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Noise Source Location Optimization

This article describes a method to determine locations of noise sources that minimize modal coupling in complex acoustic volumes. Using the acoustic source scattering capabilities of the boundary element method, predictions are made of mode shape and pressure levels due to various source locations. Combining knowledge of the pressure field with a multivariable function minimization technique, the source location generating minimum pressure levels can be determined. The analysis also allows for an objective comparison of "best/worst" locations. The technique was implemented on a personal computer for the U.S. Space Station, predicting 5–10 dB noise reduction using optimum source locations.

INTRODUCTION

Equipment within enclosed volumes excites acoustic resonances that may exceed acceptable limits. Excessive noise can result in reduced work proficiency, sleep interference, and fatigue. Acoustic resonances may also excite structural modes resulting in excessive vibration. The structural vibration itself may reradiate acoustic noise or transport excessive vibratory energy. In the case of the US Space Station, acoustic levels can excite equipment vibrations that can exceed payload microgravity limitations. This is true especially in the frequency range below 100 Hz, where strong isolated resonances exist.

Finite element method (FEM) and boundary element method (BEM) assessments are commonly used to predict acoustic or vibration modes, especially for the low frequency ranges where modal frequencies are not closely spaced. These analysis methods can provide narrow band frequency response level predictions caused by acoustic and/or mechanical forces. The analysis provides complete information of the pressure field, which includes node and antinode locations.

For acoustic volumes suitable for human occupation, FEM and BEM analyses have a practicable upper frequency limit of a few hundred hertz. This limit is due to computer resource requirements and not from theoretical limits. The limits arise from the large number of volume or surface elements required to accurately characterize the sinusoidal components of the resulting pressure field. About five elements per highest frequency wavelength are necessary. BEM and FEM techniques are preferred in the low frequency range because other methods such as statistical energy analysis (SEA), ray tracing, and simple room acoustic equations do not model the resonant mode characteristics necessary to help designers optimize equipment placement, or modify equipment operating frequencies that could efficiently excite interior resonances. In the lower frequency ranges, modes are widely spaced, allowing for the possible direct control of modal amplitude by locating contributing sources in such a manner that minimizes modal amplitudes.

This article presents a method that automatically locates low frequency sources so that the source–acoustic volume coupling efficiency is minimized. This results in a pressure field minimized for predetermined receiver locations. The

Received November 10, 1993; Accepted March 24, 1994.
Shock and Vibration, Vol. 1, No. 5, pp. 431–437 (1994)

CCC 1070-9622/94/050431-07

method also allows for an objective assessment of a potential source location versus alternate locations that would be more efficient (or inefficient). This helps the designer to quantify the quality of source locations. Another advantage of this approach is that noise reduction is accomplished without the added cost and complexity of absorption or damping materials or active control equipment.

MODE PREDICTION METHODOLOGY

The BEM method only needs to subdivide (mesh) the acoustic boundary surface, which makes modeling much simpler than for FEM techniques that must mesh both the surface and the interior volume. The BEM solution strategy usually begins with the classical Helmholtz integral:

$$C_p\phi_p = \int_s (\phi G'_{p,q} - \phi'_q G_{p,q})\, dS + 4\pi\phi_p^I$$

where ϕ = total scalar velocity potential, pressure = $ikz_0\phi$; ϕ^I = external scattering source potential at p, due to a noise source; G = free space Green's function, e^{-ikR}/R, $R_{p,q} = |p_{x,y,z} - q_{x,y,z}|$; q = any point on volume (domain) surface, S; p = initially any point on S, then any fixed receiver point interior to S; k = wave number, ω/c; G', ϕ' = derivatives with respect to unit surface normal, $\phi' = 0$ for hard boundaries $\phi' = -\{V_n\}$ where V_n = particle velocity normal to surface, S; C_p = a coefficient depending on geometry at p (see Seybert et al., 1985); $\int_s$ = surface integral over the volume boundary surface; z_0 = characteristic impedance of the medium, ρc for air. This relationship, after appropriate numerical integration, yields the relational form:

