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The use of transmissibility properties to estimate FRFs on modified structures

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Abstract. When deriving an experimental model from Frequency Response Functions (FRFs), it may happen that the measurement of certain FRFs is impossible. This may be an important issue, mainly in the field of condition monitoring and damage detection, since some points of interest may become inaccessible in operational conditions. In this circumstance, it is useful to have some tools that can provide the prediction of such dynamic information. The transmissibility concept, extended to a general multiple degree-of-freedom system, can play an important role to circumvent these situations. The authors have shown in previous works that the estimation of such FRFs can be made possible by invoking important properties associated with the transmissibility function. The objective of this work is to evaluate different sets of FRFs, estimated by using the transmissibility concept and its associated properties, in an actual continuous structure to which different patterns of structural modification are applied. A supplementary study in this work shows that some of the sets for applied forces/known responses can better estimate the FRF data.

Keywords: Transmissibility, structural modification, Frequency Response Functions

1. Introduction

The notion of transmissibility is presented in every classic textbook on vibrations, associated to the single degree-of-freedom system, when its base is moving harmonically: it is defined as the ratio between the modulus of the response amplitude and the modulus of the imposed motion amplitude. In recent years the concept has been extended to N degrees-of-freedom, relating a set of unknown responses to another set of known responses, for a given set of applied forces, as it has been presented by Ewins and Liu [1] and Varoto and McConnell [2] although with some restrictions. A general approach to the transmissibility concept was presented by Ribeiro [3].

An application where the transmissibility seems of great interest is when, in field service, one can not measure the responses at some co-ordinates of the structure. If the transmissibility matrix of the original system can be evaluated beforehand, then, by measuring in service some responses, one would be able to estimate the responses at the inaccessible co-ordinates. This is possible, since there are important transmissibility properties, as it is presented by Maia et al. [4] that provide us the necessary tools to estimate those responses.

An experimental case study was presented by Urgueira et al. [5], demonstrating that the derived properties presented in [4] allow us to estimate the frequency response functions (FRFs) data at certain inaccessible locations of various structural modified systems.

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2. Transmissibility properties

The approach proposed in [3] is based on harmonically applied forces (easy to generalize to periodic ones). If one has a vector F_A of magnitudes of the excitation forces (and /or moments) at co-ordinates A, a vector $\boldsymbol{X}_U$ of unknown response amplitudes at co-ordinates U and a vector $\boldsymbol{X}_K$ of known response amplitudes at co-ordinates K, then one can write:

$$\boldsymbol{X}_U = \boldsymbol{H}_{UA}\,\boldsymbol{F}_A \tag{1}$$

$$\boldsymbol{X}_K = \boldsymbol{H}_{KA}\,\boldsymbol{F}_A, \tag{2}$$

where $\boldsymbol{H}_{UA}$ and $\boldsymbol{H}_{KA}$ are the receptance frequency response matrices relating co-ordinates U and A, and K and A, respectively. Assuming that the number of known co-ordinates is equal or higher than the number of applied forces, i.e., $\#K \geqslant \#A$, one can eliminate $\boldsymbol{F}_A$ between (1) and (2), leading to

$$\boldsymbol{X}_U = \boldsymbol{H}_{UA}\,\boldsymbol{H}_{KA}^{+}\,\boldsymbol{X}_K \tag{3}$$

or

$$\boldsymbol{X}_U = \boldsymbol{T}_{UK}^{(A)}\,\boldsymbol{X}_K \tag{4}$$

where $\boldsymbol{H}_{KA}^{+}$ is the pseudo-inverse of $\boldsymbol{H}_{KA}$. Thus, the transmissibility matrix is defined as:

$$\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}_{UA}\,\boldsymbol{H}_{KA}^{+} \tag{5}$$

An alternative definition of transmissibility, based on the dynamic stiffness matrices of the structure has been presented by Ribeiro et al. [6]:

$$\boldsymbol{T}_{UK}^{(A)} = -\,\boldsymbol{Z}_{BU}^{+}\,\boldsymbol{Z}_{BK} \tag{6}$$

where Z is the dynamic stiffness matrix and B represents the co-ordinates where there are no applied forces. As $\boldsymbol{Z} = \boldsymbol{K} - \omega^2\boldsymbol{M}$, one can now relate the transmissibility functions to the spatial properties of the system. To make this possible, one must bear in mind that it is mandatory that both conditions regarding the number of co-ordinates be valid, i.e.,

$$\#B \geqslant \#U \text{ and } \#K \geqslant \#A \tag{7}$$

Note that the set of co-ordinates (A) where the forces can be applied do not need to coincide with the set of known responses (K). The only restriction is that – for the pseudo-inverse to exist – the number of K co-ordinates must be greater or equal than the number of A co-ordinates.

