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Optimal Location of Plate Damped Parts by Use of a Genetic Algorithm

Optimal damping of plates (or beams) partially covered by viscoelastic constrained layers is presented. The design variables are the locations and the sizes of the damped parts. The objective function is a linear combination of the first modal damping factors calculated from a specific finite element analysis. The discrete design variable optimization problem is solved using a genetic algorithm.

INTRODUCTION

The use of viscoelastic constrained layers to damp or to control the vibration level of mechanical structures is an old and successful technique particularly efficient for beams, plates, or shells. One of the most weight-effective methods of incorporating a viscoelastic material in a built-up structure is in the form of a constrained layer. The elastomer is sandwiched between the structure itself and a thin sheet of metal and bonded to both (Fig. 1). Flexural vibration causes shearing strain in the viscoelastic core that dissipates energy and thereby reduces vibrations. Marcelin, Trompette, and Smati (1992) have studied the optimal damping of beams damped in such a way that only one or several portions of the beam are covered; conventional nonlinear programming optimization was used. Here this problem is reviewed by extending the application to plates and by solving a discrete optimization problem. In the case of plates, a very small number of articles have been previously published on the subject. Theoretical and experimental studies have been presented on completely covered plates (Lu, Kilian, and Everstine, 1979; He and Ma, 1988; Shin and Maurer, 1991). Some theoretical and experimental studies are concerned with partially covered plates (Jullien and Takagami, 1981), but without an optimization step. Both partial covering and optimization are considered here: for such a purpose, special plate finite elements were built to represent the behavior of the sandwich parts and a genetic search was used to maximize the objective function. A similar question was dealt with by Hajela and Lin (1991) but only in the case of beams. The major conclusion in achieving an optimal damping is the uselessness of covering the whole plate (Jullien and Takagami, 1981). The comparison between the computational and the experimental results of Shin and Maurer (1991) shows that the proposed approach is an efficient and attractive way for solving such optimization problems.

FINITE ELEMENT (FE) MODEL

The dynamic behavior of partially covered plates is obtained from a modal model. The homoge-

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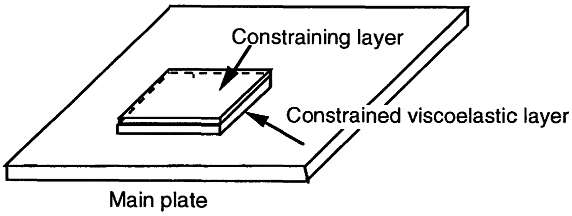


FIGURE 1 Plate damped by viscoelastic material.

neous parts of the plate are discretized by conventional C_1 FEs and the heterogeneous or sandwich ones by specific FEs designed to represent accurately the viscoelastic core shear damping effect. The finite elements of the damped and undamped parts must be as compatible as possible. Such elements have been proposed previously for beams (Trompette, Boillot, and Ravanel, 1978; Marcelin et al., 1992). They are extended here to plates.

Homogeneous Plate FE

It is a usual Kirchhoff plate bending element with four nodes. The degrees of freedom (DOF) are w, $\partial w/\partial x$, $\partial w/\partial y$ where w is the transverse plate displacement. The interpolation function w is,

$$w = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2 + a_7x^3 + a_8y^3 + a_9x^2y + a_{10}xy^2 + a_{11}x^3y + a_{12}xy^3. \tag{1}$$

In the following applications the in-plane displacements of the main plate are neglected.

Heterogeneous Plate FE

They are built on the following assumptions (Fig. 2):

- the transverse displacement w is independent of the transverse coordinate z, $w = w(x, y)$;
- the viscoelastic layer bending stiffness is negligible and the considered strains are only shear strains γ_{xz}, γ_{yz};
- the longitudinal displacements of the different parts of the plate are given by

$$u_A = u_3 + h_3(\partial w/\partial x), \quad u_B = -h_1(\partial w/\partial x) \quad \text{and}$$
$$v_A = v_3 + h_3(\partial w/\partial x), \quad v_B = -h_1(\partial w/\partial y); \tag{2}$$

- the viscoelastic core material is characterized by a complex shear modulus G that can be frequency and temperature dependent;
- the thicknesses $h_3 \ll h_1$ and $h_2 < h_1$;
- the shear strains in the elastic layers are neglected;
- the longitudinal displacements u and v of the plate medium surface are set to be 0. The validity of this approximation has been verified on numerical examples.