$$C_p\phi_p = [G']\{\phi_q\} - [G]\{\phi'_q\} + 4\pi\{\phi_p^I\} \quad (1)$$

where the matrices $[G]$ and $[G']$ represent geometric information only. If P or V_n or an impedance relationship is defined in the initial boundary conditions, then the corresponding unknown quantity can be solved. Once the $\{P\}$ and $\{V_n\}$ are known on the boundary, the pressure due to a particular source potential, ϕ_p^I, can be determined for any other location in the domain by redefining point p in the Green's function as the receiver location. Because $\{P\}$ and $\{V_n\}$ are now known on the boundary, only one row of $[G']$ and $[G]$ needs to be recomputed.

The initial full $[G]$ and $[G']$ matrices only need to be computed once. This is fortunate because most of the solution time is used to compute these initial matrices. However, the boundary surface pressure vector and receiver location point is recomputed for each new source location, defined by ϕ_p^I. The objective of the minimization procedure is to find the (x, y, z) coordinate location of ϕ^I which will minimize ϕ_p. Resonance effects are included in the formalism implicitly. Because frequency is a parameter in the equations, the solution is valid only for the particular frequency initially defined in the Green's function.

This BEM Green's function is appropriate for pure acoustic problems. However, structural BEM, structural FEM problems, or coupled structural–acoustic problems can also benefit from the optimization process. The form of the equation is not critical to the optimization procedure, but it could affect the choice of optimization algorithm depending on the smoothness of the function or other numerical factors.

The frequencies chosen for analysis will typically be those known to cause design problems. A common situation is where vibrating equipment such as a fan motor, etc., has forcing frequency components close to an acoustic or structural resonance. These frequencies will be efficiently amplified causing potential noise and vibration problems. In the initial design phase, the designer may have some knowledge of the forcing spectrum. Knowing the system response frequencies and mode shapes, the designer can avoid resonance amplification by optimizing equipment placement. The optimization technique described here does not have to be used at resonance frequencies, but seems most appropriate for those instances where resonances are a potential problem.

For simple geometries like tubes and cubes, it is not difficult to determine source locations that will minimize or maximize mode excitation. However, as the volume geometry becomes more complex, the mode shape contours become too complex to use intuition for placing sources, necessitating a more careful approach. In three dimensional problems, the nodal pressure locations are typically defined on a complicated twisting surface.

Equipment located near the antinode of an acoustic resonance will efficiently couple its energy to the resonance, resulting in much higher amplitude than source locations near an acoustic node. The effect is similar to that for structural vibrations having excitation sources located near antinodes. In this case, the mechanical response amplitude is a function of the mode shape vector times the excitation vector. Similar to structural excitation, an acoustic response analysis may be performed to determine the optimum source placement.

To illustrate the acoustic source–mode coupling effect, an example of a structural force–mode coupling is briefly described. For structural problems, a simplified equation by Crandall and McCalley (1988) describing the relationship of the structural response vector to the input force vector at a single frequency is:

$$\{x\} = \text{real}[(1/\omega_r^2 - \omega^2 + j\eta_r\omega_r\omega)V_rV_r^T/V_r^TMV_r\{d\}]$$

where ω_r = resonant frequency for mode r; V_r = eigenvector for mode r; d = vector of driving forces corresponding to a set of locations; x = vector of responses corresponding to a set of locations; M = mass matrix corresponding to a set of locations; η = damping loss factor corresponding to a set of locations. This relationship shows that the response amplitude vector $\{x\}$ is proportional to the product of input force vector $\{d\}$, and the system mode shape eigenvector, V_r. If the force only occurs at antinodes where V_r is 0, then the response will be 0. Conversely, a force at an antinode will excite the response vector more efficiently. A similar effect occurs in acoustic systems, but the effect is more complicated due to acoustic scattering at the bounding surface.

