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Random Vibration of Space Shuttle Weather Protection Systems

The article deals with random vibrations of the space shuttle weather protection systems. The excitation model represents a fit to the measured experimental data. The cross-spectral density is given as a convex combination of three exponential functions. It is shown that for the type of loading considered, the Bernoulli–Euler theory cannot be used as a simplified approach, and the structure will be more properly modeled as a Timoshenko beam. Use of the simple Bernoulli–Euler theory may result in an error of about 50% in determining the mean-square value of the bending moment in the weather protection system.

INTRODUCTION

This study deals with the random vibrations of space shuttle weather protection systems modeled as Timoshenko beams. The experimentally determined excitation, rather than assumed expression of its cross-spectral density will be utilized. Random vibrations of Timoshenko beams under simplest, "rain-under-roof" excitation, namely with both time-wise and space-wise white noise, was considered previously. Samuels and Eringen (1957) were the first authors to study random vibrations of Timoshenko beams. They concluded, for the above excitation, that the results produced by using either the Timoshenko or Bernoulli–Euler theory differed by <5%. Crandall and Yildiz (1962) studied effects of both the dynamical models utilized and of the postulated damping mechanism. They considered, as an excitation, the time-wise band-limited white noise and investigated the growth pattern of the response characteristics with the increase of the

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cutoff frequency. Banerjee and Kennedy (1985) investigated the effect of axial compression. Singh and Abdelnaser (1993) studied the effect of boundary conditions. The latter study is in agreement with the study by Samuels and Eringen (1957) in the sense that for the length-to-depth ration, L/d, above 9, the percent difference between results produced by the Timoshenko theory and by the Bernoulli–Euler theory is <5%. Therefore, one may conclude that for this case of the length-to-depth ratio one should utilize a simpler Bernoulli–Euler theory.

However, the above studies concentrated on the excitation with constant spectral density. As a result, within the modal analysis approach, only the low end of the frequency spectrum contributes significantly to the response. However, the shear deformation and rotary inertia only considerably affect the higher frequencies. Because the contribution of jth mode is inverse proportional to ω_j^4 for the viscously damped beam, the significantly affected frequencies contributed very little in the formulation of the response. This is the reason for a small numerical difference between the Bernoulli–Euler and Timoshenko beams.

However, the situation must not be such in the general case of excitation, as was first demonstrated by Elishakoff and Lubliner (1985). They considered the band-limited white noise excitation with two cutoff frequencies, namely, the lower one $\omega_{c,1}$ and the upper one $\omega_{c,2}$, i.e., the spectral density was taken constant for frequency ranges $-\omega_{c,2} \leq \omega \leq -\omega_{c,1}$ and $\omega_{c,1} \leq \omega \leq \omega_{c,2}$. Such a band-limited white noise tends to ideal white noise when $\omega_{c,1}$ tends to zero and $\omega_{c,2}$ tends to infinity. It was demonstrated that when $\omega_{c,1}$ is zero or close to zero, both the Timoshenko theory and the Bernoulli–Euler theory produce similar results. However, when $\omega_{c,1}$ increases, the higher modes become more important. Because the higher natural frequencies are reduced by the effects of shear deformation and rotary inertia, the response predicted by Timoshenko theory is in excess of that predicted by the Bernoulli–Euler theory. In such circumstances one must utilize Timoshenko beam theory. In several example cases Elishakoff and Lubliner (1985) demonstrated that the application of the Bernoulli–Euler theory would yield an error on the order of 50% or more.

In this study we consider the response of weather protection systems to random excitations. The set of consistent differential equation, as discussed by Elishakoff and Lubliner (1985) is utilized. Namely, the fourth-order derivative of the displacement with respect to time, appearing in the original Timoshenko equation, is neglected as discussed by Love (1927), Tseitlin (1961), Egle (1969), and Elishakoff and Abramovich (1992).

The typical weather protection systems for launch vehicles are made of thin corrugated metal sheets (Fig. 1), with a much larger bending stiffness in the direction perpendicular to the corrugation, namely, $I_z \gg I_y$. These sheets are supported by thin beams parallel to the O_z direction. Such a weather protection system is susceptible to excessive shear deformation. It is therefore reasonable to model such a system as a Timoshenko beam.

