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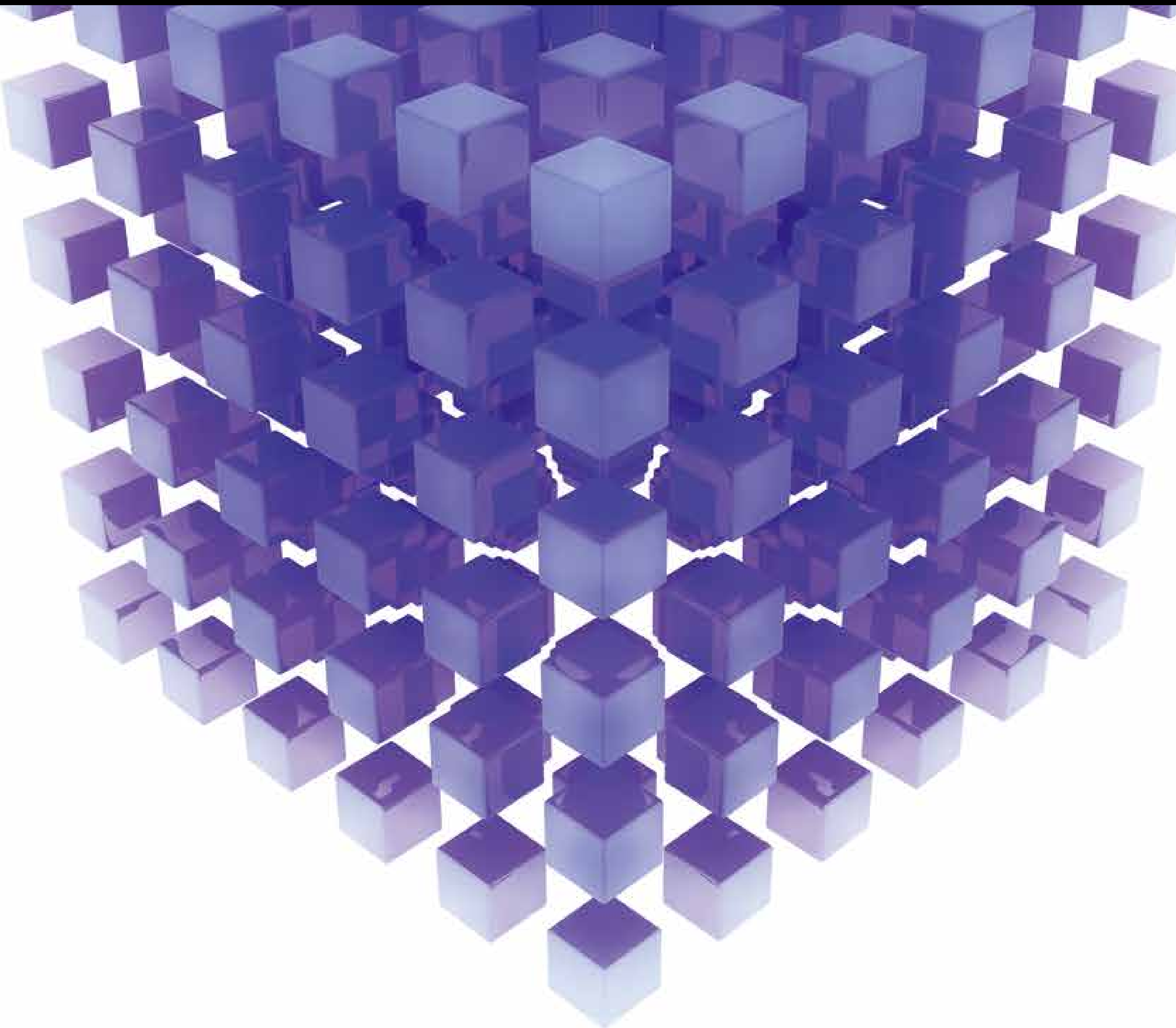
Logistics Systems Optimization under Competition

Guest Editors: Tsan-Ming Choi, Kannan Govindan, and Lijun Ma



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Editorial Board

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Editorial

Logistics Systems Optimization under Competition

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Nowadays, optimization on logistics and supply chain systems is a crucial and critical issue in industrial and systems engineering. Important areas of logistics and supply chain systems include transportation control, inventory management, and facility location planning. Under a competitive market environment, decision making for all these critical areas requires more sophisticated mathematical modeling and analysis. For example, the noncooperative and cooperative analytical game theory and computational based evolutionary algorithms are some popular tools in exploring logistics systems optimization problems under competition. Since many of these optimization problems are complex, innovative analytical models and novel algorithms will be needed in order to optimize the respective logistics systems under the competitive environment. Motivated by the importance of the topic, this special issue of this journal is compiled and it aims at publishing the timely and significant findings on scientific research in logistics systems optimization under competition. This special issue puts high emphasis on the advance of optimization methods, innovative models, and analytical explorations from an industrial engineering and operations research perspective.

After rigorous review, this special issue features twelve interesting research papers. The topics of these papers range from vehicle routing problems, reverse logistics management, channel coordination challenges, dual-channel operations, and distribution network optimization, to retail inventory ordering decisions in supply chain systems. We briefly introduce these interesting research studies in the following.

In "Optimal Routing for Heterogeneous Fixed Fleets of Multicompartment Vehicles," Q. Wang et al. develop a novel metaheuristic based search method, known as the reactive guided tabu search (RGTS) method, to solve the multicompartment vehicle routing problem (MCVRP) with heterogeneous fleet. They consider the case when there is only a single vehicle which helps to support multiple customer orders. Since finding the optimal solution of MCVRP is computationally expensive, they design a few guiding rules, which employ the searching history, to enhance the searching. They conduct numerical analysis and reveal that their proposed method significantly outperforms the classical method.

In "Evaluating Reverse Supply Chain Efficiency: Manufacturer's Perspective," motivated by the importance of environmental sustainability and remanufacturing operations, M. Kumar et al. use the well-established fuzzy data envelopment analysis (FDEA) approach to study reverse supply chain management. They conduct their analysis from the manufacturer's perspective. Technically, they convert the proposed FDEA model into a crisp linear programming optimization problem. As a result, the problem is formulated as an interval programming problem. They argue that their proposed model can help to generate robust results. They show that the ISO 14001 certification scheme only slightly helps improve the supply chain's level of environmental sustainability. Furthermore, their findings interestingly show that companies which have implemented reverse supply chain practices for a shorter period of time would actually outperform those which have implemented reverse supply chain practices for a longer duration of time.

In "Competition with Online and Offline Demands considering Logistics Costs Based on the Hotelling model," Z.-H. Hu et al. examine, via the Hotelling model, the competition effects of shops' location. To be specific, they consider two kinds of logistics costs, namely, the consumer's travelling cost for bricks-and-mortar store's demand and the seller's delivery cost for online demands. They further examine the consumer's waiting cost for online orders and highlight the importance brought by the ratio of online demand to the total demand (online plus offline).

In "Electronic Markets Selection in Supply Chain with Uncertain Demand and Uncertain Price," F. Yang et al. study the critical supply chain management problems in the presence of electronic markets. They develop some stylish analytical models to examine the optimal decision on the selection between public and private electronic markets. They consider three different scenarios: (i) the electronic market is solely used for buying, (ii) the electronic market is solely used for selling, and (iii) the electronic market is used for both selling and buying. They consider two sources of uncertainty, including demand uncertainty and price uncertainty, in their model. They derive the analytical conditions in which it is optimal for the supply chain agent to select a particular electronic market. One important finding that this study shows is that the electronic market's usage fee is a critical factor for assessing the electronic market's performance. It should be a focal point in the optimal selection and proper development of electronic market.

In "A Methodology to Exploit Profit Allocation in Logistics Joint Distribution Network Optimization," Y. Wang et al. study the logistics joint distribution network (LJDN) optimization problem. Their problem includes the optimal vehicle routes scheduling and profit allocation mechanism for multiple distribution centers. To be specific, they develop a model with an objective to minimize the total cost of the multiple distribution centers in the joint distribution network. They consider the situation in which each distribution center is assigned to serve a certain number of distribution units. They first develop and employ a novel revised particle swarm optimization (PSO) algorithm, which combines the PSO algorithm and genetic algorithm, to solve this problem. Then, they propose a cooperative game theory based model to derive the optimal profit allocation mechanism among the distribution centers.

In "Multiple Objective Fuzzy Sourcing Problem with Multiple Items in Discount Environments," F. Arikan develops a multiple criteria fuzzy sourcing problem with multiple items in discounts. He formulates the problem as a single period multiobjective mixed integer linear programming problem with fuzzy parameters on demand level and aspiration level of each objective. He employs a hybrid fuzzy approach which combines three fuzzy mathematical models to identify the solution. He argues that the fuzzy formulation makes the problem more realistic and the solution mechanism can be implemented in real world applications.

In "Impact of Heterogeneous Consumers on Pricing Decisions under Dual-Channel Competition," Y. Wei and F. Li analytically investigate the impacts brought by heterogeneous consumer behaviors on the equilibrium pricing decisions under a competitive dual-channel environment. To be specific, they consider a supply chain with one retailer and one manufacturer. The supply chain is led by the manufacturer and there are two channels, namely, the direct channel (i.e., selling directly to consumers) and the indirect channel (i.e., supplying to the retailer first). Consumers can decide which channel to make their purchases, which depends highly on the prices offered by the different channels. Owing to the complexity of the problem, they make use of an agent-based modeling and computational simulation approach to study the problem. They find that when the consumers are increasingly loyal to the indirect channel, the retailer will set a higher selling price and make more profit. They also reveal that when the consumer rationality level increases, the offered selling prices by both channels would decrease.

In "Optimal Ordering and Disposing Policies in the Presence of an Overconfident Retailer: A Stackelberg Game," Z. Wang et al. examine the behavioral decision making issue in inventory control. To be specific, they consider the case when the retailer has an overconfident behavior on the supply chain performance. They find that the retailer's overconfident behavior may not harm the supply chain provided that the level of overconfidence is less than a certain threshold. They further study the supply chain channel coordination issue with the buy-back returns contract. They prove that the buy-back returns contract will achieve the Pareto improving situation in the supply chain if the level of overconfidence is low.

In "Customized Transportation, Equity Participation, and Cooperation Performance within Logistics Supply Chains," X. Lin et al. explore the customized transportation issues in a logistics system. They develop a game-theoretic model. They analytically find that a take-or-pay supply contract cannot properly deal with the problem. They hence propose an equity participation plus simple contract scheme to help improve the performance of customized transportation. They show that, at the equilibrium, the private-type of logistic supply chains would choose a more efficient customized production level than the public-type counterpart.

In "Two-Echelon Inventory Optimization for Imperfect Production System under Quality Competition Environment," X. Lai et al. develop a novel two-echelon optimal inventory control model for a supply chain system with quality competition. They consider the situation in which the supplier's production process is imperfect and there are quality problems. They hence derive the optimal ordering policy for the buyer and the optimal shipping policy from the vendor to the buyer. They conclude that, in the supply chain system, both the supplier and the buyer may benefit from the quality improvement investment made by the supplier.

In "A Hybrid Approach to the Optimization of Multiechelon Systems," P. Sitek and J. Wikarek examine the freight transportation challenges with different distribution strategies. They propose a hybrid approach to tackle the multiechelon capacitated vehicle routing problem. They make use of mathematical programming and constraint logic programming approaches in developing the solution algorithm. They have also illustrated the implementation of the proposed approach and compared its performance with other existing mathematical programming methods.

In "The Newsvendor Problem with Different Delivery Time, Resalable Returns, and an Additional Order," F. Zeng et al. study the case when the newsvendor faces demands with different delivery times. They investigate the case in the presence of resalable returns. They also consider the case when an additional order can be placed by the newsvendor. They construct the formal analytical model and examine numerically the impacts brought by the proportion of instant delivery needs and the return rate on the optimal ordering quantity. They show that the newsvendor's expected profit decreases as the proportion of instant delivery needs and rate of returned increase.

We believe that this special issue presents many interesting and timely research studies on logistics systems optimization. The coverage is indeed comprehensive: some studies focus on manufacturing side and some on retailing side; some studies explore transportation problems and some explore inventory decisions. The valuable academic and managerial insights generated by the papers of this special issue contribute significantly to the literature. In addition, hopefully, these important research results will help motivate future research on logistics systems optimization under a competitive setting.

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Research Article

Electronic Markets Selection in Supply Chain with Uncertain Demand and Uncertain Price

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In recent years, more and more companies start online operation. Electronic market becomes a key component of some companies' strategy. Supply chain management is another key component of the strategy as being adopted by an increasing number of companies. There are many interactions between electronic market and supply chain. One of the key questions is to select one type of electronic market from the view of supply chain. This paper develops some models to explore the issue of selection between public electronic market and private electronic market in three scenarios where electronic market is used for buying, for selling, and for both selling and buying, respectively. In a public electronic market, neither the supplier nor the retailer is the owner of the electronic market. However, in a private electronic market, there is an owner that is either the supplier or the retailer. Besides demand uncertainty, we take into account the price uncertainty in electronic market. We explore the conditions under which the agent of supply chain selects one certain type of electronic market by comparing expected profits of supply chain members in different scenarios. Some sensitivity analyses are conducted to explore the impact of the customer demand, electronic market retail price, and e-market use fee on the selection of electronic market. Finally, some interesting managerial and academic insights are obtained.

1. Introduction

In recent years, electronic market (hereafter e-market) has received increasing attention from industry and more and more companies start online operation. For example, http://www.suning.com/ created his first e-commerce platform for online shopping in China in 2010. In 2013, the turnover of China B2B market was 8.2 trillion with more than 30 percent year-on-year growths. With the growth of e-commerce, e-market has been always one of the focuses in academia. Some details regarding business concept of electronic commerce can be seen in Kumar and Raheja [1].

Supply chain management is another strategy adopted by an increasing number of companies. Along with the e-commerce growth, e-market has a strong influence on supply chain management. Many papers in the literature explored the interaction between e-market and supply chain. For example, Peleg et al. [2] researched three procurement strategies to provide policies for supplier when there is an e-market. They showed that supplier can make use of e-market to get high income. Seifert et al. [3] quantified the benefits of using online spot market from a supply chain perspective. Their results demonstrated that companies using online spot market can get higher service level and higher variability in profits than companies who do not use online spot market. Choi et al. [4] coordinated the supply chain by return policy and the return products were sold at a higher price to the supplier in e-market. Their results made it clear that the expected profit in e-market is always larger than the traditional market. Ganeshan et al. [5] showed that trading products either in the electronic spot market or via derivate instruments can get optimal procurement portfolios and mitigate risk. Standing et al. [6] summarized twelve years of research on e-market places since 1997 and pointed out the relative lack of papers on the organisational implications of managing e-market places. Khanam [7] noted that e-market has several features to increase user friendliness. Ning et al. [8] discussed the coordinate markdown policy when e-market is introduced besides the traditional market and showed that e-market is a benefit for supply chain in terms of the expected profit improvement. But e-market may have several disadvantages compared with traditional market from Kacen et al.'s [9] research such as shipping and handling charges and good uncertainty. So more studies containing comprehensive characteristics on e-market are needed. Narenji et al. [10] introduced evolutionary game theory in finding the price and delivery time strategy equilibrium when supply chain is centralized and decentralized. Their conclusions demonstrated that supply chain with e-market is better than that with traditional market in most scenarios. Maity and Dass [11] helped us in making supply chain choice between traditional market and e-market from the view of media richness theory containing both product characteristics and channel characteristics. Their study showed that customers prefer e-market over other channels that have higher or lower media richness.

As it can be seen above, e-market plays an important role in supply chain management and scholars have attached great importance on it. But e-market is such a significant market that needs a more detailed analysis to dig up its huge potential. For example, as a matter of fact, both price uncertainty and demand uncertainty exist in supply chain. It is essential to take both uncertainties into consideration. However, most of the papers in this field focus only on one of the two uncertainties. For instance, Wang and Benaroch [12] only took the demand uncertainty into consideration and analyzed the decision of supply chain members who use or do not use e-market. Ghose and Yao [13] considered the uncertain price and found that the "law of one price" can prevail. There are also a few papers that take both uncertainties into account. For example, Inderfurth et al. [14] explored the channel selection with considering random price or stochastic demand in their sourcing model where price and demand are all have cumulative distribution, density function, expected value, and standard deviation.

What is more, there are different types of e-markets. For example, according to the owner who builds and operates the e-market, the e-market can be classified into three types, that is, third party-driven, buyer-driven, and seller-driven (Shon et al. [15]). Furthermore, some e-markets are used only for buying, only for selling, or for both selling and buying. The right selection of e-market is critical for a supply chain. Xing et al. [16] investigated the supply chain members' strategies in decentralized supply chain in B2B electronics market where the supplier is the owner. Truong et al. [17] showed that retailers' expectations of benefits differ significantly between different public and private market. From the point of Khanam's [7] research, the owner who drives the e-market charges the other members and the charge impacts the retail price of e-market and indirectly affects earnings. Pei et al. [18] obtained the expected profit when there is an owner in three different kinds of e-markets. Li et al. [19] identified the conditions under which the supplier would join public or private market. However, neither of the above two papers obtains the close form of the optimal order quantity and which specific kind of e-market the supplier should join.

Motivated by the above, we study the e-markets selection from the view of supply chain between public e-market and private e-market with uncertain demand and uncertain price in three different scenarios where e-market is used for buying, for selling, or for both selling and buying, respectively. The close form of the optimal order quantity of retailer and the profit of supply chain members in different e-markets is derived in the case of customer demand and retail price following uniform distribution. The comparison between expected profits of e-markets can help supply chain members trade-off between public e-market and private e-market in three scenarios. Some sensitivity analyses are also conducted to explore the impact of the customer demand, the price of e-market, and the e-market use fee on the e-market selection. In the following text, EB, ES, and EM are used to denote e-market for buying, for selling, and for both selling and buying, respectively.

The rest of this paper is organized as follows. The next section describes the notations and the assumptions of the problem. Section 3 explores the optimal strategies of the retailer when e-market is managed by a third-party manager, the supplier, and the retailer, respectively. The trade-off between public e-market and private e-market in three supply chain structures is discussed. Section 4 examines the impact of the various e-market parameters via sensitivity analysis. The last section offers concluding remarks as well as some opportunities for future research. All the proofs of the theoretical results are given in the Appendix.

2. Notations and Assumptions

We consider a two-echelon supply chain consisting of a single supplier and a single retailer. There are two kinds of markets: the traditional market and the e-market. Product is the newsvendor type with short life-cycle, short selling season, and uncertain demand. The retailer cannot make any replenishment from the traditional market during selling season due to the long lead time. However, it is assumed that the retailer can make emergency replenishment from the e-market if they can pay more. Some papers can be given to illustrate that the assumption is a good approximation of reality. For example, Fu et al. [20] studied the newsvendor model with multiple options of expediting options. They argued that various expediting options can help satisfy the demand with a different more expensive price than early order. Some other studies such as [21, 22] also pointed out that a higher price can make a shorter lead time. Moreover, due to the fleetness to deal with information in e-market, the product in e-market can be assumed as a kind of expediting options with higher price and zero lead time.

The sequence of the events is as follows.

The retailer decides his order quantity q before selling season. He can forecast the cumulative distribution function $F_d(\cdot)$ of the demand. When selling season comes, the retailer fills demand x by quantity q.

In traditional market, if the realized demand x is higher than the initial order quantity q, the retailer will face lost sales with an additional penalty cost g. If the demand is lower than q, all unsold products will be salvaged at salvage value v.

If the retailer can use the e-market, there are three cases as follows.

Case 1. When e-market is used only for buying, extra demand $x - q$ can be filled via e-market. However, if the retailer has excess inventory at the end of selling season, it could only be salvaged at unit price v in the traditional market.

Case 2. When the e-market is used only for selling, the retailer can sell products via both traditional market and e-market. All excess inventory $q - x$ could be sold at unit price p_e via the e-market. However, there might be a stock out $x - q$ since the retailer only uses traditional market to order.

Case 3. When the e-market is used for both selling and buying, the retailer can fill all the demand x via traditional market and via e-market. He can make emergency replenishment to fill the demand via the e-market. In addition, the retailer can sell all excess inventory $q - x$ at unit price p_e via the e-market.

Definitions of the notation and assumptions of this paper are presented below.

2.1. Notations

We have the following:

p_r: unit retail price in traditional market;

p_e: unit retail price in e-market, nonnegative continuous random variable;

μ_e: the mean of p_e;

$f_e(\cdot)$: probability density function of p_e;

$F_e(\cdot)$: cumulative distribution function of p_e;

x: customer demand, nonnegative continuous random variable;

μ_d: the mean of x;

$f_d(\cdot)$: probability density function of x;

$F_d(\cdot)$: cumulative distribution function of x;

q: retailer's order quantity;

c: unit production cost of supplier;

w: unit wholesale price in traditional market;

g: unit lost sales cost;

v: unit salvage value;

M: the upper production capacity limit of supplier;

m: the ratio of the e-market use fee to the gross transaction in the e-market, that is, the e-market user should pay a percentage of part of their gross transaction in e-market as the e-market use fee to the e-market owner.

2.2. Assumptions

(A1) To avoid the trial cases, it is assumed that the following inequations hold: $0 < v < c < w < p_r$, $g > 0$, $q < M$.

(A2) Demand x and e-market price p_e are two-dimension uniform (2DU) distributed. It is assumed that $x \sim U(\mu_d - r, \mu_d + r)$, $p_e \sim U(\mu_e - s, \mu_e + s)$, and $(x, p_e) \sim U(\mu_d - r, \mu_d + r; \mu_e - s, \mu_e + s)$.

(A2′) To see whether the findings under the premise of two-dimension uniform distribution still hold for more general case, we consider an extension of the model in the numerical example to the case that demand x and e-market price p_e are bivariate normal (BN). There is $(x, p_e) \sim \mathrm{BN}(\mu_d, \mu_e, \sigma_d, \sigma_e, \rho)$ where $\rho \geq 0$.

3. Model Formulation and Analysis

In this section, we develop and analyze the models in different scenarios, such as public e-market, private e-market, and e-market for buying, for selling, and for both selling and buying, respectively.

Suppose the retailer or the supplier wants to deal with lost sales or/and excess inventory via e-market. The user should pay a fee for the transaction in the e-market to the owner. For example, in a public e-market, the e-market use fee should be paid to the third party who is in charge of the e-market. In a private e-market, if the supplier is the owner, the retailer who needs to join the e-market has to pay a e-market use fee to the supplier, and vice versa.

In each model, expected profits and optimal order quantity are derived in the case that demand and retail price are uniform distribution. By comparing the expected profits of the supply chain agents in these models, we obtain the condition under which the supply chain member chooses a public e-market or a private e-market and analyze the optimal type of e-market should be built. We find that e-market use fee ratio m and parameters of uncertain price and demand are all important parameters in market selection decision-making.

3.1. Supply Chain Models in EB

3.1.1. Supply Chain Models in Public EB. If e-market is a public e-market and is used for only buying, the retailer can make an emergency order from the supplier to fill the excess demand via the e-market. Both the retailer and the supplier will pay e-market use fee to the third party that is in charge of the market.

Suppose Δq_{EB} is the emergency order of the retailer via e-market, that is, the product quantity the supplier sells in e-market besides the order quantity of retailer q_{EB}. The expected profit function of the supplier is given by

$$\begin{aligned}
\Pi_{\mathrm{EB}}^{S}&\left(q_{\mathrm{EB}} + \Delta q_{\mathrm{EB}}\right) \\
&= -c\left(q_{\mathrm{EB}} + \Delta q_{\mathrm{EB}}\right) + w q_{\mathrm{EB}} \\
&\quad + (1 - m)\int_{-\infty}^{\infty} \Delta q_{\mathrm{EB}}\, p_e f_e\left(p_e\right) dp_e \\
&= -c\left(q_{\mathrm{EB}} + \Delta q_{\mathrm{EB}}\right) + w q_{\mathrm{EB}} + (1 - m)\,\mu_e \Delta q_{\mathrm{EB}} \\
&= (w - c)\, q_{\mathrm{EB}} + \left[(1 - m)\,\mu_e - c\right] \Delta q_{\mathrm{EB}}.
\end{aligned} \quad (1)$$

It is obvious that if $m < 1 - (c/\mu_e)$, the supplier will get more profit than in traditional market and would like to join the e-market.

Then the expected profit function of the retailer is given by

$$\begin{aligned}\Pi_{\mathrm{EB}}^{R}(q) = & -wq + \int_{-\infty}^{q} p_r x f_d(x)\,dx \\ & + \int_{-\infty}^{q} v(q-x) f_d(x)\,dx + \int_{q}^{\infty} p_r q f_d(x)\,dx \\ & - \int_{q}^{\infty} dx \int_{-\infty}^{\infty} (p_e - p_r)(x-q) f_{d,e}(x,p_e)\,dp_e \\ & - \int_{q}^{\infty} dx \int_{-\infty}^{\infty} m p_e (x-q) f_{d,e}(x,p_e)\,dp_e.\end{aligned} \quad (2)$$

It is straightforward to prove that $\Pi_{\mathrm{EB}}^{R}(q)$ is concave as in [18]. By using the first- and second-order optimality conditions, we can obtain the following equation that the optimal order quantity of the retailer denoted by q_{EB}^{*} is subject to

$$w = vF_d(q_{\mathrm{EB}}^{*}) + \int_{q_{\mathrm{EB}}^{*}}^{+\infty} \int_{-\infty}^{+\infty} (1+m)\, p_e f_{d,e}(x,p_e)\,dp_e dx. \quad (3)$$

When demand and retail price in e-market are 2DU distributed, the close form of the optimal order quantity is given by

$$q_{\mathrm{EB}}^{*} = \mu_d + \frac{r(2w - v - (1+m)\mu_e)}{v - (1+m)\mu_e}. \quad (4)$$

And the optimal expected profit of the retailer is obtained as follows:

$$\begin{aligned}\Pi_{\mathrm{EB}}^{R*} = & \frac{v-(1+m)\mu_e}{4r} q_{\mathrm{EB}}^{*\,2} \\ & + \frac{1}{2r}\left[(1+m)\mu_e(\mu_d + r) - v(\mu_d - r) - 2rw\right] q_{\mathrm{EB}}^{*} \\ & + p_r\mu_d + \frac{v}{4r}(\mu_d - r)^2 - \frac{\mu_e}{4r}(1+m)(\mu_d + r)^2 \\ = & \; p_r\mu_d - w\mu_d + rv - rw + \frac{r(v-w)^2}{(1+m)\mu_e - v}.\end{aligned} \quad (5)$$

3.1.2. Supply Chain Models in Private EB

(a) Supplier as the Owner in EB. If e-market is managed by the supplier and used for only buying, the retailer can make an emergency order from the supplier to fill the excess demand via the e-market. The retailer will pay an e-market use fee to the supplier that is in charge of the e-market.

In decentralized supply chain, the profit function of the retailer and the optimal order quantity are the same with those in the public EB. However, the supplier's profit function changes into the following formulation:

$$\begin{aligned}& \Pi_{\mathrm{EB}}^{S}(q_{\mathrm{EB}} + \Delta q_{\mathrm{EB}}) \\ & \quad = -c(q_{\mathrm{EB}} + \Delta q_{\mathrm{EB}}) + w q_{\mathrm{EB}} \\ & \quad\quad + (1+m)\int_{-\infty}^{\infty} \Delta q_{\mathrm{EB}} p_e f_e(p_e)\,dp_e \\ & \quad = (w-c) q_{\mathrm{EB}} + ((1+m)\mu_e - c)\Delta q_{\mathrm{EB}}.\end{aligned} \quad (6)$$

As seen from the above, if the retailer is not the owner of the EB, the expected profit of the retailer is independent of the owner of the EB. But the supplier would prefer his own private EB to public EB if ignoring the operating costs.

(b) Retailer as the Owner in EB. If e-market is managed by the retailer and only used for buying, the retailer can make an emergency order from the supplier to fill the excess demand via the e-market. Supplier should pay the retailer e-market use fee and his expected profit function is the same with (1). The expected profit function of retailer in decentralized supply chain is given by

$$\begin{aligned}\Pi_{\mathrm{EB}}^{R}(q) = & -wq + \int_{-\infty}^{q} p_r x f_d(x)\,dx \\ & + \int_{-\infty}^{q} v(q-x) f_d(x)\,dx + \int_{q}^{\infty} p_r q f_d(x)\,dx \\ & - \int_{q}^{\infty} dx \int_{-\infty}^{\infty} (p_e - p_r)(x-q) f_{d,e}(x,p_e)\,dp_e \\ & + \int_{q}^{\infty} dx \int_{-\infty}^{\infty} m p_e (x-q) f_{d,e}(x,p_e)\,dp_e.\end{aligned} \quad (7)$$

We can derive the optimal order quantity as (8). Consider the following:

$$w = vF_d(q_{\mathrm{EB}}^{*}) + \int_{q_{\mathrm{EB}}^{*}}^{+\infty} \int_{-\infty}^{+\infty} (1-m)\, p_e f_{d,e}(x,p_e)\,dp_e dx. \quad (8)$$

If demand and retail price in e-market are 2DU distributed, the close form of optimal order quantity and expected profit of retailer are given by

$$q_{\mathrm{EB}}^{*} = \mu_d + \frac{r(2w - v - (1-m)\mu_e)}{v - (1-m)\mu_e}. \quad (9)$$

$$\begin{aligned}\Pi_{\mathrm{EB}}^{R*} = & \frac{v-(1-m)\mu_e}{4r} q_{\mathrm{EB}}^{*\,2} \\ & + \frac{1}{2r}\left[(1-m)\mu_e(\mu_d + r) - v(\mu_d - r) - 2rw\right] q_{\mathrm{EB}}^{*} \\ & + p_r\mu_d + \frac{v}{4r}(\mu_d - r)^2 - \frac{\mu_e}{4r}(1-m)(\mu_d + r)^2 \\ = & \; p_r\mu_d - w\mu_d + rv - rw + \frac{r(v-w)^2}{(1-m)\mu_e - v}.\end{aligned} \quad (10)$$

Retailer's expected profit has increased when formulation (7) minus (2) if the in two expressions are at the same value. It means that retailer would rather choose his own private e-market than public e-market in EB when operating cost is smaller than the additional income C_{EB}, where

$$C_{\mathrm{EB}} = \int_{q}^{\infty} dx \int_{-\infty}^{\infty} 2mp_e (x-q) f_{d,e}(x,p_e)\, dp_e. \tag{11}$$

If demand and retail price in e-market are 2DU distributed, the additional revenue in the e-market controlled by retailer is given by

$$C_{\mathrm{EB}} = \frac{2m\mu_e r(v-w)^2}{(\mu_e - v)(\mu_e - v + 2m\mu_e)}. \tag{12}$$

The above two models indicate that e-market owner would enjoy more profit in private EB if additional income is larger than the construction and operation cost of EB.

3.2. Supply Chain Models in ES

3.2.1. Supply Chain Models in Public ES. If e-market is managed by a third-party manager, the supplier's profit function is as follows since ES involves the retailer and the supplier cannot join ES:

$$\Pi_{\mathrm{ES}}^{S}(q_{\mathrm{ES}}) = -cq_{\mathrm{ES}} + wq_{\mathrm{ES}}. \tag{13}$$

The profit function of retailer and the optimal order quantity in decentralized supply chain are given by

$$\begin{aligned}
\Pi_{\mathrm{ES}}^{R}(q) = {} & -wq + \int_{-\infty}^{q} p_r x f_d(x)\, dx \\
& + \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} p_e (q-x) f_{d,e}(x,p_e)\, dp_e \\
& + \int_{q}^{\infty} p_r q f_d(x)\, dx - \int_{q}^{\infty} g(x-q) f_d(x)\, dx \\
& - m \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} p_e (q-x) f_{d,e}(x,p_e)\, dp_e. \\
w = {} & (p_r + g)(1 - F_d(q_{\mathrm{ES}}^{*})) \\
& + (1-m) \int_{-\infty}^{q_{\mathrm{ES}}^{*}} \int_{-\infty}^{+\infty} p_e f_{d,e}(x,p_e)\, dp_e dx.
\end{aligned} \tag{14}$$

When demand and retail price in e-market are 2DU distributed, the optimal order quantity and expected profit of the retailer are given by

$$\begin{aligned}
q_{\mathrm{ES}}^{*} &= \frac{2rw - (p_r+g)(\mu_d+r) + \mu_e(1-m)(\mu_d - r)}{(1-m)\mu_e - p_r - g}, \\
\Pi_{\mathrm{ES}}^{R*} &= \frac{1}{4r}((1-m)\mu_e - p_r - g) q_{\mathrm{ES}}^{*\,2} \\
&\quad + \frac{1}{2r}[(p_r+g)(\mu_d+r) \\
&\qquad - \mu_e(1-m)(\mu_d-r) - 2rw] q_{\mathrm{ES}}^{*} \\
&\quad + \frac{1}{4r}\left[((1-m)\mu_e - p_r)(\mu_d - r)^2 - g(\mu_d+r)^2\right] \\
&= p_r\mu_d + (1-m)\mu_e r - \mu_d w - rw \\
&\quad + \frac{r((1-m)\mu_e - w)^2}{g + p_r - (1-m)\mu_e}.
\end{aligned} \tag{15}$$

3.2.2. Supply Chain Models in Private ES. In this scenario, only the retailer is the owner of the supply chain. Then the profit function of the retailer is given by

$$\begin{aligned}
\Pi_{\mathrm{ES}}^{R}(q) = {} & -wq + \int_{-\infty}^{q} p_r x f_d(x)\, dx \\
& + \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} p_e (q-x) f_{d,e}(x,p_e)\, dp_e \\
& + \int_{q}^{\infty} p_r q f_d(x)\, dx \\
& - \int_{q}^{\infty} g(x-q) f_d(x)\, dx.
\end{aligned} \tag{16}$$

The optimal order quantity should satisfy

$$\begin{aligned}
w = {} & (p_r+g)(1 - F_d(q_{\mathrm{ES}}^{*})) \\
& + \int_{-\infty}^{q_{\mathrm{ES}}^{*}} \int_{-\infty}^{+\infty} p_e f_{d,e}(x,p_e)\, dp_e dx.
\end{aligned} \tag{17}$$

When demand and retail price in e-market are 2DU distributed, the optimal order quantity and expected profit of the retailer are given by

$$q_{\mathrm{ES}}^{*} = \frac{2rw - (p_r+g)(\mu_d+r) + \mu_e(\mu_d - r)}{\mu_e - p_r - g}, \tag{18}$$

$$\begin{aligned}
\Pi_{\mathrm{ES}}^{R*} &= \frac{1}{4r}(\mu_e - p_r - g) q_{\mathrm{ES}}^{*\,2} \\
&\quad + \frac{1}{2r}[(p_r+g)(\mu_d+r) - \mu_e(\mu_d - r) - 2rw] q_{\mathrm{ES}}^{*} \\
&\quad + \frac{1}{4r}\left[(\mu_e - p_r)(\mu_d - r)^2 - g(\mu_d + r)^2\right] \\
&= \mu_d p_r + \mu_e r - \mu_d w - rw + \frac{r(\mu_e - w)^2}{g - \mu_e + p_r}.
\end{aligned} \tag{19}$$

The retailer's additional revenue is obtained as follows:

$$C_{\mathrm{ES}} = \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} mp_e (q-x) f_{d,e}(x,p_e)\, dp_e. \tag{20}$$

If demand and retail price in e-market are 2DU distributed, the expected additional revenue when the retailer builds ES is given by

$$C_{\mathrm{ES}} = m\mu_e r + \frac{r(\mu_e - w)^2}{g - \mu_e + p_r} - \frac{r((1-m)\mu_e - w)^2}{g - (1-m)\mu_e + p_r} = \frac{m\mu_e r(g + pr - w)^2}{(g - \mu_e + p_r)(g - (1-m)\mu_e + p_r)}. \tag{21}$$

It turned out that the retailer in ES will prefer private e-markets to public e-market if $(g-\mu_e+p_r)(g-(1-m)\mu_e+p_r) > 0$ and C_{ES} is larger than the construction and operation cost of ES.

3.3. Supply Chain Models in EM

3.3.1. Supply Chain Models in Public EM. When the third party builds EM and supply chain members join EM independently, the profit of the supplier is given by

$$\Pi^S_{\mathrm{EM}}(q_{\mathrm{EM}} + \Delta q_{\mathrm{EM}}) = (w - c)\, q_{\mathrm{EB}} + [(1-m)\mu_e - c]\,\Delta q_{\mathrm{EB}}. \tag{22}$$

The retailer's expected profit is given by

$$\begin{aligned}\Pi^R_{\mathrm{EM}}(q) &= (1-m)\int_{-\infty}^{q} dx \int_{-\infty}^{\infty} p_e(q-x)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad - (1+m)\int_{q}^{\infty} dx \int_{-\infty}^{\infty} p_e(x-q)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad - wq + p_r\mu_d.\end{aligned} \tag{23}$$

The optimal order quantity q^*_{EM} should satisfy

$$w = (1-m)\mu_e + 2m\int_{q^*_{\mathrm{EM}}}^{\infty} dx \int_{-\infty}^{\infty} p_e f_{d,e}(x,p_e)\, dp_e. \tag{24}$$

When demand and retail price in e-market are 2DU distributed, the optimal order quantity and the expected profit of retailer are given by

$$q^*_{\mathrm{EM}} = \mu_d + \frac{(\mu_e - w)r}{m\mu_e}, \tag{25}$$

$$\begin{aligned}\Pi^{R*}_{\mathrm{EM}} &= -\frac{m\mu_e}{2r}{q^*_{\mathrm{EM}}}^2 + \left[\frac{m\mu_e\mu_d}{r} + \mu_e - w\right] q^*_{\mathrm{EM}} \\ &\quad + p_r\mu_d - \mu_e\mu_d - \frac{m\mu_e}{2r}(\mu_d^2 + r^2) \\ &= \mu_d(p_r - w) - \frac{m\mu_e r}{2} + \frac{r(\mu_e - w)^2}{2m\mu_e}.\end{aligned} \tag{26}$$

3.3.2. Supply Chain Models in Private EM

(a) Supplier as the Owner in EM. Since the retailer is not the owner of the supply chain, his decision making is not affected by public and private e-markets. The profit function and the optimal order quantity of the retailer are the same as that in the supply chain managed by third-party manager. The profit function of the supplier in this case is given by

$$\Pi^S_{\mathrm{EM}}(q_{\mathrm{EM}} + \Delta q_{\mathrm{EM}}) = (w - c)\, q_{\mathrm{EM}} + ((1+m)\mu_e - c)\,\Delta q_{\mathrm{EM}}. \tag{27}$$

It can be obtained from (22) and (27) that if the supplier's profit increases, he will build his private e-market ignoring the operating costs.

(b) Retailer as the Owner in EM. If the retailer is the owner in EM, the profit function of the retailer in EM is given by

$$\begin{aligned}\Pi^R_{\mathrm{EM}}(q) &= \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} p_e(q-x)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad - \int_{q}^{\infty} dx \int_{-\infty}^{\infty} p_e(x-q)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad + \int_{q}^{\infty} dx \int_{-\infty}^{\infty} mp_e(x-q)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad - wq + p_r\mu_d.\end{aligned} \tag{28}$$

When demand and retail price in e-market are 2DU distributed, $\Pi^R_{\mathrm{EM}}(q)$ is not concave and the expected profit is given by

$$\begin{aligned}\Pi^R_{\mathrm{EM}} &= \frac{m\mu_e}{4r}{q^*_{\mathrm{EM}}}^2 + \left[\mu_e - w - \frac{m\mu_e}{2r}(\mu_d + r)\right] q^*_{\mathrm{EM}} \\ &\quad + p_r\mu_d - \mu_e\mu_d + \frac{m\mu_e}{4r}(\mu_d + r)^2.\end{aligned} \tag{29}$$

The abscissa of this reverse parabola's symmetric axis is $\mu_d + r - (2r/m\mu_e)(\mu_e - w)$.

Suppose that supply is adequate, that is, $\mu_d + r < M$. If $\mu_d + r - (2r/m\mu_e)(\mu_e - w) < (\mu_d + r)/2$, the optimal order quantity is $q^*_{\mathrm{EM}} = \mu_d + r$, else the retailer would choose to buy and sell in the e-market he builds without taking any orders from traditional suppliers, that is, $q^*_{\mathrm{EM}} = 0$. Hence there is

$$q^*_{\mathrm{EM}} = \begin{cases} \mu_d + r, & m < \dfrac{4r(\mu_e - w)}{\mu_e(\mu_d + r)}, \\ 0, & \text{else.} \end{cases} \tag{30}$$

The expected profit of retailer is given by

$$\Pi^{R*}_{\mathrm{EM}} = \begin{cases} (\mu_e - w)r + (p_r - w)\mu_d, & m < \dfrac{4r(\mu_e - w)}{\mu_e(\mu_d + r)}, \\ (p_r - \mu_e)\mu_d + \dfrac{m\mu_e}{4r}(\mu_d + r)^2, & \text{else.} \end{cases} \tag{31}$$

The additional revenue of the retailer is given by

$$\begin{aligned}C_{\mathrm{EM}} &= \int_{-\infty}^{q} dx \int_{-\infty}^{\infty} mp_e(q-x)\, f_{d,e}(x,p_e)\, dp_e \\ &\quad + \int_{q}^{\infty} dx \int_{-\infty}^{\infty} 2mp_e(x-q)\, f_{d,e}(x,p_e)\, dp_e.\end{aligned} \tag{32}$$

When demand and retail price in e-market are 2DU distributed, the additional revenue of the retailer is given by

$$C_{\mathrm{EM}}=\begin{cases}(\mu_e-w)\,r+\dfrac{m\mu_e r}{2} \\ \quad -\dfrac{r(\mu_e-w)^2}{2m\mu_e}, & m<\dfrac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)}, \\ (w-\mu_e)\,\mu_d+\dfrac{m\mu_e r}{2} \\ \quad -\dfrac{r(\mu_e-w)^2}{2m\mu_e}+\dfrac{m\mu_e(\mu_d+r)^2}{4r}, & \text{else.}\end{cases} \tag{33}$$

Hence, the retailer would build his private e-market if the additional income C_{EM} is larger than the construction and operation cost of an e-market.

3.4. The Choice between EB, ES, and EM. From the analyses above, it can be found that both supplier and retailer prefer their private e-market. In this section, we intend to analyze which kind of private e-market should supplier and retailer build.

3.4.1. Supplier as the Owner. When the supplier is the owner of EB or EM, he would compare the expected profit in EB and EM. The profits are (6) and (27) if demand and retail price in e-market are 2DU distributed. Equations show that supplier's profits are determined by the optimal order quantity in (4) and (25). There is the following theorem.

Theorem 1. *If $m>1-(v/\mu_e)$, the supplier would choose EM as his own e-market. If $0<m<1-(v/\mu_e)$, the supplier would choose EB as his own e-market. If $m=1-(v/\mu_e)$, it makes no difference for the supplier to choose EB or EM.*

From Theorem 1, it can be found that the supplier's decision depends on the unit salvage value v, the unit expected retail price μ_e, and the e-market use fee ratio m. For any given m and v, the supplier's choice will change from EM to EB as the expected e-market price μ_e increases. Because as μ_e increases, shutting down the selling channel can help in increasing expected sales in EB and earn more profit in EB than that in ES. For any given μ_e and v, the supplier will choose EM and give up EB as m increases since the supplier can charge more e-market use fee from the retailer in EM.

3.4.2. Retailer as the Owner. When the retailer is the owner of EB, ES, or EM, the expected profits are as (10), (19), and (31), respectively. The retailer would compare the expected profit in EB, ES, and EM when he makes choice. There are the following theorems.

Theorem 2. *If m and other parameters satisfy one of the following three conditions, the retailer would prefer EM as his own e-market to EB:*

(1)

$$\begin{gathered}\mu_e+v-2w>0,\\ m<\frac{4wv+\mu_e^2-2v^2-w^2-2w\mu_e}{\mu_e\,(\mu_e+v-2w)}\wedge\frac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)};\end{gathered} \tag{34}$$

(2)

$$\begin{gathered}\beta^2-4\alpha\gamma>0,\\ \frac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)}<m<\frac{\mu_e-v}{\mu_e}\wedge\frac{-\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha};\end{gathered} \tag{35}$$

(3)

$$\begin{gathered}\beta^2-4\alpha\gamma<0,\\ m>\frac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)},\end{gathered} \tag{36}$$

where

$$\begin{aligned}\alpha&=\mu_d^2\mu_e^2+2r\mu_d\mu_e^2+\mu_e^2r^2,\\ \beta&=\mu_d^2\mu_e v-\mu_e^2r^2-6r\mu_d\mu_e^2-\mu_d^2\mu_e^2+5v\mu_e r^2-4w\mu_e r^2\\ &\quad+2rv\mu_d\mu_e+4rw\mu_d\mu_e,\\ \gamma&=4r\mu_d\mu_e^2-4v\mu_e r^2+4w\mu_e r^2-4rv\mu_d\mu_e-4rw\mu_d\mu_e\\ &\quad+8r^2v^2-12r^2vw+4r^2w^2+4rvw\mu_d.\end{aligned} \tag{37}$$

Theorem 3. *If m and other parameters satisfy one of the following two conditions, the retailer would choose EM as his own e-market instead of ES:*

(1)

$$\begin{gathered}m<\frac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)},\\ 2\mu_e r<\frac{r(\mu_e-w)^2}{g-\mu_e+p_r};\end{gathered} \tag{38}$$

(2)

$$m>\frac{4r\,(\mu_e-w)}{\mu_e\,(\mu_d+r)}\vee\eta, \tag{39}$$

where $\eta=4r((\mu_e-w)^2+w(\mu_d+r)-\mu_e(\mu_d-r))/(\mu_e(\mu_d+r)^2(g-\mu_e+p_r))$.

Theorem 4. *If m and other parameters satisfy one of the following two conditions, the retailer would chose EB as his own e-market instead of ES:*

(1)

$$m < \frac{\mu_e - \nu}{\mu_e}; \tag{40}$$

(2)

$$m > \frac{\mu_e + \nu}{\mu_e} - \kappa, \tag{41}$$

where $\kappa = (w - \nu)^2(p_r + g - \mu_e)/(\mu_e((\mu_e - w)^2 - (p_r + g - \mu_e)(\mu_e + 2w - \nu)))$.

From the above three theorems, it can be found that the choice of the retailer depends on the three parameters, that is, μ_e, μ_d, and m. However, the three parameters are interacted and none of them can determine the selection of the retailer alone. The interaction between e-market use fee and uncertain parameters is complicated. We explore the interaction between them by numerical experiments in the following section.

4. Sensitivity Analysis

In this section, we deploy the numerical experiment research to analyze the impacts of some parameters. The experiment data are assumed by ourselves. Although they are not real data from the reality, all of them are in reasonable range.

In the first three subsections we analyze the impact of parameters on the optimal order quantity in the case that demand and retail price in e-market are 2DU distributed. The numerical analysis in a more general case that demand x and e-market price p_e are BN distributed is also given to make a comparison with 2DU case. The changing trend of each kind of demand and e-market retail price distribution is roughly similar to that illustrated in figures. The three subsections are, respectively, about the impact of the customer demand, the impact of the retail price in e-market, and the impact of the e-market use fee. In these subsections, we analyze the model in which the supplier is the owner in EM.

In the last two subsections, we analyze parameters' impact on the selection of EB, ES, and EM when supplier or retailer builds his own e-market in the case that demand and retail price in e-market are 2DU distributed.

4.1. The Impact of the Customer Demand. Assuming that $\mu_e = 12$, $\sigma_e = 2.5$, and $\mu_d = 100$ and other parameters are set as follows: $p_r = 30, w = 10, g = 20, \nu = 6, \rho = 0.2, M = 200$, and $m = 0.2$. Figure 1 shows that there is a positive relationship between customer demand variance and order quantity. It is due to high customer demand volatility that the retailer tends to order more products in traditional channel to meet the needs. When the order quantity exceeds the actual demand, excess inventory will be sold in e-market that can also ensure the increase of retailer's profit.

Assuming that $\mu_e = 12$, $\sigma_e = 2.5$, and $\sigma_d = 25$, we can also find that there is an approximately linear relationship

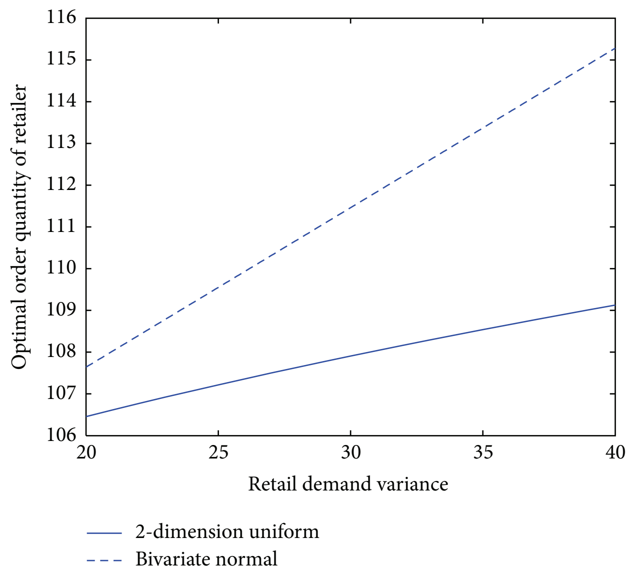

Figure 1: Changes of the order quantity with respect to the retail demand variance in EM.

between customer demand expectation and order quantity. So the figure is omitted.

In fact, the main source of retailer's profit is from traditional channel as we have supposed a lower retail price in e-market in this section. When the demand of traditional channel increases, retailer has an inevitable choice to increase order quantity to expand his income. Especially when the excess inventory can be sold in e-market, it is possible to pursuit more revenue.

4.2. The Impact of the Retail Price in E-Market. The parameters are reset as follows: $p_r = 10, w = 8, g = 6, \nu = 4, \mu_d = 60$, and $\sigma_d = 25$.

Figure 2 shows the relationship between order quantity and retail price expectation in EM. With the increase of retail price expectation, the order quantity also increases. This is because dramatic price fluctuations will make retailer's profit expectations get lower. This suggests that when a huge bubble merges in e-market, people should avoid entering into e-market.

4.3. The Impact of the Use Fee. In this part we discuss the relationship between order quantity and e-market use fee in e-market. The parameters are reset as follows: $\mu_d = 100$, $\sigma_d = 25$, $\mu_e = 12$, and $\sigma_e = 2.5$. Other parameters are the same as those in Section 4.1.

Figure 3 shows how the e-market use fee charged by e-market manager impacts order quantity in EM. It can be seen that the use fee affects order quantity negatively. When the e-market use fee ratio increases gradually, the retailer's order quantity declines slowly. Because the more e-market use fee is charged, the less the retailer trades in e-market till he quits entirely.

4.4. The Impact on the Selection of the Supplier. The parameters are set as follows: $p_r = 50, w = 30, g = 20, \nu = 20, r = 20, c = 18, \mu_d = 100, m = 0.1$, and $\mu_e = 30$.

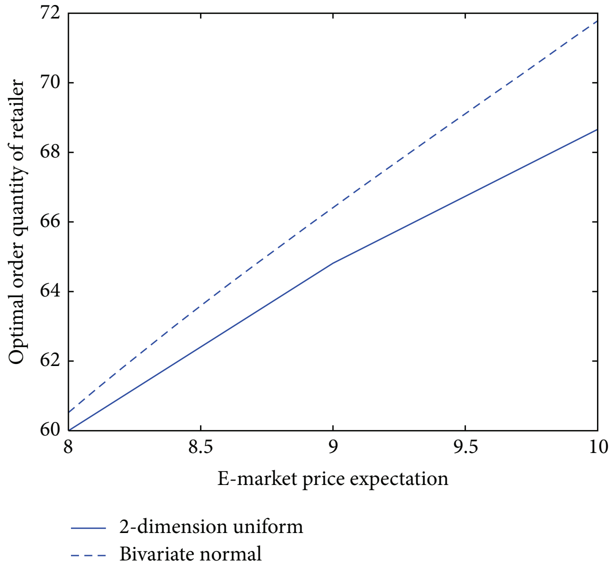

FIGURE 2: Changes of order quantity with respect to the retail price expectation in EM.

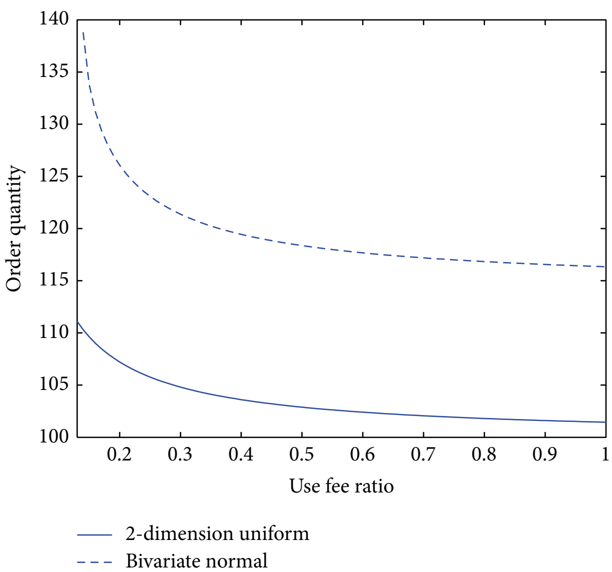

FIGURE 3: Changes of order quantity with respect to the e-market use fee ratio in EM.

Since the type of the e-market built by the supplier depends on μ_e, v, and m, here we suppose that v is a fixed amount and explore the relationship between μ_e, m, and expected profit.

Figure 4 shows the changes of the expected profit difference of the supplier with respect to μ_e and m when the supplier builds EB and EM. The horizontal grid in Figure 4 is the boundary in which the supplier chose EB or EM. The lines in $\mu_e - m$ plane are the contour line of profit difference. The contour line near $m = 0.25$ is the one that supplier's expected profit difference is zero. If the profit difference is below the zero horizon or behind the zero contour line, the expected

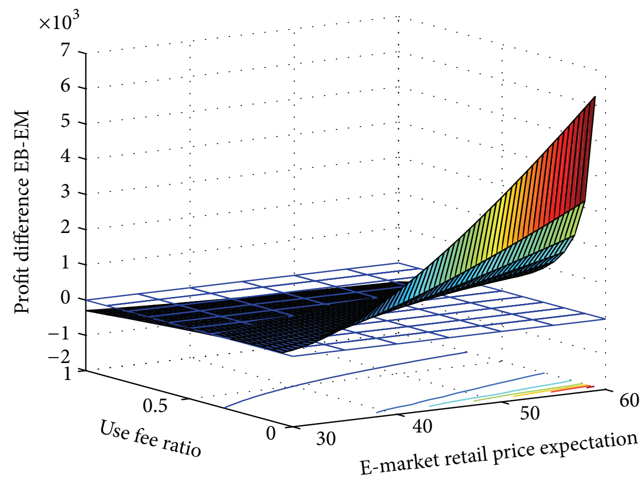

FIGURE 4: Changes of the expected profit difference of supplier in EB and EM with respect to μ_e and m.

profit in EB is less than that in EM. Hence the supplier would choose EM as his own e-market. When μ_e gets larger and m gets smaller, the supplier will build EB as he can get more profit in EB than in EM. This is because that the retailer would order fewer products in traditional market to avoid the excess inventory and transfer to electronic platform for ordering if he cannot sell goods in e-market. Thus, the supplier can sell more products in e-market which is exactly the motivation of the supplier to close the selling channel for the retailer and only build an EB market. This result agrees with Theorem 1.

4.5. The Impact on the Selection of the Retailer. The choice of the retailer is more complicated than that of the supplier according to Theorems 2, 3, and 4.

Figure 5 shows the changes of the retailer's expected profit with respect to μ_e in three different e-markets. It can be found that ES is the worst e-market under these initial conditions. There is a threshold point in EM as the retailer's expected profit is a piecewise function determined by m and $4r(\mu_e - w)/(\mu_e(\mu_d + r))$. Before this point, there is a negative relationship between profit of EM and μ_e. After this point, the profit in EM increases because the retailer can trade in EM completely. As for EB, the expected profit in it is decreasing with the retail price μ_e. The profits in three e-markets indicate that the retailer should shut down the selling channel for more profit when μ_e is in a certain interval. This is because of the fact that opening selling channel can help manage the excess inventory or obtain more revenue when e-market retail price is too large or too small. But selling channel is not an advantage if the retailer making ordering strategy when μ_e is in a certain interval.

As the expected profit in ES is very low in these numerical conditions, we only explore the changes of the expected profit in EB and EM with respect to the demand expectation and variance. Figure 6 shows the changes of the expected profit of the retailer with respect to the mean value of demand μ_d and demand variance $r^2/3$ in three different e-markets. The lines in *expectation-variance* plane are the contour line of profit

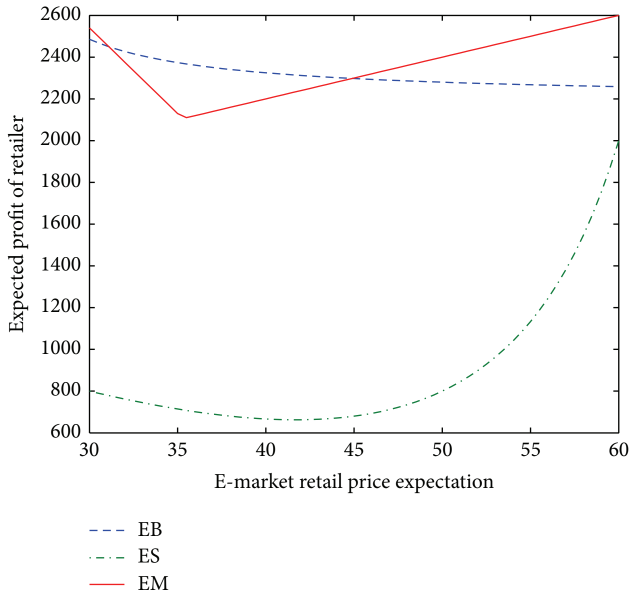

FIGURE 5: Changes of the retailer's expected profit with respect to μ_e.

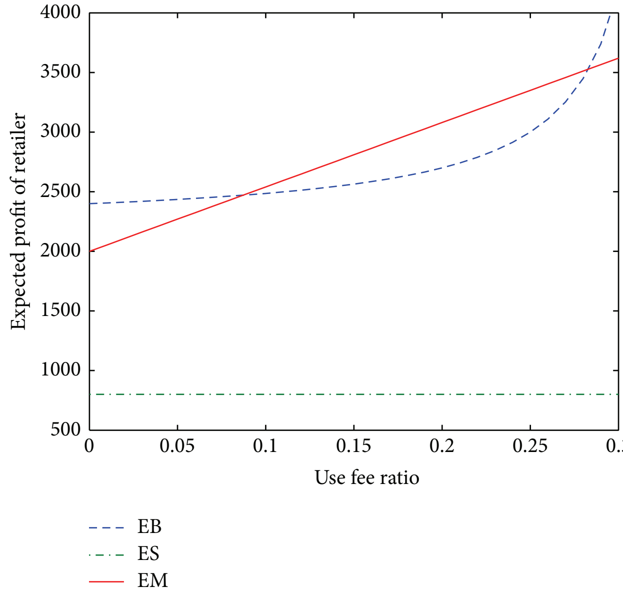

FIGURE 7: Retailer's expected profit changes with respect to e-market use fee ratio when he builds EB, ES, and EM.

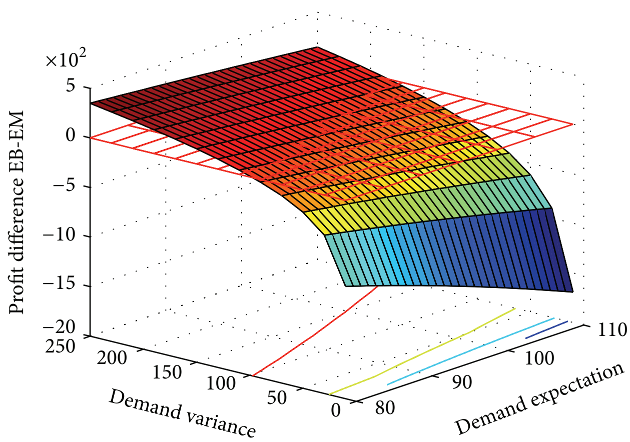

FIGURE 6: Changes of the expected profit difference of retailer in EB and EM with respect to μ_d and $r^2/3$.

difference. The contour line near *variance* = 100 is the decision boundary of the retailer. When demand expectation and demand variance are in the left side of this boundary in Figure 6, the retailer should choose EB from the calculation results. It illustrates that the choice between EB and EM mainly depends on the volatility of demand and is less affected by mean value of demand in this numerical case. If the volatility of demand is very big, the retailer should shut down the selling channel and order in traditional market according to the expected variance before selling season.

Figure 7 shows the changes of the expected profit of the retailer with respect to the e-market use fee ratio m in three different e-markets. Parameter m has no impact on the profit in ES which can also be observed in (19). There is a positive relationship between e-market use fee ratio and profit in EM. The profit function in EB is a nondecreasing convex function of m. Since the retailer is the owner of e-market, he will earn more from the supplier who sells products via e-market if m increases. The retailer should choose EM when the e-market use fee ratio is in a certain interval.

Through the above numerical examples, it can be found that the retailer's choice indeed depends on different product parameters. The key point of retailer's decision is the comparison of expected profits.

5. Conclusions

This paper develops some models to explore the issue of selection between several types of e-markets from the view point of supply chain. Besides demand uncertainty, we also take into account the price uncertainty in e-market. We explore the conditions under which the agent of supply chain selects one certain type of e-market by comparing expected profits of supply chain members in different scenarios in the case of customer demand and retail price following uniform distribution. Some sensitivity analyses are also conducted to explore the impact of the customer demand, e-market retail price, and e-market use fee on the optimal order quantity and on the selection of e-market in the case that demand x and e-market price p_e are 2DU and BN. Our results demonstrate that the e-market use fee can be an important factor for assessing the performance of an e-market, and therefore is worth taking into consideration in the building of e-market. The main contribution of our paper is to obtain the additional revenue of the owner of the e-market. The cost of operating private e-market is a complicated value that contains multiple factors with specialized knowledge. If we can use a parameter to describe the operating cost, the comprehensive trade-off between different scenarios is straightforward by comparing the operating cost and the additional revenue. In this paper, the supplier's wholesale price and the retailer's retail price are assumed to be exogenous. If the retail price is ingenuous, the selection of the e-market in supply chain is a potential topic for future research. In addition, if the retailer can return the excess inventory to the supplier, it is another valuable research problem, that is, the buying-back option, for future research.

Appendix

A. The Derivation of the Optimal Order Quantity in Different Scenarios

Proof. When demand and retail price in e-market are BN distributed, the optimal order quantity satisfies the following equations.

In public EB, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{EB}}^*\right) = \frac{(1+m)\, f_d\left(q_{\mathrm{EB}}^*\right)}{-v+(1+m)\,\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{EB}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{-w+(1+m)\,\mu_e}{-v+(1+m)\,\mu_e}. \tag{A.1}$$

As the retailer is the owner of a private EB, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{EB}}^*\right) = \frac{f_d\left(q_{\mathrm{EB}}^*\right)}{-v+\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{EB}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{-w+\mu_e}{mp_r - v+\mu_e}. \tag{A.2}$$

In public ES, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{ES}}^*\right) = -\frac{f_d\left(q_{\mathrm{ES}}^*\right)}{p_r+g-(1-m)\,\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{ES}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{p_r+g-w}{p_r+g-(1-m)\,\mu_e}. \tag{A.3}$$

As the retailer is the owner of a private ES, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{ES}}^*\right) = -\frac{f_d\left(q_{\mathrm{ES}}^*\right)}{p_r+g-\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{ES}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{p_r+g-w}{p_r+g-\mu_e}. \tag{A.4}$$

In public EM, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{EM}}^*\right) = \frac{f_d\left(q_{\mathrm{EM}}^*\right)}{\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{EM}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{-w+\mu_e(1+m)}{2m\mu_e}. \tag{A.5}$$

As the retailer is the owner of a private EM, the optimal order quantity is given by

$$F_d\left(q_{\mathrm{EM}}^*\right) = \frac{f_d\left(q_{\mathrm{EM}}^*\right)}{\mu_e}\left(2\mu_d\mu_e - 2q_{\mathrm{EM}}^*\mu_e + \rho\sigma_d\sigma_e\right) + \frac{\mu_e - w}{2\mu_e}. \tag{A.6}$$

□

Proof of Theorem 1. When the supplier is the owner of EB and EM, the difference of his optimal expected profit between EB and EM is given by

$$\Pi_{\mathrm{EB}}^{S*} - \Pi_{\mathrm{EM}}^{S*} = \frac{r(w-(1+m)\,\mu_e)^2\,(v-(1-m)\,\mu_e)}{m\mu_e\,(v-(1+m)\,\mu_e)}. \tag{A.7}$$

Notice that $(w-(1+m)\mu_e)^2 > 0$ and $v-(1+m)\mu_e < 0$, and the sign of $\Pi_{\mathrm{EB}}^{S*} - \Pi_{\mathrm{EM}}^{S*}$ is the same with $v-(1-m)\mu_e$, that is, $m-(1-(v/\mu_e))$. The proof is completed. □

Proof of Theorem 2. There are two cases when the retailer compares the profits in EB and EM.

Case 1. When $m < 4r(\mu_e - w)/(\mu_e(\mu_d + r))$, the difference of retailer's optimal expected profit between EM and EB is given by

$$\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{EB}}^{R*} = r\left(\mu_e + v - 2w\right) + \frac{r(v-w)^2}{v-(1-m)\,\mu_e}. \tag{A.8}$$

Solving inequality $\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{EB}}^{R*} > 0$, the following solution sets are obtained. □

(1) Consider the following:

$$\mu_e + v - 2w < 0, \qquad \frac{\mu_e - v}{\mu_e} < m < \frac{4wv+\mu_e^2-2v^2-w^2-2w\mu_e}{\mu_e\left(\mu_e+v-2w\right)}. \tag{A.9}$$

(2) Consider the following:

$$\mu_e + v - 2w > 0, \qquad m > \frac{\mu_e - v}{\mu_e}. \tag{A.10}$$

(3) Consider the following:

$$\mu_e + v - 2w > 0, \qquad m < \frac{4wv+\mu_e^2-2v^2-w^2-2w\mu_e}{\mu_e\left(\mu_e+v-2w\right)}. \tag{A.11}$$

Since Π_{EB}^{R*} is calculated in the condition that Π_{EB}^{R*} is concave with q_{EB}^*, $m < (\mu_e - v)/\mu_e$ must be true. And when $\mu_e + v - 2w > 0$, there is $(4wv+\mu_e^2-2v^2-w^2-2w\mu_e)/(\mu_e(\mu_e + v - 2w)) < (\mu_e - v)/\mu_e$. Therefore, conditions (1) and (2) are rejected. In this case, conditions where the retailer chooses EM as his own e-market can be formulated as follows:

(4) Consider the following:

$$\mu_e + \nu - 2w > 0,$$
$$m < \frac{4w\nu + \mu_e^2 - 2\nu^2 - w^2 - 2w\mu_e}{\mu_e(\mu_e + \nu - 2w)} \wedge \frac{4r(\mu_e - w)}{\mu_e(\mu_d + r)}. \quad (A.12)$$

Case 2. When $m > 4r(\mu_e - w)/(\mu_e(\mu_d + r))$, the difference of retailer's optimal expected profit between EM and EB is given by

$$\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{EB}}^{R*} = \frac{\alpha m^2 + \beta m + \gamma}{4r(m\mu_e + \nu - \mu_e)}, \quad (A.13)$$

where $\alpha = \mu_d^2\mu_e^2 + 2r\mu_d\mu_e^2 + \mu_e^2 r^2 > 0$.

When $\Delta = \beta^2 - 4\alpha\gamma < 0$, EM is always better than EB.

When $\Delta = \beta^2 - 4\alpha\gamma > 0$, the two solutions of $\alpha m^2 + \beta m + \gamma = 0$ are $(-\beta - \sqrt{\beta^2 - 4\alpha\gamma})/2\alpha$ and $(-\beta + \sqrt{\beta^2 - 4\alpha\gamma})/2\alpha$. The following solution set can be obtained after sovling inequality $\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{EB}}^{R*} > 0$ and condition (6) is also rejected because $m < (\mu_e - \nu)/\mu_e$.

(5) Consider the following:

$$m > \frac{4r(\mu_e - w)}{\mu_e(\mu_d + r)},$$
$$m < \min\left(\frac{\mu_e - \nu}{\mu_e}, \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}\right). \quad (A.14)$$

(6) Consider the following:

$$m > \max\left(\frac{4r(\mu_e - w)}{\mu_e(\mu_d + r)}, \frac{\mu_e - \nu}{\mu_e}, \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}\right). \quad (A.15)$$

Summarizing the two cases, the retailer should choose EM under the conditions (4) and (5) or he should choose EB as his own e-market. Thus, the proof is completed.

Proof of Theorem 3. There are also two cases when the retailer compares the profits in EM and ES.

Case 1. When $m < 4r(\mu_e - w)/(\mu_e(\mu_d + r))$, the difference of the retailer's optimal expected profit between EM and ES is given by

$$\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{ES}}^{R*} = 2\mu_e r - \frac{r(\mu_e - w)^2}{g - \mu_e + p_r}. \quad (A.16)$$

If parameters satisfy the following conditions, $\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{ES}}^{R*} > 0$. □

(1) Consider the following:

$$m < \frac{4r(\mu_e - w)}{\mu_e(\mu_d + r)}, \qquad 2\mu_e r > \frac{r(\mu_e - w)^2}{g - \mu_e + p_r}. \quad (A.17)$$

Case 2. When $m > 4r(\mu_e - w)/(\mu_e(\mu_d + r))$, the difference of retailer's optimal expected profit between EM and ES is given by

$$\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{ES}}^{R*} = w(\mu_d + r) - \mu_e(\mu_d - r) + \frac{m\mu_e(\mu_d + r)^2}{4r} - \frac{r(\mu_e - w)^2}{g - \mu_e + p_r}. \quad (A.18)$$

$\Pi_{\mathrm{EM}}^{R*} - \Pi_{\mathrm{ES}}^{R*} > 0$ results in the following condition:

$$m > \frac{4r\left((\mu_e - w)^2 + w(\mu_d + r) - \mu_e(\mu_d - r)\right)}{\mu_e(\mu_d + r)^2(g - \mu_e + p_r)}. \quad (A.19)$$

In the condition, we obtain another inequality (A.19). So the final range of in this case is condition (2) as there should be a join between and inequality (A.19).

Equivalently, in this case only the parameters satisfy condition (2) should the retailer choose EM as his own e-market.

(2) Consider the following:

$$m > \frac{4r(\mu_e - w)}{\mu_e(\mu_d + r)} \vee \eta, \quad (A.20)$$

where $\eta = 4r((\mu_e - w)^2 + w(\mu_d + r) - \mu_e(\mu_d - r))/(\mu_e(\mu_d + r)^2(g - \mu_e + p_r))$.

In the case and, we obtain condition (1) and condition (2). To sum up, a union result of the two conditions is the theorem condition.

Proof of Theorem 4. When the retailer makes a choice between EB and ES, the optimal profit difference is given by

$$\Pi_{\mathrm{EB}}^{R*} - \Pi_{\mathrm{ES}}^{R*} = r(w + \mu_e) - r(\nu - w) + \frac{r(\mu_e - w)^2}{\mu_e - g - p_r} + \frac{r(\nu - w)^2}{(1 - m)\mu_e - \nu}. \quad (A.21)$$

While Π_{EB}^{R*} and Π_{ES}^{R*} are concave with q_{EB}^* and q_{ES}^*, $m < (\mu_e - \nu)/\mu_e$ and $g + p_r - \mu_e > 0$ are true. The following constraint of m can be easily derived by $\Pi_{\mathrm{EB}}^{R*} - \Pi_{\mathrm{ES}}^{R*} > 0$.

