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Influence of Flexible Foundation on Isolator Wave Effects

This article deals with the interaction between wave effects in mounts and resonances of foundations in flexible vibration isolation systems. A new model is proposed that is represented as a rigid mass supported by two linear unidirectional isolators on a flexible foundation beam, whose closed-form solutions for transmissibility and response ratio are then obtained, with which the influence of wave effects coupled with the flexibility of the foundation on the effectiveness of isolation is discussed. The wave effects on flexible isolation systems are analyzed under various parametric conditions and compared with those in rigid systems. In addition, several special cases are presented to show the transition between various limiting cases. Some approaches to control wave effects are also proposed.

INTRODUCTION

Because today machines run at high speed with flexible components of lighter and thinner structure, some mounted on upper floors in multistory workshops, the problems for vibration and noise control are challenging. In these cases, vibration isolation may not be adequately predicted at higher frequencies, because their isolators may not behave as ideal resilient members in the high frequency range where the so-called wave effects will be apparent in their supports. The effectiveness, therefore, will be considerably reduced by approximately 20 dB as compared to those predicted in a massless spring dashpot mount. Therefore, it is necessary to consider the distributed mass and stiffness in the isolators when dealing with the problem of isolation at higher frequencies.

The wave effects in isolators has been studied theoretically and experimentally by Harrison (1952), Snowdon (1978), and Sykes (1960). However, attention was focused predominantly on the mounting itself, and theoretical models were limited to either a single-mount system or with a rigid foundation. These approaches might lead to overoptimistic, or even erroneous predictions for the efficiency of isolation for three reasons: the coupling between mounting points being neglected; the interaction between mount and foundation being overlooked; the wave effects in the mount being studied separately from the resonances of the foundation. In this study, these three factors are considered simultaneously; and closed forms for transmissibility and response ratios are derived from a new model in which the system is regarded as a rigid mass on a resilient foundation via four isolators. The influence of both wave effects and flexibility of the foundation on the efficiency of isolation under various parametric conditions were analyzed. Some new phenomena about wave effects were found and

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corresponding approaches to control them were presented.

MODEL AND EQUATIONS

Models

The model in Fig. 1(a) is the proposed parallel type isolation system, which could be used in common engineering practice, in which the ideal rigid machine M is supported by two identical mounts laid on a nonrigid foundation simulated by an end clamped beam with internal solid damping $E^* = E(1 + j\delta)$, where δ is the damping factor of the beam. The machine vibrates under the excitation of a sinusoidally varying force P. The performance of each mount is described by its complex stiffness, $K^* = K(1 + j\eta)$, where K and η are its respective stiffness and damping factor. Other isolation system models in Fig. 1(b–d) are shown for comparison.

Substructure Mobility Analysis

Considering the system consisting of three substructures, i.e., machine A, isolator system B, and flexible foundation C, we can obtain the transfer matrices of mobility for each substructure in the following forms:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \left[\begin{array}{cc|c} 1/\bar{M} + 1/(2\bar{J}) & 1/\bar{M} - 1/(2\bar{J}) & -1/\bar{M} \\ 1/\bar{M} - 1/(2\bar{J}) & 1/\bar{M} + 1/(2\bar{J}) & -1/\bar{M} \\ \hline -1/\bar{M} & -1/\bar{M} & 1/\bar{M} \end{array}\right] \tag{1}$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} \cos(n^*h)I & \mu^* \sin(n^*h)I \\ -(\mu^*)^{-1} \sin(n^*h)I & \cos(n^*h)I \end{bmatrix} \tag{2}$$

$$C = [C_{ik}]_{2\times 2},\ C_{ik} = \frac{j\omega}{M_b} \sum_{m=1}^{\infty} \frac{(\beta^*)^4}{(\beta^*)^4 - 1} \cdot \frac{\varphi_m(h_i)\varphi_m(h_k)}{\omega_m^2(1 + j\delta)} \quad (i, k = 1, 2) \tag{3}$$

where $\bar{M} = j\omega M$, $\bar{J} = j\omega J/(2b)$, J is the moment of inertia of the machine, I is a 2×2 unit matrix, $\mu^* = j\omega m/(n^*h)$, $n^* = \omega\sqrt{\rho/E^*}$, $(\beta^*)^4 = (1 + j\delta)(\omega_m/\omega)^2$, $\varphi_m(h_i)$ is the normal function of the beam, M_b is the mass of the beam, and ρ is the density of the isolator.

