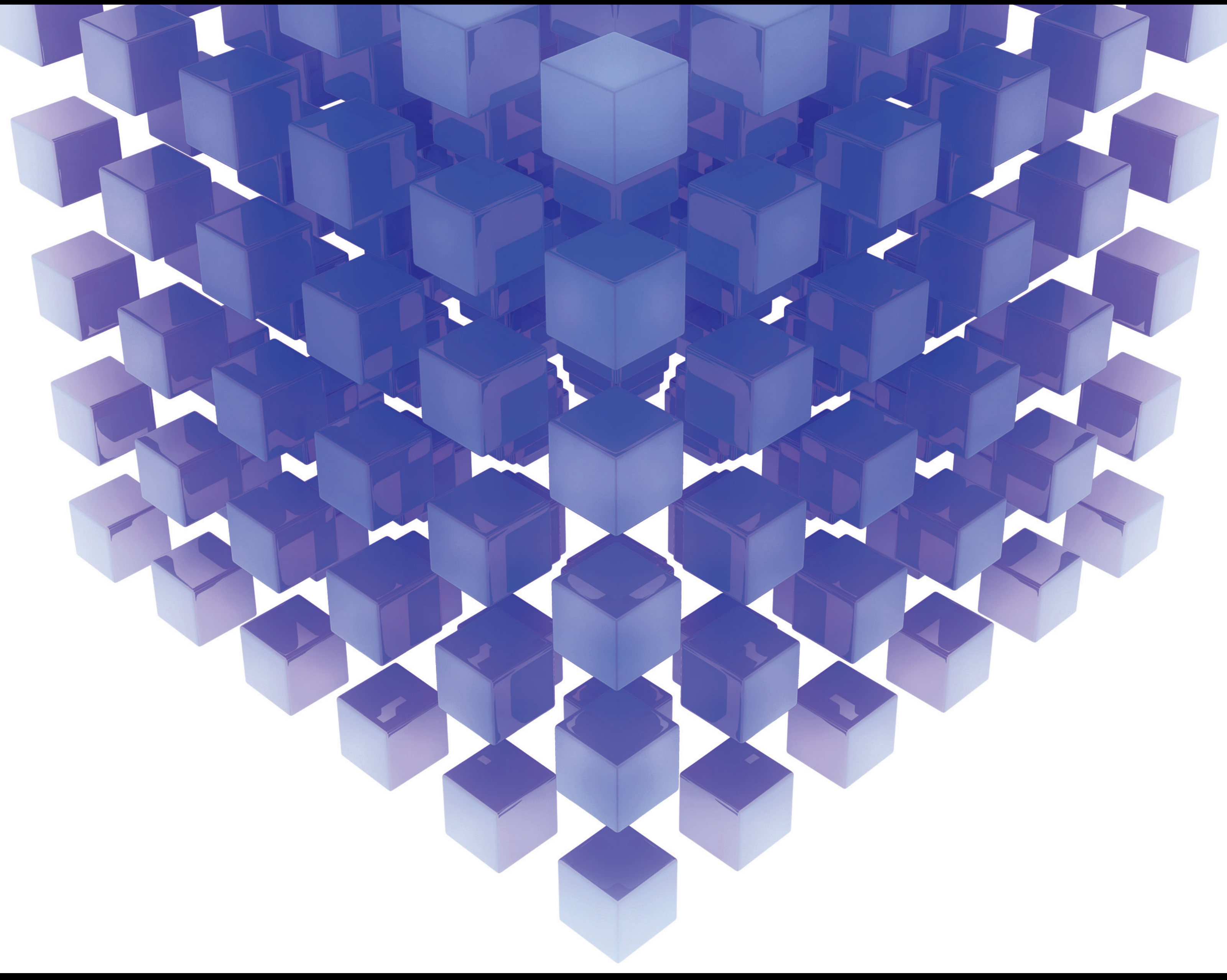
Mathematical Problems in Engineering

Building Mathematical Models for Multicriteria and Multiobjective Applications

Guest Editors: Adiel T. de Almeida, Love Ekenberg, Martin J. Geiger, Juan Carlos Leyva Lopez, and Danielle Morais



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Editorial Board

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Editorial

Building Mathematical Models for Multicriteria and Multiobjective Applications

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In our daily lives or professional settings, there are many decision problems involving multiple criteria, which may be conflicting and incommensurable. The complexity of real-world decision and the plethora of factors involved necessitate the implementation of sound theoretical frameworks for structuring decision-making processes. Multicriteria Decision Making/Aid (MCDM/A) and multiobjective methods can be highly useful for decision makers (DMs) in such tasks.

Multicriteria and multiobjective approaches provide a wide variety of methodological tools for supporting the DMs when facing real-world decision problems when designing and analyzing mathematical models, representing both the preferences of decision makers and the various characteristics of the decision problems at hand in a number of areas.

There is nevertheless still a need for further developments of multicriteria and multiobjective models and applications in order to meet an increasing demand for solving complex problems. The aim of this special issue is therefore to further expand the field and to promote and disseminate research and applications among academics and other professionals interested in theories, methodologies, and applications of multicriteria and multiobjective approaches for tackling complex problems in a wide variety of areas.

The special issue received a high number of high-quality submissions. After a rigorous peer-review process, with an acceptance rate of about 18%, ten accepted papers provide a variety of applications for real-world problems while combining theoretical methodology and mathematical analysis. The authors of these papers are from Brazil, China, Spain, Taiwan, and Turkey.

This selection of papers reflects the diversity of research studies applying different multicriteria and multiobjectives methods and includes theoretical procedures being developed for different areas as well as practical experiments for validating these.

In the area of information system, A. P. H. de Gusmão and C. P. Medeiros present a study with an additive model used to evaluate alternatives for selecting a strategic information system using FITradeoff, which is a new method for eliciting the weights of criteria. The FITradeoff has the advantage of requiring less effort from the decision-maker when expressing preferences regarding weights of criteria, minimizing the risk of inconsistent answers. The model is using data from a glass packaging factory to select a single information system from a set of systems previously identified as relevant.

Dealing with evaluation of the security level of a city, C. J. J. de Figueiredo and C. M. M. Mota show mapping locations for the occurrence of robberies applying the multicriteria method Dominance-Based Rough Set Approach. With this classification model, it is possible to understand the social dynamics and also to propose strategies against violence.

R.-C. Chen and S.-P. Suen propose a multiobjective mechanism to help retail industry practitioners to optimize the allocation of stores. The model is based on three phases. Firstly, all stores are divided into clusters using a geographic information system (GIS) and k-means clustering algorithm. Secondly, stores are strategically selected according to the requirements of the company and the attributes of the stores. And finally, it is determined which stores to close by using a neighborhood-based multiobjective genetic algorithm (NBMOGA).

Considering Earned Value Management (EVM) system, which is applied to monitor and control organizations' projects to attain high performance, S.-Y. Chou et al. employ a hybrid multiple criteria decision-making (HMCDM) procedure for achieving aspired EVM projects' performance. The proposed procedure quantifies gap indices with respect to aspiration levels of EVM based on interinfluence effects among criteria/dimensions/alternatives and systemizes the quantitative results in the context of influential network relation maps (INRM), helping managers to find routes for EVM application decisions. This study demonstrates the importance of adopting a systematic procedure to analyze interinfluenced criteria associated with the EVM application decisions.

In view of service related to environmental sector, L. Delgado-Antequera et al. design a framework to solve waste collection problems applying an Asymmetric Vehicle Routing Problem (AVRP) with side constraints and several variations. This model is based on an interactive biobjective method for minimizing the total distance travelled by all vehicles and the length of the longest route to balance the working day within the fleet of vehicles. The model helps the companies in charge of waste management in obtaining a better planning of their services.

Regarding the aerospatial field of research, L. Jia et al. propose an integrated decision model based on combination weighting and multiattribute intelligent grey target to calculate aircraft combat survivability, that is, the capability of an aircraft to avoid or withstand a man-made hostile environment.

Another paper in this research field is presented by A. Mateos and A. Jiménez-Martín, who investigate air traffic control systems while applying the multiobjective optimization simulated annealing algorithm for collision avoidance in ATM, accounting for three admissible maneuvers (velocity, turn, and altitude changes). The emphasis is on the minimization of the maneuver number and magnitude, time delays, or deviations in the leaving points. This study is especially important to improve the safety standards in free flight unstructured environments.

For the industrial sector, T. Kellegöz presents a goal programming model as well as heuristic methods based on a variable neighborhood search approach for multiobjective assembly line balancing problems with multimanned stations. This model minimizes the total number of multimanned stations as well as the total number of workers for smoothing the number of personnel at the stations.

Another strategic application concerns power projection, that is, a military term referred to the capacity of a state to apply national transportation networks for crisis response, which is a crucial element of a state's military power in modern warfare. Focused on this research field, H.-L. Bi et al. present a hub location problem in the design of power a projection network from a resilience perspective. The authors propose a biobjective and trilevel integer programming model for analyzing tradeoffs between the performances of the projection network in different situations. Besides considering the cost of the projection network in normal situations, it also analyzes the performance in presence of hub disruptions.

In the civil construction sector, P. Palha et al. deal with subcontractors' selection problems considering various attributes, such as variations in projects and type of activity. The authors present an additive sorting method for categorizing activities to be outsourced in civil construction based on the ROR-UTADIS method, including new forms of supplying preference information.

Despite the significant spread of method and applications presented herein, this special issue can only cover a small diversity of areas of the recent developments and applications in the fields. Our intention is merely to demonstrate the broad usability of multicriteria and multiobjective methods of high standards and it is our hope that it will inspire and stimulate further development of mathematical models for multicriteria and multiobjective applications in the future.

Acknowledgments

We would like to express our deepest gratitude to the authors for their contributions to this special issue and the cooperation and assistance of many reviewers, whose feedback was very useful in improving the quality of papers submitted.

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Research Article

Multiobjective Simulated Annealing for Collision Avoidance in ATM Accounting for Three Admissible Maneuvers

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Technological advances are required to accommodate air traffic control systems for the future growth of air traffic. Particularly, detection and resolution of conflicts between aircrafts is a problem that has attracted much attention in the last decade becoming vital to improve the safety standards in free flight unstructured environments. We propose using the archive simulated annealing-based multiobjective optimization algorithm to deal with such a problem, accounting for three admissible maneuvers (velocity, turn, and altitude changes) in a multiobjective context. The minimization of the maneuver number and magnitude, time delays, or deviations in the leaving points are considered for analysis. The optimal values for the algorithm parameter set are identified in the more complex instance in which all aircrafts have conflicts between each other accounting for 5, 10, and 20 aircrafts. Moreover, the performance of the proposed approach is analyzed by means of a comparison with the Pareto front, computed using brute force for 5 aircrafts and the algorithm is also illustrated with a random instance with 20 aircrafts.

1. Introduction

Cargo and air traffic (AT) congestion has experienced a general exponential growth throughout the world over the last decade. Every minute of the day, both morning and afternoon, there are about 11,000 aircrafts in the air somewhere in the world, as can be seen in real time at https://www.flightradar24.com/.

2014 was the first year in which 100,000 flights per day were exceeded. Europe's largest airports handle about 2,000 daily takeoffs and landings. This trend continues to increase gradually and estimates predict bending movements until 2030.

With the systems currently available, the air traffic control agencies are not able to efficiently manage this large increase which is taking place due to several factors as follows:

(1) Efficient use of airspace: currently, the airspace is rigidly structured and with a large number of constraints that aircrafts have to comply with. They must fly along predetermined routes through certain waypoints, which are set by the agencies of air traffic control (ATC), something that usually fails to produce optimal results. Aircrafts are not allowed to fly directly to their final destination taking advantage of favorable winds without making changes to their trajectories causing unnecessary fuel costs, which can indirectly cause increases in ticket prices. This problem is particularly evident in transoceanic routes, which are experiencing the greatest growth in demand.

(2) Increased ATC workload: AT controllers have, among other functions, to prevent collisions between aircrafts and redirect routes to avoid adverse conditions. In congested areas, such as regions near to airports called *terminal radar approach controls* (TRACONs), AT controllers often simplify their high workloads making aircraft maintain default routes outside these regions, causing delays in landings and takeoffs.

(3) Slow communication: communication is restricted to a tedious voice communication between aircraft and AT controllers, causing frequent bottleneck situations.

In view of the problems described, the aviation community has been working in recent years on a concept called *free flight*. This innovative concept allows pilots to choose their own routes, altitude, and velocity to reduce delays and manage the use of aircraft fuel more efficiently. The preferences of the pilots will be restricted only in very congested airspace areas or to prevent unauthorized entry into military areas.

Free flight is potentially possible due to the availability of technologies such as *global positioning systems* (GPS); communications data links such as *automatic dependence surveillance-broadcast* (ADS-B); detection systems and collision avoidance; and powerful computing increasingly being implemented in aircrafts.

In addition, there are several decision support tools that reduce the workload of both AT controllers and pilots and optimize their capacity, for example, the detection and resolution of conflicts in airspace sectors, landings and takeoffs management in airports, and organizational systems of the workload of AT controllers to better organize their tasks to increase productivity.

These technological advances will also allow current ATC systems to accommodate the future growth of air traffic. Algorithms to detect and solve aircraft conflicts are vital to improve the safety standards in free flight unstructured environments. These systems can be used on land by ATC or by the *flight management system* (FMS) of each aircraft.

In this paper, we focus on the development of algorithmic tools for aircraft *conflict detection and resolution* (CDR) problem. We assume that each aircraft is surrounded by cylinder representing a security virtual volume. Conflict between two aircrafts occurs when the respective aircraft security volumes overlap.

Different approaches can be found in literature to deal with collision avoidance accounting for different number and types of admissible maneuvers for aircrafts and with different solution approaches, including the use of exact solvers, simulation techniques, and metaheuristics. The work in [1] present a survey with the most important of these up to the year 2000, whereas [2] focuses on approaches from 2000 up to 2012.

One of the first approaches to deal with collision avoidance was [3]. A path planning problem among given waypoints avoiding all possible conflicts was considered aimed at minimizing the total flight time. Two mixed-integer linear programs were proposed accounting for velocity changes and angle changes as admissible maneuvers, respectively. The work in [4] proposes a three-dimensional formulation as a mixed-integer nonlinear program in which only velocity changes were admissible. CPLEX was used for the resolution in both approaches.

Simulation techniques have also been used to handle CDR problems. For instance, [5] analyzes the economic performance of a specific conflict resolution strategy based on velocity change between two aircrafts in terms of extra time and fuel consumption. The work in [6] also considers a velocity regulation problem, but from a different perspective, distinguishing between *crossing conflicts* (the wider), in which the aircrafts intersect at some point and security cylinders overlap, and *conflicts trail*, caused when an aircraft pursues another, both with different velocities.

Neural networks have been also used for performing velocity changes in CDR problems [5, 7, 8].

More recently, [9] focuses on mixed-integer optimization models based on velocity regulation. They propose to accelerate or decelerate during a specified time interval, reverting back to the original velocity once the conflict is avoided. They propose a heuristic procedure where the problem is decomposed and locally exactly solved.

Other less frequent proposals consider turn changes that lead to nonlinear optimization models. For instance, [10] proposes a two-step approach. First, a nonconvex mixed-integer nonlinear optimization is used to minimize the weighted aircraft angle variations. Then, a set of unconstrained quadratic optimization models are considered, where aircrafts are forced to return to their original flight plan as soon as possible once there is no aircraft in conflict with any other. Both an exact and an approximate resolution are proposed. In the second, the turn changes are discretized to reduce the search space.

Different metaheuristics have been proposed for solving CDR models accounting for turn changes, such as *ant colony systems* [11, 12], *genetic algorithms* [13], *variable neighborhood search* [14], and *particle swarm optimization* [15], which uses a series of waypoints the aircrafts can pass through.

A pretty realistic proposal is described in [16], wherein the acceleration variable is added to the model. It intends to solve conflicts discretizing the time remaining until it occurs at different intervals, optimizing acceleration, and velocity that should be assigned to each aircraft. A nonlinear mixed 0-1 model is used to solve the problem, which is iteratively linearized by using Taylor polynomials. This approach is then enhanced in [17], extending control to aircraft outside the aviation sector to manage, that is, taking into account those aircrafts leaving it or entering it. Moreover, they take into account the conflicts that may arise when an aircraft is climbing or descending to change altitude.

The work in [18] improves the velocity change model by adding altitude changes when necessary, for example, in head-to-head conflict situations. A multiobjective perspective is considered including objectives such as velocity variation and total number of maneuvers and forcing to return to the original flight configuration when no aircrafts are in conflict. An exactly solved mixed 0-1 linear optimization model is used, with small computational time for the execution making it suitable for real-time use.

In [19] an innovative point of view based on the choice of different strategies to avoid conflicts is proposed. An original trajectory model using B-splines is introduced together with a new semi-infinite programming formulation of the separation constraint involved in CDR problems.

In this paper we propose using *simulated annealing* to deal with a CDR problem accounting for three admissible maneuvers (velocity, turn, and altitude changes) in a multiobjective context. Specifically, the *archive simulated annealing-based multiobjective optimization algorithm* (AMOSA) has been adapted to the CDR problem accounting for objectives such as minimizing the maneuver number and magnitude, time delays, or deviations in the leaving points.

Both the possibility of performing three types of maneuvers and the multiobjective context make this paper an original contribution regarding previous works on CDR problems.

The paper is structured as follows. The mathematical modeling for the multiobjective problem under consideration is introduced in Section 2, including the identification of parameters, decision variables, and constraints and the description and modelization of the candidate objective functions for analysis. AMOSA and its adaptation to the considered CDR problem are described in Section 3. Section 4 deals with the parameter setting and the performance analysis when 5 aircrafts are considered. The parameter setting for 10 and 20 aircrafts and an example illustrating the flexibility of the proposed algorithm are provided in Section 5. Finally, some conclusions are provided in Section 6.

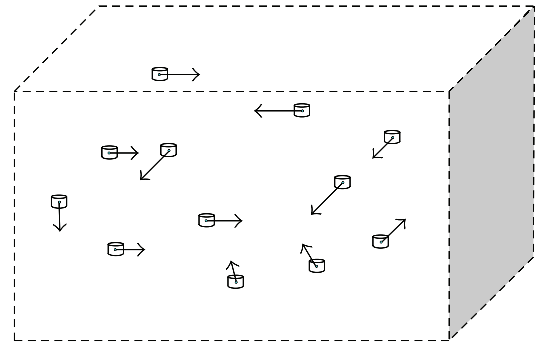

Figure 1: Aerial sector with several aircrafts.

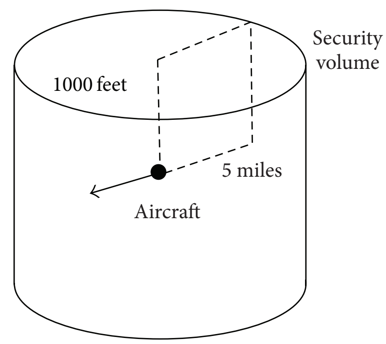

Figure 2: Security volume of an aircraft.

2. Mathematical Modeling

We assume that in a particular moment there are n aircrafts in an *aerial sector*, a cubic volume in the space managed by an AT controller; see Figure 1. We have to decide which maneuvers to perform to avoid possible collisions between them. We also assume that the decision on maneuvers will be effective until n aircrafts leave the aerial sector or until a new aircraft enters the aerial sector. At the moment a new aircraft enters the aerial sector, the analysis we propose should be again carried out to identify new maneuvers to avoid new possible collisions caused by the entering aircraft.

Moreover, we assume that the number of maneuvers performed by the aircrafts from the moment they entered the aerial sector until the moment the analysis is carried out is known. This information will be useful when analyzing the dispersion of maneuvers objective, as described afterwards.

The collision avoidance problem can be mathematically modeled as follows.

A conflict between two aircrafts occurs if the horizontal and/or vertical distances between them are smaller than some given security limits, making the security volume adopt a cylindrical form; see Figure 2. Thus, the security cylinders corresponding to any couple of aircrafts should never intersect to avoid conflicts.

In the approach we propose, we account for three types of aircraft maneuvers: *velocity change* (VC), *altitude change* (AC), and *turn change* (TC). A best type of maneuver does not exist since each maneuver has advantages and drawbacks. For example, the TC maneuver is efficient regarding fuel consumption, but it makes the aircraft leave its original trajectory, which constitutes an inconvenience.

Moreover, we consider a multiobjective perspective of the problem including the minimization of the maneuver number, magnitude and dispersion, time delays, and deviations in the leaving points.

The *dispersion of the maneuvers* objective aims at spreading the effort of all aircraft maneuvers in an attempt to avoid situations in which some aircrafts continuously perform maneuvers over time whereas other aircrafts do not do so. The *deviations in the leaving points* objective minimize the sum of the distances between the theoretical leaving points according to the initial trajectory when entering the aerial sector and the real leaving points after possible maneuver performances.

2.1. Parameters and Decision Variables. We consider n aircrafts in an aerial sector at the time t. First, parameters accounting for the *features* of the aircrafts are as follows:

$v_{\min,i}$: minimum velocity for the aircraft i.

$v_{\max,i}$: maximum velocity for the aircraft i.

$z_{\min,i}$: minimum altitude for the aircraft i.

$z_{\max,i}$: maximum altitude for the aircraft i.

$\beta_{\max,i}$: maximum variation of the angle for the aircraft i.

Secondly, initial parameters for each aircraft when entering the aerial sector (denoted by subscript ini) are as follows:

$v_{\text{ini},i}$: velocity of the aircraft i upon entry.

$z_{\text{ini},i}$: altitude of the aircraft i upon entry.

$x_{\text{ini},i}$: abscise of the aircraft i upon entry.

$y_{\text{ini},i}$: ordinate of the aircraft i upon entry.

$\alpha_{\text{ini},i}$: angle of the aircraft i with respect to the horizontal upon entry.

Next, we consider final parameters for each aircraft when leaving the aerial sector assuming that no maneuvers have been performed, that is, according to its initial trajectory when entering the aerial sector, as follows:

$x_{\text{fin},i}$: estimated abscise of the aircraft i when leaving the aerial sector.

$y_{\text{fin},i}$: estimated ordinate of the aircraft i when leaving the aerial sector.

$z_{\text{fin},i}$: estimated altitude of the aircraft i when leaving the aerial sector.

The parameters accounting for the *configuration* of the aircrafts at the time t are as follows:

v_i^t: velocity of the aircraft i at the time t.

z_i^t: aircraft i altitude at the time t.

x_i^t: aircraft i abscise at the time t.

y_i^t: aircraft i ordinate at the time t.

t_i: time necessary to arrive to the bound of the aerial sector with a constant velocity v_i^t.

α_i^t: angle with respect to the horizontal of the aircraft i at the instant t.

man_i^t: number of maneuvers performed by the aircraft i at the time t since it entered in the aerial sector.

Finally, parameters accounting for *security distances and the collision risk for aircrafts* are as follows:

$s_{\text{hor},i}$: horizontal security distance for the aircraft i.

$s_{\text{ver},i}$: vertical security distance for the aircraft i.

$c_{\text{hor,ver}}$: relative importance between vertical and horizontal risk.

According to current standards, the horizontal security distance is usually 5 nautical miles, whereas the vertical security distance is 1000 feet (see Figure 2), but other values could be used in the analysis.

Regarding the *decision variables*, as mentioned before, we propose three types of aircraft maneuvers: *velocity change* (VC), *altitude change* (AC), and *turn change* (TC). Thus, we consider three binary variables vc_i, ac_i, and tc_i for each aircraft i, pointing out whether a velocity, altitude, or turn change is performed, respectively, and a continuous variable q_i representing the magnitude (proportion) of the change performed. Note that $\text{vc}_i + \text{ac}_i + \text{tc}_i \leq 1 \ \forall i$ since each aircraft is allowed to perform, at most, one maneuver, and $q_i \in [-1, 1]$ since the changes can be negative or positive.

Consequently, a solution will consist of a vector with four elements per aircraft, identifying the maneuver performed and its magnitude.

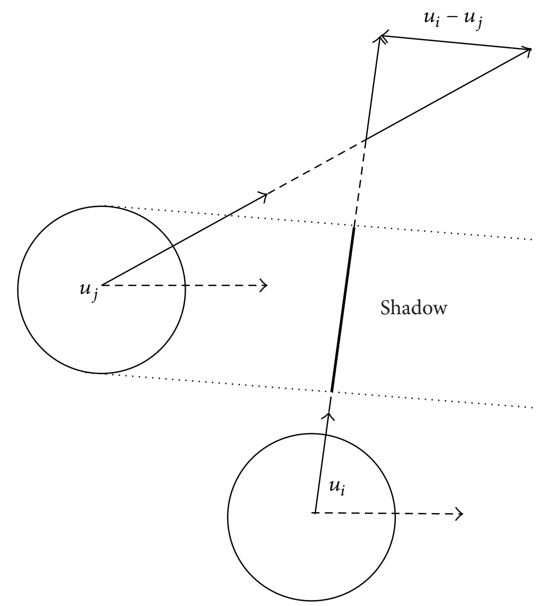

Figure 3: Detecting conflict situations.

2.2. Constraints. As a first approach, the main constraint should be avoiding conflicts between aircrafts; that is, the intersection between security cylinders should always be empty for any two aircrafts in the aerial sector under consideration. A geometric construction for detecting conflict situations is considered [3, 10]; see Figure 3.

The velocity vectors of two aircrafts i and j are

$$\vec{u}_i = \left(v_{\text{new},i} \times \cos\left(\alpha_{\text{new},i}\right), v_{\text{new},i} \times \sin\left(\alpha_{\text{new},i}\right)\right),$$
$$\vec{u}_j = \left(v_{\text{new},j} \times \cos\left(\alpha_{\text{new},j}\right), v_{\text{new},j} \times \sin\left(\alpha_{\text{new},j}\right)\right), \quad (1)$$

respectively, where $v_{\text{new},i}$ is the new velocity considering the VC maneuver and $\alpha_{\text{new},i}$ is the new angle with the horizontal plane by adding the magnitude of the TC maneuver.

The basic idea of the model comes from the construction of the relative velocity vector $\vec{u}_i - \vec{u}_j$; see Figure 3. The two straight lines parallel to the relative velocity vector and tangent to the security circle of aircraft j (dotted lines in Figure 3) define a region where the intersection with the trajectory for aircraft i is a segment named the *shadow segment*.

A *horizontal conflict* occurs if the security cylinder of aircraft i intersects the *shadow segment* generated by aircraft j or, on the contrary, since $\vec{u}_i - \vec{u}_j$ and $\vec{u}_j - \vec{u}_i$ are parallels.

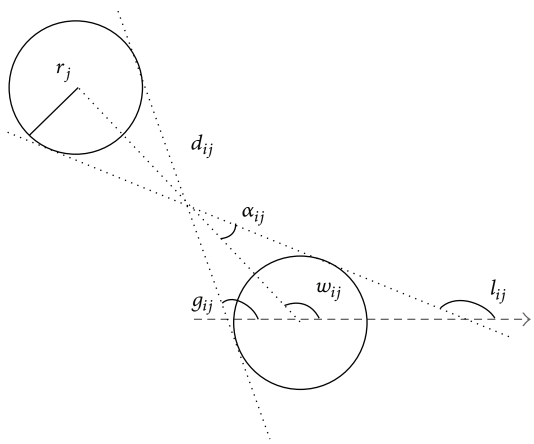

Figure 4: Main angles and distances.

Considering now the cutting planes that are tangent to both cylinders (see Figure 4) and the angles g_{ij} and l_{ij}, there is no conflict if one of the following two conditions holds:

$$\frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})} \geq \tan(l_{ij}),$$
$$\frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})} \leq \tan(g_{ij}). \quad (2)$$

Note that the functions at the left of the above expressions can cause a zero denominator. These cases are referred to as *model pathological situations* and produce unstable solutions since a conflict between two aircrafts may be erroneously determined due to the null denominator, forcing the aircrafts to crash in the worse case. This situation is detected when $|x_i - x_j| < s_{\text{hor},i} + s_{\text{hor},j}$. Therefore, variables $\alpha_{\text{new},i}$, $\alpha_{\text{new},j}$ and parameters l_{ij} and g_{ij}, which represent angles, are rotated $\pi/2$ radians when computing expressions for horizontal risk detection to overcome such pathological situation.

Vertical conflicts are detected more easily considering the security cylinders. Computing the vertical distance between two aircrafts can detect these conflicts. Figure 5 shows the modeling of the vertical conflicts.

However, very restrictive situations may occur in an aerial sector at certain times. For example, it could be very difficult to find a feasible solution without (either horizontal or vertical) conflicts when there is a high density of nearby aircrafts. Therefore, we have relaxed the collision avoidance constraint, which becomes an additional objective function from now on, as described in the next section. This allows us to better explore the solution space but, in contrast, it complicates reaching the optimal solution.

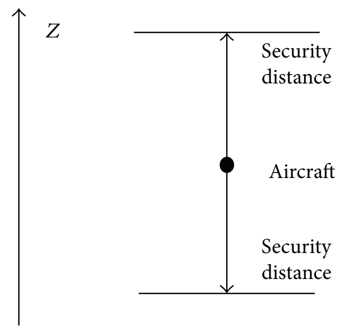

Figure 5: Vertical security distance and turn vector.

Finally, the following constraints check whether the new velocity, altitude, and turn satisfy the features of the corresponding aircraft:

$$\begin{aligned} v_{\min,i} &\leq v_{\text{new},i} \leq v_{\max,i}, \\ z_{\min,i} &\leq z_{\text{new},i} \leq z_{\max,i}, \\ \alpha_{\text{new},i} - \alpha_i &\leq \beta_{\max,i}, \end{aligned} \quad (3)$$

where $v_{\text{new},i}$, $z_{\text{new},i}$, and $\alpha_{\text{new},i}$ are the new aircraft configuration.

2.3. Objective Functions. Six different objective functions will be considered for analysis: specifically, minimizing the magnitude of maneuvers, collision risks, number of maneuvers, time delays, deviations in the leaving points, and the maneuver dispersion. Further information and the mathematical modeling of such objective functions are provided below.

2.3.1. Objective 1: Minimizing Maneuver Magnitudes. It makes sense to claim that aircrafts perform maneuvers as smoothly as possible to avoid conflicts; that is, abrupt maneuvers that may disturb passengers or even be dangerous should be avoided. According to the above, the dispersion magnitude maneuvers performed by aircrafts should be incorporated into the analysis. Moreover, a high dispersion should be penalized, that is, situations in which maneuver magnitudes are very high for some aircrafts and very low for others.

The first objective function, f_1, can be then modeled by the sum of the average maneuver magnitude and a dispersion term:

$$\min \quad f_1 = a_1 + \frac{w_1}{n}\sum_{i=1}^{n} \left| a_1 - |q_i| \right|, \quad (4)$$

where

$$a_1 = \frac{1}{n}\sum_{i=1}^{n} |q_i|, \quad (5)$$

and w_1 represents the relative importance of the dispersion term regarding the average maneuver magnitude.

2.3.2. Objective 2: Minimizing Collision Risks. This objective is the constraint we decided to relax. As cited before, this allows a better exploration of the solution space. Moreover, if we consider two solutions in which there are no conflicts we can compute a conflict risk measure for each of them accounting for the distances between all pairs of aircrafts.

There is a conflict between a couple of aircrafts if their collision risk is positive $r_{ij} > 0$; otherwise ($r_{ij} \leq 0$) there is no conflict.

We differentiate the situation in which there is at least one conflict between a couple of aircrafts and the one in which there are no conflicts to compute an average collision risk (a_2). In the first case, a_2 is computed taking into account only those $r_{ij} > 0$, avoiding that negative values nullify the positive. In the second case ($r_{ij} \leq 0$, $\forall i, j$), all r_{ij} are used to compute a_2, which is a negative value:

$$a_2 = \begin{cases} \dfrac{1}{k_1}\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} r_{ij}, & \text{if } r_{ij} \leq 0 \ \forall i, j \\ \dfrac{1}{k_2}\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} \varphi_{ij}, & \text{if } \exists \text{ at least one } r_{ij} > 0, \end{cases} \tag{6}$$

where

$$\varphi_{ij} = \begin{cases} r_{ij}, & \text{if } r_{ij} > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

and k_1 and k_2 are the numbers of elements considered in the average computations, that is, $k_1 = n!/(n-2)!4$, since all pairs of aircrafts are considered, whereas k_2 is the number of couples with conflict.

The above two situations are also considered in the modelization of objective function, f_2. The average collision risk is minimized in both cases, but in the second case ($a_2 \leq 0$), the dispersion on the collision risk values is also considered:

$$\min \quad f_2 = \begin{cases} a_2, & \text{if } a_2 > 0 \\ a_2 + \dfrac{w_2}{k_1}\displaystyle\sum_{i=1}^{n}\sum_{j=i+1}^{n} \left|a_2 - r_{ij}\right|, & \text{else.} \end{cases} \tag{8}$$

Next, we clarify how the collision risk for a couple of aircrafts is computed. The collision risk r_{ij} between two aircrafts i and j is computed in four different ways, depending on the combinations of vertical and horizontal conflicts:

$$r_{ij} = \begin{cases} r_{\text{hor},ij} + c_{\text{hor,ver}} \times r_{\text{ver},ij}, & \text{if } r_{\text{hor},ij} \leq 0 \wedge r_{\text{ver},ij} \leq 0 \\ r_{\text{hor},ij} + c_{\text{hor,ver}} \times r_{\text{ver},ij}, & \text{if } r_{\text{hor},ij} > 0 \wedge r_{\text{ver},ij} > 0 \\ r_{\text{hor},ij}, & \text{if } r_{\text{hor},ij} \leq 0 \wedge r_{\text{ver},ij} > 0 \\ c_{\text{hor,ver}} \times r_{\text{ver},ij}, & \text{if } r_{\text{hor},ij} > 0 \wedge r_{\text{ver},ij} \leq 0. \end{cases} \tag{9}$$

To compute *horizontal collision risk*, we take into account the analysis of the horizontal conflict shown in Section 2.2, which concludes that there is no horizontal conflict if one of the following two conditions holds:

$$\begin{gathered} \frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})} \geq \tan(l_{ij}), \\ \frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})} \leq \tan(g_{ij}). \end{gathered} \tag{10}$$

The *horizontal collision risk* ($r_{\text{hor},ij}$) is computed as

$$\begin{aligned} r_{\text{hor},ij} &= \min\{\gamma_{ij}, \delta_{ij}\}, \\ \gamma_{ij} &= \tan(l_{ij}) - \frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})}, \\ \delta_{ij} &= \frac{v_{\text{new},i} \times \sin(\alpha_{\text{new},i}) - v_{\text{new},j} \times \sin(\alpha_{\text{new},j})}{v_{\text{new},i} \times \cos(\alpha_{\text{new},i}) - v_{\text{new},j} \times \cos(\alpha_{\text{new},j})} - \tan(g_{ij}). \end{aligned} \tag{11}$$

Thus, it is possible to assess how far two aircrafts are to horizontally invade their respective security cylinders or, in case of conflict, to measure the intensity of invasion.

The new aircraft configuration changes or not depending on the maneuver being performed:

$$\begin{aligned} v_{\text{new},i} &= \begin{cases} v_i, & \text{if } \text{vc}_i = 0 \\ v_i + q_i(v_i - v_{\min,i}), & \text{if } \text{vc}_i = 1 \wedge q_i \leq 0 \\ v_i + q_i(v_{\max,i} - v_i), & \text{if } \text{vc}_i = 1 \wedge q_i > 0, \end{cases} \\ \alpha_{\text{new},i} &= \begin{cases} \alpha_i, & \text{if } \text{tc}_i = 0 \\ \alpha_i + q_i\beta_{\max,i}, & \text{if } \text{tc}_i = 1. \end{cases} \end{aligned} \tag{12}$$

Parameters l_{ij} and g_{ij} are computed on the basis of the angles and distances shown in Figure 4:

$$\begin{aligned} l_{ij} &= \omega_{ij} + \theta_{ij}, \\ g_{ij} &= \omega_{ij} - \theta_{ij}, \\ \omega_{ij} &= \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right), \\ \theta_{ij} &= \arcsin\left(\frac{s_{\text{hor},i} + s_{\text{hor},j}/2}{d_{ij}/2}\right). \end{aligned} \tag{13}$$

The *vertical collision risk* ($r_{\text{ver},ij}$) just consists of computing the vertical distance between aircrafts added to the vertical security distance, so that if there is no conflict, then $r_{\text{ver},ij} \leq 0$ and vice versa:

$$r_{\mathrm{ver},ij} = \begin{cases} \max\left\{z_{\mathrm{new},j} - z_{\mathrm{new},i} + s_{\mathrm{ver},j}, z_{\mathrm{new},j} - z_{\mathrm{new},i} + s_{\mathrm{ver},i}\right\}, & \text{if } z_{\mathrm{new},i} > z_{\mathrm{new},j} \\ \max\left\{z_{\mathrm{new},i} - z_{\mathrm{new},j} + s_{\mathrm{ver},i}, z_{\mathrm{new},i} - z_{\mathrm{new},j} + s_{\mathrm{ver},j}\right\}, & \text{if } z_{\mathrm{new},i} \le z_{\mathrm{new},j}, \end{cases} \quad (14)$$

where $z_{\mathrm{new},i}$ takes different values depending on whether or not this maneuver is performed and on whether the altitude is increased or decreased:

$$z_{\mathrm{new},i} = \begin{cases} 0, & \text{if } \mathrm{ac}_i = 0 \\ z_i + q_i\left(z_i - z_{\min,i}\right), & \text{if } \mathrm{ac}_i = 1 \wedge q_i \le 0 \\ z_i + q_i\left(z_{\max,i} - z_i\right), & \text{if } \mathrm{ac}_i = 1 \wedge q_i > 0. \end{cases} \quad (15)$$

2.3.3. Objective 3: Minimizing the Numbers of Maneuvers. The objective is to minimize the number of maneuvers performed by n aircrafts. This is directly connected with the AT controllers workload, since they communicate the corresponding maneuvers to the pilots. Therefore, this goal is equivalent to minimizing the controllers' workload. To model this objective function we just have to sum up the three binary variables associated to possible maneuvers of the aircrafts in the solution under consideration

$$\min \quad f_3 = \sum_{i=1}^{n}\left(\mathrm{vc}_i + \mathrm{ac}_i + \mathrm{tc}_i\right). \quad (16)$$

Note again that $\mathrm{vc}_i + \mathrm{ac}_i + \mathrm{tc}_i \le 1\ \forall i$ since each aircraft can perform at most one type of maneuver.

2.3.4. Objective 4: Minimizing Time Delays. The time an aircraft will leave the aerial sector may differ from the expected time according to its initial trajectory (when entering the aerial sector) if a VC or TC maneuver is performed. Then, the aim is now minimizing the sum of the delays for the aircrafts. Moreover, we will again take into account the dispersion of such delays.

This objective function, f_4, is computed by adding the average delay (a_4) and the dispersion of the delays:

$$\min \quad f_4 = a_4 + \frac{w_4}{n}\sum_{i=1}^{n}\left|a_4 - \left|t_{\mathrm{new},i} - t_i\right|\right|, \quad (17)$$

where $a_4 = (1/n)\sum_{i=1}^{n}\left|t_{\mathrm{new},i} - t_i\right|$ and

$$t_{\mathrm{new},i} = \frac{\sqrt{\left(x_{\mathrm{fin},i} - x_i\right)^2 + \left(y_{\mathrm{fin},i} - y_i\right)^2}}{v_{\mathrm{new},i}}, \quad (18)$$

is the time aircraft i leaves the aerial sector once the maneuvers associated to the solution under consideration are performed. Note that $t_{\mathrm{new},i} = t_i$ if an AC maneuver is performed.

In f_4, $w_4 \in [0,1]$ represents the relative importance of the dispersion term regarding the average delay, analogously to w_1 and w_2 in objective functions f_1 and f_2, respectively.

2.3.5. Objective 5: Minimizing Deviations in the Leaving Points. The point at which an aircraft will leave the aerial sector may differ from the expected point according to its initial trajectory (when entering the aerial sector) as a consequence of the maneuvers performed. As such, the aim is now minimizing the sum of the distances between both leaving points for each aircraft. Moreover, we will again take into account the dispersion of such distances.

This objective function, f_5, is computed by adding the average (a_5) and the dispersion of the distances between the original leaving point and the new leaving point once the maneuvers associated to the solution under consideration are performed:

$$\min \quad f_5 = a_5 + \frac{w_5}{n}\sum_{i=1}^{n}\left|a_5 - d_i\right| \quad \text{with } a_5 = \frac{1}{n}\sum_{i=1}^{n} d_i, \quad (19)$$

where the distance is computed as follows:

$$d_i = \sqrt{\left(x_{\mathrm{fin},i} - x_{\mathrm{fin,new},i}\right)^2 + \left(y_{\mathrm{fin},i} - y_{\mathrm{fin,new},i}\right)^2 + \left(z_{\mathrm{fin},i} - z_{\mathrm{fin,new},i}\right)^2}, \quad (20)$$

where $x_{\mathrm{fin,new},i} = \mathrm{intersection}(x_i, \alpha_{\mathrm{new},i})$, $y_{\mathrm{fin,new},i} = \mathrm{intersection}(y_i, \alpha_{\mathrm{new},i})$, and $z_{\mathrm{fin,new},i} = \mathrm{intersection}(z_i, \alpha_{\mathrm{new},i})$ are the coordinates of the new leaving point, with $\alpha_{\mathrm{new},i}$ being the new angle with respect to the horizontal of the aircraft i once the maneuvers associated to the solution under consideration are performed.

$w_5 \in [0,1]$ again represents the relative importance of the dispersion term regarding the average value (a_5).

2.3.6. Objective 6: Minimizing Maneuver Dispersion. The last objective under consideration is related to the dispersion of maneuvers over time, that is, over multiple executions of the analysis. The aim is to share the maneuver effort among the aircrafts to attempt to avoid situations in which some aircrafts continuously perform maneuvers over time, whereas other aircrafts scarcely do it.

For this, as mentioned before, we assume that a vector including the number of maneuvers performed by each aircraft from the moments they entered the aerial sector until the moment the analysis is carried out is available ($\mathrm{man}_1, \ldots, \mathrm{man}_i, \ldots, \mathrm{man}_n$). Thus, a dispersion measure can be computed as follows:

$$\min \quad f_6 = \frac{1}{n}\sum_{i=1}^{n}\left|a_6 - \mathrm{man}_{\mathrm{new}_i}\right|, \quad (21)$$

with $a_6 = (\sum_{i=1}^{n} \text{man}_{\text{new},i})/n$ being the average number of maneuvers and

$$\text{man}_{\text{new},i} = \begin{cases} \text{man}_i, & \text{if } \text{vc}_i = 0 \wedge \text{ac}_i = 0 \wedge \text{tc}_i = 0 \\ \text{man}_i + 1, & \text{if } \text{vc}_i = 1 \vee \text{ac}_i = 1 \vee \text{tc}_i = 1, \end{cases} \quad (22)$$

being the new number of maneuvers accumulated by aircraft i if the maneuvers associated to the solution under consideration are performed.

3. Multiobjective Simulated Annealing

Simulated annealing (SA) [20, 21] is a trajectorial metaheuristic which is named for and inspired by annealing in metallurgy.

An initial feasible solution is randomly generated. In each iteration a new solution y is randomly generated from the neighborhood of the current solution, $y \in N(x_i)$. If the new solution is better than the current one, then the algorithm moves to that solution ($x_{i+1} = y$); otherwise the movement to the worst solution is performed with certain probability. Note that accepting worse solutions allows for a more extensive search for the optimal solution and avoids trapping in local optima in early iterations. The probability of accepting a worse movement is a function of both a temperature factor and the change in the cost function as follows:

$$p = e^{-(f(y)-f(x_i))/T_i}, \quad (23)$$

where T_i is the temperature in the ith iteration.

The initial value of temperature (T_0) is high, which leads to a diversified search, since practically all movements are allowed. As the temperature decreases, the probability of accepting a worse movement falls. If the temperature is zero, then only better movements will be accepted, which makes simulated annealing work similar to hill climbing.

Temperature is usually kept constant for L iterations and is then decreased after multiplying by a cooling rate ($\alpha < 1$). The algorithm stops when there has been a maximum number of iterations without accepting solutions.

Metaheuristics have recently become very popular for multiobjective optimization. The aim is now to derive a good approximation of the efficient or Pareto set or, alternatively, take advantage of a decision-maker's preferences to identify a *satisficing* efficient solution. There are several reasons that explain the increasing acceptance of SA and other metaheuristics; for instance, they converge speedily to Pareto-optimal solutions, handle both discrete and continuous problems with ease, and are less susceptible to the shape of the Pareto front.

Multiobjective simulated annealing (MSA) was first proposed in [22]. The algorithm is analogous to the classical SA but now based on the concept of archiving the Pareto-optimal solutions and introducing a modification in the acceptance criteria of solutions, for which different approaches or a combination of them can be found in the literature aimed at increasing the probability of accepting nondominated solutions; see, for example, the method of Ulungu et al. (UMOSA) [23], the method of Suppapitnarm et al. (SMOSA) [24], or the *Pareto simulated annealing* (PSA) [25]. A comparison of the above methods and other methods can be found in [26].

3.1. AMOSA Method. In this paper we consider the *archive simulated annealing-based multiobjective optimization algorithm* (AMOSA) [27], which incorporates the concept of *archive* where the nondominated solutions generated are stored and determine the acceptance probability of a new solution taking into account the *domination status* of the new solution (y) with the current one (x_i), as well as those in the *archive*. For this purpose, the *amount of domination* measure is used [26] and defined as follows: given two solutions x_i and y,

$$D_{x_i,y} = \prod_{j=1}^{m} \frac{\left[f_j(x_i) - f_j(y)\right]}{R_j}, \quad (24)$$

where m is the number of objectives and R_j is the range of the jth objective.

Based on the domination status between current solution x_i and new solution y, we can face the following situations:

(1) If x_i *dominates* y and k points from the *archive* also dominate y, then $x_{i+1} = y$ with probability

$$p = e^{-D_{\text{avg}}/T}, \quad (25)$$

where

$$D_{\text{avg}} = \frac{\sum_{l=1}^{k} D_{\text{archive}_l,y} + D_{x_i,y}}{k+1} \quad (26)$$

denotes the average amount of domination of y by $(k+1)$ points, namely, the current solution (x_i) and k points of the *archive*.

(2) If y and x_i *are nondominating to each other*, then we check the domination status of y and points in the *archive*.

(a) If y is dominated by k solutions in *archive*, then $x_{i+1} = y$ with probability $p = e^{-(D_{\text{avg}}+E)/T}$, but now $D_{\text{avg}} = \sum_{l=1}^{k} D_{\text{archive}_l,y}/k$. E is a new element accounting for the possible redundancy associated to the incorporation of y to the *archive*, as explained later.

(b) If y is nondominated by all solutions in the *archive*, then $x_{i+1} = y$ and add y to the *archive*.

(c) If y dominates k solutions in the *archive*, then $x_{i+1} = y$, add y to the *archive,* and remove k dominated solutions from it.

(3) If y *dominates* x_i, then we check the domination status of y and points in the *archive*.

(a) If y is dominated by k solutions in the *archive*, we compute the minimum of the difference of domination amounts between y and the k

points ($\Delta\text{dom}_{\min}$). Then, x_{i+1} is the solution in the *archive* corresponding to $\Delta\text{dom}_{\min}$ with probability

$$p = \frac{1}{1 + e^{-\Delta\text{dom}_{\min}}}. \quad (27)$$

Else, $x_{i+1} = y$.

(b) If y does not dominate any solution in the *archive*, then $x_{i+1} = y$, and add y to the *archive*. If x_i is in *archive*, then remove it from the *archive*.

(c) If y dominates k solutions in the *archive*, then $x_{i+1} = y$, add y to the *archive,* and remove the k dominated solutions from it.

Element E in the above algorithm accounting for the redundancy would produce the inclusion of y in the archive, which is computed as follows:

$$E = \sqrt{\sum_{j=1}^{m} \left(\frac{C_j}{R_j}\right)^2}, \quad (28)$$

where C_j is the distance in each coordinate (objective) to the nearest solution to y and R_j is the range of each objective.

As a result of the incorporation of the element E, as the temperature decreases, solutions that increase the diversity in the *archive* will be accepted. This makes the algorithm tend to converge faster, whereas in the exploitation phase it tends only to improve solutions and add diversity to the Pareto front.

It is important to note that nondominated solutions are stored in the *archive* up to a maximum, MA. If the number of nondominated solutions exceeds MA, clustering is applied to reduce the size to MA [27].

3.1.1. AMOSA Adaptation to the CDR Problem. To adapt AMOSA for the resolution of the CDR problem considered in this paper, we must first identify the way solutions are represented and how the neighborhood of a solution is defined.

As mentioned in the mathematical modeling of the CDR problem, each solution consists on a vector with four elements per aircraft, three of them being binary elements representing the maneuver performed (VC, AC, or TC) and another element representing its magnitude (q_i). However, when the model was implemented, it was decided to replace maneuver elements by a single element with 4 possible values depending on the maneuver performed (VC = 1, AC = 2, TC = 3, and NM = 0), where NM means no maneuver performed. We denote by $t_i^j \in \{0, 1, 2, 3\}$ and $q_i^j \in [-1, 1]$ the type and magnitude of the maneuver performed by the aircraft j in the solution x_i, respectively.

Besides, given solution x_i, a new solution y is randomly generated from its neighborhood as follows. First, an aircraft

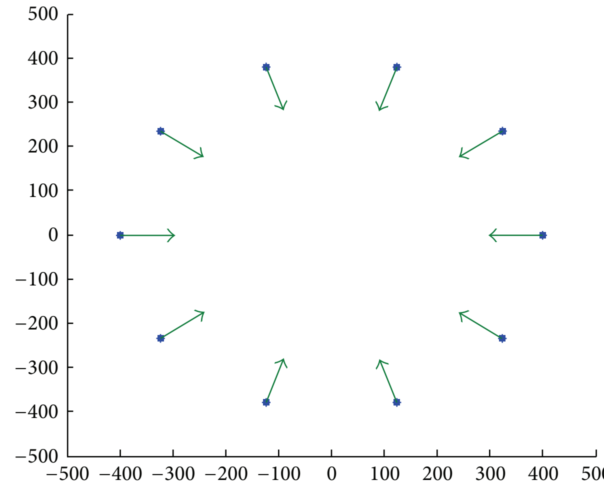

FIGURE 6: Standard instance of the CDR problem.

j is uniformly selected from the solution, then we randomly decide if the type of maneuver for this aircraft is changed:

$$t_{i+1}^j = \begin{cases} \text{mod}\,(r_2, 4), & \text{if } r_1 < \varphi \\ t_i^j, & \text{else,} \end{cases} \quad (29)$$

where φ is an algorithm parameter and $r_1, r_2 \sim U[0, 1]$.

If value 1, 2, or 3 is randomly selected, then we randomly generate the magnitude of the maneuver:

$$q_{i+1}^j = \begin{cases} q_i^j + q, & \text{if } r_4 < 0.5 \\ q_i^j - q, & \text{else,} \end{cases} \quad (30)$$

where $q, r_4 \sim U[0, 1]$.

The above expression works when $t_{i+1}^j = t_i^j$; that is, the maneuver type remains. Otherwise,

$$q_{i+1}^j = \begin{cases} q, & \text{if } r_4 < 0.5 \\ -q, & \text{else.} \end{cases} \quad (31)$$

We must check that $q_{i+1}^j \in [-1, 1]\ \forall j$.

Besides, AMOSA parameters must be fixed by analyzing the algorithm performance for a representative instance set accounting for different sizes of the problem, that is, for different numbers of aircrafts. For this, we have carried out different tests accounting for different combinations for a subset of the algorithm parameters, analyzing the algorithm performance for each combination, as described in the following sections. We must distinguish between the algorithm parameters and those associated with the nature of the CDR problem, that is, parameters related to air traffic management.

We have used a standard instance to fix parameters, also used by other approaches in the literature. The instance consists of n of aircrafts with the same altitude and speed and equidistant from the center of a circle and with direction toward the center of that circle; see Figure 6. This implies that all aircrafts have conflicts between each other, since if no change is performed they will meet in the center of the circle. This instance is the most difficult one we can face and is often used to measure the performance of new approaches.

Aircraft parameters were fixed values as close to reality as possible using as a reference the features of an *airbus A320*, that is, minimum velocity = 440 km/h, maximum velocity = 870 km/h, initial velocity = 800 km/h, minimum altitude = 10 km, maximum altitude = 15 km, initial altitude = 12 km, and maximum angle = 45 grades. Horizontal and vertical security distances are 4.0234 km and 0.1524 km, respectively. The dimension of a Spanish aerial sector was used, that is, *aerial sector radio* = 250 km. The number of maneuvers performed by the aircrafts from the moment they entered the aerial sector until the moment the analysis is carried out (man_i^t) will consist on values 0 and 1 for even and odd aircrafts; that is, one maneuver at most has been performed by aircrafts before the execution of the algorithm in time t.

The parameter values whose combinations were analyzed are initial temperature, T_0 (for 5 aircrafts it is 100, 1000, 10^4, 10^5, and 10^6, whereas for 10 and 20 aircrafts it is 10^4, 10^5, 10^6, 10^7, and 10^8); number of iterations the temperature is kept constant ($L = 5, 10, 50, 100, 200$); cooling rate ($\alpha = 0.8, 0.9, 0.95, 0.98, 0.99$); probability of maneuver change ($\varphi = 1/2$, $1/3$, $1/4$, $1/5$, and $1/6$); magnitude change ($q = 0.05, 0.1, 0.2, 0.3, 0.4$); and maximum number of iterations without accepting solutions (convergence criterion) $\{10, 20, 50, 100, 200\}$.

The following indicators were used to measure the algorithm performance:

(1) Minimum value for each of objective functions f_1 and f_2: although the aim of a multiobjective optimization algorithm is to find compromise solutions equidistantly spread around the Pareto front, this indicator can give an idea of the exploitation capacity of the algorithm.

(2) Number of nondominated solutions found: as the previous indicator, it is not a good measure of the algorithm performance, since many solutions could be found but which are not well spread around the Pareto front. However, it may provide a rough idea of the operating capability of the algorithm.

(3) Dispersion of solutions: this measure provides information about the good or bad distribution of solutions around the Pareto front. However, it could occur that the algorithm derives a set of solutions which are very well spread around the Pareto front but that set includes very few solutions. Thus, this measure should be interpreted together with the previous one. Similarly, solutions could be well distributed but far from the Pareto front.
A redundancy measure (R) is used, which consists of taking the difference between the average distance of each solution and its nearest one:

$$R = \frac{1}{p}\sum_{i=1}^{p}\left|m - d_{i,\min}\right|, \tag{32}$$

where $m = (1/p)\sum_{i=1}^{p} d_{i,\min}$ and $d_{i,\min}$ is the minimum distance of the solution i to any other solution.

(4) Dominance of solutions: it is computed as the mean of the hypervolume dominated by each solution, $\overline{H}$. Combined with the previous two metrics, it is possible to compare the performance of different executions of the algorithm with different parameters.

If we simultaneously consider the previous four indicators, it is not possible to derive the best parameter values. However, this problem can be simplified if we aggregate the metrics that measure the algorithm performance. If we consider the relative importance of indicators by means of (empirically obtained) weights, transform maximizing into minimizing objectives, normalize them, and discard atypical values when necessary, then we have the following metric:

$$\begin{aligned} m_{1,s} = {} & 0.05\frac{f_{1,s}}{\max_s\left(f_{1,s}\right)} + 0.1\frac{f_{2,s} - \min_s\left(f_{2,s}\right)}{-\min_s\left(f_{2,s}\right)} \\ & + 0.1\frac{\max_s\left(N_s\right) - N_s}{\max_s\left(N_s\right)} + 0.3\frac{R_s}{\max_s\left(R_s\right)} \\ & + 0.45\frac{\max_s\left(\overline{H}_s\right) - \overline{H}_s}{\max_s\left(\overline{H}_s\right)}, \end{aligned} \tag{33}$$

where $f_{1,s}$ and $f_{2,s}$ are the values in f_1 and f_2 for the sth execution, N_s is the number of nondominated solutions derived in the sth execution, R_s is the redundancy measure, and $\overline{H}_s$ is the mean hypervolume dominated, respectively.

A second metric represents the computation time:

$$m_{2,s} = \frac{t_{ej}^s}{\max_s\left(t_{ej}^s\right)}. \tag{34}$$

4. Parameter Setting and Performance Analysis with 5 Aircrafts

The results of several executions with different combination of parameters regarding both metrics are shown in Figure 7, whereas Figure 8 shows nondominated points (executions) in Figure 7.

Twelve nondominated points were found. The combination of parameters we chose (pointed out in red) due to its very good performance and very low computation time (0.1156 seconds) is $T_0 = 100$, $L = 5$, $\alpha = 0.99$, $q = 0.4$, $\varphi = 1/4$, and convergence iterations = 20.

The valuations in the indicators for the selected combination are $f_{1,s} = 0.1947$, $f_{2,s} = -960.7386$ (0.0012 normalized), number of nondominated solutions = 115.2 (0.0938 normalized), $R_s = 78.7474$ (0.0629 normalized), $\overline{H}_s = 60.9578$ (0.1343 normalized), and $t_{ej}^s = 0.1156$ seconds ($m_{2,s} = 0.00073$), with 3583.8 iterations carried out (0.0064 normalized).

We will now provide different figures showing the evolution of some interesting elements and the different individual objectives along the algorithm execution with the selected

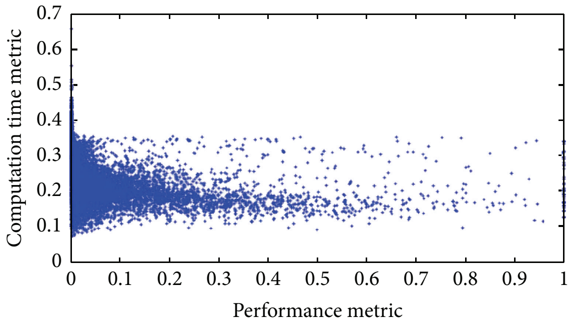

FIGURE 7: Performance metric (see (33)) and computation time metric (see (34)).

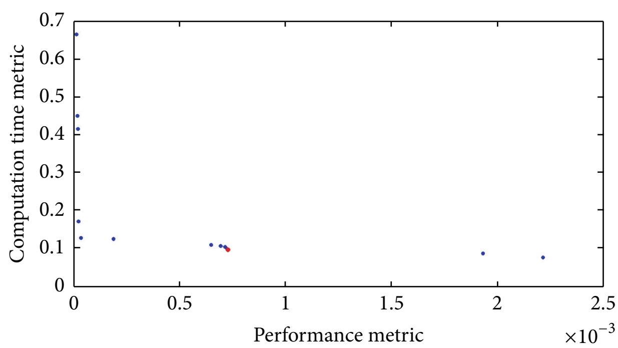

FIGURE 8: Nondominated and the selected (in red) solution regarding both metrics. Note that we want to minimize performance and computation time metrics, which are shown in Figure 7.

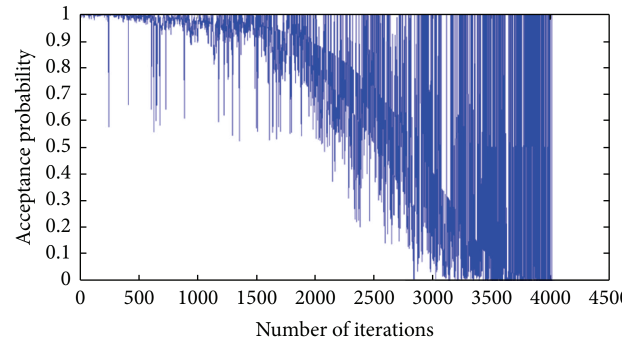

FIGURE 9: Evolution of acceptance probability of new solutions.

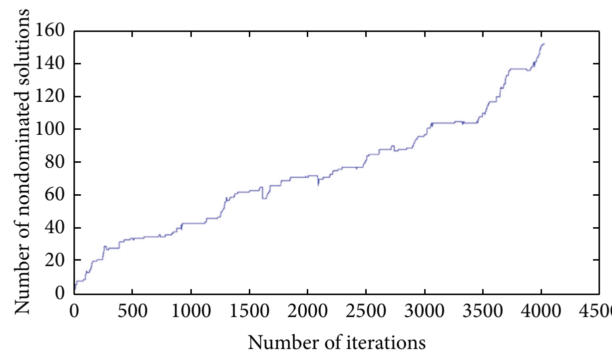

FIGURE 10: Evolution of the number of nondominated solutions.

parameter combination. Note that each objective function will be individually considered but in all cases we ensure no collision ($r_{ij} \leq 0$, $\forall i, j$).

Figure 9 shows the evolution of the acceptance probability of new solutions. At the beginning of the algorithm the acceptance probability is very close to 1. This means that in the first iterations the algorithm has a great capacity for exploration in the search space, accepting nondominated solutions that dominate the current one with any value of redundancy as well as solutions dominated by the current one. However, as the number of iterations progresses, the acceptance probability gradually decreases, leading the algorithm to an exploitation or intensification state, in which only solutions that dominate the current one or nondominated solutions with a redundancy value very small are accepted.

Figure 10 shows the evolution of the number of nondominated solutions. It is possible to see that the number of nondominated solutions increases along the execution of the algorithm. An even higher number of iterations would imply a greater number of nondominated solutions. However, not surprisingly, there is little practical difference between, for example, 100 and 5000 solutions if we assume a similar distribution of both set points around the Pareto front.