At each node the DOF are w, $\partial w/\partial x$, $\partial w/\partial y$ and the longitudinal displacements u_3 and v_3 of the constraining layer. The interpolation function for w is the same as that for the homogeneous element. The interpolation functions for u and v are those of the bilinear plane element.

The element stiffness matrix is the sum of the three contributions: bending, in-plane traction of the constraining layer, and in-plane shear of the constrained viscoelastic layer. The first two are usual, the last is obtained from the following in-plane shear strain energy:

$$E = \frac{1}{2}\int_{(A)}\int_{h_1}^{h_1+2h_2} (G\gamma_{xz}^2 + G\gamma_{yz}^2)dzdA \tag{3}$$

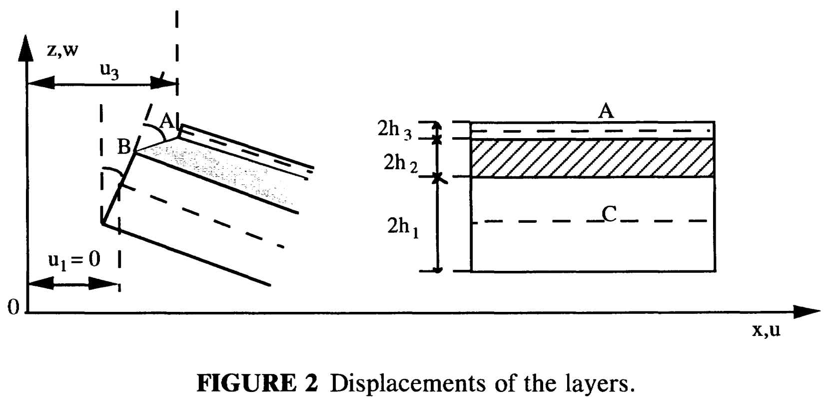


FIGURE 2 Displacements of the layers.

with:

$$\gamma_{xz} = \frac{\partial u_2}{\partial z} + \frac{\partial w}{\partial x} \quad \text{and} \quad \frac{\partial u_2}{\partial z} = \frac{u_A - u_B}{2h_2}$$

so:

$$\gamma_{xz} = \frac{1}{2h_2}\left(u_3 + d\frac{\partial w}{\partial x}\right) \quad \text{with} \quad d = h_1 + 2h_2 + h_3 \tag{4}$$

in the same manner:

$$\gamma_{yz} = \frac{1}{2h_2}\left(v_3 + d\frac{\partial w}{\partial y}\right). \tag{5}$$

Element Mass Matrix

A consistent mass matrix M is used that takes into account the bending contribution and the in-plane effects of the different parts, that is, the calculated kinetic energy is:

$$T = \frac{1}{2}\int_1 \rho_1 \dot{w}^2 d\tau + \frac{1}{2}\int_2 \rho_2 \dot{w}^2 + \dot{u}_2^2 + \dot{v}_2^2) d\tau + \frac{1}{2}\int_3 \rho_3(\dot{w}^2 + \dot{u}_3^2 + \dot{v}_3^2) d\tau. \tag{6}$$

Assembly

Due to the viscoelasticity of the core material the global dynamic equilibrium equations are simple only if the displacements are periodic: $x = x_0 e^{j\omega t}$. In this case, the viscoelastic modulus is written $G = G_0(1 + j\beta)$ and the corresponding stiffness matrix is

$$|K_v| = |K_{vr}| + j|K_{vi}|. \tag{7}$$

The dynamic behavior of the whole plate (or beam) is given by the matrix equation

$$(-|M|\omega^2 + |K_1| + j|K_2|)\{u\} = \{F\} \tag{8}$$

in which the imaginary part K_2 of the total stiffness matrix is equal to K_{vi}.

Modal Response

Frequencies and mode shapes of the undamped associated structure can be considered as a good and simple modal basis to be used for predicting the dynamic behavior of the damped structure (Trompette, Marcelin, and Schmeding, 1993).