EQUIVALENT TIMOSHENKO BEAM

For the sake of simplified analysis, a typical segment of the corrugated sheet may be substituted by an equivalent I beam as shown in Fig. 2. We equate an infinitesimal element of the web of the corrugated beam with its equivalent part on the I beam (Fig. 2),

$$y = \eta \cos \alpha \tag{1}$$

$$t_1 dy = t d\eta; \quad t_1 = \frac{t}{\cos \alpha}. \tag{2}$$

Furthermore, the shear flow in the two elements must be equal,

$$\tau t = \tau_1 t_1 \tag{3}$$

where τ and τ_1 are the shear stresses in the corrugated sheet and the I beams, respectively. Hence

$$\tau_1 = \tau \cos \alpha. \tag{4}$$

The shear strains are given by

$$\gamma = \frac{\tau}{G}; \quad \gamma_1 = \frac{\tau_1}{G_1} \tag{5}$$

where G is the shear modulus of the material in the corrugated beam, and G_1 is the equivalent shear modulus in the I beam.

Now, for the I beam web to have the same shear stiffness, the displacement u in the longitudinal direction must be the same as that of the corrugated web,

$$du = \gamma d\eta = \gamma_1 dy. \tag{6}$$

Hence

$$\gamma_1 = \frac{\gamma}{\cos\alpha} \tag{7}$$

and

$$\frac{\tau_1}{G_1} = \frac{\tau\cos\alpha}{G_1} = \frac{\tau}{G\cos\alpha}. \tag{8}$$

From Eq. (8), one obtains the equivalent shear modulus

$$G_1 = G\cos^2\alpha. \tag{9}$$

One can show that shear stress energies are equal in the two models

$$dU = \frac{1}{2}\frac{\tau_1^2}{G_1}t_1 dy = \frac{1}{2}\frac{\tau^2\cos^2\alpha}{G\cos^2\alpha}td\eta = \frac{1}{2}\frac{\tau^2}{G}td\eta. \tag{10}$$

FREE VIBRATION OF TIMOSHENKO BEAM

The equation of motion for a Timoshenko beam may be written as follows (Elishakoff and Lubliner, 1985),

$$\begin{aligned} EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\left(1 + \frac{E}{k'G_1}\right)\frac{\partial^4 w}{\partial x^2\partial t^2} \\ + \rho A\beta_0\frac{\partial w}{\partial t} - \frac{\rho IE}{k'G_1}\frac{\partial^3 w}{\partial x^2\partial t} \\ = q(x,t) + \frac{1}{k'G_1A}\left(-EI\frac{\partial^2 q}{\partial x^2} + \rho I\frac{\partial^2 q}{\partial t^2}\right) \end{aligned} \tag{11}$$

where E = modulus of elasticity, I = moment of inertia, ρ = material density, A = cross-sectional area, G_1 = equivalent shear modulus, w = displacement, x = axial coordinate, t = time, k' = shear coefficient, q = excitation, β_0 = transverse viscous damping coefficient $\beta_0 = c/\rho A$. The natural frequencies of the structure are found by letting $\beta_0 \equiv 0$, $q(x, t) \equiv 0$, and

$$w(x,t) = \psi(x)e^{i\omega t} \tag{12}$$

For a simply supported beam, the node shape can be chosen as

$$\psi_j(x) = \sin\frac{j\pi x}{L} \tag{13}$$

where j is the number of half sine waves in the x direction. The natural frequencies are computed from

$$\omega_j^2 = \frac{EI}{\rho A}\left(\frac{j\pi}{L}\right)^4\left[1 + \frac{1}{A}\left(\frac{j\pi}{L}\right)^2\left(1 + \frac{E}{k'G_1}\right)\right]^{-1}. \tag{14}$$

The shear correction factor k' is given by (Cowper, 1966)

$$k' = 10(1+\nu)(1+3m)^2/B \tag{15}$$

where ν is Poisson's ratio, and B is defined as

$$\begin{aligned} B = {} & 12 + 72m + 150m^2 + 90m^3 \\ & + \nu(11 + 66m + 135m^2 + 90m^3) \\ & + 30n^2(m + m^2) + 5\nu n^2(8m + 9m^2) \end{aligned} \tag{16}$$

$$m = \frac{2bt_f}{ht_w}, \quad t_f = t_w = t_1, \quad b = \frac{a}{2}, \quad n = \frac{b}{h}. \tag{17}$$

