Hindawi Publishing Corporation
Science and Technology of Nuclear Installations
Volume 2013, Article ID 261327, 6 pages
http://dx.doi.org/10.1155/2013/261327

Research Article

Solution of Point Reactor Neutron Kinetics Equations with Temperature Feedback by Singularly Perturbed Method

Wenzhen Chen, Jianli Hao, Ling Chen, and Haofeng Li

Department of Nuclear Energy Science and Engineering, Naval University of Engineering, Faculty 301, Wuhan 430033, China

Correspondence should be addressed to Wenzhen Chen; cwz2@21cn.com

Received 30 May 2013; Revised 27 August 2013; Accepted 29 August 2013

Academic Editor: Arkady Serikov

The singularly perturbed method (SPM) is proposed to obtain the analytical solution for the delayed supercritical process of nuclear reactor with temperature feedback and small step reactivity inserted. The relation between the reactivity and time is derived. Also, the neutron density (or power) and the average density of delayed neutron precursors as the function of reactivity are presented. The variations of neutron density (or power) and temperature with time are calculated and plotted and compared with those by accurate solution and other analytical methods. It is shown that the results by the SPM are valid and accurate in the large range and the SPM is simpler than those in the previous literature.

1. Introduction

The analysis of variation of neutron density (or power) and reactivity with time under the different conditions is an important content of nuclear reactor physics or neutron kinetics [1–7]. Some important achievements on the supercritical transient with temperature feedback with big ($\rho_0 > \beta$) or small ($\rho_0 < \beta$) reactivity inserted have been approached through the effort of many scholars [7–12]. The studies on the delayed supercritical transient with small reactivity inserted and temperature feedback are introduced in the related literature [13–15], in which the explicit function of density (or power) and reactivity with respect to time is derived mainly with decoupling method, power prompt jump approximation, precursor prompt jump approximation, temperature prompt jump approximation [10, 16], and so forth. From the detailed analysis and comparison of the results in the early and recent literature [7, 12, 14], it is found that some results have certain limit and rather big error under the particular conditions. In present work, the variation law of power, reactivity, and precursor density with respect to time at any level of initial power is obtained by the singularly perturbed method (SPM). All the results are compared with those obtained by the numerical solution which tend to the accurate solution under very small time step size [17]. It is proved that the SPM is correct and reliable and is simpler than the analytical methods by the related literature.

2. Theoretical Derivation

The point reactor neutron kinetics equations with one group of delayed neutrons are [3, 4]

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{l} n(t) + \lambda C(t), \tag{1}$$

$$\frac{dC(t)}{dt} = \frac{\beta}{l} n(t) - \lambda C(t), \tag{2}$$

where n is the average neutron density, t is the time, ρ is the reactivity, β is the total fraction of the delayed neutron, l is the prompt neutron lifetime, λ is the radioactive decay constant of delayed neutron precursor, and C is the average density of delayed neutron precursor. When multiplied with a certain coefficient, n represents the power. It is assumed that the reactor has a negative temperature coefficient of reactivity α ($\alpha > 0$) when a small step reactivity $\rho_0(<\beta)$ is inserted. Consider the temperature feedback, and the real reactor reactivity is

$$\rho = \rho_0 - \alpha T, \tag{3}$$

where T is the temperature increment of the reactor, namely, $T = T_s - T_0$, where T_s and T_0 are the instantaneous temperature and initial temperature, respectively. After the reactivity ρ_0 is inserted into the reactor the adiabatic model is still employed [3, 15]; then we have

$$\frac{dT}{dt} = K_c n(t), \tag{4}$$

where K_c is the reciprocal of thermal capacity of reactor.

