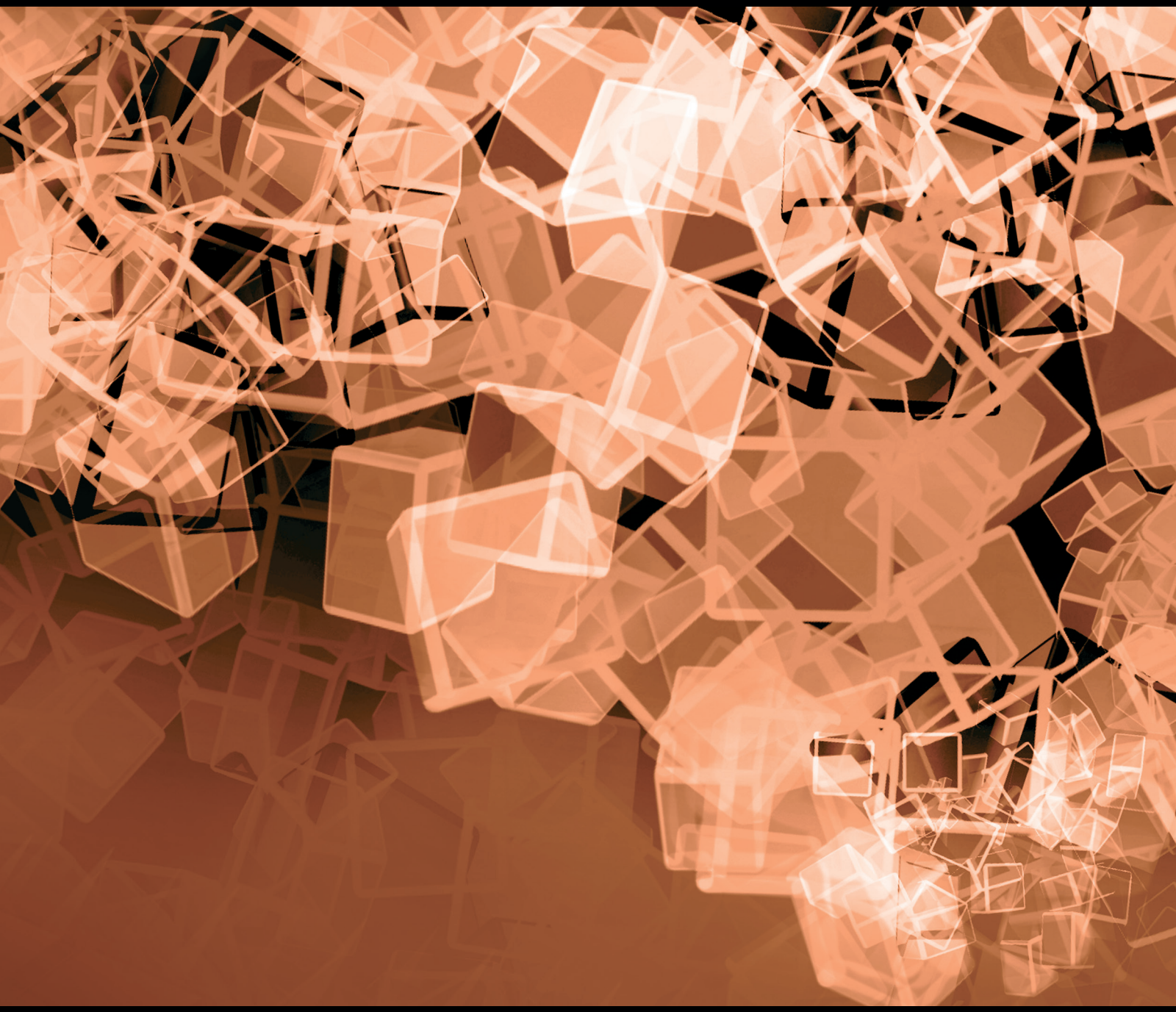
Discrete Dynamics in Nature and Society

Discrete Chaotic Dynamics for Economics and Social Science

Guest Editors: Christos K. Volos, Sundarapandian Vaidyanathan, Viet-Thanh Pham, and Esteban Tlelo-Cuautle



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Editorial Board

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Editorial

Discrete Chaotic Dynamics for Economics and Social Science

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Discrete chaotic dynamical systems deal with the maps that are extremely sensitive to their initial conditions, which are known as the butterfly effect. The discrete chaotic dynamical systems are characterized by the existence of a positive Lyapunov exponent. It is well known that even a first-order discrete dynamical system can exhibit an astonishing variety of dynamical behaviors ranging from stable fixed points to chaotic regions. Discrete chaotic dynamical systems have emerged as an important field of research in science and engineering especially in areas such as biology, economics, cryptosystems, and secure communications.

This special issue is focused upon the modeling and applications of discrete chaotic dynamics in the two fields: economics and social science. In the research, several control algorithms and techniques have been developed to study and control the discrete control systems such as active control, adaptive control, backstepping control, sliding mode control, fuzzy logic control, artificial neural networks, and evolutionary algorithms.

This special issue also focuses on control methodologies for the discrete chaotic dynamics with applications for social and financial systems. This special issue contains five papers, the contents of which are summarized as follows.

"A Mathematical Model of Communication with Reputational Concerns" by C. Huang et al. investigates a mathematical model where an expert advises a decision maker for two periods. The decision maker is initially unsure about whether the expert is biased or not. After consulting the expert on the decision problem of period one, the decision maker updates belief about the expert's bias and consults the expert on the problem of period two. The authors show that more information is delivered in the model's first period than in the one-period situation of communication.

"Analysis and Control of the Complex Dynamics of a Multimarket Cournot Investment Game with Bounded Rationality" by L. Zhao introduces a dynamic multimarket Cournot model, which is based on a specific inverse demand function. Puu's incomplete information approach, as a realistic method, is used to contract the corresponding dynamical model under this function. Therefore, some stability analysis is carried out on the model to detect the stability and instability conditions of the system's Nash equilibrium. Based on the analysis, some dynamic phenomena such as bifurcation and chaos are found. Numerical simulations are used to provide experimental evidence for the complicated behaviors of the system evolution. It is observed that the equilibrium of the system can lose stability via flip bifurcation or Neimark-Sacker bifurcation and time-delayed feedback control is used to stabilize the chaotic behaviors of the system.

"The Government Incentive Regulation Model and Pricing Mechanism in Power Transmission and Distribution Market" by H. Zhang and J. Jiang derives new results in the power transmission and distribution market. The power transmission and distribution (T&D) market's natural monopoly and individual information have been the impediment to improve the energy efficiency in the whole T&D market. In order to improve the whole social welfare, T&D market should be controlled by government. An incentive regulation

model with the target of maximizing social welfare has been studied. A list of contracts with transferring payment and quantity of T&D are given to motivate the corporation to reveal the true technical parameter and input the optimal investment. The corporate revenue, optimal investment, and effort are proved to depend on its own technical parameter. The part of incentive regulation model ends with the optimal pricing mechanism of T&D market. At the end of this paper, the authors also provide a numerical example to explain the main ideas of this research work and confirm its function graphically.

"Chaos Control on a Duopoly Game with Homogeneous Strategy" by M. Bai and Y. Gao investigates the dynamics of a nonlinear discrete-time duopoly game, where the players have homogenous knowledge on the market demand and decide their outputs based on adaptive expectation. The Nash equilibrium and its local stability are investigated. The numerical simulation results show that the model may exhibit chaotic phenomena. Quasiperiodicity is also found by setting the parameters at specific values. They demonstrate that the chaotic system can be stabilized to a stable state by using delayed feedback control method. The discussion of control strategy shows that the effect of both firms taking control method is better than that of single firm taking control method.

"Chaotic Synchronization of Modified Discrete-Time Tinkerbell Systems" by K. Ding and X. Xu studies the chaotic synchronization of chaotic modified discrete-time Tinkerbell systems. By constructing the Lyapunov function and using the linear feedback control, some synchronization criteria for modified discrete-time Tinkerbell systems are derived. The conservativeness characteristics of those synchronization criteria are compared. The effectiveness of derived results is demonstrated by six examples.

Of course, the selected topics and papers do not provide an exhaustive study of all areas of this special issue. Nonetheless, they represent rich many-faceted knowledge that we have the pleasure of sharing with the readers.

Acknowledgments

We would like to express appreciation to the authors for their excellent contributions and patience in assisting us. The hard work of all reviewers on these papers is also very greatly acknowledged.

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Sundarapandian Vaidyanathan
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Research Article

Chaotic Synchronization of Modified Discrete-Time Tinkerbell Systems

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This paper studies chaotic synchronization of modified discrete-time Tinkerbell systems. By constructing the Lyapunov function and using the linear feedback control, some synchronization criteria for modified discrete-time Tinkerbell systems are derived. The conservativeness of those synchronization criteria is compared. The effectiveness of derived results is demonstrated by six examples.

1. Introduction

Many models in engineering can be mathematically described as a Tinkerbell system, which is a classical two-dimensional discrete-time system with Tinkerbell-like trajectories. The dynamical behaviors of Tinkerbell systems have been studied in some papers (see, e.g., [1–3]).

Within the dynamical evolution of a Tinkerbell system, chaotic behaviors have attracted many researchers' interests because chaos can increase the complexity of evolution of systems and even influence the stability of systems. Therefore, the chaotic synchronization of two discrete-time Tinkerbell systems was studied in [2, 4]. It should be pointed out that the synchronization methods of chaotic discrete systems are different from those of continuous systems (see, e.g., [5–14]) because of the difference of constructing of Lyapunov functions. Recently, the classical Tinkerbell system has been expected to extend to a modified format in order to well satisfy the requirement of the normalization. However, how to achieve the chaotic synchronization of two modified discrete-time Tinkerbell systems has not been studied yet. Furthermore, a question arises as to whether the linear feedback control can be used to achieve global synchronization for modified discrete-time Tinkerbell systems.

Motivated by these two points, the synchronization of modified discrete-time Tinkerbell systems is studied in this paper. Some synchronization results for modified discrete-time Tinkerbell systems are derived by using linear feedback control. The conservativeness of those derived synchronization criteria is compared. Six examples are used to illustrate the effectiveness of our derived results.

2. Preliminaries

Consider the following modified Tinkerbell system:

$$\begin{aligned} x_1(n+1) &= w\left(\sin^2\left(x_1(n)\right) - \sin^2\left(x_2(n)\right)\right) + ax_1(n) \\ &\quad + bx_2(n), \\ x_2(n+1) &= p\sin\left(x_1(n)\right)\sin\left(x_2(n)\right) + cx_1(n) \\ &\quad + dx_2(n), \end{aligned} \tag{1}$$

where $x_1(n)$, $x_2(n)$ are state variables. Constants a, b, c, d, p, w are system parameters. The initial conditions of (1) are $x_1(0) = x_{1_0}$, $x_2(0) = x_{2_0}$.

One can construct the slave system associated with master system (1) as follows:

$$\begin{aligned} y_1(n+1) &= w\left(\sin^2(y_1(n)) - \sin^2(y_2(n))\right) + ay_1(n) \\ &\quad + by_2(n) + k_1(x_1(n) - y_1(n)) \\ &\quad + k_2(x_2(n) - y_2(n)), \\ y_2(n+1) &= p\sin(y_1(n))\sin(y_2(n)) + cy_1(n) \\ &\quad + dy_2(n) + k_3(x_2(n) - y_2(n)) \\ &\quad + k_4(x_2(n) - y_2(n)), \end{aligned} \tag{2}$$

where $y_1(n), y_2(n)$ are the state variables, and the initial conditions of (2) are $y_1(0) = y_{1_0}$, $y_2(0) = y_{2_0}$. $k_1(x_1(n) - y_1(n))$, $k_2(x_2(n) - y_2(n))$ and $k_3(x_1(n) - y_1(n))$, $k_4(x_2(n) - y_2(n))$ are controls. The gains k_1, k_2, k_3, k_4 can be derived later.

After defining two error variables

$$\begin{aligned} e_1(n) &= x_1(n) - y_1(n), \\ e_2(n) &= x_2(n) - y_2(n), \end{aligned} \tag{3}$$

one can have the following error system:

$$\begin{aligned} e_1(n+1) &= (a - k_1)e_1(n) + (b - k_2)e_2(n) \\ &\quad + w(\sin(x_1(n)) - \sin(y_1(n)))(\sin(x_1(n)) \\ &\quad + \sin(y_1(n))) - w(\sin(x_2(n)) - \sin(y_2(n))) \\ &\quad \cdot (\sin(x_2(n)) + \sin(y_2(n))), \\ e_2(n+1) &= (c - k_3)e_1(n) + (d - k_4)e_2(n) \\ &\quad + p(\sin(x_1(n))\sin(x_2(n)) \\ &\quad - \sin(y_1(n))\sin(y_2(n))). \end{aligned} \tag{4}$$

It follows from the well-known mean value theory that

$$\sin(\alpha) - \sin(\beta) = \cos(\theta)(\alpha - \beta) \tag{5}$$

for $\theta \in [\min\{\alpha, \beta\}, \max\{\alpha, \beta\}]$. Then, error system (4) can be rewritten as

$$\begin{aligned} &e_1(n+1) \\ &\quad = [a + w\cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1] \\ &\quad \cdot e_1(n) \\ &\quad + [b - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n))) - k_2] \\ &\quad \cdot e_2(n), \\ &e_2(n+1) = [c + p\cos(\theta_4)\sin(y_2(n)) - k_3]e_1(n) \\ &\quad + [d + p\sin(x_1(n))\cos(\theta_3) - k_4]e_2(n), \end{aligned} \tag{6}$$

where the initial conditions of (6) are $e_1(0) = x_{1_0} - y_{1_0}$, $e_2(0) = x_{2_0} - y_{2_0}$,

$$\begin{aligned} &\theta_1, \theta_4 \in [\min\{x_1(n), y_1(n)\}, \max\{x_1(n), y_1(n)\}], \\ &\theta_2, \theta_3 \in [\min\{x_2(n), y_2(n)\}, \max\{x_2(n), y_2(n)\}]. \end{aligned} \tag{7}$$

The stability of error system (6) is equivalent to the synchronization of (1) and (2). The purpose of this paper is to give some synchronization criteria for discrete-time systems described by (1) and (2).

3. Main Results

In this section, some synchronization criteria for discrete-time chaotic systems described by (1) and (2) will be provided.

Firstly, one can define the following Lyapunov function:

$$V(n) = |e_1(n)| + |e_2(n)|. \tag{8}$$

Theorem 1. *For two given constants $0 < \eta, \mu < 1$ such that $|w| \le \min\{\eta/2, \mu/2\}$ and $|p| \le \min\{1-\eta, 1-\mu\}$, two discrete-time chaotic systems described by (1) and (2) can achieve global synchronization, if k_1, k_2, k_3, k_4 satisfy the following inequalities:*

$$\begin{aligned} &\max\{a + 2w, a - 2w\} - \eta < k_1 \\ &\quad < \eta + \min\{a + 2w, a - 2w\}, \end{aligned} \tag{9}$$

$$\begin{aligned} &\max\{c + p, c - p\} - (1 - \eta) < k_3 \\ &\quad < 1 - \eta + \min\{c + p, c - p\}, \end{aligned} \tag{10}$$

$$\begin{aligned} &\max\{b - 2w, b + 2w\} - \mu < k_2 \\ &\quad < \mu + \min\{b - 2w, b + 2w\}, \end{aligned} \tag{11}$$

$$\begin{aligned} &\max\{d + p, d - p\} - (1 - \mu) < k_4 \\ &\quad < 1 - \mu + \min\{d + p, d - p\}. \end{aligned} \tag{12}$$

Proof. After computing the difference of $V(n)$ along with (6), one can obtain

$$\begin{aligned} &V(n+1) - V(n) \le [|a \\ &\quad + w\cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1| + |c \\ &\quad + p\cos(\theta_4)\sin(y_2(n)) - k_3| - 1]|e_1(n)| + [|b \\ &\quad - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n))) - k_2| + |d \\ &\quad + p\sin(x_1(n))\cos(\theta_3) - k_4|]|e_2(n)|. \end{aligned} \tag{13}$$

It follows from condition (9), condition (10), and $|w| \le \min\{\eta/2, \mu/2\}$, $|p| \le \min\{1-\eta, 1-\mu\}$ that

$$L_1 + L_2 < 1, \tag{14}$$

where

$$\begin{aligned} &L_1 \\ &\quad = |a + w\cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n))) - k_1|, \\ &L_2 = |c + p\cos(\theta_4)\sin(y_2(n)) - k_3|. \end{aligned} \tag{15}$$

Moreover, condition (11), condition (12), and $|w| \le \min\{\eta/2, \mu/2\}$ and $|p| \le \min\{1-\eta, 1-\mu\}$ imply that

$$L_3 + L_4 < 1, \tag{16}$$

where

$$L_3 = \left|b - w\cos(\theta_2)\left(\sin(x_2(n)) + \sin(y_2(n))\right) - k_2\right|, \quad (17)$$

$$L_4 = \left|d + p\sin(x_1(n))\cos(\theta_3) - k_4\right|.$$

It follows from (13), (14), and (16) that

$$V(n+1) - V(n) < 0, \quad \forall e_1(n), e_2(n) \neq 0, \quad (18)$$

which means the stability of error system (6); that is, two discrete-time chaotic systems described by (1) and (2) can achieve global synchronization. This ends the proof. □

Remark 2. η and μ can be any positive constants in (0, 1) such that $|w| \leq \min\{\eta/2, \mu/2\}$ and $|p| \leq \min\{1-\eta, 1-\mu\}$.

Remark 3. In [2, 4], the active control and backstepping control were used to achieve synchronization for the classical discrete-time Tinkerbell systems. For the modified discrete-time Tinkerbell system (1), the linear control is used to achieve synchronization, which is the significant difference between Theorem 1 of this paper and results in [2, 4].

If $|b - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| + |d + p\sin(x_1(n))\cos(\theta_3)| < 1$, one can have the following corollary with $k_2 = k_4 = 0$.

Corollary 4. *For any given constant $0 < \eta < 1$ such that $|w| \leq \eta/2$, $|p| \leq 1-\eta$, two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_4 = 0$ and k_1, k_3 satisfy the following inequalities:*

$$\begin{aligned} &\max\{a+2w, a-2w\} - \eta < k_1 \\ &\quad < \eta + \min\{a+2w, a-2w\}, \\ &\max\{c+p, c-p\} - (1-\eta) < k_3 \\ &\quad < 1 - \eta + \min\{c+p, c-p\}, \\ &|b| + 2|w| + |d| + |p| < 1. \end{aligned} \quad (19)$$

If $|b - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| < 1$ and $|c + p\cos(\theta_4)\sin(y_2(n))| < 1$, one can have the following synchronization criterion with $k_2 = k_3 = 0$.

Corollary 5. *Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_3 = 0$ and k_1, k_4 satisfy the following inequalities:*

$$\begin{aligned} &\max\{a+2w, a-2w\} - \nu < k_1 \\ &\quad < \nu + \min\{a+2w, a-2w\}, \\ &\max\{d+p, d-p\} - \delta < k_4 \\ &\quad < \delta + \min\{d+p, d-p\}, \\ &|b| + 2|w| < 1, \\ &|c| + |p| < 1, \end{aligned} \quad (20)$$

where

$$\begin{aligned} &\nu = 1 - (|c| + |p|), \\ &2|w| < \nu, \\ &\delta = 1 - (|b| + 2|w|), \\ &|p| < \delta. \end{aligned} \quad (21)$$

If $|b - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| + |d + p\sin(x_1(n))\cos(\theta_3)| < 1$ and $|c + p\cos(\theta_4)\sin(y_2(n))| < 1$, one can have the following synchronization criterion with $k_2 = k_3 = k_4 = 0$.

Corollary 6. *Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_2 = k_3 = k_4 = 0$ and k_1 satisfies the following inequalities:*

$$\begin{aligned} &\max\{a+2w, a-2w\} - \nu < k_1 \\ &\quad < \nu + \min\{a+2w, a-2w\}, \\ &|b| + 2|w| + |d| + |p| < 1, \\ &|c| + |p| < 1, \end{aligned} \quad (22)$$

where

$$\begin{aligned} &\nu = 1 - (|c| + |p|), \\ &2|w| < \nu. \end{aligned} \quad (23)$$

If $|a + w\cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n)))| + |c + p\cos(\theta_4)\sin(y_2(n))| < 1$, one can derive the following synchronization result with $k_1 = k_3 = 0$.

Corollary 7. *For given constant $0 < \mu < 1$ such that $|w| \leq \mu/2$ and $|p| \leq 1-\mu$, two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_1 = k_3 = 0$ and k_2, k_4 satisfy the following inequalities:*

$$\begin{aligned} &|a| + 2|w| + |c| + |p| < 1, \\ &\max\{b-2w, b+2w\} - \mu < k_2 \\ &\quad < \mu + \min\{b-2w, b+2w\}, \\ &\max\{d+p, d-p\} - (1-\mu) < k_4 \\ &\quad < 1 - \mu + \min\{d+p, d-p\}. \end{aligned} \quad (24)$$

If $|a + w\cos(\theta_1)(\sin(x_1(n)) + \sin(y_1(n)))| + |c + p\cos(\theta_4)\sin(y_2(n))| < 1$ and $|b - w\cos(\theta_2)(\sin(x_2(n)) + \sin(y_2(n)))| < 1$, one can have the following synchronization criterion with $k_1 = k_2 = k_3 = 0$.

