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Synthesis of Optimal Isolation Systems of Hand-Transmitted Vibration

In this article a procedure is presented for the analytical synthesis of optimal vibration isolation for a hand–arm system subjected to stochastic excitation. A general approach is discussed for a selected vibration isolation criterion. The general procedure is illustrated by analytical examples for different hand–arm systems described by their driving-point impedances. The influence of particular forms of excitation and the structure of the vibroisolated hand–arm systems on the resultant vibration isolation is then discussed. Some numerical examples illustrating the procedure have also been included.

INTRODUCTION

The hand–arm system (HAS) is one of the most important biomechanical structures of the human body. Vibrating tools in manufacturing processes, mining, and other branches of industry where hands and fingers grasp or push vibrating objects are the principal causes of hand injuries due to severe vibration. A wide range of vibrating objects can bring about such magnitudes of acceleration, which jointly with the exposure time produce observable injurious effects. The three principal approaches to the problem of the influence of vibration on the human body may be specified as medical, engineering, and interdisciplinary. The medical approach concerns such symptoms of the influence of vibration on the HAS as Reynauld's syndrome, perturbation in the vascular and nervous systems, elasticity, and forces developed by muscles, strength of bones, smooth functioning of articulations and joints, and peripheral neurological disorders and many other. Medical approaches usually result in general recommendations concerning vibration isolation of the HAS. The engineering approach is concerned with the improved construction of hand tools and the attenuation of the sources of vibration transmitted to hands. As a result of such improvements, the levels of harmful vibration observed by different international organizations have been steadily diminishing for more than years. These improvements have the functional character of and are applied for specific tools and strictly defined work situations and hand–arm positions. The third approach deals with the modeling of the HAS. The fundamentals of hand anatomy (bones of the phalanges, metacarpus, carpus, wrist, forearm, and arm, the tissues, and muscles) result from experiments with dummies and living people as will as the application of numerical methods of optimization. This understanding constitutes the base for biomechanical modeling of HAS. There are many biomechanical models of this system, all of which resulted from investigation of its dynamic characteristics in particular situations. The problem of severe vibration on humans has been described in detail in (Griffin, 1990). In the present article a general procedure for analytical construction

Received August 7 1995; Revised April 24 1997.
Shock and Vibration, Vol. 4, No. 4, pp. 281–291 (1997)
ISSN 1070-9622/97/$8.00

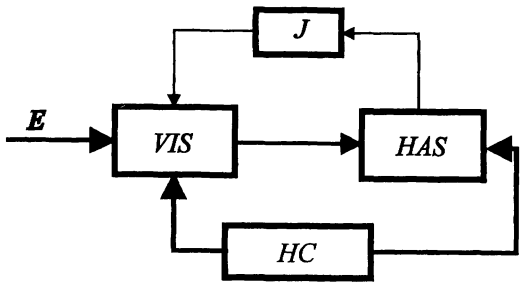

FIGURE 1 Principal factors of the problem of the vibration isolation of hand–arm and whole body systems: E, excitation; VIS, vibration isolation system; HAS, hand–arm system; HC, human controller; and J, criterion of isolation.

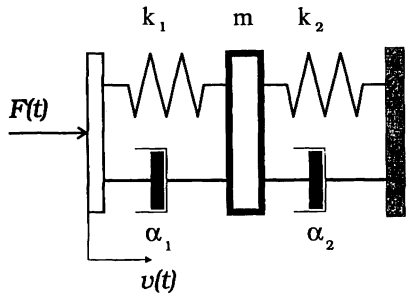

FIGURE 2 Dynamical structure of the 1.5-DOF bent hand–arm model.

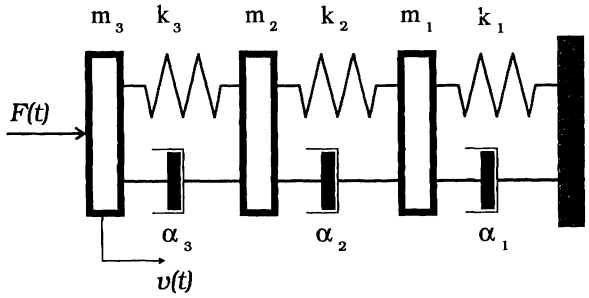

FIGURE 3 Dynamical structure of the 3-DOF hand–arm model.

of optimal vibration isolation for HAS is proposed, based on Wiener–Hopf filtration theory (Goupta and Hasdorff, 1981; Newton et al., 1957).

VIBRATION ISOLATION PROBLEM OF HAS

There are some principal factors that influence the problem of vibration isolation of hand–arm and whole-body systems. These factors are presented in block form in Fig. 1, where E represents excitation, VIS represents the vibration isolation system, HAS represents the hand–arm system, HC represents the human controller, and J represents the criterion of isolation. The block represented by J has been introduced among the physical components to underline the capital influence of the performance specifications on the VIS structure.

The most important excitations are transmitted to the hands of hand-held tool operators. These excitations can have a very diverse character. This character varies greatly depending on the kind of hand-held tool used, the shape, mass, and material of the object being polished, ground, or cut; the type and condition of the work, as well as the operator technique. The power spectral densities of acceleration of several of these hand-held tools can be found in the literature (Bovenzi et al., 1979; Griffin, 1990; Harris and Crede, 1979). Selected values of the parameters of power spectral densities of acceleration are given in Table 1.