RANDOM VIBRATION OF TIMOSHENKO BEAM

Consider a random acoustic excitation field characterized by a cross-spectral density

$$\begin{aligned} \Phi_q(x_1, x_2; \omega) = {} & S_q(\omega)\exp[-\alpha(\omega)|x_1 - x_2| \\ & - i\gamma(\omega)(x_1\ x_2)] \end{aligned} \tag{18}$$

where $S_q(\omega)$ is the spectral density at the reference point $x_1 = x_2 = x$; $\alpha(\omega)$ and $\gamma(\omega)$ are, respectively, the decay and phase functions of ω. They are determined from experimental data and are given in the Appendix. Assuming that the beam has reached the state of probabilistic stationarity, the cross-spectral densities of response for displacement and moment are, respectively, found as follows (Lin, 1976; Elishakoff, 1983):

$$\begin{aligned} \Phi_{DD}(x_1, x_2; \omega) = b^2 \int_0^L\int_0^L \Phi_q(y_1, y_2; \omega) \\ H_D(x_1, y_1; \omega) \\ H_D^*(x_2, y_2; \omega)dy_1dy_2 \end{aligned} \tag{19}$$

$$\Phi_{MM}(x_1, x_2; \omega) = b^2 \int_0^L \int_0^L \Phi_q(y_1, y_2; \omega) H_M(x_1, y_1; \omega) H_M^*(x_2, y_2; \omega) dy_1 dy_2 \quad (20)$$

where H_D and H_M are the frequency response functions of displacement and bending moment, respectively, L is the length of the beam, and b is the width of the beam.

In order to derive the function H_D, let the harmonic point loading be imposed at location y:

$$q(x, t) = \delta(x - y)e^{i\omega t}. \quad (21)$$

Thus, the responses of displacement and moment at location of x will be

$$w(x, t) = H_D(x, y; \omega)e^{i\omega t} \quad (22)$$

$$M(x, t) = H_M(x, y; \omega)e^{i\omega t}. \quad (23)$$

Substituting Eq. (22) into Eq. (11) and expending H_D in the series in terms of the mode shapes $\psi_j(x)$, we obtain the following expression for $H_D(x, y; \omega)$

$$H_D(x, y; \omega) = \sum_{j=1}^{\infty} \frac{1}{\nu_j^2} H_j(\omega)\psi_j(x)\psi_j(y) \quad (24)$$

with ν_j^2 being the norm of jth mode, namely,

$$\nu_j^2 = \int_0^L \psi_j^2(x)dx = \frac{L}{2}. \quad (25)$$

The modal frequency response function in the jth mode $H_j(\omega)$, is derived as

$$H_j(\omega) = \frac{1 + \alpha\lambda_1\lambda_2[(j\pi)^2 - \rho L^2/E\omega^2]}{\rho A[1 + \alpha\lambda_2(1 + \lambda_1)(j\pi)^2]} \left[\omega_j^2 - \omega^2 + i\omega\beta_0 \frac{1 + \alpha\lambda_1\lambda_2[(j\pi)^2}{1 + \alpha\lambda_2(1 + \lambda_1)(j\pi)^2}\right]^{-1} \quad (26)$$

with two nondimensional parameters

$$\lambda_1 = \frac{E}{k'G}, \quad \lambda_2 = \frac{I}{AL^2}. \quad (27)$$

Moreover, α is an artificial parameter, with $\alpha = 1$ corresponding to the Bresse–Timoshenko theory and $\alpha = 0$ associated with the Bernoulli–Euler theory.

Note that the relationship between the bending moment $M(x, t)$ and displacement $w(x, t)$ is

$$M(x, t) = -EI\left[w''(x, t) - \frac{\alpha}{k'GA} V_y'(x, t)\right] = -EI\left[w''(x, t) - \frac{\alpha}{k'GA} [c_0\dot{w}(x, t) + \rho A\ddot{w}(x, t) - q(y, t)\right]. \quad (28)$$

Substituting Eqs. (21)–(23) into Eq. (28), we obtain H_M as follows

$$H_M(x, y, \omega) = -EI\left\{H_D''(x, y\ \omega) + \frac{\alpha\rho}{k'} G[(\omega^2 - i\omega\beta_0)H_D(x, y, \omega) + \frac{1}{\rho A}\delta(x - y)\right\}. \quad (29)$$