Corollary 8. *Two discrete-time systems described by (1) and (2) can achieve global synchronization, if $k_1 = k_2 = k_3 = 0$ and k_4 satisfies the following inequalities:*

$$\begin{aligned} &|a| + 2|w| + |c| + |p| < 1, \\ &|b| + 2|w| < 1, \\ &\max\{d+p, d-p\} - \delta < k_4 \\ &\quad < \delta + \min\{d+p, d-p\}, \end{aligned} \tag{25}$$

where

$$\begin{aligned} &\delta = 1 - (|b| + 2|w|), \\ &|p| < \delta. \end{aligned} \tag{26}$$

Remark 9. The conservativeness of synchronization criteria is compared as follows. Corollary 6 is more conservative than Corollaries 4 and 5. Corollary 8 is more conservative than Corollaries 5 and 7. However, only one controller $k_1(x_1(n) - y_1(n))$ is used in Corollary 6. And the single controller $k_4(x_2(n) - y_2(n))$ is used in Corollary 8, which means that reducing of controllers results in the increasing of conservativeness.

4. Simulation Examples

Example 1. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.9, d = 0.5, p = 0.2, w = 0.1$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively. A chaotic attractor can be generated by the discrete-time Tinkerbell system (1) which is revealed by Figure 1.

Because $0.1 = |w| \le \min\{\eta/2, \mu/2\}$ and $0.2 = |p| \le \min\{1-\eta, 1-\mu\}$, one can choose $\eta = \mu = 0.5$. By using Theorem 1 with $\eta = \mu = 0.5$, one can obtain that $0.6 < k_1 < 1.2$, $-0.9 < k_2 < -0.3$, $0.6 < k_3 < 1.2$, and $0.2 < k_4 < 0.8$. We choose $k_1 = 1$, $k_2 = -0.8$, $k_3 = 1$, and $k_4 = 0.7$. Figures 2 and 3 give the demonstration of master Tinkerbell system (1) and slave Tinkerbell system (2), respectively. The trajectories of error system (6) are illustrated by Figures 4 and 5, respectively, which show that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 2. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.9, d = 0.1, p = 0.1, w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

Due to $0.05 = |w| \le \eta/2$, $0.1 = |p| \le 1 - \eta$, we can set $\eta = 0.5$. It is easy to see that $|b| + 2|w| + |d| + |p| = 0.9 < 1$. By using Corollary 4 with $\eta = 0.5$, one can derive that $k_2 = k_4 = 0$, $0.5 < k_1 < 1.3$, and $0.5 < k_3 < 1.3$. We choose $k_2 = k_4 = 0$, $k_1 = 1.1$, and $k_3 = 1$. The trajectories of error system (6) are illustrated by Figure 6, which reveals that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

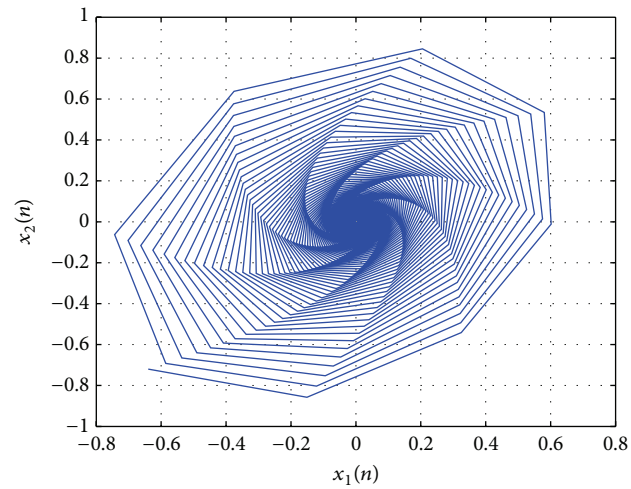

Figure 1: Chaotic attractor of discrete-time Tinkerbell system (1).

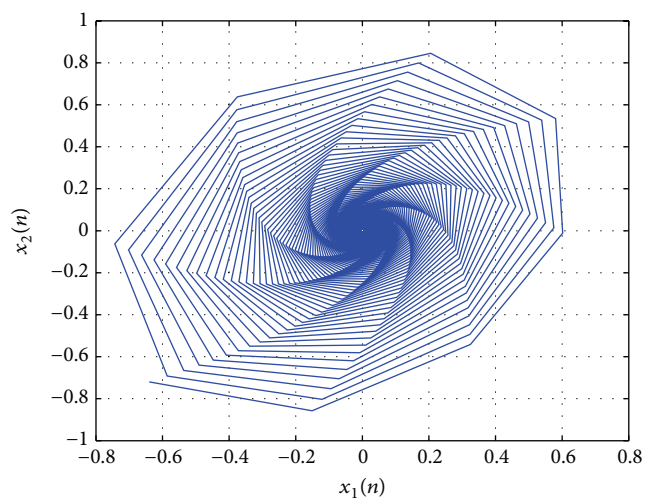

Figure 2: Master Tinkerbell system (1).

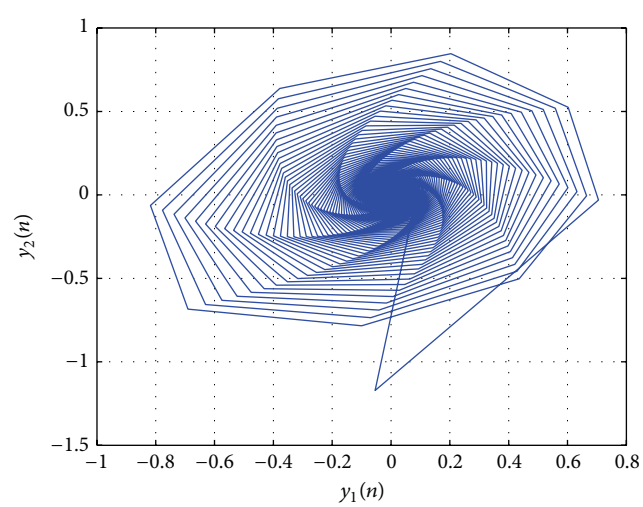

Figure 3: Slave Tinkerbell system (2).

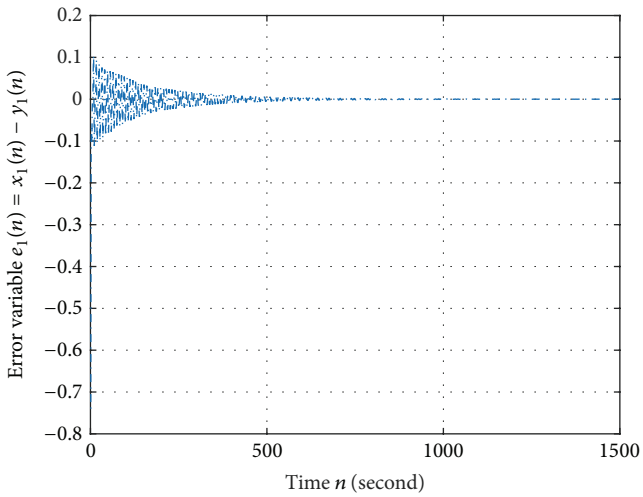

FIGURE 4: Error variable $e_1(n)$ of error system (6) with $k_1 = 1$, $k_2 = -0.8$, $k_3 = 1$, and $k_4 = 0.7$.

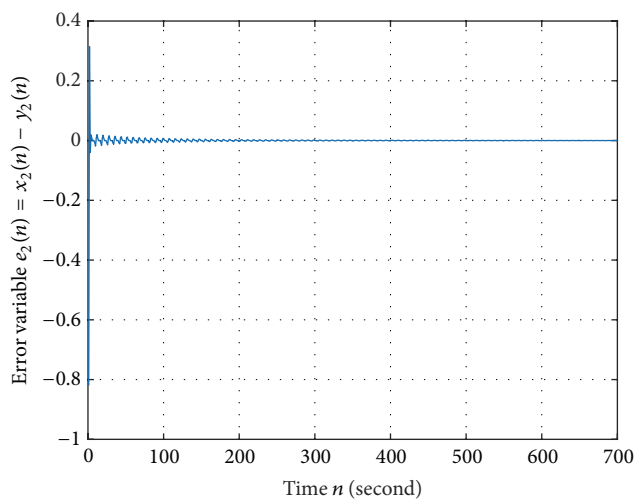

FIGURE 5: Error variable $e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = -0.8$, $k_3 = 1$, and $k_4 = 0.7$.

Example 3. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.6, d = 0.1, p = 0.1, w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

Due to $|b| + 2|w| = 0.7 < 1, |c| + |p| = 0.7 < 1$, one can set $\nu = 1 - (|c| + |p|) = 0.3 > 2|w| = 0.1$ and $\delta = 1 - (|b| + 2|w|) = 0.3 > |p| = 0.1$. By using Corollary 5 with $\nu = 0.3$ and $\delta = 0.3$, one can derive that $k_2 = k_3 = 0$, $0.7 < k_1 < 1.1$, and $-0.1 < k_4 < 0.3$. We choose $k_2 = k_3 = 0$ and $k_1 = 1$ and $k_4 = 0.2$. The trajectories of error system (6) are illustrated by Figure 7, which shows that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 4. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.9, b = -0.6, c = 0.5, d = 0.1, p = 0.1, w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

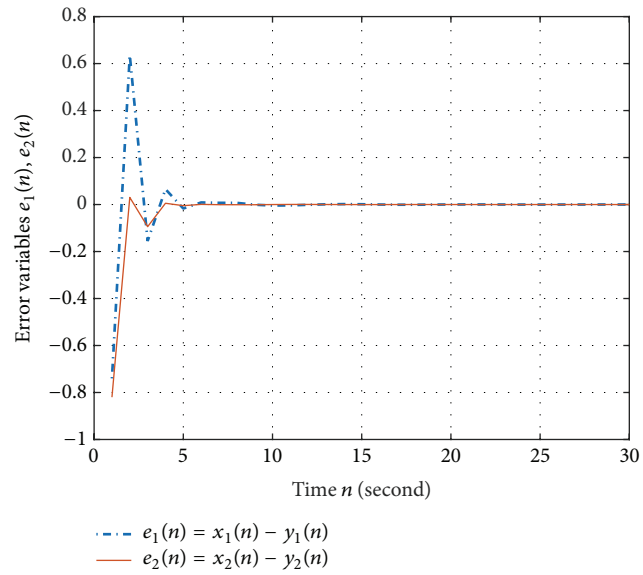

FIGURE 6: Error variables $e_1(n)$, $e_2(n)$ of error system (6) with $k_1 = 1.1$, $k_2 = 0$, $k_3 = 1$, and $k_4 = 0$.

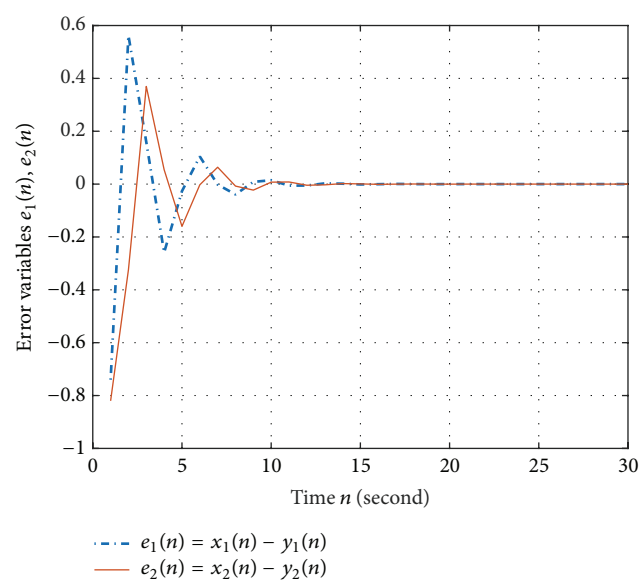

FIGURE 7: Error variables $e_1(n)$, $e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0.2$.

It is easy to see that $|b| + 2|w| + |d| + |p| = 0.9 < 1$ and $|c| + |p| = 0.6 < 1$. Let $\nu = 1 - (|c| + |p|) = 0.4 > 2|w| = 0.1$. By using Corollary 6, one can derive that $k_2 = k_3 = k_4 = 0$ and $0.6 < k_1 < 1.2$. We choose $k_2 = k_3 = k_4 = 0$ and $k_1 = 1$. The trajectories of error system (6) are illustrated by Figure 8, which demonstrates that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 5. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.4, b = -0.6, c = 0.1, d = 0.5, p = 0.2, w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

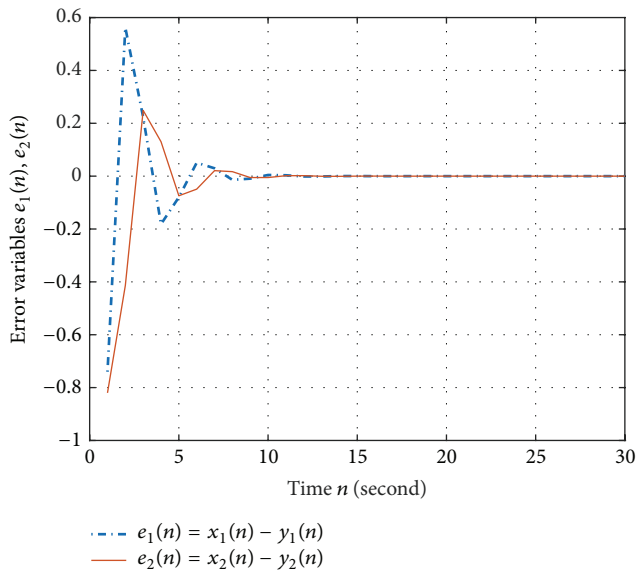

FIGURE 8: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 1$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0$.

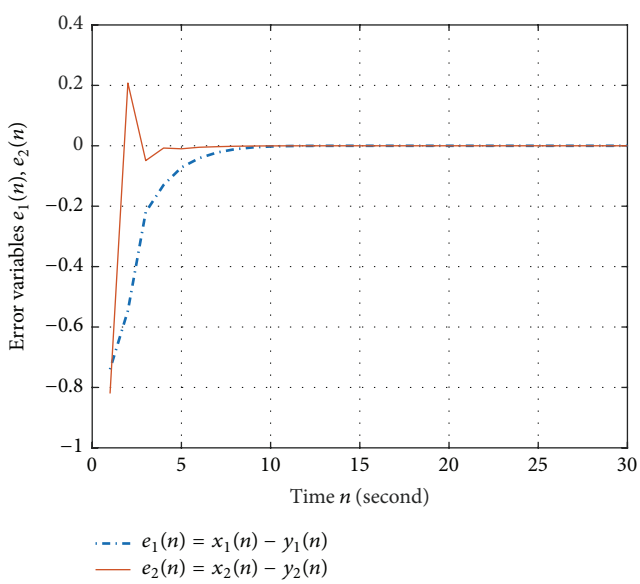

FIGURE 9: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 0$, $k_2 = -0.9$, $k_3 = 0$, and $k_4 = 0.75$.

Because $0.05 = |w| \leq \mu/2$ and $0.2 = |p| \leq 1 - \mu$, one can choose $\mu = 0.5$. It is easy to see that $|a| + 2|w| + |c| + |p| = 0.8 < 1$. By using Corollary 7 with $\mu = 0.5$, one can obtain that $k_1 = k_3 = 0$, $-1 < k_2 < -0.2$, and $0.2 < k_4 < 0.8$. We choose $k_1 = k_3 = 0$, $k_2 = -0.9$, and $k_4 = 0.75$. The trajectories of error system (6) are illustrated by Figure 9, which shows that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Example 6. Consider modified discrete-time Tinkerbell systems (1) and (2) with $a = 0.4, b = -0.6, c = 0.1, d = 0.5, p = 0.1, w = 0.05$. The initial conditions are $x_1(0) = -0.64$, $x_2(0) = -0.72$, $y_1(0) = 0.1$, and $y_2(0) = 0.1$, respectively.

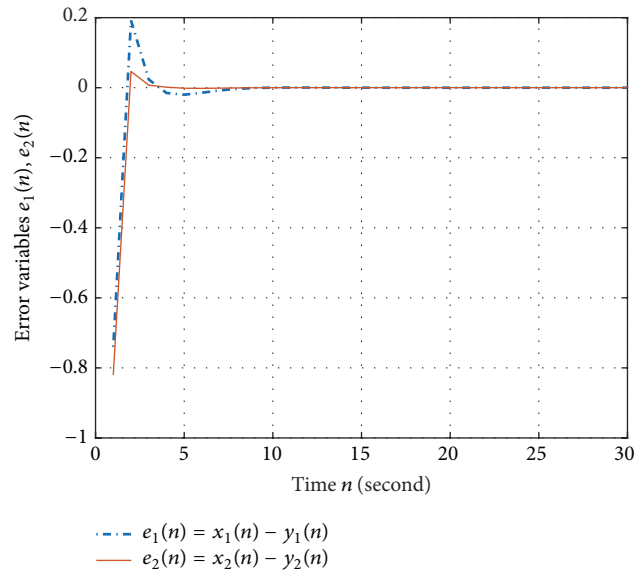

FIGURE 10: Error variables $e_1(n), e_2(n)$ of error system (6) with $k_1 = 0$, $k_2 = 0$, $k_3 = 0$, and $k_4 = 0.6$.

Notice that $|a| + 2|w| + |c| + |p| = 0.7 < 1$ and $|b| + |p| = 0.7 < 1$. Let $\delta = 1 - (|b| + |p|) = 0.3 > 2|w| = 0.1$. We can also use Corollary 8 to derive that $k_1 = k_2 = k_3 = 0$ and $0.3 < k_4 < 0.7$. We choose $k_1 = k_2 = k_3 = 0$ and $k_4 = 0.6$. The trajectories of error system (6) are demonstrated by Figure 10, which reveals that master Tinkerbell system (1) and slave Tinkerbell system (2) achieve global synchronization.

Remark 10. Corollaries 6 and 8 fail to derive conclusions for Examples 2 and 3, respectively, which can illustrate the effectiveness of the statement of Remark 9.

5. Conclusions and Future Works

We have studied synchronization of modified discrete-time Tinkerbell system. We have obtained some synchronization criteria by using linear feedback control. We have compared the conservativeness of those synchronization criteria. Six examples have been provided to illustrate the effectiveness of our results. In this paper, we only consider the linear feedback control. It should be pointed out that the control process could suffer time delays. How to use time-delayed control to achieve synchronization of modified discrete-time Tinkerbell system is our research interest in the future.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

Acknowledgments

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Research Article

Chaos Control on a Duopoly Game with Homogeneous Strategy

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We study the dynamics of a nonlinear discrete-time duopoly game, where the players have homogenous knowledge on the market demand and decide their outputs based on adaptive expectation. The Nash equilibrium and its local stability are investigated. The numerical simulation results show that the model may exhibit chaotic phenomena. Quasiperiodicity is also found by setting the parameters at specific values. The system can be stabilized to a stable state by using delayed feedback control method. The discussion of control strategy shows that the effect of both firms taking control method is better than that of single firm taking control method.

1. Introduction

An oligopoly is a market form in which a market or industry is dominated by a small number of sellers. Thus, they have power to decide the market price and the yields. The behavior of one player inevitable influences the other players' behavior. To maximize their profits, each oligopolist is likely to be aware of the actions of the others.

Cournot, in 1838, introduced the first formal theory of oligopoly [1]. In Cournot model, each oligopolist assumes that other firms hold their outputs constant, and all oligopolists select a quantity based on others' outputs to maximize profits. However, oligopoly game becomes complex when the players adopt dynamic strategy depending on their previous outputs and their rivals' outputs.

Expectations play an important role in modeling economic phenomena. A firm can choose its expectation rules to adjust its production. There exist three different firms' expectations: naive, bounded rational, and adaptive [2]. The case of where both players adopt bounded rational expectation and that of one takes bounded rational expectation and the other takes adaptive expectation have been studied by many researchers [3–10]. However, there are few literatures that focus on the case of where both players adopt adaptive expectation.

Recently, the dynamics of duopoly game has been studied by many researchers. Chaotic phenomena in duopoly game were first found by Puu [11]. Later, Kopel [12] studied the stability of the duopoly game with different demand functions and different cost functions. Chaos was also found in this paper. Agiza and Elsadany [2] studied the dynamics in the Cournot duopoly game with heterogeneous players; in which case, one player accepts naive expectation, and the other adopts bounded rational expectation. Ma and Ji [4] studied duopoly game with homogenous players in electric power industry, where both players adjust their outputs according to bounded rational rule. Sarafopoulos [5] assumed that one player accepts bounded rational expectation and the other uses adaptive expectation.

Since chaotic phenomena were found in duopoly game, there are also many researchers focusing on the chaos control in duopoly game [6, 13–17]. However, to our best knowledge, there are few literatures that focus on the efficiency of chaos control. In this paper, we explore the dynamics of a homogeneous duopoly game where both players take adaptive expectation. According to the analysis of numerical simulations, the parameters' effects on the stability of the system are obtained. To lead the system to stable state, we try to use DFC method. The efficiency of the case where only one company takes control measure and that of the case where both companies take control measure is compared.

The paper is organized as follows. In Section 2, the model and the parameters are introduced. In Section 3, we study the equilibriums and the stability of the equilibriums of the model. In Section 4, we give the numerical results about the reactions of the two players. In Section 5, we control the system to a stable situation. Section 6 gives the conclusions.

