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Response of Launch Pad Structures to Random Acoustic Excitation

The design of launch pad structures, particularly those having a large area-to-mass ratio, is governed by launch-induced acoustics, a relatively short transient with random pressure amplitudes having a non-Gaussian distribution. The factors influencing the acoustic excitation and resulting structural responses are numerous and cannot be predicted precisely. Two solutions (probabilistic and deterministic) for the random vibration problem are presented in this article from the standpoint of their applicability to predict the response of ground structures exposed to rocket noise. Deficiencies of the probabilistic method, especially to predict response in the low-frequency range of launch transients (below 20 Hz), prompted the development of the deterministic analysis. The relationship between the two solutions is clarified for future implementation in a finite element method (FEM) code.

INTRODUCTION

One purpose of taking acoustic measurements during Space Shuttle launches is their application to structural response analysis and environmental testing. The choice of a response analysis method is subject to observations drawn from these measurements. Because the acoustic pressure field is extremely complex, it does not lend itself to a simple analytical description or field idealization nor does there exist a unique response analysis method. Based on experience and observations, however, two different approaches to the response analysis emerged. The first approach, called probabilistic, is based on the classical solution of the random vibration theory as presented by Lin (1967) and Elishakoff (1983). The analysis assumes a stationary input/response relation and requires a definition of the acoustic field in terms of power spectrum and cross-power spectrum (CPS). The second approach developed at the Kennedy Space Center, called deterministic, is a recent development prompted primarily by a deficiency of the former method to accurately predict response in the low-frequency range of the launch transient, where fundamental resonances of most pad structures lie and the highest strain responses occur. Pressure response spectrum, together with correlated pressure distributions (CPDs) and pressure correlation lengths (PCLs), provide a platform for the deterministic analysis. Because both methods purport to address a class of problems that is inherently stochastic, further references to probabilistic and deterministic methods use the terms power spectrum (PS) and pressure response spectrum (PRS) approaches, respectively. The PS approach is supported by an extensive bibliography, thus, derivations are omitted. The PRS approach is presented in greater

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detail because it is a more recent development. In particular, the definition of launch acoustic field in terms of CPDs is emphasized, together with the relation between CPDs and the newly developed function, the normalized CPS (NCPS). Because a complete set of NCPSs (matrix **Nc**) required for analysis is seldom if ever available, the approximate relation $\mathbf{Nc} = C^{T} \cdot C$ between NCPSs and CPDs allows for a partial estimate of **Nc** to be made from a few pairs of launch measurements. The usefulness of each approach, however, should be judged by first evaluating how well the launch environment fits the assumptions of each model and the ease of implementation in a finite element method (FEM) code. With this in mind, solutions by the PS and PRS approaches are presented in matrix form.

RESPONSE ANALYSIS APPROACHES: A BRIEF OVERVIEW

Power Spectrum Approach

The exact solution for a steady-state response of a linear structure in a random stationary acoustic field, which may or may not be homogeneous, written in terms of PS and CPS of modal coordinates is:

$$\underset{k\times k}{\boldsymbol{\Phi}_{\mathbf{qq}}} = \underset{k\times k}{\mathbf{H}_{\mathbf{m}}} \cdot \underset{k\times n}{\mathbf{b}^{T}} \cdot \underset{n\times n}{\mathbf{dA}} \cdot \underset{n\times n}{\boldsymbol{\Theta}_{\mathbf{pp}}} \cdot \underset{n\times n}{\mathbf{dA}} \cdot \underset{n\times k}{\mathbf{b}} \cdot \underset{k\times k}{\mathbf{H}_{\mathbf{m}}^{*}} \tag{1}$$

where $\boldsymbol{\Phi}_{\mathbf{qq}}$ is the solution matrix of k modal coordinates of k normal modes and $\mathbf{H}_{\mathbf{m}}$ is the matrix of modal frequency response functions (FRFs). $\boldsymbol{\Theta}_{\mathbf{pp}}$, **b**, and **dA** are the acoustic load matrix, the matrix of n modal displacements at n loaded points due to k normal modes, and the diagonal matrix of areas at n nodal points, respectively.

Major computational problems and the complexities associated with the definition of the acoustic load matrix, $\boldsymbol{\Theta}_{\mathbf{pp}}$, in the exact solution necessitated the search for a simplified response solution. This search was also influenced by two concepts from the theory of random processes: the white noise excitation having a PS of a constant intensity, K psi^2/H, which extends from zero to an infinite frequency; and the white noise decay, which defines CPS of a white noise excitation. The variation of a white noise CPS along a single axis, x, between points x_1 and x_2 is defined by:

$$\boldsymbol{\Theta}_{\mathbf{pp}}(x_1, x_2, \omega) = K \cdot \exp(-\alpha|x_2 - x_1|) \cdot \exp(-i\beta\omega(x_2 - x_1)) \tag{2}$$