The cross-spectral densities for displacement and moment can be rewritten by substituting Eqs. (24) and (29) into Eqs. (19) and (20) as follows

$$\Phi_{DD}(x_1, x_2; \omega) = b^2 \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} H_j(\omega)H_k^*(\omega)\psi_j(x_1)\psi_k(x_2)I_{jk}(\omega) \quad (30)$$

$$\begin{aligned}\Phi_{MM}(x_1, x_2; \omega) &= b^2(EI)^2 \left\{\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} [\psi_j''(x_1)\psi_k''(x_2) \right.\\ &+ \eta^2(\omega^4 + \omega^2\beta_0^2)\psi_j(x_1)\psi_k(x_2)\\ &+ \eta(\omega^2 + i\omega\beta_0)\psi_j''(x_1)\psi_k(x_2)\\ &+ \eta(\omega_2 - i\omega\beta_0)\psi_k''(x_2\psi_j(x_1)] H_j(\omega)H_k^*(\omega)I_{jk}(\omega)\\ &+ \frac{\eta}{\rho A}\sum_k^{\infty} [\eta(\omega^2 + i\omega\beta_0)\omega_k(x_2)\\ &+ \psi_k''(x_2)] H_k^*(\omega)J_k^*(x_1; \omega)\\ &+ \frac{\eta}{\rho A}\sum_j [\eta(\omega^2 - i\omega\beta_0)\omega_j(x_1)\\ &+ \psi_j''(x_1)] H_j(\omega)J_j(x_2; \omega)\\ &\left. + \left(\frac{\eta}{\rho A}\right)^2 \Phi_q(x_1, x_2; \omega)\right\}; \quad \eta = \frac{\alpha\rho}{kG}\end{aligned} \quad (31)$$

where two integrals of I_{jk} and J_j are defined as follows:

$$I_{jk}(\omega) = \frac{1}{\omega_j^2 \nu_k^2} \int_0^L \int_0^L \Phi_q(y_1, y_2; \omega) \psi_j(y_1)\psi_k(y_2)\, dy_1\, dy_2 \tag{32}$$

and

$$J_j(x) = \frac{1}{\nu_j^2} \int_0^L \Phi_q(y, x, \omega)\psi_j(y)\, dy. \tag{33}$$

Note that the integral of $I_{jk}(\omega)$ possesses the Hermitian property.

$$\begin{aligned} I_{jk}(\omega) &= I_{kj}^*(\omega) \\ I_{jk}(-\omega) &= I_{jk}^*(\omega). \end{aligned} \tag{34}$$

Hence, the mean square values of both displacement and moment can be obtained by integration of both two spectral densities defined by Eqs. (28) and (29) over positive range of frequencies only, i.e.,

$$E[w^2(x, t)] = 2 \int_0^\infty \Phi_{DD}(x, x, \omega)\, d\omega \tag{35}$$

$$E[M^2(x, t)] = 2 \int_0^\infty \Phi_{MM}(x, x, \omega)\, d\omega \tag{36}$$

in which, the integrals I_{jk} and J_j can be replaced by their respective real parts and are evaluated in the exact form as follows:

$$\begin{aligned} I_{jk}(\omega) &= \frac{1}{\omega_j^2 \nu_k^2} \int_0^L \int_0^L \mathrm{Re}\{\Phi_q(y_1, y_2; \omega) \\ &\quad \psi_j(y_1)\psi_k(y_2)\, dy_1\, dy_2 \\ &= 4S_q(\omega) jk\pi^2 \\ &\left[2\mathrm{Re}\left\{\frac{1 + \exp[-(\bar{\alpha} + i\bar{\gamma})]}{[(\bar{\alpha} + i\bar{\gamma})^2 + (k\pi)^2][(\bar{\alpha} + i\bar{\gamma})^2 + (j\pi)^2]}\right\}\right. \\ &\left. + \bar{\alpha}\delta_{jk} \frac{1 + [\bar{\alpha}^2 + \bar{\gamma}^2](jk\pi^2)^{-1}}{[(\bar{\alpha} + i\bar{\gamma})^2 + (j\pi)^2][(\bar{\alpha} - i\bar{\gamma})^2 + (k\pi)^2]}\right] \end{aligned} \tag{37}$$