(1) Consider the following:

$$m > \frac{\mu_e + \nu}{\mu_e} - \kappa, \quad (A.22)$$

where we denote $\kappa \triangleq (w - \nu)^2(p_r + g - \mu_e)/\mu_e((\mu_e - w)^2 - (p_r + g - \mu_e)(\mu_e + 2w - \nu))$. As we do not know the value of $(\mu_e - \nu)/\mu_e$ and $((\mu_e + \nu)/\mu_e) - \kappa$, the final condition that retailer chooses EB as his own e-market is

$$m > \frac{\mu_e + \nu}{\mu_e} - \kappa, \quad (A.23)$$

or

$$m < \frac{\mu_e - \nu}{\mu_e}. \quad (A.24)$$

Then the proof is completed. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

A Methodology to Exploit Profit Allocation in Logistics Joint Distribution Network Optimization

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Logistics joint distribution network (LJDN) optimization involves vehicle routes scheduling and profit allocation for multiple distribution centers. This is essentially a combinational and cooperative game optimization problem seeking to serve a number of customers with a fleet of vehicles and allocate profit among multiple centers. LJDN routing optimization based on customer clustering units can alleviate the computational complexity and improve the calculation accuracy. In addition, the profit allocation mechanism can be realized based on cooperative game theory through a negotiation procedure by the Logistics Service Provider (LSP). This paper establishes a model to minimize the total cost of the multiple centers joint distribution network when each distribution center is assigned to serve a series of distribution units. An improved particle swarm optimization (PSO) algorithm is presented to tackle the model formulation by assigning distribution centers (DCs) to distribution units. Improved PSO algorithm combines merits of PSO algorithm and genetic algorithm (GA) with global and local search capabilities. Finally, a Shapley value model based on cooperative game theory is proposed to obtain the optimal profit allocation strategy among distribution centers from nonempty coalitions. The computational results from a case study in Guiyang city, China, suggest the optimal sequential coalition of distribution centers can be achieved according to Strictly Monotonic Path (SMP).

1. Introduction

A logistics joint distribution network (LJDN) is usually composed of several logistics facilities (e.g., logistics centers and distribution centers) and a large number of customers [1–3]. Different from single-depot logistics distribution network optimization problems, the cooperation mechanism widely exists for LJDN. How to allocate the cost savings among different logistics participants in LJDN is considered as the most critical issue during the optimization procedure. Properly optimizing LJDN not only mitigates network-wide traffic congestion and reduces negative environmental effects (i.e., energy consumption and traffic pollution), but also promotes mutual cooperation for profit maximization [4, 5]. To achieve these goals, a necessary negotiation process between multiple participants is desired, and this process can be implemented by introducing a Logistics Service Provider (LSP) to coordinate or discussing within the existing participants [6–8]. The LSP can be defined as a Logistics Service Provider that performs corresponding logistics operations on behalf of other participants [9]. LSPs are actively looking for opportunities to increase both efficiency and profit for their own clients (i.e., participants) [10, 11]. In addition, logistics companies and manufacturers tend to outsource their noncore business to a third-party for financial savings, and this raises the transportation demand and stimulates resource integration in LSP market.

LJDN optimization is a strategic and tactical procedure with multiple complicated steps, such as multiple depots (or centers) vehicle routing optimization and profit allocation procedures. Most previous studies focused on designing efficient algorithms for solving Multiple Depot Vehicle Routing Problems (MDVRP) [2, 12–14]. To ensure successful delivery and pickup services in MDVRP, the coordination among multiple depots should be taken prior to vehicle routing. This incurs an interesting issue on how to allocate profit within multiple logistics entities in a cooperative manner [15, 16]. Traditional MDVRP neglected such a profit allocation procedure by assuming that each logistic entity is willing to cooperate. This assumption oversimplifies the realistic condition where the logistics activities are profit-driven, and thus individual benefit should be incorporated during horizontal and vertical cooperation [11], which can be achieved via a profit allocation mechanism to retain a mutually beneficial relationship. The stability of cooperation relies on the rationality of profit allocation. Therefore, the rationality of profit allocation is the core of the logistics joint distribution network optimization [17] and should be taken into account in this study.

The cost savings from multiple depots vehicle routing optimization will be served as the input to allocate profit among various logistics entities. Therefore, developing a robust solution algorithm to find optimal solutions for MDVRP is necessary. However, in metropolitan logistics distribution network with thousands of customers, traditional approach for MDVRP may not be effective to cope with such a sophisticated scenario [18, 19]. To alleviate the computational complexity and improve the calculation accuracy, customer clustering should be initially applied before VRP optimization. A large logistics region can be grouped into several smaller zones where customers share certain common features (i.e., geospatial location, demand, etc.). Then, customers within each zone form a clustering unit and request service from each depot. Many researchers have proposed a variety of clustering methods to study MDVRP [14, 20–24]. Thangiah and Salhi [21] presented a generalized clustering approach based on genetic algorithm, and their genetic clustering method can be further used to solve the MDVRP. Wu et al. [22] proposed a hybrid simulated annealing algorithm to solve MDVRP, where insertion and 2-swap operators were used to manipulate customers from one cluster to another cluster. Dondo and Cerdá [23] developed a three-stage heuristic approach for the multiple depots routing problem with time windows and heterogeneous vehicles: cluster generation, cluster assignment and sequencing, and nodes sequencing within clusters (i.e., vehicle scheduling). Mirabi et al. [14] presented three hybrid heuristics to solve the MDVRP. Yücenur and Demirel [24] developed a genetic algorithm based on the clustering technique for studying the multidepot vehicle routing problem, and the cluster first-route second algorithm was proposed. In their paper, clustering procedures were used to group customers with similar characteristics to reduce the calculation complexity.

Customer clustering is usually an intermediate stage during the MDVRP optimization procedure. The distribution centers (DCs) can be assigned to a number of customer clustering units for delivery service, and this issue can be considered as a variant of the quadratic assignment problem (QAP) [25, 26]. The QAP was first introduced by Koopmans and Beckmann [27] and aimed to assign n facilities to n locations in such a way that each facility is assigned to one exact location. The goal of the QAP is to minimize the sum of the distances multiplied by the corresponding flows and the associated cost of allocating each facility to a certain location [28–30]. Within each clustered region, the mathematical programming model and solution algorithm can be further developed to solve MDVRP [12, 15, 31]. Ho et al. [13] developed two hybrid genetic algorithms based on customer clustering techniques for dealing with MDVRP. Liu et al. [32] presented a mathematical programming model and a two-phase greedy algorithm to study the full truckloads multidepot capacitated vehicle routing problem in carrier collaboration. Aras et al. [16] formulated two mixed-integer linear programming models (MILP) for selective MDVRP with pricing, and a Tabu Search on the basis of heuristic method was proposed to solve the MILP model. Bettinelli et al. [33] established an integer linear programming model and presented a branch-and-cut-and-price algorithm to solve the multidepot heterogeneous vehicle routing problem with time windows. Narasimha et al. [34] proposed an extension of ant-colony technique to solve the min-max MDVRP. Tu et al. [35] presented a bi-level Voronoi diagram-based metaheuristic to tackle the large-scale MDVRP.

Based on the aforementioned discussion, a natural thought is to incorporate the profit allocation paradigm into multiple depots' vehicle routing optimization for logistics joint distribution network. However, only a small number of relevant studies have been conducted on this research domain. Özener and Ergun [36] presented cost allocation mechanisms based on the cooperative game theory, and then a set of new properties and several cost allocation schemes were proposed to study a collaborative transportation procurement network. Krajewska et al. [37] presented the profit margins resulting from horizontal cooperation among freight carriers, which is based on the cooperative game theory for a pickup and delivery problem with time windows. The possibilities of sharing these profit margins among the partners were also discussed. Wang et al. [38] constructed two mathematical models to study the optimal allocation of the module members for given garment assembly tasks in a modular production system. Frisk et al. [17] proposed a new cost allocation method based on economic models. These models include Shapley value, the nucleolus, shadow prices, and volume weights for collaborative forest transportation. Cruijssen et al. [7] proposed a novel "supplier-initiated outsourcing" procedure to exploit synergy in transportation. Lozano et al. [39] presented a linear model to allocate the cost savings among different companies when their transportation requirements are simultaneously considered. However, the above studies suffer from the following issues. (1) The network size in most studies is relatively small. When considering the large-scale logistics distribution network, customer clustering should be adopted prior to VRP optimization for reducing the calculation complexity. Consequently, the mathematical programming model and solution algorithm should be designed to optimize the logistics distribution network based on customer clustering units rather than customers. (2) Most studies focus on investigating the mechanism of profit allocation from economic perspectives but neglect the interaction between profit allocation and VRP optimization. To the best of our knowledge, no explicit architecture was developed to explain how optimized vehicle routings affect the profit distribution among multiple logistics participants. Therefore, a reasonable profit allocation approach based on cooperative game theory should be designed and combined with MDVRP for LJDN optimization.

This study aims to construct a multiple centers logistics joint distribution network and then develops a linear programming model with a solution algorithm to optimize the network for cost savings calculation. Based on the computed cost savings, a profit allocation approach is proposed to distribute the total profit within logistics participants and determine the optimal strategy for sequential coalitions. Compared with the previous studies, the main contributions of this paper lie in the following. (1) The model formulation is first established to minimize the total cost in the LJDN optimization procedure. (2) An improved particle swarm optimization (PSO) algorithm is proposed to assign distribution centers (DCs) to distribution units and resolve the model. (3) Shapley value model is utilized to study the profit allocation among multiple distribution centers in LJDN. (4) A real-world numerical study is undertaken to demonstrate the applicability of the proposed method.

2. Logistics Joint Distribution Network

Logistics joint distribution network (LJDN) can be established through negotiation. The negotiation procedure is organized by either Logistics Service Provider (LSP) or players from the distribution network [1, 4, 40–42]. LJDN can reasonably integrate the resources together. Therefore, it can reduce the crisscross transportation phenomenon and realize information sharing. In our study, the logistics distribution network contains multiple DCs and a large amount of customer clustering units. Figure 1 presents a logistics network structure change before and after joint distribution. DCs are independent with each other before the LJDN is established. Each customer clustering unit is a group of customers with common features, such as similar temperature controlled goods and similar geographical conditions; the customer clustering unit is referred to as a distribution unit.

The logistics distribution network structure without joint distribution exhibits a nonoptimal condition as shown in Figure 1(a), where several DCs still serve those distribution units with long distances, even if these distribution units are adjacent to other peer DCs. This is probably due to the customer loyalty and market condition [4, 5]. For instance, customers may still continue to request the same service from their previous DCs for long-term cooperation relations. In addition, if no equivalent policies (e.g., door-to-door service, discount, etc.) are available from the other alternative DCs in the market, customers are more likely to accept services from previous DCs even if these DCs are far away from them. To optimize the unreasonable network structure, the cooperation between DCs should be promoted by reassigning distribution units to different DCs. Such a network structure adjustment may lead to cost savings compared with the previous "unreasonable" logistics network, and the generated profits are needed to distribute among multiple DCs. Therefore, the LJDN optimization becomes two critical issues on how to redesign the logistics network and then allocate the gained profits within multiple DCs in an effective manner. As shown in Figure 1(b), when LJDN is established, each distribution unit is reasonably assigned to its adjacent DC. Goods can be delivered among DCs by a fleet of semitrailer trucks, and each DC can perform a variety of routing plans to serve several distribution units with small trucks. LJDN optimization needs to consider the interplay among DCs and distribution units, and it is a multiconstraint combinatorial and game optimization issue. The goal of LJDN is to serve each customer timely and reduce the total cost of the entire system.

Under the actual circumstance, each distribution unit includes multiple customers, and the center of each distribution unit can be calculated as $\overline{x_j} = \sum_{o=1}^{N_j} x_{jo}/N_j$, $\overline{y_j} = \sum_{o=1}^{N_j} y_{jo}/N_j$, where the coordinate for each customer o in the distribution unit j can be expressed as (x_{jo}, y_{jo}), and the number of customers is N_j. The distance can be calculated based on each set of coordinates. For the convenience of calculation, several assumptions need to be set before LJDN is established.

(1) The flow of goods is bidirectional between DCs; this means that DCs contain both input and output flows.

(2) The customer demands are predetermined and considered relatively stable within a certain period.

(3) Semitrailer trucks are used for loading and unloading goods between the DCs, and small trucks are used for delivery between DCs and distribution units.

(4) The distribution units are served by a fleet of small trucks. Each small truck will return to its DC after serving all customers in each distribution unit. For each small truck, the total delivery distance is identical to the returning distance. It is worth noting that no vehicle routing problem (VRP) was involved to determine the sequence for both DCs and distribution units in this study.

(5) Multiple DCs are independent with each other before the LJDN is established. LJDN will be established through negotiation organized by LSP, and the ultimate optimization goal is to maximize the cost savings.

3. Related Definitions and Model Formulation

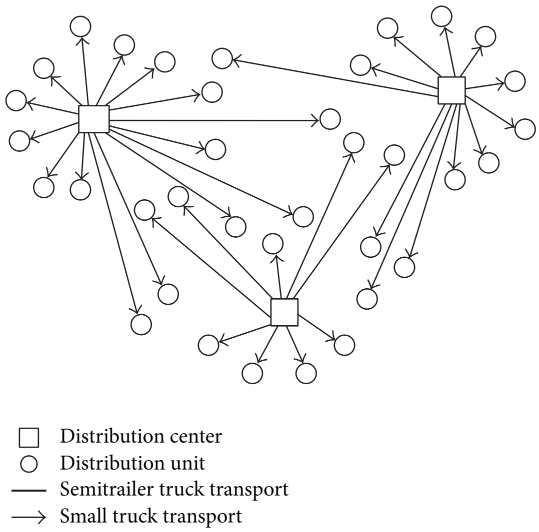
3.1. Related Definitions. Several related definitions are needed and described as follows.

$I\{i \mid i = 1, 2, 3, \ldots, m\}$ denotes the set of distribution centers; in addition, $h \in I$, and h represents a distribution center that is different from i and denoted as $h \neq i$.

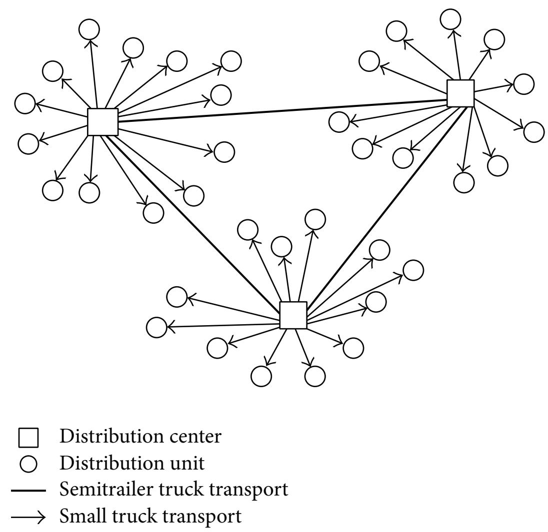
$J\{j \mid j = 1, 2, 3, \ldots, n\}$ denotes the set of distribution units in a logistics distribution network.

x_{ij} expresses the delivery quantity from the ith distribution center to the jth distribution unit.

q_j denotes the demand quantity of the jth distribution unit within one working period.



(a) Before joint distribution



(b) After joint distribution

Figure 1: Logistics joint distribution network diagram.

Q expresses the total demand quantity of all distribution units and can be expressed as $Q = \sum_j q_j$, where $j \in J$.

d_i denotes the delivery capacity of the ith distribution center within one working period.

$d_{i,h}$ expresses the delivery quality from distribution center i to h.

D denotes the total delivery capacity of the distribution centers and can be expressed as $D = \sum_i d_i$, where $i \in I$.

$B\{B_i \mid i = 1, 2, \ldots, m\}$ denotes the decision matrix, and $B_i = 0$ or 1, where $i \in I$. $B_i = 1$ denotes that the distribution center i agrees to cooperate in logistics joint distribution network. $B_i = 0$ denotes that the distribution center i refuses to cooperate in logistics joint distribution network.

$r_{i,j}$ denotes the distribution relation between DCs and distribution units, $r_{i,j} = 0$ or 1, where $i \in I$, $j \in J$, $r_{i,j} = 1$ denotes that the distribution center i will deliver goods to the distribution unit j, and $r_{i,j} = 0$ indicates that the distribution center i will not deliver goods to the distribution unit j. In addition, $\sum_{i \in I} r_{i,j} = 1$.

3.2. Model Formulation. The model formulation can be considered as the objective function to minimize the total cost when each DC is assigned to serve a series of distribution units. The notations used in the logistics joint distribution network optimization formulation are listed as follows:

LC_v: loading capacity of small truck,

LC_s: loading capacity of semitrailer truck,

C_v: fuel consumption rate of small truck (gallon/K miles),

C_s: fuel consumption rate of semitrailer truck (gallon/K miles),

ρ_v: gasoline price (dollar/gallon),

ρ_s: diesel price (dollar/gallon),

F_v: annual maintenance cost of small truck (dollar/year),

F_s: annual maintenance cost of semitrailer truck (dollar/year),

$L_{i,h}$: distance from the distribution center i to h,

$L_{i,j}$: distance from the distribution center i to the distribution unit j,

λ: variable cost coefficient of each distribution center,

K: distance unit for fuel consumption calculation,

T: number of working period,

G_i: fixed cost of the distribution center i in a working period,

ς_i: service cost (e.g., personnel cost, maintenance cost, transportation cost, etc.) of the LSP for distribution center i when cooperation is achieved in a working period.

The model formulation is composed of F_1, F_2, and F_3; these subformulations can be shown as follows.

F_1 denotes the sum of transportation cost and annual maintenance cost for each pair of distribution centers in LJDN within a working period, and it can be calculated as

$$F_1 = \sum_{i,h \in I, h \neq i} \left(\frac{d_{i,h}}{\mathrm{LC}_s} \times \frac{C_s}{K} \times \rho_s \times L_{i,h} + \frac{d_{i,h}}{\mathrm{LC}_s} \times \frac{F_s}{T} \right). \quad (1)$$

F_2 denotes the sum of transportation cost and annual maintenance cost from each distribution center assigned to each distribution unit by a fleet of small trucks within a working period; it can be calculated as

$$F_2 = \sum_{i\in I}\sum_{j\in J}\left(\frac{r_{i,j}\times x_{i,j}}{\mathrm{LC}_v}\times\frac{C_v}{K}\times\rho_v\times 2\times L_{i,j}\right) + \sum_{i\in I}\left(\frac{d_i}{\mathrm{LC}_v}\times\frac{F_c}{T}\right). \quad (2)$$

F_3 is fixed and variable costs of all distribution centers and LSP:

$$F_3 = \sum_{i\in I}\left(1-B_i\right)G_i + B_i Z_i + \lambda\times d_i. \quad (3)$$

The objective function for logistics joint distribution network optimization can be expressed as follows:

$$\text{Min} \quad F = F_1 + F_2 + F_3 \quad (4)$$

$$\text{Subject to} \quad d_i = \sum_{j\in J} r_{i,j}\times x_{i,j}, \quad \forall i\in I \quad (5)$$

$$B_i \le 1, \qquad B_i = \{0,1\}, \quad \forall i\in I \quad (6)$$

$$\sum_{j\in J} q_j = \sum_{i\in I} d_i, \quad \forall i\in I,\ \forall j\in J \quad (7)$$

$$Q = D, \quad \forall i\in I,\ \forall j\in J \quad (8)$$

$$\sum_{i\in I}\left(r_{i,j}\times x_{i,j}\right) = q_j, \quad \forall i\in I,\ \forall j\in J \quad (9)$$

$$\sum_{i\in I} r_{i,j} = 1, \quad \forall i\in I,\ \forall j\in J \quad (10)$$

$$r_{i,j} = \{0,1\}, \quad \forall i\in I,\ \forall j\in J. \quad (11)$$

The objective function (4) is to minimize the total cost of a logistics joint distribution network. Constraint (5) ensures that delivery capacity is equal to total delivery quantity at distribution center i. Constraint (6) enumerates two scenarios that the distribution centers either agree to cooperate or not. Constraint (7) and constraint (8) guarantee that the total demand quantity should equal the total delivery quantity. Constraint (9) ensures that the total delivery quantity from all distribution centers to the distribution unit j equals the demand at the distribution unit j. Constraint (10) assures that a distribution unit can only be served by one distribution center. In constraint (11), if the distribution center i serves the distribution unit j, then $r_{i,j}$ is set to 1; otherwise, $r_{i,j}$ is set to 0.

3.3. Shapley Value Model. When cooperation among multiple distribution centers in LJDN is achieved, each distribution unit is adequately assigned to its adjacent distribution center for VRP optimization. LSP is then required to allocate the gained profit due to network structure adjustment to each distribution center. To implement a fair and effective profit allocation strategy, the Shapley value model can be utilized in this study. The Shapley value model belongs to the cooperative game theory, which studies cooperative behavior by analyzing the negotiation process within a group of players in setting up a joint plan or contract of activities, such as profit allocation of collaboratively generated revenues. The Shapley value model is a method that presents a unique solution to the cost and profit allocation problem. Properly allocating profit can generate the synergy savings and be critical to any logistics cooperation [17, 39, 43, 44]. Several related notations are needed and presented as follows:

N: set of target players in a coalitional form and is called grand coalition,

S: set of coalitions from the collection of all subsets of N,

$v(S)$: values of all coalitions S,

$\phi(N,v)$: Shapley value allocated to a certain player in the coalitions,

T: subset of coalitions belonging to S,

σ: the LSP's synergy requirement,

$C_0(i)$: cost of player i without coalition,

$C(S)$: total cost in the S by LSP,

Γ: set of possible sequential coalitions in grand coalition N,

$\pi(i)$: rank of player i in sequence π,

$\eta(i,\pi,s)$: cost reduction percentages to player i on step s along sequence π.

When we set N as a finite set of players, $2^N - 1$ can denote the number of all subsets of N excluding the null set. The elements of all subsets are called *coalitions*; N is also known as the *grand coalition*. The values of all coalitions in S are mapped by a characteristic function denoted by v. The Shapley value method is to construct a vector $\phi(N,v)$ that allocates the value $v(N)$ of the grand coalition based on the values $v(S)$ of all coalitions S [45]. The Shapley value model stated in (12) expresses the profit to be allocated for player i and is on the basis of the hypothesis that the grand coalition is formed by entering each player into this coalition at a time. As the player i enters the coalition, player i can be assigned the *marginal contribution* $v(S) - v(S - \{i\})$. The Shapley value is the average expected payoff of players in a completely random procedure:

$$\phi_i(N,v) = \sum_{S\subset N;\, i\in S}\frac{(|S|-1)!\,(|N|-|S|)!}{|N|!}\left[v(S) - v(S-\{i\})\right], \quad \forall i\in N. \quad (12)$$

If S is assumed as the grand coalition, the subgame (S,v) is given by the restriction of v to 2^S-1; for all $T\subset S$, the marginal contribution can be expressed as $v(T)-v(T-\{i\})$. The Shapley value model then becomes

$$\phi_i(S,v)=\sum_{T\subset S;i\in T}\frac{(|T|-1)!\,(|S|-|T|)!}{|S|!}\left[v(T)-v(T-\{i\})\right],\quad \forall i\in S. \tag{13}$$

The Shapley value model is based on four fairness properties. The calculated cost and profit allocation strategy should satisfy the four properties including *efficiency*, *symmetry*, *dummy property,* and *additivity* [7, 17, 39]. These four properties exhibit several desirable features from a practical perspective. Both the rationality and stability of allocation can be guaranteed based on these properties.

The LSP in the current logistics network is responsible to coordinate between multiple logistics entities by providing services such as warehousing, transportation management, and negotiation. Since the players in the form of coalition are served by LSP and accumulate a certain amount of profit, the LSP needs to extract a certain amount of gained profit as a result of synergy service. This share of profit is also called *synergy requirement* and is expressed as $\sigma\in[0,1]$. When σ is set as a low value, the LSP will receive a lower prospected profit, and the players in LJDN are more likely to cooperate. The value $v(S)$ of a coalition S in the synergy game can be determined in

$$v(S)=(1-\sigma)\max\left\{\sum_{i\in S}C_0(i)-C(S),0\right\}. \tag{14}$$

In (14), $C_0(i)$ expresses the cost of player i in the absence of synergy, while $C(S)$ is the total cost of all players served by LSP in a coalition S. In addition, the synergy group can be only established when the total cost of all players in the form of coalition S is lower than the total cost of players without any coalition. Whenever $\sum_{i\in S}C_0(i)<C(S)$ happens, the players in S will not accept the LSP's service, and $v(S)$ will be set to 0.

We assume that Γ is the set of sequences in grand coalition N, and these sequences contain $|N|!$ different combinations π. $\pi(i)$ is used to express the rank of player i in sequence π. The cost reduction percentages $\eta(i,\pi,s)$ can be defined as

$$\eta(i,\pi,s)=\frac{\phi_i\left(\bigcup_{\pi(\mu)\le s,\mu\in N}\mu,v\right)}{C_0(i)},\qquad s\ge\pi(i). \tag{15}$$

In (15), the cost reduction percentages $\eta(i,\pi,s)$ are used to explain the *Strictly Monotonic Path* (SMP) in the following sections [7]. SMP is considered such a sequence where the cost reduction percentages for all committed players are monotonically increasing as each player joins the coalition. The above procedure can be illustrated in Section 4.2.

4. The Method Solving Procedure

The method solving procedure is composed of two steps: (1) improved particle swarm optimization (PSO) algorithm is used to solve the logistics joint distribution network optimization model. The aim of this step is to assign each distribution unit into each corresponding distribution center by optimizing the total cost calculated in (4); (2) Shapley value model is then utilized to allocate the gained profit among multiple distribution centers in logistics joint distribution network, where the gained profit is calculated by comparing the costs from the optimized logistics network and the initial logistics network.

4.1. Improved PSO Algorithm Design. Particle swarm optimization (PSO) is a swarm intelligence stochastic approach. PSO is inspired by social behavior of bird flocking and optimizes the local best solution according to the particle's position and velocity [46, 47]. PSO algorithm can be extended to solve the combinational optimization problems [48–50]. This paper presents an improved PSO algorithm with crossover and mutation operations from genetic algorithm (GA). The improved PSO algorithm can increase the reliability of solving the optimization model. The relevant notations can be predefined as follows:

v_γ^{t+1}: velocity of particle γ at iteration number $t+1$,

rand$(\cdot)$: a random fraction between 0 and 1,

x_γ^{t+1}: a position of particle γ at iteration number $t+1$,

fix$(\cdot)$: integer the position of each particle,

p_γ^t: individual best known position of particle γ at iteration number t,

g_γ^t: global best known position of particle γ at iteration number t,

c_1 and c_2: acceleration coefficients,

w_{int} and w_{end}: inertia weights,

V: maximum velocity,

T': total number of distribution centers,

rand int$[1,T']$: a random integer between 1 and T',

p_c: crossover probability,

p_m: mutation probability,

$S_{\max}$: maximum number of iterations for improved PSO algorithm,

Swarm size: size of particle swarm.

4.1.1. Particle Encoding Scheme and Particle State Update Operations. Particle encoding scheme and evaluation function design are the key issues in the algorithm operations [48, 51–55]. For the logistics joint distribution network optimization, the number of distribution centers and the location and delivery capacity of each distribution center are needed to be taken into account. Therefore, a two-dimensional particle encoding is presented in this study. The first dimension of the particles can be expressed as $1,2,3,\dots,j,\dots,L$, and L is the total number of distribution units. The second dimension can be expressed as the sequence number of the distribution center that is assigned to serve each distribution unit, and the second dimension code will express the position of one particle. $y_{k,j}$ is the distribution center assigned to the jth

Table 1: Two-dimensional particle encoding.

Distribution unit	1	2	$\cdots$	j	$\cdots$	L
Distribution center number	$y_{k,1}$	$y_{k,2}$	$\cdots$	$y_{k,j}$	$\cdots$	$y_{k,L}$

distribution unit in the kth particle. For example, $y_{k,j} = 1, 2, 3$ and $j = 1, 2, 3, \ldots, L$. The two-dimensional particle encoding table is shown in Table 1. $y_{4,2} = 3$ expresses that the third distribution center is assigned to the second distribution unit in the fourth particle.

The above encoding method ensures that each distribution unit is served by a certain distribution center in the joint distribution network. The initial population of particles can be generated by using the particle encoding method. Then, the initial fitness function value can be obtained based on the objective function, and record the initial individual best known position (p) and best known fitness function value. Meanwhile, the initial global best known position (g) and initial best known fitness function value also remained in the calculation procedure. To evaluate the effectiveness of improved PSO algorithm, the fitness function value should be properly defined. If the objective function is F_γ, the fitness function value can be shown as

$$Z_\gamma = \frac{1}{F_\gamma}. \tag{16}$$

Both the position and velocity information can be used to express the state of a particular particle. The next state of a particle depends on the current position and velocity. A particle state update mechanism is presented in the improved PSO algorithm procedure. The velocity and position can be updated through the following equations:

$$v_\gamma^{t+1} = \begin{cases} w \times v_\gamma^t + c_1 \times \operatorname{rand}(t) \times \left(p_\gamma^t - x_\gamma^t\right) & \\ \quad + c_2 \times \operatorname{rand}(t) \times \left(g_\gamma^t - x_\gamma^t\right) & -V \le v_\gamma^{t+1} \le V \\ -V + 2 \times V \times \operatorname{rand}(t) & \text{others} \end{cases} \tag{17}$$

$$x_\gamma^{t+1} = \begin{cases} \operatorname{fix}\left(x_\gamma^t + v_\gamma^{t+1}\right) & -V \le v_\gamma^{t+1} \le V \\ \operatorname{rand\ int}\left[1, T'\right] & \text{others.} \end{cases} \tag{18}$$

In addition, the inertia weight of each particle w can be further described as follows:

$$w = \frac{\left(w_{\text{int}} - w_{\text{end}}\right)\left(S_{\max} - t\right)}{S_{\max}} + w_{\text{end}}. \tag{19}$$

w decreases with time in (19), and w can be calculated with feedback to (17) for obtaining the new velocity and position of the particle.

4.1.2. Improved PSO Algorithm Procedure. Based on the above introduction of particle encoding scheme and particle state update operations, the improved PSO algorithm is detailed as follows.

Step 1. An integer is randomly generated as each dimension of position vector in each particle within $[1, T']$, and the initial speed vector of each particle is randomly generated within $[-V, V]$.

Step 2. Find and regenerate the unqualified particles; calculate the fitness function value Z_i of each particle by using (16).

Step 3. The fitness function value can be used as the individual best known solution P_γ, and the global optimal solution P_g can also be found.

Step 4. Execute the crossover operation based on the p_c, and calculate the fitness function of these particles; update the relevant solutions P_γ and P_g.

Step 5. Execute the mutation operations based on the p_m, and calculate the fitness function of these particles; update the relevant solutions P_γ and P_g.

Step 6. Calculate the new velocity and position of each particle based on (17), (18), and (19), and determine the number of iterations; if it exceeds the maximum number of iterations, the calculation procedure will be terminated, or return to Step 2.

Step 7. Calculate and select the optimal solution (i.e., position and fitness function value) from all feasible particles. This optimal solution will be the final result for the joint distribution network optimization.

In the course of above improved PSO algorithm, the particle swarm operations and genetic operations are reasonably combined. Therefore, it enhances the search space, provides a more robust global and local search capability, and improves the optimization capability of the proposed algorithm.

4.2. Profit Allocation Application Based on Shapley Value Model. Once the logistics joint distribution network optimization is achieved by the improved PSO algorithm, the optimal profit allocation strategy among distribution centers from nonempty coalitions can be generated. In order to understand the Shapley value model, the calculation procedure with a 3-player example is presented in Table 2 based on (13) and (14). We assume $\sigma = 0$ for calculation convenience.

Table 2: An assumptive 3-player example.

S	$\sum_{i\in S} C_0(i)$	$C(S)$	$v(S)$	$\phi(S, v)$
{A}	200	160	40	(40; ·; ·)
{B}	350	380	0	(·; 0; ·)
{C}	150	120	30	(·; ·; 30)
{A, B}	550	510	40	(40; 0; ·)
{A, C}	350	260	90	(50; ·; 40)
{B, C}	500	480	20	(·; −5; 25)
{A, B, C}	700	580	120	(63; 8; 49)

All of the possible coalitions are listed in Table 2, and all cost reduction percentages can be calculated and demonstrated in Figure 2. In order to establish the grand coalition, the LSP will have to select an effective cooperation strategy for profit allocation. The order of each player joining into a

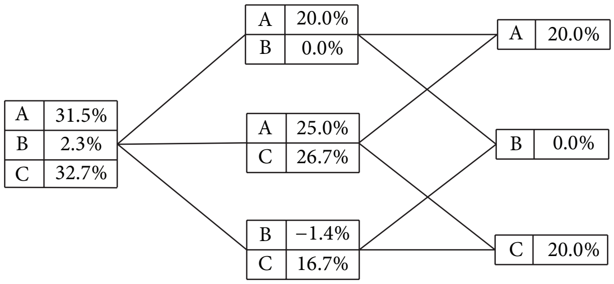


FIGURE 2: Cost reduction percentages in the 3-player example.

TABLE 3: Possible sequential coalitions for grand coalition based on SMP.

π_1 = ACB				π_2 = CAB			
Player i	A	C	B	Player i	C	A	B
$\eta(i,\pi,1)$	20.0%	—	—	$\eta(i,\pi,1)$	20.0%	—	—
$\eta(i,\pi,2)$	25.0%	26.7%	—	$\eta(i,\pi,2)$	26.7%	25.0%	—
$\eta(i,\pi,3)$	31.5%	32.7%	2.3%	$\eta(i,\pi,3)$	32.7%	31.5%	2.3%

coalition affects the magnitude of distributed profit. Figure 2 presents the diagram for cost reduction percentage changes when each player joins a coalition. A coalition can be established through the sequence π = ACB, indicating that player A joins a coalition, followed by player C, and the final coalition is formed among player A, player C, and player B. According to the definition of *Strictly Monotonic Path*, π = ACB and π = CAB are the only two suitable sequences satisfying the requirement of SMP, where their cost reduction percentages increase as each player joins the coalition. The next step is to find the most favorable sequence from these two SMPs as the optimal profit allocation strategy. The coalition procedures with cost reduction percentage for π = ACB and π = CAB are shown in Table 3. When only one player (either player A or player C) exits in the logistics network, the cost for this player A will be reduced from 200 to 160 and this player C will be reduced from 150 to 120 due to the service provided by LSP, which is equivalent to a 20% reduction. However, the cost reduction rate for π = ACB when player C joins reaches 26.7%, while this rate is only 25% when player A joins for π = CAB. This implied that player C is more likely to form a coalition with player A because player C can receive a higher cost reduction compared with another scenario that player A joins the coalition with a less profit gain (e.g., 25% cost reduction rate). Thus, the selected profit allocation strategy should be to let player A enter the logistics network, followed by player C, and player B finally joins the coalition. This leads to the grand coalition constructed among all the players.

Without loss of generality, the above illustration can be summarized as below.

Step 1. Select the diagonal values from the cost reduction percentage matrix (e.g., Table 3) in possible sequential coalitions based on SMP.

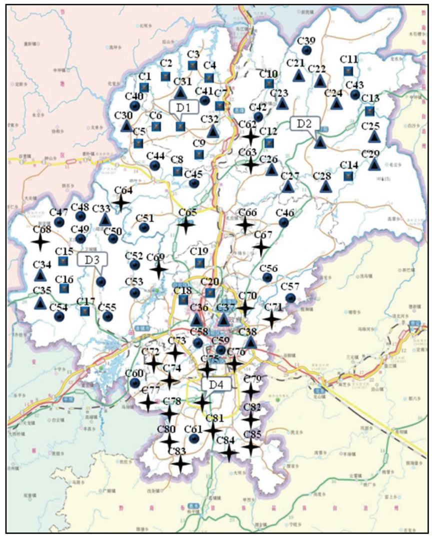


FIGURE 3: Distribution centers and units distribution diagram.

Step 2. Find the player with the maximum lowest cost reduction percentage in the selected diagonal values. If the cost reduction percentage remains the same for all possible sequential coalitions, then seek for another player with the maximal second lowest cost reduction rate. This process continues until at least one player can be found or all players have been searched.

Step 3. The selected sequential coalition will be considered as the candidate profit allocation strategy. If all players have been searched, select any sequential coalition as the candidate profit allocation strategy.

5. Implementation and Analysis

5.1. Data Source. To illustrate the applicability of the proposed methods in logistics joint distribution network optimization, a practical example in Guiyang, China, is used for the numerical study. Guiyang city is the capital of Guizhou Province and is a critical transportation hub. The locations of four DCs and 85 distribution units are demonstrated in Figure 3. 85 distribution units are, respectively, expressed as $C1, C2, C3, \ldots, C85$, and four distribution centers are, respectively, expressed as D1, D2, D3, D4. The customer units in square are assigned to D1, the customer units in triangle are assigned to D2, the circular customer units are assigned to D3, and the star-like customer units are assigned to D4. The distribution networks composed by four DCs have considerable geographical overlap shown in Figure 3, so the cooperative transportation service provided by LSP is necessary.

For the convenience of calculation, the delivery demand of each distribution unit is converted into the standard roll pallet quantity. The characteristics of four distribution centers are summarized in Table 4 including the number of customer units, periodic demand quantity (one week), and graphic symbols.

Table 4: Characteristics of four distribution centers.

DC	Number of distribution units	Periodic demand (roll pallets)	Distribution unit symbol in Figure 3
D1	20	32600	■
D2	18	30000	▲
D3	23	39500	●
D4	24	41500	✦

5.2. Improved PSO Parameter Setting and Optimization Results. In this case study, parameter settings can be determined based on previous discussion [49, 54, 55]. The parameters are given as follows.

(1) $w_{\text{int}} = 0.8$ is the initial inertia weight and $w_{\text{end}} = 0.3$ is the inertia weight of maximum evolution generation used for PSO speed calculation.

(2) $c_1 = c_2 = 2$ are the acceleration coefficients used for PSO speed calculation.

(3) $V = 4$ denotes the maximum velocity.

(4) $p_c = 0.6$ and $p_m = 0.01$ express the crossover probability and mutation probability, respectively.

(5) $S_{\max} = 1000$ is the maximum number of generations.

(6) $T' = 4$ denotes the maximal random integer.

(7) Swarm size = 100 is the number of particles used to increase the diversity of initial particle swarm.

(8) Several other parameters used in the model formulation can be set as $\lambda = 1.5$, $\text{LC}_s = 2000$, $\text{LC}_v = 200$, $C_s = 6.6$, $C_v = 3.2$, $\rho_v = 3.99$, $\rho_s = 3.97$, $F_s = 4800$, $F_v = 1600$, $K = 100$, $T = 52$, $G_1 = 1145$, $G_2 = 1791$, $G_3 = 1968$, $G_4 = 1408$, $\varsigma_1 = 725$, $\varsigma_2 = 1166$, $\varsigma_3 = 1015$, and $\varsigma_4 = 1616$.

Five working days is considered one planning period, and there are $2^4 - 1$ combinations of nonempty coalitions that can be served by the LSP. The improved PSO algorithm is implemented to adequately assign each distribution unit into each distribution center by optimizing the total cost based on empirical data. The generated profit will be then redistributed among different distribution centers using the Shapley model. The optimization result over a planning period from all coalitions is shown in Table 5.

In addition, for explanatory purposes, all distribution units affiliated with each distribution center for grand coalition are listed in Table 6.

In the next section, the cost savings due to optimized logistics joint distribution network will be allocated among different distribution centers based on Shapley value model.

5.3. Shapley Value Model Application. As previously discussed, the gained profit benefited from network optimization through LSP should be reallocated to each distribution center. The Shapley model can be utilized to fulfill this goal. For practical purposes, the synergy requirement is set to $\sigma = 0.1$ in the case. The appropriate synergy requirement value reflects the negotiation power between the LSP and players from coalitions.

All of the possible coalitions have been shown in Table 7, and all cost reduction percentages and possible coalitional sequences can be calculated and are shown in Figure 4. The last column of Figure 4 shows that three coalitions have a positive percentage gain except coalition {D4}. In other words, if D1, D2, and D3 agree to accept the service provided by LSP, then, the LSP can optimize the existing logistics operations at a lower cost, compared with the scenario where each distribution center manages its own distribution units individually. In addition, the first column of Figure 4 shows that the D1, D2, D3, and D4 can obtain a certain percentage of benefits when the grand coalition is reached. This indicates that all distribution centers collaborate with the service of LSP.

Table 5: Comparison between initial network and optimized network over a planning period.

S	Demand	Total cost for initial network	Total cost for optimized network
{D1}	32600	12639	12219
{D2}	30000	12668	12043
{D3}	39500	16475	15522
{D4}	41500	15721	15929
{D1, D2}	62600	25307	23024
{D1, D3}	72100	29114	25704
{D1, D4}	74100	28360	28136
{D2, D3}	69500	29143	25653
{D2, D4}	71500	28389	27737
{D3, D4}	81000	32196	30441
{D1, D2, D3}	102100	41782	35853
{D1, D2, D4}	104100	41028	37914
{D1, D3, D4}	113600	44835	40352
{D2, D3, D4}	111000	44864	40250
{D1, D2, D3, D4}	143600	57503	50374

Table 6: Distribution units assignment based on grand coalition.

DC	Number of distribution units
D1	C1, C2, C3, C4, C5, C6, C7, C8, C9, C30, C31, C32, C40, C41, C44, C45
D2	C10, C11, C12, C13, C14, C21, C22, C23, C24, C25, C26, C27, C28, C29, C39, C42, C43, C46, C62, C63, C66, C67
D3	C15, C16, C17, C19, C33, C34, C35, C47, C48, C49, C50, C51, C52, C53, C54, C55, C64, C65, C68, C69
D4	C18, C20, C36, C37, C38, C56, C57, C58, C59, C60, C61, C70, C71, C72, C73, C74, C75, C76, C77, C78, C79, C80, C81, C82, C83, C84, C85

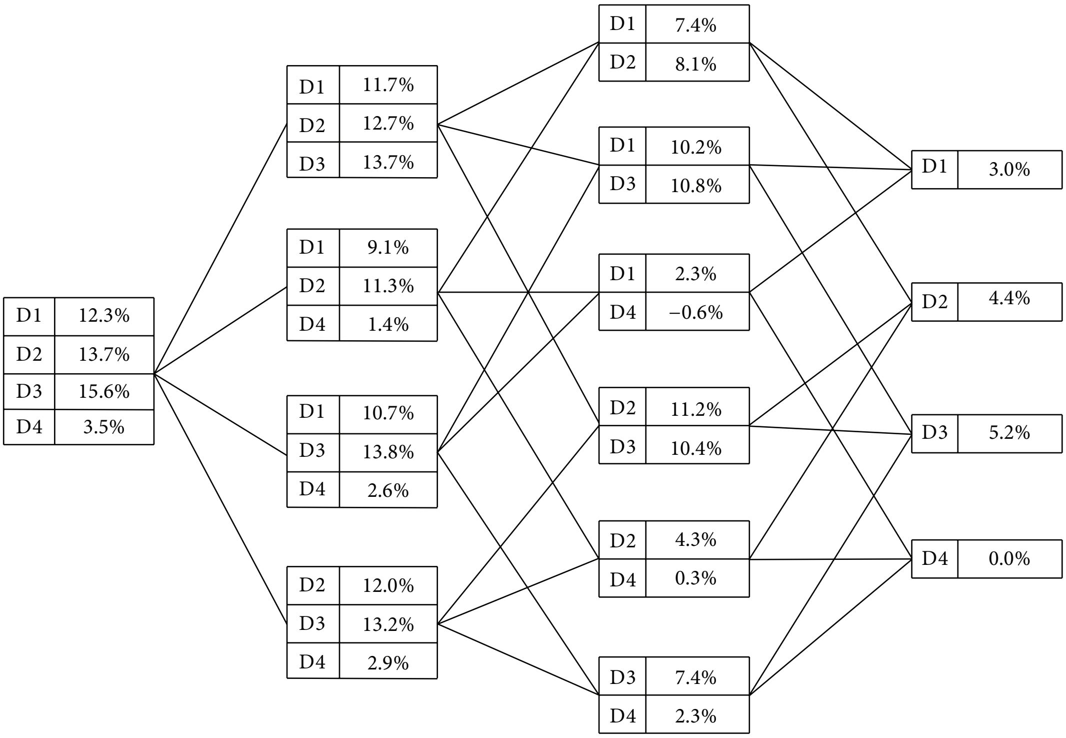


Figure 4: Cost reduction percentages for all coalitions.

Table 7: Profit allocation in joint logistics distribution network.

S	$\sum_{i\in S} C_0(i)$	$C(S)$	$v(S)$	$\phi(S, v)$
{D1}	12639	12219	378	(378; ·; ·; ·)
{D2}	12668	12043	563	(·; 563; ·; ·)
{D3}	16475	15522	858	(·; ·; 858; ·)
{D4}	15721	15929	0	(·; ·; ·; 0.0)
{D1, D2}	25307	23024	2055	(935; 1020; ·; ·)
{D1, D3}	29114	25704	3069	(1295; ·; 1774; ·)
{D1, D4}	28360	28136	202	(290; ·; ·; −88)
{D2, D3}	29143	25653	3141	(·; 1423; 1718; ·)
{D2, D4}	28389	27737	587	(·; 545; ·; 42)
{D3, D4}	32196	30441	1580	(·; ·; 1219; 361)
{D1, D2, D3}	41782	35853	5336	(1475; 1603; 2258; ·)
{D1, D2, D4}	41028	37914	2803	(1147; 1432; ·; 224)
{D1, D3, D4}	44835	40352	4035	(1347; ·; 2275; 413)
{D2, D3, D4}	44864	40250	4153	(·; 1524; 2168; 461)
{D1, D2, D3, D4}	57503	50374	6416	(1558; 1735; 2578; 545)

5.4. Sequential Coalition Selection. Based on the discussion in Section 2, the willingness of each distribution center to join a coalition depends on how the coalition is organized. It is necessary to investigate how the sequential coalition impacts the final profit allocation. Similar to the proposed approaches in Section 4.2, we will introduce an effective approach to select the optimal sequential coalition for grand coalition establishment. The LSP will be used as the coordinator, increasing the negotiation power for all distribution centers.

Possible sequential coalitions with calculated cost reduction percentage matrices are presented in Figure 4. The sequential coalitions that satisfy the condition of SMP are further selected as shown in Tables 8–11.

The cost reduction percentages in each column from Table 8 to Table 11 are strictly monotonically increasing. This implies that these coalitions all conform to SMP provided in Section 3.3. An interesting question arises on how to select the optimal cooperation strategy from the 18 possible sequences from Tables 8–11. Based on the proposed approaches in Section 4.2, we can select $\pi = \{D1, D3, D2, D4\}$ from Table 8, $\pi = \{D2, D3, D1, D4\}$ from Table 9, $\pi = \{D3, D2, D1, D4\}$ from Table 10, and $\pi = \{D4, D3, D2, D1\}$ from Table 11, as the best sequential coalitions for each table. These sequential coalitions can be represented with cost reduction percentage matrices as in Table 12.

Applying the same approach in Section 4.2 into Table 12 leads to the optimal sequential coalition as $\pi_3 = \{D3, D2, D1, D4\}$. The most feasible and beneficial cooperation strategy is described as follows. The logistics operation of D3 is initially optimized by LSP, and then a coalition is formed up between D2 and D3, followed by D1 joining the coalition. Finally, the grand coalition is established among all distribution centers. As presented in Figure 5, the cost reduction percentage is increasing for each distribution center when the coalition is updated. In reality, this strategy is favorable for both LSP and distribution centers since the high cost savings and reasonable profit allocation encourage logistics participants to cooperate with each other.

Table 8: Sequential coalitions starting from D1 for grand coalition based on SMP.

π_1 = {D1, D2, D3, D4}

Player i	D1	D2	D3	D4
$\eta(i,\pi,1)$	3.0%	—	—	—
$\eta(i,\pi,2)$	7.4%	8.1%	—	—
$\eta(i,\pi,3)$	11.7%	12.7%	13.7%	—
$\eta(i,\pi,4)$	12.3%	13.7%	15.6%	3.5%

π_2 = {D1, D2, D4, D3}

Player i	D1	D2	D4	D3
$\eta(i,\pi,1)$	3.0%	—	—	—
$\eta(i,\pi,2)$	7.4%	8.1%	—	—
$\eta(i,\pi,3)$	9.1%	11.3%	1.4%	—
$\eta(i,\pi,4)$	12.3%	13.7%	3.5%	15.6%

π_3 = {D1, D3, D2, D4}

Player i	D1	D3	D2	D4
$\eta(i,\pi,1)$	3.0%	—	—	—
$\eta(i,\pi,2)$	10.2%	10.8%	—	—
$\eta(i,\pi,3)$	11.7%	13.7%	12.7%	—
$\eta(i,\pi,4)$	12.3%	15.6%	13.7%	3.5%

π_4 = {D1, D3, D4, D2}

Player i	D1	D3	D4	D2
$\eta(i,\pi,1)$	3.0%	—	—	—
$\eta(i,\pi,2)$	10.2%	10.8%	—	—
$\eta(i,\pi,3)$	10.7%	13.8%	2.6%	—
$\eta(i,\pi,4)$	12.3%	15.6%	3.5%	13.7%

Table 9: Sequential coalitions starting from D2 for grand coalition based on SMP.

π_1 = {D2, D1, D3, D4}

Player i	D2	D1	D3	D4
$\eta(i,\pi,1)$	4.4%	—	—	—
$\eta(i,\pi,2)$	8.1%	7.4%	—	—
$\eta(i,\pi,3)$	12.7%	11.7%	13.7%	—
$\eta(i,\pi,4)$	13.7%	12.3%	15.6%	3.5%

π_2 = {D2, D1, D4, D3}

Player i	D2	D1	D4	D3
$\eta(i,\pi,1)$	4.4%	—	—	—
$\eta(i,\pi,2)$	8.1%	7.4%	—	—
$\eta(i,\pi,3)$	11.3%	9.1%	1.4%	—
$\eta(i,\pi,4)$	13.7%	12.3%	3.5%	15.6%

π_3 = {D2, D3, D1, D4}

Player i	D2	D3	D1	D4
$\eta(i,\pi,1)$	4.4%	—	—	—
$\eta(i,\pi,2)$	11.2%	10.4%	—	—
$\eta(i,\pi,3)$	12.7%	13.7%	11.7%	—
$\eta(i,\pi,4)$	13.7%	15.6%	12.3%	3.5%

π_4 = {D2, D3, D4, D1}

Player i	D2	D3	D4	D1
$\eta(i,\pi,1)$	4.4%	—	—	—
$\eta(i,\pi,2)$	11.2%	10.4%	—	—
$\eta(i,\pi,3)$	12.0%	13.2%	2.9%	—
$\eta(i,\pi,4)$	13.7%	15.6%	3.5%	12.3%

Table 10: Sequential coalitions starting from D3 for grand coalition based on SMP.

π_1 = {D3, D1, D2, D4}

Player i	D3	D1	D2	D4
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	10.8%	10.2%	—	—
$\eta(i,\pi,3)$	13.7%	11.7%	12.7%	—
$\eta(i,\pi,4)$	15.6%	12.3%	13.7%	3.5%

π_2 = {D3, D1, D4, D2}

Player i	D3	D1	D4	D2
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	10.8%	10.2%	—	—
$\eta(i,\pi,3)$	13.8%	10.7%	2.6%	—
$\eta(i,\pi,4)$	15.6%	12.3%	3.5%	13.7%

π_3 = {D3, D2, D1, D4}

Player i	D3	D2	D1	D4
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	10.4%	11.2%	—	—
$\eta(i,\pi,3)$	13.7%	12.7%	11.7%	—
$\eta(i,\pi,4)$	15.6%	13.7%	12.3%	3.5%

π_4 = {D3, D2, D4, D1}

Player i	D3	D2	D4	D1
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	10.4%	11.2%	—	—
$\eta(i,\pi,3)$	13.2%	12.0%	2.9%	—
$\eta(i,\pi,4)$	15.6%	13.7%	3.5%	12.3%

π_5 = {D3, D4, D1, D2}

Player i	D3	D4	D1	D2
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	7.4%	2.3%	—	—
$\eta(i,\pi,3)$	13.8%	2.6%	10.7%	—
$\eta(i,\pi,4)$	15.6%	3.5%	12.3%	13.7%

π_6 = {D3, D4, D2, D1}

Player i	D3	D4	D2	D1
$\eta(i,\pi,1)$	5.2%	—	—	—
$\eta(i,\pi,2)$	7.4%	2.3%	—	—
$\eta(i,\pi,3)$	13.2%	2.9%	12.0%	—
$\eta(i,\pi,4)$	15.6%	3.5%	13.7%	12.3%

Table 11: Sequential coalitions starting from D4 for grand coalition based on SMP.

	$\pi_1 = \{D4, D2, D1, D3\}$					$\pi_2 = \{D4, D2, D3, D1\}$			
Player i	D4	D2	D1	D3	Player i	D4	D2	D3	D1
$\eta(i,\pi,1)$	0.0%	—	—	—	$\eta(i,\pi,1)$	0.0%	—	—	—
$\eta(i,\pi,2)$	0.3%	4.3%	—	—	$\eta(i,\pi,2)$	0.3%	4.3%	—	—
$\eta(i,\pi,3)$	1.4%	11.3%	9.1%	—	$\eta(i,\pi,3)$	2.9%	12.0%	13.2%	—
$\eta(i,\pi,4)$	3.5%	13.7%	12.3%	15.6%	$\eta(i,\pi,4)$	3.5%	13.7%	15.6%	12.3%
	$\pi_3 = \{D4, D3, D1, D2\}$					$\pi_4 = \{D4, D3, D2, D1\}$			
Player i	D4	D3	D1	D2	Player i	D4	D3	D2	D1
$\eta(i,\pi,1)$	0.0%	—	—	—	$\eta(i,\pi,1)$	0.0%	—	—	—
$\eta(i,\pi,2)$	2.3%	7.4%	—	—	$\eta(i,\pi,2)$	2.3%	7.4%	—	—
$\eta(i,\pi,3)$	2.6%	13.8%	10.7%	—	$\eta(i,\pi,3)$	2.9%	13.2%	12.0%	—
$\eta(i,\pi,4)$	3.5%	15.6%	12.3%	13.7%	$\eta(i,\pi,4)$	3.5%	15.6%	13.7%	12.3%

Table 12: Possible sequential coalitions for grand coalition based on SMP.

	$\pi_1 = \{D1, D3, D2, D4\}$					$\pi_2 = \{D2, D3, D1, D4\}$			
Player i	D1	D3	D2	D4	Player i	D2	D3	D1	D4
$\eta(i,\pi,1)$	3.0%	—	—	—	$\eta(i,\pi,1)$	4.4%	—	—	—
$\eta(i,\pi,2)$	10.2%	10.8%	—	—	$\eta(i,\pi,2)$	11.2%	10.4%	—	—
$\eta(i,\pi,3)$	11.7%	13.7%	12.7%	—	$\eta(i,\pi,3)$	12.7%	13.7%	11.7%	—
$\eta(i,\pi,4)$	12.3%	15.6%	13.7%	3.5%	$\eta(i,\pi,4)$	13.7%	15.6%	12.3%	3.5%
	$\pi_3 = \{D3, D2, D1, D4\}$					$\pi_4 = \{D4, D3, D2, D1\}$			
Player i	D3	D2	D1	D4	Player i	D4	D3	D2	D1
$\eta(i,\pi,1)$	5.2%	—	—	—	$\eta(i,\pi,1)$	0.0%	—	—	—
$\eta(i,\pi,2)$	10.4%	11.2%	—	—	$\eta(i,\pi,2)$	2.3%	7.4%	—	—
$\eta(i,\pi,3)$	13.7%	12.7%	11.7%	—	$\eta(i,\pi,3)$	2.9%	13.2%	12.0%	—
$\eta(i,\pi,4)$	15.6%	13.7%	12.3%	3.5%	$\eta(i,\pi,4)$	3.5%	15.6%	13.7%	12.3%

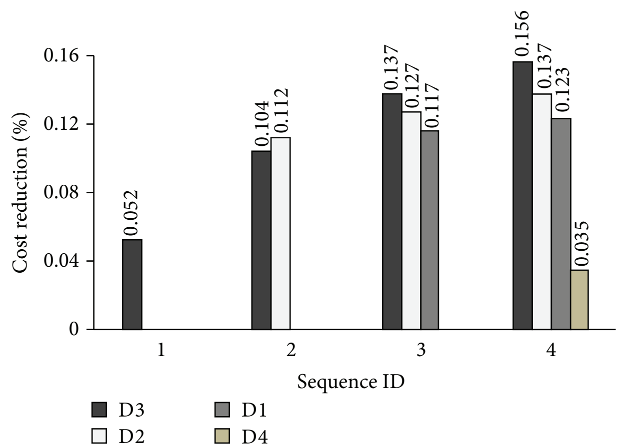


Figure 5: The cost reduction percentage diagram for the optimal sequential coalition.

The LSP receives 10% of the total savings (i.e., 713 USD per week) since the synergy requirement is set to $\sigma = 0.1$. The remaining profit is required to distribute among D1, D2, D3, and D4 by following the Shapley value model. The distribution center D3 receives the highest cost reduction percentage due to the high customer demand and unoptimized logistics network structure. Nevertheless, the cost reduction percentage for distribution center D4 is lowest. This is probably because the surrounding traffic as well as economic condition is ideal and thus leaves little room to improve the existing logistics optimization plan. Moreover, the LSP provides a higher service cost for D4 compared with other distribution centers. Consequently, the gained profit for D4 is reduced.

The synergy requirement σ varies depending on each distribution center's negotiation capability. LSP can decrease its synergy requirement for a certain distribution center if this distribution center is able to negotiate with other participants independently. Similarly, it is possible that the LSP may not able to persuade a certain distribution center (e.g., D4) to join a coalition when a specific synergy requirement is provided. In this case, the grand coalition will downgrade to $\{D1, D2, D3\}$, and the optimal sequential coalition $\pi = \{D3, D2, D1\}$ can be generated based on the similar calculation procedure in Section 4.2. If no distribution center is willing to cooperate, the LSP should reconsider its synergy plan by lowering the synergy requirement.

6. Conclusions

This paper studies the logistics joint distribution network optimization problem, where multiple DCs and customers exist and interact with each other. The LJDN can be constructed by either LSP or existing DCs in the logistics network. A novel approach is proposed to optimize the joint logistics network and allocate the gained profit among DCs. The Logistics Service Provider (LSP) bridges both the collaborative network optimization problem and profit allocation problem and distributes the cost savings to each DC from the nonempty coalition. This further reduces the complexity and enhances the robustness of designing a large-scale logistics network.

A joint distribution model formulation is initially built to optimize the total cost of nonempty coalition logistics systems. This model uses an improved PSO algorithm, and then a Shapley value model is utilized to perform the profit allocation among DCs from nonempty coalitions. Finally, the optimal sequential coalition can be obtained according to Strictly Monotonic Path (SMP) theory—all DCs can receive benefits when each DC joins the coalition. To evaluate the effectiveness of the LJDN optimization methods, a computational experiment in Guiyang city, China, was conducted. By properly adjusting the synergy requirement value σ, the results can be used to optimize the logistics distribution network and determine the optimal profit allocation strategy in a cooperative and effective fashion.

An interesting direction for further research is to optimize the multilevel logistics distribution network (MLLDN). In addition, multiple LSPs coexist in MLLDN, where each DC may be competitively served by several LSPs. In this case, the new profit allocation model needs to be established by reconsidering the ownership of each DC. To prevent a certain LSP from dominating the market, the heterogeneity of synergy requirement values should be incorporated to improve the profit allocation model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding to the publication of this paper.

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Research Article

A Hybrid Approach to the Optimization of Multiechelon Systems

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In freight transportation there are two main distribution strategies: direct shipping and multiechelon distribution. In the direct shipping, vehicles, starting from a depot, bring their freight directly to the destination, while in the multiechelon systems, freight is delivered from the depot to the customers through an intermediate points. Multiechelon systems are particularly useful for logistic issues in a competitive environment. The paper presents a concept and application of a hybrid approach to modeling and optimization of the Multi-Echelon Capacitated Vehicle Routing Problem. Two ways of mathematical programming (MP) and constraint logic programming (CLP) are integrated in one environment. The strengths of MP and CLP in which constraints are treated in a different way and different methods are implemented and combined to use the strengths of both. The proposed approach is particularly important for the discrete decision models with an objective function and many discrete decision variables added up in multiple constraints. An implementation of hybrid approach in the $\mathrm{ECL^iPS^e}$ system using Eplex library is presented. The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) and its variants are shown as an illustrative example of the hybrid approach. The presented hybrid approach will be compared with classical mathematical programming on the same benchmark data sets.

1. Introduction

In the modern freight transportation there are two main distribution strategies: direct shipping and multiechelon distribution. In the direct shipping, vehicles, starting from a depot, bring their freight directly to the destination, while in the multiechelon systems, freight is delivered from the depot to the customers through an intermediate point.

The majority of multiechelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels [1].

In recent years multiechelon systems have been introduced in different areas:

(i) logistics enterprises and express delivery service companies under competitions;

(ii) hypermarkets and supermarkets products distribution;

(iii) multimodal freight transportation;

(iv) supply chains;

(v) delivery in logistic competition;

(vi) E-commerce and home delivery services under competitions;

(vii) city and public logistics.

The vast majority of models of optimization in freight transportation and logistics industry have been formulated as the mixed integer programming (MIP) or mixed integer linear programming (MILP) problems and solved using the operations research (OR) methods [2]. Their structures are similar and proceed from the principles and requirements of mathematical programming (MP) [2, 3].

Unfortunately, high complexity of decision-making models and their integer nature contribute to the poor efficiency of OR methods. Therefore a new approach to solving these problems was proposed. As the best structure for the implementation of this approach, a declarative environment was chosen [4, 5].

It seems that better results will be obtained by the use of the declarative constraint programming paradigms (CP/CLP) especially in modeling. The CP-based environments have advantage over traditional methods of mathematical modeling in that they work with a much broader variety of interrelated constraints and allow producing "natural" solutions for highly combinatorial problems.

The main contribution of this paper is hybrid approach (mixed CP with MP paradigms) to modeling and optimization of the Multi-Echelon Capacitated Vehicle Routing Problems or the similar vehicle routing problems. In addition, some extensions and modifications to the standard Two-Echelon Capacitated Vehicle Routing Problems (2E-CVRP) are presented.

The paper is organized as follows. In Section 2 the literature related to Multi-Echelon Vehicle Routing Problems has been reviewed. Next section is about our motivation and contribution. In Section 4 the concept of hybrid approach to modeling and solving and the solution hybrid framework have been presented. Then, the general description of Multi-Echelon Vehicle Routing Problems and mathematical model of 2E-CVRP has been discussed. Finally test instances for 2E-CVRP with extension variants and some computational results were discussed in Section 6.

2. Literature Review

The Vehicle Routing Problem (VRP) is used to design an optimal route for a fleet of vehicles to serve a set of customers' orders (known in advance), given a set of constraints. The VRP is used in supply chain management in the physical delivery of goods and services. The VRP is of the NP-hard type.

Nowadays, the VRP literature offers a wealth of heuristic and metaheuristic approaches, which are surveyed in the papers of [6, 7] because exact VRP methods have a size limit of 50–100 orders depending on the VRP variant and the time-response requirements.

There are several variants and classes of VRP like the capacitated VRP (CVRP), VRP with Time Windows (VRPTW), and Dynamic Vehicle Routing Problems (DVRP), sometimes referred to as Online Vehicle Routing Problems and so forth [6].

Different distribution strategies are used in freight transportation. The most developed strategy is based on the direct shipping: freight starts from a depot and arrives directly to customers. In many applications and real situations, this strategy is not the best one and the usage of a multiechelon and particular two-echelon distribution system can optimize several features as the number of the vehicles, the transportation costs, loading factor, and timing.

In the literature the multiechelon system and the two-echelon system in particular refer mainly to supply chain and inventory problems [1]. These problems do not use an explicit routing approach for the different levels, focusing more on the production and supply chain management issues. The first real application of a two-tier distribution network optimizing the global transportation costs is due to [8] and is related to the city logistics area. They developed a two-tier freight distribution system for congested urban areas, using small intermediate platforms, called satellites (intermediate points for the freight distribution). This system is developed for a specific situation and a generalization of such a system has not already been formulated. The complete mathematic model of The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) with the solution for sample test data in the classical approach has been proposed by [7], complemented with the method for boosting the computing efficiency (see Section 5).

The increasing role of supply chains and their urban parts evokes a need to focus greater attention on this issue in modeling and efficient optimization methods, in particular.

3. Motivation and Contribution

Based on [2, 5–7, 9–11] and our previous work [3, 4, 12] we observed some advantages and disadvantages of both (CP/MP) paradigms.

An integrated approach of constraint programming/constraint logic programming (CP/CLP) and mixed integer programming/mixed integer linear programming (MIP/MILP) can help to solve optimization problems that are intractable with either of the two methods alone [13–15]. Although Operations Research (OR) and Constraint Programming (CP) have different roots, the links between the two environments have grown stronger in recent years.

Approaches known from the literature are based mostly on the division of the main problem into sub-problems and iteratively solving each of them in the proper CP/CLP or MP/MILP technique. This is usually a collection of many local optimization points of feasible solutions. Other approaches are based on a "blind" transformation for the CLP to the MILP model. In most cases, this results in an explosion of the number of constraints and variables, which has a negative impact on the effectiveness of optimization. In the proposed hybrid approach, a very important element is the transformation of the initial problem and its solution in the field of domains, which takes place in CP/CLP environment. Then the converted and "slimmed down" problem is solved in the MILP environment, thus creating a global approach to optimization [14, 16].

Both MIP/MILP and finite domain CP/CLP involve variables and constraints. However, the types of the variables and constraints that are used, and the way the constraints are solved, are different in the two approaches [13, 15].

MIP/MILP relies completely on linear equations and inequalities in integer variables; that is, there are only two types of constraints: linear arithmetic (linear equations or inequalities) and integer (stating that the variables have to take their values in the integer numbers). In finite domain CP/CLP, the constraint language is richer. In addition to linear equations and inequalities, there are various other constraints such disequalities, nonlinear and symbolic (*alldifferent*, *disjunctive*, *cumulative*, etc.) constraints. In both MIP/MILP and CP/CLP, there is a group of constraints that can be solved with ease and a group of constraints that are difficult to solve. The easily solved constraints in MIP/MILP are linear equations and inequalities over rational numbers.

Integer constraints are difficult to solve using mathematical programming methods and often the real problems of MIP/MILP make them NP-hard.

In CP/CLP, domain constraints with integers and equations between two variables are easy to solve. The system of such constraints can be solved over integer variables in polynomial time. The inequalities between two variables, general linear constraints (more than two variables), and symbolic constraints are difficult to solve, which makes real problems in CP/CLP NP-hard. This type of constraints reduces the strength of constraint propagation. As a result, CP/CLP is incapable of finding even the first feasible solution.

Both environments use various layers of the problem (methods, the structure of the problem, data) in different ways. The approach based on mathematical programming (MIP/MILP) focuses mainly on the methods of optimization and, to a lesser degree, on the structure of the problem. However, the data is completely outside the model. The same model without any changes can be solved for multiple instances of data. In the approach based on constraint programming (CP/CLP), due to its declarative nature, the methods are already built-in. The data and structure of the problem are used for its modelling in a significantly greater extent.

To use so much different environments and a variety of functionalities such as modeling, optimization, and transformation, the declarative approach was adopted.

The motivation and contribution behind this work were to create a hybrid method for constrained decision problems modelling and optimization instead of using mathematical programming or constraint programming separately.

It follows from the above that what is difficult to solve in one environment can be easy to solve in the other.

Moreover, such a hybrid approach allows the use of all layers of the problem to solve it. In our approach, to modelling and optimisation, we proposed the environment, where:

(i) knowledge related to the problem can be expressed as linear, logical, and symbolic constraints;

(ii) the optimization models solved using the proposed approach can be formulated as a pure model of MIP/MILP or of CP/CLP, or it can also be a hybrid model;

(iii) the problem is modelled in the constraint programming environment by CLP-based predicates, which is far more flexible than the mathematical programming environment/very important for decision-making problems under competitions;

(iv) transforming the decision model to explore its structure has been introduced by CLP-based predicates;

(v) constrained domains of decision variables, new constraints, and values for some variables are transferred from CP/CLP into MILP/MIP/IP by CLP-based predicates;

(vi) optimization is performed by MP-based environment.

As a result, a more effective hybrid solution environment for a certain class of decision and optimization problems (2E-CVRP or similar) was obtained.

4. A Hybrid Solution Framework for Capacitated Vehicle Routing Problems (HSFCVRP)

Both environments have advantages and disadvantages. Environments based on the constraints such as CLPs are declarative and ensure a very simple modeling of decision problems, even those with poor structures if any. In the CLP a problem is described by a set of logical predicates. The constraints can be of different types (linear, nonlinear, logical, binary, etc.). The CLP does not require any search algorithms. This feature is characteristic of all declarative backgrounds, in which modeling of the problem is also a solution, just as it is in Prolog, SQL, and so on. The CLP seems perfect for modeling any decision problem.

Numerous MP models of decision-making have been developed and tested, particularly in the area of decision optimization. Constantly improved methods and mathematical programming algorithms, such as the simplex algorithm, branch and bound, and branch-and-cost, have become classics now [2].

The proposed method's strength lies in high efficiency of optimization algorithms and a substantial number of tested models. Traditional methods when used alone to solve complex problems provide unsatisfactory results. This is related directly to different treatment of variables and constraints in those approaches (Section 3).

This schema of the hybrid solution framework for Capacitated Vehicle Routing Problems (HSFCVRP) and the concept of this framework with its predicates (P1–P7) are presented in Figure 1. The names and descriptions of the CLP predicates and the implementation environment are shown in Table 1.

From a variety of tools for the implementation of the CP/CLP, ECL^{i}PS^e software [4, 12] of constraint programming applications. ECL^{i}PS^e contains several constraint solver libraries, a high-level modelling and control language, interfaces to third-party solvers, an integrated development environment, and interfaces for embedding into host environments. ECL^{i}PS^e was used to model the problem, transform it, and search for a domain solution by constraint propagation. This solution was then the basis for the final MP model, developed in the Eplex library [9] of the ECL^{i}PS^e environment. Since ECL^{i}PS^e version 5.7, standalone Eplex have become the standard. The previous lib(eplex), which loads Eplex with the range bounds keeper and the IC variant have now been phased out, so users of these old variants must now move to using standalone Eplex. The Eplex library allows MP/MIP/MILP problems to be modelled in ECL^{i}PS^e and solved (optimized) by an external MP solver.