where α and β are constants that, presumably, may be adjusted to fit experimental data. Terms containing α and β are magnitude and phase expressions of white noise decay. The concept of white noise decay and the possibility of an analytical definition appeared to have facilitated the definition of $\boldsymbol{\Theta}_{\mathbf{pp}}$ in Eq. (1). Recent research shows that the definition of $\boldsymbol{\Theta}_{\mathbf{pp}}$ can be simplified even when the acoustic field is not a white noise (of constant intensity K) but is characterized by a frequency-dependent PS given by $S_p(f)$. Then the matrix $\boldsymbol{\Theta}_{\mathbf{pp}}$ may be normalized by a scalar function $S_p(f)$, so that:

$$\underset{n\times n}{\boldsymbol{\Theta}_{\mathbf{pp}}} = S_p(f) \cdot \underset{n\times n}{Nc} \tag{3}$$

where **Nc** is the matrix of normalized CPS (NCPS). The concept of such normalization may be applied to any type of acoustic field. In a homogeneous acoustic field, main diagonal elements of **Nc** are all constant, real, and equal to 1.0. Phase relations in **Nc** remain the same as in $\boldsymbol{\Theta}_{\mathbf{pp}}$. If the field is assumed to have white noise decay, then off-diagonal elements are complex analytical functions and are given by the right-hand side of Eq. (2), except for the K term. Definition of both PSs and NCPSs is influenced by data processing intervals and locations of sensors relative to the plume (vertical/horizontal direction arrays; see Figs. 1–4).

It is important to note two limitations of a white noise decay field. First, the magnitude is not a function of frequency; and second, the phase is a linear function of frequency with a slope being proportional to the relative distance $x_2 - x_1$ in Eq. (2). Contrary to the assumptions of a white noise decay model (Fig. 5), Shuttle measurements indicate a strong dependence of the magnitude of **Nc** on the frequency (Fig. 6) and a somewhat nonlinear dependence of phase on relative distance. The power spectrum approach is based on the assumption of a steady-state input/response relation. In reality, the actual launch environment is a transient with a duration that for some frequencies (below 20 Hz) does not con-

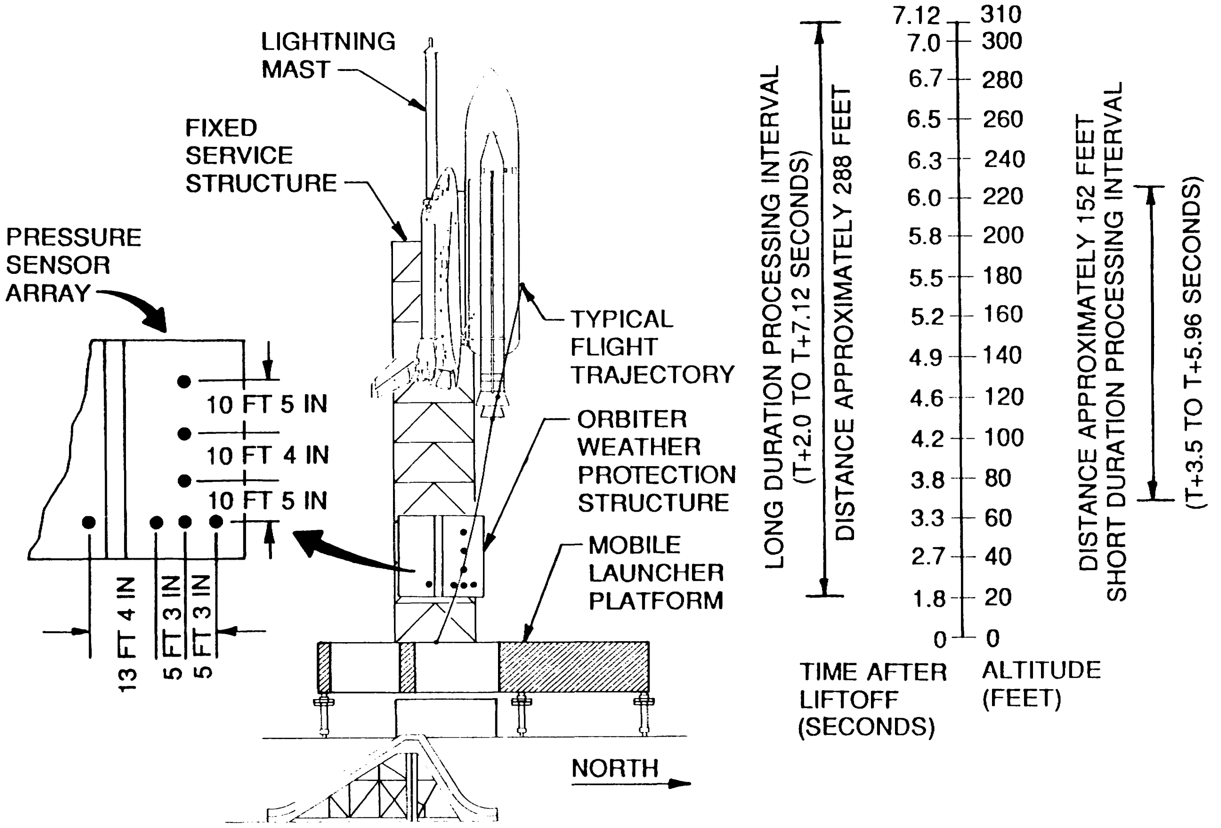

FIGURE 1 Shuttle launch showing the two-dimensional sensor array.

tain a sufficient number of cycles to induce a full resonance. This significant drawback leads to overestimation of structural responses and, in practice, predicted overloads/failures that never materialized.