Figure 11 shows the evolution of the minimum value for f_1 that ensures no collision. We can see that it improves from 0.4 to 0.19. This means that aircrafts in the solution with minimum value for f_1 would be forced to perform an average magnitude change maneuver of 20%, assuming that the variance between them is not taken into account ($w_1 = 0$) in this case.

Figure 12 shows the evolution of the minimum values for f_2. The average collision risk ($w_2 = 0$) improves from a value close to -1 to value -30. This means that we go from a situation in which though the average risk collision is negative, there are some collisions, to another safer situation in which there are no collisions and slack in the security distances. This low risk has been reached on a solution in which several turn change (TC) maneuvers have been performed, since this maneuver more effectively reduces the risk.

Regarding the evolution of the minimum values for f_3, the minimum and maximum number of maneuvers throughout the execution are 4 and 5, respectively; that is, less than 4 maneuvers cannot be performed to avoid collisions. Note that in the CDR instance under consideration the aircrafts have the same altitude and speed and are equidistant from the center of a circle and with direction toward the center of that circle and that minimum number of maneuvers which ensure no collision is 4.

If we now pay attention to the evolution of the minimum values for f_4 that ensures no collision, the minimum value achieved for time delay is 0, since a solution involving only altitude change maneuvers was found. Note also that we have assumed that maneuvers are instantly performed.

Figure 13 shows the evolution of the minimum values for f_5 that ensures no collision. We can see that the distances

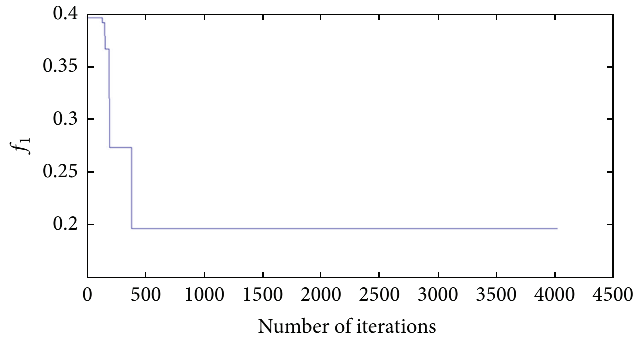

FIGURE 11: Evolution of the minimum value for maneuver magnitudes (f_1).

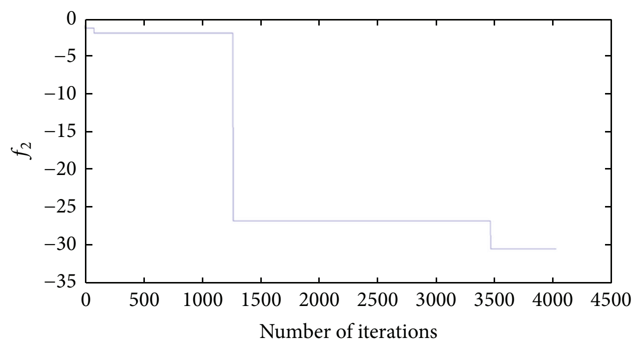

FIGURE 12: Evolution of the minimum value for collision risks (f_2).

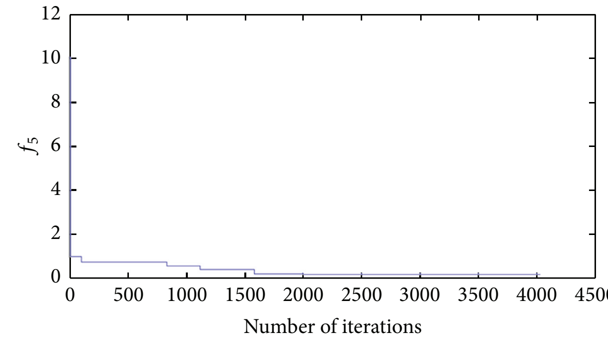

FIGURE 13: Evolution of the minimum value for deviations in the leaving points (f_5).

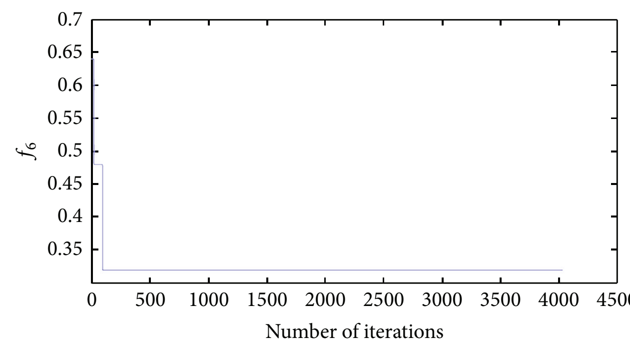

FIGURE 14: Evolution of the minimum values for maneuver dispersion (f_6).

between the points at which aircrafts will leave the aerial sector and the expected points according to their initial trajectory (when entering the aerial sector) decrease from 10 km to almost 0. This means that a solution in which the aircraft maneuvers are only velocity changes (VC) has not been found, since this maneuver ensures that the real and estimated leaving points are the same.

Finally, Figure 14 shows the evolution of the minimum values for f_6 that ensures no collision. Maneuver dispersion begins at 0.65 and decreases to a value close to 0.17. Note that in the parameters setting we assumed that the number of maneuvers performed by the 5 aircrafts from the moment they entered the aerial sector until the moment the analysis is carried out was $\{1, 0, 1, 0, 1\}$; that is, aircrafts 1, 3, and 5 had previously performed one maneuver.

As mentioned before, when we analyzed f_3, at least 4 maneuvers have to be performed to avoid collisions, leading to three possible dispersion values, the lowest being 0.17.

Next, we will analyze the performance of the proposed algorithm by comparing the set of nondominated solutions derived from it with the real Pareto front, which can be computed using brute force for this instance with 5 aircrafts having conflicts between each other.

Of course, we discretized maneuver magnitude to 200 values in the range $[-1, 1]$; that is, a step of 0.01 is considered. Besides, we considered all the possible combinations of maneuvers, 4^5. Only those solutions for which there was no conflict were stored, leading to 258719616 solutions (14,326 Gbs). This set was iteratively reduced, leading to 6641 nondominated solutions. Clearly, we obtained an approximation of the real Pareto front since a discretization in the maneuver magnitudes was done.

The metric for comparison is the relationship of the hypervolume dominated by each nondominated solution derived by the algorithm with hypervolume dominated by the solution derived using brute force closest to the first, among those that dominate it:

$$m = \sum_{i=1}^{m} \frac{h_{\text{alg},i}}{h_{\text{opt,min}}}, \tag{35}$$

where $h_{\text{opt,min}} = \min_{j=1}^{n} \{h_{\text{opt},j}, h_{\text{alg},i}\}$ and $h_{\text{opt},j}$ are the solutions that dominate $h_{\text{alg},i}$.

We carried out 100 executions of the algorithm with the optimal parameter values identified above for 5 aircrafts. The algorithm performance (on average) is 0.25 worse regarding Pareto front. If a higher number of values were considered in the discretization of the maneuver magnitude a higher number of nondominated would be obtained and the performance would approach Pareto front. Figures 15–19 show an example of nondominated solutions derived by the algorithm (in red) and using brute force (in blue) for the different objectives with respect to f_2, the collision risks. In these figures, we can see how the nondominated solutions derived by the algorithm are spread around the Pareto front.

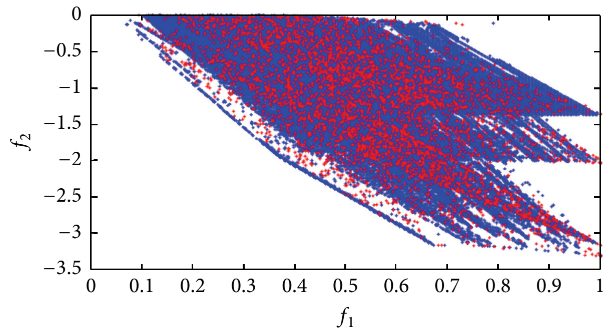

FIGURE 15: Pareto front and its approximation when the objectives shown on figure are maneuver magnitudes (f_1) and collision risks (f_2).

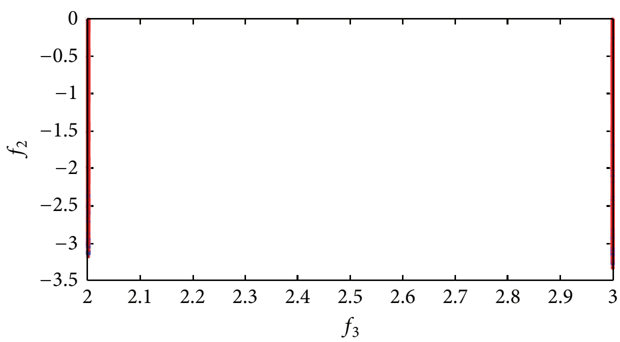

FIGURE 16: Pareto front and its approximation when the objectives shown on figure are numbers of maneuvers (f_3) and collision risks (f_2).

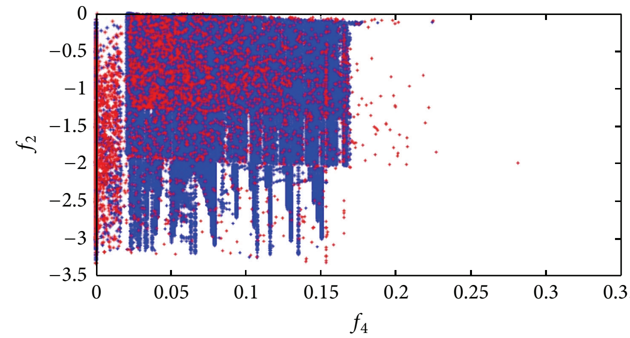

FIGURE 17: Pareto front and its approximation when the objectives shown on figure are time delays (f_4) and collision risks (f_2).

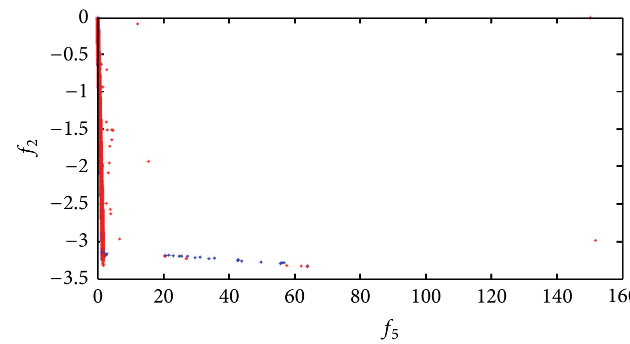

FIGURE 18: Pareto front and its approximation when the objectives shown on figure are deviations in the leaving points (f_5) and collision risks (f_2).

5. Parameter Setting for 10 and 20 Aircrafts and Illustrative Example

In this section we first identify the best combination of parameter values for 10 and 20 aircrafts, analogously as in the previous section for 5. Then, we use a randomly generated instance with 20 aircrafts to illustrate the algorithm and its flexibility.

First, we consider 10 aircrafts and the possible values for T_0: $\{10^4, 10^5, 10^6, 10^7, 10^8\}$. Figure 20 shows the values regarding the performance and computation time metrics for several executions, whereas nondominated points (executions) are shown in Figure 21.

The two points located at the right were discarded since they have a very high execution time for a tiny variation of performance with respect to the others. The following combination of parameters was selected: $T_0 = 10000$, $L = 5$, $\alpha = 0.8$, $t = 0.05$, $\varphi = 1/6$, and convergence iterations = 100.

The valuations in the different indicators by the selected combination are $f_{1,s} = 0.2776$, $f_{2,s} = -154.9622$ (0.0092 normalized), number of nondominated solutions = 48.8000 (0.1025 normalized), $R_s = 32.1758$ (0.0498 normalized), $\overline{H}_s = 607.0908$ (0.0329 normalized), and $t^s_{ej} = 0.0964$ seconds ($m_{2,s} = 0.0019$), with 1912 iterations carried out (0.0019 normalized).

Regarding 20 aircrafts, the values regarding the performance and computation time metrics for several executions are shown in Figure 22, whereas nondominated points (executions) are shown in Figure 23.

The combination of parameters chosen, see Figure 23, was the following: $T_0 = 10000$, $L = 5$, $\alpha = 0.8$, $t = 0.1$, $\varphi = 1/5$, and convergence iterations = 200.

As mentioned before, the most difficult instance has been used to identify the combination of parameter values that yields the best performance of the algorithm. As such, once that optimal parameter set has been identified we will illustrate the algorithm for another randomly generated instance with 20 aircrafts; see Figures 24 and 25.

The algorithm is then executed with the optimal parameters set, leading to a set of nondominated solutions. If only one solution has to be implemented we need to incorporate the preferences of a decision-maker (DM) to derive a *compromise* (nondominated) solution. Specifically, we assume that the DM only wants to minimize the delay (f_4) and the deviations in the leaving points (f_5), with weights 1000 and 1/4, respectively; that is, we consider the following expression:

$$f = 1000 \times f_4 + \frac{1}{4} \times f_5. \quad (36)$$

The compromise solution for the previous weighted function is (VC, 0.43, NM, –, VC, 0.26, TC, 0.07, NM, –,

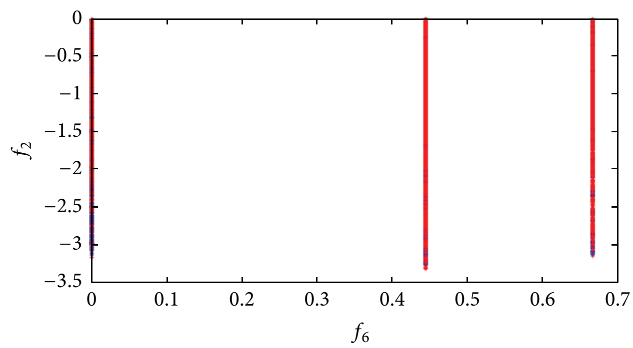

Figure 19: Pareto front and its approximation when the objectives shown on figure are maneuver dispersion (f_6) and collision risks (f_2).

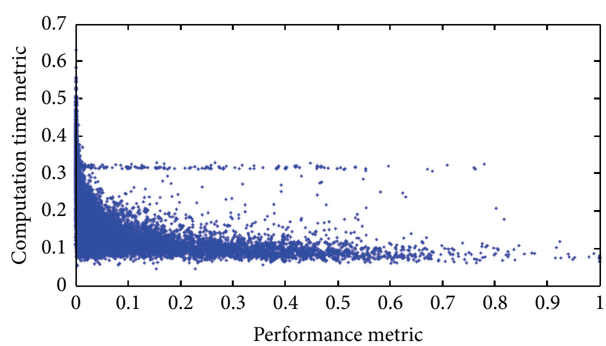

Figure 20: Performance and computation time metrics for 10 aircrafts.

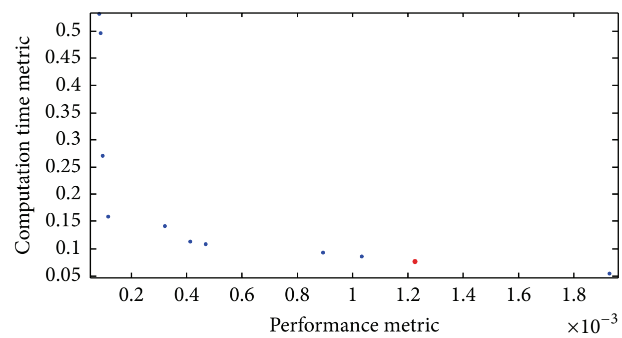

Figure 21: Nondominated and the selected (in red) solution for 10 aircrafts.

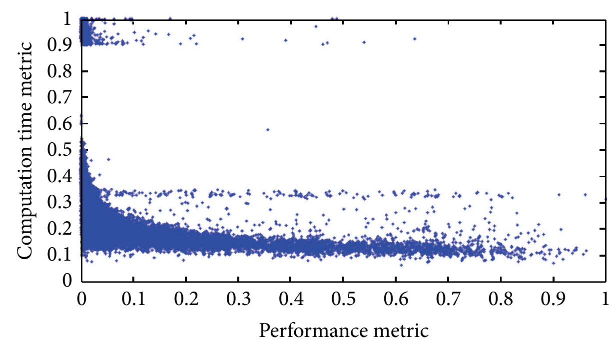

Figure 22: Performance and computation time metrics for 20 aircrafts.

VC, −0.08, AC, 0.95, VC, 0.07, VC, −0.27, VC, −0.49, VC, −0.13, TC, 0.39, VC, 0.47, NM, –, TC, −0.24, TC, 0.83, TC, −0.06, VC, −0.48, AC, 0.88, VC, −0.38) with objective values $f_1 = 0.381293$, $f_2 = -152.273331$, $f_3 = 17.0$, $f_4 = 0.013386$, $f_5 = 81.067757$, and $f_6 = 2.75$.

Let us pay attention to the aircraft in blue in Figures 24 and 25, whose initial configuration is position = (116.67, −47.70, 12.15), entry point = (255.00, −90.91, 12), estimated leaving point = (−255.00, 68.41, 12), current velocity = 833.84 km/h, initial velocity = 900.00 km/h, current angle = 2.83 grades, initial angle = 3.21 grades, maneuvers performed since it entered the aerial sector = 4, elapsed time since it entered the aerial sector = 0.16, and estimated time traveling in the aerial sector = 0.59.

The compromise solution implies that this aircraft must perform a velocity change maneuver (VC) of magnitude 0.43. That is, its velocity will increase by 43%. This is consistent, since the current velocity before applying the algorithm was 833.84 km/h, and the initial velocity when it entered the aerial sector was 900 km/h. It could be interpreted as the aircraft increases its velocity to try to counteract the delay it would cause if we maintain the current velocity. Therefore, the aircraft configuration changes after running the algorithm and now the current velocity is 904.91 km/h and maneuvers performed so far are 5.

The developed algorithm is very flexible since objectives can individually or in combination be optimized. For example, if we now want to optimize f_6 (minimizing maneuver dispersion), the following compromise solution is reached: (NM, –, NM, –, NM, –, NM, –, NM, –, NM, –, AC, −0.20, AC, 0.09, TC, 0.57, NM, –, TC, 0.74, AC, −1.00, NM, –, AC, 0.07, VC, −0.25, NM, –, AC, 0.68, VC, −0.06, TC, 0.78, NM, –), with objective values $f_1 = 0.444926$, $f_2 = -151.998688$, $f_3 = 10.0$, $f_4 = 0.013348$, $f_5 = 86.590012$, and $f_6 = 2.36$.

We realize how, regardless of the maneuver performed, the algorithm tends to homogenize the number of maneuvers performed by the aircrafts. The minimum value for f_6 is 2.36 in comparison with value 2.75 obtained where f_4 and f_5 were optimized. The maneuvers performed before running the algorithm were (4, 9, 9, 10, 7, 8, **3**, **4**, **3**, 9, **1**, **2**, 6, **1**, **3**, 9, **8**, **4**, **4**, 10), whereas maneuvers performed later are (4, 9, 9, 10, 7, 8, **4**, **5**, **4**, 9, **2**, **3**, 6, **2**, **4**, 9, **9**, **5**, **5**, 10). Highlighted in bold are those aircrafts that have performed a maneuver.

6. Conclusions

We have dealt with the conflict detection and resolution (CDR) problem in air traffic management accounting for three admissible maneuvers (velocity, turn, and altitude changes). Moreover, a multiobjective context has been considered, accounting for the minimization of the maneuver number and magnitude, collision risks, time delays, deviations in the leaving points, and maneuver dispersion.

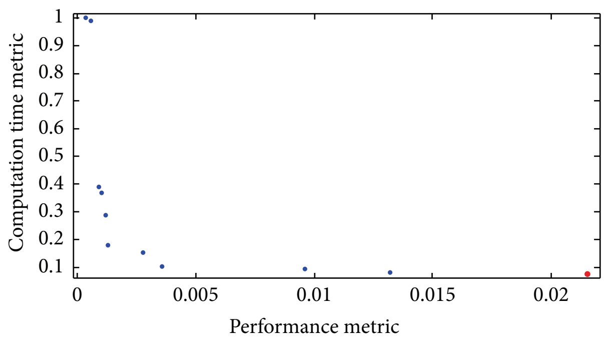

FIGURE 23: Nondominated and selected (in red) solutions for 20 aircrafts.

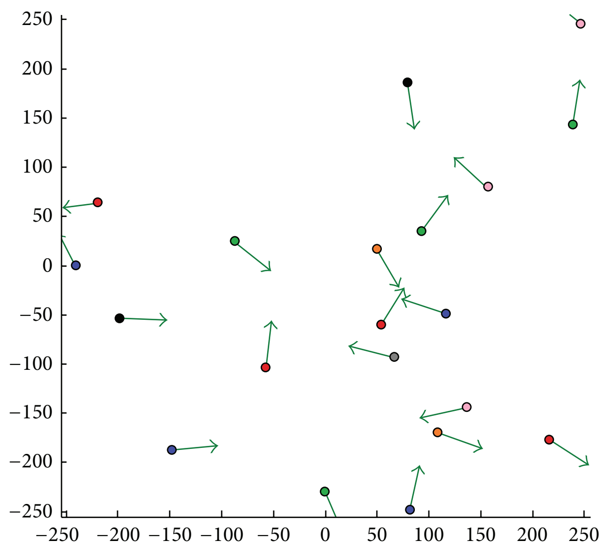

FIGURE 24: Random instance with 20 aircrafts, Plane *XY*.

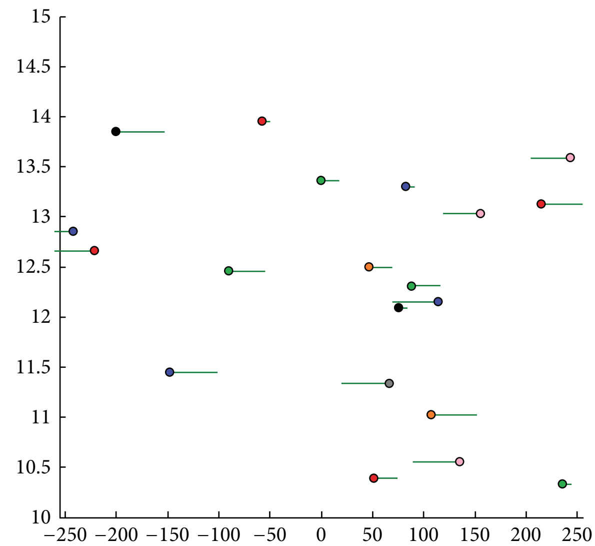

FIGURE 25: Random instance with 20 aircrafts, Plane *XZ*.

Both the possibility of performing three types of maneuvers and the multiobjective context make this paper an original contribution regarding previous works on CDR problems.

We have adapted the *archive simulated annealing-based multiobjective optimization algorithm* to solve the CDR problem and conducted a thorough analysis of the effect of parameters in the resolution based on some instances of the problem, which led to a subset of optimal values for such parameters. The results derived by a discretized version of the algorithm for 5 aircrafts were compared with the real Pareto front computed using brute force, showing a good approximation. Moreover, a more complex random instance with 20 aircrafts has been considered to illustrate the algorithm and show its flexibility.

As future research work, we propose improving and extending some of the model features and its resolution. We have assumed that the altitude changes are instantaneous. In general, the planes are far away enough from each other so that the time required to perform an altitude change is negligible. However, in some cases the algorithm could fail to detect a conflict.

Another improvement could be allowing aircrafts to perform more than one maneuver simultaneously (e.g., VC and TC at once). This would mean better performances in some objectives. Moreover, the evaluation objectives, the most time-consuming part in the algorithm, could be parallelized to improve execution times to come near to output real-time solutions.

Finally, the good performance of the approach proposed has been proven. However, its performance should be compared with the application of other metaheuristics for the specific CDR problem under consideration.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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Research Article

A Model for Selecting a Strategic Information System Using the FITradeoff

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This paper arose from the perceived need to make a contribution towards assessing a strategic information system by using a new method for eliciting the weights of criteria. This is considered one of the most complex and important stages in multicriteria models. Multicriteria models have been proposed to support decisions in the context of information systems given that problems in this field deal with many conflicting criteria. The new procedure for eliciting the weights of the criteria has the advantage of requiring less effort from the decision-maker and, thus, the risk of inconsistent answers is minimized. Therefore, a model based on this new procedure is proposed and applied using data from a glass packaging factory that needs to select a single information system from a set of systems previously identified as relevant. The results obtained are consistent both with the performance of alternatives and with the additive model used to evaluate the alternatives.

1. Introduction

As observed by Arvidsson et al. [1], information systems (IS) have a strategic role within organizations since they are used to bring about strategic intent. Industrial production managers, service providers, and business administrators invest in IS with a view to obtaining greater efficiency, agility, and security in their operations. Although the market offers a variety of IS which serve different areas within organizations, it is not possible, in most organizations, to invest simultaneously in all the systems that they require. Usually, the resources (whether these have to do with financial resources, time, workforce, infrastructure, and so on) required for these investments are scarce.

Due to the relevance of this subject, academic researchers have devoted special attention to selecting IS, which is one of the three strands within IS strategy research, identified by Chen et al. [2]:

(i) IS and business strategy alignment: where models to provide a strategic alignment between IS and business strategy [3] and models to analyze the positive impact of this alignment [4] are developed.

(ii) Strategic IS planning to identify portfolios of systems: where the aim is to define a set of systems that jointly contribute to the organization [5, 6]. In some models, depending on the objective and particularities of the organization, the aim is not to define a portfolio but simply to select or to rank the systems [7].

(iii) To use a specific system and assess its contribution towards adding to competitive advantage [1, 8].

Regarding the second strand, a large number of intuitive and analytical models have evolved over the last twenty or so years to assist decision-makers (DMs) in evaluating IS projects [9]. According to Zandi and Tavana [9], the main methodologies used for selecting and prioritizing IS projects can be divided into single-criterion cost benefit analysis, multicriteria scoring models and ranking methods, and portfolio methods. A subjective committee is also identified by Chen [10] as a class of methodologies used for IS project selection. The single-criterion cost benefit analysis is illustrated in [11], where Research and Development (R&D) projects are evaluated, and in [12], where R&D projects are selected by using an expected utility approach. The use of multicriteria scoring models and ranking methods for IS project selection is demonstrated in [13], which uses a multiobjective decision model, and in [14], where an analytic network process and goal programming are applied. Currently, the use of multicriteria methods designed to handle portfolio problems has gained special attention within the context of IS. de Almeida and Vetschera [15] propose a method to correct a problem on scale transformations in the PROMETHEE V method, which can be used to support portfolio decision problems. Other papers demonstrate the use of a portfolio method such as [16]. A fuzzy set approach for R&D portfolio selection is proposed by [17].

Initially models that evaluate the financial return on investment in IS were proposed to assist the selection of IS projects. However, the process for doing so requires several conflicting objectives to be analyzed but these cannot be measured monetarily. Thus, multicriteria decision-making/aiding (MCDM/A) methods are gaining in importance due to their inherent ability to judge different alternatives and to rank them.

The aim of this paper is to put forward a contribution in the field of strategic IS selection by using the FITradeoff (Flexible and Interactive Tradeoff) method, proposed by de Almeida et al. [18]. The application presented here was conducted using data from IS investments in a factory of one of the largest manufacturers of glass packaging in the world. These data were also used in an application presented by de Gusmão et al. [19] when the ELECTRE IV method was used by different DMs to rank the alternatives and, at the end of the model, the ranks were aggregated. Here, the alternatives are ranked according to an additive model in MAVT (Multiattribute Value Theory) scope, using the FITradeoff method for eliciting scaling constants or weights of criteria. The results of the two applications cannot be compared since the first application is conducted using a noncompensatory method while the value function of MAVT is compensatory. However, in both cases, the issue of defining the weights of criteria is undertaken (automatically by the method used) so as to minimize the effort that DMs need to make with regard to this.

This paper is organized as follows: Section 2 discusses the importance of multicriteria models in many instances of decision-making and presents an overview of the FITradeoff procedures. The model proposed is briefly presented in Section 3 and applied in Section 4, which discusses details of the steps of the model. The results area is also analyzed in Section 4, thereby allowing the DM to test the robustness of the model. Finally some conclusions and insights that may prompt future research are considered.

2. Background

2.1. Multicriteria Models. When planning investments in IS, not only the financial implications of such investment, but also other objectives such as competitive advantage, market share, and future growth need to be assessed. Thus, MCDM/A methods are gaining in importance since they allow different alternatives to be evaluated and ranked or one subset to be selected [20, 21]. These methods consider several points of view, characterized as conflicting issues, thereby enabling an integrated assessment of the problem in question to be made [22].

According to Vincke [23], MCDM/A can be considered a set of methods developed to support decision problems faced by organizations and individuals. The problems can include selecting suppliers [24], planning maintenance [25, 26], IS selection [20], and evaluating critical technology for generating energy [27].

According to de Almeida et al. [18], defining the weight of criteria, in multicriteria decision, mainly in additive models, is considered a problem either because the DM does not understand the meaning of weight or because the DM does not have sufficient knowledge and information to define the weights. Due to some major difficulties and challenges in procedures for eliciting weights that have been identified and considering that the tradeoff elicitation procedure—the procedure most used in additive models—has a strong axiomatic foundation besides some inconsistencies, de Almeida et al. [18] propose the FITradeoff method.

Unlike traditional procedures, in the flexible elicitation procedure, the DM does not need to provide imprecise or incomplete information *a priori*. In the flexible elicitation process, there is an attempt to minimize the effort that the DM must make, when compared with considering the traditional procedures. Thus it is expected that less inconsistency occurs during the process [18].

The search for ways to make it easier to determine the parameters required in decision-making, including the weights of the criteria, is not something new. The ELECTRE TRI Assistant method, proposed by Mousseau et al. [28], for example, requires from the DM much less cognitive effort. In this case, the parameters are defined indirectly using holistic information given by the DM through assignment examples, which are alternatives assigned by the DM to categories according to his/her comprehensive preferences. The use of assignment examples makes sense since the ELECTRE TRI is a multicriteria model whose goal is to assign each alternative to one of the categories which are predefined. Also, Mousseau and Dias [29] propose a slight adaptation of the valued outranking relation used in ELECTRE III and ELECTRE TRI. Although the modified outranking relation keeps the complexity of inferring the weights and cutting level, the veto thresholds are inferred easily.

The last two examples refer to methods of outranking. Regarding additive models, Melo Brito et al. [30], for example, present the application of a multicriteria methodology to support the selection of repair contracts in a context where information is imprecise, that is, when it is not possible to assign precise values to importance parameters of the criteria used for contract selection, but unlike the FITradeoff there is no elicitation procedure. A decision support system (DSS) was proposed by de Almeida et al. [31] to support a DM to establish the weights of criteria in a multicriteria decision problem by using a flexible elicitation procedure.

2.2. Overview of the FITradeoff Procedure. The usual notations adopted in elicitation of weights processes are used [32, 33]:

(i) $X = \{x_1, x_2, \ldots, x_n\}$ represents the vector of consequences of an alternative, considering all i ($i = 1, \ldots, n$) criteria.

(ii) k_i represents the weight for the criteria i.

(iii) $v_i(x_i)$ represents the value function of the consequences x_i for the I criteria.

Thus, according to Keeney and Raiffa [32] and Keeney [34], $v(x)$ aggregates the value functions $v_i(x_i)$ as

$$v(x) = \sum_{i=1}^{n} k_i v_i(x_i). \quad (1)$$

Assuming that

$$\sum_{i=1}^{n} k_i = 1, \quad k_i \geq 0, \quad (2)$$

it is well known that, in additive models, the most appropriate denomination for k_i is as a scaling constant and not as a weight, but, as in [18], in this paper the term weight will be used for the sake of simplification.

The procedure for applying the FITradeoff, like that for the traditional model, is divided into two parts:

(i) Obtaining the orders of the weights k_i, using the preference P.

(ii) Obtaining the values of k_i, using the indifference relation I.

The first part allows an n-dimension weight space (φ_n) to be defined which is given by

$$\varphi_n = \left\{ (k_1, k_2, k_3, \ldots, k_n) \mid k_1 > k_2 > k_3 > \ldots > k_n; \ \sum_{i=1}^{n} k_i = 1; \ k_i \geq 0 \right\}. \quad (3)$$

In this notation, and from now on, it was assumed, as assumed in de Almeida et al. [18], that the criteria are ordered from the most relevant to the least relevant.

The second part is now begun and it is in this part that the difference between the procedure for the traditional model and the FITradeoff is seen. In the FITradeoff, it is not necessary for the DM to define an exact value (x_i^I), which would denote the outcome of criterion i for which the indifference is obtained between consequences, whereas the traditional method requires this. In the FITradeoff, this procedure requires the DM to specify a range from x_i' to x_i'' that represents, respectively, the upper and lower limit that x_i^I can assume. Thus, given any criterion i, the following relations can be established:

$$v_i(x_i') > \frac{k_{i+1}}{k_i}, \qquad v_i(x_i'') < \frac{k_{i+1}}{k_i}. \quad (4)$$

Thus, as shown in [18], as a result of the second part, a new weight space (φ_n^s) may be obtained, which is a subspace of (3), in which all the valid relations of type (4) are considered:

$$\varphi_n^s = \left\{ \begin{array}{c} (k_1, k_2, k_3, \ldots, k_n) \mid k_1 > k_2 > k_3 > \ldots > k_n; \ \sum_{i=1}^{n} k_i = 1; \ k_i \geq 0 \\ k_1 v_1(x_1'') < k_2 < k_1 v_1(x_1'); \ldots; \\ \vdots \\ k_{n-1} v_1(x_{n-1}'') < k_n < k_{n-1} v_1(x_{n-1}') \end{array} \right\}. \quad (5)$$

For more details on the definition of x_i^I, see Keeney and Raiffa [32] and Keeney [34].

The FITradeoff is operationalized by a DSS, which includes the following stages [18]:

(1) Evaluating the intracriteria.

(2) Ranking the weights of the criteria.

(3) Attempting to solve the problem using the available set of weights.

(4) Evaluating the DM's preferences.

At the end of Stage (3), a check is made on whether or not a unique solution has been obtained, that is, if an optimal alternative has been identified. The DSS classifies the alternatives in three situations: potentially optimal, dominated, or optimal. If a single solution is not found, the DM goes on to the next stage, namely, that of evaluating the DM's preferences which can be divided into four steps:

(4.1) Setting values for testing the distribution of weights.

(4.2) Asking the DM to state his/her preferences.

(4.3) Computing LPP.

(4.4) Finalization.

These four steps constitute the main stage of the FITradeoff [18]. The aim of using the heuristic presented in Stage (4.1) is to compute the value of x_i, thereby minimizing the number of questions to the DM. The output of this step, and the input for Stage (4.2), is a new set of values for x_i' and x_i''. Based on this new set of values, the DM, in Stage (4.2), has three options: to see partial results; not to proceed in the system; or to proceed with a view to making a choice. This choice consists of defining if there is a preference or an indifference relation between two consequences. If there is a preference relation, the DM has to signal the preferred consequence. Depending on the consequence chosen, either x_i' or x_i'' assumes the x_i value. In the case of indifference, x_i^I assumes the x_i value.

Having made this choice, Stage (4.3) is started and a Linear Program Problem (LPP) model is run [18]. It is important to note that this LPP model is also applied in Step (3) assuming $v_i(x_i') = 1$ and $v_i(x_i'') = 0$ for all criteria i. This LPP has the following objective function:

$$\underset{k_1,k_2,\ldots,k_n}{\text{Max}} \sum_{i=1}^{n} k_i v_i\left(x_{ij}\right), \quad j = 1, 2, \ldots, m. \tag{6}$$

Thus, the aim is to find an alternative j, from the set of m alternatives, that has the maximum value given in (1) in accordance with the weight of criteria space given by (5). Therefore, it is necessary to consider some constraints in the LPP. The relations (4) are introduced as constraints on how strict inequality is avoided.

Also, what should be considered to solve the problem (the optimal alternative) is that the maximum value of the alternative j should be greater than (or equal to) any other alternative in the subset. Thus, the following constraint has to be considered:

$$\sum_{i=1}^{n} k_i v_i\left(x_{ij}\right) \geq \sum_{i=1}^{n} k_i v_i\left(x_{iz}\right), \quad z = 1, 2, \ldots, m, \; z \neq j. \tag{7}$$

This LPP runs until an optimal alternative is found. If this does not happen, the dominated alternatives are eliminated and the process is started again, from Step (4.1). Now, just the alternatives identified as potentially optimal are considered in the subsequent steps.

3. Model Proposed

As already explained, the aim of this paper is to put forward a contribution in the field of strategic IS selection by using FITradeoff and based on the data presented in [19]. Thus, based on the framework presented in [19] and on the general procedure of the FITradeoff, the model proposed can be structured as shown in Figure 1.

First of all, the IS that are identified as relevant for the organization and also the criteria that will be used to evaluate the IS must be defined. This step can be supported, as proposed in [19], by some methods, for example, using the BSC (Balanced Score Card), as a result of which, from the strategic viewpoint of the organization, objectives and also some criteria can be identified. The identification of strategic criteria is particularly important when decisions are being made about IS. The model allows working with either continuous or discrete criteria.

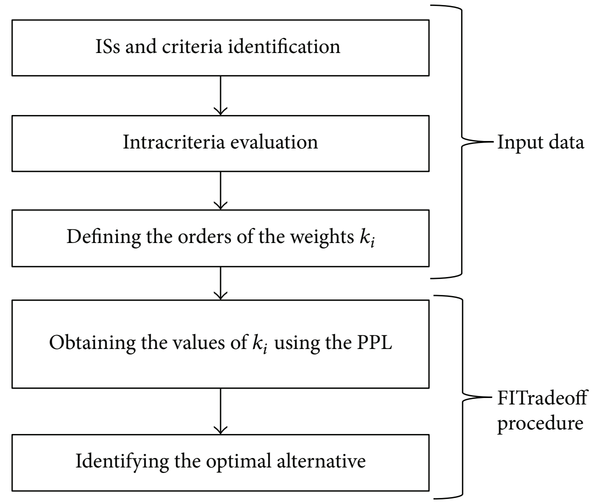

Figure 1: Model proposed for selection using FITradeoff.

Thereafter, the intracriteria are evaluated. Here, it is important to emphasize that, in the FITradeoff, consequences x_i may be dealt with monotonically by either increasing or decreasing them [18].

As explained in the previous section, the FITradeoff is divided into two parts, the ultimate goal of which is to determine the values of k_i, for which the global evaluation of the alternatives is maximized. Using the language of the DSS constructed by de Almeida et al. [18] to operationalize the FITradeoff, the aim is to find an alternative (IS) from the set of alternatives that has the maximum value given in (1) and considering the weight space given by (5). Since the DSS conducts the elicitation process in a flexible way, only essential information is required from the DM.

In the following section, the proposed model is applied in the context of IS selection and the steps used are discussed in greater detail.

4. An Application of IS Selection Using FITradeoff

The data from a factory of one of the largest manufacturers of glass packaging in the world were used to validate the model proposed. These data were used before when applying ELECTRE IV in a decision group problem presented in [19]. The biggest difference between the two models resides in the method used to evaluate the IS. In the model proposed by [19], ELECTRE IV was used to prioritize IS, so the application was conducted as a ranking problem. However, in the model proposed in this paper, an additive mode, MAVT, is used, that, by also using FITradeoff, enables the best IS according to the criteria considered to be selected. So, in this paper, the problem is regarded as a multicriteria problem about choice. To do so, the value judgment of a single DM, expert in the field, is taken into consideration. In addition, this application enables the contribution of the model in the field of IS selection to be assessed. Moreover, it is also important to evaluate this kind of problem from different perspectives as it stands to reason that these methods (ELECTRE IV and MAVT) have specific assumptions that need to be validated before they are used and also to bear in mind that their results cannot be compared.

The organization identified and analyzed, in the process of managing its IS, some IS projects but, due to lack of resources, cannot invest in them all. Therefore, it needs to select which one to invest in, first. Considering the strategic importance of the IS for the organization, they should be assessed in the light of strategic criteria, which could be obtained directly from the DM or by a method such as the BSC.

In 2011, four IS projects were identified as relevant for the manufacturer, namely,

(i) CSS: the aim of this IS is to store and manage customer's complaints in a database, thus allowing the main problems to be classified, solutions to be prioritized, and responses to clients to be generated;

(ii) CRS: it serves to create routes using digital maps that lead to delivery times and the cost of fuel and spare parts being reduced and to deploying software so that loads can be assembled quickly;

(iii) SAP: its aim is to complete the implementation of SA, thus ensuring greater agility and consistency in providing monthly reports to the Executive Board;

(iv) EL: this is needed to develop employees and to empower and train new leaders.

The criteria, the same as those used in [19], which were defined using the BSC method, are cost reduction (CR), improving functional qualifications (IFQ), improving customer relationships (ICR), improving the quality of products and services (IQPS), and improving the efficiency and control of processes (IECP).

According to the criterion CR, the alternatives are assessed considering their contribution to reducing the company's total costs, including workforce costs, transportation costs, and time savings. The aim of criterion IFQ is to assess the contribution of the system to improving functional qualifications. The contribution to improving the quality of products and services offered by the company is evaluated by using the third criterion. In the same way, the criterion IECP enables an evaluation to be made of how systems influence the efficiency gain. All the criteria are to be maximized.

In the second step of the proposed model, in which the intracriteria are evaluated, the four alternatives are assessed for all criteria using a 5-point Likert scale. In order to obtain initial references for the criteria weights, the traditional tradeoff procedure was applied. The order obtained was $k_{\mathrm{CR}} > k_{\mathrm{IFQ}} > k_{\mathrm{ICR}} > k_{\mathrm{IQPS}} > k_{\mathrm{IECP}}$. Thus, the first three steps of the DSS were run and three alternatives were identified as nondominated alternatives (CCS, CRS, and EL). This result means that more than one solution was found, assuming $v_i(x_i') = 1$ and $v_i(x_i'') = 0$ for all criteria i in the LPP model.

Then Step (4.1) is started and the DM was requested to supply information. In Step (4.2), two alternatives (A and B) were presented to the DM. The criterion "cost reduction" has the greater weight of the two criteria being analyzed, is given the value indicated for x_i, and is assumed by Alternative A. Alternative B represents the best consequence for the criterion "efficiency and control of processes," the weight of which is less than that of the criterion "cost reduction." If the DM prefers Alternative A, x_i assumes x_i'' value. If B is chosen, x_i assumes x_i' value. Also, if the DM is indifferent to the alternatives, then x_i assumes x_i^I value. Thereafter, the LPP model is run until it presents the same three nondominated alternatives.

Table 1: Cycles of the process.

Cycle	Consequence A	Consequence B	DM's answer	Nondominated
1	3 of CR	IECP	A	CSS, CRS, EL
2	3 of CR	IFC	B	CSS, CRS, EL
3	3 of IFQ	ICR	A	CSS, CRS, EL
4	4 of ICR	IECP	B	CSS, CRS, EL
5	4 of IQPS	IECP	A	CSS, CRS, EL
6	4 of CR	IFC	A	CSS, CRS
7	2 of IFQ	ICR	I	CRS

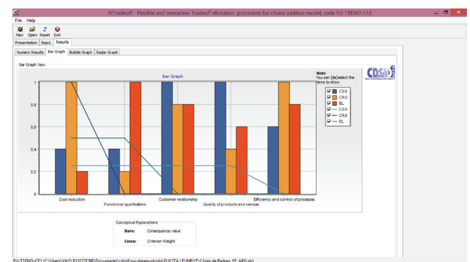

Figure 2: Result of the LPP model in the first cycle.

Figure 2 shows the result of the LPP model from the first cycle.

Finally, after seven cycles, the CRS system is identified as the unique solution. The partial results from the DSS are presented in Table 1.

For each output of the LPP model, a bar graph is provided. For example, the bar graph presented after the sixth cycle is illustrated in Figure 3.

To exemplify how the consequences are presented to the DM in each cycle, Figure 4 illustrates the question that the DSS runs in the seventh cycle.

The great advantage of this procedure is its flexibility with regard to analyzing the alternatives. It does so by producing bar graphs during the process. These display the weights of the criteria which are identified according to the nondominated alternatives. The DM's answer in the seventh cycle, for example, was based on information from Figure 4.

The DSS of FITradeoff was applied using the software available, on request, at http://fitradeoff.org/.

4.1. Analyses of the Results. Alternative CRS has the best performance for criteria CR and IECP, which is highlighted by the orange bars in the graph of Figure 3, and alternative

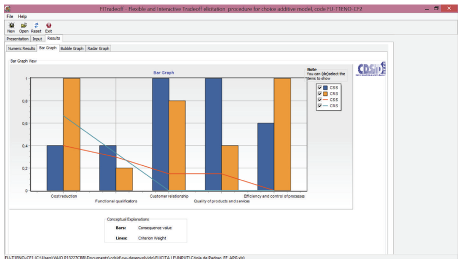

Figure 3: Result of the LPP model after the sixth cycle.

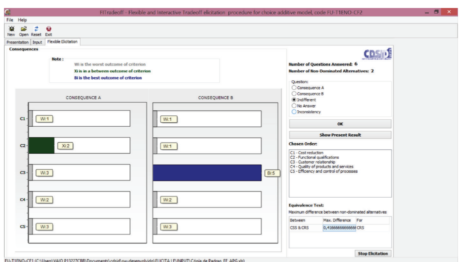

Figure 4: The performance of consequences A and B in the seventh cycle.

CCS has the best performance for the criteria ICR and IQPS, which is highlighted by the blue bars in the graph. Thus these alternatives persist for several cycles as nondominated alternatives. In fact these alternatives have a balanced global performance, which makes them preferred in an additive model such as MAVT. In [19], the application made using ELECTRE IV presented SAP as the alternative to be prioritized. The difference between these results was expected not only due to the fact that the alternatives that present a balanced performance are prioritized in additive models, but also because the application presented in [19] was conducted by a decision-making group while the application presented here takes into account the value of judgment of a single DM.

Also, although the problem was not solved in the first three steps and the subsequent steps needed to be undertaken, it was nevertheless possible to reach the best alternative setting in a few cycles. This corroborates that the goal of the method had been achieved, which is to minimize the effort that the DM requires to make.

5. Conclusions

Investments in IS are strategic for many organizations. Thus decision in this area requires the evaluation of both quantitative and qualitative criteria. Therefore, the use of multicriteria decision models was proposed to support the DM or a group of DMs in selecting IS that have the potential to contribute most to an organization's business.

In this paper a model, based on the FITradeoff method, is proposed. It supports defining which is the best IS to invest in, from a set of alternatives and a set of strategic criteria. The model is validated using data from a glass packaging factory, which identified some IS relevant in 2011. However, due to the lack of resources, it needed to identify the most critical IS.

For the application, the DSS of the FITradeoff is used. This DSS interacts easily and flexibly with the DM. The bar graphs provided by the software also enable the DM to analyze the performance of the alternatives and weights in each cycle.

The results obtained are consistent with input data and the range of the criteria weights obtained initially with the traditional tradeoff procedure. The effort required and the time spent on this by the DM are consistent with the objectives targeted by the method.

Thus, the FITradeoff consists of a more sophisticated additive model that uses a more sophisticated approach to conduct an evaluation that uses multicriteria, as in the case of IS assessment. The need to compile portfolios from the alternatives, in this kind of problem, may well prompt future research into how to improve this method.

Competing Interests

The authors declare that they have no competing interests.

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Research Article

A Classification Model to Evaluate the Security Level in a City Based on GIS-MCDA

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The aim of this paper is to map the most favorable locations for the occurrence of robberies in the Brazilian city through the multicriteria method Dominance-Based Rough Set Approach. Considering the city divisions with alternatives and evaluating by several spatial criteria, a decision-maker is building a preference model with based previous knowledge. Next, decision rules induced from preference information are introduced to the spatial environment to get the results. The decision rules can be seen as conditional part (represented by criteria) and decision part (assignment to decision classes). The rules classify all the alternatives according to security level. Moreover, the rules help to understand the social dynamics of the city and to assist in the proposition of strategies against violence.

1. Introduction

The issues of public safety and violence are often discussed because they directly affect each person living in society. In general, understanding and explaining the occurrence of violence require significant efforts to collect information on the issue. Information such as crime rates and socioeconomic variables associated with the population is important for the development of new research [1, 2]. Then the data can be used to create strategic options that will help to combat violence.

For Andresen [3], field studies of violence require more than discrete data. It is necessary to assess the evolution of violence over time and space for decision-making in public security. According to Elmes and Roedl [4], the Geographic Information System (GIS) is an important tool to support this decision-making process and formulate strategies to combat crime.

In the literature, the use of GIS in the field of criminality has already been reported in several different contexts. Various studies show themes and applications such as the identification of crime spatial patterns [4], spatial diversity of crimes [5], spatial correlation between crime and inequality [2, 6], and simulation and agent-based models for exploring crime patterns [7]. However, our intention is to provide an alternative technique, using GIS within the context of crime. This technique explores several factors that can help to understand violence.

Therefore, by evaluating different areas as alternatives and the impact of the multiple criteria with respect to violence, we used Multicriteria Decision-Making (MCDM). The primary objective of MCDM is to assist a decision-maker (DM) in choosing, ordering, or sorting a given set of two or more alternative criteria [8]. Moreover, several studies demonstrate the importance of the MCDM in many research fields [9–11]. In our case, we also considered the features of the spatial information supported by GIS. In the literature, there are very few studies of the combined use of MCDM and GIS to criminality. Gurgel and Mota [12] presented a GIS-MCDM model to prioritize regions for allocation resources considering several criteria; and in [13] a multicriteria approach was proposed aimed at setting police patrol sectors.

The focus of this study is to build a GIS-MCDM model to assess the level of security (increase in crime) in a city. The present study can be divided into two main contributions. First, we discuss the classification of the spatial alternatives to evaluate the level of security and its relationship to criminality using a GIS-MCDM approach. Second, we discuss how the results of the model can be used in the formulation of security policies and which criteria are most important.

The rest of the paper is divided as follows. Section 2 presents background information illustrating the importance of GIS-MCDM. Section 3 presents the MCDM method used in the application. In Section 4, we present a model of the problem and describe the procedures used. In Section 5, we apply and discuss the results that were obtained. In Section 6, we present our conclusions and perspectives for future studies.

2. Background on GIS-MCDM

The MCDM approach assists in building an aggregation model based on the preference information sourced by the DM [14]. For Dyer et al. [15], since the 1980s MCDM methods have been implemented in computer systems to support the decision-making process and have subsequently been called Decision Support Systems (DSS). In recent years, authors have explored MCDM methods in different applications [16–18].

Specifically, there has been a growth in the use of GIS with MCDM because of the development computerized systems and improved access by users [19]. This combination is important because spatial decision problems typically involve several alternatives assessed by conflicting multiple criteria. Also, the evaluation is conducted by a DM or group of decision-makers [20]. Afterwards the concept of GIS-based Multicriteria Decision-Making (GIS-MCDM) emerged [19].

In the recent literature, several studies have combined the decision process using GIS and MCDM such as choosing a better route for vehicles [21], identifying sustainable sites for environmental conservation [20, 22], sorting regions to implement energy mixes [23], allocating industries [24], and suitable land use [25].

We also observed that authors have evaluated specific combination MCDM methods with GIS. The adaptation of GIS-MCDM needs to create a synergy that might facilitate the aggregation of information. In this sense, [20, 26, 27] have presented studies using the importance of scales to assess alternatives based on DM preferences or situations involving uncertainty.

On the contrary, authors have shown more traditional methods including both compensatory and noncompensatory methods. In the compensatory methods, AHP was integrated into the GIS environment [28, 29]. Noncompensatory methods present practical applications with ELECTRE and other outranking methods [22, 30]. Also, there are software packages to facilitate the generation of recommendations to decision-makers [30].

Thus, the possibilities are extensive for combining the GIS-MCDM approach to support decision-making processes that directly involve the spatial use. However, there are a few studies that use GIS-MCDM for the public security field [12, 13]. Thus, the gap related with GIS-MCDM and criminality becomes a motivator for building a model to solve specific problems in public safety.

3. DRSA Method

In this section, we present Dominance-Based Rough Set Approach (DRSA) method that was integrated with GIS. We choose a method that allows using the set of reference examples (real or fictitious) for aggregating the information from preference obtained with the DM. Thereafter, each reference alternative of the set is allocated in preordered classes [31, 32]. To arrive in results, the DRSA method consider the preference model in the form of a set of "IF…THEN…" decision rules discovered from the data by inductive learning [14, 32].

Highlighted DRSA method, the absence in weights, and preference thresholds used by DM avoiding a high cognitive effort are required. The reference examples are used as input to get DM's preference information. Moreover, there is an interactive construction between the DM and the analyst. The rules are transparent and easy to interpret for the DM and give arguments to justify and explain the decision.

3.1. Notations Used. Let set of alternatives be finite, discrete, and nonempty $n, A = \{a_1, a_2, \dots, a_n\}$.

Let a finite, discrete, and nonempty set of alternatives $A^* = \{a_1^*, \dots, a_k^*, \dots\}$ assuming $A^* \subseteq A$, called the set of reference examples where the DM wishes to express his/her preferences for a given problem.

Also, let a collection of finite and nonempty set of criteria $m, C = \{c_1, c_2, \dots, c_m\}$, and each alternative has an evaluation criterion $c_m(a_k^*)$ for all $a_k^* \in A^*$. Thus, for two alternatives a_1^* and $a_2^* \in A^*$, we have $c_m(a_1^*) \gtrsim c_m(a_2^*)$ which means that "a_1^* is at least as good in relation to a_2^* when compared with criteria c_m", representing a weak preference relation between both alternatives pairs [32]. We also assume that these criteria are preference ordered with two types: cost criteria (the smaller the better) and gain criteria (the greater the better).

In addition $\mathrm{Cl} = \{\mathrm{Cl}_t,\ t \in T\}$, with $T = \{1, \dots, j\}$, such that $a_1^* \in A^*$ must belong to one and only one class $\mathrm{Cl}_t \in \mathrm{Cl}$. Each class is called decision class. Assuming too that these classes are ordered for all and any r and $s \in T$, such as $r > s$, the actions included in Cl_r are preferred over the actions contained in Cl_s. The sets to be approximated are called upward and downward unions of decision classes, respectively (see (1) and (2)). Consider

$$\mathrm{Cl}_t^{\geq} = \bigcup_{s \geq t} \mathrm{Cl}_s, \tag{1}$$

$$\mathrm{Cl}_t^{\leq} = \bigcup_{s \leq t} \mathrm{Cl}_s, \quad t = 1, \dots, j. \tag{2}$$

It is assumed that for each evaluation of the alternatives with respect to criteria having a strictly monotonicity relationship with decision class, we can define the dominance relation according to [32]. Let $P \subseteq C$ be a subset of condition criteria; we can say that a_1^* *dominates* a_2^* in the condition criteria space (denoted by $a_1^* D_p a_2^*$) if $a_1^* \gtrsim a_2^*\ \forall c \in P$. Assuming, without loss of generality, that the domains of the criteria are numerical and that they are ordered so that the preference increases with the value, we can say that $a_1^* D_p a_2^*$

is equivalent to $a_1^* \geq a_2^* \ \forall c \in P$, $P \subseteq C$. The analogous definition holds in the decision class space [32].

In DRSA, the granules of knowledge used for approximation are dominance cones that are defined as follows in objects that are dominating and dominated, respectively, with respect to P:

$$D_P^+ a_1^* = \{a_2^* \in A^* : a_2^* D_P a_1^*\}, \tag{3}$$

$$D_P^- a_1^* = \{a_2^* \in A^* : a_1^* D_P a_2^*\}. \tag{4}$$

Finally, the upper and lower approximations of unions of decision classes with respect to P are calculated as follows:

(i) The P-upper approximation of $\mathrm{Cl}_t^{\geq}$: $\overline{P}(\mathrm{Cl}_t^{\geq}) = \{a_1^* \in A^* : D_P^-(a_1^*) \cap \mathrm{Cl}_t^{\geq} \neq \emptyset\}$.

(ii) The P-lower approximation of $\mathrm{Cl}_t^{\geq}$: $\underline{P}(\mathrm{Cl}_t^{\geq}) = \{a_1^* \in A^* : D_P^+(a_1^*) \subseteq \mathrm{Cl}_t^{\geq}\}$.

(iii) The P-upper approximation of $\mathrm{Cl}_t^{\leq}$: $\overline{P}(\mathrm{Cl}_t^{\leq}) = \{a_1^* \in A^* : D_P^+(a_1^*) \cap \mathrm{Cl}_t^{\leq} \neq \emptyset\}$.

(iv) The P-lower approximation of $\mathrm{Cl}_t^{\leq}$: $\underline{P}(\mathrm{Cl}_t^{\leq}) = \{a_1^* \in A^* : D_P^-(a_1^*) \subseteq \mathrm{Cl}_t^{\leq}\}$.

Finally, the P-boundaries (doubtful regions) of the unions $\mathrm{Cl}_t^{\geq}$ and $\mathrm{Cl}_t^{\leq}$ are defined, respectively, as follows:

$$Bn_P\left(\mathrm{Cl}_t^{\geq}\right) = \overline{P}\left(\mathrm{Cl}_t^{\geq}\right) - \underline{P}\left(\mathrm{Cl}_t^{\geq}\right), \tag{5}$$

$$Bn_P\left(\mathrm{Cl}_t^{\leq}\right) = \overline{P}\left(\mathrm{Cl}_t^{\leq}\right) - \underline{P}\left(\mathrm{Cl}_t^{\leq}\right). \tag{6}$$

To evaluate the results using the sample of the reference examples, the DRSA apply the accuracy of approximation. For any $t \in T$ and for any $P \subseteq C$ the accuracy is defined as $\mathrm{Cl}_t^{\geq}$ and $\mathrm{Cl}_t^{\leq}$ by P as the respective ratios (see (7) and (8)). Consider

$$\alpha_P\left(\mathrm{Cl}_t^{\geq}\right) = \frac{\mathrm{card}\left(\underline{P}\left(\mathrm{Cl}_t^{\geq}\right)\right)}{\mathrm{card}\left(\overline{P}\left(\mathrm{Cl}_t^{\geq}\right)\right)}, \tag{7}$$

$$\alpha_P\left(\mathrm{Cl}_t^{\leq}\right) = \frac{\mathrm{card}\left(\underline{P}\left(\mathrm{Cl}_t^{\leq}\right)\right)}{\mathrm{card}\left(\overline{P}\left(\mathrm{Cl}_t^{\leq}\right)\right)}. \tag{8}$$

From the accuracy approximation we can obtaining the quality approximation (see (9)). It expresses the ration of all P-correctly sorted reference examples to all reference examples in the table. For every minimal $P \subseteq C$ we define such that $\gamma_P(\mathrm{Cl}) = \gamma_C(\mathrm{Cl})$ is called a reduct of Cl and denoted by $\mathrm{RED}_{\mathrm{Cl}}(P)$. The intersection of all of the reducts is called the core and denoted by $\mathrm{CORE}_{\mathrm{Cl}}$:

$$\begin{aligned}\gamma_P(\mathrm{Cl}) &= \frac{\mathrm{card}\left(A^* - \left(\bigcup_{t\in T} Bn_P\left(\mathrm{Cl}_t^{\leq}\right)\right)\right)}{\mathrm{card}\left(A^*\right)} \\ &= \frac{\mathrm{card}\left(A^* - \left(\bigcup_{t\in T} Bn_P\left(\mathrm{Cl}_t^{\geq}\right)\right)\right)}{\mathrm{card}\left(A^*\right)}.\end{aligned} \tag{9}$$

The decision rules are the final of the DRSA method and are divided in two parts: condition and decision, where the condition part specifies the values assumed by one or more criteria and the decision part specifies an assignment to one decision class [33].

4. Development of GIS-MCDM Model for Public Safety

The present study shows the usefulness of the GIS-MCDM approach, using DRSA method. In this the section we expose the steps of the GIS-MCDM model for public safety and application performed on a real problem.

4.1. Steps of the GIS-MCDM. The integration between the DRSA method with spatial data is made in two systems. All evaluations for choosing the reference examples are prepared in a GIS environment, which avoids the decision table to realize the same procedure. Moreover, the visualization of data becomes better understood by the DM. However, to execute DRSA, we used the free software called jMAF (available at http://idss.cs.put.poznan.pl/).

The construction of the model comprises two integrated processes. The first is the selection of the reference examples using the maps, which contains the numeric values for each criterion (layers). Each layer is a set of alternatives evaluated by one criterion. The DM chooses the same subset of alternatives considering all criteria. Next, each alternative is allocated to only one predefined class. These procedures are performed in ArcGis 10.1 and exported to the second step.

In second procedure, the alternatives are evaluated for each criterion and each alternative is allocated in only one decision class [34]. Furthermore, we may get the results in relation to $\overline{P}(\mathrm{Cl}_t^{\geq})$ and $\underline{P}(\mathrm{Cl}_t^{\geq})$, as well as $\overline{P}(\mathrm{Cl}_t^{\leq})$ and $\underline{P}(\mathrm{Cl}_t^{\leq})$ for decision classes, and we may obtain the decision rules that are used to map the alternatives. Thereafter, the rules that are exported come back to ArcGis 10.1 and are implemented on the Python environment to classify all the alternatives. The decision rules are divided in the condition criteria part (IF) and decision classes part (THEN). This permits the interactive decision process to be with the DM. Because if the DM does not agree with the results, he/she can change the $A^* \subseteq A$ set. Figure 1 shows the flowchart with the procedures.