Because the boundary reflects (scatters) incident source pressure waves, some volume excitation will always occur, even if the source is located identically at a node. This is because the entire volume surface reradiates scattered pressure waves back into the interior. This causes the entire surface to behave as a distributed source rather than just the initial source point. A consequence of this behavior is that a point source will always generate a significant modal pressure field amplitude. This characteristic is different than for structural modes where only local direct forces affect the structure response, and is one reason why using only intuition for source placement optimization may not result in expected results for acoustic problems.

MINIMIZATION PROCEDURE

Choosing a frequency for optimization comes from knowledge of the system's eigenfrequencies initially determined from an acoustic FEM model, which solved for the lowest frequencies of interest. Alternatively, using a technique called the dual reciprocity method (DRM) (Brebbia and Ciskowski, 1991) or the particular integral method (PIM, Banerjee et al., 1988), BEM matrices can be generated that are solved directly for eigenvalues and eigenmodes. An alternate but slower procedure is to solve a BEM model over a range of closely spaced frequencies, noting the response amplitudes and identifying these as resonant peaks.

The optimization procedure uses the BEM method to compute the interior pressures caused by a scattering source, ϕ^I, located within the volume. A trial source location initiates the optimization process. Through successive iterations, new source locations are evaluated for resulting pressures at a particular receiver location of interest, p. The algorithm used to evaluate the new trial location is Powell's multivariable function minimization procedure (Press et al., 1986). The variables are the three coordinates (x, y, z), and the function minimized is a set of BEM routines based on Eq. (1), which solves the acoustic model at a particular receiver location. The technique is efficient for this problem because the pressure field is smooth, although nonlinear. Powell's method does not require an explicit global gradient function to be efficient. This characteristic is useful in the current situation because determining the gradient function of the pressure field everywhere would be difficult and very computationally expensive.

Powell's method seems well suited for minimization problems where the solution tends to be located somewhere along the nodal surface. These surfaces often vary quickly along one or two dimensions, but slowly in a third dimension. That is, the nodal surface tends to be a long twisting pressure valley along which a minimization algorithm must follow before reaching an optimum. After finding the valley floor, the Powell method follows it in a direction which is changing slowly. Paradoxically, this often gives the quickest solution because it does not waste time going up and down valley sides on the way to reaching an optimum location.

During the minimization search, the procedure usually finds a nodal surface close to the initial starting position, and then moves along that surface until an optimum location is found. This optimum location is the location on the nodal surface that couples least efficiently when producing a pressure field at the predefined receiver location. This process is similar to finding the minimum of a simple one dimensional function, except in this case the function is dependent on the three spatial variables (x, y, z). This is why a multivariable function minimization procedure is needed. The solution vector consists of the (x, y, z) source location for the ϕ^I coordinates resulting in a function (pressure) minimum at the fixed point, p. The iteration process can be summarized with the following steps:

1. Define initial parameters:
   a. choose a fixed receiver location, p, where the minimization is desired;
   b. choose an initial trial source location for ϕ^I;
   c. define numeric or geometric limits for a valid source region;
   d. compute full $[G]$, $[G']$ matrices, these are computed only once;
   e. compute initial $\{P\}$ from initial source location per Eq. (1).
2. Perform source optimization iteration:
   a. call subroutine Powell to minimize $p(x, y, z)$ (see Press et al., 1986).
   b. Powell uses result from function call to FUNC:
      - determine if (x, y, z) location is valid, if invalid location, return a penalty;
      - compute initial boundary pressure due to ϕ^I;
      - solve for resulting ϕ and ϕ' on boundary using full $[G\}$ and $[G'\}$ from Eq. (1);
      - compute resulting pressure at p using one row of $[G]$ and $[G']$.
   c. If delta pressure at receiver location, p, is still too large, go to step a.
      - otherwise, optimum is found (minimum) . . . STOP.
3. Repeat step 2 to find maximum by reversing sign of the pressure.