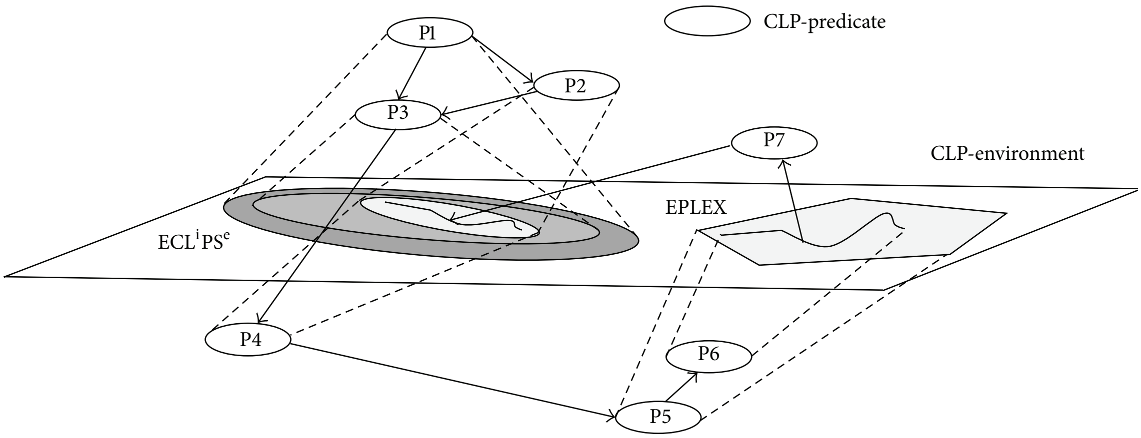

Figure 1: The scheme of the hybrid solution framework for Capacitated Vehicle Routing Problems (HSFCVRP).

Table 1: Description of CLP predicates.

Predicate	Description
P1 CLP environment	The implementation of the model in CLP, the term representation of the problem in the form of predicates.
P2 CLP environment	The transformation of the original problem aimed at extending the scope of constraint propagation. The transformation uses the structure of the problem. The most common effect is a change in the representation of the problem by reducing the number of decision variables and the introduction of additional constraints and variables, changing the nature of the variables, and so forth.
P3 CLP environment	Constraint propagation for the model: constraint propagation is one of the basic methods of CLP. As a result, the variable domains are narrowed, and in some cases, the values of variables are set, or even the solution can be found.
P4 CLP environment	Generation by the AG: (i) the model for mathematical programming: generation performed automatically using CLP predicate; (ii) additional constraints on the basis of the results obtained by predicate P3; (iii) domains for different decision variables and other parameters based on the propagation of constraints. Transmission of this information in the form of fixed value of certain variables and/or additional constraints to the MP. Merging files generated by predicate AG into one file. It is a model file format in MP format.
P5 EPLEX environment	Finding the consistent area based on information from the CLP.
P6 EPLEX environment	The solution of the model from the P4 by MP solver.
P7 EPLEX environment	Solution transfer from EPLEX to CLP (predicate *eplex_get(vars,Zm)*)

5. Two-Echelon Capacitated Vehicle Routing Problem as an Illustrative Example

The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) is an extension of the classical Capacitated Vehicle Routing Problem (CVRP) where the delivery depot-customers pass through intermediate depots (called satellites). As in CVRP, the goal is to deliver goods to customers with known demands, minimizing the total delivery cost in the respect of vehicle capacity constraints. Multiechelon systems presented in the literature usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels [7, 8].

In 2E-CVRP, the freight delivery from the depot to the customers is managed by shipping the freight through intermediate depots. Thus, the transportation network is decomposed into two levels (Figure 2): the 1st level connecting the depot (d) to intermediate depots (s) and the 2nd one connecting the intermediate depots (s) to the customers (c). The objective is to minimize the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate depots are considered, while the timing of the deliveries is ignored.

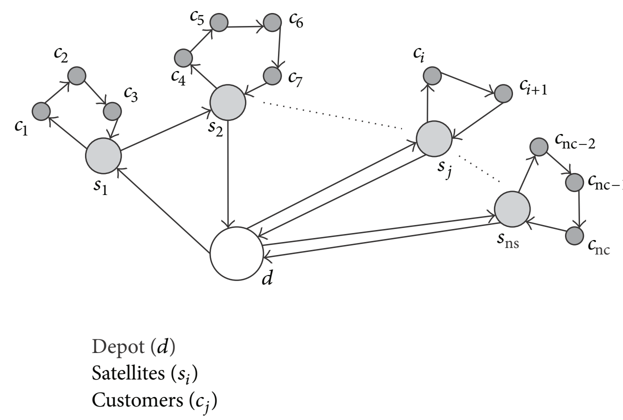

Figure 2: Example of 2E-CVRP transportation network.

From a practical point of view, a 2E-CVRP system operates as follows (Figure 2):

(i) freight arrives at an external/first/base zone, the depot, where it is consolidated into the 1st-level vehicles, unless it is already carried into a fully loaded 1st-level vehicles;

(ii) each 1st-level vehicle travels to a subset of satellites that will be determined by the model and then it will return to the depot;

(iii) at a satellite, freight is transferred from 1st-level vehicles to 2nd-level vehicles.

5.1. Mathematical Model. The formal mathematical model (MILP) was taken from [7]. Table 2 shows the parameters and decision variables of 2E-CVRP. Figure 2 shows an example of the 2E-CVRP transportation network for this model:

$$\min \sum_{i,j\in V_0\cup V_s}\left(c_{i,j}\cdot X_{i,j}\right)+\sum_{k\in V_s}\sum_{i,j\in V_s\cup V_c}\left(c_{i,j}\cdot Y_{k,i,j}\right) + \sum_{k\in V_s}\left(s_k\cdot Ds_k\right) \tag{1}$$

$$\sum_{i\in V_s} X_{0,i}\le M_1 \tag{2}$$

$$\sum_{j\in V_s\cup V_0, j\ne k} X_{j,k}=\sum_{i\in V_s\cup V_0, i\ne k} X_{k,i} \quad \text{for } k\in V_s\cup V_0 \tag{3}$$

$$\sum_{k\in V_s}\sum_{j\in V_c} Y_{k,k,j}\le M_2 \tag{4}$$

$$\sum_{i\in V_c, j\in V_c} Y_{k,i,j}=\sum_{i\in V_c, j\in V_c} Y_{k,j,i} \quad \text{for } k\in V_s \tag{5}$$

$$\sum_{i\in V_0\cup V_s, i\ne j} Q^1_{i,j}-\sum_{i\in V_s, i\ne j} Q^1_{j,i} = \begin{cases} Ds_j & j \text{ is not the deport} \\ \sum\limits_{i\in V_c} - d_i & \text{otherwise}\end{cases} \quad \text{for } j\in V_s\cup V_0 \tag{6}$$

$$Q^1_{i,j}\le k_1\cdot X_{i,j} \quad \text{for } i,j\in V_s\cup V_0,\ i\ne j \tag{7}$$

$$\sum_{i\in V_s\cup V_c, i\ne j} Q^2_{k,i,j}-\sum_{i\in V_c, i\ne j} Q^2_{k,j,i} = \begin{cases} Z_{k,j}d_j & j \text{ is not a satelite} \\ -D_j & \text{otherwise}\end{cases} \quad \text{for } j\in V_c\cup V_S,\ k\in V_s \tag{8}$$

$$Q^2_{k,i,j}\le k_2\cdot Y_{k,i,j} \quad \text{for } i,j\in V_s\cup V_c,\ i\ne j,\ k\in V_s \tag{9}$$

$$\sum_{i\in V_s} Q^1_{i,V_0}=0 \tag{10}$$

$$\sum_{j\in V_c} Q^2_{k,j,k}=0 \quad \text{for } k\in V_s \tag{11}$$

$$Y_{k,i,j}\le Z_{k,j} \quad \text{for } i\in V_s\cup V_c,\ j\in V_c,\ k\in V_s \tag{12}$$

$$Y_{k,i,j}\le Z_{k,j} \quad \text{for } i\in V_s,\ j\in V_c,\ k\in V_s \tag{13}$$

$$\sum_{i\in V_s\cup V_c} Y_{k,i,j}=Z_{k,j} \quad \text{for } k\in V_s,\ j\in V_c,\ i\ne k \tag{14}$$

$$\sum_{i\in V_s} Y_{k,j,k}=Z_{k,j} \quad \text{for } k\in V_s,\ j\in V_c,\ i\ne k \tag{15}$$

$$\sum_{i\in V_s} Z_{i,j}=1 \quad \text{for } j\in V_c \tag{16}$$

$$Y_{k,i,j}\le\sum_{l\in V_s\cup V_0} X_{k,l} \quad \text{for } k\in V_s,\ i,j\in V_c \tag{17}$$

$$Y_{k,i,j}\in\{0,1\},\qquad Z_{k,l}\in\{0,1\} \quad \text{for } k\in V_s,\ i,j\in V_s\cup V_c,\ l\in V_c \tag{18}$$

$$X_{k,j}\in Z^+ \quad \text{for } k,j\in V_s\cup V_0 \tag{19}$$

$$Q^1_{i,j}\ge 0 \quad \text{for } i,j\in V_s\cup V_0; \qquad Q^2_{k,i,j}\ge 0 \quad \text{for } i,j\in V_s\cup V_c,\ k\in V_s \tag{20}$$

$$\sum_{i,j\in S_c} Y_{k,i,j}\le\left|S_c\right|-1 \quad \text{for } S_c\subset V_c,\ 2\le\left|S_c\right|\le\left|V_c\right|-2 \tag{21}$$

$$Q^2_{k,i,j}\le\left(k_2-d_j\right)\cdot Y_{k,i,j} \quad \text{for } i,j\in V_c,\ k\in V_s \tag{22}$$

$$Q^2_{k,i,j}-\sum_{l\in V_s} Q^2_{k,j,l}\le\left(k_2-d_j\right)\cdot Y_{k,i,j} \quad \text{for } i,j\in V_c,\ k\in V_s \tag{23}$$

$$Ds_k=\sum_{l\in V_c}\left(d_j\cdot Z_{k,j}\right) \quad \text{for } k\in V_s. \tag{24}$$

The objective function minimizes the sum of the routing and handling operations costs. Constraints (3) ensure, for $k=V_0$, that each 1st-level route begins and ends at the depot, while when k is a satellite, impose the balance of vehicles entering and leaving that satellite. Constraints (5) force each 2nd-level route to begin and end to one satellite and the balance of

TABLE 2: Summary indices, parameters, and decision variables.

Symbol	Description
	Indices
n_s	Number of satellites
n_c	Number of customers
$V_0 - [v_0]$	Deport
$V_s = \{v_{s1}, v_{s2}, \ldots, v_{sn}\}$	Set of satellites
$V_c = \{v_{c1}, v_{c2}, \ldots, v_{cn}\}$	Set of customers
	Parameters
M_1	Number of the 1st-level vehicles
M_2	Number of the 2nd-level vehicles
K_1	Capacity of the vehicles for the 1st level
K_2	Capacity of the vehicles for the 2nd level
d_i	Demand required by customer i
$c_{i,j}$	Cost of the arc(i, j)
s_k	Cost of loading/unloading operations of a unit of freight in satellite k
	Decision variables
$X_{i,j}$	An integer variable of the 1st-level routing is equal to the number of 1st-level vehicles using arc(i, j)
$Y_{k,i,j}$	A binary variable of the 2nd-level routing is equal to 1 if a 2nd-level vehicle makes a route start from satellite k and go from node i to node j and 0 otherwise
$Q^1_{i,j}$	The freight flow arc(i, j) for the first level
$Q^2_{k,i,j}$	The freight arc(i, j) where k represents the satellite where the freight is passing through.
$Z_{k,j}$	A binary variable that is equal to 1 if the freight to be delivered to customer j is consolidated in satellite k and 0 otherwise

TABLE 3: Summary indices, parameters, and decision variables for transformed model.

Symbol	Description
	Indices
n_s	Number of satellites
n_c	Number of customers
Ts	Number of possible routes from depot to satellites (CLP-determined)
Tc	Number of possible routes from satellites to customers (CLP-determined)
i	Satellite index
l	Depot-satellite route index
j	Customer index
k	Satellite-customer route index
M_1	Number of the 1st-level vehicles
M_2	Number of the 2nd-level vehicles
	Input parameters
s_s	Cost of loading/unloading operations of a unit of freight in satellite s
D_j	Demand required by customer j
Pc_k	Total demand for route k (CLP-determined)
Ks_l	Route l cost (CLP-determined)
Kc_k	Route k cost (CLP-determined)
$U_{l,i}$	If i is located on route l $U_{l,i} = 1$, otherwise $U_{l,i} = 0$
$W_{k,s}$	If satellite or receipient s is located on route k $W_{k,s} = 1$, otherwise $W_{k,s} = 0$
K_1	Capacity of the vehicles for the 1st level
	Decision variables
Y_l	If the tour takes place along the route l from the route set generated for level 1, then $Y_l = 1$, otherwise $Y_l = 0$
X_k	If the tour takes place along the route k from the route set generated for level 2, then $X_k = 1$, otherwise $X_k = 0$
	Computed quantities
Ps_l	Total demand for route l

Table 4: Decision variables and constraints before i after transformation.

Before transformation	After transformation	Description
		Decision variables
$X_{i,j}$, $Q^1_{i,j}$	X^T_l	Transformation of decision variables level 1 from the arc model arc(i, j) to the route model (l).
$Y_{k,i,j}$, $Q^2_{k,i,j}$, $Z_{k,j}$	Y^T_k	Transformation of decision variables level 2 from the arc model arc(i, j) to the route model (k).
		Constraints
(1)	(T1)	Objective function after transformation, different decision variables, the same in terms of the essence and functionality.
(2)	(T7)	Number of 1-type resources (CLP-determined)
(3)	—	Supply balance equation for 1-level nodes is unnecessary after transformation. This is a result of the route model to which particular vehicles are allocated.
(4)	(T2)	Number of 2-type resources (CLP-determined)
(5)	—	Vehicle balance equation for level 2 is unnecessary after transformation. This is a result of the route model to which particular vehicles are allocated.
(6)	(T4)	Supply balance for satellites.
(7)	(T6)	Number of tours for level 1 resulting from the capacity of vehicles.
(8)	—	Supply balance constraint for recipients is not required. In the route model, the supply volume is calculated for the route.
(9)	—	Supply volume constraint resulting from the vehicle capacity is unnecessary for level 2. The routes are generated only for the allowable capacities.
(10)	—	No return loads from satellite to depot (10). The routes are generated so as to automatically ensure this.
(11)	—	No return loads from the customer to satellite (11). The routes are generated so as to automatically ensure this.
(12), (13)	—	No $z_{k,j}$ variable after transformation.
(14)–(16)	(T3)	No overlapping deliveries to customers.
(17)	—	This is ensured by the route model.
(18)–(20)	(T8), (T9)	Integer and binary
(21)–(23)	—	Additional constraints are not necessary in the model with routes.

vehicles entering and leaving each customer. The number of the routes in each level must not exceed the number of vehicles for that level, as imposed by constraints (2) and (4). The flows balance on each network node is equal to the demand of this node, except for the depot, where the exit flow is equal to the total demand of the customers and for the satellites at the 2nd-level, where the flow is equal to the demand (unknown) assigned to the satellites which provide constraints (6) and (8). Moreover, constraints (6) and (8) forbid the presence of subtours not containing the depot or a satellite, respectively. In fact, each node receives an amount of flow equal to its demand, preventing the presence of subtours. Consider, for example, that a subtour is present between the nodes i, j, and k at the 1st level. It is easy to check that, in such a case, any value does not exist for the variables $Q^1_{i,j}$, $Q^1_{j,k}$, and $Q^1_{k,i}$, satisfying the constraints (6) and (8). The capacity constraints are formulated in (7) and (9), for the 1st-level and the 2nd-level, respectively. Constraints (10) and (11) do not allow residual flows in the routes, making the returning flow of each route to the depot (1st-level) and to each satellite (2nd-level) equal to 0. Constraints (12) and (13) indicate that a customer j is served by a satellite k ($Z_{k,j} = 1$) only if it receives freight from that satellite ($Y_{k,i,j} = 1$). Constraint (16) assigns each customer to one and only one satellite, while constraints (14) and (15) indicate that there is only one 2nd-level route passing through each customer and connect the two levels. Constraint (17) allows the start of a 2nd-level route from a satellite k only if a 1st-level route has served it. Constraints from (17) to (20) result from the character of the MILP-formulated problem. Additional constraints were introduced by [7] to increase the solution search efficiency. They strengthen the continuous relaxation of the flow model. In particular, authors in [7] used two families of cuts, one applied to the assignment variables derived from the subtour elimination constraints (edge cuts) and the other based on the flows. The edge-cuts explicitly introduce the well-known subtours elimination constraints derived from the TSP (Traveling Sales Problem). They can be expressed as constraint (21). The inequalities explicitly forbid the presence in the solution of subtours not containing

Table 5: The results of numerical examples for 2E-CVRP.

E-n13-k4	HSFCVRP (P3)		MP + Edge-Cuts (P2)		MP (P1)	
	T	Fc	T	Fc	T	Fc
E-n13-k4-01	17,36	280	600*	280	600*	280
E-n13-k4-02	17,22	286	600*	286	600*	286
E-n13-k4-03	15,39	284	600*	284	600*	284
E-n13-k4-04	10,09	218	44	218	65	218
E-n13-k4-05	9,58	218	48	218	108	218
E-n13-k4-06	11,05	230	78	230	154	230
E-n13-k4-07	9,16	224	39	224	64	224
E-n13-k4-08	13,03	236	46	236	75	236
E-n13-k4-09	13,22	244	67	244	93	244
E-n13-k4-10	14,08	268	107	268	183	268
E-n13-k4-11	18,91	276	159	276	600*	276
E-n13-k4-12	20,38	290	600*	290	600*	290
E-n13-k4-13	15,14	288	600*	288	600*	288
E-n13-k4-14	9,53	228	29	228	67	228
E-n13-k4-15	9,38	228	42	228	86	228
E-n13-k4-16	11,48	238	61	238	90	238
E-n13-k4-17	10,38	234	40	234	64	234
E-n13-k4-18	10,28	246	52	246	79	246
E-n13-k4-19	11,30	254	78	254	126	254
E-n13-k4-20	12,14	276	76	276	487	276
E-n13-k4-21	15,11	286	600*	286	600*	286
E-n13-k4-22	9,97	312	600*	312	600*	312
E-n13-k4-23	15,36	242	51	242	50	242
E-n13-k4-24	14,39	242	54	242	92	242
E-n13-k4-25	10,38	252	67	252	121	252
E-n13-k4-26	12,19	248	36	248	67	248
E-n13-k4-27	12,02	260	51	260	69	260
E-n13-k4-28	24,09	268	53	268	65	268
E-n13-k4-29	17,11	290	83	290	94	290
E-n13-k4-30	15,00	300	104	300	136	290
E-n13-k4-31	16,27	246	61	246	84	246
E-n13-k4-32	10,28	246	100	246	600*	246
E-n13-k4-33	15,17	258	93	258	123	258
E-n13-k4-34	11,00	252	48	252	55	252
E-n13-k4-35	8,92	264	40	264	52	264
E-n13-k4-36	11,11	272	97	272	138	272
E-n13-k4-37	16,06	296	109	296	213	296
E-n13-k4-38	16,69	304	124	304	600*	304
E-n13-k4-39	12,58	248	58	248	65	248
E-n13-k4-40	11,50	254	27	254	38	254
E-n13-k4-41	16,19	256	58	256	79	256
E-n13-k4-42	14,20	262	58	262	74	262
E-n13-k4-43	14,34	262	62	262	64	262
E-n13-k4-44	15,28	262	40	262	41	262
E-n13-k4-45	15,14	262	32	262	55	262
E-n13-k4-46	11,42	280	135	280	600*	280
E-n13-k4-47	12,20	274	95	274	142	274
E-n13-k4-48	13,17	280	76	280	257	280
E-n13-k4-49	11,16	280	79	280	117	280

Table 5: Continued.

E-n13-k4	HSFCVRP (P3)		MP + Edge-Cuts (P2)		MP (P1)	
	T	Fc	T	Fc	T	Fc
E-n13-k4-50	12,30	280	63	280	83	280
E-n13-k4-51	14,97	280	48	280	62	280
E-n13-k4-52	15,30	292	63	292	98	292
E-n13-k4-53	12,33	300	66	300	150	300
E-n13-k4-54	14,28	304	94	304	600*	304
E-n13-k4-55	14,19	310	216	310	600*	310
E-n13-k4-56	17,05	310	60	310	162	310
E-n13-k4-57	14,13	326	221	326	600*	326
E-n13-k4-58	9,17	326	78	326	600*	326
E-n13-k4-59	12,02	326	56	326	112	326
E-n13-k4-60	13,91	326	42	326	68	326
E-n13-k4-61	12,20	338	600*	338	600*	338
E-n13-k4-62	10,05	350	79	350	365	350
E-n13-k4-63	11,92	350	83	350	239	350
E-n13-k4-64	10,13	358	122	358	600*	358
E-n13-k4-65	12,94	358	219	358	600*	358
E-n13-k4-66	11,91	400	600*	400	600*	400

*Calculations stopped after 600 s, the feasible value of the objective function.
Fc: the optimal value of the objective function.

Table 6: The results of numerical examples for 2E-CVRP with logical constraints.

E-n13-k4	Fc	T	C	V	exCustomer*
E-n13-k4-01	284	15,36	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-07	240	7,16	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-11	290	16,91	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-20	280	13,14	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-26	270	10,72	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-32	270	10,88	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-33	276	14,124	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-40	284	11,23	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-46	308	11,12	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9
E-n13-k4-54	334	14,28	21	788	2,3; 2,4; 2,6; 2,7; 1,8; 1,9

*Pairs of customers that cannot be served on one route.

the depot, already forbidden by constraint (8). The number of potential valid inequalities are exponential, so that each customer reduces the flow of an amount equal to its demand d_i-constraints (22) and (23).

5.2. Model Transformation. One of the most important features that characterize the hybrid approach is the ease of modeling and transformation of the problem. The transformation is usually used to reduce the size of the problem and increase the efficiency of the search for a solution. In this case the transformation is based on the transition from arc to the route notation. During the transformation in the CLP the TSP, traveling salesman problem, is repeatedly solved and only the best routes in terms of costs are generated. In the process of transformation, the capacity vehicles constraints and those resulting from the set of orders are taken into account at both first and second level. For 2E-CVRP variants, time and logic constraints are also included.

The obtained optimization model after the transformation (T1)–(T9) has different decision variables (Table 3) and different constraints than those in the MILP (1)–(24). Some of the decision variables are redundant; other variables are subject to aggregation. This results in a very large reduction in their number. Decision variables before and after the transformation are shown in Table 4. The transformation also reduces or eliminates some of the constraints of the model (Table 4):

$$\min \sum_{k=1}^{Tc} \left(Y_k^T \cdot Kc_k\right) + \sum_{l}^{Ts} X_l^T \cdot Ks_l \tag{T1}$$

$$\sum_{k=1}^{Tc} Y_k^T \leq M_2 \tag{T2}$$

Table 7: (a) The results of numerical examples for 2E-CVRP-TW (hard windows). (b) The results of numerical examples for 2E-CVRP-TW (soft windows, penalty = 30).

(a)

E-n13-k4	T 40	50	60	70	80	90	100	110	130	150	160
E-n13-k4-01	—	—	—	—	—	—	280	280	280	280	280
E-n13-k4-07	—	224	224	224	224	224	224	224	224	224	224
E-n13-k4-11	—	—	304	276	276	276	276	276	276	276	276
E-n13-k4-20	—	294	280	276	276	276	276	276	276	276	276
E-n13-k4-26	—	248	248	248	248	248	248	248	248	248	248
E-n13-k4-32	—	—	262	246	246	246	246	246	246	246	246
E-n13-k4-33	—	258	258	258	258	258	258	258	258	258	258
E-n13-k4-40	—	284	284	254	254	254	254	254	254	254	254
E-n13-k4-46	—	—	308	308	280	280	280	280	280	280	280
E-n13-k4-54	—	—	—	324	304	304	304	304	304	304	304

(b)

E-n13-k4	T 40	50	60	70	80	90	100	110	130	150	160
E-n13-k4-01	358	354	346	346	310	310	280	280	280	280	280
E-n13-k4-07	270	224	224	224	224	224	224	224	224	224	224
E-n13-k4-11	306	306	304	276	276	276	276	276	276	276	276
E-n13-k4-20	366	294	280	276	276	276	276	276	276	276	276
E-n13-k4-26	292	248	248	248	248	248	248	248	248	248	248
E-n13-k4-32	322	278	262	246	246	246	246	246	246	246	246
E-n13-k4-33	336	258	258	258	258	258	258	258	258	258	258
E-n13-k4-40	344	284	284	254	254	254	254	254	254	254	254
E-n13-k4-46	344	310	308	308	280	280	280	280	280	280	280
E-n13-k4-54	342	334	334	324	304	304	304	304	304	304	304

$$\sum_{k=1}^{Tc} Y_k^T \cdot W_{k,j} = 1 \quad \text{for } j = 1, \ldots, n_c \tag{T3}$$

$$\sum_{k=1}^{Tc} X_k \cdot W_{k,i} \cdot Pc_k = \sum_{l}^{Ts} Ps_l \cdot U_{l,i} \quad \text{for } i = 1, \ldots, n_s \tag{T4}$$

$$\sum_{i=1}^{n_s} \sum_{k=1}^{Tc} Y_k^T \cdot W_{k,i} \cdot Pc_k = \sum_{l}^{Ts} Ps_l \tag{T5}$$

$$X_l^T \cdot K_1 \geq Ps_l \quad \text{for } l = 1, \ldots, Ts \tag{T6}$$

$$\sum_{l=1}^{Ts} X_l^T \leq M_1 \tag{T7}$$

$$Y_k^T \in \{0, 1\} \quad \text{for } k = 1, \ldots, Tc \tag{T8}$$

$$X_l^T \in C \quad \text{for } l = 1, \ldots, Ts. \tag{T9}$$

6. Computational Tests: Two-Echelon Capacitated Vehicle Routing Problem

For the final validation of the proposed hybrid approach, the benchmark data for 2E-CVRP was selected. 2E-CVRP, a well described and widely discussed problem, corresponded to the issues to which our approach was applied.

The instances for computational examples were built from the existing instances for CVRP [17] denoted as E-n13-k4. All the instance sets can be downloaded from the website [18]. The instance set was composed of 5 instances with 1 depot, 12 customers, and 2 satellites. The full instance consisted of 66 instances because the two satellites were placed over twelve customers in all 66 possible ways (number of combinations: 2 out of 12). All the instances had the same position for depot and customers, whose coordinates were the same as those of instance E-n13-k4. The instances differed in the choice of two customers who were also satellites (En13-k4-4, En13-k4-5, En13-k4-6, En13-k4-12, etc.).

Numerical experiments were conducted for the same data in three runs. The first run was a classical implementation of models (1)–(20) and its solution in the MP-based environment (P1). The second run used the same environment for models (1)–(24) with additional edge-cuts (P2). In the next run the models (1)–(20) were transformed (T1)–(T9) and solved in the proposed hybrid solution framework (P3). The calculations were performed using a computer with the following specifications: Intel(R) Core(TM) 2, 2 × 2, 40 GHZ RAM 2 GB. The analysis of the results for the

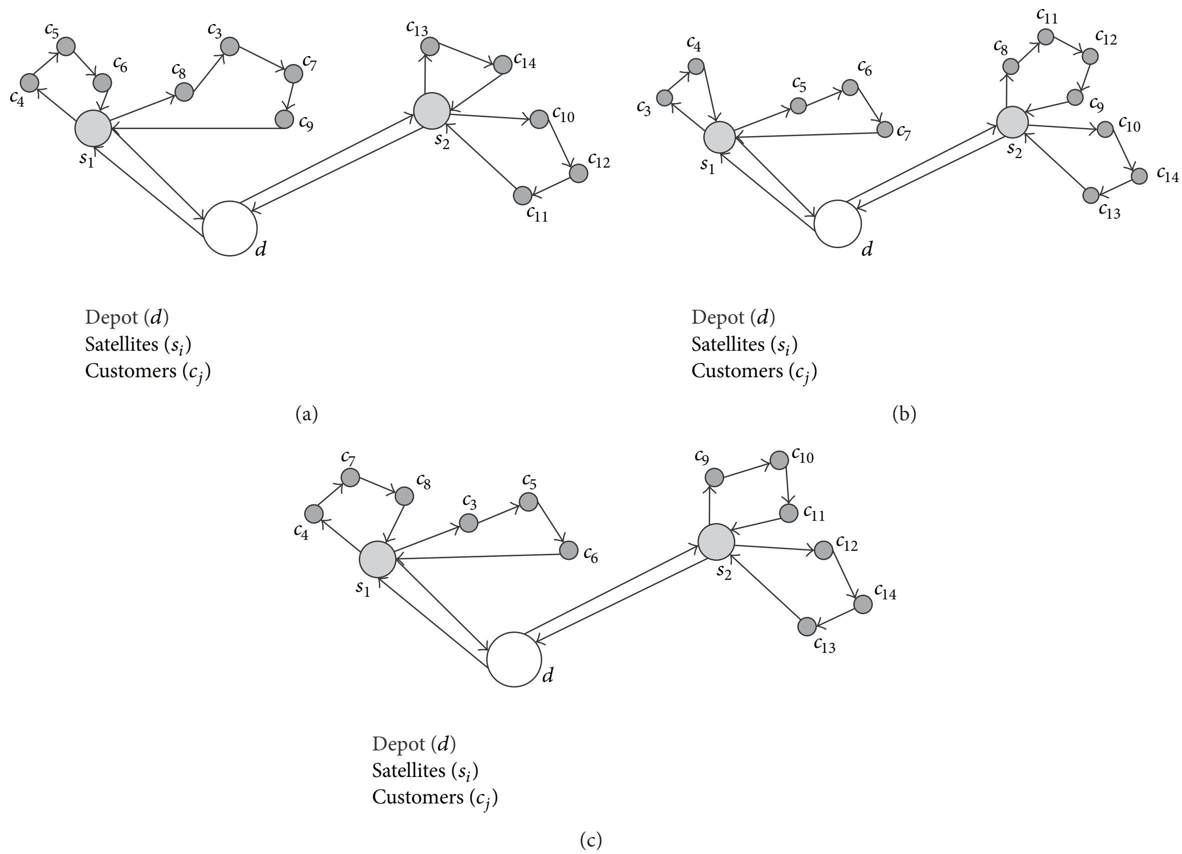

Figure 3: (a) Example of 2E-CVRP transportation network for E-n13-k4-20 instance. (b) Example of 2E-CVRP transportation network for E-n13-k4-20 instance with logic constraints. (c) Example of 2E-CVRP-TW transportation network for E-n13-k4-20 instance.

benchmark instances demonstrates that the hybrid approach may be a superior approach to the classical mathematical programming. For all the examples, the solutions were found 4–40 times faster than they are in the classical approach (Table 5). In many cases the calculations ended after 600 s as they failed to indicate that the solution was optimal. The number of constraints (C) and decision variables/integer ($V/\text{int}\,V$) was, for example, P1, P2, and P3, respectively, C = 1262, V = 744/368 for P1, C = 1982, V = 744/368 for P2, and C = 21, V = 1082/1079 for P3. Thus, the combinatorial spaces ($V \times C$) for illustrative examples were

(i) P1 $\approx$ 1 000 000;

(ii) P2 $\approx$ 1 400 000;

(iii) P3 $\approx$ 22 700.

The logical relationship between mutually exclusive variables was taken into account, which in real-world distribution systems means that the same vehicle cannot transport two types of selected goods or two points cannot be handled at the same time.

Those constraints result from technological, marketing, sales safety or competitive reasons. Only declarative application environments based on constraint satisfaction problem (CSP) make it possible to implement of this type of constraint. Table 6 presents the results of the numerical experiments conducted for 2E-CVRPs with logical constraints relating to the situation where two delivery points (customers) can be handled separately but not together in one route.

The final stage of the research was to optimize Two-Echelon Capacitated VRP with Time Windows (2E-CVRP-TW). This problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered. The time windows can be hard or soft. This variant of the 2E-CVRP is extremely important in a competitive environment.

In the first case the time windows cannot be violated, while in the second one if they are violated a penalty cost is paid. 2E-CVRP-TW has been implemented in a hybrid environment. This was followed by the optimization problem under the time constraints (time windows). There have been experiments with both windows hard and windows soft. The results are shown in Table 7(a) and Table 7(b). The impact of these constraints on the value of the objective function can be clearly seen. For instance E-n13-k4-20 also shows graphically the optimal way of delivery (see Figure 3(a)) as well as the impact on the route of logical constraints (Figure 3(b)) and the time window (Figure 3(c)).

7. Conclusion and Discussion on Possible Extension

The efficiency of the proposed approach is based on the reduction of the combinatorial problem and using the best properties of both environments. The hybrid approach (Table 3) makes it possible to find solutions in the shorter time.

In addition to solving larger problems faster, the proposed approach provides virtually unlimited modeling options with many types of constraints. Therefore, the proposed solution is recommended for decision-making problems under competitions and that has a structure similar to the presented models (Section 5). This structure is characterized by the constraints and objective function in which the decision variables are added together.

Further work will focus on running the optimization models with nonlinear and other logical constraints, multi-objective, uncertainty, and so on, in the hybrid optimization framework. The planned experiments will employ HSFCVRP for Two-Echelon Capacitated VRP with Satellites Synchronization, 2E-CVRP with Pickup and Deliveries, and other VRP issues in Supply Chain Sustainability [19] and other routing problems [20].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

The Newsvendor Problem with Different Delivery Time, Resalable Returns, and an Additional Order

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In a B2C scenario, the retailer is confronted with two kinds of demand. One requires an immediate delivery after placing an order, while the other prefers a delayed shipment due to some personal reasons. Considering demands for different delivery time, we explore a newsvendor model with resalable returns and an additional order to optimize the procurement decision under a stochastic demand distribution. The impact of the proportion of the instant delivery needs and the return rate on the order quantity and the expected profit is illustrated through numerical tests. It is shown that the expected profit decreases as the ratios of immediate delivery needs and returned products increase. Besides, if the sum of the percentage of the instant delivery needs and the return rate is less than 1, the expected profit is always greater than the result if the sum of them is equal to or greater than 1. Management implications are also discussed.

1. Introduction

Consider the following scenario. Kevin places an order for a free-size navy-blue hoodie online on Monday. In order to get the hoodie as soon as possible, he prefers the immediate delivery. Meanwhile, Jason orders the same hoodie in the same online shop. However, because of a business trip, he asks for his hoodie being sent at least five days later. The next day, Kevin receives his hoodie as scheduled, but it turns out that the color seems so much darker than the one shown on the Internet that he decides to return it instantly. Afterwards, the online shop sends the hoodie that Kevin returned to Jason on Saturday, and Jason is quite satisfied with the product and the timing.

In this scenario, what Kevin and Jason experience is quite prevalent nowadays. With the booming of E-commerce, the number of consumers shopping online shoots up. Comparing to the in-store shopping, however, online consumers are more likely to return products due to a lack of seeing, touching, and trying the commodities, which results in the return rate ranging from 18% to 74% [1]. Since consumers return products due to their personal reasons, most of the returns are in good condition and can be resold directly once received by the e-tailer. In this sense, managing the returns becomes an integral part of e-tailers' supply chain management. For instance, when deciding the procurement quantity at the very beginning, the B2C retailer must take the quantity of resalable returns into account. Additionally, as shown in the previous example, another difference between the online shopping and the in-store shopping is that, in traditional brick-and-mortar shopping, consumers can get the product instantaneously once it is paid. When consumers are shopping on the Internet, they are free to choose an instant delivery or a delayed delivery. In this case, the B2C retailer is confronted with two kinds of demand, which complicate the process of decision-making about the replenishment quantity. One demand prefers the immediate delivery after placing an order, while the other asks for the delayed shipment. Since the sales in former case may be lost when there is a shortage, the retailer needs to order and decide the procurement quantity at the beginning of the sales period, which is used to meet the instant delivery needs. As to the consumers requiring the delayed delivery, the retailer can satisfy them with the returns arriving before the end of the selling period, the remaining stocks of the initial replenishment (if any), and a second order. Based on the abovementioned reasons, both the quantity of resalable returns and the demands for either immediate or delayed delivery have a great influence on inventory management of B2C retailers. Thus, Internet retailers' flexibility in deciding the order quantity is of vital importance, which aims at maximizing their profit as well as achieving an optimal service level.

Since returns play such an important role in deciding order quantity, much of the existing literature dealing with procurement decisions considers the case with returns that need recovery, remanufacture, or recycle [2, 3]. They mainly analyze the impact of the quality and quantity of the returned goods and the lead-time to refurbish them on the decision-making about the order quantity. Generally, this kind of problem focuses on the traditional business because when shopping at the physical store, consumers can have a better overall idea about the product's size, texture, appearance, and performance before purchasing. Hence, the ratio of returns due to quality problems or the end of service life is much larger than the ratio of returns due to personal preference [4]. Besides, most of the former returns need refurbishing, remanufacturing, or recovery. So, in the background of traditional business, the research usually focuses on returned products with quality problem, which is refurbished or recovered before reselling to the market.

In contrast with consumers shopping at the physical store, online consumers evaluate the commodity through pictures displayed on the website. Therefore, they return products mostly because what they get eventually is different from what they saw on the Internet. The ratio of returns as good as new constitutes a large amount of total returns [5]. When deciding the order quantity, the quantity of the resalable returns should be taken into account. So, another stream of literature modeling procurement decisions has emerged, analyzing the effect of the quantity of resalable returns [1, 5, 6] and the return policy [7–10] on the order quantity of Internet retailers.

In the context of E-commerce, not only do the returns affect the replenishment decision, but also demands for different delivery time have influence on it. Online consumers can choose between receiving the product as soon as possible and delaying the delivery time, which is the unique feature of online shopping differing from in-store shopping. To the best of our knowledge, few literatures consider the different delivery time. Hence, we extend existing research to the case considering demands for different delivery options as well as the returns.

Previous literature models a procurement decision with returns from the perspectives of either the single period or the multiperiod, depending on the properties of the products in the research. In contrast with the products in the multiperiod model, the goods in a single period model are highly seasonal or more perishable [11]. In the context of e-business, commodities with short period and seasonal characteristics such as apparel and personal electronics account for most of the sales [12], which leads us to formulate the problem based on a single period model (also known as the newsboy or newsvendor model).

Comparing to the classic newsvendor model that only one order is placed before the beginning of the sales period, a recourse case is proposed where an additional replenishment takes place during or at the end of the period [13–15]. Research on the single period problem with a second order points out that an additional order chance can effectively improve the accuracy of demand forecasting and increase revenue. However, existing research dealing with a second procurement ignores the effect of returns. Besides, research considers returns without taking an additional order into account. Thus, we extend the classic newsvendor model to the one considering both resalable returns and an additional order simultaneously.

For brevity, the following novel contributions differentiate our model from the abovementioned literature. First of all, in a B2C scenario, the delivery time uncertainty is considered. We distinguish the demand asking for immediate delivery from the demand requiring delayed shipment and analyze their impact on the procurement decisions and the expected profit of the B2C retailer. Additionally, we extend the newsboy problem to the model considering both resalable returns and a second order simultaneously. Considering demands for different delivery time, resalable returns, and an additional replenishment, we formulate the problem based on the newsvendor model to analyze the order quantity that not only maximizes Internet retailers' profit but also optimizes their service level.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature and points out the characteristics of the paper. Section 3 delineates the problem in detail. Section 4 introduces the notation and formulates the expected profit function of the B2C retailer. In Section 5, a brief numerical illustration is presented, from which some observations and management implications are deduced. Last but not least, Section 6 summarizes the findings of the paper and shows some extensions.

2. Literature Review

Research concentrating on procurement decisions has been increasing rapidly. There are three streams of research related to our problem. The first stream focuses on used returns that need to be refurbished before reselling. Providing the demand is known and returns are stochastic; Ferrer [16] presents a remanufacturing procedure with only one collection site. Robotis et al. [17] extend Ferrer's system to two collection sites storing two distinct returned products, respectively. Based on the system proposed by Robotis et al., Webster and Weng [11] examine a system with two collection sites and one refurbishing site. Not only do they derive the optimal production and procurement quantities maximizing the profit of the entire supply chain, but also they figure out under what condition it is better to use only one of the collection sites. Additionally, since timely quality information is of vital importance, Teng et al. [10] demonstrate a quality classification scheme for the automotive industry. In a similar vein, Ferguson et al. [4] further explore the value of classification according to different quality levels. In order to timely evaluate the quality of returns, Panagiotidou et al. [2] study the sampling inspection in a remanufacturing system, which improves the procurement decision. The aforesaid literature mainly concentrates on returns from offline consumers. Because they can have a better understanding of products before purchasing, offline consumers usually return products because of quality problems. So, the quality and quantity of returns and the lead-time to remanufacture them are considered. In contrast, we concentrate on the intact returns, the quality of which is as good as the new. There is no need to recover the returned products and they can be resold after a quick quality check.

Returns requiring refurbishing or recovery take the lion's share of total returns in the background of traditional business. However, since online consumers make purchase decisions mostly based on picture on the Internet, their likelihood to return intact products is high due to the fact that the item they receive is usually far from what they expected. In most cases, the items they return have no difference from brand new commodities. So a second group of literature deals with this kind of returned products, which can be resold instantly. According to the values of the cost coefficients, Vlachos and Dekker [5] first study the optimal replenishment strategies with uncontrollable return flows. Under the condition of various return options, they derive different optimal order quantities. Mostard and Teunter [1] relax two hypotheses in the research of Vlachos and Dekker, which assume that commodities can be returned only once and the proportion of the returns is fixed. In the model of Mostard and Teunter, products can be returned more than once. Furthermore, in consideration of the variability in the percentage of returns, they figure out a closed-form equation determining the optimal order quantity. Last but not least, using real data from a mail order enterprise, they compare the procurement quantity deduced from their model to those presented by Vlachos and Dekker and to the original order quantity of the company. Most of the earlier research uses historical data to forecast the mean and standard deviation of demand. In practice, however, sales data of highly seasonal products is difficult to obtain. Mostard et al. [6] extend the newsvendor problem to a distribution-free one with resalable returns. Even though the existing literature analyzes the impact of resalable returns on procurement decisions, few of them take an additional order into consideration at the same time, which can significantly improve the accuracy of demand forecast and increase profit.

Comparing to the classic newsvendor model, where the order can only be placed once before the sales period, Gallego and Moon [13] analyze a recourse case based on the distribution-free newsvendor problem. The unsatisfied demand is assumed to be deterministic and can be met by an additional replenishment at the end of the season. Kodama [18] discusses a sophisticated situation with returns to manufacturer in the case of surplus and a second order in the case of stock-out. Two cases are considered. One is the demand that occurs once only at a particular point of the period. The other is the demand that follows a general demand pattern. Similarly, Khouja [14] extends the model of Gallego and Moon to the case that a ratio of the unmet demand is lost immediately and the remainder is satisfied by an emergency order. In contrast, H. S. Lau and A. H. L. Lau [15] introduce the additional order quantity as a second decision variable, which is determined during the midseason compared to the first order quantity determined at the start of the sales period. The coordination between these two decision variables is of vital importance. Previous research considering an additional order ignores the effect of returns. Also, as mentioned, research dealing with returns does not analyze the influence of a second order.

So far, few literatures consider both resalable returns and a second order as we do. Besides, most of the abovementioned research explores the problem in the setting of offline business and refers to the same method in the context of e-business. But consumers are free to choose either an instant delivery or a delayed delivery while shopping online, which differentiates the e-shopping from the in-store shopping. The B2C retailer should take the impact of different delivery time on inventory into consideration when deciding the replenishment quantity at the very beginning. Therefore, based on a newsvendor model, we consider demands for different delivery options, returns, and a second order simultaneously and formulate and optimize the expected profit of the B2C retailer.

3. Problem Description

In the setting of E-commerce, the demand x for a single product is stochastic, which follows a distribution function $F(x)$ and a probability density function $f(x)$. Since the delivery time can be customized to the needs of consumers, demand is classified into two types by the different delivery time that consumers require. Consumers preferring the immediate delivery after placing an order account for α percent (according to historical data, α is a constant) of the total demand, while the proportion of consumers asking for a delayed delivery because of some personal reasons is $(1-\alpha)$. Before the sales period, the vendor needs to estimate the demand asking for instant delivery and to decide the initial order quantity Q_1, the only decision variable in the model. The retailer purchases at the price c_p per unit ordered and sells at the price p per unit sold. All the commodities replenished before the selling season are used to meet the instant delivery needs. Due to forecast errors, this kind of demand is lost at the cost c_s per shortage once there is a stock-out. Otherwise, the remaining stocks are used to satisfy the demand, preferring delayed delivery. Meanwhile, returns arriving before the end of the selling season are assumed to be of the same quality as new products and resold to meet the delayed delivery needs at the same price. Consumers receive a full refund if products are returned. Since the number of returned products is uncontrollable, we assume that the returns comprise β percent of total sales. The vendor pays at the unit cost c_m to collect, inspect, repack, and restock the returned products. Compared to the classic newsboy model, if the demand requiring delayed delivery is higher than the remaining stocks of the first procurement (if any) and the returned products, according to the unmet needs, an additional order is allowed in our model. Of course, the second order at the end leads to higher purchase cost $\tilde{c_p}$. At the end of the season, unsold products are disposed at the unit cost c_d without salvage value.

4. Notation and Models

Notation

x: Demand

Q_0: First order quantity

Q_1: Second order quantity

α: Proportion of demand requiring instant delivery $(1 < \alpha < 0)$

β: Proportion of returns $(1 < \beta < 0)$

p: Unit selling price

c_p: First order purchase cost per product

$\widetilde{c_p}$: Second order purchase cost per product

c_s: Penalty cost per shortage

c_d: Disposal cost per product

c_m: Return management cost per returned product

π: Profit

${Q_0}^*$: Optimal order quantity

$E\pi(Q_0)$: Expected profit for the order quantity Q_0.

The problem described in the former part has only one decision variable: the first order quantity Q_0. Q_1 is placed at the end of the season after observing the unsatisfied demand. Formulating the expected profit for the replenishment quantity Q_0, we derive the optimal order quantity ${Q_0}^*$. We begin to formulate and optimize the model from the situation without returns, based on which the model considering returns is formulated and optimized.

4.1. Model without Returns

Case A ($\alpha x \geq Q_0, Q_1 = (1-\alpha)x$). The profit of the B2C vendor when the first order quantity is Q_0 and the demand is x is

$$\pi(Q_0, x) = pQ_0 + p(1-\alpha)x - c_pQ_0 - \widetilde{c_p}Q_1 - c_s(\alpha x - Q_0). \tag{1}$$

Case B ($\alpha x < Q_0$, $Q_1 = \max\{(1-\alpha)x - (Q_0 - \alpha x), 0\}$). The profit of the B2C vendor when the first order quantity is Q_0 and the demand is x is

$$\begin{aligned}\pi(Q_0, x) = {} & p\alpha x + p(1-\alpha)x - c_pQ_0 - \widetilde{c_p}Q_1 \\ & - c_d \max\{(Q_0 - \alpha x) - (1-\alpha)x, 0\}.\end{aligned} \tag{2}$$

Let

$$H(Q_0, x) = p\min\{\alpha x, Q_0\} + p(1-\alpha)x - c_pQ_0, \tag{3}$$

$$G_1(Q_0, x) = \widetilde{c_p}Q_1 + c_s(\alpha x - Q_0), \tag{4}$$

$$G_2(Q_0, x) = \widetilde{c_p}Q_1 + c_d\max\{(Q_0 - \alpha x) - (1-\alpha)x\}. \tag{5}$$

Combining (3), (4), and (5) gives

$$\pi(Q_0, x) = \begin{cases} H(Q_0, x) - G_1(Q_0, x) & x \geq \dfrac{Q_0}{\alpha} \\ H(Q_0, x) - G_2(Q_0, x) & \text{otherwise.}\end{cases} \tag{6}$$

The expected profit of the B2C retailer is

$$\begin{aligned}E\pi(Q_0, x) = {} & \int_0^\infty H(Q_0, x) f(x)\,dx \\ & - \int_0^{Q_0/\alpha} G_2(Q_0, x) f(x)\,dx \\ & - \int_{Q_0/\alpha}^\infty G_1(Q_0, x) f(x)\,dx.\end{aligned} \tag{7}$$

Taking the derivative of (3), (4), and (5),

$$\begin{aligned}&\frac{d}{dQ_0}\int_0^\infty H(Q_0, x) f(x)\,dx \\ &\quad = \int_0^{Q_0/\alpha} \frac{d}{dQ_0} p(\alpha x - Q_0) f(x)\,dx + (p - c_p) \\ &\quad = -pF\left(\frac{Q_0}{\alpha}\right) + (p - c_p),\end{aligned} \tag{8}$$

$$\begin{aligned}&\frac{d}{dQ_0}\int_{Q_0/\alpha}^\infty G_1(Q_0, x) f(x)\,dx \\ &\quad = \int_{Q_0/\alpha}^\infty \frac{d}{dQ_0} c_s(\alpha x - Q_0) f(x)\,dx \\ &\qquad - \widetilde{c_p}(1-\alpha)\frac{Q_0}{\alpha} f\left(\frac{Q_0}{\alpha}\right)\frac{1}{\alpha} \\ &\quad = -c_s\left[1 - F\left(\frac{Q_0}{\alpha}\right)\right] - \widetilde{c_p}(1-\alpha)\frac{Q_0}{\alpha} f\left(\frac{Q_0}{\alpha}\right)\frac{1}{\alpha},\end{aligned} \tag{9}$$

$$\begin{aligned}&\frac{d}{dQ_0}\int_0^{Q_0/\alpha} G_2(Q_0, x) f(x)\,dx \\ &\quad = \frac{d}{dQ_0}\int_0^{Q_0/\alpha} \left[\widetilde{c_p}(x - Q_0)^+ + c_d(Q_0 - x)^+\right] f(x)\,dx \\ &\quad = \widetilde{c_p}\left(\frac{Q_0}{\alpha} - Q_0\right) f\left(\frac{Q_0}{\alpha}\right)\frac{1}{\alpha} \\ &\qquad + \int_0^{Q_0} \frac{d}{dQ_0} c_d(Q_0 - x) f(x)\,dx \\ &\qquad + \int_{Q_0}^{Q_0/\alpha} \frac{d}{dQ_0}\widetilde{c_p}(x - Q_0) f(x)\,dx \\ &\quad = \widetilde{c_p}\left(\frac{Q_0}{\alpha} - Q_0\right) f\left(\frac{Q_0}{\alpha}\right)\frac{1}{\alpha} + c_dF(Q_0) \\ &\qquad - \widetilde{c_p}\left[F\left(\frac{Q_0}{\alpha}\right) - F(Q_0)\right].\end{aligned} \tag{10}$$

Combining (8), (9), and (10) gives

$$\begin{aligned}\frac{d}{dQ_0}E\pi(Q_0,x) &= -pF\left(\frac{Q_0}{\alpha}\right)+(p-c_p)+c_s\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]\\ &\quad - c_dF(Q_0)+\widetilde{c_p}\left[F\left(\frac{Q_0}{\alpha}\right)-F(Q_0)\right]\\ &= (p+c_s-c_p)-(p+c_s-\widetilde{c_p})F\left(\frac{Q_0}{\alpha}\right)\\ &\quad -(c_d+\widetilde{c_p})F(Q_0).\end{aligned} \tag{11}$$

So $(d^2/d^2Q_0)E\pi(Q_0,x) = -(p+c_s-\widetilde{c_p})f(Q_0/\alpha)(1/\alpha)-(c_d+\widetilde{c_p})f(Q_0)$.

Because $(d^2/d^2Q_0)E\pi(Q_0,x) < 0$, $E\pi(Q_0,x)$ is strictly concave.

Let $(d/dQ_0)E\pi(Q_0,x) = 0$.

Denote $r = (p+c_s-\widetilde{c_p})/(p+c_s+c_d)$ and $1-r = (c_d+\widetilde{c_p})/(p+c_s+c_d)$.

Then, $(p+c_s-c_p)-(p+c_s+c_d)[rF(Q_0/\alpha)+(1-r)F(Q_0)] = 0$.

As $F(x)$ is a cumulative distribution function (of course nondecreasing function), we can derive

$$\begin{gathered}(p+c_s-c_p)-(p+c_s+c_d)\\ \times\left[rF\left(\frac{Q_0}{\alpha}\right)+(1-r)F\left(\frac{Q_0}{\alpha}\right)\right]\le 0,\\ (p+c_s-c_p)-(p+c_s+c_d)\left[rF(Q_0)+(1-r)F(Q_0)\right]\ge 0.\end{gathered} \tag{12}$$

The optimal order quantity ${Q_0}^*$ belongs to a closed interval

$${Q_0}^* \in \left[\alpha F^{-1}\left(\frac{p+c_s-c_p}{p+c_s+c_d}\right), F^{-1}\left(\frac{p+c_s-c_p}{p+c_s+c_d}\right)\right]. \tag{13}$$

4.2. Model with Returns

Case A ($\alpha x \ge Q_0$, $Q_1 = \max\{(1-\alpha)x-\beta[Q_0+(1-\alpha)x],0\}$). The profit of the B2C vendor when the first order quantity is Q_0 and the demand is x is

$$\begin{aligned}\pi(Q_0,x) = {} & pQ_0+p(1-\alpha)x-(p+c_m)\beta\left[Q_0+(1-\alpha)x\right]\\ & - c_s(\alpha x-Q_0)-c_pQ_0-\widetilde{c_p}Q_1\\ & - c_d\max\{\beta\left[Q_0+(1-\alpha)x\right]-(1-\alpha)x,0\}.\end{aligned} \tag{14}$$

Case B ($\alpha x < Q_0$, $Q_1 = \max\{(1-\alpha)x-\beta x-(Q_0-\alpha x),0\}$). The profit of the B2C vendor when the first order quantity is Q_0 and the demand is x is

$$\begin{aligned}\pi(Q_0,x) = {} & p\alpha x+p(1-\alpha)x-(p+c_m)\beta x-c_pQ_0-\widetilde{c_p}Q_1\\ & - c_d\max\{\beta x+(Q_0-\alpha x)-(1-\alpha)x,0\}.\end{aligned} \tag{15}$$

Let

$$\begin{aligned}H(Q_0,x) = {} & p\min\{\alpha x-Q_0,0\}+p(1-\alpha)x-(p+c_m)\beta x\\ & - c_s\max\{\alpha x-Q_0,0\}+(p-c_p)Q_0,\end{aligned} \tag{16}$$

$$\begin{aligned}G_1(Q_0,x) = {} & \widetilde{c_p}Q_1-(p+c_m)(Q_0-\alpha x)\beta\\ & + c_d\max\{\beta\left[Q_0+(1-\alpha)x\right]-(1-\alpha)x,0\},\end{aligned} \tag{17}$$

$$G_2(Q_0,x) = \widetilde{c_p}Q_1+c_d\max\{\beta x+(Q_0-\alpha x)-(1-\alpha)x,0\}. \tag{18}$$

Combining (16), (17), and (18) gives

$$\pi(Q_0,x) = \begin{cases} H(Q_0,x)-G_1(Q_0,x) & x \ge \dfrac{Q_0}{\alpha}\\ H(Q_0,x)-G_2(Q_0,x) & \text{otherwise.}\end{cases} \tag{19}$$

The expected profit of the B2C retailer is

$$\begin{aligned}E\pi(Q_0,x) = {} & \int_0^\infty H(Q_0,x)f(x)\,dx\\ & - \int_0^{Q_0/\alpha} G_2(Q_0,x)f(x)\,dx\\ & - \int_{Q_0/\alpha}^\infty G_1(Q_0,x)f(x)\,dx.\end{aligned} \tag{20}$$

Taking the derivative of (16), (17), and (18),

$$\begin{aligned}&\frac{d}{dQ_0}\int_0^\infty H(Q_0,x)f(x)\,dx\\ &\quad = \int_0^{Q_0/\alpha}\frac{d}{dQ_0}p(\alpha x-Q_0)f(x)\,dx\\ &\qquad + \int_{Q_0/\alpha}^\infty \frac{d}{dQ_0}\left[-c_s(\alpha x-Q_0)\right]+(p-c_p)\\ &\quad = \int_0^{Q_0/\alpha}-pf(x)\,dx+\int_{Q_0/\alpha}^\infty c_sf(x)\,dx+(p-c_p)\\ &\quad = -pF\left(\frac{Q_0}{\alpha}\right)+c_s\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]+(p-c_p)\\ &\quad = (p+c_s-c_p)-(p+c_s)F\left(\frac{Q_0}{\alpha}\right),\end{aligned} \tag{21}$$

$$(*)=\frac{d}{dQ_0}\int_{Q_0/\alpha}^{\infty}G_1(Q_0,x)f(x)\,dx$$
$$=\frac{d}{dQ_0}\int_{Q_0/\alpha}^{\infty}\left\{\widetilde{c_p}\left[(1-\alpha)x-\beta\left(Q_0+(1-\alpha)x\right)\right]^+\right.$$
$$+c_d\left[\beta\left(Q_0+(1-\alpha)x\right)-(1-\alpha)x\right]^+$$
$$\left.-\beta\left(p+c_m\right)\left(Q_0-\alpha x\right)\right\}f(x)\,dx$$
$$=g(Q_0)+\int_{Q_0/\alpha}^{\infty}\frac{d}{dQ_0}$$
$$\times\left\{\widetilde{c_p}\left[(1-\alpha)x-\beta\left(Q_0+(1-\alpha)x\right)\right]^+\right.$$
$$+c_d\left[\beta\left(Q_0+(1-\alpha)x\right)-(1-\alpha)x\right]^+$$
$$\left.-\beta\left(p+c_m\right)\left(Q_0-\alpha x\right)\right\}f(x)\,dx$$
$$=g(Q_0)-\beta\left(p+c_m\right)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]$$
$$+\int_{Q_0/\alpha}^{\infty}\frac{d}{dQ_0}\left\{\widetilde{c_p}\left[(1-\alpha)(1-\beta)x-\beta Q_0\right]^+\right.$$
$$\left.+c_d\left[\beta Q_0-(1-\alpha)(1-\beta)x\right]^+\right\}f(x)dx, \tag{22}$$

where $g(Q_0)=-(1/\alpha)f(Q_0/\alpha)\{\widetilde{c_p}[(1-\alpha)(Q_0/\alpha)-\beta(Q_0/\alpha)]^+ + c_d[\beta(Q_0/\alpha)-(1-\alpha)(Q_0/\alpha)]^+\}$ and

$$(**)=\frac{d}{dQ_0}\int_{Q_0/\alpha}^{\infty}G_2(Q_0,x)f(x)\,dx$$
$$=-g(Q_0)+\int_0^{Q_0/\alpha}G_2(Q_0,x)f(x)\,dx. \tag{23}$$

(i) Consider $\alpha<1-\beta$ and then $\beta/(1-\beta)(1-\alpha)<1/\alpha$:

$$(*)=g(Q_0)-\beta\left(p+c_m\right)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]$$
$$+\int_{Q_0/\alpha}^{\infty}\frac{d}{dQ_0}\widetilde{c_p}\left[(1-\alpha)(1-\beta)x-\beta Q_0\right]f(x)\,dx$$
$$=g(Q_0)-\beta\left(p+c_m\right)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]$$
$$-\beta\widetilde{c_p}\left[1-F\left(\frac{Q_0}{\alpha}\right)\right]$$
$$=g(Q_0)-\beta\left(p+c_m+\widetilde{c_p}\right)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right], \tag{24}$$

$$(**)=-g(Q_0)+\int_{Q_0/(1-\beta)}^{Q_0/\alpha}\frac{d}{dQ_0}\widetilde{c_p}\left[(1-\beta)x-Q_0\right]f(x)\,dx$$
$$+\int_0^{Q_0/(1-\beta)}\frac{d}{dQ_0}c_d\left[Q_0-(1-\beta)x\right]f(x)\,dx$$
$$=-g(Q_0)-\widetilde{c_p}\left[F\left(\frac{Q_0}{\alpha}\right)-F\left(\frac{Q_0}{1-\beta}\right)\right]+c_dF\left(\frac{Q_0}{1-\beta}\right)$$
$$=-g(Q_0)-\widetilde{c_p}F\left(\frac{Q_0}{\alpha}\right)+\left(\widetilde{c_p}+c_d\right)F\left(\frac{Q_0}{1-\beta}\right). \tag{25}$$

Combining (21), (24), and (25) gives

$$\frac{d}{dQ_0}E\pi(Q_0,x)$$
$$=\frac{d}{dQ_0}\int_0^{\infty}H(Q_0,x)f(x)\,dx-(*)-(**)$$
$$=\left(p+c_s-c_p\right)-\left(p+c_s\right)F\left(\frac{Q_0}{\alpha}\right)+\beta\left(p+c_m+\widetilde{c_p}\right)$$
$$-\left[\beta\left(p+c_m+\widetilde{c_p}\right)-\widetilde{c_p}\right]F\left(\frac{Q_0}{1-\beta}\right)$$
$$=\left(p+c_s-c_p\right)+\beta\left(p+c_m+\widetilde{c_p}\right)$$
$$-\left[p+c_s-\widetilde{c_p}+\beta\left(p+c_m+\widetilde{c_p}\right)\right]F\left(\frac{Q_0}{\alpha}\right)$$
$$-\left(\widetilde{c_p}+c_d\right)F\left(\frac{Q_0}{1-\beta}\right). \tag{26}$$

Let $(d/dQ_0)E\pi(Q_0,x)=0$.

Denote $r=(p+c_s-\widetilde{c_p}+\beta(p+c_m+\widetilde{c_p}))/(p+c_s+c_d+\beta(p+c_m+\widetilde{c_p}))$ and $1-r=(\widetilde{c_p}+c_d)/(p+c_s+c_d+\beta(p+c_m+\widetilde{c_p}))$.

Then, $(p+c_s-c_p)+\beta(p+c_m+\widetilde{c_p})-[p+c_s+c_d+\beta(p+c_m+\widetilde{c_p})][rF(Q_0/\alpha)+(1-r)F(Q_0/(1-\beta))]=0$.

As $F(x)$ is a cumulative distribution function (of course nondecreasing function), we can derive

$$\left(p+c_s-c_p\right)+\beta\left(p+c_m+\widetilde{c_p}\right)$$
$$-\left[p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)\right]$$
$$\times\left[rF\left(\frac{Q_0}{\alpha}\right)+(1-r)F\left(\frac{Q_0}{\alpha}\right)\right]\leq 0,$$
$$\left(p+c_s-c_p\right)+\beta\left(p+c_m+\widetilde{c_p}\right)$$
$$-\left[p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)\right]$$
$$\times\left[rF\left(\frac{Q_0}{1-\beta}\right)+(1-r)F\left(\frac{Q_0}{1-\beta}\right)\right]\geq 0. \tag{27}$$

The optimal order quantity ${Q_0}^*$ belongs to a closed interval

$$ {Q_0}^* \in \left[\alpha F^{-1}\left(\frac{p+c_s-c_p+\beta\left(p+c_m+\widetilde{c_p}\right)}{p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)}\right),\right. \left.(1-\beta)F^{-1}\left(\frac{p+c_s-c_p+\beta\left(p+c_m+\widetilde{c_p}\right)}{p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)}\right)\right]. \quad (28) $$

(ii) Consider $\alpha \geq 1-\beta$ and then $\beta/(1-\beta)(1-\alpha) \geq 1/\alpha$:

$$ \begin{aligned} (*) &= g(Q_0) - \beta(p+c_m)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right] \\ &\quad + \int_{\beta Q_0/(1-\beta)(1-\alpha)}^{\infty} \frac{d}{dQ_0}\widetilde{c_p}\left[(1-\alpha)(1-\beta)x-\beta Q_0\right] \times f(x)\,dx \\ &\quad + \int_{Q_0/\alpha}^{\beta Q_0/(1-\beta)(1-\alpha)} \frac{d}{dQ_0}c_d\left[\beta Q_0-(1-\alpha)(1-\beta)x\right] \times f(x)\,dx \\ &= g(Q_0) - \beta(p+c_m)\left[1-F\left(\frac{Q_0}{\alpha}\right)\right] - \beta\widetilde{c_p}\left[1-F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right)\right] \\ &\quad + \beta c_d F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right) - F\left(\frac{Q_0}{\alpha}\right) \\ &= g(Q_0) - \beta\left(p+c_m+\widetilde{c_p}\right) + \beta(p+c_m-c_d)F\left(\frac{Q_0}{\alpha}\right) \\ &\quad + \beta\left(\widetilde{c_p}+c_d\right)F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right), \end{aligned} \quad (29) $$

$$ \begin{aligned} (**) &= -g(Q_0) + \int_0^{Q_0/\alpha} \frac{d}{dQ_0}c_d\left[Q_0-(1-\beta)x\right]f(x)\,dx \\ &= -g(Q_0) + \int_0^{Q_0/\alpha} c_d f(x)\,dx = -g(Q_0) + c_d F\left(\frac{Q_0}{\alpha}\right). \end{aligned} \quad (30) $$

Combining (21), (29), and (30) gives

$$ \begin{aligned} &\frac{d}{dQ_0}E\pi(Q_0,x) \\ &= \frac{d}{dQ_0}\int_0^{\infty} H(Q_0,x)f(x)\,dx - (*) - (**) \\ &= \left(p+c_s-c_p\right) - (p+c_s)F\left(\frac{Q_0}{\alpha}\right) + \beta\left(p+c_m-\widetilde{c_p}\right) \\ &\quad - \beta(p+c_m-c_d)F\left(\frac{Q_0}{\alpha}\right) \\ &\quad - \beta\left(\widetilde{c_p}+c_d\right)F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right) - c_d F\left(\frac{Q_0}{\alpha}\right) \\ &= \left(p+c_s-c_p\right) + \beta\left(p+c_m-\widetilde{c_p}\right) \\ &\quad - \left[p+c_s+c_d+\beta(p+c_m-c_d)\right]F\left(\frac{Q_0}{\alpha}\right) \\ &\quad - \beta\left(\widetilde{c_p}+c_d\right)F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right). \end{aligned} \quad (31) $$

Let $(d/dQ_0)E\pi(Q_0,x) = 0$.

Denote $r = (p+c_s+c_d+\beta(p+c_m-c_d))/(p+c_s+c_d+\beta(p+c_m+\widetilde{c_p}))$ and $1-r = \beta(\widetilde{c_p}+c_d)/(p+c_s+c_d+\beta(p+c_m+\widetilde{c_p}))$.

Then, $(p+c_s-c_p)+\beta(p+c_m-\widetilde{c_p})-[p+c_s+c_d+\beta(p+c_m+\widetilde{c_p})][rF(Q_0/\alpha)+(1-r)F(\beta Q_0/(1-\beta)(1-\alpha))] = 0$.

As $F(x)$ is a cumulative distribution function (of course nondecreasing function), we can derive

$$ \begin{aligned} &\left(p+c_s-c_p\right) + \beta\left(p+c_m-\widetilde{c_p}\right) \\ &\quad - \left[p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)\right] \\ &\quad \times \left[rF\left(\frac{Q_0}{\alpha}\right) + (1-r)F\left(\frac{Q_0}{\alpha}\right)\right] \geq 0, \\ &\left(p+c_s-c_p\right) + \beta\left(p+c_m-\widetilde{c_p}\right) \\ &\quad - \left[p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)\right] \\ &\quad \times \left[rF\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right) + (1-r)F\left(\frac{\beta Q_0}{(1-\beta)(1-\alpha)}\right)\right] \\ &\leq 0. \end{aligned} \quad (32) $$

The optimal order quantity ${Q_0}^*$ belongs to a closed interval

$$ {Q_0}^* \in \left[\frac{(1-\beta)(1-\alpha)}{\beta}F^{-1}\left(\frac{p+c_s-c_p+\beta\left(p+c_m-\widetilde{c_p}\right)}{p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)}\right),\right. \left.\alpha F^{-1}\left(\frac{p+c_s-c_p+\beta\left(p+c_m-\widetilde{c_p}\right)}{p+c_s+c_d+\beta\left(p+c_m+\widetilde{c_p}\right)}\right)\right]. \quad (33) $$

5. Numerical Test

The purpose of the numerical illustration is twofold. The primary objective is to numerically analyze the impact of α and β on the optimal order quantity ${Q_0}^*$ and the expected profit $E\pi(Q_0)$. The second purpose is to determine the management implications of the models. Even though it is

Table 1

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.1	0.1	72.354	9.508	85.576	0.294	516.528
	0.2	64.302	9.442	75.532	0.270	448.543
	0.3	56.238	9.367	65.569	0.245	380.628
	0.4	48.173	9.283	55.697	0.218	312.724
	0.5	40.118	9.186	45.932	0.188	244.755
	0.6	32.092	9.073	36.292	0.156	176.621
	0.7	24.118	8.935	26.805	0.122	108.174
	0.8	16.045	8.758	17.516	0.085	40.316
	0.9	8.506	8.506	8.506	0.140	−80.561

Table 2

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.2	0.1	72.187	19.017	85.576	0.294	517.516
	0.2	64.136	18.883	75.532	0.270	449.523
	0.3	56.147	18.734	65.569	0.245	381.169
	0.4	48.227	18.566	55.697	0.218	312.401
	0.5	40.172	18.373	45.932	0.188	244.436
	0.6	32.110	18.146	36.292	0.156	176.516
	0.7	24.188	17.870	26.805	0.122	103.911
	0.8	17.516	17.516	17.516	0.268	−61.653
	0.9	7.630	7.561	17.012	0.140	−82.703

Table 3

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.3	0.1	72.317	28.525	85.576	0.294	516.748
	0.2	64.191	28.325	75.532	0.270	449.198
	0.3	56.275	28.101	65.569	0.245	380.411
	0.4	48.191	27.849	55.697	0.218	312.617
	0.5	40.118	27.559	45.932	0.188	244.607
	0.6	33.368	27.219	36.292	0.156	135.444
	0.7	26.805	26.805	26.805	0.385	−43.170
	0.8	15.658	15.327	26.274	0.268	−66.316
	0.9	6.676	6.616	25.518	0.140	−84.865

Table 4

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.4	0.1	72.391	38.034	85.576	0.294	516.306
	0.2	64.173	37.766	75.532	0.270	449.306
	0.3	56.238	37.468	65.569	0.245	380.614
	0.4	48.662	37.131	55.697	0.218	303.337
	0.5	42.935	36.745	45.932	0.188	155.762
	0.6	36.292	36.292	36.292	0.494	−25.047
	0.7	23.836	22.976	35.741	0.385	−50.831
	0.8	13.421	13.137	35.033	0.268	−71.128
	0.9	5.723	5.671	34.024	0.140	−87.027

Table 5

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.5	0.1	72.354	47.542	85.576	0.294	516.525
	0.2	64.357	47.208	75.532	0.270	446.975
	0.3	57.921	46.835	65.569	0.245	343.753
	0.4	52.597	46.414	55.697	0.218	174.256
	0.5	45.932	45.932	45.932	0.594	−7.235
	0.6	32.030	30.243	45.365	0.494	−36.370
	0.7	19.863	19.147	44.676	0.385	−59.026
	0.8	11.180	10.948	43.791	0.268	−75.940
	0.9	4.767	4.726	42.530	0.140	−89.190

Table 6

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
0.6	0.1	73.208	57.051	85.576	0.294	502.487
	0.2	67.676	56.649	75.532	0.270	374.505
	0.3	62.330	56.202	65.569	0.245	192.663
	0.4	55.697	55.697	55.697	0.687	10.301
	0.5	40.101	36.745	55.118	0.594	−23.272
	0.6	25.634	24.195	54.438	0.494	−49.095
	0.7	15.888	15.317	53.611	0.385	−67.221
	0.8	8.945	8.758	52.549	0.268	−80.752
	0.9	3.818	3.780	51.037	0.140	−91.352

impossible to derive an optimal solution to both the model without returns and the model with returns, fortunately, it is possible to do so through a numerical illustration. Without losing generality, we assume that the demand follows a normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 10$. In all numerical tests in this section, the selling price, the penalty cost, the first order purchase cost, the second order purchase cost, the disposal cost, and the return management cost per unit are $p = 4$, $c_s = 1$, $c_p = 3$, $\widetilde{c_p} = 3.1$, $c_d = 1$, and $c_m = 1$, respectively. MATLAB is used to obtain the closed interval of $Q_0{}^*$ and the value of $Q_0{}^*$ with a margin of error $\delta = 0.01$. We consider different combinations of α and β, under which the optimal replenishment quantity and the expect profit change correspondingly. Tables 1, 2, 3, 4, 5, 6, 7, 8, and 9 present the optimal procurement quantity and the expected profit when α and β change from 0.1 to 0.9, respectively.

