Calculation of Lead-Iron Double-Layer Thickness for Gamma-Ray Shielding by MATLAB Program

1. Introduction

Although nuclear technology is very useful for researches and industries, maintenance of nuclear reactor is necessary for the safety reason. International Commission on Radiological Protection (ICRP) has suggested that the limitation of radiation exposure dose for occupational radiation worker is 20 mSv/year [1]. During the maintenance period, Thailand Institute of Nuclear Technology (TINT) has informed that the gamma radiation intensity of a bolt taken from the nuclear reactor was about 100 mSv/hr, whereas photon energy was 1.3325 MeV that was similar to Co-60 source. Consequently, this is considered as a radioactive waste. To evaluate the radiation safety due to the nuclear interactions, one of the best ways is the use of Monte Carlo simulation [2–4]. According to Monte Carlo approach, statistical processing time is prolonged at 13 hrs partly due to the complex geometry design [5, 6]. To avoid wasting of time, MATLAB software is certainly introduced to calculate the thickness of materials. This thickness will be used as an input variable in Monte Carlo simulation to reduce wasting of time. There are four advantages for MATLAB. First, this software is able to plot the contour graph to analyze the optimal thickness of materials. This contour-plotting technique is easy and more suitable than 3D surface plotting technique for finding the optimum solution [7]. Second, there are a variety of commands used in for-loop and if-clause processes [8] which are important components for the optimization model. Third, output data stored in matrix format after mathematical processing is very simple for the next mathematical process. Last, the command patterns in calculation for processing data are easily written.

Here we have performed the calculations for shielding materials of lead and iron layers for a 1.3325 MeV and 100 mSv/hr gamma-ray by MATLAB software. This design is comprised of three parts. The first and second parts are the logarithmic interpolation for mass attenuation coefficient and the bilogarithmic interpolation for exposure buildup factor, respectively. These parts are compared with the standard reference database from National Institute of Standards and Technology (NIST) [9] and American National Standard (ANS-6.4.3) for checking a validation [10]. The third part is to apply this optimization model to determine the suitable
2. Simulation Procedures

2.1. Mathematical Format for Optimization. Objective function was defined as the gamma attenuation of double-layer materials in a narrow (Figure 1(a)) and board beam (Figure 1(b)) [11].

To achieve 1000 times the radiation attenuation of container as informed by TINT and to follow the radiation dose limitation as informed by ICRP, our objective function was limited at 0.1 mSv/hr or

\[ f(x_1, x_2) = I = I_0 B_1 B_2 e^{-\mu_1 x_1 - \mu_2 x_2} \leq 0.1 \]

\[ \begin{cases} B_1 B_2 = 1, & \text{for a narrow beam,} \\ B_1 B_2 > 1, & \text{for a board beam,} \end{cases} \]

where \( I_0 \) and \( I \) were the initial and transmitted radiation intensity (mSv/hr), respectively. \( \mu_1 \) and \( \mu_2 \) were the linear attenuation coefficient (cm\(^{-1}\)) of lead and iron, respectively. \( B_1 \) and \( B_2 \) were the exposure buildup factor of lead and iron, respectively. Mass attenuation coefficient \( (\mu/\rho) \) was calculated by (2) according to logarithmic interpolation (LI) as shown in Figure 2(a) [12, 13]. Exposure buildup factor \( (B) \) was computed by (3)–(5) according to bilogarithmic interpolation (BI) modified from bilinear interpolation as seen in Figure 2(b) [14, 15].

\[
\frac{\mu}{\rho} = \left(\frac{\mu/\rho}_1 \right) (\log E_2 - \log E) + \left(\frac{\mu/\rho}_2 \right) (\log E - \log E_1)
\]

\[
B = \left(\frac{A_1}{\log E_2 - \log E} + \frac{A_2}{\log E - \log E_1}\right)
\]

\[
A_1 = \frac{(\log PD_2 - \log PD) + (\log PD - \log PD_1)}{\log PD_2 - \log PD_1}
\]

\[
A_2 = \frac{(\log PD_2 - \log PD) + (\log PD - \log PD_1)}{\log PD_2 - \log PD_1}
\]

\[
PD_1 = \mu_1 \cdot x_1,
\]

\[
PD_2 = \mu_2 \cdot x_2.
\]
Constrained functions were divided into three parts: material thickness, weight, and cost. 3D and 2D configuration of containers were illustrated in Figures 3 and 4, respectively.

For economical reason, constrained function of the double-layer thickness of cylindrical container was limited at 30 cm or

\[ g_1(x_1, x_2) : c_{air} + x_1 + x_2 \leq 30. \]  \hspace{1cm} (7)

For technical reason, constrained function of the container weight was limited at 1,000 kg or

\[ g_2(x_1, x_2) : \frac{2\pi}{1000} \left\{ \rho_1 c_{air}^3 + \rho_1 \left[ (x_1 + c_{air})^3 - c_{air}^3 \right] \right\} + \rho_3 \left[ (x_1 + x_2 + c_{air})^3 - (x_1 + c_{air})^3 \right] \leq 1000, \]  \hspace{1cm} (8)

where \( \rho_1, \rho_2, \rho_3 \) were the density of air, lead, and iron (g/cm\(^3\)), respectively. \( c_{air} \) was the radius of the gap inside a container.

For economical reason, constrained function of the container’s cost was limited at 800 USD or

\[ g_3(x_1, x_2) : \frac{4.4\pi}{1000} \left\{ A\rho_1 \left[ (x_1 + c_{air})^3 - c_{air}^3 \right] \right\} + B'\rho_3 \left[ (x_1 + x_2 + c)^3 - (x_1 + c_{air})^3 \right] \leq 800, \]  \hspace{1cm} (9)

where \( A, B' \) were the cost of lead at 0.97 USD/lb as of April 21, 2017 [16] and iron at 0.028 USD/lb as of January 31, 2017 [17], respectively.