Considering the conditions for force equilibrium and motion compatibility at the junction, we can obtain the force transmitted to the foundation from the isolators as follows:

$$Q = \{Q_1, Q_2\}^T = -[B_{12} + A_{11} \cdot B_{11} + B_{22} \cdot C + A_{11}B_{12} \cdot C]^{-1} \cdot A_{12} \cdot P. \tag{4}$$

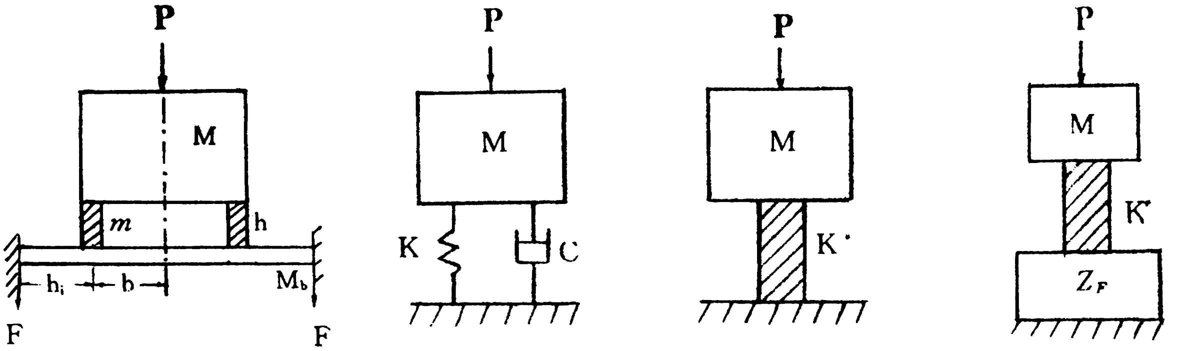

FIGURE 1 Models of vibration isolation systems: (a) current model; (b) classical model; (c) Harrison's model; (d) Snowdon's model.

As Eq. (4) relates the force Q acting on the foundation at each mounting point to the known exciting force P, it could be taken as a measurement of the transmissibility of both mounts and the overall system.

By introducing Eqs. (1), (2), and (3) into (4), we obtain:

$$T_1 = \left|\frac{Q_1}{P}\right| = \frac{1}{2}|C_0 - A^*[C_0 + S_0/(\gamma_I n^* h)] - \gamma_I(n^* h)S_0|^{-1}. \tag{5}$$

The total transmissibility is

$$T = |(Q_1 + Q_2)/P| = 2T_1 \quad \text{(symmetricity)}. \tag{6}$$

The response ratio of the system may be derived in the same manner. Noticing that when machine A is directly laid on the foundation, the four-pole parameters for the isolator system B will be changed into

$$B_{11} = B_{22} = I_{2\times2}, \quad B_{12} = B_{21} = O_{2\times2}. \tag{7}$$

Accordingly, the force directly transmitted to the foundation can be expressed as

$$Q_u = \{Q_{u1}, Q_{u2}\}^T = -[A_{11} + C]^{-1} \cdot A_{12} \cdot P. \tag{8}$$

To calculate the response ratio, one should first obtain the responses of the foundation at the mounting points with and without isolators. By using the transfer matrix method, we obtain the required responses

$$V = CQ, \tag{9a}$$

$$V_u = CQ_u. \tag{9b}$$

Therefore, the response ratio of the system can be derived as

$$R_i = \left|\frac{V_i}{V_{ui}}\right| = \left|\frac{1 - A^*}{C_0 - A^*[C_0 + S_0/(\gamma_I n^* h)] - \gamma_I(n^* h)S_0}\right| \quad (i = 1, 2), \tag{10}$$

$$R_1 = R_2 = 2|1 - A^*|T_1 \quad \text{(symmetricity)}, \tag{11}$$

where $C_0 = \cos(n^* h)$, $S_0 = \sin(n^* h)$, $\gamma_I = M/(2m)$, and

$$A^* = \frac{M}{M_b}\sum_{m=1}^{\infty}\frac{\varphi_m^2(h_1)}{(\beta^*)^4 - 1}.$$