$$\begin{aligned} J_j(x, \omega) &= \frac{1}{\nu_j^2} \int_0^L \mathrm{Re}\{\Phi_q(y, x; \omega)\}\psi_j(y)\, dy \\ &= 2S_q(\omega)\mathrm{Re}\left\{\frac{1}{(\bar{\alpha} - i\bar{\gamma})^2](j\pi)^2}\right. \\ &[(\bar{\alpha} - i\bar{\gamma})\sin(j\pi\xi) - j\pi \cos(j\pi\xi) + j\pi e^{-(\bar{\alpha}-i\bar{\gamma})\xi}] \\ &+ \frac{1}{(\bar{\alpha} + i\bar{\gamma})^2](j\pi)^2}[(\bar{\alpha} - i\bar{\gamma})\sin(j\pi\xi) \\ &\left. + j\pi \cos(j\pi\xi) + (-1)j\pi e^{(\bar{\alpha}+i\bar{\gamma})(\xi-1)}]\right\}, \end{aligned} \tag{38}$$

where nondimensional parameters $\bar{\alpha}$ and $\bar{\gamma}$ are defined as follows:

$$\bar{\alpha} = \alpha(\omega)L; \quad \bar{\gamma} = \gamma(\omega)L; \quad \xi = x/L. \tag{39}$$

NUMERICAL EXAMPLE AND DISCUSSION

A thin corrugated metal sheet, the typical weather protection system for launch vehicles, (Fig. 1) is chosen as an example for application. We chose an element of a sheet (Fig. 2) and modeled it as an equivalent I beam, with properties $I = 0.0247$ in.^{4}, $A = 0.264$ in.^{2}, $b = 1.1$ in. Thus two nondimensional parameters in Eq. (27) become $\lambda_1 = 8.852$; $\lambda_2 = 3.3 \times 10^5$.

Table 1 lists the natural frequencies of the beam within both the Timoshenko theory, and the Bernoulli–Euler theory approximations. It can be seen, as expected, that the values of natural frequencies of the Timoshenko beam are less than those of the Bernoulli–Euler beam, and for the higher order modes there is a bigger difference between two approximations.

Figures 3 and 4 portray the exact spectral densities of the displacement and the moment at the midsection of the beam. The solid curve presents results associated with the Bernoulli–Euler theory the dashed one denotes the results obtained using the Timoshenko beam theory. Only odd-numbered modes contribute to the response, due to sinusoidal mode shapes and the calculations performed for the middle cross section of the beam. It is shown that within the same range of frequencies (0–1000 Hz), the spectral density of response for the Timoshenko beam has more peaks than that for the Bernoulli–Euler beam. In other words, for the Timoshenko beam there are generally more modal contributions to the re-

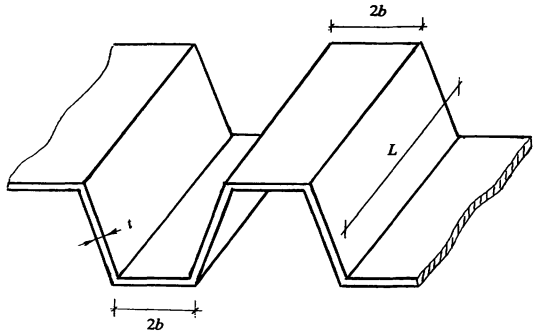

FIGURE 1 A corrugated metal sheet.

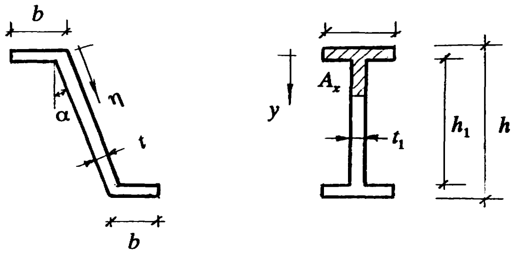

FIGURE 2 A typical segment of the corrugated sheet and the equivalent I beam.