3.1. Mass Attenuation Coefficient and Exposure Buildup Factor.

To investigate a validation of MATLAB, we firstly check \( \mu/\rho \) according to condition 1 (red frame from Figure 6). For instance, when \( E_{input} = 0.15 \) MeV, we get \( \mu/\rho = 0.143600 \) cm\(^2\)/g from MATLAB that is the same number as informed by NIST [9]. On the other hand, we check \( B \) according to condition 1 (red frame from Figure 7). For example, when we have \( PD_{input} = 5 \) mfp and \( E_{input} = 20 \) MeV, we get \( B = 5.22 \) from MATLAB that is the same number as ANS-6.4.3 [10]. This number also gives better results than invariant embedding method (IEM) [18].

In our study, if we have \( E_{input} = 1.3325 \) MeV according to C-60 and \( I_0 = 100 \) mSv/hr, the mass attenuation coefficient and exposure buildup factor are calculated in Table 1 and they are summarized in Figure 8.

3.2. Optimal Double-Layer Thickness. To understand the effects of container thickness on the container weight and cost, the space or void inside the container (\( c_{air} \)) calculated by (7) is varied from 0 to 30 cm. The results are shown in Figure 9. It is observed that container’s weight and cost calculated by (8)-(9) tend to increase if we increase the gap inside the container.

However, when we assume \( c_{air} = 7.0 \) cm that is possibly suitable for keeping the radioactive waste, the optimal thickness analysis on the contour graph is presented in Figure 10. This optimal solution is identified as an intersection between the objective function \( f(x) \) and the all constrained functions \( g(x) \) [7]. Coordinate points on this identified line (red arrow in Figure 10) are therefore taken to calculate the weight and cost of container as shown in Table 2.

Finally, the optimal double-layer thickness selected from the identified line is \( x_1 = 3.2 \) cm (lead) and \( x_2 = 17.0 \) cm (iron). Thus, container’s weight and cost are 994.30 kg and 167.30 USD, respectively.

To sum up, this optimization model concerns two merits. One is that the users are able to modify whatever the parameters (i.e., photon energy, initial intensity, material types and thicknesses, and material weight and cost) are to obtain the optimal thickness of materials of interests. Another one is that the simulation time is very fast just 10 sec.

4. Conclusions

From optimization model of the double-layer shielding design and selection of lead and iron cylindrical container by MATLAB software to store the radioactive waste at 1.3325 MeV and 100 mSv/hr, we have found that the mass attenuation coefficient and exposure buildup factor were 0.056601 cm\(^2\)/g for lead, 0.051862 cm\(^2\)/g for iron and 4.7316 for lead, 1.6681 for iron, respectively. These numbers were the same as the standard reference database. The double-layer thickness selected from the analysis on contour plotting was 3.2 cm for lead and 17.0 cm for iron to achieve 1000 times the radiation attenuation of container (0.1 mSv/hr). The total container weight and cost from these designed materials were 994.30 kg and 167.30 USD, respectively.
Figure 4: 2D configuration of container: (a) top view and (b) side view.

Figure 5: Overview flowchart of plotting a contour graph.

Table 1: Calculated mass attenuation coefficient ($\mu/\rho$) and exposure buildup factor ($B$) at $E_{\text{input}} = 1.3325$ MeV. PD is calculated from (6).

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\rho$ (gm/cm$^3$)</th>
<th>$\mu/\rho$ cm$^2$/g</th>
<th>$\mu$ cm$^{-1}$</th>
<th>$x$ cm</th>
<th>PD</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 = lead</td>
<td>11.35</td>
<td>0.0566</td>
<td>0.6424</td>
<td>21.0</td>
<td>13.4904</td>
<td>4.7316</td>
</tr>
<tr>
<td>2 = iron</td>
<td>7.874</td>
<td>0.0518</td>
<td>0.4083</td>
<td>2.0</td>
<td>0.8166</td>
<td>1.6681</td>
</tr>
</tbody>
</table>

Table 2: Container's weight and cost from the optimal solution by MATLAB.

<table>
<thead>
<tr>
<th>$c_{\text{air}}$ (cm)</th>
<th>$x_1$ (cm)</th>
<th>$x_2$ (cm)</th>
<th>Weight (kg)</th>
<th>Cost (USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7.0</td>
<td>11.0</td>
<td>808.40</td>
<td>404.60</td>
</tr>
<tr>
<td>7</td>
<td>6.4</td>
<td>12.0</td>
<td>838.80</td>
<td>356.50</td>
</tr>
<tr>
<td>7</td>
<td>5.7</td>
<td>13.0</td>
<td>860.00</td>
<td>305.00</td>
</tr>
<tr>
<td>7</td>
<td>5.1</td>
<td>14.0</td>
<td>893.80</td>
<td>266.10</td>
</tr>
<tr>
<td>7</td>
<td>4.5</td>
<td>15.0</td>
<td>929.40</td>
<td>231.30</td>
</tr>
<tr>
<td>7</td>
<td>3.7</td>
<td>16.0</td>
<td>943.90</td>
<td>188.50</td>
</tr>
<tr>
<td>7</td>
<td>3.2</td>
<td>17.0</td>
<td>994.30</td>
<td>167.30</td>
</tr>
</tbody>
</table>
Figure 6: Flowchart of mass attenuation coefficient ($\mu/\rho$) calculation.

Figure 7: Flowchart of exposure buildup factor ($B$) calculation.
Conflicts of Interest

The authors declare no conflicts of interest.

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References