The power spectral densities of hand-tool excitations have irregular shape and are difficult to approximate by rational functions of the complex variable s. The HAS itself is a very complex, active biological system and there have been many attempts to measure its dynamical characteristics (Bystroem et al., 1978; Daikoku and Ishikawa, 1990; Dieckman, 1957; Kuhn, 1953; Liang and Griffin, 1996; Meltzer et al., 1980; Reynolds and Keith, 1977; Wasiliew, 1972; Wood et al., 1978). It is very difficult to build a general model of such a system. The existing biomechanical models are mostly lumped parameter, passive, and finite degree of freedom mechanical systems. They are of functional character and their structure is obtained from measurements of the mechanical impedance and the corresponding transmissibility function. Among the models of HAS one can find very simple ones (Dieckman, 1957; Kuhn, 1953), as well as more complex dynamical structures (Daikoku and Ishikawa, 1990; Fritz, 1991; Meltzer, 1979; Meltzer et al., 1988; Reynolds and Keith, 1977). In this article two different structures of HAS models have been considered. These structures are presented correspondingly in Figs. 2 and 3. The structure of Fig. 2 was adopted (Wasiliew, 1972) as a bent hand–arm model. The model presented in Fig. 3 was later adopted as the HAS general model (Daikoku and Ishikawa, 1990; Meltzer, 1979; Meltzer et al., 1988; Reynolds and Keith, 1977).

A number of criteria are used in the vibration isolation of the HAS. Recent international (ISO 5349, 1986) and Polish (Normy Polskie, 1992) hygienic standards introduce velocity and the root mean square and peak values of acceleration as principal parameters for estimation of vibration levels that can be supported without risk by operators of hand-held tools and

Table 1. Approximate Parameters of Power Spectral Density of Acceleration of Chosen Hand Tools

Tool	Frequency Bandwith (Hz)	Mean Value of PSD $[\frac{(ms^{-2})^2}{Hz}]$	Max. Value Frequency (Hz)	Max. Value of PSD $[\frac{(ms^{-2})^2}{Hz}]$
Hand-held portable grinder	11–1000	0.003	60	2.0
Front and rear handles of a small chain saw	15–1000	0.5	120	125 175
Chipping tool used for weld destruction	1–10000	10	14	300

other vibrating objects. It should be stated, however, that there are no fixed criteria applicable to all possible situations and work positions with hand-held tools. Disorders associated with hand-transmitted vibration and the efficiency of tools are two contradictory factors that should be taken into account in the establishement of each criterion. The minimization of the absolute acceleration and the relative displacement for a 1-degree of freedom (DOF) kinematically excited system is the best example of such a contradiction. The criterion of isolation will have, in each application, a functional character and will depend on the design principles concerning the VIS.

In hand-held tools or other vibrating objects the isolating systems are applied as internal (Bar et al., 1989) or external (Basista, 1991) parts of the principal construction. External isolators, whether passive or active, are built between the tool or other vibrating object and the handle. These external devices (usually rubber mountings between the handle assembly and the power unit) may also include provision for heating the handle by either exhaust gases or electricity. Internal VIS are the results of the global conception and design of a tool that can have material, passive, or state control character. The present state of VIS is the result of increased interest by tool manufacturers, primarily a result of pressure from tool users and the general increase in knowledge of the injurious character of vibrating equipment.

FORMULATION OF PROBLEM

General Assumptions

As discussed previously, the problem of isolation of hand-transmitted vibrations is very complex. Consequently, each analytical approach is based on some

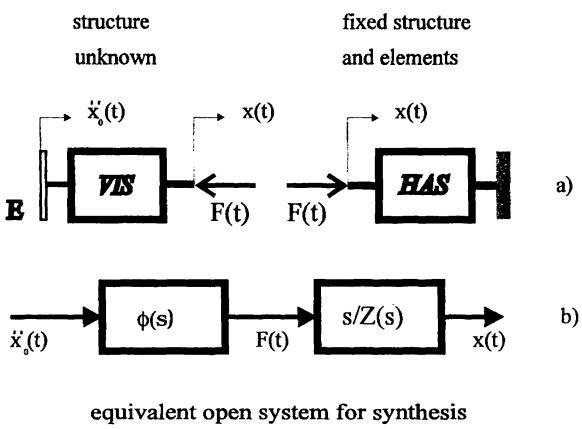

FIGURE 4 Block diagram of the hand–arm and the vibration isolation system.

simplifications and assumptions. In Fig. 4(a) the general schemes of the HAS and VIS are presented. The HAS is represented by the driving-point impedance $Z(s)$, where s is a complex variable. In Fig. 4(b) an equivalent open system is presented. In this article the following assumptions have been adopted:

- The HAS is considered to be a linear, lumped parameter system described by its driving-point mechanical impedance.
- The excitations are taken as random, normal, and ergodic processes.
- Only the unidirectional motion of the system is considered.
- The influence of the HAS on the source of vibration is considered as negligible.

