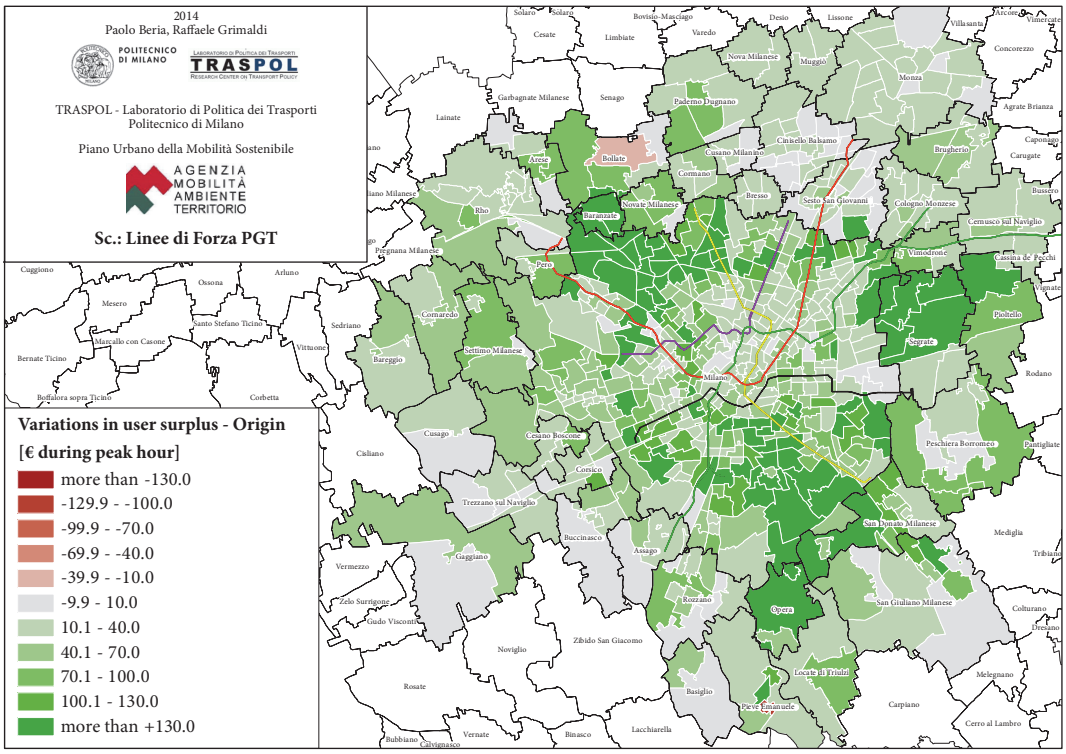


Figure 6: Variation in consumer surplus due to the 6 new high-performance public transport lines envisaged by the PGT (€ in the morning peak hour; our elaboration).

4.2. The Rail Connection between Malpensa Terminal 2 and the Simplon Line. The project presented by the Lombard railway company (FerrovieNord S.p.A.) and the airport manager of Milano Malpensa and Milano Linate (SEA S.p.A.) proposes an extension of the existing rail tracks from Malpensa airport's terminal 2 to Gallarate railway station, located on the Simplon rail line. The aim of this new connection is to increase the number of direct services, both national and international, stopping at the airport. Due to the relocation of actual services on the new tracks, the project allows the release of capacity on the Simplon rail axis, thus enabling the regional government to increase frequencies of some suburban services. This project was the very first application of the regional and national guidelines, recently issued [21, 22].

Similar to what was presented for the previous case, for the CBA assessment, we interfaced the transport model, developed by the Lombardy Region, with the evaluation procedure, developed with a database and spreadsheet software. In this case, the transport model was developed quite recently (it was used for the first time in 2015 to evaluate the Regional Program on Mobility and Transport). For this reason, in the preliminary evaluation phase, a higher effort was devoted to debug and calibration procedures (see below). These operations were made possible by the geographical visualizations of surplus variation effects through the interactions with a GIS software (see Section 4.2.2).