Combining (3) and (4) results in

$$\frac{d\rho}{dt} = -\alpha K_c n(t). \tag{5}$$

Substituting (2) into the derivative of (1) with respect to t yields

$$l\frac{d^2n}{dt^2} = (\rho - \beta)\frac{dn}{dt} + n\frac{d\rho}{dt} + \lambda\beta n - l\lambda^2 C. \tag{6}$$

Substituting λC obtained from (1) into (6) and simplifying it yields

$$\frac{d^2n}{dt^2} = \left(\frac{\rho - \beta}{l} - \lambda\right)\frac{dn}{dt} + \frac{n}{l}\frac{d\rho}{dt} + \frac{\lambda\rho n}{l}. \tag{7}$$

The transient process is supposed to begin at $t = 0$ and $\rho(0) \neq 0$ [15, 17], so the initial conditions can be given as $\rho(0) = \rho_0$, $n(0) = n_0$, $(dn/dt)|_{t=0} = \rho_0 n_0/l$, $(d^2n/dt^2)|_{t=0} = ((\rho_0 - \beta)/l)(\rho_0 n_0/l) - (\alpha K_c n_0^2/l)$, where n_0 is the initial neutron density (or power).

For $|(\rho - \beta)/l| \gg \lambda$, $\beta/l \gg \lambda$ [3], (1) and (2) are stiff equations. According to the singularly perturbed method [18], solution of $n(t)$ includes the inner solution $n_f(t)$ in inner part and the outer solution $n_l(t)$ in outer part. In this paper both inner and outer solutions are approximated to be zero order:

$$n(t) = n_f(t) + n_l(t). \tag{8}$$

The initial conditions are

$$n_0 = n_{f0} + n_{l0}, \tag{9}$$

$$n'(0) = n_f'(0) + n_l'(0) = \frac{\rho_0 n_0}{l}, \tag{10}$$

$$n''(0) = n_f''(0) + n_l''(0) = \frac{\rho_0 - \beta}{l}\frac{\rho_0 n_0}{l} - \frac{\alpha K_c n_0^2}{l}, \tag{11}$$

where n_{f0} is the initial value of inner solution and n_{l0} is the initial value of outer solution.

Substituting (8) into (7) and (5), respectively, yields

$$\begin{aligned}\frac{d^2(n_f + n_l)}{dt^2} &= \left(\frac{\rho - \beta}{l} - \lambda\right)\frac{d(n_f + n_l)}{dt} + \frac{(n_f + n_l)}{l}\frac{d\rho}{dt} \\ &\quad + \frac{\lambda\rho(n_f + n_l)}{l}, \\ \frac{d\rho}{dt} &= -\alpha K_c\left[n_f(t) + n_l(t)\right].\end{aligned} \tag{12}$$

In the outer part the inner solution attenuates to zero, and dn_f/dt and d^2n_f/dt^2 can be neglected, namely, $dn_f/dt \approx 0$ and $d^2n_f/dt^2 \approx 0$; therefore, in the outer part (12) can be simplified as follows:

$$\frac{d^2n_l}{dt^2} = \frac{\rho - \beta}{l}\frac{dn_l}{dt} + \frac{n_l}{l}\frac{d\rho}{dt} - \lambda\frac{dn_l}{dt} + \frac{\lambda\rho n_l}{l}, \tag{13}$$

$$\frac{d\rho}{dt} = -\alpha K_c n_l. \tag{14}$$

Because $n_l(t)$ varies slowly and $l \approx 10^{-4}$ s, compared to other terms, the term on the left side of (13) can be neglected, and then we have

$$(\beta - \rho + \lambda l)\frac{dn_l}{dt} = \left(\frac{d\rho}{dt} + \lambda\rho\right)n_l. \tag{15}$$

Combining (14) and (15) results in

$$\frac{dn_l}{d\rho} = \frac{\alpha K_c n_l - \lambda\rho}{\alpha K_c(\beta - \rho + \lambda l)}. \tag{16}$$

Integrating (16) subjective to the initial conditions $\rho(0) = \rho_0$ and $n_l(0) = n_{l0}$ yields

$$n_l = \frac{\lambda(\rho_0^2 - \rho^2)}{2\alpha K_c(\beta - \rho + \lambda l)} + \frac{(\beta - \rho_0 + \lambda l)}{(\beta - \rho + \lambda l)}n_{l0}. \tag{17}$$

Substituting (17) into (14) leads to

$$-\frac{d\rho}{dt} = \frac{\lambda(\rho_0^2 - \rho^2) + 2\alpha K_c(\beta - \rho_0 + \lambda l)n_{l0}}{2(\beta - \rho + \lambda l)}. \tag{18}$$

With the initial conditions $\rho(0) = \rho_0$, the solution of (18) is

$$t = \frac{\beta + \lambda l}{\lambda\rho_1}\ln\left(\frac{\rho_1 - \rho}{\rho_1 + \rho}\right)\left(\frac{\rho_1 + \rho_0}{\rho_1 - \rho_0}\right) + \frac{1}{\lambda}\ln\left[\frac{(\rho_1^2 - \rho_0^2)}{(\rho_1^2 - \rho^2)}\right], \tag{19}$$

where $\rho_1 = \sqrt{\rho_0^2 + 2\alpha K_c(\beta - \rho_0 + \lambda l)n_{l0}/\lambda}$.