Key observations that pointed toward possible limitations of the power spectrum approach and led to the development of the pressure response spectrum are: overpredictions resulting from the steady-state response solution applied

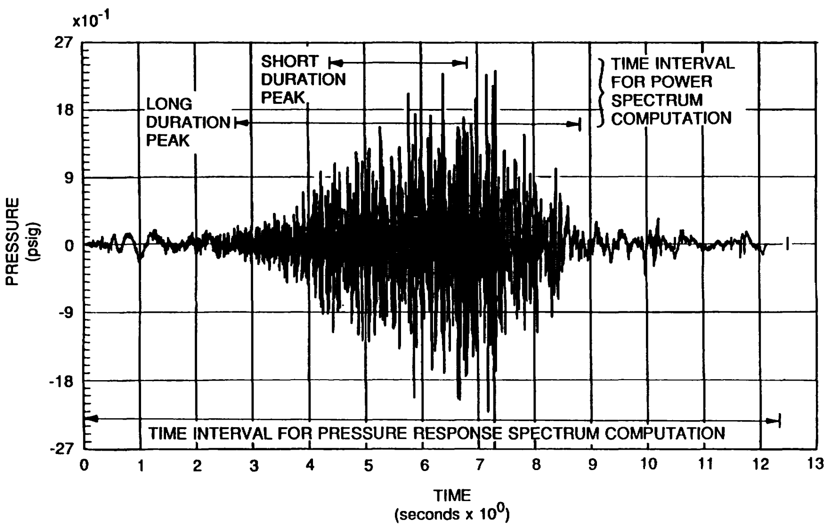

FIGURE 2 Shuttle data: short transient with random pressure amplitudes.

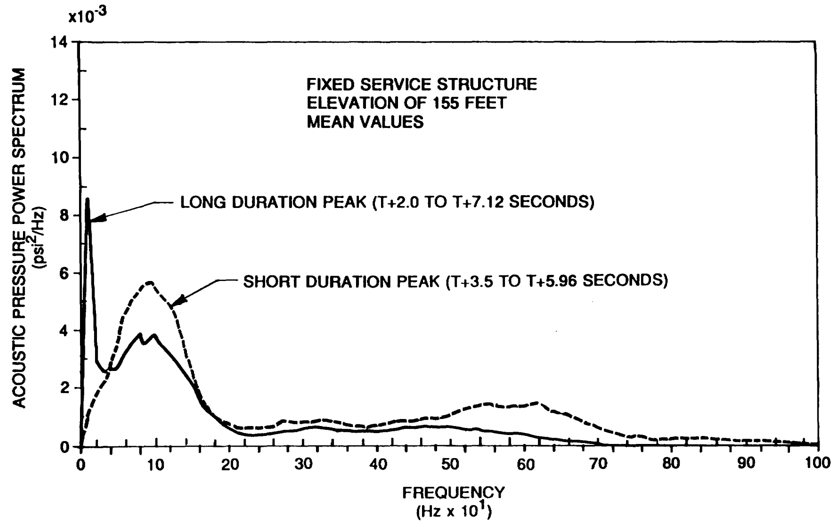

FIGURE 3 Variation of liftoff pressure power spectrum.

to a transient input; difficulty in the definition of a unique PS from the measured launch transient; and the invalidity of assumed pressure correlations such as white noise decay, etc.

Pressure Response Spectrum Approach

The basic premise behind the concept of the PRS is that a total structural response consists of uncoupled responses in individual structural vibration modes. Therefore, the response in a mode can be obtained by integrating the equation of motion for that mode in the time domain. If the time history of a generalized modal load is known, the integration is possible, and it does not matter whether or not the generalized modal load is a random transient. In this integration process, the input is the incident pressure time history; however, the definition of a corresponding generalized modal load contains elements of a random response analysis and is uniquely related to the PS of the generalized modal load [contained in Eq. (1)].