Mathematical Description of HAS

In the majority of cases the biomechanical models of the HAS were designed on the basis of the

driving-point impedance measurements. These models were considered as lumped parameter, passive, and minimum-phase systems. The driving-point impedance of such systems can be presented in general form as a proper rational function in descending powers of s:

$$Z(s) = \frac{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}, \quad (1)$$

where the realizability conditions can be stated as follows:

1. All numerator, a_n, and denominator, b_m, coefficients should be nonnegative.
2. The highest degree of the numerator polynomial n should not exceed that of the denominator polynomial m by more than unity. The same applies to the lowest degrees of the numerator and denominator polynomials.
3. The poles and zeros of $Z(s)$ should all lie in the complex left-half of the s plane.
4. The zeros on the imaginary axis of the s plane should be simple and not multiple.
5. The real part of $Z(s)$ with $i\omega$ substituted for s, should be nonnegative for any value of ω.

Power Spectral Densities of Excitation

The form of the excitations generated by hand-held tools varies greatly according to the kind of tool, its source of energy and weight, the work position, and the shape, mass, and material of the object being polished, ground, or cut. In this study a general design procedure illustrated by some application examples will be presented. The excitations $\ddot{x}_0(t)$ will be described by means of selected power spectral densities. Two kinds of excitation accelerations were considered: $\ddot{x}_0(t)$ taken as a white noise with the power spectral density

$$S_{\ddot{x}_0}(s) = \frac{\sigma^2}{2\pi}, \quad (2)$$

and $\ddot{x}_0(t)$ taken as a narrow-band random process with the power spectral density,

$$S_{\ddot{x}_0}(s) = \frac{\gamma\sigma^2}{\pi}\frac{\Omega^2 - s^2}{(\Omega^2 + s^2)^2 - 4\gamma^2 s^2}, \quad (3)$$

where σ^2 [m^2s^{-4}], γ (s^{-1}), Ω (s^{-1}) are known parameters.

These two power spectral densities can be considered as two extreme cases of excitation between harmonic (single frequency) and pure random signals. The very first approximation of most power spectral densities of real vibration sources can be obtained by combining and shaping of the densities given by (2) and (3). Due to the great variety of existing power spectral densities, this approach allows estimation of the extreme references for the optimum hand–arm VIS.

Criterion of Vibration Isolation

As the criterion of synthesis of the optimal VIS, the following expression was assumed:

$$J = \sigma_\delta^2 + \lambda\sigma_{\ddot{x}}^2 = \min, \quad (4)$$

where σ_δ^2 is the mean square value of the relative displacement $\delta(t) = x(t) - x_0(t)$, $\sigma_{\ddot{x}}^2$ is the mean square value of the acceleration of the point of contact between HAS and VIS, and λ is a Lagrangian multiplier. The mean square values σ_y^2 can be calculated as follows:

$$\sigma_y^2 = \frac{1}{i}\int_{-i\infty}^{i\infty} |H_y(s)|^2 S_{\ddot{x}_0}(s)\, ds. \quad (5)$$

In (5) the index y denotes, respectively, δ and $\ddot{x}$ and $H_\delta(s)$, $H_{\ddot{x}}(s)$ are the transfer functions between the variables $x(t) - x_0(t)$, $\ddot{x}(t)$, and $\ddot{x}_0(t)$. Criterion (4) is a particular case of the more general criteria proposed (Książek, 1994, 1996) for a sitting human operator.

DETERMINATION OF OPTIMUM VIS WITH STRUCTURE A PRIORI UNKNOWN

General Approach

As shown in Fig. 4, the force $F(t)$ is a force of contact between a hand–arm structure and a VIS. To obtain the optimal VIS, the Wiener–Hopf theory has been applied. Let

$$\phi(s) = F(s)\ddot{x}_0^{-1}(s), \quad (6)$$

$$Z(s) = \frac{F(s)}{\dot{x}(s)}, \quad (7)$$

where $\dot{x}(s)$, $\ddot{x}_0(s)$, and $F(s)$ are the Laplace transforms of random time functions $\dot{x}(t)$, $\ddot{x}_0(t)$, and $F(t)$, respectively; $\phi(s)$ is the function describing the optimal VIS that does not have any poles in the right-hand side of the s plane; and $Z(s)$ is the driving-point impedance of the HAS as a function of s.

The corresponding transfer functions and the criterion of vibration isolation (4) can be written as

$$H_{(x-x_0)/\ddot{x}_0}(s) = \frac{s\phi(s) - Z(s)}{s^2 Z(s)}, \quad (8)$$

$$H_{x/x_0}(s) = \frac{s\phi(s)}{Z(s)}, \tag{9}$$

$$J(\phi(s), \lambda) = \frac{1}{i}\int_{-i\infty}^{+i\infty}\left[\left|\frac{s\phi(s) - Z(s)}{s^2 Z(s)}\right|^2 + \lambda\left|\frac{s\phi(s)}{Z(s)}\right|^2\right] S_{\ddot{x}_0}(s)\, ds. \tag{10}$$

Introducing

$$G(s) = \frac{s}{Z(s)} \tag{11}$$

and supposing that the spectral density can be factorized in the following way,

$$S_{\ddot{x}_0}(s)s^{-4} = S_0\psi(s)\psi(-s), \tag{12}$$

criterion (4) reduces to

$$J(\phi(s), \lambda) = \frac{1}{i}\int_{-i\infty}^{+i\infty}\left[|\phi(s)G(s) - 1|^2 + \lambda s^4|\phi(s)G(s)|^2\right] S_0\psi(s)\psi(-s)ds. \tag{13}$$

