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A Space-Frequency Data Compression Method for Spatially Dense Laser Doppler Vibrometer Measurements

When spatially dense mobility shapes are measured with scanning laser Doppler vibrometers, it is often impractical to use phase-separation modal parameter estimation methods due to the excessive number of highly coupled modes and to the prohibitive computational cost of processing huge amounts of data. To deal with this problem, a data compression method using Chebychev polynomial approximation in the frequency domain and two-dimensional discrete Fourier series approximation in the spatial domain, is proposed in this article. The proposed space-frequency regressive approach was implemented and verified using a numerical simulation of a free-free-free-free suspended rectangular aluminum plate. To make the simulation more realistic, the mobility shapes were synthesized by modal superposition using mode shapes obtained experimentally with a scanning laser Doppler vibrometer. A reduced and smoothed model, which takes advantage of the sinusoidal spatial pattern of structural mobility shapes and the polynomial frequency-domain pattern of the mobility shapes, is obtained. From the reduced model, smoothed curves with any desired frequency and spatial resolution can be produced whenever necessary. The procedure can be used either to generate nonmodal models or to compress the measured data prior to modal parameter extraction.

INTRODUCTION

Experimental modal analysis (EMA) can be viewed as a technique for reducing experimental data obtained from standard dynamic tests using a modal model. The most frequently used methodology (see for instance Ewins, 1984 consists of measuring frequency response functions (FRFs), and then estimating the parameters of a modal model, either directly from the FRFs or from their inverse Fourier transforms, the impulse response functions. These methods are known as phase-separation methods, in contrast with the phase-resonance methods, where a set of sinusoidal forces are amplitude and phase tuned to excite each mode separately. From the reduced modal model, any FRF can be synthesized with any desired frequency resolution at any of the measured degrees of freedom of the structure.

When using scanning laser Doppler vibrome-

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ters (LDVs), one usually measures the relative magnitudes and phases of the outward velocities at each surface location while exciting the structure harmonically (Li et al., 1993). Using the force signal as a reference, this is equivalent to measuring one frequency line of the FRFs. The outward velocity field measured in this way over the structure surface at each frequency is called a *mobility shape*.

Storing and processing with commercial EMA software hundreds of those mobility shapes, each one with thousands of elements, is prohibitive. To be able to use phase-separation EMA techniques, it is absolutely necessary to compress the data. This is a new situation in EMA applications, which is not likely to occur when using conventional instrumentation. Dippery et al. (1994) recently proposed multi-input/multi-output (MIMO) modal test data compression techniques based on the singular value decomposition of the FRF matrix, which are not applicable to single-reference LDV tests.

Furthermore, there are practical limitations to the use of EMA, which are generally due to the modal model itself. Whenever the number of modes in the frequency range of interest is too high and/or the modes are strongly coupled by damping, applying EMA is not practical. This is the case, for instance, of shell modes of aircraft fuselages (Li et al., 1993) and acoustic modes in cavities (Lyon, 1975).

When EMA is not applicable, there are two other possible approaches. One consists of using the FRFs directly, which is referred to as using a *response model*. When using spatially dense laser measurements, response models become too cumbersome due to the high spatial resolution. The other is the statistical approach, the so-called statistical energy analysis (SEA) developed by Lyon (1975). SEA is an approximate method that works reasonably well when the number of modes is very high. This leaves uncovered a frequency range where there are too many modes for EMA but too few for SEA.

Halvorsen et al. (1991) recently investigated new nonmodal identification methods applicable in this intermediate frequency range; but, again, their methods are only suitable for MIMO modal tests because they are based on the singular value decomposition of the FRF matrix.

Arruda (1993) proposed a spatial modal parameter estimation technique suitable for LDV measurements but only applicable to lightly damped structures. In this study, the possibility of compressing spatially dense mobility shapes measured with LDVs for either applying phase-separation modal estimation methods or storing reduced response models is investigated. The proposed space-frequency regressive approach is described and verified using a numerical simulation example of a free-free-free-free rectangular aluminum plate. A set of mobility shapes was numerically simulated by modal superposition. To make the simulation more realistic, mode shapes, obtained experimentally using a scanning LDV, were used to synthesize the mobility shapes.