A given transient pressure time history, $p(t)$,

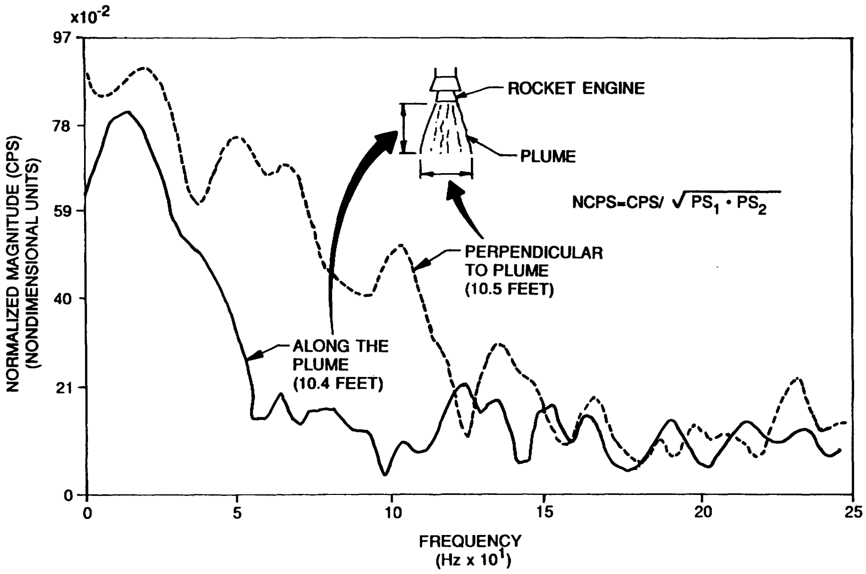

FIGURE 4 Variation of normalized cross-power spectrum along the rocket plume axis.

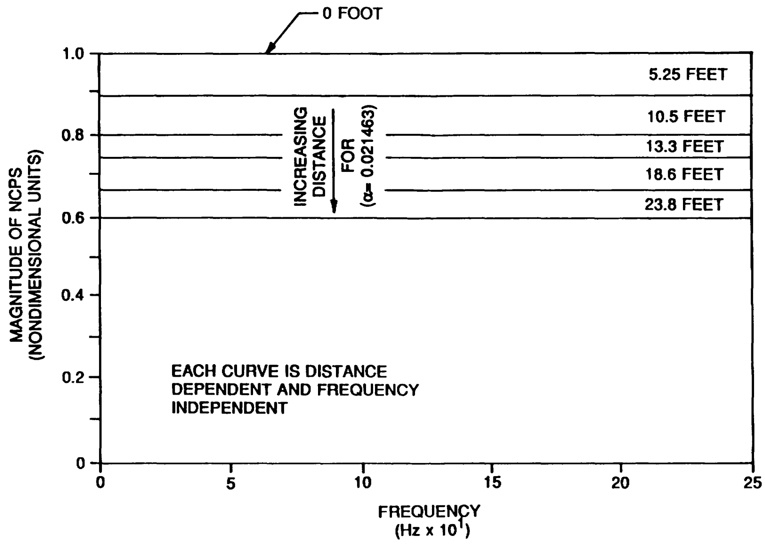

FIGURE 5 White noise decay model.

has a PS, $S_p(f)$, which is derivable by a normalized averaging procedure and does not require an assumption of a stationary random process. An examination of two NCPSs based on past Shuttle launches for "short" and "long" processing intervals indicates similarity, regardless of $p(t)$ being a transient. Therefore, functions derivable from NCPSs, such as PCLs and CPDs, may be assumed to be time invariant for the duration of the launch pressure transient. Consequently, for a structural vibration mode, the product of a normal modal displacement and a corresponding CPD is also time invariant. This product, when integrated over the area of the structure, defines a generalized modal load for a constant and unit $p(t)$. Thus, for a time variable $p(t)$, the generalized modal load for a jth mode is proportional to $p(t)$ and may be written as follows:

$$GL_j(t) = AJ_j \cdot p(t) \tag{4}$$

where AJ_j is the above-mentioned integrated product, independent of time. The PS of a generalized modal load is given by:

$$Sgl_j(f) = (AJ_j)^2 \cdot S_p(f) \tag{5}$$

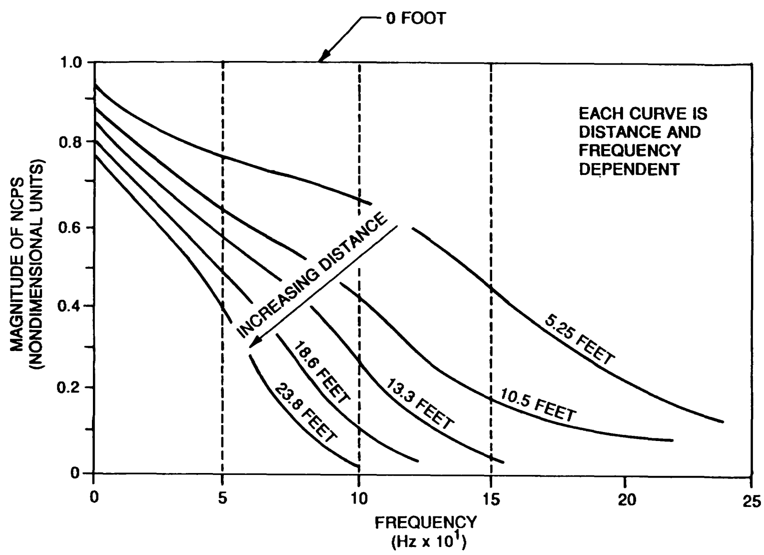

FIGURE 6 Launch-induced acoustic field model (horizontal direction).