5. An Application with Real Data

In this section, we present the results of the application using real data. As follows, the model is performed with other subsets of the reference examples. Finally, we bring a discussion about the impact on the security policies.

5.1. Results of the Model GIS-MCDM. We performed an application using real data in the city of Recife, the state capital of Pernambuco, the second most populous city in Northeastern Brazil. According to Ratton et al. [35], Recife was very violent, but the violence decreased because of to the Pact for Life program, which was established in 2007 by the Government of Pernambuco. The Pact for Life program aimed to suppress the violence that plagued the state by using laws to punish crime and strategies to the violence. However,

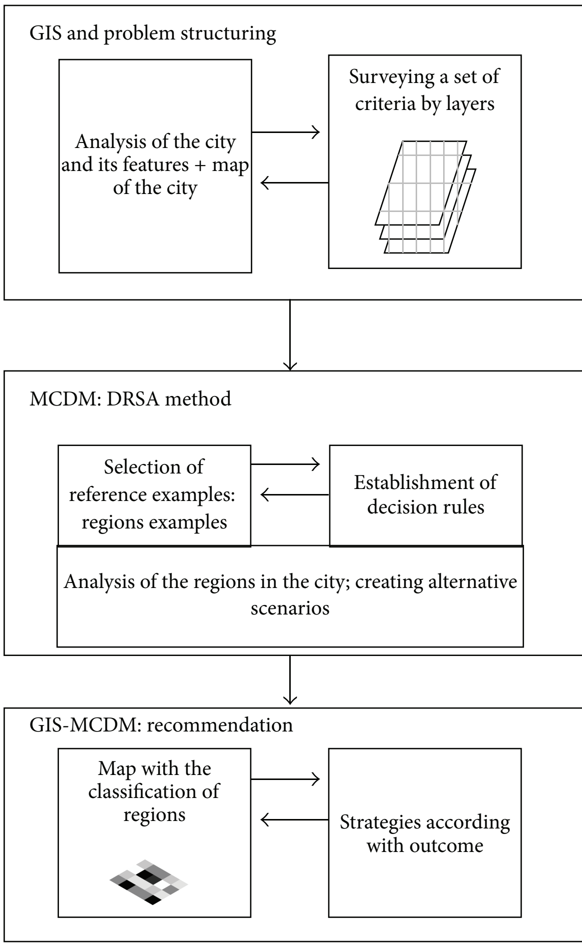


Figure 1: Flowchart used in the construction of the GIS-MCDM model.

there are still many challenges to be overcome in the program to improve it before it can fully benefit society [35].

Recife city has a spatial division called the Human Development Units (HDUs). Each unit is an alternative and is evaluated according to the criteria used in this problem. The territory that was used to perform this application is composed of 62 alternatives. Figure 2 shows the localization of Recife city.

To apply the proposed approach, different criteria were built with the base in factors that have been indicated by authors of specialized literature that explain how there are the increases in violence. In our case, these criteria explain how the increase or decrease in the occurrence influence robbery. About the issue, Andresen [3] and Levitt [36] provided detailed discussion of how related factors such as unemployment and low income affect the occurrence of crimes. For instance, a person who had no chance of working becomes motivated to commit crime in terms of disadvantaged conditions in which he or she lives. On the other hand, Fajnzylber et al. [37] and Frank et al. [5] state that there are many different factors that influence the occurrence of robbery, such as socioeconomic factors, physical environment, and demographic density, and that these factors end up being disaggregated into various peices of information related to these features.

We also check that the issue of crime is treated directly within spatial context [4, 6]; therefore, there is a motivation to use the factors that can be used to map areas that are more or less secure. In this sense, there are studies that aggregate information about the occurrence of crime to evaluate the safety level as expressed above. However, there is no preference aggregation of the DM or other criteria built by DM, with [12, 13] being an exception. Consequently, this study serves to identify areas where there are more crimes. Then, we select the approach GIS-MCDM to create results-based preference model.

Based on these factors exposed, we raised criteria that can be taken into account to evaluate the city in relation to the safety level. In our case, the criteria have a relation with robbery and are presenting a preference ordered. For the city of Recife, we consider a total of five criteria that influence the occurrence of this type of crime; these are described in Table 1, and the 2005 Atlas of Human Development in Recife was used as the data source.

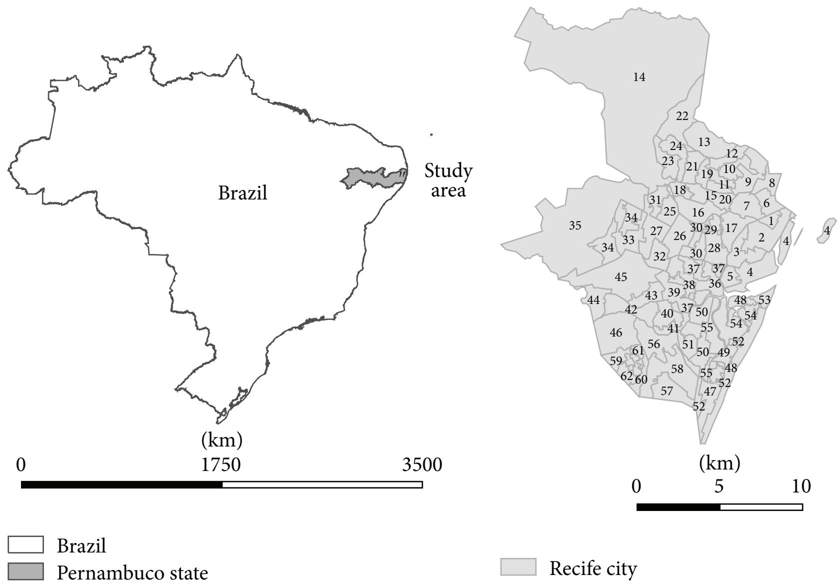
When applying MCDM methods there is a difficulty in determining the contribution of each criterion in the problem (or even the relative importance). Therefore, in the DRSA method, this step of inserting information concerning the criteria is not used, and every criterion has equal importance. However, as this is a classification problem, it is necessary to assign each class a level of preference, and, in our case, this reflects the level of safety with which it is associated.

Given the criteria, five classes (Cl) were determined according to the preferences of the DM: $\mathrm{Cl}_{\text{Very high}} > \mathrm{Cl}_{\text{High}} > \mathrm{Cl}_{\text{Moderate}} > \mathrm{Cl}_{\text{Low}} > \mathrm{Cl}_{\text{Very low}}$. Thus $\mathrm{Cl}_{\text{Very high}}$ is a place with a low incidence of robberies and a very high level of security while $\mathrm{Cl}_{\text{Very low}}$ is a place with a high incidence of robberies and a very low level of security.

The exploration of the map in the initial model phase enables the choice of reference examples in the map of Recife city. Each alternative has information about the criteria and is displayed on the map to DM. In Figure 3 we present alternatives evaluated by each criterion. The values were grouped by ArcGis (this function can be called Natural breaks). Next, the data is exported in table format (.txt). We established the initial classification decision table, where the rows contain the alternatives chosen by the DM and columns contain the criteria. A decision class evaluated by the DM is listed in the final column. In Table 2, we list reference examples that were used.

Firstly, DRSA method was applied using the data from Table 2. After the quality of the results was evaluated. According to the definition in Section 3.1, an accuracy of approximation was equal to 1 in both $\mathrm{Cl}_t^{\geq}$ and $\mathrm{Cl}_t^{\leq}$ for all the decision classes. The quality of approximation was also equal to 1. Consequently, the reference examples are suitable to obtain precise classification and have a strong ability that will be used in the classification of the other alternatives.

The outcomes reveal two reducts: $\mathrm{RED}_{\mathrm{Cl}}$ (Gini, Infrastructure, Demographic density, and Education); $\mathrm{RED}_{\mathrm{Cl}}$ (Gini, Income, Infrastructure, and Demographic density). Therefore, the $\mathrm{CORE}_{\mathrm{Cl}}$ is represented by the following criteria: Gini



(a) (b)

Figure 2: (a) Brazil and Pernambuco state and (b) Recife city with alternatives.

Income Gini index Infrastructure Education Demographic density

Figure 3: Alternatives considering each criterion.

Table 1: Criteria used with descriptive values.

Criterion	Maximum value	Minimum value	Mean	Standard deviation	Preference	Definition
Income, R$* (by person)	1863.64	86.15	378.66	400	Gain	The lower the income of the person the greater the chances of the person committing a crime
Gini index	0.72	0.40	0.52	0.05	Cost	Measuring the distribution of the income; if less, then people have equal distribution
Infrastructure (bathroom and piped water%)	99.59	48.53	83.57	12.40	Gain	The precarious condition makes the place prone to crime
Education (years)	13	4	7.36	2.36	Gain	The better education conditions decrease the chances of people getting involved in crime
Demographic density per km^2	28422	355	12390	6617	Cost	The population increase makes the environment more propitious for making off after the crime

*For each US$1.00 equal R$3.80.

Table 2: Reference examples evaluated by criteria and decision class.

HDU code	Gini	Income	Infrastructure	Demographic density per km^2	Education	Decision class
16	0.50	1353.42	94.74	6436	11.73	1
19	0.47	126.0	75.00	23956	5.51	1
21	0.45	141.47	78.81	18506	5.97	1
15	0.52	902.38	97.31	10888	10.58	2
20	0.47	158.00	78.00	28220	6.12	2
7	0.50	893.13	94.45	8796	11.09	3
35	0.55	187.10	78.24	1516	6.55	3
45	0.50	143.00	89.00	1930	5.52	3
2	0.50	616.00	98.00	3927	10.27	4
3	0.50	868.60	99.00	6577	11.20	4
48	0.61	1,864.00	96.00	9887	11.77	4
4	0.60	169.00	67.00	1893	5.64	5
50	0.47	571.00	95.00	6739	10.09	5

Table 3: Decision rules generated by the DOMLEM.

Rule ID	Rule description	Class	Number of supporting objects
Rule 1	IF (gini ≤ 0.47) THEN	At least very high	2
Rule 2	IF (inc ≥ 1353.42) THEN	At least very high	1
Rule 3	IF (gini ≤ 0.47) THEN	At least high	3
Rule 4	IF (inc ≥ 902.38) THEN	At least high	2
Rule 5	IF (inc ≥ 893.13) THEN	At least moderate	3
Rule 6	IF (demog ≤ 1516) THEN	At least moderate	1
Rule 7	IF (gini ≤ 0.50) & (demog ≤ 1930) THEN	At least moderate	1
Rule 8	IF (edu ≥ 10.27) THEN	At least low	6
Rule 9	IF (infr ≥ 89) & (demog ≤ 1930) THEN	At least low	1
Rule 10	IF (infr ≤ 67.00) THEN	At most very low	1
Rule 11	IF (gini ≥ 0.47) & (demog ≥ 6739) & (edu ≤ 10.09) THEN	At most very low	1
Rule 12	IF (gini ≥ 0.60) THEN	At most low	2
Rule 13	IF (gini ≥ 0.50) & (inc ≤ 868.6) & (demog ≥ 3927) THEN	At most low	3
Rule 14	IF (gini ≥ 0.47) & (inc ≤ 571.00) & (demog ≥ 6739) THEN	At most low	2
Rule 15	IF (gini ≥ 0.47) & (inc ≤ 893.13) THEN	At most moderate	8
Rule 16	IF (gini ≥ 0.50) THEN	At most high	6
Rule 17	IF (demog ≥ 28220) THEN	At most high	1

Index, Infrastructure, and Demographic density. These are the three criteria that are adequate to explain the decision, according to the DRSA method.

Then, we performed the jMAF system to create the decision rules by the DOMLEM algorithm. Using the algorithm we can obtain 17 deterministic certain decision rules from 13 reference examples in total. Those decision rules represent the certain knowledge. The certain rules are originated from P-lower approximation of $\underline{P}(\mathrm{Cl}_t^{\geq})$ and $\underline{P}(\mathrm{Cl}_t^{\leq})$ of the union class. The decision rules can be implanted directly into a GIS environment to generate the results to all the alternatives and those presented in Table 3.

Given the decision rules, we can compare the remaining alternatives that do not belong in Table 2. The results are presented in the form of map, where the darker alternatives need more attention, because they have a lower level of security. In Figure 4 the following are the classification in the all alternatives for the Recife city.

Therefore, we can draw some conclusions about the results:

(i) HDU 52 is classified as *at most very low*. Therefore, it is a place with high chances for robbery occurrence. When we compare criteria evaluations with Rules 10 and 11, we can see that the alternative complies with both rules. Also, HDU 52 has a Gini Index equal to 0.72, demonstrating a high social inequality among its inhabitants.

(ii) The alternatives 10 and 11 are encircled by other alternatives that were classified as being more prone

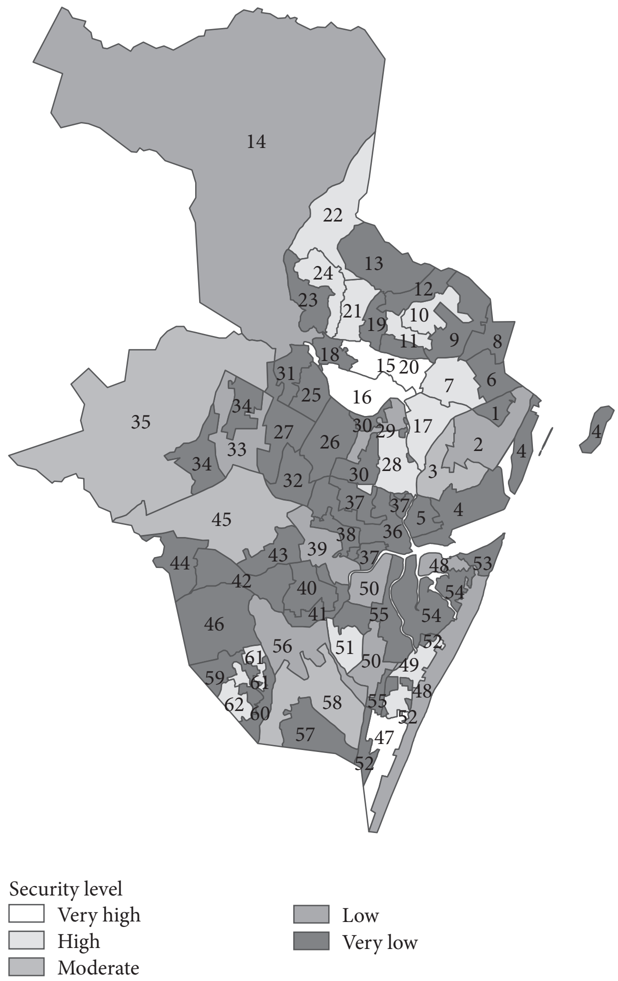


Figure 4: The classification using GIS-MCDM model for security public.

to robbery, because the criminal proliferation varies over space and time. Some alternatives fit certain rules while other alternatives are explained by other kinds of rules.

(iii) The spatial proximity of the alternatives is also highlighted. The HDUs that present darker color are neighboring other alternatives with the same color, because of the proximity along the criteria values.

5.2. Discussion. Let us now consider the change in the reference examples as an option for DM and discuss the results in the case involving a security problem. Each alternative is described in terms of the decision rules and rules have both representation and recommendation tasks. Then, we modified the data presented in Table 2 resulting in new decision rules, but these alterations maintain the readiness to interpret the results according to the DRSA method [14]. We also kept the same number of the reference examples. Either some examples that belong in Table 2 were changed by new examples or they just altered the class. Consequently, we want to check the impact presented by the new results.

The most important argument for the proposal of several rules is the fact that these rules may be applied in different results. However, it is important to discover if the alternatives swapped their security level. For instance, HDU 33 was allocated in Cl_{Low}, as can be seen in Figure 4, which takes into account Rules 8, 12, and 13. After the new application of the model, this alternative was assigned to $Cl_{Very\ low}$ class. Now, the rules that are classified in Cl_{Low} modified their values and HDU 33 complies with condition rules that classify $Cl_{Very\ low}$. On the contrary, for HDUs 17 and 28, both changed from Cl_{High} to $Cl_{Very\ high}$ class. Now, these alternatives can be considered of high level of security. The results are presented in Figure 5, which shows that some alternatives changed to new classes.

Moreover, the other remarks consider the decision rules and the remaining results. In the second result, there was only reduct and only $CORE_{Cl}$ (Gini, Infrastructure, and Demographic density). The number of rules decreased to 15, while the quality and accuracy were kept with the same values. Therefore, the examples were also suitable to explain all the data of the problem. That decision rule highlighted other combinations. For instance, IF (gini $\geq$ 0.47) & (inc $\leq$ 893.13) THEN *At most Moderate* passed to IF (income $\geq$ 893.13) & (demog $\leq$ 1516) THEN *At most Moderate*.

The second part of the section is about the problem of public security and GIS-MCDM. Reference examples in which the DM is given options by MCDM models have been widely reported [31, 38–40]. A multicriteria approach with predefined information enables the DM to know which examples give him/her the opportunity to be more secure. In our case, using the map is most favorable because the DM does not look for alternatives in a table which may represent your preference information. We also avoid other MCDM methods that use weights and preference thresholds to aggregate the preference.

The results of this study can be used to propose strategies to help the police and to enhance public security. The DM may be interested either in increasing police effectiveness or in planning public polices for improvements in certain regions (i.e., a specific set of the HDU). In the second situation, the DM may have a focus on building new schools and developing infrastructure. However, such situation would involve more than one DM, implying the problem of aggregation of group preferences; this situation is not investigated in this paper.

The criteria that are socioeconomic indicators show the discrimination of the values with respect to the security level. Alternatives allocated into Cl_{Low} have a mean year of education of 6.47 and 6.97 for first and second results for the Education criteria, respectively. However, HDUs classified in Cl_{High} and $Cl_{Very\ high}$ have a mean year of education of 8.63. These mean years present the following results: places more prone to robbery have citizens with little education. The same situation occurs for the criteria: Gini index, Income, and Infrastructure. HDUs with security decreased also present adverse values with relation to all the alternatives (Table 1). The most interesting is the Gini index (means of 0.54 for the alternatives allocated in $Cl_{Very\ low}$) because it serves as inequality indicator.

Figure 5: Comparing two results: (a) first results and (b) results using other reference examples.

Another interesting point is to observe how a modification in the reference examples may affect the result. In other words, the DM might want to check how the preference information affects the results through a more general outcome. With respect to the DRSA method, a HDU passing from one class to another is because of the new decision rules. However, we may also see the sensibility of the alternatives and use the results within security strategies. For instance, an alternative that passed to a better class no longer receives attention from the Government. Instead, there was just a change into another class, which yet requires resources to establish a secure place. Therefore, the comparison between changes of the classes is an important consideration. Figure 6 shows each situation.

Still, the second results of the model allow making two conclusions: the intention of the DM to change the set of the reference examples to apply the DRSA method or the participating of more DMs. In the first case, the DM is interested in using the other examples motivated by preference information. This fact will be necessary in case he/she does not agree with the decision rule. The second case will be the participating new DM. However, there is a problem in how to aggregate the preference for a specific situation in public security.

6. Conclusion

In this paper, we discuss the use of a GIS-MCDM approach in order to get a simple and intuitive explanation of the results. We integrate the DRSA method with the GIS tool to evaluate the safety level in Recife, Brazil. The connection in GIS-MCDM was motivated because the spatial information is available, which encourages processes that select soft reference examples. Also, the construction of the model was performed in two steps: (1) selection and evaluation of the reference examples analyzed on the spatial shape using DRSA method and (2) applying the decision rules in GIS tool to generate the final recommendation.

The two contributions expected as the study's objectives were achieved. Firstly, the feasibility of adapting the GIS-MCDM system and creating a classification problem in a georeferenced environment for the final decision was determined. These results are useful for decision-making and planning to solve public security problems through a proper implementation of the results obtained, which can give a final recommendation for the current decision problem. Then, we checked the impacts caused by the reference examples chosen and they are useful in security policies. Also, alternatives

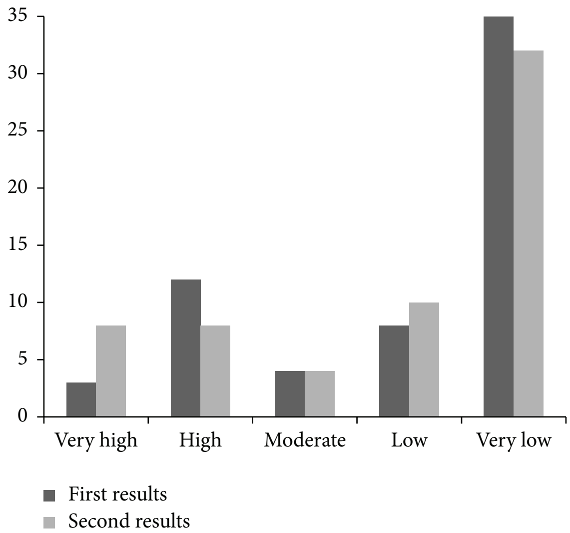


Figure 6: Distribution of HDUs class for the two results.

classified as having low level of security have the worst condition based on relation of the criteria.

Finally, the work in view of the junction between GIS and MCDM does not exhaust the possibilities of study in the field of public safety. Other studies would result in group decision-making using methodologies presented in [33, 41, 42] or adopt other multicriteria approach that use combined methods to facilitate the preference aggregation [14].

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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Research Article

A Three-Phase Multiobjective Mechanism for Selecting Retail Stores to Close

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To operate a successful and growing business, a retail store manager has to make tough decisions about selectively closing underperforming stores. In this paper, we propose using a three-phase multiobjective mechanism to help retail industry practitioners determine which stores to close. In the first phase, a geographic information system (GIS) and k-means clustering algorithm are used to divide all the stores into clusters. In the second phase, stores can be strategically selected according to the requirements of the company and the attributes of the stores. In the third phase, a neighborhood-based multiobjective genetic algorithm (NBMOGA) is utilized to determine which stores to close. To examine the effectiveness of the proposed three-phase mechanism, a variety of experiments are performed, based partly on a real dataset from a stock-list company in Taiwan. Results from the experiments show that the proposed three-phase mechanism can help efficiently decide which store locations to close. In addition, the neighborhood radius has a considerable influence on the results.

1. Introduction

Retail is one of the largest industries in the world [1, 2]. While there is still room for the retail industry to expand, retail store operators are facing some intense pressure [3]. The pressure is straightforward. As more and more competitors enter this industry, the operating profit is considerably shrunk and, thus, some operators have to close some underperforming stores and/or withdraw from the industry [3, 4].

To close retail stores, some important factors must be considered. The first important factor is profit. In order to ensure that the company can make a profit, the manager has to close some underachieving stores that no longer meet the company's requirements. The second critical factor to be considered is the distance customers need to drive or walk to get to a store. One of the most important factors affecting customer store choice is the travelling time [5]. If the distance is too long, the customer may choose alternatives from other brands. The third factor to be considered is the percentage of the company's employees who quit as a result of store closures. Employees have to change their work locations once a store is closed. Many of the employees are part-time students, who tend to find jobs near their schools. If the distance from the new work location to the school is too far, they are very likely to quit the job, which will waste a huge amount of training costs and cause instability in store operations. Thus, to reduce employee turnover rate, the closure decision should shorten the possible movement distance as much as possible. Another influencing factor is concerned with the strategy of the firm [6]. Some stores are strategically retained even if their profits are low. These stores may include, for instance, a store used for training or historical or symbolic store for the firm.

For several decades, one selection problem concerning retail stores has been focusing on selecting suitable store locations to open [7–15]. To the best of our knowledge, little literature presents any mechanism to decide which stores to close or discusses the effects of closing on the retailer's performance [16–18]. To help managers solve the problem mentioned above, this study presents a novel mechanism which consists of three phases. In the first phase, retail stores are grouped based on their geographic distribution. In the second phase, strategically desirable stores are selected and they are kept

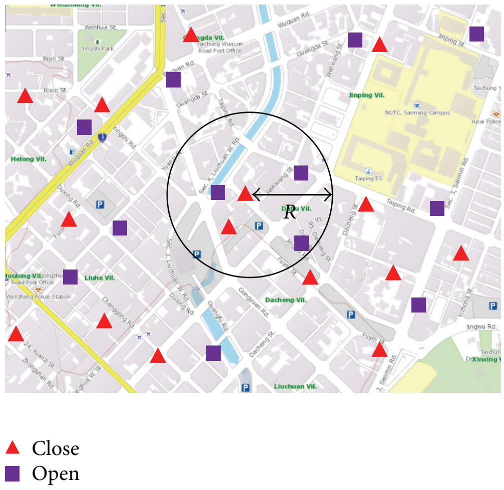


Figure 1: Nearby stores of a *selected* (to-close) store.

open. The goal of the third phase is to optimally determine which of the remaining stores in the second phase to close.

The remainder of this paper is organized as follows. In Section 2, a brief description of the store selection problem as well as modeling is presented. In Section 3, the proposed mechanism is introduced. Results and discussion are presented in Section 4. In Section 5, conclusions are drawn.

2. The Store Selection Problem

In this section, we start with a brief description and a presentation of the mathematical formulation for the problem.

For ease of description, some variables are first defined. Let $S = \{x_i \mid i = 1, \ldots, s_0\}$ be a set of retail stores, where s_0 is the total number of stores a firm possesses. Suppose that a percentage of stores have to keep open. The percentage is designated as p and $0 \le p < 1$. For example, if p is 90%, then 10% of stores are to close. For $i = 1, \ldots, s_0$, we let $x_i = 0$ if store i is selected to close. Otherwise, x_i is set to be 1. To save space, if a store is selected to close, we call it a *selected* store; otherwise, the store is kept open. Some employees working at the *selected* stores have to change their working places and they have to move a distance from the original store to the new one. Similarly, customers have to travel a distance if a store nearer to them is closed. The farther the movement distance is, the more likely it would be for an employee to quit the job or for a customer to select alternatives from other brands. To evaluate the total movement distance of store i caused by closure, we define nearby stores of a *selected* store i (as the center) as those stores which are located within a circle region with a radius R, as shown in Figure 1. Since several stores may exist within the neighborhood of store i, we designate N_{ij} as the jth nearby store of store i. If the jth nearby store of store i is a *selected* one, $N_{ij} = 0$; otherwise, $N_{ij} = 1$. The total number of nearby stores for store i is n_i.

There are two objectives for store closure decisions. The first objective is to minimize the total profit loss, which is defined as the profit difference before and after closure. Note that the loss is based on profit rather than revenue. An increase in revenue may cause a loss in profit because of an increased cost. Consequently, a profit-based objective is a more feasible one for closure decisions. On the other hand, to decrease sales loss caused by lost customers and to reduce the rate at which employees quit their jobs, the total distance from the *selected* store to its nearby stores is minimized. The mathematical formulation of this problem can thus be given by the following.

Objectives are

$$\begin{aligned} &\text{Minimize} \quad Z_1 = \sum_{i=1}^{s_0} (V_i - C_i)(1 - x_i) \\ &\text{Minimize} \quad Z_2 = \sum_{i=1}^{s_0} \sum_{j=1}^{n_i} (1 - x_i) d_{ij} (1 - N_{ij}), \end{aligned} \tag{1}$$

subject to

$$\sum_{i=1}^{s_0} x_i = \begin{cases} s_0 p & \text{if } s_0 p \text{ is an integer,} \\ \operatorname{Int}(s_0 p) + 1 & \text{otherwise,} \end{cases} \tag{2}$$

$$\sum_{j=1}^{n_i} N_{ij} \ge 1 \quad i = 1, \ldots, s_0, \tag{3}$$

$$x_i \in \{0, 1\} \quad i = 1, \ldots, s_0, \tag{4}$$

where V_i is the revenue of store i, C_i is the cost of store i, d_{ij} is the distance from store i to its jth nearby store, and

$$x_i = \begin{cases} 0 & \text{if store } i \text{ is } \textit{selected} \text{ to close,} \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

Equation (2) states that the total number of stores selected to open is equal to a specific number, which is equal to $s_0 p$ if $s_0 p$ is an integer, or the number is equal to $\operatorname{Int}(s_0 p) + 1$ if $s_0 p$ is not an integer. Equation (3) requires that at least one of the nearby stores of a *selected* store i remains open, ensuring that the customers can find at least an alternative open store within a distance R.

3. Three-Phase Mechanism

3.1. The Decision Framework. There are three basic phases for solving the selection problem, which we illustrate in Figure 2 and then discuss in more detail. In each phase, there are some influencing factors.

Phase I: Clustering Stores. When the number of stores involved is large, it is reasonable for an operator to divide all the retail stores into several clusters, based on the objectives of the company. For example, Seven-Eleven Japan builds a cluster of around 50 to 60 stores supported by a distribution center. Such clustering allows Seven-Eleven Japan to operate an efficient system [20]. One commonly used clustering scheme is based on location. Stores located in close proximity to each other are aggregated, using a geographic information system (GIS) [21, 22] or other clustering techniques [23].

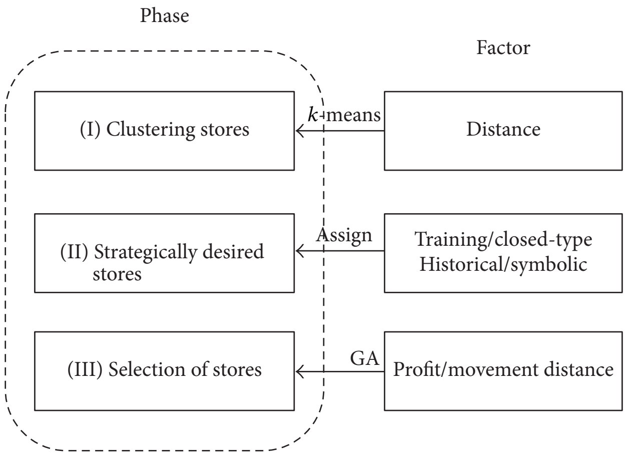


Figure 2: A three-phase decision framework.

Table 1: The values of x_i for the six stores in the example. For reasons for open/close decisions, please refer to Phase II, Section 3.1.

Store	1 ($i = 1$)	2 ($i = 2$)	3 ($i = 3$)	4 ($i = 4$)	5 ($i = 5$)	6 ($i = 6$)
Reason	(3)	(1)	(4)	(4)	(2)	(3)
x_i	1	1	1	0	1	1

Among the popular clustering algorithms, k-means [24, 25] is one of the simplest techniques.

Phase II: Strategically Desired Stores. In this phase, some stores are strategically kept. This phase aims to achieve the strategic objectives of the firm. An additional benefit of this phase is the reduction in the amount of computational time. The higher the number of stores, the more the computational time needed.

The possible reasons for strategic stores may include the following:

(1) Stores for training: personnel at all levels should be trained to meet the firm's needs to provide products and services of a required quality. Therefore, some stores used for training should be kept.

(2) Closed-type stores: this kind of store is located in a closed area, such as a campus or a military camp. The stores generally are not open to the public.

(3) Historic or symbolic stores: for instance, the first retail store opened in an area or a symbolic store at a famous tourist attraction will be retained.

(4) Others: this point refers to those stores subject to very strict contracts or local regulations [26].

Phase III: Selection of Stores. In this phase, optimization of store selection is achieved based on the company's objectives. The most important objective for many companies is profit. They tend to minimize the total profit deficit. Another important consideration is distance. A shorter movement distance caused by closing stores can reduce the turnover rate of employees. In addition, the customer can drive or walk to a store with a shorter distance.

In this paper, we use a genetic algorithm, which has been successfully applied to many fields [19, 27–34], to optimize store selection. The detail is depicted in the following subsection.

3.2. Method of Solution

3.2.1. Using k-Means Algorithm for Clustering in Phase I. k-means is one of the most popular clustering algorithms, in which items are moved among sets of clusters until the desired set is reached [25]. k-means is most suited for separating convex clusters [35, 36]. An additional advantage of using k-means algorithm is that we can use its centroid in a cluster as the location of the distribution center. To cluster stores using k-means, the input data, including the number of clusters k, the location coordinates of stores, and the convergence criterion, should be given. The value of k depends on the needs of the manager. Generally, k varies from three to six. In this paper, the algorithm is organized as follows:

Input:

k //Number of desired clusters
$D = \{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\}$ //Set of store points

Output:

K //Set of clusters

k-**means algorithm**:

Assign initial values for means; $m_1, m_2, \ldots, m_k$
Repeat
Assign each item (x_i, y_i) to the cluster which has the closest mean;
Calculate new mean for each cluster;
Until convergence criteria are met (All the values between two iterations are the same).

3.2.2. Deciding Strategically Desired Stores in Phase II. Some stores can be selected to remain open or to close before going to the third phase, in order to reduce computation time. For easy understanding, examples are given as follows. Suppose that a store chain has six stores to be strategically kept open or closed, as shown in Table 1. The scenarios are as below.

Store 1. This store is the first one to be opened for the store chain. This store is chosen to keep open. Thus, the decision variable x_1 is set to be 1, as shown in Table 1.

Store 2. The store is established mainly for training. This store will not be closed even if profit is low. Consequently, x_2 is set to be 1.

Store 3. This store is subjected to a strict contract. Even if the store is closed, the store owner has to pay rent. As a result, store 3 will be open in the next period.

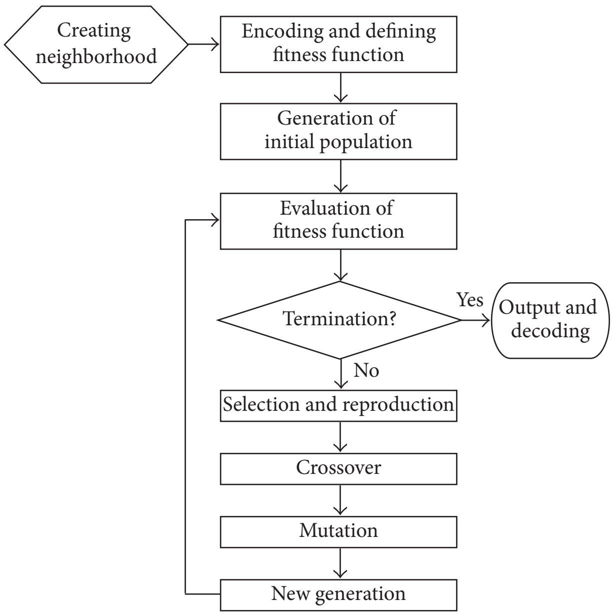


Figure 3: The flowchart for the genetic algorithm.

Store number	1	2	3	4	5	6	7	8	9	10	11	12	13	$\cdots$	s_n
Close or not	0	1	0	0	1	0	0	0	1	0	0	1	0	$\cdots$	0

Figure 4: Representation of a chromosome.

Store 4. The lease term of the store is ended and the store owner wants to terminate the lease. The store is forced to close. Therefore, $x_4 = 0$.

Store 5. This store is located within an important military region. The store is established for the sake of service and will always be kept open. Therefore, $x_5 = 1$.

Store 6. This store is located in a famous tourism region. It is chosen to remain open, no matter what the level of profit is.

A summary of the values of decision variable x_i for these six stores is shown in Table 1.

3.2.3. Using Genetic Algorithm to Optimally Select Closing Stores in Phase III. A neighborhood-based multiobjective genetic algorithm (NBMOGA) is developed to solve the selection problem. The flowchart for the GA is illustrated in Figure 3 and the details are described in the following subsections.

(1) Encoding and Initial Population. A binary encoding is employed, as shown in Figure 4. The sequence number stands for the store number. Since the number of stores is s_n after strategic selection for the nth cluster, there are s_n genes in a chromosome. The value in the gene is either 1 or 0, where 0 means the store is selected to close and 1 means the store is to be kept open.

A random method is employed to generate the initial population.

(2) Evaluation of Fitness Function. The chromosomes are evaluated based on the total profit deficit and the total movement distance. Nondominated fitness values are recorded and their corresponding solutions are chosen as the Pareto ones. In this paper, the scheme utilized to find Pareto solutions is a rank-based approach [19]. Suppose that an individual G_h at generation t is dominated by $q_h^{(t)}$ individuals in the present generation. Then, the rank of the individual at generation t can be expressed as [33]

$$\operatorname{rank}\left(G_h, t\right) = 1 + q_h^{(t)}. \tag{6}$$

All the nondominated individuals are assigned rank 1, as illustrated in Figure 5. Take chromosome A as an example. Its fitness value is $(X_{\mathrm{A}}, Y_{\mathrm{A}})$. If no other chromosomes exist within the rectangular regions $0 \le Z_1 \le X_{\mathrm{A}}$ and $0 \le Z_2 \le Y_{\mathrm{A}}$, the rank of the chromosome is assigned as 1. Likewise, the rank of chromosome B is 1, since no chromosomes are within the regions $0 \le Z_1 \le X_{\mathrm{B}}$ and $0 \le Z_2 \le Y_{\mathrm{B}}$. All the chromosomes ranked as 1 are collected into the Pareto set. The Pareto set is updated at each generation. As evolution continues, the Pareto set will gradually shift toward exploiting the nondominated points in the criteria space. The process is continued until the end condition is reached.

(3) Selection. The binary tournament selection method [28] is employed to select the fitter individuals. Tournament selection is similar to rank selection in terms of selection pressure, but it is computationally more efficient and more amenable to parallel implementation [28]. The process is as follows. First, two chromosomes are randomly chosen from the population. The fitter one is selected to become parent A and the process is repeated to select another parent B. Subsequently, the GA combines parents A and B to generate the offspring.

(4) Crossover. The uniform crossover method is employed to generate offspring. The method is to generate a mask, which has a length equal to the chromosome and is composed of a random set of binary numbers. Figure 6 illustrates the uniform crossover scheme. Where there is a "1" in the mask, the value of a gene is copied from parent A; otherwise, the gene is copied from parent B. The offspring contain a mixture of genes from each parent. For example, the value of the first cell in the mask is "0," indicating that the value of the first gene for the offspring is copied from parent B. Therefore, the value assigned will be "1." The procedure is continued until all of the values of the genes are copied.

(5) Mutation. Three genes in the chromosome are randomly selected and their values are changed, as illustrated in Figure 7. The advantage of using three-gene mutation is that it increases the diversity of chromosomes and then increases the possibility of finding optimal solutions.

(6) Adjustment. Since the percentage of open stores is required to be equal to the preassigned value p, some of the gene values should be adjusted if the percentage is different from the required value. The GA program will check the percentage after genetic operations. If the actual percentage

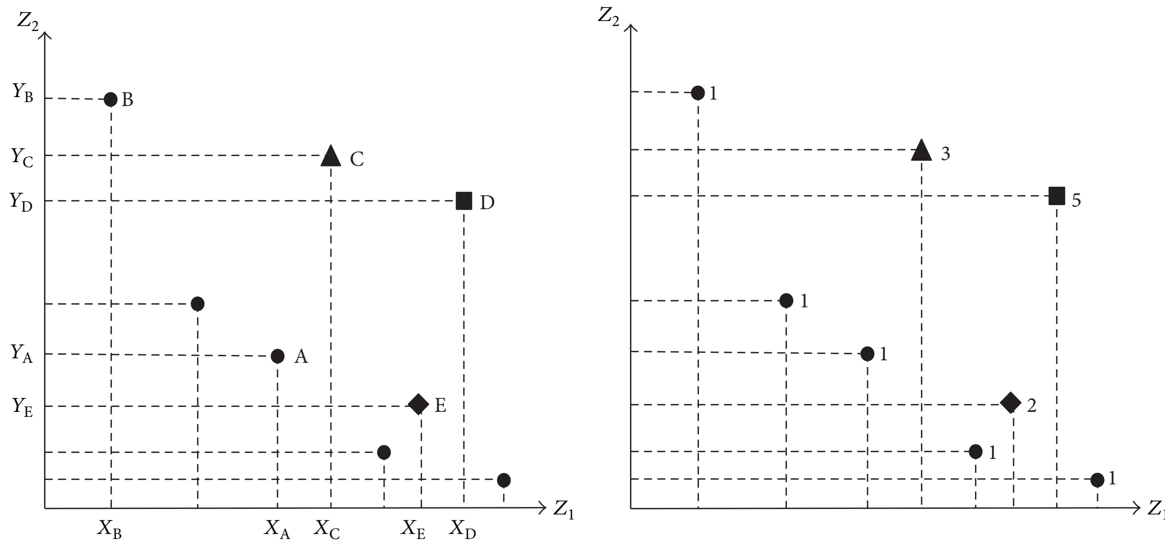


FIGURE 5: The ranking in the present scheme. All the nondominated individuals are assigned rank 1 [19].

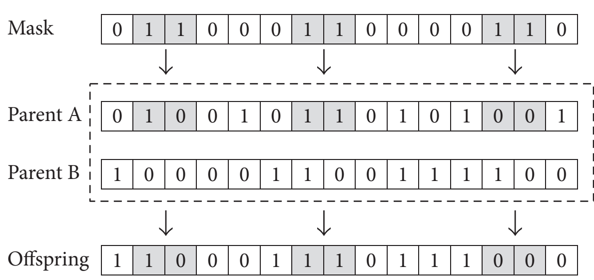


FIGURE 6: Illustration of uniform crossover.

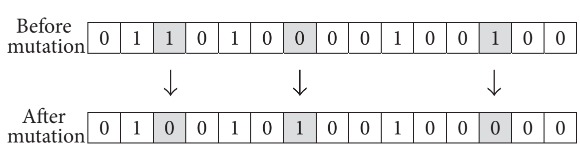


FIGURE 7: Illustration of a three-point mutation method.

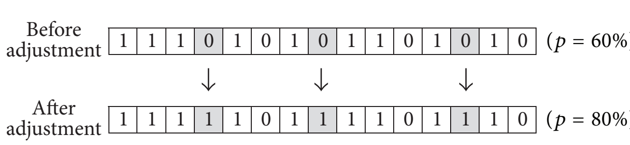


FIGURE 8: Illustration of adjustment.

exceeds p, some of the unselected stores will become *selected*, while some of the *selected* stores will become unselected, if the actual percentage is smaller than p. The process is repeated until the percentage of *selected* stores is equal to p, as shown in Figure 8.

(7) Replacement. The tournament selection method is used to replace the worst chromosomes. First, chromosome A is randomly chosen from the population of the previous generation. Then, the chromosomes in the next generation are compared with chromosome A. If there is any chromosome better than chromosome A, the replacement is done. The process is repeated until all the chromosomes in the next generation are compared.

(8) Termination. The program is run until a preassigned generation number is reached.

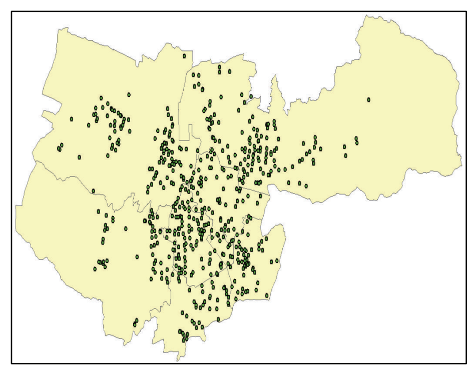
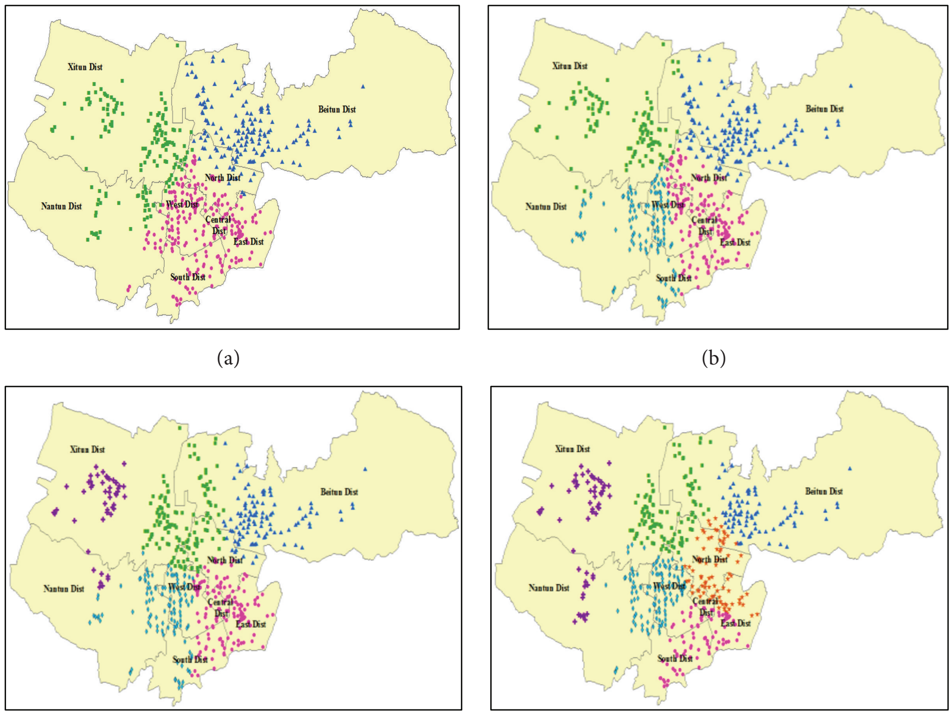


FIGURE 9: The distribution of retail stores.

4. Results and Discussion

To evaluate the effectiveness of the proposed approach, a variety of experiments were performed using a reference case. Part of the dataset is from a famous chain store sector in Taichung City, Taiwan. The reference case includes 550 stores. The distribution is depicted in Figure 9. The neighborhood radius is set to 1 km and the generation number is set to be 50,000 at the reference case.

The GA program used in this study was coded with Visual Studio C++. The program was run on an Intel(R)Core(TM)i7-2600 CPU @ 3.40 GHz with a 4.00 GB RAM. The operating system is Windows 7.



(a) (b) (c) (d)

Figure 10: (a) The results of clustering with $k = 3$. (b) The results of clustering with $k = 4$. (c) The results of clustering with $k = 5$. (d) The results of clustering with $k = 6$.

4.1. The Clustering Result. The stores were clustered using k-means algorithm [24, 25]. The number of clusters k varied from three to six. The clustering results are shown in Figure 10. In the region with lower store density, the stores are easier to cluster. However, when store density is higher, it is difficult to divide the stores in an intuitional way and, thus, an efficient tool is needed. Results from this study show that the clustering tool developed by this study can efficiently cluster stores. Combined with GIS, the clustering results are easily observed and facilitate decision-making by the manager.

4.2. The Results of Store Selection. In this paper, we use GA (NBMOGA) to optimize store selection. At the reference case, k was set to be 4. Each case was run ten times and the best results are kept and compared. The GA parameters, including generation number, mutation rate, crossover rate, and population number, were tested. A variety of experiments show that the generation number = 50,000, the mutation rate = 0.05, the crossover rate = 0.9, and the population number = 30 can obtain stable solutions.

In this paper, a rank-based scheme is employed to find Pareto solutions. We have used this scheme to solve some multiobjective problems and the results from GA are compared well with an exact algorithm [19, 31]. The results from the two previous studies [19, 31] show that the GA scheme we used in this study has good performance when it is compared with the Brutal-Force Method (BFM).

The optimized results are shown in Figures 11(a)–11(d). The points marked by green color (triangle) are stores selected to close, while those marked by red color (square) are to remain open. Since the retail network is so complex, it is very difficult to select stores for shuttering by simple rules only. Though the network is quite complex, the system developed in this paper is rather easy to use and can help the manager in efficient simulation. It requires only seconds to find solutions, even when the number of stores is as high as 550. One clear advantage of the genetic algorithm is that, by its very nature, we are able to produce a number of feasible solutions, thus facilitating discussion on the merits of various decisions and supporting multiobjective decision-making [34].

The interrelationship between stores is complex. For example, if a store is strategically retained to be open, what will its influence be on the other stores? To facilitate this issue, the GA program is designed to be easy to use. As shown on the left side of Figure 12, if a store is selected to be strategically retained, just by setting the value of the attribute in the input data to be "1," then the store will be retained by the program (see the output result on the right side of Figure 12).

4.3. The Influence of Opening Percentage. To observe the influence of the opening percentage on the solutions, the opening percentage p is changed from 70% to 95%, with an increment of 5%. R is set to be 2.5 km. Figures 13(a)–13(c) show the results. The points marked by green (triangle) are stores selected to close, while those marked by red (square) are retained as open. As the value of p is changed from 70% to 95%, more stores located in the suburbs are selected for

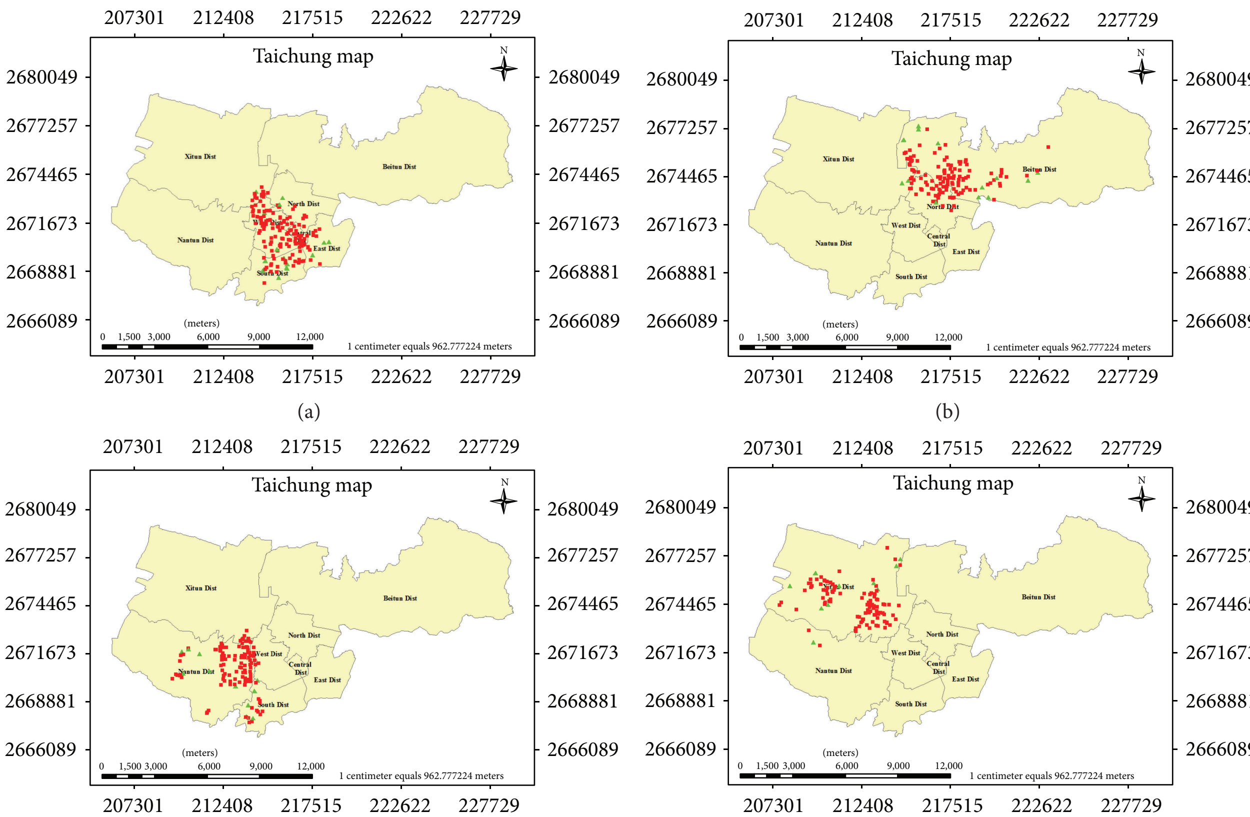


Figure 11: (a) The store selection results for cluster 1. (b) The store selection results for cluster 2. (c) The store selection results for cluster 3. (d) The store selection results for cluster 4.

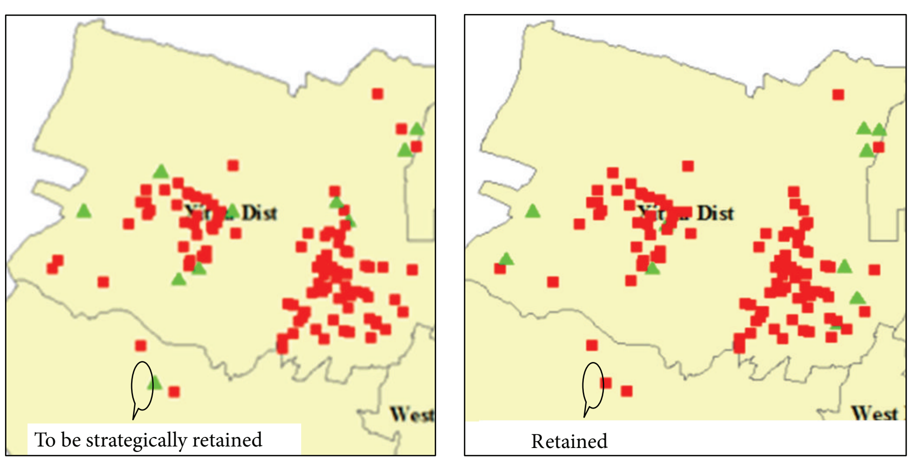


Figure 12: The result and its influence for a strategically retained store.

closure. The reason is that the revenue of the stores located in the suburbs is generally less than the revenue of those located in the city center.

To facilitate the observation of results, the Pareto solutions are presented in Figure 14. As the total movement distance increases, the total profit loss decreases. Figure 14 shows this trend. As expected, a higher value of p has a lower profit loss.

4.4. The Influence of Neighborhood Radius. The neighborhood radius R is varied from 1 km to 3 km, with an increment of 1 km to see its influence on the results. The selected

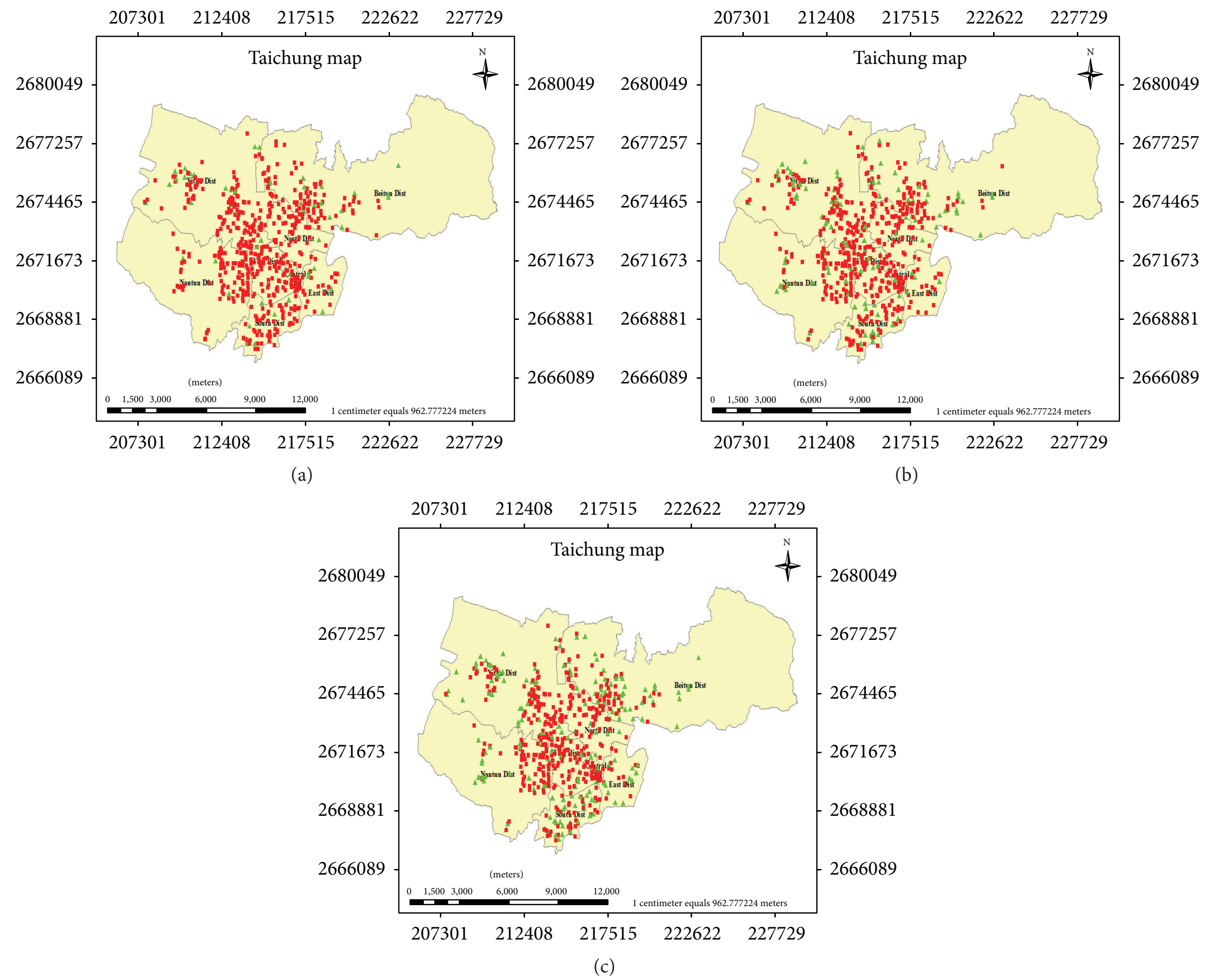


FIGURE 13: (a) The results for opening percentage $p = 90\%$. (b) The results for opening percentage $p = 80\%$. (c) The results for opening percentage $p = 70\%$.

locations where stores are to close are marked with a green triangle. As R is increased, the selected stores gradually move from the city center to the suburbs, as we can see from Figures 15(a)–15(c). This is an important finding that would help managers make the right decisions. Using the system developed in this paper, the managers can easily simulate and may effectively control their profit loss if stores are to close.

The variation of the total profit loss with the total movement distance is shown in Figure 16. As the total profit loss decreases, the total movement distance tends to increase.

Special note must be taken that a higher value of R may cause more lost customers, which, in turn, causes more lost profit. A previous survey [11] shows that when the distance increases, the percentage of willingness to walk to buy decreases, as displayed in Figure 17. When the distance is longer than 150 m, more than 20% of people are possibly not willing to walk to another store [11].

4.5. The Influence of Store Number. To investigate the influence of the number of stores on the performance of the GA program, a lot of experiments with different numbers of stores are performed. The number of stores is changed from 200 to 1,500. The results are shown in Table 2 and Figure 18. The experimental results are based on 10 trials. As the number of stores increases, the average computation time increases in a linear manner, indicating that the GA program is time-efficient. In addition, the coefficient of variation (C_v) is small, showing that the GA program is quite stable.

5. Conclusions

Retail store operators are facing intense pressure. As competitors flood the marketplace, operating profit is significantly reduced. Given such a circumstance, store operators have to think about closing some underperforming stores. In this paper, we propose a system based on a three-phase decision framework to address this issue. In the first phase, k-means algorithm and geographic information system (GIS) are first used to divide the stores into some regions. In the second phase, some stores can be strategically retained based on the

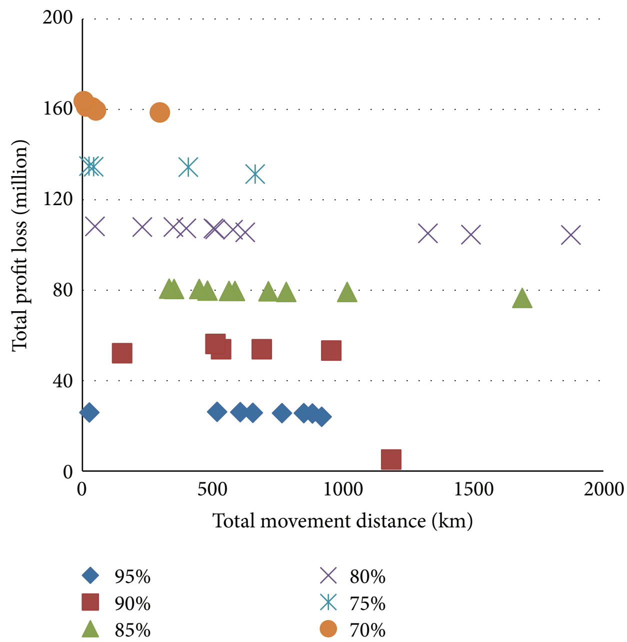


Figure 14: The variation of profit loss with total movement for different opening percentage.