Given that the expected effects mainly spread at the regional level, model zoning, user characteristics, and transport modes presented a higher level of aggregation in comparison with the previous case.

In detail, the transport model provided the generalised cost components of each origin–destination (OD) pair for four different travel purposes (work, study, leisure, and business trips) and, at the first logit stage, the transport modes considered were *private* and *public*. The number of passengers, for every mode on each OD pair, referred to daily movements inside the Lombardy Region and between Lombardy and Switzerland. In parallel, we developed a simplified transport model on a spreadsheet, to include it in the evaluation of other high-speed rail passengers originating from other Italian regions. For the characteristics of attracted users, in this secondary model, we considered a unique travel purpose (leisure) and car (as driver), car (as passenger), coach, HS train (plus shuttle bus interchange), and HS train (plus Malpensa express train interchange) as available transport modes. This model was calibrated using a Customer Satisfaction survey on Malpensa airport users realized in 2016 for SEA S.p.A.

In both models, each segment of the simulated mobility is associated with a set of calibrated β coefficients (5), including the values of time (per trip purpose and per mode). To ensure results comparability, in the simplified model, we considered the same value of time for leisure purposes included in the regional transport model. Differently from the elaboration presented in the previous case study, no active modes were included in these simulations. Consequently, we were able to use directly generalised costs provided by models, thus ensuring the highest level of consistency with them. In addition, models give the quantities, in terms of daily passengers, for every mode on each OD pair, allowing us to calculate the users changing modes. Finally, model outputs, as in the previous case, also included data used to correct the monetary transfers in the CBA.

Overall, the output of the regional model consists of a database file with tables of attributes (generalised cost), quantities (amount of users), and transfers (fares, tolls, and taxes) per 238,210 origin-destination pairs and 8 (2×4) mode and travel purpose combinations. By means of SQL procedures, we estimated the surplus variation using the *RoH* method. Once the core calculations were done per OD pair, mode, and travel purpose, we proceeded with the aggregation of the results in 1,455 origin and destination zones, using the same zoning of the model. The same thing has been done for all attributes, quantities, and transfers. The simplified model is based on 50 origin-destination pairs corresponding to the relation between Malpensa airport and the main provincial capitals (and surrounding hinterlands) outside of the Lombardy Region; the final output referred to 2 (2×1) mode and travel purpose combinations; once the surplus calculation was done for each of the initial 5 transport modes, the results were aggregated in the private and public mode to ensure comparability with the regional model.

After the aggregation, the obtained data of both models have been interfaced with spreadsheet software for the CBA, and limited to regional model data, with a GIS software to produce cartographical representations of the main variables.

In total, eight final scenarios have been evaluated through this procedure. However, even if this number is definitely lower than the total scenario evaluated for the mobility plan, we take even more care to develop ad hoc debug procedures within the automation process.

This effort was necessary to allow a perfect interaction between the *RoH* method and the regional transport model output. In fact, while logit alone allows the introduction of "out-of-scale" generalised costs for nonexisting options (typically putting 9.999 in the generalised cost components), the *RoH* does not. In other words, both *reference* and *postproject* scenario have to be "feasible" for every existing combination of transport modes and travel purpose. Otherwise, if that alternative became feasible due to the project implementation, the *RoH* method will return unrealistic user surplus values.

This was initially the case of some OD pairs in the Lombardy transport model within the public mode (the use of public transport was capped to a maximum travel time of 2 hours), which were "not existing" (with a conventional 9.999 GC associated) before the project and "existing" (with a realistic GC associated) after. This produced wrong results that were pointed out thanks to the transposition of the results on a GIS software (other anomalies were detected in relation to incoherent railway station connectors and simulated service tariffs).