The equation of motion for a jth mode (omitting the subscript for brevity) in terms of generalized modal coordinate, q, is:

$$\ddot{q} + 2\xi\Omega\dot{q} + \Omega^2 q = (AJ/M) \cdot p(t) \quad (6)$$

where M, Ω, and ξ are parameters of the jth mode, generalized modal mass, circular resonance frequency, and modal damping, respectively. The equation of motion, Eq. (6), is solved (integrated) for a variable $q/(AJ/M)$, with zero initial conditions and an array of frequencies, $f = \Omega/2\pi$, to include resonances of all modes of interest. Similar to well-known shock spectra, only the peak (maximax) values are retained in the solution. Because a solution corresponds to each of the assumed frequencies, $q = q(f)$, for presentation purposes the plotted variable on response spectra plots is:

$$Y(f) = q(f)/(AJ/M/\omega^2) \quad (7)$$

Then, in applications, the maximax value of $q(f)$ is obtained from the plotted quantity $Y(f)$, as follows:

$$q(f) = Y(f) \cdot (AJ/M/\omega^2) \quad (8)$$

using values of AJ, M, and $\omega = \Omega$ for a particular mode of interest. When response coordinates are computed for k modes, it is convenient to arrange them as main diagonal elements of a $k \times k$ maximax response matrix $\mathbf{Q}$. The major difficulty in obtaining a response solution by means of response spectra is in the computation of the AJ-factor associated with each generalized modal load. Research at the Kennedy Space Center addressed this problem and provided diagrams of J-coefficients for a few types of beam structures (corresponding A is a normalizing constant equal to a beam span). An illustration of response spectra to input pressures covering the entire time history ($T = 0.0$ to $T = 12.286$ s) and computed from a sample of 12 launches is shown in Fig. 7. The exact computation of AJ-factors and the interrelationship between the PS and PRS approaches of response analysis is clarified next, with the aid of a newly developed function (NCPSs).

RESPONSE ANALYSIS METHODS: A RELATIONSHIP

Definition of Acoustic Excitation

The definition of a generalized modal load history presented in Eqs. (4) and (5) in an intuitive and heuristic manner may appear to lack the same rigor as the theory of the PS approach. Actually, the relation between PS and PRS approaches is much closer than it appears to be, although the results of analysis by each method may differ. An interrelation between the two is obtained by the comparison of PSs of generalized modal loads defined by each type of solution. For the PS approach, from Eqs. (1) and (3), these PSs are defined by the main diagonal elements of the

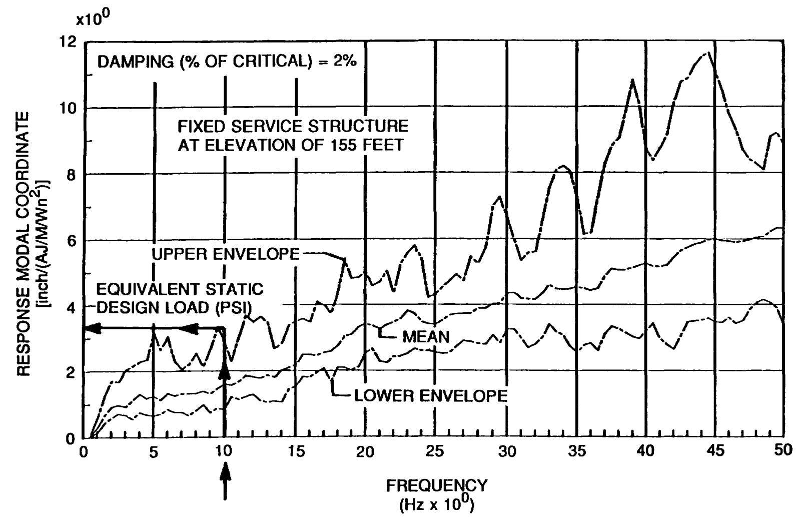

FIGURE 7 Liftoff pressure response spectrum to rocket noise.

matrix:

$$\underset{k\times n}{[\ \mathbf{b}^T} \cdot \underset{n\times n}{\mathbf{dA}} \cdot \underset{n\times n}{\mathbf{Nc}} \cdot \underset{n\times n}{\mathbf{dA}} \cdot \underset{n\times k}{\mathbf{b}\ ]} \cdot S_p(f) \tag{9}$$

where $S_p(f)$ is a scalar multiplier with a physical meaning of an acoustic pressure PS. The matrix in brackets is a complex Hermitian function of frequency, since its component matrix **Nc** is Hermitian and frequency dependent. A corresponding deterministic definition of generalized modal load PSs is given in Eq. (5). Note that each AJ_j term in Eq. (5) is a function of a discrete resonance frequency of the jth normal mode because a corresponding CPD has also similar characteristics.