Observations from Tables 1–8 are as follows.

(a) Both $Q_0{}^*$ and $E\pi(Q_0)$ decrease as β increases.

(b) $E\pi(Q_0)$ is small or negative when $\alpha + \beta \geq 1$, but it is positive when $\alpha + \beta < 1$. Namely, when $\alpha + \beta < 1$, the expected profit $E\pi(Q_0)$ is always greater than the expected profit under the circumstance of $\alpha + \beta \geq 1$.

6. Conclusion and Extensions

In this paper, we present a model for a single period problem with resalable returns and an additional order. Moreover, two kinds of demand, demand for instant delivery and demand for delayed delivery, are considered. The model is resolved with the purpose of maximizing the expected profit. The closed interval of the optimal order quantity is derived, which shows many interesting characteristics. The optimal order

Table 7

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
	0.1	77.516	66.559	85.576	0.294	400.373
	0.2	72.194	66.091	75.532	0.270	211.836
	0.3	65.569	65.569	65.569	0.773	27.590
	0.4	48.096	41.773	64.980	0.687	−12.321
0.7	0.5	30.089	27.559	64.304	0.594	−42.452
	0.6	19.226	18.146	63.511	0.494	−61.822
	0.7	11.918	11.488	62.546	0.385	−75.416
	0.8	6.712	6.569	61.307	0.268	−85.564
	0.9	2.861	2.835	59.543	0.140	−93.514

Table 8

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
	0.1	82.104	76.068	85.576	0.294	230.786
	0.2	75.532	75.532	75.532	0.854	44.654
	0.3	66.703	43.712	74.936	0.773	−3.333
	0.4	32.087	27.849	74.263	0.687	−41.543
0.8	0.5	20.061	18.373	73.491	0.594	−61.635
	0.6	12.817	12.097	72.584	0.494	−74.548
	0.7	7.951	7.659	71.481	0.385	−83.610
	0.8	4.473	4.379	70.065	0.268	−90.376
	0.9	1.908	1.890	68.049	0.140	−95.676

Table 9

α	β	$Q_0{}^*$	Q_a	Q_b	r	$E\pi(Q_0)$
	0.1	85.576	85.576	85.576	0.929	61.513
	0.2	81.309	37.766	84.974	0.854	9.206
	0.3	74.972	21.856	84.303	0.773	−43.749
	0.4	16.040	13.924	83.546	0.687	−70.771
0.9	0.5	10.034	9.186	82.677	0.594	−80.817
	0.6	6.411	6.049	81.657	0.494	−87.274
	0.7	3.972	3.829	80.416	0.385	−91.805
	0.8	2.237	2.190	78.823	0.268	−95.188
	0.9	0.954	0.945	76.555	0.140	−97.838

quantity is affected by the values of α and β mostly, which in turn affect the expected profit of the B2C retailer.

In order to analyze the impact of the proportion of the immediate delivery needs and the return rate on the optimal order quantity and the expected profit, an approximate method is also used to estimate the changes of the optimal procurement quantity and the corresponding expected profit. Even though it is impossible to derive an optimal solution to both the model without returns and the model with returns, fortunately, it is possible to do so through a numerical illustration.

Through the numerical test, we find that when $\alpha + \beta < 1$, the expected profit $E\pi(Q_0)$ is always greater than the expected profit under the circumstance of $\alpha + \beta \geq 1$. In this sense, it is better to keep the sum of α and β under 1 so as to profit. If the sum of α and β is equal to or more than 1, the retailer can consider changing the return policy in order to reduce the cost. For instance, the consumers only get partial refund once they return the goods due to their personal reasons. Additionally, the retailer can charge consumers some management fees for the returned products. Furthermore, it is better to keep the return rate under a specific level since the expected profit $E\pi(Q_0)$ increases as the return rate β decreases. Provided that the return rate of some products continues to be high, there must be some problem with the product itself. Thus, the retailer can think about pulling them off the shelves. Another interesting finding is that the optimal order quantity $Q_0{}^*$ is more affected by the proportion of returns β than the ratio of the immediate delivery needs α. Therefore, accurately estimating the return rate is important when deciding the first order quantity.

According to the existing research, the expected demand as well as its distribution function undoubtedly influences the order quantity and inventory management [19–21]. On the basis of the newsvendor model, however, instead of the impact of the demand estimation, we mainly focus on the impact of the proportion of the instant delivery needs and the return rate on the order quantity and the expected profit. Following the hypothesis of the newsvendor model, we assume that the demand x for a single product is stochastic, which follows a distribution function $F(x)$ and a probability density function $f(x)$. In the future, we will take the demand forecasting into consideration by using data from the beginning of the sales period.

Further research extensions include freeing the second order quantity as a decision variable. When taking the timing of returns into consideration, we can extend the single period model to a multiperiod problem. For example, returned products arriving after the end of the first selling season can be resold at the next sales period.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Impact of Heterogeneous Consumers on Pricing Decisions under Dual-Channel Competition

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This paper studies impact of heterogeneous consumer behavior on optimal pricing decisions under dual channel supply chain competition, which consists of one manufacturer and one retailer. The manufacturer is market leader with two sales channels: one is direct channel facing consumers directly and the other is indirect channel facing the retailer. Consumers decide whether to buy and from which channel to buy products. Purchasing decisions are based on considerations of prices posted on different channels, preference or loyalty to specific channels, and degree of rationality in decision-making process. Due to the complexity of heterogeneous consumer decision behavior, traditional mathematical analysis to the pricing problem becomes quite challenging. An agent-based modeling and simulation approach is then proposed and implemented. Simulation results reveal that consumer behavior influences both prices and profits. When consumers are increasingly loyal to the retailing channel, the retailer can make a higher selling price and more benefits. On the other hand, when consumers are increasingly loyal to the direct channel, the number of purchases from the direct channel increases and the manufacturer is better off. It is also interesting to note that as rationality level increases, selling prices for both channels slightly decrease.

1. Introduction

With rapid growth of e-commerce manufacturers have seen online channel as an addition to the existing retailers with brick-and-stone stores, which on one side can reduce their operating costs and, on the other side, can potentially increase market coverage and further enhance interactions with ending consumers. Retailers, however, being aware of the competition of the direct sales channel(s), start to complain about those "stolen sales" to the manufacturers. Such scenario is often called "channel conflict." From consumers' perspective, they are more than happy to welcome such dual channel competition, so as to expect lower prices and better service flexibility.

Under dual channel competition, consumers now make their purchasing decisions in a sophisticated manner. First they need to decide whether to buy and then decide from which channel to buy products. Quite a few factors influence their decisions: prices posted in different channels, preference or loyalty to specific channels, degree of rationality in decision-making process, and so forth. Our research interest focuses on the following. What are the impacts of those factors on consumer choice? How the manufacturer and the retailer make their selling prices under dual channel competition? Will the retailer and the manufacturer always be better off if considering consumer behavior?

In specific, we consider impacts of consumer loyalty and bounded rationality on consumers' decision. Brand loyalty means that people will purchase company A's products or services even if company B's products or services are cheaper and/or of a higher quality, if they have brand loyalty to company A [1]. Similarly, we extend brand loyalty definition to channel loyalty, based on observations that aged people prefer to purchasing from brick-and-stone stores instead of e-platforms while young people do on the contrary. Partial rationality or irrationality in the other part of their actions is called bounded rationality [2]. People with bounded rationality fail to find an optimal choice maybe because they lack the ability and resources to arrive at the optimal solution.

This paper considers a dual channel supply chain consisting of one manufacturer and one retailer. Manufacturer is the market leader and makes his pricing decisions for both direct-sales channel and indirect channel. Retailer is the follower and then decides her retailing price accordingly. Existing work has revealed that to deal with channel conflict, manufacturer needs to carefully make pricing decisions for different channels. However, with channel loyalty and bounded rationality behavior considered, a simple numerical example indicates that the concavity (unimodality) of manufacturer's profit is lost (see Figure 3), which becomes a challenge to traditional mathematical modeling approach.

Facing the intrinsic complexity of the optimization problem, this paper proposes an agent-based simulation approach to handle it. Agents are modeled to simulate heterogeneous consumers' decision-making process as well as decision-making of the manufacturer and the retailer. Individual consumer's behavior is described as an intelligent agent's interaction and reaction to its environment. Two reasons support our selection. One is that multiagent model provides a natural description of the system. In the model, the manufacturer is represented as an agent who set the wholesale price and online price; the retailer agent publishes its retail price; and each potential consumer agent individually assesses the market situation and makes purchase decision. The other reason comes from that agent behavior is heterogeneous and nonlinear, which matches the description of consumer who differs in valuation and preference of channel selection.

Simulation results reveal that consumer behavior influences both prices and profits. With more consumers loyal to retail channel, the retailer sets a higher price and gains a higher profit. On the other side, when more consumers are loyal to online channel, the number of purchases from online channel increases and the manufacturer benefits from it. It is also interesting to note as rationality level increases, selling prices for both channels slightly decrease.

The remainder of this paper is organized as follows. Section 2 discusses related work. Section 3 describes model of the dual-channel supply chain system. Section 4 proposes an agent-based simulation and optimization algorithm. Section 5 conducts simulation and summarizes finding. Section 6 concludes the whole paper.

2. Related Work

Two streams of work closely relate to our research problem: one is study on dual channel supply chain system and the other is consumer behavior which was studied intensively in marketing field and extended to operations management area recently.

There is growing literature in supply chain management investigating channel conflicts. To name a few, Yan [3] analyzes differentiated branding strategy and profit sharing mechanism when manufacturer adds a direct channel to its existing retailing channel. Hua et al. [4] analyze impacts of delivery lead time and consumer acceptance of a direct channel on the manufacturer's and the retailer's pricing strategies. Chen et al. [5] discuss dual channel management with service competition.

Consumer behavior has been studied intensively in marketing literature and behavioral science, such as consumer loyalty [1], loss aversion [6], long-tail phenomenon [7], and decoy effect [8]. Recently researchers in operations management have started to include the behavior of decision making into operational models and study its impact on the operational and supply chain decisions. Su [9] uses discrete choice model to describe bounded rationality of decision-maker in newsvendor models. The results are verified with experimental observations which are not able to explain with perfect rationality. Wang and Webster [10] use loss aversion to model decision-maker's behavior in a single-period newsvendor problem. Their results show that if shortage cost is not negligible, then a loss-averse newsvendor may order more than a risk-neutral newsvendor. In a similar newsvendor setting, Li [11] analyzes the impacts of reference point in loss-aversion's value function. Assuming that private valuations of consumers are random distribution with heavy tails, Ibragimov and Walden [7] analyze the optimal pricing strategies for the monopolist producer.

Papers mentioned above mostly adopt analytical modeling approach to mathematically find the optimal pricing decisions. However, when complexity of the problem studied is increasing it becomes quite challenging to achieve analytically results. Researchers turn to multiagent simulation to handle supply chain problems. An agent-based approach can effectively model and simulate complicated interactions among players in supply chain systems, for example, decisions making, information sharing, competition and cooperation, and so forth.

Long and Zhang [12] indicate the advantages of agent-based approach in handling system dynamics, uncertainty, and partial information sharing. They propose an integrated framework for agent-based inventory-production-transportation modeling and distributed simulation of supply chains. Hua et al. [13] investigate how bankruptcy occurs and propagates in supply chain networks. They use agent-based approach to model complicating decision-making process such as horizontal competition among retailers, order allocation strategies of retailers, and wholesale price of manufacturers. Liang and Huang [14] model different inventory systems in a supply chain, where agents are coordinated to control inventory and minimize the total cost of supply chain by sharing information and forecasting knowledge. Other work includes T. Zhang and D. Zhang [8], which exhibit the emergent decoy effect phenomenon in the market. Li and Huang [15] study a dual-channel supply chain pricing problem, in which the value the consumer derives from of the product is assumed to be triangular distributed according to different type of product.

However, studies addressing influence of consumer behavior on a dual-channel supply chain are quite few. This is because competition among channels in supply chain system already makes the problem difficult to handle with mathematical modeling approach; it is even challenging

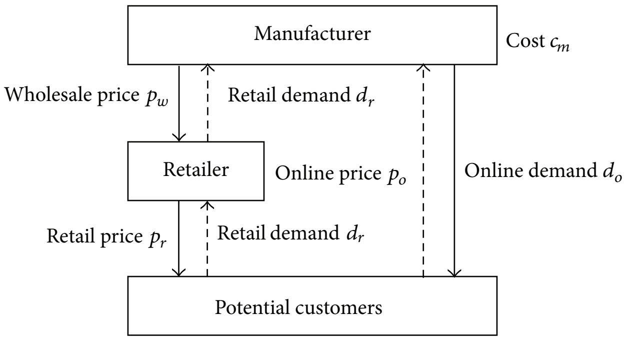


Figure 1: The dual-channel supply chain system.

with the consideration of nonlinear function of consumer behavior. This paper contributes to the literature in two aspects. First, we consider heterogeneous consumer behavior in a dual-channel competition supply chain system. Second, we propose an agent-based modeling approach to solve the current problem and obtain inspiring insights.

3. The Dual-Channel Supply Chain Model

We consider a single-period, single product supply chain system under dual channel competition. The supply chain system consists of one manufacturer and one retailer. The manufacturer has a traditional sales channel, that is, an indirect channel facing the brick-and-stone retailer, as well as a new direct channel facing the ending consumer, that is, online channel. This is a very typical dual-channel supply chain setting and has been studied widely in the literature (e.g., [5, 15]).

Figure 1 illustrates the model setting. The manufacturer sells the product to the retailer by the indirect channel at wholesale price p_w and to the ending consumers by direct online channel at price p_o. Unit cost for producing the products is c_m. The retailer also faces the ending consumer and decides retailing price p_r. To avoid the case in which the retailer buy products from the direct channel instead from the manufacturer, we assume $p_w \leqslant p_o$.

Consumer decides whether to buy products or not. If he decides to buy he also needs to choose from which channel to buy: the online channel or the retailer. Denote by d_r the aggregate demand at the retailer channel and d_o the aggregate demand at the online channel. Both the manufacturer and the retailer make their pricing decisions to maximize their own profits.

Consider a Stackelberg game where the manufacturer is the leader and the retailer is the follower. The sequence of events is as follows. Before the selling season, the manufacturer sets the prices of p_w and p_o. The retailer then decides p_r based on the given wholesale price p_w. During the selling season, consumer decides whether or not to buy the product and from which channel to buy it. At the end of the selling season, profits of both the manufacturer and the retailer are calculated. Implicitly we assume there is no capacity constraint of the manufacturer; that is, all orders will be satisfied.

Given the wholesale price p_w offered by the manufacturer, the retailer determines the retailing price p_r so as to maximize her profit, π_r:

$$\pi_r = d_r \cdot (p_r - p_w). \quad (1)$$

Projecting the retailer's best response, the manufacturer needs to carefully determine the pricing decisions so as to maximize his own profit, π_m, which constitutes two parts: net revenue from demand d_r and net revenue from demand d_m:

$$\pi_m = d_r \cdot (p_w - c_m) + d_o \cdot (p_o - c_m). \quad (2)$$

3.1. Consumer Behavior. Facing the multiple sales channels, each consumer needs to decide whether to buy the product and from which channel to buy. This paper considers heterogeneous consumer in the following sense.

First, each consumer has his own valuation of the product, denoting by v_i, with i the index of consumer i, ($i = 1, 2, \ldots, N$). Here N is the market size. We assume that v_i is uniformly distributed from 0 to 1. Normally to say, consumer has willingness to buy the product only if the utility is nonnegative; that is, $u_i \geq 0$. Further denote by $u_{i,r}$ the utility of consumer i if purchasing from retailer channel and $u_{i,o}$ the utility of consumer i if purchasing from online channel.

Second, different consumers have different preferences to the sales channels, to which we refer channel loyalty. Assume that α_r $(0 < \alpha_r < 1)$ proportion of consumers is loyal to the retailer channel, which means that those consumers only purchase from the retailer though the price posted on the direct channel is even less. That is, purchasing occurs when utility $u_{i,r}$ is larger than or equal to 0; that is, $u_i = u_{i,r} = v_i - p_r \geq 0$, while $u_{i,o} = 0$. Similarly α_o $(0 < \alpha_o < 1)$ proportion of consumers is loyal to direct channel, who only purchase from direct channel disregarding the price on the indirect channel; that is, $u_i = u_{i,o} = v_i - p_o \geq 0$, and $u_{i,r} = 0$. The rest $1 - \alpha_r - \alpha_o$ proportion of consumers is so-called "switchers," who are flexible to purchase from either channel.

Among those switchers, consumers might also have different valuations to different channels. Usually, we have $u_{i,r} = v_i - p_r$. However, empirical studies indicate that consumers perceive less utility for products purchased from online channel, often at a discounted value [15]; that is, $u_{i,o} = \theta v_i - p_o$, where $0 < \theta < 1$. The discounted ratio θ is different for varied products. For example, books ($\theta = 0.904$) are more acceptable as compared to food items ($\theta = 0.784$) [15].

Given $u_{i,r}$ and $u_{i,o}$, switcher i then needs to decide which channel to purchase or not. General to say, consumer will select the channel whichever provides the larger utility. Such decision is based on so-called perfect rationality. However, as indicated by [16] consumers make choice decisions with bounded rationality because of lack of information, hassle cost, access to resources, and so forth. Following [16] we assume β $(0 < \beta < 1)$ proportion of consumers is rational, while the rest $(1 - \beta)$ proportion of consumers is bounded rational.

We employ multinomial logit (MNL) choice model to describe channel selection choice under bounded rationality, similar as in [9]. By bounded rationality, we only consider

TABLE 1: Classification of consumer types.

Consumer type	Loyalty			Rationality		Fraction to the total population
	Indirect channel (α_r)	Direct channel (α_o)	Acceptable to both channels ($1-\alpha_r-\alpha_o$)	Perfect rationality (β)	Bounded rationality ($1-\beta$)	
Type-1	Y*	—*	—	—	—	α_r
Type-2	—	Y	—	—	—	α_o
Type-3	—	—	Y	Y	—	$(1-\alpha_r-\alpha_o)\cdot\beta$
Type-4	—	—	Y	—	Y	$(1-\alpha_r-\alpha_o)\cdot(1-\beta)$

*"Y" denotes "Yes"; "—"denotes "not applicable."

consumers whose utility is positive. That is, if consumers' utility is negative or zero, they are rational enough not to purchase from either channel. Switcher i then chooses channel j with probability φ_{ij}, $j=\{r,o\}$, with

$$\varphi_{ij}=\frac{\exp\left(u_{i,j}\right)}{\exp\left(u_{i,r}\right)+\exp\left(u_{i,o}\right)}. \tag{3}$$

In addition,

$$\varphi_{io}=\frac{\exp\left(\theta v_i-p_o\right)}{\left[\exp\left(\theta v_i-p_o\right)+\exp\left(v_i-p_r\right)\right]}. \tag{4}$$

For each switcher i, $\varphi_{io}+\varphi_{ir}=1$. Based on the description of channel loyalty, channel valuation differences, and levels of rationality, consumers can then be classified into six types, as summarized in Table 1. Figure 2 depicts different decision-making process for these six types of consumers.

3.2. Demand for Different Channels. From Table 1 and Figure 2, we are now able to summarize probability of demand at the retailer side, $\text{Prob}(d_r)$:

$$\begin{aligned}
&\text{Prob}\left(d_r\right)\\
&=\alpha_r\cdot\text{Prob}\left(v_i\geq p_r\right)+\left(1-\alpha_r-\alpha_o\right)\\
&\quad\cdot\{\beta\cdot[\text{Prob}\left(v_i-p_r\geq 0,\ \theta v_i\geq p_o,\ v_i-p_r>\theta v_i-p_o\right)\\
&\quad\quad+\text{Prob}\left(v_i-p_r\geq 0,\ \theta v_i<p_o\right)]\\
&\quad+\left(1-\beta\right)\cdot[\varphi_{ir}\cdot\text{Prob}\left(v_i\geq p_r,\ \theta v_i\geq p_o\right)\\
&\quad\quad+\text{Prob}\left(v_i\geq p_r,\ \theta v_i<p_o\right)]\}.
\end{aligned} \tag{5}$$

The first item comes from Type-1 consumers, who are loyal to the retailer channel and purchase from the retailer channel once their valuation is larger than posted price. There are two parts in the second item, with the first one from Type-3 consumers and the second one from Type-4 consumers.

Similarly, probability of demand at the online channel, $\text{Prob}(d_o)$, can be calculated by

$$\begin{aligned}
&\text{Prob}\left(d_o\right)\\
&=\alpha_o\cdot\text{Prob}\left(v_i\geq p_o\right)+\left(1-\alpha_r-\alpha_o\right)\\
&\quad\cdot\{\beta\,[\text{Prob}\left(v_i-p_r\geq 0,\ \theta v_i\geq p_o,\ \theta v_i-p_o\geq v_i-p_r\right)\\
&\quad\quad+\text{Prob}\left(v_i<p_r,\ \theta v_i\geq p_o\right)]\\
&\quad+\left(1-\beta\right)[\varphi_{io}\cdot\text{Prob}\left(v_i\geq p_r,\ \theta v_i\geq p_o\right)\\
&\quad\quad+\text{Prob}\left(v_i<p_r,\ \theta v_i\geq p_o\right)]\}.
\end{aligned} \tag{6}$$

Probability of no purchase is then

$$\begin{aligned}
&\text{Prob}\left(\text{no purchase}\right)\\
&=\alpha_r\cdot\text{Prob}\left(v_i<p_r\right)+\alpha_o\cdot\text{Prob}\left(v_i<p_o\right)\\
&\quad+\left(1-\alpha_r-\alpha_o\right)\cdot\text{Prob}\left(v_i<p_r,\ \theta v_i<p_o\right).
\end{aligned} \tag{7}$$

3.3. Objective Functions. The retailer's objective is then to find the optimal retailing price p_r to maximize the expected net revenue where $E[\pi_r]=N\cdot\text{Prob}(d_r)\cdot(p_r-p_w)$. Similarly, the manufacturer then needs to decide the optimal p_w and p_o to maximize the expected net revenue, where $E[\pi_m]=N\cdot\text{Prob}(d_r)\cdot(p_w-c_m)+N\cdot\text{Prob}(d_o)\cdot(p_o-c_m)$. To clarify, this paper uses net revenue and profit interchangeably hereafter.

Notice that even for a uniformly distributed valuation v_i, the probability of demands at each sales channel is quite complex due to the heterogeneity of consumer's behavior. This makes the maximization of net revenues of both the retailer and the manufacturer quite challenging in terms of mathematical approach. A simple example illustrates that the net revenues of both manufacturer and retailer are not concave or unimodal in their pricing decisions. This motivates us to consider alternative optimization approach to handle the current problem.

Example 1. Let $p_w=0.2$, $\alpha_r=0.3$, $\alpha_o=0.3$, $\beta=0.6$, $\theta=0.9$. Figure 3(a) plots the net revenue of the retailer with p_r the horizontal axis (given $p_o=0.4$). Figure 3(b) plots the net revenue of the manufacturer with p_o the horizontal axis (given $p_w=0.4$).

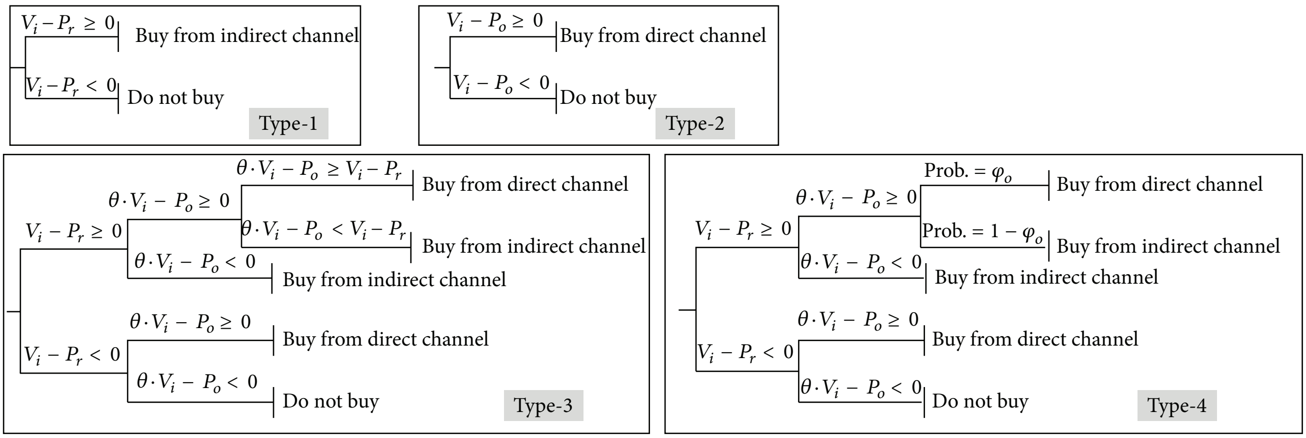


FIGURE 2: Decision trees for different types of consumers.

4. An Agent-Based Model and Simulation Algorithm

4.1. An Agent-Based Model. Agent-based modeling has been proved to be an effective pragmatic approach to analyze supply chain design and management problems [6]. In the dual-channel supply chain system we establish three types of agents:

(i) manufacturer agent: the manufacturer is modeled as an autonomous agent who sets his wholesale price p_w and online price p_o and calculates his profit based on consumer purchasing behavior;

(ii) retailer agent: the retailer agent is modeled as an autonomous agent who decides her retail price p_r to maximize her revenue;

(iii) consumer agents: consumers are modeled as a collection of N consumer agents. Each consumer agent i, $i \in [1, \ldots, N]$, differs in channel loyalty, channel valuations as well as levels of rationality. Agent i makes his decision of purchase and channel of purchase based on the net utility of $u_{i,r}$ and $u_{i,o}$.

4.2. Simulation Algorithm. Simulation steps for the Stackelberg game are as follows.

(1) The manufacturer agent first announces p_w and p_o, and the retailer agent announces p_r.

(2) Based on the prices posted, channel loyalty, utility preference for different channels, and rationality levels, each consumer agent makes his own purchasing decision.

(3) The manufacturer agent and the retailer agent collect profits from N consumer agents, separately.

Pricing decisions of the manufacturer (retailer) are made based on the best response to the action of the retailer (manufacturer) to maximize his own expected profit, that is, (2) (or (1)). In game theory, the best response is the strategy which produces the most favorable outcome for a player. Such a process is so-called best response dynamics [17] and has been widely used in solving gaming problems in computer-based modeling setting [18]. Pseudocode of the algorithm is developed and proposed in Algorithm 1. We enumerate pricing decisions in their feasible value sets; that is, $p_w \in [0, 1.0)$, $p_o \in [p_w, 1.0)$, $p_r \in [p_w, 1.0)$. Parameter $\varepsilon = 0.005$ is the computing step which determines the precise of the results.

5. Simulation Results and Analysis

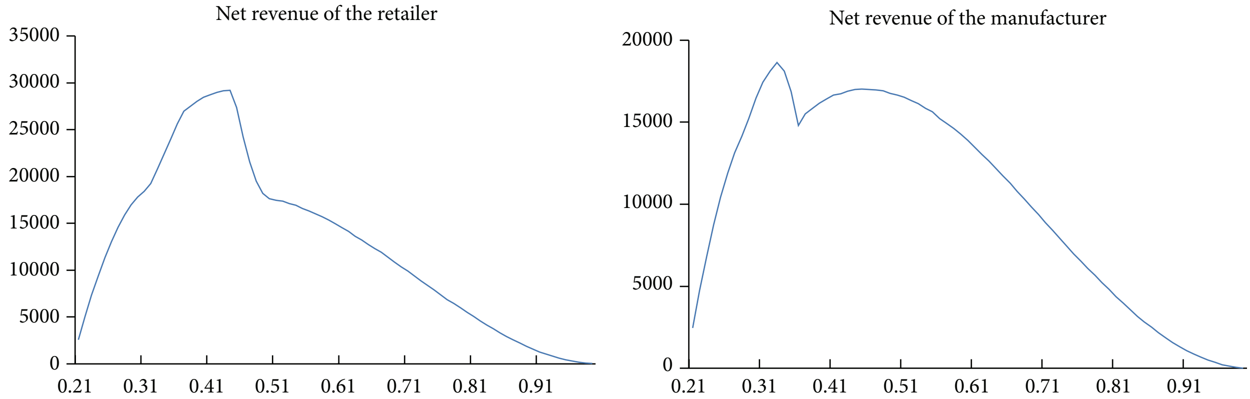
5.1. Verification and Validation. Considering agent-based modeling is "bottom-up" instead of "top-down" which traditional mathematical modeling approach often employs, we need first to verify and validate the established agent model.

To do so, we establish a base scenario. If the simulation results of the base scenario match theoretical results, we then validate the agent model.

Base Scenario. Set market size $N = 10{,}000$ and product cost $c_m = 0.20$. Suppose that all consumers are perfectly rational and they are acceptable to both sales channels. In addition, purchasing from online channels makes no difference from retail channel. That is, $\alpha_r = \alpha_o = 0$, $\beta = 1$, $\theta = 1$. Based on these assumptions theoretical results of optimal prices can be easily achieved.

Figure 4 depicts the simulation results of the optimal net revenue of both the manufacturer and the retailer with respect to the wholesale price p_w. We observe that when $p_w = 0.4$ the manufacturer's net revenue achieves maximum, with $\pi_m = 0.19764$ and $\pi_r = 0.04941$. This is consistent with theoretical results in [15], where $p_o^* = p_w^* = 0.40$, $\pi_m^* = 0.20$, and $\pi_r^* = 0.05$.

Repeat the simulation for 100 times, and we can depict in Figure 5 the Quantile-quantile plot of π_m. It shows that the simulation results of 100 replications ($M = 100$) are normally distributed, which indicates that setting M to 100 is fairly enough.



(a) (b)

Figure 3: (a) Nonconcavity of retailer's net revenue. (b) Nonconcavity of manufacturer's net revenue.

```
FUNCTION StackelbergGame ()
ε = 0.005;
finalPw = finalPr = finalPo = 0.0;
finalπm = finalπr = 0.0;
FOR (P_w = 0.0; P_w < 1.0; P_w +=ε) {
    bestPo = bestPr = P_w;
    bestπm = bestπr = 0.0;
    FOR (P_o = P_w; P_o < 1.0; P_o +=ε) {
        maxπr = maxπm = maxPr = 0.0;
        FOR (P_r = P_w; P_r < 1.0; P_r +=ε) {
            (πm, πr) = simulation (P_w, P_r, P_o);
            IF (maxπr < πr) {
                maxπr = π_r, maxπm = π_m;
                maxPo = P_o;
            }
        }
        IF (bestπm < maxπm) {
            bestπm = maxπm, bestπr = maxπr;
            bestPr = maxPr, bestPo = P_o;
        }
    }
    IF (finalπm < bestπm) {
        finalπm = bestπm, finalπr = bestπr;
        finalPw = P_w, finalPr = bestPr, finalPo = bestPo;
    }
}
RETURN (finalPw, finalPr, finalPo, finalπm, finalπr);
```

Algorithm 1: Pseudocode for the dual-channel Stackelberg game.

5.2. Simulation Results and Analysis. We are now ready to run agent-based simulation to investigate the optimal pricing decisions. We take particular interests in

(1) impact of channel loyalty on demands, pricing decisions, and net revenues;

(2) impact of rationality (bounded rationality) on demands, pricing decisions, and net revenues.

Set market size $N = 500{,}000$, product cost $c_m = 0.20$. We first establish a benchmark case (Scenario A) by setting $\alpha_r = 0.30$, $\alpha_o = 0.30$, $\beta = 0.60$, $\theta = 0.90$. Table 2 shows the optimal solutions. The optimal pricing decisions for the manufacturer is $p_w^* = 0.60$, and $p_o^* \approx p_w^*$. The optimal retailing price is higher. There are about 43.2% consumers that will buy from the direct online channel.

Next, we simulate three special scenarios and report the results in Table 2. Scenario B changes $\beta = 1.00$ while keeping the other parameters unchanged, which means all the consumers in the market are "perfectly rational"—Type-1, Type-2, and Type-3. Scenario C sets $\alpha_r = \alpha_o = 0$, which

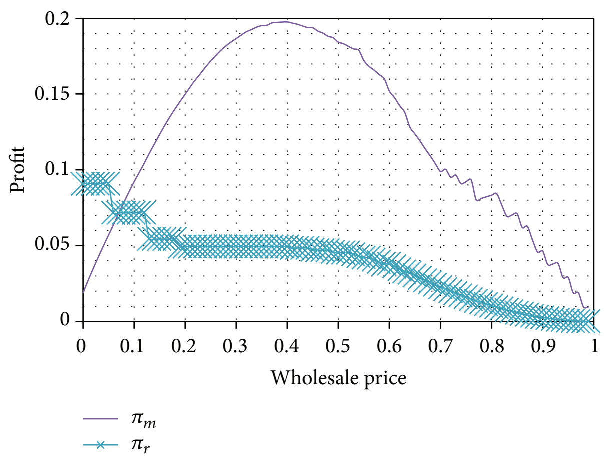


FIGURE 4: Simulation results of the base scenario.

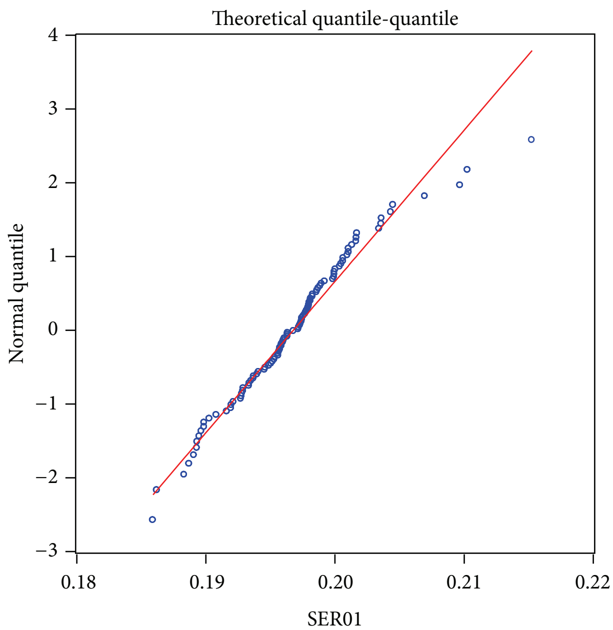


FIGURE 5: Quantile-quantile plot of π_m.

TABLE 2: Optimal solutions under four scenarios.

	p_w	p_o	p_r	d_r	d_o	π_m	π_r
Scenario A	0.60	0.61	0.68	97,461	74,159	69,389.59	7,796.88
Scenario B	0.55	0.55	0.61	136,053	67,692	71,310.75	8,163.18
Scenario C	0.57	0.57	0.63	148,635	36,615	68,542.50	8,918.10
Scenario D	0.57	0.57	0.63	185,197	0	68,522.89	11,111.82

implies all consumers are switchers and no channel loyalty—Type-3 and Type-4. Scenario D further eliminates the impact of bound rationality by setting $\alpha_r = 0.00$, $\alpha_o = 0.00$, $\beta = 1.00$, $\theta = 0.90$, which means only Type-4 consumers in the market.

Comparing results of Scenario B to Scenario A, we find as consumers become more rational, both retailing price and online price decrease while profits of both manufacturer and retailer increase. Comparing Scenario C to Scenario A, we find that when there is no channel loyalty, selling prices for both channels decrease while demand from the retailing channel increases much and the retailer enjoys a higher profit level. Comparing Scenario D to Scenario C, we find that when no behavior pattern exists, the retailer is happy because all consumers approach her for purchasing. Although online price is lower than retailing price, considering the discounted value of online product, consumers still prefer to retail channel. Such scenario is also observed in [13].

We change values of α_r, α_o, and β separately, so as to observe impacts of each parameter. Table 3 summarizes optimal solutions for varying parameters, including optimal pricing decisions, net revenues of the manufacturer, the retailer, and the total supply chain, as well as the number of purchases from each channel.

Based on Table 3, we summarize our observations as follows.

Impact of Channel Loyalty. We observe that the impact of loyalty to retail channel and online channel is quite different. As α_r increases, p_r^* and p_o^* exhibit an increasing trend; however, p_w^* does not show clear pattern. The number of purchases from online channel decreases, while the number of retail channels does not change much. The total number of purchases decreases as well. As a result, the profit of the retailer increases while the profit of the manufacturer decreases. The manufacturer should then adjust their marketing strategy when the ratio of consumers who are loyal to retail channel is large, for example, offering promotion or deeper discounts at online channels, to prevent their profits reduced.

As α_o increases, p_r^*, p_w^*, and p_o^* do not change much. The number of purchases from online channel increases, while purchase from retail channel decreases significantly. This explains the reason why sales at retail channel decrease significantly when consumers get used to online shopping. The manufacturer tends to get benefit from the increasing online sales while the retailer may be harmed. For retailers, they should then increase marketing promotion to drag consumers back to the traditional retailing channel when more consumers get used to online shopping.

Impact of Rationality. Notice that β denotes the degree of rationality. As β increases, p_r^*, p_w^*, and p_o^* turn to be slightly decreasing. As a result, the number of purchasing for both channels increases. When rationality level is low, which means consumers is possible to select the online channel instead of retail channel no matter what the selling price is. The retailer is then less motivated to lower the price to attract consumers. When rationality level is high, the retailer can use pricing decisions to attract consumers to buy from her channel. The same argument goes for the online channel pricing decisions. We further see that the manufacturer benefits from higher rationality level; however, the retailer does not turn to be obvious pattern of increasing or decreasing with respect to the rational level. This is because the retailer's profit comes from the price difference of p_r^* and p_w^* and the number of purchases from the retail channel, with one factor decreasing while the other factor increasing with

Table 3: Optimal solutions for varying parameters.

α_r	$p_w{}^*$	$p_r{}^*$	$p_o{}^*$	$\pi_S{}^*$	$\pi_r{}^*$	$\pi_m{}^*$	#_total[1]	#_Retailer[2]	#_Online[3]
0.2	0.57	0.64	0.57	77780.62	6921.18	70859.44	191512	98874	92638
0.25	0.6	0.67	0.6	77232.76	6749.96	70482.8	176207	96428	79779
0.3	0.6	0.68	0.61	77186.47	7796.88	69389.59	171620	97461	74159
0.35	0.62	0.7	0.63	75598.64	7568.88	68029.76	160409	94611	65798
0.4	0.59	0.69	0.62	76672.47	9823.5	66848.97	166181	98235	67946
0.45	0.58	0.69	0.62	76770.26	11061.82	65708.44	166026	100562	65464
0.5	0.58	0.7	0.63	76089.79	11914.44	64175.35	160790	99287	61503
α_o	$p_w{}^*$	$p_r{}^*$	$p_o{}^*$	$\pi_S{}^*$	$\pi_r{}^*$	$\pi_m{}^*$	#_total	#_Retailer	#_Online
0.2	0.61	0.68	0.61	76234.88	7638.19	68596.69	167309	109117	58192
0.25	0.58	0.66	0.59	77817.58	8568.08	69249.5	180309	107101	73208
0.3	0.6	0.68	0.61	77186.47	7796.88	69389.59	171620	97461	74159
0.35	0.61	0.69	0.62	76608.14	7113.28	69494.86	167581	88916	78665
0.4	0.59	0.68	0.61	77324.73	7606.62	69718.11	174167	84518	89649
0.45	0.58	0.68	0.61	77846.11	7824.3	70021.81	176510	78243	98267
0.5	0.6	0.7	0.63	76783.22	6889.6	69893.62	167350	68896	98454
β	$p_w{}^*$	$p_r{}^*$	$p_o{}^*$	$\pi_S{}^*$	$\pi_r{}^*$	$\pi_m{}^*$	#_total	#_Retailer	#_Online
0.6	0.6	0.68	0.61	77186.47	7796.88	69389.59	171620	97461	74159
0.65	0.58	0.66	0.59	77584.35	8141.52	69442.83	180668	101769	78899
0.7	0.61	0.68	0.61	77269.64	6997.69	70271.95	171395	99967	71428
0.75	0.59	0.66	0.59	77962.76	7328.3	70634.46	181114	104690	76424
0.8	0.58	0.65	0.58	78466.33	7556.43	70909.9	186605	107949	78656
0.85	0.57	0.64	0.57	78652.51	7745.71	70906.8	191640	110653	80987
0.9	0.57	0.64	0.57	78730.87	7798.91	70931.96	191708	111413	80295

[1]"#_total" means total number of purchases.
[2]"#_Retailer" means number of purchases from retailer channel.
[3]"#_Online" means number of purchases from online channel.

rationality level. However, the manufacturer can benefit by adding an online channel, considering the total number of purchases is increased much more than the slightly decrease of online selling price and wholesale price.

As indicated in above discussion, in a Stackelberg game, the manufacturer as a dominant takes advantage by introducing online channel to increase his profit. On the contrary, the retailer could get benefit from cultivating consumer loyalty to the retail channel.

6. Conclusions

This paper addresses impact of consumer behavior on pricing decisions under dual-channel competition. Consumers are heterogeneous and each of them makes his own purchasing decision according to prices posted on both channels, channel loyalty, utility preference for different channels, and degree of rationality in decision-making process. Results of agent-based simulation show that consumer behavior influences both prices and profits. With more consumers loyal to retail channel, the retailer set a higher price and gains a higher profit. On the other side, when more consumer loyal to online channel, the number of purchases from online channel increases and the manufacturer benefits from it. It is also interesting to note as rationality level increases, selling prices for both channels slightly decrease.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Customized Transportation, Equity Participation, and Cooperation Performance within Logistics Supply Chains

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Customized transportation has received growing concerns by researchers and practitioners in recent years. Despite the fact that one consignor often holds partial ownership of its carrier within a supply chain, the existing interpretations behind them remain relatively unexplored. Based on the game models, we find that a simple take-or-pay contract is not likely to solve the low-efficient customized production problem, and equity participation mechanism plus simple contract may improve the cooperation performance of customized transportation. In the case of the owner-managed carrier, only when purchasing at par can it be ensured to obtain the socially optimal customization investment, but when purchasing at premium or discount, the optimal partial ownership selected by consignor cannot motivate the carrier to make the most efficient customization investment. With the optimal solutions, we also provide a theoretic foundation for calculating the optimal partial ownership and for interpreting why the interfirm share-holding ratios of the member-firms within the familial-type logistic supply chains are much larger than the ratios within the public-type logistic supply chains. Finally, our results show that the familial-type logistic supply chains may choose more efficient customized production level than public-type logistic supply chains.

1. Introduction

Customized production is a production strategy focused on the broad provision of personalized products and services, mostly through modularized product/service design, flexible processes, and integration between supply chain members. Recent years have witnessed growing concerns about customized transportation services within logistic supply chains. The customization relationships among two or more member-firms form a complex network, in which there exists quite thorough and formal cooperation pointed out by Choi [1]. Strategic cooperation usually occurs in the shape of subcontracts, or manufacturing and marketing interfaces, so that cooperative arrangement for technical sharing, technical transfer, and customization service has become one of the most important cooperation ways in the last two decades [2].

According to Richardson [3], since two parties must not only take promised responsibilities for their future cooperation behaviors, but also should be provided with equivalent guarantee, then long-term contracts or equity participation schemes become the quite formal cooperation ways. Many of the interfirm research and development (R&D) cooperation in the United States among small biotechnology firms and established pharmaceutical companies are cemented by equity participation [4].

In China, it is well documented that many manufacturers (consignor) hold minority equity ownership in their transportation service suppliers (carrier), and, thereafter, these transportation service providers make substantial investments, such as refrigerating equipment for retaining freshness, or integrated ERP systems used as tracking parcel that Choi et al. suggested in [5], and provide customized transportation service for their downstream consignor [5, 6], just as the similar cases of Japanese auto industry described by Aoki et al. [7, 8]. In fact, these features have been suggested as some keys to the success of Chinese logistic industry. For the above mentioned cooperation transactions, why is equity participation adopted rather than long-term contract? That is exactly the paradox in transaction efficiency from different cooperative arrangements that we want to investigate.

In reality, due to the complexity, and uncertainty of future affairs and traders' bound rationality, it is impossible or implausible to list all the related details of contract when taking transaction cost into consideration. Since incomplete contracts cannot resolve the "*hold-up*" problem of opportunists and discounts its value [9], under the backgrounds of opportunism [10], assets specificity [11], incomplete contracts, and future's uncertainty [12], various organizational interventions, such as vertical integration, exchange hostages, shifting property rights, and designing an authority relationship, are proposed to solve the "*hold-up*" problem, which can prevent two parties from breaking a contract.

Cooperative transaction among member-firms within logistic supply chain may lose efficiency because of *ex post* opportunistic behavior caused by specific investment resulting likely from customized production or outside options. Williamson [13], Klein et al. [14] believe that specific investment does not likely acquire the optimal efficiency with the anticipation of *ex post* opportunistic behaviors and vertical integration instead of spot market can avoid or reduce opportunistic behaviors caused by assets specificity. The final choice is made based on the comparison of transaction costs of two types of cooperative mechanisms.

However, integration (internal transaction) is not always more outperformed than market transaction, which means that it would lead to inefficiency when all productive activities are arranged in one integrated organization. Why cannot market relationship among independent entities be replaced fully by integration? Williamson's replacement model provides the following reasons for avoiding integration. That is, if firms manufacture inputting products by themselves, economies of scale and economies of scope may not be achieved. Otherwise, when the degree of assets specificity is low, governance cost of internal organizations will be larger than that of market organizations [13]. When analyzing the disadvantage of governance cost of internal organizations, related interpretations of loss of control rights and malfunction of selective optimal intervention are convincing.

Based on the criticism and inheritance of existent theories, Milgrom and Roberts [15] propose that the cost of political activities within one organization is also an important obstacle for concentration of control rights, which interprets that internal transaction within integration organizations is not always better than market transaction among entities in view of nonmarket transaction costs. The limitations of integration organizations indicate that a single mode of integration among parent firm and its member-firms is not optimal, and the concurrence of various bonding relationships within supply chains is a feasible choice of decision-makers when constructing an up-down stream relationship.

With growing cooperation transactions within supply chains, share-holding arrangement (partial ownership) becomes one of the most important cooperative schemes. However, there are few theoretical explanations aiming at equity participation phenomena, especially within logistic supply chain [2, 16]. Conventional wisdom is that partial ownership, by reducing opportunism, helps to promote the bond between upstream firms and downstream firms through inspiring greater specific investment and customized production level [9–12]. So, without contractual remedies, equity participation may perhaps play a role instead. However, under symmetric information, equity participation by the downstream firm in an upstream supplier has no effect on the payoffs when two parties bargain to allocate the benefits of specific investment [11]. In view of this, it remains unclear how partial ownership promotes the bond.

In the related models, Bolton and Whinston [17] and Rajan and Zingales [18] discuss how to make optimal ownership allocation in details. However, they only make some analyses about efficiency loss within integration organization, but do not measure enhanced efficiency resulting from cooperation mechanism of partial ownership. Different from their works, this paper focuses on demonstrating what roles partial ownership plays in customized production used to motivate investors, and simultaneously, probing into why the complete ownership is not possible to be optimal.

Dasgupta and Tao [19] provide some theoretical interpretations for partial ownership phenomena, based on the models about selective investment decision made by the upstream firms. They consider two types of investment, in which specific investment levels differ. Given potential income of investment to be random, when the downstream firm holds partial ownership of the upstream firm, the upstream firm would choose more efficient types of investment than the simple contract.

Compared with innovative mechanism design theory of ownership suggested by Aghion and Tirole [20], Dasgupta's theoretical interpretation on partial ownership is more profound, as they not only agree on the viewpoint that the relative bargaining power of the trade parties will affect the efficiency of ownership allocation but also install the bargaining power parameter into the models and obtain the relevance between the related parameters and the magnitude of partial ownership. When the downstream firm takes equity participation ratio into account, the optimal partial ownership is not only of the credible commitment that can boost the upstream firm to choose a high specific investment level, but also of the mechanism arrangement in order to maximize the benefits of the downstream firm. The equity participation ratio decision must tradeoff these two mechanisms, which means that the downstream firm maximizing its own benefits does not have to choose equity participation ratio that can motivate the upstream firm to make the complete specific investment.

Hence, viewing customization level as a variable chosen by the upstream firm may meet the realistic condition within logistic supply chains better. In this paper, the potential income of customization investment is assumed to increase with customized production level, which accords with rational principle and is our logic starting point of research.

In addition, the existence of partial ownership may affect customization decision and the success probability of obtaining outside opportunity income slightly. On the one hand, partial ownership could lower the diligence expenditure of the upstream firm's entrepreneurs. On the other hand, it has the supervisory effects on them imposed by the downstream firm which was neglected by Dasgupta and Tao [19], but is taken into account in our paper. Then, in fact, partial ownership won't change the diligence expenditure of decision-makers evidently. Once investment and customized production are made, both the investment income and the outside option income are supposed to be realizable. This is why we incorporate customization level, partial ownership, and outside opportunity into the same analytical framework, which become a key point that distinguishes our research from other customization literatures.

In addition to analyzing the case of the owner-managed upstream firms, our model also involves the case of public-firms managed by professional managers. Obviously, the decision-makers of these two kinds of firms have so many differences between their customization decision behaviors that we should probe into them separately. From this angle of view, we can demonstrate why the familial-type logistic supply chains may choose more efficient customized production level than public-type logistic supply chains.

The remaining portion of this paper is organized as follows. Section 2 outlines the basic model and analyzes the limitations of the simple contract. Section 3 probes into the problem of equity participation of the owner-managed carrier. Section 4 discusses the first-best partial ownership of the publicly traded carrier. Section 5 analyses the effects of the optimal ownership strategy on the efficiency of customized transportation. Finally, conclusion is made in Section 6.

2. The Customized Transportation Models

2.1. Model Settings. Assuming that the member-firms within the logistic supply chain are respectively one carrier and one consignor in a transportation trade, the two parties can make the specific investment for customized transportation bilaterally or unilaterally. Without loss of generality, we assume that the carrier can make the unilateral specific investment and customized transportation for consignor, which can increase the goods' value of the consignor. For example, if the carrier invests in refrigerating equipment or ant-vibration appliance for consignor's goods in the great risk of decoy or brittleness, then this customized transportation would increase the goods' value and consignor would make a larger profit.

The carrier can either be owner-managed firms or public-firms run by professional managers. Here, we call the owners of owner-managed firms as entrepreneur and call the decision-maker of the publicly traded firm as the manager. We assume that the consignor is run by professional managers in the interest of the stockholders. To be mentioned, the owner-managed consignor will not affect our analyses. Unless specially pointed out, carrier is seen as an owner-managed one in our analyses. In addition, all players are assumed to be risk-neutral.

For simplicity of the analyses and the generalization of models, three stages of game are considered; that is, $t = 0, 1, 2$. At $t = 0$, the consignor offers to buy a fraction $r \in [0, 1]$ of the transportation carrier at price $P(r)$, and the entrepreneur can accept or reject the offer.

At $t = 1$, the entrepreneur chooses the customization level parameter a based on the potential value $V(a)$ and the cost function $C(a)$ of customized transportation, where $a \in [0, 1]$. Larger a means the higher customized transportation level for carrier. In order to grasp the nature of customization investment behavior, we assume, the value of transportation service $V(a)$ and its investment in customized transportation, for example, $C(a)$, are convexly increasing in a. For simplifying analyses and without loss of generality, we adopt the following function shape: $V(a) = e^a g$ and $C(a) = \beta a^2$, where g represents the general transportation service value without customized investment (in the case of $a = 0$) and β is a constant expressing the needed investment in the case of complete customization service. In addition, we also suppose that the net revenue function of customized production $[V(A) - C(A)]$ is concave, which ensures that there exists the optimal a in our analyses. That is, $\forall a \in [0, 1]$ and $r \in [0, 1]$, and inequalities $e^a g > 2\beta a$ and $e^a g < 2\beta$ hold.

At $t = 2$, $V(a)$ and $C(a)$ are realized. $V(a)$ and $C(a)$ are common knowledge, but they cannot be verified, so the contract cannot be consigned on it.

Then, we consider that at $t = 0$, the carrier and the consignor sign a simple take-or-pay contract, and (P_T, P_N) is the payoff portfolio the consignor will pay to the carrier in two cases when trade in the contract occurs or not. At $t = 2$, the carrier makes renegotiation with the consignor bilaterally. If the renegotiation between them fails, carrier will trade with other consignor, and the potential value of the customization will be reduced to $g(1 - a)$.

If the relative bargaining power of the consignor is assumed to be λ, then that of the upstream carrier is $(1 - \lambda)$, where $\lambda \in [0, 1]$. We incorporate the analyses of surplus allocation in the cases of trade or nontrade into the framework of the Nash bargaining solutions. By backward deductive method, since (P_T, P_N) can be renegotiated at $t = 2$, it means (P_T, P_N) should obey the constraints of the Nash bargaining solutions.

Under what condition could simple contract (P_T, P_N) acquire the optimal incentive outcome? And why can equity participation arrangement enhance more efficient customization level? These paradoxes are what we are interested in.

2.2. Simple Contract and Customization Level Choice. Given simple contract (P_T, P_N) and the equity participation ratio $\mathbf{r}$ of consignor, the payoff matrix of two trading parties is expressed as in two cases when the negotiation either triumphs (denoted by T) or fails (denoted by N) as following:

$$\begin{array}{ccc} & \text{Carrier} & \text{Consignor} \\ T & (1-r)\left[P_T - C(a)\right] & V(a) - P_T + r\left[P_T - C(a)\right] \\ N & (1-r)\left[P_N + g(1-a) - C(a)\right] & r\left[P_N + g(1-a) - C(a)\right] - P_N \end{array} \tag{1}$$

In the game model of the repeated bargains, the Nash bargaining solutions require that contract (P_T, P_N) satisfies

$$\max \left\{(1-r)\left[P_T - C(a)\right] - (1-r)\left[P_N + g(1-a) - C(a)\right]\right\}^{1-\lambda}$$

$$*\left\{V(a)-P_T+r\left[P_T-C(a)\right]\right.$$
$$\left.-r\left[P_N+g(1-a)-C(a)\right]+P_N\right\}^{\lambda}. \tag{2}$$

The first order condition of P_T demands

$$\frac{(1-\lambda)(1-r)}{(1-r)\left[P_T-P_N-g(1-a)\right]} + \frac{\lambda(r-1)}{V(a)-P_T+r\left[P_T-P_N-g(1-a)\right]+P_N}=0. \tag{3}$$

Then

$$P_T=\frac{\left[e^a-r(1-a)+\left(1-a-e^a\right)\lambda\right]g}{1-r}+P_N. \tag{4}$$

Using payoff matrix, we know that when transaction occurs, the revenues of carrier and consignor are $(1-r)[P_T-C(a)]$ and $(V(a)-P_T+r[P_T-C(a)])$ respectively, expressed by $S_T(r,P_N)$ and $B_T(r,P_N)$. So we have

$$\begin{aligned} S_T(r,P_N)&=\left[e^a-r(1-a)+\left(1-a-e^a\right)\lambda\right]g\\ &\quad+(1-r)\left[P_N-\beta a^2\right],\\ B_T(r,P_N)&=\left[e^a\lambda+(1-a)(r-\lambda)\right]g\\ &\quad-(1-r)P_N-r\beta a^2. \end{aligned} \tag{5}$$

The socially optimal customization level choice must satisfy

$$\max_a\left[V(a)-C(a)\right]. \tag{6}$$

The first order condition demands

$$\frac{e^a}{a}=\frac{2\beta}{g}. \tag{7}$$

That is, the first-best customization level parameter $a_{\text{fb}}=\arg[e^a/a=2\beta/g]$. But, if $r=0$, can (P_T,P_N) make sure that the entrepreneur chooses a_{fb}?

For simplicity of the analyses, let $P_N=0$, and then it demands that the manager chooses a^* to maximize carrier's revenue as follows:

$$a^*=\arg\max_a S_T(r,P_N). \tag{8}$$

Then, we have

$$a^*=\arg\max\left[\frac{e^a}{a}=\frac{\lambda-r}{(1-\lambda)a}+\frac{2(1-r)\beta}{(1-\lambda)g}\right]. \tag{9}$$

When $r=0$, above equation can be changed into

$$a^*=\arg\max\left[\frac{e^a}{a}=\frac{\lambda}{(1-\lambda)a}+\frac{2\beta}{(1-\lambda)g}\right]. \tag{10}$$

Apparently, from $\lambda\in[0,1]$, we can deduce

$$\frac{\lambda}{(1-\lambda)a}+\frac{2\beta}{(1-\lambda)g}\geq\frac{2\beta}{g}. \tag{11}$$

The function $[V(a)-C(a)]$ is concave, so

$$a^*\leq a_{\text{fb}}. \tag{12}$$

It means, when the entrepreneur of the upstream carrier has the whole bargaining power ($\lambda=0$), simple contract (P_T,P_N) can induce the entrepreneur to choose the optimal customization level. It is not difficult to understand that, when the entrepreneur with an overwhelming bargaining power can obtain all of investment surplus, he or she will make the choice of the fist-best customization level. It is consistent with the interpretation of Choi et al. [19–22] that the incomplete contract can also lead to efficient outcomes under some special conditions.

But when $\lambda>0$, (P_T,P_N) contract cannot ensure that the entrepreneur would make the efficient customization investment. Then, how many ration of equity participation can result in efficient outcomes?

If $a^*=a_{\text{fb}}$, then $V'(a^*)=C'(a^*)$. This means

$$\frac{\beta l}{g}=\frac{\lambda-r}{(1-\lambda)a}+\frac{2(1-r)\beta}{(1-\lambda)g}. \tag{13}$$

Obviously, only when $r=\lambda$, may (14) hold. Since $P_T=V(a_{\text{fb}})$, then $(P_T,P_N)=(V(a_{\text{fb}}),0)$.

Integrating the above analyses, we can conclude the following.

Proposition 1. *(i) Only when the entrepreneur of carrier occupies all of bargaining power (e.g., $\lambda=1$) can simple contract $(V(a_{fb}),0)$ result in the socially optimal customization level chosen by carrier; (ii) but if $0<\lambda<1$, the efficient customization investment outcome can be obtained only through equity participation $r=\lambda$ plus simple contract $(V(a_{fb}),0)$.*

Proposition 1 does not mean the consignor d must make the decision of equity participation $r=\lambda$ at $t=0$, because as a rational entity, maximizing its benefit, the consignor would not only consider the customization efficiency of the upstream carrier but also make benefit-cost analyses of equity participation. Hence, the optimal equity participation ratio need not satisfy the condition under which the manager would choose the socially optimal customization level. In next section, we will discuss the problem about optimal equity participation ratio.

3. Equity Participation by Consignor in Owner-Managed Carrier

This section aims at obtaining the comparative static outcome affecting the equity participation ratio factors, so we will install the cost items related to the equity participation in order to obtain the internal angle solution about the optimal partial ownership.

3.1. Optimal Equity Participation Ratio. Given r, the entrepreneur will choose a^* to maximize the customization investment revenue; that is

$$a^*=\arg\max_a S_T(r). \tag{14}$$

Given a chosen by carrier at $t = 1$, the manager of consignor would, at $t = 0$, decide on optimal equity participation ratio r to maximize its total net income. As we all know, under the mechanism of partial ownership the consignor can get a lot of benefits from customized transportation, but must pay for obtaining partial ownership of carrier. The reason why complete vertical integration is not always the optimal choice lies in the fact that the integration cost is likely to exceed the added-value benefits of customized transportation.

Assuming that other outside income of the upstream carrier is π_0, we express $P(r)$ that the consignor pays for obtaining an equity ratio r of the carrier in the following equation:

$$P(r) = (1+\theta)\left[(S_T(o)+\pi_0) - (S_T(r) + (1-r)\pi_0)\right]. \quad (15)$$

In this equation, $(S_T(o)+\pi_0)$ term represents total income of entrepreneur when $r = 0$, and $(S_T(r) + (1-r)\pi_0)$ is the correspondent revenue that entrepreneur earns when equity participation ratio is r. The surplus between these two terms can be regarded as the real value of the equity fraction r of the upstream carrier. Let θ denote the average premium (discount) price coefficient at which the consignor acquires a fraction r of the carrier, where $\theta > 0$ represents purchasing at premium, $\theta = 0$ represents purchasing at par, and $\theta < 0$ represents purchasing at discount. In order to accord with the actual scenarios of equity fight, we install θ-coefficient into the formula to calculate $P(r)$.

Let π_d denote net income of the downstream consignor; then

$$\pi_d = B_T(r) + r\pi_0 - P(r). \quad (16)$$

Substituting $B_T(r)$ and $P(r)$ into above equation, we get

$$\pi_d = e^a g - \beta a^2 + \theta S_T(r) - \theta\pi_0 r - (1+\theta) S_T(0). \quad (17)$$

Anticipating that the entrepreneur chooses a^* based on equity participation ratio r, that is, $a^* = a^*(r)$, the manager of the downstream consignor will try to acquire optimal equity ratio r^* for maximizing π_d; that is

$$r^* = \arg\max_r \pi_d. \quad (18)$$

The first order condition demands

$$\frac{\partial a^*}{\partial r}\left[V'(a^*) - C'(a^*)\right] + \theta\left[C(a^*) - g(1-a^*)\right] - \theta\pi_0 = 0. \quad (19)$$

Without loss of generality, suppose $\beta a_{\mathrm{fb}}^2 - g(1-a_{\mathrm{fb}}) \le 0$; we will discuss the results in the cases of different θ-values.

3.2. The Effects of θ on a^* and r^*

(1) $\theta > 0$, which means purchasing at premium.

By (9), we know

$$\frac{\partial^2 S_T(r)}{\partial (a^*)^2} < 0. \quad (20)$$

Moreover, $\partial^2 S_T(r)/\partial a^* \partial r = 2\beta > 0$.

Thus, $\partial a^*/\partial r = (-\partial^2 S_T(r)/\partial a^*\partial r)/(\partial^2 S_T(r)/\partial (a^*)^2) > 0$.

Since $V'(a^*) - C'(a^*) < 0$, when θ is sufficiently smaller, we have

$$\frac{\partial \pi_d}{\partial r|_{r=0}} > 0, \qquad \frac{\partial \pi_d}{\partial r|_{r=\lambda}} = \theta\left[C(a_{\mathrm{fb}}) - (1-a_{\mathrm{fb}})g - \pi_0\right] < 0. \quad (21)$$

Thus, the optimal partial ownership $r^* \in (0, \lambda)$, and we can get $a^* > a_{\mathrm{fb}}$.

If θ is sufficiently larger, $\forall r \in [0,1]$, $\partial\pi_d/\partial r < 0$, then $r^* = 0$.

(2) $\theta = 0$, which means purchasing at par.

Obviously, for (19) to hold, it requires

$$V'(a^*) - C'(a^*) = 0. \quad (22)$$

Then, $r^* = \lambda$ and $a^* = a_{\mathrm{fb}}$.

(3) $\theta < 0$, which means purchasing at discount.

Evidently, when the value of the upstream fabric supplier is underestimated, the consignor will buy as much equity as possible to maximize its total net income. Although over-incentive effects of equity may lead to customized production level exceeding the social optimality, and the increase of the equity investment revenue renders the manager of the consignor to purchase the share of r^* greatly exceeding social optimal level; that is, $r^* > \lambda$. Besides, when the share-holding ratio of the downstream consignor exceeds the entrepreneur's, the customization level of carrier should equal the social optimal one. So, we have

$$a^* \begin{cases} = a_{\mathrm{fb}} & \text{if } r^* \ge \dfrac{1}{2}, \\ < a_{\mathrm{fb}} & \text{if } r^* < \dfrac{1}{2}. \end{cases} \quad (23)$$

If θ is sufficiently negative, then $r^* = 1$ and $a^* = a_{\mathrm{fb}}$. Notably, we regard θ as a constant in this paper, which does not affect our analyses about the optimal equity participation ratio.

But what we must pay attention to is that, with r vibrating, average discount (or premium) price parameter θ may also be changeable. The exact magnitude of the optimal customization level a and partial ownership r will depend on θ-value.

The three-dimension chart (see Figure 1) shows that, when θ drops from 0.4 to −0.4, $\pi_d(a, r)$, the equilibrium path demonstrates an increasing trend, and the optimal customization level a is increasing in r. With equity purchase shifting from discount price to premium price, the drop of optimal equity participation ratio will result in the customization transportation level and the net income of consignor decreasing simultaneously.

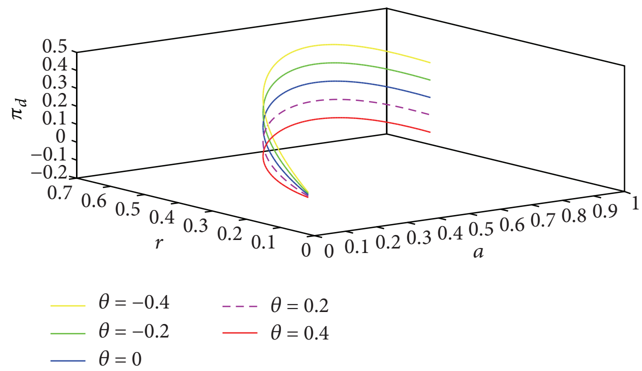

Figure 1: The relationship chart among π_d, a, and r, given $\lambda = 0.5$, $\beta = 2$, and $g = 2$.

Concluding the above analyses, we get the following.

Proposition 2. *(i) If θ is a sufficiently small positive value, $C(a_{fb}) - g(1-a_{fb}) \leq 0$, the consignor holds the optimal partial equity of the carrier $r^* \in (0, \lambda)$, and the customization level which the entrepreneur chooses is below social optimal level; that is, $a^* < a_{fb}$. If θ is sufficiently larger, choosing $r^* = 0$ will be beneficial for the consignor.*

(ii) In the case when purchasing at par ($\theta = 0$), $r^* = \lambda$ and $a^* = a_{fb}$ hold, and the efficient customization level can be obtained.

(iii) In the case when purchasing at discount, $r^* > \lambda$ and $a^* > a_{fb}$ hold, and optimal partial ownership can motivate the entrepreneur to choose the socially optimal customization type.

Proposition 2 demonstrates that only when purchasing at par can optimal equity participation mechanism make sure that the most efficient outcome is obtained. When purchasing at premium or discount price, the distortion effects of wealth transfer will make the customization outcome brought up by optimal partial ownership deviate from social optimal investment type.

According to Proposition 2, we can interpret why the average equity ratios Chinese consignors hold of carriers' equity ownership exceed the corresponding value of American firms, under the condition of the same technology parameters. In China, majority equity ownership of the firms is under the control of individual investors or institution investors, the exchange ratio of the stock is very low, and the exchange of stock is often solved by private negotiation [22]. Generally, in this case, $P(r)$ will not deviate seriously from its real value, and θ is likely to be bigger than zero slightly. But in the United States, whose capital markets are very mature, equity ownership of firms is widely dispersed, the public shareholders hold the majority equities, the ownership r is obtained by exchange in the stock market, and the bids of many trade entities lead to θ-value exceeding zero greatly [23–25]. Therefore, when other technical parameters are not changed, the mechanism that the greater θ-value would lead to the smaller r^* will result in the interfirm equity participation ratio in China to be obviously higher than that of the same industry in the United States.

4. Equity Participation by the Consignor in the Public-Type Carrier

In this section, we will expand our analyses of the optimal partial ownership to the public carrier operated by professional managers. When the carrier is not owner-managed but is a public-firm, the analyses of last section do not apply, completely, in that professional managers in the interest of the stockholders will not be affected directly by the dispersive equities, when considering investment income. Even so, our analyses below demonstrates that partial ownership still affects the customization choice of managers, and a conclusion similar to the last section will be obtained.

For simplicity of analyses, besides the assumption that the manager of carrier is risk neutral, we also assume that there exists no manager's moral hazard about reward compensating mechanism and endeavor choice problem. Assume that rewards of manager are a minority part of the whole profit of carrier, which ensures that manager makes decisions in the interest of the stockholders. Based on these assumptions, we can attain the bargaining outcome below.

Lemma 3. *Given* $\mathbf{r}$ *and* λ*, let* $P_N = 0$*, then the consignor pays*

$$\widehat{P}_T = \frac{(1-\lambda)e^a g + (\lambda - r) g(1-a)}{1-r} \tag{24}$$

to the public-type carrier.

And the incomes obtained by the two trade parties are, respectively,

$$\begin{gathered}\widehat{S}_T(r) = \frac{(1-\lambda)e^a g + (\lambda - r) g(1-a)}{1-r} - \beta a^2, \\ \widehat{B}_T(r) = \lambda g e^a - (\lambda - r) g(1-a) - r\beta a^2.\end{gathered} \tag{25}$$

From the payoff matrix of two trade parties, we can obtain $\widehat{P}_T = P_T$; that is, no matter what type the carrier belongs to, the payoffs of the consignor are the same. There exist differences between two types of carrier' trade income function. But for the consignor, the function form of the trade income remains unchanged.

Observing that the manager of the carrier will choose the customization level $\widehat{a}^*$ to maximize its utility, given the manager's risk neutrality, we know

$$\widehat{a}^* = \arg\left[\frac{\partial \widehat{S}_T(r)}{\partial a} = 0\right]. \tag{26}$$

Given r, the manager would choose $\widehat{a}^*(r)$ to maximize the earnings of the carrier; then decision-makers of consignor can decide on equity participation ratio to maximize its net income $\widehat{\pi}_d$. Here, we adopt the form below to describe $\widehat{\pi}_d$:

$$\widehat{\pi}_d = \widehat{B}_T(r) + r\pi_0 - \widehat{P}(r), \tag{27}$$

where $\widehat{P}(r)$ denotes the price at which the consignor buys a fraction r of equity of the carrier. Assuming that the faction r of equity is purchased through tender offers, the real value

$\delta(r)$ of the carrier based on the partial ownership r can be expressed as

$$\delta(r) = \widehat{S}_T(r) + \pi_0. \tag{28}$$

$\widehat{P}(r)$ can be denoted as follows:

$$\widehat{P}(r) = (1+\theta) r\delta(r). \tag{29}$$

Therefore, the total net income of the consignor can be described as

$$\widehat{\pi}_d = \widehat{B}_T(r) + r\pi_0 - r(1+\theta)\left[\widehat{S}_T(r) + \pi_0\right]. \tag{30}$$

The optimal partial ownership $\widehat{r}^*$ must satisfy the first-order condition

$$\frac{\partial \widehat{\pi}_d}{\partial r \mid r = \widehat{r}^*} = 0. \tag{31}$$

We further have

$$\begin{aligned}&\left(\frac{\partial \widehat{a}^*}{\partial r}\right)\left[V'(\widehat{a}^*) - C'(\widehat{a}^*)\right] - \theta \widehat{S}_T(r) - \theta\pi_0 \\ &\quad - \frac{(1+\theta r)(1-a)g}{(1-r)^2} = 0.\end{aligned} \tag{32}$$

Obviously, when $\theta > 0$ and is sufficiently small, $\widehat{r}^* \in (0,\lambda)$. Similar to the conclusions of the last section, the values of $\widehat{a}^*$ and $\widehat{r}^*$ will be adjusted to correspond to positive or negative θ-value. In this paper we will only analyze the case of $\theta > 0$.