SPACE-FREQUENCY REGRESSION

Modal parameter estimation may be interpreted as a particular space-frequency (or space-time) regression of measured FRFs, one which uses a modal model (usually with a nonproportional viscous damping model). From a set of $N_i \times N_o \times N_\omega$ complex FRF values, where N_i is the number of input forces (or references), N_o the number of output velocities (or responses), and N_ω the number of frequency lines, a reduced set of $(N_o + 1)N$ complex modal parameter values is estimated, where N is the number of estimated complex modes. From this reduced model, any FRF between the measured degrees of freedom may be synthesized (reciprocity), with any desired frequency resolution. The reduction is achieved as $N \ll N_\omega$.

Starting from this regressive viewpoint of modal parameter estimation, an alternative formulation may be sought where the modal model is discarded. This may be necessary because either N is too large or the available modal parameter estimation methods fail to work in the presence of strongly coupled modes.

The proposed frequency-spatial regression generates a reduced, smoothed model of the measured data that reproduces measured data as well as interpolates in frequency and space.

Spatial Domain Regression

The data compression in the spatial domain can be done using a two-dimensional (2-D) Fourier series, which takes advantage of the sinusoidal pattern of the mobility shapes of the structural surfaces sufficiently far from the boundaries. This is a well-known property of the wave equation solution for solids when the near-field effects are neglected.

For 2-D mobility shapes mapped over a rectangular grid, say H_{mn}, one could think of using the 2-D discrete fourier transform (DFT). The difficulty in using the DFT is due to the fact that its implicit *periodization* introduces high-frequency components that account for the sharp edges present in the *wrapped-around* data. This phenomenon is known as *leakage*. In the data smoothing process, leakage is prejudicial, as it causes distortion of the low-pass filtered data. The usual way to reduce leakage is windowing, but this technique is not suitable in the case of finite length, spatial domain data. To overcome the leakage problem, the proposed technique consists of representing the data by a 2-D regressive discrete Fourier series (RDFS) proposed by Arruda (1992), which will be briefly reviewed here. Unlike the DFT, in the RDFS the original length of the data is not assumed to be equal to the signal period nor is the number of frequency lines assumed to be equal to the number of data points, i.e.,

$$H_{mn} = \sum_{k=-p}^{p} \sum_{l=-q}^{q} Z_{kl} W_{\mathcal{M}}^{mk} W_{\mathcal{N}}^{\ln} + \varepsilon^{mn}; \quad m = 0, M-1; \quad n = 0, N-1, \tag{1}$$

where H_{mn} represents the discretized data with constant resolution Δx and Δy, $W_{\mathcal{M}} = \exp(\imath 2\pi/\mathcal{M})$, $W_{\mathcal{N}} = \exp(\imath 2\pi/\mathcal{N})$, and ε_{mn} accounts for the noise and higher frequency contents of $\mathbf{H}$. The length of the data in x is $M\Delta x$ but the period of the RDFS is $\mathcal{M}\Delta x > M\Delta x$. The data reduction is achieved because $p \ll M$ due to the expected low wavenumber of the surface. In the y direction $\mathcal{N}\Delta y > N\Delta y$ and $q \ll N$. The $M \times N$ data in $\mathbf{H}$ are represented by a $(2p + 1) \times (2q + 1)$ complex matrix $\mathbf{Z}$ of elements Z_{kl}.

The RDFS is an approximation instead of an interpolation of H_{mn}. The Euler–Fourier coefficients cannot be calculated by the DFT. Rewriting Eq. (1) in matrix form:

$$\mathbf{H} = \mathbf{W}_{\mathcal{M}}\mathbf{Z}\mathbf{W}_{\mathcal{N}} + \varepsilon. \tag{2}$$