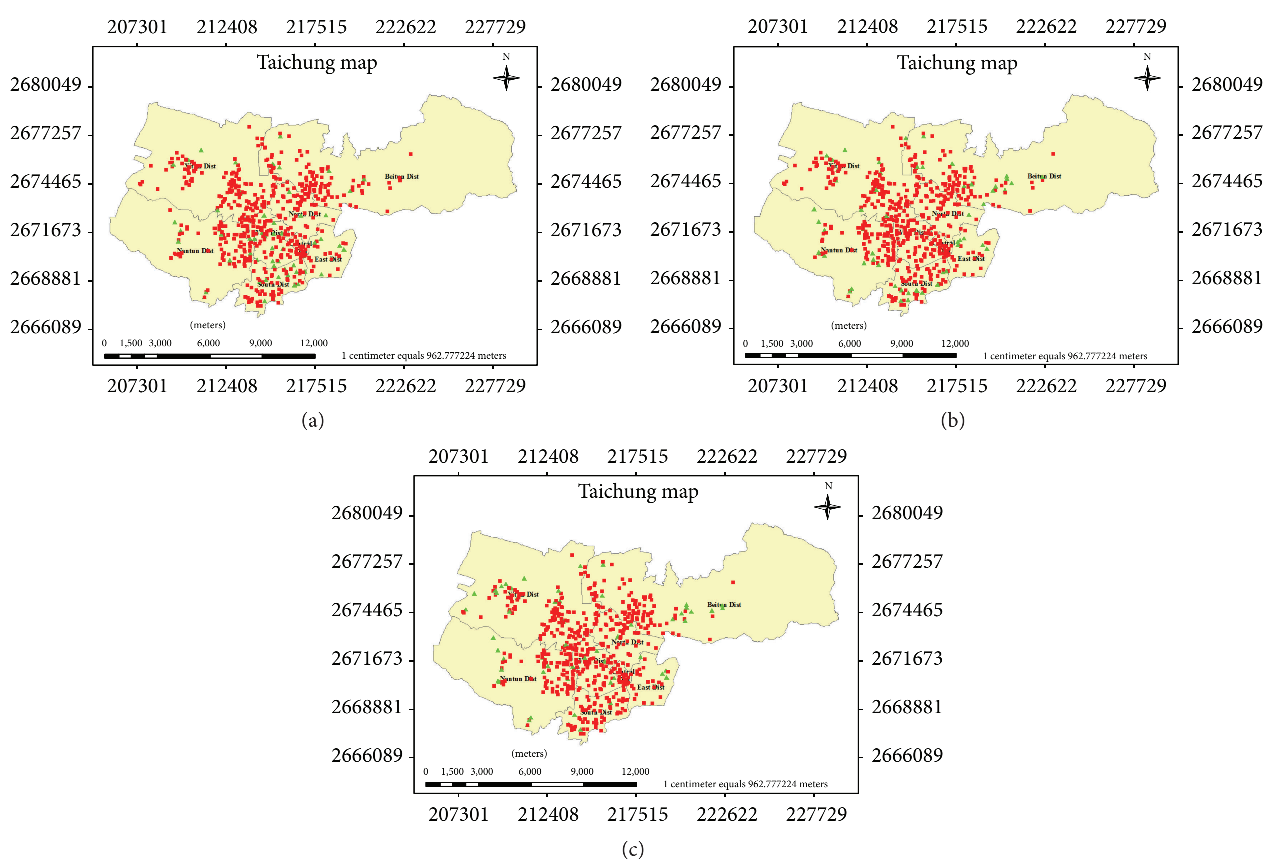


Figure 15: (a) The results for neighborhood radius $R = 1$ km. (b) The results for neighborhood radius $R = 2$ km. (c) The results for neighborhood radius $R = 3$ km.

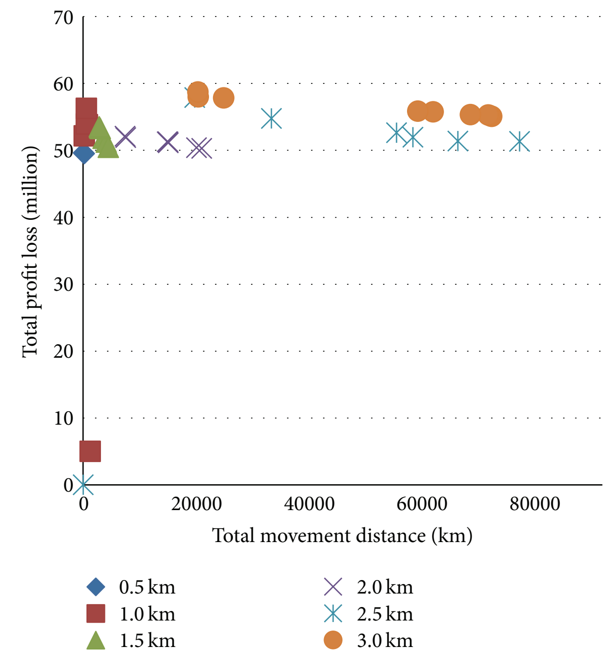


FIGURE 16: The variation of total profit loss with total movement distance for different neighborhood radius R.

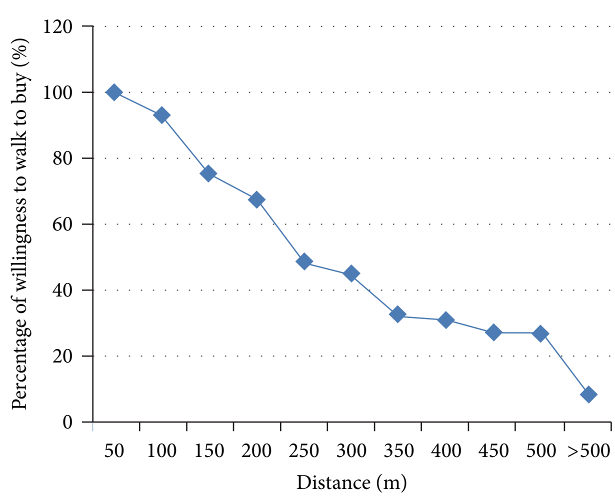


FIGURE 17: The variation of percentage of willingness to walk to buy with distance [11].

requirements of the company. Then, a new neighborhood-based multiobjective genetic algorithm is employed to decide which stores are to close. To examine the effectiveness of the proposed framework, some real data provided by a stock-list company are used to execute some experiments. To investigate the influence of the number of stores on the performance of the GA program, a lot of experiments with different numbers of stores are performed. The experimental results are based on 10 trials.

Results from this study show that the proposed system can help decide store locations efficiently. As the number of stores increases, the average computation time increases in

TABLE 2: The variation of average (AVG) computation time with the number of stores.

Number of stores	200	400	600	800	1000	1200	1500
AVG	1.550	3.042	4.115	5.351	6.575	7.588	9.620
C_v	0.0158	0.0190	0.0104	0.0166	0.0106	0.0133	0.0185

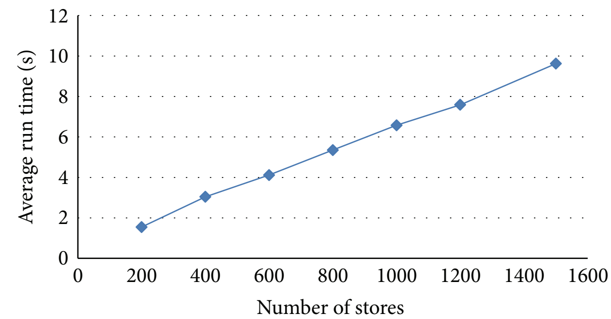


FIGURE 18: The variation of the average computation time with different numbers of stores.

a linear manner, indicating that the GA program is time-efficient. Moreover, the coefficient of variation is small, showing that the GA program is quite stable. As neighborhood radius is increased, the stores selected for closure gradually move from the city center to the suburbs. Special note must be taken that a higher value of neighborhood radius could cause more lost customers, since people are generally not willing to walk too far to find an alternative store when a store is closed.

A two-step approach, with clustering first and performing GA next, may result in suboptimal solutions. To get optimal solutions, integrated approaches are encouraged to be employed in the future. In this study, the interaction between different store chains is ignored. More studies are also needed to discuss the interaction between them and even collaboration with each other.

Nomenclature

Superscripts

t: tth generation.

Subscripts

0: Original
h: hth individual, see (6)
i: ith retail store
j: jth nearby store of a *selected* (to-close) store
n: The nth cluster
C_i: The cost of store i
d_{ij}: The distance from store i to its jth nearby store
G_h: The hth individual chromosome, see (6)
k: The number of clusters
K: Set of clusters

N_{ij}: The jth nearby store of store i
p: The percentage of stores that have to keep open
$q_h^{(t)}$: The number of individuals that dominate the hth individual at the tth generation, (6)
R: The neighborhood radius (see Figure 1)
s_0: The total number of stores before strategic selection
s_n: The number of stores after strategic selection for the nth cluster
S: A set of retail stores
V_i: The revenue of store i
x_i: Decision variable: if store i is to close, then $x_i = 0$; otherwise, $x_i = 1$
Z_1: Objective 1: minimizing the total profit loss
Z_2: Objective 2: minimizing the total travel distance caused by store closure.

Competing Interests

The authors declare that they have no competing interests.

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Research Article

A Novel Hybrid MCDM Procedure for Achieving Aspired Earned Value Project Performance

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A better-performing project gains more subsequent businesses. Many organizations worldwide apply an earned value management (EVM) system to monitor and control their projects' performance. However, a successful EVM application requires handling multiple interinfluenced criteria with feedback effects for decision-making and continuous improvements throughout the application life cycle. The conventional decision approaches assume that preferences between criteria are independent and put their focuses on decision-making. This study employs a hybrid multiple criteria decision-making (HMCDM) method to devise a novel procedure to fulfil the deficiencies. The proposed procedure enables us to evaluate interinfluence effects and gap indices among criteria/dimensions/alternatives and then systemize the evaluation results in a context of influential network relation map (INRM). The INRM provides managers with visual information to find a route in making application decisions, while identifying critical gaps for continuous improvements. A numerical example is presented to illustrate the applicability of the proposed procedure. The results show that, by employing the HMCDM method, the proposed procedure can provide organizations with a foundation to ensure that the aspired EVM application outcomes are achieved at different levels within an organization.

1. Introduction

A project is "a temporary endeavor undertaken to transform limited resources into a unique product, service, or result," in order to satisfy the needs of society, users, and customers [1]. A better-performing project gains more subsequent businesses and is ultimately of strategic importance to an organization [2, 3]. To attain high performances, many organizations worldwide apply an earned value management (EVM) system to monitor and control their projects [4, 5]. A successful EVM application enables us to produce reliable performance indices at initial stages of a project, as early as 15 to 20 percent of the project process [6, 7], thus allowing organizations to understand project health, predict future trends, and take required control actions to minimize deviations, thereby attaining the aspired performances throughout the project life cycle [8–10].

According to Kim et al. [11], EVM application can be formulated as a multiple criteria decision-making (MCDM) problem, which requires experts to analyze a set of interinfluenced application criteria with feedback effects throughout the application process. Some of these criteria include the following: using information systems to report project progress in an accurate and timely manner [11]; using a project management process to break down the project scope and organizational structure [7]; training stakeholders in the effective use of EVM [12]; and providing ongoing efforts to improve the application of EVM [10]. According to Fleming and Koppelman [7], a lack of accurate understanding of the above-mentioned interinfluenced criteria can lead to a series of shortcomings in the implementation of an EVM. Kwak and Anbari [5] have also noted that without adequate analysis upfront, even after an application decision has been conceived and implemented, unanticipated efforts will be required to solve new problems as the implementation proceeds. These studies demonstrate the importance of adopting a systematic procedure to analyze interinfluenced criteria associated with the EVM application decision. Additionally, to obtain aspired application outcomes, continuous improvements should be also early considered in order to prevent the selected decisions from producing negative outcomes [2, 13, 14].

However, according to the literature review of this study, most traditional MCDM approaches assume that the preferences between decision variables are independent and put their emphasis on evaluation and selection of decision alternatives without addressing practical means to implement required improvements [15–20]. Yet, as discussed previously, EVM application requires a decision approach that addresses these issues. Consequently, this study employs a hybrid multiple criteria decision-making (HMCDM) method to devise a novel procedure to fulfil the above-mentioned deficiencies. The HMCDM method contains a decision-making trial and evaluation (DEMATAEL) technique [21], a DEMATEL-based analytical network procedure (DANP) [22], and a modified multicriteria optimization and compromise solution (ViseKriterijumska Optimizacija I Kompromisno Rešenje in Serbian; VIKOR) method [23]. This combined approach was introduced by Tzeng [17] as a new trend of decision-making. Recently, it has been successfully applied in different business fields to solve and improve complex and interdependent real-world problems [18, 22, 24–27] and is thus examined in this study.

The proposed novel procedure uses experts' judgments to model interdependent EVM application problems with a decision framework considering improvement requirements. The procedure then employs the HMCDM method to quantify gap indices with respect to aspiration levels of EVM application based on interinfluence effects among factors/dimensions/alternatives within the decision framework. Finally, the HMCDM method systemizes the quantitative results in the context of influential network relation maps (INRM). The INRM helps managers find a route for EVM application decisions, while identifying critical gaps for prior improvements throughout the life of the decisions implementation. A numerical example is presented to illustrate how the proposed procedure operates in practice. The results show that, by employing the HMCDM method, the proposed procedure can provide organizations with a foundation to ensure that the aspired EVM application outcomes are achieved at different levels within an organization. The remainder of this paper is organized as follows. In Section 2, the EVM literature is reviewed in relation to the proposed procedure; in Section 3, essential concepts of the HMCDM model are presented and the proposed procedure is introduced; in Section 4, a numerical example showing the applicability of the proposed procedure is presented, and main findings are discussed; conclusions are provided in the final section.

2. The Literature Review about Earned Value Management

This section briefly reviews research literature associated with EVM application and then identifies the dimensions and factors/criteria for establishing a decision framework in formulating the proposed procedure for pursuing the aspiration levels of EVM application through better decision-making and continuous improvements.

The EVM was originally developed as a technique by the United States Department of Defense (DoD) in the 1960s to manage the financial aspects of major acquisition projects. In 1967, the DoD adopted the 35 standardized EVM managerial criteria, defined by the United States Air Force as the Cost Schedule Control System Criteria (C/SCSC). This regulatory system was used by the DoD and its contractors to monitor and control various projects over the next three decades [7]. In 1996, the National Defense Industrial Association reduced the EVM criteria to a total of 32, which were formally accepted by ANSI/EIA in 1998 in their publication of the ANSI/EIA 748-98 standard, known as EVM system [6]. During the following year, the Project Management Institute (PMI) adopted EVM as a managerial tool and technique to monitor projects, as stated in its publication titled A Guide to the Project Management Body of Knowledge (PMBOK®Guide) and subsequently described in a separate publication, Practice Standards for Earned Value Management. These publications and the promotion of EVM principles, including their regulation, standardization, and simplification, have led to increasing interest in the use and development of innovative applications of EVM among organizations and experts worldwide [3, 5, 6, 8, 11].

However, while EVM has been widely accepted as one of the most pragmatic systems for managing project performances in both public and private organizations, the studies have also noted that the development of EVM elements and the wide acceptance of EVM do not in themselves guarantee that the EVM application will be successful for projects in all organizations [2, 6]. Some of the common issues arising in projects managed through EVM in different organizations, including the U.S. government and its subsidiary agencies, include overbudgeting, schedule delays, and unsatisfactory performance [5, 11]. These phenomena indicate that even in organizations with long-term operational experience, the implementation of EVM can result in deviations from organizations' aspiration level [7]. Hence, the subject of the effective EVM application requires further study to assist organizations in obtaining intended outcomes. In particular, organization must enable us to assess the current capability of each subordinate unit to understand whether EVM application decisions could eventually help the unit to better manage project performance. What application factors should be in place for each unit to apply EVM and to avoid the need for unintended efforts during the implementation of EVM decisions? Furthermore, if the EVM application is justified as inappropriate, then how can each unit improve its weakness to facilitate benefits through EVM application in the future?

According to the American National Standards Institute/ Electric Industries Association, a reliable EVM application should consider 32 criteria belonging to five categories: (1) organization, (2) planning and budgeting, (3) accounting, (4) analysis and revision, and (5) data maintenance [28]. Fleming and Koppelman [7], who conducted research on many software projects, proposed ten "must-haves" that are required to fully grasp and apply the critical earned value concept in enhancing the management of all types of projects in an industry. These ten "must-haves" require the complete definition of a project's scope of work using a work breakdown structure (WBS) at the outset of project planning as well as through the continuous management of all changes during project execution. Another study by Kwak and Anbari [5] based on the National Aeronautics and Space Association (NASA) indicated that key success factors for the implementation of EVM included the early introduction of EVM, the full involvement of users, and consistent communication with all stakeholders. Lipke [10] argued that the elements required for executing projects and facilitating continuous improvement are necessary ingredients for EVM application to ensure successful project outcomes. These studies have provided useful information for understanding the factors influencing the successful EVM application from different perspectives but lack an integrated or systematic procedure for analyzing level of readiness of these factors when making application and improvement decisions.

Stratton [12] proposed a five-step mature model of earned value management to enhance the quality and use of EVM within an organization. This model can be linked to the ANSI/ESI standard 748 to create assessment matrices that help users to evolve an EVM within their own organizations and to assess the relative strengths of various EVM applications. This study has focused on developing a systematic procedure for analyzing effectively EVM implementation while assuming the independence of the factors in the assessment matrices. This assumption conflicts with the real-world application situations discussed in many other studies [3, 5, 10].

A more comprehensive study by Kim et al. [11] used surveys mailed to 2,500 individuals and on-site case studies conducted within six organizations and concluded that approximately 40 interactive factors in four dimensions (the EVM user, the EVM methodology, the implementation process, and the project environment) could influence significantly the EVM application in four ways: (1) accepting the concept, (2) applying EVM by project managers and team members, (3) enabling projects to be completed within constraints and with satisfactory performance, and (4) bringing overall satisfaction to users of this methodology. The study concluded by proposing an implementation framework to assist both industrial and government agencies applying EVM more effectively for different sizes and types of projects. However, the proposed model and framework were qualitative in nature and did not provide a systematic mean to quantitatively analyze interrelated effects among the dimensions/factors for application decisions and management actions.

According to the literatures discussed above, the factors/criteria influencing the effective EVM application can be grouped into four dimensions: the EVM user, the EVM methodology, the implementation process, and the project environment. Each dimension contains respective factors, as shown in Table 1. In the next section, a novel procedure based on the HMADM method is proposed to evaluate and analyze these interdependent application dimensions/factors in relation to the selection and improvement of application decisions, with the goal of obtaining aspiration levels of EVM application.

3. The Proposed Procedures for Obtaining the Aspiration Levels of EVM Application

To explain the proposed procedure, this section first briefly introduces the essential concepts related to the HMCDM model that combines the following elements: DEMATEL technique, DEMATEL-base ANP, and modified VIKOR; subsequently, this section discusses how the model is employed to develop the proposed procedure.

The HMCDM model was proposed by Tzeng [17], who combined new concepts and techniques to handle complicate and dynamic real-world problems. First, the HMCDM model employs the DEMATEL technique to quantify interinfluence effects among decision variables and visualize the effects on an influential network relation map (INRM). The DEMATEL technique was developed by the Battelle Geneva Institute in 1972 for assessing and solving complex groups of problems [29]. This technique used Boolean operation and Markov Process to quantify cause and effect relationships on each dimension/criterion within a system (or subsystem). Quantitative values results are then systemized on a single map showing degree and direction that each dimension/criterion can influence each other and to the overall system performance [30]. The interinfluence values of DEMATEL can not only help managers gain valuable information for understanding specific societal problems, but also be further used with other methods to obtain more precise weighting values and gap indices in dealing with the real-world decision and improvement problems [21, 31]. Second, this model provides a procedure known as DANP that applies a basic concept of the analytic network procedure (ANP) to transform the interinfluence value of DEMATEL into influential weights (IWs) for prioritizing decision variables. ANP was proposed by Saaty [32] to address interdependence and feedback among the factors, dimensions, or alternatives associated with a decision-making problem. However, ANP assigns identical weights for each cluster per group on the normalized supermatrix, neglecting the influence in different degree. DANP used DEMATEL technique to adjust the ANP equal weighting assumption for better communication of real interdependent situations and improvement alternatives and decisions [22, 31]. These features avoid the assumption of traditional decision models, such as AHP, TOPSIS, path analysis, and SEM, that the value creation criteria are independently and hierarchically structured, thereby enabling interdependent decision situations to be viewed as decision process and outcomes [18].

Third, this model adopts the principle of "aspiration levels" [33] to replace the traditional max/min approach [15, 34], through a modified VIKOR method, when choosing

TABLE 1: Evaluation factors and dimensions.

Dimensions/factors	Descriptions
EVM users (D_1)	
Experience (C_1)	Experience in using EVMS
Training (C_2)	Training at school and on-job training to understand how to use EVMS
Administrative capabilities (C_3)	Administrative expertise of project managers
Technical capabilities (C_4)	Technical expertise of project managers
Changes in work contents (C_5)	Acceptance of power shift after implementing EVMS
EVM methodology (D_2)	
WBS (C_6)	Using work breakdown structure (WBS) details project scopes
CPM (C_7)	Using the Critical Path Method (CPM) as scheduling tool of projects
IPT (C_8)	Using Integrated Project Team (IPT) facilitates understanding among project participants
Computer system (C_9)	Using automated computer system as part of EVMS process
Integrated project management (C_{10})	Using a project management system including EVMS
Implementation process (D_3)	
Open communication (C_{11})	Open communications among project team players including customers
Sufficient resources (C_{12})	Provision of sufficient resources in the EVMS process
Top-down approach (C_{13})	Top management perceives EVMS as a pragmatic way in managing project effectively
Integrated change control system (C_{14})	Using separated office to handle required changes justified by EVMS
Continuous improvement (C_{15})	Providing ongoing efforts to improve application of the EVMS
Project management environment (D_4)	
Colleague-based work environment (C_{16})	A colleague-based project management environment as opposed to bureaucratic culture
Ownership of EVM to lower level project managers (C_{17})	Flexibility allowed lower level project managers
Risk free (C_{18})	Allowing project players to select their own form of EVMS use within a general framework
Culture (C_{19})	A strong trust and supportive culture in which project is performed
Regulations (C_{20})	Complete regulations for implementing EVMS

a relatively good solution from existing alternatives. This feature produces the size of performance gaps to aspiration levels on each criterion/dimension/alternative, thus enabling managers to use a single value for both decision-making and continuous improvements [25]. The VIKOR method was proposed by Opricovic [35] to solve problems that involve incommensurable and conflicting factors. Originally, this method focused on analyzing a set of alternatives and selecting a compromise solution closest to the ideal state [34]. The ideal state was defined as a set of maximum/minimum values relating to each benefit/cost criterion among all alternatives. However, these traditional compromises can entail "choosing the best among inferior options/alternatives": that is, pick the best apple in a barrel of rotten apples; thus, the traditional procedure has to entail "improving" the potential solutions [18]. Hence, Tzeng [17] proposed the modified VIKOR method to replace the maximum/minimum approach with "aspired-worst" by setting $f_j^* = 10$ and $f_j^- = 0$ as the aspiration level and the worst level, respectively, for criterion j, if performance scores with measuring range are from 0 to 10 in questionnaires of each criterion as complete dissatisfaction/bad $\leftarrow 0, 1, 2, \ldots, 4, 5, 6, \ldots, 8, 9, 10 \rightarrow$ extreme satisfaction/good. Recently, this method has been used to aid decision makers in identifying critical gaps in need of further improvement [36, 37].

Combining all these concepts and techniques, the HMCDM model allows managers to avoid "choosing the best among inferior options/alternatives," (i.e., avoiding "picking the best apple among a barrel of rotten apples") [17]. More importantly, the HMCDM model extends the evaluation and selection of decision functions to include identification of critical gaps for continuous improvement over the life of decision implementation [24, 27, 37]. The detailed descriptions, notations, and computational processes can be found in [17, 19, 26, 38].

This study applies the HMCDM model to devise a novel procedure for obtaining aspiration levels of EVM application through four main stages: (1) form an expert team,

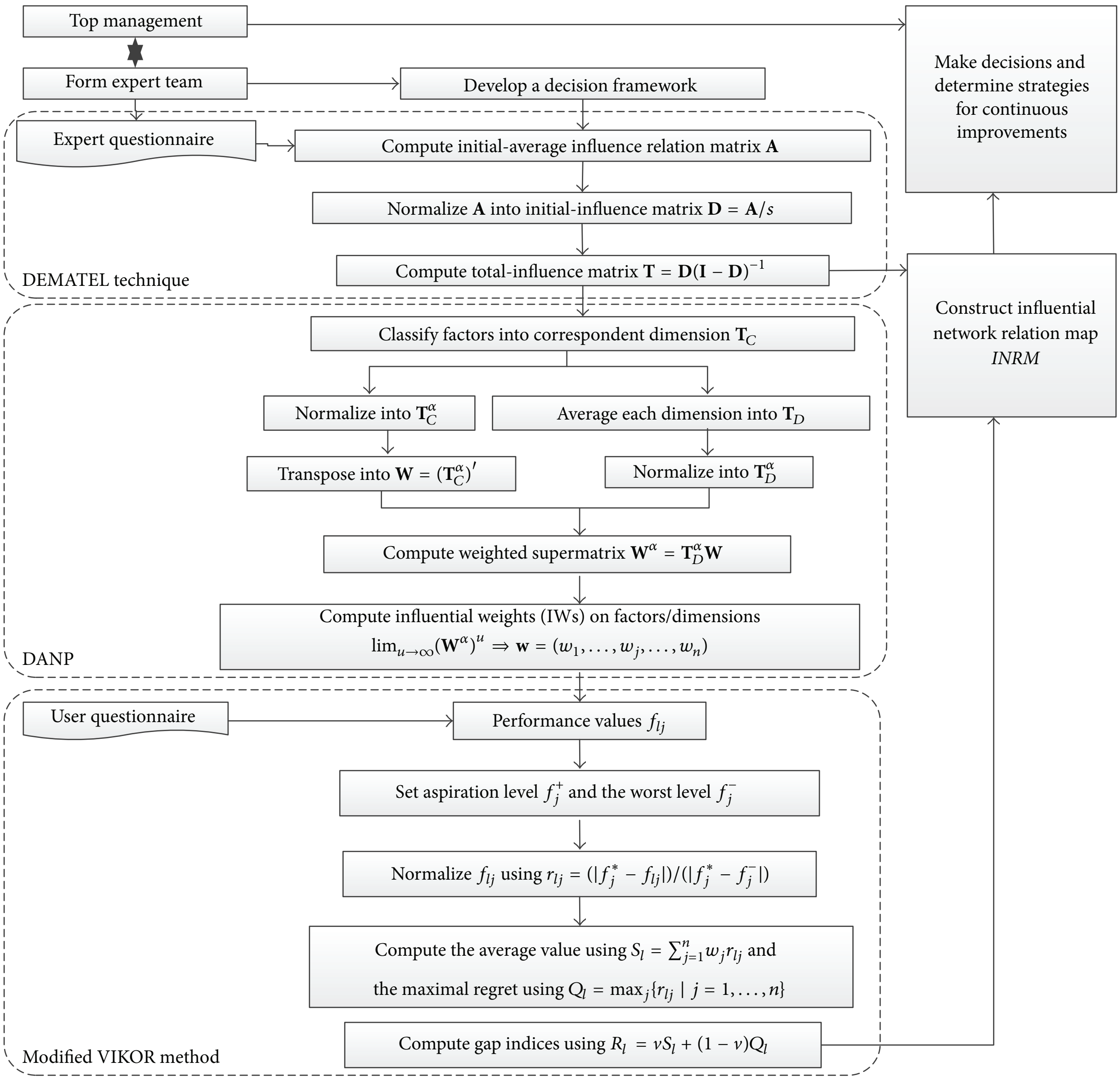

Figure 1: A graphical representation of the proposed procedure.

(2) develop a decision framework, (3) systemize and visualize decision information using HMCDM model, and (4) make application decisions and determine improvement strategies based on INRM. A graphical representation of our procedure is depicted in Figure 1.

As shown in Figure 1, the proposed procedure first forms an expert team (ET) through a top management committee according to the predetermined qualifications. Second, the ET identifies influencing criteria to develop a novel decision framework (Figure 2) which considers both the decision-making and continuous improvements associated with an interrelated decision problem. The decision framework developed in this stage is different from traditional ones which only consider decision-making. Third, based on the decision framework, the procedure uses the HMCDM model to evaluate, systemize, and visualize decision and improvement information including the following: computing interinfluence effects using the DEMATEL technique; computing influential weights using DANP; computing gap indices using modified VIKOR method; and, lastly, systemizing the decision information obtained from the previous steps on the visualized DEMATEL's INRM, showing preference of alternatives and how much improvement is required for each criterion and dimension associated with each alternative. Finally, referring to the INRM, the ET gains valuable information to finalize application decisions with top management and stakeholders, while determining strategies for continuous improvements in achieving the

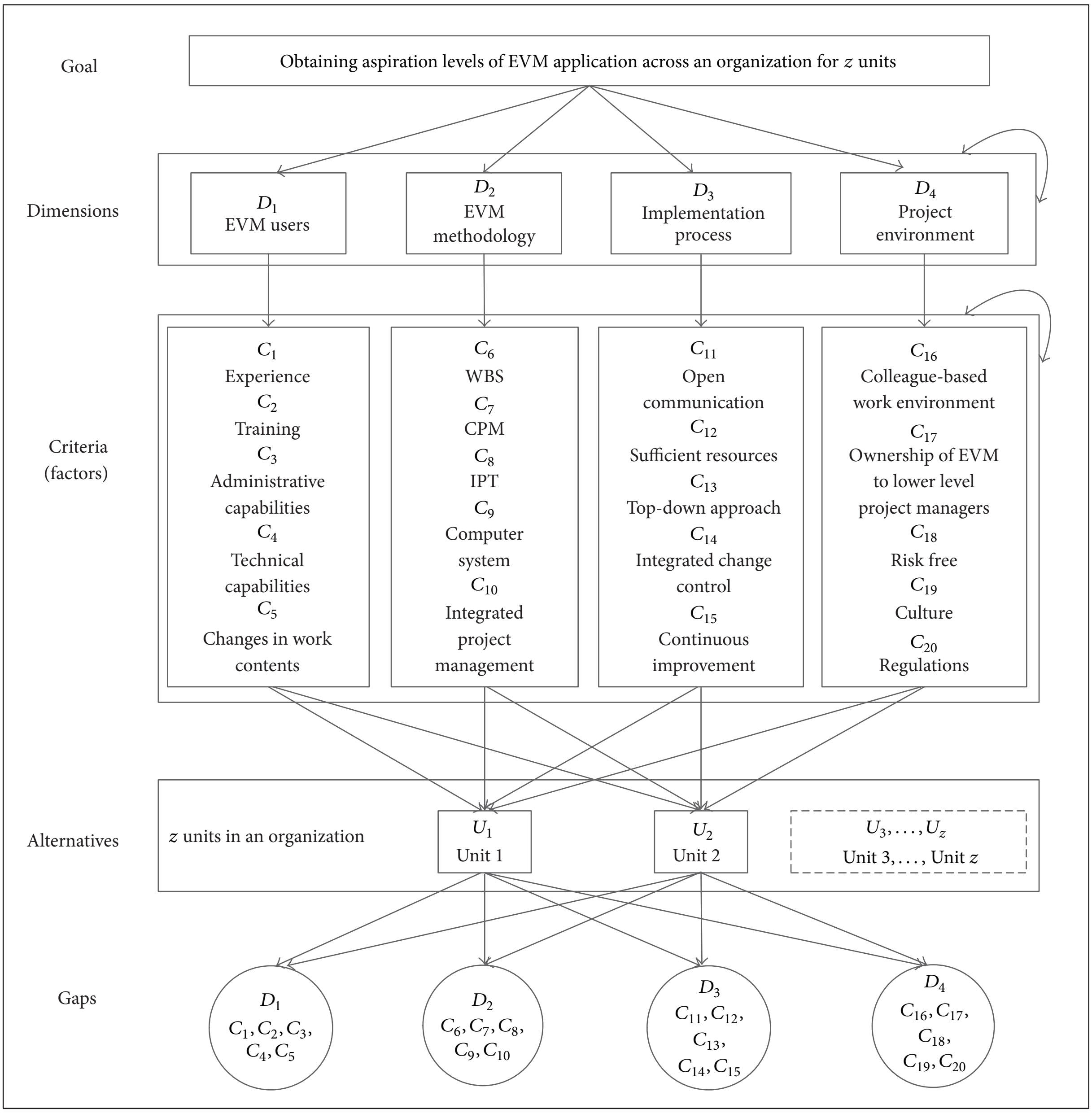

Figure 2: The decision framework for EVM application.

aspired EVM application outcomes in an organization. In the next section, a numerical example is presented to illustrate how the proposed procedure operates in practice.

4. A Numerical Example to Illustrate the Proposed Procedure

In this section, we use an empirical example from a defense organization to illustrate the application of the proposed procedure to a real-world problem. To preserve confidentiality, all data related to the example have been transformed into equivalent units by normalization, which does not compromise the analysis or gap measurement for each factor and dimension and overall alternatives in order to reach the desired aspiration levels.

4.1. Problem Descriptions. The Ministry of National Defense (MND) of a country has been experiencing difficulties obtaining sufficient defense funding during the economic recession and is consequently considering whether to apply EVM to its acquisition units to sustain superior defense capacities with limited resources by ensuring better regulation of the performance and progress of its projects. However, the MND has many acquisition units. As a result of the multisourcing strategy adopted by the MND to acquire its projects from manufacturers in the U.S., Europe, and the domestic market, each unit exhibits certain differences in infrastructure for the management of the projects from different sources. These differences have made EVM application in the MND more complicated than in organizations with mature or identical project management infrastructures for their subordinates. To better manage this complicated situation, the MND required a comprehensive and systematic evaluation to analyze, select, and improve the appropriate decisions that would enable the aspired EVM application outcomes to be achieved in the different units. The MND therefore applied the proposed procedure in a pilot project, to assess two units and obtain satisfactory outcomes.

4.2. Application of the Procedure. Here, we illustrate the stepwise process by which the MND applied our procedure to obtain application decisions and improvement strategies to assist subordinate units in determining how to accept and use EVM to manage project performances with aspired results.

4.2.1. Form a Team. The MND formed an ET with seven experts, one from each of following sectors: acquisition, technology, manufacturing, logistics, end users, procurement, and finance. All experts were selected based on their proficiency in relation to EVM, as assessed by a top management MND committee according to a set of predetermined qualifications.

4.2.2. Develop a Novel Decision Framework. In this stage, the ET members identify 20 influencing factors as evaluation criteria in 4 dimensions and develop a decision framework as shown in Figure 2.

In Figure 2, the highest level of the decision framework is the goal: obtaining aspiration levels of EVM application across MND for two acquisition units (two alternatives), denoted by U_1 and U_2, where two units also represent the alternatives to be evaluated at the fourth level of the decision framework. The second and third levels contain dimensions and factors (groups of interinfluence factors), used to evaluate the alternatives. The fifth and final levels include the gaps for each dimension and factor to be measured in terms of how to reach aspiration levels through continuous improvements.

4.2.3. Systemize and Visualize Decision Information Using HMCDM Model. In this stage, the ET members first employed the DEMATEL technique to evaluate the interinfluence effects among 20 factors within the DF and averaged the results in an initial-average 20-by-20 matrix $\mathbf{A} = [a_{ij}]_{20\times 20}$ (Table 2).

The initial-average matrix was further normalized as an initial-influence matrix $\mathbf{D}$ (Table 3), using

$$\mathbf{D} = \frac{\mathbf{A}}{s} = \left[d_{ij}\right]_{n\times n}, \tag{1}$$

where $s = \max(\max_{1\leq i\leq n}\sum_{j=1}^{n} a_{ij}, \max_{1\leq j\leq n}\sum_{i=1}^{n} a_{ij})$.

Subsequently, through matrix operation using (2), a total-influence matrix $\mathbf{T}$ was obtained as in Table 4. In Table 4, all factors in $\mathbf{T}$ were further classified into the corresponding dimensions as matrix $\mathbf{T}_C$, and each dimension was averaged to obtain matrix $\mathbf{T}_D$:

$$\mathbf{T} = \mathbf{D}(\mathbf{I}-\mathbf{D})^{-1}, \quad \text{when } \lim_{u\to\infty}\mathbf{D}^u = [0]_{n\times n}, \tag{2}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{D} = [d_{ij}]_{n\times n}$, $0 \leq d_{ij} < 1$, $0 < \sum_{j=1}^{n} d_{ij} \leq 1$, $0 < \sum_{i=1}^{n} d_{ij} \leq 1$. If the summation of at least one column or one row (but not all) is equal to one, then we can guarantee that $\lim_{u\to\infty}\mathbf{D}^u = [0]_{n\times n}$.

In matrix $\mathbf{T}$, the inconsistency rate (IR) of the evaluation results from all experts was only 2.70%, which is less than 5%. This result implied that the inclusion of an additional expert in this study would not influence the findings and that the significant confidence level is 97.30%.

According to Table 4, the ET employed DANP to compute the influential weights (IWs) for the dimensions and factors. During this process, the matrices $\mathbf{T}_C$ and $\mathbf{T}_D$ obtained through DEMATEL were normalized as $\mathbf{T}_C^{\alpha}$ and $\mathbf{T}_D^{\alpha}$, and then we transposed matrix $\mathbf{T}_C^{\alpha}$ into an unweighted supermatrix $\mathbf{W} = (\mathbf{T}_C^{\alpha})'$. Subsequently, $\mathbf{T}_D^{\alpha}$ was multiplied by $\mathbf{W}$ to obtain a weighted supermatrix $\mathbf{W}^{\alpha} = \mathbf{T}_D^{\alpha}\mathbf{W}$, as shown in Table 5, and finally multiplied by $\mathbf{W}^{\alpha}$ until it converged into IWs for factors and dimensions, as shown in Table 6.

As shown in Table 6, the ET generally agreed that, in terms of the IWs of DANP, all dimensions and factors have the similar level of importance for effective EVM application. However, the DEMATEL results (Table 4) provide managers with additional information to justify the level of interinfluence among factors/dimensions to achieve the aspired EVM application.

After the DANP steps, the ET administered a questionnaire to collect the opinions of users at different units regarding the outcomes that their units can achieve through EVM application based on their current operational capabilities. Typically, the main components of the questionnaire can be designed as shown in Table 7 set scores to evaluate the respective performance outcomes on a scale from 1 to 5: "N/A (1)," "A (2)," "AU (3)," "AUP (4)," and "AUPS (5)."

In this case, 18 and 20 respondents in U_1 and U_2 were interviewed, respectively. The ET averaged all responses as performance value f_{lj} and then set the worst value $f_j^- = 1$ and the aspiration level (best value), $f_j^* = 5$. Subsequently, the modified VIKOR method was employed to compute the gap indices through using (3)~(6). The computational results are summarized in Table 8:

$$r_{lj} = \frac{\left(\left|f_j^* - f_{lj}\right|\right)}{\left(\left|f_j^* - f_j^-\right|\right)}. \tag{3}$$

$$S_l = \sum_{j=1}^{n} w_j r_{lj}, \quad l = 1, 2, \ldots, m, \tag{4}$$

where w_j is the IWs of the factor from DANP:

$$Q_l = \max_j \left\{r_{lj} \mid j = 1, 2, \ldots, n\right\}, \quad l = 1, 2, \ldots, m. \tag{5}$$

$$R_l = v\left(S_l\right) + (1-v)\left(Q_l\right), \tag{6}$$

Table 2: The initial-average matrix **A** obtained through the DEMATEL.

A	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
C_1	0.000	2.857	2.857	3.429	3.143	3.286	2.857	3.000	2.714	2.571	2.714	2.143	3.571	3.571	3.429	3.000	2.714	3.000	2.571	2.429
C_2	2.714	0.000	3.143	3.857	2.857	3.429	3.429	3.429	2.571	3.286	2.429	1.857	2.571	2.857	3.286	2.714	2.571	3.000	2.714	2.143
C_3	2.429	2.000	0.000	2.143	2.714	2.286	2.286	2.571	2.000	2.571	2.571	2.429	2.286	2.571	2.571	2.143	2.714	2.857	2.286	2.143
C_4	3.143	2.286	2.143	0.000	2.286	3.429	3.286	3.429	3.000	2.571	2.286	2.143	2.571	3.000	3.000	2.857	3.143	3.286	2.714	2.143
C_5	2.857	2.286	2.571	2.429	0.000	2.429	1.571	3.000	2.143	2.857	3.000	2.143	2.571	3.286	3.286	3.000	2.714	2.857	3.286	3.000
C_6	3.000	2.429	2.857	2.571	2.714	0.000	3.143	3.143	2.571	3.429	3.286	3.000	3.000	3.429	3.286	3.143	3.143	3.000	2.714	2.857
C_7	2.286	2.286	2.286	2.714	2.000	3.143	0.000	3.286	2.714	3.000	2.571	2.571	2.143	2.429	2.429	2.143	2.143	2.000	2.143	2.000
C_8	2.857	2.429	2.429	2.714	2.714	3.429	3.143	0.000	2.429	3.000	3.571	2.714	3.143	3.143	3.143	2.857	2.571	3.000	3.143	2.286
C_9	2.857	3.286	3.429	3.429	2.286	3.571	3.286	3.571	0.000	3.571	2.429	3.143	2.429	3.000	3.286	2.714	2.571	2.286	2.429	2.143
C_{10}	2.857	2.429	2.429	2.714	2.571	3.286	3.143	3.143	2.571	0.000	2.857	2.000	2.571	3.429	2.857	2.714	2.429	2.571	2.857	2.286
C_{11}	3.000	3.286	2.714	3.143	2.857	2.857	3.000	3.286	2.000	3.286	0.000	3.000	2.714	3.571	3.571	3.000	3.143	3.143	3.286	2.714
C_{12}	2.000	3.143	2.714	3.000	2.714	2.429	2.571	3.143	2.571	2.857	3.286	0.000	2.429	2.857	3.571	2.714	2.714	2.714	2.714	2.429
C_{13}	2.429	3.143	2.571	2.714	2.857	2.714	2.571	2.571	2.286	2.714	2.857	3.286	0.000	3.571	2.857	3.571	3.000	2.857	3.000	2.429
C_{14}	2.143	2.286	3.000	3.143	2.429	2.857	2.286	3.000	2.571	2.571	2.857	2.429	2.286	0.000	3.143	2.143	2.143	2.286	2.286	2.286
C_{15}	3.286	3.429	3.000	3.286	2.857	2.571	2.571	2.857	2.143	2.857	3.286	2.714	2.571	3.429	0.000	2.714	2.143	2.857	2.571	2.286
C_{16}	3.000	2.571	2.714	2.571	3.000	2.714	2.857	2.571	2.857	3.000	3.143	2.571	2.143	3.143	3.143	0.000	3.429	2.714	3.286	2.286
C_{17}	3.286	3.000	2.714	3.000	2.714	2.714	2.429	3.286	2.143	2.714	2.857	2.714	2.571	2.857	3.000	3.286	0.000	2.857	2.714	2.571
C_{18}	3.429	3.286	3.000	3.571	3.143	3.000	2.857	3.143	2.429	3.000	3.286	2.571	2.714	3.143	3.429	3.000	3.286	0.000	3.286	2.286
C_{19}	3.143	2.571	3.000	2.857	3.143	2.429	2.429	3.286	1.857	2.714	3.571	2.286	3.000	2.714	3.429	3.000	3.000	3.714	0.000	2.857
C_{20}	2.429	2.857	3.000	2.571	2.429	2.143	2.143	2.429	1.857	2.429	2.714	2.286	2.286	2.571	2.857	2.429	2.286	2.571	3.000	0.000

Table 3: The initial-influence matrix **D** obtained through the DEMATEL.

D	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
C_1	0.000	0.048	0.048	0.058	0.053	0.055	0.048	0.050	0.046	0.043	0.046	0.036	0.060	0.060	0.058	0.050	0.046	0.050	0.043	0.041
C_2	0.046	0.000	0.053	0.065	0.048	0.058	0.058	0.058	0.043	0.055	0.041	0.031	0.043	0.048	0.055	0.046	0.043	0.050	0.046	0.036
C_3	0.041	0.034	0.000	0.036	0.046	0.038	0.038	0.043	0.034	0.043	0.043	0.041	0.038	0.043	0.043	0.036	0.046	0.048	0.038	0.036
C_4	0.053	0.038	0.036	0.000	0.038	0.058	0.055	0.058	0.050	0.043	0.038	0.036	0.043	0.050	0.050	0.048	0.053	0.055	0.046	0.036
C_5	0.048	0.038	0.043	0.041	0.000	0.041	0.026	0.050	0.036	0.048	0.050	0.036	0.043	0.055	0.055	0.050	0.046	0.048	0.055	0.050
C_6	0.050	0.041	0.048	0.043	0.046	0.000	0.053	0.053	0.043	0.058	0.055	0.050	0.050	0.058	0.055	0.053	0.053	0.050	0.046	0.048
C_7	0.038	0.038	0.038	0.046	0.034	0.053	0.000	0.055	0.046	0.050	0.043	0.043	0.036	0.041	0.041	0.036	0.036	0.034	0.036	0.034
C_8	0.048	0.041	0.041	0.046	0.046	0.058	0.053	0.000	0.041	0.050	0.060	0.046	0.053	0.053	0.053	0.048	0.043	0.050	0.053	0.038
C_9	0.048	0.055	0.058	0.058	0.038	0.060	0.055	0.060	0.000	0.060	0.041	0.053	0.041	0.050	0.055	0.046	0.043	0.038	0.041	0.036
C_{10}	0.048	0.041	0.041	0.046	0.043	0.055	0.053	0.053	0.043	0.000	0.048	0.034	0.043	0.058	0.048	0.046	0.041	0.043	0.048	0.038
C_{11}	0.050	0.055	0.046	0.053	0.048	0.048	0.050	0.055	0.034	0.055	0.000	0.050	0.046	0.060	0.060	0.050	0.053	0.053	0.055	0.046
C_{12}	0.034	0.053	0.046	0.050	0.046	0.041	0.043	0.053	0.043	0.048	0.055	0.000	0.041	0.048	0.060	0.046	0.046	0.046	0.046	0.041
C_{13}	0.041	0.053	0.043	0.046	0.048	0.046	0.043	0.043	0.038	0.046	0.048	0.055	0.000	0.060	0.048	0.060	0.050	0.048	0.050	0.041
C_{14}	0.036	0.038	0.050	0.053	0.041	0.048	0.038	0.050	0.043	0.043	0.048	0.041	0.038	0.000	0.053	0.036	0.036	0.038	0.038	0.038
C_{15}	0.055	0.058	0.050	0.055	0.048	0.043	0.043	0.048	0.036	0.048	0.055	0.046	0.043	0.058	0.000	0.046	0.036	0.048	0.043	0.038
C_{16}	0.050	0.043	0.046	0.043	0.050	0.046	0.048	0.043	0.048	0.050	0.053	0.043	0.036	0.053	0.053	0.000	0.058	0.046	0.055	0.038
C_{17}	0.055	0.050	0.046	0.050	0.046	0.046	0.041	0.055	0.036	0.046	0.048	0.046	0.043	0.048	0.050	0.055	0.000	0.048	0.046	0.043
C_{18}	0.058	0.055	0.050	0.060	0.053	0.050	0.048	0.053	0.041	0.050	0.055	0.043	0.046	0.053	0.058	0.050	0.055	0.000	0.055	0.038
C_{19}	0.053	0.043	0.050	0.048	0.053	0.041	0.041	0.055	0.031	0.046	0.060	0.038	0.050	0.046	0.058	0.050	0.050	0.062	0.000	0.048
C_{20}	0.041	0.048	0.050	0.043	0.041	0.036	0.036	0.041	0.031	0.041	0.046	0.038	0.038	0.043	0.048	0.041	0.038	0.043	0.050	0.000

Table 4: The total-influence matrix $\mathbf{T}$ for factors $\mathbf{T}_C$ and for dimensions $\mathbf{T}_D$ obtained through DEMATEL.

$\mathbf{T}(\mathbf{T}_C)$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
C_1	0.379	0.414	0.419	0.450	0.416	0.441	0.415	0.459	0.369	0.431	0.438	0.377	0.411	0.472	0.475	0.425	0.412	0.429	0.418	0.365
C_2	0.416	0.361	0.416	0.450	0.405	0.437	0.418	0.459	0.361	0.435	0.427	0.366	0.389	0.454	0.465	0.414	0.403	0.422	0.413	0.355
C_3	0.351	0.335	0.307	0.361	0.345	0.357	0.341	0.380	0.300	0.362	0.367	0.321	0.328	0.383	0.388	0.345	0.347	0.359	0.347	0.303
C_4	0.410	0.386	0.388	0.376	0.384	0.424	0.403	0.445	0.357	0.411	0.412	0.360	0.377	0.442	0.447	0.404	0.400	0.414	0.401	0.344
C_5	0.395	0.377	0.386	0.405	0.338	0.398	0.367	0.427	0.335	0.405	0.413	0.350	0.368	0.436	0.441	0.396	0.384	0.398	0.400	0.349
C_6	0.432	0.412	0.424	0.443	0.415	0.394	0.424	0.467	0.371	0.450	0.453	0.395	0.407	0.476	0.479	0.433	0.424	0.434	0.425	0.376
C_7	0.354	0.345	0.349	0.375	0.339	0.376	0.309	0.397	0.316	0.374	0.372	0.328	0.331	0.387	0.391	0.350	0.343	0.351	0.350	0.305
C_8	0.418	0.401	0.405	0.433	0.403	0.437	0.413	0.404	0.359	0.431	0.445	0.380	0.398	0.458	0.464	0.417	0.404	0.422	0.420	0.357
C_9	0.423	0.418	0.426	0.449	0.401	0.444	0.421	0.467	0.324	0.445	0.432	0.391	0.392	0.461	0.471	0.419	0.408	0.416	0.414	0.359
C_{10}	0.398	0.381	0.386	0.411	0.382	0.414	0.393	0.432	0.344	0.362	0.413	0.351	0.370	0.440	0.437	0.394	0.382	0.395	0.395	0.340
C_{11}	0.438	0.431	0.427	0.458	0.423	0.446	0.428	0.476	0.367	0.453	0.407	0.400	0.408	0.484	0.490	0.436	0.429	0.442	0.440	0.379
C_{12}	0.390	0.398	0.396	0.422	0.389	0.406	0.390	0.439	0.348	0.414	0.425	0.323	0.373	0.438	0.454	0.400	0.392	0.403	0.399	0.347
C_{13}	0.406	0.406	0.402	0.427	0.400	0.420	0.399	0.440	0.352	0.421	0.428	0.383	0.342	0.459	0.453	0.422	0.405	0.415	0.413	0.355
C_{14}	0.363	0.356	0.371	0.393	0.357	0.383	0.357	0.405	0.323	0.379	0.388	0.336	0.344	0.360	0.415	0.362	0.354	0.368	0.363	0.319
C_{15}	0.415	0.407	0.405	0.432	0.397	0.414	0.396	0.440	0.347	0.419	0.431	0.371	0.381	0.453	0.404	0.405	0.388	0.411	0.402	0.349
C_{16}	0.413	0.396	0.403	0.423	0.401	0.418	0.402	0.438	0.359	0.424	0.431	0.371	0.376	0.450	0.456	0.364	0.410	0.411	0.415	0.351
C_{17}	0.416	0.401	0.401	0.428	0.395	0.417	0.394	0.447	0.347	0.418	0.425	0.372	0.381	0.444	0.452	0.415	0.354	0.412	0.405	0.354
C_{18}	0.447	0.433	0.434	0.467	0.430	0.451	0.428	0.476	0.376	0.452	0.461	0.395	0.410	0.480	0.491	0.439	0.434	0.395	0.442	0.374
C_{19}	0.424	0.405	0.416	0.437	0.412	0.423	0.404	0.458	0.351	0.428	0.447	0.375	0.398	0.454	0.470	0.421	0.412	0.435	0.372	0.367
C_{20}	0.362	0.359	0.366	0.379	0.351	0.366	0.349	0.390	0.307	0.371	0.380	0.328	0.339	0.395	0.404	0.361	0.351	0.366	0.369	0.278

$\mathbf{T}_D$	D_1	D_2	D_3	D_4
D_1	0.387	0.397	0.404	0.386
D_2	0.401	0.399	0.413	0.389
D_3	0.404	0.403	0.406	0.392
D_4	0.408	0.404	0.415	0.388

Note: where t_{ij}^{p} and t_{ij}^{p-1} denote the average influence of factor i on j according to $p = 7$ and $p - 1 = 6$ experts, respectively, and $n = 20$ denotes the number of factors; thus, the results above are significant at a significant confidence level of 97.30% in gaps which is greater than the 95% level used to test for significance, that is, $\mathrm{IR} = (1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n}(|t_{ij}^{p} - t_{ij}^{p-1}|/t_{ij}^{p}) \times 100\% = 2.7\%\ (0.027)$, and significant confidence level = 1 − IR = 97.30%.

where $l = 1, 2, \ldots, m$, v is presented as the weight of the strategy of maximum group utility (priority improvement) and $1 - v$ is the weight of individual regret.

As shown in Table 8, the gap indices for alternatives U_1 and U_2 are 0.520 and 0.739, respectively. These values revealed the gap size that each unit would need to be improved to reach the aspiration level. These values imply that the EVM application with required continuous improvements would enhance performance of the acquisition projects in U_1; however, the EVM application may not help U_2 to enhance the performance of projects unless the current operational capabilities of U_2 are further improved.

Additionally, the ET developed the INRM with the use of the results of the DEMATEL and the modified VIKOR method (Tables 4 and 8). During this process, using Table 4, the ET computed the degree of total influence that a factor exerted on the other factors (sum of each row), r_i, and the degree of total influence that a factor received from the other factors (sum of each column), c_i. The ET also derived $r_i + c_i$, indicating the degree of the central role that respective dimension/factor i plays in the system, and $r_i - c_i$, indicating the degree of net influence that respective dimension/factor i contributes to the system. If $r_i - c_i$ is positive, then the dimension/factor i affects other dimensions/factors and, if $r_i - c_i$ is negative, then the dimension/factor i is influenced by other dimensions/factors. The results were summarized as shown in Table 9.

In Table 9, the degree of the central role $(r_i + c_i)$ of the EVM users (D_1), the EVM methodology (D_2), the implementation process (D_3), and the project management environment (D_4) are 3.174, 3.201, 3.243, and 3.171, respectively. These values indicate that all members of the ET generally agreed that all 4 dimensions play a central role in achieving the MND's EVM application at aspiration levels. However, among the 4 dimensions, the degree of net influence $(r_i - c_i)$ on the project management environment (D_4) is 0.060, and an emphasis on this dimension is the basic requirement for the MND to apply EVM in managing projects effectively. This finding also implies that if the project management environment is not well established, EVM application would be affected negatively. Table 9 also contains the interinfluence effects on factors, showing valuable indications for better

Table 5: The weighted supermatrix $\mathbf{W}^{\alpha}$ derived from DANP.

$\mathbf{W}^{\alpha}$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
C_1	0.045	0.050	0.051	0.052	0.051	0.051	0.050	0.051	0.050	0.051	0.051	0.049	0.050	0.050	0.051	0.051	0.051	0.051	0.051	0.050
C_2	0.049	0.043	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.050	0.050	0.050	0.049	0.050	0.049	0.050	0.049	0.049	0.050
C_3	0.049	0.050	0.044	0.049	0.050	0.050	0.050	0.049	0.050	0.049	0.049	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.050	0.051
C_4	0.053	0.054	0.052	0.047	0.052	0.052	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.054	0.053	0.052	0.053	0.053	0.053	0.053
C_5	0.049	0.049	0.050	0.049	0.044	0.049	0.048	0.049	0.047	0.049	0.049	0.049	0.049	0.049	0.049	0.050	0.049	0.049	0.050	0.049
C_6	0.053	0.052	0.052	0.052	0.052	0.047	0.053	0.053	0.053	0.053	0.052	0.051	0.052	0.052	0.052	0.051	0.052	0.052	0.051	0.051
C_7	0.050	0.050	0.049	0.050	0.048	0.050	0.043	0.050	0.050	0.050	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049
C_8	0.055	0.055	0.055	0.055	0.056	0.055	0.056	0.049	0.055	0.055	0.055	0.055	0.054	0.055	0.055	0.054	0.055	0.055	0.055	0.055
C_9	0.044	0.043	0.044	0.044	0.044	0.044	0.044	0.044	0.038	0.044	0.042	0.044	0.043	0.044	0.043	0.044	0.043	0.043	0.043	0.043
C_{10}	0.051	0.052	0.053	0.051	0.053	0.053	0.053	0.053	0.053	0.046	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052
C_{11}	0.052	0.052	0.053	0.052	0.053	0.053	0.053	0.053	0.052	0.053	0.047	0.053	0.052	0.053	0.053	0.053	0.053	0.053	0.054	0.053
C_{12}	0.045	0.045	0.046	0.045	0.045	0.046	0.047	0.046	0.047	0.045	0.046	0.041	0.047	0.046	0.046	0.046	0.046	0.045	0.045	0.046
C_{13}	0.049	0.048	0.047	0.048	0.047	0.047	0.047	0.048	0.047	0.047	0.047	0.047	0.042	0.047	0.047	0.046	0.047	0.047	0.048	0.047
C_{14}	0.056	0.055	0.055	0.056	0.056	0.055	0.055	0.055	0.055	0.056	0.056	0.055	0.056	0.049	0.056	0.056	0.055	0.055	0.054	0.055
C_{15}	0.056	0.057	0.056	0.056	0.056	0.056	0.056	0.056	0.057	0.056	0.057	0.057	0.056	0.057	0.050	0.056	0.056	0.056	0.056	0.056
C_{16}	0.051	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.051	0.050	0.051	0.045	0.051	0.051	0.050	0.050
C_{17}	0.049	0.049	0.050	0.050	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.048	0.050	0.044	0.050	0.049	0.049
C_{18}	0.051	0.052	0.052	0.052	0.051	0.050	0.050	0.051	0.050	0.050	0.051	0.051	0.050	0.051	0.051	0.051	0.051	0.046	0.052	0.051
C_{19}	0.050	0.050	0.050	0.050	0.051	0.049	0.050	0.051	0.050	0.050	0.051	0.050	0.050	0.050	0.050	0.051	0.050	0.051	0.045	0.051
C_{20}	0.044	0.043	0.044	0.043	0.044	0.044	0.044	0.043	0.043	0.043	0.044	0.044	0.043	0.044	0.044	0.043	0.044	0.043	0.044	0.039

understanding critical elements in EVM application in different units within MND.

Based on Tables 8 and 9, the INRM was developed as shown in Figure 3. Taking the dimensions as an example (on the top center in Figure 3), the x-coordinate is the degree of central role $r_i + c_i$, and the y-coordinate is the degree of net influence $r_i - c_i$. First, we marked the coordinates of the EVM users (D_1), the EVM methodology (D_2), the implementation process (D_3), and the project management environment (D_4), which are $(3.174, -0.025)$, $(3.204, -0.001)$, $(3.243, -0.034)$, and $(3.171, 0.060)$, respectively. The process then referred to Table 4 to determine the arrow directions based on the degree of total influence between each dimension. For instance, according to Table 4, the degree of total influence of EVM users (D_1) on the project management environment (D_4) is 0.386; conversely, the degree of total influence of the project management environment (D_4) on EVM users (D_1) is 0.408. The arrow direction is then drawn from project management environment (D_4) to EVM users (D_1) because 0.408 is greater than 0.386. Likewise, the influential directions among all the dimensions and factors are determined and depicted accordingly. Additionally, the ET marked the gap indices on the INRM for factors/dimensions with respect to each alternative based on Table 8.

As shown in Figure 3, the INRM quantified and systemized the gap indices and the degree and direction of interinfluence effects among 20 factors within 4 dimensions associated with the aspired EVM application in the MND. Therefore, it helps managers easily analyze EVM application situations that are essential to make better application decisions. For example, the visualized interinfluence effects at the dimensional level on the INRM (on the top center in Figure 3) revealed that the project management environment (D_4) and the EVM methodology (D_2) were prerequisites for qualified EVM users (D_1) to implement an effective process (D_3) to achieve the aspired application outcome. When adopting the same approach, systematic information associated with decisions to accomplish the aspired EVM application can be realized comprehensively.

4.2.4. Make Application Decisions and Determine Improvement Strategies. In this stage, the ET arranged a series of meetings chaired by the MND's top management, including representatives from related functional divisions. All of the participants reviewed Tables 1–9 and, with reference to the INRM, discussed application situations for each unit, and which factors or dimensions should be prioritized for improvements. The participants also discussed the affordability and availability of the resources required for potential improvements. The eventual outcome of these meetings was to apply EVM at U_1 and to delay its application in U_2 until the dimensions, factors, and/or overall gaps for that unit could be improved to a level below 0.500. Additionally, the participants determined the improvement strategies to be adopted, including allocation of the priority of and responsibility for a set of improvement activities. For instance, according to the size of the gap to the aspiration on the dimensions in Table 8, the ET classified the respective dimensional levels for U_1 and U_2 in descending order as follows: U_1: $\{D_4\ (0.518) \succ D_3\ (0.488) \succ D_1\ (0.408) \succ D_2\ (0.395)\}$; and U_2: $\{D_4\ (0.753) \succ D_1\ (0.653) \succ D_3\ (0.633) \succ D_2\ (0.600)\}$. These values revealed that the

Table 6: The influential weights obtained through DANP.

	Influential weights for factors (C_j)/dimensions (D_j)																			
Factors	C_1 0.050	C_2 0.049	C_3 0.050	C_4 0.053	C_5 0.049	C_6 0.052	C_7 0.049	C_8 0.055	C_9 0.043	C_{10} 0.052	C_{11} 0.053	C_{12} 0.045	C_{13} 0.047	C_{14} 0.055	C_{15} 0.056	C_{16} 0.050	C_{17} 0.049	C_{18} 0.051	C_{19} 0.050	C_{20} 0.043
Dimensions			D_1 0.250					D_2 0.251					D_3 0.256					D_4 0.243		

Table 7: Sample questionnaire responses.