By (32), we get

$$\begin{aligned}\frac{\partial^2 \widehat{\pi}_d}{\partial r \partial \lambda} &= \left[\frac{\partial \widehat{a}^*}{\partial r}\right]\left[V''(\widehat{a}^*) - C''(\widehat{a}^*)\right]\left[\frac{\partial \widehat{a}^*}{\partial \lambda}\right] - \frac{1+\beta r}{(1-r)^2} \\ &\quad * \left[-(1-\widehat{a}^*)V(\widehat{a}^*) - (1-\lambda)V(\widehat{a}^*)\left(\frac{\partial \widehat{a}^*}{\partial \lambda}\right)\right. \\ &\quad \left. + (1-a)(1-\lambda)V(\widehat{a}^*)\left(\frac{\partial \widehat{a}^*}{\partial \lambda}\right)\right] \\ &\quad + \left[\frac{\partial^2 \widehat{a}^*}{\partial r \partial \lambda}\right]\left[V'(\widehat{a}^*) - C'(\widehat{a}^*)\right] + \frac{\beta(1-\widehat{a}^*)V(\widehat{a}^*)}{1-r}.\end{aligned} \tag{33}$$

By $\partial \widehat{S}_T(r)/(\partial a \mid a = \widehat{a}^*) = 0$, we know

$$\frac{\partial \widehat{a}^*}{\partial r} > 0, \qquad \frac{\partial \widehat{a}^*}{\partial \lambda} < 0, \qquad \frac{\partial^2 \widehat{a}^*}{\partial r \partial \lambda} < 0. \tag{34}$$

Then, in (33), all the right-hand terms exceed zero; thus,

$$\frac{\partial^2 \pi_d}{\partial r \partial \lambda} > 0. \tag{35}$$

Since $\mathrm{Sign}(\partial r^*/\partial\lambda) = \mathrm{Sign}(\partial^2 \widehat{\pi}_d/\partial r \partial \lambda)$, we can get

$$\frac{\partial \widehat{r}^*}{\partial \lambda} > 0, \qquad \frac{\partial \widehat{r}^*}{\partial \pi_0} < 0. \tag{36}$$

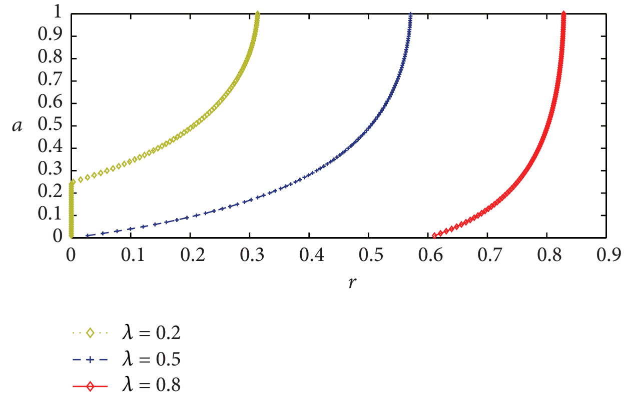

FIGURE 2: The relationship chart between a and r, given $\beta = 2$, $g = 1.2$.

Given the values of λ, β, and g, we can find out the relationship between a and r through numerical simulation, in Figure 2.

Our numerical result shows that parameters a and r are increasing function of λ, and a increases nonlinearly in r.

From the above analyses, we can conclude the following.

Proposition 4. *Assuming that the carrier is a public-firm managed by the manager in the interest of the share-holders, when purchasing at premium price, the optimal equity participation ratio $\widehat{r}^*$ by the consignor increases in λ, but decreases in π_0.*

5. Discussions about Customized Production Efficiency and Policy Implications

The theory of optimal ownership structure extracts the distillate of two stream academic ideas, the financial structure theory and the managerial motivation theory, which have been agreed on in economic literature [19]. For example, Jensen and Meckling [26] pointed out, that agent costs caused by dilution of ownership are derived from the fact that the incentive of inside controllers cannot keep track with that of the owner's. On the other hand, many academic papers demonstrate that an outside artificial person who holds a major part of ownership has a positive effect on the firm's value [10, 17, 26–28]. These papers emphasized the supervising function of the outside artificial persons' share-holders to the firm's managers. Compared with these existing articles, we provide a theoretical interpretation why outside artificial persons hold partial ownership under the circumstance of vertical transaction relationship. Our result is if the downstream firm holds partial ownership of the upstream firm, it may function as a bonding mechanism, which improves the performance of two parties. It means that, compared with $r = 0$, the mechanism of partial ownership improves the efficiency of customization service.

However, can the optimal partial ownership result in social optimal customized transportation level?

As we know, no matter what type the carrier is, either owner-managed one or a public-firm managed by managers, $r^* = \lambda$ or $r^* = 0.5$ is the necessary condition bringing in social optimal customization level. If the carrier is an owner-managed one, only when the purchasing of partial ownership at par price occurs may choosing $r^* = \lambda$ be a rational decision for the consignor aiming at maximizing its total net income. But purchasing at premium price is dominant in reality; if calculated under general case of $\lambda = 0.5$, the optimal partial ownership r^* is less than λ, not large enough to motivate the decision-maker of the carrier to choose the most efficient customized production level. The marginal return obtained by the consignor through enlarging equity participation ratio will be offset by marginal costs of purchasing the equity ownership, and premium price distorting effects lead to the efficiency loss of the carrier's customization investment ($r^* < 0.5$).

On the opposite, when purchasing at discount, overmotivation leads to $r^* > \lambda$, and the customization efficiency loss may still occur. Therefore, we hold that, the effect of wealth transfer in the purchase of equity ownership is the main reason that leads to efficiency loss of customized production, which shows our theory about partial ownership different from the entrepreneur endeavor choice interpretation by Dasgupta and Tao, but similar to the conclusion of Aghion and Tirole's that partial ownership arrangement can motivate special investment, but cannot solve the investors' underinvestment problem totally.

However, when the carrier is public-managed, even if there exist $\theta = 0$ and the optimal partial ownership $\hat{r}^* < \lambda$, social optimal customization level cannot be obtained because the benefit of the consignor does not keep consistent with the trade parties' common benefit. When $\theta > 0$, $\hat{r}^*$ will become smaller and the higher loss degree of the customization investment efficiency will occur. Obviously, under the second-order condition constraint that we can get the optimal solution satisfying (32), the larger the average premium price parameter is, the smaller $\hat{r}^*$ is, the lower customized production efficiency becomes, and the greater social welfare loss will be, which holds true in the case of owner-managed carrier. As for the case of $\theta < 0$, it accords with the aforementioned analyses, that is, the overmotivation of equity ownership may lead to customized production efficiency loss.

In China, the state-owned equity reform in logistic industry just began, with unclear ownership partition, and many social functions assumed by firms are not peeled off. The high premium-price acquisitions may make carriers to choose $r^* = 0$, which leads to the undermotivation of the customization investment of consignors and the social optimal customized transportation cannot be realized. Therefore, in the developing process of logistic supply chains in China, it is necessary that the logistic firms should become the entities that can be self-managed, self-constricted, and self-motivated, and the burden of the social functions will lead to the loss of the transportation efficiency, which are all what we must pay attention to.

In addition, when all technical parameters keep constant, comparing optimal partial ownership in the cases of two different types of carrier, we can discover that, in theory, the ratio of optimal equity participation by the consignor in the owner-managed carrier should exceed the ratio in the public-managed firm; that is, $r^* > \hat{r}^*$. It provides a theoretic foundation for us to interpret that the interfirm mutual shareholding ratios of the member-firms within the familial-type logistic supply chains are much larger than those ratios within the public-type logistic supply chains. At the same time, it also means that the customized production efficiency of the former is higher than that of the latter. Although there is room for improvement, we still firmly believe that the success of Chinese logistic industry should be mainly attributed to the advantage of owner-managed efficiency.

To be mentioned, the mechanism of partial ownership undoubtedly improves the cooperative efficiency of two trade parties. Although social welfare level this arrangement provides is not optimal, the cooperative mechanism helps to enhance Pareto improvement of the return of two trade parties, compared with simple contract system.

6. Conclusions

The phenomena that one consignor holds equity ownership of one carrier within logistic supply chain are often observed. Despite its importance, the existing interpretations for interfirm partial ownership scheme remain relatively unexplored. In this paper, a theoretical explanation for equity participation arrangement existing between member-firms within logistic supply chain is provided under the background of customized production.

Based on the models in which we view the parameter for customization level as the selective variable of the carrier, we figure out that the simple contract cannot solve the low-efficient customized production problem. Equity participation mechanism together with simple contract can improve the efficiency of customized transportation. The partial ownership mechanism supported by customized production plays a role as a bond in keeping the relational transactions among member-firms, but not a role as the efficiency-enhancing mechanism resulting from outside artificial persons' supervision in some literature about equity ownership structure.

What is more, on the basis of keeping the logic deductive consistency, we obtain the outcome of the optimal partial ownership affected by the relative bargaining power λ of the consignor and the other outside income π_0 of the carrier, under two cases of owner-managed firm type and public-firm type, respectively, and show why the familial-type logistic supply chains may choose more efficient customized production level than public-type logistic supply chains.

To be mentioned, our theory provides few cases in which we can verify the interfirm equity participation ratio. Although many important conclusions we obtain seem to be consistent with some evidences, there is a need for further study, such as analyzing bilateral cross-shareholding case and considering the relationship between bargaining power and equity participation ratio through extending our models.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Optimal Ordering and Disposing Policies in the Presence of an Overconfident Retailer: A Stackelberg Game

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This paper investigates the impact of the retailer's overconfident behavior on supply chain performance. We start with a basic model on the rational newsvendor model and investigate the retailer's optimal ordering decision and expected profit. Next, we extend the basic model and introduce an overconfident retailer. We find that the retailer's overconfident behavior does not necessarily damage the supply chain compared with the basic model when the overconfident level does not exceed a threshold. We also design the cooperation and buyback mechanism and conduct numerical analysis to compare the manufacturer's and retailer's expected profits and real profits with those in the basic newsvendor model. It can achieve Pareto improvement in the supply chain when the overconfident level is low. When the retailer's overconfident level exceeds a threshold, the retailer's ordering decision cannot make the whole supply chain sustainable development.

1. Introduction

As a classical model in MS/OM, the newsvendor problem has been deeply explored; however, in reality the actual ordering of the newsvendor always deviates from the theoretically optimal result, and the actual ordering fluctuates around the theoretically optimal ordering quantity. Most scholars use the classical economic assumption that the decision-maker is a rational man, while he/she often has different decision-making behavior, and it is also illustrated in psychology that people cannot be entirely rational and often make irrational decisions [1, 2].

In recent years, decision-making has been becoming a hot issue along with the fact that consumer behavior was introduced into operational management. Scholars have proved the bias between cognitive reflection and decision-making by experiment and the decision-makers are prone to risk preferences, fairness preferences, eye tracking social preferences, overconfidence, and other preferences (see [3–7]). Among them, overconfident behavior becomes one of the hot spots and affects the operation management of the enterprise seriously. For instance, the high volume-trading problem [8], the bankruptcy because of the crazy mergers made by CEO [9], and the presence of oil or gas predicted by young geologist too confidently led to loss of millions of dollars [10]. Overconfidence has been explored deeply in the field of behavioral psychology, behavioral finance, and behavioral management but rarely in the field of management science and operations management. Croson et al. [11] first introduce overconfident behavior in supply chain management.

Based on the classical newsvendor model, we focus on the order variance when the decision-makers are overconfident and rational, respectively. Considering the cooperation and buyback mode, this paper illustrates how the retailer's overconfident behavior affects her ordering decision and the profit of each member in the whole supply chain and how the manufacturer responds to encourage the overconfident retailer to decide the systematic optimal ordering quantity. The remainder of this paper is organized as follows. Section 2 reviews the relevant literature, and Section 3 describes the basic problem. Sections 4 and 5 discuss the basic model and overconfident retailer model, respectively. Section 6 discusses two kinds of decision-making mechanisms, the cooperation mode and buyback mode, and reports the results of an extensive numerical study, and Section 7 concludes this paper with a discussion of the results.

2. Literature Review

There are two primary streams of research that relate to our analysis: the literature on newsvendor problem and overconfident behavior. The newsvendor problem is a classic supply chain management issue which has been proposed in the fifties of the last century and has received wide attention in academia. Arrow et al. [12] analyzed the famous critical fractile solution for the newsvendor problem; after that, many scholars regard maximizing the expected profit as a decision objective and use the expected utility theory and mathematical model to explore the newsvendor problem. For example, Whitin [13] develops the newsvendor model where the demand is price dependent and follows a uniform distribution; moreover, he finds the optimal order which depends on the price variance under the objective of maximizing the expected profit. Thakkar et al. [14] investigated the optimal ordering quantity under the objective of maximizing the expected investment return and found wider application compared with the expected maximum. Khouja [15] classified the newsvendor problem into 11 categories, made a literature review on the predecessors' research, and provided recommendations for future research. For recent reviews on the newsvendor model, refer to Petruzzi and Dada [16] and Qin et al. [17]. None of these papers consider the decision-makers' decision behavior.

More recently, a stream of research has emerged that explored decision-makers' behavior. For example, Eeckhoudt et al. [18] studied the risk-aversion behavioral decision-makers and found that decision-makers' ordering quantity is always lower than the optimal ordering quantity. Many scholars have used experimental evidence to verify the existence of decision deviation. Schweitzer and Cachon [4] explain why retailers' order differs from the expected profit-maximizing quantity and define the high (low) profit environment and find that decision-makers' ordering quantity is less than (greater than) the optimal ordering quantity under the high (low) profit environment by experiments. Benzion et al. [19] test participants' ordering decisions and show that the decision-makers have behavior preferences. In these behavioral preferences, the overconfident behavior is particularly prevalent and potentially harmful behavior [20]. Although many of these papers consider behavior preferences, few papers address the overconfident behavior in supply chain management.

Overconfident behavior is quite mature in psychology and has progress in the field of behavioral finance and behavioral management [21, 22]. But in the field of supply chain inventory management, the research on overconfidence is rare. Moore and Healy [23] define three types of overconfidence: overestimation, overplacement, and overprecision. People overestimate their actual capabilities, overplace their position among their colleagues or peers, and believe they are better than others. They believe that their predictions are more accurate than they actually are. This paper is also based on the third definition. Croson et al. [11] summarize the previous experimental study on human behavior, citing the third definition [23] overprecision, and use salvage costs and price adjustments to revise the overconfident retailer's order decisions. Ren and Croson [24] and Proeger and Meub [25] provide experiment evidence for overconfidence as social bias. Unlike the present analysis, these papers do not analyze that the retailer's and manufacturer's profits can or cannot increase jointly after the implementation of the contract.

In this paper, we analyze the overconfident behavior in the traditional supply chain. In this setting, we consider a two-echelon supply chain that consists of one single manufacturer and one single retailer who is overconfident on the market demand. We discuss a basic model and then introduce an overconfident retailer. In addition, we also design the cooperation and buyback mechanisms and conduct a numerical analysis to compare the manufacturer's and retailer's expected profits and real profits with those in the basic newsvendor model.

3. Problem Descriptions

Consider a two-echelon supply chain, which is composed of one single manufacturer and one single retailer who has overconfident behavior on the market demand, and the manufacturer is the leader and the retailer is the follower in the supply chain. To be specific, the manufacturer is a risk-neutral decision-maker and does not have overconfident behavior, and the retailer is also a risk-neutral decision-maker and has overconfident behavior. The chronology of the event is that the manufacturer produces products and sells them to the retailer in wholesale price w, and the retailer sells these products to the market in selling price p. The manufacturer can predict the market demand X accurately, and the demand X follows a distribution with mean μ and variance σ^2 [11]. The market demand can be expressed as $X = \sigma\varepsilon + \mu$, where ε is a random variable, whose mean is 0 and variance is 1, and denote the CDF and PDF by $F(x)$ and $f(x)$.

For the overconfident retailer, there is a deviation between the forecast demand and the actual demand. Thus, the cumulative distribution function is $F_a(x_a)$ and the probability density function is $f_a(x_a)$. Refer to the definition of overconfidence defined by Moore and Healy [23], the forecast demand mean μ_a is not equal to the actual demand mean μ for the retailer overestimates the market environment and her ability. On the other hand, the overconfident retailer's demand variance σ_a^2 is less than or equal to the actual demand variance σ^2. Without loss of generality, the overconfident retailer's forecast demand can be expressed as

$$X_a = \sigma_a\varepsilon + \mu_a = (1-a)\,\sigma\varepsilon + (1+a)\,\mu, \tag{1}$$

where a indicates the retailer's overconfident level, and $0 \leq a \leq 1$. When $a = 0$, the retailer has no overconfident behavior; when $a = 1$, we have $X_a = 2\mu$ and the retailer behaves in an extremely overconfident manner and believes that the demand is constant. The closer a is to 1, the more intense overconfidence the retailer behaves.

In the following discussion, let c represent unit production cost. For unsold products, s_R and s_M represent unit salvage value that is disposed of by the retailer and manufacturer, respectively.

It is assumed that the above distribution functions are first-order differentiable and have strict monotone inverse function; these parameters satisfy the following relationship:

$$c < w < p, \qquad s_R < s_M < c, \tag{2}$$

where the first constraint is to ensure the wholesale price is higher than the manufacturer's production cost and the retail price is higher than the wholesale price, in order to guarantee the retailer's and manufacturer's profits. The second constraint indicates that the residual value of the product is lower than the manufacturer's production cost, and the residual value of the product in the retailer is less than that in the manufacturer, which illustrates that the manufacturer has the advantage in disposing the unsold products.

4. Basic Model

In this section, we first discuss a basic model where the manufacturer and the retailer are risk-neutral and rational. We assume the market demand of the product is X, the demand distribution function $F(x)$ is continuous and differentiable and is strictly increasing, the average demand is $E(x) = \mu$, and the variance is $\mathrm{Var}(x) = \sigma^2$. The retailer's ordering quantity is Q. When the market demand satisfies $X < Q$, the sale quantity is $S(Q) = X$; otherwise, the sale quantity is $S(Q) = Q$. Thus, the expected sale quantity is $S(Q) = \min\{Q, X\} = Q - \int_0^Q F(x)dx$, and the expected unsold quantity is $I(Q) = Q - S(Q) = \int_0^Q F(x)dx$.

The retailer's and the manufacturer's profits are given by

$$\begin{aligned}\pi_R^N &= pS(Q) + s_R I(Q) - wQ \\ &= (p-w)Q - (p-s_R)\int_0^Q F(x)\,dx,\end{aligned} \tag{3}$$

$$\pi_M^N = wQ - cQ. \tag{4}$$

The profit of the supply chain is given by

$$\begin{aligned}\pi^N &= \pi_R^N + \pi_M^N \\ &= (p-c)Q - (p-s_R)\int_0^Q F(x)\,dx.\end{aligned} \tag{5}$$

The objective of the retailer is to choose Q^* to maximize her profit. Besides, the expected profit function π_R^N has a maximum because the second-order condition $\partial^2\pi/\partial Q^2 < 0$. Solving the first-order derivative of (3) with respect to Q, we can obtain

$$F\left(Q^{N^*}\right) = \frac{p-w}{p-s_R}. \tag{6}$$

Thus, $Q^{N^*} = F^{-1}((p-w)/(p-s_R))$. Q^{N^*} is the retailer's optimal ordering quantity in the newsvendor model, which is consistent with the results of Pasternack [26].

5. Overconfident Retailer Model

In this section, we consider an overconfident retailer who has an overconfident behavior on the market demand. In this case, the retailer's ordering quantity is Q_a. When the market demand satisfies $X_a < Q_a$, the sale quantity is $S_a(Q_a) = X_a$; otherwise, $S_a(Q_a) = Q_a$. Therefore, $S_a(Q) = \min\{Q_a, X_a\}$. It can be rewritten as

$$\begin{aligned}S_a(Q_a) &= \int_0^{Q_a} x_a f_a(x_a)\,dx_a + Q_a\int_{Q_a}^{\infty} f_a(x_a)\,dx_a \\ &= Q_a - \int_0^{Q_a} F_a(x_a)\,dx_a.\end{aligned} \tag{7}$$

The expected unsold quantity is $I_a(Q_a) = Q_a - S_a(Q_a) = \int_0^{Q_a} F_a(x_a)dx$.

In order to facilitate comparison with the expected profit under the basic model, we examine the relationship on the optimal ordering quantity between the rational retailer and the overconfident retailer in Theorem 1.

Theorem 1. *The overconfident retailer's optimal ordering quantity is*

$$Q_a^* = (1-a)\,Q^{N^*} + 2a\mu. \tag{8}$$

Proof. As discussed above, the rational retailer's optimal ordering quantity is $Q^{N^*} = F^{-1}[(p-w)/(p-s_R)]$, and we define $(p-w)/(p-s_R) = \rho$; that is, $\rho = F(Q^{N^*})$.

Note that because $X = \sigma\varepsilon + \mu$, $X_a = \sigma_a\varepsilon + \mu_a = (1-a)\sigma\varepsilon + (1+a)\mu$, and X and X_a follow the relationship that $X_a = (1-a)X + 2a\mu$, we have $F_a(x_a) = P((1-a)X + 2a\mu \le x_a) = F[(x_a - 2a\mu)/(1-a)]$.

As a result, the overconfident retailer's optimal ordering quantity is $Q_a^* = F_a^{-1}[(p-w)/(p-s_R)] = F_a^{-1}(\rho)$.

And it can be rewritten as $\rho = P(X_a \le Q_a^*) = P((1-a)X + 2a\mu \le Q_a^*) = F[(Q_a^* - 2a\mu)/(1-a)]$.

Finally, we have $F(Q^{N^*}) = F[(Q_a^* - 2a\mu)/(1-a)]$; that is, $Q_a^* = (1-a)Q^{N*} + 2a\mu$. □

Theorem 1 presents the overconfident retailer's optimal ordering policy, and based on this result, we can derive some properties of the optimal ordering policy as follows.

Corollary 2. *When $Q^{N^*} < 2\mu$, $Q_a^* > Q^{N^*}$; otherwise, $Q_a^* \le Q^{N^*}$.*

Proof. According to the result of Theorem 1 that $Q_a^* = (1-a)Q^{N^*} + 2a\mu$, we can obtain $Q_a^* - Q^{N^*} = a(2\mu - Q^{N^*})$, so it is easy to obtain the conclusion of Corollary 2. □

Subsequently, we consider the manufacturer's and retailer's expected profits separately. When the overconfident level is a, the retailer's expected profit can be expressed as

$$\begin{aligned}\pi_R^{a^*} &= E\{-wQ_a + pS_a(Q_a) + s_R I_a(Q_a)\} \\ &= (p-w)Q_a^* - (p-s_R)\int_0^{Q_a^*} F_a(x_a)\,dx_a,\end{aligned} \tag{9}$$

where $x_a = (1-a)x + 2a\mu$.

Making variable substitution to (9) according to (8), it is easy to see

$$\begin{aligned}&\int_0^{Q_a^*} F_a(x_a)\,dx_a \\ &= \int_{-2a\mu/(1-a)}^{Q^{N^*}} F_a[(1-a)x + 2a\mu]\,d[(1-a)x + 2a\mu] \\ &= (1-a)\int_0^{Q^{N^*}} F(x)\,dx.\end{aligned} \tag{10}$$

Bringing (8) and (10) into (9), we can get

$$\begin{aligned}\pi_R^{a^*} = &\ (p-w)(1-a)Q^{N^*} - (p-s_R)(1-a) \\ &\times \int_0^{Q^{N^*}} F(x)\,dx + 2a\mu(p-w).\end{aligned} \tag{11}$$

In the same logic, the manufacturer's expected profit is

$$\begin{aligned}\pi_M^{a^*} &= E\{wQ_a - cQ_a\} \\ &= (w-c)(1-a)Q^{N^*} + 2a\mu(w-c).\end{aligned} \tag{12}$$

Under the conditions where the retailer disposes of the unsold products, the total profit of the supply chain is $\pi^{a^*} = \pi_M^{a^*} + \pi_R^{a^*}$.

After simplifying, we can easily obtain $\pi^{a^*} = (p-c)(1-a)Q^{N^*} - (p-s_R)(1-a)\int_0^{Q^{N^*}} F(x)dx + 2a\mu(p-c)$.

Based on the optimal profits of the rational and overconfident retailers, we can derive the following results.

Corollary 3. *When $Q^{N^*} > 2\mu$, the expected profit of the whole supply chain in the overconfident newsvendor model is less than that in the basic newsvendor model, and the expected profit of the whole supply chain decreases with a. When $Q^{N^*} < 2\mu$, with a increasing, the expected profit of the whole supply chain in the overconfident newsvendor model has two cases: it is either higher or higher first and then lower than that in the basic newsvendor model.*

Proof. Compared with the basic newsvendor model, the real profit difference is

$$\begin{aligned}\Delta\pi = \pi^{N^*} - \pi^{a^*} &= (p-c)\left(Q^{N^*} - Q_a^*\right) \\ &\quad - (p-s_R)\int_{Q_a^*}^{Q^{N^*}} F(x)\,dx \\ &= a(p-c)\left(Q^{N^*} - 2\mu\right) - (p-s_R) \\ &\quad \times \int_{(1-a)Q^{N^*}+2a\mu}^{Q^{N^*}} F(x)\,dx.\end{aligned} \tag{13}$$

Therefore, solving the first-order derivative of $\Delta\pi$ with respect to a, we can obtain

$$\begin{aligned}\frac{\partial\Delta\pi}{\partial a} &= (p-c)\left(Q^{N^*} - 2\mu\right) + (p-s_R)\left(2\mu - Q^{N^*}\right) \\ &\quad \times F\left((1-a)Q^{N^*} + 2a\mu\right) \\ &= (p-s_R)\left(Q^{N^*} - 2\mu\right)\left[\frac{p-c}{p-w}F\left(Q^{N^*}\right) - F\left(Q_a^*\right)\right].\end{aligned} \tag{14}$$

It is clear that $p - s_R > 0$ and $1 < (p-c)/(p-w)$. When $Q^{N^*} > 2\mu$, we have $Q_a^* < Q^{N^*}$. Since $F(Q)$ is an increasing function, this implies that $\partial\Delta\pi/\partial a$ is absolutely bigger than 0; that is, $\Delta\pi$ is increasing in a. In particular, when $a = 0$, we have $\Delta\pi = 0$. Therefore, the profit difference is increasing in a.

When $Q^{N^*} < 2\mu$, we have $Q_a^* > Q^{N^*}$ and $\partial\Delta\pi/\partial a = (p - s_R)(Q^{N^*} - 2\mu)[((p-c)/(p-w))F(Q^{N^*}) - F(Q^{N^*} + (2\mu - Q^{N^*})a)]$ and when a is low, there is r which made the equation $rF(Q^{N^*}) = F(Q^{N^*} + (2\mu - Q^{N^*})a)$ set-up and satisfied $1 < r < (p-c)/(p-w)$; that is, $\partial\Delta\pi/\partial a$ is always less than zero; when $a = 1$, $F(Q^{N^*} + (2\mu - Q^{N^*})a) = F(2\mu)$, and if $(p-c)/(p-w)F(Q^{N^*}) > F(2\mu)$, $\partial\Delta\pi/\partial a$ is less than zero, and the profit difference decreases with a; if $(p-c)/(p-w)F(Q^{N^*}) < F(2\mu)$, $\partial\Delta\pi/\partial a$ is first less and then greater than zero and the profit difference is first decreasing and then is increasing in a. □

From Theorem 1 and Corollaries 2 and 3, we find that if $Q^{N^*} < 2\mu$, then $dQ_a^*/da = 2\mu - Q^{N^*} > 0$; thus, we have $\pi_M^a > \pi_M$ and $\pi_R^a < \pi_R$, whereas if $Q^{N^*} > 2\mu$, $dQ_a^*/da = 2\mu - Q^{N^*} < 0$; thus, $\pi_M^a < \pi_M$, $\pi_R^a < \pi_R$. In other words, the bigger a is, the bigger the ordering quantity deviates from the optimal one and results in lower profit.

Just as analyzed in Section 2, Gervais et al. [22], Glaser and Weber [8], and Malmendier and Tate [9] have provided enough evidences in financial field. We find that the overconfident behavior does not necessarily damage the supply chain, if the manufacturer can take advantage of the retailer's overconfident behavior properly, such as subsidy or profit sharing mechanism which can improve the supply chain performance effectively. However, in the decentralized decision, both the retailer and the manufacturer maximize their own profits, and their decisions cause a double marginalization effect. What is worse, the overconfident behavior inevitably makes the retailer's profit losses.

We know that $Q_a^* < Q^{N^*}$ when $Q^{N^*} > 2\mu$; besides, the bigger w is, the bigger Q_a^* deviates from Q^{N^*} and the lower the retailer's profit is; when $Q^{N^*} < 2\mu$, then $Q_a^* > Q^{N^*}$, and increasing w leads to $\pi_R^{a^*} < \pi_R^{N^*}$. Therefore, the manufacturer should sell products to the retailer with low wholesale price, which could reduce the retailer's cost and then promote the retailer's ordering quantity and profit.

6. Disposing Mechanism on Salvage Products

Based on the analysis of the impact of the retailer's overconfident behavior on the supply chain, as a leader in the supply chain, the manufacturer should take effective measures to promote the retailer's ordering decisions. Herz et al. [27], Choi et al. [28], and Govindan et al. [29] have done relevant research on supply chain coordination. Xu et al. [30] and Huang et al. [31] also analyzed such practical problems. In this section, we try to design and analyze two coordination mechanisms: cooperation mode and buyback mode.

6.1. Cooperation Mode. In this mode, we consider the manufacturer and retailer cooperate to dispose of the unsold products to realize a "win-win" situation. It is assumed that $s_M > s_R$, which means that the manufacturer has the advantage in the disposing of the unsold products; therefore, the manufacturer disposes of the unsold products at the end of selling season. We also assume that the allocation rate of the residual value is λ. For unsold product, the manufacturer's benefit is $\lambda(s_M - s_R)$ and the retailer's benefit is $s_R + (1-\lambda)(s_M - s_R)$.

We assume that the manufacturer knows the retailer's overconfident level, and to manufacturer, the cost of the disposing of the unsold products is zero. Similar to the basic newsvendor model, in the corporation mode, the retailer believes her expected profit is

$$\begin{aligned}\pi_R^{ac} &= E\left\{pS(Q) + \left[s_R + (1-\lambda)(s_M - s_R)\right] I(Q) - wQ\right\} \\ &= (p-w)Q^{ac} - \left[p - s_R - (1-\lambda)(s_M - s_R)\right] \\ &\quad \times \int_0^{Q^{ac}} F_a(x_a)\, dx_a.\end{aligned} \tag{15}$$

Solving the first-order derivative of (15) with respect to Q^{ac}, we can derive the optimal ordering quantity as follows

$$Q^{ac^*} = F_a^{-1}\left[\frac{p-w}{p - s_R - (1-\lambda)(s_M - s_R)}\right]. \tag{16}$$

From (16), the ordering quantity is increasing in $1-\lambda$. Therefore, a proper incentive can instruct the retailer to choose a proper ordering quantity.

The retailer believes that her profit is maximal when the optimal ordering quantity is (16). However, it is up to the actual market demand. Therefore, her real expected profit is

$$\begin{aligned}\pi_R^{ac^*} &= (p-w)Q^{ac^*} - \left(p - s_R - (1-\lambda)(s_M - s_R)\right) \\ &\quad \times \int_0^{Q^{ac^*}} F(x)\, dx.\end{aligned} \tag{17}$$

By the same logic, the manufacturer's real expected profit is

$$\pi_M^{ac^*} = (w-c)Q^{ac^*} + \lambda(s_M - s_R)\int_0^{Q^{ac^*}} F(x)\, dx. \tag{18}$$

And the real expected profit of the whole supply chain is

$$\pi^{ac^*} = (p-c)Q^{ac^*} - (p - s_M)\int_0^{Q^{ac^*}} F(x)\, dx. \tag{19}$$

Since $\partial\pi^{ac^*}/\partial Q^{ac^*} = 0$, we can obtain that $Q^{ac^*} = F^{-1}[(p-c)/(p-s_M)]$, which is not necessarily equal to (16). Here we mark that $Q^{Nc^*} = F^{-1}[(p-c)/(p-s_M)]$. When the retailer chooses Q^{Nc^*}, the profit of the whole supply chain is maximized. Furthermore, from (19), the bigger the ordering quantity deviates from Q^{Nc^*}, the bigger the profit deviates from the maximum profit.

Compared with the basic newsvendor model, the profit difference is

$$\begin{aligned}\Delta\pi = \pi^{N^*} - \pi^{ac^*} &= (p-c)\left(Q^{N^*} - Q^{ac^*}\right) \\ &\quad - (p - s_R)\int_0^{Q^{N^*}} F(x)\, dx + (p - s_M) \\ &\quad \times \int_0^{Q^{ac^*}} F(x)\, dx.\end{aligned} \tag{20}$$

Corollary 4. *When $Q^{N^*} > 2\mu$, then $Q^{Nc^*} > Q^{N^*} > Q_a^*$ and $Q^{Nc^*} > Q^{ac^*} > Q_a^*$; when $Q^{N^*} < 2\mu$, then $Q^{ac^*} > Q_a^* > Q^{N^*}$ and $Q^{Nc^*} > Q^{N^*}$.*

Proof. For any Q^{N^*}, we have $Q^{Nc^*} = F^{-1}[(p-w)/(p-s_M)] > F^{-1}[(p-w)/(p-s_R)] = Q^{N^*}$ and $Q^{ac^*} > Q_a^*$. When $Q^{N^*} > 2\mu$, we can obtain $Q^{N^*} > Q_a^*$ from Corollary 2 and then obtain $Q^{Nc^*} > Q^{N^*} > Q_a^*$.

Here we mark that $Q' = F_a^{-1}[(p-w)/(p-s_M)]$; then, $Q' > Q^{ac^*}$. It is easy to see that $Q' = (1-a)Q^{Nc^*} + 2a\mu$ which is similar to the proof of Theorem 1. When $Q^{Nc^*} > 2\mu$, we have $Q^{Nc^*} > Q'$ and $Q^{Nc^*} > Q^{ac^*}$, and then $Q^{Nc^*} > Q^{ac^*} > Q_a^*$. When $Q^{N^*} < 2\mu$, we obtain $Q_a^* > Q^{N^*}$ from Corollary 2; therefore, $Q^{ac^*} > Q_a^* > Q^{N^*}$. □

Corollary 5. *When $Q^{N^*} > 2\mu$, then $\pi^{Nc^*} > \pi^{ac^*}$, and the bigger a is, the bigger the expected profit of the whole supply chain deviates from the maximum profit.*

Proof. In the case of $Q^{N^*} > 2\mu$, when the retailer chooses Q^{Nc^*}, the manufacturer can coordinate supply chain to maximize his profit; therefore, we have $\pi^{Nc^*} > \pi^{ac^*}$. Here we mark that the rational optimal ordering quantity is $Q^{c^*} = F^{-1}((p-w)/(p-s_R-(1-\lambda)(s_M-s_R)))$ and the profit is π^{c^*}; thus, $\pi^{c^*} - \pi^{ac^*} = (p-c)(Q^{c^*} - Q^{ac^*}) - (p-s_M)\int_{Q^{ac^*}}^{Q^{c^*}} F(x)dx$. According to the proof of Corollary 3, we concluded that the profit of the whole supply chain decreases with a. Since π^{Nc^*} and π^{c^*} are fixed, we draw the conclusion that the bigger a is, the bigger the expected profit of the whole supply chain deviates from the maximum profit. □

Compared with the basic newsvendor model, the residual value of the unit product in the manufacturer is more than that in the retailer in the cooperation mode. Moreover, in the specific market environment, the manufacturer can make Q^{ac^*} closer to Q^{Nc^*} through adjusting the size of λ.

Apparently, the retailer's ordering quantity is affected mainly by the retailer's overconfident level a, demand mean μ, and the variance σ^2. In this section, we provide a numerical study to examine the impacts of the retailer's overconfident behavior on the optimal ordering quantity and profits. We assume that the market demand follows a normal distribution $N \sim (60, 150^2)$, $p = 24$, $w = 9$, $c = 6$, $s_M = 5$, and $s_R = 4$, and we calculate that the retailer's optimal ordering quantity is 161.1735, the retailer's profit is 637.9868, the manufacturer's profit is 483.5204, and the profit of the supply chain is 1121.5072 under the basic newsvendor model. We first analyze the circumstance of $Q^{N^*} > 2\mu$, and then let $\lambda = 0.5$, and the specific dates are listed in Table 1.

From Table 1, the result that Q_a^* and Q^{ac^*} decrease with a may be different from our expectations. Many psychologists and scholars give explanations, such as Schweitzer and Cachon [4] who found that when the market is in a high-profit environment, decision-makers tend to order less and when the market is in a low-profit environment, decision-makers tend to order more. In addition, π_a^* deviates from π^{N^*}, which also examines the validity of Corollary 3. For a fixed a, if λ decreases, then the retailer increases her ordering quantity, making the profit of the whole supply chain increase. Therefore, increasing the incentive properly can improve the performance of the supply chain, but it does not ensure that profits of the retailer and the manufacturer increase jointly.

Figure 1 shows an illustration of how π^{ac^*}, π^{N^*}, and π_a^* vary with λ and a. In cooperation mode, when the retailer's overconfident level is low, we have $\pi^{ac^*} > \pi^{N^*}$. This can be explained as follows. The main reason is that the manufacturer has the advantage in disposing of the unsold products compared with the retailer, which made the increased profit through cooperation larger than the loss caused by the ordering quantity deviation, and the retailer increases her ordering quantity when motivated by the manufacturer which made the total profit increase.

The results of Figures 2 and 3 indicate the impact of the overconfident level on expected profits of the retailer and manufacturer under different allocation rate λ. To be specific, when the overconfident level a is low, for example, when $a < 0.4$, the expected profits of the retailer and manufacturer increase, which achieves the Pareto improvement, compared with the basic newsvendor model. For a fixed a, $\pi_R^{ac^*}$ and π^{ac^*} increase with the allocation rate $1-\lambda$, while $\pi_M^{ac^*}$ decreases with it. Besides, when a exceeds a threshold, the expected profits of the retailer and the manufacturer become smaller; thus, the supply chain profit is less than that in the basic newsvendor model. Therefore, from the manufacturer's perspective, when a is low, he can offer incentives to the retailer. However, when a increases, he should reduce the allocation proportion $1-\lambda$ to the retailer to ensure that their expected profit is no less than $\pi_M^{N^*}$.

Next, we use another example to illustrate the impact of the overconfident behavior on the supply chain. We discuss the scenario of $Q^{N^*} < 2\mu$. In this example, the market demand follows the normal distribution $X \sim N(60, 20^2)$ and the values of other variables remain unchanged. The retailer's optimal quantity is 73.4898 and profit is 773.0422 and the manufacturer's profit is 220.4694 and the profit of the supply chain is 993.5116 under the basic newsvendor model. Let $\lambda = 0.5$ and specific data values are listed in Table 2.

Table 1: The optimal quantity and expected profits under different overconfident level.

a	Q_a^*	π_a^*	Q^{ac^*}	π^{ac^*}	$\pi_R^{ac^*}$	$\pi_M^{ac^*}$
0	161	1121	170	1243	684	559
0.1	157	1108	165	1225	683	542
0.2	152	1095	160	1207	682	525
0.3	148	1081	155	1187	679	508
0.4	144	1066	150	1167	675	491
0.5	140	1050	145	1145	671	474
0.6	136	1034	140	1122	665	457
0.7	132	1016	135	1098	658	440
0.8	128	998	130	1073	649	423
0.9	124	979	125	1047	640	406
1	120	960	120	1020	630	390

Table 2: The optimal quantity and expected profits under different overconfident level.

a	Q_a^*	π_a^*	Q^{ac^*}	π^{ac^*}	$\pi_R^{ac^*}$	$\pi_M^{ac^*}$
0	73	993	74	1014	781	232
0.1	78	1004	79	1026	778	248
0.2	82	1009	83	1034	770	263
0.3	87	1009	88	1038	758	279
0.4	92	1006	92	1039	744	295
0.5	96	1001	97	1038	727	310
0.6	101	994	101	1035	709	326
0.7	106	986	106	1032	689	342
0.8	110	978	110	1028	670	358
0.9	115	969	115	1024	650	374
1	120	960	120	1020	630	390

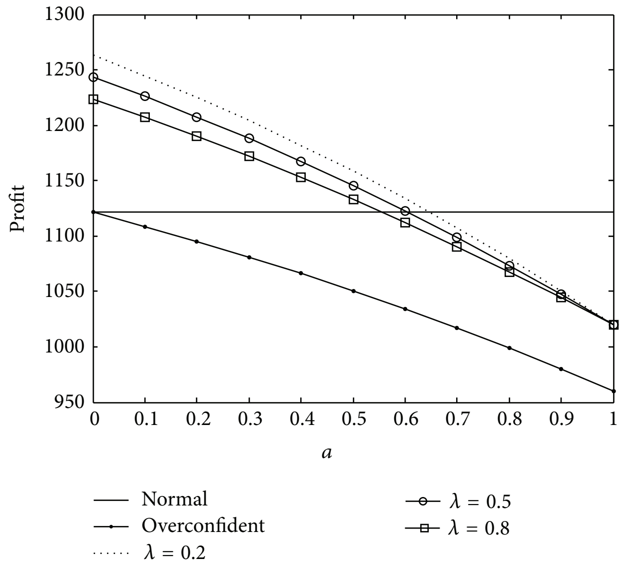

Figure 1: The expected profit of the supply chain under different allocation rate λ.

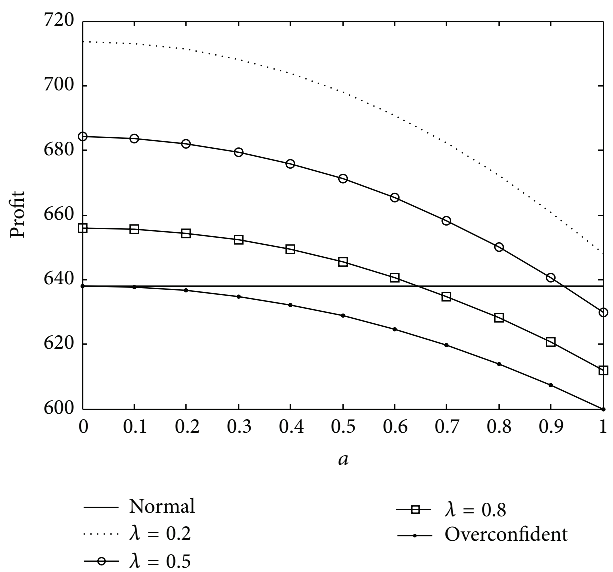

Figure 2: The retailer's expected profit under different allocation rate λ.

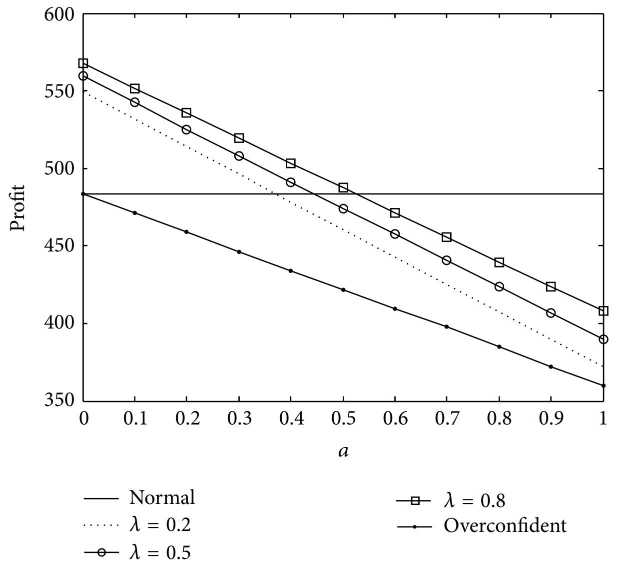

Figure 3: The manufacturer's expected profit under different allocation rate λ.

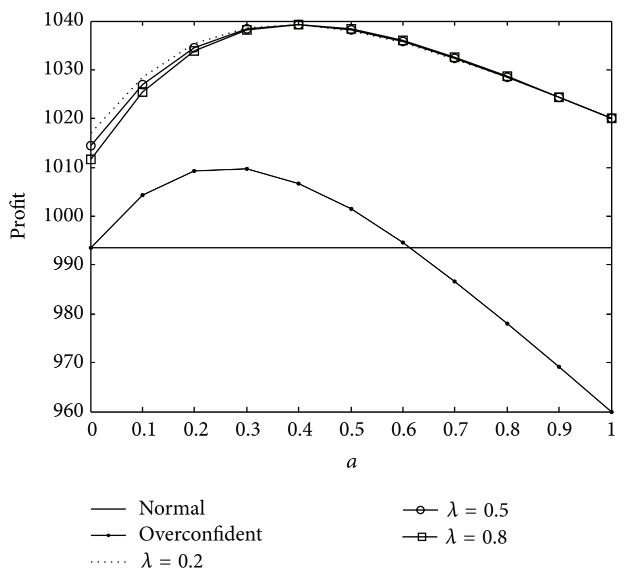

Figure 4: The expected profit of the supply chain under different allocation rate λ.

From Table 2, the equation $\pi_M^{ac^*} > \pi_M^{N^*}$ implied that the manufacturer can obtain more profits when the retailer is overconfident. Besides, Q_a^* increases and π_a^* increases first and then decreases with a. These results mean that the retailer's overconfident behavior does not necessarily damage the supply chain. Meanwhile, if the manufacturer offers proper incentive to the retailer, the whole supply chain can achieve Pareto improvement. For example, when $\lambda = 0.5$ and a is in a reasonable range, $\pi_M^{ac^*} > \pi_M^{N^*}$ and $\pi_R^{ac^*} > \pi_R^{N^*}$ hold. However, when a exceeds a threshold, the manufacturer's incentive has little impact on the retailer's ordering decision and the retailer's overconfident behavior brings more losses to the supply chain. In particular, when $a = 1$, the manufacturer's incentive cannot impact the retailer's ordering decision.

In order to illustrate the profit variance, we take different λ as presented in Figures 4–6.

We can see from Figure 4 that when $\lambda = 1$, the manufacturer does not provide any incentives, and with the increase of a, the profit of the whole supply chain is higher first and then lower than that in the basic newsvendor model, which examines the validity of Corollary 3. When the overconfident level a is in a reasonable range, the profit of the whole supply chain increases with the allocation rate $1-\lambda$; when a exceeds a threshold, the profit of the whole supply

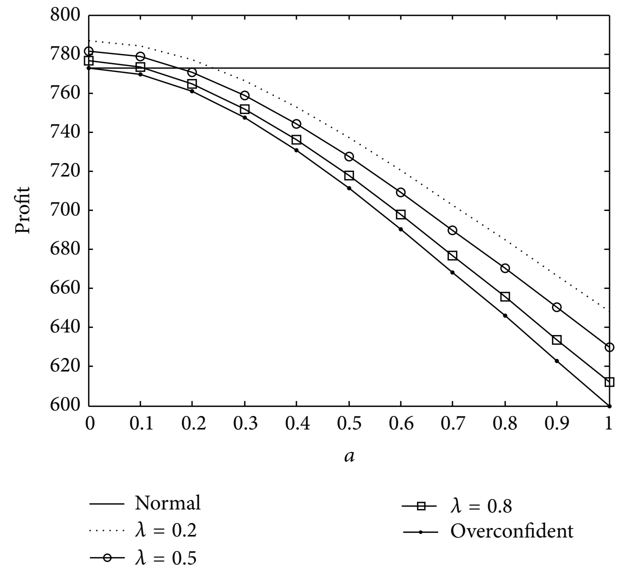

FIGURE 5: The retailer's expected profit under different allocation rate λ.

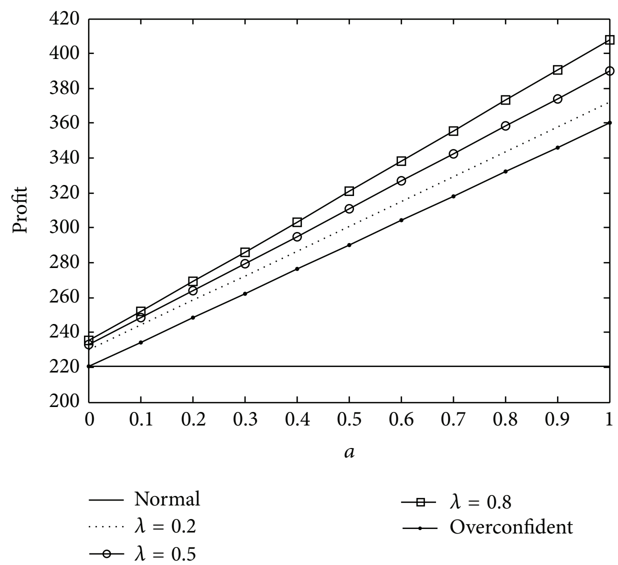

FIGURE 6: The manufacturer's expected profit under different allocation rate λ.

chain has a downward trend, and the change of λ has little impact on the profit. From Figures 5 and 6, we have $\pi_M^{ac^*} > \pi_M^{N^*}$, regardless of the change of a; that is, the manufacturer can achieve Pareto improvement. We observe that a lower allocation rate λ allows a higher overconfident level a. When a is low, the incentive can make up for the losses caused by more orders, so $\pi_R^{ac^*} > \pi_R^{N^*}$; when a is high, the retailer's order deviates from the optimal order.

It is worth pointing that the demand variance has a tremendous impact. For example, when the demand distribution follows $X \sim N(60, 75^2)$ and the other variables remain unchanged, whatever a is, the manufacturer's profit is higher than that in the basic newsvendor model, so when the manufacturer transfers all the residual value to the retailer, that is, $\lambda = 0$, we have calculated that the manufacturer's and retailer's profit increase whatever a is compared with basic newsboy model, which can achieve the Pareto improvement and examines the validity of Corollary 3.

6.2. Buyback Mode. In this mode, the manufacturer designs a buyback contract to encourage the retailer to order more products, the manufacturer buys back the unsold products and the buyback price is b, and $b > s_R$, which is to ensure that the buyback price is higher than the value of the retailer's disposals. It is easy to see that, compared with the basic newsvendor model, the retailer can get more profits when the ordering quantity is the same.

When $b = 0$, the manufacturer does not take back unsold products; when $b = s_R$, the retailer's expected profit is the same no matter who deals with unsold products; when $b = s_M$, the manufacturer transfers all the benefit from the unsold products to the retailer; when $b = w$, the retailer has no ordering risk.

In the Stackelberg game, the manufacturer is the leader and the retailer is the follower. The chronology of the game event is given as follows. Firstly, before the selling season, the manufacturer informs the retailer about the wholesale price and the buyback price. Secondly, the retailer determines her ordering quantity Q based on this information. At the end of the selling season, the manufacturer pays b for the unsold products to the retailer.

Similar to the basic model, the retailer's real expected profit is $\pi_R^{ab} = E\{pS^b(Q^{ab}) + bI^b(Q^{ab}) - wQ^{ab}\}$.

It can be simplified as $\pi_R^{ab} = (p - w)Q^{ab} - (p - b)\int_0^{Q^{ab}} F(x)dx$.

From the above equation, the optimal ordering quantity is $Q^{ab^*} = F_a^{-1}[(p - w)/(p - b)]$. Q^{ab^*} is decreasing in w and increasing in b. That is, the retailer hopes for a lower wholesale price and a higher buyback price.

In the buyback mode, the manufacturer's real expected profit is $\pi_S^{ab^*} = (w-c)Q^{ab^*} + (s_M - b)\int_0^{Q^{ab^*}} F(x)dx$ and the real expected profit of whole supply chain is $\pi^{ab^*} = (p - c)Q^{ab^*} - (p - s_M)\int_0^{Q^{ab^*}} F(x)dx$.

Compared with the basic newsvendor model, the profit difference is

$$\begin{aligned}\Delta\pi = \pi^{N^*} - \pi^{ab^*} &= (p - c)\left(Q^{N^*} - Q^{ab^{\nu}}\right) \\ &\quad - (p - s_R)\int_0^{Q^{N^*}} F(x)\,dx \\ &\quad + (p - s_M)\int_0^{Q^{ab^*}} F(x)\,dx.\end{aligned} \tag{21}$$

Therefore, the manufacturer can adjust the wholesale price w and the buyback price b to make the retailer's ordering decision closer to the optimal ordering quantity and then make the whole supply chain more efficient. If $b < s_R$,

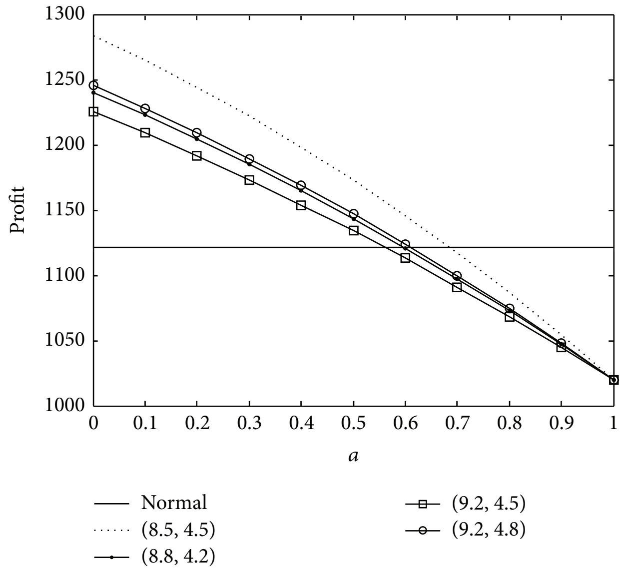

Figure 7: The expected profit of the supply chain under different w and b.

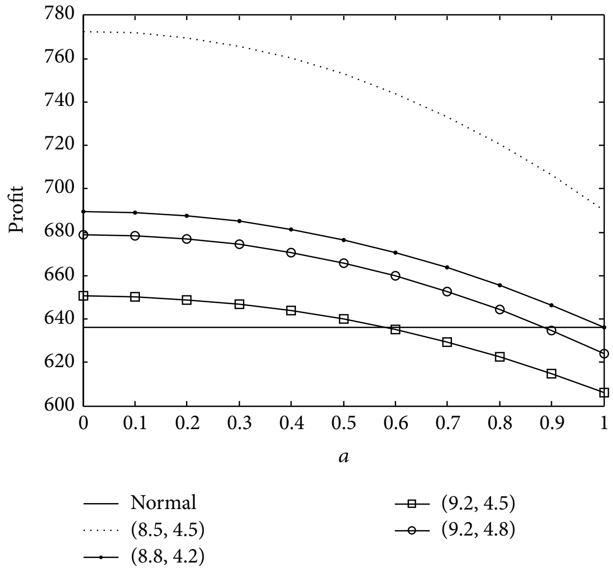

Figure 8: The retailer's expected profit under different w and b.

the retailer does not sell unsold products to the manufacturer, so the manufacturer must keep $b > s_R$.

First, we discuss the scenario that $Q^{N^*} > 2\mu$. The demand follows the normal distribution $X \sim N(60, 150^2)$, and the values of other variables remain the same except w and b. Furthermore, the profits of the whole supply chain, the retailer, and the manufacturer are shown in Figures 7–9, respectively.

From Figure 7, the lower the wholesale price is and the higher the buyback price is, the greater the total profit is. This is because of the fact that a lower w can encourage the retailer to increase her ordering quantity. We find it difficult to make the ordering quantity $F^{-1}[(p-w)/(p-b)]$ equal to $F^{-1}[(p-c)/(p-s_M)]$. Therefore, in order to increase the profit of the whole supply chain, the manufacturer should offer incentives (such as improving the buyback price and reducing the wholesale price) on the premise that his profit is higher than that in the basic model.

From Figures 8 and 9, when the incentives are larger, that is, when w is smaller and b is bigger, apparently, the retailer's profit increases. As is shown in Figure 8, when $w = 8.5$ and $b = 4.5$, the retailer's profit and the profit of the whole supply chain are significantly higher than other cases, but the manufacturer's profit is lower than other cases.

Note that as long as a is in a reasonable range, the supply chain can be coordinated. However, when a exceeds a threshold, the whole supply chain cannot realize the Pareto improvement, and the reason is that since the manufacturer's improvement measures have weaker impact on the retailer, he considers his own profit and does not offer the retailer any incentive for avoiding or reducing the loss.

Next, we discuss the scenario of $Q^{N^*} < 2\mu$. The demand follows the normal distribution $N \sim (60, 20^2)$, and the values of other variables remain the same except w and b. The profits of the whole supply chain, the retailer, and the manufacturer are shown in Figures 10–12, respectively.

Figure 10 illustrates when a is low, increasing the incentive can increase the profit of the whole supply chain, and when a is high, the incentive measure has little effect. Therefore, the manufacturer does not need to take more measures to support the retailer when a is high.

The results of Figures 11 and 12 indicate the impact of the overconfident level on expected profits of the retailer and the manufacturer under different (w, b). First of all, the manufacturer's profit increases with a, and this is because the retailer's ordering quantity increases with a. To achieve Pareto improvement, when a is low, the manufacturer does not need to offer too many incentives; when a is not very high, the manufacturer should offer more incentives to ensure that the retailer's profit is higher than that in the basic model. When a exceeds a threshold, if the manufacturer sets $1-\lambda$ to a very high value, then the manufacturer's profit is less than that in the basic model; on the other hand, if the manufacturer sets $1-\lambda$ to a low value, the retailer's profit is less than that in the basic model.

We address the differences between this paper and Croson et al. [11]. First, different from the buyback contract and the wholesale price contract in Croson et al. [11], we analyze the cooperation mode and buyback mode. In this paper, the buyback price b is larger than 0, and strictly speaking, b is larger than the salvage value s_R. However, in their paper, the buyback price can be lower than 0. If it is negative, the retailer does not sell unsold products to the manufacturer. Secondly, in our paper, the manufacturer can make the retailer and the manufacturer achieve Pareto improvement by adjusting b and w. They do not consider the change of the retailer's and manufacturer's profit after the implementation of the contract.

6.3. Results Analysis. The comparison of the two modes: in the cooperation mode, the manufacturer and retailer can better exchange information and views; meanwhile, it enables

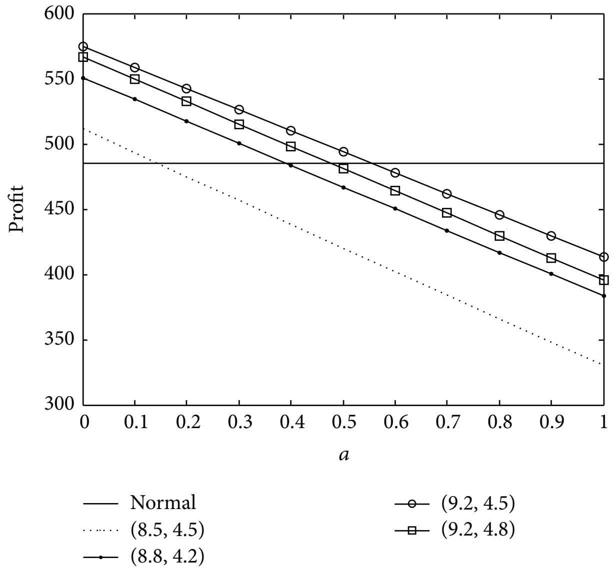

Figure 9: The manufacturer's expected profit under different w and b.

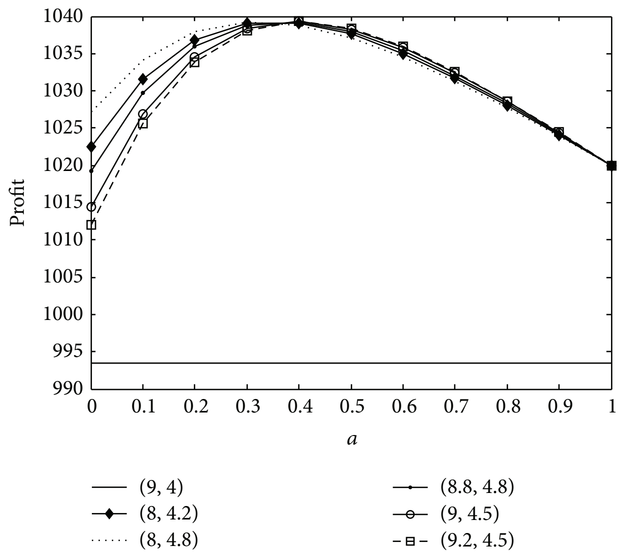

Figure 10: The expected profit of the supply chain under different w and b.

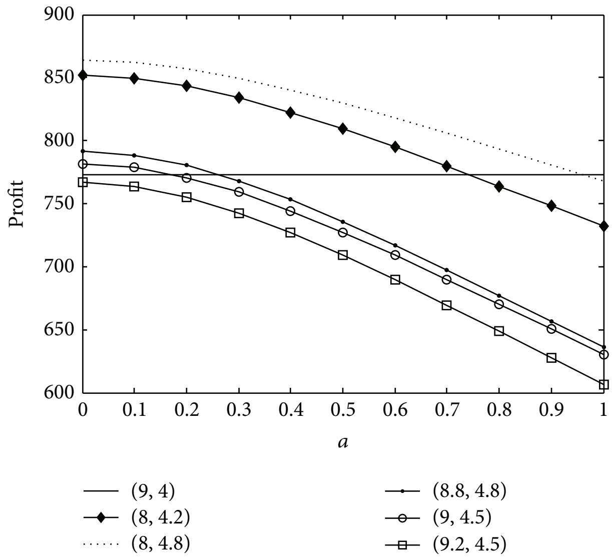

Figure 11: The retailer's expected profit under different w and b.

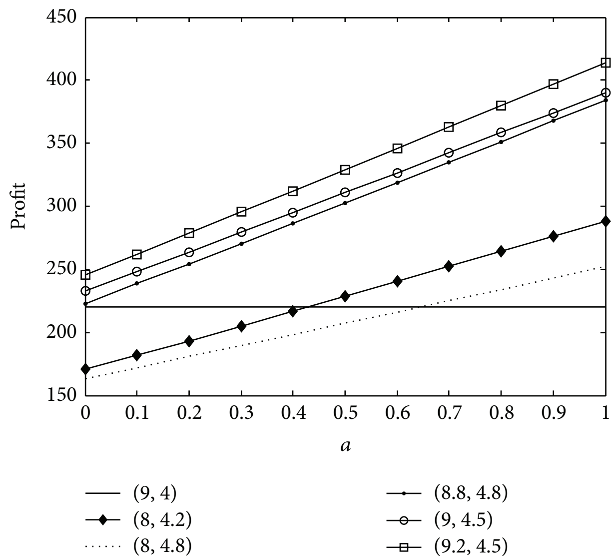

Figure 12: The manufacturer's expected profit under different w and b.

the retailer to listen to the manufacturer's recommendations, which is useful for reaching the Pareto improvement by reallocating the residual value of the unsold products; while in the buyback mode, the manufacturer can adjust the wholesale price and the buyback price, and the range of b is larger than $s_R + (1-\lambda)(s_M - s_R)$. In the buyback mode, in the scenario of $Q^{N^*} > 2\mu$, when a is high, the manufacturer can reduce the wholesale price w. In the scenario of $Q^{N^*} < 2\mu$, when a is high, the manufacturer can increase the wholesale price w. In the cooperation mode, the manufacturer can do little when a is high.

In the wholesale price contract, only w can be changed, while in the buyback contract, not only w but also b can be changed. In the wholesale price contract, the manufacturer may realize Pareto improvement by reducing w when a is low. In the buyback mode, w can be increased or reduced, and the supply chain can realize the Pareto improvement when a is low, and the manufacturer can also gain more profits by changing (w, b), and it is difficult in the wholesale price contract. As the retailer's overconfident behavior may cause more ordering, the retailer must dispose of them by herself in the wholesale price contract; thus, we take the buyback contract to share the supply chain risk. In short, compared with the wholesale price contract and the cooperation mode, the buyback mode is the optimal choice for the manufacturer.

7. Conclusions

Overconfident behavior has largely been overlooked in the supply chain management literature. This paper attempts to fill the gap. Our premise is that the retailer has overconfident behavior on the market demand. In this setting, we start with a basic model and analyze the retailer's optimal ordering decision. In the model of overconfident newsvendor, we analyze the overconfident retailer's optimal ordering decision and compare the optimal ordering decisions in the traditional and overconfident newsvendor models. We find that when $Q^{N^*} > 2\mu$, the expected profit of the whole supply chain in the overconfident newsvendor model is less than that in the basic newsvendor model, and with the retailer's overconfidence level increasing, the expected profit of the whole supply chain decreases. When $Q^{N^*} < 2\mu$, with the retailer's overconfidence level increasing, the expected profit of the whole supply chain in the overconfident newsvendor model has two cases: it is either higher or higher first and then lower than that in the basic newsvendor model. In other words, overconfident behavior does not necessarily damage the supply chain.

In terms of ordering deviation and profit losses caused by the retailer's overconfident behavior, we discuss two mechanisms: the cooperation mode and the buyback mode. Besides, we reallocate the residual value of the unsold products in them creatively in the cooperation mode and analyze the ordering decision by the Stackelberg game in the two modes. Moreover, we document the results of a numerical study to further illustrate the effects of the overconfident level on the ordering quantity and profits. We find that when a is low, the supply chain can achieve Pareto improvement by reasonable incentive. When a exceeds a threshold, the whole supply chain cannot realize the Pareto improvement, and at this time since the manufacturer's improvement measures have weaker impact on the retailer, the retailer's ordering decision cannot make the whole supply chain sustainable development. Therefore, it is harmful for the supply chain when the overconfident level is high.

There are several directions for future research. First, the manufacturer needs to design contracts to manage the retailer's overconfident behavior and coordinate supply chain and it would be an important topic in the future. Besides, in practice, the manufacturer generally franchises more than one retailer to sell their products; thus, considering two or more retailers to compete in selling could present interesting opportunities for future research.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Two-Echelon Inventory Optimization for Imperfect Production System under Quality Competition Environment

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This paper develops two integrated optimization models of two-echelon inventory for imperfect production system under quality competition environment, in which the vendor's production process is assumed to be imperfect, and JIT delivery policy is implemented to ship product from the vendor to the buyer. In the first model, product defect rate is fixed, and, in the second model, quality improvement investment is function of defect rate. The optimal policies of ordering quantity of buyer and shipment from vendor to buyer are obtained to minimize the expected annual total cost of vendor and buyer. Numerical examples are used to demonstrate the effectiveness and feasibility of the models. Sensitivity analysis is taken to analyze the impact of demand, production rate, and defect rate on the solution. Implications are highlighted in that both the vendor and the buyer can benefit from the vendor's investing in quality improvement.

1. Introduction

Nowadays, quality is a very important competition weapon; manufacturing firms must produce perfect goods in a perfect production system to compete with rivals. However, imperfect production condition exists in reality, such as imperfect supply system, imperfect machine maintenance, imperfect process, and imperfect workforce. Facing this imperfect production condition, firms on one hand need to carry out continuous improvement strategy to improve the production system and on the other hand need to make appropriate operation decision considering the imperfect production condition. This paper focuses on how to make the appropriate inventory decision for a two-echelon supply chain consisting of single vendor and single buyer, currently considering imperfect production condition of machine inspection and quality improvement strategy.

The traditional economic production quantity (EPQ) model assumes that the production process is perfect and no imperfect items are produced. However, in practice, the production facility is not failure-free and product quality is also not always perfect. In practice, the imperfect items would be rejected, repaired, and reworked, and thus extra costs are incurred. Several researches have been undertaken to study inventory models with imperfect quality. Rosenblatt and Lee [1] studied an imperfect production process with optimizing production cycle time. Their result shows that the optimal production cycle is shorter than that of the classical economic manufacturing quantity model. Salameh and Jaber [2] developed a classical economic order quantity model for items with imperfect quality. Wee et al. [3] extended Salameh and Jaber's [2] model with the permission of shortage. Cárdenas-Barrón [4] studied an economic production quantity model with rework process at a single-stage manufacturing system with planned backorders. Chung [5] revisited the work of Cárdenas-Barrón [4] and developed the sufficient and necessary condition for the existence of the solution.

Chang and Ho [6] revised Wee et al.'s [3] model and derived the exact solution to the optimal inventory model for items with imperfect quality and shortage backordering. J.-T. Hsu and L.-F. Hsu [7] developed an economic order quantity (EOQ) model with imperfect quality items, inspection errors, shortage backordering, and sales returns. The model presented a closed form solution to the optimal order size, the maximum shortage level, and the optimal order/reorder point. Khouja and Mehrez [8] developed an economic production lot size model with imperfect quality and variable production rate. Sana [9] extended Khouja and Mehrez's [8] model and investigated an economic production lot size model in an imperfect production system, in which the production facility may shift from an "in-control" state to an "out-of-control" state at any random time.

Besides the EPQ and EOQ models with imperfect items and processes, in the past, vendor-buyer integrated inventory management related to imperfect quality also has been studied. Huang [10] presented an integrated vendor-buyer inventory model for imperfect quality items. Goyal et al. [11] developed a simple approach for determining an optimal integrated vendor-buyer inventory policy for an item with imperfect quality. Huang [12] developed a model to determine an optimal integrated vendor-buyer inventory policy for flawed items in a JIT manufacturing environment. Ouyang et al. [13] proposed three methods to determine defect rate (crisp, fuzzy, and mixture of statistic and fuzzy) in an integrated vendor-buyer inventory model involving defective items. J.-T. Hsu and L.-F. Hsu [14, 15] developed a mathematical model to determine an integrated vendor-buyer inventory policy, where the vendor's production process is imperfect and produces a certain number of defective items.

In practice, product quality is usually related to the state of the production process. When the production process is in control state, the items would be in high quality level (perfect). As time goes on, the process state may deteriorate and imperfect items are produced. In recent years, some authors have studied production-inventory models with process inspection. Marek [16] considered the problem of optimization of a quality inspection process and presented a solution of optimal inspection operations in a production process. However, the inspection process is not relevant to the imperfect quality. Lee and Rosenblatt [17] developed an EMQ (economic manufacturing quantity) model of joint control of production cycles or manufacturing quantities and maintenance by inspection. Giri and Dohi [18] considered inspection process for imperfect production process where the process state shifts randomly. During each production run, the process is monitored through inspections to assess its state. J. T. Hsu and L. F. Hsu [19] developed an integrated vendor-buyer production-inventory model for items with imperfect quality and inspection errors. The production process is imperfect and produces a certain number of defective items. At the same time, the buyer's quality screening process is not perfect either. This model derives the optimal solution to integrated total annual cost. Khan et al. [20] adopted an approach similar to Salameh and Jaber [2] to study an optimal production/order quantity with imperfect processes. J.-T. Hsu and L.-F. Hsu [21] pointed out a contradiction between Lee's [22] model and their assumption and developed a modified model. Yoo et al. [23] studied the imperfect production and inspection processes in a stable production and inventory system. They developed the imperfect quality inventory models for various inspection options. Avinadav and Perlman [24] considered a batch production process that can be either stable or unstable, in which inspection is performed offline after production of the batch is completed. Chung [25] developed an integrated two-stage production-inventory deteriorating model for replenishment policy and inspection plan. Khan et al. [26] developed an integrated vendor-buyer inventory model accounting for quality inspection errors at the buyer's end.

Additionally, some researchers have studied the inventory models incorporating the issue of investment in product quality improvement. Porteus [27] developed a model that captured a relationship between quality and lot size and discussed three options for investing in quality improvements. Hong [28] incorporated joint investment in setup reduction and process quality improvement into a production system with imperfect production processes, where he assumed that setup reduction and process quality are functions of capital expenditure. Lee [22] developed a cost/benefit model for supporting investment strategies about inventory and preventive maintenance in an imperfect production system. Hou [29] considered an EPQ model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure. They studied the effects of an imperfect production process on the optimal production cycle time after capital investment strategies and process quality improvements are adopted. Yoo et al. [30] examined an imperfect production and inspection system and analyzed the solutions for different investment strategies.

