B. C. Huang

Department of Engineering
Mechanics
Tongji University
Shanghai 20092, China

K. M. Lam

A. Y. T. Leung

Y. K. Cheung

Department of Civil and Structural
Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong

Equivalent Modal Damping Ratios of a Composite Tube-Type Tall Building to Dynamic Wind Loading

This article discusses computing methods of the equivalent modal damping ratio for a composite tube-type tall building. The equivalent damping ratio of a composite structure under wind-induced vibrations encompasses the combined effects from the different values of damping, mass, and stiffness of the external and internal tubes. A detailed method incorporating Rayleigh damping and the transfer matrix method is proposed for the determination of the equivalent modal damping ratio. A simplified method for engineering applications is also proposed. Results for a 40-story building are presented exemplifying the computation and relationships between the properties of the external and internal tubes. © 1996 John Wiley & Sons, Inc.

INTRODUCTION

For composite tube-type tall buildings, different materials are used for the internal and external tubes. As a result, the damping ratios of the internal and external tubes are different. For wind-induced vibrations of tall buildings, the first or first two modes are predominant. In engineering applications, modal superposition adopting Rayleigh damping presents itself as a simple method for the analysis of dynamic responses of tall buildings to wind action (Saul and Peyrot, 1976). When applied to a tall building of composite tube-in-tube

structure, it is necessary to determine equivalent modal damping ratios. However, there has not been a quantitative analytical method to obtain the equivalent modal damping ratios, which are related to the distributions of stiffness and mass of the internal and external tubes.

In this article we present two methods for the determination of equivalent damping ratios of a composite tube-in-tube tall building. The first one is a detailed method based on the transfer matrix method and numerical integration procedure. The second is a simplified method suited to engineering applications.

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DAMPING MATRIX

Composite tube-in-tube structures are not uniform systems. There are different damping behaviors and mechanisms for the internal and external tubes. Assume that each tube is represented by a damping matrix of its own. The damping matrix of the whole structure can be represented as:

$$[C] = [C]_F + [C]_w \quad (1)$$

where $[C]$ is the damping matrix of the whole structure and $[C]_w$, $[C]_F$ are the damping matrices of the internal and external tube, respectively. Each matrix is proportional to the mass and stiffness matrices of the corresponding tube as follows:

$$[C]_w = a_w[M]_w + b_w[K]_w \quad (2a)$$

$$[C]_F = a_F[M]_F + b_F[K]_F \quad (2b)$$

where

$$a_w = \frac{2\omega_{1w}\omega_{2w}}{\omega_{1w}^2 - \omega_{2w}^2} (\zeta_{2w}\omega_{1w} - \zeta_{1w}\omega_{2w}) \quad (3a)$$

$$a_F = \frac{2\omega_{1F}\omega_{2F}}{\omega_{1F}^2 - \omega_{2F}^2} (\zeta_{2F}\omega_{1F} - \zeta_{1F}\omega_{2F}) \quad (3b)$$

$$b_w = \frac{2}{\omega_{1w}^2 - \omega_{2w}^2} (\zeta_{1w}\omega_{1w} - \zeta_{2w}\omega_{2w}) \quad (3c)$$

$$b_F = \frac{2}{\omega_{1F}^2 - \omega_{2F}^2} (\zeta_{1F}\omega_{1F} - \zeta_{2F}\omega_{2F}). \quad (3d)$$

In Eqs. (2) and (3), $[M]_w$, $[M]_F$, $[K]_w$, $[K]_F$ represent the mass matrices and stiffness matrices of the internal and external tubes; ω_{1w} , ω_{1F} , ω_{2w} , ω_{2F} are the first two modal frequencies; and ζ_{1w} , ζ_{1F} , ζ_{2w} , ζ_{2F} are the first two modal damping ratios of the internal and external tubes. The subscripts w and F denote the internal and external tubes, respectively. It should be noted that the first two vibrational modes are considered in this analysis.

Rayleigh damping is employed in this analysis. The advantage is that the damping ratio is a commonly used parameter in engineering applications and Rayleigh damping allows the damping effect to be related directly to the damping ratio. A number of physical mechanisms have been proposed for structural damping in engineering structures. Essentially, it is agreed that the principal mechanism involves frictions among the molecules in the structure materials. Based on internal

viscous damping, structural damping is expressed in the form of a damping matrix. It is proposed here that this damping matrix is suitable not only for analysis of the whole structural system, but also for individual substructures and elements. The damping matrix proposed is proportional to the stiffness matrix. Some other works on structural damping include Liang and Lee (1991) and Leung (1988) that deal with the dynamic substructure method with light or heavy damping.