Table 1 Natural Frequencies of Weather Protection System Modeled as Bernoulli–Euler Beam or Timoshenko Beam

	Natural Frequencies, f_i(Hz)	
i	Timoshenko Beam	Bernoulli–Euler Beam
1	1.739829	1.742619
2	6.926167	6.970474
3	15.46189	15.68357
4	27.19260	27.88190
5	41.91663	43.56546
6	59.39734	62.73426
7	79.37590	85.38831
8	101.5836	111.5276
9	125.7528	141.1521
10	151.6251	174.2619
11	178.9584	210.8569
12	207.5304	250.9370
13	237.1413	294.5025
14	267.6140	341.5533
15	298.7939	392.0891
16	330.5479	446.1104
17	362.7619	503.6167
18	395.3398	564.6084
19	428.2004	629.0853
20	461.2764	697.0474
21	494.5116	768.4949
22	527.8599	843.4274
23	561.2834	921.8451
24	594.7518	1003.748
25	628.2399	1089.137
26	661.7276	1178.010
27	695.1992	1270.369
28	728.6419	1366.213
29	762.0460	1465.542
30	795.4041	1568.356
31	828.7104	1674.656
32	861.9608	1784.441
33	895.1523	1897.712
34	929.2831	2014.467
35	961.3519	2134.708
36	994.3586	2258.434
37	1027.303	2385.645
38	1060.186	2516.341
39	1093.008	2650.523
40	1125.770	2788.190

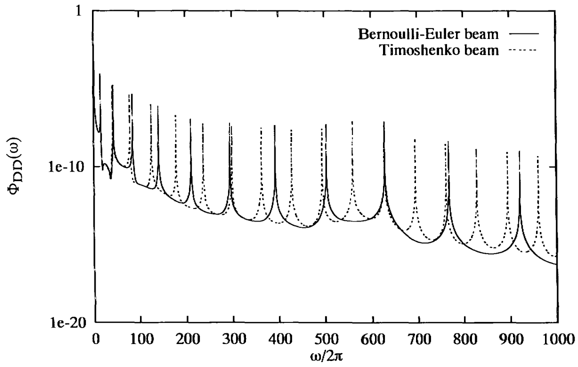

FIGURE 3 Spectral densities of the displacement at the midpoint of the beam.

sponse of the structure due to the increased modal density of natural frequencies. For the spectral density of displacement (Fig. 3) the first peak is the largest within both beam theories. The difference of values of the first peak for two beams is very small. Hence the value of areas under the spectral density curves, or the mean square values of displacement are very close, with attendant difference constituting about 1.2%.

However, for the spectral density of moment (Fig. 4), the higher order modes dominate in formulating the response of the structure. This is because there is a high peak around 585 Hz in the loading spectral density $S_q(\omega)$ (Fig. 5), and in addition there appears a factor j^2k^2 in the expression for H_M. Therefore the contribution of higher modes is significant. The difference between the values of spectral densities calculated via the Timoshenko and the Bernoulli–Euler theories, and their associated contribution to the mean square value of moment is remarkable. The application of the Bernoulli–Euler theory yields 5.5524 $(\text{lb} \cdot f \times \text{in.})^2$. Use of Timoshenko theory results

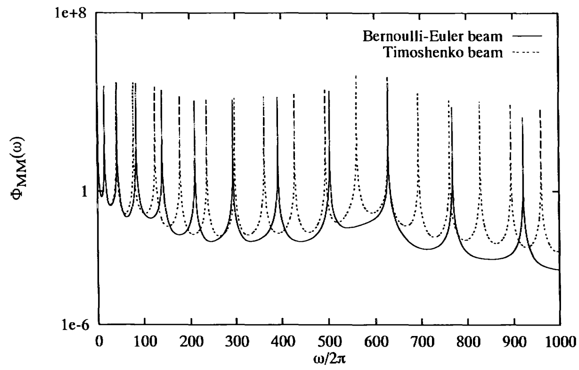

FIGURE 4 Spectral densities of the bending moment at the midpoint of the beam.

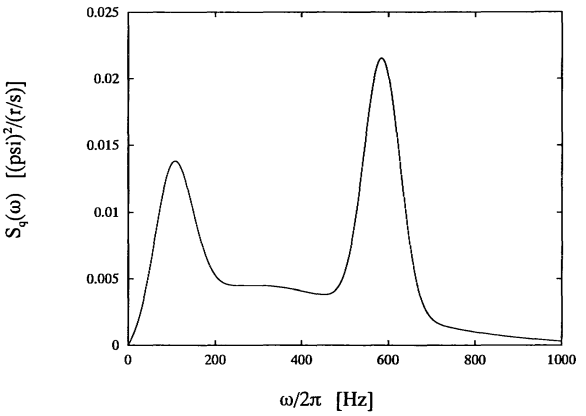

FIGURE 5(a) Spectral density of acoustic loading.