Our aim is to find $\phi(s)$ that minimizes J and that has no poles in the right-hand side of the s plane (for stability and realizability reasons). The method used to this end is a technique of variational calculus (Goupta and Hasdorff, 1981; Książek, 1978). Assume that $\phi(s)$ is subjected to a small variation, so that

$$\phi(s) = \bar{\phi}(s) + \varepsilon\eta(s). \tag{14}$$

Then J becomes

$$J(\varepsilon) = J + \varepsilon(\delta J). \tag{15}$$

To minimize J we minimize $J(\varepsilon)$ as $\varepsilon \to 0$. This will be the case if

$$\left.\frac{\delta J}{\delta\varepsilon}\right|_{\varepsilon=0} = 0. \tag{16}$$

Substituting Eqs. (14) and (15) into (13) and applying (16) we obtain

$$\frac{1}{i}\int_{-i\infty}^{+i\infty}[\bar{\phi}(s)\psi(s)\psi(-s)D(s)D(-s) - G(-s)\psi(s)\psi(-s)]\eta(-s)ds + \frac{1}{i}\int_{-i\infty}^{+i\infty}[\bar{\phi}(-s)\psi(s)\psi(-s)D(s)D(-s) - G(s)\psi(s)\psi(-s)]\eta(s)\, ds = 0, \tag{17}$$

where

$$D(s)D(-s) = G(s)G(-s)\left(1 + \lambda s^4\right). \tag{18}$$

Letting $s = -s$ in the second integral we note that (17) reduces to

$$\int_{-i\infty}^{+i\infty}[\bar{\phi}(s)D(s)D(-s)\psi(s)\psi(-s) - G(-s)\psi(s)\psi(-s)]\eta(-s)\, ds = 0. \tag{19}$$

Now, one has to find a realizable $\bar{\phi}(s)$ that is a solution of (19). Let

$$D^+(s) = [G(s)G(-s)(1 + \lambda s^4)]_+. \tag{20}$$

Decomposing

$$\frac{G(-s)\psi(s)}{D(-s)} = \left[\frac{G(-s)\psi(s)}{D(-s)}\right]_+ + \left[\frac{G(-s)\psi(s)}{D(-s)}\right]_-, \tag{21}$$

where $[\ ]_+$ and $[\ ]_-$ represent the parts that have poles in the left-hand side and the right-hand side of the s plane, respectively. In view of the condition of realizability of $\bar{\phi}(s)$, formula (19) can be written as

$$\int_{-i\infty}^{+i\infty}\left[D^+(s)\psi^+(s)\bar{\phi}(s) - \left[\frac{G(-s)\psi(s)}{D^-(s)}\right]_+\right]\psi(-s)D^-(s)\eta(-s)\, ds = 0. \tag{22}$$

Equation (22) is satisfied if

$$\bar{\phi}(s) = \frac{1}{D^+(s)\psi^+(s)}\left[\frac{G(-s)\psi(s)}{D^-(s)}\right]_+. \tag{23}$$

When the driving-point impedance $Z(s)$ of (1) is a minimum-phase function, one obtains from formula (23)

$$\bar{\phi}(s) = \frac{1}{R^+(s)G^+(s)\psi^+(s)}\left[\frac{\psi(s)}{R^-(s)}\right]_+, \tag{24}$$

where $R^+(s)$ is given as

$$R^+(s) = As^2 + Bs + C, \tag{25}$$

with

$$A = \lambda^{0.5}, \quad B = 2^{0.5}\lambda^{0.25}, \quad C = 1. \tag{26}$$

Taking into account Eq. (11) and introducing

$$E^+(s) = \frac{1}{R^+(s)s\psi^+(s)}\left[\frac{\psi(s)}{R^-(s)}\right]_+, \tag{27}$$

formula (24) reduces to

$$\bar{\phi}(s) = Z^{+}(s)E^{+}(s). \tag{28}$$

The functions $R^{+}(s)$, $E^{+}(s)$, $\bar{\phi}(s)$ should be more precisely written as $R^{+}(s, \lambda)$, $E^{+}(s, \lambda)$, $\bar{\phi}(s, \lambda)$ because they are functions of s and the Lagrange multiplier λ, as yet unknown. The mean square values σ_{δ}^{2} and $\sigma_{\ddot{x}}^{2}$ expressed by Eq. (5) are evaluated in terms of $\bar{\phi}(s)$. The multiplier λ should be selected according to the following condition

$$\sigma_{\ddot{x}}^{2} \leqslant \sigma_{\ddot{x}\text{permissible}}^{2}. \tag{29}$$

For a known value of λ, the optimum function $\bar{\phi}(s)$ is also known and the mean square values σ_{δ}^{2} can be evaluated.