ω_i and ϕ_i, $i = 1, n$ are the undamped frequencies and corresponding mode shapes obtained from the matrix equation:

$$(-|M|\omega^2 + |K_1| + |K_1|)\{x\} = \{0\}. \tag{9}$$

Performing the usual transformation $\{x\} = |\Phi|\{q\}$, and premultiplying by $|\Phi|^T$, Eq. (10) is obtained for free vibrations

$$(-|\mu|\omega^2 + |\kappa_1| + j|\kappa_2|)\{q\} = \{0\}. \tag{10}$$

Because of the orthogonality of the modes, $|\mu|$ and $|\kappa_1|$ are diagonal matrices, but not $|\kappa_2|$. Generally for beams and plates the frequencies are well separated, so the full matrix can be considered as diagonal dominant. In these conditions the modal system (10) is the sum of n uncoupled equations. It follows that in a modal response, a good approximation of the structural loss factor may be easily calculated from each equation of (10).

OPTIMIZATION METHOD

Objective Function

The objective function to maximize is the modal damping factor of a single mode i or a linear combination of modal damping factors for several i.

$$\max \eta = \mathrm{SE}_{Vi}/\mathrm{SE}_i \quad \text{or} \quad \sum_i a_i \mathrm{SE}_{Vi}/\mathrm{SE}_i. \tag{11}$$

In (11) SE_{Vi} is the elastic strain energy stored in the viscoelastic material when the structure vibrates on its ith undamped mode, SE_i is the corresponding elastic strain energy of the entire composite structure, and a_i is a weighting coefficient. Because all the matrices are diagonal and the mode shapes are nonvariable during the optimization process, the calculation of the damping factors is obviously straightforward.

$$\eta = \phi_i^T|K_2|\phi_i/\phi_i^T|K_1|\phi_i = \kappa_{2i}/\mu_i\omega_i^2 \quad \text{or} \quad \sum_i a_i \ldots \tag{12}$$

It is assumed in (12) that the structural damping introduced by the elastic materials can be neglected because it is invariant during the optimization step. As the design variables are the locations and the dimensions of the viscoelastic parts the optimization problem is obviously a discrete one in which several derivatives have no sense. So to maximize the objective function η, a genetic algorithm is used (Goldberg, 1989; Hajela 1990).

Genetic Algorithm (GA)

This algorithm is a search procedure based on the mechanics of natural genetics and natural selection. It combines an artificial "survival of the fittest" with genetic operators abstracted from nature to form a mechanism that is suitable for a variety of search problems. To achieve their breadth and robustness, GA differs from traditional methods of search and optimization in a number of ways: it works with a coding of parameter set, not with the parameters themselves, and searches from a population of points, not from a single point; finally it requires objective function values, not derivative information; and uses probabilistic transition rules, not deterministic ones.

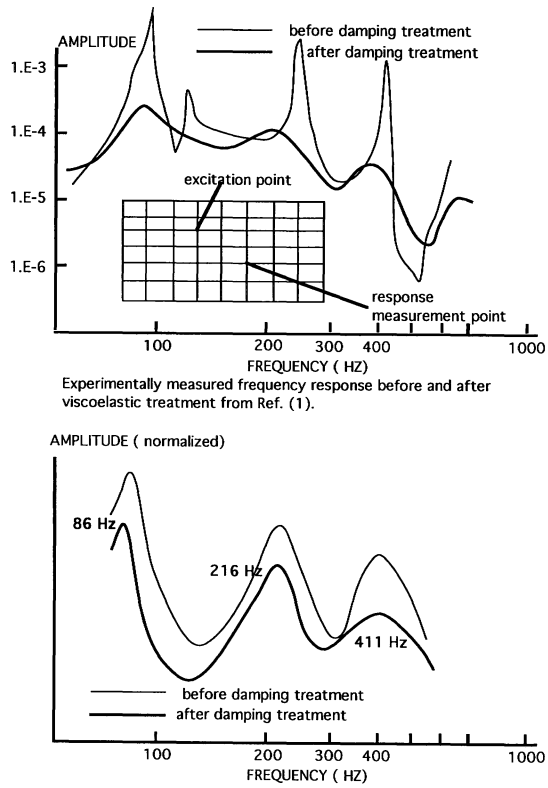
GA requires also the parameter set of the optimization problem to be coded as a finite length string. Here heterogeneous plate elements are coded by 1 and homogeneous plate elements by 0, so a design point, "a chromosome," is an n bit binary number in which n is the number of finite elements.