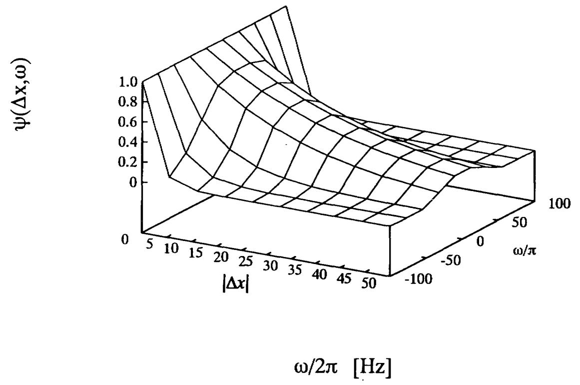

FIGURE 5(b) Variation of the decay function $\psi(\Delta x, \omega) = e^{-\alpha(\omega)|\Delta x|}$.

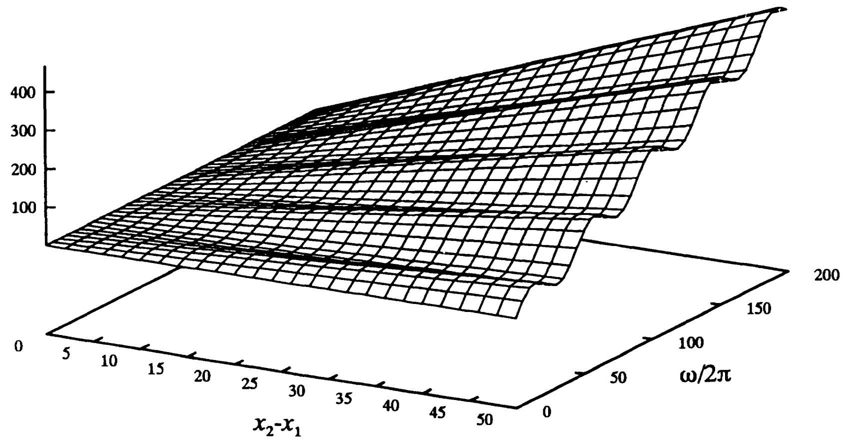

FIGURE 5(c) Phase function $\gamma(\omega)(x_2 - x_1)$.

11.3205 $(\mathrm{lb.}_f \times \mathrm{in.})^2$, with attendent difference of about 50%.

The value of mean square for the Timoshenko beam is almost twice that for the Bernoulli beam. This implies that choosing the Bernoulli–Euler beam model may not be safe for the design of the weather protection systems. As a result the refined Timoshenko beam model rather than the simple Bernoulli–Euler theory should be chosen for the space shuttle weather protection structure under the acoustic loading.

APPENDIX

The model of cross-spectral density for acoustic loading utilized in this study is given by Eq. (18) where the spectral density $S_q(\omega)$ [Fig. 5(a)] is approximated by the following expression

$$S_q(\omega) = \sum_{j=1}^{3} a_j \left|\frac{\omega}{\omega_j}\right| \exp\left[-\frac{b_j^2 - (1 - c_j)^2}{2(1 - c_j)}\right]; \qquad \text{(A.1)}$$

$$\omega_j = 2\pi f_j; \quad b_j = \left(\frac{\omega}{\omega_j} - c_j\right)$$

with parameters of:

j	1	2	3
a_j[(psi)²/(r/s)]	1.1×10^{-2}	4.5×10^{-3}	1.9×10^{-2}
f_j(Hz)	105	290	585
c_j	0.8	−1.0	0.995

Decay function $\alpha(\omega)$ is defined as

$$\alpha(\omega) = 0.03 + 5.0 \times 10^{-5} \frac{\omega}{2\pi}\left(\left|\frac{\omega}{2\pi} - 22\right|\right) \qquad \text{(A.2)}$$

and has a width dimension of [1/ft.]. The experimentally evaluated function $\psi(\Delta x, \omega) = \exp(-\alpha(\omega)|\Delta x|)$ is depicted in Fig. 5(b). Phase function $\gamma(\omega)$ reads

$$\gamma(\omega) = 0.77 \sin\left(0.14 \frac{\omega}{2\pi}\right) + 0.045 \frac{\omega}{2\pi}, \quad \text{(A.3)}$$

having a dimension [degree/ft.]. The function $\gamma(\omega)(x_1 - x_2)$ is given in Fig. 5(c).

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