In the inner part $n_l(t)$ is assumed to be constant $n_l(t) = n_l(0) = n_{l0}$, and $dn_l(t)/dt \approx (dn_l(t)/dt)|_{t=0}$, $d^2n_l(t)/dt^2 \approx (d^2n_l(t)/dt^2)|_{t=0}$, so (12) can be simplified as follows, respectively:

$$\begin{aligned}\frac{d^2n_f}{dt^2} + n_l''(0) &= \left(\frac{\rho - \beta}{l} - \lambda\right)\left(\frac{dn_f}{dt} + n_l'(0)\right) \\ &\quad + \frac{(n_f + n_{l0})}{l}\left(\frac{d\rho}{dt} + \lambda\rho\right),\end{aligned} \tag{20}$$

$$\frac{d\rho}{dt} = -\alpha K_c\left[n_f(t) + n_{l0}\right]. \tag{21}$$

In addition, the temperature feedback is not fast enough to affect the reactivity, and in the inner part it is assumed that $\rho(t) = \rho_0$, $d\rho(t)/dt \approx -\alpha K_c n_0$. From (20) we can get

$$l\frac{d^2 n_f}{dt^2} = (\rho_0 - \beta - \lambda l)\frac{dn_f}{dt} + (\lambda\rho_0 - \alpha K_c n_0)(n_f + n_{l0}) + (\rho_0 - \beta - l\lambda) n_l'(0) - n_l''(0) l. \tag{22}$$

The solution of (22) is

$$n_f = -n_{l0} - \frac{(\rho_0 - \beta - l\lambda) n_l'(0) - n_l''(0) l}{(\lambda\rho_0 - \alpha K_c n_0)} + C_1 \exp\left(\frac{1}{2l} \times \left((\rho_0 - \beta - l\lambda) + \sqrt{(\rho_0 - \beta - l\lambda)^2 + 4l(\lambda\rho_0 - \alpha K_c n_0)}\right) t\right) + C_2 \exp\left(\frac{1}{2l} \times \left((\rho_0 - \beta - l\lambda) - \sqrt{(\rho_0 - \beta - l\lambda)^2 + 4l(\lambda\rho_0 - \alpha K_c n_0)}\right) t\right). \tag{23}$$

The fast varying part $n_f(t)$ is assumed to attenuate to zero in the inner part, so $C_1(t) = 0$ and

$$-n_{l0} - \frac{(\rho_0 - \beta - l\lambda) n_l'(0) - n_l''(0) l}{(\lambda\rho_0 - \alpha K_c n_0)} = 0. \tag{24}$$

Then we can get the fast varying part $n_f(t)$ in the inner part as follows:

$$n_f = n_{f0} \exp\left(\frac{1}{2l} \times \left((\rho_0 - \beta - l\lambda) - \sqrt{(\rho_0 - \beta - l\lambda)^2 + 4l(\lambda\rho_0 - \alpha K_c n_0)}\right) t\right). \tag{25}$$

From (25) we have

$$n_{f0}' = \frac{n_{f0}}{2l}\left((\rho_0 - \beta - l\lambda) - \sqrt{(\rho_0 - \beta - l\lambda)^2 + 4l(\lambda\rho_0 - \alpha K_c n_0)}\right). \tag{26}$$