2. The Model

We assume that the duopoly players produce homogenous goods which are perfect substitutes and offer goods at discrete-time periods. Therefore, the duopoly players face the same market demands. They both choose adaptive expectation rule to decide the amounts of the goods in the next period as their response strategy. Assume that the total demand is reciprocal to price p; therefore, the reverse demand function is

$$p = \frac{1}{q_1 + q_2}, \tag{1}$$

where q_i $(i = 1, 2)$ represents the quantity that firm i produced. The cost of firm i contains two parts: one is constant cost d_i and the other is variable cost. The per units cost corresponding to the variable cost is constant, and the value is c_i. Then, the total cost of firm i is

$$C_i(q_i) = c_i q_i + d_i. \tag{2}$$

Now, we present a simple case which will be used to modify players' expectation latter. In this case, both firms have the conception that the other firm will produce at time $t+1$ as it did at time t. Then, they will decide their outputs based on the amounts their rival player did at time t. Then, the expectation net profits of firm 1 and firm 2 at time $t+1$ can be expressed as

$$\begin{aligned} \pi_1 &= p * q_1 - C_1(q_1) \\ &= \frac{q_1(t+1)}{q_1(t+1) + q_2(t)} - c_1 q_1(t+1) - d_1, \\ \pi_2 &= p * q_2 - C_2(q_2) \\ &= \frac{q_2(t+2)}{q_1(t) + q_2(t+1)} - c_2 q_2(t+1) - d_2. \end{aligned} \tag{3}$$

The marginal expectation profits of firm 1 and firm 2 at (q_1, q_2) are as follows:

$$\begin{aligned} \frac{\partial \pi_1}{\partial q_1} &= \frac{q_2(t)}{(q_1(t+1) + q_2(t))^2} - c_1, \\ \frac{\partial \pi_2}{\partial q_2} &= \frac{q_1(t)}{(q_1(t) + q_2(t+1))^2} - c_2. \end{aligned} \tag{4}$$

The firms can make maximize profits when the marginal profits are zero. Then, the reaction outputs of the two firms with respect to their competitor's last outputs are

$$\begin{aligned} q_1(t+1) &= q_1(q_2) = \sqrt{\frac{q_2(t)}{c_1}} - q_2(t), \\ q_2(t+1) &= q_2(q_1) = \sqrt{\frac{q_1(t)}{c_2}} - q_1(t). \end{aligned} \tag{5}$$

In order to make sure that q_i $(i = 1, 2)$ is nonnegative, the reaction functions are modified as

$$q_1(t+1) = \begin{cases} \sqrt{\dfrac{q_2}{c_1}} - q_2, & 0 < q_2 \le \dfrac{1}{c_1}, \\ 0, & q_2 > \dfrac{1}{c_1}, \end{cases} \tag{6}$$

$$q_2(t+1) = \begin{cases} \sqrt{\dfrac{q_1}{c_2}} - q_1, & 0 < q_1 \le \dfrac{1}{c_2}, \\ 0, & q_1 > \dfrac{1}{c_2}. \end{cases} \tag{7}$$

On the condition that the two firms have adaptive expectation, the firms compute their outputs with weights between their own last outputs and their reaction outputs $q_i(q_j)$ $(i \neq j)$. Then, the outputs they produce in the next period are as follows:

$$\begin{aligned} q_1(t+1) &= (1 - v_1) * q_1(t) + v_1 * q_1(q_2), \\ q_2(t+1) &= (1 - v_2) * q_2(t) + v_2 * q_2(q_1), \end{aligned} \tag{8}$$

where $q_1(q_2)$ and $q_2(q_1)$ are as (6) and (7), respectively, and v_1 and v_2 are weights on firms' reaction outputs, respectively. We focus on the dynamics of system (8) in the next section.

3. Equilibrium and Stability

By setting $q_i(t+1) = q_i(t)$, $(i = 1, 2)$ in map (8), the equilibrium output points of the dynamic duopoly game can be obtained as the nonnegative solutions of the algebraic system:

$$\begin{aligned} -q_1(t) + \sqrt{\frac{q_2(t)}{c_1}} - q_2(t) &= 0, \\ -q_2(t) + \sqrt{\frac{q_1(t)}{c_2}} - q_1(t) &= 0. \end{aligned} \tag{9}$$

The system has two equilibriums: one is $E_1^*(0, 0)$ and the other is $E_2^*(q_1^*, q_2^*) = (c_2/(c_1 + c_2)^2, c_1/(c_1 + c_2)^2)$. E_1^* has no practical significance because both the outputs of two firms are zero. Hence, we just investigate Nash equilibrium E_2^*.

The Jacobian matrix of the two-dimensional map (8) at equilibrium E_2^* is

$$J(E) = \begin{bmatrix} 1 - v_1 & \dfrac{1}{2} v_1 \sqrt{\dfrac{1}{c_1 q_2^*}} - 1 \\ \dfrac{1}{2} v_2 \sqrt{\dfrac{1}{c_2 q_1^*}} - 1 & 1 - v_2 \end{bmatrix}. \tag{10}$$

According to Agiza and Elsadany [3], equilibrium E_2^* is locally stable if the following conditions are held:

$$\begin{aligned} 1 - T + D &> 0, \\ 1 + T + D &> 0, \\ 1 - D &> 0, \end{aligned} \tag{11}$$

where

$$\begin{aligned} T &= 2 - v_1 - v_2, \\ D &= v_1 v_2 - v_1 - v_2 - \frac{1}{4} v_1 v_2 \sqrt{\frac{1}{c_1 q_2^* c_2 q_1^*}} + \frac{1}{2} v_1 \sqrt{\frac{1}{c_1 q_1^*}} \\ &\quad + \frac{1}{2} v_2 \sqrt{\frac{1}{c_2 q_1^*}} \\ &= v_1 v_2 - v_1 - v_2 - \frac{1}{4} \frac{v_1 v_2 (c_1 + c_2)^2}{c_1 c_2} + \frac{1}{2} \frac{v_1 (c_1 + c_2)}{c_1} \\ &\quad + \frac{1}{2} \frac{v_2 (c_1 + c_2)}{c_2}. \end{aligned} \tag{12}$$

By substituting T and D into (11), the stable conditions of equilibrium E_2^* become

$$\begin{aligned} &2(c_1 + c_2)(v_1 c_2 + v_2 c_1) - v_1 v_2 (c_1 - c_2)^2 - 4 c_1 c_2 > 0, \\ &2(c_1 + c_2)(v_1 c_2 + v_2 c_1) - v_1 v_2 (c_1 - c_2)^2 \\ &\quad - 8 c_1 c_2 (v_1 + v_2) + 12 c_1 c_2 > 0, \\ &-2(c_1 + c_2)(v_1 c_2 + v_2 c_1) + v_1 v_2 (c_1 - c_2)^2 \\ &\quad + 4 c_1 c_2 (v_1 + v_2 + 1) > 0. \end{aligned} \tag{13}$$

In view of (10), the eigenvalues associated with the equilibrium $E_2^*(q_1^*, q_2^*)$ are

$$\begin{aligned} \lambda_1 &= \frac{T + \sqrt{T^2 - 4D}}{2}, \\ \lambda_2 &= \frac{T - \sqrt{T^2 - 4D}}{2}. \end{aligned} \tag{14}$$

Now, suppose that

$$\begin{aligned} T^2 - 4D &< 0, \\ |\lambda_i| &= 1, \quad i = 1, 2. \end{aligned} \tag{15}$$

Then, $D = 1$; namely,

$$\begin{aligned} &v_1 v_2 - v_1 - v_2 - \frac{1}{4} \frac{v_1 v_2 (c_1 + c_2)^2}{c_1 c_2} + \frac{1}{2} \frac{v_1 (c_1 + c_2)}{c_1} \\ &\quad + \frac{1}{2} \frac{v_2 (c_1 + c_2)}{c_2} = 1. \end{aligned} \tag{16}$$

It follows from (16) that

$$v_1 = \frac{1 + v_2 - v_2 (c_1 + c_2)/2c_2}{v_2 - 1 - v_2 (c_1 + c_2)^2 / 4c_2 + (c_1 + c_2)/2c_2} = v_1^*. \tag{17}$$

Then, (q_1^*, q_2^*, v_1^*) is a candidate for Neimark-Sacker bifurcation point [18].

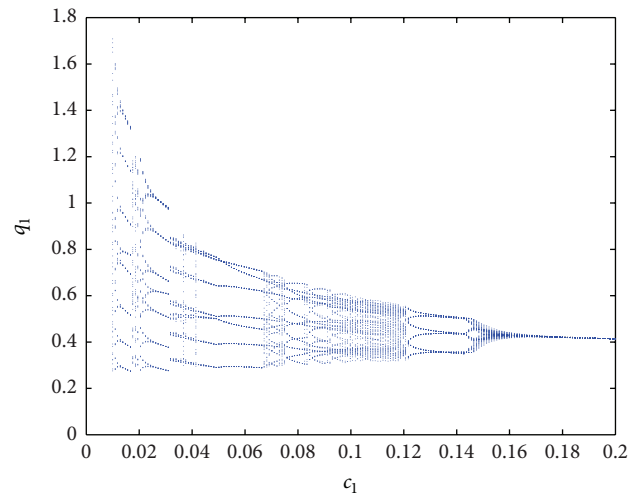

Figure 1: Bifurcation diagram with respect to parameter c_1 against variable q_1 with 500 iterations of map (8) for $v_1 = v_2 = 0.5$, $c_2 = 2$.

4. Numerical Simulations

In this section, numerical simulation results corresponding to model (8) are presented. Bifurcation diagrams and phase portraits with respect to different parameters are used to show complex dynamical behaviors of the duopoly model.

Figure 1 presents the bifurcation diagram of system (8) with respect to parameter c_1 (per unit cost of firm 1) against variable q_1 for $v_1 = v_2 = 0.5$, $c_2 = 2$. It is seen that the system is in periodic state for $c_1 > 0.18$. As c_1 decreases, periodic motion and chaotic motion occur alternatively. And the system is driven to chaos through quasiperiodic route. The main region where the system appears chaotic behavior is the range with $c_1 \in [0.082, 0.121]$. There exist many period orbits, such as period 7 orbit with $c_1 \in [0.051, 0.066]$ and period 8 orbit with $c_1 \in [0.014, 0.017]$, in model (8). For $c_1 > 0.18$, the stable output of firm 1 decreases with per unit cost c_1 increasing, which is consistent with the fact that the comparative superiority of firm 1 decreases as per unit cost c_1 increasing.

Now, set $v_2 = 0.7$, $c_1 = 0.14$, and $c_2 = 2$. It follows from (17) that $v_1^* \approx 0.2958$. Figure 2 shows the bifurcation diagram of map (8) with respect to parameter v_1. At $v_1^* \approx 0.2958$, a Neimark-Sacker bifurcation occurs. Nash equilibrium E_2^* is locally stable for $v_1 < 0.2985$ and losses its stability for $v_1 > 0.2985$. The system evolves from Nash equilibrium E_2^* into chaos with parameter v_1 (weight on firm 1's reaction function) increasing, through the mechanism of quasiperiodic route.

Figure 3 depicts the trajectories of outputs of firm 1 and firm 2 in the phase space (q_1, q_2) for $v_1 = 0.5$, $v_2 = 0.5$, $c_2 = 2$, and the initial point (0.4, 0.4). The red point is Nash equilibrium. A periodic attractor with $c_1 = 0.13$ is shown in Figure 3(b), while a quasiperiodic attractor with $c_1 = 0.12$ is shown in Figure 3(c). For $c_1 = 0.19$, the outputs of firm 1 and firm 2 converge to the Nash equilibrium, which is shown in Figure 3(a).

Figure 4 depicts the trajectories of outputs of firm 1 and firm 2 in the phase space (q_1, q_2) for $v_2 = 0.7$, $c_1 = 0.14$,

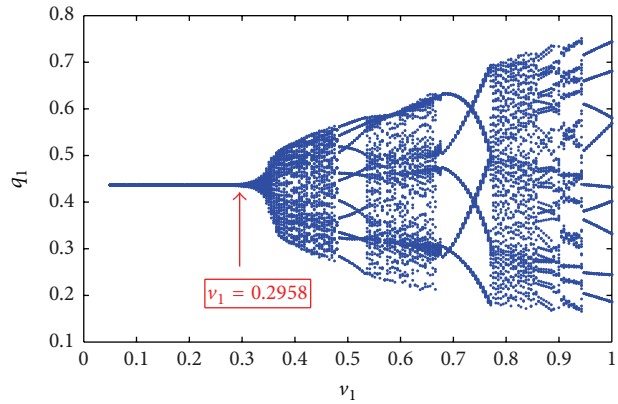

Figure 2: Bifurcation diagram with respect to parameter v_1 against variable q_1 with 500 iterations of map (8) for $v_2 = 0.7$, $c_1 = 0.14$, $c_2 = 2$.

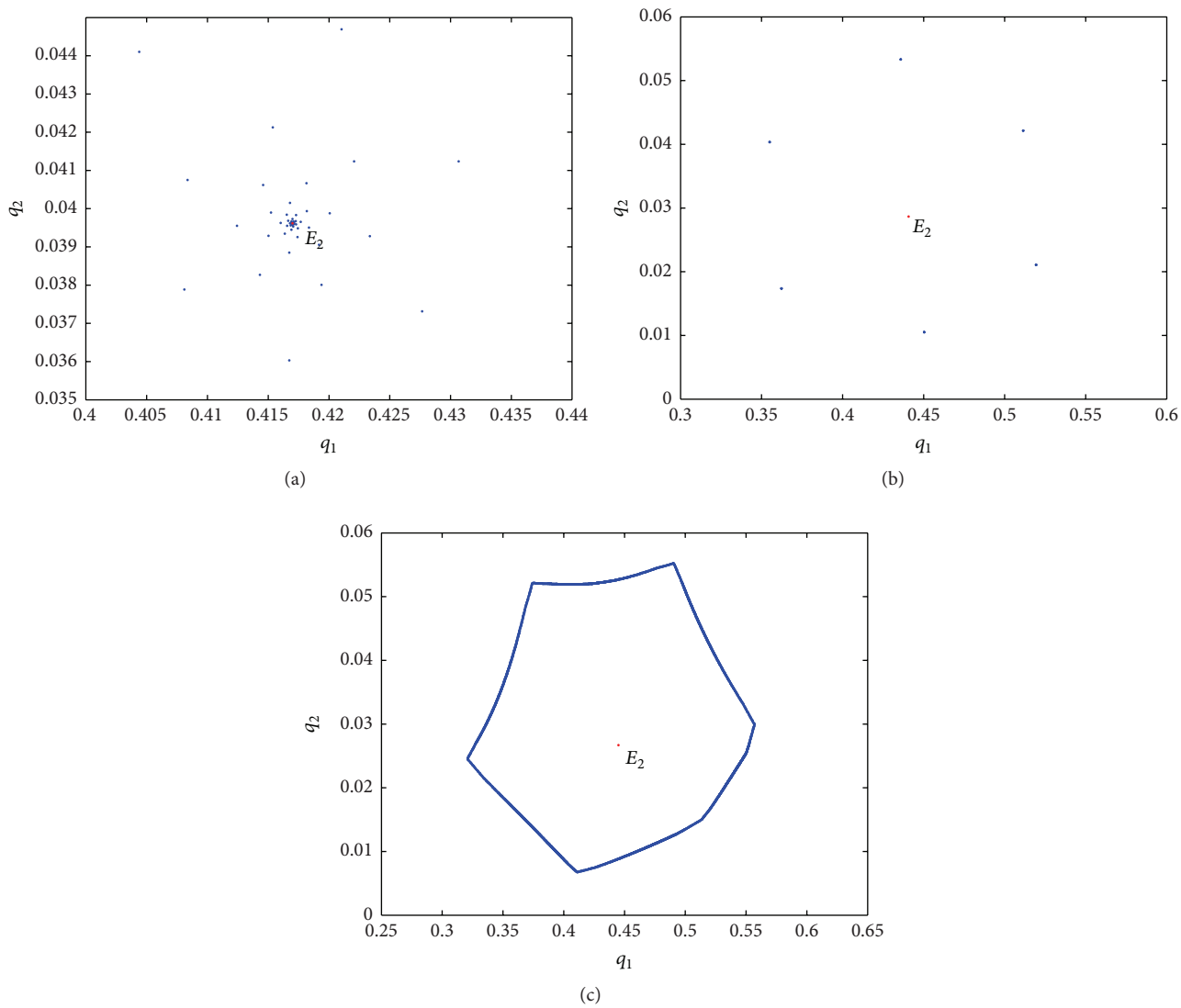

Figure 3: (a) Phase portrait of map (8) for $v_1 = 0.5$, $v_2 = 0.5$, $c_1 = 0.19$, $c_2 = 2$. (b) Phase portrait of map (8) for $v_1 = 0.5$, $v_2 = 0.5$, $c_1 = 0.13$, $c_2 = 2$. (c) Phase portrait of map (8) for $v_1 = 0.5$, $v_2 = 0.5$, $c_1 = 0.12$, $c_2 = 2$.

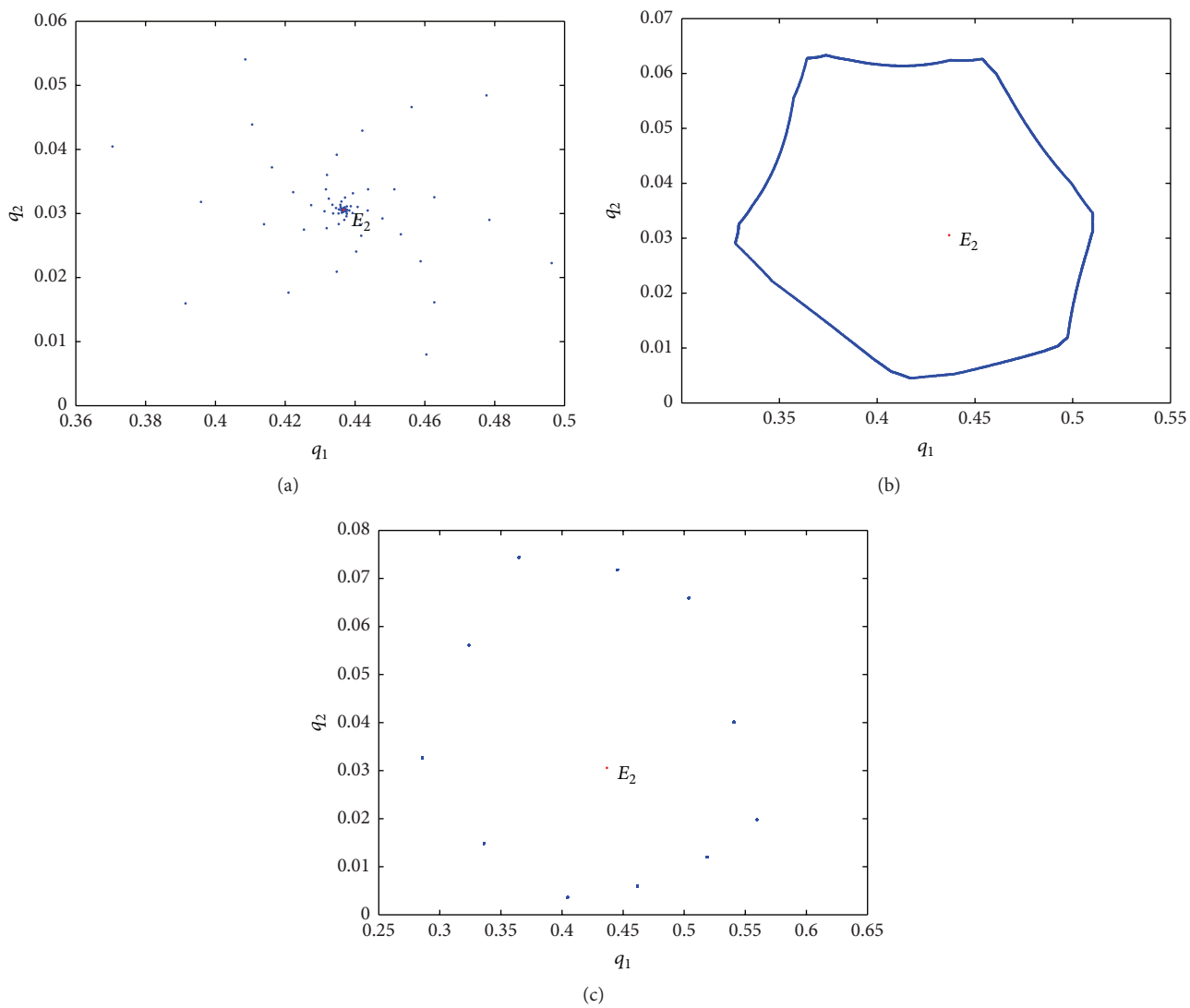

FIGURE 4: (a) Phase portrait of map (8) for $v_1 = 0.3$, $v_2 = 0.7$, $c_1 = 0.14$, $c_2 = 2$. (b) Phase portrait of map (8) for $v_1 = 0.37$, $v_2 = 0.7$, $c_1 = 0.14$, $c_2 = 2$. (c) Phase portrait of map (8) for $v_1 = 0.48$, $v_2 = 0.7$, $c_1 = 0.14$, $c_2 = 2$.

$c_2 = 2$, and the initial point (0.4, 0.4). The red point is Nash equilibrium. For $v_1 = 0.3$, the outputs of firm 1 and firm 2 converge to the Nash equilibrium, which is shown in Figure 4(a). Figure 4(b) shows a quasiperiodic attractor with $v_1 = 0.37$ and Figure 4(c) shows a periodic attractor with $v_1 = 0.48$.

5. Chaos Control in the Dynamic Output System

Any firm does not want to adjust its outputs too frequently and largely. So it is important for firms to control their outputs to a stable process. Chaos control has been studied by many researchers since chaos has been found in economy [6, 13–17]. In DFC (delayed feedback control) method [19], the most common control function is

$$u_t = K\left(q(t) - q(t-1)\right). \tag{18}$$

We consider the case that only firm 1 adopts control function to make the system stable, while firm 2 does not realize that it will generate chaos in the future on the condition that firm 1 does not take any measure to prevent this case. Then, system (8) can be rewritten as

$$\begin{aligned} q_1(t+1) &= (1 - v_1) * q_1(t) + v_1 * q_1(q_2) + u_{1,t}, \\ q_2(t+1) &= (1 - v_2) * q_2(t) + v_2 * q_2(q_1), \end{aligned} \tag{19}$$

where $u_{1,t} = K_1(q_1(t) - q_1(t-1))$.

To examine the effects of the control function, we compare the output process of firm 1 before and after adding the control function. Set $v_1 = 0.85$, $v_2 = 0.7$, $c_1 = 0.2$, and $c_2 = 2$ in model (8), which may exhibit chaos. The bifurcation diagram with respect to the control parameter K_1 is given in Figure 5. The system is stable for $K_1 < -0.22$. For $K_1 = -0.3$, the outputs processes of firm 1 before and after using control function are shown in Figure 6. It is seen that the control function effectively leads the outputs to a stable state.