The subroutine, Powell, calls a generic function named FUNC. If this was a simple single variable function being minimized, then FUNC would simply define an equation, such as a polynomial. In this situation, FUNC is a suite of subroutines, eventually returning a single value of pressure at point p.

Graphical Determination of Invalid Trial Points

The penalty returned by FUNC for invalid trial points external to a valid volume requires a little care to avoid numerical "shock." Too large a penalty drives the solution point too far from the last location, sometimes causing unstable oscillations. A formula that has shown good results is to let the penalty be proportional to the highest expected pressure and a distance measured from the geometric center to the current trial point location. The distance can be defined to the center of the valid locus of source points. This formula tends to create a pressure "basin" in which a stable solution can be found:

$$\text{penalty} = F * P_{\max}(\text{dist} + 1.0)^e$$

where F = a multiplier on the order of 1.0; e = exponent on distance factor, on the order of 1.0; $P_{\max}$ = largest expected value for pressure at receiver location; dist = magnitude of distance from trial location to geometric center. Function minimization is typical for functions having well-defined numeric limits, for example a polynomial over a range of [0, 1]. A function defined within a unit cube could have bounds of (0, 1) for all three coordinates. For more complicated and realistic structures, simple numeric limits are difficult to describe. As trial locations are evaluated, it is sometimes necessary to eliminate (delete) trial locations for ϕ^I occurring exterior to the physical volume, forcing the solution back into the interior. This attempt to find locations exterior to the volume is due to the form of the Green's function that shows pressure decreasing inversely to distance from the source. Consequently, solutions tend to drift away from the geometry to infinity. For simple volume geometries, applying numeric bounds is straightforward. However, for general convex shapes, no simple numeric bounds can be specified. Consequently, a graphics based procedure was developed to detect when trial points occurred external to the volume, and apply a numerical penalty whenever such a point occurred.

The personal computer (PC) technique for identifying external points uses three orthogonal projections of the geometry plotted to the monitor screen. An example is shown in Fig. 1. For any valid trial point, all three geometry projec-

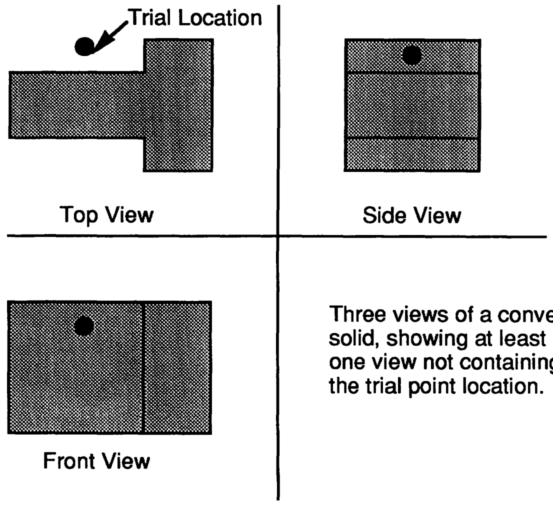

FIGURE 1 An example of a convex solid where a potential new trial source location occurs external to the acoustic volume. In at least one view, the point is located outside the projection. This can be detected by determining the background color at the plotted trial point location.