Factors	States of outcome N/A	A	AU	AUP	AUPS	Scores
Experience (C_1)	x					1
Training (C_2)				x		2
Administrative capabilities (C_3)			x			3
Technical capabilities (C_4)				x		4
Changes in work contents (C_5)					x	5

Note: "N/A" not available as score 1; "A" accepted as score 2; "AU" accepted and used as score 3; "AUP" accepted, used, and enhanced performance as score 4; "AUPS" accepted, used, and enhanced performance and satisfied all users as score 5.

Table 8: Gaps indices obtained through the modified VIKOR method.

Dimension/factor	Influential weights (IWs)		Performance values		The size of gap to aspiration level	
	Local	Global	U_1	U_2	U_1	U_2
EVM users (D_1)	*0.250*				*0.408*	*0.653*
Experience (C_1)	0.201	0.050	3.350	1.944	0.413	0.764
Training (C_2)	0.196	0.049	3.750	2.667	0.313	0.583
Administrative capabilities (C_3)	0.198	0.050	3.550	2.722	0.363	0.569
Technical capabilities (C_4)	0.210	0.053	3.200	2.778	0.450	0.556
Changes in work contents (C_5)	0.195	0.049	3.000	1.833	0.500	0.792
EVM methodology (D_2)	*0.251*				*0.395*	*0.600*
WBS (C_6)	0.206	0.052	4.000	2.833	0.250	0.542
CPM (C_7)	0.196	0.049	3.500	1.722	0.375	0.819
IPT (C_8)	0.218	0.055	3.300	2.889	0.425	0.528
Computer system (C_9)	0.173	0.043	3.100	3.056	0.475	0.486
Integrated project management (C_{10})	0.207	0.052	3.200	2.500	0.450	0.625
Implementation process (D_3)	*0.256*				*0.488*	*0.633*
Open communication (C_{11})	0.205	0.053	3.000	3.056	0.500	0.486
Sufficient resources (C_{12})	0.178	0.045	2.950	2.056	0.513	0.736
Top-down approach (C_{13})	0.184	0.047	3.300	2.444	0.425	0.639
Integrated change control system (C_{14})	0.215	0.055	3.350	2.500	0.413	0.625
Continuous improvement (C_{15})	0.218	0.056	2.650	2.278	0.588	0.681
Project management environment (D_4)	*0.243*				*0.518*	*0.753*
Colleague-based work environment (C_{16})	0.206	0.050	3.000	1.722	0.500	0.819
Ownership of EVM to lower level project managers (C_{17})	0.201	0.049	2.900	2.278	0.525	0.681
Risk free (C_{18})	0.208	0.051	2.800	1.833	0.550	0.792
Culture (C_{19})	0.206	0.050	2.750	2.389	0.563	0.653
Regulations (C_{20})	0.178	0.043	3.200	1.722	0.450	0.819
Gap indices					*0.520*	*0.739*

project management environment (D_4) was a problem that arose for both U_1 and U_2. In addition, with reference to the INRM, D_4 (3.171, 0.060) was located in the cause group; thus, improvements in the project management environment (D_4) would have the greatest effects in terms of improving the other dimensions and the selected application decisions. Furthermore, the INRM (Figure 3) showed that all five factors under the project management environment (D_4) also belonged to the cause group: the colleague-based work environment, C_{16} (16.132, 0.091); ownership of EVM by lower level project managers, C_{17} (15.913, 0.241); being risk free, C_{18} (16.817, 0.616); culture, C_{19} (16.310, 0.305); and regulations, C_{20} (14.095, 0.245). These values suggested that all factors under the project management environment (D_4) should be accorded top priority for improvement and that the MND should be able to achieve the strongest improvement effects. Additionally, with the cross-referencing of Table 8 and the INRM, the factors needing prior improvements in the respective units were as follows: U_1: {sufficient resources (C_{12}) and open communication (C_{11}) in the dimension of

Table 9: The total influence given and received on dimensions and factors obtained through DEMATEL.

Dimension/factor	r_i	c_i	$r_i + c_i$	$r_i - c_i$
EVM users (D_1)	*1.574*	*1.600*	*3.174*	*−0.025*
Experience (C_1)	8.416	8.046	16.463	0.370
Training (C_2)	8.265	7.821	16.086	0.445
Administrative capabilities (C_3)	6.928	7.925	14.853	−0.997
Technical capabilities (C_4)	7.984	8.418	16.402	−0.434
Changes in work contents (C_5)	7.768	7.785	15.552	−0.017
EVM methodology (D_2)	*1.602*	*1.602*	*3.204*	*−0.001*
WBS (C_6)	8.532	8.265	16.796	0.267
CPM (C_7)	7.040	7.850	14.890	−0.810
IPT (C_8)	8.269	8.745	17.014	−0.476
Computer system (C_9)	8.382	6.914	15.296	1.467
Integrated project management (C_{10})	7.819	8.287	16.106	−0.467
Implementation process (D_3)	*1.605*	*1.639*	*3.243*	*−0.034*
Open communication (C_{11})	8.660	8.393	17.053	0.266
Sufficient resources (C_{12})	7.946	7.272	15.218	0.674
Top-down procedure (C_{13})	8.148	7.523	15.671	0.625
Integrated change control system (C_{14})	7.298	8.826	16.124	−1.528
Continuous improvement (C_{15})	8.067	8.948	17.015	−0.881
Project management environment (D_4)	*1.615*	*1.555*	*3.171*	*0.060*
Colleague-based work environment (C_{16})	8.111	8.020	16.132	0.091
Ownership of EVM to lower level project managers (C_{17})	8.077	7.836	15.913	0.241
Risk free (C_{18})	8.716	8.101	16.817	0.616
Culture (C_{19})	8.308	8.003	16.310	0.305
Regulations (C_{20})	7.170	6.925	14.095	0.245

implementation process (D_3)} and U_2: {experience (C_1) in the dimension of EVM use (D_1), sufficient resources (C_{12}) in the dimension of implementation process (D_3)}. These factors are classified as part of the cause group, and the size of their gaps is greater than that of the other factors. In a similar fashion, the improvement strategies were determined accordingly.

4.3. Discussions and Implications. Several critical results were derived from the above-described numerical example and from the discussion with the ET members concerning the EVM application. First, according to the DEMATEL results (Tables 5, 9 and Figure 3), the interdependent relationships among 20 factors and 4 dimensions can influence the aspired EVM application outcomes. This finding is consistent with the arguments made by many studies that a set of interinfluenced criteria would significantly influence the effective EVM application and ultimately project performance [5, 11]. However, using the DEMATEL technique can analyze, systemize, and visualize these interdependencies in a single picture, thus revealing the degree and direction of interinfluence effects that each dimension and factor would exert on one another and on the aspired EVM application outcomes. Consequently, for users to be satisfied with the use of EVM to enhance their project performance, organizations require a deep understanding of these interrelationships when making application decisions. Additionally, using the DEMATEL technique can help managers to better analyze and understand interdependent application situations in detail.

Second, according to the results from the modified VIKOR method with the IWs of the DANP (Table 8), decisions regarding the MND's application of EVM may differ for different units in terms of their capabilities in the management of different projects. The results confirm that the development of EVM elements and the wide acceptance of EVM worldwide may not guarantee that EVM application will be successful for all projects in all organizations. In other words, organizations will use a systematic procedure to thoroughly analyze application situations at different levels when making suitable application decisions for all units within an organization. The members of the ET emphasized the fact that the numerical results from the modified VIKOR method and the DANP were essential for the MND, which had no prior experience in applying the EVM and encountered many different application situations in each subordinate unit. If the HMCDM procedure had not been used, the application decisions would have been identical for all units once top management had made the decision to apply EVM.

Third, according to the DANP results (Table 7), among the 20 factors, continuous improvement (C_{15}), an integrated change control system (C_{14}), and an integrated product team (IPT) (C_8) are prioritized as the top three factors with IWs of 0.056, 0.055, and 0.055, respectively. This result echoes the findings obtained from the previously reviewed studies, indicating that the EVM application is not merely the delivery of a system in an organization [11]. Rather, there is considerable potential for improvement, which includes

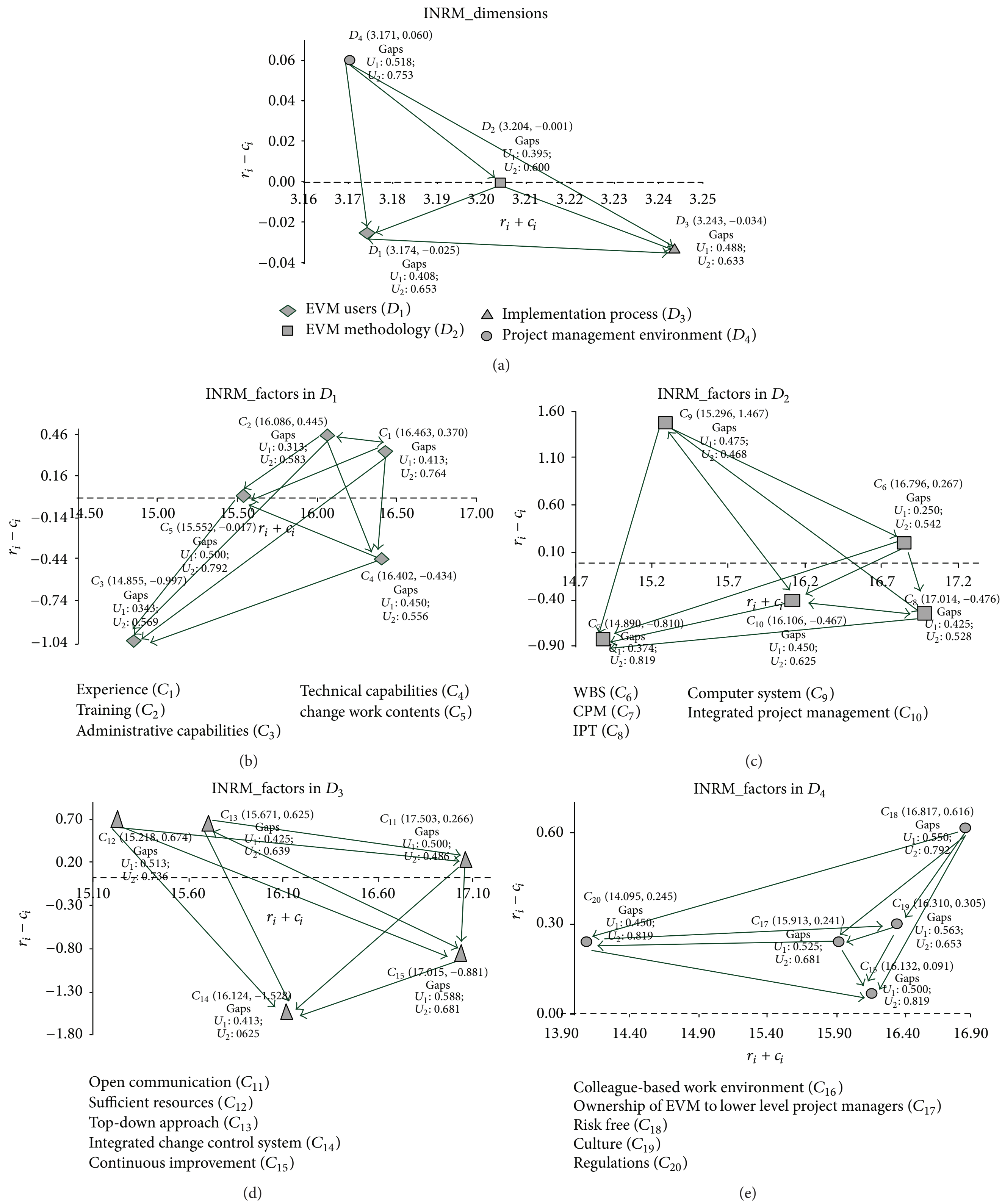

Figure 3: The INRM.

continuing to identify weaknesses in EVM and regard them as opportunities for improvements [5]. Additionally, according to the results of the modified VIKOR method (Table 8), each dimension/factor can create different sizes of gaps to impact aspired EVM application in each acquisition unit (alternative). However, the proposed procedure based on the HMCDM model, combining the DEMATEL technique, the DANP, and the modified VIKOR method, enables a cross-functional team to analyze capability gaps with respect to dimensions/factors of respective application units. Analyzing these gaps is useful in developing strategies to enable each application unit to take the most influential improvement actions to facilitate the EVM application decisions and to ensure the aspired results.

Finally, based on the above example, we argue that without the full support and participation of the various units within an organization, the proposed approach could not have been applied in the pragmatic manner described above. In particular, in the MND case, it is essential to have a small ET (with five to seven members) that includes genuine experts with full authorization from the top management to handle the application project on a full-time basis. “Genuine experts” refer to experts who are committed to taking the appropriate actions when rendering their opinions and judgments regarding the EVM application. In addition, the end users who apply the EVM must have progressive intentions to pursue performance improvement in their projects. Overall, the EVM application is not an easy task; indeed, it involves an array of interdependent variables that influence the application processes and outcomes. This example, however, has demonstrated that the procedure based on the HMCDM model combining the DEMATEL technique, the DANP, and the modified VIKOR method can not only better address application problems, but also easily identify critical factors that are highly influential in solving EVM application problems to achieve the aspiration level.

5. Conclusions

Although EVM has been widely accepted and applied to manage project performance in different types of organizations worldwide, many studies have indicated that a set of interdependent application factors can influence the EVM application process and outcomes. This study proposed a novel procedure, based on the HMCDM method, enabling organizations to obtain aspired outcomes through better decision-making and continuous improvements over the life of the application process.

A numerical example was used to demonstrate the applicability of the proposed procedure. The results showed the following merits of this study: (1) it alone measures the interinfluence effects and gap indices to support decision-making and continuous improvements in pursuing aspired EVM application outcomes; (2) the traditional concept of “effective EVM application” is extended from “illustrating of success factors and analysis framework for decision-making” to “analyzing, selecting, and improving selected decisions over application life cycle”; and (3) managers obtain a visualized route showing decision information at different levels within a decision framework, allowing EVM application to be adapted to different application situations existing within the organization. These merits indicate that the proposed procedure can provide a significant foundation for ensuring that aspiration levels of EVM application are achieved at different levels in an organization.

This study has several limitations. First, the dimensions and factors used to establish the decision framework for the proposed procedure were obtained from a limited review of the literature; thus, this study may have excluded other potential influences on the decision process associated with the effective EVM application. Further research could use other approaches, such as interviews or case studies, to select additional factors and explore the differences and similarities between these approaches. Second, the conclusions drawn are based on a case from a national defense organization. Thus, future research could apply our procedure to other cases, such as organizations in the private sector, to examine our procedure across a wider range of application situations, thus making comparisons to gain additional insights into the usefulness of the proposed procedure. Finally, the improvement strategies determined from our procedure are a set of strategic guidelines. Future research can identify substantial improvement activities. This work can be characterized as an MODM problem, and future research can adopt the DINOV method with a changeable objective and decision spaces to obtain more valuable improvement outcomes. These limitations provide directions for future research to broaden the applicability of the proposed procedure.

Competing Interests

The authors declare that they have no competing interests.

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Research Article

An Interactive Biobjective Method for Solving a Waste Collection Problem

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The aim of this paper is to propose a framework in order to solve the real-world waste collection problem in a city of southern Spain modeled as an Asymmetric Vehicle Routing Problem (AVRP) with side constraints and several variations. In this problem, not only are vehicle capacity and temporal constraints considered but multiple trips are also allowed. Furthermore, two objectives will be considered: the minimization of the total distance and the balance of the working day. Finally, in order to select a single solution among all efficient (or nondominated) solutions, an interactive method is designed using reference points.

1. Introduction

In the public sector, lots of benefits are derived from a good decision-making process. This is why in the last few years the authorities have devoted an increasing attention on this topic to solve this kind of problems in the most satisfying and efficient way. In our particular case, a real-world waste collection problem (WCP) is proposed, where labour, economic, social, and environmental aspects are considered. This essential service involves large operational costs, and so researchers strive to reduce these costs through improving the routing of waste collection, as well as the determining the most suitable location of disposal facilities (or containers) or minimizing the number of vehicles employed. The complexity of this situation increases in the presence of multiple objectives that must be satisfied simultaneously, when satisfying the particular constraints of the problem.

An early paper on waste collection was published by [1] and focused on waste collection activities in New York City (USA). They were interested in the design of routes in accordance with feasible combinations of days for the collection of containers for exactly a preset number of times. The objective was to service all containers assigned each day and to minimize the overall routing cost. In particular, they explored a variety of routing procedures and addressed the problem in two different ways: they clustered first and then optimized each of the routes; and they routed first and then partitioned this giant tour into feasible routes. Numerous researchers have since then studied similar models and have developed a variety of methods to solve the WCP so the evolution of the number of studies on waste has greatly increased over the years. Recently, other approaches have been published in this field. This is the case of [2], [3], or [4], in which a multiobjective tabu search is proposed for urban waste collection problems.

In this paper, we will solve the biobjective real-world waste collection problem in a city of southern Spain modeled as an Asymmetric Vehicle Routing Problem (AVRP) with side constraints and several variations. In the considered area, the waste is daily collected. The network road of the city can be modeled as a directed graph $G = (V, A)$, where V is the set of nodes and A is the set of arcs. Each node $i \in V$ represents a container and each arc $(i, j) \in A$ is the shortest path between two containers $(i, j \in V)$. One-way streets are represented with an arc and two-way streets are represented with two different arcs (each representing one of the two possible directions). Additionally, we consider a special node called the *depot* (which in fact is also the *landfill*) and denoted as node 0, where vehicles are parked and emptied.

In this problem, in contrast to most of the models described in the literature, various types of costs are considered: the *travel cost*, c_{ij}, that is proportional to the distance (or length) of the corresponding arc $(i, j) \in A$; the *service cost*, s_{ij}, associated to each arc, $(i, j) \in A$, that represents the time needed to go from container $i \in V$ to container $j \in V$; and the *dumping cost*, λ, that is the time spent by a vehicle when it is dumping at the landfill. Each container $i \in V$ has associated a quantity, q_i, of waste that must pick it up. Thus, two conflicting objectives are considered in this paper: (1) minimizing the total distance travelled by all vehicles and (2) minimizing the length of the longest route in order to balance the working day within the fleet of vehicles. Regarding the constraints, the total time cost of each vehicle is bounded by the working day. Also, the quantity of waste collected must be considered, since our problem is capacity constrained by the tare of the truck.

A solution of this problem consists of designing K routes where each route must start and finish at the depot and the time spent performing each route cannot exceed the available working time, W. Each vehicle, with a limited capacity Q, must perform a *route* that will be defined as a sequence of T *trips*. Therefore, a *vehicle's route* is defined as the total course driven per day from departing from the depot, until returning back to the depot.

This problem is related to the well-known family of the Asymmetric Capacitated Vehicle Routing Problem (ACVRP); see [5] or [6]. Although our problem and the ACVRP are connected, the real-situation proposed makes us include other constraints. For instance, the classical ACVRP constructs only one simple trip while we allow the construction of routes with multiple trips; another variation comes since ACVRP imposes only capacity constraints and we also need to include time constraints. Finally, the ACVRP considers just one objective, the minimization of the total distance. In contrast, we consider also the minimization of the longest route.

Given all the above, an interactive algorithm for a biobjective multistart algorithm based on reference point is implemented. In general, interactive multiobjective methods assume that decision-maker (DM) is able to provide consistent feedback regarding the preferences to be included in the resolution process in order to guide the search towards certain areas of the Pareto front. To accomplish a set of efficient solutions a multistart algorithm is implemented using the ε-constraint method and then the interactive phase is used to guide the search and select the most preferable solutions by the DM.

As we can see in the literature, many different heuristic and metaheuristic strategies (see [7–9]) have been developed and applied in the resolution of problems with multiple objectives and several constraints. These methodologies are mainly defined by the search method (local search, scatter search, tabu search, and so on) used to create the initial set of efficient solutions. However, not many papers apply interactive methods in order to obtain solution of multiobjective routing problems. Furthermore, a reduced number of these publications are directly related to the waste collection problem (see [10–12]), where the authors design a tool which allows the user to manipulate the database in the routes-construction phase. Visual alternative is presented to the managers using a Geographic Information System (GIS) which helps to the decision-maker to select the closest solution to its requirements.

The rest of the paper is organized as follows. Section 2 presents the proposed algorithm for the solution of the considered problem, and Section 3 provides the computational results applied on the real-world WCP described. Finally, Section 4 summarizes the paper and discusses future work.

2. The Proposed Algorithm

In this section, a competitive multistart algorithm is proposed to solve the considered problem. This algorithm is an iterative process, in which each iteration consists of two phases: the first one (called *construction phase*) in which a feasible solution is generated and the second one (called *local search phase*) in which it tries to improve the solution quality by using a set of neighbourhoods. As a consequence, each iteration produces a locally optimal solution. The best overall solution is selected as a result of the whole procedure.

The proposed multistart algorithm has the following characteristics. It is a Memory-less, Randomized, and Build-from-scratch algorithm. According to [13], Memory-less means that there are no elements that are common to certain previously generated solutions, Randomized means that the starting solutions are randomly generated, and Build-from-scratch means that none of the elements remain fixed from one generation to another.

As we are solving a biobjective problem, the well-known ε-constraint method is used. The traditional ε-constraint method optimizes one of the objectives, when the others are introduced into the constraint space for guaranteeing that basic requirements are satisfied (see [14], [15], or [16]). This method is appropriate for a combinatorial problem where non-big-size Pareto front is expected and suitable for evolutionary algorithms (see [17]). Also, the cost of running multiple executions for the related single-objective problem treated is not elevated, since the algorithm proposed is significantly fast although a mutiobjective extension of the methodology will be investigated in the future introducing specific constructive and local searches for each of the objectives. Other improved ε-constraint methods especially suitable for Multiobjective Integer Programming (MOIP) problems have been proposed in [18–20].

With regard to the problem treated, the minimization of the distance is included as the primary objective and the minimization of the longest route is included in the form of an inequality constraint in order to accomplish a set of efficient solutions.

Once the set of efficient solutions is obtained an interactive approach is designed. We focus on interactive approaches because they are effective methods to deal with multiobjective problems without having an excessive computational effort. We use an interactive scheme which gradually leads to the areas of the efficient set that are more attractive to the DM. Thus, on one hand, computational effort can be reduced and on the other hand, the DM is assisted in choosing the best solution according to his/her preferences.

2.1. Construction Phase. In the construction phase a feasible solution must be obtained. We have designed a multistart algorithm to solve the WCP in which a new algorithm has been employed to construct the initial routes. The algorithm is based on the Mole and Jameson insertion heuristic (see [21]), known as the *extra cost* combined with a more elaborate algorithm, called the *regret* heuristic (see [22]).

Recall that the *extra cost function*, $f^{(1)}$, computes the change in the objective function when inserting each unserved container, $j \in J$, at the best position, i, in route r_k. In mathematical terms, $f^{(1)}(j, r_k, i) = c_{r_k(i-1)j} + c_{jr_k(i)} - c_{r_k(i-1)r_k(i)}$, where $r_k(i)$ denotes the node in the ith position on route r_k. The algorithm selects the node with the lowest value of the extra cost function; that is, the extra cost function should be minimized. This strategy adds containers to routes while the capacity of a vehicle or the working-time constraint is not exceeded. However, the *regret function*, $f^{(2)}$, computes the difference in cost when inserting each unserved container, j, into the best route and when inserting it into the second-best route (obviously in its best position). More formally, $f^{(2)}(j, r_k, i) = f^{(1)}(j, r_{k_2}, i_{k_2}) - f^{(1)}(j, r_k, i)$, where r_k denotes the route with the lowest extra cost and r_{k_2} denotes the route with the second-lowest extra cost. In contrast with the extra cost function, the regret function should be maximized.

Even if the extra cost function is easier to implement and faster than the regret function, a problem related to the extra cost function is that it often postpones the insertion of the most difficult nodes (relatively large values of $f^{(1)}$) until the last iterations. In this regard, the algorithm retains very few alternatives for their insertion. The regret function strives to circumvent the problem by incorporating a kind of forecasting information when selecting the node to be inserted.

In this paper, the extra cost function and the regret function are mixed in order to keep the best features of these both functions. The new proposed algorithm will be called *mixture of regret and extra cost*. This heuristic inserts the first α% nodes using the regret function and the last $(1-\alpha)$% nodes using the extra cost function, where $\alpha \in [0, 1]$.

It is needed to emphasize that, as first step of the algorithm, all routes are initialized by including one of the most difficult nodes to service. That is, we select the K nodes furthest away from the depot and from each other. That is, in order to select the most difficult containers to serve, the next function, $g^{(0)}$, is considered, which computes the distance between unserved containers, J, and served containers. In mathematical terms, given the set of served containers, S, this function is computed for each unserved container, j, as $g^{(0)}(j) = \sum_{s \in S \cup \{0\}} \min\{c_{sj}, c_{js}\}$. A deterministic strategy would select the node with the maximum value of $g^{(0)}$. Instead, this approach constructs a list with a percentage β of the best candidates (according to the considered function) and then a node of the list is selected at random and all the involved variables are updated. The process is maintained until the K routes are initialized with only one node. Once all routes are initialized, the remaining unserved nodes are included using the function $g^{(1)}$ (i.e., minimizing $f^{(1)}$ or $-f^{(2)}$ according to the considered inclusion strategy).

Algorithm 1 shows the pseudocode of the constructive procedure. As input parameters, the algorithm receives the graph, G, the number of vehicles, K, β, the parameter of the initial nodes, and the parameter of the mixture α which control the balance of the randomness (line (1)) and returns a feasible set of routes (line (2)). The algorithm starts by initializing the set of routes with the highest cost, that is, the most difficult containers to serve (lines (4) to (9)).

The proposed multistart construction fails to guarantee the feasibility of the obtained solution (although these details have not been introduced into the pseudocode for the sake of simplicity). Specifically, due to the randomness in the construction of the initial solution, it could happen that one or a few nodes cannot be inserted at any of the already built routes, because of the capacity or time constraint. Then some nodes might remain unvisited, so one of the basic statements of the vehicle routing problem is not satisfied. Thus, when the solution is not feasible, it is discarded and the procedure triggers a new iteration to construct a new solution. Only feasible solutions are submitted to the local search phase.

2.2. Local Search Phase. Due to the size of the optimization problem under consideration, relatively fast procedures are needed in order to attain a solution within a reasonable computing time. However, potential solutions to the local search phase also need to be provided. To this end, the aforementioned constructive procedure builds very diverse solutions. Therefore, the local search procedure will explore the solution space starting from very different solutions.

The WCP presents a search space with an enormous quantity of local optima. Therefore, traditional local search methods (based only on one neighbourhood) fails to perform well on this kind of problem. For this reason, we propose a Variable Neighbourhood Descent (VND) to overcome these difficulties; see [23] or [24]. In VND, several different neighbourhoods are explored, in this case from the largest and slowest to evaluate to the smallest and fastest. The process iterates over each neighbourhood while the solution is improved, performing local searches until a local optimum is found in each neighbourhood. Only strictly better solutions are accepted after each neighbourhood search. In VND, the returned solution is a local optimum in each neighbourhood, $\mathcal{N}_m$, with $1 \leq m \leq m_{\max}$ different neighbourhoods. Therefore, the global optimum is likely to be found earlier than when considering only one neighbourhood.

The pseudocode of the local search is shown in Algorithm 2 where a nested strategy is considered. It has only two input arguments: an initial solution r (line (2)) and the number of neighbourhoods, $m_{\max}$. Our VND algorithm uses three neighbourhood structures.

(1) Neighbourhood structure $\mathcal{N}_1$: a chain of λ consecutive nodes from the route r_i are included in route r_j, where $i \neq j$, and a chain of μ consecutive

Input: $G = (V, A)$, K, β, α;
Output: $r = (r_k)_{k=1,\ldots,K}$ feasible set of routes;
Initialize: $S = \{0\}$, $J = V \setminus \{0\}$, $k \leftarrow 1$, $r \leftarrow \emptyset$, $f^* \leftarrow \infty$;
repeat
  For all $j \in J$ compute $g^{(0)}(j)$;
  Define the list of seeds: $\{j \in J \mid g^{(0)}(j) \geq g^{(0)}_{\max} - \beta(g^{(0)}_{\max} - g^{(0)}_{\min})\}$;
  Select j^* at random from the list of seeds;
  $r_k \leftarrow r_k \cup \{j^*\}$, $S \leftarrow S \cup \{j^*\}$, $J \leftarrow J \setminus \{j^*\}$, $k \leftarrow k + 1$;
until $k = K + 1$;
$r \leftarrow r$, $J \leftarrow J$;
while $J \neq \emptyset$ **do**
  while the capacity or the working day of the K vehicles is not exceeded **do**
    For all $j \in J$ compute $g^{(1)}(j, k, i)$;
    Select j^* such as $g^{(1)}(j^*, k, i) < g^{(1)}(j, k, i)$;
    $r_k \leftarrow r_k \cup \{j^*\}$, $J \leftarrow J \setminus \{j^*\}$;
  end while
end while
return r

Algorithm 1: Construction phase.

Define the set of neighbourhood structures $\mathcal{N}_m$, $m = 1, \ldots, m_{\max}$
Input: r feasible solution;
Initialize: $m \leftarrow 1$;
repeat
  local search: $r' \leftarrow \arg\min \mathcal{N}_m(r)$;
  if $f(r') < f(r)$ **then**
    $r \leftarrow r'$ and $m \leftarrow 1$;
  else
    $m \leftarrow m + 1$;
  end if
until $m = m_{\max} + 1$;
return r

Algorithm 2: VND.

nodes from the route r_j are included in route r_i. This is an interroute movement that selects all pairs of possible routes and checks which nodes can be swaped between the routes. This is repeated until all pairs of routes have been considered and the best movement is performed.

(2) Neighbourhood structure $\mathcal{N}_2$: a node is moved from one route to another, that is, this is an interroute operator. This exchange evaluates the possibility of moving one node belonging to a route r_i in the best position of any other route r_j, where $i \neq j$, in an attempt to improve the total distance. We select one route and we check which nodes can be inserted from another route. This is repeated until all possibilities have been considered and the best movement is performed.

(3) Neighbourhood structure $\mathcal{N}_3$: two consecutive nodes are exchanged within a route, that is, this is an intraroute movement in order to reduce the current route distance.

The procedure starts by obtaining a local optimum r' with respect to the first neighbourhood. Instead of abandoning the search (as a local search procedure), VND then resorts to the following neighbourhood searching for an improvement. If an improvement is found, the search starts again by considering the first neighbourhood (which implies setting $m = 1$). Otherwise, VND explores the next neighbourhood by increasing m (until $m_{\max}$ is reached).

2.3. Interactive Phase. The DM's preferences have to be included to select a solution within that set and few studies have addressed this issue in a metaheuristic context. We use an interactive procedure based on reference points although many other valid methods can be used such as the utility function method, lexicographic method, goal programming, weighting methods, and ε-constraint (see the book [25] for an excellent review about this topic). Reference-point-based methods are one of the most natural ways of expressing preference information in solutions. It eases the process to the DM, who wants to reach them for the objective functions. Then, the primary aim of the method is to reduce

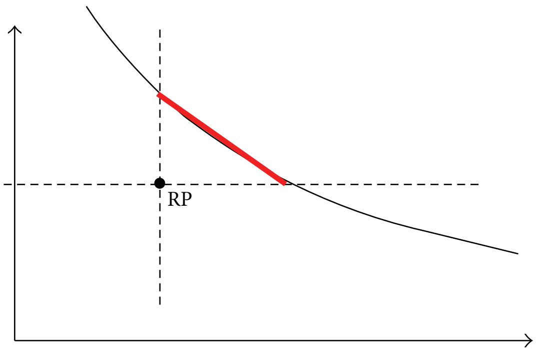

Figure 1: Unfeasible reference point.

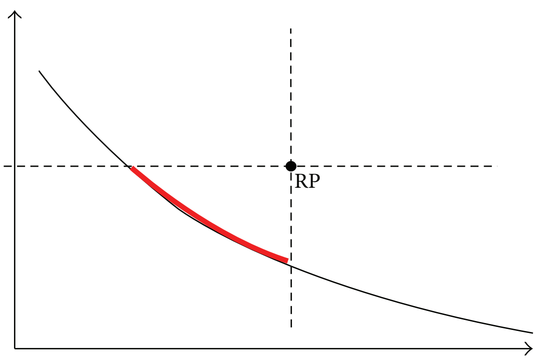

Figure 2: Feasible reference point.

the size of the approximation of the efficient frontier, using the information iteratively supplied by the DM. Then, the DM must provide the desired values for each objective (i.e., a reference point RP). Thus, the main idea is to project this point on the efficient frontier to obtain the efficient point closer to this reference point (converting the original MO problem into a single-objective optimization problem). However, our method provides a set of efficient solutions in the area where this projected solution lies, rather than a single efficient solution. When the aim is to minimize two objectives, this approach (using the reference point) will define the area of the efficient frontier where the interesting solutions are expected to be. Also, an interesting advantage comes from the fact that it is not a matter of fact that the reference point is feasible or not. This is illustrated in Figures 1 and 2 where the whole frontier is reduced to the highlighted portion. The iterative process continues until the DM reaches the best compromise regarding the best solution according to his/her preferences.

The proposed interactive process is based on the g-dominance concept proposed by [26], but, in our case, the efficient frontier obtained is not modified along the interactive phase. The g-dominance method is easy to include to any multiobjective metaheuristic strategy used, since it does not imply the modification of the main architecture of the specific search engine adopted. Apart from this, it might be applied a posteriori, to filter an area of the Pareto front, or included in the algorithm looking for its convergence to the desired zone. Additionally, this method allows the modification of the

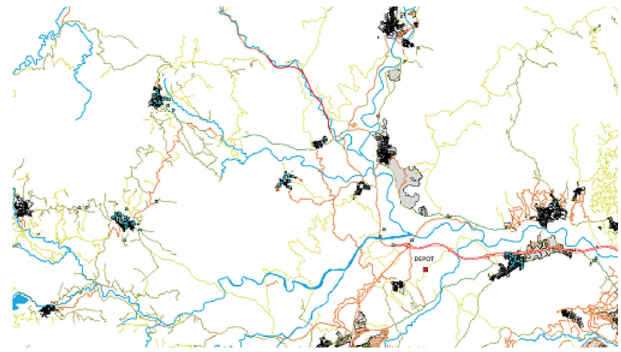

Figure 3: Localization of containers.

reference point during the searching process. On the other hand, one of its main disadvantage is that any of the reference point methods requires the knowledge of the ideal and anti-ideal point, which is not always easy to obtain.

As shown above, at each iteration, a representative subset of efficient routes, according to the offered reference point RP, is shown to the DM. If the DM does not feel satisfied with any of these solutions, he/she can modify the reference point in order to refine his/her preferences.

3. Computational Results

In this section we show the computational results when solving the WCP by using the proposed iterative algorithm. The code has been implemented using the programming language C# in Visual Studio 2010. The program was run in a Samsung Series 5 NP535U3C notebook; Windows 7 Home Premium (64-bit); CPU: AMD Dual-Core A6-4455M Accelerated Processor; speed (GHz): 2.1 GHz; CPU Cache: 1 MB; and System Memory: 4 GB.

This problem deals with four trucks which are used to satisfy the collection of 214 rear-loading containers daily. The containers are located in an area at the south of Spain and are represented in Figure 3 by the use of a GIS.

This problem belongs to the set of Capacited Vehicle Routing Problems (CVRP), since each truck is limited to collect up to 10,220 kg. Moreover, an additional constraint on the duration of the routes, of 37.5 hours per week, has been taking into consideration, in order to satisfy the work-day requirements of the company in charge. And the following objective functions are considered:

(1) f_1: minimize the total distance driven by all of the vehicles.

(2) f_2: minimize the longest route performed within the set of vehicles.

The complete algorithm was run taking into account the two objective functions previously defined and the respective constraints of the problem. Then, a set of 31 efficient Pareto points were identified. Table 1 contains a sample of these solutions, where Sol. 1 is the solution with the lowest value for function f_1 and the greatest value for f_2 and Sol. 2 is the solution with the greatest value for f_1 and the lowest value

Table 1: Sample of solutions.

	f_1	f_2
Sol. 1	252,040	111,920
Sol. 2	345,415	87,246
Sol. 3	255,680	94,599
Sol. 4	290,307	90,737
Sol. 5	308,501	88,061

Table 2: Output after the interaction phase.

	f_1	f_2
Sol. 1	284,944	93,621
Sol. 2	285,698	93,423
Sol. 3	286,181	92,835
Sol. 4	287,412	92,244
Sol. 5	288,924	92,024
Sol. 6	289,102	91,641
Sol. 7	289,323	91,063
Sol. 8	289,703	90,954
Sol. 9	290,307	90,737

Table 3: Routes of the final solution selected.

	Length (m)	Remaining capacity (kg)
Route 1	75,103.68	64.19
Route 2	92,835.88	141.20
Route 3	92,423.95	99.47
Route 4	25,817.74	2,957.11

for function f_2. It can be seen that choosing Sol. 2 instead of Sol. 1 leads to a 37.05% deterioration in f_1 and a 22.05% improvement in f_2. At this point, the procedure required information from the DM about his desired level for each of the two objectives of the problem. To do this, the optimal values of each objective function (ideal points) and a sample of 3 intermediate efficient solutions of the efficient frontier were shown (Sol. 3, Sol. 4, and Sol. 5).

The DM set his desired value for each objective of the reference point; initially RP = (270000, 90000). Using this data, the software obtained a group of 23 efficient solutions. A new sample of five solutions was shown to the DM, and he decided to continue the interaction process by determining a new reference point: RP = (285000, 91000), which leads to a set of 9 efficient solutions. After this iteration, the DM analyzed the last set reached (Table 2), choosing the third solution as the final one. This solution appeared to satisfy the preferences of the manager.

This solution obtained through the interactive process has a total length of 286,181.3 meters, where the longest route implies 92,835.88 meters of it, and it involves four routes. Having a deeper look into the results, the performance of each route is shown in Table 3. With a total distance of 75,103 meters, 10,156 kg loaded from 53 containers visited, Figure 4 displays the tour followed by the first route, whereas the value of the second objective corresponds to the second route (see Figure 5), with a total length of 92,835 meters, 54 containers

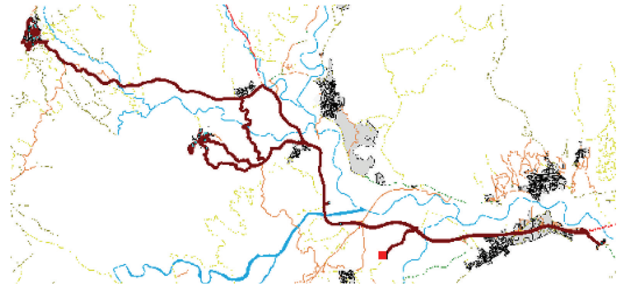

Figure 4: Route 1 of the final solution selected by the DM.

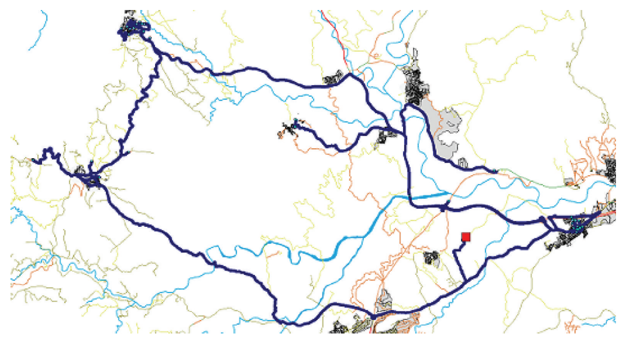

Figure 5: Route 2 of the final solution selected by the DM.

visited, and a remaining capacity of 141 kg in the truck. The third route visits 58 containers, loading a 10,121 kg which implies a truck capacity almost full (see Figure 6). Finally, the last and shortest route is displayed in Figure 7. Its length corresponds to the tour that collects 48 containers. Almost 50 km differentiates this route from the second shortest route and its remaining capacity of 2,957 kg.

On one hand, the shortest route coincides with the one ending the process with more capacity available in the truck. On the other hand, the difference between the lengths of the two longest routes is around 400 meters, which is not a very significant distance and the truck is almost full in both cases. However, the third longest route almost achieves the maximum capacity of the truck, so that might have stopped from obtaining a longer route by introducing more containers to collect. These four routes were presented to the DM through the images showed in Figures 4–7, which were generated using the GIS.

4. Conclusions and Future Research

In this paper we address a difficult real-world problem that is the waste collection problem in a city of the southern Spain. This problem was tackled because the company in charge of the waste in the city, was interested in attaining a better planning of this service. One reason for addressing the problem is that, instead of considering a classical objective function, the minimization of the total distance travelled by all vehicles, an additional objective function is also considered. This new objective function is the minimization of the longest route in order to get a balanced set of routes. In this way, a biobjective

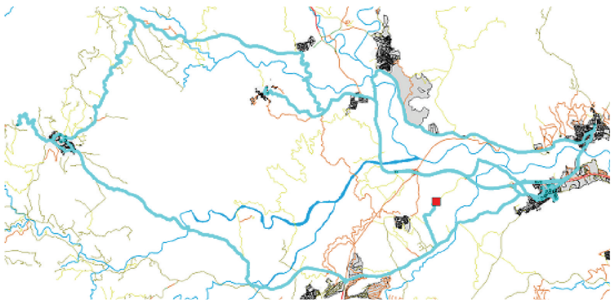

Figure 6: Route 3 of the final solution selected by the DM.

Figure 7: Route 4 of the final solution selected by the DM.

waste collection problem is addressed. Another important issue is that in most real-world problems more than one cost is associated with each arc; in our particular case, both distance and time costs are considered. Specifically, in the problem of this paper, distance costs are considered in both objective functions and the time costs are included as a constraint of the working time of each vehicle.

The most important feature of this paper is the proposal of a new procedure specifically designed to solve this biobjective real-world problem, modeled as an Asymmetric Capacitated Vehicle Routing Problem with several variations. First, a new algorithm is implemented capable of managing this difficult problem but emphasis is placed on the simplicity, speed, and effectiveness of the algorithm. These characteristics are crucial due to the size of the real-world problem. Additionally, an interactive procedure is also implemented to show different according to DM's preferences.

As future work, we would like to test the proposed framework over larger instances. Also, it could be interesting to improve the propose algorithm to get the set of efficient solution in a single run avoiding the use of single-objective formulations, that is, removing the multiple executions of the ε-constraint method. Besides, we would like to improve the interactive phase and to compare it with other methodologies in order to make the whole process easier to the DM.

Competing Interests

The authors declare that they have no competing interests.

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Research Article

Balancing Lexicographic Multi-Objective Assembly Lines with Multi-Manned Stations

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In a multi-manned assembly line, tasks of the same workpiece can be executed simultaneously by different workers working in the same station. This line has significant advantages over a simple assembly line such as shorter line length, less work-in-process, smaller installation space, and less product flow time. In many realistic line balancing situations, there are usually more than one objective conflicting with each other. This paper presents a preemptive goal programming model and some heuristic methods based on variable neighborhood search approach for multi-objective assembly line balancing problems with multi-manned stations. Three different objectives are considered, minimizing the total number of multi-manned stations as the primary objective, minimizing the total number of workers as the secondary objective, and smoothing the number of workers at stations as the tertiary objective. A set of test instances taken from the literature is solved to compare the performance of all methods, and results are presented.

1. Introduction

Assembly lines are mostly installed for producing products in high volumes and usually include automatic material handling system [1], tools, workers, and more than one workpiece. Therefore, in fact, assembly lines are designed by arranging all of their components. One of most important problems in this arrangement is to group tasks into stations. In this problem, there are some constraints, such as precedence restrictions between task pairs and cycle time constraints for stations. This grouping process is carried out for optimizing some objectives by fulfilling all of restrictions. The optimization problem of this process is called assembly line balancing problem (ALBP).

ALBPs which are strongly NP-hard [2] can be classified based on different criteria, such as line layout (straight lines, U lines, etc.), the number of models produced (single model, multimodel, mixed model, etc.), duration of tasks (deterministic, stochastic, etc.), the number of workers for each station (traditional lines, two sided lines, multi-manned lines, etc.), and the number of objectives considered (single objective, multi-objective, etc.). The reader is referred to the recent paper by Sivasankaran and Shahabudeen [3] for a comprehensive survey and classification of line balancing problems. Also, other well-crafted surveys can be found in papers by Lusa [4], Becker and Scholl [5], and Erel and Sarin [6].

In practice, products produced in assembly lines have different characteristics based on size, the number of tasks, demand structure, duration of tasks, and so forth. Therefore, different products may require different line structures which are briefly mentioned above. For example, consider the assembly processes of a computer keyboard and a bus. It is obvious that the production of the computer keyboard requires less number of tasks than the one of bus. Also, based on sum of task times, the keyboard has smaller flow time than the bus. On the other hand, more than one worker can perform tasks on the same bus workpiece while only one worker is usually allowed to perform tasks on the same keyboard workpiece. Therefore, for decreasing the flow time, tool cost, work-in-process cost, space used for the assembly line, and so forth, assembly lines with multi-manned workstations may be more suitable to produce large size products like bus in practice. Consider the problem with

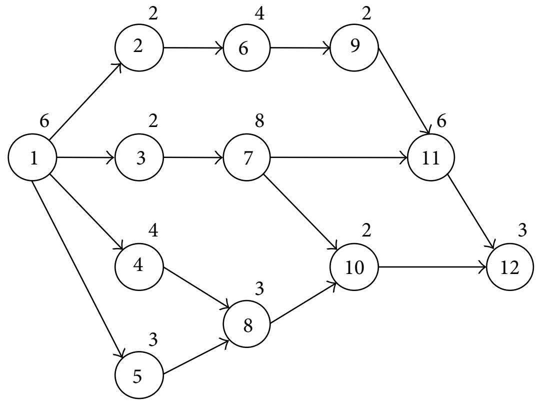
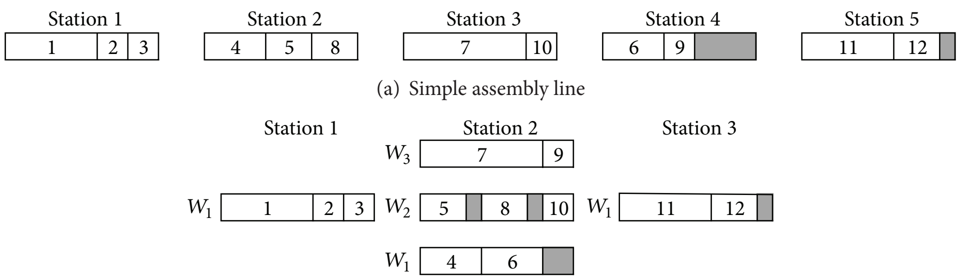


Figure 1: Precedence diagram for illustrative problem.



(a) Simple assembly line

(b) Multi-manned assembly line

Figure 2: Optimal solutions of simple and multi-manned assembly line balancing problems.

cycle time Ct = 10 and precedence diagram given in Figure 1. By using the sum of task times $t_{\text{sum}} = 45$, the lower bound value for the total number of stations for simple assembly line [15] and also for the total number of workers for multi-manned assembly line [7, 8] can be calculated as follows:

$$\text{LB}_1 = \left\lceil \frac{t_{\text{sum}}}{\text{Ct}} \right\rceil = \left\lceil \frac{45}{10} \right\rceil = 5. \quad (1)$$

For simple assembly line balancing problem with minimizing the number of stations (SALBP1), the optimum solution is given in Figure 2(a). As is seen from this figure, at least 5 stations are required to produce the goods with desired properties. There are 5 workpieces (and workers) in the line since there is a workpiece (a worker) at each one of these stations. For multi-manned assembly line balancing problem (MALBP) with minimizing the number of stations, the optimum solution is presented in Figure 2(b). In MALBP, determining a station for each task is necessary but not sufficient. To address a MALBP solution, a worker for each task and a sequence of tasks for each worker must also be obtained. As is seen from Figure 2, the multi-manned line requires less number of stations than the simple line. Hence, in multi-manned lines, same products may be produced by using less layout space, work-in-process, flow time, and so forth.

In MALBP, each task is allowed to be performed by only one worker. Also, any worker performs at most one task at a time. To the best of authors' knowledge, the first study about MALBP was done by Dimitriadis [9]. He defined the problem and proposed a heuristic method for solving it. Then, Becker & Scholl [7] developed a mixed integer programming (MIP) model and a branch & bound (BB) method for optimally solving MALBP with mounting places. In the same study, they also described how the proposed BB is used as a heuristic for large size problem instances. For solving the mixed-model MALBP of real life tractor assembly line, Cevikcan et al. [10] proposed a heuristic method executed phase by phase in five steps. The first three steps are devoted to balance the line, and the remaining two steps are used for sequencing models and transferring workers in daily operations. For solving MALBP with minimizing both number of workers and multi-manned stations in the same order, Fattahi et al. [11] proposed a MIP model and a heuristic method based on ant colony optimization. In their comparison study, they showed their heuristic's superiority over the heuristic by Dimitriadis [9]. For solving MALBP with task times dependent on the number of workers at stations, Yazdanparast and Hajihosseini

[12] proposed a MIP model based on the model of Fattahi et al. [11]. Kellegöz and Toklu [8] considered MALBP with minimizing the number of workers and proposed a BB algorithm for optimally solving them. In terms of both CPU times and quality of feasible solutions found, comparison results showed that their BB method has better performance than the revised one of the BB method proposed by Becker and Scholl [7]. Kazemi and Sedighi [13] developed a MIP model, a genetic algorithm based heuristic and a particle swarm optimization based heuristic for cost-oriented mixed-model MALBP. They solved several test instances to illustrate their methods' performance. For solving MALBP with minimizing the total number of workers, Kellegöz and Toklu [14] recently proposed a new MIP formulation, a constructive heuristic based on priority rules and an improvement heuristic based on genetic algorithms. By using benchmark instances, they illustrated the performance of methods they proposed.

The purpose of this study is twofold: (1) to present a goal programming mathematical formulation for multi-objective MALBP and (2) to develop metaheuristic methods based on the variable neighborhood search (VNS) approach for solving medium and large size instances. To the best of authors' knowledge, this study is the first attempt to solve multi-objective MALBP by using the goal programming approach and VNS algorithm. The objectives considered for MALBP in this study are as follows: (1) minimizing the total number of multi-manned stations (line length) as the primary objective, (2) minimizing the total number of workers as the secondary objective, and (3) smoothing the number of workers at stations as the tertiary objective. The remainder of this paper is organized as follows. In Section 2, the multi-objective MALBP is defined. The goal programming formulation for multi-objective MALBP is presented in Section 3. Then, the proposed metaheuristic algorithms are presented in Section 4. The computational study and its results are reported in Section 5. At last, in Section 6, some conclusions and future research directions are discussed.

2. Multi-Objective Assembly Line Balancing Problems with Multi-Manned Stations

A multi-objective MALBP could be described as follows. There is a set I of n tasks all of them have to be performed for getting the final product. Also, some tasks' operations cannot be started before the completion of some of the remaining tasks' operations. In other words, there are precedence relationships within some task pairs. The assembly line which is built to produce this homogenous product has a set J of m stations placed on a straight line. Due to product's characteristics, each station is allowed to have more than one worker. However, the number of workers simultaneously performing different tasks at the same station cannot be greater than the value $M_{\max}$. Some reasons for this limitation may be as follows [8, 9, 11]:

(1) There are $M_{\max}$ tasks having no precedence relationship between each other and they can be performed at the same station.

(2) Due to some characteristics of the product or stations, at most $M_{\max}$ workers are able to perform tasks on the same workpiece.

All the workers used in the line are identical and equally equipped. Each task is indivisible and has to be performed for a predefined deterministic time by only one worker in the line. Each worker cannot perform more than one task at a time. However, different workers of the same station may perform different tasks simultaneously. In this case, it is also ensured that relationships between simultaneous task operations are satisfied.

The decision maker has three goals as follows (in order of priority).

Goal 1. There is no certain limitation on the line length (the number of stations on the line), but the line should not have more than $L_{\max}$ stations.

Goal 2. There is no certain limitation on the total number of workers in the line, but there should not be more than $TM_{\max}$ workers in the line.

Goal 3. There is no certain limitation, but the line should have the same number of workers in each station in the line.

It is assumed that the decision maker is not able to determine precisely the relative importance of the goals. Thus, he ranks these goals from the most important (goal 1) to least one (goal 3). As a result, the decision maker first tries to satisfy goal 1 as close as possible. Then, for all alternative optimal solutions to goal 1, he tries to find the optimal solution for goal 2, and so forth. Note that while trying to satisfy the second goal as close as possible, it is ensured that the deviation from goal 1 remains at its optimal level. Also, while coming as close as possible to meeting goal 3, it is ensured that the deviations from goals 1 and 2 remain at their optimal levels. Thus, for example, while providing goal 3, it is ensured that an additional worker that makes the second deviation worse is not used.

The reason of determining target values for the first two goals may be that the decision maker wants to make arrangement on his current line. In this case, these target values may be determined by the available space and line workforce. If there is no line active, then target values for these goals may be set to zero or their lower bound values.

Smoothness index based on station times (sum of task times assigned to a station) is calculated in the following manner [15]:

$$\mathrm{SI}_{\mathrm{st}} = \sqrt{\sum_{j=1}^{m} \left(TS_{\max} - TS_j\right)^2}, \tag{2}$$

where m is the number of opened stations in the line, TS_j is the station time of station j, and $TS_{\max}$ is the maximum station time determined as $TS_{\max} = \max_{1\le j\le m}\{TS_j\}$. Using the same approach, for multi-manned assembly lines, smoothness index based on the number of workers at stations can be calculated as follows:

$$\mathrm{SI}_{\mathrm{wr}} = \sqrt{\sum_{j=1}^{m}\left(TW_{\max} - TW_j\right)^2}, \quad (3)$$

where TW_j is the number of workers used at opened station j and $TW_{\max}$ is the maximum number of used workers at a station opened in the line, which is determined as $TW_{\max} = \max_{1\le j\le m}\{TW_j\}$. The perfect balance is indicated by smoothness index value 0. In this research, it is assumed that the last goal is satisfied by minimizing the worker based smoothness index of the line.

3. Goal Programming Formulation

The mathematical model of MALBP proposed by Fattahi et al. [11] is developed here to generate the goal programming formulation of the considered multi-objective MALBP. To the best of authors' knowledge, the term "goal programming" was introduced by Charnes and Cooper [16]. For a good description of this approach, the reader is referred to the well-known operations research textbook by Winston [17].

3.1. Notations

Parameters

Ct: Cycle time,

t_i: Duration of task i,

I: Set of tasks; $I = \{1, 2, \ldots, n\}$,

J: Set of stations; $J = \{1, 2, \ldots, n\}$, where n is a valid upper bound for the number of stations [11],

K: Set of workers at a station; $K = \{1, 2, \ldots, M_{\max}\}$,

$P(i)$: Set of immediate predecessors of task i,

$P_a(i)$: Set of all predecessors of task i,

$S(i)$: Set of immediate successors of task i,

$S_a(i)$: Set of all successors of task i,

ψ, Ω, Φ: Large positive numbers,

$L_{\max}$: Target value of the number of stations opened in the line,

$TM_{\max}$: Target value of the total number of workers used in the line.

Decision Variables

st_i: Start time of task i,

$$x_{ijk} = \begin{cases} 1, & \text{if task } i \text{ is assigned to the worker } k \text{ at station } j \\ 0 & \text{otherwise,} \end{cases}$$
$$y_{ih} = \begin{cases} 1, & \text{at the same worker, if task } i \text{ is performed before tash } h \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

w_j: Number of workers used at station j,

mw: The maximum of number of workers of stations,

$mw = \max_j\{w_j\}$,

$$\alpha_{jk} = \begin{cases} 1, & \text{if station } j \text{ is opened, and it has } (mw - k) \text{ workers} \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$
$$u_j = \begin{cases} 1, & \text{if station } j \text{ is opened} \\ 0 & \text{otherwise,} \end{cases}$$

ll: Total number of stations opened in the line (line length),

d_1^+, d_1^-: Positive and negative deviations from the target value $L_{\max}$ of the number of stations opened in the line,

d_2^+, d_2^-: Positive and negative deviations from the target value $TM_{\max}$ of the total number of workers used in the line,

d_3^+, d_3^-: Positive and negative deviations from the target value zero of the smoothness index value of the line (without square root).

3.2. Mathematical Model. The proposed mixed integer goal programming model is as follows.

3.2.1. Objective Function. Consider

$$\text{lexmin}\left\{d_1^+, d_2^+, d_3^+\right\}. \tag{6}$$

3.2.2. Model Constraints

General Constraints. Consider

$$\sum_{j\in J}\sum_{k\in K} x_{ijk} = 1, \quad \forall i \in I, \tag{7}$$

$$\text{Ct}\cdot\sum_{j\in J}\sum_{k\in K}\left((j-1)\cdot\left(x_{hjk}-x_{ijk}\right)\right) + \text{st}_h + t_h \le \text{st}_i, \quad \forall i \in I,\ h \in P(i), \tag{8}$$

$$\text{st}_i + t_i \le \text{Ct}, \quad \forall i \in I, \tag{9}$$

$$\begin{gathered}\text{st}_h + \Psi\cdot\left(1-x_{hjk}\right) + \Psi\cdot\left(1-x_{ijk}\right) + \Psi\cdot\left(1-y_{ih}\right) \ge \text{st}_i + t_i, \\ \forall i \in I,\ h \in \left\{r \mid r \in I - \left(P_a(i)\cup S_a(i)\right),\ i<r\right\},\ j \in J,\ k \in K,\end{gathered} \tag{10}$$

$$\begin{gathered}\text{st}_i + \Psi\cdot\left(1-x_{hjk}\right) + \Psi\cdot\left(1-x_{ijk}\right) + \Psi\cdot y_{ih} \ge \text{st}_h + t_h, \\ \forall i \in I,\ h \in \left\{r \mid r \in I - \left(P_a(i)\cup S_a(i)\right),\ i<r\right\},\ j \in J,\ k \in K.\end{gathered} \tag{11}$$

Constraints for Goal 1. Consider

$$\sum_{j\in J}\sum_{k\in K} j\cdot x_{ijk} \le ll, \quad \forall i \in I, \tag{12}$$

$$ll + d_1^- - d_1^+ = L_{\max}. \tag{13}$$

Constraints for Goal 2. Consider

$$\sum_{k\in K} k\cdot x_{ijk} \le w_j, \quad \forall i \in I,\ j \in J, \tag{14}$$

$$\sum_{j\in J} w_j + d_2^- - d_2^+ = TM_{\max}. \tag{15}$$

Constraints for Goal 3. Consider

$$mw \ge w_j, \quad \forall j \in J, \tag{16}$$

$$\Omega\cdot u_j \ge \sum_{i\in I}\sum_{k\in K} x_{ijk}, \quad \forall j \in J, \tag{17}$$

$$\Phi\cdot\left(1-u_j\right) + \sum_{k\in k}\left(k\cdot\alpha_{jk}\right) + w_j \ge mw, \quad \forall j \in J, \tag{18}$$

$$\sum_{k\in K}\alpha_{jk} \le 1, \quad \forall j \in J, \tag{19}$$

$$\sum_{j\in J}\sum_{k\in K}\left(k^2\cdot\alpha_{jk}\right) + d_3^- - d_3^+ = 0. \tag{20}$$

Sign Restrictions. Consider

$$x_{ijk} \in \{0,1\}, \quad \forall i \in I,\ j \in J,\ k \in K, \tag{21}$$

$$y_{ih} \in \{0,1\}, \quad \forall i \in I,\ h \in \left\{r \mid r \in I - \left(P_a(i)\cup S_a(i)\right),\ i<r\right\}, \tag{22}$$

$$\alpha_{jk} \in \{0,1\}, \quad \forall j \in J,\ k \in K, \tag{23}$$

$$u_j \in \{0,1\}, \quad \forall j \in J, \tag{24}$$

$$w_j \ge 0, \quad \text{and integer } \forall j \in J, \tag{25}$$

$$\text{st}_i \ge 0, \quad \forall i \in I, \tag{26}$$

$$d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, ll, mw \ge 0. \tag{27}$$

Constraints (7), (9)–(11) are directly taken from the model which is proposed by Fattahi et al. [11]. Constraint (8) is written by using the clock time approach [15]. The proposed mixed integer goal programming model has multiple objectives given in Expression (6). The first goal is that the number of opened stations should not exceed predetermined value $L_{\max}$. As indicated earlier, the variable d_1^+ corresponds to overachievement of this target value. Therefore, the first objective aims to minimize this positive deviation. The second objective aims to minimize the positive deviation of total number of workers used in the line from predetermined target value $TM_{\max}$. At last, the third objective aims to minimize the positive deviation of smoothness index on the basis of number of workers at stations (without square root). Note that the target value of smoothness index is equal to zero. Constraint (7) ensures that each task is assigned to only one worker of the line. Constraint (8) ensures that all precedence relationships among tasks are satisfied. This constraint can also be written as follows:

$$\begin{aligned}&\text{Ct}\cdot\sum_{j\in J}\sum_{k\in K}(j-1)\cdot x_{hjk} + \text{st}_h + t_h \\ &\quad\le \text{Ct}\cdot\sum_{j\in J}\sum_{k\in K}(j-1)\cdot x_{ijk} + \text{st}_i, \\ &\qquad\qquad \forall i \in I,\ \forall h \in P(i).\end{aligned} \tag{28}$$

Constraint (9), cycle time constraint, ensures that each task is completed before the end of the cycle time. If tasks i and h which have no precedence relationship between each other are assigned to the same worker of the same station, then one of Constraints (10) and (11) becomes active (with an integer value $\Psi \ge \text{Ct}$). While the activation of Constraint (10) by using $y_{ih} = 1$ ensures $\text{st}_h \ge \text{st}_i + t_i$, the activation of Constraint (11) by using $y_{ih} = 0$ ensures $\text{st}_i \ge \text{st}_h + t_h$. Constraints (12) and (13) are used to formulate the first goal. Constraint (12) assigns proper value to variable ll. Constraint

(13) is the goal programming constraint of the number of multi-manned stations with deviational variables. Having d_1^+ with the value zero ensures that the total number of stations is $L_{\max}$ or less. The set of Constraints (14) and (15) models the second goal. Constraint (14) assigns proper value to variable w_j. Constraint (15) with deviational variables is the goal constraint of the total number of workers used in the line. While the value of d_2^+ is reduced to zero, the number of workers used in the line decreases towards the value $TM_{\max}$. The last set having Constraints (16)–(20) corresponds to the third goal. Constraint (16) ensures that the value of mw is equal to or greater than the number of workers used in each station. Constraint (17) with an integer value $\Omega \geq n$ ensures that if station j has at least one assigned task (i.e., it is opened), then $u_j = 1$. Constraints (18) and (19) are used to assign proper value to each α_{jk} variable. Constraint (18) with an integer value $\Phi \geq M_{\max}$ is active when the corresponding station is opened. For station j, note that Constraint (19) does not allow more than one α_{jk} variable to have the value 1. When Constraints (18) and (19) with the third objective are examined together, it can be seen that all α_{jk} variables of unopened station j will have the value zero. Constraint (20) is the goal programming constraint of the smoothness index on the basis of number of workers at stations. Note that the first term of this constraint computes the sum of squared deviations (from the maximum of number of workers at stations, mw) of the number of workers of stations. While the value of d_3^+ is reduced to zero, the smoothness index value decreases towards zero. This constraint is written based on the fact that minimizing the value of a positive valued function also minimizes the value of the square root of this function. Constraints (21)–(25) show the integer variables, and Constraints (26) and (27) indicate that all the remaining variables are nonnegative continuous.