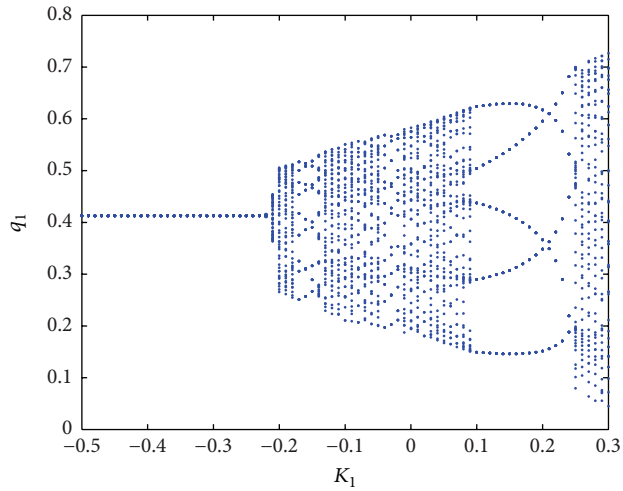

FIGURE 5: Bifurcation diagram with respect to parameter K_1 against variable q_1 with 500 iterations of map (19) for $v_1 = 0.85$, $v_2 = 0.7$, $c_1 = 0.2$, $c_2 = 2$.

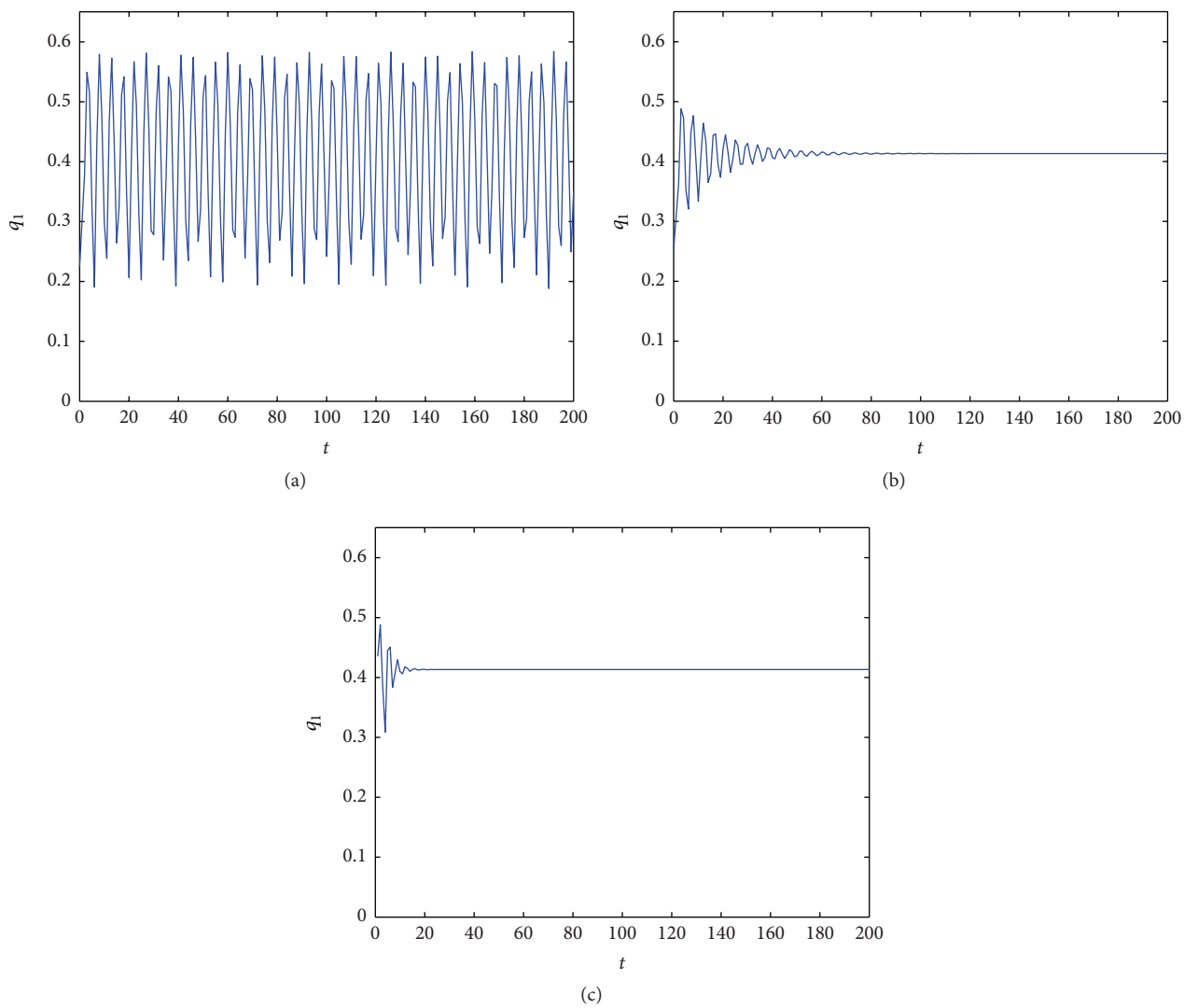

FIGURE 6: (a) Time series of output of firm 1 before using control function. (b) Time series of output of firm 1 after only firm 1 uses control function. (c) Time series of output of firm 1 after both firms use control function.

Now, consider the case that both firms realize that they will generate chaos in the future if they do not take any measure to control it. Therefore, both firms use control function to stabilize their outputs. Then, system (8) can be rewritten as

$$\begin{aligned} q_1(t+1) &= (1-v_1) * q_1(t) + v_1 * q_1(q_2) + u_{1,t}, \\ q_2(t+1) &= (1-v_2) * q_2(t) + v_2 * q_2(q_1) + u_{2,t}, \end{aligned} \tag{20}$$

where $u_{1,t} = K_1(q_1(t)-q_1(t-1))$ and $u_{2,t} = K_2(q_2(t)-q_2(t-1))$. The outputs of firm 1 are shown in Figure 6(c) for $K_1 = -0.3$ and $K_2 = -0.3$. By comparing Figure 6(b) with Figure 6(c), it is convenient to conclude that the effect of both firms taking control method is better than that of single firm taking control method.

6. Conclusion

In this paper, we have investigated a Cournot duopoly model where both the players decide their outputs weighting on their own previous outputs and the optimal outputs on the condition that their rival produces as their previous step. The Nash equilibrium and its local stability were analyzed. The numerical simulations show that the changes of marginal cost c_1 and weight factor v_1 may lead the Nash equilibrium to be unstable and the system into chaotic state. The system can quickly arrive at the Nash equilibrium by taking DFC method with a suitable controlling parameter. The effect of both firms taking control method is better than that of single firm taking control method.

In view of the fact that firms may not produce any goods if they suffer losses, it is needed to take firms' profits into consideration in modeling the outputs of firms. This problem will be investigated in our future research.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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Research Article

Analysis and Control of the Complex Dynamics of a Multimarket Cournot Investment Game with Bounded Rationality

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A dynamic multimarket Cournot model is introduced based on a specific inverse demand function. Puu's incomplete information approach, as a realistic method, is used to contract the corresponding dynamical model under this function. Therefore, some stability analysis is carried out on the model to detect the stability and instability conditions of the system's Nash equilibrium. Based on the analysis, some dynamic phenomena such as bifurcation and chaos are found. Numerical simulations are used to provide experimental evidence for the complicated behaviors of the system evolution. It is observed that the equilibrium of the system can lose stability via flip bifurcation or Neimark-Sacker bifurcation and time-delayed feedback control is used to stabilize the chaotic behaviors of the system.

1. Introduction

The dynamical behaviors of oligopoly games are complex because every oligopolistic producer in each period must consider not only its own decision but also the reactions of all other competitors. Cournot competition is an economic model used to describe the competition between some companies on the amount of output they will produce [1]. Thus a generalization of this game to the case of two markets is done. It is shown that the resulting dynamics is quite variable. In the classic model, each participant uses a naïve expectation to suppose that the opponents' output keeps the same level as the previous period and adopts an output strategy to maximize the expected profit. Many researchers have analyzed the system stability and the complex phenomena in Cournot oligopoly games with this kind of expectation [2–9]. In an early work [10], a kind of bounded rationality is assumed for the dynamical Cournot game, where each producer does not have complete knowledge of the market and updates its production by the local profit maximization method.

In recent years, a great amount of work has been done on the dynamical Cournot games with homogeneous or heterogeneous expectations. Bounded rationality in the marginal profit method is assumed to all producers in the models considering homogeneous expectation [10–13]. The models with heterogeneous expectations (naïve, boundedly rational, or adaptive) have been discussed in many other works [14–20].

In this study, a dynamic multimarket Cournot model is introduced based on a specific inverse demand function. The main purpose of our work is to formulate a novel model, which puts investment decision as a substitute for output adjustment into the dynamical Cournot game. In the model, all producers are also assumed to have bounded rationality and make their investment decisions in line with the marginal profit in the previous period. Meanwhile each firm will increase its investment if it perceives a growing marginal profit and decreases its investment if the perceived marginal profit is decreased. During a local adjustment process, this novel dynamical Cournot game aims to develop the equilibrium or demonstrate complex dynamic behaviors.

This paper is organized as follows. In Section 2, we model the dynamical game played by players with bounded rationality. In Section 3, we discuss the existence and local stability of the equilibrium points for the system. In Section 4, we show the dynamic features of this system with numerical simulations, including bifurcation diagram, phase portrait, and sensitive dependence on initial conditions. In Section 5, time-delayed feedback control is used to stabilize the chaotic behaviors of the system.

2. The Multimarket Cournot Model

In this model, the products are near substitutes but different in the quality levels. So, firms can charge different prices for different markets. Suppose we have n firms ($n > 2$) that compete in two interrelated markets A and B. For the price in the market, we consider Bowley [21] who has introduced the demand function

$$p_i = \psi_i - q_i - \theta \sum_{j \neq i} q_j, \tag{1}$$

where $0 \leq \theta \leq 1$. We also suppose that the production cost function of each firm takes a specific inverse demand function; [22–24] used this form in their work. Billand et al. [25] used the same demand function with $\theta = 1$. Let the demand in market A for firm i be

$$p_{Ai} = \psi_i - a_i \left(q_{Ai} + \sum_{j \neq i} q_{Aj} \right), \tag{2}$$

where p_{Ai} is the firm i's price in markets A, q_{Ai} is the firm i's quantity and the constant $\psi_i > 0$, $a_i > 0$ for market A. The demand in market B for firm i is

$$p_{Bi} = \beta_i - b_i \left(q_{Bi} + \sum_{j \neq i} q_{Bj} \right), \tag{3}$$

where p_{Bi} is the firm i's price in markets B, q_{Bi} is the firm i's quantity and the constant $\beta_i > 0$, $b_i > 0$ for market B.

The cost function of the firm is

$$C_i \left(q_{Ai}, q_{Bi} \right) = c_i \left(q_{Ai} + q_{Bi} \right)^2. \tag{4}$$

Hence the profit of firm i is given by

$$\pi_i = p_{Ai} q_{Ai} + p_{Bi} q_{Bi} - C_i \left(q_{Ai} + q_{Bi} \right)^2. \tag{5}$$

A standard approach to generalize static games to dynamic ones is called the bounded rationality approach [3, 5]. The corresponding dynamical system for the static multimarket Cournot model is

$$\begin{aligned} q_{Ai}(t+1) &= q_{Ai}(t) + \gamma_{Ai}\left(q_{Ai}(t)\right) \frac{\partial \pi_i}{\partial q_{Ai}(t)}, \\ &\qquad i = 1, 2, \ldots, n, \\ q_{Bi}(t+1) &= q_{Bi}(t) + \gamma_{Bi}\left(q_{Bi}(t)\right) \frac{\partial \pi_i}{\partial q_{Bi}(t)}, \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

Here, we study the monopoly case. In this case there is only one firm ($n = 1$) and two markets. We use the speeds (rates) of adjustment as linear functions of the quantities. For instance, we take $\gamma_{Ai}(q_{Ai}(t)) = \gamma_A(q_{Ai}(t))$ and $\gamma_{Bi}(q_{Bi}(t)) = \gamma_B(q_{Bi}(t))$ as an example, where γ_A and γ_B are constants, $a_1 = a$, $\psi_1 = \psi$, $\beta_1 = \beta$, and $b_1 = b$. With all the assumptions above, we get firm profit in period t as follows:

$$\begin{aligned} \pi\left(q_A, q_B\right) &= q_A \left[\psi - a q_A\right] + q_B \left[\beta - b q_B\right] \\ &\quad - c \left(q_A + q_B\right)^2. \end{aligned} \tag{7}$$

By differentiating $\pi(q_A, q_B)$, we obtain firm marginal profit with respect to its investment in period t at the markets A and B, respectively, as follows:

$$\begin{aligned} \frac{\partial \pi}{\partial q_A} &= \psi - 2 q_A (a + c) - 2 c q_B, \\ \frac{\partial \pi}{\partial q_B} &= \beta - 2 q_B (b + c) - 2 c q_A. \end{aligned} \tag{8}$$

According to the theory of Ding [26], hence, we get the dynamic system as follows:

$$\begin{aligned} q_A(t+1) &= q_A(t) + \gamma_A q_A(t) \\ &\quad \cdot \left[\omega \left(\psi - 2(a+c) q_A(t) - 2 c q_B(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\psi - 2(a+c) q_A(t-1) - 2 c q_B(t-1)\right)\right] \\ q_B(t+1) &= q_B(t) + \gamma_B q_B(t) \\ &\quad \cdot \left[\omega \left(\beta - 2(b+c) q_B(t) - 2 c q_A(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\beta - 2(b+c) q_B(t-1) - 2 c q_A(t-1)\right)\right], \end{aligned} \tag{9}$$

where $0 \leq w \leq 1$ is a weight coefficient assigned to the nondelayed period t and $1 - w$ is assigned to the delayed period $t - 1$.

Let $I_1(t) = q_A(t-1)$, $I_2(t) = q_B(t-1)$; we obtain a four-dimensional discrete dynamic system:

$$\begin{aligned} q_A(t+1) &= q_A(t) + \gamma_A q_A(t) \\ &\quad \cdot \left[\omega \left(\psi - 2(a+c) q_A(t) - 2 c q_B(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\psi - 2(a+c) I_1(t) - 2 c I_2(t)\right)\right] \\ q_B(t+1) &= q_B(t) + \gamma_B q_B(t) \\ &\quad \cdot \left[\omega \left(\beta - 2(b+c) q_B(t) - 2 c q_A(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\beta - 2(b+c) I_2(t) - 2 c I_1(t)\right)\right] \\ I_1(t+1) &= q_A(t) \\ I_2(t+1) &= q_B(t). \end{aligned} \tag{10}$$

System (10) describes a duopoly game played by boundedly rational players making decision in a process of dynamical investment in the two markets. In the following sections, we are to investigate the dynamical properties of this model.

3. Analysis of the Equilibrium Points and Stability

Let $q_\xi(t+1) = q_\xi(t)$ and $I_\xi(t+1) = I_\xi(t)$ $(\xi = A, B)$ in system (10); then we get

$$\begin{aligned} &q_A(t)\left[\omega\left(\psi - 2(a+c)q_A(t) - 2cq_B(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\psi - 2(a+c)I_1(t) - 2cI_2(t)\right)\right] = 0 \\ &q_B(t)\left[\omega\left(\beta - 2(b+c)q_B(t) - 2cq_A(t)\right)\right. \\ &\quad \left. + (1-\omega)\left(\beta - 2(b+c)I_2(t) - 2cI_1(t)\right)\right] = 0 \\ &I_1(t) = q_A(t) \\ &I_2(t) = q_B(t). \end{aligned} \tag{11}$$

Solving equations in (11), we obtain four equilibrium states of dynamics (see (10)), which are listed as follows:

$$\begin{aligned} E_0 &= (0,0,0,0), \\ E_1 &= \left(0, \frac{\beta}{2(b+c)}, 0, \frac{\beta}{2(b+c)}\right), \\ E_2 &= \left(\frac{\psi}{2(b+c)}, 0, \frac{\psi}{2(b+c)}, 0\right), \\ E^* &= \left(q_A^*, q_B^*, I_1^*, I_2^*\right) = \left(\frac{-\beta c + \psi b + c\psi}{2(bc+ab+ac)},\right. \\ &\quad \frac{a\beta + \beta c - \psi c}{2(bc+ab+ac)}, \frac{-\beta c + \psi b + c\psi}{2(bc+ab+ac)}, \\ &\quad \left.\frac{a\beta + \beta c - \psi c}{2(bc+ab+ac)}\right). \end{aligned} \tag{12}$$

In order to make these equilibrium points have economic meaning, we only consider the nonnegative cases. Since β, ψ, b, and c are positive parameters, E_1, E_2, and E^* are all positive provided that

$$\psi b + c\psi - \beta c > 0, \tag{13a}$$

$$a\beta + \beta c - \psi c > 0. \tag{13b}$$

In the following, all the nonnegativity conditions (13a) and (13b) are assumed. E_0, E_1, and E_2 are all boundary equilibriums and E^* is a unique interior equilibrium. Next, we will analyze the stability of the equilibrium.

3.1. Stability of the Boundary Equilibriums. To investigate the local stability of an equilibrium (q_A, q_B, I_1, I_2) of system (9), we work out its Jacobian matrix J:

$$J(q_A, q_B, I_1, I_2) = \begin{pmatrix} 1+\gamma_A N_1 & -2c\omega\gamma_A q_{A1}(t) & -2(1-\omega)(a+c)\gamma_A q_{A1}(t) & -2c\gamma_A q_{A1}(t)(1-\omega) \\ -2\omega c\gamma_B q_{B1}(t) & 1+\gamma_B N_2 & -2c\gamma_B q_{B1}(t)(1-\omega) & -2(b+c)\gamma_B q_{B1}(t)(1-\omega) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{14}$$

where

$$\begin{aligned} N_1 &= \omega\left(\psi - 4(a+c)q_{A1}(t) - 2cq_{B1}(t)\right) \\ &\quad + (1-\omega)\left(\psi - 2(a+c)I_1(t) - 2cI_2(t)\right), \\ N_2 &= \omega\left(\beta - 4(b+c)q_{B1}(t) - 2cq_{A1}(t)\right) \\ &\quad + (1-\omega)\left(\beta - 2(b+c)I_2(t) - 2cI_1(t)\right). \end{aligned} \tag{15}$$

An equilibrium (q_A, q_B, I_1, I_2) will be locally asymptotically stable if all the eigenvalues (real or complex) of the Jacobian matrix $J(q_A, q_B, I_1, I_2)$ lie inside the unit disk; that is, $|\lambda| < 1$ holds for any eigenvalue λ of $J(q_A, q_B, I_1, I_2)$. An equilibrium (q_A, q_B, I_1, I_2) will be unstable if there is an eigenvalue λ of $J(q_A, q_B, I_1, I_2)$ such that $|\lambda| > 1$.

Proposition 1. *The boundary equilibrium E_0 is an unstable equilibrium.*

Proof. Taking the expression of equilibrium E_0 into (14), we get the Jacobian matrix at E_0 as follows:

$$J(E_0) = \begin{pmatrix} 1+\psi\gamma_A & 0 & 0 & 0 \\ 0 & 1+\psi\gamma_B & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{16}$$

which has four eigenvalues: $\lambda_1 = 1+\psi\gamma_A$, $\lambda_2 = 1+\psi\gamma_B$, $\lambda_3 = 0$, and $\lambda_4 = 0$. Because ψ, γ_A, and γ_B are positive parameters, it can be seen that $\lambda_1, \lambda_2 > 1$ does not satisfy E_0 point stability condition. So equilibrium E_0 is unstable. □

Proposition 2. *The boundary equilibriums E_1 and E_2 are both unstable.*

Proof. At the boundary equilibrium point E_1, the Jacobian matrix (see (14)) is given by

$$J(E_1) = \begin{pmatrix} 1+\gamma_A\left(\dfrac{b\psi+\psi c-c\beta}{b+c}\right) & 0 & 0 & 0 \\ -\dfrac{\beta\gamma_B\omega c}{b+c} & 1-2\beta\gamma_B\omega & -\dfrac{\beta c\gamma_B(1-\omega)}{b+c} & -2\beta\gamma_B(1-\omega) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{17}$$

By simple calculation, we get four eigenvalues of the matrix $J(E_1)$:

$$\begin{aligned} &\lambda_1 = 0, \\ &\lambda_2 = 1+\gamma_A\left(\frac{b\psi+\psi c-c\beta}{b+c}\right), \\ &\lambda_{3,4} \\ &\quad = \frac{1}{2}\left(1-\beta\gamma_B\omega \pm \sqrt{1-4\beta\gamma_B+2\beta\gamma_B\omega+\beta^2\gamma_B^2\omega^2}\right). \end{aligned} \tag{18}$$

Obviously $\lambda_2 > 1$, so equilibrium E_1 is unstable. A similar approach shows that E_2 is unstable too. □

3.2. Stability of the Interior Equilibrium. Now we consider the asymptotical stability of the interior equilibrium E^*. The Jacobian matrix J at $E^* = (q_A^*, q_B^*, I_1^*, I_2^*)$ takes its formats

$$J(E^*) = \begin{pmatrix} 1-\gamma_A\omega K_1 & -2c\gamma_A q_A^*\omega & -2\gamma_A q_A^*(1-\omega)(a+c) & 2c\gamma_A q_A^*(1-\omega) \\ -2c\gamma_B q_B^*\omega & 1-\gamma_B\omega K_2 & -2c\gamma_B q_B^*(1-\omega) & -2\gamma_B q_B^*(1-\omega)(b+c) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{19}$$

where

$$\begin{aligned} K_1 &= 2cq_A^* + \frac{(a\psi b+\psi ca-c\beta a)}{(bc+ab+ac)}, \\ K_2 &= 2cq_B^* + \frac{(a\beta b+\beta cb-\psi bc)}{(bc+ab+ac)}. \end{aligned} \tag{20}$$

If $f(\lambda)$ denotes the characteristic polynomial of the Jacobian matrix $J(E^*)$, then

$$f(\lambda) = \lambda^4 + g_1\lambda^3 + g_2\lambda^2 + g_3\lambda + g_4. \tag{21}$$

By calculation, we get

$$\begin{aligned} g_1 &= \frac{1}{Z}\left[-2Z + \left((a+c)\gamma_A X + (b+c)\gamma_B Y\right)\omega\right], \\ g_2 &= \frac{1}{Z}\left[Z + \left((a+c)\gamma_A X + (b+c)\gamma_B Y\right)(1-2\omega) \right. \\ &\quad \left. + XY\gamma_A\gamma_B\omega^2\right], \\ g_3 &= \frac{1}{Z}(\omega-1)\left[(a+c)\gamma_A X + (b+c)\gamma_B Y \right. \\ &\quad \left. - 2XY\gamma_A\gamma_B\omega\right], \\ g_4 &= \frac{1}{Z}XY\gamma_A\gamma_B(1-\omega)^2, \end{aligned} \tag{22}$$

where $Z = ab+ac+bc$, $X = \psi b+\psi c-\beta c$, and $Y = \beta a-\psi c+\beta c$.