The least-squares solution is given by

$$\mathbf{Z} = (\mathbf{W}_{\mathcal{M}}^T\mathbf{W}_{\mathcal{M}})^{-1}\mathbf{W}_{\mathcal{M}}^T\mathbf{H}\mathbf{W}_{\mathcal{N}}^T(\mathbf{W}_{\mathcal{N}}\mathbf{W}_{\mathcal{N}}^T)^{-1}, \tag{3}$$

where the matrices to be inverted have a very small size, $(2p + 1) \times (2p + 1)$ and $(2q + 1) \times (2q + 1)$, respectively. The smoothed data $H^{(s)}$ may be obtained from

$$\mathbf{H}^{(s)} = \mathbf{W}_{\mathcal{M}}\mathbf{Z}\mathbf{M}_{\mathcal{N}}, \tag{4}$$

where $\mathbf{W}_{\mathcal{M}}$ and $\mathbf{W}_{\mathcal{N}}$ can be calculated for the desired spatial resolution.

The reduction of the data is achieved as $\mathbf{Z}$ represents the data using only $(2p + 1)(2q + 1)$ values, instead of the original MN values. The formulation of the RDFS for nonequally spaced data given by Arruda (1992) can be used in place of the formulation above when the mobility shapes are mapped over a nonregular, arbitrary grid.

Frequency-Domain Regression

The dependence of the FRF amplitudes upon frequency is approximated using polynomial regression. Orthogonal Chebychev polynomials are used in a way that is similar to the orthogonal polynomial modal parameter estimation method formulated by Shih et al. (1988). However, instead of curve fitting the FRFs directly, the RDFS coefficients, taken as virtual measurement stations, are curve fitted.

Because the mobility shapes used here are spatial distributions of FRF frequency lines, the RDFS coefficients are, in fact, linear combinations of these FRFs. Therefore, the RDFS coefficients as a function of the frequency will have the same poles as the original FRFs. This allows us to write:

$$\mathbf{Z}(\omega) = \frac{\sum_{k=0}^{r} \mathbf{a}_k \phi_k(\imath\omega)}{\sum_{k=0}^{s} \mathbf{b}_k \phi_k(\imath\omega)}, \tag{5}$$

where ϕ_k is the kth Chebycheff polynomial, $\mathbf{a}_k$ is the kth polynomial matrix coefficient (of the same dimensions as $\mathbf{Z}$), $\mathbf{b}_k$ is the kth denominator polynomial coefficient, a scalar for single excitation, and $\imath = \sqrt{-1}$. Equation (5) may be arranged into a least-squares problem and easily solved for $\mathbf{a}_k$ and $\mathbf{b}_k$, which form the final reduced model. Making, without loss of generality, $\mathbf{b}_s = 1$, one can write

$$\sum_{k=0}^{s-1} \mathbf{Z}(\omega)^H \phi_k^* \mathbf{b}_k^* - \sum_{k=0}^{r} \phi_k^* \mathbf{I} \mathbf{a}_k^H = -\phi_s^* \mathbf{Z}(\omega)^H \tag{6}$$

where $\mathbf{I}$ is the identity matrix, ^{H} denotes the complex conjugate transpose of a matrix, and * denotes the complex conjugate of a scalar. Arranging Eq. (6) in matrix form for ω varying produces the linear system of equations

$$\begin{bmatrix} \mathbf{Z}^H\phi_0^*\mathbf{Z}^H\phi_1^* \cdots \mathbf{Z}^H\phi_{s-1}^* - \phi_0^*\mathbf{I} - \phi_1^*\mathbf{I} \cdots -\phi_r^*\mathbf{I} \\ \vdots \end{bmatrix} \begin{Bmatrix} \mathbf{b}_0^H \\ \mathbf{b}_1^H \\ \vdots \\ \mathbf{b}_{s-1}^H \\ \mathbf{a}_0^H \\ \mathbf{a}_1^H \\ \vdots \\ \mathbf{a}_r^H \end{Bmatrix} = \begin{Bmatrix} -\phi_s^*\mathbf{Z}^H \\ \vdots \end{Bmatrix}. \tag{7}$$

It is important to mention that the frequency ω should be scaled so that the frequency range is always [0, 1]. This improves the condition of the least-squares problem.