3.3. Solving the Proposed Model. In the proposed model, there are three goals as follows (in order of importance):

$$\begin{aligned} &\text{Minimize } G_1 = d_1^+ \text{ (Highest priority)} \\ &\text{Minimize } G_2 = d_2^+ \text{ (Mid-level priority)} \\ &\text{Minimize } G_3 = d_3^+ \text{ (Lowest priority)}. \end{aligned} \tag{29}$$

This model is solved by considering one goal at a time, starting with goal 1 and terminating with goal 3. In a solution process, it is ensured that a lower priority goal never degrades any higher priority objective function value. For doing this in a solver, the current solution is added as a new constraint to problems with lower priorities. For example, consider d_1^* is the optimum objective value of the first problem. Then, the constraint $d_1^+ = d_1^*$ is added to the second problem. After solving the second problem, if the optimum objective value is d_2^*, then both constraints $d_1^+ = d_1^*$ and $d_2^+ = d_2^*$ are added to the last problem. After solving the last problem, the optimum solution of the goal programming model is found [18].

For all alternative optimal solutions to the highest-level objective, the solution which optimizes the second level is found and so on. That is, deviations from the goals are minimized in order of priority. Thus, the resulting solution is called the lexicographical minimum [19].

4. Proposed Variable Neighborhood Search Heuristics

VNS was introduced by Mladenović [20]. It combines local search with systematic changes of neighborhood for escaping from local optimum in the descent [21]. The basic characteristic of VNS is that it searches over several neighborhood structures in the solution space. Hence, it reduces the solution space into smaller subspaces.

The basic VNS procedure [22] is given in Algorithm 1. At each iteration of VNS search process, there is an incumbent solution which is a local optimum, and VNS searches in solutions which are in a far neighborhood of the current incumbent solution. From the current neighborhood, it randomly determines a solution (shaking phase), applying local search to this solution. If the resulting solution is better than the current incumbent solution, then the search moves to this new solution. If not, a farther neighborhood structure (after the farthest one, the nearest structure) is used to go on the search process.

Contrary to other metaheuristics based on local search methods, VNS does not follow a trajectory but explores increasingly distant neighborhoods of the current incumbent solution and jumps from this solution to a new one if and only if an improvement has been made [21]. VNS heavily relies upon the following observations [23]:

(i) A local minimum with respect to one neighborhood structure is not necessarily a local minimum for an another neighborhood structure.

(ii) A global minimum is a local minimum with respect to all possible neighborhood structures.

(iii) For many problems, local minima with respect to one or several neighborhoods are relatively close to each other.

For solving the multi-objective MALBP with VNS, following components of VNS have to be developed: (1) solution coding and encoding schemes, (2) the method used for generating initial solution, (3) neighborhood structures used in shaking phase, (4) local search procedure, and (5) the method used to compare solutions (objective function). Thus, in the next subsections, these components for solving multi-objective MALBP are described in detail.

4.1. Representing Solutions (Coding). Solutions of the multi-objective MALBP are represented by using permutations of tasks' priorities. In other words, for the problem with n tasks, let $(p_{i_1}, p_{i_n}, \ldots, p_{i_n})$ be a permutation of the value n. Then

Initialization
(i) Select the set of neighborhood structures N_k, $k = 1, 2, \ldots, k_{\max}$, that will be used in the search;
(ii) Find an initial solution x;
(iii) Choose a stopping condition;
Main step: Repeat the following sequence until the stopping condition is met:
(1) Set $k \leftarrow 1$;
(2) Until $k = k_{\max}$, repeat the following steps:
(a) *Shaking*: Generate a point x' at random from the kth neighborhood of x ($x' \in N_k(x)$);
(b) *Local search*: Apply some local search method with x' as initial solution; denote x'' the so obtained local optimum.
(c) *Move or not*: If this local optimum is better than the incumbent, move there ($x \leftarrow x''$), and continue the search with N_1 ($k \leftarrow 1$); otherwise, set $k \leftarrow k + 1$;

Algorithm 1: The basic VNS procedure.

Procedure - Build Solution
begin
(1) *BestSol* $\leftarrow$ *null*;
(2) Create initial line configuration, *LConf*;
(3) *CurrSol* $\leftarrow$ assign tasks to line with configuration *LConf*;
(4) **if** (*CurrSol* is better than *BestSol*) **then**
(4.1) *BestSol* $\leftarrow$ *CurrSol*;
(4.2) *BaseConf* $\leftarrow$ get line configuration of *CurrSol*;
(4.3) *LR* $\leftarrow$ *list* of *BestSol*'s stations having more than one used worker;
end if;
(5) **if** (*LR* = {}) **then**
(5.1) **return** *BestSol*;
end if;
(6) *s* $\leftarrow$ first element in *LR*;
(7) Delete the first element in *LR*;
(8) *LConf* $\leftarrow$ *BaseConf*;
(9) Decrease the number of workers at station *s* of *LConf* by 1;
(10) Go to Step (3);
end;

Algorithm 2: Procedure of solution construction process.

the priority values for tasks $i_1, i_2, \ldots, i_n$ are $p_{i_1}, p_{i_n}, \ldots, p_{i_n}$, respectively. These priority values are used to determine which one of available tasks is selected for assignment at each step. This approach which is called priority based coding is one of commonly used structures for other types of assembly line balancing problems in the literature [24, 25].

4.2. Building Solutions (Encoding). For building a solution based on priority values, a restricted version of the constructive heuristic proposed by Kellegöz and Toklu [14] is used. The procedure of building solution used in this study is given in Algorithm 2.

Second step in the procedure generates an initial line configuration and sets it to the current configuration *LConf*. A line configuration is a vector of the same length as the maximum number of stations, and the element i in this vector determines how many workers are available at the station i of the line. The initial line configuration has the same number of workers at stations with the best one of solutions in the set IS. If the solution has no worker at a station, then the corresponding element in the line configuration is set to $M_{\max}$. For comparing solutions, the method described in the next subsection is used. The set IS includes $M_{\max}$ solutions, and the solution in sth element is created by the following way. First, a line configuration having the number of s workers in each one of n stations is created. As indicated earlier, n is the valid upper bound value for the number of stations. Then, the method described in the following paragraph is used to assign tasks to stations. In this manner, it is aimed to start the solution building process from a good line configuration.

Assigning tasks to a line with a known line configuration is carried out in Steps (2) and (3). First, an empty station with the number of workers which is determined from the line configuration is opened. At each assignment step, a task having the following properties (checked in the same order) is selected: (1) it is not assigned yet, (2) all its predecessors are already assigned, (3) it can be completed before the end of cycle time, (4) it can be started earlier than other tasks having previous properties, and (5) it has greatest priority value within tasks having previous properties. The selected task is assigned to the worker who can start the processing of this task earlier. If more than one worker has the same minimum start time, tie is broken by selecting the worker having the smallest index. If there are unassigned tasks, and none of these tasks cannot be assigned to the current station, then a new empty station having the number of workers determined from the line configuration is opened.

After getting the current solution *CurrSol*, it is compared with the best solution *BestSol* found in the solution building process. The method which is used to compare solutions is presented in the next subsection. If the current solution is better than the best solution, then (1) the best solution is updated, (2) the line configuration *BaseConf* is created, and (3) the list *LR* is determined. The line configuration *BaseConf* is determined by using the newly found best solution. This line configuration has n stations and includes the same number of workers at each opened station of the best solution (active part of the configuration) and $M_{\max}$ workers at each one of other stations which is not opened in the best solution (passive part of the configuration). The list *LR* which includes best solution's stations which have more than one worker (multi-manned) is used in solution restriction process. This list includes stations based on minimum worker load-related strategy proposed by Kellegöz and Toklu [14]. According to this strategy, stations in *LR* are decreasingly ordered according to their minimum worker's load.

As is seen from the solution building procedure presented in Algorithm 2, if the list *LR* has no element, then the solution building process is ended. If not stopped, then the configuration restriction process is performed by using Steps (6)–(9). First, the first element in the list *LR* is determined (s), and then it is deleted from the list *LR*. After this process, the current line configuration *LConf* is set to the configuration *BaseConf*. Then, the procedure decreases the number of workers at station s of *LConf* by 1. By this way, it is aimed to reduce the number of workers used in the line which is the secondary objective of the considered MALBP. Note that the main function for searching the best solution based on all objectives is performed by VNS based heuristics proposed.

For illustrating the solution construction process, consider the priority permutation (5, 12, 3, 10, 9, 1, 6, 2, 7, 11, 4, 8) for the problem instance with cycle time 10, $M_{\max} = 3$, and precedence diagram given in Figure 1. All three target values are considered zero. In this case, three different line configurations (a row vector with $n = 12$ elements) are considered for determining initial line configuration: (1) each element is set to the value 3 (i.e., $M_{\max}$), (2) each element is set to the value 2 (i.e., $M_{\max} - 1$), and (3) each element is set to the value 1 (i.e., $M_{\max} - 2$). In order to illustrate the task assignment process, the assignment process for the first line configuration is summarized in Table 1. Resulting solutions for the determination process of the initial line configuration are presented in Figure 3. As is seen from this figure, resulting solutions have 3, 4, and 5 stations, respectively. Also, they have 7, 6, and 5 workers and 4, 2, and 0 worker based smoothness indexes (without square root), respectively. When the solution comparison method given in the next subsection is considered, it is concluded that the best of these three solutions is the one given in Figure 3(a). This solution has 3, 3, and 1 workers for stations 1, 2, and 3, respectively. Hence, the initial line configuration is determined as (3, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3).

After creating the current solution and updating the best solution, the base line configuration *BaseConf* is determined as (3, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3) and the list *LR* is formed as (Station 1, Station 2). Note that Station 3 is not added to this list since it has only one used worker. After executing Steps (6)–(9) of the procedure, the configuration *LConf* is determined as (2, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3), and the list *LR* is updated as (Station 2). The assignment process for this *LConf* configuration and the resulting solution are given in Table 2 and Figure 4(a), respectively. In this case, the current solution has the same number of stations with the best solution and less number of workers than the best solution. Therefore, according to the solution comparison method given in the next subsection, this newly generated solution is better than the best solution. Thus, after updating the best solution, the base line configuration *BaseConf* and the list *LR* are set to (2, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3) and (Station 1, Station 2), respectively. After executing Steps (6)–(9) of the procedure, the configuration *LConf* is determined as (1, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3), and the list *LR* is updated as (Station 2). The resulting solution for this configuration is given in Figure 4(b). As is seen from this figure, the resulting solution is again better than the best one. Thus, the best solution is updated. Then, the base line configuration *BaseConf* and the list *LR* are set to (1, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3) and (Station 2), respectively. After executing Steps (6)–(9) of the procedure again, the configuration *LConf* is determined as (1, 2, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3), and the list *LR* is updated as {}. The resulting solution for this configuration is given in Figure 4(c). As is seen from this figure, the resulting solution is not better than the best one. The process is stopped since there is no element in the current list *LR*.

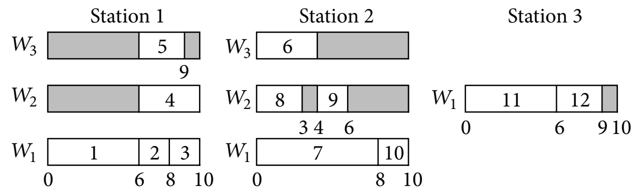
4.3. Comparing Solutions. A simple comparison method is not applicable for comparing solutions since there is more than one objective in the considered problem. Comparison method has to include all objectives with the same priority order. Thus, for comparing two different solutions (Sol_1 and Sol_2), the algorithm which has the flowchart given in Figure 5 is used. Notations s_1 and s_2 represent the number of stations opened in Sol_1 and Sol_2, respectively. Also, notations m_1 and m_2 represent the total number of workers used in Sol_1 and Sol_2, respectively, and notations ns_1 and ns_2 represent worker based smoothness index of Sol_1 and Sol_2, respectively. As is seen from Figure 5, if both solutions have the total

Table 1: Task assignment process for the line configuration (3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3).

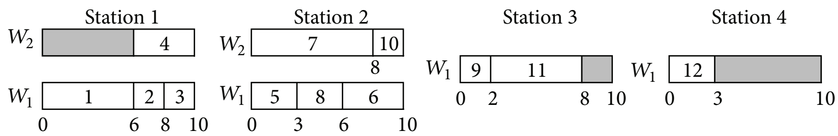
Step	Station index (man count)	Task with no predecessor (priorities)	Selected task	Start time	Man index	Notes
1	1 (3)	1 (5)	1	0	1	—
2		2 (12), 3 (3), 4 (10), 5 (9)	2	6	1	—
3		3 (3), 4 (10), 5 (9), 6 (—)	4	6	2	Task 6 cannot be completed before the end of cycle time.
4		3 (3), 5 (9), 6 (—)	5	6	3	Task 6 cannot be completed before the end of cycle time.
5		3 (3), 6 (—), 8 (—)	3	8	1	Tasks 6 and 8 cannot be completed before the end of cycle time.
6		6 (—), 7 (—), 8 (—)	—	—	—	(i) Tasks 6, 7, and 8 cannot be completed before the end of cycle time. (ii) New station is opened.
7	2 (3)	6 (1), 7 (6), 8 (2)	7	0	1	—
8		6 (1), 8 (2)	8	0	2	—
9		6 (1), 10 (—)	6	0	3	Task 10 cannot be started at time 0.
10		9 (7), 10 (—)	9	4	2	Task 10 cannot be started at time 4.
11		10 (11), 11 (—)	10	8	1	Task 11 cannot be completed before the end of cycle time.
12		11 (—)	—	—	—	(i) Task 11 cannot be completed before the end of cycle time. (ii) New station is opened.
13	3 (3)	11 (4)	11	0	1	—
14		12 (8)	12	6	1	—

Table 2: Task assignment process for the line configuration (2, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3).

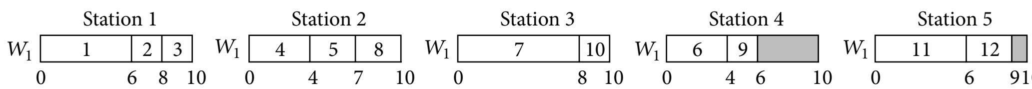
Step	Station index (man count)	Task with no predecessor (priorities)	Selected task	Start time	Man index	Notes
1	1 (2)	1 (5)	1	0	1	—
2		2 (12), 3 (3), 4 (10), 5 (9)	2	6	1	—
3		3 (3), 4 (10), 5 (9), 6 (—)	4	6	2	Task 6 cannot be completed before the end of cycle time.
4		3 (3), 5 (—), 6 (—)	3	8	1	Tasks 5 and 6 cannot be completed before the end of cycle time.
5		5 (—), 6 (—), 7 (—)	—	—	—	(i) Tasks 5, 6, and 7 cannot be completed before the end of cycle time. (ii) New station is opened.
6	2 (3)	5 (9), 6 (1),7 (6)	5	0	1	—
7		6 (1), 7 (6), 8 (—)	7	0	2	Task 8 cannot be started at time 0.
8		6 (1), 8 (—)	6	0	3	Task 8 cannot be started at time 0.
9		8 (2), 9 (—)	8	3	1	Task 9 cannot be started at time 3.
10		9 (7), 10 (—)	9	4	3	Task 10 cannot be started at time 4.
11		10 (11), 11 (—)	10	8	1	Task 11 cannot be completed before the end of cycle time.
12		11 (—)	—	—	—	(i) Task 11 cannot be completed before the end of cycle time. (ii) New station is opened.
13	3 (1)	11 (4)	11	0	1	—
14		12 (8)	12	6	1	—



(a) Solution for the line configuration (3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3)

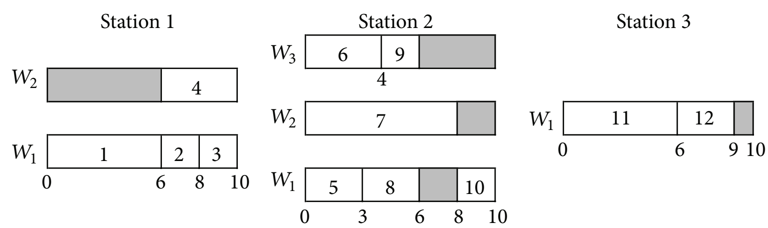


(b) Solution for the line configuration (2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2)

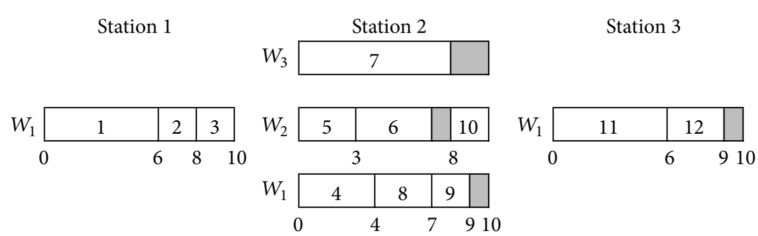


(c) Solution for the line configuration (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)

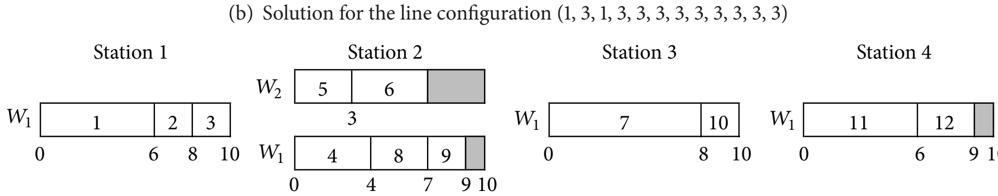
FIGURE 3: Resulting solutions for the determination of the initial line configuration.



(a) Solution for the line configuration (2, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3)



(b) Solution for the line configuration (1, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3)



(c) Solution for the line configuration (1, 2, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3)

FIGURE 4: Illustration of the solution building process.

number of opened stations less than or equal to the target value $L_{\max}$, or both solutions have the same total number of opened stations, then it is decided that there is no difference between two solutions according to the total number of opened stations. The same approach is applied for testing solutions based on both the total number of used workers and worker based smoothness indexes. Note that the target value for smoothness index is always zero, and the check is carried out in the same order with objectives. If there is no difference between two solutions based on three criteria, then

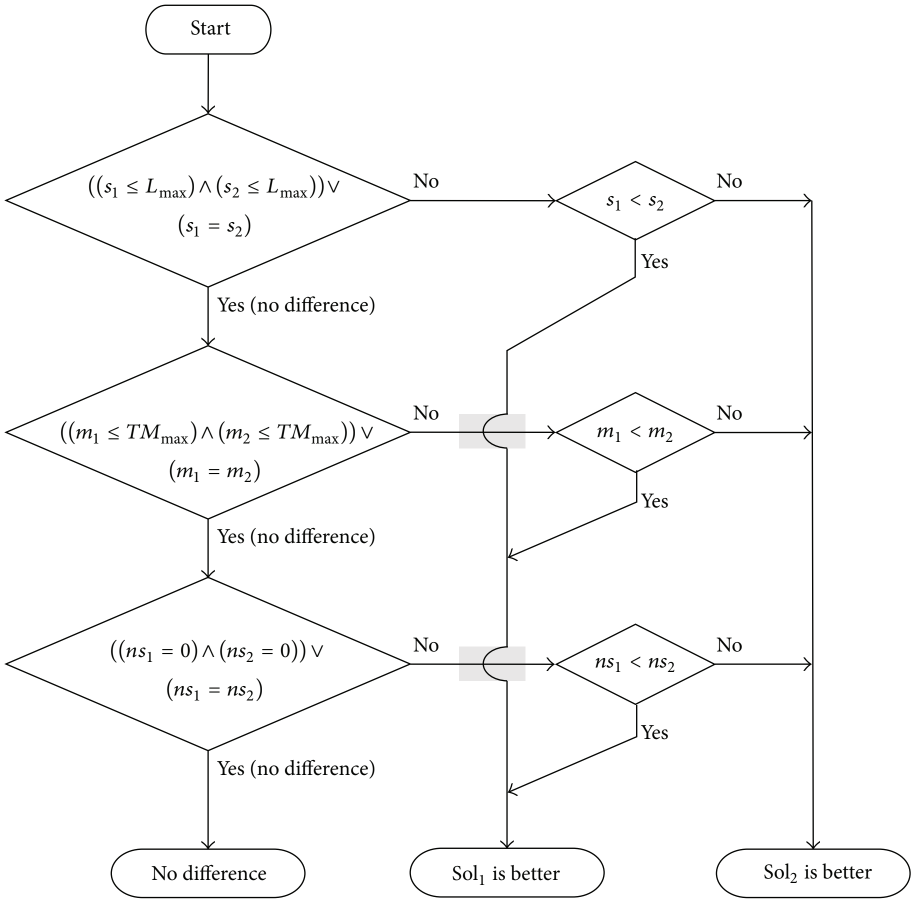


Figure 5: Flowchart of procedure comparing two solutions.

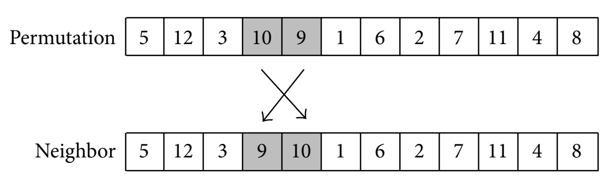


Figure 6: Illustration for producing a neighbor from API neighborhood.

it is decided that there is no difference between two solutions. If there is a difference based on one of three criteria, then it is concluded that the solution having less value based on this criterion is better.

4.4. Neighborhood Structures. For generating neighbors, the following two methods are used in this paper:

(i) adjacent pairwise interchange (API),

(ii) subset randomization (SR).

In a permutation, two adjacent digits are interchanged to generate an API neighbor. This process is illustrated in Figure 6. There are $n-1$ neighbors in API neighborhood of a permutation of the value n [26].

For generating a random neighbor by using SR mechanism, a subset of digits is first selected randomly. Then, unselected digits are copied to the same positions of the neighbor. Last, selected digits are randomly copied to empty positions of the neighbor. For SR structure, it is obvious that there is more than one neighborhood based on the number of digits selected randomly. For generating a neighbor different from its origin, at least two digits have to be selected. Let k and SR^k be the number of randomly selected digits and the neighborhood structure with parameter k, respectively. For a permutation of the value n, the interval of the parameter k is $2 \leq k \leq n$. The SR^2 ($k = 2$) method generates neighbors from general pairwise interchange neighborhood (PI) with probability 0.5. The remaining permutations are the same with their origins. Therefore, for the origin solution x, $SR^2 = PI \cup \{x\}$ is valid. On the other hand, the SR^n method generates random permutations of n. An illustrative example for the method SR^3 is presented in Figure 7.

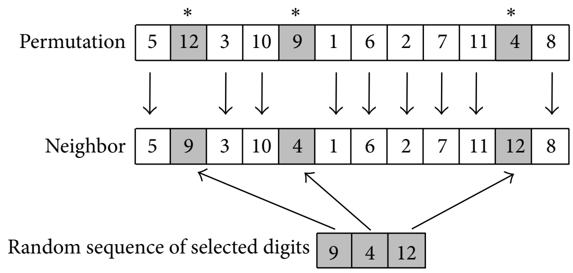


Figure 7: Illustration for producing neighbor from SR^3 neighborhood.

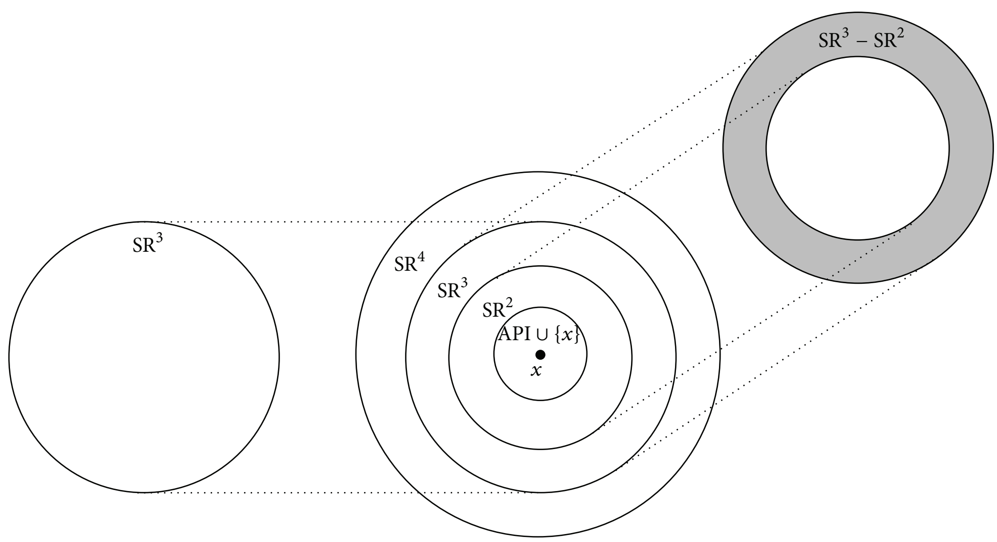


Figure 8: Relationship between neighborhoods.

If only $k-1$ digits of a neighbor which is generated from SR^k has different values from its origin, then this neighbor is also an element of SR^{k-1}. In this case, this neighbor has the same value with its origin at only one of selected k digits. Likewise, if a neighbor has the same values with its origin at two of selected k digits, then this neighbor is also an element of both SR^{k-1} and SR^{k-2}. Thus, the relationship $SR^{k-1} \subset SR^k$ is valid. This relationship is illustrated in Figure 8. If a neighbor $x' \in SR^k$ has a different value from its origin x at each one of k digits, then $x' \notin SR^{k-1}$ is valid. In this case, $x' \in (SR^k - SR^{k-1})$ is also valid. As a result, in neighborhood generation process, ensuring that each one of k positions has different value from its origin also ensures that the generated neighbor is an element of $SR^k - SR^{k-1}$.

4.5. Variations of VNS for Multi-Objective PMALBP. For solving large size problem instances, getting a good initial solution is important. For these instances, local search algorithms often require substantial amounts of running time. Hence, Hansen and Mladenović [27] introduced a new version of VNS called Reduced VNS (RVNS). For decreasing computation time, RVNS heuristic omits the local search step of the basic VNS algorithm. In this paper, RVNS heuristic is also considered.

Hansen et al. [22] declared that decomposition might be worthwhile in heuristics for very large problem instances since the performance of VNS depends in general on the local search subroutine used. Based on this approach, they proposed a new VNS based algorithm called Variable Neighborhood Decomposition Search (VNDS). VNDS decomposes the problem space by fixing all but k attributes in the local search phase for a given solution. For the considered problem in this paper, the solution attributes are priority values of tasks. Thus, priority values of tasks which are not changed in the shaking phase are determined, and the same priority values are fixed in the local search phase. Due to this fixation, the local search phase is carried out by using API structure on a restricted part of the solution (restricted API). For the SR^3 neighbor solution given in Figure 7, all but priority values of tasks 2, 5, and 11 are fixed. Thus, local search phase is applied based on only these three digits. The restricted API neighborhood for this SR^3 solution is given in Figure 9.

In brief, we developed three different heuristics based on VNS variations for solving the multi-objective MALBP.

Table 3: Properties of VNS variations.

Property	VNS	RVNS	VNDS
Initial solution	(i) Generate random. (ii) Apply local search with API structure.	(i) Generate random. (ii) Apply local search with API structure.	(i) Generate random. (ii) Apply local search with API structure.
Shaking phase	$SR^2 - (API \cup \{x\})$ $SR^3 - SR^2$ $\vdots$ $SR^{K_{max}} - SR^{K_{max}-1}$	$SR^2 - (API \cup \{x\})$ $SR^3 - SR^2$ $\vdots$ $SR^{K_{max}} - SR^{K_{max}-1}$	$SR^2 - (API \cup \{x\})$ $SR^3 - SR^2$ $\vdots$ $SR^{K_{max}} - SR^{K_{max}-1}$
Local search phase	API	—	Restricted API
Value of the parameter K_{max}	$\max\{2, \lceil 0.5 \cdot n \rceil\}$	$\max\{2, \lceil 0.7 \cdot n \rceil\}$	$\max\{2, \lceil 0.8 \cdot n \rceil\}$

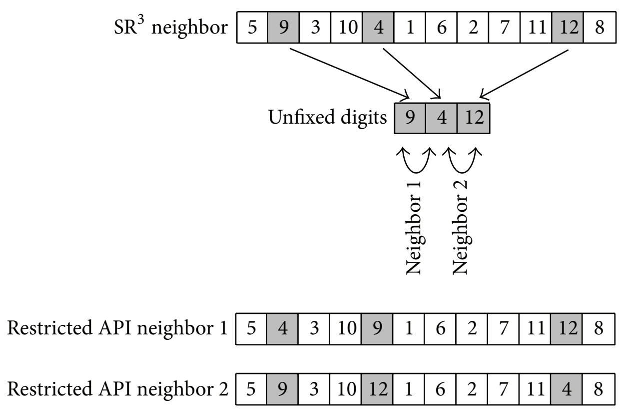


Figure 9: Generating neighbors in the local search phase of VNDS.

These heuristics are summarized in Table 3. For initiating proposed algorithms with a good incumbent solution, local search with API structure is applied to a solution which is generated randomly.

As is seen from Table 3, all methods use only one parameter, K_{max}, which is the top level of SR neighborhood structures used in the heuristic's shaking phase. For identifying the value of this parameter for each method, we carried out preliminarily experiment by using different size preliminary test instances. The K_{max} parameter value is set to each one of $\max\{2, \lceil r \cdot n \rceil\}$ values where n is the number of tasks in the problem instance and r is a control parameter getting a value from the domain $\{0.1, 0.2, \ldots, 1.0\}$. By using each one of r values, ten different K_{max} values are determined based on the size of considered instance. For each parameter value for an instance, each one of VNS, RVNS, and VNDS is run five times. After examining the results of each method, a superior level of the K_{max} parameter is determined. These selected levels are also presented in Table 3.

5. Computational Study

In order to evaluate the proposed MIP and heuristics, experiments were conducted by using benchmark instances. All heuristics were coded in C# language and compiled in the programming software Microsoft Visual C# 2010 Express Edition. MIP models of instances were solved by using the solver IBM ILOG CPLEX (version 12.5.1.0). All experiments were run on a PC with i3 predecessor, 6 GB RAM, and 64 bit Microsoft Windows 7 operating system.

Benchmark instances were generated by the following way. Test problems (task times and precedence relationships) with cycle times were given from the literature. Three small size data sets are P11, P21, and P30. They have 11, 21, and 30 tasks and are obtained from the papers by Jackson [28], Mitchell [29], and Sawyer [30], respectively. Moreover, some medium and large size data sets were also used in experiments. These data sets are P45 [31], P70 [32], P83 and P111 [33], and P148 [34]. Two different levels, 2 and 4, were considered for the parameter M_{max} (admitted maximum number of workers at a station). For each data set, three different values for the cycle time parameter were obtained from the literature. In the value determination process for target value parameters of instances, it was aimed to compare the performance of proposed methods based on the following manners:

(i) minimizing the number of stations, the number of workers, and smoothness index in the same order,

(ii) minimizing the number of workers, and smoothness index in the same order due to given target value of the number of stations,

(iii) minimizing smoothness index due to given target values of the number of stations and workers.

The method which was used to determine target values is presented in the next subsections. For an instance with n tasks, each heuristic was run for $3 \cdot n$ CPU seconds. Also, for every experimental instance, each heuristic was applied 5 times. Each execution of MIP models has a time limit of 3,600 CPU seconds.

5.1. Minimizing the Number of Stations, the Number of Workers, and Smoothness Index in the Same Order. A target value for a variable means that the decision maker desires

Table 4: Results for small and medium size instances for minimizing the number of stations, the number of workers, and smoothness index in the same order.

Problem	Ct	M_{max}	VNS result	RVNS result	VNDS result	MIP result	MIP CPU
P11	7	2	6[a]; 8[b]; 4[c]	6; 8; 4	6; 8; 4	6; 8; 4	0.16; 0.14; 0.33
		4	5; 9; 10	5; 9; 10	5; 9; 10	5; 9; 10	0.14; 0.11; 0.08
	10	2	4; 5; 3	4; 5; 3	4; 5; 3	4; 5; 3	0.14; 0.11; 0.28
		4	3; 6; 5	3; 6; 5	3; 6; 5	3; 6; 5	0.19; 0.11; 0.09
	21	2	2; 3; 1	2; 3; 1	2; 3; 1	2; 3; 1	0.11; 0.09; 0.22
		4	2; 3; 1	2; 3; 1	2; 3; 1	2; 3; 1	0.16; 0.17; 0.2
P21	14	2	7; 8; 6	7; 8; 6	7; 8; 6	7; 8; 6	0.62; 0.75; 0.37
		4	7; 8; 6	7; 8; 6	7; 8; 6	7; 8; 6	0.87; 0.52; 0.89
	21	2	4; 6; 2	4; 6; 2	4; 6; 2	4; 6; 2	0.92; 0.64; 0.58
		4	4; 6; 2	4; 6; 2	4; 6; 2	4; 6; 2	2.18; 1.94; 0.86
	35	2	3; 3; 0	3; 3; 0	3; 3; 0	3; 3; 0	0.67; 1.17; 0.2
		4	3; 3; 0	3; 3; 0	3; 3; 0	3; 3; 0	1.47; 1.94; 0.33
P30	25	2	8; 14; 2	8; 14; 2	8; 14; 2	8; 14; 2	9.1; 56.94; 1.36
		4	8; 14; 2	8; 14; 2	8; 14; 2	8; 14; 2	10.64; 122.27; 6.3
	30	2	7; 12; 2	7; 12; 2	***6.6*; 12; *1.2***	6; 12; 0	103.49; 80.65; 0.89
		4	6; 12; 10	**6; 12; *8***	6; 12; 10	6; 12; 0	16.66; 2822.2; 62.54
	41	2	***4.8*; 8.2; 1.4**	***4.8*; 8; 1.6**	5; 8; 2	4; 8; 0	1741.97; 358.52; 10.05
		4	4; 9; 3	**4; *8.8*; 3.6**	4; 9; 3	4; 8; No	26.52; 594.38; 2979.15
P45	57	2	6; 10; 2	6; 10; 2	6; 10; 2	6[int]; —; —	3600.24; —; —
		4	5; 10; 7	5; 10; 7	5; 10; 7	5; 10[int]; —	121.4; 3479.07; —
	110	2	3; 6; 0	3; 6; 0	3; 6; 0	3; 6[int]; —	315.87; 3284.37; —
		4	3; 6; 0	3; 6; 0	3; 6; 0	3; 6[int]; —	215.03; 3385.43; —
	184	2	2; 3; 1	2; 3; 1	2; 3; 1	2; 4[int]; —	167.22; 3433.02; —
		4	2; 3; 1	2; 3; 1	2; 3; 1	2; 3[int]; —	99.17; 3501.32; —
P70	176	2	12; 22; 2	12; 22; 2	**12; *21.6*; 2.4**	No; —; —	3600.27; —; —
		4	9; 22.8; 6.2	9; 23; 5.6	**9; *22.4*; 6.6**	70[int]; —; —	3601.36; —; —
	364	2	6; 10; 2	6; 10; 2	6; 10; 2	No; —; —	3600.27; —; —
		4	**4; *10.8*; 2.8**	4; 11; 1	4; 11; 1	No; —; —	3600.32; —; —
	468	2	4; 8; 0	4; 8; 0	4; 8; 0	No; —; —	3600.25; —; —
		4	3; 9; 0	**3; *8.8*; 0.2**	3; 9; 0	3; 9[int]; —	1168.64; 2432.65; —

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).
[int]Integer solution was found within a given CPU time limit.
No: no solution was found within a given CPU time limit.
—: phase was not performed since previous phases had consumed the given CPU time.

this variable to have a value not greater than the target value. In this case, for minimization of a variable, there is no difference between values equal to or smaller than the target value. From this point on, for minimizing the number of stations, the number of workers, and smoothness index in the same order, all target values are set to zero. Results for all heuristics and MIP models are given together in Table 4. Results of heuristics in this table are presented based on mean values. Note that results of MIP models are also equal to deviation values since all target values are equal to zero. For each instance, heuristic results are compared with each other based on the comparison method given in Figure 5. As is seen from Table 4, there is no difference between all methods for instances of P11 and P21 since all heuristics found optimal solutions. For one instance (with Ct = 41 and M_{max} = 4) in the problem group P30, the MIP formulation was not able to find the optimal solution. Also, the remaining instances (P45 and P70) could not be solved by using the MIP formulation. For example, for the instance P45 with Ct = 57, and M_{max} = 2, the MIP formulation was only able to find an integer solution for goal 1 within the given CPU time limit. Hence, the remaining phases for the remaining

Table 5: Results for large size instances for minimizing the number of stations, the number of workers, and smoothness index in the same order.

Problem	Ct	M_{max}	VNS result	RVNS result	VNDS result
P83	5048	2	10[a]; 16[b]; 4[c]	10; 16; 4	10; 16; 4
		4	**9; *17.2*; 12**	9; 17.4; 14.4	9; 17.4; 13.2
	6842	2	7.6; 12.4; 2.8	7.4; 12.6; 2.2	7.6; 12.4; 2.8
		4	7; 13; 5.8	**7; 13; *5.4***	7; 13; 7.6
	7571	2	6.2; 11; 1.4	**6; 11; 1**	**6; 11; 1**
		4	6; 11; 1	6; 11; 1	6; 11; 1
P111	5755	2	15.8; 28.4; 3.2	***15.6*; 28.2; 3**	15.8; 28.2; 3.4
		4	12.2; 29.4; 36	***12*; 29.4; 42.2**	***12*; 30; 41.6**
	8847	2	11; 18; 4	11; 18; 4	11; 18; 4
		4	10; 18.2; 27.4	**10; *18*; 21.2**	**10; *18*; 22**
	10743	2	8.4; 15; 1.8	**8; 15; 1**	**8; 15; 1**
		4	7; 15.6; 28.4	**7; *15.4*; 24.4**	**7; *15.4*; 25.2**
P148	434	2	7; 14; 0	7; 14; 0	7; 14; 0
		4	5; 14; 13.2	***4.4*; 14; 2.4**	4.8; 14; 1.6
	626	2	5; 10; 0	5; 10; 0	5; 10; 0
		4	3; 10; 2	3; 10; 2	3; 10; 2
	805	2	4; 8; 0	**4; *7.8*; 0.2**	4; 8; 0
		4	2.6; 8; 0.6	***2*; 8; 0**	2.2; 8; 0.2

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).

goals were not carried out (sign "–"). For the instance P45 with Ct = 57, and M_{max} = 4, the MIP formulation was able to find the optimum solution for goal 1 within 121.4 CPU seconds. Therefore, phase 2 was started. In phase 2, only an integer solution could be found within the given CPU time limit (121.4 + 3479.07 = 3600.47 CPU seconds). Hence, the last phase was not carried out. The overall evaluation of the results found in this group is as follows. All heuristics were able find optimal solutions for 14 instances whose optimal solutions were found by the MIP formulation. There are 8 instances for which heuristics showed different performance. For 2 of them (25%), VNS was able to find better solutions than at least one of other heuristics. RVNS and VNDS found better solutions for 4 (50%) and 3 (37.5%) instances, respectively.

Results for large size instances are presented in Table 5. There are 18 instances in this group. All heuristics found the same solution for each of 7 instances. For one of the remaining 11 instances, VNS was able find better solution than at least one of other two heuristics (9%). On the other hand, RVNS found better solutions for 10 instances (91%). Lastly, VNDS has better performance than at least one of other heuristics for 5 instances (46%). For these 5 instances, note that RVNS found the same solutions with VNDS. Hence, VNDS outperformed only VNS for this group of instances.

5.2. Minimizing the Number of Workers and Smoothness Index in the Same Order due to Given Target Value of the Number of Stations. As stated earlier, there is no difference between values which are in the desired interval determined by a target value for a variable. For example, if the target value for the number of stations is set 4, then values 2, 3, and 4 are not superior against each other. On the other hand, in this case, these values are superior against values greater than 4, and the value 5 is superior against values greater than 5 and so forth. For comparing the performance of heuristics according to the minimization of the number of workers and smoothness index in the same order, a target value of the number of stations for each instance was determined. Care has been taken to ensure that all heuristics could reach target values. Hence, for an instance, a target value of the number stations was set to the maximum of 15 values found in Section 5.1. Note that each of three heuristics was applied 5 times for an instance. Due to goals of minimizing the number of workers and smoothness index, target values for these goals were set to zero.

The results of the computational experiments for small and medium size instances are presented in Table 6. As is seen from this table, instances having more than 30 tasks could not be solved by the MIP formulation. For instances in sets P11, P21, and P45, there is no difference between heuristics according to their results. For the problem group P30, VNS and VNDS produced better solutions for one instance and two instances, respectively. A similar observation may be done for the problem group P70 since VNDS emerges as the best method for this group. Shortly, for 6 medium size

Table 6: For a given target value of the number of stations, results for small and medium size instances for minimizing the number of workers, and smoothness index in the same order.

Problem	Ct	M_{max}	Target # stations	VNS result	RVNS result	VNDS result	MIP		
							d_1^+	Result	CPU
P11	7	2	6	6[a]; 8[b]; 4[c]	6; 8; 4	6; 8; 4	0	6; 8; 4	0.12; 0.16; 0.33
		4	5	5; 9; 10	5; 9; 10	5; 9; 10	0	5; 9; 10	0.16; 0.11; 0.09
	10	2	4	4; 5; 3	4; 5; 3	4; 5; 3	0	4; 5; 3	0.11; 0.16; 0.28
		4	3	3; 6; 5	3; 6; 5	3; 6; 5	0	3; 6; 5	0.09; 0.13; 0.11
	21	2	2	2; 3; 1	2; 3; 1	2; 3; 1	0	2; 3; 1	0.13; 0.13; 0.22
		4	2	2; 3; 1	2; 3; 1	2; 3; 1	0	2; 3; 1	0.08; 0.19; 0.2
P21	14	2	7	7; 8; 6	7; 8; 6	7; 8; 6	0	7; 8; 6	0.59; 0.75; 0.38
		4	7	7; 8; 6	7; 8; 6	7; 8; 6	0	7; 8; 6	2.22; 0.53; 0.92
	21	2	4	4; 6; 2	4; 6; 2	4; 6; 2	0	4; 6; 2	0.94; 0.66; 0.56
		4	4	4; 6; 2	4; 6; 2	4; 6; 2	0	4; 6; 2	2.57; 2.01; 0.84
	35	2	3	3; 3; 0	3; 3; 0	3; 3; 0	0	3; 3; 0	0.61; 1.17; 0.17
		4	3	3; 3; 0	3; 3; 0	3; 3; 0	0	3; 3; 0	2.36; 1.75; 0.28
P30	25	2	8	8; 14; 2	8; 14; 2	8; 14; 2	0	8; 14; 2	13.12; 56.16; 1.37
		4	8	8; 14; 2	8; 14; 2	8; 14; 2	0	8; 14; 2	6.04; 122.3; 6.29
	30	2	7	7; 12; 2	7; 12; 2	**6.6; 12; *1.2***	0	7; 12[int]; —	7.55; 3592.52; —
		4	6	6; 12; 10	6; 12; 10	6; 12; 10	0	6; 12; 0	16.37; 3052.52; 67.28
	41	2	5	5; 8.2; 1.8	4.8; 8.2; 1.4	**4.6; *8*; 1.2**	0	5; 8; 0	12.61; 467.86; 42.62
		4	4	**4; *8.8*; 2.4**	4; 9; 3	4; 9; 3	0	4; 8; No	10.94; 639.06; 2950.07
P45	57	2	6	6; 10; 2	6; 10; 2	6; 10; 2	0	6; 10[int]; —	253.77; 3346.47; —
		4	5	5; 10; 7	5; 10; 7	5; 10; 7	0	5; 10[int]; —	73.91; 3526.56; —
	110	2	3	3; 6; 0	3; 6; 0	3; 6; 0	0	3; 6[int]; —	54.09; 3546.15; —
		4	3	3; 6; 0	3; 6; 0	3; 6; 0	0	3; 6[int]; —	95.38; 3505.09; —
	184	2	2	2; 3; 1	2; 3; 1	2; 3; 1	0	2; 3[int]; —	39.28; 3560.96; —
		4	2	2; 3; 1	2; 3; 1	2; 3; 1	0	2; 3[int]; —	60.23; 3540.24; —
P70	176	2	12	12; 22; 2	12; 22; 2	12; 22; 2	No	No; —; —	3600.29; —; —
		4	9	**9; *22.8*; 10.8**	9; 23; 6	**9; *22.8*; 10**	0	9; 22[int]; —	497.19; 3104.08; —
	364	2	6	6; 10; 2	6; 10; 2	6; 10; 2	No	No; —; —	3600.27; —; —
		4	4	4; ***11***; 1	4; 11.2; 0.8	**4; *11*; 1**	0	4; 12[int]; —	398.3; 3202.97; —
	468	2	4	4; 8; 0	4; 8; 0	4; 8; 0	No	No; —; —	3600.28; —; —
		4	3	3; 9; 0	**3; *8.8*; 0.2**	**3; *8.8*; 0.2**	0	3; 9[int]; —	357.02; 3244.26; —

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).
[int]Integer solution was found within a given CPU time limit.
No: no solution was found within a given CPU time limit.
—: phase was not performed since previous phases had consumed the given CPU time.

instances for which heuristics showed different performance, VNDS produced better solutions than at least one of other two heuristics.

For large size instances, heuristics' results are given in Table 7. For 7 of 18 instances in this group, all three heuristics obtained the same results. For 9 of the remaining 11 instances (82%), RVNS heuristic outperformed at least one of other two heuristics.

5.3. Minimizing Smoothness Index due to Given Target Values of the Number of Stations and Workers. The same method with the one for generating target values for the number of stations was used to obtain target values for the number of workers. The target value for the number of workers for an instance was set to the maximum of 15 values obtained by heuristics for the number of workers in Section 5.1.

The results for small and medium size instances are presented in Table 8. The following observations can be made from this table. By using the MIP formulation, instances with target values may be optimally solved more easily since more instances are solved optimally. Also, in this group, there are only 4 instances for which heuristics showed

Table 7: For a given target value of the number of stations, results for large size instances for minimizing the number of workers, and smoothness index in the same order.

Problem	Ct	$M_{\max}$	Target # stations	VNS result	RVNS result	VNDS result
P83	5048	2	10	10[a]; 16[b]; 4[c]	10; 16; 4	10; 16; 4
		4	9	**9; *17.4*; 13.6**	9; 17.6; 13.4	9; 17.6; 13.4
	6842	2	8	8; 12; 4	8; 12; 4	8; 12; 4
		4	7	7; 13; 3.6	**7; *12.8*; 3.8**	7; 13; 8
	7571	2	7	6.4; 11; 1.8	**6; 11; *1***	6.2; 11; 1.4
		4	6	6; 11; 3	**6; 11; *1***	**6; 11; *1***
P111	5755	2	16	15.8; 28.2; 3.4	**16; *28*; 4**	16; 28.6; 3.4
		4	13	13; 29; 33.2	13; 28.8; 23.2	**13; *28.6*; 39.8**
	8847	2	11	11; 18; 4	11; 18; 4	11; 18; 4
		4	10	10; 18; 36	**10; 18; *20.4***	10; 18; 21.2
	10743	2	9	8.8; 15; 2.6	**8.4; 15; *1.8***	9; 15; 3
		4	7	7; 15.8; 16.8	**7; *15.4*; 25.2**	7; 16; 15.4
P148	434	2	7	7; 14; 0	7; 14; 0	7; 14; 0
		4	5	5; 14; 8	**5; 14; *1***	5; 14; 3.2
	626	2	5	5; 10; 0	5; 10; 0	5; 10; 0
		4	3	3; 10; 2	3; 10; 2	3; 10; 2
	805	2	4	4; 8; 0	4; 8; 0	4; 8; 0
		4	3	2.6; 8; 0.6	**2; 8; *0***	2.4; 8; 0.4

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).

different performance. For 3 of these 4 instances (75%), RVNS obtained better solutions than other two heuristics. For the remaining instance, VNDS was superior against other two heuristics.

Similar observations can be made for large size instances whose results are given in Table 9. As is seen from this table, VNS was not able to outperform other heuristics even for a single instance. On the other hand, for 10 instances, at least one of other two heuristics outperformed VNS. For these 10 instances, RVNS obtained the same (for 3 instances) or better results (for 5 instances) than VNDS. For the remaining 2 instances, VNDS outperformed RVNS.

6. Conclusions and Future Works

In this research, multi-objective MALBP is considered. First, a MIP model based on goal programming approach is developed. Then, after defining the problem, three different variations of VNS algorithm are proposed to solve multi-objective MALBP heuristically. Experiment study is performed based on benchmark instances ranging in size from small to large for different target values of problem parameters.

Based on well-known goal programming approach, this study is the first one attempting to define the line balancing problem with multi-manned stations and to propose solution methods to solve it. After analyzing the results of experimental study, it has been seen that only small size instances could be solved optimally. On the other hand, if the instance has target values attainable, then its optimal solution may be found more easily. For small size instances, it has been seen that there is no significant difference between proposed VNS variations. However, for medium and especially large size instances, RVNS mostly outperforms other two heuristics.

A possible future research direction is to develop mathematical programming models for other goal, such as task based smoothness index and cost. Also, to the best of authors' knowledge, there is no study on the MALBP which has layout different from straight layout, such as U line and parallel line. Therefore, there is a need to develop solution algorithms for MALBP having different line layouts.

Competing Interests

The author declares that there are no competing interests.

Table 8: For given target values of the number of stations and workers, results for small and medium size instances for minimizing smoothness index.

Problem	Ct	$M_{\max}$	Target		VNS result	RVNS result	VNDS result	MIP			
			# stations	# workers				d_1^+	d_2^+	Result	CPU
P11	7	2	6	8	6[a]; 8[b]; 4[c]	6; 8; 4	6; 8; 4	0	0	6; 8; 4	0.14; 0.08; 0.33
		4	5	9	5; 9; 10	5; 9; 10	5; 9; 10	0	0	5; 9; 10	0.16; 0.08; 0.09
	10	2	4	5	4; 5; 3	4; 5; 3	4; 5; 3	0	0	4; 5; 3	0.13; 0.11; 0.31
		4	3	6	3; 6; 5	3; 6; 5	3; 6; 5	0	0	3; 6; 5	0.08; 0.08; 0.09
	21	2	2	3	2; 3; 1	2; 3; 1	2; 3; 1	0	0	2; 3; 1	0.11; 0.08; 0.23
		4	2	3	2; 3; 1	2; 3; 1	2; 3; 1	0	0	2; 3; 1	0.09; 0.08; 0.19
P21	14	2	7	8	7; 8; 6	7; 8; 6	7; 8; 6	0	0	7; 8; 6	0.58; 0.27; 0.39
		4	7	8	7; 8; 6	7; 8; 6	7; 8; 6	0	0	7; 8; 6	2.22; 0.83; 0.91
	21	2	4	6	4; 6; 2	4; 6; 2	4; 6; 2	0	0	4; 6; 2	0.92; 0.28; 0.58
		4	4	6	4; 6; 2	4; 6; 2	4; 6; 2	0	0	4; 6; 2	2.62; 0.59; 0.89
	35	2	3	3	3; 3; 0	3; 3; 0	3; 3; 0	0	0	3; 3; 0	0.58; 1.61; 0.17
		4	3	3	3; 3; 0	3; 3; 0	3; 3; 0	0	0	3; 3; 0	2.06; 1.2; 0.28
P30	25	2	8	14	8; 14; 2	8; 14; 2	8; 14; 2	0	0	8; 14; 2	13.03; 0.81; 1.37
		4	8	14	8; 14; 2	8; 14; 2	8; 14; 2	0	0	8; 14; 2	6.02; 4.57; 6.3
	30	2	7	12	7; 12; 2	**6.8; 12; *1.6***	7; 12; 2	0	0	7; 12; 0	7.55; 1.29; 8.18
		4	6	12	6; 12; 8	6; 12; 10	**6; 12; *6***	0	0	6; 12; 0	15.12; 1.3; 62.39
	41	2	5	9	5; 9; 1	5; 9; 1	5; 9; 1	0	0	5; 8; 0	11.73; 1.36; 142.48
		4	4	9	4; 9; 3	**4; 8.8; *2.4***	4; 9; 3	0	0	4; 8; 0	10.17; 2.96; 2977.37
P45	57	2	6	10	6; 10; 2	6; 10; 2	6; 10; 2	0	No	6; No; —	253.24; 3346.86; —
		4	5	10	5; 10; 7	5; 10; 7	5; 10; 7	0	0	5;10;No	73.68; 106.18; 3420.32
	110	2	3	6	3; 6; 0	3; 6; 0	3; 6; 0	0	0	3; 6; 0	53.95; 2.28; 2.23
		4	3	6	3; 6; 0	3; 6; 0	3; 6; 0	0	0	3; 6; 0	95.22; 7.64; 23.46
	184	2	2	3	2; 3; 1	2; 3; 1	2; 3; 1	0	0	2; 3; No	39.22; 527.08; 3033.8
		4	2	3	2; 3; 1	2; 3; 1	2; 3; 1	0	0	2; 3; 1[int]	60.23; 2601.69; 938.55
P70	176	2	12	22	12; 22; 2	12; 22; 2	12; 22; 2	No	—	No; —; —	3600.27; —; —
		4	9	23	9; 23; 6	**9; 23; *5.2***	9; 23; 9.4	0	0	9; 23; No	496.6; 121.28; 2982.48
	364	2	6	10	6; 10; 2	6; 10; 2	6; 10; 2	No	—	No; —; —	3600.27; —; —
		4	4	12	4; 12; 0	4; 12; 0	4; 12; 0	0	0	4; 12; 0	398.05; 66.35; 75.41
	468	2	4	8	4; 8; 0	4; 8; 0	4; 8; 0	No	—	No; —; —	3600.3; —; —
		4	3	9	3; 9; 0	3; 9; 0	3; 9; 0	0	0	3; 9; 0	355.39; 62.01; 137.83

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).
[int]Integer solution was found within a given CPU time limit.
No: no solution was found within a given CPU time limit.
—: phase was not performed since previous phases had consumed the given CPU time.

Table 9: For given target values of the number of stations and workers, results for large size instances for minimizing smoothness index.

Problem	Ct	$M_{\max}$	Target		VNS result	RVNS result	VNDS result
			# stations	# workers			
P83	5048	2	10	16	10[a]; 16.2[b]; 3.8[c]	**10; *16*; 4**	**10; *16*; 4**
		4	9	18	9; 18; 13	9; 18; 12.6	**9; 18; *12.2***
	6842	2	8	12	8; 12; 4	8; 12; 4	8; 12; 4
		4	7	13	7; 13; 7.6	**7; 13; *1***	7; 13; 7.6
	7571	2	7	11	6.6; 11; 2.2	**6.4; 11; *1.8***	**6.4; 11; *1.8***
		4	6	11	6; 11; 1	6; 11; 1	6; 11; 1
P111	5755	2	16	29	16; 29; 3	16; 29; 3	**15.8; 29; *2.6***
		4	13	29	13; 29; 15.6	**13; 29; *14.8***	13; 29; 23
	8847	2	11	18	11; 18; 4	11; 18; 4	11; 18; 4
		4	10	18	10; 18; 21.2	**10; 18; *20.4***	10; 18; 28.4
	10743	2	9	15	8.4; 15; 1.8	**8.2; 15; *1.4***	8.8; 15; 2.6
		4	7	17	7; 17; 9.4	**7; 17; *5.6***	7; 17; 7.8
P148	434	2	7	14	7; 14; 0	7; 14; 0	7; 14; 0
		4	5	14	5; 14; 11.4	**5; 14; *1***	**5; 14; *1***
	626	2	5	10	5; 10; 0	5; 10; 0	5; 10; 0
		4	3	10	3; 10; 2	3; 10; 2	3; 10; 2
	805	2	4	8	4; 8; 0	4; 8; 0	4; 8; 0
		4	3	8	3; 8; 1	**2; 8; *0***	2.2; 8; 0.2

[a]Mean number of stations.
[b]Mean number of workers.
[c]Mean smoothness index (without square root).

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Research Article

A Biobjective and Trilevel Programming Model for Hub Location Problem in Design of a Resilient Power Projection Network

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Hubs disruptions are taken into account in design of a resilient power projection network. The problem is tackled from a multiple criteria decision-making (MCDM) perspective. Not only the network cost in normal state is considered, but also the cost in the worst-case situation is taken into account. A biobjective and trilevel integer programming model is proposed using game theory. Moreover, we develop a metaheuristic based on tabu search and shortest path algorithm for the resolution of the complex model. Computational example indicates that making tradeoffs between the performances of the network in different situations is helpful for designing a resilient network.

1. Introduction

Power projection is a term used in military to refer to the capacity of a state to apply national transportation network to rapidly and effectively deploy and sustain forces in and from multiple dispersed locations to respond to crises, to contribute to deterrence, and to enhance regional stability [1]. This ability is a crucial element of a state's military power in modern warfare. It can not only guarantee the victory of the war but also help to achieve the military strategic objectives. Generally, in the process of power projection, the troops are firstly consolidated at transportation hubs by means of motorization march and then they are shipped between hubs links using different modes of transportation (highway, railway, air transport, etc.). Finally they are deployed to dispersed locations through motorization march according to the mission demands. The ability to project power relies on safe and efficient transportation network. Hub location-allocation strategies are critical issues in design of the projection network, which have significant influence on the efficiency and survivability of the network.

Traditional studies dealing with the hub location problem in transportation network usually locate hub facilities and allocate spokes to those hubs in order to minimize the total transportation cost in normal state [2–4]. However, due to the military characteristics of power projection, the network faces intentional attacks from the enemy or interdictor. Some important hub facilities would be disrupted in wartime. In practice, a network would not be completely failed even though one or more nodes have become invalid. The network continues to serve the remaining connected components and would recover through emergency management measures, such as traffic rerouting. The ability of a network to withstand and reduce the impact of disruptions is called resilience [5], which is a key indicator of the network survivability.

In recent years, a number of works have been done emphasizing resilience of transportation systems. Ip and Wang [6] introduced network resilience as a function of the number of reliable paths between all node pairs. Zhao et al. [7] developed a resilience metrics including availability, connectivity, and accessibility in a military supply network. Resilience of a transportation network in Miller-Hooks et al. [8] is defined as the expected fraction of demand that can be satisfied after disaster with budget constraints. Konak and Bartolacci [9] incorporated network resilience into network designs using a stochastic genetic approach. Janić [10] dealt with estimating the resilience of an air transport network affected by a large-scale disruptive event. More relevant works can be found in the review paper of Reggiani et al. [11].

Dealing with the hub location problem in design of a projection network, not only the transportation costs in normal state should be considered, but also the ability of withstanding and reducing the impact of disruptions in wartime should be taken into account. Hence, the hub location problem in power projection network is a multiple criteria decision-making (MCDM) problem from the resilience perspective. An MCDM problem refers to a situation in which there are at least two alternative courses of action to choose from and this choice is driven by the desire to meet multiple, often conflicting, objectives [12]. There are different classifications of MCDM problems and methods. The classification proposed by Pardalos et al. [13] distinguishes four categories: (1) multiobjective mathematical programming, (2) multiattribute utility theory, (3) outranking relations approach, and (4) preference disaggregation approach. Generally, when the criteria are nonconflicting, an MCDM problem can be solved by combining multiple objectives into one objective using a specific method such as the weighted sum model (WSM) and the analytic hierarchy process (AHP) [14]. On the contrary, when the criteria are conflicting, multiobjective optimization based on evolutionary algorithms is often used to identify the Pareto frontier [15]. Methods such as multiobjective simulated annealing (MOSA) [16], Pareto iterated local search (PILS) [17], and others have been developed. For a review of MCDM, see de Almeida et al. [18].