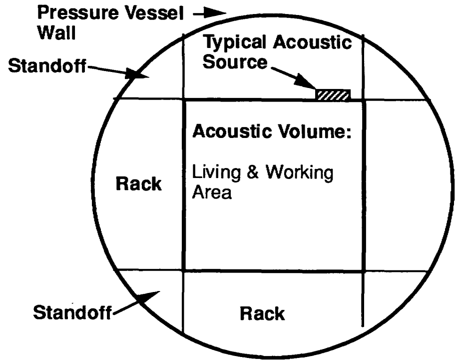

FIGURE 2 Typical Station module cross section, showing the acoustic volume and location of sound sources within equipment racks.

tions will contain the trial point when it is projected into the orthogonal views. If a point is external to the physical geometry volume, it will be outside one of the orthogonal projections. This fact can be detected by color coding the projection regions using a graphics fill algorithm. A valid projection point will only map onto a valid color. If the point is outside a projection, it is invalid and a numerical penalty is applied in the Powell algorithm, forcing the point back into the interior of the volume. This technique works for any convex solid, but will not work for concave shapes such as a cup. This is because a point inside the air volume of a cup will always appear within the projections, even when it is not within a solid cup volume. For volumes of this type, the volume must be broken into simpler objects.

Valid source regions could be specified that are more restricted than the whole volume. For example, due to physical design restrictions, the allowable location range may be limited to a small surface or subvolume region. In this case, limits may be easier to specify using conventional numeric values.

APPLICATION

The method has been applied to a simple Space Station BEM model. Typical computer time using a 33 MHz 80486 PC is about 3 min to determine both minimum and maximum locations for a single frequency mode and trial point. In general, the total pressure is the sum of both direct and scattered pressure fields. For this optimization, only the scattered (resonant) field contribution was optimized because this portion is affected by mode amplitude. The relative contribution of the two fields is a function of air and surface absorption. At frequencies below about 100 Hz, the Station is resonant field dominated for receiver locations more than 1 m from the source.

Figure 2 shows a typical cross section of the Space Station. In this case, the problem sources are along any of the four sides, but in general this is not a necessary factor. Ventilation ducts at several locations along all interior sources are known to contain frequencies that can couple efficiently with acoustic modes. Because the average absorption value is small, <0.1 at low frequencies, mode responses are amplified. Figure 3

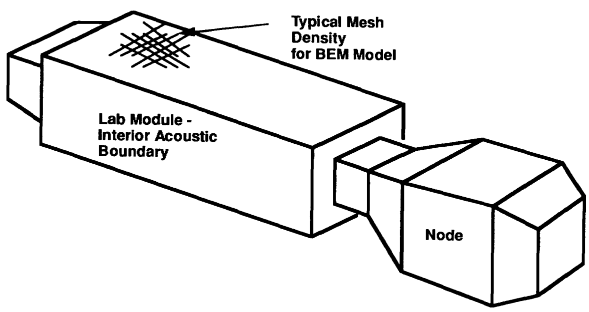

FIGURE 3 Hidden line schematic of the Space Station boundary element model used to determine acoustic eigenvalues.

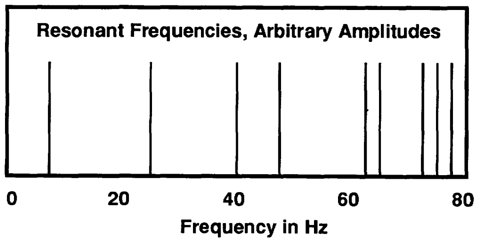

FIGURE 4 Frequencies of the first few eigenmodes for the Space Station.

shows the enclosed acoustic volume for the Station model, and the small mesh required for a complete high frequency analysis. This model is valid up to about 300 Hz. The model used in the PC optimization was much simpler because only the fourth mode was optimized. Figure 4 shows the spacing of the first few eigenmodes derived from the full Station FEM model. DRM eigenvalue results are equivalent to within approximately 1% of FEM eigenvalue analysis results.

Table 1 shows minimum and maximum pressure level results and corresponding source locations. The source location used in this case is typical of a wall mounted air duct. Significant reductions are possible by moving the source by only a few feet. The difference between best and worst source locations is about 33 dB. The distance moved from the initial trial guess was about 1.3 m. System designers can use this data to optimize equipment locations. Additionally, for a predetermined source location, a comparison to optimum can be objectively performed to assess the merits of relocation.