In addition, the output force F transmitted to the termination of the flexible foundation can be used to define the overall transmissibility T_0, which describes a companion force transmissibility across the entire system. Based on the superposition principle, T_0 is then derived as

$$T_0 = \left|\frac{2F}{P}\right| = \left|\sum_{i=1}^{\infty}\frac{4\theta_i}{n_i l}\bar{\beta}^*\varphi_i(h_1)\right| T, \tag{12}$$

and the overall transmissibility T_e is defined as

$$T_e = \left|\frac{F}{P}\right| = \frac{1}{2}T_0, \tag{13}$$

where $\varphi_i(x) = ch(n_i x) - \cos(n_i x) - \theta_i[sh(n_i x) - \sin(n_i x)]$ and $\theta_i = [ch(n_i l) - \cos(n_i l)]/[sh(n_i l) - \sin(n_i l)]$. This quantity differs significantly from either T_1 or R_i in Eqs. (5) and (10).

Case Study

Four special cases of the above-mentioned general equations are discussed below.

Classical Model. If the exciting frequencies are much lower than the fundamental frequencies of both isolator and the flexible foundation, the wave effects then may not be apparent in the mounts. Moreover, as the foundation being comparatively rigid in general, the distributed mass of isolators can be neglected, and the mobilities of the foundation can be taken as zero. The model for this case is shown in Fig. 1(b). Let $m \to 0$, $C_{ij} \to 0$ $(i, j = 1, 2)$, then Eq. (6) reduces to

$$T = \sqrt{(1 + \eta^2)/[(1 - \Omega^2)^2 + \eta^2}, \tag{14}$$

where $\Omega = \omega/\omega_0$, $\omega_0 = \sqrt{2K/M}$. This is identical to the formula derived based on the classical theory.

Harrison's Model. When a machine is mounted on a very stiff and heavy foundation (such as vibration-free concrete blocks on the ground floor), the resonances of the foundation may not be taken into account. In this case, $M_b \to \infty$, thus $C_{ij} \to 0$, and $A^* \to 0$. Therefore, Eq. (5) reduces to

$$T = |\cos(n^* h) - \gamma_I(n^* h)\sin(n^* h)|^{-1}. \tag{15}$$

This equation coincides with Harrison's (1952) formula.

Snowdon's Model. For machines mounted on upper floors of workshops, the interaction between machine and its foundation should be considered. If wave effects of the isolators are neglected, i.e., $m \to 0$, thus $\gamma_1 \to \infty$, $n^*h \to 0$, then we find Eqs. (5) and (10) are simplified to, respectively,

$$T_1' = \frac{1}{2}\frac{1}{|1 - A^* - (\Omega^*)^2|} \tag{16}$$

$$R_1' = \left|\frac{1 - A^*}{1 - A^* - (\Omega^*)^2}\right|. \tag{17}$$

These are the solutions studied by Snowdon (1973), which is also a special case ($N = 2$, $\mu = 0.5$) in another study (Xiong et al., 1990).

Single Mounting System. Snowdon (1978) also studied wave effects in isolators for a simple mounting system in which the coupling between different mounting points was obviously ignored. Here $b \to 0$, and noticing that

$$C_{11} = C_{12} = C_{21} = C_{22} = 1/Z_F,$$
$$A_{11} = A_{22} = 1/(j\omega M) = 1/Z_M,$$
$$A_{12} = A_{21} = -1/Z_M$$
$$B_{ij} = a_{ij} \quad \text{(four-pole parameters)} \quad (i, j = 1, 2).$$

We can express Eqs. (5) and (10) in the following ways

$$T' = \left|\frac{Z_F}{(\alpha_{11} + \alpha_{21}Z_M)Z_F + \alpha_{12} + \alpha_{22}Z_M}\right|, \tag{18}$$

$$R' = \left|\frac{Z_F + Z_M}{(\alpha_{11} + \alpha_{21}Z_M)Z_F + \alpha_{12} + \alpha_{22}Z_M}\right|, \tag{19}$$

where Z_F and Z_M are the impedance of foundation and machine, respectively. It can be seen that Snowdon's formulas (1973) are also a special case of the present model shown in Fig. 1(d).

RESULTS AND DISCUSSION

To reveal the influences of wave effects coupled with resonances of a flexible foundation on the effectiveness of vibration isolation, representative results were numerically evaluated under various parametric conditions. The transmissibility and the response ratio are plotted in terms of frequency ratio $\Omega(\omega/\omega_0)$ on a decibel scale, in which ω_0 is the natural frequency of the system with a rigid foundation. The results for several special cases are also given to show their relation and comparison with the present results, which are shown in Figs. 2–8 where the dashed line represents the values obtained through classical theory [i.e., case (1)].