Combining (9), (11), (13), (24), and (26) results in

$$n_{f0} = \frac{2l(\alpha K_c n_0 - \lambda\rho_0) n_0/(\rho_0 - \beta - l\lambda)^2 - 2\rho_0 n_0/(\rho_0 - \beta - l\lambda)}{2l(\alpha K_c n_0 - \lambda\rho_0)/(\rho_0 - \beta - l\lambda)^2 - \left(1 - \sqrt{1 + 4l(\lambda\rho_0 - \alpha K_c n_0)/(\rho_0 - \beta - l\lambda)^2}\right)},$$
$$n_{l0} = \frac{2\rho_0 n_0/(\rho_0 - \beta - l\lambda) - n_0\left(1 - \sqrt{1 + 4l(\lambda\rho_0 - \alpha K_c n_0)/(\rho_0 - \beta - l\lambda)^2}\right)}{2l(\alpha K_c n_0 - \lambda\rho_0)/(\rho_0 - \beta - l\lambda)^2 - \left(1 - \sqrt{1 + 4l(\lambda\rho_0 - \alpha K_c n_0)/(\rho_0 - \beta - l\lambda)^2}\right)}. \tag{27}$$

Substituting (27) into (25) and (17) can get $n_f(t)$ and $n_l(t)$; then the neutron density (or power) will be obtained by (8).

Combining (1) and (2) results in

$$\frac{d(n + C)}{dt} = \frac{\rho}{l} n. \tag{28}$$

Eliminating the time variable in (5) and (28) leads to

$$\frac{d(n + C)}{d\rho} = -\frac{\rho}{\alpha K_c l}. \tag{29}$$

Integrating (29) with the initial conditions $n(0) = n_0$, $C(0) = \beta n_0/\lambda l$ and $\rho(0) = \rho_0$ yields

$$C(t) = \frac{\rho_0^2 - \rho^2}{2\alpha K_c l} + \left(1 + \frac{\beta}{\lambda l}\right) n_0 - n(t). \tag{30}$$

Equations (8), (17), (25), (27), and (30) are the new analytical expressions derived by this paper.

3. Calculation and Analysis

The PWR with fuel ^{235}U is taken as an example with parameters $\beta = 0.0065$, $l = 0.0001$ s, $\lambda = 0.0774$ 1/s, $K_c = 0.05$ K/MW·s, and $\alpha = 5 \times 10^{-5}$ 1/K [8, 13]. For the reactor with the initial power 1 MW, while reactivity $\rho_0 = 0.5\beta$ and $\rho_0 = 0.8333\beta$ is inserted, respectively, the variations of reactivity, temperature, power with time, and power with reactivity are presented in Figures 1, 2, 3, and 4. The curves with smaller change are for $\rho_0 = 0.5\beta$ and the curves with larger change are for $\rho_0 = 0.8333\beta$. The solid line notes the accurate solution by the best basic function method with

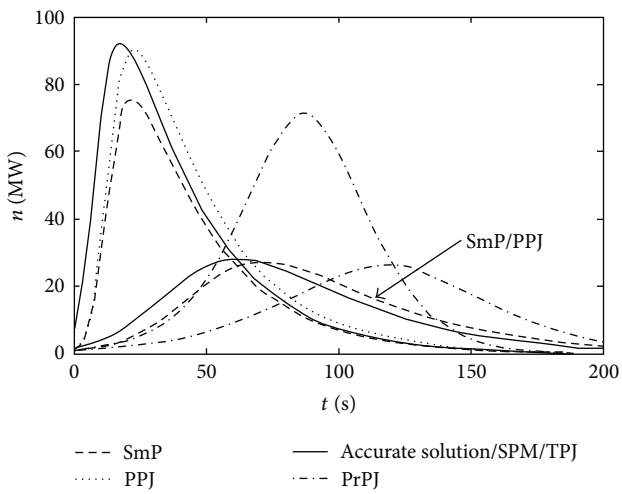

Figure 1: Variation of output power with time while inserting step reactivity $\rho_0 = 0.5\beta$ and $\rho_0 = 0.8333\beta$.