The purpose of this paper is to extend Huang's [12]. First, in this paper, we consider process inspections during production run, while Huang's model does not consider this; second, we extend Huang's model to another model for consider quality improvement investment. The main contributions of this paper lie in the fact that we extended the previous study on the vendor-buyer inventory model by considering the imperfect production condition and quality improvement investment and provided implications for practitioners in inventory decision.

The rest of the paper is organized as follows. In Section 2, notations and assumptions are first presented and then the proposed model of single-vendor single-buyer inventory for defect items is formulated; again, the proposed model is extended to consider capital investment in quality improvement. In Section 3, numerical examples and sensitivity analysis are given. Finally, conclusions and future research directions are given in Section 4.

2. Formulation of the Model

In this paper, we assume a supply chain comprised of a vendor (manufacturer) and buyer (retailer); the vendor produces product and delivers it to a buyer (retailer). An equal lot size policy is adopted. The vendor's production process is assumed to be imperfect and a fraction of defective items are produced during a production run. The machine always starts in an in-control state but may shift to the out-of-control state at any random time and produce some defective items. To reduce the number of defective items, the vendor performs periodic machine inspections during a production run and the 100% quality screening for defective items is conducted by the buyer. Further, we extend the model to consider capital investment in quality improvement by vendor. The capital investment is assumed to follow the Porteus [27] logarithmic investment function.

The questions addressed in this paper are as follows: what is the optimal inventory policy for the integrated single-vendor single-buyer system with imperfect items? And what is the optimal investment strategy for quality improvement? In order to answer these questions, in this paper, we construct two integrated production-inventory models. In the first model, defect rate is assumed to be a fixed value, and, in the second model, we extend the first model to consider the vendor's investment in quality improvement, and investment is function of defect rate, and then the vendor's cost will include capital investment.

2.1. Assumptions and Notations. The following assumptions are used throughout this paper for formulation of the problem.

(1) The supply chain system consists of a single vendor and a single buyer for trading a single product.

(2) The vendor's production rate is constant and greater than the buyer's demand rate.

(3) The vendor's production system is imperfect. It always starts in an in-control state but may shift to the out-of-control state at any random time and produce some defective items.

(4) The vendor process performs periodic inspections during a production run. At each inspection if the machine is found in out-of-control state, then restoration is done. Otherwise, preventive maintenance is performed to enhance system reliability.

(5) The production process restoration cost is proportional to the detection delay time.

(6) After process restoration, the machine becomes as good as new.

(7) Process inspection and restoration times are negligible.

(8) The buyer performs a 100% quality screening for delivered products. The screening rate is much higher than the customer demand rate.

(9) In the second model, the vendor conducts a capital investment to improve product quality; the investment cost is considered part of the total cost.

Notations

Parameters

P: Production rate of the vendor

D: Annual demand of the buyer

S_v: Vendor's setup cost per production run

S_b: Buyer's ordering cost per order

h_v: Vendor's unit holding cost

h_b: Buyer's unit holding cost

C_0: Process inspection cost

C_1: Preventive maintenance cost

r: Machine restoration cost per unit detection delay time

T_i: ith process inspection time, $i = 1, 2, \ldots, m$; $(T_1, T_2, \ldots, T_m)$ is the inspection time sequence

T: Time interval between two successive deliveries to the buyer

T_m: Vendor's production time in a cycle

N_i: Number of defective items produced in the time interval $[T_{i-1}, T_i]$, $i = 1, 2, \ldots, m$; $T_0 = 0$

R_i: Process restoration cost in the time interval $[T_{i-1}, T_i]$, $i = 1, 2, \ldots, m$

τ: Elapsed time of a shift from the "in-control" state to the "out-of-control" state in vendor production process

$f(\cdot)$: Probability density function of the time to process shift from in-control state to out-of-control state

$F(\cdot)$: Probability distribution function of the time to process shift from in-control state to out-of-control state

d: Unit screening cost for defective items

k: Unit penalty cost for defective items

x: Quality screening rate for defective items

A: Buyer's transportation cost per shipment

α_0: The original percentage of defective items before investment

α: Percentage of defective items produced when process is in "out-of-control" state

η: Fractional opportunity cost

δ: The percentage of decrease in defective items per dollar increase in quality improvement investment.

Decision Variables

t_i: ith inspection interval for process in the vendor; that is, $t_i = T_i - T_{i-1}$ for all $i = 1, 2, \ldots, m$; $T_0 = 0$

n: Number of shipments per lot from the vendor to the buyer, a positive integer

m: Number of process inspections during each production run

Q: Shipment size from the vendor to the buyer.

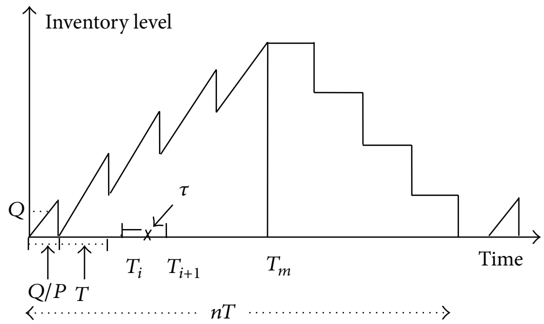

FIGURE 1: Production inventory of vendor.

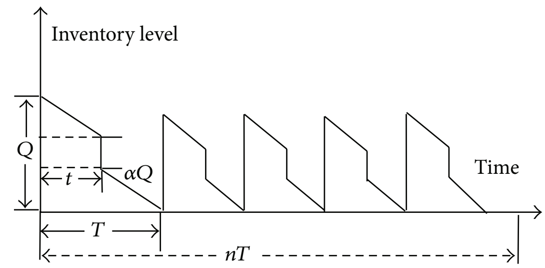

FIGURE 2: Inventory level variation of buyer in a cycle.

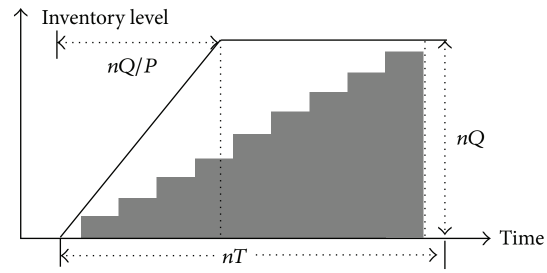

FIGURE 3: Inventory accumulation of vendor in a cycle.

2.2. Mathematical Model. The vendor's production-inventory pattern is shown in Figure 1. Since the delivery of product follows a JIT small lot size delivery policy, each time one delivery quantity Q is shipped to buyer, the production rate is P, and the number of deliveries during one production cycle is n, the production cycle time is T, and we suppose that m times process inspections are carried out during a production run. Therefore, process inspections are performed at times $T_1, T_2, \ldots, T_m$. At each process inspection if the machine is found in out-of-control state, then restoration is done. Otherwise, preventive maintenance is performed to enhance system reliability.

Each time, a lot of size Q is delivered from the vendor to the buyer, and it is assumed that each lot contains a percentage of defective items, α. After the items are delivered to the buyer, the buyer performs a 100% quality screening to identify the defective items with a screening rate of x and discards the defective items at the end of screening process. A typical configuration of the buyer's inventory level fluctuation is shown in Figure 2, where T is the order cycle length, αQ is the number of defective items withdrawn from inventory, and t is the total screening time of Q units.

2.2.1. The Integrated Decision of the Vendor and the Buyer

(1) The Vendor's Cost per Unit Time. Figure 3 shows the accumulation of vendor's inventory in a cycle. The shaded rectangles are the total inventory delivered to the buyer. Following the method in Huang [12], the vendor's holding inventory area equals the sum of the areas of triangle and rectangle minus shaded area.

Thus, the vendor's inventory holding cost per unit time can be obtained as

$$\begin{aligned}\text{Holding cost} \\ &= h_v \left\{ nQ\left(\frac{Q}{P} + (n-1)T\right) - \frac{nQ(nQ/P)}{2} \right. \\ &\qquad \left. - T\left[Q + 2Q + \cdots + (n-1)Q\right] \right\} \\ &= \frac{h_v n Q^2}{2}\left\{\frac{2-n}{P} + \frac{(n-1)(1-\alpha)}{D}\right\}.\end{aligned} \tag{1}$$

We define τ as the elapsed time of a shift from the "in-control" state to "out-of-control" state, and τ is a random variable. Inspections are undertaken at time T_i and either restoration or preventive maintenance is carried out at each inspection; therefore, $(t_i - \tau)$ is the detection delay time in the time interval $[T_{i-1}, T_i]$, $i = 1, 2, \ldots, m$, and during this delay period some defective items are produced. The vendor's restoration cost R_i in $[T_{i-1}, T_i]$ is given by

$$R_i = \begin{cases} 0, & \text{if } \tau \geq t_i, \\ r(t_i - \tau), & \text{if } \tau < t_i. \end{cases} \tag{2}$$

Thus, the vendor's expectation restoration cost in the time interval $[T_{i-1}, T_i]$ is $E[R_i] = \int_0^{t_i} r(t_i - \tau) f(\tau) d\tau$, and the vendor's expected total restoration cost is

$$\sum_{i=1}^{m} E[R_i] = \sum_{i=1}^{m} \int_0^{t_i} r(t_i - \tau) f(\tau) d\tau. \tag{3}$$

The number of defective items (N_i) produced in the time interval $[T_{i-1}, T_i]$ is

$$N_i \approx \begin{cases} 0, & \text{if } \tau \geq t_i \\ \alpha P(t_i - \tau), & \text{if } \tau < t_i \end{cases} \tag{4}$$

and the expected number of defective items produced in the time interval $[T_{i-1}, T_i]$ is $E(N_i) = \int_0^{t_i} \alpha P(t_i - \tau) f(\tau) d\tau$. The vendor's defective items production cost or penalty cost is $k\sum_{i=1}^{m} E(N_i)$, total inspection cost is mC_0, and preventive maintenance cost is $\sum_{i=1}^{m} C_1 \overline{F}(t_i)$.

The vendor's total cost in a cycle is the sum of the setup cost, inspection cost, holding cost, defective items penalty cost, restoration cost, and preventive maintenance cost. We can write the total cost of the vendor as

$$\begin{aligned}\mathrm{ETC}_V(t_i,n,Q) &= \frac{1}{nT}\left\{S_v + mC_0 + \frac{h_v n Q^2}{2}\right.\\ &\quad\times\left(\frac{2-n}{P}+\frac{(n-1)(1-\alpha)}{D}\right) + k\sum_{i=1}^{m}E(N_i)\\ &\quad\left.+\sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau + \sum_{i=1}^{m}C_1\overline{F}(t_i)\right\}.\end{aligned} \tag{5}$$

(2) The Buyer's Cost per Unit Time. The buyer's total cost in a cycle is the sum of ordering cost, holding cost, transportation cost, and quality screening cost. The buyer's holding cost per unit time is

$$\frac{h_b}{T}\left(\frac{Q(1-\alpha)T}{2}+\frac{\alpha Q^2}{x}\right) = h_b\left[\frac{Q(1-\alpha)}{2}+\frac{\alpha D Q}{x(1-\alpha)}\right]. \tag{6}$$

The buyer's ordering cost per unit time $= S_b/nT = S_bD/n(1-\alpha)Q$. The buyer's transportation cost per unit time is $A/T = AD/(1-\alpha)Q$. The buyer's quality screening cost per unit time is $dQ/T = dD/(1-\alpha)$. Thus, we can write the expected total cost of the buyer per unit time as

$$\begin{aligned}\mathrm{ETC}_B(n,Q) &= h_b\left[\frac{Q(1-\alpha)}{2}+\frac{\alpha D Q}{x(1-\alpha)}\right]+\frac{S_bD}{n(1-\alpha)Q}\\ &\quad+\frac{AD}{(1-\alpha)Q}+\frac{dD}{(1-\alpha)}.\end{aligned} \tag{7}$$

(3) The Integrated Vendor-Buyer Inventory Model. Using (5) and (7), the expected total cost of the integrated inventory system can be obtained as

$$\begin{aligned}\mathrm{ETC}(t_i,n,Q) &= \mathrm{ETC}_V(t_i,n,Q)+\mathrm{ETC}_B(n,Q)\\ &= \frac{1}{nT}\left\{S_v + mC_0 + \frac{h_v n Q^2}{2}\right.\\ &\quad\times\left(\frac{2-n}{P}+\frac{(n-1)(1-\alpha)}{D}\right)\\ &\quad+k\sum_{i=1}^{m}\int_0^{t_i}\alpha P(t_i-\tau)f(\tau)\,d\tau\\ &\quad+\sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau\\ &\quad\left.+\sum_{i=1}^{m}C_1\overline{F}(t_i)\right\}+h_b\left[\frac{Q(1-\alpha)}{2}+\frac{\alpha DQ}{x(1-\alpha)}\right]\\ &\quad+\frac{S_bD}{n(1-\alpha)Q}+\frac{AD}{(1-\alpha)Q}+\frac{dD}{(1-\alpha)}.\end{aligned} \tag{8}$$

Putting $T=(1-\alpha)Q/D$ in (8), we obtain

$$\begin{aligned}\mathrm{ETC}(t_i,n,Q) &= \frac{D}{n(1-\alpha)Q}\\ &\quad\times\left\{S_v + mC_0 + \frac{h_v n Q^2}{2}\right.\\ &\quad\times\left(\frac{2-n}{P}+\frac{(n-1)(1-\alpha)}{D}\right)\\ &\quad+k\sum_{i=1}^{m}\int_0^{t_i}\alpha P(t_i-\tau)f(\tau)\,d\tau\\ &\quad\left.+\sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau+\sum_{i=1}^{m}C_1\overline{F}(t_i)\right\}\\ &\quad+h_b\left[\frac{Q(1-\alpha)}{2}+\frac{\alpha DQ}{x(1-\alpha)}\right]\\ &\quad+\frac{S_bD}{n(1-\alpha)Q}+\frac{AD}{(1-\alpha)Q}+\frac{dD}{(1-\alpha)}.\end{aligned} \tag{9}$$

In the above equation, although the decision variable n is an integer, we can slack it as continuous variable, and then we take the derivate of the above equation to obtain the solution which is shown in the following steps.

For convenience of formulation, we define

$$\begin{aligned}G(\cdot) &= k\sum_{i=1}^{m}\int_0^{t_i}\alpha P(t_i-\tau)f(\tau)\,d\tau\\ &\quad+\sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau+\sum_{i=1}^{m}C_1\overline{F}(t_i),\end{aligned} \tag{10}$$

and then, taking the first derivative of ETC with respect to n, we have

$$\begin{aligned}\frac{\partial \mathrm{ETC}}{\partial n} &= -\frac{D}{n^2(1-\alpha)Q}\left\{S_v+mC_0+S_b+G(\cdot)\right\}\\ &\quad+\frac{D}{(1-\alpha)}\frac{h_vQ}{2}\left(\frac{1}{D}-\frac{1}{P}\right).\end{aligned} \tag{11}$$

The total cost function ETC is convex in n, since it is easy to see that

$$\frac{\partial^2 \mathrm{ETC}}{\partial n^2} = \frac{2D}{n^3(1-\alpha)Q}\left\{S_v+mC_0+S_b+G(\cdot)\right\}>0 \quad \forall n\geq 1. \tag{12}$$

Taking the first derivative of ETC with respect to Q, we have

$$\begin{aligned}\frac{\partial \mathrm{ETC}}{\partial Q} &= -\frac{D}{n(1-\alpha)Q^2}\left\{S_v+mC_0+S_b+An+G(\cdot)\right\}\\ &\quad+\frac{(2-n)Dh_v}{(1-\alpha)P}+\frac{(n-1)h_v}{2}+h_b\left(\frac{1}{2}+\frac{\alpha D}{x(1-\alpha)}\right).\end{aligned} \tag{13}$$

For fixed values of n and t_i, ETC can also be shown to be convex in Q, since

$$\frac{\partial^2 \mathrm{ETC}}{\partial Q^2} = \frac{2D}{n(1-\alpha)Q^3}\{S_v + mC_0 + S_b + An + G(\cdot)\} > 0. \tag{14}$$

Taking the first derivative of ETC with respect to t_i, we have

$$\frac{\partial \mathrm{ETC}}{\partial t_i} = \frac{D}{n(1-\alpha)Q}\frac{\partial G(\cdot)}{\partial t_i},$$
$$\frac{\partial G(\cdot)}{\partial t_i} = k\alpha P\int_0^{t_i} f(\tau)\,d\tau + r\int_0^{t_i} f(\tau)\,d\tau - C_1 f(t_i). \tag{15}$$

Therefore,

$$\frac{\partial \mathrm{ETC}}{\partial t_i} = \frac{D}{n(1-\alpha)Q}\left\{(k\alpha P + r)\int_0^{t_i} f(\tau)\,d\tau - C_1 f(t_i)\right\},$$

$$\frac{\partial^2 \mathrm{ETC}}{\partial t_i^2} = \frac{D}{n(1-\alpha)Q}\left\{(k\alpha P + r) f(t_i) - C_1 f'(t_i)\right\}, \quad i = 1,\ldots,m,$$

$$\begin{aligned}\frac{\partial^2 \mathrm{ETC}}{\partial n\partial Q} = \frac{D}{n^2(1-\alpha)Q^2}\Bigg\{ & S_v + mC_0 + S_b \\ & + k\sum_{i=1}^{m}\int_0^{t_i}\alpha P(t_i-\tau) f(\tau)\,d\tau \\ & + \sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau) f(\tau)\,d\tau\Bigg\} \\ & + \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right) > 0,\end{aligned}$$

$$\begin{aligned}&\frac{\partial^2 \mathrm{ETC}}{\partial n^2}\cdot\frac{\partial^2 \mathrm{ETC}}{\partial Q^2} - \left(\frac{\partial^2 \mathrm{ETC}}{\partial n\partial Q}\right)^2 \\
&= \big(4D^2(S_v + mC_0 + S_b + G(\cdot)) \\
&\quad \times (S_v + mC_0 + S_b + G(\cdot) + An)\big)\left(n^4(1-\alpha)^2Q^4\right)^{-1} \\
&\quad - \frac{D^2}{n^4(1-\alpha)^2Q^4} \\
&\quad \times \left[S_v + mC_0 + S_b + G(\cdot) + \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right)\right]^2 \\
&= \frac{D^2}{n^4(1-\alpha)^2Q^4}\Bigg\{4(S_v + mC_0 + S_b + G(\cdot)) \\
&\quad \times (S_v + mC_0 + S_b + G(\cdot) + An) \\
&\quad - \Bigg[S_v + mC_0 + S_b + G(\cdot) \\
&\quad + \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right)\Bigg]^2\Bigg\} \\
&> \frac{D^2}{n^4(1-\alpha)^2Q^4} \\
&\quad \times \Bigg\{\Big[2(S_v + mC_0 + S_b + G(\cdot))\Big]^2 \\
&\quad - \left[S_v + mC_0 + S_b + G(\cdot) + \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right)\right]^2\Bigg\} \\
&= \frac{D^2}{n^4(1-\alpha)^2Q^4} \\
&\quad \times \left[3(S_v + mC_0 + S_b + G(\cdot)) + \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right)\right] \\
&\quad \cdot \left[S_v + mC_0 + S_b + G(\cdot) - \frac{h_v}{2}\left(1 - \frac{D}{P(1-\alpha)}\right)\right] > 0.
\end{aligned} \tag{16}$$

Note that if $(k\alpha P + r)f(t_i) - C_1 f'(t_i) > 0$, then we have $\partial^2\mathrm{ETC}/\partial t_i^2 > 0$, $\partial^2\mathrm{ETC}/\partial n^2 > 0$, $\partial^2\mathrm{ETC}/\partial Q^2 > 0$, and $(\partial^2\mathrm{ETC}/\partial n^2)\cdot(\partial^2\mathrm{ETC}/\partial Q^2) - \left(\partial^2\mathrm{ETC}/\partial n\partial Q\right)^2 > 0$ implying that, for any given value of n, the total cost function is convex. Therefore, there exists a unique value of Q that minimizes (8).

The optimal lot size is given by

$$Q^*(n) = \sqrt{\frac{D(S_v + mC_0 + S_b + An + G(\cdot))}{n(1-\alpha)M(\cdot)}}, \tag{17}$$

where

$$M(\cdot) = \frac{(2-n)Dh_v}{(1-\alpha)P} + \frac{(n-1)h_v}{2} + h_b\left(\frac{1}{2} + \frac{\alpha D}{x(1-\alpha)}\right). \tag{18}$$

Note that the condition $(k\alpha P + r)f(t_i) - C_1 f'(t_i) > 0$ is clearly satisfied for uniform distribution, since $(k\alpha P + r) > 0$, $f(t_i) > 0$, $f'(t_i) = 0$.

From the optimality condition $\partial\mathrm{ETC}/\partial t_i = 0$, we can get

$$(k\alpha P + r)\int_0^{t_i} f(\tau)\,d\tau - C_1 f(t_i) = 0. \tag{19}$$

Using (19), we find out the optimal value of t_i^* as follows. Equation (19) can be extended as follows:

$$\begin{aligned}(k\alpha P + r)\int_0^{t_1} f(\tau)\,d\tau - C_1 f(t_1) = 0,\\
(k\alpha P + r)\int_0^{t_2} f(\tau)\,d\tau - C_1 f(t_2) = 0,\\
\vdots\\
(k\alpha P + r)\int_0^{t_n} f(\tau)\,d\tau - C_1 f(t_n) = 0.\end{aligned} \tag{20}$$

If $f(x)$ follows uniform probability distribution, that is,

$$f(x) = \begin{cases}\dfrac{1}{b}, & \text{if } 0 < x < b\\ 0, & \text{otherwise.}\end{cases} \tag{21}$$

From (19) we can obtain

$$t_1^* = t_2^* = \cdots = t_n^* = \frac{C_1}{k\alpha P + r}. \tag{22}$$

This result shows that the optimal machine inspection interval of each time is a fixed value, which is dependent on the parameters of the machine maintenance and inspection. In other words, when the inspection interval of each time is constant, the total cost is minimum. Since production run is nQ/P, we can obtain $m = nQ/Pt_i^*$.

In order to determine the optimal n that minimizes $\mathrm{ETC}(n, Q)$, the following procedure can be implemented.

Algorithm 1. Consider the following.

Step 1. For a range of values of n, determine the corresponding $Q^*(n)$ using (17) and compute $\mathrm{ETC}(n, Q^*(n))$ by substituting $Q^*(n)$ into (9).

Step 2. Derive the optimal value of n, denoted by n^*, such that $\mathrm{ETC}(n, Q^*(n)) \le \mathrm{ETC}(n-1, Q^*(n-1))$ and $\mathrm{ETC}(n, Q^*(n)) \le \mathrm{ETC}(n+1, Q^*(n+1))$.

Once we derive out n^*, the optimal size of a production batch can be obtained by $n^*Q^*(n)$.

2.2.2. The Integrated Model with Capital Investment for Quality Improvement. We now suppose that the vendor invests some capital in order to reduce the number of defective items produced. We assume a logarithmic investment function as $I(\alpha) = (\eta/\delta)\ln(\alpha_0/\alpha)$ [27], where α_0 is the defect rate before quality improvement, η means the fractional opportunity cost, and δ means the percentage of decrease in defective items per dollar increase in investment.

In this situation, the new expected total cost of the integrated model equals the total cost of the system without investment (i.e., the first model) plus the capital investment by the vendor.

Then, the expected total cost of the integrated model can be obtained as

$$\begin{aligned}
\mathrm{ETC}_I &= \frac{D}{n(1-\alpha)Q}\Bigg\{S_v + mC_0 + \frac{h_v nQ^2}{2} \times \left(\frac{2-n}{P} + \frac{(n-1)(1-\alpha)}{D}\right) \\
&\quad + k\sum_{i=1}^{m}\int_0^{t_i}\alpha P(t_i-\tau)f(\tau)\,d\tau + \sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau + \sum_{i=1}^{m}C_1\overline{F}(t_i)\Bigg\} \\
&\quad + h_b\left[\frac{Q(1-\alpha)}{2} + \frac{\alpha DQ}{x(1-\alpha)}\right] \\
&\quad + \frac{S_bD}{n(1-\alpha)Q} + \frac{AD}{(1-\alpha)Q} + \frac{dD}{(1-\alpha)} + \frac{\eta}{\delta}\ln\frac{\alpha_0}{\alpha},
\end{aligned} \tag{23}$$

where α_0 is the original defect rate, a fixed value. α is the defect rate after investment; in this model, it is a decision variable; that is, we need to determine the optimal defect rate after quality improvement investment.

In order to obtain the optimal investment, taking the first derivative of ETC_I with respect to α, we have

$$\begin{aligned}
\frac{\partial \mathrm{ETC}_I}{\partial\alpha} &= \frac{D}{n(1-\alpha)^2Q}\Bigg\{S_v + mC_0 + \frac{h_v nQ^2}{2}\frac{2-n}{P} \\
&\quad + \sum_{i=1}^{m}\int_0^{t_i} r(t_i-\tau)f(\tau)\,d\tau + \sum_{i=1}^{m}C_1\overline{F}(t_i)\Bigg\} \\
&\quad + \frac{Dk}{n(1-\alpha)^2Q}\sum_{i=1}^{m}\int_0^{t_i} P(t_i-\tau)f(\tau)\,d\tau \\
&\quad - \frac{h_bQ}{2} + \frac{h_bDQ}{x(1-\alpha)^2} + \frac{S_bD}{n(1-\alpha)^2Q} \\
&\quad + \frac{AD}{(1-\alpha)^2Q} + \frac{dD}{(1-\alpha)^2} - \frac{\eta}{\delta\alpha}.
\end{aligned} \tag{24}$$

Taking the second derivate of the total cost with respect to Q, n, t_i, it is clear that the ETC_I is still a convex function in Q, n, t_i (since these operations are similar to those operations in the first model, we omit them).

To obtain the optimal solution, we adopt the iterative algorithm proposed by Ben-Daya and Hariga [31]. Using this algorithm, one can find the optimal solution in the following procedure.

Algorithm 2. Consider the following.

Step 1. Set $\mathrm{ETC}_I^* = \infty$, $n = 1$.

Step 2. Set $\alpha = \alpha_0$ and compute Q_0 from (17).

Step 3. Compute α from (24) to zero. If $\alpha \ge \alpha_0$, set $\alpha = \alpha_0$. Update t_i with α from (19).

Step 4. Compute Q from (17) using α, t_i. If $|Q - Q_0| = 0$, compute ETC_I and go to Step 5. Otherwise, set $Q = Q_0$ and go to Step 3.

Step 5. If $\mathrm{ETC}_I^* \ge \mathrm{ETC}_I$, set $\mathrm{ETC}_I^* = \mathrm{ETC}_I$, $Q^* = Q$, $\alpha^* = \alpha$, and $n = n + 1$ and go to Step 2. Otherwise, $n^* = n - 1$ and stop.

Note that $\partial\mathrm{ETC}_I/\partial\delta < 0$ and $\partial\mathrm{ETC}_I/\partial\alpha_0 > 0$, which indicate that an increase in δ leads to a more reduction in the number of defective items per dollar increase in investment.

Table 1: Optimal values of n and m for integrated vendor-buyer model.

n	m	$Q^*(n)$	$\mathrm{ETC}(n, Q^*(n))$
1	14	147.08	950.84
2	22	109.14	829.69
3	27	91.12	793.71
4	32	79.73	781.86
5^*	36^*	71.57^*	779.83^*
6	39	65.32	782.51
7	42	60.32	787.67
8	45	56.18	794.21
9	47	52.69	801.51
10	50	49.69	809.24

*The optimal solution.

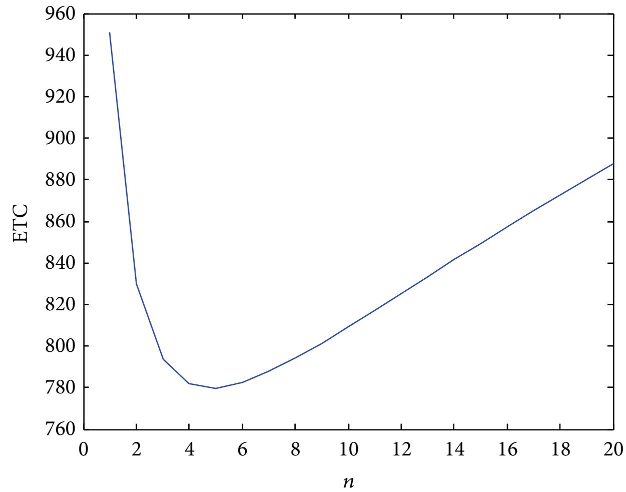

Figure 4: The behavior of ETC and n.

3. Numerical Examples and Managerial Implications

For numerical study, we consider a single-vendor single-buyer system; the following parameter values are set: production rate $P = 320$, demand rate $D = 100$, setup cost of vendor $S_v = 300$, ordering cost of buyer $S_b = 100$, inventory holding cost of vendor $h_v = 2$, inventory holding cost of buyer $h_b = 5$, transportation cost per shipment from vendor to buyer $A = 25$, the screening rate for defective items in buyer $x = 215$, per unit screening cost $d = 0.5$, machine restoration cost per unit detection delay time for vendor $r = 5$, process inspection cost and preventive maintenance cost $C_0 = 2$ and $C_1 = 15$, the per unit penalty cost for defective item $k = 30$, and the original percentage of defective items without quality improvement investment $\alpha = 0.05$. We also suppose that the time of the process shifts from in-control state to out-of-control state follows uniform probability distribution; that is,

$$f(x) = \begin{cases} \dfrac{1}{b}, & \text{if } 0 < x < b \\ 0, & \text{otherwise.} \end{cases} \quad (25)$$

3.1. Optimization Solution without Quality Improvement Investment. In this case, when $b = 1$, we obtain the results as given in Table 1. Figure 4 shows that the total cost function ETC is convex in n. We solve this problem by using Algorithm 1. The optimal solution is $n^* = 5$, $Q^* = 71.5798$, $t_i^* = 0.003$, and $m^* = 36$, and the integrated average total cost is 779.8304. The optimal size of a production batch is 357.899.

3.2. Optimization Solution with Quality Improvement Investment. Now, we consider the second one: integrated model with capital investment for reduction of defective items. In the first model, we additionally consider the parameter values $\alpha_0 = 0.05$, $\eta = 2$, and $\delta = 0.02$. Table 2 shows that when the original percentage of defective items is high, the effect of investment in quality improvement will be more obvious. This means that the total cost can be reduced more. This benefit will attract vendor to make an investment to reduce the defective items and make more profit.

From the numerical studies, it is shown that the optimal shipment size from the vendor to the buyer and the percentage of defective items decrease after quality investment. However, which one is better for the buyer?

Consider

$$\mathrm{ETC}_B(n, \alpha, Q) = h_b\left[\frac{Q(1-\alpha)}{2} + \frac{\alpha D Q}{x(1-\alpha)}\right] + \frac{S_b D}{n(1-\alpha)Q} + \frac{AD}{(1-\alpha)Q} + \frac{dD}{(1-\alpha)}. \quad (26)$$

Assume the difference between the total costs of buyer in the integrated model without and with quality improvement investment is $\Delta\mathrm{ETC}_B = \mathrm{ETC}_{1B}(\alpha, Q) - \mathrm{ETC}_B(\alpha, Q)$.

When $\Delta\mathrm{ETC}_B(\alpha, Q) < 0$, the investment is better for the buyer; otherwise there is no benefit for the buyer with investment.

Table 3 shows the comparison of annual cost for the buyer with and without investment. The numerical results show that the more the vendor invests in quality improvement, the more the benefit the buyer can obtain from the buyer's investment.

3.3. Sensitivity Analysis. In this subsection, we conduct sensitivity analysis for three important parameters: demand rate, production rate, and defect rate of product. We analyze how these three parameters impact the solution (in here, we take the first model as analysis basis).

(1) The Effect of Demand on Optimization Solution. We set the change range of demand rate from 100 to 200. Under this change range, the total cost increases from 779.8304 to 1313.5496 and the optimal shipment size increases from 71.5798 to 163.4827. The optimal number of shipments per lot from the vendor to the buyer decreases from 5 to 3; the optimal number of machine inspections during each production

Table 2: A comparison of the integrated model with and without investments.

	Model without investment		Model with investment		α	Cost difference	
α_0	Q^*	ETC^*	Q^*	ETC_I^*		ΔTC	$\%TC$[a]
0.15	189.06	1466.58	178.62	1224.23	0.093	242.35	16.52
0.20	237.14	1879.69	169.81	1201.90	0.081	677.78	36.05
0.25	318.91	2354.98	162.86	1186.33	0.072	1168.65	49.62
0.30	369.23	2871.18	157.33	1176.33	0.064	1694.85	59.02
0.35	421.13	3476.24	146.23	1243.30	0.100	2232.93	64.23

[a]Note: $\%TC = (ETC^* - ETC_I^*)/ETC^*$.

Table 3: A comparison for the buyer with and without investments[a].

	Model without investment		Model with investment		α	Cost difference	
α_0	Q^*	$ETC_B^*(\alpha, Q)$	Q^*	$ETC_{1B}(\alpha, Q)$		ΔTC	$\%ETC_B^*$
0.15	189.06	559.96	178.62	524.36	0.093	35.60	6.35
0.20	237.14	693.11	169.81	501.79	0.081	191.31	27.60
0.25	318.91	926.47	162.86	484.26	0.072	442.20	47.73
0.30	369.23	1099.13	157.33	470.37	0.064	628.75	57.20
0.35	421.13	1301.41	146.23	448.96	0.100	852.45	65.50

[a]Note: $\%ETC_B^* = (ETC_B^*(\alpha, Q) - ETC_{1B}(\alpha, Q))/ETC_B^*(\alpha, Q)$.

Table 4: Effect of demand rate on the optimal solution.

D	n^*	m^*	Q^*	ETC^*
100	5	36	71.57	779.83
120	4	37	93.93	889.78
140	4	44	109.24	997.26
160	4	50	126.00	1105.95
180	3	44	148.07	1211.13
200	3	49	163.48	1313.54

Table 5: Effect of production rate on the optimal solution.

P	n^*	m^*	Q^*	ETC^*
280	5	37	74.70	779.96
300	5	36	72.99	779.78
320	5	36	71.57	779.83
340	5	35	70.39	780.00
360	5	35	69.37	780.26

run increases first and then decreases. The result is shown in Table 4. Summarily, demand increases will cause the increase in total cost and ordering quantity.

(2) The Effect of Production Rate on Optimal Solution. For analyzing the effect of production rate on optimal solution, we set the production rate increases from 280 to 360. Table 5 shows the result.

From Table 5, we can see that, with the increase in production rate, the integrated total cost increases; at the same time, the optimal shipment size decreases, and the optimal number of machine inspections during each production run decreases. But the optimal number of shipments per lot from the vendor to the buyer remains unchanged.

Table 6: Effect of α on the optimal solution.

α	n^*	m^*	Q^*	ETC^*
0.01	6	5	46.55	573.46
0.03	6	20	55.20	670.02
0.05	5	36	71.57	779.83
0.07	4	52	92.49	900.95
0.10	3	76	126.91	1099.41

(3) The Effect of Defect Rate on Optimal Solution. We then analyze the effect of defect rate on optimal solution. From Table 6, one can see that, with an increase in the percentage of defective items, there is an increase in the number of inspections during each production run. This is intuitively true because when the vendor tries to reduce the defective items, more process inspections are needed during a production run. At the same time, the number of shipments per lot from the vendor to the buyer becomes less and the shipment size increases.

3.4. Managerial Implications. From the analytical model and numerical examples demonstration, we can conclude the following managerial implications for practitioners.

(1) Integration is a synchronization strategy in supply chain management; this paper demonstrates the benefit of integration optimization of production inventory of single vendor and single buyer in the supply chain. In practice, the vendor and the buyer should take joint action to optimize their operations.

(2) Our study reveals that quality improvement investment in the vendor not only benefits the vendor, but also benefits the buyer. Investment can reduce total cost of the vendor and the buyer; furthermore, the more the investment of quality improvement of the vendor is, the higher the benefit the buyer gains. This implies that quality improvement has significant impact on the vendor and the buyer inventory optimization. Therefore, inventory manager of the vendor should pay attention to the quality issue of stocked items.

4. Conclusions

This paper investigates two integrated single-vendor single-buyer inventory models, in which the vendor's production process is assumed to be imperfect, and JIT delivery policy is implemented to ship product from the vendor to the buyer. We analyze the two cases: one case is that the vendor has no quality improvement investment; the second case is that the vendor invests to improve quality.

We obtain the optimal number of shipments per lot from the vendor to the buyer and optimal machine inspections interval during each production run. We also calculate the optimal size of a production batch and the optimal shipment size from the vendor to the buyer.

The sensitivity of the key model-parameters, demand, production, and defect rate is examined. Analysis reveals that an increase in demand rate causes increase in the number of process inspections and decrease in the production lot size. It is also shown that an increase in the production rate causes an increase in shipment size but a decrease in the number of machine inspections, while the production lot size does not change. Further, we consider that the vendor carries out a capital investment for reduction of defective items. After investment, the percentage of defective items reduces. The investment can cause both the optimal shipment size and the integrated total cost decrease. Furthermore, the numerical results show that when the original percentage of defective items is high, the effect of investment becomes more obvious.

The future research of this paper can follow these directions. First, we can incorporate the machine reliability into the models and analyze how it affects the integrated decision. Second, the models can be extended to consider setup cost reduction in production and variable shipment size. Third, the models can also be extended to consider multiple buyers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Multiple Objective Fuzzy Sourcing Problem with Multiple Items in Discount Environments

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The selection of proper supply sources plays a vital role to maintain companies' competitiveness. In this study a multiple criteria fuzzy sourcing problem with multiple items in discount environment is considered as a multiple objective mixed integer linear programming problem. Fuzzy parameters are demand level and/or aspiration levels of objectives. Three objective functions are minimization of the total production and ordering costs, the total number of rejected units, and the total number of late delivered units, respectively. The model is developed for the all-units discount scheme. For the incremental discount and volume discount environment, modification requirements of the model are mentioned. The previously proposed interactive fuzzy approach combined with three fuzzy mathematical models is employed to obtain most satisfactory solution which is also a nondominated one. This study provides a realistic mathematical model and promising solution strategy to multiple item-single period sourcing problem in discount environment. Consideration of fuzziness makes the obtained nondominated solution implementable for the real cases.

1. Introduction

In many industries, raw material costs, components, and parts purchased from suppliers constitute a major amount of the overall production costs. The suppliers having the required performance levels in price, delivery, quality, and service strictly improve both cost and operating effectiveness of companies. Hence, the selection of proper suppliers plays a vital role to maintain companies' competitiveness. Selection decision of suppliers determine "how many and which suppliers are the best, *sourcing*," and "how much should be purchased from each selected supplier, *the order allocation among the suppliers*." The sourcing and order allocation is a multicriteria decision making problem which includes both qualitative and quantitative factors some of which may conflict. Dickson [1] identified 23 criteria that have been considered by purchasing managers in various supplier selection problems. In addition to multicriteria nature of the problem, informational vagueness because of the tangible and intangible factors of supplier selection problems is another significant fact which must be taken into account to reach effective and implementable solutions.

Another important aspect related with the real world supplier selection problems is to encounter price discounts offered by suppliers in order to encourage the buyers to select them and/or to buy larger quantities [2]. Although there are various kinds of discount contracts in practical applications, they can be investigated under two headings (Table 1) based on the physical and the informational flows in a supply chain [3]. According to the physical flow, the most common discount schemes are business volume discount and quantity discount depending on the volume being purchased and depending on the order size, respectively. Based on the informational flow in a supply chain, Sirias and Mehra [3] mentioned that lead time-dependent discount scheme is a promising strategy.

Business volume discount is used when more than one item is to be purchased. In this scheme supplier offers discounts on the total price of sales volume, not on the quantity or variety of the products purchased.

In the context of quantity discount, the sales volume of a product does not affect the prices and discounts of the other products [4]. In this scheme, single item or multiple items can be considered. Three common types of quantity discount

Table 1: Classification of discount schemes.

Discounts based on physical flows in a supply chain	Reference
Business volume discount	[7, 8]
Quantity discount	(1) Linear quantity discount [5, 6] (2) Cumulative-all-units quantity discount (AQD) (3) Noncumulative-incremental quantity discount (IQD) Detailed literature see in [3]
Bundling	[9–11]
Deferred rebates based on total value of order Deferred rebates based on the order quantity Marginal discounts based on the total value of the order	[12]
Discounts based on informational flows in a supply chain	
Lead-time dependent discount	Detailed literature see in [3]

strategies are linear quantity discount, all-units discount, and incremental-units discounts [5, 6]. The linear quantity discount strategy assumes that each supplier offers a linearly declining per unit price within the capacity range. Related with this strategy Burke et al. [5] mentioned that, for the case where each supplier has adequate capacity to meet the aggregate requirement of the customer, then the single sourcing will be the best decision for the customer when the objective function is to minimize the total costs. Hence in the literature quantity discount strategy has been often considered as if it can be either cumulative (all-units) or noncumulative (incremental). In all-units quantity discount (AQD), once an order size exceeds a predetermined quantity threshold, the suppliers can offer a discount that lowers the wholesale price on every unit purchased. There may be many such intervals, defined by the threshold values which are called price-break quantities. With a number of price breaks, the unit discounted price decreases as the quantity level increases. In incremental quantity discount (IQD), the discount rates are incrementally applied over the volumes for each interval; thus different prices are applied to the units belonging to different price breaks.

According to the physical flow another discount scheme is freight rate discount scheme offered by shippers [13, 14]. While quantity discount considers unit price, freight rate discount considers unit shipment and it can be offered according to all units (called all weights) or incremental strategies [15].

Apart from the business volume discount and quantity discounts, in the lead time-dependent discount scheme, supplier(s) offer a number of time intervals during the planning horizon to fulfill orders where in each interval more lead time corresponds to a price discount. A practical situation of such lead time dependent discount schemes can be found in chemical, semiconductor, and automotive industries where suppliers and customers engage in flexible quantity contracts that envision different pricing and capacity availabilities depending on lead times [16].

In this study the sourcing and order allocation problem is considered as a multiple objective decision making problem with vague parameters in price discount environment. Three objective functions are minimization of the total purchasing and ordering costs, the total number of rejected units, and the total number of late delivered units, respectively. Fuzziness stems from the problem parameters which are demand level and/or aspiration levels of objectives. Fuzzy parameters are described with linear membership functions according to the decision maker (DM)'s preference. All-units discount scheme is considered in the main mathematical model. For the incremental discount and volume discount environment, modification requirements of the model are mentioned. Three fuzzy mathematical models are employed separately to obtain most satisfactory solution which is also a nondominated one. The fuzzy decisions are defined by the add operator [17] in two of the models and in the third fuzzy model it is defined by the augmented max-min operator [18, 19].

The remainder of the paper is organized as follows. In Section 2, literature is reviewed in detail and the contribution of this study is revealed. In Section 3, the considered multiple objective supplier selection model is explained in crisp form. In Section 4, after preliminary definitions of fuzzy mathematical programming, fuzzy additive model [17], augmented max-min model [18, 19], and Chen and Tsai's [20] fuzzy model for the considered supplier selection problem are defined. In Section 5 the interactive solution methodology which combines these three fuzzy mathematical models is summarized. In Section 6, the solutions of each fuzzy model are presented for an illustrative example problem based on the interactive methodology. Conclusions and future directions appear in the final section.

2. Literature Review

Supplier selection problem has been a focus area of research since 1960. Reviews of supplier selection criteria and methods can be found in review studies which belong to Weber et al. [21], Degraeve et al. [22], De Boer et al. [23], Aissaoui et al. [4], and Ho et al. [24]. A detailed classification and review of qualitative techniques for supply chain planning under uncertainty can be found in Peidro et al.'s [25] review paper. Recently, Chai et al. [26] reviewed the studies on the application of the decision making techniques for supplier selection from 2008 to 2012.

The quantitative techniques for supplier evaluation and selection can be categorized into three classes [27]: (1) multiple attribute decision making, (2) mathematical programming models, and (3) intelligent approaches. Multiple attribute techniques commonly employed to select and/or rank suppliers (e.g., [28]) or to determine of the weights of the objectives (e.g., [29, 30]). In the former case to allocate orders among suppliers, mathematical programming has been utilized additionally.

Among the quantitative techniques, mathematical programming models have been extensively used for the sourcing and order allocation problem. The models can be either single period that do not consider inventory management or multi-period models which consider the inventory management by lot-sizing and scheduling orders. Single period models come handy in make-to-order manufacturing where custom parts are critical meanwhile common parts are efficiently managed by material requirement planning methods. The custom parts need to be requisitioned with each customer order and hence the custom parts inventory needs not to be considered [31].

Since Weber and Current [32] introduced a multiobjective mixed integer programming model for supplier selection and order allocation among the selected suppliers, several authors [29, 33–37] proposed multiple objective programming models to this problem. Objective functions are constructed based on four basic supplier evaluation criteria as price, quality, customer service, and delivery [38].

In literature a few of the studies have addressed both multiple objectives and fuzziness in the problem. Arikan [38–40] investigated and classified the fuzzy multiple objective sourcing and order allocation studies based on mathematical modelling in the literature, which have employed five solution approaches which are Zimmermann's [41] max-min approach [27, 42–44]; Tiwari et al.'s [17] additive model [27, 45–50]; fuzzy goal programming with weights [30]; fuzzy programming with modified fuzzy or operator [51]; and sequential quadratic programming [52]. Arikan [38] mentioned the disadvantages of these methods in detail: sequential quadratic programming does not consider objectives simultaneously. Zimmermann's max-min approach does not guarantee a nondominated solution. Although the rest of the approaches guarantee a nondominated solution, none of them is interested in a balanced solution. Tiwari et al.'s additive model maximizes achievement levels in total and the solution may include zero level achievement(s). Then an unbalanced fuzzy optimal solution is obtained [53]. Fuzzy goal programming with weights and fuzzy programming with modified fuzzy or operator do not prohibit the unbalanced solution case either. To handle all these criticisms, Arikan [38] employed Lai and Hwang's augmented max-min approach [18, 19] to fuzzy multiple objective supplier selection problem to obtain a solution which is both a nondominated and balanced one. In multiple objective decision making problems, obtaining "a best compromise solution" is an important matter of fact. The best compromise solution is the most preferred solution by the DM in the nondominated solution set [39]. For the real cases, preferences of DM(s) can be various and they may not point out a balanced solution. From this point of view, Arikan [39] proposed a two-phased approach which utilizes Chen and Tsai's [20] fuzzy model to satisfy the DM's preferred achievement levels. Utilizing Chen and Tsai's fuzzy model solely may result in "no feasible solutions" when a DM requires a very high desirable achievement degree for each fuzzy goal. In the two-phased approach, in order to prevent infeasible solution case the additive model solution is represented to the DM during the selection process of the minimum acceptable achievement levels. Afterwards Arikan [40] combined the previous works [38, 39] to accommodate an efficient interactive approach in which augmented max-min model [18, 19], additive or weighted additive model [17], and Chen and Tsai's [20] fuzzy model are employed interactively to obtain a nondominated solution to reach the DM's preferences [40]. Although this fuzzy interactive approach was generated based on the fuzzy multiple objective supplier selection problem requirements, it can be employed for any type of multiple objective programming problems by considering four different cases based on the sources of fuzziness.

Among the above-mentioned studies which have addressed both multiple objectives and fuzziness in the problem, Amid et al.'s study [46] is the only one which considers price discounts. However, it is concentrated on a single item sourcing problem and utilized Tiwari et al.'s [17] weighted additive model which may result in an unbalanced solution.

Discounts are fundamental pricing strategies in numerous industries. Hence there is a substantial amount of literature related to the discount problem. One of the early cornerstone papers in the area is Dolan's [54] in which quantity discounts schemes are classified and analyzed to provide a guideline to managers. Benton and Park [55] classified the literature on determining the lot size under quantity discount. Recently Setak et al. [56] review the literature based on the supplier selection and order allocation models for the period 2000–2010 years. They classified supplier selection models with and without discount environments.

Three steams of literature which are related to this current study are multiple criteria, fuzziness, and discount strategies. Herein, the literature is restricted with the studies concentrated on fuzzy multiple objective sourcing and order allocation problem in the presence of discount schemes as mixed integer programming model. Both single and multiple period models are investigated. In the literature the following seven studies (Table 2) consider fuzzy multiobjective supplier selection problem in the presence of discount schemes as mixed integer programming problem: Nazari-Shirkouhi et al.'s [57], Kang and Lee's [58], Razmi and Maghool's [59], Amid et al.'s [46], and Torabi and Hassini's [16, 60] studies and Arikan's [2] study.

Razmi and Maghool [59] solved a multiple item-multiple period multiple objective programming model by the lexicographic approach. In other words they considered each objective one at a time. The models utilized in the other four studies considered objectives simultaneously and generated nondominated solutions: Kang and Lee [58] considered a single item-multiple period model. Amid et al. [46] considered a single item-single period model. Both Kang and Lee [58] and Amid et al. [46] performed Tiwari et al.'s [17] weighted additive model which may result in unbalanced solutions. Torabi and Hassini [16, 60] considered multiple item-multiple period models and suggested interactive approaches in which augmented max-min model and Werner's fuzzy or model are utilized. Although augmented max-min model gives a balanced solution, for some real cases DM may not be interested in a balanced solution, and reaching his/her preferred minimum achievement levels is more important for him/her as long as the solution is a nondominated one. Fuzzy model with Werner's fuzzy or operator has $\gamma \in [0, 1]$ parameter which represents the compensation level. When $\gamma = 0$, the model becomes equivalent to the additive model; when $\gamma = 1$, then the model becomes equivalent to Zimmermann's max-min

Table 2: Fuzzy multiobjective supplier selection studies and their properties (updated version of Table 1 in [2]).

Author, year, reference	The current study	Arikan (2014) [2]	Nazari-Shirkouhi et al. (2013) [57]	Kang and Lee (2010) [58]	Razmi and Maghool (2010) [59]	Amid et al. (2009) [46]	Torabi and Hassini (2009) [16]	Torabi and Hassini (2008) [60]
Discount scheme(s)	All units (incremental, volume)	All units	All units	All units	All units, incremental, volume	All units	Lead time-dependent	Lead time-dependent
Period	Single	Single	Single	Multiple	Multiple	Single	Multiple	Multiple
Item	Multiple	Single	Multiple	Single	Multiple	Single	Multiple	Multiple
Sources of fuzziness	Aspiration levels of objectives and demand level	Aspiration levels of objectives and demand level	Aspiration levels of objectives	Aspiration levels of objectives and fuzzy triangular numbers in FAHP	Fuzzy capacity and demand levels	Aspiration levels of objectives and demand level	Aspiration levels of objectives and capacities and demand levels	Aspiration levels of objectives demand, capacity, quality, and service levels
Objectives	To minimize (1) the total production and ordering costs, (2) the total number of rejected units, (3) the total number of late delivered units	To minimize (1) the total monetary cost, (2) the total number of rejected units, (3) the total number of late deliveries	To minimize (1) the total purchasing and ordering costs, (2) the total number of defective units, (3) the total number of late delivered units	(1) To minimize the total cost (2) To maximize the yield rate (3) To fix the replenishment to a desired rate	(1) To minimize the total purchasing cost (2) To maximize the total value of purchasing	To minimize (1) the total purchasing cost, (2) the number of rejected items, (3) the number of late delivered units	To minimize (1) the total cost of logistics, (2) the total value of purchasing, (3) the number of defective items, (4) the late deliveries of purchased items	To minimize (1) the total cost of logistics, (2) the total value of purchasing
Solution approach	Interactive approach which utilizes Tiwari et al.'s [17] additive fuzzy model; Lai and Hwang's [18, 19] augmented max-min model; Chen and Tsai's [20] fuzzy model	Two phased additive approach [39] which utilizes Tiwari et al.'s [17] additive fuzzy model; Chen and Tsai's [20] fuzzy model	Interactive approach including fuzzy goal programming representation with piecewise linear membership functions, max-min operator, and fuzzy add operator	Two models constructed: (1) Zimmermann max-min approach [41] (2) Tiwari et al.'s [17] weighted additive fuzzy model	Fuzzy constraints were converted to crisp constraints and the augmented ε-constraint method is performed in which objectives are considered lexicographically	Tiwari et al.'s [17] weighted additive fuzzy model	Interactive approach including fuzzy goal programming, the weighted average method for defuzzification, and fuzzy model defined by Werners' [61] fuzzy or operator	Interactive approach including the weighted average method for defuzzification, Lai and Hwang's augmented max-min model [18, 19], and fuzzy model defined by Werners (1988) fuzzy or operator
Software	GAMS	GAMS	GAMS	LINGO	GAMS	LINDO/LINGO	GAMS	GAMS

model [41]. Determination of gamma parameter makes the model implementation harder [38]. Arikan [2] considered a single item-single period multiple sourcing problem in the presence of all-units discount scheme and solved it by the two-phased approach proposed in [39]. In that study [2], a mixed integer mathematical model based on Amid et al.'s [46] study is solved to obtain the DM's preferred achievement levels.

In the literature there is no multiple item-single period fuzzy model except for the Nazari-Shirkouhi et al.'s [57] to our knowledge. In their study, fuzziness stems from the goals attained to objectives whereas demand levels were considered in crisp sense. They represented fuzzy goals mathematically by piecewise linear membership functions. Nazari-Shirkouhi et al. [57] utilized a two-phase fuzzy goal programming approach. The detailed classification and criticism of the existed fuzzy goal programming approaches in the literature can be found in Arikan's [62]. Nazari-Shirkouhi et al.'s [57] approach is the goal programming version of Lee and Li's [53] approach. Lee and Li [53] developed a two-phase approach to improve the feasible but dominated solution yielded by min operator. In the first phase of the approach, Zimmerman's max-min approach is used. In the second phase of the approach a fuzzy model by using arithmetical average aggregating operator for the same problem is constructed with a lower bound restriction which is the solution of the first fuzzy model. The approach requires two phases to obtain a nondominated solution. In the second phase fuzzy additive model utilizes with a lower bound restriction which is the solution of the first fuzzy model. Although that lower bound restriction prevents the occurrence of an unbalanced solution, the approach does not satisfy the DM's preferred achievement levels, surely if there exist.

In this study, fuzzy sourcing and order allocation problem with multiple items and multiple suppliers in discount environment is considered. The aim of the study is to reach a nondominated solution to satisfy the DM's preferred achievement levels. The detailed investigation of the literature shows that the studies were disregarded to satisfy the DM's preferred achievement levels for the mentioned problem. In that case, previously proposed interactive approach [40] comes handy not only to reach the aim of this study but also to consider the different cases for the fuzzy parameter occurrences and the DM's different bias on the nondominated solution. Another important fact related with the interactive approach is that each fuzzy model utilized in the approach guarantees a nondominated solution separately. Interactive steps of the approach let the DM incorporate (modify) his/her preferences during the solution process. The significantly important traits of the DM such as experience, knowledge, ability, and foresight can be included into the decision process by interactivity even when the analyst and the DM are not the same person. The mathematical model of the considered problem is constructed as a linear 0-1 mixed integer programming model based on [57] with additional constraints and variations for different discount schemes. Fuzziness stems from fuzzy demand levels for each or some product(s). Objectives' aspirations can be fuzzy or crisp. Fuzziness is defined mathematically by using linear membership functions. Fuzzy demand(s) are assumed to be triangular number(s). Based on the considered interactive solution procedure [40], the DM preferred achievement levels are satisfied with a nondominated solution.

3. Multiobjective Supplier Selection Model with Price Breaks

In this study a linear 0-1 mixed integer model based on [57] is considered for supplier selection problems with multiple products and the all-units discount scheme. The proposed model restricts the number of suppliers employed by the buyer with a maximum number quota for each product as well. The model is as follows:

$$\text{Min } z_1 = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} P_{ijk}x_{ijk} + \sum_{i\in I}\sum_{j\in J} F_{ij}y_j, \tag{1}$$

$$\text{Min } z_2 = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} q_{ijk}x_{ijk}, \tag{2}$$

$$\text{Min } z_3 = \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} t_{ijk}x_{ijk} \tag{3}$$

subject to

$$\sum_{j\in J}\sum_{k\in K} x_{ijk} = D_i, \quad \forall i; \tag{4}$$

$$y_{ijk} \le x_{ijk} \le C_{ij}y_{ijk}, \quad \forall i, j, k; \tag{5}$$

$$V_{ijk-1}y_{ijk} \le x_{ijk} \le V_{ijk}y_{ijk}, \quad \forall i, j, k, \tag{6}$$

$$\sum_{k\in K} y_{ijk} = w_{ij}, \quad \forall i, \forall j, \tag{7}$$

$$\sum_{j\in J} w_{ij} \le N_i, \quad \forall i, \tag{8}$$

$$y_j \le \sum_{i}\sum_{k} y_{ijk} \le my_j \quad \forall j, \tag{9}$$

$$\sum_{k\in J} y_{ijk} \le 1, \quad \forall i, j, \tag{10}$$

$$\sum_{j\in J} e_{ij} \sum_{k\in K} x_{ijk} \ge E_i D_{\tilde{i}}, \quad \forall i; \tag{11}$$

$$\sum_{j\in J} s_{ij} \sum_{k\in K} x_{ijk} \ge S_i D_{\tilde{i}}, \quad \forall i; \tag{12}$$

$$\sum_{j\in J} r_{ij} \sum_{k\in K} x_{ijk} \ge R_i D_{\tilde{i}}, \quad \forall i; \tag{13}$$

$$x_{ijk} \ge 1 \quad \forall i, j, k \text{ and integer}, \tag{14}$$

$$y_j, y_{ijk}, w_{ij} \in \{0, 1\}, \quad \forall i, \forall j, \forall k, \tag{15}$$

where i is index for products, j is index for suppliers, and k is index for quantity discount levels.

Decision variables:

x_{ijk} is the number of units of product i ordered from the jth supplier at discount level k;

y_{ijk} is 1 if product i is ordered from supplier j at discount level k; 0, otherwise;

w_{ij} is 1 if product i is ordered from supplier j; 0, otherwise;

y_j is 1 if at least one product is provided from supplier j; 0, otherwise.

Parameters:

D_i is the aggregate demand of the product i from all suppliers over a fixed planning period;

P_{ijk} is the per unit net purchase cost of product i from supplier j at discount level k;

F_{ij} is the fixed ordering cost for product i from supplier j;

q_{ijk} is the percentage of the rejected units of product i delivered by the supplier j, at discount level k;

t_{ijk} is the percentage of the late units of product i delivered by the supplier j, at discount level k;

C_{ij} is the capacity of jth supplier for product i;

V_{ijk} is the maximum purchased volume of product i from the jth supplier at kth discount level;

N_i is the maximum number of suppliers that can be selected for product i;

E_i is the lower bound of quota flexibility required by ith product;

S_i is the lower bound of service level required by ith product;

R_i is the lower bound of rating value required by ith product;

e_{ij} is the flexibility of supplier quota allocation of jth supplier for ith product;

s_i is the service level required by ith product;

r_i is the rating value required by ith product.

Objective function (1) minimizes total purchasing and fixed ordering costs. Objective function (2) minimizes the total number of rejected units, which serves in maximizing total quality of purchased items. Objective function (3) minimizes the total number of late delivered units, which serves in maximizing the service level of purchased items. Constraint (4) ensures that the overall demand is satisfied. Constraint (5) means that order quantity of each supplier should be equal to or less than its capacity. Constraint (6) ensures that the number of units purchased from supplier j is placed in the appropriate discount interval for each item. Constraints (7) and (8) restrict the number of suppliers employed by the buyer with a maximum number quota for each product. Constraint (9) means all the products purchased from the same supplier are placed in one order. It is calculated for the fixed ordering cost in objective (1). Constraint (10) ensures only one discount level is eventually used for the amount if product i is purchased from supplier j. Constraints (11)–(13) represent that the quota flexibility, service level, and rating values must exceed a given level. Constraint (14) prohibits negative orders. Constraint (15) presents binary integer variables.

The following two modifications on the above model ((1)–(15)) are needed when suppliers offer incremental and volume discount schemes, respectively. In both cases considering suppliers' offers, the buyer makes decision to minimize the total purchase cost in addition to the other two objectives defined formerly subject to the same constraint set.

When suppliers offer incremental discount scheme, the above formulation can be modified with formulation (16) instead of (1):

$$\operatorname{Min} z_1 = \sum_i \sum_j \left(\sum_{m=1}^{k-1} p_{ijm} \left(V_{ijm} - V_{ij,m-1} \right) + p_{ijk} \left(x_{ijk} - V_{ij,k-1} \right) \right) + \sum_{i \in I} \sum_{j \in J} F_{ij} y_j, \tag{16}$$

where $V_{ijk-1} y_{ijk} \le x_{ijk} \le V_{ijk} y_{ijk}$, $\forall i$, $\forall j$, $\forall k$.

If the incremental discount scheme is offered by suppliers, then the company may determine the lower bounds for purchasing quantities with the following additional constraint (17) which enforces the minimum purchasing quantities assigned to each supplier for each product are greater than or equal to the percentage (u) of demand of the respective product. The percentage reflects the DM's preferences based on the business relationship with suppliers and the company [36]:

$$\sum_{k \in K} x_{ijk} \ge u D_i, \quad \forall i. \tag{17}$$

When suppliers offer a volume discount schemes, the above formulation can be modified with formulation (18) instead of (1):

$$\operatorname{Min} z_1 = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \left(1 - d_{ijk} \right) v_{ijk} + \sum_{i \in I} \sum_{j \in J} F_{ij} y_j, \tag{18}$$

where $\sum_i \sum_k v_{ijk} = \sum_i \sum_k p_{ijk} x_{ijk}$ $\forall j$.

4. Preliminary Definitions

Consider the fuzzy multiple objective programming problem (19) with l fuzzy objective functions and s fuzzy constraints:

$$\begin{aligned} &\text{Find} && x \\ &\text{s.t.} && c_k x \mathrel{\tilde{\le}} z_k \quad k \in I \\ &&& a_r x \cong b_r \quad r \in T \\ &&& x \in X, \end{aligned} \tag{19}$$

where

(i) X is a set of deterministic linear constraints and sign restrictions;

(ii) $c_k x = \sum_{i=1}^{n} c_{ki} x_i$ $k = 1, \ldots, l$; $a_r x = \sum_{i=1}^{n} a_{ri} x_i$ $r = 1, \ldots, s$;

(iii) for $k \in I$, z_k is the imprecise aspiration level for the kth objective function;

(iv) $z_k \in [z_k^L, z_k^U]$ denote the imprecise lower and upper, respectively, bounds for the kth fuzzy objective function. $b_r \in [b_r - \Delta_{rL}, b_r + \Delta_{rU}]$ denote the imprecise lower and upper, respectively, bounds for the rth fuzzy constraints.

According to fuzzy mathematical programming, each fuzzy objective and constraint are defined in terms of fuzzy subsets with the appropriate membership functions denoted by $\mu_k(c_k x)$ for $k \in I$ and $\mu_r(a_r x)$ for $r \in T$, respectively. Assuming that membership functions are linear, mathematical definitions are given in (20) and (21). Equation (20) represents linear monotone decreasing membership function $\mu_k(c_k x)$ for minimization type objectives with fuzzy aspiration levels and expression (21) is a triangular membership function $\mu_r(a_r x)$ for constraints:

$$\mu_k(c_k x) = \begin{cases} 1 & \text{if } c_k x \le z_k^L \\ \dfrac{z_k^U - c_k x}{z_k^U - z_k^L} & \text{if } z_k^L \le c_k x \le z_k^U \\ 0 & \text{if } c_k x \ge z_k^U, \end{cases} \qquad \forall k \in I, \tag{20}$$

$$\mu_r(a_r x) = \begin{cases} 0 & \text{if } a_r x \le b_r - \Delta_{rL} \\ \dfrac{a_r x - (b_r - \Delta_{rL})}{\Delta_{rL}} & \text{if } b_r - \Delta_{rL} \le a_r x \le b_r \\ \dfrac{b_r + \Delta_{rU} - a_r x}{\Delta_{rU}} & \text{if } b_r \le a_r x \le b_r + \Delta_{rU} \\ 0 & \text{if } a_r x \ge b_r + \Delta_{rU}, \end{cases} \qquad \forall r \in T. \tag{21}$$

4.1. Fuzzy Additive Model. Fuzzy additive model based on Tiwari et al.'s [17] study for the multiple objective programming model (19) is given in (22). Variables denoted by λ_k and λ_r represent achievement levels of fuzzy objective functions and fuzzy constraints, respectively:

$$\begin{aligned} \max \quad & \frac{\left(\sum_{k=1}^{l} \lambda_k + \sum_{r=1}^{s} \lambda_r\right)}{(l+s)} \\ \text{s.t.} \quad & \lambda_k \le \mu_k(c_k x) \quad k \in I \\ & \lambda_r \le \mu_r(a_r x) \quad r \in T \\ & \lambda_i, \lambda_r \in [0,1] \quad k = 1, \dots, l; \ r = 1, \dots, s; \\ & x \in X. \end{aligned} \tag{22}$$

Fuzzy weighted additive model [17] has the same constraints with the following objective function (23):

$$\max w_k \sum_{k=1}^{l} \lambda_k + w_r \sum_{r=1}^{s} \lambda_r, \tag{23}$$

where w_k and $w_r \in [0,1]$ and they denote relative weights for each fuzzy criterion, and $\sum_k w_k + \sum_r w_r = 1$ for $k = 1, \dots, l$; $r = 1, \dots, s$.

4.2. Augmented Max-Min Model. Augmented max-min model (24) based on Lai and Hwang's [18, 19] approach is adapted for model (19) as follows:

$$\begin{aligned} \max \quad & \lambda + \frac{\left\{\sum_{k=1}^{l} \mu_k(c_k x) + \sum_{r=1}^{s} \mu_r(a_r x)\right\}}{(l+s)} \\ \text{s.t.} \quad & \lambda \le \mu_k(c_k x), \quad k = 1, 2, \dots, l \\ & \lambda \le \mu_r(a_r x) \quad r = 1, 2, \dots, s \\ & x \in X \\ & \lambda \in [0,1]. \end{aligned} \tag{24}$$

Variable λ represents the minimum satisfaction degree and is defined as in (25):

$$\min_{k,r} \{\mu_k(c_k x), \mu_r(a_r x)\}, \quad \text{for } k = 1, 2, \dots, l; \ r = 1, 2, \dots, s. \tag{25}$$

4.3. Chen and Tsai's Fuzzy Model. Chen and Tsai's [20] fuzzy model is adopted for model (19) in (26). Variables denoted by λ_k and λ_r represent achievement levels of fuzzy objective functions and fuzzy constraints, respectively, defined in model (22). Parameters α_k and α_r represent the minimum acceptable achievement levels for the kth objective, rth constraint, respectively, determined by the DM. This model allows the DM to assign explicitly a desirable achievement degree for each fuzzy aspiration [20]:

$$\begin{aligned} \max \quad & \frac{\left(\sum_{k=1}^{l} \lambda_k + \sum_{r=1}^{s} \lambda_r\right)}{(l+s)} \\ \text{s.t.} \quad & \lambda_k \le \mu_k(c_k x) \quad k \in I \\ & \lambda_r \le \mu_r(a_r x) \quad r \in T \\ & \lambda_k \ge \alpha_k, \quad k = 1, \dots, l \\ & \lambda_r \ge \alpha_r \quad r = 1, \dots, s \\ & x \in X \\ & \alpha_k, \alpha_r, \alpha_p, \lambda_i, \lambda_r, \lambda_p \in [0,1] \\ & k = 1, \dots, l; \quad r = 1, \dots, s; \\ & x \in X. \end{aligned} \tag{26}$$

5. Interactive Solution Methodology [40]

To solve the multiple objective sourcing problem with multiple items in discount environment, previously proposed interactive algorithm [40] is performed based on the assumptions of the problem. The flowchart of the interactive approach is modified for the current study and included in the appendix. Case (b) and case (c) mentioned in the flowchart are adequate for the assumptions of the problem in this study. Surely, the other two cases can be considered based on the problem requirements. In the current study it is assumed that fuzzy demand level(s) are fuzzy triangular number(s). The followed steps are summarized below.

(1) Construct the multiple objective model ((1) to (15)).

(2) Determine fuzzy parameters as aspiration levels of objectives and/or fuzzy demand level(s).

(3) If both aspiration levels of objectives and/or demand level(s) are fuzzy then construct the linear or piecewise linear membership functions for each fuzzy goal (20) and/or each fuzzy demand (21).

(4) If the model has only fuzzy demand level(s) for each or some product(s), considering $b_r - \Delta_{rL}$ and $b_r + \Delta_{rU}$ one at a time, solve the mathematical model defined with (1) to (15) and find the ideal solutions for each objective. Construct two payoff tables. Determine the lower and upper bounds for each objective. Ask the DM whether bounds are acceptable. Update bounds as s(he) wishes. Construct linear/piecewise linear membership function for each objective.

(5) If there are equal or numerically close relative weights for fuzzy aspirations go to step (6); otherwise go to step (7).

(6) Construct and solve augmented max-min model (24). Present the fuzzy optimal solution(s) to the DM. If the DM is satisfied then stop. Else, go to step (8).

(7) If there are unequal relative weights for fuzzy aspirations then construct and solve fuzzy weighted additive model (23). Present the fuzzy optimal solution(s) to the DM. If the DM is satisfied then stop. Else go to step (8).

(8) Ask the DM for the minimum acceptable aspiration level for each fuzzy objective and/or fuzzy constraint. During the determination process, the DM will take aspiration levels obtained in step (6)/(7) as references for minimum acceptable aspiration levels. Construct and solve Chen and Tsai's fuzzy model (26). Present the fuzzy optimal solution(s) to the DM. If the DM is satisfied then stop. Else if the fuzzy model parameters are both fuzzy aspirations of objectives and fuzzy right hand side constants go to step (3).

6. Illustrative Example

The following example problem is from Nazari-Shirkouhi et al.'s study [57] for sourcing and order allocation problem with multiple items and multiple suppliers. The aim of the example problem is to select appropriate suppliers based on three pur-chasing criteria which are price, quality, and service level for supplying two products. Each supplier offers "all units" price breaks. The prices are in the three price levels for each supplier. The percentage of the rejected items and late deliveries, suppliers' capacities, and quantity levels with offered prices are provided in Table 3. The original data includes quota flexibility, service level, and rating values of each supplier for each item (Table 4). To solve the model, Nazari-Shirkouhi et al. [57] performed an experimentation in which objectives are unified by using weighted sum of normalized objectives.

In this current study we modified the data set with fuzzy aspirations and fuzzy demand levels. The approach based on the payoff tables proposed in [40] is utilized for the determination of the lower and upper bound during the construction of the membership functions. The fuzzy model with the obtained bounds maintains the solution occurring in the feasible region of the main problem. Furthermore it gives a Pareto optimal solution.

Step (1): multiple objective sourcing and order allocation model is constructed ((1) to (15)).

Step (2-3): fuzzy triangular number for fuzzy demand of each product is defined with $[b_r - \Delta_{rL}, b_r, b_r + \Delta_{rU}]$.

Step (4): considering lower bounds and upper bounds of demand levels, one at a time, the corresponding mathematical model defined with (1) to (15) in Section 3 is constructed and ideal solutions for each objective are found. Constructed payoff tables are Tables 5 and 6.

The lower and upper bounds for each objective according to payoff tables are determined as $(z_1^L, z_1^U) = (21135, 28810)$, $(z_2^L, z_2^U) = (41.60, 68.60)$, $(z_3^L, z_3^U) = (34.60, 57.40)$. Bounds are assumed as acceptable for the DM. The related membership functions are constructed by using (20) equations with the mentioned bounds of aspirations. Bounds are assumed as acceptable for the DM. Go to step (5).

Step (5-6): it is assumed that there are equal or numerically close relative weights for fuzzy aspirations, and then the solution of the constructed augmented max-min model (24) for the illustrative example is given in Table 5. For this case it is assumed that the DM is not satisfied with aspiration levels mentioned in Table 7 column 2 to go to step (8).