By equating (5) and (9) and cancelling $S_p(f)$ appearing on each side of the equation:

$$[AJ_j(f = f_j)]^2 = \mathrm{diag}_{jj}[\mathbf{b}^T \cdot \mathbf{dA} \cdot \mathbf{Nc} \cdot \mathbf{dA} \cdot \mathbf{b}]_{@f=f_j} \tag{10}$$

where diag_{jj} designates a jth element on the main diagonal of the matrix in brackets, computed at the frequency f_j of the jth mode resonance. Thus, each approach (PS or PRS) defines a generalized modal load and its PS through identical computations.

Factors $AJ(f)$ in Eq. (10) define a vibroacoustic coupling between a structure through its modal matrix **b** and an acoustic field characterized by its matrix **Nc** of NCPSs. This definition of $AJ(f)$ is a smooth and continuous function of frequency. Beyond this common point at Eq. (10), the PS and PRS approaches of response analysis diverge and take different paths.

Concept of Pressure Correlations

Utilization of response spectra, $Y(f)$, in the computation of maximax values of modal coordinates, $q(f)$, defined by Eq. (8), requires a knowledge of $AJ(f)$-factors for each normal mode of vibrating structure. Equation (10) provides the necessary theoretical relation. Practical calculations of $AJ(f)$ must be made from the definition of a generalized modal load by means of a CPD, which leads to Eq. (4). The CPD is a function of PCL, and the PCL is a function of frequency. The concept of PCLs, CPDs, and their relation to NCPSs is presented next because it is a measure of correlation between pressures in an acoustic field.

A PCL defines an extent of effective (correlated) pressures that contribute to the total magnitude of a generalized modal load of a structural vibration mode excited at its resonance frequency. Two mutually perpendicular PCLs centered at the reference point and positioned parallel to the surface of the planar structure define a pressure correlation area (PCA). Within the PCA, there exists a CPD centered on the reference point. In turn, a CPD facilitates a computation of AJ-factors and the definition of generalized modal loads by means of AJ-factors. A PCL is simply computed from two measurements at a distance D_s, using either the NCPS or two functions, coherence (COH) and the phase from the CPS. The direction of a PCL is defined by the positions (locations) of source measurements.

$$\begin{aligned}\mathrm{PCL}(f) = 2 \cdot \pi \cdot D_s/\arccos\{2 \cdot (A_p^{0.5}) \\ \cdot [(\mathrm{COH} - NL)^{0.25}] \\ \cdot \cos(\mathrm{phase}/2) - 1\}\end{aligned} \tag{11}$$

where A_p defines the effect of a homogeneous or nonhomogeneous field on $\mathrm{PCL}(f)$ and where NL defines the COH noise level. PCLs are an explicit function of frequency and direction. PCLs also define an extent of CPD over the surface of the structure. A CPD defines the correlation between pressures at a point within the correlation area and the center of the CPD. For the computation of generalized modal loads, the values of PCLs and the corresponding CPD are important at the mode resonance frequency only. Along the direction of a $\mathrm{PCL}(f)$, the corresponding CPD is assumed to vary as a cosine function:

$$\mathrm{CPD}(f, x) = \{1 + \cos[2 \cdot \pi \cdot x/\mathrm{PCL}(f)]\}/2 \tag{12}$$

where x is the distance from the center of the distribution, with the constraint:

$$-\mathrm{PCL}/2 \le x \le \mathrm{PCL}/2$$

A similar definition of $\mathrm{CPD}(f, y)$ may also be obtained for the y-direction. Then, for a planar structure, a CPD over an area is defined as the product of CPDs in two perpendicular directions, as follows:

$$\mathrm{CPD}(f, x, y) = \mathrm{CPD}(f, x) \cdot \mathrm{CPD}(f, y) \tag{13}$$

An illustration of the CPD center placement and its effect on computed AJ-coefficient for the first mode of a three-span continuous beam is