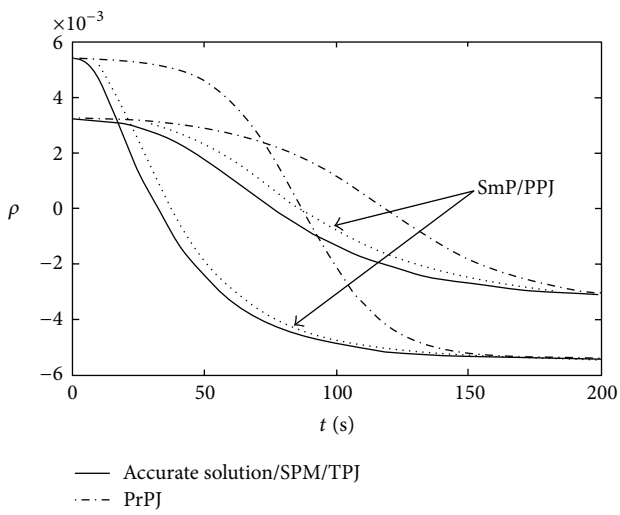

Figure 2: Variation of total reactivity with time while inserting step reactivity $\rho_0 = 0.5\beta$ and $\rho_0 = 0.8333\beta$.

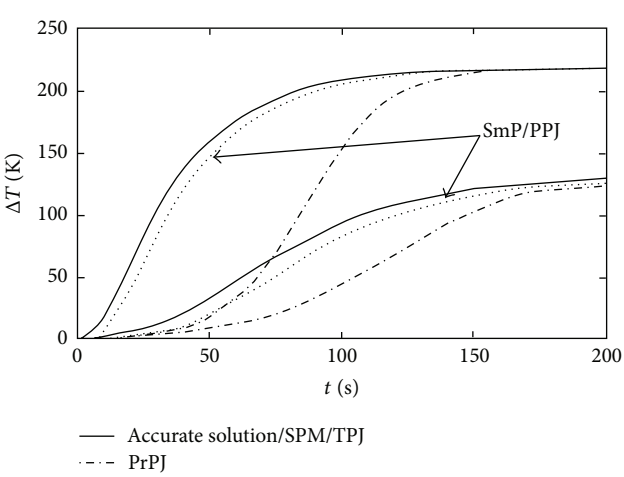

Figure 3: Variation of temperature rise of reactor with time while inserting step reactivity $\rho_0 = 0.5\beta$ and $\rho_0 = 0.8333\beta$.

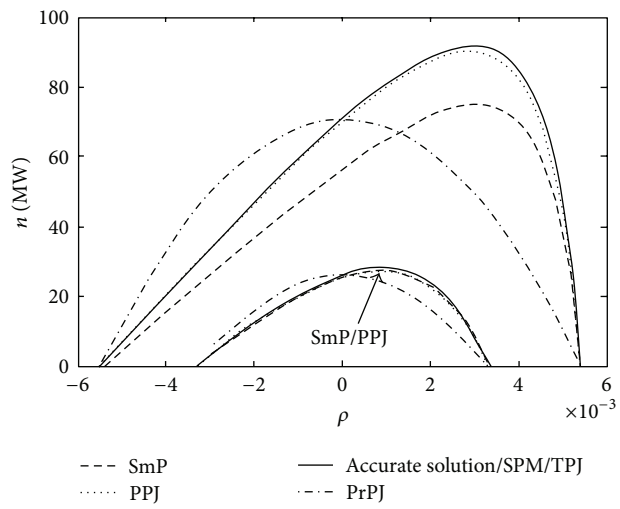

Figure 4: Variation of output power with reactivity while inserting step reactivity $\rho_0 = 0.5\beta$ and $\rho_0 = 0.8333\beta$.

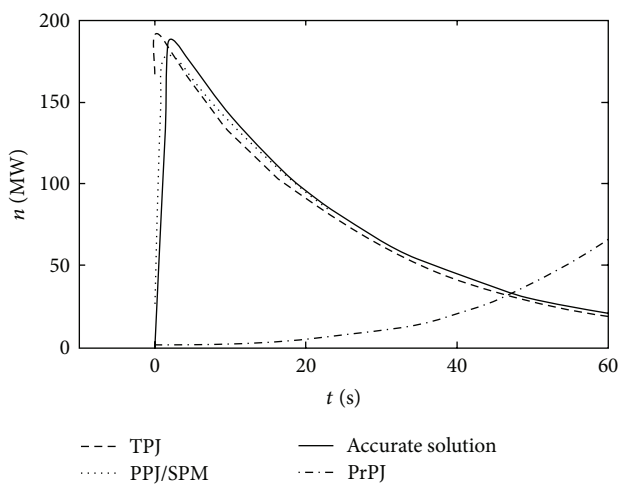

Figure 5: Variation of output power with time while inserting step reactivity 0.995β.