For all the roots of the polynomial $f(\lambda)$ (the eigenvalues of the Jacobian matrix $J(E^*)$) to lie inside the unit disk, Schur-Cohn Criterion [27] gives the necessary and sufficient conditions as

(i) $f(1) > 0$;

(ii) $(-1)^4 f(-1) > 0$;

(iii) the determinants of the 1×1 matrices $M_1^{\pm}$ and the 3×3 matrices $M_3^{\pm}$ are all positive, where

$$\begin{aligned} M_1^{\pm} &= 1 + g_4, \\ M_3^{\pm} &= \begin{pmatrix} 1 & & \\ g_1 & 1 & \\ g_2 & g_1 & 1 \end{pmatrix} \pm \begin{pmatrix} 0 & 0 & g_4 \\ 0 & g_4 & g_3 \\ g_4 & g_3 & g_2 \end{pmatrix}. \end{aligned} \tag{23}$$

In our model, we have $f(1) = 1 + g_1 + g_2 + g_3 + g_4 = XY\gamma_A\gamma_B/Z$, $1 \pm g_4 = 1 \pm (1/Z)XY\gamma_A\gamma_B(1-\omega)^2$; then, from the nonnegativity conditions (13a) and (13b), we know that the first condition $f(\lambda) > 0$ and $1 \pm g_4 > 0$ must hold. Consequently, we conclude that the interior equilibrium point E^* of system (10) is asymptotically locally stable if it meets the following conditions:

$$f(-1) = 1 - g_1 + g_2 - g_3 + g_4 = \frac{4Z + 2\left[(a+c)X\gamma_A + (b+c)Y\gamma_B\right](1-2\omega) - XY\gamma_A\gamma_B(2\omega-1)^2}{Z} > 0,$$
$$\mathrm{Det}\left(M_3^{\pm}\right) > 0, \tag{24}$$

where $\mathrm{Det}(M)$ represents the determinant of matrix M.

4. Numerical Simulation

In this section, numerical simulations show how the system evolves under different levels of parameters, especially with adjustment speeds γ_A and γ_B. In all the numerical simulations, the other parameters are fixed: $a = 1$, $b = 0.6$, $\omega = 0.6$, $c = 0.4$, $\psi = 5$, and $\beta = 4$.

So, we will use some numerical simulations to show the complicated behavior of the model (stability, period-doubling bifurcation, and chaos).

Figure 1 shows the bifurcation diagram of quantities q_A and q_B with the adjustment speed γ_A while the other parameters are constant and have taken the value $\gamma_B = 0.6$. This figure shows that the equilibrium points q_A and q_B are locally stable for $\gamma_A < 0.0755$.

Figure 2 shows the bifurcation diagram of the quantities q_A and q_B with the adjustment speed γ_B while the other parameters are constant and have taken the value $\gamma_A = 0.5$. This figure shows that the equilibrium points q_A and q_B are locally stable for $\gamma_B < 0.443$, and such a complicated process (period-doubling bifurcation, Neimark-Sacker bifurcation, periodic window, etc.) continues to lead the system to chaos.

The observations from Figures 1 and 2 show that when γ_A and γ_B are increasing, it shows more complex dynamic phenomena by appearing in the process of the evolution of Neimark-Sacker bifurcation and cycle window.

In Figure 3, the nonlinear dynamic system (10) can be showed from three dimensions of strange attractor, and strange attractor shows in the system space which is comprised by three arbitrary state variables.

Figure 4 shows the sensitivity of system (when loosing stability) to initial conditions, with $(q_{A1}(0), q_{B1}(0), I_1(0), I_2(0)) = (1, 2, 1, 2)$ and $(q_{A1}(0) + 0.0001, q_{B1}(0), I_1(0), I_2(0))$. Figure 4 shows the difference among the different orbits with slightly deviated initial values which builds up rapidly after a number of iterations, although their initial states are indistinguishable.

In Figures 5(a), 5(b), and 5(c), the bifurcation diagrams are plotted for the weight coefficient assigned w when γ_B is fixed as 0.6. Figure 5(a) is for $\gamma_A = 0.105$, Figure 5(b) is for $\gamma_A = 0.515$, and Figure 5(c) is for $\gamma_A = 0.615$. Figure 5(a) shows a reverse period-doubling bifurcation, while Figures 5(b) and 5(c) combine reverse period-doubling bifurcation and reverse Neimark-Sacker bifurcation. The three figures show that the system tends toward stability with the increased weight coefficient, which also shows that a high residual rate has a positive effect on the system stability.

5. Chaos Control

From the numerical simulations, the adjustment rate and the weight coefficient have great influence on the stability of system (10). If the model parameters fail to locate into the stable region required, the behaviors of the dynamics will be much complicated. In a real economic system, chaos is not desirable and will be not expected, and it is needed to be avoided or controlled so that the dynamic system would work better. In this section, we use the time-delayed feedback control (e.g., [18, 28]) to control system chaos. We modify the first equation of system (10) by intercalating a controller $k(q_{A1}(t) - q_{A1}(t+1))$ as a small perturbation, where $k > 0$ is a controlling coefficient. Then the controlled system is given by

$$\begin{aligned}
q_A(t+1) &= q_A(t) + \gamma_A q_A(t) \\
&\quad \cdot \left[\omega\left(\psi - 2(a+c)q_A(t) - 2cq_B(t)\right)\right. \\
&\quad \left. + (1-\omega)\left(\psi - 2(a+c)I_1(t) - 2cI_2(t)\right)\right] \\
&\quad + k\left(q_A(t) - q_A(t+1)\right), \\
q_B(t+1) &= q_B(t) + \gamma_B q_B(t) \\
&\quad \cdot \left[\omega\left(\beta - 2(b+c)q_B(t) - 2cq_A(t)\right)\right. \\
&\quad \left. + (1-\omega)\left(\beta - 2(b+c)I_2(t) - 2cI_1(t)\right)\right] \\
I_1(t+1) &= q_A(t) \\
I_2(t+1) &= q_B(t).
\end{aligned} \tag{25}$$

It is easy to see that the new system (25) has the same equilibriums as system (10) and it takes the following equivalent form:

$$\begin{aligned}
q_A(t+1) &= q_A(t) + \frac{1}{1+k}\gamma_A q_A(t) \\
&\quad \cdot \left[\omega\left(\psi - 2(a+c)q_A(t) - 2cq_B(t)\right)\right. \\
&\quad \left. + (1-\omega)\left(\psi - 2(a+c)I_1(t) - 2cI_2(t)\right)\right], \\
q_B(t+1) &= q_B(t) + \gamma_B q_B(t) \\
&\quad \cdot \left[\omega\left(\beta - 2(b+c)q_B(t) - 2cq_A(t)\right)\right. \\
&\quad \left. + (1-\omega)\left(\beta - 2(b+c)I_2(t) - 2cI_1(t)\right)\right] \\
I_1(t+1) &= q_A(t) \\
I_2(t+1) &= q_B(t).
\end{aligned} \tag{26}$$

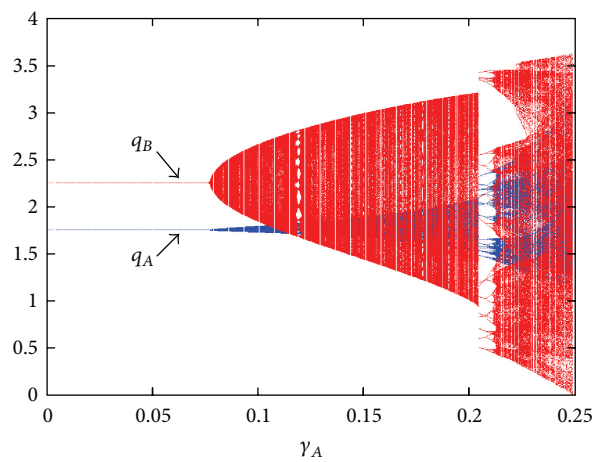

Figure 1: Bifurcation diagram for the quantities q_A and q_B with the adjustment speed γ_A.

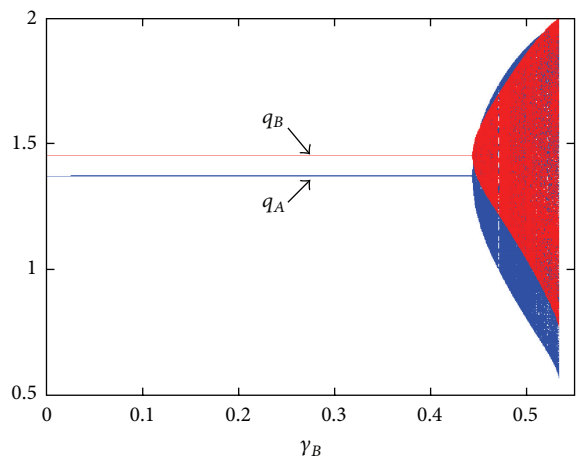

Figure 2: Bifurcation diagram for the quantities q_A and q_B with the adjustment speed γ_B.

The Jacobian matrix of the controlled system (26) is given by

$$J = \begin{pmatrix} 1 + \dfrac{\gamma_A N_1}{1+k} & -\dfrac{2c\omega\gamma_A q_A(t)}{1+k} & -\dfrac{2(1-\omega)(a+c)\gamma_A q_A(t)}{1+k} & -\dfrac{2c\gamma_A q_A(t)(1-\omega)}{1+k} \\ -2\omega c\gamma_B q_B(t) & 1+\gamma_B N_2 & -2c\gamma_B q_B(t)(1-\omega) & -2(b+c)\gamma_B q_B(t)(1-\omega) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{27}$$

where

$$\begin{aligned} N_1 &= \omega\left(\psi - 4(a+c)q_{A1}(t) - 2cq_{B1}(t)\right) \\ &\quad + (1-\omega)\left(\psi - 2(a+c)I_1(t) - 2cI_2(t)\right), \\ N_2 &= \omega\left(\beta - 4(b+c)q_{B1}(t) - 2cq_{A1}(t)\right) \\ &\quad + (1-\omega)\left(\beta - 2(b+c)I_2(t) - 2cI_1(t)\right). \end{aligned} \tag{28}$$

As has been shown in Figures 1 and 2, chaotic behavior of system (10) occurs when all the model parameters take their values as $(a, b, c, \psi, \beta, \gamma_A, \gamma_B, \omega) = (1, 0.6, 0.4, 5, 4, 0.55, 0.6, 0.55)$.

Using this group of parameters values, we obtain the Jacobian matrix (see (26)) at the interior equilibrium as follows.

As shown in Figure 1, the controlled system near the adjustment coefficient $\gamma_A = 0.081$ is entered into a state of chaos. When the model parameter selection, the initial

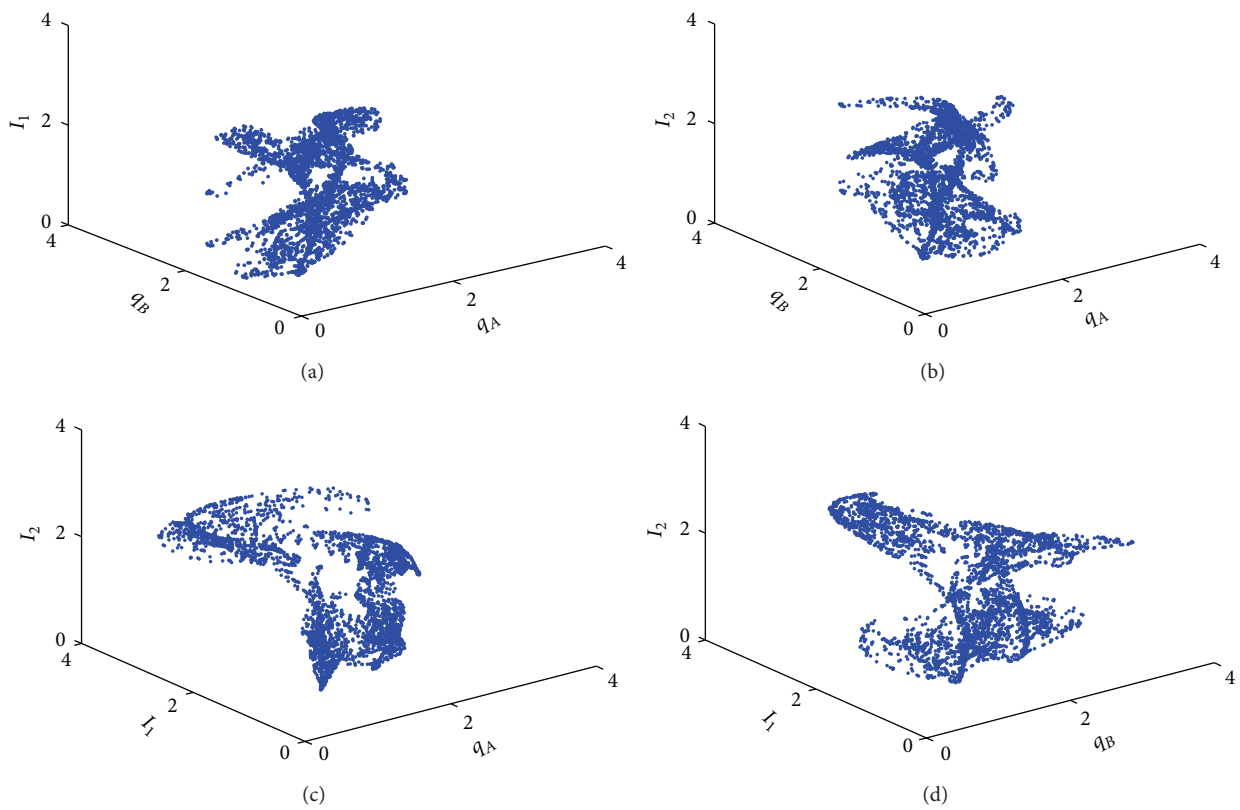

Figure 3: The three-dimensional strange attractors of the dynamical system (10).

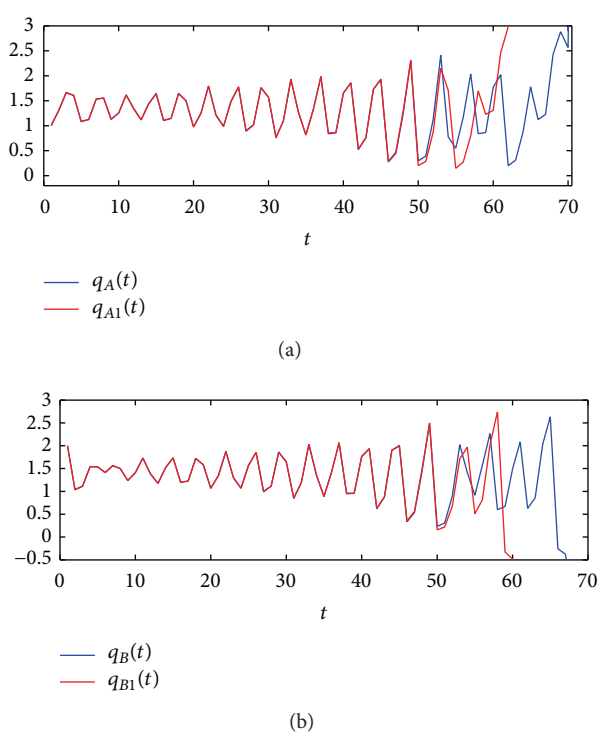

Figure 4: Sensitive dependence for the dynamical system (10) on initial conditions.

conditions $(q_{A1}(0), q_{B1}(0), I_1(0), I_2(0)) = (1, 2, 1, 2)$, and system (10) show the messy chaos phenomenon, the external variable parameter γ_A is in a stable area. Analysis under the controlled system (26) in this parameter values of the chaos represents the stability of the new system (26). The parameter values in the system use (26) the Jacobian matrix as follows:

$$J = \begin{pmatrix} 1 + \frac{0.517}{1+k} & -\frac{0.242}{1+k} & -\frac{0.693}{1+k} & -\frac{0.198}{1+k} \\ -0.528 & -2 & -0.432 & -1.08 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (29)$$

Corresponding characteristic polynomial calculates the matrix $P(\lambda) = \lambda^4 + v_1\lambda^3 + v_2\lambda^2 + v_3\lambda + v_4$, with the coefficient of theory, as follows:

$$\begin{aligned} v_1 &= \frac{k + 0.483}{1 + k}, \\ v_2 &= \frac{-0.92k - 1.38878}{1 + k}, \\ v_3 &= \frac{-1.08k - 0.461448}{1 + k}, \\ v_4 &= \frac{0.662904}{1 + k}. \end{aligned} \quad (30)$$

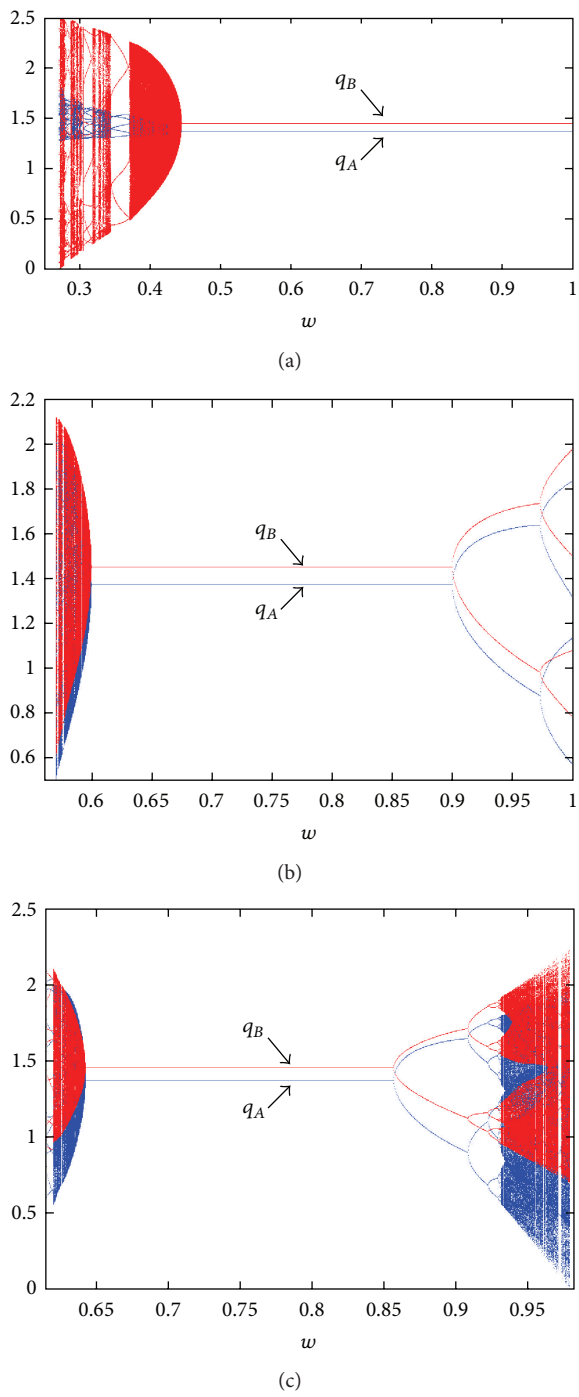

Figure 5: Bifurcation diagrams of dynamic system (10) with the weight coefficient assigned w.

According to Schur-Cohn stability criterion, when the Jacobi matrix of the characteristic polynomial $P(\lambda)$ of the coefficients satisfy the following conditions:

$$\begin{aligned} &\left|1+v_2+v_4\right| > v_1+v_3 \\ &\left|1-v_4^2+v_2v_4-v_2v_4^2+v_1v_3v_4-v_3^2\right| \\ &\quad > v_2-v_2v_4+v_4-v_4^3-v_1v_3+v_1^2v_4, \end{aligned} \tag{31}$$

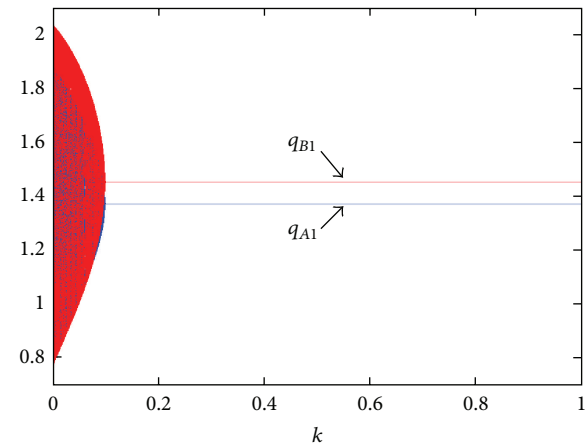

Figure 6: Bifurcation diagrams of system (26) with the controlling factor k.

the entire characteristic root of the matrix is less than 1, which shows that dynamic system (26) is to take the set of parameter values under stable, so the controlled system (10) of chaos control tends to be a stable orbit. From the stability conditions, we get that all the eigenvalues of the matrix will lie inside the unit disk by providing $k > 0.0917$. When $k > 0.0917$, the controlled system (26) will be asymptotically locally stable.