If the order of the polynomial of the denominator is s and the order of the numerator is r, the order of the reduced model will be (for a single reference) $(2p + 1) \times (2q + 1) \times r + s$. We used $r = s - 1$ in the examples shown here. The interpolation of the RDFS coefficients from the polynomial coefficients is immediate, with any arbitrary frequency resolution. It must be mentioned here that it is possible to proceed with the computation of the structure eigenvalues and eigenvectors at this point. For this purpose, a companion matrix can be built with the computed polynomial coefficients, as explained in detail by Shih et al. (1988).

Proposed Procedure

A prototype software was developed to verify the proposed formulation. The algorithm may be summarized as follows:

1. Apply a robust data smoothing technique to eliminate outliers from measured mobility shapes.
2. Apply the RDFS to each mobility shape.
3. Curve fit the RDFS coefficients as functions of the frequency (using them as virtual measurement stations).
4. Obtain the polynomial coefficient matrices $\mathbf{a}_k$ and scalars $\mathbf{b}_k$, which form the reduced model.
5. Synthesize mobility shapes at any desired frequency with arbitrary spatial resolution, or any desired FRF at any location within the scanned area with arbitrary frequency resolution from the reduced model.

The robust data smoothing technique used to remove the noise spikes common in LDV measurements was the well-known median filter (see for instance Huang, 1981). The removal of the outliers is important because they can bias the RDFS approximation, which is made in a least-squares sense.

The number of wavenumber lines in the RDFS may be determined by visual inspection of the smoothed mobility shape compared to the measured one or by an error norm criterion. The number of frequency lines is increased until the error criterion is satisfied.

The determination of the order of the polynomial is more involved. Although orthogonal Chebychev polynomials are used, the orthogonality does not insure, in this case, a diagonal matrix in the linear system of equations that must be solved to determine the polynomial coefficients, Eq. (7). The Chebychev polynomials improve the condition of the matrix to be inverted, but do not insure its full rank. If the order of the polynomial increases too much (in our case this happened above order 12 or higher), the system may become numerically unstable and the polynomial approximation may fail.

SIMULATION EXAMPLE

To illustrate the use of the proposed space-frequency data compression method, a numerical simulation of a free-free-free-free rectangular aluminum plate was used. To make the simulation more realistic, the mode shapes used to synthesize the mobility shapes were obtained experimentally.

A 16.75 × 18 and 0.125 in. thick aluminum plate was hung from its two upper corners (larger sides vertical) by fish lines to approximate the free-free-free-free boundary condition. The first five modes of the plate were obtained with a single-input, phase-resonance method consisting of exciting the plate with a sinusoidal force using an electromagnetic shaker and looking for the frequencies where the velocities measured with the LDV are all in phase with the force signal. At those frequencies the mobility shape was assumed to be approximately equal to the mode shape, which is a reasonable assumption for low-damped structures with well-separated modes. The measured natural frequencies were approximately: 54.5, 72.7, 96.6, 128.39, and 135.28 Hz. Figure 1 shows a scheme of the experimental setup.

The detailed description of this approximate modal analysis method using an LDV is given by Sun et al. (1993). The measurement grid was 70 $\times$ 65. No correction for the angle between the laser beam direction and the normal to the plate surface was needed, because the LDV head was placed at approximately 10 ft away from the plate. Given the plate dimensions, the angular error was very small, under 0.2%.

Figure 2 shows the first five mode shapes measured with the LDV. The mode shapes were filtered using a median filter with a 25-point cell. A set of mobility shapes was synthesized by mode superposition using these first five modes. The synthesis was made using the formulation of the FRF of a proportionally damped structure:

$$H_{ij}(\omega) = \sum_{k=1}^{s} \frac{\psi_{ik}\psi_{jk}}{\omega_k^2 - \omega^2 + \imath 2\xi\omega\omega_k} \tag{8}$$

where ψ_{ik} is the ith element of the kth mode shape vector. The excitation station was kept constant, $j = 1$, while the measurement stations varied, $i = 1, MN$. For each frequency ω, the elements of the vector H_{ij}, $j = 1$, $i = 1, MN$ were arranged in matrix form following the mapping that was