FORMULATION OF EQUATION OF MOTION

A composite-type tall building with a steel framed outside tube and a reinforced concrete inside tube is considered. The building is assumed symmetrical about the two horizontal axes and torsional motion is neglected.

For the external tube, the equivalent plate method of Coull and Bose (1975) is used. Flexural and shear deformation of the elements are included in the equivalent shear rigidity. The internal tube is modeled as bending and shear bars. The mode in which the internal tube works in coordination with the external tube is shown in Fig. 1.

Under dynamic wind loading $F(i, t)$ on each story, the equation of motion in the along-wind direction is:

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{F\} \quad (4)$$

FIGURE 1 Analysis model of tube-in-tube system.

in which $\{y\}$, $\{\dot{y}\}$, $\{\ddot{y}\}$ are the along-wind displacement, velocity, and acceleration vectors; $[M]$ is the inertia matrix of the tube-in-tube system, with a diagonal matrix for the mode shown in Fig. 1; and $[C]$, $[K]$ are the damping matrix and stiffness matrix of the tube-type building. The stiffness matrix $[K]$ can be expressed as:

$$[K] = [K]_w + [K]_F \quad (5)$$

where $[K]_w$, $[K]_F$ are the statistically condensed stiffness matrices of the internal and external tubes, respectively. It is shown from Eq. (4) that mode coupling is caused by the damping matrix $[C]$. So the usual mode-superposition method cannot be adopted to solve the equation. The response must be obtained by integrating these equations simultaneously. The Newmark β method is employed in this study.

ANALYSIS OF STIFFNESS MATRIX BY TRANSFER MATRIX METHOD

According to the model and hypotheses mentioned above, a transfer relationship and a transfer matrix can be obtained to relate the forces at the upper and lower ends of the i th story of the tall building (Fig. 2).

$$\begin{bmatrix} U \\ R \end{bmatrix}_{w,i+1} = [T]_{w,i} \begin{bmatrix} U \\ R \end{bmatrix}_{w,i} + [P]_{w,i} \quad (6a)$$

$$\begin{bmatrix} U \\ R \end{bmatrix}_{F,i+1} = [T]_{F,i} \begin{bmatrix} U \\ R \end{bmatrix}_{F,i} + [P]_{F,i} \quad (6b)$$

$$[T]_{wi} = \begin{bmatrix} 1 & h_i & \frac{h_i^2}{2E_{wi}I_{wi}} & \frac{h_i^3}{6E_{wi}I_{wi}} - \frac{h_i}{G_{wi}A_{wi}} \\ 0 & 1 & \frac{h_i}{E_{wi}I_{wi}} & \frac{h_i^2}{2E_{wi}I_{wi}} \\ 0 & 0 & 1 & h_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7a)$$

$$[T]_{Fi} = \begin{bmatrix} 1 & h_i & \frac{h_i^2}{2E_{Fi}I_{Fi}} & \frac{h_i^3}{6E_{Fi}I_{Fi}} - \frac{1}{D_{Fi}} \\ 0 & 1 & \frac{h_i}{E_{Fi}I_{Fi}} & \frac{h_i^2}{2E_{Fi}I_{Fi}} \\ 0 & 0 & 1 & h_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7b)$$

FIGURE 2 Transfer matrix model of structure element.

In Eq. (6), the associated vectors are defined by:

$$\begin{bmatrix} U \\ -\frac{U}{R} \end{bmatrix}_{w(F),i+1} = \begin{bmatrix} y & \vartheta & M & V \end{bmatrix}_{w(F),i+1}^T$$

$$\begin{bmatrix} U \\ -\frac{U}{R} \end{bmatrix}_{w(F),i} = \begin{bmatrix} y & \vartheta & M & V \end{bmatrix}_{w(F),i}^T$$

$$[P]_{w(F),i} = [0 \quad 0 \quad 0 \quad \Delta V]_{w(F),i}^T$$

The symbols used in Eqs. (6) and (7) are shown in Fig. 2. E_{wi} , E_{Fi} and I_{wi} , I_{Fi} represent the modulus of elasticity and the moments of inertia of the internal and external tubes; I_{wi} is obtained by dividing the shear wall into a number of sections. Formulation of I_F follows Coull and Bose (1975); A_w is the shear area of internal tube (webs only) and D_F is the shear rigidity of the equivalent external tube.

A unit force ($\Delta V_i = 1$) is applied to the internal or external tube when using the transfer matrix method. It is exerted to each story in succession. Hence the horizontal displacement at each story is obtained, and is represented as a column of $n \times n$ flexibility matrices. The stiffness matrices are the inversion of these flexibility matrices $[F]$.

$$[K]_w = [F]_w^{-1} \quad (8a)$$

$$[K]_F = [F]_F^{-1} \quad (8b)$$

In comparison with the usual method, the present analysis allows the flexural and shear deforma-

tions to be considered at the same time. It also allows the tube-in-tube structure with variation along height to be analyzed using the transfer matrix. $[\mathbf{F}]$ may be formed rapidly and easily. The matrices of order 4 need to be multiplied each time. These are clear advantages of this model and method.