In Figure 6, it is obviously observed that, with the control coefficient k increasing, the system gradually gets out of chaos and periodic windows and achieves stability when $k > 0.0917$. When $k = 1.5$ and $k = 3.5$, Figures 7 and 8 show the stable behaviors of the orbits of the controlled system beginning from the initial state $(x_1(0), x_2(0), I_1(0), I_2(0)) = (1, 2, 1, 2)$. These two graphs can be found in the stable region, the feedback gains strength value, and the more chaotic behavior can quickly control the stable orbit.

6. Conclusion

In this study we have taken into consideration a dynamic Cournot game based on quadratic cost function which is built with the same assumption of bounded rationality. We discuss the stability of each equilibrium solution by using the nonlinear system. Numerical simulations are used to provide experimental evidence for the complicated evolution behaviors of the system. It proves that the chaotic behavior of the system can be controlled. It proves that parameters play an important role in the stability of the economy systems based on analysis of dynamical behaviors of the established game models. It shows that the weight coefficient assigned can enhance the stability of systems. If each firm renews its investment strategy too fast or too slow, the systems will postpone converging to equilibrium and may respond intricately, including bifurcation, chaos, and initial state of sensitivity. Meanwhile, time-delayed feedback control could be used to stabilize the chaotic behaviors of the dynamic systems.

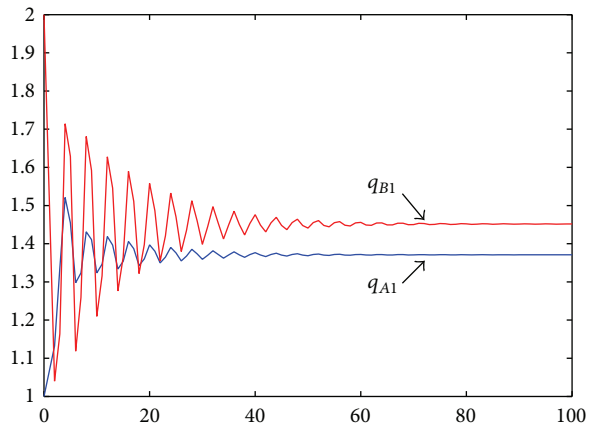

Figure 7: Evolution for the dynamic system (26) with the controlling factor $k = 1.5$.

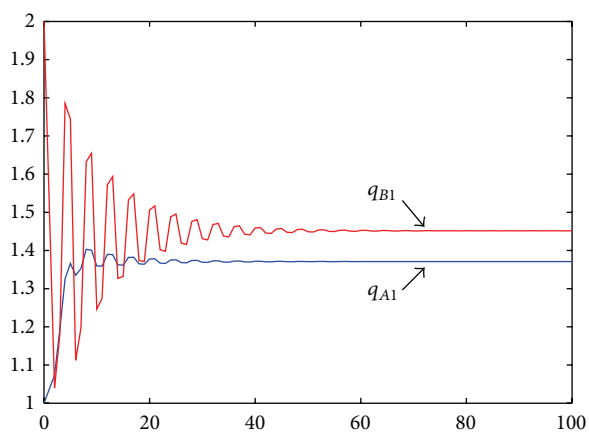

Figure 8: Evolution for the dynamic system (26) with the controlling factor $k = 3.5$.

Competing Interests

The author declares that they have no competing interests.

Acknowledgments

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Research Article

The Government Incentive Regulation Model and Pricing Mechanism in Power Transmission and Distribution Market

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The power transmission and distribution (T&D) market's natural monopoly and individual information have been the impediment to improving the energy efficiency in the whole T&D market. In order to improve the whole social welfare, T&D market should be controlled by government. An incentive regulation model with the target of maximizing social welfare has been studied. A list of contracts with transferring payment and quantity of T&D are given to motivate the corporation to reveal the true technical parameter and input the optimal investment. The corporate revenue, optimal investment, and effort are proved to depend on its own technical parameter. The part of incentive regulation model ends with the optimal pricing mechanism of T&D market. At the end of this paper, we give a numerical example to explain our research and confirm its function graphically.

1. Introduction

Power transmission and distribution (T&D) corporations play an important role in energy efficiency improving, mainly due to the natural monopoly and their individual information [1]. Power transmission pricing mechanism receives more and more attention in recent years, since power transmission is a very special service [2]. It is related to not only the future of the transmission grid but also the reformation of power pricing [3]. But the power transmission grid is naturally monopolistic. In this situation, government has to regulate the power transmission market to prevent inefficiency and protect social welfare. To formulate reasonable T&D, pricing mechanism is the key to the power reformation in China [4, 5]. The problems in power transmission and distribution market are seriously blocking the power reform in China [4]. So it is of great significance to research the regulation and pricing mechanism in T&D market and obtain some sensible conclusions.

The former researches in regulation of T&D market mainly focus on the cost regulation [6–8]. Joskow [9] concludes the researches about T&D regulation mechanism published in the economic journal; most of these researches formulate regulation mechanism based on the corporate cost, effort, and the asymmetric information in quality. Kristiansen [10] shows three kinds of T&D pricing mechanisms based on incentive regulation model: the Wangensteen model, the optimal power flow model, and the Hogan model; nevertheless, only Hogan model is the incentive regulation model in economics.

Both theory and practice have shown that the current regulation prices of T&D market cannot properly address several vital problems such as asymmetric information and low efficiency [11, 12]. Vogelsang [13] focuses on two-part tariffs where the variable part would reflect congestion charges (and ancillary services), while the fixed part would reflect capacity costs. Vogelsang [14] shows that performance-based regulation (PBR) is influenced by the Bayesian and non-Bayesian incentive mechanisms. Further, he points out that Bayesian incentives are impractical, and, from their properties, they can be combined with practical non-Bayesian mechanisms for application to transmission pricing. Hogan et al. [2] point out that certain investments in transmission upgrades cause negative network effects on other transmission links, so that capacity is multidimensional. So they study the problems with a new model (HRV model) which combines the merchant (long-run financial rights to transmission) and regulatory (incentive regulation hypothesis) approaches in a setting

with price-taking electricity generators and loads. In the model, there are two types of price index weights: chained Laspeyres weights and idealized weights. Laspeyres weights have shown good economic properties under well-behaved, stable cost, and demand conditions, while idealized weights correspond to perfectly predicted quantities and possess strong efficiency properties. Hogan et al. [15] prove the HRV model practicable. Rosellón and Weigt [16] obtain the result suggesting that a new incentive mechanism, based on HRV regulatory approach, is generally suited as an incentive tool for network extensions.

The T&D market price usually abnormally soars and fluctuates mainly due to the dissatisfaction with the requirement of the corporate incentive compatibility [17–21]. So, Joskow [22] suggests incentive regulation theory and addresses that incentive regulation theory implies that the adverse selection and moral hazard problems resulting from the regulators' information disadvantages are best handled by offering firms a menu of cost contingent incentive contracts. Wang et al. [23] study a model, with which government can reveal enterprise's cost successfully and make out consumer types effectively. Cai and Ye [24] research government incentive regulation model under asymmetric information and obtain the optimal price of power transmission and distribution. Osorio and Sauma [1] study two models of the principal agent bilevel type in T&D market, and they point that, under the traditional regulatory frameworks, T&D corporations have disincentive to promote energy efficiency.

In summary, to analyze how to reduce the effect of asymmetric information and incent the corporation to invest and improve the efficiency by researching the incentive regulation model is useful and urgent. In this paper, we consider regulation and pricing mechanism based on government incentive regulation model [25] and obtain the optimal price of power with economic mechanism design theory [26, 27]. The major contributions of this paper are as follows:

(1) Based on government incentive regulation model, we research government regulation and pricing mechanism in T&D market. As the regulator, government focusing on the social welfare offers a menu of contracts about the regulation policy and the pricing mechanism to T&D corporations.

(2) It encourages corporation to not only reveal the accurate technical parameter but also make optimal investment. In this paper, we consider the technical parameter as the private information of corporation, and investment lets the efficiency be higher (reducing the technical parameter). So government obtains this private information and encourages corporation to invest by some helpful policy.

2. Premise of the Model

2.1. Assumptions. In this paper, we consider 6 assumptions between government and the T&D corporation.

Assumption 1. The corporation and government have different targets. The corporation is eager for more profit; however, government as the regulator must maximize the social welfare. Government pays transferring payment to observe accurate technical parameter β of the corporation, meanwhile encouraging the corporation to invest and improve efficiency.

Assumption 2. We designate $Mpq/((1-G)NA)$ as the utility to the consumers if the quantity of power transmission and distribution is q, based on Social Welfare and Comprehensive Evaluation of Electric Power Universal Service [28] and Social Welfare Function Construction of Electric Power Universal Service Based on Expected Utility Theory and Prospect Theory [29].

Assumption 3. Neglect the loss in the process of power transmission and distribution, and we assume p and q follow the following function:

$$p = a - bq. \tag{1}$$

Assumption 4. Consider moral hazard and adverse selection in motivation. The corporation keeps the technical parameter β as its private information, and it can improve efficiency by increasing investment $I(e)$ (the cost of effort); as the result, the unit cost can be reduced by e.

The output cost of the corporation can be shown as below [25, 30]:

$$C = (\beta - e)\, q, \tag{2}$$

where $\beta \in [\underline{\beta}, \overline{\beta}]$ and high β means inefficient technical level. $F(\beta)$ means the absolutely continuous distribution function and its density is $f(\beta)$. While $\beta \in [\underline{\beta}, \overline{\beta}]$, $f(\beta) > 0$ and the monotone hazard rate $d[F(\beta)/f(\beta)]/d\beta \geq 0$, for most distributions, such as uniform distribution, normal distribution, logarithmic distribution, exponential distribution, and Laplace distribution, satisfy the condition.

Here we designate e as the moral hazard parameter, and it is related to the type of the corporation; it is represented in $e = e(\beta)$. $I(e)$ is the cost of effort the corporation made or it can be seen as the investment for improving the efficiency, and it is constrained by $I' > 0$, $I'' > 0$, $I''' \geq 0$, and $I(\beta) = +\infty$.

Assumption 5. The output cost of the corporation C, the cost of effort $I(e)$, and the quantity of transmission and distribution q are visible to government.

Assumption 6. The corporation has its accurate information of technical parameter, while government just knows it belongs to $[\underline{\beta}, \overline{\beta}]$. When the corporation announces the technical parameter $\widehat{\beta}$, it selects one from a series of contracts $\{T(\beta), q(\beta)\}$ given by government.

2.2. The Sequence of Government and the Corporation. The game sequence is shown in Figure 1:

(1) The corporation obtains its accurate information of technical parameter β according to its experience and historical data.

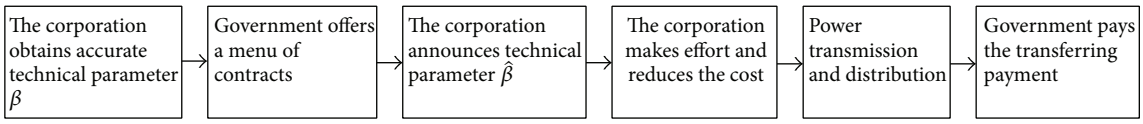

Figure 1: The game sequence between government and the corporation.

(2) Based on all information it can get, government formulates a series of contracts $\{T(\beta), q(\beta)\}$ with the target of maximizing the social welfare.

(3) The corporation announces the technical parameter $\hat{\beta}$ after researching the contracts and selects one.

(4) The two participants, government and the corporations, sign up the contract.

(5) The corporation invests $I(e)$ and reduces the unit cost to the level $c = \beta - e$.

(6) Government pays the transferring payment $T(\hat{\beta})$. The contract ends.

3. The Model and Pricing Mechanism

In this model, we consider the target of government, the expected profit of the corporation, the incentive constraints, and the participant constraints.

3.1. The Target of Government and the Corporation. Government aims at the social welfare, including both the utility of consumers and the utility of the corporation. The utility of consumers is the difference between total utility and the cost they pay, as it is shown below:

$$\frac{Mp(q)q}{(1-G)NA} - (1+\lambda)T, \tag{3}$$

where T is the transferring payment from government and $\lambda > 0$ is the shadow cost of public funds.

The utility of the corporation is formed by three terms: the transferring payment (T), the output cost (C), and the cost of effort (I):

$$U = T - C - I. \tag{4}$$

We can directly obtain the social welfare function from (3) and (4):

$$J_0 = \left[\frac{Mp(q)q}{(1-G)NA} - (1+\lambda)T\right] + (T - C - I). \tag{5}$$

Further, the final incentive target of government can be written as

$$S_0 = \int_{\underline{\beta}}^{\overline{\beta}} \left\{\left[\frac{Mp(q)q}{(1-G)NA} - (1+\lambda)T\right] + (T - C - I)\right\} dF(\beta). \tag{6}$$

It is the total social welfare from various types of corporations.

3.2. The Incentive Constraints to the Corporation. Government and the corporation sign up the contract based on the technical parameter $\hat{\beta}$ announced by the corporation. Then, the utility of the corporation can be expressed by the accurate technical parameter β and the announced technical parameter $\hat{\beta}$:

$$U(\hat{\beta}, \beta) = T(\hat{\beta}) - I(\beta - C(\hat{\beta})) - C(\hat{\beta}). \tag{7}$$

Thus, we obtain the incentive constraints when each of $\beta_1, \beta_2 \in [\underline{\beta}, \overline{\beta}]$, as they are shown below:

$$\begin{aligned} &T(\beta_1) - I(\beta_1 - C(\beta_1)) - C(\beta_1) \\ &\quad \geq T(\beta_2) - I(\beta_1 - C(\beta_2)) - C(\beta_2), \\ &T(\beta_2) - I(\beta_2 - C(\beta_2)) - C(\beta_2) \\ &\quad \geq T(\beta_1) - I(\beta_2 - C(\beta_1)) - C(\beta_1). \end{aligned} \tag{8}$$

3.3. The Participant Constraints to the Corporation. In order to achieve the target, government should not let the utility of the corporation be less than its lowest profit (normalized to 0 here) to keep the corporation working:

$$U(\hat{\beta}, \beta) \geq 0. \tag{9}$$

3.4. Model Based on Government Incentive Regulation. According to all the analyses above, the problem can be formulated as follows:

$$\begin{aligned} \{P_1\} \quad S = \max_{\{T(\cdot), q(\cdot)\}} \quad & \int_{\underline{\beta}}^{\overline{\beta}} \left\{\left[\frac{Mp(q)q}{(1-G)NA} - (1+\lambda)T\right] + (T - C - I)\right\} dF(\beta) \\ \text{s.t.} \quad & (8) \ (9). \end{aligned} \tag{10}$$

Let us consider the problem. From (8), we get

$$\int_{\beta_1}^{\beta_2}\int_{C(\beta_1)}^{C(\beta_2)} I''(x-y)\,dx\,dy \geq 0. \tag{11}$$

Because $I'' > 0$ (Assumption 4), (11) indicates that $C(\beta)$ is the nondecreasing function of β; in other words, $\dot{C}(\beta) \geq 0$.

For the best profit, the corporation may give simulated technical parameter β, so we take the envelope theorem into (7) to maximize the utility of the corporation; we get

$$\dot{U}(\beta) = -I'(\beta - C(\beta)) \leq 0. \tag{12}$$

In addition, the more utility the corporation obtains, the more payment the government will pay. Therefore, government lets $U(\overline{\beta}) = 0$, and we can get $U(\beta)$ from (12):

$$U(\beta) = \int_{\beta}^{\overline{\beta}} I'\left(e\left(\widetilde{\beta}\right)\right) d\widetilde{\beta}. \tag{13}$$

Equation (13) shows the information rent obtained by the corporation with lower technical parameter or high effective technical level. Accordingly, the information rent government paid willingly is

$$\begin{aligned}\int_{\underline{\beta}}^{\overline{\beta}} U(\beta)\,dF(\beta) &= \int_{\underline{\beta}}^{\overline{\beta}}\int_{\beta}^{\overline{\beta}} I'\left(e\left(\widetilde{\beta}\right)\right) d\widetilde{\beta}\,dF(\beta)\\ &= \int_{\underline{\beta}}^{\overline{\beta}} \frac{F(\beta)}{f(\beta)} I'(e(\beta))\,dF(\beta).\end{aligned} \tag{14}$$

Thus, the target of government equation (10) can be rewritten as

$$\begin{aligned}S = \max_{\{C(\cdot),q(\cdot),U(\cdot),I(\cdot)\}} \int_{\underline{\beta}}^{\overline{\beta}} \Bigg[&\frac{Mp(q)\,q}{N(1-G)A} - (1+\lambda)C\\ &- (1+\lambda)I - \lambda U\Bigg] dF(\beta).\end{aligned} \tag{15}$$

By replacing (4) and (14) into (15), we can reformulate the problem as

$$S = \max_{\{e(\cdot),q(\cdot)\}} \int_{\underline{\beta}}^{\overline{\beta}} \left[\frac{M(a-bq(\beta))\,q(\beta)}{N(1-G)A} - (1+\lambda)\cdot(\beta - e(\beta))\,q(\beta) - (1+\lambda)I(e(\beta)) - \lambda\frac{F(\beta)}{f(\beta)} I'(e(\beta))\right] dF(\beta) \tag{16}$$

$$\text{s.t.}\quad \dot{e}(\beta) \leq 1. \tag{17}$$

Equation (17) is equivalent to $\dot{C}(\beta) \geq 0$.

We designate J as the integral part of (16); thus,

$$\frac{\partial^2 J}{\partial q^2} = -2b\frac{M}{N(1-G)A} \leq 0. \tag{18}$$

So we can obtain the optimal q from $\partial J/\partial q = 0$, as it is shown below:

$$q^* = \frac{a}{2b} - \frac{(1+\lambda)(\beta - e(\beta))(1-G)NA}{2bM}. \tag{19}$$

In addition, since $I''' \geq 0$ (Assumption 4), J is concave. Neglect the constraint equation (17); we obtain the following formula by rewriting $\partial J/\partial e = 0$:

$$I'(e(\beta)) = q(\beta) - \frac{\lambda}{1+\lambda}\cdot\frac{F(\beta)}{f(\beta)}\cdot I''(e(\beta)). \tag{20}$$

Furthermore, find the derivative of β from (20); we get

$$\dot{e}(\beta) = -\frac{(NA/(2bM))\cdot(1+\lambda)(1-G) + (\lambda/(1+\lambda))\cdot I''(e(\beta))\cdot d(F(\beta)/f(\beta))/d\beta}{I''(e(\beta)) + (\lambda/(1+\lambda))[F(\beta)/f(\beta)]\,I'''(e(\beta))}. \tag{21}$$

It is obvious that $\dot{e}(\beta) < 0$, so that it does not contradict (17).

From (20), we get $e^*(\beta)$. And the optimal information rent and the transferring payment follow the following formulations:

$$\begin{aligned}U^*(\beta) &= \int_{\beta}^{\overline{\beta}} I'\left(e\left(\widetilde{\beta}\right)\right) d\widetilde{\beta},\\ T^*(\beta) &= U^*(\beta) + I(e^*(\beta)) + C^*(\beta),\end{aligned} \tag{22}$$

where $C^*(\beta) = \beta - e^*(\beta)$ is strictly increasing function. From this, it is noted that $C^*(\cdot)$ can be replaced by its inverse function $\beta = \beta^*(C)$. Thus, the optimal transferring payment from government can be expressed as

$$\begin{aligned}T^*(C) = &\int_{\beta}^{\overline{\beta}} I'\left(e^*\left(\widetilde{\beta}(C)\right)\right) d\widetilde{\beta} + I(e^*(\beta^*(C)))\\ &+ [\beta^*(C) - e^*(\beta^*(C))]\cdot\Bigg[\frac{a}{2b}\end{aligned}$$

$$- \frac{(1+\lambda)\left(\beta^*(C) - e^*\left(\beta^*(C)\right)\right)(1-G)NA}{2bM}\Bigg], \tag{23}$$

where the output cost C is visible for the regulator, government.

4. Results Analyses

Proposition 1. *Government regulates the corporation with these contracts $\{T^*(C), q^*\}$, which can make the corporation not only show its accurate technical parameter β but also make more effort $e(\beta)$. The contracts $\{T^*(C), q^*\}$ are represented in*

$$T^*(C) = \int_{\beta}^{\overline{\beta}} I'\left(e^*\left(\widetilde{\beta}(C)\right)\right) d\widetilde{\beta} + I\left(e^*\left(\beta^*(C)\right)\right) + \left[\beta^*(C) - e^*\left(\beta^*(C)\right)\right] \cdot q^*, \tag{24}$$

$$q^* = \frac{a}{2b} - \frac{(1+\lambda)\left(\beta - e(\beta)\right)(1-G)NA}{2bM}. \tag{25}$$

Equation (24) shows that the transferring payment from government consists of three parts: the compensation for output cost $[\beta^*(C) - e^*(\beta^*(C))] \cdot q^*$, the compensation for the investment $I(e^*(\beta^*(C)))$, and the information rent $\int_{\beta}^{\overline{\beta}} I'(e^*(\widetilde{\beta}(C)))d\widetilde{\beta}$. In the model based on government incentive regulation, the quantity of power transmission and distribution q^* is different when the technical parameter β and the effort $e(\beta)$ change. In addition, there is no distortion of the quantity of power transmission and distribution q^*.

Proposition 2. *The corporation receives different information rent $U^*(\beta)$ due to its technical parameter, and $U^*(\beta) = \int_{\beta}^{\overline{\beta}} I'(e(\widetilde{\beta}))d\widetilde{\beta}$.*

Information rent means the rest of the transferring payment and the cost. From $U^*(\beta)$, we conclude that the information rent is directly related to the technical parameter and the marginal cost for effort to improve efficiency.