The genetic algorithm used is as described by Goldberg (1989). It is a simple one, three genetic operators abstracted from nature are used: reproduction, crossover, and mutation. There are no particular limitations and constraints. An example using a sharing mechanism will be presented in the next section; this concept is introduced in order to simultaneously locate relative optima in a multimodal design space. The approach is based on the concept of shared resources among distinct sets of populations. Each such set converges to one relative optimum. The principle of sharing is implemented by degrading the fitness of each design in proportion to the number of designs located in its neighborhood through the use of sharing functions (Goldberg and Richardson, 1987; Hajela and Lin, 1992).

RESULTS

Four Examples

EXAMPLE 1: At first, tests were conducted on the example of Shin and Maurer (1991), which concerns an entirely covered plate. The purpose was to compare the results given by the present work (using our particular FE) and those obtained by Shin and Maurer (1991) by both experimental and theoretical methods in the case of this simple damped plate. The dimensions of the plate were the following: length, L = 762 mm; width, b = 355.6 mm; h_1 = 9.525 mm; h_2 = 1.5875 mm; h_3 = 6.35 mm. The other characteristics of the materials are given by Shin and Maurer (1991). It is noted that the assumptions over the thicknesses $h_3 \ll h_1$ and $h_2 < h_1$ are not so well verified; nevertheless the comparison between the experimental results of Shin and Maurer (1991) and the present computational results given in Fig. 3 shows that the proposed approach is efficient in predicting the dynamic response of plates that are damped with a thin viscoelastic constrained layer.



Calculated frequency response before and after treatment

FIGURE 3 Experiments and theory results.

EXAMPLE 2: The second example was a very simple one and showed the possibility of a mesh refinement strategy during the genetic algorithm. It was concerned with a square plate, dimensions of which were: L = 6 m; b = 6 m; h_1 = 0.003 m; h_2 = 0.0015 m; h_3 = 0.0002 m.

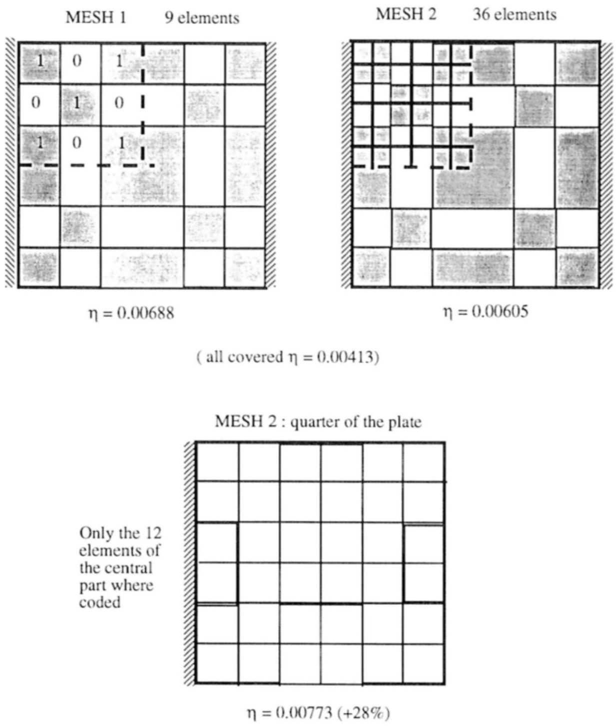


FIGURE 4 Clamped-clamped plate mesh refinement.

The viscoelastic material shear modulus G_0 was equal to 126.7 MPa, the loss factor $\beta = 1$ (both were constant), and the specific gravity of the viscoelastic material was 2500 kg/m^3. The elastic material was aluminum.

Nine plate elements were used to model a quarter of the plate. The optimization was conducted for the first clamped–clamped mode. Only a quarter of the plate was studied; the length of the chromosome was 9, the population size 10, the maximum number of generations was 10, the crossover probability was 8.0E-01 and the mutation probability was 6.0E-02. All the results of this case are given in Fig. 4.