Our contribution attempts to analyze the hub location problem in power projection network in view of game theory. We establish a biobjective and trilevel integer programming model, which can be used to make tradeoffs between the performance of the network in normal and interdicted state. A multiobjective tabu search approach based on shortest path algorithm is developed to solve the model. A computational example is presented to illustrate the model and algorithm. The paper ends with some concluding remarks.

2. Problem Description

In the process of power projection, multiple modes of transportation are used to deliver personnel and equipment, which belongs to intermodal transport. Intermodal transport is characterized by the combination of the advantages of rail and road, rail for long distances and large quantities, and road for collecting and distributing over short or medium distance [19]. In this contribution, the motorized march process of power projection can be regarded as the road mode in the intermodal transport. And the backbone transport is usually served by railway or by highway. Commonly, the structure of a power projection network is hub-spoke distribution paradigm [20], in which all traffic moves along spokes connected to the hub at the center. In the power projection network, the starting and end points of the troops can be viewed as nonhub nodes, while the rally points can be regarded as the hub nodes.

It is clear that hubs have higher connectivity than nonhub nodes. In contrast to the failure of nonhubs and links, the failure of hubs has more impact on the resilience of the network [21]. Therefore, in this paper, we focus on the issue of hub location strategy dealing with probable hub disruptions in design of a power projection network. When parts of the hub nodes are disrupted and cannot be restored in limited time, common resilience management actions are rerouting the traffic flows through the remaining connected hubs. Traffic flows rerouting can ensure the completion of projection mission. However, the cost and time of power projection will increase. Since the transportation time would greatly exceed the scheduled time if we reroute the traffic flow, we consider the resilience of the power projection network from a cost increasing perspective without time restraint. In this paper, the resilience of the power projection network is defined as follows [5]:

$$R = \frac{C_{\text{before}}}{C_{\text{after}}}, \tag{1}$$

where R is the resilience of the power projection network, C_{before} is the transportation costs of the network in normal situation, and C_{after} is the transportation costs of the network when some hub nodes have been disrupted.

When dealing with the design of hub location strategy in power projection network, on the one hand, we need to consider minimize to total cost of the network in the normal state, that is Minimize(C_{before}); on the other hand, we hope that the increased cost of the network would not be much higher once some hub nodes are disrupted, that is, Minimize(C_{after}). However, in view of game theory, the enemy may select the hubs for attack which result in the greatest damage to the network [22], that is, Maximize(C_{after}). Therefore, at the beginning of the design we should take the worst-case into account [23], that is, Minimize(Maximize(C_{after})). Besides, after the interdiction, traffic rerouting should follow the least-cost principle; in other words, each origin-destination (O-D) pair is served by its least-cost route via the remaining hubs. Thus, there is $C_{\text{after}} = \text{Minimize}(C')$, where C' is the transportation cost of the network served by the remaining hubs after attack.

From what is mentioned above, we can conclude that the hub location strategy in power projection networks for resilience perspective is a biobjective and trilevel integer programming problem.

3. Mathematical Model

3.1. Mathematical Hypothesis. The power projection procedure can be viewed as the intermodal transportation process in hub-spoke network and the basic assumptions are as follows.

(1) The hub network is complete with the presence of a direct hub link between each hub pair. And not all the hubs would be disrupted at the same time.

(2) There is a cost discount factor using hub links due to economies of scale.

(3) A nonhub node can be allocated to more than one hub node and direct links between nonhub nodes are not permitted.

(4) The distance and transportation cost satisfy the triangle inequality, so that a shipment between any O-D pairs may travel at most two hub nodes.

(5) The hubs and links are uncapacitated.

(6) A hub facility loses all its capacity if it was disrupted. However, the traffic demand from or to this node would not be eliminated. That is, the hub node becomes a nonhub node after disruption.

(7) The enemy would select the hubs for attack which result in the greatest damage to the network, and we should take the worst-case into account in design.

(8) When some hub nodes are disrupted, traffic rerouting should follow the least-cost rule.

3.2. Model Parameters. Notations employed in the model are introduced as follows:

$N = \{1, 2, \ldots, n\}$: set of all nodes,

H: set of hubs,

p: number of hubs,

q: number of disrupted hubs, where $q < p$,

$s = \{1, 2\}$: set of transportation modes between hub pairs, where $s = 1$ for road transportation and $s = 2$ for railway transportation,

w_{ij}: traffic demand from origin node $i \in N$ to destination node $j \in N$, where $w_{ii} = 0$,

F_k: fixed cost of opening and operating a hub at node k,

c_{ij}^s: unit transportation cost from node i to node j using transportation mode s, where $c_{ii}^s = 0$,

$\widehat{c}_{km}^s$: drayage and operating cost of one unit transportation via hubs j and m, where $\widehat{c}_{kk}^s = 0$,

$C_{ijkm}^s = c_{ik}^1 + \widehat{c}_{km}^s + \alpha^s c_{km}^s + c_{mj}^1$: unit transportation cost from origin node i to destination j via hubs k and m using mode s,

α^s: transportation cost discount factor between hubs using mode s.

3.3. Decision Variables. There are five sets of decision variables in the model:

$$
\begin{aligned}
y_k &= \begin{cases} 1, & \text{if node } k \text{ is a hub;} \\ 0, & \text{otherwise,} \end{cases} \\
z_k &= \begin{cases} 1, & \text{if node } k \text{ is disrupted;} \\ 0, & \text{otherwise,} \end{cases} \\
X_{ijkm}^s &= \begin{cases} 1, & \text{if flow from } i \text{ to } j \text{ via hub pair } (k, m) \text{ using mode } s; \\ 0, & \text{otherwise,} \end{cases} \\
U_{ijkm}^s &= \begin{cases} 1, & \text{if flow from } i \text{ to } j \text{ via hub pair } (k, m) \text{ using mode } s \text{ after } q\text{-disruption;} \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
\tag{2}
$$

3.4. Biobjective and Trilevel Integer Programming Model. Based on the hypothesis and notations mentioned above, the hub location strategy in power projection networks form resilience perspective is modeled as

$$
\begin{aligned}
\text{Minimize}\quad & M_1 \\
&= \sum_{i\in N}\sum_{j\in N}\sum_{k\in N}\sum_{m\in N}\sum_{s\in S} w_{ij} X_{ijkm}^s C_{ijkm}^s \\
&\quad + \sum_{k\in N} F_k y_k
\end{aligned}
\tag{3}
$$

$$\text{Minimize}\quad M_2 \tag{4}$$

$$\text{subject to}\quad \sum_{k\in N} y_k = p \tag{5}$$

$$\sum_{k\in N}\sum_{m\in N}\sum_{s\in S} X_{ijkm}^s = 1, \quad \forall i, j \in N \tag{6}$$

$$X_{ijkm}^s \le y_k, \quad \forall i, j, k, m \in N,\ s \in S \tag{7}$$

$$X_{ijkm}^s \le y_m, \quad \forall i, j, k, m \in N,\ s \in S \tag{8}$$

$$y_k, X_{ijkm}^s \in \{0, 1\} \quad \forall i, j, k, m \in N,\ s \in S, \tag{9}$$

where

$$M_2 = \text{Maximize} \quad C' + \sum_{k \in N} F_k z_k \tag{10}$$

$$\text{subject to} \quad \sum_{k \in N} z_k = q \tag{11}$$

$$z_k \in \{0,1\} \quad \forall k \in H \tag{12}$$

$$\forall i, j, k, m \in N \tag{13}$$

$$\text{Minimize} \quad C' = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in N} \sum_{s \in S} w_{ij} U^s_{ijkm} C^s_{ijkm} \tag{14}$$

$$\text{subject to} \quad \sum_{k \in N} \sum_{m \in N} \sum_{s \in S} U^s_{ijkm} = 1, \quad \forall i, j \in N \tag{15}$$

$$U^s_{ijkm} \le y_k (1 - z_k), \quad \forall i, j, k, m \in N, \; s \in S \tag{16}$$

$$U^s_{ijkm} \le y_m (1 - z_m), \quad \forall i, j, k, m \in N, \; s \in S \tag{17}$$

$$U^s_{ijkm} \in \{0,1\} \quad \forall i, j, k, m \in N, \; s \in S. \tag{18}$$

The mathematical model above is a biobjective and trilevel integer programming. The first level program has two objective functions from the network designer's perspective, one of which seeks to minimize the total network cost in the normal situation presented by objective function (3), while the other seeks to minimize the total network cost in the worst-case (after q-disruption) presented by objective function (4). In the objective function (3), the first term in right hand is the network transportation cost; the second term is the fixed costs of hubs. Constraint (5) requires that there are p hubs to be located. Constraints (6) refer to the choice of a specific arc for interhub travel, which ensure that each O-D pair (i, j) is assigned to exactly one using one transportation mode between hubs. Constraints (7) and (8) guarantee that each O-D pair can only be assigned to an existing hub pair. Constraints (9) are binary requirements.

The second level program constitutes a worst-case scenario presented by objective function (10). In the worst case, the enemy (or attacker) would choose the hubs for attack which maximize the total network cost. The first term in the objective function (10) calculates the transportation cost after q-disruption; the second term calculates the value of the q hubs which are interdicted. Constraint (11) implies that there are q hubs disrupted. Constraints (12) are integrity constraints.

The third level program is to determine the least-cost route for each O-D pair in the existing network after q hubs failure. The objective function is presented by (14) from the users' perspective. If there are q hubs disrupted, traffic flows in the network would be rerouted through the remaining hubs, as presented by constraints (15). Constraints (16) and (17) prevent an assignment to a hub that has been disrupted. Constraints (18) are binary requirements.

4. Multiobjective Tabu Search Algorithm

The model above is a combinatorial problem with multiple objectives and multilayer planning. It is very difficult to solve using quantitative methods. General practices are using the heuristic algorithms [24]. In comparison with other heuristic algorithms, tabu search [25, 26] is a well-known approach which can efficiently overcome local optimality entrapment through simulating the process of human thinking. It guides the search to explore the feasible region using a short- and a long-term memory of previously visited solutions. This procedure can easily find the best solution and escape from local optima. In this paper, we develop a heuristic approach based on Floyd shortest path algorithm and tabu search to solve the model. Due to the nested relations in the model, feasible solutions can be constructed through three steps. First, a set of p nodes are randomly generated as the hubs in normal situation. Second, all the possibly disrupted cases are enumerated for the current hub set. Then, for each case, the traffic assignment of all O-D pairs to the remaining hubs is determined by shortest path algorithm.

4.1. Floyd Shortest Path Algorithm. During each iteration of the tabu search procedure, assignment problems of all O-D pairs to the given hubs are needed to determine the calculation of the objective functions (3), (10), and (14). The Floyd shortest path algorithm [27] can be employed to get the overall solutions efficiently.

Let H be the set of hubs. The weight matrix of the network is denoted as $D = (d_{ij})_{n \times n}$. There is

$$d_{ij} = \begin{cases} \min\left\{\hat{c}^1_{ij} + \alpha^1 c^1_{ij}, \hat{c}^2_{ij} + \alpha^2 c^2_{ij}\right\} & \forall i, j \in H, \; i \ne j \\ c^1_{ij} & \forall i \in H, \; j \notin H \vee i \notin H, \; j \in H \\ \infty & \forall i, j \notin H. \end{cases} \tag{19}$$

Denote $R = (r^{(k)}_{ij})_{n \times n}$ as the route matrix of the network using vertices only from the set $\{1, 2, \ldots, k\}$ as intermediate points along the way, where $r^{(0)}_{ij} = j$ for $k = 0$.

Then, basic steps for Floyd shortest path algorithm are as follows.

(1) Let $D^{(0)} = D$.

(2) Calculate $D^{(k)} = (d^{(k)}_{ij})_{n \times n}$ $(k = 1, 2, \ldots, n)$, where

$$d^{(k)}_{ij} = \min\left[d^{(k-1)}_{ij}, d^{(k-1)}_{ik} + d^{(k-1)}_{kj}\right]. \tag{20}$$

Calculate $R^{(k)} = (r_{ij}^{(k)})_{n\times n}$ $(k = 1, 2, \ldots, n)$, where

$$r_{ij}^{(k)} = \begin{cases} r_{ij}^{(k-1)}, & \text{if } d_{ij}^{(k-1)} \le d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \\ r_{ik}^{(k-1)}, & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)}. \end{cases} \quad (21)$$

(3) The elements in $D^{(n)} = (d_{ij}^{(n)})_{n\times n}$ are the unit transportation cost of the shortest path between all pairs of vertices. The allocation relationship between nonhub nodes and hub nodes can be determined by the route matrix $R^{(n)} = (r_{ij}^{(n)})_{n\times n}$. The shortest path from i to j is $i \to r_{ij}^{(n)} \to r_{ji}^{(n)} \to j$.

Using the unit transportation cost matrix of the shortest path in the specific network, we can easily calculate the transportation cost terms in the objective functions (3), (10), and (14), respectively.

4.2. Constitution of Solution and Neighborhood. For the first level program, denote the set of all nodes as $N = [1, 2, \ldots, n]'$. After randomizing the sequence of the set, the first p nodes in the new set are selected as the set of hubs in the normal situation, denoted by H. The neighborhood $N(H)$ is constituted by a swap of a hub node with a nonhub node. For $n = 5$, $p = 2$, and randomized set of nodes $[2, 4, 1, 5, 3]'$, the set of hubs can be selected as $[2, 4]'$. Its neighborhood consists of $[1, 4, 2, 5, 3]'$, $[5, 4, 1, 2, 3]'$, $[3, 4, 1, 5, 2]'$, $[2, 1, 4, 5, 3]'$, $[2, 5, 1, 4, 3]'$, and total $2 \times (5 - 2) = 6$ elements. The optimization procedure is carried out by tabu search.

For the second level program, given the set of hubs H in normal situation, we can enumerate all the possibly disrupted cases in the worst-case scenario. Suppose that $[1\ \ 2\ \ 3\ \ 4\ \ 5]$ is the current set of hubs and there are two hubs disrupted after attack. Then, there are 10 possible combinations of remaining set of hubs, that is, $[1\ \ 2\ \ 3]$, $[1\ \ 2\ \ 4]$, $[1\ \ 2\ \ 5]$, and so forth. The optimization procedure in this level program can be realized by comparing the objective values of all the possible combinations.

The third level program is a facility assignment and path selection problem. The optimal solution can be obtained by the Floyd shortest path algorithm mentioned before.

4.3. Nondominated Solutions Set. The two objectives of the first level program are sometimes conflicting. Generally, the optimization result for the program is not a unique solution but a nondominated solutions set (or Pareto optimal solutions set). Here, a solution, H_1, is said to dominate another solution, H_2, if H_1 is not worse than H_2 in all objectives and it is strictly better than H_2 in at least one objective. A solution is said to be nondominated (or Pareto optimal) if it is not dominated by other solutions in the feasible solution space. All nondominated solutions for the multiobjective problem are called nondominated solutions set or Pareto optimal solution set.

4.4. Fitness Function. The objective functions (3) and (4) are used to be the fitness functions for tabu search procedure. However, in each iteration, only one objective function is randomly selected as the fitness function for the current solution. This idea is motivated by the approach proposed by Kulturel-Konak et al. [28].

4.5. Move Rule. The acceptance of a pairwise exchange between a hub node and a nonhub node is called a move. First, the candidate solutions in the neighborhood of the current solution are sorted by their value of fitness. According to the fitness, the best candidate that is not tabu, or if tabu it satisfies the aspiration criterion, is accepted as the next move for the iteration.

4.6. Tabu List, Tabu Tenure, and Tabu Frequent. During the iterations, the node exchanges are recorded in a short-term memory, called tabu list. The same exchanges are forbidden for a certain number of iterations, called tabu tenure. In this paper, the tabu tenure is set as $l = \sqrt{n(n-p)}$. The update of the tabu list follows the rule of first in first out and later in later out. In order to avoid the local optimum, the search needs to restart from a different initial solution for several times. Long-term search memory is used to guide the new initial solution away from the nodes with highly visited frequency, called node frequency.

4.7. Aspiration Criterion. The aspiration criteria for the tabu search are as follows: (1) a tabu move can be accepted if it generates a solution which is not dominated by any of the current nondominated solutions; (2) if all the candidate solutions are tabu and there are no candidates that dominate the current nondominated solutions, then the best candidate is selected as the new current solution.

4.8. Framework of the Heuristic Algorithm. Based on what is mentioned above, the framework of the heuristic algorithm is as follows.

Step 1. Initialize Pareto set and tabu list as empty, set the *Max Run* times of the search and the *Max Iteration* numbers in each run. The search in each run terminates after "*Max Count*" numbers of nonimproving moves. The node frequency is set as *Node Frequency* = 0.

Step 2. Select p nodes randomly as initial hubs set H^0, solve the shortest path problem, and calculate the total network cost M_1^0 in normal situation. For the hubs set H^0, enumerate all the possibly disrupted cases in the q-disrupted hubs scenario. For each case, the shortest path problem is solved by the third level program (14), and the network cost for the case is obtained. The maximum cost of the cases is substituted into formula (4) and then we can get the total network cost M_2^0 in the worst-case scenario. Add $\{H_0, M_1^0, M_2^0\}$ into the Pareto solutions set. Set the initial hubs set as the current solution.

Step 3. Generate a random value ε from the $[0\ \ 1]$ uniform distribution. If $\varepsilon \le 0.5$, choose formula (3) as the fitness function; otherwise, select formula (4) as the fitness function.

Table 1: Serial number and coordinates of the projection nodes.

Coordinates	Serial number														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
X	50	100	250	300	350	450	550	650	670	730	750	780	800	850	920
Y	900	300	600	200	400	500	50	530	360	200	790	650	460	800	920

Step 4. Generate the neighborhood set for the current solution. All solutions in the neighborhood are sorted according to their fitness. The first 10 solutions are selected as the candidate solutions.

Step 5. If the best candidate dominates some solutions in the Pareto set, turn to Step 9; otherwise, turn to Step 6.

Step 6. If all candidates are tabu, select the best candidate as the current solution; otherwise, select the best nontabu candidate as the current solution. Check the domination relationship between the current solution and the solutions in the Pareto set. If the current solution is not be dominated by any solution in the Pareto set, turn to Step 7; otherwise, turn to Step 8.

Step 7. Update the current solution, tabu list and tabu frequency. The iteration numbers *Iteration* + 1. Add the solution into the Pareto set, *Count* = 0. Turn to Step 10.

Step 8. Update the current solution, tabu list and tabu frequency. Increment the count of iteration numbers, *Iteration* + 1. Increment the count of consecutive nonimproving moves, *Count* + 1. Turn to Step 10.

Step 9. Take the candidate solution as the current solution and add it into the Pareto set. Remove the dominated solutions from the Pareto set. Update the tabu list and tabu frequency, *Iteration* + 1, *Count* = 0.

Step 10. If *Iteration* < *Max Iteration* and *Count* < *Max Count*, return to Step 3 for next iteration; otherwise, turn to Step 11.

Step 11. The search restarts from a new initial solution ignoring the nodes with high tabu frequency. Return to Step 2.

Step 12. If the run times reach the maximum limit, terminate the search and output the Pareto set.

5. Illustrated Example

5.1. Computational Data. Traditional researches on the hub location problem in transportation network usually use the instances from the AP data set [29], American CAB data set [24], or Turkish network data set [4], and so forth. However, these data sets only include the basic data of the distance and demand flow between cities. Besides, all of these data sets only consider the single transportation mode. Since there are no specific cases and data for the research

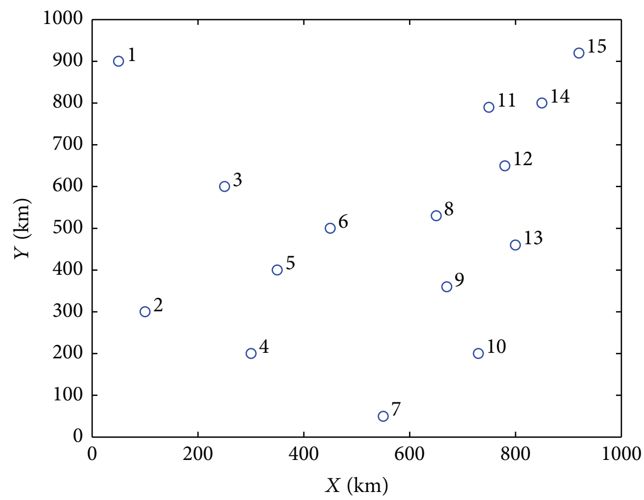

Figure 1: Spatial distribution of the projection nodes.

on hub location problem in power projection network, the illustrated computational data in this paper are generated using simulation method. While generating the data, we set some rules in order to make the data conform to the realities of real world logistic networks. For example, the distance between nodes should satisfy the triangle inequality. The unit transportation cost between two nodes through a road link is typically higher compared to the rail link. The rail shipment transit cost typically exceeds that of road shipment. Parameter values in the model are set using the experiences and methods from related literatures [2, 3].

Suppose that there are 15 projection nodes; the horizontal and vertical coordinates are sampled randomly from $[0, 1000]$, as shown in Table 1. The spatial distribution of the projection nodes is shown as Figure 1.

The distance matrix is obtained using the Euclidean distance between nodes; that is, $d_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$. Suppose that the ground transportation cost for is 0.6 ¥/t·km and ground transportation speed is 70 km/h. The railway transportation cost is set as 0.2 ¥/t·km and the railway speed is set as 60 km/h. The traffic demands between O-D pairs are generated from the uniform distribution $[200, 600]$. The traffic flow matrix is tabulated in Table 2.

The unit transit and operating cost for highway transportation between nodes are generated by sampling from a uniform distribution according to 5%–10% of the ground transportation cost, as shown in Table 3. The unit transit and operating cost of railway transportation are set as twice the cost of highway for the same nodes pair.

Table 2: Traffic flow matrix between nodes (unit: t).

Nodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	312	278	563	357	210	469	535	589	223	380	433	375	418	260
2	312	0	433	246	223	592	314	438	585	274	277	337	573	356	309
3	278	433	0	252	374	237	446	204	429	516	294	379	428	225	399
4	563	246	252	0	539	402	312	499	295	583	448	440	269	236	302
5	357	223	374	539	0	430	524	362	595	236	328	405	224	490	423
6	210	592	237	402	430	0	501	244	244	308	410	589	484	325	317
7	469	314	446	312	524	501	0	579	224	434	314	531	276	377	357
8	535	438	204	499	362	244	579	0	259	390	563	421	213	222	522
9	589	585	429	295	595	244	224	259	0	590	230	435	366	324	306
10	223	274	516	583	236	308	434	390	590	0	524	343	229	436	564
11	380	277	294	448	328	410	314	563	230	524	0	542	585	472	361
12	433	337	379	440	405	589	531	421	435	343	542	0	221	322	432
13	375	573	428	269	224	484	276	213	366	229	585	221	0	268	312
14	418	356	225	236	490	325	377	222	324	436	472	322	268	0	386
15	260	309	399	302	423	317	357	522	306	564	361	432	312	386	0

Table 3: Transit and operating cost for highway transportation between nodes (unit: ¥/t).

Nodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	28	21	38	26	28	56	25	34	58	30	38	50	48	43
2	28	0	16	11	16	21	26	27	22	38	38	24	37	41	33
3	21	16	0	13	9	8	31	13	19	24	30	23	30	30	40
4	38	11	13	0	9	13	14	18	19	21	36	29	18	37	40
5	26	16	9	9	0	7	23	10	12	19	33	27	20	26	25
6	28	21	8	13	7	0	19	12	8	22	20	17	17	26	21
7	56	26	31	14	23	19	0	24	16	13	24	35	22	41	34
8	25	27	13	18	10	12	24	0	9	14	16	8	10	11	17
9	34	22	19	19	12	8	16	9	0	7	24	17	8	22	24
10	58	38	24	21	19	22	13	14	7	0	22	23	9	21	27
11	30	38	30	36	33	20	24	16	24	22	0	8	10	4	13
12	38	24	23	29	27	17	35	8	17	23	8	0	9	5	17
13	50	37	30	18	20	17	22	10	8	9	10	9	0	11	15
14	48	41	30	37	26	26	41	11	22	21	4	5	11	0	5
15	43	33	40	40	25	21	34	17	24	27	13	17	15	5	0

The fixed cost of opening and operating a hub is generated by sampling from uniform distribution [100, 120]; the unit is ten thousand ¥, as tabulated in Table 4.

The hub-to-hub transportation cost discount factor using highway is set as $\alpha^1 = 0.8$, and the factor using railway is set as $\alpha^2 = 0.6$.

5.2. Computational Results and Analysis. Suppose that there are $p = 5$ hubs to be located in normal situation and there would be $q = 2$ hubs disrupted in the worst-case scenario. The test was run on an Intel Core i5 2.2 GHz and 8 GB RAM computer. The tabu tenure was set as 7, and the tabu frequency was set as 5. The relationship between the size of the Pareto set and the number of iterations in a run of tabu search procedure is shown in Figure 2. As Figure 2 shows, the size of the Pareto solutions set no longer changes after 43 iterations, indicating that the algorithm converges. Computational experience herein shows that the search procedure converges after 50 iterations in each run. Thus, we set the maximum iterations in a run of tabu search as 50. In order to avoid the local optimum, the search procedure restarts from a new initial solution for 20 times. The computational problem was solved after 25.4 s CPU time. There are 6 nondominated solutions in the Pareto set, as tabulated in Table 5.

As shown in Table 5, for solution 1 which selects [1 5 8 10 14] as the hubs set, although this instance has the minimum network cost in normal situation, it has the maximum network cost in the worst-case state. In other words, the network has a poor resilience for solution 1. For solution 3 which locates [1 5 8 9 14] as hubs set, it is 10.8% better than solution 1 in the worst-case cost but only 1.7% worse in the normal cost. Solution 3 has the maximum efficiency-cost ratio. Solution 6 which chooses [3 5 8 9 11] as the hubs set has the best network resilience

Table 4: Fix cost of opening and operating hubs (unit: ten thousand ¥).

Nodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Cost	119	113	109	103	115	112	109	118	108	103	113	113	106	116	120

Table 5: Pareto solutions set for the computational example.

Solution	Selected hubs	Normal cost M_1	Worst-case cost M_2	Increase in M_1 (%)	Decrease in M_2 (%)	Efficiency-cost ratio	Network resilience R
1	$[1\ 5\ 8\ 10\ 14]$	2.2581×10^7	3.2656×10^7	—	—	—	0.6914
2	$[1\ 5\ 8\ 10\ 11]$	2.2850×10^7	3.2626×10^7	1.2	0.9	0.75	0.7004
3	$[1\ 5\ 8\ 9\ 14]$	2.2972×10^7	2.9126×10^7	**1.7**	**10.8**	**6.35**	0.7887
4	$[1\ 5\ 8\ 9\ 11]$	2.3242×10^7	2.9096×10^7	2.9	10.9	3.76	0.7988
5	$[3\ 5\ 8\ 9\ 14]$	2.3474×10^7	2.9052×10^7	4.0	11.0	2.75	0.8080
6	$[3\ 5\ 8\ 9\ 11]$	2.3806×10^7	2.9022×10^7	5.4	11.1	2.06	0.8203

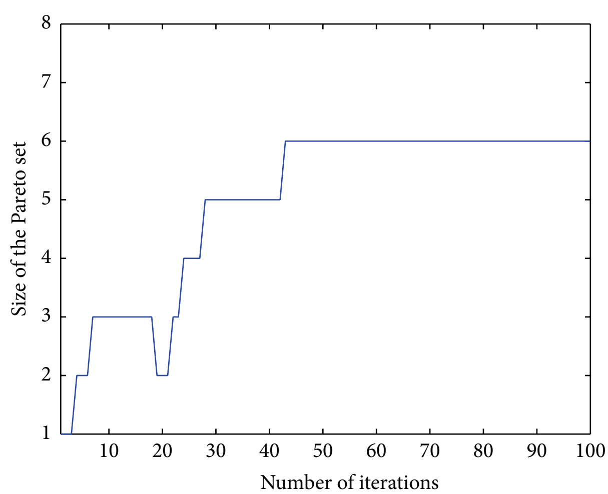

Figure 2: Convergence curve of the tabu search procedure.

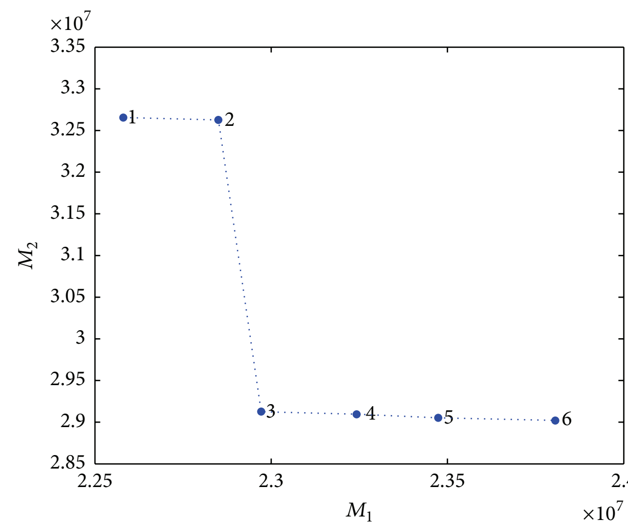

Figure 3: Tradeoff curve for the hub location strategy.

and the minimum cost in worst-case scenario. However, it imposes on the network an excessive cost in normal situation. Therefore, decision-makers can select different hub location strategies from the Pareto set according to their specific requirements. The tradeoff curve for the hub location strategy is depicted in Figure 3. In general, solution 1 is suitable for the optimum decision-making and the case of lighter threat from the enemy. Solution 6 is suitable for the pessimism decision-making and the case of serious threat from the enemy. Since solution 3 is much better than solution 1 in terms of the worst-case cost but not much worse in terms of the normal cost, it is a more reasonable hub location strategy for the design of a resilient power projection network.

6. Conclusion

The ability of power projection is a crucial element of a state's military power. The projection procedure relies on safe and efficient transportation network. In this paper, we focus the issue on the hub location problems in design of power projection network from a resilience perspective. Not only the cost of the projection network in normal situation is considered, but also the performance in presence of hubs disruptions is taken into account. A biobjective and trilevel integer programming model is proposed for the problem in view of game theory. The model can help to make tradeoffs between the performances of the projection network in different situations. A heuristic approach based on multiobjective tabu search and Floyd shortest path algorithm is developed to solve the model. Computational example shows that the results can provide some useful references for the hub location decision in design of a resilient power projection network.

The intermodal transport in power projection in this paper only considers the modes of road and rail. More transport modes can be taken into account in the future research. In our analysis, it is assumed that the enemy may select the hubs for attack which result in the greatest damage to the network. It is worthwhile to extend the model to cases in which the hubs disruptions following a probability distribution.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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Research Article

Aircraft Combat Survivability Calculation Based on Combination Weighting and Multiattribute Intelligent Grey Target Decision Model

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Aircraft combat survivability is defined as the capability of an aircraft to avoid or withstand a man-made hostile environment, which has been increasingly important. In order to give a rational calculation of aircraft combat survivability, an integrated method based on combination weighting and multiattribute intelligent grey target decision model is proposed. Firstly, an evaluation index system containing susceptibility index and vulnerability index as well as their subindexes is established. Then a multiattribute intelligent grey target decision model is introduced. A combination weighting method is brought up based on a modified AHP (analytic hierarchy process) method and the entropy method, offering a rational weight for various indexes. Finally, utilize the multiattribute intelligent grey target decision model to assess the aircraft combat survivability of aircraft, verified by a practical case of five aircraft. The results show that the proposed method is effective and has a great value in engineering application, which will provide useful references for other projects' evaluation.

1. Introduction

Survivability is defined as the capability of a system, including its crew, to avoid or withstand a hostile environment without suffering an abortive impairment of its ability to accomplish its designated mission [1]. For the aircraft combat survivability, Robert E. Ball defined it as the capability of an aircraft to avoid or withstand a man-made hostile environment. With the development of precision guided weapons, particularly radar-guided missiles and infrared-guided missiles, aircraft combat survivability is becoming increasingly important. The American Department of Defense has taken aircraft combat survivability as a basic design criterion. For example, the Navy MIL-HDBK-2069-1997 *aircraft survivability* stipulates that the survivability criterion should be carried out throughout the cycle life. The newest combat aircraft, F/A-18E/F Super Hornets, F/A-22 Raptor, and F-35 Lighting II, to name a few, has adopted survivability strengthening measures from the initial research phases.

Usually survivability can be subdivided into susceptibility and vulnerability, referring to the inability of an aircraft to avoid and withstand the man-made hostile environment, respectively. Aircraft combat survivability can be also defined as the probabilistic values that the aircraft would survive in man-made hostile environment, with the antithesis killability. The more susceptible and vulnerable the aircraft in the hostile environment, the more killable and lower survivable the aircraft.

Wang et al. construct an analytic model for aircraft survivability assessment based on the theory of stochastic duel considering the encounter process [2]. Konokman et al. carry out the aircraft survivability analysis considering vulnerability against fragmenting warhead threat [3]. Li et al. propose a vulnerability modeling and computation method based on product structure and CATIA and assess the effects of redundant technology [4]. Erlandsson and Niklasson argue a five-state survivability model, including undetected state, detected state, tracked state, engaged state, and hit state [5]. Shi et al. build an aircraft antagonistic model and a warfare model based on the agent theory [6]. These simulation methods bring a great deal of calculation and complex procedure,

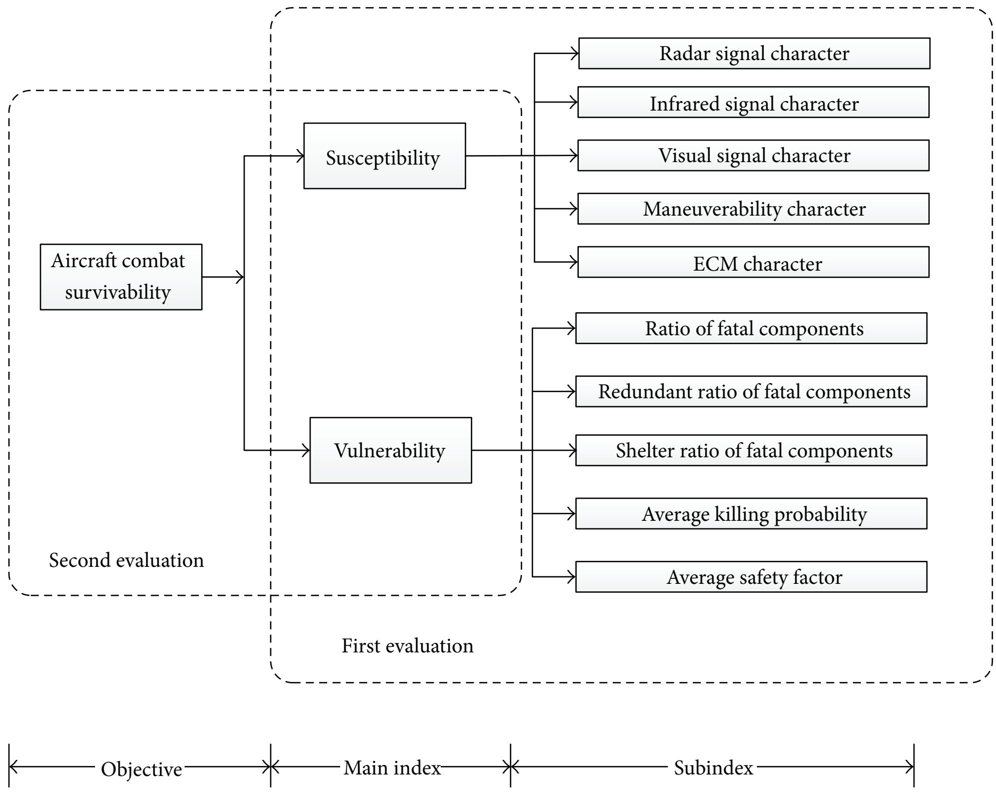

Figure 1: Aircraft combat survivability evaluation index system.

which impose restrictions on the application especially in the initial research without accurate data.

Multicriteria decision method is a useful replacement through ranking and selecting a finite number of alternative plans [7]. Grey target theory is the application of nonuniqueness principle in decision theory, which has been used in many good fields. A new multiattribute intelligent grey target decision model is introduced into the aircraft combat survivability analysis. In this paper, firstly, the aircraft combat survivability evaluation index system is established containing susceptibility index and vulnerability index. Secondly, a multiattribute intelligent grey target decision model is established based on the index system. Then, a combination weighting method is brought up based on a modified AHP (analytic hierarchy process) method and the entropy method, offering the combination weight to the multiattribute intelligent grey target decision model. In the end, a numerical example containing five aircraft is given, proving that this method is practicable and effective.

2. Aircraft Combat Survivability Evaluation Index System

Survivability can be subdivided into susceptibility and vulnerability. So the index system contains susceptibility index and vulnerability index. The index is established in hierarchy. The main index is susceptibility index and vulnerability index. The subindexes of susceptibility contain radar signal character, infrared signal character, visual signal character, maneuverability character, and ECM (electronic countermeasure) character, while the subindexes of vulnerability contain ratio of fatal components, redundant ratio of fatal components, shelter ratio of fatal components, average killing probability, and average safety factor, as is shown in Figure 1. The details are as follows.

2.1. Aircraft Susceptibility Subindexes. Considering the character of the aircraft, radar signal character, infrared signal character, visual signal character, maneuverability character, and ECM character are brought out.

Radar Signal Character. Radar signal is the most important signal character of the aircraft for locating, identifying, and tracking. Radar is one of the most lethal threats for aircraft. For example, early warning radars can provide airborne target information out of hundreds of miles, while ground control interceptor can even provide an accurate target location. From a simplified form of radar range equation in (1), we can see that RCS (radar cross section) is the one and only controllable parameter for the aircraft:

$$R_{\max}^4 = \underbrace{\left(\frac{P_r C_t^2 \lambda^2}{(4\pi)^2} \cdot \frac{1}{S_{\min} L}\right)}_{\text{Radar Capability}} \underbrace{(\sigma)}_{\text{RCS}} \underbrace{F_1^2 F_2^2}_{\text{Atmospheric propagation}}, \quad (1)$$

where $R_{\max}$ is the maximum range at which an aircraft can be detected, C_t is the characteristic parameter of radar which varies for different type, and F_1 and F_2 are the propagation loss of signal emission and return.

So, we choose radar cross section as the evaluation index denoted by the sign S_{11} with the unit square meter m^2.

Infrared Signal Character. Infrared signal is another important signal character of the aircraft. With the development of stealth and antistealth techniques of radars, the aircraft's RCS has decreased significantly. However, compared with the environment, infrared signal is still significant even for the stealth aircraft. In the war area for 20 years, about 90% of airplanes were damaged by the infrared-guided missile [8]. Nowadays, IRST (infrared search and track) system, FLIR (forward looking infrared) system, and infrared-guided missiles have been significant threats by accurately locating aircraft. For point source infrared detector, the infrared range equation under uniform background can be written as $R \propto \sqrt{I}$, where R is the range at which an aircraft can be detected and I is the aircraft's infrared radiation intensity. For most aircraft, the engines are the largest sources of thermal energy. So, we choose turbine inlet temperature as the evaluation index denoted by the sign S_{12} with the unit Kelvin K.

Visual Signal Character. Visual signal is another important factor in determining overall aircraft detectability [9]. Air combat in visual range is still essential and visual acquisition before launch may be required. Contrails, engine exhaust glow, cockpit lighting, and luminescence may provide visual cues. Here, we choose size factor, which is closely related to the probability of visual detection as the evaluation index denoted by the sign S_{13} with the unit meter m. Size factor is defined as

$$S_{13} = \sqrt[3]{\frac{3 * L * H * W}{4 * \pi}}, \tag{2}$$

where L is the length of aircraft, H is the height of aircraft, and W is the wingspan of aircraft.

Maneuverability Character. Maneuverability is an effective means of defense for the aircraft against detection and attack. Supersonic maneuver is now the standard of new generation aircraft. Supersonic maneuver gives an aircraft a lower susceptibility and a higher survivability. Maneuverability character can be defined with the maximum allowable overload $n_{y\max}$, the maximum steady turn overload n_{cir}, and specific excess power SEP as is shown in the following:

$$S_{14} = n_{y\max} + n_{\text{cir}} + \text{SEP} \times \frac{9}{300}. \tag{3}$$

Electronic Countermeasure Character. Electronic countermeasure plays an important role in modern military affairs, including active and passive jamming, which is an effective mean to decrease aircraft susceptibility and enhance aircraft survivability. Electronic countermeasure equipment contains omnidirectional radar warning equipment, radar chaff dispensing device, infrared jammer, and infrared-guided missiles. However, for electronic countermeasure capability, we can only give a fuzzy value. Electronic countermeasure character can be denoted by the sign S_{15} with dimensionless unit. A typical value can be read in Table 1.

Table 1: Electronic countermeasure subindexes of the aircraft.

Number	Airborne electronic countermeasure equipment	S_{15}
1	Omnidirectional radar warning equipment	1.05
2	Ibid + passive jamming dispensing device	1.10
3	Ibid + active infrared and electromagnetic jammer	1.15
4	Ibid + missile approach warning device	1.20

Table 2: Aircraft vulnerability subindexes and definition.

Subindexes	Sign	Definition
Ratio of fatal components	S_{21}	$S_{21} = \frac{N_F}{N}$
Redundant ratio of fatal components	S_{22}	$S_{22} = \frac{N_R}{N_F}$
Shelter ratio of fatal components	S_{23}	$S_{23} = \sum_{i=1}^{N_F} \frac{N_{Si}}{N_F}$
Average killing probability	S_{24}	$S_{24} = \sum_{i=1}^{N_F} \frac{P_{Ki}}{N_F}$
Average safety factor	S_{25}	$S_{25} = \sum_{i=1}^{N_F} \left(\frac{n_{Di}}{n_{Di,\max}} \right)$

2.2. Aircraft Vulnerability Subindexes. Aircraft vulnerability refers to the inability of an aircraft to withstand the man-made hostile environment, which lies on ratio of fatal components, redundant ratio of fatal components, shelter ratio of fatal components, average killing probability, and average safety factor. These factors can be defined as is shown in Table 2, where N_F is the numbers of fatal components, N is the numbers of whole components, and N_S is the numbers of redundant fatal components. N_{Si} is the redundant degree of the ith fatal component. P_{Ki} is the killing probability of the ith component while the aircraft is hit. n_{Di} and $n_{Di,\max}$ are the designed overload and maximum overload of the ith fatal component, respectively.

Although there are clear equations for the calculation of aircraft vulnerability subindexes, the value of every parameter is comparatively subjective for different definition such as "fatal" and different granularity analysis.

3. A Multiattribute Intelligent Grey Target Decision Model

Here we introduce a multiattribute intelligent grey target decision model proposed by Liu et al. [10]. This model takes the situation of the shoot and miss of the bull's eye of the objective's effect value and vector based on four kinds of uniform effect measures.

3.1. Problem Description. Assume that $A = \{a_1, a_2, \ldots, a_n\}$ is the event set, $B = \{b_1, b_2, \ldots, b_m\}$ the countermeasure set, and $S = \{s_{ij} = (a_i, b_j) \mid a_i \in A,\ b_j \in B\}$ the decision set. $\mu_{ij}^{(k)}$ is the effect value of s_{ij} in objective k, referring to the similar level or the removed level between the sample and the critical sample.

Assume that $d_1^{(k)}$ and $d_2^{(k)}$ are the upper and lower critical value s_{ij} in objective k. Then $S^1 = \{r \mid d_1^{(k)} \le r \le d_2^{(k)}\}$ is the one-dimension grey target and $\mu_{ij}^{(k)} \in [d_1^{(k)}, d_2^{(k)}]$ is the pleased effect in objective k. Multidimensional grey target can be discussed in the same way. Details are shown in [10].

3.2. Uniform Effect Measures. Considering the decision objectives with different meaning, different dimension, and/or different nature, the effect value of the objective should be transferred to the uniform effect measures.

Assume that u_{i0j0}^k is the critical value of objective k, and then the grey objective decision of k is designed as follows:

$$u_{ij}^k \in \begin{cases} \left[u_{i0j0}^k, \max\limits_i \max\limits_j \left\{u_{ij}^k\right\}\right], & k \in \text{BTO} \\ \left[\min\limits_i \min\limits_j \left\{u_{ij}^k\right\}, u_{i0j0}^k\right], & k \in \text{CTO} \\ \left[A - u_{i0j0}^k, A + u_{i0j0}^k,\right], & k \in \text{MTO}, \end{cases} \tag{4}$$

where $k \in$ BTO means that objective k belongs to benefit type objective, $k \in$ CTO means that objective k belongs to cost type objective, and $k \in$ MTO means that objective k belongs to moderate type objective (same as what follows). And A is the moderate value of the moderate type objective.

For the decision objective of benefit type and cost type, the effect measures r_{ij}^k can be shown as

$$r_{ij}^k = \begin{cases} \dfrac{u_{ij}^k - u_{i0j0}^k}{\max_i \max_j \left\{u_{ij}^k\right\} - u_{i0j0}^k}, & k \in \text{BTO} \\ \dfrac{u_{i0j0}^k - u_{ij}^k}{u_{i0j0}^k - \min_i \min_j \left\{u_{ij}^k\right\}}, & k \in \text{CTO}. \end{cases} \tag{5}$$

For the decision objective of moderate type, the effect measure can be divided as the upper effect measure and the lower effect measure according to the scale of effect measure r_{ij}^k, which can be shown as follows:

$$r_{ij}^k = \begin{cases} \dfrac{u_{ij}^k - A + u_{i0j0}^k}{u_{i0j0}^k}, & u_{ij}^k \in \left[A - u_{i0j0}^k, A\right], \text{ lower effect measure} \\ \dfrac{A + u_{i0j0}^k - u_{ij}^k}{u_{i0j0}^k}, & u_{ij}^k \in \left[A, A + u_{i0j0}^k\right], \text{ upper effect measure.} \end{cases} \tag{6}$$

The effect measure of the decision objective of benefit type reflects the similar level between the sample and the biggest sample as well as the removed level between the sample and the critical sample. The effect measure of the decision objective of cost type reflects the similar level between the sample and the smallest sample as well as the removed level between the sample and the critical sample. The lower effect measure of the decision objective of moderate type reflects the level between the sample less than moderate value A and the lower critical sample. The upper effect measure of the decision objective of moderate type reflects the level between the sample greater than moderate value A and the upper critical sample.

The decision objective of benefit type is a type of objective with an expectance the bigger the better or the more the better. The decision objective of cost type is a type of objective with an expectance the smaller the better or the less the better. The decision objective of moderate type is a type of objective with an expectance neither too big nor too small or neither too many nor too few.

The miss of the bull's eye under different conditions can be shown as

$$\begin{gathered} u_{ij}^k < u_{i0j0}^k \quad k \in \text{BTO}, \\ u_{ij}^k > u_{i0j0}^k \quad k \in \text{CTO}, \\ u_{ij}^k < A - u_{i0j0}^k \\ \text{or } u_{ij}^k > A + u_{i0j0}^k \\ k \in \text{MTO}. \end{gathered} \tag{7}$$

The effect measure r_{ij}^k is satisfied with the following requirements.

(1) r_{ij}^k is dimensionless; (2) r_{ij}^k is a uniform variable, namely, $r_{ij}^k \in [-1, 1]$. (3) The greater the r_{ij}^k, the more ideal the effect.

Thus in order to satisfy the standardization, the selection of critical value u_{i0j0}^k usually satisfies the following:

$$\begin{aligned} & u_{ij}^k \geq -\max_i \max_j \left\{u_{ij}^k\right\} + 2 * u_{i0j0}^k, \quad k \in \mathrm{BTO}, \\ & u_{ij}^k \leq -\min_i \min_j \left\{u_{ij}^k\right\} + 2 * u_{i0j0}^k, \quad k \in \mathrm{CTO}, \\ & u_{ij}^k \geq A - 2 * u_{i0j0}^k \\ \text{or } & u_{ij}^k \leq A + 2 * u_{i0j0}^k, \\ & \qquad\qquad k \in \mathrm{MTO}. \end{aligned} \tag{8}$$

Assume that ω_k is the decision weight of objective k and $\sum_{k=1}^{s} \omega_k = 1$. Thus $r_{ij} = \sum_{k=1}^{s} \omega_k * r_{ij}^k$ will be the synthetic effect measures of decision approach s_{ij} and $R = \{r_{ij}\}$ will be the matrix of synthetic effect measures of decision set S.

The synthetic effect measure r_{ij} is satisfied with the following requirements.

(1) r_{ij} is dimensionless; (2) r_{ij} is a uniform variable, namely, $r_{ij} \in [-1, 1]$. (3) The greater the r_{ij}, the more ideal the effect.

While $r_{ij} \in [-1, 0]$ means the miss of the bull's eye and $r_{ij} \in [0, 1]$ means the hit of the bull's eye, through the comparison of the value of r_{ij}, we can judge the performance of a_{i0}, b_{j0}, and s_{i0j0} according to the definition shown as follows.

Definition 1. b_{j0} is the best decision of event a_i if $\max_{1 \leq j \leq m}\{r_{ij}\} = r_{ij0}$; a_{i0} is the best event corresponding with the decision b_{j0} if $\max_{1 \leq i \leq n}\{r_{ij}\} = r_{i0j}$; s_{i0j0} is the best decision approach if $\max_{1 \leq i \leq n,} \max_{1 \leq j \leq m}\{r_{ij}\} = r_{i0j0}$.

3.3. Algorithm Steps. Algorithm steps of the multiattribute intelligent grey target decision model are as follows.

Step 1. Form the decision set $S = \{s_{ij} = (a_i, b_j) \mid a_i \in A,\ b_j \in B\}$.

Step 2. Confirm the decision objective $k = 1, 2, \ldots, s$.

Step 3. Confirm the decision weight of objective k ω_k ($k = 1, 2, \ldots, s$).

Step 4. Form the matrix $U^k = \{u_{ij}^k\}$ of effect measures of decision set S.

Step 5. Set the critical value of the objective.

Step 6. Obtain the uniform matrix $R^k = \{r_{ij}^k\}$ of effect measures.

Step 7. Obtain the uniform matrix $R = \{r_{ij}\}$ of synthetic effect measures.

Step 8. Confirm the best decision b_{j0} or the best decision approach s_{i0j0} according to the definitions.

Table 3: Exponential scale the golden section.

Exponential scale	One factor compared to another
1.000	Equally important
1.618	Slightly more important
2.618	More important
4.236	Greatly more important
6.854	Absolutely more important
Reciprocal	Reversed scale

4. Combination Weighting Method

In Step 3, the decision weight of objective k ω_k is needed. In this paper, according to requirements of the evaluation of aircraft combatant survivability, the evaluation system consists of various criteria with unequal importance, which has a biggish influence on the evaluation. So a rational weighting is necessary. In order to elicit weights, a combination weighting method is brought up based on a modified AHP (analytic hierarchy process) method and the entropy method, offering the subjective weight and the objective weight, respectively.

4.1. Analytic Hierarchy Process Based on Exponential Scale. The AHP is a proven decision-making tool integrating the quantitative analysis and qualitative analysis together, considering the quantitative weight and qualitative weight, into the decision-making process. The AHP has been used widely in a far-ranging field, especially in solving complex decision-making problems with numerous criteria. The AHP establishes a hierarchical structure showing the relationship of the target, criteria, and index, using the decision matrix. The AHP can be broken into five steps as building a hierarchy, making comparisons, calculating weights, checking consistency, and producing the result. The detailed steps are shown in [11].

Dr. Saaty developed a 9-point scale in the pairwise comparisons, which states whether one factor compared to another is important or not. Assign value of 1 to equally important and values of 3, 5, 7, and 9 to slightly more important, more important, greatly more important, and absolutely more important. Values of 2, 4, 6, and 8 are reserved for intermediate values. The 9-point scale gives the difference of importance, but the ratio of importance in the pairwise comparisons is what we need according to the AHP. So the AHP based on exponential scale is built. Introduce the ratio of importance α into the comparison as the exponential scale. Replace the values of $1, 3, 5, 7,$ and 9 with the values of $1, \alpha, \alpha^2, \alpha^3,$ and α^4. Assume α meets the rule of ladder by leaps, namely, $a^k = a^{k-1} + a^{k-2}$, where $k \in \{2, 3, 4, 5\}$. It turns out to be that $a = 1.618$, which satisfied the golden section of importance. The reciprocal scale is achieved when comparing the factors in the opposite direction; that is, if A is more important than B (2.618), B could be said to be less important than A (1/2.618). The exponential scale is shown in Table 3.

For the given matrix M through pairwise comparisons, the vector of weights ω^* is the normalized eigenvector of the matrix associated with the largest eigenvalue, $\lambda_{\max}$, using

TABLE 4: Susceptibility subindexes of different aircraft.

Aircraft	S_{11}	S_{12}	L	H	W	S_{13}	$n_{y\max}$	n_{cir}	SEP	S_{14}	S_{15}
A	12.7	1672	19.43	5.63	13.05	6.985	9.0	7.5	265	25.33	1.15
B	5.8	1503	14.36	5.20	9.13	5.460	9.0	8.6	238	25.53	1.05
C	4.9	1672	15.04	5.09	10.01	5.677	9.0	7.5	290	26.17	1.15
D	5.0	1756	18.4	4.88	13.6	6.631	7.0	6.0	245	21.17	1.20
E	0.1	1922	19.05	5.39	13.56	6.927	9.0	9.0	330	29.00	1.20

the equation $M\omega^* = \lambda_{\max}\omega^*$. Thus after calculating and checking of consistency in the matrix, the result is produced.

4.2. Entropy Weight Method. Entropy, according to Shannon's information theory, reflects the degree of disorder of information [12]. In the information system, the smaller the entropy value, the greater the degree of order, and the greater the amount of information. The entropy weight method is an effective method calculating the objective weight, just correlating with the data distribution of the evaluation matrix, without relying on the subjective preference of decision-maker.

Suppose that there is a matrix $M = [m_{ij}]_{n\times m}$ $(i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, m)$ with evaluating indexes counted m and evaluating objects counted n. Matrix $P = [p_{ij}]_{n\times m}$ is the normalized matrix of M. So the entropy of the jth index is defined as

$$E_j = -\frac{1}{\log n}\sum_{i=1}^{n} p_{ij}\log p_{ij}. \tag{9}$$

Then the entropy weight of jth index is defined as

$$\omega_j' = \frac{\left(1 - E_j\right)}{\left(m - \sum_{j=1}^{m} E_j\right)}. \tag{10}$$

4.3. Algorithm Steps. The AHP based on exponential scale and entropy weight method are both available in processing the weight; however they have their own advantages and disadvantages. The combination weighting method can evade the disadvantages. The effective combination of subjective weight and objective weight reconciles the expert's preference for the index and the decrease of fuzzy random color, thus producing an advantage of both weighting methods. So the combination weighting method produces a scientific and rational weight.

The combination weight can be defined as

$$\omega_j = \lambda\omega_j^* + (1-\lambda)\,\omega_j', \tag{11}$$

where ω_j is the combination weight of the jth index; ω_j^* is the subjective weight given by the AHP based on exponential scale; ω_j' is the objective weight given by the entropy weight method; λ is the share of subjective weight in the combination weight.

5. Evaluation of Aircraft Survivability

5.1. Data Collection and Pretreatment. Through literature review, data of five aircraft is acquired in susceptibility subindexes, which is shown in Table 4.

But for the vulnerability subindexes, as can be seen ever, which are comparatively subjective, we cannot give a precise value. The vague set is an effective way to solve this problem [13]. For the restriction of the length of this paper, the details will not be discussed here. We know that linguistic variables are available in certainty. Seven-grade linguistic variables $S = \{\text{VG}, \text{G}, \text{FG}, \text{M}, \text{FP}, \text{P}, \text{VP}\}$ presented in vague values are shown in Table 5.

However, with vague value and precise value in one evaluation, it is hard to handle. Here we can transfer the useful information of vague value into a precise value through a score function. We know that a vague value, such as [0.4, 0.7], can be interpreted as "the vote for a resolution is 4 in favor, 3 against, and 3 abstentions [13]." Score function can give a certain value of the vague set correlated with a certain value of the fuzzy set, through the undefined and unascertained information of the vague set.

A proved score function contrived by Guo et al. [14] is given as

$$S = \begin{cases} t_A(x_i), & t_A(x_i) + f_A(x_i) = 1 \\ t_A(x_i) + \dfrac{\pi_A(x_i)}{2} + \left(t_A(x_i) - f_A(x_i)\right)\dfrac{\pi_A(x_i)}{2} & 0 < t_A(x_i) + f_A(x_i) < 1 \\ -1, & t_A(x_i) + f_A(x_i) = 0, \end{cases} \tag{12}$$

where $t_A(x_i) + f_A(x_i) = 1$ means that $\pi_A(x_i) = 0$; that is, the information is absolutely confirmed, so $S = t_A(x)$. While $t_A(x_i) + f_A(x_i) = 0$ means that $\pi_A(x_i) = 1$, that is, the information is absolutely unascertained, so $S = -1$.

Table 5: Seven-grade linguistic variables of vague value.

Grade	Typical vague values	Abstention
Very good (VG)	[1, 1]	0
Good (G)	[0.8, 0.9]	0.1
Fairly good (FG)	[0.6, 0.8]	0.2
Medium (M)	[0.5, 0.5]	0
Fairly poor (FP)	[0.2, 0.4]	0.2
Poor (P)	[0.1, 0.2]	0.1
Very poor (VP)	[0, 0]	0

Table 6: Vulnerability subindexes of different aircraft.

Subindexes	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}
A	[0.2, 0.4]	[0.8, 0.9]	[0.8, 0.9]	[0.2, 0.4]	[0.6, 0.8]
B	[0.5, 0.5]	[0.6, 0.8]	[0.6, 0.8]	[0.5, 0.5]	[0.5, 0.5]
C	[0.2, 0.4]	[0.6, 0.8]	[0.6, 0.8]	[0.2, 0.4]	[0.5, 0.5]
D	[0.1, 0.2]	[1, 1]	[0.6, 0.8]	[0.1, 0.2]	[0.8, 0.9]
E	[0.1, 0.2]	[0.8, 0.9]	[0.8, 0.9]	[0.1, 0.2]	[0.6, 0.8]

Through the seven-grade linguistic variables of vague value, we give vulnerability subindexes of different aircraft the value shown in Table 6.

Thus the subindexes can be normalized as is shown in Table 7. For vulnerability subindexes, a certain value should be given by score function and then the vector normalization method is used. The type of index is also given. Here for the subindexes, there are only too types, where "+" represents benefit type and "−" represents cost type.

5.2. Calculation of Weight. According to AHP based on exponential scale, the subjective weight can be calculated through the introduced five steps. Pairwise comparisons are quite important. Each of the subfactors will need to be compared to each other. There are two approaches generally available. The first is comparing the factors of susceptibility and vulnerability, followed by comparing each subfactor under each factor separately. The other approach is comparing each subfactor to every other one directly, which involves more comparisons. This approach would be difficult and lead to an uncertain result. For instance, trying to compare radar signal character to ratio of fatal components would be difficult, as they are so dissimilar. So the first approach will be used.

Three matrixes of pairwise comparisons given by experts are established. Matrix P is the comparison of susceptibility index and vulnerability index. Define susceptibility index as more important than vulnerability index, thus producing the two by two matrix. P_1 and P_2 are the comparison of subindexes of susceptibility and vulnerability, respectively:

$$P = \begin{bmatrix} 1.000 & 2.618 \\ \dfrac{1}{2.618} & 1.000 \end{bmatrix}, \tag{13}$$

$$P_1 = \begin{bmatrix} 1.000 & 1.618 & 4.236 & 2.618 & 1.618 \\ \dfrac{1}{1.618} & 1.000 & 2.618 & 1.618 & 1.618 \\ \dfrac{1}{4.236} & \dfrac{1}{2.618} & 1.000 & \dfrac{1}{1.618} & \dfrac{1}{1.618} \\ \dfrac{1}{2.618} & \dfrac{1}{1.618} & 1.618 & 1.000 & 1.000 \\ \dfrac{1}{1.618} & \dfrac{1}{1.618} & 1.618 & 1.000 & 1.000 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 1.000 & 1.618 & 1.618 & 2.618 & 4.236 \\ \dfrac{1}{1.618} & 1.000 & 1.000 & 1.618 & 2.618 \\ \dfrac{1}{1.618} & 1.000 & 1.000 & 1.618 & 1.618 \\ \dfrac{1}{2.618} & \dfrac{1}{1.618} & \dfrac{1}{1.618} & 1.000 & 2.618 \\ \dfrac{1}{4.236} & \dfrac{1}{2.618} & \dfrac{1}{1.618} & \dfrac{1}{2.618} & 1.000 \end{bmatrix}. \tag{14}$$

The next steps are calculating weights and checking consistency. The calculation of P is $\omega_I^* = (0.7236, 0.2764)$, and the calculation of P_1 and P_2 is as follows:

$$\omega_1^* = (0.3553, 0.2399, 0.0916, 0.1483, 0.1649),$$
$$\omega_2^* = (0.3496, 0.2160, 0.1993, 0.1502, 0.0848). \tag{15}$$

Reference [11] gives the method of checking consistency, using the equation $\mathrm{CR} = \mathrm{CI}/\mathrm{RI}$, where $\mathrm{CI} = (\lambda_{\max} - n)/(n - 1)$, RI is the random consistency index, $\lambda_{\max}$ is maximum eigenvalue of the matrix, and n is the number of factors. The consistency of matrixes P_1 and P_2 is 0.0063 and 0.0168, respectively, which is less than 10 percent; that is, the matrix is consistent enough and can be used for calculating results.