Proposition 3. *The optimal investment for improving efficiency is represented in*

$$I'\left(e(\beta)\right) = q(\beta) - \frac{\lambda}{1+\lambda} \cdot \frac{F(\beta)}{f(\beta)} \cdot I''\left(e(\beta)\right). \tag{26}$$

Under the optimal regulation mechanism, the T&D corporations should put optimal effort to improve the efficiency. And the optimal effort can be seen as the optimal investment. From Proposition 3, the optimal investment is influenced by the technical parameter (different corporation has different optimal investment level), the distribution of the technical parameter, and the shadow cost.

Proposition 4. *In the model based on government incentive regulation, the price of power transmission and distribution is shown below:*

$$p^* = \frac{a}{2} + \frac{(1+\lambda)\left(\beta - e(\beta)\right)(1-G)NA}{2M}. \tag{27}$$

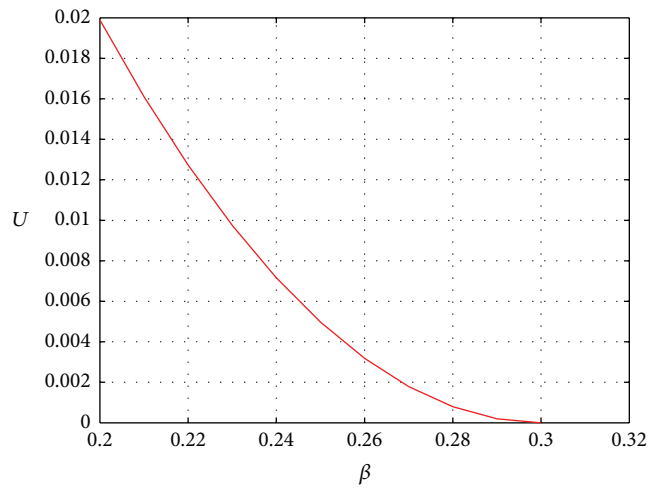

Figure 2: Diagram of the corporate information rent.

From Proposition 4, we confirm that Gini coefficient, coefficient of social welfare, shadow cost of public funds, and quantity of people in certain area and their average income including demand play important roles in the price of power transmission and distribution.

5. Numerical Example

We make a numerical example to explain our model lively. Consider β has a uniform distribution over $[0.2, 0.3]$ and its distribution function is $F(\beta) = 10\beta - 1$; meanwhile, we designate $I(e) = e^2$, $\lambda = 0.08$, and $G = 0.47$ [31]; the software Matlab 7.1 is used to simulate the case.

5.1. Main Results. Figure 2 shows that the information rent U increases with the technical parameter β. Especially when $\beta = 0.3$, U turns to be 0. In order to reveal the accurate technical parameter of the corporation and encourage it to make an effort, government must pay some information rent to the corporation with low technical parameter (high technical level). The T&D corporation cares more about the information rent, which can be seen as the net profit. The information rent goes down when the technical parameter increases in Figure 2, in agreement with Proposition 2 without any surprise.

The most important decision content of the regulation mechanism between the government and the T&D cooperation is the transferring payment and the quantity (in Proposition 1). Figures 3 and 4 show the changing tendency of the transferring payment over β and the advantage in technical parameter $\overline{\beta} - \beta$. Figure 4 shows if $\overline{\beta} - \beta \in [0, 0.02]$, the transferring payment changes in the same direction with $\overline{\beta} - \beta$; while $\overline{\beta} - \beta \in [0.02, 0.10]$, they change in the opposite direction. That is because the compensation for output cost is more than the compensation for cost of effort when $\beta \in [0.20, 0.28]$. So government tends to the corporation with low technical parameter (high technical level) more.

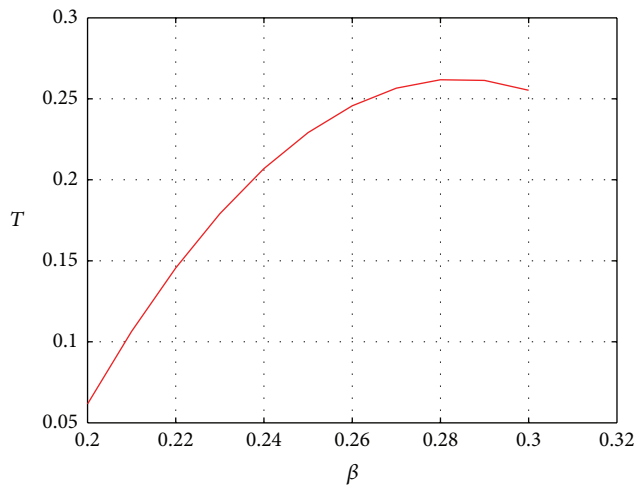

Figure 3: Diagram of the transferring payment in β.

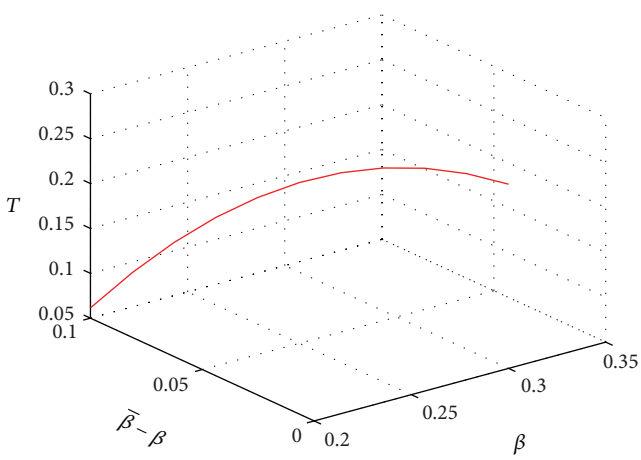

Figure 4: Diagram of the transferring payment in β and $\overline{\beta} - \beta$.

Furthermore, we designate $\beta = 0.25$, $a = 1.2$, $b = 0.3$, $M = 4450$, $A = 7383$, and $N = 2.42$ [32] and simulate the social welfare J (15). As Figure 5 shows, for certain technical parameter β, government, aiming at maximizing the social welfare, must pay reasonable transferring payment to encourage the corporation to make optimal effort and reach optimal quantity.

5.2. Sensitivity Analyses. We perform several sensitivity analyses. The results of the sensitivity analyses regarding the transferring payment, the social welfare, and the optimal effort of the T&D corporation are discussed below.

The sensitivity analyses for social welfare versus a, which can be seen as the market scale, are performed by replacing by −50%, −25%, +25%, and +50% one by one and keeping the remaining parameters, as shown in Table 1. The social welfare significantly changes with the market scale. And when the market scale goes up (+50%, +25%), the social welfare changes much more than when it goes down (−50%, −25%), for example, 0.6391 versus 0.3572 when $\beta = 0.22$, or 0.51 versus 0.2281 when $\beta = 0.26$.

Table 1: Sensitivity analyses for social welfare versus a.

Technical parameter	*Basement* and changes of a	Social welfare
	+50%	+0.6391
	+25%	+0.2843
$\beta = 0.22$	*1.2*	*0.4060*
	−25%	−0.2139
	−50%	−0.3572
	+50%	+0.5746
	+25%	+0.2521
$\beta = 0.24$	*1.2*	*0.3006*
	−25%	−0.1815
	−50%	−0.2926
	+50%	+0.5100
	+25%	+0.2197
$\beta = 0.26$	*1.2*	*0.1997*
	−25%	−0.1493
	−50%	−0.2281
	+50%	+0.4454
	+25%	+0.1875
$\beta = 0.28$	*1.2*	*0.1031*
	−25%	−0.1170
	−50%	−0.1635

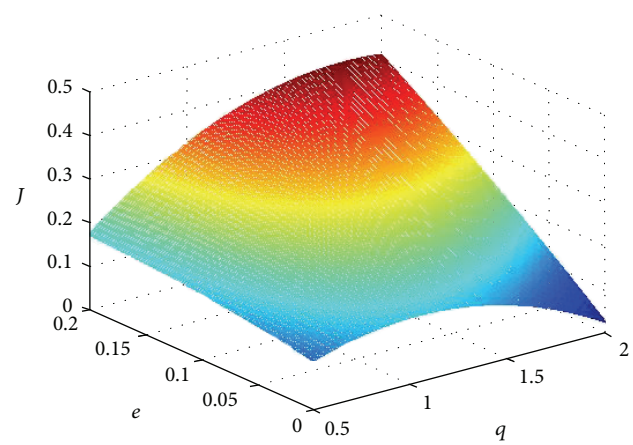

Figure 5: Diagram of social welfare in corporate effort and quantity.

Figure 6 shows the diagram for the optimal effort versus shadow cost with uncertainty, when the T&D corporate technical parameter $\beta = 0.25$. And it is obvious that as the shadow cost λ increases (or in other words, the public funding becoming expensive), the T&D corporation should put larger optimal effort (e^* becomes larger). And the optimal effort increases by the shadow cost linearly in Figure 6. But the shadow cost influences the second-order derivative of the efficiency investment ($I''(e)$) directly and then impacts the optimal effort indirectly, which can be seen in Proposition 3. And the linear influence for the optimal effort versus shadow cost is explained by the quadratic relationship between the efficiency investment I and the effort e.

The optimal effort of the T&D corporation decreases by the electricity coefficient of social welfare M (shown in

Table 2: Sensitivity analyses for social welfare versus M.

Technical parameter	*Basement* and changes of M	Optimal effort	Price	Social welfare
0.22	+50%	−0.5100	+0.0158	+0.1922
	+25%	−0.0306	+0.0142	+0.0880
	4450	*0.1591*	*0.6699*	*0.4060*
	−25%	+0.0511	−0.0549	−0.0558
	−50%	+0.1533	−0.2821	−0.0179
0.24	+50%	−0.0383	−0.0168	+0.2108
	+25%	−0.0323	−0.0066	+0.0993
	4450	*0.1194*	*0.7386*	*0.3006*
	−25%	+0.0383	−0.0124	−0.0764
	−50%	+0.1149	−0.1254	−0.0955
0.26	+50%	−0.0256	−0.0495	+0.2266
	+25%	−0.0154	−0.0273	+0.1086
	4450	*0.0796*	*0.8073*	*0.1997*
	−25%	+0.0255	+0.0300	−0.0922
	−50%	+0.0766	+0.0313	−0.1496
0.28	+50%	−0.0128	−0.0822	+0.2398
	+25%	−0.0077	−0.0481	+0.1162
	4450	*0.0398*	*0.8760*	*0.1031*
	−25%	+0.0128	+0.0725	−0.1032
	−50%	+0.0383	+0.1880	−0.1801

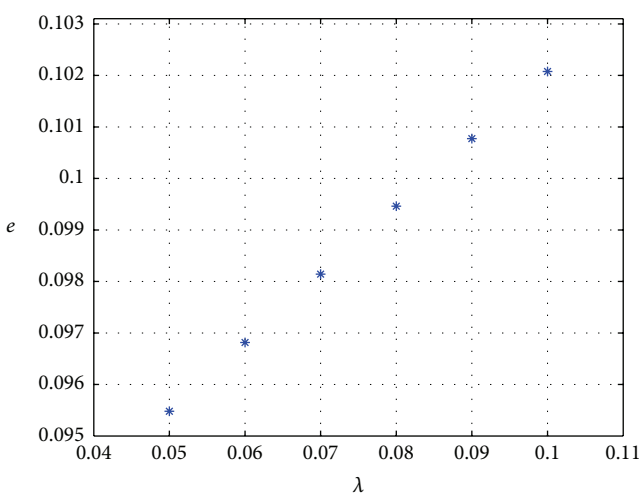

Figure 6: Diagram for the optimal effort versus shadow cost with uncertainty.

Table 2). When M goes up 50% or 25%, the optimal effort changes more than when M goes down 50% or 25%. The optimal price of T&D market varies differently by M, according to the technical parameter. When $\beta = 0.22$, the price increases by M's increase; when $\beta = 0.24$, the price decreases whatever M goes up or down; when $\beta = 0.26$ or $\beta = 0.28$, the price decreases by M's increase. Interestingly, the social welfare also varies differently by M. When $\beta = 0.22$, the social welfare always increases whatever M increases or decreases; when

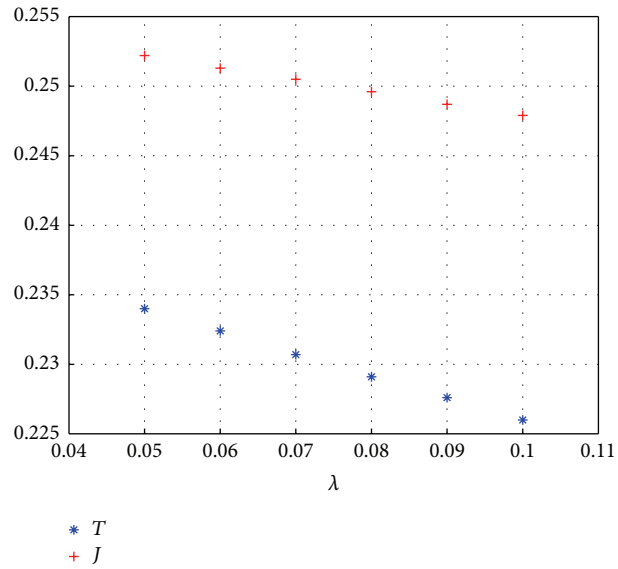

Figure 7: Diagram for transferring payment and social welfare versus shadow cost with uncertainty.

$\beta = 0.24$, $\beta = 0.26$, and $\beta = 0.28$, the social welfare has the same trend with M.

Nevertheless, when the technical parameter $\beta = 0.25$, both the transferring payment and the social welfare decrease by the shadow cost, as shown in Figure 7. And the transferring payment goes a little sharper than the social welfare. That is to say, the loss of the T&D corporation due to the shadow cost increasing (the transferring payment goes down) will not be compensated from the social welfare partly. The drop of the social welfare is shared by the public and the T&D corporation.

More interestingly, the transferring payment and the social welfare decrease also by the average income, as shown in Figure 8. The social welfare goes much sharper than the transferring payment. Even the social welfare becomes smaller than the transferring payment, when the average income is bigger than 2.7. The public may pay more for the electricity usage, when they are rich enough (average income big enough). So the loss of the T&D corporation due to the average income increasing will be compensated from the public mostly.

6. Conclusions

Power transmission and distribution is naturally monopolistic due to the nondivision and scale economy of the power network. Government regulates the power corporation to protect social welfare from monopolistic price. Reasonable pricing mechanism is more important to the reformation in Chinese power market. In this paper, we use this efficient tool, government incentive regulation model, to research the problems resulting from the regulators' information disadvantages. And the conclusions we obtain are shown below:

(1) This model is proved efficient to reveal the corporate private information and encourage it to invest to

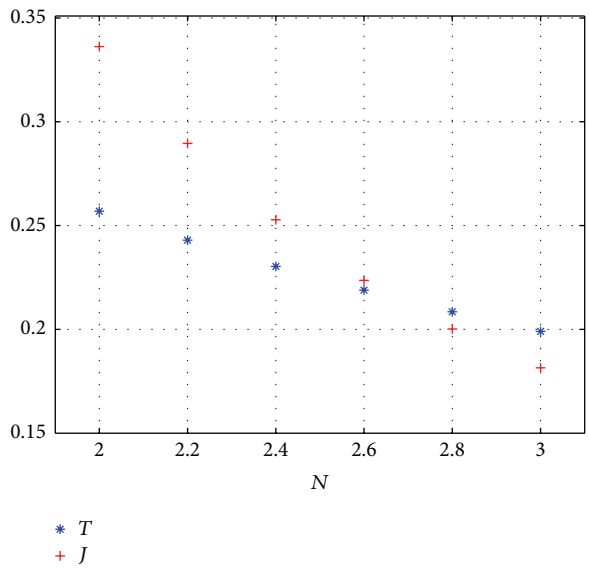

Figure 8: Diagram for transferring payment and social welfare versus average income with uncertainty.

improve the power transmission and distribution efficiency. Government achieves its purpose by a menu of contracts to regulate the corporation.

(2) Government makes the compensation policy which includes the compensation for output cost, the compensation for the effort, and the information rent. In addition, the information rent is relevant to corporate technical parameter.

(3) We find the optimal price of power transmission and distribution. Many factors are related to the price including demand, such as the average income of people, the quantity of people, the Gini coefficient, the shadow cost of public funds, and the coefficient of social welfare.

It is important to note that this paper is modeled based on the Chinese P&D market, so the conclusions are given with the basis of Chinese situation. Of course, the government incentive regulation model is also suitable for other P&D markets regulated by government, for example, California [1], UK [8], and Iberian [17]. And the conclusions also make sense in other P&D markets. Above all, in this paper, we only consider the government's regulation in T&D market, but there is also the market power effect during the T&D pricing. So next we will add the market effect in our model.

Parameters

q: Quantity of power transmission and distribution
β: Technical parameter (high β means inefficient technical level)
e: Effort made by the corporation
I: Investment of the corporation (the cost of effort)
U: Utility of the corporation
C: Output cost of the corporation
T: Transferring payment from government
p: Price of power transmission and distribution
A: Quantity of people in a certain area
N: Average income of people in a certain area
G: Gini coefficient
λ: Shadow cost of public funds
M: Coefficient of social welfare.

Competing Interests

The authors declare that they have no competing interests.

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Research Article

A Mathematical Model of Communication with Reputational Concerns

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We investigate a mathematical model where an expert advises a decision maker for two periods. The decision maker is initially unsure about whether the expert is biased or not. After consulting the expert on the decision problem of period one, the decision maker updates belief about the expert's bias and consults the expert on the problem of period two. We find that more information is delivered in the model's first period than in the one-period situation of communication.

1. Introduction

This paper studies a mathematical model involving a decision maker and an expert with two periods. In each period, the decision maker needs to make a decision but does not know which one is the best. There is an expert who knows the optimal decisions but may be biased in favor of decisions that are different from the ideal ones of the decision maker. Initially unsure about whether the expert is biased or not, the decision maker consults the expert for the ideal decision in period one. After receiving the expert's advice, the decision maker makes the first-period decision and updates belief about the expert's bias. Then the decision maker consults the expert for the optimal decision in period two. We study whether the expert delivers more information on the optimal decision in period one to the decision maker, compared to the case where the decision maker consults the expert only in period one.

Many situations in real life are captured by the model described above. For instance, a patient usually has less information about his illness than a doctor does. So the former consults the latter. However, the latter may prefer the former to buy expensive medicine or to take medical procedures, even when the patient's disease is mild and these medicine and procedures are unnecessary. The patient is often uncertain about whether the doctor has such preferences. But he can form some belief about the doctor's preferences based on the doctor's former prescriptions. The doctor expects that former prescriptions will affect the patient's belief of the doctor's preferences and influence future communications between the patient and the doctor. Will the doctor give more accurate prescriptions in period one, compared to the case where the doctor advises the patient only in period one?

We find that the expert communicates more information to the decision maker about the optimal decision in period one. This is because if the decision maker believes that the expert is more likely to be unbiased at the end of period one, the expert's payoff will be higher in period two. In period one, the expert engages less in misreporting information, in order to increase the probability that the decision maker believes that the expert is unbiased at the end of period one. As a result, more information is communicated between the expert and the decision maker in period one.

There are several papers that study similar issues as the current paper does. In these papers, the decision maker is uncertain about the expert's bias and the expert is concerned about establishing a reputation for being unbiased. There is another strand of literature studying the case where experts

observe signals about the state with different accuracies and each expert prefers to be perceived as having accurate information. In Ottaviani and Sørensen [1], it is shown that experts with reputational concerns for having accurate information typically do not wish to tell the truth. In Bourjade and Jullien [2], the expert cannot misreport the information but can conceal the information. In the current paper, the expert is not concerned with establishing a reputation for having accurate information since it is assumed that all experts have perfect information about the state. In Sobel [3], the expert communicates with the decision maker repeatedly. The expert may be biased in favoring a particular decision. In Benabou and Laroque [4], a model similar to the one in Sobel [3] is studied. Li [5] studies the information transmission between the expert and the decision maker through an intermediary. All papers mentioned above assume that the unbiased expert truthfully reports information. In this paper, the unbiased expert may lie in order to enhance his reputation.

The paper that is closest to the current paper is that of Morris [6], who also studies a two-period information transmission model between a decision maker and an expert. It is found that the unbiased expert may send a report different from the observed signal in order to enhance the expert's reputation for being unbiased. Our study is different from that of Morris [6] in the following ways. First, we assume that the expert can perfectly observe the state. Second, there is a continuum of possible states and decisions in the current paper, whereas, in Morris [6], only two states and decisions are possible.

The remaining part of the paper proceeds as follows. Section 2 builds a two-period model of strategic information transmission between a decision maker and an expert. In addition, it characterizes an equilibrium in period two of the model and an equilibrium in period one. Section 3 compares the ex ante expected payoff of the decision maker in period one of the model to that of the decision maker when the decision maker consults the expert in only one period. Section 4 concludes and discusses a direction for future work.

2. Model

There are two periods. A decision maker (hereafter DM) needs to take a decision in each period. The decision is to choose a real number in the interval $[0,1]$. In period $t \in \{1,2\}$, after DM takes decision $a_t \in [0,1]$, DM receives some payoff, which depends on the underlying state in period t denoted as s_t. Assume that the payoff of DM in period t is

$$-(a_t - s_t)^2. \tag{1}$$

Apparently, the payoff is higher when the decision a_t is closer to s_t. Therefore, the decision that gives DM the highest payoff in period t (hereafter the optimal decision of DM in period t) is equal to s_t. However, DM does not know s_t except that it is uniformly distributed over $[0,1]$ and s_1 is distributed independently of s_2.