White Noise Acceleration Excitation

For white noise excitation, from Eqs. (4), (12), and (21) we have

$$\left[\frac{\psi(s)}{R^{-}(s)}\right]_{+} = \frac{Ks + L}{s^{2}}, \tag{30}$$

where the parameters K and L must be calculated from Eq. (26) and the following system of algebraic equations

$$\begin{aligned} KA + U &= 0, \\ -KB + W + LA &= 0, \\ KC - LB &= 0, \\ LC &= 1. \end{aligned} \tag{31}$$

Formula (27) then leads to

$$E^{+}(s) = \frac{Ks + L}{s(As^{2} + Bs + C)}. \tag{32}$$

Finally, the expression for the optimum function $\bar{\phi}(s)$ of (28) takes the form

$$\begin{aligned} \bar{\phi}(s) = {} & \frac{(Ks + L)}{s(As^{2} + Bs + C)} \\ & \times \frac{(a_n s^{n} + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}{(b_m s^{m} + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0)}. \end{aligned} \tag{33}$$

Narrow-Band Acceleration Excitation

In the case of narrow-band acceleration excitations one obtains from Eqs. (5), (12), and (21)

$$\left[\frac{\psi(s)}{R^{-}(s)}\right]_{+} = \frac{Ms^{3} + Ns^{2} + Ps + Q}{s^{2}(s^{2} + 2\gamma s + \Omega^{2})}. \tag{34}$$

Then formula (27) gives

$$E^{+}(s) = \frac{Ms^{3} + Ns^{2} + Ps + Q}{s(\Omega + s)(As^{2} + Bs + C)}. \tag{35}$$

The parameters M, N, P, Q are to be calculated from the following system of algebraic equations:

$$\begin{aligned} MA + V &= 0, \\ -MB + NA + 2\gamma V + T &= 0, \\ MC - NB + PA + V\Omega^{2} + 2\gamma T &= 0, \\ NC - PB + QA + T\Omega^{2} &= 0, \\ PC - QB &= 1, \\ QC &= \Omega. \end{aligned} \tag{36}$$

Finally, the expression for the optimum function $\bar{\phi}(s)$ takes the form

$$\begin{aligned} \bar{\phi}(s) = {} & \frac{(Ms^{3} + Ns^{2} + Ps + Q)}{s(As^{2} + Bs + C)(\Omega + s)} \\ & \times \frac{(a_n s^{n} + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0)}{(b_m s^{m} + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0)}. \end{aligned} \tag{37}$$

OPTIMAL VIS FOR SELECTED HAND–ARM MODELS

Mass and Double Kelvin–Voigt Hand–Arm Model

The model of the hand–arm composed of one mass and two Kelvin–Voigt systems shown in Fig. 3 is represented by the impedance with the following parameters:

$$\begin{aligned} a_3 &= m\alpha_1, \\ a_2 &= mk_1 + \alpha_1\alpha_2, \\ a_1 &= \alpha_1 k_2 + \alpha_2 k_1, \\ a_0 &= k_1 k_2, \\ b_3 &= m, \\ b_2 &= \alpha_1 + \alpha_2, \\ b_1 &= k_1 + k_2, \\ b_0 &= 0. \end{aligned} \tag{38}$$

White Noise Excitation. In this case the power spectral density is given by Eq. (2). Based on formulae (28), (30), and (32), the function $\bar{\phi}(s)$ takes the form

$$\bar{\phi}(s) = \frac{(Ks+L)(\alpha_1 s + k_1)}{s(As^2+Bs+C)} \times \frac{(ms^2+\alpha_2 s + k_2)}{[ms^2+(\alpha_1+\alpha_2)s+k_1+k_2]}. \tag{39}$$

Narrow-Band Noise Excitation. For the power spectral density given by Eq. (3), the function $\bar{\phi}(s)$ takes the form

$$\bar{\phi}(s) = \frac{(Ms^3+Ns^2+Ps+Q)}{s(As^2+Bs+C)(\Omega+s)} \times \frac{(\alpha_1 s+k_1)(ms^2+\alpha_2 s+k_2)}{[ms^2+(\alpha_1+\alpha_2)s+k_1+k_2]}. \tag{40}$$

3-DOF Model of HAS

The 3-DOF HAS model is presented in Fig. 3. For this model the parameters of the driving-point impedance given by (1) take the form