In comparison with the SUMP case, the Appraisal Summary Table has been split between financial and economic performances. The two analyses differ for how they treat user surplus and externality components (included in the economic CBA, but not in the financial one). Net Present

Value (NPV) and Internal Rate of Return (IRR) have been calculated for both parts, while the Net Benefit over Investment Ratio (NBIR) and the Benefit Cost Ratio (B/C) are indicators only for the latter. In this stage, no budget constraint is foreseen and all actions are included. When, at latter stages, the available budget will be defined, these constraints can be managed consistently using appropriate algorithms [41]. Lastly, a separate tool for Risk Analysis was implemented to test the *critical variables* identified by the Sensitivity Analysis (for each scenario, investment costs, user surplus, externalities, and demand growth trends were tested). The term "critical variable" refers to those components for which a variation of 1% of their value leads to a percentage variation equal to or greater than the VANe [21, 22].

4.2.1. Scenarios. Previous to the assessment phase, public actors and stakeholders defined a set of 8 scenarios differing in terms of simulated demand (based on, resp., a *conservative* and *optimistic* growth in air passengers for the period 2016–2025), feasible rail projects and, consequently, different networks and supply.

In particular, in order to ensure faster connections between Milan and the airport, and thus possibly attracting both new demand and shifting the existing one from airport shuttle buses, decision-makers hypothesized to drop one of the two Milan stops (Milano Porta Garibaldi) from the airport express train.

Besides the main aim of ensuring a faster and more frequent connection with the airport, another political goal was to demonstrate quantitatively the necessity of redesigning or even deleting part of a correlated rail project, named "Potenziamento Rho-Gallarate e Raccordo Y." due to potential interferences in train circulations caused by the foreseen Y-shaped flat junction. This flat junction was approved by the national government in 2007 and inserted into the Strategic Infrastructure Program (PIS). The approval process did not require either an assessment or preliminary design phase due to the special condition allowed to the so-called "Legge Obiettivo", abrogated in 2016 (for critical aspects related to "Legge Obiettivo" projects see [42, 43]).

On the contrary, airport managers were mainly focused on enlarging the catchment area of the airport through adding, to the already forecasted suburban and regional supply [44], other market oriented rail services connecting the main regional and provincial Italian capitals.

Table 3 lists the scenarios that emerged from the consultation phase, with their main characteristics.

Each scenario was constructed starting from a "question," posed by decision-makers, which summarizes the different contexts in which the project could be realized or not. From a methodological point of view, all those services that, entirely or for part of their path, generate user benefits not directly connected to the project under assessment were considered invariants. Likewise, only the infrastructural investments necessary to ensure the supply remodelling envisaged were considered. Other projects, even if not yet realized, have been eventually considered as invariants between the *reference* and *postproject* scenarios.

4.2.2. Outputs and Results. In order to make the decision of the regional government more transparent and informed, the outputs of the assessment have been presented in a homogeneous and detailed way. Documents and analyses clarify all the relevant facts that should back the decisions, not limited to the sole aggregate CBA indicators (NPV, NBIR, B/C, and IRR). As for the previous case, results were presented through an *Appraisal Summary Table* (Figure 7) and a *Book of Maps* (Figure 8).

A particular care was taken with the scenario comparison, in order to enrich the undergoing debate between decision-makers and stakeholders. Interestingly, results underline a series of critical aspects, common for all scenarios, that none of the actors had foreseen before.

First, the hypothesized network reconfiguration was considerably beneficial for regional users that move between the regional capital and its hinterland, but—unexpectedly—airport users were associated with a *loss* of surplus (from −0.01 to −0.04 €/day per passenger) and with a shift from public transport to private car.

Thanks to the cartographical visualization of the phenomenon (Figure 8), it was clear how this particular result was a direct consequence of the new path envisaged for Malpensa express service. In fact, according to regional OD matrix data, almost 67% of users direct to and coming from the airport are generated/attracted by Milan. Thus, ensuring a faster connection from Milano Centrale station through cancelling the intermediate stop of Milano Porta Garibaldi implicates an increase in travel time, and eventually in the number of interchanges, for those users located in the southwest and northwest of Milan. Only those located in northeast areas obtain a reduction in travel time (for remaining zones, thanks to the existing metro network, the accessibility of these two stations are equivalent). Overall, the net effect for Milanese users is a *loss* in surplus in respect to the airport connection.