There is an expert who observes s_t at the beginning of period t (throughout the paper, we use "he" to denote the expert and "she" to denote DM). However, the ideal decision

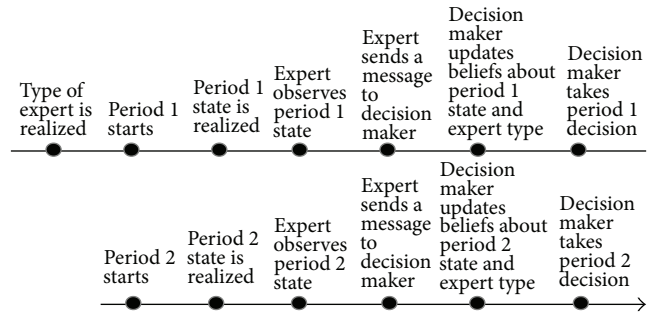

Figure 1: Timeline of the two-period communication model.

of the expert may be different from that of DM. We assume that, in period t, the payoff received by the expert when DM takes decision a_t is

$$-(a_t - s_t - \beta)^2. \tag{2}$$

The expert can be of two types. For one type of the expert, $\beta = 0$ and the expert's ideal decision in period t is the same as that of DM. We call this type of the expert "unbiased." For the other type, $\beta = b > 0$ and the expert's ideal decision is higher than that of DM by b. We call this type of the expert "biased." The expert knows his own type, but DM does not. Initially, DM believes that the expert is of either type with equal probabilities. In addition, the expert's type is distributed independently of the state in each period.

In each period, DM communicates with the expert about the state of that period before making a decision. The communication process is as follows. After observing s_t, the expert sends a report $m_t \in [0,1]$ to DM. We assume that the expert can send any report in $[0,1]$, which can be different from the state. Also, the expert does not incur any cost in sending a report. After receiving the report from the expert, DM updates her belief about the distribution of the state in that period and her belief about the expert's type. Then DM takes a decision a_t. The timeline of the game between the expert and DM is shown in Figure 1.

We assume that DM and the expert do not discount their future payoffs. The total payoff of DM is the sum of her payoffs in two periods, which is

$$u_{\mathrm{DM}}(a_1, s_1, a_2, s_2) = -(a_1 - s_1)^2 - (a_2 - s_2)^2, \tag{3}$$

and the total payoff of the expert is

$$u_E(a_1, s_1, a_2, s_2) = -(a_1 - s_1 - \beta)^2 - (a_2 - s_2 - \beta)^2. \tag{4}$$

A strategy of the unbiased expert in period t is a mapping from the state space $[0,1]$ to the space of possible reports $[0,1]$, denoted as K_t. A strategy of the biased expert is another mapping from the state space to the space of possible reports, denoted as K_t'. A strategy of DM is a mapping from the space of possible reports to the space of decisions, denoted as J_t. A belief of DM about the state in period t after receiving a report m_t from the expert is a probability distribution of s_t conditional on m_t, denoted as $F(s_t \mid m_t)$. A belief of DM about the type of the expert after receiving a report m_t is

a probability that the expert is unbiased given m_t, denoted as $\lambda(m_t)$.

The equilibrium concept that we use in this paper is *perfect Bayesian Nash equilibrium* (hereafter the equilibrium). An equilibrium consists of a strategy for each type of the expert, a strategy of DM, a belief of DM about the state, and a belief of DM about the type of the expert. To be an equilibrium, these strategies and beliefs must satisfy the following conditions: the strategy of each type of the expert maximizes the expert's expected payoff, given the strategy of DM; DM's strategy maximizes her expected payoff, given DM's beliefs about the state and the type of the expert; DM's beliefs about the state and the type of the expert are derived from the strategies of two types of the expert using Bayes' rule.

We first characterize an equilibrium in period two for an arbitrary belief of DM at the beginning of this period regarding the type of the expert. The proofs of formal results are relegated to the appendix.

Lemma 1. *If DM believes at the beginning of period two that the expert is unbiased with probability λ and the possible bias of the expert is not too large, in particular, $b \leq 1/4$, there exists an equilibrium in period two where each type of the expert sends two possible reports m_2^* and $m_2'^*$. The biased (unbiased, resp.) expert sends report m_2^* if and only if period-two state is less than $\bar{s}_{2b}^*$ ($\bar{s}_2^*$). Otherwise, they send report $m_2'^*$. DM takes decision a_2^* ($a_2'^*$) after receiving report m_2^* ($m_2'^*$). a_2^*, $a_2'^*$, $\bar{s}_{2b}^*$, and $\bar{s}_2^*$ are given by*

$$\begin{aligned} a_2^* &= \frac{1}{4} - (1-\lambda)\, b, \\ a_2'^* &= \frac{3}{4} - (1-\lambda)\, b, \\ \bar{s}_{2b}^* &= \frac{1}{2} - (2-\lambda)\, b, \\ \bar{s}_2^* &= \frac{1}{2} - (1-\lambda)\, b. \end{aligned} \tag{5}$$

We can calculate the expected payoff of the biased expert in period two before he observes the state in period two. Since period-2 state is distributed uniformly over $[0,1]$, the state will be less than $\bar{s}_{2b}^*$ with probability $\bar{s}_{2b}^*$. In this case, the biased expert will send report m_2^* and DM will take decision a_2^*. In addition, period-2 state will be higher than $\bar{s}_{2b}^*$ with probability $1-\bar{s}_{2b}^*$. In this case, the biased expert will send report $m_2'^*$ and DM will take decision $a_2'^*$. Therefore, the biased expert's expected payoff in period two before he observes the state in period two is

$$\begin{aligned} &\bar{s}_{2b}^* E_{s_2}\left[-\left(a_2^* - s_2 - b\right)^2 \mid s_2 \leq \bar{s}_{2b}^*\right] \\ &\quad + \left(1-\bar{s}_{2b}^*\right) E_{s_2}\left[-\left(a_2'^* - s_2 - b\right)^2 \mid s_2 \geq \bar{s}_{2b}^*\right]. \end{aligned} \tag{6}$$

By substituting a_2^*, $a_2'^*$, and $\bar{s}_{2b}^*$ with their respective values, the biased expert's expected payoff in period two is

$$-\frac{1}{2}(2-\lambda)^2 b^2 - \frac{1}{48}. \tag{7}$$

Similarly, we can show that the unbiased expert's expected payoff in period two before the expert observes period-two state is

$$-\frac{1}{2}(1-\lambda)^2 b^2 - \frac{1}{48}. \tag{8}$$

It is straightforward to see that the expected payoffs of biased and unbiased experts are both strictly increasing in λ for $0 \leq \lambda \leq 1$. This implies that both types of the expert prefer DM to believe that the expert is unbiased with a higher probability at the beginning of period two. Therefore in period one, both types of the expert have incentives to manipulate the belief of DM about the type of the expert, in order to receive a higher expected payoff in period two.

Next, we consider the game between DM and the expert in period one. We have the following conclusion.

Proposition 2. *When the possible bias of the expert is not too large, in particular, $b \leq 1/4$, there exists an equilibrium in period one. In equilibrium, there are two possible reports m_1^* and $m_1'^*$ sent by each type of the expert. The biased (unbiased) expert sends report m_1^* if and only if period-one state is less than $\bar{s}_{1b}^*$ ($\bar{s}_1^*$). Otherwise, they send report $m_1'^*$. When receiving report m_1^* ($m_1'^*$), DM's posterior belief about the type of the expert is λ_1^* ($\lambda_1'^*$) and DM takes decision a_1^* ($a_1'^*$). m_1^*, $m_1'^*$, $\bar{s}_{1b}^*$, $\bar{s}_1^*$, λ_1^*, and $\lambda_1'^*$ are given by the solution to the following system of equations:*

$$\begin{aligned} &-\left(a_1 - \bar{s}_{1b} - b\right)^2 - \frac{1}{2}\left(2-\lambda_1\right)^2 b^2 \\ &\quad = -\left(a_1' - \bar{s}_{1b} - b\right)^2 - \frac{1}{2}\left(2-\lambda_1'\right)^2 b^2, \\ &-\left(a_1 - \bar{s}_1\right)^2 - \frac{1}{2}\left(1-\lambda_1\right)^2 b^2 \\ &\quad = -\left(a_1' - \bar{s}_1\right)^2 - \frac{1}{2}\left(1-\lambda_1'\right)^2 b^2, \\ &\lambda_1 = \frac{\bar{s}_1}{\bar{s}_1 + \bar{s}_{1b}}, \\ &a_1 = \frac{1}{2}\frac{\bar{s}_1}{2} + \frac{1}{2}\frac{\bar{s}_{1b}}{2}, \\ &\lambda_1' = \frac{1-\bar{s}_1}{\left(1-\bar{s}_1\right) + \left(1-\bar{s}_{1b}\right)}, \\ &a_1' = \frac{1}{2}\frac{1+\bar{s}_1}{2} + \frac{1}{2}\frac{1+\bar{s}_{1b}}{2}. \end{aligned} \tag{9}$$

In Figure 2, we depict variables a_1^*, $a_1'^*$, $\bar{s}_{1b}^*$, $\bar{s}_1^*$, λ_1^*, and $\lambda_1'^*$ for different values of b.

3. Comparison with No Reputational Concerns

In this section, we consider the following question: whether the ex ante expected payoff of DM when the expert has reputational concerns is higher than that of DM when the expert does not have such concerns. In particular, we

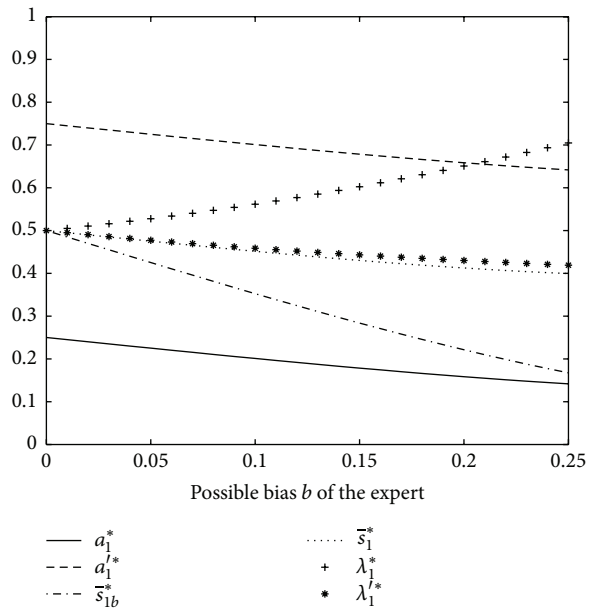

Figure 2: Equilibrium variables in period one for different possible biases of the expert.

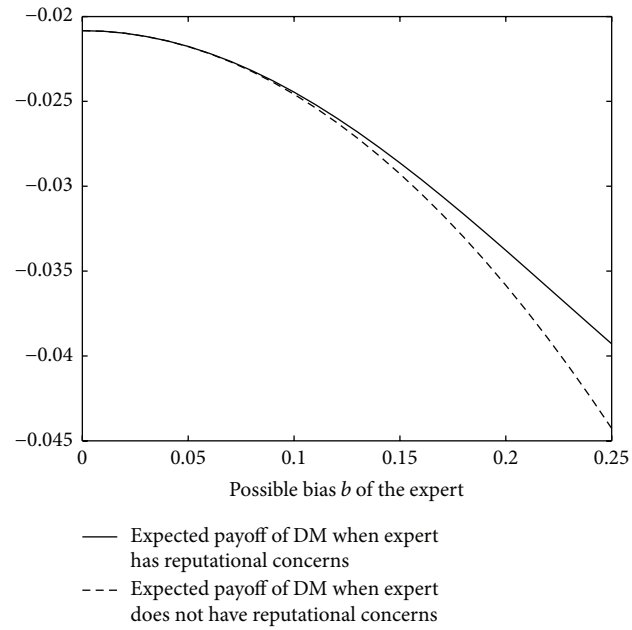

Figure 3: Ex ante expected payoff of DM when the expert is concerned with reputation versus that of DM when the expert does not have such concerns.

compare the ex ante expected payoff of DM in period one of our model with that of DM in a model where the expert communicates with DM in only one period and therefore has no reputational concerns.

Consider a model where there is only one period and DM consults the expert about the state only once. Other aspects of the model are the same as the model in Section 2. Since there is only one period, DM's belief about the type of the expert at the end of the period does not affect the expected payoff of the expert. In this sense, the expert has no reputational concerns.

Note that the analysis of the model is identical to the analysis of period-two game between the expert and DM in Section 2, except that DM's belief about the expert's type before receiving any report from the expert is 1/2, instead of λ as in Section 2. By following identical arguments as in Section 2, we can characterize an equilibrium:

$$\begin{aligned} a_1^n &= \frac{1}{4} - \frac{1}{2b}, \\ a_1'^n &= \frac{3}{4} - \frac{1}{2b}, \\ \bar{s}_{1b}^n &= \frac{1}{2} - \frac{3}{2b}, \\ \bar{s}_1^n &= \frac{1}{2} - \frac{1}{2b}. \end{aligned} \tag{10}$$

The ex ante expected payoff of DM before receiving the expert's report can be calculated as follows:

$$\begin{aligned} &\frac{1}{2}\left\{\bar{s}_{1b}^n E_{s_1}\left[-\left(a_1^n - s_1\right)^2 \mid s_1 \le \bar{s}_{1b}^n\right]\right. \\ &\quad \left. + \left(1 - \bar{s}_{1b}^n\right) E_{s_1}\left[-\left(a_1'^n - s_1\right)^2 \mid s_1 > \bar{s}_{1b}^n\right]\right\} \\ &+ \frac{1}{2}\left\{\bar{s}_1^n E_{s_1}\left[-\left(a_1^n - s_1\right)^2 \mid s_1 \le \bar{s}_1^n\right]\right. \\ &\quad \left. + \left(1 - \bar{s}_1^n\right) E_{s_1}\left[-\left(a_1'^n - s_1\right)^2 \mid s_1 > \bar{s}_1^n\right]\right\}, \end{aligned} \tag{11}$$

which can be shown to be

$$-\frac{3}{8}b^2 - \frac{1}{48}. \tag{12}$$

By replacing variables a_1^n, $a_1'^n$, $\bar{s}_{1b}^n$, and $\bar{s}_1^n$ by a_1^*, $a_1'^*$, $\bar{s}_{1b}^*$, and $\bar{s}_1^*$, respectively, in (11), we can calculate the ex ante expected payoff of DM in period one of the model in Section 2 where the expert has reputational concerns.

In Figure 3, we draw the ex ante expected payoff of DM when the expert has reputational concerns and that of DM when the expert does not have such concerns. Both ex ante payoffs are drawn for different possible biases of the expert. From the figure, we can see that DM's ex ante expected payoff when the expert has reputational concerns is no less than that of DM when the expert does not have such concerns. When the possible bias of the expert is large, the former payoff is strictly greater than the latter one. This result indicates the beneficial effect of the expert's reputational concerns on the expected payoff of DM, especially when the expert can possibly have a large bias.

4. Conclusion

The current paper studies a mathematical model in which an uninformed decision maker consults an informed expert about the optimal decision. The expert may be biased in

favoring a decision that is different from the ideal decision of the decision maker. It is shown that there is an equilibrium where the expert is concerned with his reputation for being unbiased, which is the probability that the expert is unbiased as perceived by the decision maker. Different from previous models where the expert is concerned with his reputation for being unbiased, the decision and state spaces are both continuous in the current paper. We show that, from an ex ante point of view, the decision maker's expected payoff is higher when the expert cares about his reputation than that of DM when the expert does not have reputational concerns.

The paper focuses on an equilibrium of the model where there are only two possible reports sent by the expert and two possible decisions made by the decision maker. The main reason for this is tractability. When the possible bias of the expert is small, we conjecture that there exists an equilibrium with more than two possible reports and more than two possible decisions. Characterizing such an equilibrium is more involved and is left for future research.

Appendix

A. Proof of Lemma 1

Suppose that, at the beginning of period two, DM believes that the expert is unbiased with probability λ. Consider the problem that the biased expert faces when observing that the period-two state is s_2. Suppose that the expert faces a choice between sending two possible reports denoted as m_2 and m_2'. Suppose further that DM takes decision a_2 (a_2') when receiving report m_2 (m_2') from the expert and $a_2 < a_2'$. Therefore, the biased expert's second-period payoff is $-(a_2 - s_2 - b)^2$ (resp., $-(a_2' - s_2 - b)^2$) by sending report m_2 (resp., m_2'). It is straightforward to verify that the former payoff is strictly greater than the latter one if and only if s_2 is less than cutoff value $\bar{s}_{2b}$, where

$$\bar{s}_{2b} = \frac{\left(a_2 + a_2'\right)}{2} - b. \tag{A.1}$$

Similarly, the unbiased expert's period-two payoff by sending report m_2 is greater than that by sending report m_2', if and only if the second-period state s_2 is less than cutoff value $\bar{s}_2$, where

$$\bar{s}_2 = \frac{\left(a_2 + a_2'\right)}{2}. \tag{A.2}$$

Given the strategies of biased and unbiased experts as described above, DM updates her belief about period-two state after receiving a report from the expert. After DM receives report m_2, DM will infer that, with probability λ, the expert is unbiased; therefore period-two state is less than $\bar{s}_2$, and with probability $1 - \lambda$, the expert is biased; therefore period-two state is less than $\bar{s}_{2b}$. Given DM's belief about the state after receiving report m_2, DM's optimal decision is equal to the expected value of period-two state. In other words, $a_2 = E_{s_2}[s_2 \mid m_2]$. We have that DM's optimal decision when receiving report m_2 is

$$a_2 = \lambda \frac{\bar{s}_2}{2} + (1 - \lambda) \frac{\bar{s}_{2b}}{2}. \tag{A.3}$$

In a similar fashion, we can find that after receiving report m_2' DM's best decision is

$$a_2' = \lambda \frac{1 + \bar{s}_2}{2} + (1 - \lambda) \frac{1 + \bar{s}_{2b}}{2}. \tag{A.4}$$

From solving the above four equations, we can get the values of a_2^*, $a_2'^*$, $\bar{s}_2^*$, and $\bar{s}_{2b}^*$ as stated in the lemma. In order for the above solution to represent an equilibrium in period two, cutoff states $\bar{s}_{2b}^*$ and $\bar{s}_2^*$ must both lie between 0 and 1. Therefore, the possible bias of the expert must not be too large; in particular, $b \leq 1/(4 - 2\lambda)$. We consider a situation where the possible bias of the expert is small enough such that an equilibrium as represented by the above solution exists even when DM believes that the expert is biased for sure ($\lambda = 0$). In order words, $b \leq 1/4$.

B. Proof of Proposition 2

Consider two possible decisions of DM in period one, a_1 and a_1', where $a_1 < a_1'$. Suppose that DM takes decision a_1 (a_1') when receiving report m_1 (m_1'). In addition, suppose that DM believes that the expert is unbiased with probability λ_1 (λ_1') when receiving report m_1 (m_1').

Consider the problem of the biased expert when he observes that period-one state is s_1. If the expert sends report m_1, the expert's expected payoff, including the expected payoff in period two, is

$$-\left(a_1 - s_1 - b\right)^2 - \frac{1}{2}\left(2 - \lambda_1\right)^2 b^2 - \frac{1}{48}. \tag{B.1}$$

If the biased expert sends report m_1', the expert's expected payoff is

$$-\left(a_1' - s_1 - b\right)^2 - \frac{1}{2}\left(2 - \lambda_1'\right)^2 b^2 - \frac{1}{48}. \tag{B.2}$$

It is straightforward to show that there exists a period-one state $\bar{s}_{1b}$ satisfying that the expected payoff of the biased expert by sending report m_1 is equal to the expert's expected payoff by sending report m_2 when period-one state is $\bar{s}_{1b}$. We have the following equation:

$$\begin{aligned} &-\left(a_1 - \bar{s}_{1b} - b\right)^2 - \frac{1}{2}\left(2 - \lambda_1\right)^2 b^2 \\ &\quad = -\left(a_1' - \bar{s}_{1b} - b\right)^2 - \frac{1}{2}\left(2 - \lambda_1'\right)^2 b^2. \end{aligned} \tag{B.3}$$

In addition, the former payoff is higher than the latter payoff if and only if period-one state is less than $\bar{s}_{1b}$. Similarly, there exists a period-one state $\bar{s}_1$ satisfying that the expected payoff of the unbiased expert by sending report m_1 is equal

to the expert's expected payoff by sending report m_1' when period-one state is $\bar{s}_1$. We have that

$$\begin{aligned} &-(a_1-\bar{s}_1)^2-\frac{1}{2}(1-\lambda_1)^2 b^2 \\ &\quad=-\left(a_1'-\bar{s}_1\right)^2-\frac{1}{2}\left(1-\lambda_1'\right)^2 b^2. \end{aligned} \tag{B.4}$$

In addition, the former payoff is higher than the latter payoff if and only if period-one state is less than $\bar{s}_1$.

When receiving report m_1, by Bayes' rule, DM believes that the expert is unbiased with probability

$$\lambda_1=\frac{\bar{s}_1}{\bar{s}_1+\bar{s}_{1b}}, \tag{B.5}$$

and DM's optimal decision is

$$a_1=E_{s_1}\left[s_1 \mid m_1\right]=\frac{1}{2}\frac{\bar{s}_1}{2}+\frac{1}{2}\frac{\bar{s}_{1b}}{2}. \tag{B.6}$$

In a similar fashion, when receiving report m_1', DM believes that the expert is unbiased with probability

$$\lambda_1'=\frac{1-\bar{s}_1}{\left(1-\bar{s}_1\right)+\left(1-\bar{s}_{1b}\right)}, \tag{B.7}$$

and DM's optimal decision when receiving report m_1' is

$$a_1'=E_{s_1}\left[s_1 \mid m_1'\right]=\frac{1}{2}\frac{1+\bar{s}_1}{2}+\frac{1}{2}\frac{1+\bar{s}_{1b}}{2}. \tag{B.8}$$

Competing Interests

The authors declare that they have no competing interests.

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