The transmissibility matrix in *MDOF* systems may be compared to a "black box", relating different sets of dynamic responses on the same structure, but without taking into account its dynamic properties. A kind of transmissibility model of the structure is defined, like the response one. In contrast, the transmissibility functions are strictly local in nature, being functions of input-output frequency response functions.

It has been shown by Maia et al. [4], that there are two important properties for the transmissibility matrix of MDOF systems. These properties are basically connected to the conservation of the transmissibility matrix even if some modifications are made in the mass and stiffness values of the original system:

Property 1: The values of the transmissibility matrix do not change if some modification is made on the mass values of the system where the dynamic loads can be applied – subset A.

Property 2: The values of the transmissibility matrix do not change if some modification is made on the stiffness of a spring connecting co-ordinates of subset A – (where the dynamic loads can be applied).

These properties can be easily understood observing the derivation of the transmissibility matrix; Eqs (1) to (5).

As it can be seen, vector $\boldsymbol{F}_A$ is eliminated to obtain Eq. (3). This means that a possible modification on the mass values related to coordinates A, will not affect the transmissibility matrix. The same is true for modifications on stiffness values, provided they are made among coordinates of the set A.

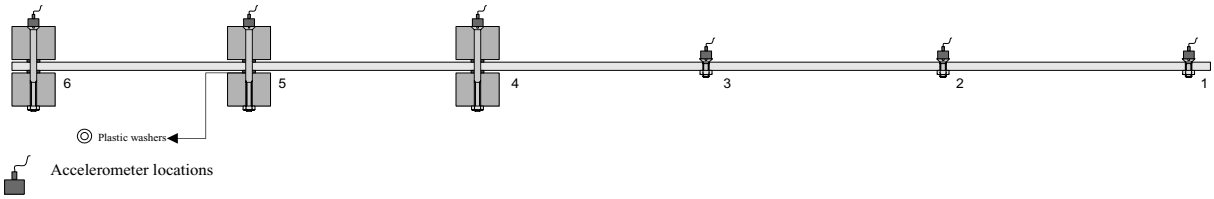


Fig. 1. Original structure.

Stiffness Modified Structure (SM)

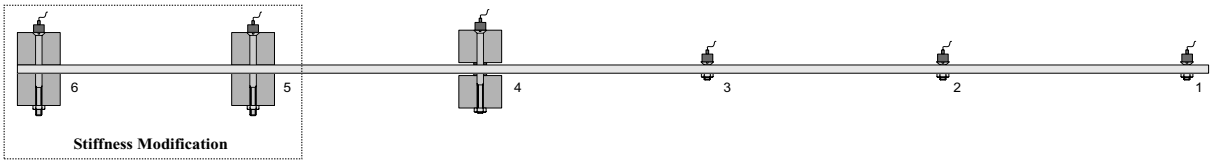


Fig. 2. System with stiffness modification.

From Eq. (5), one has $\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}_{UA}\,\boldsymbol{H}_{KA}^{+}$. Thus, for the modified system, one has (in the conditions stated in properties 1 and 2):

$$\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}'_{UA}\boldsymbol{H}'^{+}_{KA} \tag{8}$$

Therefore,

$$\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}_{UA}\,\boldsymbol{H}_{KA}^{+} = \boldsymbol{H}'_{UA}\,\boldsymbol{H}'^{+}_{KA} \tag{9}$$

Thus, if the receptance matrix relating co-ordinates K and A of the modified system ($\boldsymbol{H}'_{KA}$) is known, the receptance matrix (H'_{UA}) relating co-ordinates U and A can be derived as:

$$\boldsymbol{H}'_{UA} = \boldsymbol{T}_{UK}^{(A)}\,\boldsymbol{H}'_{KA} \tag{10}$$

3. Experimental case study

3.1. Original structure

The experimental case study is mainly formed by a simulated "free-free" suspended steel beam (1000 × 32 × 6 mm) to which six equal steel blocks (32 × 32 × 20 mm) were attached at co-ordinates 4, 5 and 6, as illustrated in Fig. 1. This structure is herein referred to as the original structure, and it was built in such a way to perform simple mass and stiffness modifications at some co-ordinates/regions. As it can be seen, the blocks are initially attached to the structure via a screw, having a plastic wash between each block and the beam. By removing the washes the two blocks are compressed directly to the beam surface, leading to an increase in the local stiffness.

3.2. Modified structures

In order to prove that it is possible to estimate the FRFs of modified systems, provided that the transmissibility matrix of the original system is known, some modifications have been made in the original structure, according to properties 1 and 2.

The stiffness modification made in the original structure corresponds to an increase in the stiffness properties between co-ordinates 5 and 6, by the removal of the plastic washers (Fig. 2).