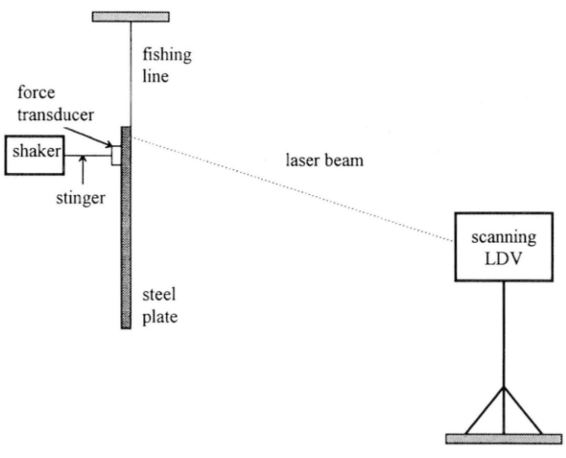

FIGURE 1 Scheme of the experimental setup.

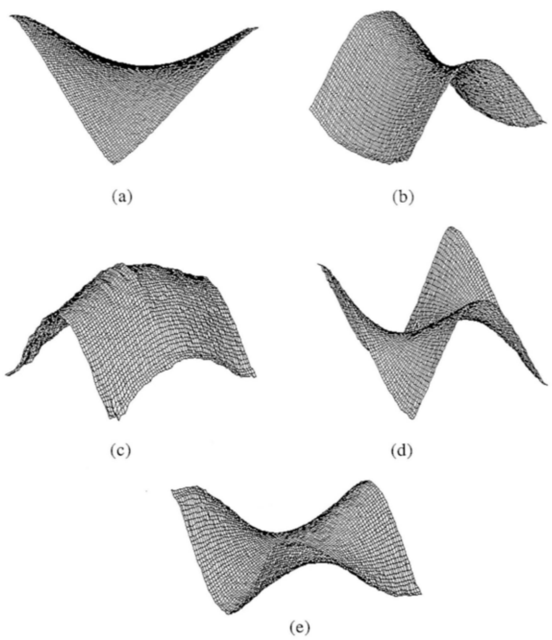

FIGURE 2 First five mode shapes of the rectangular plate measured with a scanning LDV. (a) mode 1; (b) mode 2; (c) mode 3; mode 4; mode 5.

used when measuring the mode shapes with the LDV, thus producing a mobility shape H_{mn}, $m = 0, M - 1$, $n = 0, N - 1$. In the example plate, $M = 70$ and $N = 65$. The simulated FRFs are unscaled; each mode shape was normalized with unitary maximum amplitude. The decibel scale was used when plotting the FRF magnitudes and the reference value is 1.

The proposed space-frequency data compression method was applied to the synthesized mobility shapes in the frequency range 40–108 Hz with a 2-Hz resolution.

Figure 3 shows a typical comparison of an FRF synthesized using the measured modal parameters with the corresponding FRF synthesized using the reduced model. The FRF synthesized using the compressed model was computed with a finer frequency resolution of 0.5 Hz. It can be observed that it interpolates the simulated FRF values almost exactly. The average normalized error between simulated and synthesized FRFs was below 5% for all the FRFs used in the data compression process; its mean value was 3.2%.

Figure 4 shows a typical comparison of a mobility shape synthesized using the measured modal parameters with the corresponding mobility shape synthesized using the reduced model. Both the real and imaginary parts of the mobility shapes are shown. The synthesized surfaces are

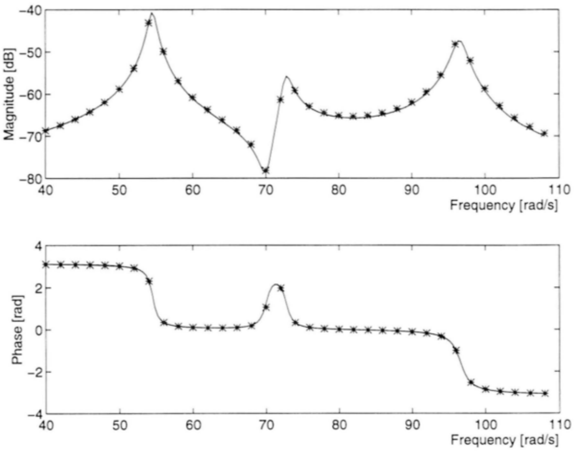

FIGURE 3 Typical comparison of simulated and synthesized FRFs with excitation at grid location $m = 0$, $n = 0$, and response at grid location $m = 4$, $n = 4$. (a) Magnitude; (b) Phase. *** simulated using measured mode shapes; —synthesized using the compressed model.