Based on the theoretical results, both the wave effects in the flexible isolation system and the coupling effects of the related system on transmissibility and response ratio will be discussed.

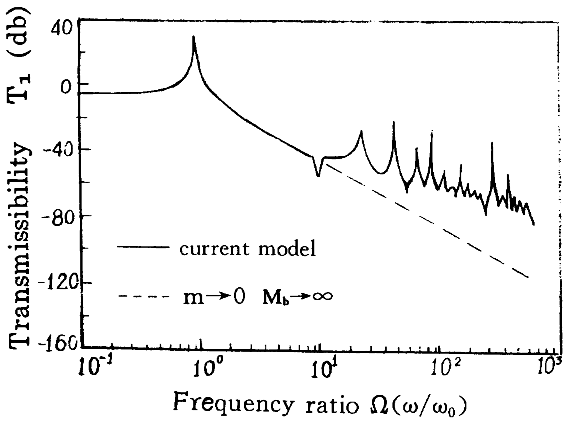

FIGURE 2 Wave effects in a flexible isolation system: $\gamma_l = 50$, $\eta = 0.2$, $\gamma = 0.01$, $\lambda = 10$, $M/M_b = 10$.

Wave Effects in Flexible Isolation System

It can be seen from Figs. 2 and 3 that when the exciting frequency becomes relatively high,

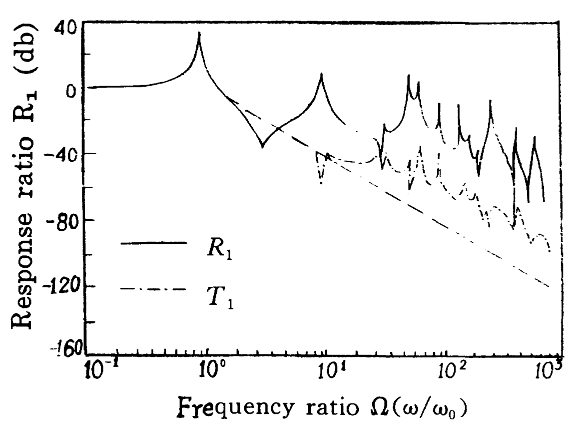

FIGURE 3 Comparison of R_1 and T_1: $\gamma_l = 100$, $\eta = 0.05$, $\delta = 0.01$, $\lambda = 10$, $M/M_b = 10$.

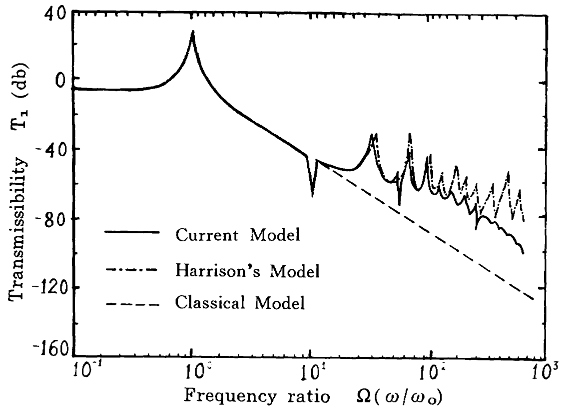

FIGURE 4 Transmissibilities in three isolation models: $\gamma_l = 100$, $\eta = 0.05$, $\delta = 0.01$, $M/M_b = 10$, $\lambda = 10$.

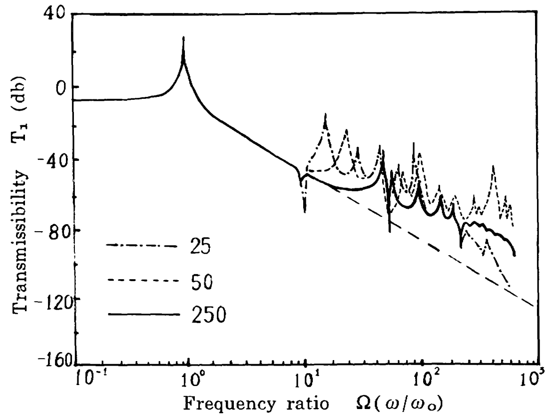

FIGURE 6 Wave effects in a flexible system influenced by mass ratio γ_l: $\eta = 0.05$, $\delta = 0.01$, $M/M_b = 10$, $\lambda = 10$.

standing waves will occur in the isolator. The peaks in R and T are not only influenced by self-resonances of the flexible foundation, but also are more significantly influenced by wave effects in the isolators, which may be troublesome and lead to poor performance. It has been observed that the higher the frequency, the greater the deviation of the transmissibility or the response ratio curves from that predicted by the classical theory (dashed line). This demonstrates that the classical theory, which was based on the assumption of massless resilient element for isolators, is not valid in the higher frequency range.