Step (7): assume that there are unequal weights for fuzzy aspirations; then the solution of the constructed fuzzy weighted additive model (23) for the illustrative example is given in Table 8. Weight vector is defined for three different cases as w^1, w^2, w^3. Assume that DM is not satisfied; then go to step (8).

Step (8): fuzzy additive model solution (22) in Table 7 column 3 can be presented to the DM as references for his/her minimum acceptable aspiration levels which are assumed as (0.5, 0.5, 0.5) for α_k, $k = 1, 2, 3$, and (0.5) for α_r $r = 1, 2, 3, 4, 5$ as the first case. Then Chen and Tsai's fuzzy model (24) is constructed and solved (Table 7 columns four and five). Minimum acceptable aspiration levels attained by the DM are assumed as (0.7, 0.5, 0.5) for α_k, $k = 1, 2, 3$, and (0.5, 0.7, 0.5, 0.5, 0.7) for α_r $r = 1, 2, 3, 4, 5$ as the second case. Then Chen and Tsai's fuzzy model (14) is constructed and solved (Table 7 columns six and seven).

Table 3: Data for the illustrative example (modified from [57]).

Product i	Fuzzy demand $[b_r - \Delta_{rL}, b_r, b_r + \Delta_{rU}]$	Supplier j	F_{ij}	q_{ij} (%)	t_{ij} (%)	C_{ij}	$P_{ij1}, P_{ij2}, P_{ij3}$ ($)	$V_{ij0}, V_{ij2}, V_{ij3}$
1	[750, 700, 650]	1	800	4	1	1300	18, 17.5, 17	0, 100, 200
		2	750	3	2	1100	17, 16.5, 16	0, 120, 220
		3	600	4	1	1000	15, 14.5, 14	0, 150, 300
		4	650	3	2	900	16, 15.5, 15	0, 90, 180
2	[650, 600, 550]	1	800	4	2	1400	6.5, 6, 5.5	0, 80, 170
		3	600	2	3	1400	4, 3.5, 3	0, 60, 190
		4	650	1	2	1400	5, 4.5, 4	0, 90, 210
3	[500, 450, 400]	2	750	2	2	1300	10, 9.5, 9	0, 75, 180
		3	600	5	1	1200	11, 10.5, 10	0, 60, 130
4	[450, 400, 350]	1	800	2	3	1000	8, 7.5, 7	0, 100, 180
		2	750	1	3	1000	12, 11.5, 11	0, 150, 300
		3	600	1	4	1100	10, 9.5, 9	0, 90, 160
		4	650	0	2	800	13, 12.5, 12	0, 150, 240
5	[430, 380, 330]	1	800	2	1	1200	6, 5.5, 5	0, 120, 220
		2	750	2	2	1300	5, 4.5, 4	0, 100, 200

Table 4: Quota flexibility, service level, and rating data for the illustrative example [57].

Product i	Supplier j	E_i (%)	S_i (%)	R_i (%)	e_{ij} (%)	s_{ij} (%)	r_{ij} (%)
1	1	2	4	1	3	94	92
	2		3	2	2	90	95
	3		4	1	2	94	90
	4		3	2	5	94	96
2	1	2	4	2	3	91	95
	3		2	3	3	95	96
	4		1	2	4	96	96
3	2	2	2	2	5	95	91
	3		5	1	4	95	92
4	1	3	2	3	4	92	93
	2		1	3	4	92	93
	3		1	4	1	95	92
	4		0	2	1	94	93
5	1	2	2	1	4	91	90
	2		2	2	4	96	91

Table 5: Payoff table (demand level chosen as $b_r - \Delta_{rL}$).

	$z_1(x_1{}^*)$	$z_2(x_2{}^*)$	$z_3(x_3{}^*)$
$x_1{}^*$	$21135^{\dagger}$	23915	25100
$x_2{}^*$	58.90	$41.60^{\ddagger}$	57.60
$x_3{}^*$	47	45.80	$34.60^{\#}$

Table 6: Payoff table (demand level chosen as $b_r + \Delta_{rU}$).

	$z_1(x_1{}^*)$	$z_2(x_2{}^*)$	$z_3(x_3{}^*)$
$x_1{}^*$	25660	28623.5	$28810^{\dagger}$
$x_2{}^*$	62.50	54.1	$68.60^{\ddagger}$
$x_3{}^*$	$57.40^{\#}$	54.41	45.60

The algorithm is terminated with the assumption that the DM is satisfied with the current fuzzy optimal solution. If it is assumed that DM is not satisfied, then we should go to step (4). It will be carried on steps of the algorithm.

Models (23), (24), and (26) are utilized for the illustrative example and fuzzy optimal solutions are obtained by GAMS computer programming package. Their summaries which are given in Tables 7 and 8 constitute nondominated solutions. When fuzzy additive model solution is investigated, it is observed that the third objective which minimizes the total number of late delivered units and serves in maximizing the service level of purchased items has the least achievement level as 0.496. This is because the additive model maximizes achievement levels in total. On the other hand augmented max-min model solution maximizes the minimum achievement level as well and it gives more balanced solution. In Table 8 weighted additive model solutions are presented. To determine relative weights is another multiple attribute decision making problem. In this example problem for three different weight sets is considered. In a realistic point of view the DM may not prefer achievement levels less than 0.5. In that case the procedure will carry on with step (8). After presenting additive (/weighted additive) or augmented max-min model solutions to the DM, Chen and Tsai's fuzzy model provides a preferred solution.

7. Conclusions and Future Direction

In this study a multiple sourcing supplier selection problem is considered as a multiple objective linear programming problem. The literature is reviewed for the studies which handle supplier selection by fuzzy multiple objective mathematical programming and they are investigated according to their solution approaches. The detailed literature reviews in the author's previous studies [38–40] lead the construction of

Table 7: Solution summaries to the illustrative example with linear membership functions.

	Augmented max-min model	Additive model	Chen and Tsai's fuzzy model			
			α_k/α_r		α_k/α_r	
Min z_1	24778	24850	0.5	24860	0.7	23436
Min z_2	54.41	51.10	0.5	51.20	0.5	55.01
Min z_3	45.43	46.01	0.5	46.00	0.5	46.00
$\mu_1(c_1x)$	0.525	0.510		0.515		0.700
$\mu_2(c_2x)$	0.526	0.648		0.644		0.503
$\mu_3(c_3x)$	0.525	0.496		0.500		0.500
$\mu_1(a_1x)$	1.000	1.000	0.5	1.000	0.5	0.500
$\mu_2(a_1x)$	1.000	1.000	0.5	1.000	0.7	1.000
$\mu_3(a_2x)$	1.000	1.000	0.5	1.000	0.5	0.960
$\mu_4(a_3x)$	1.000	1.000	0.5	1.000	0.5	0.500
$\mu_5(a_1x)$	1.000	1.000	0.5	1.000	0.7	1.000

Table 8: Solution summaries to the illustrative example for fuzzy weighted additive model.

	Additive model Case 1	Additive model Case 2	Additive model Case 3
Min z_1	23850	25500	26000
Min z_2	55.01	54.60	47.10
Min z_3	48.30	42.60	49.10
$\mu_1(c_1x)$	0.646	0.431	0.366
$\mu_2(c_2x)$	0.500	0.519	0.796
$\mu_3(c_3x)$	0.399	0.649	0.364
$\mu_1(a_1x)$	1.000	1.000	1.000
$\mu_2(a_1x)$	1.000	1.000	1.000
$\mu_3(a_2x)$	1.000	1.000	1.000
$\mu_4(a_3x)$	1.000	1.000	1.000
$\mu_5(a_1x)$	1.000	1.000	1.000

Case 1 w^1 (0.25, 0.10, 0.15, 0.10, 0.10, 0.10, 0.10, 0.10).
Case 2 w^2 (0.10, 0.10, 0.25, 0.10, 0.10, 0.10, 0.10, 0.10).
Case 3 w^3 (0.10, 0.25, 0.10, 0.10, 0.10, 0.10, 0.10, 0.10).

the current study's literature. However they are not interested in discount strategies. Herein the literature is also reviewed for the discount strategies. A clear classification is constructed for the discount schemes mentioned in the literature (Table 1) with references. The studies which handle sourcing problem by fuzzy multiple objective mathematical programming in discount environment are also investigated and criticized based on their solution approaches (Table 2).

Detailed review reveals that there is no multiple item-single period fuzzy model for multiple objective sourcing problem in discount environment except for the Nazari-Shirkouhi et al.'s [57] in the literature. The current study proposes an enhanced mathematical model by additional constraints for the quota flexibility, service level, and rating values for each product, by adding modification requirements of the model for the different discount schemes and by considering fuzziness not only of aspiration levels of objectives but also of demand levels. Furthermore the interactive solution approach utilized in the current study gives opportunities which go beyond to all other approaches employed for the problem, because it [40] comes handy not only to reach a nondominated solution which satisfies the DM's preferred achievement levels but also to consider the different cases for the fuzzy parameter occurrences and the DM's different bias on the nondominated solution.

This study provides a realistic multiple objective sourcing and order allocation model which considers three objective functions as minimization of costs, maximization of quality, and maximization of on-time delivery with fuzzy aspiration levels, respectively, and/or fuzzy demand. To keep the business relationship with suppliers the model contains restrictions on the number of suppliers employed by the buyer with a maximum number quota for each product. When suppliers offer incremental and volume discount schemes, the required modifications on the model are also investigated. The model is employed to construct fuzzy mathematical models which give nondominated solutions. Each possible fuzzy parameter is represented mathematically by using a linear membership function. For fuzzy demand levels, fuzziness is defined by triangular membership functions which can be isosceles or unbalanced. In the utilized interactive procedure three well known fuzzy mathematical models are employed based on the problem requirements. Both fuzzy additive and augmented max-min models give nondominated solutions. Augmented max-min model solution is balanced additionally. Chen and Tsai's fuzzy model with the interactive steps of the solution approach gives an opportunity to the DM to obtain her/his own preferred achievement levels for the objectives and for demand levels. Consideration of fuzziness makes the obtained nondominated solution implementable for the real cases.

In the relatively scarce studies which try to deal with the sourcing and order allocation problem with multiple items, multiple criteria, and fuzzy parameters in discount environment simultaneously, this study provides a realistic mathematical model and promising strategies to satisfy the purchasing managers' (DM's) preferences which can be a balanced solution or there can be preferred achievements for each of the fuzzy aspirations to be satisfied, as long as the obtained result is a nondominated solution.

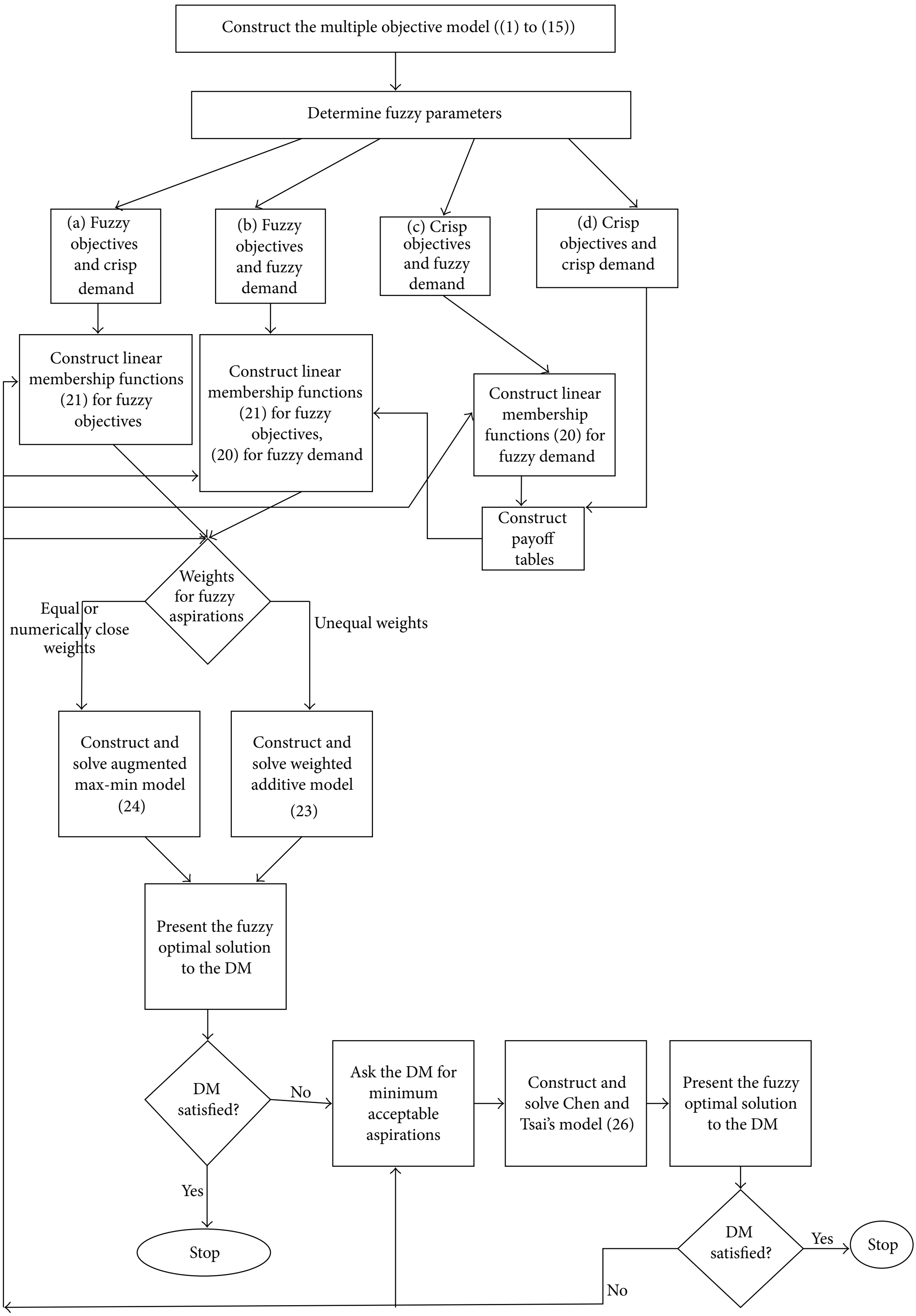

Figure 1: Flowchart of the interactive approach from [40] with modifications for the current study.

For the planned future work, fuzziness in each supplier's capacity is going to be considered. In that case sourcing and order allocation problem will become a nonlinear programming problem.

Appendix

See Figure 1.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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Research Article

Competition with Online and Offline Demands considering Logistics Costs Based on the Hotelling Model

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Through popular information technologies (e.g., call centers, web portal, ecommerce and social media, etc.), traditional shops change their functions for servicing online demands while still providing offline sales and services, which expand the market and the service capacity. In the Hotelling model that formulates the demand effect by considering just offline demand, the shops in a line city will locate at the center as a the result of competition by games. The online demands are met by the delivery logistics services provided by the shops with additional cost; the consumers' waiting time after their orders also affects their choices for shops. The main purpose is to study the effects of the following aspects on the shops' location competition: two logistics costs (consumers' travelling cost for offline demands and the shops' delivery logistics cost for online demands), the consumers' waiting cost for online orders, and the ratios of online demands to the whole demands. Therefore, this study primarily contributes to the literature on the formulation of these aspects by extending the Hotelling model. These features and effects are demonstrated by experiments using the extended Hotelling models.

1. Introduction

People usually visit and buy what they want in supermarkets, shopping centers, and plaza, whereas, accompanied by the information technology and Internet, online business has an increasing tendency. In 2009, the online retail sales of USA grew 11%, while all retail sales only grew 2.5% [1]. About 154 million people had the experiences of online shopping, contributing to online retail sales of $155 billion or 6% of total retail sales. It is forecasted that the online sales of USA will keep growing at 10% as an annual rate through 2014 to $250 billion. In Western Europe, online retail sales are expected to grow by 11% per year, going from €68 billion in 2009 to €114.5 billion in 2014. The integration of online and offline undoubtedly brings enterprises grand profits and provides multichannels for consumers.

Under the merging of online business, there are more and more people shopping on the Internet. Without going outside, people can enjoy the service from shops or other providers all over the city. Compared to the offline business, shops may suffer an additional cost, delivering cost. According to the Hotelling model [2], competitive shops often locate in the center of a line city, and they are closed to each other. However, with the development of online business, orders may come from people all over the city and should be met in the shortest time. Obviously, the locations deduced by the Hotelling model will impose great costs on the shops for online orders. Concentrated distribution, which works well in the past time, can no longer operate perfectly for the following reasons. First, remote consumers will wait for longer time to get the ordered goods because of long traveling time, which may cause low satisfaction degrees; second, long distance makes large delivery logistics cost, which carries burdens on shops; third, shops may increase the price to sustain a good profit, which may lose some consumers and form a vicious cycle. This work extends the Hotelling model for the offline and online demands.

This work extends the Hotelling model to study the effects of combination of offline and online demands on the competition of shops. The traditional Hotelling model has formulated the location game considering the pricing problem and the consumers' travelling logistics costs. For online demands, the consumers may choose shops based on the prices and the waiting time after orders, although the consumers do not pay for the travelling costs by themselves. However, the shops will pay for the delivery cost for servicing the consumers that order goods online. Therefore, the shop's profit should be subtracted by the logistics cost that relates to the orders and the distances between the shops and consumers. Therefore, the indifferent location computed in the side of consumers and the profit computed in the side of shops challenge the extension of the Hotelling model. Following these clues, new models are developed for analyzing the effects of logistics cost deduced by online demands on the shops' location competition.

Considering the new situation that the combination of online and offline business promotes great success, this paper extends the Hotelling model. The logistics costs, the shops' delivery times, and the consumers' waiting times are incorporated into the extended models. In the fields including online sales and competition, this study contributes to the literature in the following points. First, based on offline business, this work considers the model combining offline and online business. Second, the logistics costs and the online demands are considered in assessing the location competition game between shops. Third, the Hotelling model is extended by simultaneously considering offline and online demands and the shops' logistics cost for delivery.

The rest of the study is organized as follows. In Section 2, there are the studies related to online and offline business, competition issues in logistics systems, and the applications of the Hotelling model. Then, in Section 3 the problem is described and the problem is formulated in Section 4. The developed models are then demonstrated and analyzed in Section 5. Finally, we conclude the study in Section 6.

2. Related Studies

2.1. Online and Offline Business. Since the early 1990s, many manufacturers and retailers have incorporated the Internet into their multichannel strategy and devoted considerable resources to building the online channel. Chu et al. [1] thought that the Internet substantially reduces search costs and grants easy access to product and price information. Online shopping involves no travel, product carrying, or restrictions on shopping hours, offering great accessibility, convenience, and time saving. In contrast, offline shopping allows physical examination of the products, interpersonal communication, and instant gratification but involves high travel costs and search costs and often has restrictions on shopping hours, especially in countries with strong labor laws. Shankar et al. [3] claimed that the online business offers more opportunities for interactive and personalized marketing than offline business. Moreover, the online consumers can compare alternatives easily especially for functional products and services. Kwon and Lennon [4] suggested the synergy between online and offline operations generated by the seamless integration between the two channels. Kwon and Lennon believed that the synergy enriches the consumers' experiences with the retailer, strengthens the brand image of the retailer, and cultivates consumer loyalty in both channels.

The offline and online business impose complex effects on the consumers' loyalty. Loureiro and Roschk [5] devised a model that compares the offline and online shops and regards the consumers' ages as a moderator. In the offline context, positive emotions predict loyalty among younger consumers but not among older ones; in the online context, the effect of graphics design on loyalty is stronger for younger consumers than for older ones. Kwon and Lennon [4] investigated the effect of the interplay between a multichannel retailer's offline and online brand images on consumers' online perceived risk and online loyalty within the framework of a theory of cognitive dissonance. Shankar et al. [3] proposed a conceptual framework and developed hypotheses about the effects of the online medium on consumer satisfaction and loyalty and on the relationships between satisfaction and loyalty. The results showed that loyalty to the shops is higher when the service is chosen online than offline. Besides, loyalty and satisfaction have a reciprocal relationship such that each positively reinforces the other, and this relationship between overall satisfaction and loyalty is further strengthened online.

The consumers' behaviors in the online business context are examined by many researchers. Kollmann et al. [6] identified relevant shopping motives in multichannel environments. The empirical analysis suggested that the degree of consumers' convenience orientation in contrast to the degree of risk aversion and service orientation encourages the online channel over the offline channel. Chu et al. [1] studied the moderating effects of household (e.g., shopping frequency) and product (e.g., sensory nature) characteristics on household brand loyalty, size loyalty, and price sensitivity across online and offline channels for grocery products. Households are more brand-loyal, more size-loyal but less price sensitive in the online channel than in the offline channel. The differences between online and offline business in brand loyalty and price sensitivity are large for light online shoppers and smallest for heavy online shoppers. The online and offline differences in brand loyalty, size loyalty, and price sensitivity are large for food products and for sensory products. Ahn et al. [7] explored some online and offline features of Internet shopping malls and their relationships with the acceptance behaviors of consumers, and they provide a domain-specific, integrative approach in evaluating the quality and antecedents of user acceptance for Internet shopping malls. Technology acceptance model [8] is valid in predicting the acceptance of the Internet shopping malls and that online and offline features have positive effects on the user acceptance. Both online and offline features have greater effects on the usefulness, attitude, and intention to use than either online or offline features separately.

Grewal et al. [9] provided an overview of findings from past research in both offline and online domains and presents an organizing framework, as well as an agenda to advance the research. The issues on the online and offline pricing and promotional strategies, as well as coordinating these strategies, are highlighted. This work also incorporated the pricing strategy in studying the location competition affected by logistics costs.

2.2. Competition in Logistics. The competitions involved in the operations and management of the supply chains are extensively examined in literature. Farahani et al. [8] considered that the competitive supply chain network design affects its cost as well as its performance and asserted that supply chains compete to capture market shares for themselves. Barney et al. [10] proposed a resource-based competition method to assess and determine the important resources and operations of a firm. In this strategic management framework, firms are viewed as entities which possess and control complex resources. The resources are scarce or unique, substitutable, and durable, which are thought to contribute to the firm's competitive advantage. Moreover, the logistic service can be regarded as a special resource so that it is also important in the firms' decisions. Papachristos [11] concerned the market competitions between new and remanufactured products and explored the environmental impact of competition on the shift from products to services with a system dynamics model. Rezapour and Farahani [12] designed the network structure of a competitive supply chain based on a bilevel model. The price and service levels competition in market are considered in the model of stochastic price and service demands with the presence of existing and external rivals. The supply chain network is assumed to be set at first and could be adjusted for new prices and services. Nagurney et al. [13] developed a game theory model for supply chain competition in time-sensitive markets where consumers are sensitive to the delivery time. An algorithm is devised to solve the model with equilibrium conditions.

The location game is a typical kind of competition between firms. Gao et al. [14] considered the location-then-price game as a two-stage game of n players on the graph which is abstracted from the spread of the firms in the real world. Based on the best response dynamics and the multinomial logit model, the price and location equilibria are built and shares of n players are computed by price equilibrium. The location and price equilibria are derived for the competitive airlines in Russian and Chinese airline market. Godinho and Dias [15] addressed a discrete competitive location game in which two decision makers have to decide simultaneously where to locate their services without knowing the decisions of each other. Obviously, the decisions made by one firm will affect the payoffs of another. A model and algorithmic approach are designed to calculate the Nash equilibria in both preferential rights overbidding consuming patterns for franchisees and franchisers. The results are advantageous for the franchiser if the overbidding is possible. Fotakis and Tzamos [16] studied the facility location games where a number of facilities are set in a metric space based on the locations reported by the strategic agents. The agents minimize their connection cost to the facilities; however, the locations may be misreported by the agents. The model based on the winner-imposing mechanisms is designed to produce the mechanisms for the agents and facilities under the presumption that no agent can benefit from misreporting his location. A continuous metric space is also considered for the oblivious winner-imposing mechanisms. Some papers consider the competitions in transportations during the logistics' activities. For example, Reyes [17] studied the transshipment problem based on the Shapley value concept from the cooperative game theory by considering logistics cost and quality.

This work studied the location competition between shops with offline and online demands by considering logistics costs. The shops can gain competition advantage by improving logistics services.

2.3. Hoteling Model. Since Hotelling [2] developed the two-stage game of spatial competition model, a lot of relevant literatures are gradually emerging. Considering that no equilibrium price solution exists when both firms are not far enough from each other, d'Aspremont et al. [18] replaced the linear transportation cost function with quadratic function, such that a price equilibrium solution exists everywhere. d'Aspremont concluded that the two firms gathering in the center will make the equilibrium price zero. Another way to solve the equilibrium problem is to change the city's shape to circular [19] or square [20]. By introducing consumer differentiation, increasing the number of firms, and relaxing the assumption of demand elasticity and product differentiation, the Hotelling model is extended by Tabuchi and Thiees [21], Palma et al. [22], Eaton [23], and Shaked and Sutton [24]. The researches on the applications of the Hotelling model can be divided into two classes. One solves the problem of selecting location and pricing for the product. Wey and Hong [25] extended the Hotelling model for determining the locations and the optimal numbers of plants for the purpose of using rice straws. Gao et al. [14] applied the Hotelling model to the study of the competition in airline market, where airlines first decide plane allocation and then choose ticket prices. The existence of equilibrium in pure strategies on the graph for the case of multiple plays is extensively discussed. Using distance to measure product features like quality, for firms, the Hotelling model is used to decide the product quality and brand positioning and so forth. Blosh and Manceau [26] added advertising investment in the Hotelling model and asserted that advertising makes consumers prefer products, while also decreasing the price. Therefore, advertising investment is not always necessary for firms. Based on the Hotelling model, Mariñoso [27] proved that endogenous switching costs will increase the market competition, so that the firms should produce compatible brand products.

To investigate the effects of the business mode combining offline and online demands on the location competition, the Hotelling model is extended by the following ways. First, in the consumer aspects, for online orders, the consumers' waiting cost is formulated to determine the indifferent location; second, the profit of a firm is subtracted by the delivery logistics cost for the online demands; third, the offline and online demands are considered in the integrated Hotelling model for studying the effect of the ratio of online demand on the competition.

3. Problem Description

Hotelling's linear city model was developed by Harold Hotelling in his article "Stability in Competition" in 1929 [2]. In this model he introduced the notions of locational equilibrium in a duopoly in which two firms have to choose their location considering consumers' distribution and transportation costs. Initially, the model was developed as a game in which firms first choose a location and after a selling price for their products. To set their business in the best location to maximize profits, the firms have to evaluate three key variables: competitors' location, consumers' distribution, and transportation costs. This model includes two different approaches. First, the static approach consists of a single stage, where firms choose their location and prices simultaneously; second, the dynamic one considers that the price is set after determining the location. The model is based on a linear city that consists of only a single straight street. For easy comprehension, the Hotelling model is sometimes explained by using the example of a beach where two ice cream stands are trying to decide their best locations.

3.1. Static Approach. In a beach going from west to east, of size $[0,1]$, where consumers are distributed evenly, two identical ice cream stands (A and B) with a marginal cost of production, $c > 0$, try to determine their best location. A is located at a distance a from point 0, while B is located at a distance b from point 1. Both ice cream stands offer the exact same ice creams, and therefore consumers' utility will be given only by the price of the ice cream and the distance to the stand. Despite differences in prices (P_1 and P_2), the stand with the lowest prices will not necessarily attract all the demands since consumers consider the distance to the stands. If both stands' prices are equal, the differential factor is how close consumers are to each stand. All consumers located to the left of a would go to stand A, and all consumers located to the right of $(1-b)$ would go to stand B. The remaining consumers, located between both stands, would go to whichever is the closest. Here, x is the exact middle of that beach, so consumers to its left would go to stand A, while consumers to its right would go to stand B. Two conditions are necessary for profits to be positive and maximized in both stands: selling prices must be higher than marginal costs; $a \neq 1-b$. This second condition implies that both stands cannot be located at the same point exactly in the middle of the beach. If this position is indifferent for the consumers, each stand would decrease its prices to attract consumers, and thus they would enter into a price war. If they had different marginal costs, the stand with the highest marginal cost would be in a clear disadvantage and would end up exiting the market, as the other stand would be able to push the prices further down and so attract all consumers. Depending on how prices are set, the prices of both stands are equal to their marginal costs. In this approach the key factor for product differentiation is location. Each stand will therefore set prices that will be higher than their marginal costs and will choose a location other than the middle of the beach.

3.2. Dynamic Approach. The dynamic approach has two stages. At first, the stands choose their location and then the prices are set. With regard to the previous approach two additional assumptions are introduced: both stands have equal marginal production cost; total cost for consumers depends on the price of the goods and the distance to the selected stands (the unit travelling cost is t). The point of division between the areas served by these two stands is determined by the condition that at a certain point it is indifferent for consumers to choose either one of the stands. After equating and equalizing, (1) is got. By solving it, the demand functions of the two stands are (2):

$$p_1 + t(x-a) = p_2 + t((1-b)-x), \tag{1}$$

$$\begin{gathered} D_1 = x = \frac{p_2 - p_1}{2t} + \frac{(1-b)+a}{2}, \\ D_2 = 1 - x. \end{gathered} \tag{2}$$

By using backward induction the subgame perfect Nash equilibrium can be found where both firms maximize their profits.

The dynamic approach has two opposite effects. First, an incentive for both stands to locate at the center of the beach increases their market share by reaching out the greatest amount of consumers, in what is known as the demand effect. And, an incentive for both stands to locate at opposite extremes exists, in what is considered to be the strategic effect. While the first effect will reduce differentiation between the stands, the second one will increase it.

This study considers the demand effect of two shops in a line city on their locations. However, activated by the offline and online sales patterns in the fast food industry, this work considers additional online orders based on the Hotelling model. The two shops sell goods at their locations when consumers come to their shops while they also accept online orders and deliver the goods to the consumers. However, the shops will overtake the delivery logistics costs. In the above static and dynamic approaches, when the consumers order the goods offline by going to the shops, the consumers themselves must undertake the travelling cost (t). Therefore, in the proposed problem in this study, as the main difference to the above Hotelling models, for online demands the shops undertake the logistics cost (delivery), and for offline demands the consumers undertake the logistics costs (travelling). Under the demand effect, two shops will locate at the center of the city. The effect of the consideration of online demands and logistics on the models is focused on in this work.

4. Formulation

4.1. Baseline Model Only considering Offline Demands

4.1.1. Shops Locating at the Ends of the Line City. In this analysis, p_i represents the price of shop i, $i = 1, 2$. The demand of shop i at p_i is denoted by $D_i(p_1, p_2)$. The point x is the indifferent location of the consumer to buy from shop 1 or shop 2. And the unit travel cost for consumer is denoted by t.

Assumption 1. The commodities from both shops are homogeneous; that is, the commodity cost c is the same for both shops.

Assumption 2. Without loss of generality, assume that each consumer purchases a single commodity at once.

Assumption 3. Shops in the line city are profit-oriented sellers; that is, profits of the shops should be positive.

Based on Assumptions 1 and 2, the propositions below are proposed.

Proposition 4. *A consumer chooses the less cost shop in the offline baseline situation; that is, the consumer on the left of x chooses shop 1 and on the right chooses shop 2.*

Proposition 4 suggests an economical consumer behavior that consumer prefers the less cost commodity. In the offline mode, consumers have to pay for the price of the commodity and the travel cost to the shop (no matter shop 1 or 2). Assuming that the travel cost is linear with the distance, x is obtained by

$$p_1 + tx = p_2 + t(1-x). \tag{3}$$

Proposition 5. *In the offline baseline situation, the Nash equilibrium exists in the profits of the shops and the equilibrium profits are $\pi_1^{off} = \pi_2^{off} = t/2$.*

In the context of the line city model, x is between the positions 0 and 1. Hence, the demand $D_i(p_1, p_2)$ is determined as

$$\begin{aligned} D_1(p_1, p_2) &= x = \frac{p_2 - p_1 + t}{2t}, \\ D_2(p_1, p_2) &= 1 - x = \frac{p_1 - p_2 + t}{2t}. \end{aligned} \tag{4}$$

According to the price p_i and $D_i(p_1, p_2)$, the profit π_i^{off} of shop i is evaluated by

$$\begin{aligned} \pi_1^{\text{off}}(p_1, p_2) &= (p_1 - c) D_1^{\text{off}}(p_1, p_2) \\ &= \frac{1}{2t}(p_1 - c)(p_2 - p_1 + t), \\ \pi_2^{\text{off}}(p_1, p_2) &= (p_2 - c) D_2^{\text{off}}(p_1, p_2) \\ &= \frac{1}{2t}(p_2 - c)(p_1 - p_2 + t). \end{aligned} \tag{5}$$

To get a positive profit as defined in Assumption 3, π_i^{off} is positive so that p_i is higher than c. We get the first order conditions of (5) on the price in the pricing game between shops 1 and 2 as

$$\begin{aligned} \frac{\partial \pi_1^{\text{off}}}{\partial p_1} &= p_2 + c + t - 2p_1, \\ \frac{\partial \pi_1^{\text{off}}}{\partial p_2} &= p_1 + c + t - 2p_2. \end{aligned} \tag{6}$$

Moreover, the second order conditions of (5) are

$$\frac{\partial^2 \pi_1^{\text{off}}}{\partial p_1^2} = \frac{\partial^2 \pi_1^{\text{off}}}{\partial p_2^2} = -2. \tag{7}$$

So the Nash equilibrium profits exists where (8) is satisfied:

$$\begin{aligned} \frac{\partial \pi_1^{\text{off}}}{\partial p_1} &= p_2 + c + t - 2p_1 = 0, \\ \frac{\partial \pi_1^{\text{off}}}{\partial p_2} &= p_1 + c + t - 2p_2 = 0. \end{aligned} \tag{8}$$

And the equilibrium prices and profits of both shops are

$$\begin{aligned} p_1^* &= p_2^* = c + t, \\ \pi_1^{\text{off}} &= \pi_2^{\text{off}} = \frac{t}{2}. \end{aligned} \tag{9}$$

4.1.2. Shops Locating at the Arbitrary Positions of the Line City. In the following, the shops can locate at any arbitrary positions of the line city. The location of shop 1 is $a \geq 0$, while the location of shop 2 is $1 - b$.

Assumption 6. Without loss of generality, it is assumed that $1 - a - b \geq 0$ (the shop 1 is on the left of the shop 2).

Considering consumer's quadratic travelling cost (the traveling cost is td^2, where d is the traveling distance from the consumer position to the shop stand), then the indifferent position x is obtained by

$$p_1^* + (x-a)^2 t = p_2^* + (1 - x - b)^2 t. \tag{10}$$

Proposition 7. *In the arbitrary position situation, the Nash equilibrium exists in the profits of the shops, where $p_1^* = c + t(1 - a - b)(1 + ((a-b)/3))$ and $p_2^* = c + t(1 - a - b)(1 + ((b-a)/3))$. p_1^* and p_2^* are the equilibrium prices of shops 1 and 2, respectively.*

As the result of (10), we get the value of x by

$$x = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}. \tag{11}$$

Similar to the baseline city situation, the demands of both shops are derived as

$$\begin{aligned} D_1(p_1, p_2) &= x = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}, \\ D_2(p_1, p_2) &= 1 - x = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)}. \end{aligned} \tag{12}$$

Because the positions of the shops are flexible, (12) is more complex than (4). Demand in arbitrary position situation is determined by five parameters, that is, positions a, b, prices p_1, p_2, and travel cost t. The profits are derived as

$$\begin{aligned}\pi_1(p_1,p_2) &= (p_1-c)D_1(p_1,p_2)\\ &= (p_1-c)\left(a+\frac{1-a-b}{2}+\frac{p_2-p_1}{2t(1-a-b)}\right),\\ \pi_2(p_1,p_2) &= (p_2-c)D_2(p_1,p_2)\\ &= (p_2-c)\left(b+\frac{1-a-b}{2}+\frac{p_1-p_2}{2t(1-a-b)}\right).\end{aligned} \tag{13}$$

Analyzing the first-order and second-order derivatives of (13) on the prices, the results are presented as

$$\begin{aligned}\frac{\partial\pi_1}{\partial p_1} &= a+\frac{1-a-b}{2}+\frac{p_2-2p_1}{2t(1-a-b)}+\frac{c}{2t(1-a-b)},\\ \frac{\partial\pi_2}{\partial p_2} &= b+\frac{1-a-b}{2}+\frac{p_1-2p_2}{2t(1-a-b)}+\frac{c}{2t(1-a-b)},\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial^2\pi_1}{\partial p_1^2} &= \frac{-2}{2t(1-a-b)},\\ \frac{\partial^2\pi_2}{\partial p_2^2} &= \frac{-2}{2t(1-a-b)},\end{aligned} \tag{15}$$

According to Assumption 6, the values of the first-order derivatives (see (14)) are varying from positive to negative while the values of the second-order derivatives (see (15)) are negative. So the Nash equilibrium exists (see (16)):

$$\frac{\partial\pi_1}{\partial p_1}=\frac{\partial\pi_2}{\partial p_2}=0. \tag{16}$$

The prices at equilibrium are solved as

$$\begin{aligned}p_1^*(a,b) &= c+t(1-a-b)\left(1+\frac{a-b}{3}\right),\\ p_2^*(a,b) &= c+t(1-a-b)\left(1+\frac{b-a}{3}\right).\end{aligned} \tag{17}$$

In the above solution, when the first-order derivative is derived for the profit to the price, the results will be different for the conditions that p_1, p_2, a, b are independent of each other; namely,

$$\frac{\partial a}{\partial p_1}=\frac{\partial b}{\partial p_1}=\frac{\partial p_2}{\partial p_1}=0. \tag{18}$$

When $a=b=0$, the shop 1 locates at 0 while the shop 2 locates at 1; when $a=1-b$, two shops locate at the same position. The equilibrium prices under these two conditions are (19) and (20) individually:

$$p_1^*(0,1)=p_2^*(0,1)=c+t, \tag{19}$$

$$p_1^*(a,1-a)=p_2^*(a,1-a)=c. \tag{20}$$

4.2. Extension Model for Online Consumer Demands. Based on the models in Section 4.1 for only considering offline demands, this section develops models for the online demands when considering the shops' delivery logistics costs. Similar to Section 4.1, two levels are considered, first for the shops locating at the ends of the line city and second for deciding locations for the shops. Two additional parameters are introduced for the online demands and delivery logistics: the cost of waiting for a unit of time is denoted by c^w; and the distribution cost for travelling for a unit of time is denoted by c^d.

4.2.1. Shops Locating at the Ends of the Line City

Proposition 8. *In the online shopping situation, a consumer prefers the less cost shop based on the commodity cost and penalty costs of waiting; that is, if consumers are on the left of the indifferent position $x^{on}=(p_2-p_1)/c^w+1/2$, they choose shop 1, while others choose shop 2.*

In the perspective of consumers, they choose online shops not according to their travelling costs to the shops, but according to the penalty costs deduced by waiting times. Therefore, the indifferent position x should satisfy (21) and is derived as

$$p_1+c^w(x^{\text{on}})^2=p_2+c^w(1-x^{\text{on}})^2, \tag{21}$$

$$x^{\text{on}}=\frac{p_2-p_1}{c^w}+\frac{1}{2}. \tag{22}$$

Similar to (4), the demands for the shops are derived as

$$\begin{aligned}D_1^{\text{on}} &= \frac{p_2-p_1}{c^w}+\frac{1}{2},\\ D_2^{\text{on}} &= \frac{p_1-p_2}{c^w}+\frac{1}{2}.\end{aligned} \tag{23}$$

However, different from the offline mode, the travel cost for the consumer is substituted by the delivery logistics costs for the shops. Therefore, the profits functions of the shops are different (comparing to (5)), as shown in

$$\begin{aligned}\pi_1^{\text{on}} &= \int_0^{x^{\text{on}}}\left(p_1-c-c^d x\right)dx\\ &= p_1x-cx-\frac{c^d x^2}{2}\Big|_0^{(p_2-p_1)/c^w+(1/2)}\\ &= p_1\left(\frac{p_2-p_1}{c^w}+\frac{1}{2}\right)-c\left(\frac{p_2-p_1}{c^w}+\frac{1}{2}\right)\\ &\quad-\frac{c^d}{2}\left(\frac{p_2-p_1}{c^w}+\frac{1}{2}\right)^2,\end{aligned}$$

$$\begin{aligned}\pi_2^{\text{on}} &= \int_{x^{\text{on}}}^{1}\left(p_2 - c - c^d(1-x)\right)dx \\ &= p_2 x - cx - c^d x + \left.\frac{c^d x^2}{2}\right|_{(p_2-p_1)/c^w+(1/2)}^{1} \\ &= \left(p_2 - c - c^d\right)\left(\frac{p_1-p_2}{c^w}+\frac{1}{2}\right) \\ &\quad + \frac{c^d}{2}\left(1-\left(\frac{p_2-p_1}{c^w}+\frac{1}{2}\right)^2\right).\end{aligned} \tag{24}$$

Proposition 9. *In the online shopping situation, the Nash equilibrium exists in the profits of the shops where equilibrium prices are* $p_1^* = p_2^* = c + (c^w/2) + (c^d/2)$.

As seen in (24), the profits should be subtracted by the delivery logistics cost (see $c^d x$ and $c^d(1-x)$ in the equations). When p_1, p_2, c, c^d are independent of each other, the first-order derivatives of the profits on the prices are shown in

$$\begin{aligned}\frac{\partial\pi_1^{\text{on}}}{\partial p_1} &= \frac{1}{2} - \frac{2p_1}{c^w} + \frac{p_2}{c^w} + \frac{c}{c^w} + \frac{c^d}{c^w}\left(\frac{p_2-p_1}{c^w}+\frac{1}{2}\right), \\ \frac{\partial\pi_2^{\text{on}}}{\partial p_2} &= \frac{1}{2} - \frac{2p_2}{c^w} + \frac{p_1}{c^w} + \frac{c}{c^w} + \frac{c^d}{c^w}\left(\frac{p_1-p_2}{c^w}+\frac{1}{2}\right).\end{aligned} \tag{25}$$

It is found that the second-order derivatives of the profits on the prices are negative, because c^w and c^d are both positive in (26). So the optimal prices which are equilibrium prices for both shops under the maximization of the profits are derived as

$$\frac{\partial^2\pi_1^{\text{on}}}{\partial p_1^2} = \frac{\partial^2\pi_2^{\text{on}}}{\partial p_2^2} = -\frac{2}{c^w} - \frac{c^d}{(c^w)^2}, \tag{26}$$

$$\begin{aligned}p_1^* &= c + \frac{c^w}{2} + \frac{c^d}{2}, \\ p_2^* &= c + \frac{c^w}{2} + \frac{c^d}{2}.\end{aligned} \tag{27}$$

4.2.2. Shops Locating at Arbitrary Positions of the Line City

Proposition 10. *In the online shopping situation where shops are at arbitrary positions, the Nash equilibrium exists in the profits of the shops.*

In the following, the shop locations are decisions and can locate at arbitrary positions of the line city. Then, the indifferent position x is derived by (28) and (29). The derivation processes in (30)–(33) are similar to (11)–(17) but the analytic expressions are more complex:

$$p_1 + c^w\left(x^{\text{on}} - a\right)^2 = p_2 + c^w\left(1 - x^{\text{on}} - b\right)^2, \tag{28}$$

$$x^{\text{on}} = a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}, \tag{29}$$

$$\begin{aligned}D_1^{\text{on}} &= x^{\text{on}} = a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}, \\ D_2^{\text{on}} &= 1 - x^{\text{on}} = b + \frac{1-a-b}{2} + \frac{p_1-p_2}{2c^w(1-a-b)},\end{aligned} \tag{30}$$

$$\begin{aligned}\pi_1^{\text{on}} &= \int_0^a\left(p_1 - c - (a-x)c^d\right)dx \\ &\quad + \int_a^{x^{\text{on}}}\left(p_1 - c - c^d(x-a)\right)dx \\ &= (p_1 - c)\left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right) \\ &\quad - a^2c^d + c^d a^2 \\ &\quad - c^d\left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right)^2 \times 2^{-1}, \\ \pi_2^{\text{on}} &= \int_{x^{\text{on}}}^{1-b}\left(p_2 - c - c^d(1-b-x)\right)dx \\ &\quad + \int_{1-b}^{1}\left(p_2 - c - c^d(x-(1-b))\right)dx \\ &= (p_2 - c)\left(\frac{1-a+b}{2} - \frac{p_2-p_1}{2c^w(1-a-b)}\right) \\ &\quad + \left(c^d b - c^d\right) \\ &\quad \times\left(1 - b - \left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right)\right) \\ &\quad + \left(c^d(1-b)^2\right. \\ &\quad\quad \left. - c^d\left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right)^2\right) \times 2^{-1} \\ &\quad + \left(c^d - c^d b\right)b - \frac{c^d - c^d(1-b)^2}{2},\end{aligned} \tag{31}$$

$$\begin{aligned}\frac{\partial\pi_1^{\text{on}}}{\partial p_1} &= \left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right) \\ &\quad - \frac{(p_1-c)}{2c^w(1-a-b)} + \frac{c^d}{2c^w(1-a-b)} \\ &\quad \times\left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right), \\ \frac{\partial\pi_2^{\text{on}}}{\partial p_2} &= \left(\frac{1-a+b}{2} - \frac{p_2-p_1}{2c^w(1-a-b)}\right) - \frac{(p_2-c)}{2c^w(1-a-b)} \\ &\quad + \frac{\left(c^d b - c^d\right)}{2c^w(1-a-b)} - \frac{c^d}{2c^w(1-a-b)} \\ &\quad \times\left(a + \frac{1-a-b}{2} + \frac{p_2-p_1}{2c^w(1-a-b)}\right),\end{aligned} \tag{32}$$

$$p_1^* = \left(c^w (1-a-b)(3-a-3b) + 3c + \frac{c^d}{2}(b+a+1) + \frac{c^d\left(2c + c^d b - c^d\right)}{2c^w(1-a-b)} \right) \cdot \left(\frac{c^w(1-a-b)}{3c^w(1-a-b)+c^d} \right),$$

$$p_2^* = 2c^w(1-a-b) + 2c + c^d b - c^d - \left(c^w (1-a-b)(3-a-3b) + 3c + \frac{c^d}{2}(b+a+1) + \frac{c^d\left(2c + c^d b - c^d\right)}{2c^w(1-a-b)} \right) \cdot \left(\frac{c^w(1-a-b)}{3c^w(1-a-b)+c^d} \right). \quad (33)$$

4.3. Model for Online and Offline Demands

Proposition 11. *In the mixed mode of online and offline, the profit is the combination of the online and offline profits according to a rate of online consumers and offline consumers, and the Nash equilibrium exists.*

The shops finally considered simultaneously serve for offline and online demands. A demand has the possibility of c^{on} to order goods by online methods such that the possibility of offline demand is $(1 - c^{\mathrm{on}})$. Here, the online and offline demands are independent of each other; the introduction of online channel will not increase the total demand. Based on the models in Sections 4.1 and 4.2, the profit model for online and offline demands is derived as

$$\pi_1 = c^{\mathrm{on}}\pi_1^{\mathrm{on}} + \left(1 - c^{\mathrm{on}}\right)\pi_1^{\mathrm{off}},$$

$$\pi_2 = c^{\mathrm{on}}\pi_2^{\mathrm{on}} + \left(1 - c^{\mathrm{on}}\right)\pi_2^{\mathrm{off}},$$

where,

$$\pi_1^{\mathrm{on}} = \int_0^a \left(p_1 - c - c^d x\right) dx + \int_a^{x^{\mathrm{on}}} \left(p_1 - c - c^d (x-a)\right) dx \quad (34)$$

$$\pi_2^{\mathrm{on}} = \int_{x^{\mathrm{on}}}^{1-b} \left(p_2 - c - c^d (1-b-x)\right) dx + \int_{1-b}^{1} \left(p_2 - c - c^d (x-1+b)\right) dx$$

$$\pi_1^{\mathrm{off}} = x^{\mathrm{off}} p_1$$

$$\pi_2^{\mathrm{off}} = \left(1 - x^{\mathrm{off}}\right) p_2.$$

When c^{on} is fixed, the equilibrium prices and profits can be derived by combining the results in Sections 4.1 and 4.2.

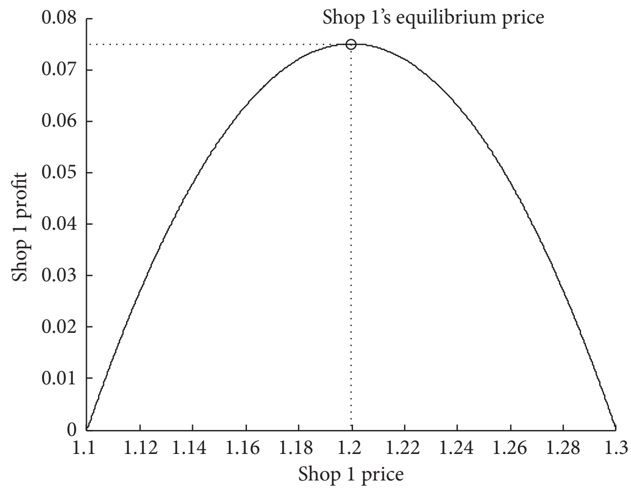

Figure 1: Relation between price and profit for shop 1 when shop 2 uses equilibrium price.

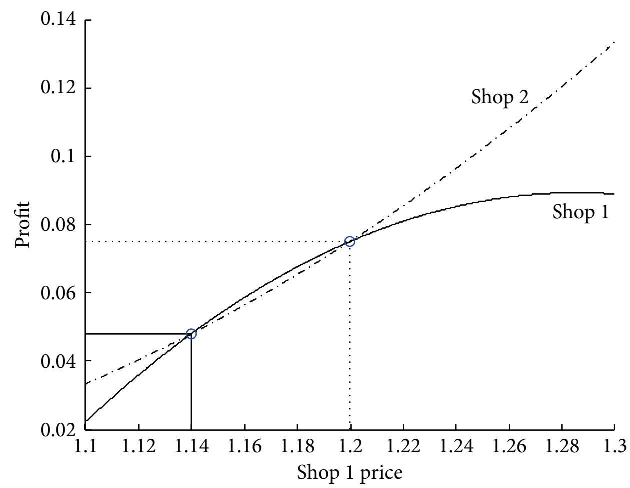

Figure 2: Profit curves of two shops when shop 2 decides its price following pricing of shop 1.

5. Experimental Results

In the following, the models in Sections 4.2.1, 4.2.2, and 4.3 are demonstrated.

5.1. Online Shops Locating at the Ends of the Line City. The following experiments use the settings $c = 1$, $c^w = 0.2$, and $c^d = 0.2$. Figure 1 presents the fact that the shop 1's profit varies with the price when the shop 2's price is the equilibrium price. It is seen that when the shop 1 deviates from the equilibrium price, its profit falls.

Figure 2 presents the profit curves for the two shops when the shop 2 determines its price following the pricing decision of shop 1. Changing the price from the equilibrium price by the shop 1 will incur the profit increase for shop 2 when shop 2 undertakes the following pricing strategy. Under this strategy, when shop 1 decreases its price to get competitive position in the price interval (1.14 to 1.2), shop 1 can get minor advantage

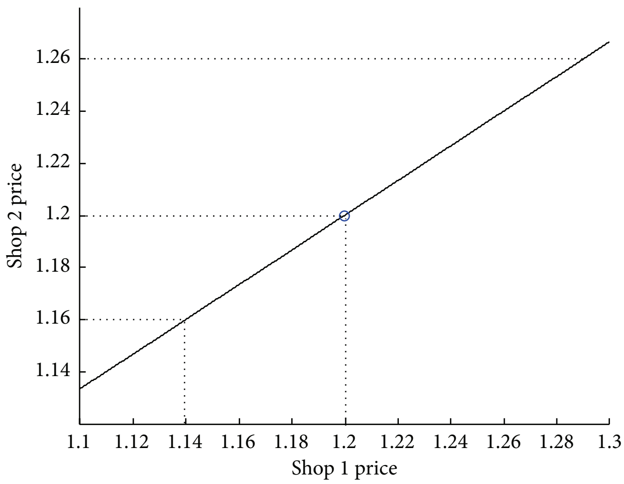

Figure 3: Relation between prices of two shops when shop 2 decides its price following shop 1.

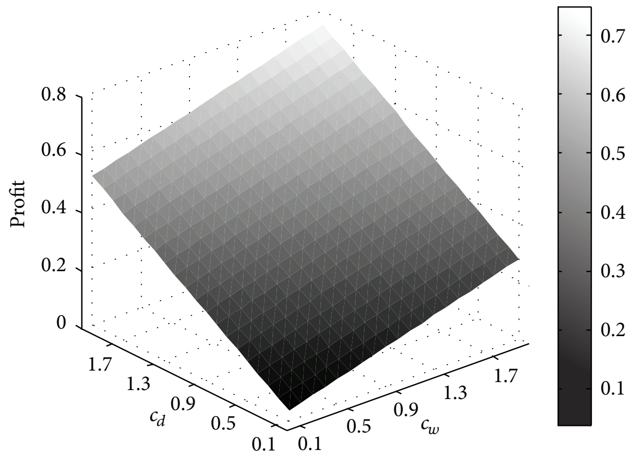

Figure 5: Impacts of varying unit waiting and delivery logistics costs on equilibrium profit.

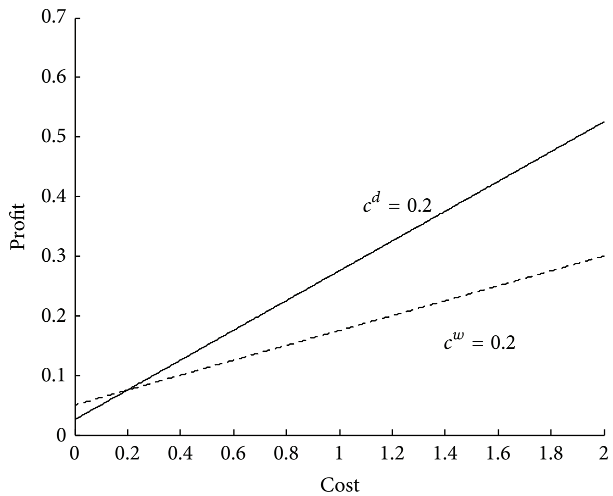

Figure 4: Equilibrium profits varying with waiting cost and delivery logistics cost.

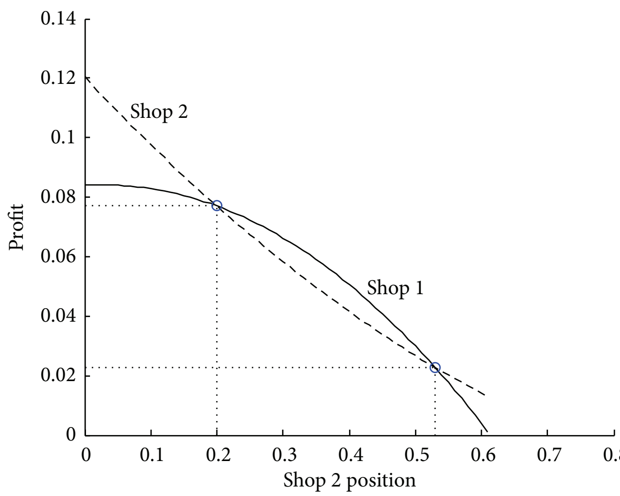

Figure 6: Equilibrium profits varying with shop 1's location when shop 2 locates at 0.2 ($b = 0.2$).

comparing to shop 2 while its profit also decreases. In the other intervals (from 1.1 to 1.14 and from 1.2 to infinite values), shop 1 will lose its competitive capability. Figure 3 further depicts the fact that the shop 2's optimal price is linear to the shop 1's price under the above described conditions.

Figure 4 presents the result of sensitivity analysis when the equilibrium profit varies with the cost parameters (c^w and c^d). In the experiment, one cost varies when the other is set to 0.2. In this study, the delivery logistics cost is linear to the travelling time from the shops to consumers, whereas the waiting cost is linear to the quadric of the waiting time (which is the time travelling from the shop to the consumer). Therefore, the effect of varying unit delivery logistics cost on the equilibrium profit is larger than that of varying the unit waiting cost. The impacts of varying the unit waiting and delivery logistics costs on the equilibrium profit are shown in Figure 5.

5.2. Online Shops Locating at Arbitrary Positions of the Line City. Under the demand effect, the two shops will locate at the center of the line city. However, the delivery logistics cost will "pull" the shops from the center. Figure 6 presents the shops' equilibrium profits varying with shop 2's location when shop 1 locates at 0.2 ($a = b = 0.2$ indicates the equilibrium locations for the two shops). When shop 1 moves to shop 2, their profits all decrease, although it seems that shop 1 earns more than shop 2. When shop 1 moves to shop 2, shop 2 with the fixed location chooses to decrease its price for attracting consumer demands, and the equilibrium price curves of the two shops are presented in Figure 7.

The profit of shop 1 is shown in Figure 8 with variable locations of shops 1 and 2. When the shop moves towards the center, the profit decreases until the shop is close enough to another shop. And there is an interesting phenomenon revealed by the experiment that when the location of one shop is fixed, the other shops moving toward the center

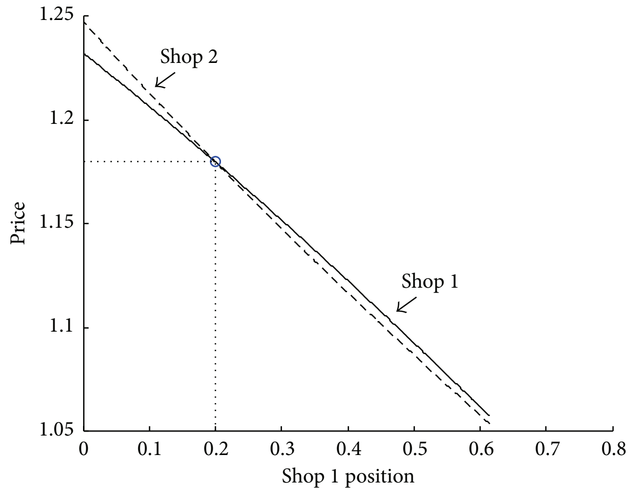

Figure 7: Equilibrium prices varying with shop 1's location when shop 2 locates at 0.2 ($b = 0.2$).

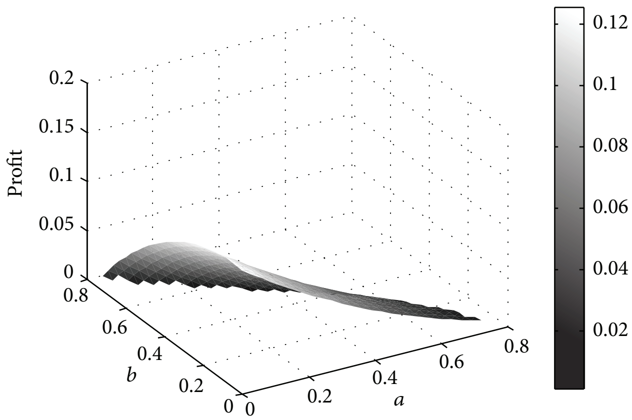

Figure 8: Shop 1's equilibrium profit varying with locations of two shops.

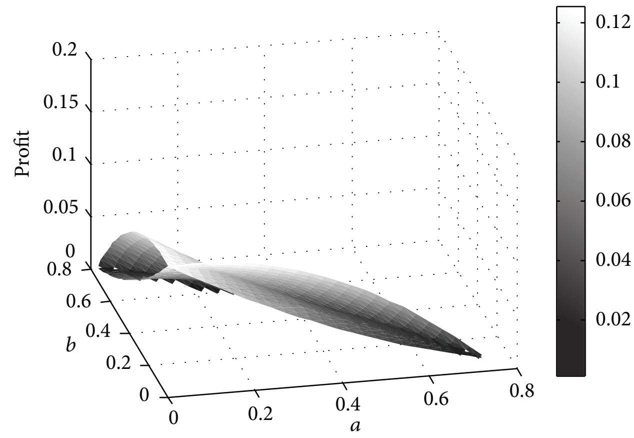

Figure 9: Equilibrium profit varying with locations of two shops.

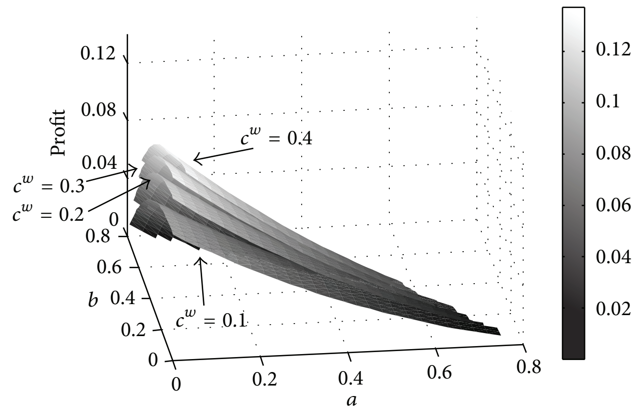

Figure 10: Equilibrium profit varying with shop 1's location (a) and unit waiting cost (c^w).

promote the profit of the fixed shop. When the shops move close to each other, the prices arise because the shops do not necessarily attract additional consumers by decreasing the prices. This variety of price will also benefit another fixed shop and leads to the peak point in the surface in Figure 8. Furthermore, when the production cost changes and the profit does not change, they can be the same as seen in Figure 8, because both shops sell homogeneous products.

Figure 9 shows the profit surfaces of both shop 1 and shop 2 from two angles of view. The intersection of the two surfaces is the Nash equilibrium profit line associated with the shops, and it is formed when the distances of the shops to the two ends of the city are equal. When the relative locations of the shops are not equal, the shop at disadvantageous location could move to the equilibrium location to get more profit and simultaneously decrease the competitor's profit.

Figure 10 depicts the equilibrium profit varying with the shops' locations and the unit waiting cost, while Figure 11 presents the equilibrium profit varying with the shops' locations and the unit delivery logistics cost. As a comparison result, the effect of unit waiting cost on the profit is bigger than the unit delivery logistics cost. The profit can be improved by decreasing delivery logistics cost and increasing the delivery service quality (which helps to decrease the waiting cost).

5.3. Shops Locating at Arbitrary Positions for Mixed Online and Offline Demands. In the following demonstration of the model considering online and offline demands, the default parameters are set as follows: the unit delivery logistics cost for online demand, $c^d = 0.2$; the unit waiting cost for online demand, $c^w = 0.2$; the unit production cost of goods, $c = 1$; the ratio of online demand, $c^{on} = 0.5$; the unit travelling cost of consumers for offline demand, $t = 0.2$.

Figures 12, 13, and 14 reveal the effects of locations and ratios of online demands or unit waiting costs on the equilibrium profits. Increasing the ratio of online demand or unit waiting cost improves the equilibrium profits.

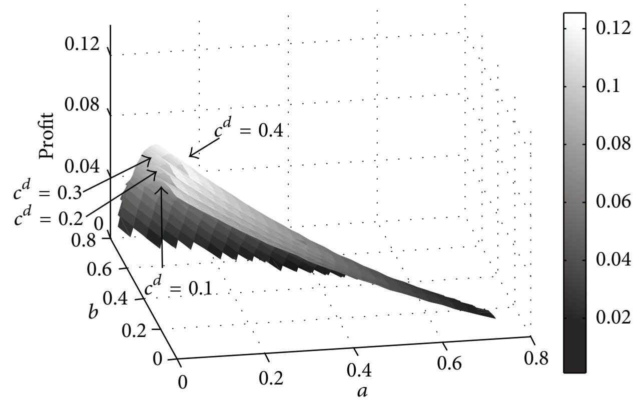

Figure 11: Equilibrium profit varying with shop 1's location (a) and unit delivery logistics cost (c^d).

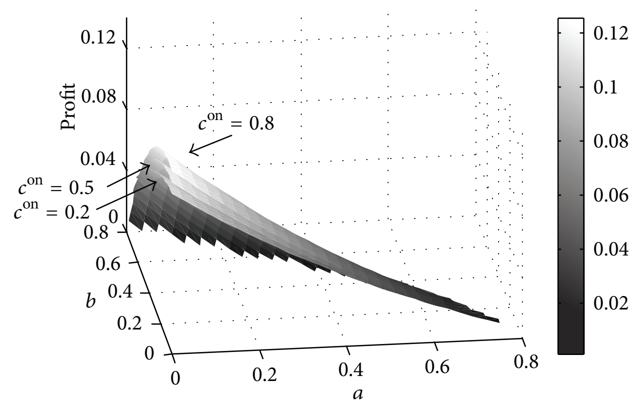

Figure 13: Equilibrium profit varying with shops' locations for three ratios of online demands ($c^{on} = 0.2, 0.5, 0.8$).

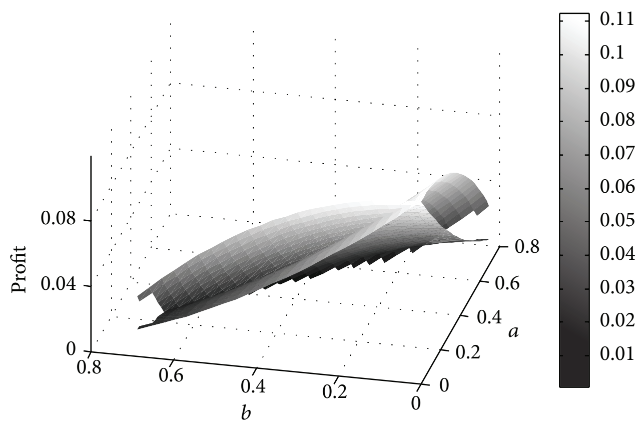

Figure 12: Equilibrium profit varying with shops' locations ($c^{on} = 0.5$).

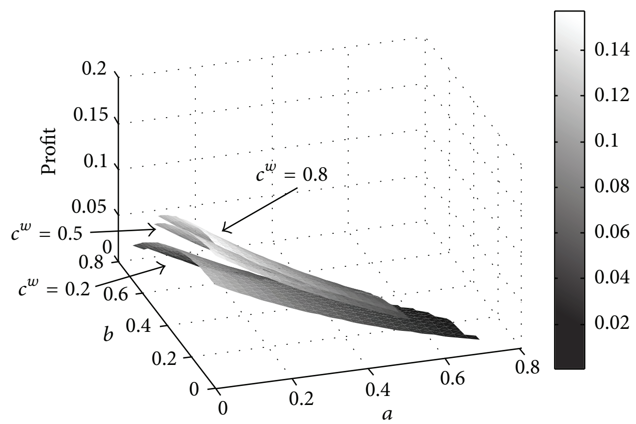

Figure 14: Equilibrium profit varying with shops' locations for three unit waiting costs ($c^w = 0.2, 0.5, 0.8$).

6. Conclusion

Traditional merchants are facing offline and online demands that are challenging the business and service modes. Activated by the new business mode of combining offline and online sales in the fast food industry, this study examines the effects of the mode on the competition based on the Hotelling model. Under the demand effect, the shop stands will locate at the center in a line city in the Hotelling model that only considers the consumers' logistics cost traveling to the shops. In the extended model for the offline and online demands, the shops undertake the delivery logistics cost for the online orders, whereas the consumers' waiting cost will affect their choices of shops called. Therefore, the main contributions of this study involve the following features: considering online and offline demands in an extended Hotelling model, considering consumers' travelling cost to shops for offline demand and shops' delivery logistics cost for online demand, and considering the effect of consumers' waiting cost for online orders on the indifferent shop location. As for the future research direction, first, the parameters used in the study should be further verified by empirical studies. Second, this study does not consider the demands dependent on the online sales mode, which is a practical feature of online and offline business. Third, the formulation for line city could be extended for real urban scenarios.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Evaluating Reverse Supply Chain Efficiency: Manufacturer's Perspective

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The paper aims to illustrate the use of fuzzy data envelopment analysis (DEA) in analyzing reverse supply chain (RSC) performance from the manufacturer's perspective. By using an alternative α-cut approach, the fuzzy DEA model was converted into a crisp linear programming problem, thereby altering the problem to an interval programming one. The model is able to obtain precise and robust efficiency values. An investigation was also made between the obtained efficiency scores and certain relevant background information of the companies. The study revealed that while ISO 14001 certification usually ensures an environmentally friendly supply chain network, companies which have implemented RSC techniques since a longer duration do not necessarily have a more efficient supply chain in general.

1. Introduction

Individual companies are no longer in a position to compete as distinct self-sufficient entities, thus forcing them to focus their attention towards supply chains. In the current competitive environment, the ability of a business to succeed will depend on the ability of the manufacturer to integrate itself into the intricate network of the supply chain. It is noted that a supply chain is not a chain of companies with one-to-one, business-to-business relationships but a network of multiple companies and relationships. The operations in supply chains and logistics are part of today's most significant commercial activities as they are pivotal for businesses to remain viable. The two main goals of managing a supply chain are to realize where a particular company stands among its competitors and how it can be developed further.

To understand how proficiently a company is performing in a supply chain, we measure its efficiency. Traditionally, the efficiency of a supply chain was measured by taking the ratio of revenue over the total supply chain operational cost. Nonetheless, there are several additional objectives of a business apart from monetary benefits. These objectives depend on several factors, namely, societal factors, environmental factors, technological factors, and so forth. Incorporating all these multiple factors in the form of inputs and outputs to determine efficiency is difficult without the right tool. Therefore, finding the efficiency while incorporating multiple inputs and outputs for a reverse supply chain (RSC) is even more challenging, as this is not something traditionally done.

The aim of RSC is to help in collecting, recycling, reusing, or disposing end-of-life products. It is crucial to recapture the value of products and to provide methods for proper disposal [1]. The process is usually very complex, with many individual players involved in it (such as customers, collectors, recyclers, etc.). Progress in RSC is slow as it is not usually considered as a system which could return sufficient value for the resources involved. Thus, it becomes very important to efficiently manage this chain. Efficiency measurement of RSC remains to be a relatively underdeveloped and unexplored area [2]. Instead of focusing on the entire RSC, this paper intends to measure the efficiency of RSC from a manufacturer's perspective. Thus, only data obtained from manufacturers have been considered.

The main contribution of this study is to pioneer the evaluation of RSC efficiency by primarily using data envelopment analysis (DEA). DEA was first proposed by Charnes et al. [3] and is a nonparametric method of efficiency analysis for associating units relative to their best peers (efficient frontier). Mathematically, it is a linear programming-based methodology for evaluating the relative efficiencies of a set of decision making units (DMUs). DEA accepts multiple inputs and outputs, making it ideal for estimating the efficiency of a complex structure like the supply chain which involves many attributes. However, one major point of concern is that observed values of the input and output data in real-world RSC scenarios are often not precise. Imprecise evaluation may be the result of incomplete and unavailable information. Therefore, various fuzzy methods for dealing with this impreciseness in DEA have been proposed. In this paper, a fuzzy version of the CCR model (please refer to Section 3) is utilized to obtain the RSC efficiencies of different companies from a manufacturer's point of view. Other background information of the respective companies has been collected. A comparison of this data has given greater insights into the reason why certain companies have not performed well in implementing an efficient RSC system.

In general, the paper is structured as follows. Section 2 reviews the efficiency measurement for RSC. Section 3 deals with the explanation of the fuzzy DEA model used to measure RSC efficiency. In Section 4, the overall model is presented, carefully explaining the reasons behind selecting its performance attributes. Next, Section 5 explains the data collection process. Following this, Section 6 presents and discusses the results obtained from a real-world application of the model. Finally, in Section 7, conclusions are drawn from the paper and future research directions are highlighted.

2. Literature Review

Prahinski and Kocabasoglu [4] defined RSC as the effective and efficient management of the series of activities required to retrieve a product from a customer and either dispose it or recover its value. The importance of understanding RSC has increased due to several reasons. As mentioned by Trebilcock [5], the amount of product returns has risen over 50% of sales for many industries. End-of-life take-back laws and environmental conscious consumers have pressured companies to take responsibility for the disposal of their products which could contain hazardous substances. Additionally, landfill capacity has become more and more limited and expensive, and thus recycling end-of-life products has become more viable [6]. Despite its importance, RSC is usually not a firm's core business because getting top managers' attention and measuring its performance are a difficult task [7].