Secondly, the travel time reduction of 6 minutes for HS passengers proved to be not sufficient to promote a sensible modal shift in favour of the public transport mode (Figure 9). In fact, the increase in user surplus is almost completely related to the reduction of the number of interchanges (the ratio of $\beta^{\text{Interchange}}$ on β^{Cost} shows that the perceived cost for each interchange is equal to 50 €, suggesting that this typology of users considers a direct service more attractive than a connection similar in respect to travel time, but that implies even just an interchange). Consequently, on those OD pairs where a direct bus connection already exists, few users are attracted from the new HS service.

Lastly, both R_25|P_25s1 and R_25|P_25s2 scenarios, with a B/C ratio, respectively, of 1.07 and 1.22, show a limited improvement in consumer conditions in respect to project costs. Thus, it was not entirely possible to validate the thesis of the public decision-maker on the effectiveness of the first solution in comparison with the latter. However, this result is partially misleading because, due to the preliminary stage of project design, evaluations utilized feasible train timetables only for the new services envisaged. On the contrary, possible interferences with the already existing services were not considered.

Table 3: Project scenarios.

Scenario code	R_18*	P_18*	R_25*	P_25s1*	P_25s2*	P_25s0*	P_30*
OD matrix horizon	2015	2015	2025**	2025**	2025**	2025	2025**
Infrastructure components							
Rail track T2 - Gallarate		x		x			x
Y flat junction						x	
Y split-level junction					x		
Fourth rail track Rho-Parabiago			x	x	x	x	x
Fourth rail track Parabiago- Gallarate							x
Forecasted rail services and frequency							
Malpensa Express (Milan – Saronno – Terminal 2)	30′		30′			60′	
Malpensa Express (Milano – Saronno – Terminal 2 - Gallarate)		30′					
Malpensa Express (Milano – Rho – Gallarate – Terminal 1)				30′			15′
Malpensa Express (Milano – Rho – Y junction – Terminal 2)					30′	60′	
Regional train (Bergamo – Saronno – Gallarate)	60′		60′		60′	60′	
Regional train (Bergamo – Saronno – Gallarate – Terminal 1)		60′		60′			60′
Suburban train S9 (Albairate – Saronno)	30′	30′					
Suburban train S9 (Albairate – Saronno – Busto Arsizio)			30′	30′	30′	30′	
Suburban train S9 (Albairate – Saronno – Busto Arsizio – Gallarate – Terminal 1)							30′
Suburban train S15 (Milan – Parabiago)			30′	30′	30′	30′	
Suburban train S15 (Milan – Parabiago – Gallarate – Terminal 1)							30′
HS train (*Multiple Origins* - Milan – Saronno – Terminal 2)			Spot			Spot	
HS train (*Multiple Origins* - Milan – Gallarate – Terminal 1)				Spot			Spot
HS train (*Multiple Origins* - Milan – Rho – Y junction – Terminal 2)					Spot	Spot	

*R stands for *state of the art* scenario, while P is associated to *post-project* scenario; **in these scenarios, both *conservative* and *optimistic* demand was considered.

From this analysis, decision-makers decided to allow the progress of the infrastructural project to the next design phase and, at the same time, to prepare further checks aimed at optimizing network supply.

5. Conclusions

Cost-benefit analysis and public expenditure assessment techniques are broadly studied in literature. However, some relevant aspects of evaluation, especially related to the practical application in complex cases and to the effective inclusion of CBA in the policy design process, remain unsolved.

The primary objective of this paper is to give indications about how to correctly evaluate, using Cost-Benefit Analysis, complex urban mobility plans and infrastructural projects. This need is more and more actual, given the increasing shift of mobility planning practices [1] from a single infrastructure to complex and consistent urban plans, to take advantage of the benefits of *policy packaging* [2].

To reach this goal, once a transport model is available, it is convenient to extract the needed data from the chosen model and to adopt, depending on whether nonadditive policies are included in the simulation or not, either the *Rule of Half* or *logsum* methods for consumer surplus calculation.