The mass modification made in the original structure by increasing the mass associated to coordinate 6 as it is present in Fig. 3.

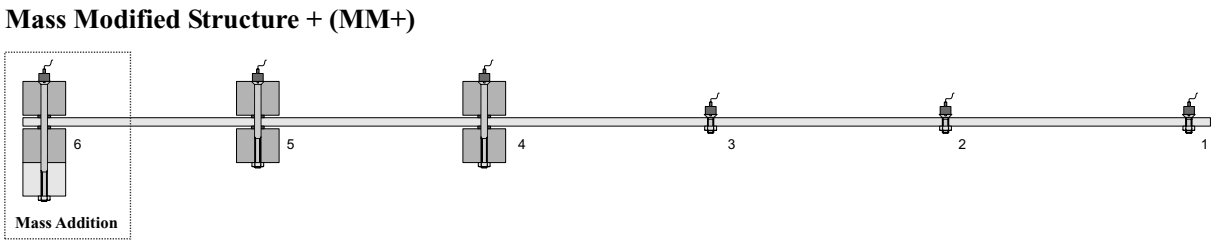
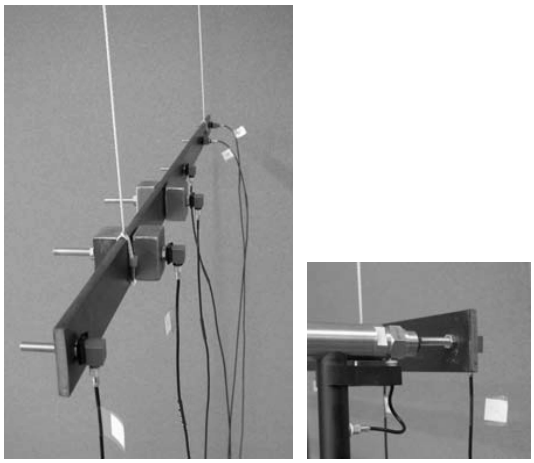


Fig. 3. System with mass modification (mass addition).



(a) (b)

Fig. 4. (a) Suspension and measurement set-up; (b) Excitation set-up.

3.3. Experimental set-up and data assessment

The test structure was suspended by two inextensible cables, simulating a "free-free" condition, as it is presented in Fig. 4(a).

The sets of co-ordinates corresponding to the response and excitation data are presented in Fig. 1. The excitation points of the structure are located at the opposite side of the measurement points as it can be shown in Fig. 4(b).

The excitation forces were applied through an impact hammer (B&K type 8202) with a plastic tip, Fig. 4(b). The response measurements were collected via six uni-axial accelerometers (B&K type 4507/4508). The FRF data was acquired and processed by a B&K type 2035 analyser with eight channels, allowing for the simultaneous acquisition of the six responses and one force. The sensitivity of the force transducer is $3.84\ pc/N$ and for all the accelerometers is 100 m$v/\ g$. Each pair of accelerometer/force was calibrated by using a block with a known mass.

The transient and the exponential window functions have been used, respectively, for the force and for the response measurements, since an impact hammer test was performed. All of the FRFs were measured in the range 0–400 Hz using a frequency resolution of 0.5 Hz.

3.4. Estimation of FRFs using the transmissibility concept

Although with this experimental test structure it is possible to apply forces and measure responses in all co-ordinates it will be assumed, as a first case study, that the *known* (☆) and *unknown* (○) responses are associated with the co-ordinates $K = 2$, 4, 6 and $U = 1$, 3, 5, for both original and modified structures, as it can be seen in Fig. 5. The unknown responses are estimated by using the original transmissibility data and then are compared with the actual measured ones.

So, the response vectors associated to known and unknown responses are given by

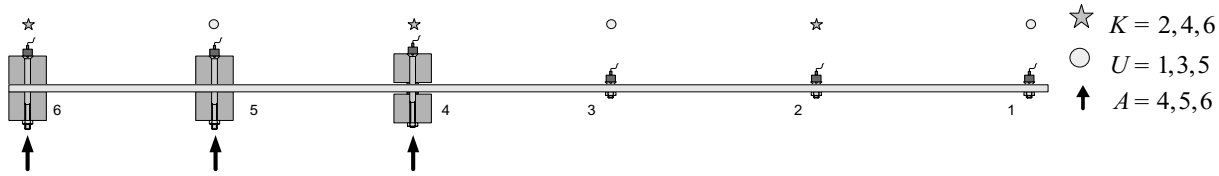


Fig. 5. Co-ordinates associated to known and unknown responses and to the applied loads.