MODEL OF DYNAMIC WIND LOADING

The fluctuating wind forces in the along-wind direction are:

$$F(i, t) = \rho \mu_D(i) \bar{U}(i) u(t) A(i) \quad (9)$$

in which ρ is density of the air; $\mu_D(i)$ is the drag coefficient; $A(i)$ is the windward area of the building; $\bar{U}(i)$, $u(t)$ are mean and fluctuating wind velocities, where $u(t)$ was modeled by Davenport (1961) and Vaicaitis et al. (1975) as:

$$u(t) = \sqrt{2} \sum_{k=1}^n [2S(\omega_k) \Delta\omega]^{1/2} \cos(\omega_k t + \phi_k) \quad (10)$$

where $S(\omega_k)$ is the Davenport spectrum of the fluctuating wind.

EQUIVALENT DAMPING RATIOS

In general, two equivalent principles should be considered in the analytical computation of the equivalent damping ratios. The first is the maximum acceleration of the top story, and the next is the base overturning moment. The former may ensure human comfort under wind loading. The latter is important in the design of the structural strength of the whole building. The base overturning moment is calculated from the equivalent static loads. In other words, the maximum acceleration of the top or the overturning moment with the equivalent damping ratios should be equal to the practical values of the structure. The Rayleigh damping matrix, which is related to equivalent damping ratios ζ_e , is given by

$$[\mathbf{C}] = a[\mathbf{M}] + b[\mathbf{K}] \quad (11)$$

where $[\mathbf{M}]$, $[\mathbf{K}]$ are the mass and stiffness matrix of the composite tube. The formulae for the proportional factors a and b are similar to Eq. (3), but the damping ratios and frequencies are ζ_{e1} , ζ_{e2} and ω_1 , ω_2 of the composite tube.

A SIMPLIFIED METHOD

Consider two single degree of freedom systems shown in Fig. 3. The equation of motion with the two tubes working in coordination with each other in the horizontal direction is

$$(M_1 + M_2)\ddot{y} + (C_1 + C_2)\dot{y} + (K_1 + K_2)y = F(t) \quad (12a)$$

or

$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = F(t)/(M_1 + M_2) \quad (12b)$$

in which

$$2\zeta\omega = (C_1 + C_2)/(M_1 + M_2) \quad (13a)$$

$$\omega^2 = (K_1 + K_2)/(M_1 + M_2). \quad (13b)$$

It is assumed that

$$C_1 = 2\zeta_1\omega_1 M_1 \quad (14a)$$

$$C_2 = 2\zeta_2\omega_2 M_2 \quad (14b)$$

where ζ_1 , ζ_2 are the critical damping ratios of the M_1 , M_2 systems.

Substituting Eq. (14a) and (14b) into Eq. (13a), we get

$$\zeta = \zeta_1 \sqrt{\alpha\beta} + \zeta_2 \sqrt{(1-\alpha)(1-\beta)} \quad (15)$$

where

$$\alpha = K_1/(K_1 + K_2)$$

$$\beta = M_1/(M_1 + M_2).$$

Equation (15) establishes the relationship between the equivalent damping ratio ζ of the whole

FIGURE 3 Model of two-mass system.

system and the damping ratios ζ_1, ζ_2 of two single degree of freedom systems (see Fig. 3). This is to say that Eq. (15) can be used to analyze the equivalent damping ratio of the whole system for any desired values of ζ_1 and ζ_2 .

For the composite tube-type building, it is important that the parameters ζ, ζ_1, ζ_2 in Eq. (15) are taken as equivalent damping ratio ζ_{ej} , and damping ratios ζ_{wj}, ζ_{Fj} , respectively, of the internal and external tube, and M_1, M_2 the generalized mass M_{wj}, M_{Fj} , and K_1, K_2 the generalized stiffness K_{wj}, K_{Fj} of the internal and external tubes for any mode j . Hence Eq. (15) is rewritten as

$$\zeta_{ej} = \zeta_{Fj} \sqrt{\alpha_{Fj} \beta_{Fj}} + \zeta_{wj} \sqrt{(1 - \alpha_{Fj})(1 - \beta_{Fj})} \quad (16)$$

in which ζ_{Fj}, ζ_{wj} are the damping ratios for mode j of the external and internal tubes; and

$$\alpha_{Fj} = M_{Fj}/M_j \quad (17a)$$

$$\beta_{Fj} = K_{Fj}/K_j. \quad (17b)$$

M_j, M_{Fj} are the generalized masses of the tube-in-tube and external tube for the j th mode and K_j, K_{Fj} are the generalized stiffness of the tube-in-tube and external tube for the j th mode:

$$K_j = M_j \omega_j^2$$

$$K_{Fj} = M_{Fj} \omega_{Fj}^2.$$

The computation of the damping ratio ζ_{ej} of the tube-type building by Eq. (16) is very simple. The results may be obtained rapidly once the frequency and vibration mode are obtained for mode j of the whole and external tubes.