Another objective of the paper deals with the enrichment of the outputs to allow a better evaluation. Thanks again to the integration with a transport model, it is possible to provide a geographical and social distributive analysis, spatially depicting the effects on consumer surplus, and of envisaged actions, policies, and projects. This tool could potentially be applied also to spatially represent other variables (such as safety or

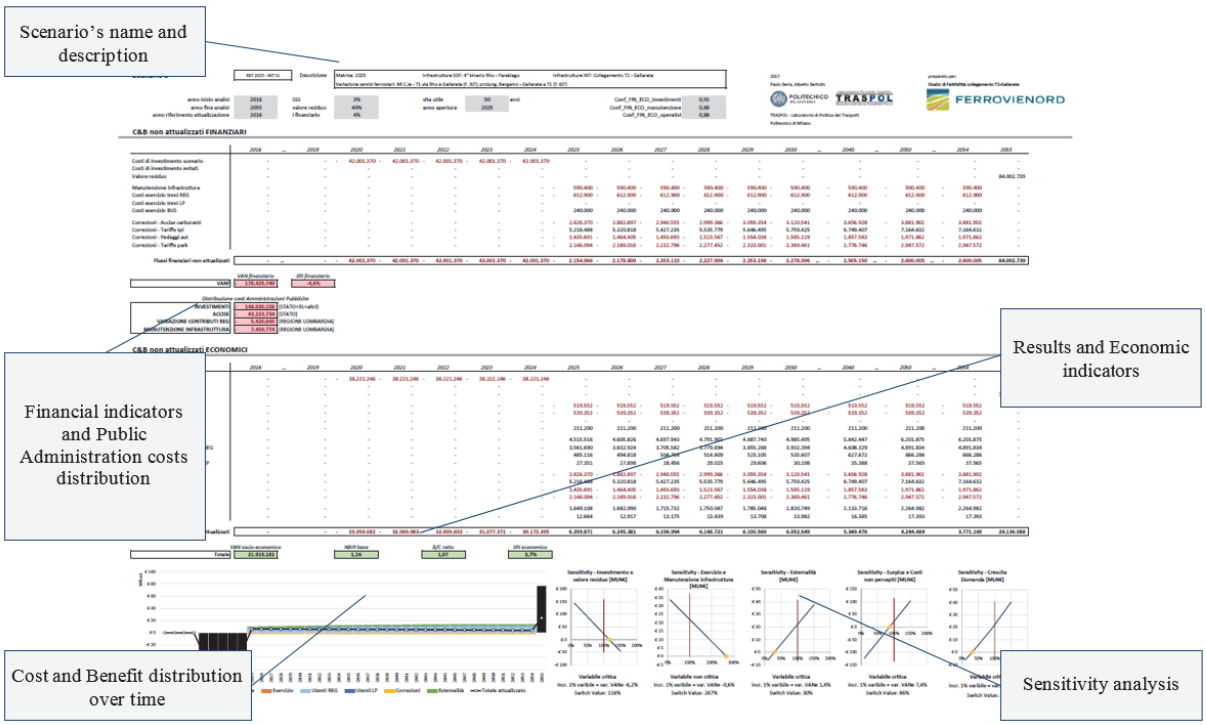
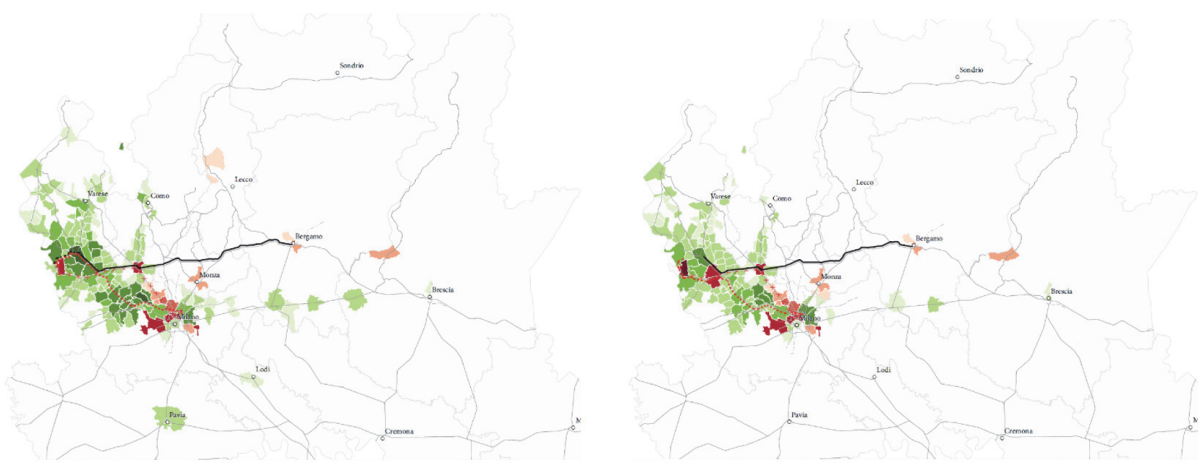