$$\begin{aligned}
a_6 &= m_1 m_2 m_3,\\
a_5 &= m_1 m_3(\alpha_2+\alpha_3) + m_1 m_2 \alpha_3 + m_3 m_2(\alpha_1+\alpha_2),\\
a_4 &= m_1 m_3(c_2+c_3) + m_1 m_2 c_3 + m_1\alpha_3(\alpha_2+\alpha_3)\\
&\quad + (\alpha_1+\alpha_2)[m_3(\alpha_2+\alpha_3)+m_2\alpha_3]\\
&\quad + m_3 m_2(c_1+c_2) - m_3\alpha_2^2 - m_1\alpha_3^2,\\
a_3 &= m_1\alpha_3(c_2+c_3) + m_1(\alpha_2+\alpha_3)c_3\\
&\quad + (\alpha_1+\alpha_2)[m_3(c_2+c_3)\\
&\quad + m_2 c_3 + \alpha_3(\alpha_2+\alpha_3)]\\
&\quad + (c_1+c_2)[m_3(\alpha_2+\alpha_3)+m_2\alpha_3]\\
&\quad - 2m_3 c_2\alpha_2 - \alpha_3\alpha_2^2 - 2m_1 c_3\alpha_3 - (\alpha_1+\alpha_2)\alpha_3^2,\\
a_2 &= m_1 c_3(c_2+c_3) + (\alpha_1+\alpha_2)\\
&\quad \times [\alpha_3(c_2+c_3)+(\alpha_2+\alpha_3)c_3]\\
&\quad + (c_1+c_2)[m_3(c_2+c_3)+m_2 c_3+\alpha_3(\alpha_2+\alpha_3)]\\
&\quad - m_3 c_2^2 - c_3\alpha_2^2 - 2c_2\alpha_2\alpha_3\\
&\quad - m_1 c_3^2 - \alpha_3^2(c_1+c_2) - 2c_3\alpha_3(\alpha_1+\alpha_2),\\
a_1 &= (\alpha_1+\alpha_2)c_3(c_2+c_3)\\
&\quad + (c_1+c_2)[\alpha_3(c_2+c_3)+(\alpha_2+\alpha_3)c_3]\\
&\quad - \alpha_3 c_2^2 - 2c_2\alpha_2 c_3 - (\alpha_1+\alpha_2)c_3^2\\
&\quad - 2c_3\alpha_3(c_1+c_2),\\
a_0 &= c_3(c_2+c_3)(c_1+c_2) - c_3 c_2^2 - (c_1+c_2)c_3^2,\\
b_4 &= m_1 m_2,\\
b_3 &= m_1(\alpha_2+\alpha_3) + m_2(\alpha_1+\alpha_2),\\
b_2 &= m_1(c_2+c_3) + \alpha_1(\alpha_2+\alpha_3)\\
&\quad + \alpha_2\alpha_3 + m_2(c_1+c_2),\\
b_1 &= \alpha_1(c_2+c_3) + c_1(\alpha_2+\alpha_3) + \alpha_2 c_3 + c_2\alpha_3,\\
b_0 &= c_1(c_2+c_3) + c_2 c_3.
\end{aligned} \tag{41}$$

White Noise Excitation. In this case one obtains

$$\bar{\phi}(s) = \frac{(Ks+L)}{s^2(As^2+Bs+C)} \times \frac{(a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)}{(b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0)}. \tag{42}$$

Narrow-Band Noise Excitation. In this case the function $\bar{\phi}(s)$ obtained from (34) and (37) is given by

$$\bar{\phi}(s) = \frac{(Ms^3+ns^2+Ps+Q)}{s^2(As^2+Bs+C)(\Omega+s)} \times \frac{(a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)}{(b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0)}. \tag{43}$$

NUMERICAL RESULTS

General Data

For comparison purposes, in this paragraph the functions $\bar{\phi}(s)$ given by (39), (40), (42), and (43) will be presented graphically. The numerical data have been taken correspondingly for 3-DOF hand–arm (Meltzer et al., 1980) and mass-double Kelvin–Voigt (Wasiliev, 1972) system models. The following parameters of the power spectral density for the narrow-band excitation were assumed: $\sigma^2 = 1\ \mathrm{m}^2 s^{-4}$, $\gamma = 0, 1\ \mathrm{s}^{-1}$. The three following sets of Ω were considered: $\Omega = 40\ \mathrm{s}^{-1}$, $\Omega = 100\ \mathrm{s}^{-1}$, and $\Omega = 300\ \mathrm{s}^{-1}$. $\Omega = 40\ \mathrm{s}^{-1}$ corresponds to the minimal value of the impedance magnitude of the 3-DOF model of Meltzer et al. The values of $\Omega = 100, 300\ \mathrm{s}^{-1}$ are correspondingly the minimum and the maximum values of the impedance magnitudes of the Wasiliev model. To enable quantitative estimation of the influence of Lagrangian multipliers on the magnitude of $\phi(s)$, two different values of λ were chosen, $\lambda = 0.0001, 10{,}000\ \mathrm{s}^{-4}$.

The parameters A and B for the values λ chosen are given in Table 2. The parameters K, L for white noise excitation and M, N, P, Q for narrow-band noise excitation are presented correspondingly in Tables 3 and 4.

It can be shown easily from (8) and (9) that for

$$\phi(s) = s^{-1} Z(s), \tag{44}$$

Table 2. Parameters A and B

λ (s^4)	A (s^2)	B (s)
0.0001	0.01	0.14142
10000	100	14.142

Table 3. White Noise Excitation

λ (s^4)	K (s)	L (—)
0.0001	0.1414	1.0
10000	14.14	1.0

Parameters K and L for λ chosen.

Table 4. Narrow-Band Noise Excitation

Ω (s^{-1})	λ (s^4)	M (s^2)	N (s)	P (—)	Q (s^{-1})
40	0.0001	0.0041796	0.027845	6.6568	40
	10000	0.3535	0.0957	565.68	40
100	0.0001	0.00151478	0.010416	15.142	100
	10000	0.14152	0.038304	1415.2	100
300	0.0001	0.0004825	0.003433	43.426	300
	10000	0.0471516	0.001276	4243.6	300

Parameters M, N, P, and Q for λ and Ω.