The final structure (genetic solution) was coded by 101010101 for the quarter plate. The damping factor increased from 0.00413 (for the whole covered plate) to 0.00688 for the optimal solution.

From the preceding results it can be pointed out that some blocks of binary digits do not change in the chromosomes. This observation can be associated to an FE mesh refinement strategy. These blocks are not coded and optimization is performed only on the variable parts of the chromosomes. In this manner, the dimensions of the chromosomes become less important when the mesh is refined.

With this mesh refinement strategy, the preceding example is studied again (Fig. 4); results (mesh 2) are improved to a proportion of 28%.

Example 3: The third example was concerned with a free–free rectangular plate dimensions of which were: $L = 6$ m; $b = 5$ m; $h_1 = 0.003$ m; $h_2 = 0.0015$ m; $h_3 = 0.0002$ m. There was a concentrated mass of 500 kg on the plate (Fig. 5).

The shear modulus G_0, the loss factor β (both were constant), and the specific gravity of the viscoelastic material were those of the preceding example.

Thirty plate elements were used to model the plate. The optimization was conducted for the first and second free–free modes. In this example, the convergence of the process depending on the probability of mutation was examined. It was shown that a small value of the probability of mutation ensured a fast convergence but sometimes far from the maximum value (see convergence results on Fig. 6 with probabilities of mutation values 0.0033 and 0.0). It seems difficult therefore to determine an optimal strategy to ensure the best convergence law.

In this example, two designs were considered: either all or a part (6) of the elements can be covered. In this case, the string length is 30, but the five first strings determine the location of the first element, the following five other strings determine the location of the second element, and so on.

Example of optimal string when all the elements may be covered (mode 1):

101010011101101100011111101010

that means that elements, 1, 3, 5, 8, 9, 10, 12, 13, 15, 16, 20, 21, 22, 23, 24, 25, 27 29 are covered.

Example of optimal string when only six elements are covered (mode 1):

11110 01011 11011 11101 00001 01101

that means that elements 15 (11110), 26 (01011), 27 (11011), 23 (11101), 16 (00001), 22 (01101) are covered.

Figure 5 shows the results when all the elements may be covered, and Fig. 6 when only six elements may be covered.

Example 4: The last example is concerned with the same plate and is described in Fig. 7. This example shows the utility of a sharing mechanism as described in the preceding section; four

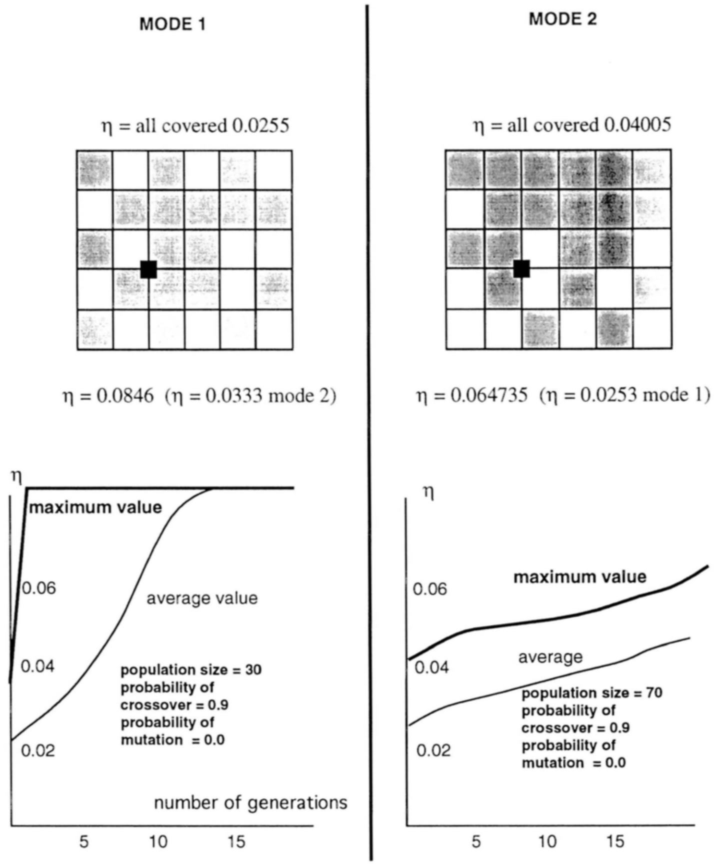


FIGURE 5 FREE-FREE rectangular plate with a concentrated mass (30 FE for the whole plate); all the elements may be covered; string length = 30.