On the other hand, the results obtained from different theories are almost the same in the low frequency range due to isolators continuously behaving as lumped elements. That is to say, the simple classic theory is still applicable with remarkable accuracy for the prediction of isolation in the low frequency ranges.

Figure 3 shows a comparison of the R_l curve with the T_l curve. We found that both levels will increase due to wave resonances. However, the resonances in the R_l curve are much more obvious than those in the T_l curve, meaning that the response ratio is more sensitive to the changes of system parameters than that in transmissibility. From this point of view, response ratios are often suggested to evaluate the effectiveness of isolation for systems with flexible foundations.

Effects on Standing-Wave Frequency

The frequency at which the standing-wave resonance occurs depends strongly on the dynamic

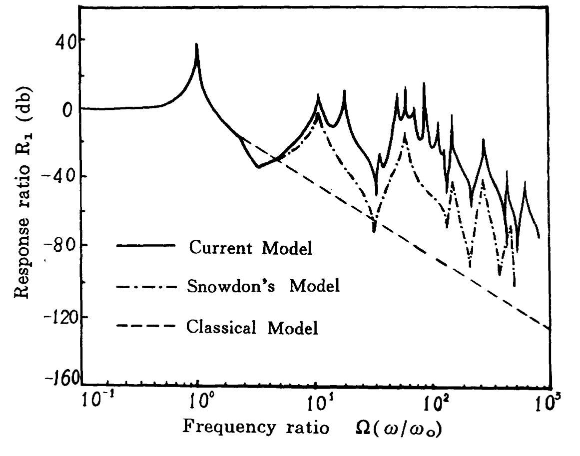

FIGURE 5 Comparison of R in three models: $\gamma_l = 25$, $\eta = 0.05$, $\delta = 0.01$, $M/M_b = 10$, $\lambda = 10$.

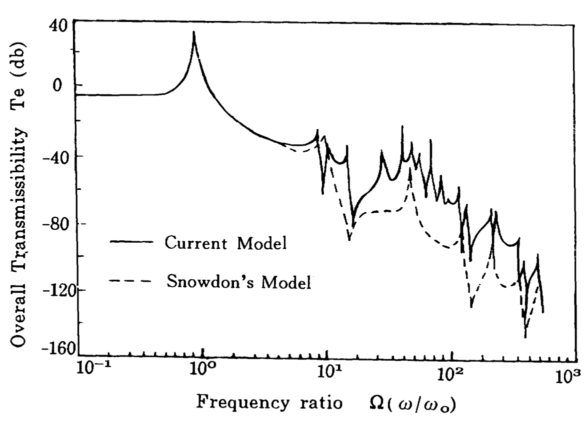

FIGURE 7 Comparison of T_e with T_l' for $\gamma_l = 25$: $\eta = 0.05$, $\delta = 0.01$, $M/M_b = 10$, $\lambda = 10$.

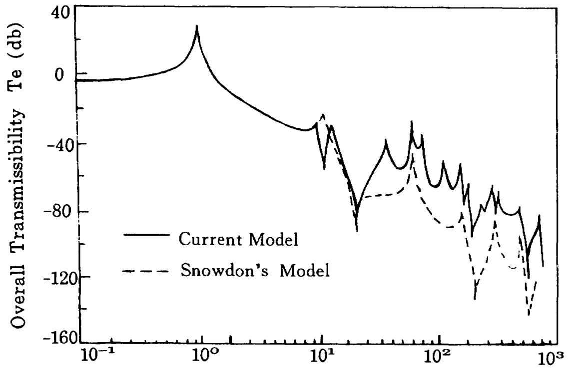

FIGURE 8 Comparison of T_e with T_l' for $\gamma_l = 100$, with other parameters unchanged.

characteristics of the foundation. Comparing different curves in Fig. 4, it is obvious that wave effects will occur when $\omega/\omega_0 > 10$. Moreover, we find that the standing wave frequencies shift to the lower frequency range as compared with those predicted by Harrison (1952). This shows the flexibility of the foundation causing an earlier occurrence of wave effects, which may directly influence the reliability of the isolation design.