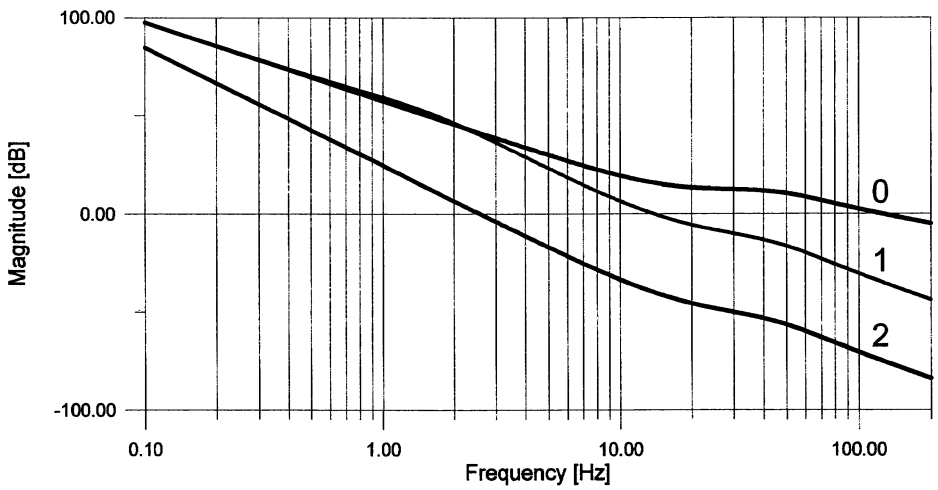

FIGURE 5 The magnitudes of $\phi(s)$ for the bent hand model and the white noise acceleration excitation: 0, without an isolation system; 1, with an isolation system ($\lambda = 0.0001\ s^4$); 2, with an isolation system ($\lambda = 10{,}000\ s^4$).

an HAS is rigidly connected with the source of excitation. In this case there is no isolation system between the source of vibration and the HAS.

Numerical Data and Results for a Bent Hand–Arm Model

The data concerning the HAS model (Wasiliev, 1972) for an elbow angle of 45° and grip force of 200–500 N are as follows: $m = 3{,}6$ kg, $k_1 = 270{,}000\ \mathrm{kg\,s^{-2}}$, $k_2 = 33{,}000\ \mathrm{kg\,s^{-2}}$, $\alpha_1 = 660\ \mathrm{kg\,s^{-1}}$, $\alpha_2 = 560\ \mathrm{kg\,s^{-1}}$. The numerical values of the parameters of (37) have been calculated on the basis of these data. In Fig. 5 the magnitudes of $\phi(s)$ are presented for white noise excitation and for the two values of λ chosen. Figure 6 shows the magnitudes of $\phi(s)$ for narrow-band noise excitation and two values of λ and Ω.

Numerical Data and Results of 3-DOF Hand–Arm Model

The data concerning the HAS model according to Meltzer et al. (1980) are: $m_1 = 3{,}47$ kg, $k_1 = 0{,}7910^4\ \mathrm{kg\,s^{-2}}$, $\alpha_1 = 289\ \mathrm{kg\,s^{-1}}$, $m_2 = 1{,}84$ kg,

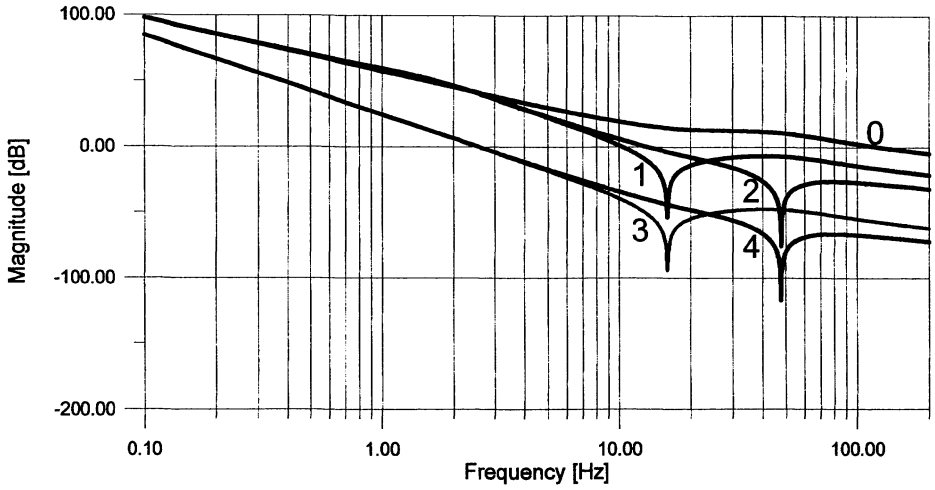

FIGURE 6 The magnitudes of $\phi(s)$ for the bent hand model and the narrow-band noise acceleration excitation: 0, without an isolation system; 1, with an isolation system ($\lambda = 0.0001\ s^4$, $\Omega = 100\ s^{-1}$); 2, with an isolation system ($\lambda = 0.0001\ s^4$, $\Omega = 300\ s^{-1}$); 3, with an isolation system ($\lambda = 10{,}000\ s^4$, $\Omega = 100\ s^{-1}$); 4, with an isolation system ($\lambda = 10{,}000\ s^4$, $\Omega = 300\ s^{-1}$).