So the subjective weight of all subindexes is

$$\omega^* = (0.2571, 0.1736, 0.0663, 0.1073, 0.1193, 0.0966, 0.0597, 0.0551, 0.0415, 0.0234). \tag{16}$$

The entropy weight can be calculated using the entropy weight method as

$$\omega' = (0.1601, 0.0810, 0.1045, 0.0815, 0.0804, 0.1210, 0.0820, 0.0812, 0.1210, 0.0873). \tag{17}$$

Table 7: The normalization of the subindexes.

Subindexes	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}
Type	−	−	−	+	+	−	+	+	−	+
A	0.813	0.437	0.491	0.443	0.447	0.405	0.462	0.494	0.405	0.480
B	0.371	0.393	0.383	0.447	0.408	0.779	0.387	0.413	0.779	0.324
C	0.314	0.437	0.399	0.458	0.447	0.405	0.387	0.413	0.405	0.324
D	0.320	0.459	0.466	0.370	0.466	0.179	0.523	0.413	0.179	0.574
E	0.006	0.503	0.486	0.507	0.466	0.179	0.462	0.494	0.179	0.480

Then the combination weight ω can be calculated in (11), where $\lambda = 0.5$ as follows:

$$\omega = (0.2086, 0.1273, 0.0854, 0.0944, 0.09985, 0.1088, 0.07085, 0.06815, 0.08125, 0.05535). \tag{18}$$

5.3. Multiattribute Intelligent Grey Target Decision. Here we give an evaluation of aircraft survivability. The evaluation of aircraft survivability is the event a_1, so $A = \{a_1\}$. The aircraft A, B, C, D, and E are the countermeasures b_1, b_2, b_3, b_4, b_5, so $B = \{b_1, b_2, b_3, b_4,$ and $b_5\}$. Thus we can form the decision set $S = \{s_{11}, s_{12}, s_{13}, s_{14}, s_{15}\}$.

As is shown above, the susceptibility subindexes and vulnerability subindexes of different aircraft are chosen as the decision objective $k = 1, 2, \ldots, 10$. For the decision objective of benefit type, the critical value is $u^k_{i0j0} = 1922$, $k = 1, 2, 3, 6, 9$. For the decision objective of cost type, the critical value is $u^k_{i0j0} = 0.5$, $k = 4, 5, 7, 8, 10$. The decision weight of objective k has been given above. The matrix $U^k = \{u^k_{ij}\}$ of effect measures of decision set S is formed according to the collected data. Thus the uniform effect measures matrix $R^k = \{r^k_{ij}\}$ is obtained with the defined effect measure for benefit type objective and effect measure for cost type objective, shown as follows:

$$R^k = \begin{bmatrix} 0.0000 & 0.5476 & 0.6190 & 0.6111 & 1.0000 \\ 0.5967 & 1.0000 & 0.5967 & 0.3962 & 0.0000 \\ 0.0000 & 1.0000 & 0.8577 & 0.2321 & 0.0380 \\ 0.5313 & 0.5568 & 0.6386 & 0.0000 & 1.0000 \\ 0.6667 & 0.0000 & 0.6667 & 1.0000 & 1.0000 \\ 0.6234 & 0.0000 & 0.6234 & 1.0000 & 1.0000 \\ 0.5577 & 0.0000 & 0.0000 & 1.0000 & 0.5577 \\ 1.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.6234 & 0.0000 & 0.6234 & 1.0000 & 1.0000 \\ 0.6234 & 0.0000 & 0.0000 & 1.0000 & 0.6234 \end{bmatrix}. \tag{19}$$

Through the equation $r_{ij} = \sum_{k=1}^{s} \omega_k * r^k_{ij}$, the uniform matrix $R = \{r_{ij}\}$ of synthetic effect measures can be obtained as $R = [0.4533, 0.3795, 0.5237, 0.6138, 0.7383]$. Then according to the definitions, the best countermeasure is b_5; that is, the aircraft E is the best decision and has the best aircraft survivability. Through the ranking of r_{ij}, we can give a ranking of aircraft, $E > D > C > A > B$.

A rational weight is essential. An opposite example with equal weight for every subindex is given. The weight is 0.1. For the restriction of the length of this paper, we just list the result of the uniform matrix of synthetic effect measures, $R = [0.8782, 0.7764, 0.7527, 0.8742, 0.8393]$. Here, the aircraft A is the best decision and has the best aircraft survivability. This result is quite different. And the difference of every aircraft is little, which is hard to give a rational evaluation.

With the calculated weight according to AHP based on exponential scale and the entropy weight method, we can just calculate the susceptibility of the aircraft using the same method. According to the algorithm steps, the uniform matrix of synthetic effect measures can be obtained as $R_S = [0.8244, 0.8136, 0.8304, 0.8106, 0.7580]$. Then according to the definitions, the best countermeasure is b_3; that is, the aircraft C is the best decision and has the best aircraft susceptibility. Through the ranking of r_{ij}, we can give a ranking of aircraft, $C > A > B > D > E$. However, this result is so different with the aircraft survivability. Some reasons can be listed. Firstly, aircraft susceptibility and aircraft survivability are different in definitions and evaluation subindexes. Then, although turbine inlet temperature is a good evaluation index for the infrared signal character, different from RCS, turbine inlet temperature is not enough especially for the new generation aircraft. Turbine inlet temperature is not the sole influence factor of infrared signal character. Infrared suppressing functions such as stealth materials and thermal isolation have no influence on turbine inlet temperature but have great influence on infrared signal character. This is the future work to establish a more rational index system.

Then we calculate the vulnerability of the aircraft. The combination weight has been given in the former section. The uniform matrix of synthetic effect measures is obtained as $R_V = [0.7300, 0.2279, 0.5395, 0.9248, 0.9183]$. Then the best countermeasure is b_4; that is, the aircraft D is the best decision and has the best aircraft vulnerability. The ranking of aircraft vulnerability is $D > E > A > C > B$.

6. Conclusions

With the development of precision guided weapons, aircraft combat survivability has been increasingly important. Multicriteria decision method is a useful method through ranking and selecting a finite number of alternative plans. In this paper, in order to give a rational evaluation of aircraft combat survivability, a new multiattribute intelligent grey target decision model is introduced. Conclusions can be shown as follows:

(1) The aircraft combat survivability evaluation index system is established in hierarchy containing susceptibility index and vulnerability index and their subindexes, which is essential to aircraft survivability.

(2) The multiattribute intelligent grey target decision model is introduced, which not only can be used in the evaluation of aircraft combat survivability, but also can be used in any similar evaluation questions.

(3) Then a combination weighting method is brought up based on a modified AHP (analytic hierarchy process) method and the entropy method, offering the combination weight to the multiattribute intelligent grey target decision model, which can evade the disadvantages. This method produces a scientific and rational weight, improving the accuracy and objectivity of the evaluation, which can be used in any evaluation requiring a rational weight.

(4) In the end, a numerical example containing five aircraft is given, proving that this method is practicable and effective. The result of more numerical calculation can provide more details, to find the most important influencing factors, which can be used in the program design of new aircraft and modified design of the old aircraft.

However, we did not consider the real states in the combat, such as detected, tracked, or hit, as well as the influence of aircraft combat capability. So future work will be carried out on in directions: (1) establish a more rational evaluation system and find more rational subindexes, such as the representation of infrared signal character; (2) consider the real states in the combat, the threats, and aircraft combat capability to give capability of an aircraft to avoid or withstand a man-made hostile environment.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Research Article

A Model for Sorting Activities to Be Outsourced in Civil Construction Based on ROR-UTADIS

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The subcontractor's selection problem is currently treated as a supply chain problem with a prequalification procedure to balance the main objectives of the client: cost, quality, and time. Unfortunately, most of the selection processes are analysed under the same methodology without considering that variations in project, type of activity, and other attributes should affect the chosen method. To provide a novel form of treating subcontractor's selection, we proposed an additive sorting method to categorize activities to be outsourced in civil construction based on ROR-UTADIS method, which is a modification of the UTADIS method that includes new forms of supplying preference information. It was applied in the construction of a brewery in Brazil. It was perceived that the method is applicable and intuitive for decision makers, even though there are quite a few points to be taken, analysed to avoid misclassification.

1. Introduction

The choice of contractors and subcontractors is problematic studied by several authors over the years due to the clients' difficulty to achieve the best value for invested money since the construction industry has several construction companies with different credibility, sizes, and quality. Reference [1] made a literature review on contractor selection practice in the United Kingdom and realized that the main criteria evaluated by the clients are time, cost, and quality, which vary in priority relation for each client through a trade-off process. The main problem is that some of these criteria are usually subjective and probabilistic, driving the process to a multicriteria decision analysis (MCDA) approach. Therefore, the client plays the role of the decision maker (DM) and needs to analyse his final objectives to have his criteria weighted and reach the best decision.

To solve the contractor selection problem [2] proposed some prequalification criteria, such as contractors' organization, financial considerations, management resources, past experience, and past performance, which are analysed using Multiattribute Analysis. Later the selection is made based on tenderer evaluation by applying Multiattribute Utility Theory (MAUT) [3] in more specific criteria. These criteria were built hierarchically, and each of them had subcriteria to achieve the analysis. Sönmez et al. [4] also advocated for the prequalification of contractors along with evidential reasoning to solve the multicriteria decision-making (MCDM) problem of selection when there are uncertainty and imprecision in the decision process. Lam et al. [5] also proposed a prequalification of contractors for selection based on Support Vector Machine to classify the companies into two classes based on agreed criteria. If the client decides to apply a prequalification analysis, he still needs to carry out a proper selection, which can be driven in the same way of supplier selection's methodologies. In literature, one can find several methodologies.

There are models in which one can find the combination of TOPSIS with other tools, such as fuzzy decision-making approach, [6] proposed a model to calculate the fuzzy positive solutions and fuzzy negative solutions simultaneously, and [7] proposed a framework for supplier selection that includes linear programming. Also, [8] combines FAHP with FTOPSIS, [9] combined a fuzzy DEMANTEL model to evaluate the criteria for cause or effect, and [10] used the tools to propose a group decision model. Finally, [11] evaluated green suppliers by using linguistic preferences, [12] used hierarchical fuzzy TOPSIS, and [13] built a model to select green suppliers in Brazil. There are models that combine Minkowski distance and grey number operations [14] and interval data [15].

Models based on FAHP can also be found, such that a Fuzzy hierarchical TOPSIS model is proposed with parametric considerations to avoid violations of the TOPSIS method [16]. Still on this consideration, [17] proposed a model to incorporate the method considerations of benefits, opportunities, costs, and risks, [18] combines FAHP with MAUT, [19] applies the FAHP to assess contractor selection criteria in the group environment, and [20] extended the method to D numbers. Still on fuzzy sets, it can be found along with goal programming for a single DM [21] and to group decision [22] or with ELECTRE III for group decision [23] or green supplier selection [24].

There are combinations of ELECTRE with fuzzy approach [25] and Atanassov interval-valued intuitionistic fuzzy sets [26]. There is analysis combining multiattribute decision-making with intuitionistic fuzzy sets [27]. One can also find in literature models combining MAUT with linear programming (LP) for group decision-making [28], models to aggregate crisp values into interval-valued intuitionistic fuzzy sets for group decision-making [29], or the development of new fuzzy aggregation operators for intuitionistic fuzzy in the supplier selection context [30]. In the context of supplier selection, one can find an approach with fuzzy inhomogeneous multiattribute for group decision-making [31] and the combination of VIKOR method with fuzzy sets for group decision-making and applying linear programming for choosing the best supplier [32]. Reference [33] proposed a group decision-making method based on entropy measure with VIKOR method and [34] proposed a compromise solution method for group decision considering both conflicting qualitative and quantitative criteria. Reference [35] proposed an integration of evaluation of criteria under MAUT, but with the use of the ELECTRE method to avoid the rigid axioms of the prior method.

The propositions found in literature are either to prequalify the subcontractors or to select them, but none of them considered that, during the whole cycle of the project, several selections will occur and they may involve different sums of money, risks, qualities, and necessities. If all selections are analysed through the same procedure, it is likely to adopt a methodology that could be either too strict or too loose. Gonçalo and Alencar [36] presented a model in which they apply PROMSORT to sort the activities and materials to be hired or bought into classes, by analysing its strategic impact on the company's goals. Similar to this last article, we propose to apply a sorting procedure, but our goal is to allow the DM to apply different methodologies to subcontractors' selection that will be more appropriate to each class of assignment, since all activities will be hired at some stage of the project. Thus, we propose a model to categorize the activities based on ROR-UTADIS [37] and allow the DM to apply different subcontractors' selection methodologies.

The sorting procedures can be either case-based or elicited. ROR-UTADIS is one of the case-based methods. These methods can consider an underlying additive function, as is the case of UTADIS [38, 39] and its modifications or decision rules to classify the alternatives, as is the case of Dominance-Based Rough Sets Approach (DBSA) [40]. The original Rough Sets Theory could not consider DM's preference information; it was based on "if…then…" rules, that the DM could build, applying them to actions/objects to obtain preference relations. One could achieve a recommendation through the exploitation of these relations. The original rough sets approach required the binary relations defined on it to have some properties, such as being reflexive, transitive, and symmetric, making it difficult to apply to obtain preference information of a DM, but the DBSA relaxed the properties requiring them to be reflexive and transitive. It was also proved that Sugeno Integral cannot be used in DBSA because it requires the set to be single-graded and the sorting problem using DBSA needs more than one grade on the evaluation scale [40]. The DBSA is not directly used in any of the additive sorting methods, but the lower and upper thresholds approximations used for classification in this method resemble the necessary and possible assignments found in the case-based methods with underlying additive function $\text{UTADIS}^{\text{GMS}}$ [41] and ROR-UTADIS [37]. Then, this study uses ROR-UTADIS to make the classification of civil construction (CC) activities in different categories of analysis.

This paper is structured into six sections; in the next section, we introduce the current scenario of additive sorting methods, explaining their developments and drawbacks. In Section 3 we present a brief review of the methodology applied in this paper. Section 4 is devoted to explaining the proposed model and its importance to the civil construction (CC). Section 5 presents the selected context of the application and the information required to run the analysis, as well as the results. In Section 6 we discuss the results presented in the previous section. Finally, in Section 7 we conclude with final remarks.

2. Additive Sorting Methods

Sorting is problematic, where the DM needs to assign alternatives to predefined classes, which are defined in an ordinal way; classification, on the other hand, is problematic to assign alternatives to predefined nominal classes, that is, nonordered classes. Then, in this study, we propose an application of a sorting method. The researchers in this area show that these methods are important to solve real-world problems, such as performing medical diagnosis through classification of patients into disease groups, assigning personnel to appropriate occupation groups based on their qualifications, credit risk assessment, failure prediction, and so forth. The development of MCDA techniques started with discriminant analysis and recently is based on operations research and artificial intelligence techniques [42]. Sorting models for group decision-making have been also developed [43].

In [42] there is a description of the two aspects that involves sorting methodology: the form of criteria aggregation and the method applied to define preferential information. Regarding aggregation, one can find three types: outranking, as in ELECTRE TRI; the utility function, as in UTADIS ([38, 39]); and the simple discriminant functions, which differ from the MAUT methodology because it cannot be considered a preference model. One can elicit preference information directly or indirectly.

An example of direct elicitation is the methodology proposed by [44]. The method consists of defining classes that are not ordered and their criteria. The classification is dichotomous and built through the application of SMARTS method. The name of the method is Multiple Criteria Classification (MCC), once the classes may consider different criteria for its classification. It provides four types of classification: (a) alternatives do not override each other and are all classified; (b) one or more alternatives override each other and are all classified; (c) alternatives do not override each other, and the classification is incomplete, and (d) alternatives override each other and the classification is incomplete.

Case-based sorting methods are indirect forms of definition of preference information that require the presentation of a set of hypothetical or real cases, and the DM is supposed to assign each of them to one predefined class, with such information possible to calibrate the parameters to reflect the DM's preferences. Examples of case-based sorting methods are UTADIS and its variations, DBSA and case-based distance model. The drawback of this approach is that only a few judgments produce the DM's preference and therefore can be consistent with several sets of parameters. To calculate errors and minimize misclassification most of those methods apply linear programing to calculate holistically the parameters. The authors in [45] state that there are three possible outcomes in the solution of this error identification:

(i) The minimization problem has a unique solution with value larger than zero, showing inconsistencies in the preference information.

(ii) The solution is unique and equal to zero; thus there is only one possible solution.

(iii) There are several solutions equal to zero, showing that there are several profiles compatible with the information provided by the DM.

Chen et al. [46] present a proposal of a case-based distance model to solve sorting problematic. It was designed based on an ABC analysis of stock-keeping units in an enterprise. Instead of analysing it based only on annual dollar usage, other criteria were included. It uses Euclidean distances to do the calculations, once the DM can easily understand it. At the beginning of the process, the DM has to define if the criteria increase in preference or decrease and the variability of each criterion, with the information of each alternative on each criterion, is possible to formulate the problem that will be solved using linear programming.

The UTADIS method is a sorting and interactive method, in which the global utility model or additive utility function and class thresholds are calculated through linear programming. Thus, as a result, all parameters of the DM's preference information are calculated, such as the difference between the marginal utilities of two successive values of subintervals and the threshold to ensure the classification and the errors of misclassification. Furthermore, [47] built software known as PREFDIS (PREFerence DIScrimination). It incorporates the original UTADIS method and its variations: UTADIS I, to incorporate distances of correctly classified alternatives from the utility thresholds; UTADIS II, based on mixed integer programming formulation to minimize misclassifications; and UTADIS III that combines the two other variants. The system allows the DM to model nonmonotone preference and includes a postoptimality phase to verify other optimal and suboptimal solutions. Authors of [48] carried out an extensive experimental investigation on UTADIS to shed light on some critical issues regarding the stability of the sorting model developed through preference disaggregation analysis. To correct these problems, they proposed a heuristic (HEUR2), which was tested using Monte Carlo simulation and subjected to an ANOVA.

Köksalan and Özpeynirci [49] presented a modification on UTADIS to diminish the misclassification problems that occurs even when considerable information is available. This methodology does not try to estimate the parameters of an additive utility function. Instead, they impose some restrictions in the linear programming to show if a selected alternative may be assigned to a class. It assumes an additive utility function and takes into account the restrictions created by the DM's assignments to try to place alternatives into categories.

Cai et al. [50] proposed other modification called PUTADIS. It is a progressive approach to assigning alternatives to ordered categories, considering two types of imprecise information and allowing the DM to provide preference information in an interactive form. The global utility function of the DM is built and updated using a heuristic algorithm and later a mixed integer linear programming model is applied in order to identify inconsistencies on preference information. If there is any inconsistency, it is presented to the DM in order to review his assignments. When it is consistent, then three mixed integer linear programs provide the fittest category and a range of possible categories.

Greco et al. [41] modified UTADIS into UTADIS$^{\mathrm{GMS}}$ bringing to the original method two kinds of assignment to classes: the necessary and the possible assignment. These assignments are computed through linear programming and are based on reference examples. The necessary assignments are the ones that for sure belong to the assigned class and the possible ones are those that could be assigned to two or more classes. The proposition allows the DM to provide imprecise assignment examples (interval assignments) considering the confidence levels of information; in such a case the method expresses the results as ranges of classes that correspond to different confidence levels. It allows the use of nondecreasing marginal value functions, instead of piecewise linear marginal value functions, and when the set of assignments is inconsistent, then the DM is required to analyse more reference alternatives. Reference [51] extended the methods UTA^{GMS} and UTADIS^{GMS} to group decision, calling them UTA^{GMS}-GROUP and UTADIS^{GMS}-GROUP, respectively. The concepts of necessary and possible assignments are extended to the DMs and the space investigated is the consensus and disagreement among DMs.

Greco et al. [52] extended UTADIS^{GMS} introducing the concept of the representative value function in robust ordinal regression with the aim of considering complete sets of instances of a preference model compatible with the information provided by the DM; it means that the representativeness of a selected value function is understood in the sense of robustness preoccupation. This method was also extended to group decision in [53] with the same considerations of the method proposed by [50], but considering the representative value function in robust ordinal regression.

Kadziński and Tervonen [54] presented a new approach for multiple criteria sorting problems considering a set of preference model instances compatible with the disaggregation of preferences. The analysis was made using PREFDIS [47], and the possible and necessary assignments were made using robust ordinal regression (ROR). The analysis was enriched with class acceptability indices adapted from Stochastic Multicriteria Acceptability Analysis (SMAA), to analyse the alternatives that were classified as possible, to decide to which class it should be assigned. The ROR approach is the notion of assignment-based weak preference relations and, analogously to the assignments, new necessary and possible assignment-based relations were established, as well as an estimative of assignment based on pairwise outranking indices. Later on the ROR-UTADIS model presented in [37] introduced an assignment-based pairwise comparison to the disaggregation process in which the information provided by the DM comes in the form of imprecise statements referring to the desired assignments for pairs of alternatives, but without assigning the reference alternatives to any concrete class.

Since the ROR-UTADIS method is a holistic additive method that allows the DM to provide more imprecise information, it was selected for this application. It is important for the method to be additive because the DM treats the problem in a compensatory rationality. This method also allows the DM to provide imprecise information when he is not confident about his preference information. Finally, we believe that the extra preference information will decrease number of preference profiles, requiring fewer iterations to reach a recommendation.

3. Review on ROR-UTADIS

Kadziński et al. [37] developed a method called ROR-UTADIS, which holistically calculates all parameters using disaggregation of preferences. The objective is to calculate value functions that are compatible with the reference alternatives. The information provided is incomplete, indirect, and imprecise, guaranteeing, thus, the interactivity and flexibility of the procedure. The information provided may be in the form of assignment examples, assignment-based pairwise comparisons, or desired class cardinalities. The first two options are related to the reference alternatives, and the last is related to the whole set of alternatives. The following mathematical models were built to reproduce preference of the DMs, which can specify information for all types of preference information or provide only the pieces of information he feels comfortable with.

The notation used is as follows.

(i) $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ is a finite set of n alternatives.

(ii) $A^R = \{a^*, b^*, \ldots\}$ is a finite set of reference alternatives, assuming $A^R \subseteq A$.

(iii) $G = \{g_1, g_2, \ldots, g_j, \ldots, g_m\}$ is a finite set of m evaluation criteria, $g : A \to \mathbb{R}\ \forall j \in J = \{1, 2, \ldots, m\}$.

(iv) $X_j = \{x_j \in \mathbb{R} : g_j(a_i) = x_j,\ a_i \in A\}$ is the set of all different evaluations of g_j, $j \in J$, and its preferential direction is strictly crescent; thus we assume without loss of generality that greater values on $g_j(a_i)$, imply a better performance of alternative a_i on criterion j.

(v) $x_j^1, x_j^2, \ldots, x_j^{n_j(A)}$ are the ordered values of X_j, $x_j^k < x_j^{k+1}$, $k = 1, 2, \ldots, n_j(A) - 1$, where $n_j(A) = |X_j|$ and $n_j(A) \leq n$.

(vi) $g_{j,*}$ and g_j^* are the lower and upper bounds for the performance scale g_j; if they are not designated a value, it can be assumed that they are equal to the worst and best performances of existing alternatives; that is, $g_{j,*} = x_j^1$ and $g_j^* = x_j^{n_j(A)}$.

(vii) Let $C_1, C_2, \ldots, C_p$ be p predefined preference-ordered classes, where C_{h+1} is preferred to C_h, $h = 1, 2, \ldots, p - 1, h \in H, H = \{1, 2, \ldots, p\}$.

The preferences of the DMs are presented using an additive value function:

$$U(a) = \sum_{j=1}^{m} u_j\left(g_j(a)\right) = \sum_{j=1}^{m} u_j(a). \tag{1}$$

The basic sets of constraints are constructed to guarantee the marginal value functions u_j and they are monotone, nondecreasing and occur in the interval $[0, 1]$. There are two sets of constraints: one for the threshold-based procedure (TH) and another to the example-based procedure (EX). This paper only presents the threshold-based procedure equations, except for the base equations, once this was the selected procedure. Consider

$$E_{\mathrm{EX}}^{\mathrm{BASE}}\begin{cases}[M1]\ u_j\left(x_j^k\right)-u_j\left(x_j^{k-1}\right)\geq 0, & k=2,\ldots,n_j(A),\ j=1,\ldots,m,\\ [M2]\ u_j\left(x_j^1\right)\geq u_j\left(g_{j,*}\right), & u_j\left(x_j^{n_j(A)}\right)\leq u_j\left(g_j^*\right),\\ [N1]\ u_j\left(g_{j,*}\right)=0, & j=1,\ldots,m,\ \sum_{j=1}^{m}u_j\left(g_j^*\right)=1.\end{cases}\tag{2}$$

Due to the limitations on holistic judgment caused by general monotonic marginal value functions, they should be substituted for piecewise linear functions. Thus for each $u_j = 1,\ldots,m$, a number of characteristic points (γ_j) have to be defined; to divide the intervals $[g_{j,*}; g_j^*]$ into $\gamma_j - 1$ equals subintervals with the endpoints $g_j^s = g_{j,*} + (g_j^* - g_{j,*}) * s - 1/\gamma_j - 1$, $s = 1,\ldots,\gamma_j$. Modifying $[M1]$ and $[M2]$ to adapt to the piecewise linear equations, the set of constraints is as follows:

$$E_{\mathrm{EX}}^{\mathrm{BASE}}\begin{cases}[M1]\ u_j\left(g_j^s\right)-u_j\left(g_j^{s-1}\right)\geq 0, & s=1,\ldots,\gamma_j,\ j=1,\ldots,m,\\ [M2]\ u_j\left(x_j^k\right)=u_j\left(g_j^{s-1}\right)+\dfrac{\left(u_j\left(g_j^s\right)-u_j\left(g_j^{s-1}\right)\right)\left(x_j^k-g_j^{s-1}\right)}{\left(g_j^s-g_j^{s-1}\right)} & \forall x_j^k\in\left[g_j^{s-1};g_j^s\right],\ j=1,\ldots,m,\ k=2,\ldots,n_j(A),\\ [N1]\ u_j\left(g_{j,*}\right)=0, & j=1,\ldots,m,\ \sum_{j=1}^{m}u_j\left(g_j^*\right)=1.\end{cases}\tag{3}$$

The representation of the preference of the DM regarding the threshold-based procedure is through a vector (U,t), where U represents the utility and t the threshold vector $t = \{t_1,\ldots,t_{p-1}\}$ that separates the p ordered classes, defined in such a way that t_{h-1} and t_h are the lower and upper threshold of class C_h. The basic set of constraints under this configuration is

$$E_{\mathrm{TH}}^{\mathrm{BASE}}\begin{cases}t_1\geq\varepsilon,\ t_{p-1}\leq 1-\varepsilon,\\ t_h-t_{h-1}\geq\varepsilon, & h=2,\ldots,p-1,\\ E_{\mathrm{EX}}^{\mathrm{BASE}}.\end{cases}\tag{4}$$

The threshold-based sorting model is defined by $(U,\mathbf{t})\in(U,\mathbf{t})^R$ and $a\rightarrow C_h$ if and only if $U(a)\in[t_{h-1},t_h[$.

In the assignment examples the reference alternatives ($a^*\in A^R\subseteq A$) are assigned to a range of classes; hence, the desired assignment is

$$a^*\longrightarrow\left[C_{L^{\mathrm{DM}}(a^*)},C_{R^{\mathrm{DM}}(a^*)}\right],\tag{5}$$

where $[C_{L^{\mathrm{DM}}(a^*)},C_{R^{\mathrm{DM}}(a^*)}]$ is an interval of contiguous classes; if $L^{\mathrm{DM}}(a^*) = R^{\mathrm{DM}}(a^*) = h$ for some $h\in H$, then the assignment is said to be precise and imprecise otherwise:

$$L^U(a)=\mathrm{Max}\left\{\{1\}\cup\left\{L^{\mathrm{DM}}(a^*):U(a^*)\leq U(a),\ a^*\in A^R\right\}\right\},$$

$$R^U(a)=\mathrm{Min}\left\{\{p\}\cup\left\{R^{\mathrm{DM}}(a^*):U(a^*)\geq U(a),\ a^*\in A^R\right\}\right\}.\tag{6}$$

To guarantee that the assignment example a^*, which is assigned by the DM to a class range $[C_{L^{\mathrm{DM}}(a^*)},C_{R^{\mathrm{DM}}(a^*)}]$, is neither worse than the lower threshold of class $C_{L^{\mathrm{DM}}(a^*)}$ nor worse than the upper threshold of class $C_{R^{\mathrm{DM}}(a^*)}$, the problem is submitted to constraints (7) as follows:

$$E_{\mathrm{TH}}^{AE}\begin{cases}\forall a^*\in A^R\longrightarrow\left[C_{L^{\mathrm{DM}}(a^*)},C_{R^{\mathrm{DM}}(a^*)}\right]:\\ [TAE_1]\ U(a^*)\geq t_{L^{\mathrm{DM}}(a^*)-1},\\ [TAE_1]\ U(a^*)+\varepsilon\leq t_{R^{\mathrm{DM}}(a^*)}.\end{cases}\tag{7}$$

Different from the assignment examples, which consist in holistic judgments calculated through recommendations provided by the DM, the assignment-based pairwise comparison is composed of two reference alternatives, $(a^*,b^*)\in A^R\times A^R$, and pairwise comparison between them with imprecise judgment. It is represented as $a^* \underset{\geq k,\mathrm{DM}}{\overset{\rightarrow}{>}} b^*$, meaning that a^* and b^* are separated by at least k classes and $a^* \underset{\leq l,\mathrm{DM}}{\overset{\rightarrow}{>}} b^*$ meaning that a^* and b^* are separated by at most l classes. To guarantee these restrictions, the problem is submitted to the following constraints E_{TH}^{PCL} and E_{TH}^{PCU}:

$$E_{\mathrm{TH}}^{PCL}\begin{cases}\forall a^*, b^* \in A^R : a^* \underset{\geq k,\mathrm{DM}}{\succ\rightarrow} b^*: \\ \text{for } h = 1,\ldots,p-k: \\ [TPCL_1]\ U(a^*) \geq t_{h+k-1} - Mv_h^{PCL}(a^*, b^*), \\ [TPCL_2]\ U(b^*) + \varepsilon \leq t_h + Mv_h^{PCL}(a^*, b^*), \\ [TPCL_3]\ \sum_{h=1}^{p-k} v_h^{PCL}(a^*, b^*) = p-k-1, \\ [TPCL_4]\ v_h^{PCL}(a^*, b^*) \in \{0,1\}, \qquad h = 1,\ldots,p-k. \end{cases}$$

$$E_{\mathrm{TH}}^{PCU}\begin{cases}\forall a^*, b^* \in A^R : a^* \underset{\leq l,\mathrm{DM}}{\succ\rightarrow} b^*: \\ \text{for } h = 0,\ldots,p-l-1: \\ [TPCU_1]\ U(a^*) + \varepsilon \leq t_{h+l+1} + Mv_h^{PCU}(a^*, b^*), \\ [TPCU_2]\ U(b^*) \geq t_h - Mv_h^{PCU}(a^*, b^*), \\ [TPCU_3]\ \sum_{h=0}^{p-l-1} v_h^{PCU}(a^*, b^*) = p-l-1, \\ [TPCU_4]\ v_h^{PCU}(a^*, b^*) \in \{0,1\}, \qquad h = 0,\ldots,p-l-1. \end{cases} \tag{8}$$

It is also possible for the DM to impose requirements concerning class cardinalities, which is common in real-world sorting problems, such as classification of journals or graduation courses. As a solution, one may include the following constraints, requiring that C_h has at least $N_{h,\mathrm{DM}}^{\min}$ and at most $N_{h,\mathrm{DM}}^{\max}$ alternatives, with $N_{h,\mathrm{DM}}^{\min} \leq N_{h,\mathrm{DM}}^{\max}$:

$$E^{CC}\begin{cases}[CL]\ \sum_{a\in A} v_h(a) \geq N_{h,\mathrm{DM}}^{\min}, \\ [CU]\ \sum_{a\in A} v_h(a) \leq N_{h,\mathrm{DM}}^{\max}, \\ [CV]\ v_h(a) \in \{0,1\}, \qquad a \in A,\ h \in H. \end{cases} \tag{9}$$

The DM can use constraint $[CL]$ or $[CU]$ to model the class cardinality of class C_h and the information can be either some alternatives or a frequency. Also, it is necessary also to apply the set of constraints $E_{\mathrm{TH}}^{\mathrm{CC}}$:

$$E_{\mathrm{TH}}^{CC}\begin{cases}\forall a \in A,\ h = 1,\ldots,p: \\ [TC_1]\ U(a) \geq t_{h-1} - M(1 - v_h(a)), \\ [TC_2]\ U(a) + \varepsilon \leq t_h + M(1 - v_h(a)), \\ [TC_3]\ \sum_{h=1}^{p} v_h(a) = 1, \qquad \forall a \in A. \end{cases} \tag{10}$$

To verify if the set of preference model instances (pairs $((U,t)^R$ or the value functions U^R)) compatible with the information provided by the DM is not empty, the following problem is considered:

$$\begin{aligned} \text{Maximize:}\quad & \varepsilon, \\ \text{subject to}\quad & E^{\mathrm{SORT}}, \end{aligned} \tag{11}$$

where $E^{\mathrm{SORT}} = E_{\mathrm{TH}}^{\mathrm{BASE}} \cup E_{\mathrm{TH}}^{AE} \cup E_{\mathrm{TH}}^{PCL} \cup E_{\mathrm{TH}}^{PCU} \cup E^{\mathrm{CC}} \cup E_{\mathrm{TH}}^{\mathrm{CC}}$.

The problem is solved applying mixed integer linear programming and if the solution is not empty, then E^{SORT} is feasible and $\varepsilon^* > 0$, where ε^* is the solution of the former equation. If this occurs, it means that the pieces of preference information could be reproduced; if it cannot happen, then the assumed preference model needs to be reviewed. To apply corrections in the model a binary variable should be added in each piece of preference information, especially the ones where the DM could not provide confident information.

Since the model works with a recommendation rather than a solution and the compatible recommendations depend on which piece of preference information is selected, it is important to submit the information to robustness and sensitivity analysis. Thus, the DMs are obliged to confront their value systems, providing insights into the process.

The sets of possible assignments are the ones in which at least one compatible model instance exists that assigns alternative a to class C_h and the necessary assignments are the ones where all model instances assign alternative a to class

C_h. The possible assignment of $a \in A$ to class C_h, $h \in H$, can be verified through the set of constraints $E^{\mathrm{TH}}(a \rightarrow^{P} C^h)$ and the necessary assignment through the set of constraints $E^{\mathrm{TH}}(a \rightarrow^{N} C^h)$:

$$E^{\mathrm{TH}}\left(a \longrightarrow^{P} C^h\right) \begin{cases} [TP_1] & U(a) \geq t_{h-1}, \quad \text{if } h \geq 2, \\ [TP_2] & U(a) + \varepsilon \leq t_h, \quad \text{if } h \leq p-1, \\ [TP_3] & E^{\mathrm{SORT}} \end{cases}$$

$$E^{\mathrm{TH}}\left(a \longrightarrow^{N} C^h\right) \begin{cases} [TN_1] & U(a) + \varepsilon \leq t_{h-1} + M \cdot v_1, \quad \text{if } h \geq 2, \\ [TN_2] & U(a) \geq t_h - M \cdot v_2, \quad \text{if } h \leq p-1, \\ [TN_3] & v_1 + v_2 = 1, \quad \text{if } 1 \leq h \leq p-1, \\ [TN_4] & v_1, v_2 \in \{0,1\}, \\ [TN_5] & E^{\mathrm{SORT}}. \end{cases} \tag{12}$$

The sets of possible assignment-based preference relations $a \gtrsim^{\rightarrow,P} b$ are true if a is assigned to a class at least as good as the class of b for at least one compatible model instance, and the necessary assignment-based preference relation $a \gtrsim^{\rightarrow,N} b$ holds if a is assigned to a class at least as good as the class of b for all compatible model instances. These constraints allow direct comparisons between alternatives concerning their sorting recommendations. The relation $a \gtrsim^{\rightarrow,P} b$ is verified through the set of constraints $E_h^{\mathrm{TH}}(a \gtrsim^{\rightarrow,P} b)$ and relation $a \gtrsim^{\rightarrow,N} b$ through the set of constraints $E_h^{\mathrm{TH}}(a \gtrsim^{\rightarrow,N} b)$:

$$E_h^{\mathrm{TH}}\left(a \gtrsim^{\rightarrow,P} b\right) \begin{cases} [TPAR_1] & U(a) \geq t_{h-1}, \quad \text{if } h \geq 2, \\ [TPAR_2] & U(b) + \varepsilon \leq t_h, \quad \text{if } h \leq p-1, \\ [TPAR_3] & E^{\mathrm{SORT}} \end{cases}$$

$$E_h^{\mathrm{TH}}\left(a \gtrsim^{\rightarrow,N} b\right) \begin{cases} [TNAR_1] & U(b) \geq t_h, \\ [TNAR_2] & U(a) + \varepsilon \leq t_h, \\ [TNAR_3] & E^{\mathrm{SORT}}. \end{cases} \tag{13}$$

To consider $N_h^{\min}$ and the $N_h^{\max}$ as minimal and maximal cardinality of class C_h, the mixed integer linear programming problems need to be solved:

$$\begin{aligned} \text{Minimize/Maximize:} \quad & \sum_{a \in A} v_h(a), \\ \text{subject to} \quad & E^{\mathrm{SORT}}, \\ & \text{where } v_h(a) \in \{0,1\}. \end{aligned} \tag{14}$$

4. Sorting Model for Categorizing Activities in Civil Construction

In the CC context, outsourcing activities is common practice. During the project cycle, several hiring processes are open and closed, and they have different sizes and schedules. They have to be wisely managed to avoid losses and liabilities, and the selection must have a structured methodology, considering that there are a great variety of types of activities, risks, and contract size during this cycle. There will be contracts that will require more attention during its conduction, and others will not, due to their simplicity. The application in this paper consists of a sorting problematic: categorizing all activities with ROR-UTADIS to allow the DM to apply different methods to select subcontractors, improving his administration over the subcontractors.

In this kind of project, it is possible to have more than one DM, but this model does not evaluate the aggregation of DM's preferences. Thus, only one DM in the contractor atmosphere will manage the project, which is the director, and he has total autonomy to make all the decisions in the project he manages. His objectives are aligned with the ones of the company he works for that has a decentralized structure, requiring its president to be completely absent from the projects. Besides being legally and technically responsible for the project, the director has to conquer the project, satisfy the client and manage the relationship between the two companies. Moreover, he is involved in all parts of the process and has to take responsibility for the consequences achieved, so he will be willing to keep the corporate image, reach the best profitability possible in the project, and satisfy and win the client for a long-term relationship.

Other actors will be directly or indirectly involved in the process, such as the analyst, stakeholders, and specialists. There is one actor present in the literature [55], which is not part of this process: the client, as the one that works as an intermediate in the process between the DM and the analyst,

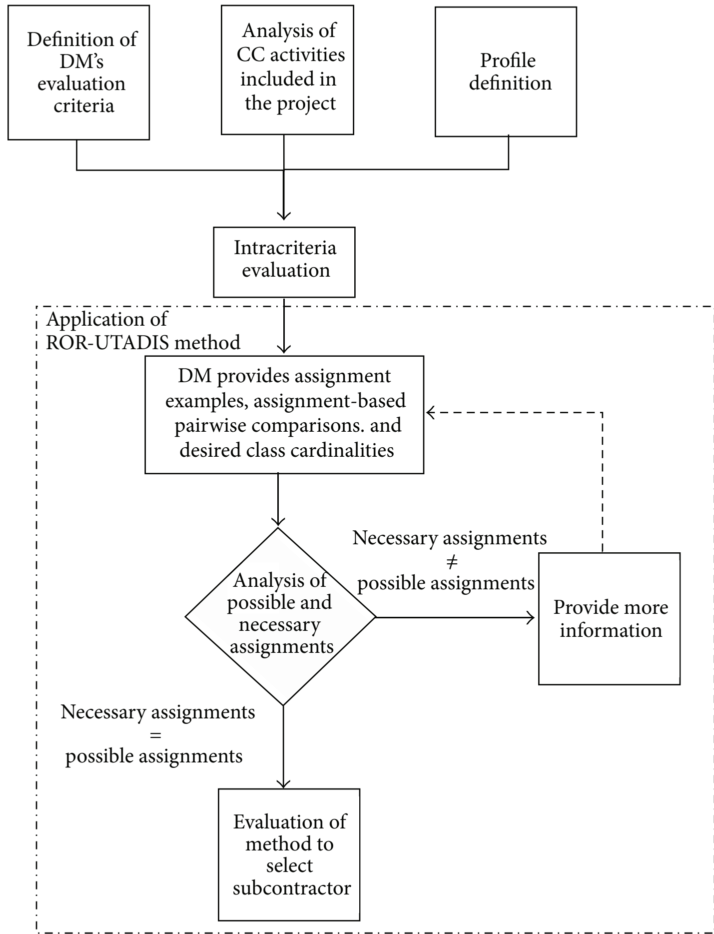

Figure 1: Flow of the sorting model for categorizing subcontractors' activities based on ROR-UTADIS.

when the DM is an actor usually absent from the decision process for any reason. However, in this situation the DM will be involved full-time in the decision process, resulting in the exclusion of this actor. The analyst will work on structuring the problems, the bidding procedures, hiring the subcontractors, and managing the relationship between them and the contractor. The stakeholders will be the Ministry of Labour, the Federal Government, the City Hall, the state, the population, environmental agencies, and the client, which can be either public or private. In this problem it is private, therefore present in all steps of the project. There may be specialists hired during the project but will not be involved in the sorting procedure.

Figure 1 shows the flow of the proposed model to categorize subcontractors in civil construction. The analyst has to structure the alternatives, by verifying which civil construction activities will be outsourced. After that he might help the DM to verify his objectives, to relate them to attributes, and evaluate the alternatives. Both actors build the classes' profiles. After this process, the analyst has to create an evaluation matrix of the alternatives and finally, with the help of a Decision Support System (DSS) built in MATLAB, the DM provides his preference information. He will keep providing information until the model converges to one unique solution. In the following section a problem context and its description, as well as the descriptions of the stages of the presented model, are presented.

5. Sorting Model Application

The problem context is the CC of a brewery in Pernambuco State, Brazil, in a cost plus contract. Therefore, there is only an estimation of how much it will cost, which is expected to be US$ 70,000,000.00 (seventy million dollars). Once it is a cost plus contract, several activities have to be outsourced, instead of being carried out by the contractor itself. Thus, many subcontractors might be hired. The objectives of the director, who plays the role of DM, are to preserve the company's image, by avoiding liabilities, problems with population and Ministry of Labour and guaranteeing the safety of the workers related to the project. Those can be achieved by hiring reliable companies, verifying if they are financially healthy and if they work according to the safety laws. The other two objectives are to reach the best profitability that will only be possible enlarging the contract's objective, which can be reached with the third objective of winning the client, attained by delivering quality, low cost and finishing the project in the time agreed between contractor and client. The application

Table 1: Alternatives analysed in the brewery project.

Alternatives	Description
Air conditioning	Activity includes the supply of air conditioning system and installation
Concrete	Concrete supply according to mix informed by the contractor and including installation of the concrete batching plant inside the project area and availability of concrete mixer truck and concrete pump in the quantity required
Containers	Activity includes the supply of containers for the building site and installation (electricity and logic)
Heavy equipment	Activity includes the supply of heavy equipment as planned by the contractor and preventive and corrective maintenance
Molds, shoring, and scaffolding	The activity includes the supply of molds, shoring, and scaffolding, as well as shoring and scaffolding projects for the activities and projects presented by the contractor
Gypsum liner and partition	The activity includes the supply and installation of gypsum liners and partitions
Continuous Flight Auger Stake (CFA Stake)	The activity includes equipment supply, activity, and dynamic and static load reports
Hydroseeding	The activity includes material and equipment necessary to the hydroseeding growth
Waterproofing	The activity includes material, equipment, and tests to prove the waterproofing
Asphalt paving	The activity includes equipment and materials necessary for the asphalt pavement of the designated area
Concrete paving	The activity includes equipment and guarantee for the concrete paving activity; the material is on behalf of the contractor
Precast concrete	The activity includes precast concrete adaptation project, material, equipment, transportation, and installation of concrete elements
Food supply	The activity includes food supply for the personnel designated by the contractor in the building site's dining hall
Property security	The activity includes 24 h of armed property security and patrol
Vegetal suppression	The activity includes equipment and licenses to do the vegetal suppression
Earthwork	The activity includes equipment and personnel for the earthwork according to the project provided by the contractor
Transportation of personnel	The activity includes equipment to transport personnel from and to their residencies in the beginning and by the end of each business's hours according to the routes provided by the contractor

was analysed using the threshold-based procedure and, to calculate the results and create an interactive environment, the whole procedure was performed using MATLAB.

5.1. Definition of Alternatives (Activities Included in the Brewery Construction). There were several activities to be outsourced for the construction; some were known to be low risk and low cost activities but required a long-term relation; others were short-term relations but involved high costs or high risks or both. Thus, it was important to list all the alternatives. Table 1 presents all the activities of the project to be hired through a bidding procedure.

5.2. Evaluation of DM's Criteria. The DM expressed his objectives to formulate the sorting procedure based on his preferences; this information was used to define seven criteria to evaluate the alternatives. Table 2 presents these criteria. Three of them are cardinal, and the others are presented in qualitative scale, which are ordered in direction of preference and coded in numbers that were later converted to cardinal scale.

5.3. Profiles. The sorting problematic consists of evaluating several alternatives to sort them into classes; for this purpose it is necessary to analyse which classes are considered. In this problem, the main reason to sort the alternatives into classes is to avoid liabilities by managing carefully the contracts more likely to present those questions. There are three predefined classes to classify the alternatives: high impact (C3), medium impact (C2), and low impact (C1) activities.

The high impact activities are the ones that represent high costs in the project and directly affect the schedule of the project and the client's perception of the activities carried out and their quality. These activities influence the histogram of personnel and guarantee of the project. The contractor has to accompany the subcontractors' personnel due to the long permanence in the building site and interaction with other companies and contractor's labour. It may be a specialized activity and have only a few suppliers or even be a monopoly, so these selection processes have to be carefully accompanied.

The medium impact activities, even though they may consist of a long-term relation, have many available suppliers, and if a delay comes up, it can be recovered by contracting another company to work on the activity simultaneously. It strongly affects the client's satisfaction with the project's accomplishment. There are also some preoccupations with liabilities, once they usually consist of long-term relations. The low impact activities do not have to be hired with the same rigor as the high impact and medium impact contracts,

Table 2: Criteria used to analyse the brewery project.

Criteria	Description	Scale	Min/max
Cost	Expected cost to accomplish the activity, calculated before the bidding procedure	Monetary	Max
Activity duration	Duration expected on schedule for the accomplishment of the activity	Days	Max
Number of suppliers	Suppliers' quantity in order to verify easiness to find companies to carry out the activity	Unit	Min
Available resources	Qualitative criteria in order to evaluate the availability of resources in the market (local or surrounding areas)	Qualitative (1 to 5) (1) No available resources near the CC site, all resources must be taken from another city centre (2) No availability of materials or equipment near the CC site, but the local labour can be trained for qualification purposes (3) Qualified labour near the CC site, but materials and equipment must be taken from another city centre (4) Availability of equipment and materials but labour must be trained (5) All resources are available	Min
Risk exposition	Proxy criteria in order to evaluate through qualitative criteria the risk exposition analysing technical responsibility and labour permanence in the building site	Qualitative (1 to 5) (1) No labour permanence is required and subcontractor is not technically responsible for the activity (equipment renting without activity) (2) No labour permanence is required and subcontractor is not technically responsible for the activity, but some visits are necessary (equipment renting without operator but with maintenance) (3) Labour permanence is required, but subcontractor is not technically responsible for the activity (equipment renting with activity, steel cutting and bending, etc.) (4) No labour permanence is required, but subcontractor is technically responsible for the activity (projects and project evaluation) (5) Labour permanence is required, and subcontractor is technically responsible for the activity	Max
Necessity of maintenance	Qualitative criteria in order to evaluate the necessity of maintenance to attain the activity	Qualitative (1 to 4) (1) Maintenance is not necessary (2) Only corrective maintenance is necessary (3) Only preventive maintenance is necessary (4) Preventive and corrective maintenance are necessary	Max
Interaction with other activities	Qualitative criteria in order to evaluate the interaction with other activities to take into account security risks and extension of other activities' duration	Qualitative (1 to 5) (1) Activity does not occur in building site (2) Activity occurs in the building site but without interaction with other activities or teams (3) Activity is carried out inside the building site and in contact with other outsourcers' teams and contractor's teams, but without interaction with the activities (4) Activity is carried out inside the building site, without contact with other outsourcers' teams and contractor's teams, but with interaction with the activities (5) Activity is carried out inside the building site and in contact with other outsourcers' teams and contractor's teams and interacts with the activities	Max

since they represent short-time relations with low costs. Usually they do not affect the client's satisfaction. They rarely consist of a source of liability, neither related to problems with labour, nor related with tribute. If there is a contract to make the relation official, the document obligations are relaxed.

5.4. Application of ROR-UTADIS. The sorting model proposed was applied in the data presented in the previous items. As explained in Section 4, we chose to use the ROR-UTADIS method [37], which is a modification of the UTADIS method [38, 39]. This method was selected due to its compensatory

TABLE 3: Alternative evaluation matrix.

Alternatives	Criteria						
	Cost (US$) ($g_1$)	Activity duration (days) (g_2)	Number of suppliers (g_3)	Available resources (g_4)	Risk exposition (g_5)	Necessity of maintenance (g_6)	Interaction with other activities (g_7)
Concrete	4,000,000.00	360.00	5.00	3.00	5.00	4.00	5.00
Concrete paving	700,000.00	60.00	4.00	1.00	5.00	4.00	5.00
Continuous Flight Auger (CFA) Stake	700,000.00	180.00	12.00	1.00	5.00	4.00	4.00
Hydroseeding	110,000.00	15.00	2.00	1.00	5.00	4.00	2.00
Earthwork	1,800,000.00	90.00	6.00	5.00	5.00	4.00	5.00
Precast concrete	2,700,000.00	270.00	2.00	5.00	4.00	1.00	4.00
Transport of personnel	1,500,000.00	360.00	4.00	5.00	3.00	4.00	3.00
Molds, shoring, and scaffolding	150,000.00	360.00	4.00	4.00	5.00	1.00	5.00
Heavy equipment	400,000.00	360.00	6.00	4.00	3.00	3.00	5.00
Food supply	1,200,000.00	360.00	5.00	2.00	3.00	1.00	3.00
Gypsum liner	35,000.00	21.00	2.00	1.00	5.00	1.00	2.00
Asphalt paving	90,000.00	4.00	2.00	5.00	5.00	4.00	2.00
Property security	500,000.00	360.00	4.00	5.00	3.00	1.00	3.00
Vegetal suppression	19,000.00	15.00	2.00	5.00	5.00	1.00	4.00
Containers	370,000.00	360.00	5.00	1.00	1.00	1.00	1.00
Waterproofing	25,000.00	60.00	2.00	5.00	5.00	1.00	2.00
Air conditioning	70,000.00	90.00	3.00	3.00	3.00	1.00	3.00

TABLE 4: Standardized alternative evaluation matrix.

Alternatives	Criteria						
	g_1	g_2	g_3	g_4	g_5	g_6	g_7
Concrete	1.0000	1.00	0.2	0.3333	1	0.8	1
Concrete paving	0.1750	0.17	0.25	1	1	0.8	1
Continuous Flight Auger (CFA) Stake	0.1750	0.50	0.08333	1	1	0.8	0.8
Hydroseeding	0.0275	0.04	0.5	1	1	0.8	0.4
Earthwork	0.4500	0.25	0.1667	0.2	1	0.8	1
Precast concrete	0.6750	0.75	0.5	0.2	0.8	0.2	0.8
Transport of personnel	0.3800	1.00	0.25	0.2	0.6	0.8	0.6
Form system, shoring, and scaffolding	0.0375	1.00	0.25	0.25	1	0.2	1
Heavy equipment	0.1000	1.00	0.1667	0.25	0.6	0.6	1
Food supply	0.3000	1.00	0.2	0.5	0.6	0.2	0.6
Gypsum liner	0.0088	0.06	0.5	1	1	0.2	0.4
Asphalt paving	0.0225	0.01	0.5	0.2	1	0.8	0.4
Property security	0.1250	1.00	0.25	0.2	0.6	0.2	0.6
Vegetal suppression	0.0048	0.04	0.5	0.2	1	0.2	0.8
Containers	0.0925	1.00	0.2	1	0.2	0.2	0.2
Waterproofing	0.0063	0.17	0.5	0.2	1	0.2	0.4
Air conditioning	0.0175	0.25	0.3333	0.3333	0.6	0.2	0.6

nature, allowing criteria to trade-off. As a holistic procedure, there is no need to do a profile characterization, once the restrictions imposed by the preference information supplied by the DM are used to calculate the parameters. The evaluation matrix is presented in Table 3.

In order to apply the first set of restrictions that plays the role of calculating the utility of each alternative, the alternatives must be previously standardized. The marginal value functions were then calculated with the first set of restrictions (E_{TH}^{BASE}) that uses the boundaries and piecewise linear information to do the calculations. Table 4 presents the standardized values but in this table the alternatives are ordered by the value of marginal utility function in a descending order.

Table 5: Assignment examples.

Class	First iteration	Second iteration	Third iteration
C1	Vegetal suppression		
C2	Heavy equipment	Transport of personnel, food supply	
C3	Continuous Flight Auger (CFA) Stake		Earthwork

Table 6: Assignment-based pairwise comparisons.

Symbols	First iteration	Symbols	Second iteration	Symbols	Third iteration
11	Hydroseeding $\underset{\geq 1,\mathrm{DM}}{\overset{\rightarrow}{>}}$ gypsum liner			31	Earthwork $\underset{\geq 1,\mathrm{DM}}{\overset{\rightarrow}{>}}$ precast concrete
12	Containers $\underset{\geq 0,\mathrm{DM}}{\overset{\rightarrow}{>}}$ air conditioning				
13	Concrete $\underset{\geq 2,\mathrm{DM}}{\overset{\rightarrow}{>}}$ asphalt paving				
14	Gypsum liner $\underset{\leq 1,\mathrm{DM}}{\overset{\rightarrow}{>}}$ asphalt paving				

Table 7: Desired class cardinalities.

Class	All iterations	
	$N_{h,\mathrm{DM}}^{\min}$	$N_{h,\mathrm{DM}}^{\max}$
C1	3	9
C2	3	9
C3	3	9

The DM provided the system with different types of preference information. In Table 5 it is possible to verify three assignment examples in the first iteration, such as Continuous Flight Auger (CFA) Stake for class 3. Table 6 presents the assignment-based pairwise comparisons; in the first iteration four examples were provided; for example, hydroseeding is considered better than gypsum liner by at least one class. Moreover, in Table 7 it is possible to verify the class cardinalities, which were kept equal in all iterations. The set of compatible instances of the preference model is not empty; therefore, the presented preference statements are consistent. As presented in the third section, once the mixed linear programming converges to a solution, the model can recommend possible and necessary assignments. Table 8 presents the possible assignments for each iteration. In the first iteration four alternatives were assigned to class three that deserves more attention; only one alternative was assigned to the second class, and it was the exemplary assignment, six alternatives were assigned directly to class one, and six alternatives had imprecise assignments, requiring the process to be powered with more information.

In Table 8 the iterations are based on the first recommendation, to diminish the impreciseness of the process. In the second iteration, only two more exemplary alternatives were supplied both to the second class since this was the class with more vagueness on its information. The last iteration was made considering one more exemplary assignment and an assignment-based pairwise comparison. It is important to notice that the assignment-based pairwise comparison is not used to rank alternatives but to create relations among them, to calibrate the model.

Once the DM provided all information, it was possible to reach a final solution, with all alternatives necessarily assigned to one class. Thus, we found a complete preorder, where the alternatives concrete, concrete paving, CFA Stake, hydroseeding, and earthwork were indifferent to each other and strictly preferable to the ones assigned to classes one and two. Alternatives transport of personnel, precast concrete, form system, shoring and scaffolding, heavy equipment, food supply, and gypsum liner were indifferent among them and strictly preferable to alternatives asphalt paving, property security, vegetal suppression, containers, waterproofing, and air conditioning, which created the last set of indifferences.

6. Discussion of Results

In the application of this model, the results presented were mostly coherent with the expectations of the DM, except for three alternatives, one in each class. The hydroseeding was assigned to class three due to its performances in criteria available resources, risk exposition, and necessity of maintenance. However, usually the DM would considered class one or two because the cost is too low, and it does not affect the schedule of the project strongly; besides it is a very quick activity that can be carried out at any time of the project. The precast concrete was sorted as class two, due to its performance in the criteria number of suppliers, available resources, and necessity of maintenance. However, as critical activity that influences the whole schedule of the project, the cost and its operations involve many risks the DM, usually, would handle as class three activity, to avoid future liabilities. The last one was property security that was sorted as class one due to the influence of the criteria cost, number of suppliers, available resources, and necessity of maintenance; the DM expected to have it categorized as class three. The model is important to enrich the DM's vision of each activity. The fact that the DM did not agree with the result in some issues shows that the categorization is important to avoid the wrong management in each contract, giving a perspective of aspects he usually would not perceive and spending time in the analysis of an activity that would not bring greater problems.

Table 8: Possible assignments.

Alternatives	First iteration	Second iteration	Third iteration
Concrete	C3	C3	C3
Concrete paving	C3	C3	C3
CFA Stake	C3	C3	C3
Hydroseeding	C3	C3	C3
Earthwork	C2-C3	C2-C3	C3
Transport of personnel	C2-C3	C2-C3	C2
Precast concrete	C2-C3	C2-C3	C2
Form system, shoring, and scaffolding	C2-C3	C2	C2
Heavy equipment	C2	C2	C2
Food supply	C1-C2	C2	C2
Gypsum liner	C1-C2	C2	C2
Asphalt paving	C1	C1	C1
Property security	C1	C1	C1
Vegetal suppression	C1	C1	C1
Containers	C1	C1	C1
Waterproofing	C1	C1	C1
Air conditioning	C1	C1	C1

The model proposed is applicable and important to verify the output data. Among the holistic models, this one has a very important feature of allowing the DM to provide imprecise information and pairwise comparison through the relative position in the classification whenever he/she feels safe about the information, which greatly improves the generation of preference information. It is easy for the DM to understand the process, even when he is unable to understand the calculations.

However, some issues should be considered in the data analysis. Usually, the sorting process is not compensatory once the categorization of alternatives usually has some acceptability threshold in each criterion that sometimes is called veto and sometimes is called a discordance index. When applying a holistic method, it is not possible to elicit those limits and it is even more impossible to apply them, because the sorting is made considering a class threshold and ignoring criterion thresholds. Also, in the method applied, the marginal utilities are calculated based on the boundaries and piecewise linear information; it is not possible to include in the procedure the elicitation of constant scales the trade-off among criteria. Thus, some preference information is unavoidably lost during this process. To take into account this consideration it would be necessary to elicit the constant scales before the standardization and include it in the standardization process that could be rather difficult for the DM to understand and apply properly.

The behaviour verified in the first two cases presented could be caused by the lack of preference information on scales constants. That would balance the differences among the criteria and reflect the subtleness of the information provided, once marginal utility values of these alternatives are placed in the threshold limits of the classes they were assigned to. In the last alternative, maybe if the criteria threshold had been elicited, the classification proposed was more similar to the expectations of the DM.

7. Conclusion

In this paper, we presented an application of the ROR-UTADIS method [37] to the context of subcontractor's selection in the construction industry, where, instead of proposing a new methodology to run the selection, we propose to categorize the services prior to the definition of the selection methodology. The model presented results that were coherent with the ones expected, based on previous experience, becoming clear that it can be applied to contexts different than the one presented by the authors. The main motivation to apply this model and not the others is due to its robustness and compatibility with fewer profiles, allowing the information to converge in only a few iterations. This process is easy for the DM to provide information, due to the disaggregation of preference, but have some limitations, since it is harder to elicit scale constants and include them in the analysis.

The inclusion of additional preference information, apart from helping to create stronger restrictions to the problem, even if it allows the DM to consider imprecise assignments, such as assigning one single alternative to more than one class, also gives the DM more confidence in the analysis. Because sometimes he may not be sure about which class to assign a reference alternative but needs to limit the class cardinalities or can compare alternatives, enriching the process with information that usually he would not be able to provide.

Future works may focus on developing an additive method considering direct elicitation of preference information, to include in the analysis constant scale elicitation and correct minor misclassification problems that might come up, due to the compensatory nature of the methodology.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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