In examining the accuracy of Eq. (16), we analyzed a number of 2 degree of freedom systems with different parameters. We found that in all cases, the equivalent damping ratio computed from Eq. (16) agrees favorably with the result obtained from the detailed numerical integration method. An example of a 40-story building will be used to illustrate the methods.

Illustrative Example

The composite tube-in-tube building consisting of 40 stories, with two districts along its height, is shown in Fig. 4. The story heights are 4.5 m (1–5) and 3.5 m (6–40), respectively. The total height is 145 m. The cross sections of the beam, columns, and internal tube are varied along the

FIGURE 4 Plan of example tall buildings (dimensions in meters).

height step by step. Their data are omitted here. The computed results are as follows:

1. example of fluctuations of accelerations of the top with time, which is shown in Fig. 5;
2. example of fluctuations of base overturning moment with time, which is also shown in Fig. 5.
3. The equivalent damping ratios: For the detailed method, assuming $\zeta_{F1} = \zeta_{F2} = 0.01$,

Table 1. Equivalent Modal Damping Ratios for First Mode ζ_{e1}

ζ_{F1}	ζ_{w1}	Detailed Method		Simplified Method
		(1)	(2)	
0.01	0.020	0.01128	0.01322	0.01175
	0.025	0.01301	0.01473	0.01262
	0.030	0.01473	0.01622	0.01349
	0.035	0.01644	0.01771	0.01437
	0.040	0.01816	0.01922	0.01524
	0.045	0.01988	0.02071	0.01612
	0.050	0.02159	0.02236	0.01700
	0.055	0.02330	0.02401	0.01787
0.02	0.020	0.02000	0.02000	0.02000
	0.025	0.01959	0.02159	0.02088
	0.030	0.02132	0.02306	0.02175
	0.035	0.02303	0.02460	0.02262
	0.040	0.02474	0.02621	0.02350
	0.045	0.02646	0.02769	0.02437
	0.050	0.02820	0.02934	0.02534
	0.055	0.02991	0.03099	0.02621

Note. (1) and (2) represent the equivalent damping ratios ζ_{e1} determined with the criteria of equal maximum acceleration at the top floor and of equal base overturning moment, respectively.

FIGURE 5 Example of time fluctuations of acceleration at top floor and base overturning moment.

0.02 and $\zeta_{w1} = \zeta_{w2} = 0.02$ to 0.05, results on the maximum dynamic acceleration at the top and the base overturning moment are respectively shown in Table 1. In addition, the equivalent damping ratios ζ_{e2} are slightly higher than ζ_{e1} , and are not shown in Table 1.

4. Results of the simplified method: When the equivalent damping ratios for the j th mode are analyzed, the frequency and mode shape of the whole structure and the external tube structure for mode j are computed, and further the corresponding generalized mass and stiffness are obtained. The results for the first mode are shown in Table 1. These results coincide essentially with the detailed method.

SUMMARY AND CONCLUSIONS

In general, the longer the natural period of the structure, the more obvious are the effects of damping to dynamic responses of the structure under wind forces. It is particularly important for the damping ratio to be determined correctly.

Two methods for analyzing the equivalent damping ratio of a composite tube-in-tube building are presented.

Rayleigh damping and the transfer matrix method are adopted and used to analyze buildings with varying sections along the height. The flexural and shear deformation can also be considered.

The simplified method to determine equivalent damping ratios of a composite tube-type building presented here is suitable for engineering design. The results are quite satisfactory, and can be obtained rapidly after the dynamic properties of the whole and external tube are determined, respectively.

The equivalent damping ratio is related to the distribution of mass and stiffness between the internal and external tubes. Its value is near to the damping ratio of the external tube when the mass and stiffness of the external tube are larger, such as in the present example. When the mass and stiffness of the internal and external tubes are roughly equal, the equivalent damping ratio is approximately equal to the average of the two damping ratios.

The methods to obtain equivalent modal damp-

ing ratios are different. They can also be derived from the dynamic behaviors of the structure, such as the complex frequencies. This will be discussed in a future article.

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