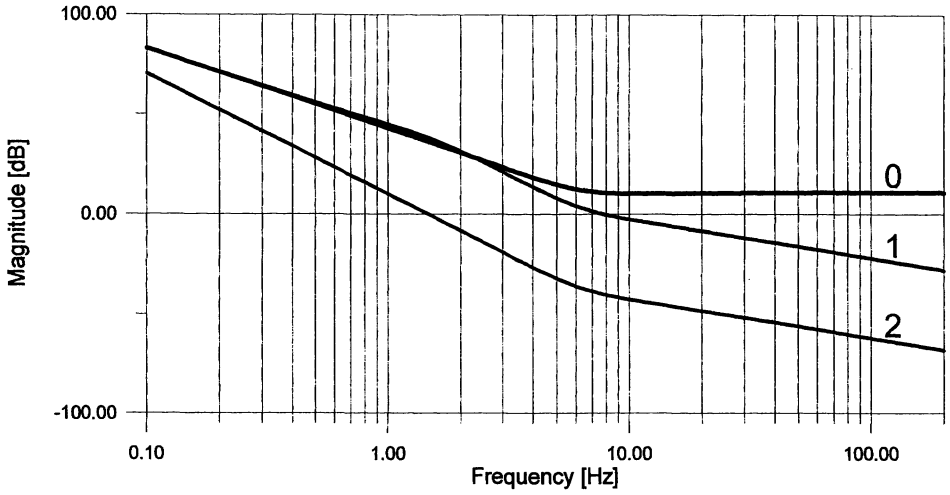

FIGURE 7 The magnitudes of $\phi(s)$ for the 3-DOF hand model and the white noise acceleration excitation: 0, without an isolation system; 1, with an isolation system ($\lambda = 0.0001\ s^4$); 2, with an isolation system ($\lambda = 10{,}000\ s^4$).

$k_2 = 2{,}3510^4\ \mathrm{kg\,s^{-2}}$, $\alpha_2 = 184\ \mathrm{kg\,s^{-1}}$, $m_3 = 0{,}11$ kg, $k_3 = 11{,}110^4\ \mathrm{kg\,s^{-2}}$, $\alpha_3 = 334\ \mathrm{kg\,s^{-1}}$. In Fig. 7 the magnitudes of $\phi(s)$ for white noise excitation and the two values of λ are presented. Figure 8 shows the magnitudes of $\phi(s)$ for narrow-band noise excitation and the two values of λ and Ω.

CONCLUDING REMARKS

The analytical procedure of the synthesis of an optimum vibration isolation system presented in this article can be extended to an arbitrary vibroisolated object described by its driving-point impedance. The final results will depend on the form of the criterion, excitation, and driving-point impedance. For criterion (4), transfer functions (8) and (9) depend on the parameters of the excitation and the form of the criterion. As has been demonstrated in Figs. 5–8, the magnitudes of the optimal $\phi(s)$ are lower than the magnitude of $\phi(s)$, corresponding to a rigid junction between the HAS models and the sources of excitations. The differences increase with increasing values of the Lagrangian multiplier λ. For very small values of λ, $\lambda = 10^{-4}\ s^4$ in the range of frequencies below 2.5 Hz, the values of the corresponding magnitudes of $\phi(s)$

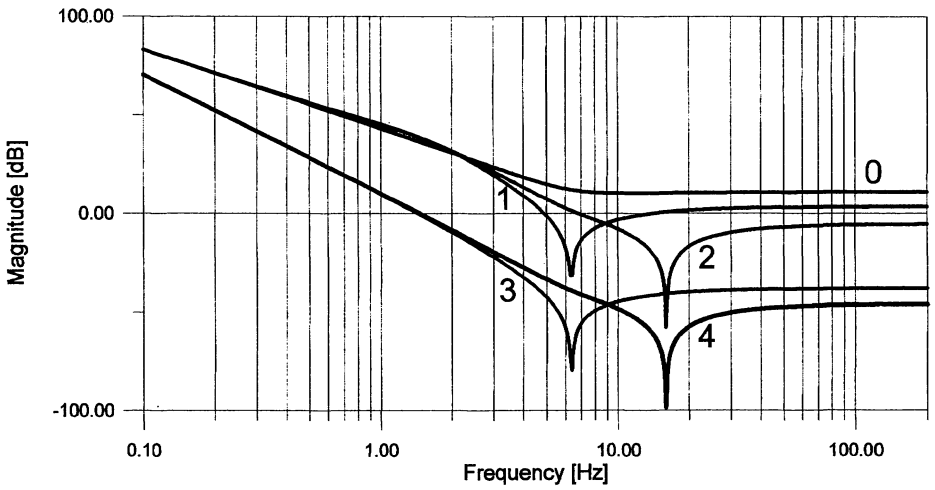

FIGURE 8 The magnitudes of $\phi(s)$ for the 3-DOF hand model and the narrow-band noise acceleration excitation: 0, without an isolation system; 1, with an isolation system ($\lambda = 0.0001\ s^4$, $\Omega = 100\ s^{-1}$); 2, with an isolation system ($\lambda = 0.0001\ s^4$, $\Omega = 300\ s^{-1}$); 3, with an isolation system ($\lambda = 10{,}000\ s^4$, $\Omega = 100\ s^{-1}$); 4, with an isolation system ($\lambda = 10{,}000\ s^4$, $\Omega = 300\ s^{-1}$).

are comparable. It can be shown from formulae (8), (9), (5), and (4) that the magnitudes of $\phi(s)$ directly determine the effectiveness of the synthesized system. The lower the values of $|\phi(s)|$, the greater determine the effectiveness of the isolation system. The advantages of the optimal VIS are particularly visible in the regions of the resonant frequencies of HAS and for the dominant frequencies of excitations.

ACKNOWLEDGEMENT

This study was supported by Polish Scientific Research Committee, PB-0297-P4-92-03.

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