For measuring the efficiency of a supply chain, it is essential to obtain the input and output data values. In the case of the forward supply chain, the data required are usually available as they have traditionally been used by companies to keep track of their goods or services. These data are critical as most businesses make use of them as a measure of growth and market share. Thus, measuring the efficiencies of forward supply chains can be done by normal DEA or other methods as the data are normally available.

Now consider the case of RSC. The importance of RSC has not reached the maturity level of forward supply chain. Recycling or reusing of goods is not as common as production of goods. So it is difficult to find companies which have recycled for more than 10 years, even though they could have existed for thrice as long. Even in the case that recycling is a very important aspect in an industry (say paper industry), the informatics involved in RSC is not as high as that of forward supply chain. In addition, for some cases like measuring the impact of business activities on the environment, it may not be possible to obtain all required data with the required preciseness [8].

A literature review related to the performance measurement of RSC has been performed by Taticchi et al. [9] and they called for a more structured approach to evaluate its performance. In addition, there are limited existing performance measurement tools which have been applied in RSC and they may not be adequate to fully assess its efficiency. Some tools which have been utilized in applications related to RSC performance evaluation are, for example, analytical hierarchy process [10, 11], life cycle assessment [12], activity-based costing [13], balanced scorecard [14], and fuzzy logic [15].

There are very few papers published in which RSC performance has been evaluated directly. It is observed that the measurement of RSC efficiency by nonfuzzy methods is difficult due to the lack of precise numerical data required. Another observation on previous studies is the analyses were done on a single supply chain instead of multiple supply chains. Therefore, there is a need for a more robust tool to handle the unavailability of precise data and to benchmark multiple supply chains. One tested method in case of unavailability of data is the use of linguistic data. So in this condition, data are measured by qualitative values which have ordinal relations. For example, words like "Excellent," "Good," and "Average" could be used to describe certain attributes, and from this we can conclude that "Excellent" is better than "Good" which is in turn better than "Average."

It is also observed that fuzzy DEA has been developed over the past few years and has been proven to successfully measure efficiencies [16–21]. One of the biggest advantages of using fuzzy DEA to solve RSC models is the use of linguistic data to achieve precise and robust efficiency values. Thus, in this paper, a fuzzy version of the CCR model has been applied to analyze RSC performance. This has also allowed the researchers to obtain data from more companies as the information requested is basically linguistic. In addition, different types of companies can be approached, which could differ by the type of product, the monetary scale, and other factors.

3. Mathematical Model

This paper utilizes DEA as the base model for RSC evaluation. The first DEA model is called CCR model which was developed by Charnes et al. [3]. DEA is a mathematical model used to measure the relative efficiencies of a group of homogenous organizations (commonly referred to as decision making units or DMUs in the DEA literature) that convert multiple inputs into multiple outputs. It is favorable for performing benchmarking analysis because it does not need any prior weight to be assigned to the measures and thus reducing subjective or biased judgment on the results.

Assume that there are n DMUs ($\mathrm{DMU}_1, \mathrm{DMU}_2, \ldots, \mathrm{DMU}_n$) to be evaluated. Varying amounts of m different inputs are consumed by each DMU to produce s different outputs. The jth DMU (DMU_j) consumes m inputs x_{ij} ($i = 1, \ldots, m$) to produce s outputs y_{rj} ($r = 1, \ldots, s$). In the following model, the crisp vectors of input and output values of the current DMU under evaluation, DMU_p, are represented by x_{ip} ($i = 1, \ldots, m$) and y_{rp} ($r = 1, \ldots, s$), respectively. The CCR model is defined as follows:

$$\begin{aligned} \text{Max} \quad & W_p = \sum_{r=1}^{s} \left(u_r y_{rp}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} \left(v_i x_{ip}\right) = 1 \\ & \sum_{r=1}^{s} \left(u_r y_{rj}\right) - \sum_{i=1}^{m} \left(v_i x_{ij}\right) \le 0 \quad \forall j \\ & u_r, v_i \ge 0 \quad \forall i, r. \end{aligned} \tag{1}$$

Model (1) requires precise information and data, which may not be available in all cases, but this does not end the modeling of a complex system. Generally, the lack of quantitative data required to solve model (1) is made up by obtaining qualitative data. As it is not feasible to incorporate qualitative data into this model directly, this model is to be modified. In other words, a fuzzy approach seems ideal for such a complex system. The method proposed by Saati et al. [22] has been implemented to introduce fuzziness into the CCR model. The CCR model with fuzzy data can be written as follows:

$$\begin{aligned} \text{Max} \quad & W_p = \sum_{r=1}^{s} \left(u_r \widetilde{y}_{rp}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} \left(v_i \widetilde{x}_{ip}\right) = \widetilde{1} \quad \forall i \\ & \sum_{r=1}^{s} \left(u_r \widetilde{y}_{rj}\right) - \sum_{i=1}^{m} \left(v_i \widetilde{x}_{ij}\right) \le 0 \quad \forall j \\ & u_r, v_i \ge 0 \quad \forall i, r, \end{aligned} \tag{2}$$

where ~ is used to denote fuzziness.

Triangular fuzzy numbers are extensively used as fuzzy inputs. Thus, the fuzzy input in model (2) is in the form of $\widetilde{x} = (x^m, x^l, x^u)$ with $x^l \le x^m \le x^u$. Similarly, the fuzzy output is in the form of $\widetilde{y} = (y^m, y^l, y^u)$ with $y^l \le y^m \le y^u$.

Let $\widetilde{x}_{ij} = (x_{ij}^m, x_{ij}^l, x_{ij}^u)$ and $\widetilde{y}_{rj} = (y_{rj}^m, y_{rj}^l, y_{rj}^u)$. The previous model is updated as follows:

$$\begin{aligned} \text{Max} \quad & W_p = \sum_{r=1}^{s} u_r \left(y_{rp}^m, y_{rp}^l, y_{rp}^u\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i \left(x_{ip}^m, x_{ip}^l, x_{ip}^u\right) = \left(1, 1^l, 1^u\right) \quad \forall i \\ & \sum_{r=1}^{s} u_r \left(y_{rj}^m, y_{rj}^l, y_{rj}^u\right) - \sum_{i=1}^{m} v_i \left(x_{ij}^m, x_{ij}^l, x_{ij}^u\right) \le 0 \quad \forall j \\ & u_r, v_i \ge 0 \quad \forall i, r. \end{aligned} \tag{3}$$

Model (3) represents a possibilistic linear programming problem. One widely used method for solving this type of problem is the α-cut approach. Thus, incorporating the α-cut approach changes the model as follows:

$$\begin{aligned} \text{Max} \quad & W_p = \sum_{r=1}^{s} u_r \left[\alpha y_{rp}^m + (1-\alpha) y_{rp}^l, \alpha y_{rp}^m + (1-\alpha) y_{rp}^u\right] \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i \left[\alpha x_{ip}^m + (1-\alpha) x_{ip}^l, \alpha x_{ip}^m + (1-\alpha) x_{ip}^u\right] \\ & \qquad = \left[\alpha + (1-\alpha) 1^l, \alpha + (1-\alpha) 1^u\right] \quad \forall i, \\ & \sum_{r=1}^{s} u_r \left[\alpha y_{rj}^m + (1-\alpha) y_{rj}^l, \alpha y_{rj}^m + (1-\alpha) y_{rj}^u\right] \\ & \qquad - \sum_{i=1}^{m} v_i \left[\alpha x_{ij}^m + (1-\alpha) x_{ij}^l, \alpha x_{ij}^m + (1-\alpha) x_{ij}^u\right] \\ & \qquad \le 0 \quad \forall j \\ & u_r, v_i \ge 0 \quad \forall i, r. \end{aligned} \tag{4}$$

Model (4) can be solved in an efficient and robust manner by incorporating variables in the intervals which are capable of satisfying the set of constraints and maximizing the objective function at the same time. Thus, the final model is transformed to:

$$\begin{aligned} \text{Max} \quad & W_p = \sum_{r=1}^{s} \left(\overline{y}_{rp}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{m} \left(\overline{x}_{ip}\right) = 1 \\ & \sum_{r=1}^{s} \left(\overline{y}_{rj}\right) - \sum_{i=1}^{m} \left(\overline{x}_{ij}\right) \le 0 \quad \forall j, \\ & v_i \left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^l\right) \\ & \qquad \le \overline{x}_{ij} \le v_i \left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^u\right) \quad \forall i, j, \\ & u_r \left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^l\right) \\ & \qquad \le \overline{y}_{rj} \le u_r \left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^u\right) \quad \forall r, j, \\ & u_r, v_i \ge 0 \quad \forall i, r. \end{aligned} \tag{5}$$

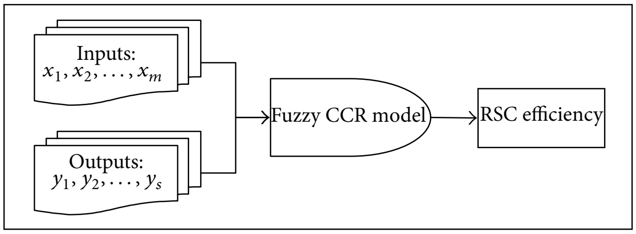

Figure 1: Conceptual framework for RSC evaluation.

Model (5) is now a parametric programming model with α being a parameter between 0 and 1, thereby converting it into a parametric linear programming problem. Now for each α value, there exists an optimum solution. Thus, the efficiency scores may change due to the value of α. It is quite noticeable that lower α values help in providing a greater range, thereby helping in better optimization for any particular DMU in general. So one could expect the value of the efficiency score to decrease or remain constant with increasing α. This point will be further discussed in Section 6.

Once the data are entered into the model, it generates the efficiency score of the RSC. As the efficiency has been evaluated from the manufacturer's perspective, it gives an idea of how well the manufacturer has adjusted into the supply chain. Lower efficiency implies that the manufacturer could focus more on recycling or reusing aspects of its business. Higher efficiency implies that the manufacturer has adjusted properly into the supply chain, and hence it is not only performing well individually but also helps in the overall supply chain to be more efficient. The result of this model is twofold. It helps in providing a reliable and robust efficiency score from linguistic data which would otherwise be difficult to calculate because of lacking numerical information. It also helps in effective comparison of companies which are not of similar production type, as the input of linguistic data has ensured a similar comparison ratio as opposed to numerical data.

4. Overall Model

Figure 1 shows the conceptual framework for evaluating RSC efficiency. Firstly, the data are collected which represent the input and output values of a particular DMU. The efficiency is calculated from the manufacturer's perspective. Hence, it is of great importance that the inputs and outputs have been formulated such that the manufacturer answers these based on its performance in the field of interest. Following the triple bottom-line framework in sustainability [23] and a critical review on the literature, the developed input and output measures cover the major aspects of a company's RSC, namely, environmental, economic, societal, and operational aspects, investment, and relationship between supply chain parties. These aspects ensure that the study covers diverse elements of RSC, thus making the study more relevant. It is also important to note that most of these performance attributes will not have been documented or measured quantitatively by a manufacturer. Hence, linguistic data are collected from each manufacturer. This proves to be very useful as normally, attributes like societal measures do not have any measuring yardstick. With the use of linguistic data, the attributes have managed to accept many diverse factors into one model, thus making it more realistic. Each of these performance attributes is described below.

4.1. The Input Variables

x_1: Operational and Technological Cost. This performance attribute is mainly concerned with the cost required in operating the recycling or reusing process. It also includes the cost of acquiring the necessary technology. From the manufacturer's point of view, it shows how much capital has been used to implement RSC, hence making it an input variable [24].

x_2: Investment in Environmental Initiatives and Certification. Chien and Shih [25] evaluated environmental performance by considering environmental policies and measures, among other factors. This performance attribute is mainly concerned with the amount of money that the manufacturer has invested to make the RSC more environmentally friendly by rolling out pertinent initiatives and obtaining formal certification such as ISO 14001. This is an input because it is the expenditure that has to be spent in order to have an efficient RSC.

x_3: Relationship between RSC Parties. This performance attribute is concerned with the smooth functioning of the RSC as a whole [4]. The stronger the relationship between parties is, the easier it is for the supply chain to function properly. Thus, it is considered as an important performance attribute for the model. It is chosen as an input variable because it is an enabler for an efficient RSC.

4.2. The Output Variables

y_1: Environmental—Reduction of Impact. It is the performance attribute which measures the RSC "greenness." This mainly shows the supply chain's ability to reduce energy consumption, emission, pollution, and waste disposal as given in Papadopoulos and Giama [26]. From the manufacturer's perspective, the impact on the environment is directly related to the approaches adopted in manufacturing. Thus, it is considered as an output variable, as it depends directly on the manufacturer's actions.

y_2: Economic—Cost Saving and Profit Generation. Zhu and Sarkis [27] mentioned that economic performance is the most important factor for companies that wish to implement environmental management practices. It is the most widely used performance attribute to measure the efficiencies of various profit-oriented organizations. It mainly deals with the money saved and profit generated by the manufacturer in implementing RSC techniques. It is considered as an output variable as it shows how well the manufacturer has succeeded in implementing RSC techniques.

y_3: Societal—Quality of Life. This performance attribute mainly concerns the health and wellness of people involved in the supply chain, namely, the customers, recyclers, manufacturers, and so forth, as clearly emphasized by Wan and Chan [28]. Apart from the aforementioned, the society at large is affected by the actions of the company. This attribute is considered as an output variable, mostly because it gives an idea of how well the company has managed to function smoothly without adversely affecting the society at large.

Once the data have been obtained, they are then structured and entered into the model (discussed in Section 3). This model calculates precise and robust efficiency scores based on the parameter α. It is able to accept different linguistic data and is also able to give various values of efficiency for different values of α, thereby separating the truly efficient RSCs from the rest.

5. Data Collection

Testing the model in a real-world environment is important for the study. A survey was designed to collect input and output variables data from different companies. The companies were not pertinent to any particular industry. Apart from the aforementioned data, information like product type, number of employees in the company, duration (number of years) for which RSC practices have been adopted, and environmental certification received by the company was also collected. The purpose of this information was to make more in-depth insights as to how the efficiencies are affected by these factors, thereby improving the value of the study. Normally, supply chain efficiencies measured by DEA are more comparable for similar companies, as it measures the relative performances of DMUs which are similar units. However, the use of fuzzy DEA has crossed this barrier, thereby improving the reach of the study to more diverse types of companies, while keeping the efficiencies comparable. As the survey aimed to provide the manufacturer's perspective on RSC, the data from diverse companies helped in analyzing various supply chains from a single perspective. Data collection was done by mailing.

A questionnaire was developed especially for this purpose (please refer to the Appendix). It was evident at the beginning of the study that exact values of the required data would not be easily accessible or in some cases not available. It also became clear that asking for such data which were not accessible might result in very few responses, thereby reducing the response rate significantly. So questions were accompanied by check boxes with answers like "Very High," "High," "Normal," "Low," and "Very Low." Thus, the answers were completely linguistic. In other words, the data collected were fuzzy.

The questionnaire survey was carried out between July and August 2013. The companies were chosen from the Federation of Malaysian Manufacturers Directory. It was evident that not all companies have a functional RSC. Thus, the authors contacted the companies via telephone to check whether they have a proper RSC in place. A total of 70 companies in the Johor region were contacted. Only 44 of these companies satisfied the criterion. Only then, questionnaires were sent to the Managing Directors or Plant Managers of these companies. It is noted that these companies were not producing similar goods. We received responses from 21 companies, which made the response rate a commendable 47.7%. This is mainly due to the linguistic nature of the data required, without which certainly the responses would be lesser and the study would not have been possible. Of these 21 returned questionnaires, 20 were filled completely to the level required for the study. Hence, 20 responses from different types of companies were used in the study. The number of DMUs satisfies the rules of thumb established by Golany and Roll [29], Bowlin [30], and Dyson et al. [31], about the minimum number of DMUs needed for a DEA analysis.

The answers were then assigned with values such as 10, 30, 50, 70, and 90. Specifically, 10 corresponds to the answer for "Very Low" and 90 corresponds to "Very High," with 50 being the moderate value. The $(x_{ij}^m, x_{ij}^l, x_{ij}^u)$ and $(y_{rj}^m, y_{rj}^l, y_{rj}^u)$ values of every input and output for each DMU have been determined. The maximum range of values was set to be ±5 of the original value. The reason for this is that it allows a range which should provide enough variation for the optimum value to exist but at the same time prevent any overlap between any two neighboring values. For instance, there is no chance for any permissible value of α that the values of Very Low and Low are the same. At most, Very Low may reach the value of 15 and Low may reach the value of 25, maintaining a minimum gap of 10 throughout. Clearly, the real value of this range also depends on α, as explained in the model in Section 3. As the value of the parameter α is chosen, the efficiency score is obtained. $\alpha = 0$ corresponds to the maximum range and $\alpha = 1$ corresponds to the efficiencies being calculated by mean values of the data without considering left or right spreads.

The data from the responses were put into the fuzzy CCR model and solutions were obtained using a MATLAB program written based on the model. The results are presented in the next section.

6. Results and Discussion

Table 1 shows the efficiencies of different DMUs for different α values. An efficiency value of 1 means a DMU is efficient in its RSC, while a score less than 1 indicates a DMU is less efficient. Of the 20 DMUs, 9 are efficient when $\alpha = 0$ and $\alpha = 0.25$, 8 are efficient when $\alpha = 0.5$, 7 are efficient when $\alpha = 0.75$, and only 3 are efficient when $\alpha = 1$. The number of efficient DMUs decreases as α increases. Generally, it can be observed that the efficiency value for every DMU decreases (or remains constant) with the increase of α. This point was raised in Section 3. The explanation is very straightforward. With a decrease in the spread of value when increasing α, the value is confined to a smaller range till it finally becomes one value (mean value) when $\alpha = 1$. Thus, by increasing the α value, there is a good chance that the optimum data value gets removed from the range, thereby causing the value to be less optimum and consequently achieving a lower efficiency score.

Table 1: Efficiencies of different DMUs for different α values.

DMU	α 0	0.25	0.50	0.75	1
D_1	1.0000	1.0000	1.0000	1.0000	0.9600
D_2	1.0000	1.0000	1.0000	1.0000	1.0000
D_3	0.4720	0.3921	0.3261	0.2672	0.2171
D_4	0.5891	0.5062	0.4298	0.3630	0.3027
D_5	0.5666	0.4873	0.4148	0.3482	0.2867
D_6	0.6151	0.5204	0.4371	0.3625	0.2975
D_7	1.0000	1.0000	1.0000	1.0000	0.9400
D_8	0.6899	0.6054	0.5254	0.4494	0.3771
D_9	0.2667	0.2264	0.1874	0.1509	0.1160
D_{10}	1.0000	1.0000	1.0000	1.0000	1.0000
D_{11}	0.6220	0.5292	0.4466	0.3748	0.3111
D_{12}	1.0000	1.0000	1.0000	1.0000	0.9600
D_{13}	0.5560	0.4772	0.4051	0.3407	0.2800
D_{14}	1.0000	1.0000	0.7299	0.5257	0.3800
D_{15}	0.4418	0.3636	0.2980	0.2414	0.1924
D_{16}	0.9085	0.7425	0.5926	0.4635	0.3520
D_{17}	0.4117	0.3569	0.3045	0.2543	0.2064
D_{18}	1.0000	1.0000	1.0000	1.0000	0.8600
D_{19}	1.0000	1.0000	1.0000	1.0000	1.0000
D_{20}	1.0000	1.0000	1.0000	0.9672	0.7544

For D_2, D_{10}, and D_{19}, their efficiency values remain as 1. Thus, the efficiency values have not been affected for these DMUs. For D_1, D_7, D_{12}, D_{18}, and D_{20}, their efficiency values start at 1 for $\alpha = 0$, but by around $\alpha = 0.75$ or so, their efficiencies reduce. These DMUs represent the most efficient ones in terms of their RSCs. There are also several DMUs whose efficiencies lie around the 0.5 region. For DMUs whose efficiencies lie below this, their RSCs are highly inefficient. It might be because these DMUs have not considered the environmental, economic, societal, and other aspects of their companies and have therefore performed in an inefficient manner.

Table 2 shows the background information which was also collected from the manufacturers. The average value of efficiency was obtained for the different values of efficiency obtained from different α values. In addition, rankings have been given by sorting the average efficiency scores (see the last column of Table 2). Those DMUs which have low rankings would need a radical change in their RSC in order to become more efficient.

Based on Table 2, some additional insights linking the background information of the companies with the average efficiency scores can be attained. Firstly, it was expected that ISO certification would provide a major edge to the companies in terms of their superior RSC practices, thereby significantly boosting their efficiency scores. Of the 20 companies, 9 of them have an ISO 14001 certification. The average of the efficiencies of the ISO rated companies was 0.7057. The average of the efficiencies of the others was 0.6107. Clearly, ISO rated companies have a better efficiency than those not rated. However, if the companies are analyzed individually, then one would notice that ISO certification does not guarantee a company to be efficient in RSC. For example, D_3 and D_4 perform below average even though they are ISO certified. This could be because they have utilized more inputs (cost and investment) rather than producing more outputs, to attain the certification. Nevertheless, in average, ISO certification helps to provide a better and more efficient RSC.

The other factor taken into consideration was the duration for which the companies have implemented RSC techniques to improvise their business model. It was thought that the longer the duration, the higher the efficiency score in general. However, apart from a few exceptions, companies with lesser experience have had higher efficiency scores. This was unexpected.

To explain this observation, it could be that the companies which have started implementing RSC techniques recently are bound to have superior technological machinery to perform effective recycling or reusing and therefore have had greater success in achieving more efficient supply chains. It is possible that manufacturers which are old-timers in the field have not managed to update their techniques and machinery or have simply focused on some other aspects of their business model. This might indicate that RSC has lost its momentum in the organizations which have long implementation history and thus they are not performing well. Hence, it seems justifiable that the companies which have recently started utilizing RSC techniques have performed efficiently.

Table 2: Data for background information of each DMU and average efficiency (average of efficiencies obtained from various α values).

DMU	Average efficiency	ISO certification	Years since RSC was implemented	Efficiency ranking
D_1	0.9920	No	2	4
D_2	1.0000	No	4	1
D_3	0.3349	Yes	3	17
D_4	0.4382	Yes	6	14
D_5	0.4207	No	10	15
D_6	0.4465	No	4	13
D_7	0.9880	Yes	12	6
D_8	0.5294	Yes	5	11
D_9	0.1895	No	9	20
D_{10}	1.0000	Yes	2	1
D_{11}	0.4567	Yes	6	12
D_{12}	0.9920	Yes	13	4
D_{13}	0.4118	No	5	16
D_{14}	0.7271	No	2	9
D_{15}	0.3074	No	9	18
D_{16}	0.6118	Yes	4	10
D_{17}	0.3068	No	5	19
D_{18}	0.9720	No	3	7
D_{19}	1.0000	Yes	3	1
D_{20}	0.9443	No	8	8

It is worth noting that in terms of other background information of the companies (i.e., type of industry and number of employees), no particular insight could be inferred between these factors and the average efficiency scores.

In terms of managerial implications, the model is applicable to evaluate RSC performance from a manufacturer's perspective. For each DMU and for every α value, the model is able to consolidate multiple input and output measures into one efficiency score. The model does not need precise data and it serves as a useful benchmarking tool to evaluate the RSC performances of various organizations. The results could help the DMUs under evaluation to have an overview on how well their RSC is performing in relation to others. Consequently, they could strategize and take appropriate actions to improve their RSC efficiency which will enable them to be more sustainable environmentally, economically, and socially.

7. Conclusions

Supply chain management is one of the most discussed topics in the commercial literature. Measuring the efficiency of a supply chain is a heavily researched topic. Despite this, there are very few papers which have attempted to measure RSC efficiency. This area remains to be one of the least developed areas in supply chain studies. In this paper, DEA has been used to measure RSC performance from a manufacturer's perspective. Since precise data would have been difficult to obtain, a fuzzy version of the CCR model was used to solve the problem. It was converted into a parametric programming model with α being the parameter. A questionnaire survey was conducted and data from 20 companies were collected. The data were then analyzed using the fuzzy CCR model and the efficiency scores for various α values were obtained. It was observed that the efficiency scores reduce with increasing α values. This was expected as with increasing α values, the range of inputs reduces, thereby reducing the chance that the optimized input is chosen. In addition, the efficiency scores were used to allot efficiency ranks.

The paper then compared the organizational background information with the rankings to get some additional insights. It was observed that in average, the companies certified with ISO 14001 performed slightly better than those without the certification. This was expected as ISO certification ensures a lesser impact on the environment, which is one of the performance attributes of the model. In addition, it was found that, in general, companies with lesser years of RSC implementation could perform better than those which have implemented it for a longer period.

The model is applicable to evaluate RSC performance from a manufacturer's perspective. By providing different efficiency scores for each α value, managers can have an overview on their RSC performance and have flexibility to judge the results based on their experience. The next stage of this research will be extending the evaluation to other parties in the entire RSC. It would be more challenging as the number of input and output measures will be higher, and thus the amount of data to be collected will be greater and the programming will be more complicated. Another avenue for future research is to use the model in a longitudinal study to measure RSC performance before and after certain initiatives or interventions. Lastly, researchers can work on the model to measure the performance of a complete closed-loop supply chain that includes both the forward and reverse chains.

Appendix

Questionnaire

Part A: Background Information

(1) Type of industry (with reference to product): ____________________

(2) Number of employees in your company: ________________

(3) How long has your company implemented reverse supply chain activities such as collecting end-of-life products, recycling and reprocessing them? ____________ Years

(4) What certification or award has your company received in reverse supply chain management (such as ISO 14001 certification)? ______________________________

Part B: Reverse Supply Chain Performance Evaluation Measures

(1) How much operational and technological cost is involved in your company's reverse supply chain activities (such as recycling and reprocessing)?

☐ Very High

☐ High

☐ Normal

☐ Low

☐ Very Low

(2) How much financial investment is involved in your company's environmental initiatives (such as awareness campaign, advertisement and certification)?

☐ Very High

☐ High

☐ Normal

☐ Low

☐ Very Low

(3) How is the relationship between your company and other parties that are involved in the reverse supply chain?

☐ Very Strong

☐ Strong

☐ Normal

☐ Weak

☐ Very Weak

(4) How much have the reverse supply chain activities helped the environment (in terms of reducing pollution, carbon footprint and energy consumption)?

☐ Very Significant

☐ Significant

☐ Average

☐ Insignificant

☐ Very Insignificant

(5) How much have the reverse supply chain activities helped the economy (in terms of cost saving and profit generation)?

☐ Very Significant

☐ Significant

☐ Average

☐ Insignificant

☐ Very Insignificant

(6) How much have the reverse supply chain activities contributed to the society (in terms of improving quality of life, health and wellness)?

☐ Very Significant

☐ Significant

☐ Average

☐ Insignificant

☐ Very Insignificant

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

Optimal Routing for Heterogeneous Fixed Fleets of Multicompartment Vehicles

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We present a metaheuristic called the reactive guided tabu search (RGTS) to solve the heterogeneous fleet multicompartment vehicle routing problem (MCVRP), where a single vehicle is required for cotransporting multiple customer orders. MCVRP is commonly found in delivery of fashion apparel, petroleum distribution, food distribution, and waste collection. In searching the optimum solution of MCVRP, we need to handle a large amount of local optima in the solution spaces. To overcome this problem, we design three guiding mechanisms in which the search history is used to guide the search. The three mechanisms are experimentally demonstrated to be more efficient than the ones which only apply the known distance information. Armed with the guiding mechanisms and the well-known reactive mechanism, the RGTS can produce remarkable solutions in a reasonable computation time.

1. Introduction

In areas such as delivery of fashion apparel, petroleum distribution, chemical transportation, food delivery, and waste recycling, due to the characteristics of the products, vehicles with multiple separated compartments are indispensable for delivering to (or picking up from) clients who require delivery (or pick-up) of more than one (incompatible) type of product. Routing the vehicles in such multicompartments environments has gained some attention in the vehicle routing problem (VRP) circle [1–3] and it is called the "multicompartment VRP" (MCVRP).

A fixed homogeneous fleet is often assumed in the classic VRP, and also in the MCVRP. However, many researchers (e.g., [4–9]) claim that the fleet should be assumed heterogeneous because of the following reasons. Firstly, even though the fleet is very likely homogeneous in the inception of a distribution company, after a long time in business, the company tends to own heterogeneous vehicles as a result of vehicle market changes (they would buy different vehicles at different costs) and the different depreciation rates of vehicles [10]. Secondly, it may be wiser for the company to possess/operate a heterogeneous fleet for business flexibility since it can meet different needs of clients.

Heterogeneous fleets are particularly common in multicommodity multicompartment transportation because the physical structure of multicompartment vehicles is more complex than ordinary ones. For example, in the transportation of apparel products, products with different styles and packaging are usually delivered at the same time in one vehicle. Based on the product characteristics, some products are hanged on flexible swing rods and some are packed in boxes. Therefore, the vehicle is reorganized to form multiple separated "compartments" for each type of product. Moreover, products such as gas, chemical, and food require not only separation in multiple compartments, but also special treatments (e.g., temperature, pressure requirements, etc.) during transportation, which imply potential technical differences between vehicles. Therefore, it is natural to consider multicompartment transportation under the setting of heterogeneous fleet.

We formalize the heterogeneous fixed fleet multi-compartment vehicle routing problem (HFFMCVRP) in this article and propose a tabu search-based algorithm called the Reactive Guided Tabu Search (RGTS), which allows a guiding mechanism to collaborate with the well-known reactive mechanism [11], to tackle this problem. The guiding mechanisms employ constantly-updated search history of the appearance of arcs in solutions and the objective values of those solutions in the search to overcome the problem of a large amount of local optima and an induced high diversification. We also change the ways of using the information and compare the resulting different guiding mechanisms to see the delicacy. Experiments on generated HFFMCVRP instances show that (1) the collaboration of guiding mechanism and reactive mechanism significantly improves the efficiency of tabu search, (2) the guiding mechanisms utilizing search history in different ways have similar efficiency, but they all overwhelm the guiding mechanisms without utilizing history information.

In the rest of this paper, we review the literature on two closely related topics, the MCVRP and the HFFVRP, in Section 2. A formal description of the HFFMCVRP is given in Section 3. The outline and details of our proposed algorithm are provided in Section 4. In Section 5, we report computational results and justify the algorithm. Section 6 gives conclusions.

2. Related Literature

Much of the literature regarding the MCVRP focuses on dispatching different petroleum products using tank trucks with isolated compartments (e.g., [12–15]). For environmental issues, multicompartment vehicles are also used in the collection of source-separated waste streams (e.g., [16–18]). Muyldermans and Pang [2] apply a guided local search algorithm to evaluate the cost savings of cocollection over single collection in waste collection. El-Fallahi et al. [1] point out another application involving the distribution of cattle food to farms, and they first use the term "multicompartment VRP" to classify such problems. Believing that "the problem is harder than the VRP and only very small instances can be solved by commercial MIP solvers," they develop a memetic algorithm (MA) and a tabu search (TS) algorithm to tackle the problem. The MA and TS are evaluated in instances that were generated from the capacitated VRP benchmark instances. We note that although very few MCVRP literature directly focuses on the transportation of chemical and fashion products, similar to other VRP, the MCVRP is a common problem for these and many other areas (see, e.g., [19–21]). Back to the MCVRP, Derigs et al. [3] comprehensively introduce a solver suite of heuristic components covering a broad range of alternative methods for construction, local search, large neighborhood search, and metaheuristics to solve the MCVRP. Comparisons between these components are made in order to determine the best composition of the solver suite. However, in the MCVRP instances generated by bisecting classic VRP instances, these heuristics all failed to produce MCVRP solutions of negligible gaps with respect to the corresponding best-known VRP solutions (the instance-generating process guarantees that a feasible VRP solution is also feasible to the MCVRP). This suggests that the MCVRP is difficult and there is still room for improvement. Besides, these papers are creditable in that they are devoted to the problem commonly found in multi-commodity distribution, but there is one thing missing, that is, the heterogeneous fleet. Taillard [4] first addresses the heterogeneous fleet routing issue; noticing that the feasibility check and the cost evaluation of a move require more additional effort in the HFFVRP than in the homogeneous VPRs, he uses a column generation method combined with adaptive memory programming (AMP) embedded in a TS, called the heuristic column generation (HCG), to solve the problem. Though coped with powerful solver CPLEX, the HCG fails to produce most of the BKS (best-known solution) unveiled later by other HFFVRP metaheuristics mentioned below.

Tarantilis et al. [22] propose a list-based threshold accepting algorithm (LBTA), and Tarantilis et al. [23] develop a backtracking adaptive threshold accepting algorithm (BATA). Both LBTA and BATA belong to the class of threshold accepting (TA) algorithms [24]. TA explores the solution search space by allowing uphill moves using a threshold value in order to escape premature local optima. In BATA, the threshold value is not only lowered, but it is also raised—or backtracked—depending on failure to find a new solution using the former value; while in LBTA, a list of values is continually updated and the maximum value is used to update the threshold during the search. Tarantilis et al. [25] establish a guided TS (GTS) in which "bone" (chain of nodes) operations rather than node or edge operations are considered. The neighborhood search is then guided by a mechanism that continually modifies the objective function through penalty along undesired edges chosen during iterations. Li et al. [5] propose a record-to-record travel (RTR) algorithm, denoted as the HRTR, for the HFFVRP. The HRTR alternately applies downhill moves and record-to-record travel moves in the inner loop, while in the outer loop the current solution is perturbed by a procedure of removing and reinserting nodes in order to break through the local traps. Li et al. [26] develop a heuristic in which a multistart adaptive memory programming (MAMP) at each iteration constructs multiple provisional solutions from the elite pool. This pool is continuously upgraded by a modified TS. Path relinking is also integrated as an intensification strategy to enhance the performance of the MAMP. Brandão [27] designs an elaborated TS that includes several techniques such as shaking/perturbation, adaptive parameters updating, postoptimization, and strategic oscillation in order to maximize the robustness.

Being the MCVRP or the HFFVRP, or other rich VRP, the algorithms for them are designed to be more and more complicated and specific to cope with the increasing complexities of the rich VRP. Intrinsically, the HFFMCVRP is more complex than the MCVRP and the HFFVRP since it carries the characteristics of both of them, which leaves one little choice but metaheuristics when trying to tackle the HFFMCVRP. Therefore, we propose the RGTS for this task.

3. Description of the HFFMCVRP

Before we present the algorithm, we describe the HFFMCVRP first.

The HFFMCVRP is defined on the undirected network $G = (L, E)$. Vehicles of the same type share the same characteristics. Customers may place several orders requesting different products, and the orders can be served separately; that is, the customers can be visited more than once. The objective of the HFFMCVRP is to seek the routes with the minimal total transportation cost to serve all the customer orders by assigning the orders to compartments in different vehicles such that all capacities and incompatibilities constraints are met. The HFFMCVRP can be formulated as an integer program as follow:

$$\min \sum_{v\in V}\sum_{i\in L}\sum_{j\in L} \mathrm{cost}^v d_{ij} b_{ijv} \tag{1}$$

s.t.

$$\sum_{j\in L_c} b_{0jv} \leqslant 1, \quad v \in V, \tag{2}$$

$$\sum_{i\in L} b_{ilv} = \sum_{j\in L} b_{ljv}, \quad v \in V,\ l \in L, \tag{3}$$

$$u_{0v} = 1, \quad v \in V, \tag{4}$$

$$(n+1)\sum_{i\in L} b_{ijv} \geqslant u_{jv}, \quad v \in V,\ j \in L_c, \tag{5}$$

$$b_{ijv}\left(u_{iv} - u_{jv} + 1\right) = 0, \quad v \in V,\ i \in L,\ j \in L_c, \tag{6}$$

$$\sum_{v\in V}\sum_{j\in L_c} b_{0jv} f_m(v) \leqslant \left|V_m\right|, \quad m \in M, \tag{7}$$

$$\sum_{o\in O} q_o x_{ovc} \leqslant Q_c, \quad v \in V,\ c \in C_v, \tag{8}$$

$$\sum_{v\in V}\sum_{c\in C_v} x_{ovc} \leqslant 1, \quad o \in O, \tag{9}$$

$$\sum_{o\in S_j^o}\sum_{c\in C_v} x_{ovc} \leqslant |O| \sum_{i\in L} b_{ijv}, \quad v \in V,\ j \in L_c, \tag{10}$$

$$\sum_{o\in S_p^o} x_{ovc} \leqslant |O|\, y_{pvc}, \quad p \in P,\ v \in V,\ c \in C_v, \tag{11}$$

$$y_{pvc} = 0, \quad (p, c) \in I_2,\ v \in V,\ c \in C_v, \tag{12}$$

$$y_{pvc} + y_{qvc} \leqslant 1, \quad (p, q) \in I_1,\ v \in V,\ c \in C_v, \tag{13}$$

$$b_{ijv} \in \{0, 1\}, \quad i \in L,\ j \in L,\ v \in V, \tag{14}$$

$$u_{iv} \in \{0, 1, \ldots, n+1\}, \quad i \in L,\ v \in V, \tag{15}$$

$$x_{ovc} \in j\,\{0, 1\}, \quad o \in O,\ v \in V,\ c \in C_v, \tag{16}$$

$$y_{pvc} \in \{0, 1\}, \quad p \in P,\ v \in V,\ c \in C_v. \tag{17}$$

Objective (1) aims at minimizing the total travel cost. Constraint (2) ensures that all vehicles depart at most once from depot 0. Constraint (3) ensures that if a vehicle arrives at a location, then it must leave from this location afterward. Constraints (2)-(3) make sure that a vehicle tour must always start and end at depot 0. Constraint (4) enforces the location of depot 0 at position 1. Constraint (5) makes sure that if node j is never visited by vehicle v, its position will be $u_{jv} = 0$. Constraint (6) imposes the condition that the position of node j is just higher than the position of node i, if vehicle v travels from i to j. Constraints (4)–(6) complete the subtour elimination. Constraint (7) sets a limit on the number of vehicles available, where $f_m(v) = 1$ if vehicle v is of type m, otherwise 0. Constraint (8) states that the goods loaded into compartment c on vehicle v must not exceed the compartment capacity Q_c. Constraint (9) ensures that each order is assigned to exactly one compartment of a vehicle. Constraint (10) imposes the restriction that a vehicle must visit customer j if any orders from j are assigned to the vehicle. Constraint (11) makes sure that the total order number of a product in a compartment cannot exceed the total order number of a product. Constraints (12)-(13) describe the incompatibilities and (14)–(17) indicate the decision variables.

4. The Proposed RGTS Algorithm

In our RGTS algorithm, we first obtain an initial feasible solution through a simple procedure. This solution is subsequently improved by a number of well-known operators in the TS framework. In order to seek better quality solutions, we applied a reactive mechanism and a guided mechanism to strengthen the TS.

4.1. Constructing the Initial Solution. First, a void route is generated for each vehicle in the fleet (e.g., for a fleet consisting of three type-A vehicles and two type-B vehicles, five dedicated routes denoted as "A, A, A, B, B" will be generated). Then, following Procedure 1, a *feasible* initial solution can be generated (not necessarily, but for all our tests, it happened to be so) (see Figure 1).

Procedure 1. Construction of the initial solution is as follows.

(1) Sort all the orders in ascending order according to the demand.

(2) While there are unserved orders, find the closest one $\overline{o}$ to the depot. According to the relationship I_2, find $\overline{c} = \arg\min\{Q_c - q_{\overline{o}} \mid Q_c \geqslant q_{\overline{o}}\}$, and assign $\overline{o}$ to $\overline{c}$ (i.e., add $\overline{o}$ to the end of route $l_{\overline{c}}$ of $\overline{c}$).

(3) One by one, insert the sorted orders into route $l_{\overline{c}}$ using least-cost insertion while maintaining route feasibility.

(4) Update Q_c and repeat steps (2)-(3) until all orders are served.

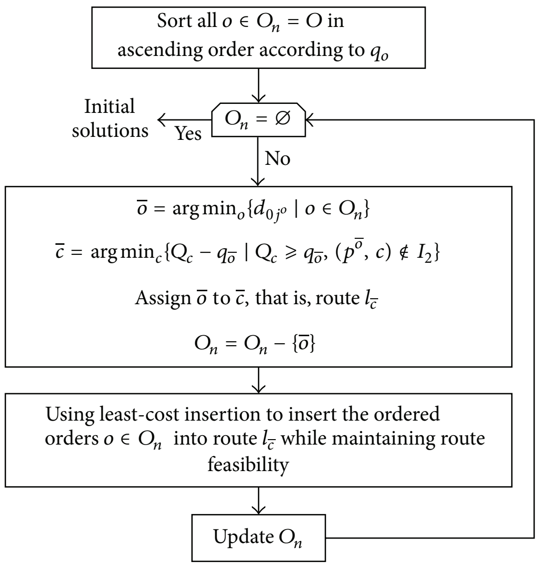

Figure 1: Flow chart of Procedure 1.

4.2. Operators and Neighborhood Reduction Strategy. Operators define the local search moves. We implement several well-known operators. Specifically, for an interroute operation, we use *2-opt**, *1-0 move* (*interrelocate*), *1-1 move* (*interswap*), and *2-1 move* (*three-point move*), while for an innerroute operation, we use *swap*, *relocate*, and *2-opt*. Note that the nodes in a route now mean order nodes, not customer nodes.

Due to the high complexity of the VRPs, even metaheuristics may require a significant amount of computation when solving medium/large-scale problems. Accordingly, there is nearly a consensus (e.g., [5, 28]) that one should implement the so-called neighborhood reduction strategy to focus on desirable neighborhoods so as to reduce computation. In our mechanism, a nontabu move is allowed only if it connects edge (o_i, o_j) such that the owner of o_i, customer i, is a neighbor of the owner of o_j, customer j; that is,

$$d_{ij} \leqslant \delta \cdot \text{avg}_i, \qquad d_{ij} \leqslant \delta \cdot \text{avg}_j, \tag{18}$$

where δ is a small number used to define the limit of neighborhoods and avg_i is the average distance of all edges (of customer nodes) starting from i.

4.3. Guiding Mechanism. A guiding mechanism is a systematic method that continuously identifies low-quality features and tries to reject them by penalizing them in the objective function so as to guide the search into unexplored domains of the solution space. In routing problems or shortest path problems, it is common to define long edges as low-quality features. A long edge is more likely to be selected according to a utility function and then to be penalized by adding an extra value to its original distance; as a result, this edge can be highly possibly avoided in future search. Voudouris and Tsang [29] first introduce a guiding mechanism into the local search when solving the traveling salesman problem. Every time the local search falls in a local optima S^*, the edge $\zeta = (i^*, j^*)$ in S^* corresponding to the maximum value of utility function $U(i, j) = d_{ij}/(1 + p_{ij})$ is penalized as $d'_\zeta = d_\zeta + p_\zeta \cdot \lambda$, where p_ζ indicates the times that edge ζ has been penalized and λ is an experimentally valued parameter that defines the strength of penalties. Tarantilis et al. [25] introduce another guiding mechanism into the tabu search (the guided tabu search, GTS) and successfully applied it to the HFFVRP. A parameter *guidFreq* is used to switch on/off the guiding mechanism; that is, during every *guidFreq* iterations, an edge in the current solution is selected according to utility function $U(i, j) = d_{ij}/[(1 + p_{ij}) \cdot \text{AVG}_{ij}]$ and penalized as $d'_\zeta = d_\zeta + p_\zeta \cdot \lambda \cdot \text{AVG}_\zeta$, where AVG_{ij} is the average length of all edges beginning from locations i and j in the edge set E and d'_ζ is only used for $2 \times$ *guidFreq* iterations, and the original one is then substituted back. This allows some self-correction in the algorithm because a "bad" edge can have a second chance to become "good," since "bad" and "good" are defined by the somewhat naïve utility function. Tarantilis et al. [25] claim that their utility function considers not only the distance d_{ij}, but also the relative positions of i and j according to the rest of the customer population (i.e., AVG_{ij}), which leads to a more balanced edge selection.

Edge selection determined by utility function plays a key role in the guiding mechanism because a poor selection would turn guiding into misleading. In the above two guiding mechanisms, such misleading may be avoided by decreasing the utility function values of edges that have been penalized many times before. That is, the edges of higher values of p_{ij} have smaller values of $U(i, j)$, so the selection will not be restricted on a small set of edges. However, except for p_{ij}, other factors in those utility functions (viz., AVG_{ij} and d_{ij}) are predetermined characteristics of the problems, thus containing no information about the evolution of the search that might serve as useful memories. To utilize such renewable information about the evolution of the search, we introduce a matrix $Appr_{(n'+1)\times(n'+1)}$ to collect information about the appearance of edges in sound solutions. At the beginning, every $Appr_{ij}$ is set to one and is updated each time an operator returns a local optima s^*; that is, for every edge in s^*, if $f(s_b) < f(s^*)$, renew $Appr_{ij} = Appr_{ij} + f(s_b)/f(s^*)$, otherwise $Appr_{ij} = \min\{Appr_{ij} - f(s_b)/f(s^*), 1\}$, where s_b is the incumbent best-found solution. By applying $Appr_{(n'+1)\times(n'+1)}$, we are able to propose a more "just" utility function which utilizes historic search information to evaluate the edges better. Note that in step (2) of Procedure 2, we reset $Appr_{(n'+1)\times(n'+1)}$ to its default state, a unit matrix, if the search is considered to be in the chaos. Thus, it allows further self-correcting opportunity. It also serves as an important link between the guiding mechanism and the reactive mechanism.

Three utility functions are given in (19). They all contain the matrix *Appr* but differ in using other predetermined characteristics of the problems, d_{ij} and AVG_{ij} (note that distance between two orders is distance between their corresponding customers). Function U_1 can be considered as the most comprehensive function, while U_2 as the purest and U_3 as the modest. In all utility functions, the more often the edge

(i, j) appears in good solutions, the higher the value of $Appr_{ij}$ is, leading to a smaller value of $U(i, j)$, hence a less possibility of edge (i, j) being selected and penalized;

$$\begin{aligned} U_1(i, j) &= \frac{d_{ij}}{(1 + p_{ij}) \cdot \mathrm{AVG}_{ij} \cdot Appr_{ij}}, \\ U_2(i, j) &= \frac{1}{(1 + p_{ij}) \cdot Appr_{ij}}, \\ U_3(i, j) &= \frac{d_{ij}}{(1 + p_{ij}) \cdot Appr_{ij}}. \end{aligned} \tag{19}$$

Note that we use the standard penalty term $d'_\zeta = d_\zeta + p_\zeta \cdot \lambda$ for all utility functions. Our proposed guiding mechanism is sketched out in Procedure 2 (see Figure 2).

Procedure 2. The guiding mechanism is as follows.

s^* is the local optima returned by an operator

(1) Update $Appr_{n\times n}$

(2) If $guid = guidFreq$

select edge $\zeta = (i^*, j^*) = \arg\max_{(i,j)\in s^*} U(i, j)$,

penalize edge by $d'_\zeta = d_\zeta + p_\zeta \cdot \lambda$ for $2 \times guidFreq$ iterations

$guid = 0$;

else

$guid = guid + 1$.

4.4. Reactive Mechanism. The reactive TS (RTS) is first proposed by Battiti and Tecchiolli [11]. The primary feature of their RTS is the reactive mechanism for adapting the tabu tenure, or tabu list size TL, to the evolution of the search in order to free the search trajectory from a limited part of the search space, instead of entirely avoiding closed search cycles or repetitions. The RTS has been successfully applied to the VRP, with a few modifications (e.g., [30–32]). They demonstrate the robustness of the RTS and the limiting effect of parameter changes. Therefore, we apply the standard routine of the RTS with an additional modification: turning on/off the neighborhood reduction strategy and resetting a matrix used in the guiding mechanism (step (2) of Procedure 3) to default state. The reactive mechanism is described in Procedure 3 and the parameters are described in parameters section (see Figure 3).

Procedure 3. The reactive mechanism is as follows.

s^* is the local optima returned by an operator.

(1) $TLLastCh = TLLastCh + 1$; $itr = itr + 1$.

If $f(s^*) \in LTL$,

$ReptP = itr$; $itr = 0$; $RCounter = RCounter + 1$;

else

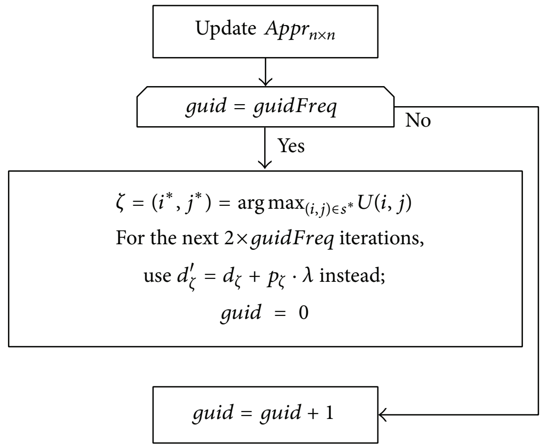

Figure 2: Flow chart of Procedure 2.

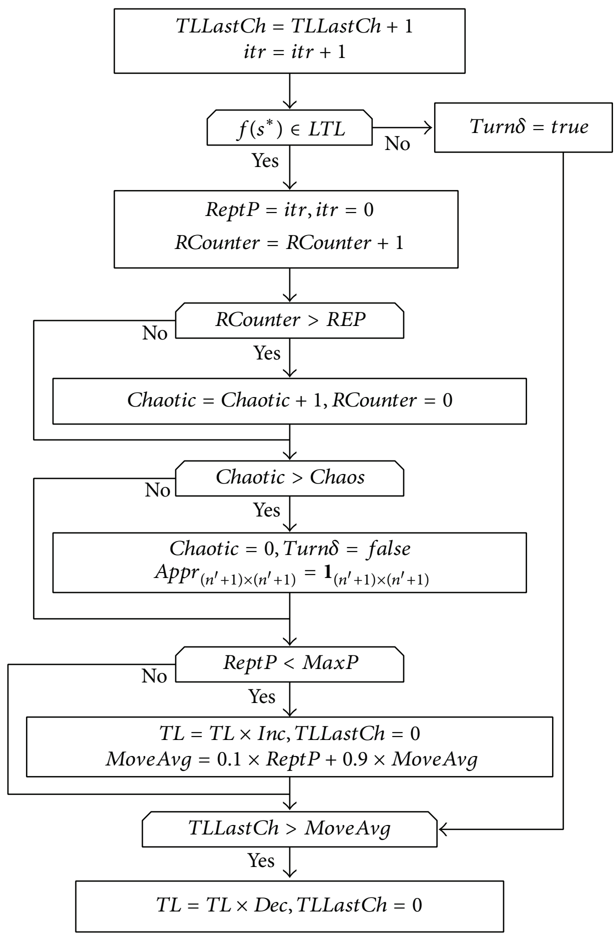

Figure 3: Flow chart of Procedure 3.

$Turn\delta = true$; go to (4).

(2) If $RCounter > REP$,

$Chaotic = Chaotic + 1$; $RCounter = 0$;

if $Chaotic > Chaos$,

$Chaotic = 0; Turn\delta = false; Appr_{(n'+1)\times(n'+1)} = \mathbf{1}_{(n'+1)\times(n'+1)}$.

(3) If $ReptP < MaxP$,

$TL = TL \times Inc; TLLastCh = 0;$

$MoveAvg = 0.1 \times ReptP + 0.9 \times MoveAvg$.

(4) If $TLLastCh > MoveAvg$,

$TL = TL \times Dec; TLLastCh = 0$.

In Procedure 3, note that $f(s) \in LTL$ implies that we use another list LTL containing the objective values of former best-found solutions to help detect repetition; namely, a repetition is detected if $f(s^*)$ has been reached before, even if s^* is a new solution with different structure. In other words, the "repetition" is a harsh one here. In addition to the dynamic change of tabu tenure, the second feature of RTS is the "escape procedure," which is activated when the search is considered to be drowning in chaos, that is, when $Chaotic > Chaos$. Unlike the escape procedure of Battiti and Tecchiolli [11], which consists of a series of random moves, we use $Turn\delta$ as a switch to turn off the neighborhood reduction strategy in order to free the search from a restricted area of solution space; the neighborhood reduction strategy is turned back on itself if the repetition is not detected in the next iteration. Moreover, the matrix used in the guiding mechanism is set to the default state; that is, $Appr_{(n'+1)\times(n'+1)} = \mathbf{1}_{(n'+1)\times(n'+1)}$, so as to provide more opportunities of self-correction mentioned in last subsection. Now we present the framework of our algorithm in Figure 4.

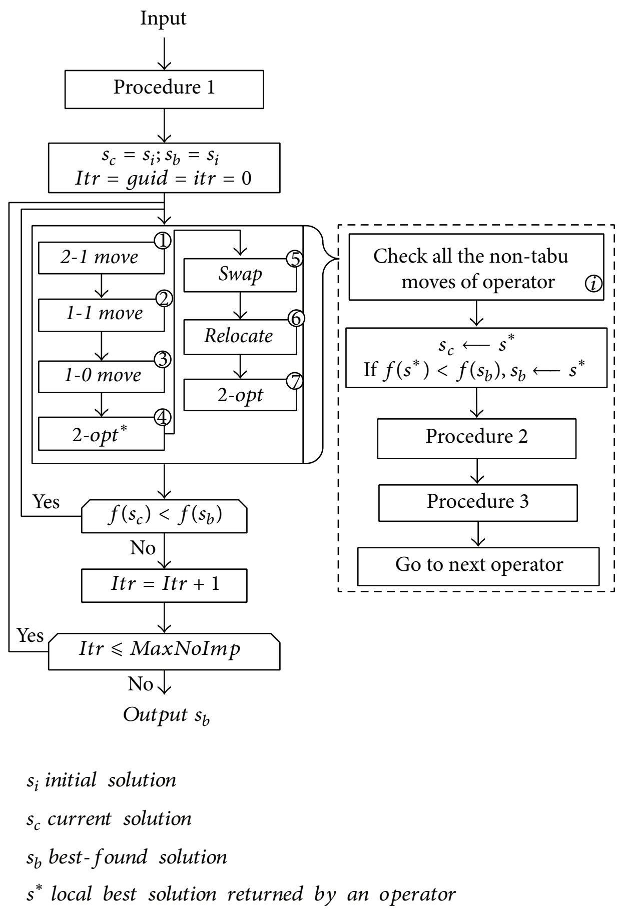

Figure 4: Framework of the RGTS.

5. Computational Results

Our proposed algorithm was programmed in Visual Studio 2008 C# and executed on a laptop with Intel (R) Pentium (R) Dual CPU at 1.32 GHz, 1 GB of RAM in Windows XP. To the best of our knowledge, no instances are publicly available for the HFFMCVRP; therefore, we generate test instances based on the existing HFFVRP instances, which themselves are generated from classic VRP instances.

5.1. The HFFMCVRP Instances Sets. The well-known HFFVRP benchmark instances set, the "T-8" set, in Taillard [4], numbered from 13 to 20, is modified to generate our instances. The generating procedure is basically the same as the ones used in El-Fallahi et al. [1] and Muyldermans and Pang [2], which can be summarized as creating two compartments of same capacity or bisecting the capacity of each vehicle and the demand of each customer. As a result, there are two products; each client orders the same amounts of them (consider complementary or correlated products, e.g., two types of petroleum); each vehicle has two compartments; product $i = 1, 2$ must be delivered in compartment $i = 1, 2$ of whichever vehicle. The characteristics of the generated instances (called TC-8) are summarized in Table 1.

The benefit of such generating procedure is that the BKSs of T-8 can serve as benchmarks or target solutions for algorithms solving TC-8 since feasible T-8 solutions are also feasible TC-8 solutions (the two orders of every customer are shipped together). Besides, the TC-8, though not real life instances, may be representative of them to some extent. For example, in medium sized cities such as Dalian in China, a major oil company typically has around 100 regular customers and 10–20 vehicles, which may be represented by instances 17–20, while the number of customers and number of vehicles in apparel products delivery, food delivery, and waste recycling can be small or big, dependent on the size of the related company/sector (e.g., supermarket or store, common solid waste, or toxic waste). In such cases, instances 13–16 are somewhat representative.

Intuitively, the bisection of a T-8 instance with n customer nodes results in a TC-8 instance with $n' = 2n$ order nodes—in other words, $2n^2 + n$ edges in its corresponding graph while there are only $(n^2 + n)/2$ edges in the graph of the T-8 instance. This fact indicates that a TC-8 instance should have a much larger search space and more local traps than its corresponding T-8 instance.

5.2. Parameter Tuning. Like many other metaheuristics, the RGTS employs a variety of parameters that require a tuning procedure in order to achieve the optimum balance between computational effort and solution quality. Observing the

TABLE 1: Characteristics of TC-8 derived from T-8.

TC-8	n'	Type A Property A	Type B Property B	Type C Property C	Type D Property D	Type E Property E	Type F Property F
13	100	10, 1.0, 4	15, 1.1, 2	20, 1.2, 4	35, 1.7, 4	120, 2.5, 2	200, 3.2, 1
14	100	60, 1.0, 4	80, 1.1, 2	150, 1.4, 1	—	—	—
15	100	25, 1.0, 4	50, 1.6, 3	80, 2, 2	—	—	—
16	100	20, 1.0, 2	40, 1.6, 4	70, 2.1, 3	—	—	—
17	150	25, 1.0, 4	60, 1.2, 4	100, 1.5, 2	175, 1.8, 1	—	—
18	150	10, 1.0, 4	25, 1.3, 4	50, 1.9, 2	75, 2.4, 2	250, 2.9, 1	400, 3.2, 1
19	200	50, 1.0, 4	100, 1.4, 3	150, 1.7, 3	—	—	—
20	200	30, 1.0, 6	70, 1.7, 4	100, 2, 3	—	—	—

n': number of orders; Type A–F: vehicle types; Property A–F: capacity of each compartment, variable cost per unit of vehicles, and number of vehicles available.

success of RTS in several cases [30–33], we boldly follow the similar parameter settings regarding the reactive mechanism as found in literature. To be prudent, we also carried out extensive tests and finally reached the ones in Table 2. Note that the initial value of TL is set to be $n'/2$ rather than "1" in standard RTS to avoid meaningless adjustment of tabu list size during early-stage iterations. As for the parameters regarding the guiding mechanism, we varied them within relatively wide ranges and tested RGTS's performance on five randomly generated instances. More specifically, the candidate values of $guidFreq$ are $\{1, 2, \ldots, 10\}$, and the candidate values of λ are $\{0.1, 0.2, \ldots, 1\}$. Other parameters, after intensive tests, are summarized in Table 2.

TABLE 2: Parameter settings of RGTS.

Parameters	Value
REP	3
Chaos	3
Inc	1.1
Dec	0.9
MaxP	50
δ	1.0
MaxNoImp	50
GuidFreq	5
λ	0.1
TL	$n'/2$

5.3. Results on TC-8. We have defined three different utility functions above, and now we want to see how they affect the efficiency of the guiding mechanism before we test the RGTS. We conduct comparisons between algorithms with different guiding mechanisms (i.e., different utility functions and penalty terms) and without reactive mechanisms, and the results are shown in Table 3. The first row lists different guiding mechanisms, where "VT" represents the mechanisms using the utility function and penalty term proposed by Voudouris and Tsang [29] and "T" represents the one proposed by Tarantilis et al. [25], while "U1," "U2," and "U3" are mechanisms with utility functions U_1, U_2, and U_3, respectively, and the standard penalty term. Other aspects such as the construction procedure, the operators, and the algorithmic routines are defined in Section 4. "OV" stands for the best objective values reached by the algorithms and "CPU" indicates the required running times in seconds.

Table 3 shows that mechanisms "VT" and "T" are outperformed by "U1," "U2," and "U3," which all take *Appr* into account. This suggests that utilizing the search history information does improve a guiding mechanism's efficiency. However, the results obtained by "U1," "U2," and"U3" are quite close, indicating that once search history information is used to help select undesired edges, edge distance does not matter that much. In fact, it is the "U2" without taking account of any distance information but only search history information returns the best results. After all, edges with long distance are not necessarily undesirable. From now on, we use "U2" without further statements.

To visualize the diversification effect and the effectiveness of the guiding mechanism, we provide Figure 5 to demonstrate the progress of TS with (black line) and without (grey line) the use of the guiding mechanism for Problem 19 of TC-8. It is obvious that without the guiding mechanism, the basic TS (to keep it running longer, the $MaxNoImp$ is set to 100) is trapped after a few iterations and is unable to escape. In contrast, with the guiding mechanism the algorithm seems to be skillful in avoiding local optima and finally leads to a better solution. Similar results are observed in tests of other problems.

The HFFMCVRP is so complex that even the algorithm with "U2" is unable to unveil TC-8 solutions very close to the best-known solutions of the HFFVRP benchmark T-8. So we allow the guiding mechanisms to collaborate with the well-known reactive mechanism (RGTS) and see if such collaboration results in repulse or intimacy between these two mechanisms. In Table 4, we compare RGTS with its reduced versions, that is, TS without either guiding or reactive mechanisms (TS), TS with only reactive mechanism (TS-R), and TS with only guiding mechanism (TS-G).

Table 4 clearly supports that the two mechanisms can coexist and supplement each other. The collaboration of two mechanisms yields the most (average 4.86%) improvement over the basic TS. Note that compared to the BKS of T-8 (see Table 5), the RGTS produces solutions to TC-8 with an average gap 4.06%. Though it is not a small gap, we content with it since TC-8 has quite different structure from T-8

Table 3: Comparisons of guiding mechanisms on TC-8.

TC-8	VT		T		U1		U2		U3	
	OV	CPU	OV	CPU	OV	CPU	OV	CPU	OV	CPU
13	1711.68	44	1637.85	58	**1588.97**	93	**1588.97**	99	**1588.97**	85
14	735.16	52	**650.91**	77	713.75	95	718.90	73	713.75	106
15	1059.11	62	1054.19	65	**1043.57**	89	1076.14	95	**1043.57**	91
16	1248.98	41	1207.07	56	1175.01	64	**1173.31**	82	1263.66	41
17	1170.58	255	**1145.69**	572	1169.44	166	1169.44	241	1169.44	237
18	2033.98	538	2085.99	452	2036.12	248	**1972.22**	612	2007.50	610
19	1220.33	1018	1224.31	1170	**1197.53**	2167	1200.56	1095	**1197.53**	1746
20	1675.80	359	1711.13	837	1711.00	573	**1651.32**	1116	1662.58	605
Average	1356.95	296	1339.64	410	1329.42	436	**1318.86**	426	1330.88	440

Table 4: Comparison of RGTS and its reduced versions on TC-8.

TC-8	TS		TS-R			TS-G			RGTS		
	OV	CPU	OV	Gap	CPU	OV	Gap	CPU	OV	Gap	CPU
13	1588.97	81	1686.08	6.11	45	1588.97	0.00	99	**1560.97**	**−1.76**	155
14	730.07	103	652.13	−10.68	206	718.9	−1.53	73	**625.06**	**−14.38**	148
15	1076.15	118	1077.11	0.09	84	1076.14	0.00	95	**1025.76**	**−4.68**	168
16	1225.80	82	1203.39	−1.83	68	1173.31	−4.28	82	**1168.25**	**−4.69**	80
17	1169.44	304	1156.89	−1.07	475	1169.44	0.00	241	**1114.63**	**−4.69**	725
18	2036.12	329	2013.31	−1.12	421	1972.22	−3.14	612	**1939.85**	**−4.73**	908
19	1197.53	1978	1212.17	1.22	790	1200.56	0.25	1095	**1186.70**	**−0.90**	2001
20	1680.11	589	1666.95	−0.78	964	1651.32	−1.71	1116	**1628.49**	**−3.07**	1345
Average	1338.02	448	1333.50	−1.01	381	1318.86	−1.30	426	**1281.21**	**−4.86**	691

Gap (%): gap with respect to the solutions found by the basic TS without reactive and guided mechanisms.

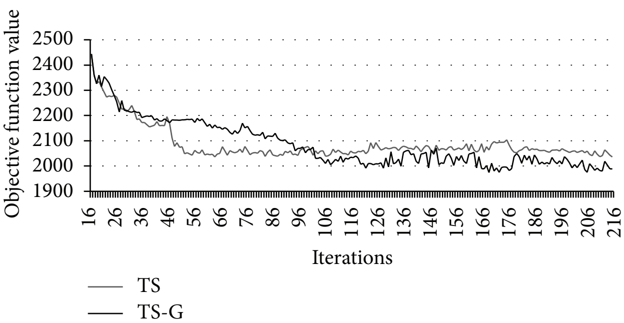

Figure 5: Diversification effect of the guiding mechanism.

and is far more complex than T-8; moreover, the BKSs of T-8 are found by several specifically designed algorithms and parameter settings. However, incontestably, there is room for further improvement on the algorithm for the HFFMCVRP.

Since our HFFMCVRP instance set TC-8 is generated from the HFFVRP benchmark set T-8, we wonder how well does our HFFMCVRP algorithm perform in solving T-8. Hence, we apply the RGTS to T-8 with little modification and the same parameter settings, even though it is not intentionally designed for the HFFVRP. According to Table 5, it is fair to say that RGTS is robust, because it unveiled solutions of an average 2.06% gap with regard to the BKSs of T-8 found by four remarkable algorithms. Note that similar gaps can be observed in the MCVRP literature (see [1–3]); that is, the designed-for-MCVRP algorithms can produce VRP solutions of above 1% gap with respect to the BKSs of the classic VRP in relatively long running times. El Fallahi et al. [1] claimed that the VRP solution is hard to find because of its special structure in a MCVRP context: the two products for each customer must be regrouped in the same trip. Since the HFFVRP is more difficult than the classic VRP because of the heterogeneous fleet, we reckon that 2.06% is expected and acceptable.

6. Conclusions

In this paper, we studied the HFFMCVRP, which is a rich vehicle routing problem commonly found in the practice of multicommodities (e.g., oil, food, fashion, and waste collection) distribution where multicompartment vehicles are indispensable due to safety and health issues. Heterogeneous fleet is almost universal in realistic logistic, and more so when it comes to multicompartment vehicle as the vehicles may have more technical differences. Hence, the HFFMCVRP successfully captures the essence of real life multicommodities distribution. However, not enough attention, which it deserves, has been paid to it.

We propose a hybrid algorithm RGTS to solve the HFFMCVRP. The RGTS systematically integrates a reactive mechanism and a guiding mechanism within the TS framework.

Table 5: Comparison of RGTS with HFFVRP algorithms on T-8.

T-8	BKS of T-8	HCG[a]		HRTR[b]		Guided TS[c]		TSA[d]		RGTS		Gap
		Gap	CPU	Gap	CPU	Gap	CPU	Gap	CPU	OV	CPU	
13	**1517.84**	0.014	473	**0**	358	**0**	116	**0**	56	1534.72	26	1.11
14	**607.53**	1.335	575	**0**	141	**0**	92	**0**	55	630.38	57	3.76
15	**1015.29**	0.155	335	**0**	166	**0**	79	**0**	59	1025.87	48	1.04
16	**1144.94**	0.796	350	**0**	188	**0**	130	**0**	94	1146.38	61	0.10
17	**1061.96**	0.926	2245	**0**	216	**0**	153	**0**	206	1071.82	102	0.93
18	**1823.58**	2.554	2876	**0**	366	**0**	252	**0**	198	1860.01	111	2.00
19	**1117.51**	**0**	5833	0.253	404	0.253	327	0.253	243	1172.40	301	4.91
20	**1534.17**	1.669	3402	**0**	447	**0**	479	**0**	302	1574.24	220	2.61
Average	**1227.85**	2.614	2011	1228.21	286	1228.21	203	1228.21	152	1251.98	115	2.06
Average Gap	0.000	0.931	—	0.032	—	0.032	—	0.032	—	2.060	—	—

[a]HCG: heuristic column generation by Taillard [4] (SucnSparc workstation 50 MHz).
[b]HRTR: record-to-record travel algorithm for HFFVRP by Li et al. [5] (Athlon 1 GHz).
[c]Guided TS: guided tabu search by Tarantilis et al. [25] (Pentium IV 2.4 GHz).
[d]TSA: tabu search algorithm by Brandão [27] (Pentium M 1.4 GHz).

The guiding mechanism is capable of utilizing historic information continuously fed from the search evolution to help more impartially select edges for penalization. Experiments have shown that compared to classic guiding mechanisms employing only predetermined information (i.e., edge distance), our mechanism provided the best results on all generated instances. Moreover, once search history information is used to help select undesired edges, edge distance information may not matter that much. After all, edges with long distance are not necessarily undesirable. To enhance its efficiency, the well-known reactive mechanism is also incorporated in RGTS, and experiments supported that two mechanisms can coexist and supplement each other. The collaboration of two mechanisms yielded an average 4.86% improvement over the basic TS.

For the future research, except for algorithmic issues such as further improvements in the RGTS and designing exact algorithms, one may consider a loading subproblem in the distribution of multiple products in multiple compartments, especially when the compartments are flexible and the products are special (e.g., in shape, form, etc.).

Notations

L: Node set $\{0, 1, \ldots, n\}$ including one depot (node 0)
L_c: Client set, that is, $L \setminus \{0\}$
E: Edge set, that is, $\{(i, j) \mid i, j \in L\}$
d_{ij}: Distance of edge $(i, j) \in E$, $d_{ii} = 0$, $d_{ij} = d_{ji}$, $d_{ij} + d_{jl} \geqslant d_{il}$
V: Vehicle set
M: Vehicle type set
V_m: Subset of V containing only vehicles of type $m \in M$
cost^v: Variable cost per unit distance of vehicle $v \in V$
C_v: Compartment set of vehicle $v \in V$
Q_c: Capacity of compartment $c \in C_v$, $v \in V$
P: Product set
O: Order set
j^o: Customer who places order $o \in O$, $j^o \in L_c$
p^o: Product type of order $o \in O$, $p^o \in P$
q_o: Quantity of order $o \in O, q_o > 0$
S_j^o: Orders placed by customer $j \in L_c$, that is, $\{o \in O \mid j^o = j\}$
S_p^o: Orders for product $p \in P$, that is, $\{o \in O \mid p^o = p\}$
I_1: Set of products incompatibilities, $I_1 \subseteq P \times P$, that is, $(p, q) \in I_1$, indicates that products p and q must not be delivered together in the same compartment
I_2: Set of incompatibilities between products and compartments, $I_2 \subseteq P \times C$, that is, $(p, c) \in I_2$, means that products p must not be delivered in compartment c
b_{ijv}: $= 1$ if vehicle v travels from node i to j, otherwise $= 0$
x_{ovc}: $= 1$ if order o is assigned to compartment $c \in C_v$, otherwise $= 0$
y_{pvc}: $= 1$ if product p is assigned to compartment $c \in C_v$, otherwise $= 0$
u_{iv}: The position of node i in the tour of vehicle v, $u_{iv} = 0$ indicates that node i is never visited by vehicle v.

Parameters

TL: Tabu list
LTL: Long tabu list for detecting repetition
$Turn\delta$: Switch of the neighborhood reduction strategy
$RCounter$: Counter for repetitions
REP: Constant threshold for $RCounter$
$Chaotic$: Counter for $RCounter > REP$
$Chaos$: Constant threshold for $Chaotic$
Inc: Percentage increase for TL

Dec: Percentage decrease for *TL*
ReptP: Number of iterations between two consecutive repetitions
MaxP: Constant threshold for *ReptP*
TLLastCh: Number of iterations since the last change of *TL*
MoveAvg: Changing threshold for *TLLastCh*
itr: Counter used to calculate *ReptP*
GuidFreq: Frequency of guiding mechanism
MaxNoImp: Maximum number of consecutive iterations allowed when the best-found solution has not been improved.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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