It is further revealed that the lower the stiffness of the foundation, the earlier the wave effects will occur when other parameters remain unchanged. In Table 1, the frequency ratio $\omega_W^{(1)}/\omega$ (the first standing-wave frequency to the exciting frequency) varies with $\lambda(\lambda = \omega_1/\omega_0$, ω_1 is the fundamental frequency of the foundation) and is shown for mass ratios $\gamma_l = 25$ and 200.

Effectiveness of Isolation

The influence of a flexible foundation coupled with wave effects over the efficiency of isolation can be observed from Fig. 5. The figure shows that the interaction between these two factors makes the peaks of the curve pronounced and closer to each other, seriously reducing the effectiveness of the high frequency isolation. Only in the lower frequency range ($\Omega < 10$) do the predicted values produced by the three models coincide well. On the other hand, the results obtained from the three models differ from each other by about 20–50 db at higher frequencies. This demonstrates the interaction between wave effects in the mounts and the flexibility of the foundation. It is suggested that these two factors should be taken into account for accurate prediction of the efficiency of high frequency isolation.

Table 1. Influence of Flexible Foundation on the First Standing—Wave Frequency $\omega_w^{(1)}/\omega$

	λ		
γ_l	10	15	Rigid
25	16.050	16.151	17.195
200	44.195	44.415	44.504

Effect of Foundation Internal Damping

Although many pronounced wave resonances can be suppressed reasonably well by internal damping of the rubber mounts, the performance of mounts becomes less effective with the increase of frequency. Figures 4–8 show the effects of internal damping of the foundation on the suppression of wave effects. With sufficient damping in the foundation, the wave effects may be smoothed and may cease to exhibit any obvious resonance peaks. Also, increase of the damping of the foundation will decrease the vibrational level of the foundation and suppress the resonance peaks at standing-wave frequencies. When high-speed running machines mounted on upper floors or decks supported by relatively flexible and lightly damped structures, the compound system will be liable to vibration. In this case, it is important for the foundation to be specially treated with damping material to increase its damping capacity to facilitate the control of wave effects.

Effect of Mass Ratio

The ratio of machine mass to the mass of the isolator is also an important parameter in determining the standing—wave frequencies. Figures 6–8 show how the mass ratio γ_l influences transmissibilities when other system parameters remain unchanged. With the decrease of mass ratio γ_l, the resonance peaks shift toward the lower frequency range. Consequently, the smaller the value of γ_l, the lower the frequency at which the first wave resonance occurs, and the more apparent the T curve deviation from the prediction of classic theory. Moreover, the occurrence of wave resonance is of less concern as γ_l becomes larger. Wave effects calculations from Eq. (15) are also plotted for different mass ratios of γ_l = 25 and 100, which are shown in Figs. 7 and 8. The overall transmissibility curves obtained from Snowdon's (1973) theory are redrawn for comparison. Again, the curves show how the levels to which T_e are increased by the wave resonances depends upon the value of mass ratio γ_I. As shown in the figures, the occurrence of wave effects become less and scattered as γ_I becomes larger. Therefore, it is proposed that isolators be used as near to their maximum rated load as possible to make γ_I a relatively large quantity.

CONCLUSIONS

1. Wave resonance peaks and the levels of the transmissibility and the response ratio are much higher in a flexible isolation system as compared to those of traditional system models. For accurate prediction of the effectiveness of vibration isolation at higher frequencies, the interaction between wave effects in isolators and the flexibility of foundations should be considered simultaneously.
2. Wave effects are influenced by the characteristics of the isolator, the frequency of the vibration source, mass ratio, and, more importantly, the impedance of the foundation. For high-speed running machines on flexible foundations, the standing-wave frequencies are lower than those predicted by rigid foundation theory. The earlier occurrence of wave resonances will directly affect the reliability of isolation design.
3. The increase of the foundation damping can not only decrease vibration responses of elastic foundation, but also can suppress the resonance peaks at standing-wave frequencies.
4. With a decrease of mass ratio, the wave resonance peaks become more pronounced and shift toward the lower frequency range. It is proposed that isolators be used as near to their maximum rated loads as possible to prevent earlier occurrence of wave effects and decrease the magnitude of the transmissibility and response ratio.

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