Figure 5 shows results of optimization in a higher frequency range where the four loudest sources were optimized. A complication in the

Table 1. Results of a Station Source Location Optimization

Description	Values
Receiver location, (x, y, z)	(0.5, 2.0, 2.0)
Initial source location	(4.4, 1.5, 1.5)
Initial receiver pressure	116.1 dB
Minimum found at	(5.7, 1.8, 0.5)
Minimum function	84 dB
Maximum found at	(3.9, 1.8, 1.8)
Maximum function	117 dB
Difference between max and min	33 dB
PC CPU run time	1:07

This was analyzed for the fourth mode at 49 Hz.

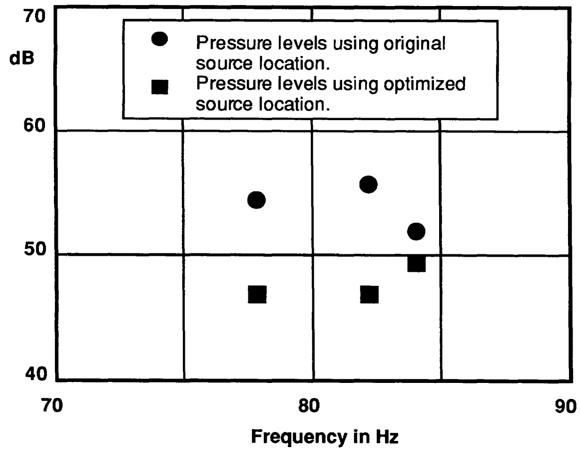

FIGURE 5 Calculated pressure levels for the Space Station at selected frequencies, showing the benefit of moving source locations.

analysis in this frequency range is the modal overlap occurring due to increased modal density. Despite the added complexity, significant reductions can be expected, on the order of 5–8 dB.

FUTURE REFINEMENTS

A useful extension of this technique will be to perform simultaneous optimization for several modes. In many applications in the speech frequency range, several resonances will exist in the lower bands of interest. Rather than try to optimize for each frequency separately, the band levels could be optimized simultaneously. This could tend to extend the useful frequency range to higher speech frequencies or larger volumes. The application could also easily be extended to apply to several receiver locations where the minimization would be performed in some average or location weighted sense.

This application concentrated on finding a local minimum close to a single nodal surface. Global minima should be determined for eigenmodes with two or more nodal surfaces. Finding a global function minimum is more difficult, although methods suitable for computer implementation have been investigated by Frank (1991). The methods concentrate on implementing efficient search strategies within the domain, usually initiated with a randomly selected array of trial starting locations. To implement these methods, one would replace the optimization algorithm used in this investigation with the new procedure.

REFERENCES

Banerjee, P., Ahmad, S., and Wang, H., 1988, "A New BEM Formulation for the Acoustic Eigenfrequency Analysis," *International Journal for Numerical Methods in Engineering,* Vol. 26, pp. 1299–1309.

Brebbia, C. A., and Ciskowski, R. D., Editors, 1991, *Boundary Element Methods in Acoustics,* Computational Mechanics Publications, Elsevier Applied Science, New York.

Crandall, S. H., and McCalley, R. B., 1988, "Matrix Methods of Analysis," in Harris, C., *Shock and Vibration Handbook,* 3rd ed., McGraw–Hill, Chap. 28, pp. 28–35.

Frank, P. D., 1991, "Optimization and Search Methods for Multidisciplinary Design," Boeing Computer Services Applied Math and Statistics Report, AMS-TR-172.

Press, Flannery, Teukolsky, and Vetterling, 1986, *Numerical Recipes,* Cambridge Univ. Press, Chap. 10, pp. 294–300. Cambridge.

Seybert, A. F., Soenarko, B., and Shippy, D. J., 1985, "An Advanced Computational Method for Radiation and Scattering of Acoustic Waves in Three Dimensions," *Journal of the Acoustic Society of America,* Vol. 77, pp. 362–368.