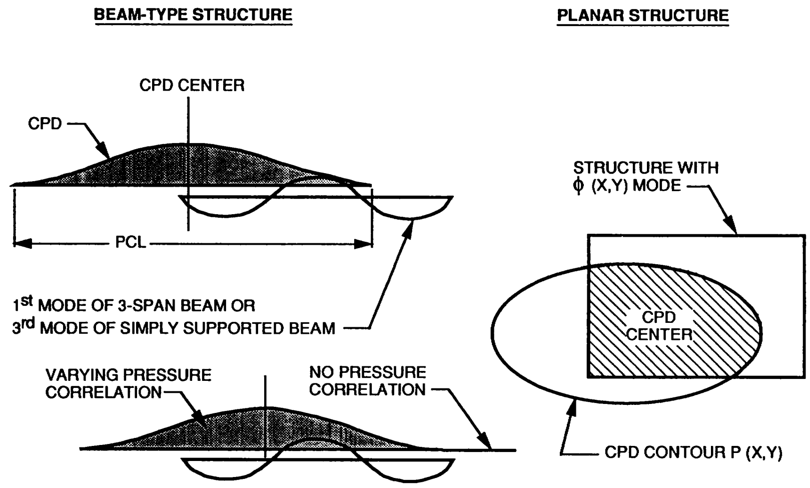

FIGURE 8 Load position and its effect on vibroacoustic coupling (*AJ*).

shown in Fig. 8. The position of the CPD center must be optimized in order to yield a maximum design value of *AJ*. For commonly encountered structures (plates, multispan beams, etc.), there are tables and diagrams of computed *J*-factors, often squared *J*-factors that are known as joint acceptance. The value of *A* corresponding to a joint acceptance must be defined; it is usually set equal to the area of a plate or to the span of a beam. *AJ* in integral form for a two-dimensional case is defined as:

$$AJ(fr, xc, yc) = \int_A \mathbf{b}(x, y) \cdot \mathrm{CPD}(fr, x, y) \cdot \mathbf{dA} \tag{14}$$

(at fr = resonance frequency) where xc and yc are the coordinates of the CPD center, and $\mathbf{b}(x, y)$ defines normal mode displacements perpendicular to the x, y plane.

Approximation of the NCPS Matrix

An important relation between a CPD and the matrix of NCPSs, when both are defined within the same region of an acoustic field, is discussed next. Because the matrix of NCPSs, **Nc**, as defined by Eq. (3), is a complex Hermitian matrix, each element of **Nc** is a function of frequency having a hidden third dimension. If the imaginary component of NCPSs is disregarded [i.e., using Re(NCPS) only or assuming zero phase], then the matrix **Nc** becomes symmetric and it can be formally decomposed into a dot product of two matrices:

$$\underset{n \times n}{\mathbf{Nc}} = \underset{n \times m}{\mathbf{C}^T} \cdot \underset{m \times n}{\mathbf{C}} \tag{15}$$

Because the dimension m of the matrix **C** is not defined, a simplification of **Nc** computations appears to be possible if $1 \le m \le n$. Thus, a decomposition by Eq. (15) should be attempted.

Next, Eq. (10) is presented for the case of a single mode, where the modal vector **b** has a dimension $n \times 1$, thus defining $[AJ]^2$ for that mode. Substituting Eq. (15) into (10), results in:

$$\underset{1 \times 1}{[AJ]^2} = \underset{1 \times n}{\mathbf{b}^T} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times m}{\mathbf{C}^T} \cdot \underset{m \times n}{\mathbf{C}} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times 1}{b} \tag{16}$$

Because dA is a diagonal matrix, the right-hand side of Eq. (16) becomes a dot product of the matrix $[\mathbf{C} \cdot \mathbf{dA} \cdot \mathbf{b}]$ premultiplied by its own transpose, as follows:

$$AJ \cdot AJ = [\underset{m \times n}{\mathbf{C}} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times 1}{\mathbf{b}}]^T \cdot [\underset{m \times n}{\mathbf{C}} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times 1}{\mathbf{b}}] \tag{17}$$

A representation of the integral in Eq. (14) by an equivalent matrix form is given by:

$$AJ(fr, xc, yc) = \underset{1 \times n}{\mathbf{C1}} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times 1}{\mathbf{b}} \tag{18}$$

Equation (18) also follows from Eq. (17) when the dimension m in matrix $\mathbf{C}$ is set to one so that $\mathbf{C} = \mathbf{C1}$ (the equality is assumed in the frequency plane fr, where fr is the resonance frequency).

The discussion of the deterministic method to this point provides an insight into the computations of AJ-factors and into the relation between a CPD and the matrix of NCPSs. If the entire matrix $\mathbf{Nc}$ were available, computed AJ-factors using Eq. (10) would be an exact maximum. Because the total matrix of NCPSs is seldom if ever available, one must use intuitive simplifications for approximating $\mathbf{Nc}$ by:

$$\underset{n \times n}{\mathbf{Nc}} = \underset{n \times 1}{\mathbf{C}^T} \cdot \underset{1 \times n}{\mathbf{C}} \tag{19}$$

In summary, the complex NCPSs can be approximated by much simpler CPDs. Elements of both matrices are functions of frequency (the hidden third dimension). Because Eqs. (15) and (19) cannot be solved for $\mathbf{C}$ when $m = 1$, the exact maximum of an AJ-factor computed by Eq. (18) is lost in the process of approximation. This loss does not appear to be too large a price to be paid for the advantages offered by the response spectra solution in a transient environment where only approximations are possible. The main advantage of the PCL concept is that, for the entire frequency range of interest, PCLs can be computed from only a few NCPSs. The computational accuracy is enhanced by restricting the PCLs to remain within a range of 3 to 7 distances between source measurements. PCLs and CPDs are easy to visualize (Fig. 8). Moreover, the general trend of PCL variation with the distance from the vehicle exhausts can be accurately defined. At a constant frequency, PCLs are characteristically shorter the closer the measurements are to the vehicle, and vice versa. This allows for a broader range of reliable extrapolations from PCLs than when attempting to visualize a variation of CPSs with location. The benefits derivable from a transportable $\mathbf{Nc}$ matrix are significant.