very small step size [17]. The results of this paper and the accurate solution as well as the temperature prompt jump (TPJ) method in the literature [10] are almost the same and hard to distinguish in Figures 1–4. The short dashed-dot line and long dashed-dot line represent the results of precursor prompt jump (PrPJ) method in the literature [9] and the small parameter (SmP) method in the literature [15], respectively. The dashed line notes the results of power prompt jump (PPJ) method in the literature [16]. The difference is caused by the approximate treatment to obey the methods of PrPJ, SmP, PPJ, and SmP. Furthermore the variation in the vicinity of prompt supercritical process is also calculated and is shown in Figures 5 and 6. The correct results cannot be obtained by the small parameter method (SmP) in the vicinity of prompt supercritical process and are not shown in Figures 5 and 6.

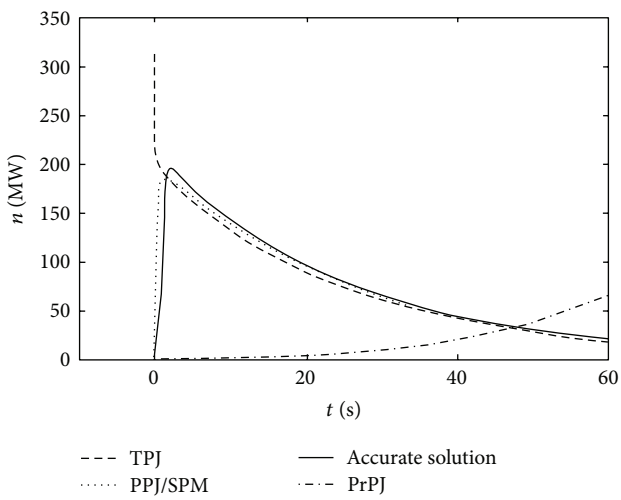

Figure 6: Variation of output power with time while inserting step reactivity 0.998β.

(1) From Figures 1–6 it can be concluded that very good results cannot be obtained by the precursor prompt jump (PrPJ) method to calculate the delayed supercritical progress with small step reactivity and temperature feedback.

(2) For small step reactivity, the results by the small parameter (SmP) method are close to those by the power prompt jump (PPJ) method and are better than those by the precursor prompt jump (PrPJ) method, but the accuracy of results by the small parameter method decreases with the increase of the reactivity inserted. The power is negative when the small parameter method is used to calculate the transient process in the vicinity of prompt supercritical state. From Figures 1–4, it can be seen that the small parameter method is more suitable for the calculation of reactivity and temperature increase than for that of power.

(3) The results are quite precise using the power prompt jump (PPJ) method for the delayed supercritical process, but the main problem compared to the accurate solution is that some displacement exists along time axis. Furthermore it should be pointed out that each power peak value obtained by the precursor prompt jump (PrPJ) method, power prompt jump (PPJ) method, or small parameter (SmP) method is lower than that obtained by the accurate solution or singularly perturbed method (SPM) see Figure 1.

(4) From Figures 1–4 it can be also found that the temperature prompt jump method (TPJ) and the singularly perturbed method (SPM) in this paper are the two most precise methods for the delayed supercritical process with small step reactivity and temperature feedback. However from Figures 5 and 6 it can be seen that as the reactivity inserted increases to the vicinity of prompt supercritical process, the total discrepancy of power by the TPJ method is larger than that by the SPM or PPJ method, and the irrelevant phenomena that the power jumps at first and then decreases monotonously from the peak will appear in the TPJ method as shown in Figure 6.

4. Conclusions

The analytical expressions of power (or neutron density), reactivity, the precursor power (or density), and temperature increase with respect to time are derived for the delayed supercritical process with small reactivity ($\rho_0 < \beta$) and temperature feedback by the singularly perturbed method. Compared with the results by the accurate solution and other methods in the literature, it is shown that the singularly perturbed method (SPM) in this paper is valid and accurate in the large range and is simpler than those in the previous literature. The method in this paper can provide a new theoretical foundation for the analysis of reactor neutron dynamics.