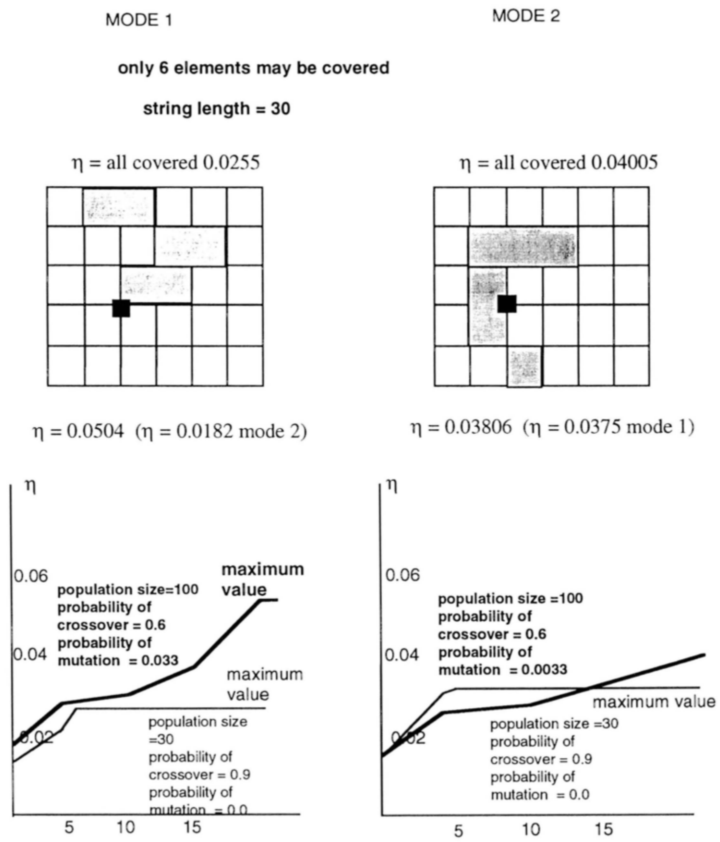


FIGURE 6 FREE-FREE rectangular plate with a concentrated mass (30 FE for the whole plate).

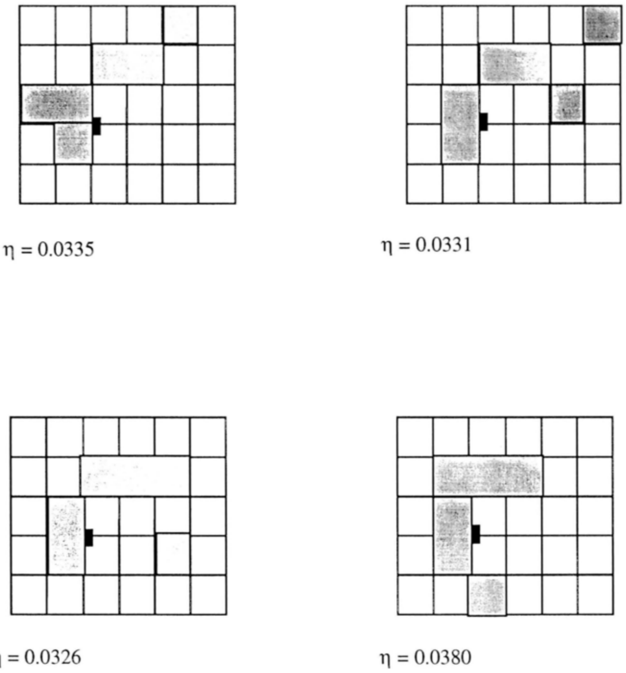


FIGURE 7 Sharing solutions.

different solutions are simultaneously obtained in using this sharing mechanism.

CONCLUSION

The optimization of damping of plates by constrained viscoelastic layers, when only one or several portions of the plate are covered, was considered. Applications may concern several domains such as aeronautics, automobile, sports, and building industries. From a more theoretical point of view it is obvious that this study can be extended to the damping of shells without too many difficulties.

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