Response Solutions Using CPDs

Instead of a complete $\mathbf{Nc}$ matrix that is valid for many locations and distances from rocket exhaust, one may realistically have only a few pairs of measurements. Thus, the approximate decomposition of Re($\mathbf{Nc}$) matrix by a single row of the $\mathbf{C}$ matrix containing a CPD [Eq. (19)] may become the only resource to either check an existing acoustic field model or to derive another. If the concept of CPDs were incorporated into the pressure spectrum approach, Eq. (1) would have the following form:

$$\begin{aligned} \underset{k \times k}{\boldsymbol{\Phi}_{qq}} = (\underset{k \times k}{\mathbf{H}_m} \cdot \underset{k \times n}{\mathbf{b}^T} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times 1}{\mathbf{C}^T} \cdot \\ \underset{1 \times n}{\mathbf{C}} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times k}{\mathbf{b}} \cdot \underset{k \times k}{\mathbf{H}_m}) \cdot S_p(f) \end{aligned} \tag{20}$$

All terms of Eq. (20) were defined previously except for matrix $\mathbf{C}$, which contains CPDs as a function of frequency in the "hidden third dimension." Practical computations are governed by the frequency array in $\mathbf{H}_m$, which contains resonances of selected modes; thus, the array in matrix $\mathbf{C}$ must correspond to $\mathbf{H}_m$. Power spectrum of response modal coordinates, diagonal elements of $\boldsymbol{\Phi}_{qq}$, must be integrated in the frequency domain to obtain mean square responses. Presently, only a few FEM codes are capable of handling a variety of correlated random excitation. However, it is difficult to model totally uncorrelated or partially correlated excitation (typical of Shuttle near-field acoustics) using existing codes. A PRS approach for peak values of modal coordinates [derived from Eq. (8)] in a matrix form similar to Eq. (20) may be obtained from:

$$\begin{aligned} \underset{k \times k}{\mathbf{Q}} = \mathrm{diag}[\underset{k \times n}{\mathbf{C}(fk)} \cdot \underset{n \times n}{\mathbf{dA}} \cdot \underset{n \times k}{\mathbf{b}}] \cdot \\ \underset{k \times k}{\mathrm{CMP}} \cdot \underset{k \times k}{\mathbf{Y}(fk)} \end{aligned} \tag{21}$$

where $\mathbf{Q}$ is a $k \times k$ diagonal matrix of maximax response modal coordinates q(peak), diag designates that only main diagonal elements of the matrix in brackets need be computed, and $\mathbf{C}(fk)$ is the matrix containing a CPD corresponding to the resonance frequency, fk, of each kth mode. The definition of $\mathbf{C}(fk)$ requires a different specification of position of the center of CPD for each mode and a corresponding PCL. CMP is a $k \times k$ diagonal matrix of generalized modal compliances, and $\mathbf{Y}(fk)$ is a $k \times k$ diagonal matrix containing values of response spectra at kth mode resonance frequency, fk. It should be noted that Eq. (21) does not contain the hidden third dimension because each frequency-dependent CPD is now contained in rows of $\mathbf{C}(fk)$. The simplification of input and computational requirements in Eq. (21) relative to Eq. (20) is significant. There is a definite price for simplification: multiple runs must be made by varying the position of the CPD center in $\mathbf{C}(fk)$ in order to ensure that computed response in each mode is an absolute maximum.

CONCLUSIONS

The experience gained from Shuttle launches has led to significant developments in the applied analysis methodology. Efforts to characterize rocket noise resulted in newly developed functions (NCPSs, PCLs, etc.) as main descriptors defining an acoustic field, while CPDs provided a graphic illustration of vibroacoustic coupling and allowed a computation of *AJ*-coefficients for any type of structure. Search for an alternate solution was necessary because the structural overloads and failures predicted by state-of-the-art methods did not materialize. Presently, the existing database of NCPSs, PCLs, etc., is small and insufficient to support a full-scale design effort. Therefore, theoretical developments supported by measurements should continue. Both PS and PRS approaches are provided in matrix form, suitable for implementation into existing FEM programs. It is recommended that both solutions be implemented because, within a certain frequency range, each may have an advantage over another. Within the low-frequency range (0–20 Hz) of launchpad structure resonances, the PRS approach is simpler and more accurate than the PS approach.

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