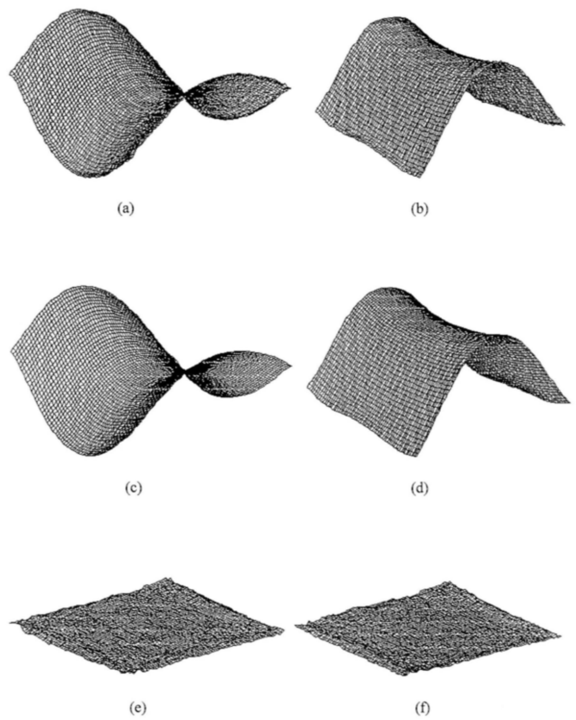

FIGURE 4 Typical comparison of simulated and synthesized mobility shapes, at 80 Hz. Simulated using measured mode shapes; (a) real; (b) imaginary. Synthesized using the compressed model; (c) real; (d) imaginary. Difference between simulated and synthesized; (e) real; (f) imaginary.

smoother due to the spatial filtering effect of the RDFS approximation, but it can be observed that the mobility shape information is well preserved. The average normalized error for the 17 mobility shapes in the frequency range 40–108 Hz with a 4-Hz interval was below 9%; its mean value was 2.3%.

The original data consisted of 35 mobility shapes, each with 4550 complex FRF values. Using the RDFS with $p = q = 3$ and polynomials of order $s = 5$, the reduced model consists of only 300 complex values. The reduction coefficient achieved in this case was 530 times, but could be even larger if mobility shapes with higher spatial resolution had been used.

The good agreement between the original FRFs and mobility shapes and corresponding functions synthesized using the compressed model, of which the results in Figures 3 and 4 are typical, shows that the regressive model represents the measured data adequately. Besides, the reduced model can produce smoothed curves with any desired frequency and spatial resolutions.

CONCLUSIONS

A space-frequency regression method using Chebychev polynomials and 2-D discrete Fourier series approximation was proposed. The method may be used to compress spatially dense mobility shapes measured with scanning laser Doppler vibrometers. Large reduction rates, easily over 1,000, can be obtained without significant loss of information, and with the benefit of data smoothing.

With the proposed method, a reduced and smoothed model can be obtained, which takes advantage of the sinusoidal spatial pattern of structural mobility shapes and of the polynomial frequency-domain pattern of the FRFs. The reduced model can be stored economically, and later used to synthesize smoothed mobility shapes or FRFs with any desired spatial and frequency resolution. It can also make the modal parameter extraction from spatially dense mobility shapes measured with LDVs viable.

The technique was implemented and verified using a numerical simulation example of a free-free-free-free rectangular aluminum plate, where the mobility shapes were synthesized using modal superposition. The mode shapes used in the simulation were obtained experimentally, which makes the example more realistic.

An issue that deserves further investigation is the optimization of the number of wavenumber lines of the RDFS and of the order of the polynomial. Some kind of quantitative measurement of the preservation of information in the compressed model could be used to determine the optimal order of both the RDFS and the polynomial approximations. Also, the application of the proposed method to a realistic experimental example will require the on-line implementation of the proposed method, such that the measured operating shapes can be compressed and stored as they are measured with the scanning LDV.

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