Figure 7: An example of Appraisal Summary Table organisation.



(a) (b)

Figure 8: Total variation in user surplus. (a) Scenario R_25|P_25s1; (b) scenario R_25|P_25s2.

environmental benefits). However, in order to carry out this type of analysis, a deeper interaction with the transport model is needed. In particular, information on the different paths between OD pairs have to be collected from the fourth step of the transport model ("route assignment" phase).

A second possible output to effectively represent and communicate the results of the evaluation is the *Appraisal Summary Table*. Both tools, by improving results readability, can help politicians, policy-makers, stakeholders, and citizens to correctly understand project and plan effects and in general improve the transparency and the awareness of the choices taken.

In particular, through the description of two operative case studies, we show evidence on how these two tools can provide a significant support to decision-makers: by quantitatively demonstrating how, in mature networks, smaller actions have a systematically higher efficiency than large and expensive projects (SUMP case study) and by unearthing possible critical aspects of the envisaged network supply (Malpensa airport rail connection case study).

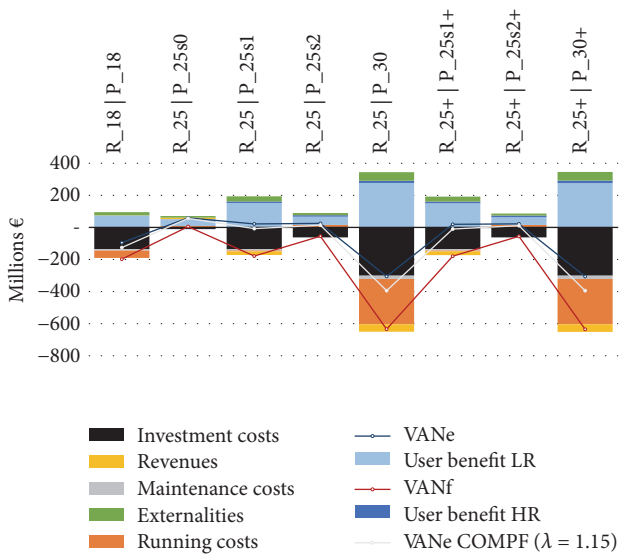


Figure 9: CBA compact indicators and costs distribution.

Lastly, these tools can be also a part of a debug procedure of the transport model itself, thus providing hints on how to return more consolidated results from the simulation point of view.

Additional Points

Highlights. (i) Traditional CBA can also be applied to the evaluation of complex transport plans. (ii) We revised the ways to calculate consumer surplus: generalised costs comparison, Rule of Half, and *logsum*. (iii) We comment on advantages and limits of both methods, suggesting when *logsum* has to be preferred to the Rule of Half. (iv) We show how CBA can be integrated with transport models. (v) Interaction with decision-makers can be improved by using spatial distributive analysis.

The attribution of the different sections is the following: Sections 1, 2, 3, and 4.1 have been written by Paolo Beria and Raffaele Grimaldi and were originally included in the working paper Beria & Grimaldi [23]. Alberto Bertolin is also the author of the remaining sections.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper. Since September 2017, Raffaele Grimaldi has been employed at Autorità di Regolazione dei Trasporti (the Italian transport regulation authority), but this article presents the results of former research and does not represent the views of the authority.

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