Acknowledgments

This research is supported by the National Natural Science Foundation of China (Project no. 11301540) and the Natural Science Foundation of Naval University of Engineering.

References

[1] H. P. Gupta and M. S. Trasi, "Asymptotically stable solutions of point-reactor kinetics equations in the presence of Newtonian temperature feedback," *Annals of Nuclear Energy*, vol. 13, no. 4, pp. 203–207, 1986.

[2] H. Van Dam, "Dynamics of passive reactor shutdown," *Progress in Nuclear Energy*, vol. 30, no. 3, pp. 255–264, 1996.

[3] D. L. Hetrick, *Dynamics of Nuclear Reactors*, American Nuclear Society, La Grange Park, Ill, USA, 1993.

[4] W. M. Stacey, *Nuclear Reactors Physics*, Wiley-Interscience, New York, NY, USA, 2001.

[5] A. A. Nahla and E. M. E. Zayed, "Solution of the nonlinear point nuclear reactor kinetics equations," *Progress in Nuclear Energy*, vol. 52, no. 8, pp. 743–746, 2010.

[6] G. Espinosa-Paredes, M.-A. Polo-Labarrios, E.-G. Espinosa-Martínez, and E. D. Valle-Gallegos, "Fractional neutron point kinetics equations for nuclear reactor dynamics," *Annals of Nuclear Energy*, vol. 38, no. 2-3, pp. 307–330, 2011.

[7] A. E. Aboanber and A. A. Nahla, "Solution of the point kinetics equations in the presence of Newtonian temperature feedback by Padé approximations via the analytical inversion method," *Journal of Physics A*, vol. 35, no. 45, pp. 9609–9627, 2002.

[8] A. E. Aboanber and Y. M. Hamada, "Power series solution (PWS) of nuclear reactor dynamics with newtonian temperature feedback," *Annals of Nuclear Energy*, vol. 30, no. 10, pp. 1111–1122, 2003.

[9] W. Z. Chen, B. Zhu, and H. F. Li, "The analytical solution of point-reactor neutron-kinetics equation with small step reactivity," *Acta Physica Sinica*, vol. 50, no. 8, pp. 2486–2489, 2004 (Chinese).

[10] H. Li, W. Chen, F. Zhang, and L. Luo, "Approximate solutions of point kinetics equations with one delayed neutron group and temperature feedback during delayed supercritical process," *Annals of Nuclear Energy*, vol. 34, no. 6, pp. 521–526, 2007.

[11] T. Sathiyasheela, "Power series solution method for solving point kinetics equations with lumped model temperature and feedback," *Annals of Nuclear Energy*, vol. 36, no. 2, pp. 246–250, 2009.

[12] Y. M. Hamada, "Confirmation of accuracy of generalized power series method for the solution of point kinetics equations with feedback," *Annals of Nuclear Energy*, vol. 55, pp. 184–193, 2013.

[13] Q. Zhu, X.-L. Shang, and W.-Z. Chen, "Homotopy analysis solution of point reactor kinetics equations with six-group delayed neutrons," *Acta Physica Sinica*, vol. 61, no. 7, Article ID 070201, 2012.

[14] S. D. Hamieh and M. Saidinezhad, "Analytical solution of the point reactor kinetics equations with temperature feedback," *Annals of Nuclear Energy*, vol. 42, pp. 148–152, 2012.

[15] A. A. Nahla, "An analytical solution for the point reactor kinetics equations with one group of delayed neutrons and the adiabatic feedback model," *Progress in Nuclear Energy*, vol. 51, no. 1, pp. 124–128, 2009.

[16] W. Chen, L. Guo, B. Zhu, and H. Li, "Accuracy of analytical methods for obtaining supercritical transients with temperature feedback," *Progress in Nuclear Energy*, vol. 49, no. 4, pp. 290–302, 2007.

[17] H. Li, W. Chen, L. Luo, and Q. Zhu, "A new integral method for solving the point reactor neutron kinetics equations," *Annals of Nuclear Energy*, vol. 36, no. 4, pp. 427–432, 2009.

[18] Z. Q. Huang, *Kinetics Base of Nuclear Reactor*, Peking University Press, Beijing, China, 2007 (Chinese).