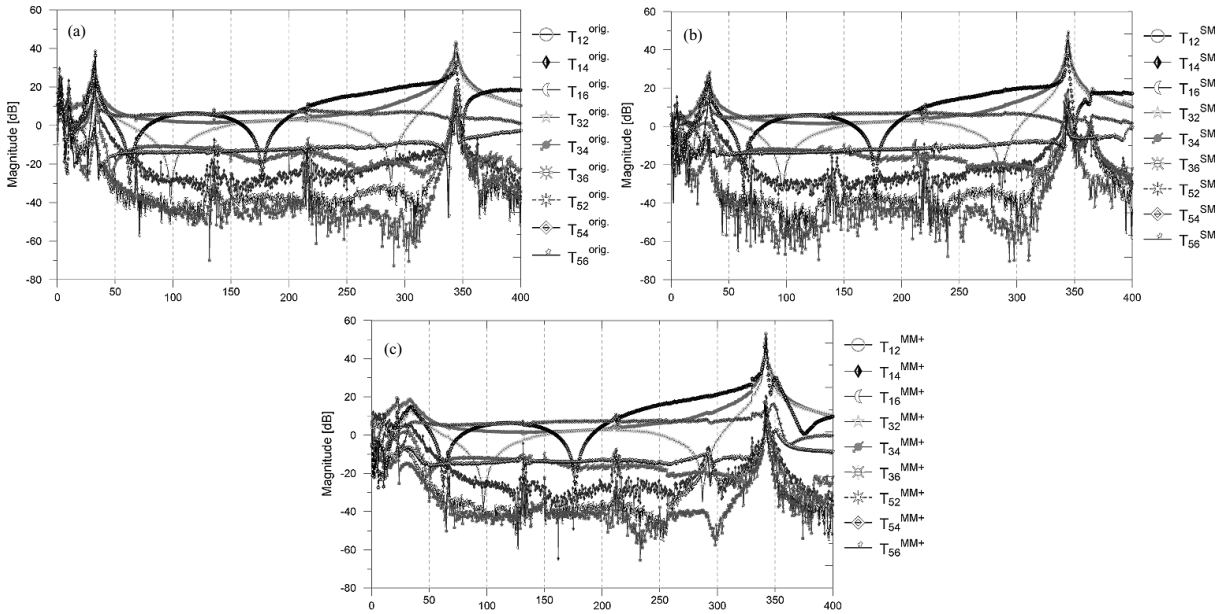


Fig. 6. Transmissibilities obtained for (a) original structure, (b) stiffness modified structure (SM) and (c) mass modified structure (MM+).

$$\boldsymbol{X}_K^T = \left\{ X_2 \; X_4 \; X_6 \right\}^T \text{ and } \boldsymbol{X}_U^T = \left\{ X_1 \; X_3 \; X_5 \right\}^T \tag{11}$$

The vector $\{F_A\}^T$ contains the loads which can be applied (even if some of them are null in certain cases)

$$\boldsymbol{F}_A^T = \left\{ F_4 \; F_5 \; F_6 \right\}^T \tag{12}$$

According to Eq. (5) and considering the above-defined subsets, the transmissibility matrix is given by:

$$\begin{bmatrix} T_{12}^{(A)} & T_{14}^{(A)} & T_{16}^{(A)} \\ T_{32}^{(A)} & T_{34}^{(A)} & T_{36}^{(A)} \\ T_{52}^{(A)} & T_{54}^{(A)} & T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{34} & H_{35} & H_{36} \\ H_{54} & H_{55} & H_{56} \end{bmatrix} \begin{bmatrix} H_{24} & H_{25} & H_{26} \\ H_{44} & H_{45} & H_{46} \\ H_{64} & H_{65} & H_{66} \end{bmatrix}^+ \tag{13}$$

In Fig. 6 all transmissibility functions, are presented for the original and for the modified structures (SM and MM+).

Each of the transmissibility curves shown in Fig. 6(a), should coincide with the corresponding ones in Fig. 6(b) and Fig. 6(c), according to properties 1 and 2 previously mentioned. This conclusion is difficult to extract by only observing those graphs. This fact shows that it is necessary to create a simple and clear way to understand the main differences in the transmissibility behaviour. Similarly to the Norm Indicator for the receptance matrix, developed by Almeida et al. [7], the Transmissibility Indicator (TI) is developed in the present work by using the real and imaginary part of each element $T_{rs}(\omega)$ pertaining to the transmissibility matrix $\boldsymbol{T}_{UK}^{(A)}(\omega)$:

$$T_{rs}(\omega) = \text{Re}\ (T_{rs}(\omega)) + i \cdot \text{Im}\ (T_{rs}(\omega)) \tag{14}$$

The Transmissibility Indicator (TI) can be defined as:

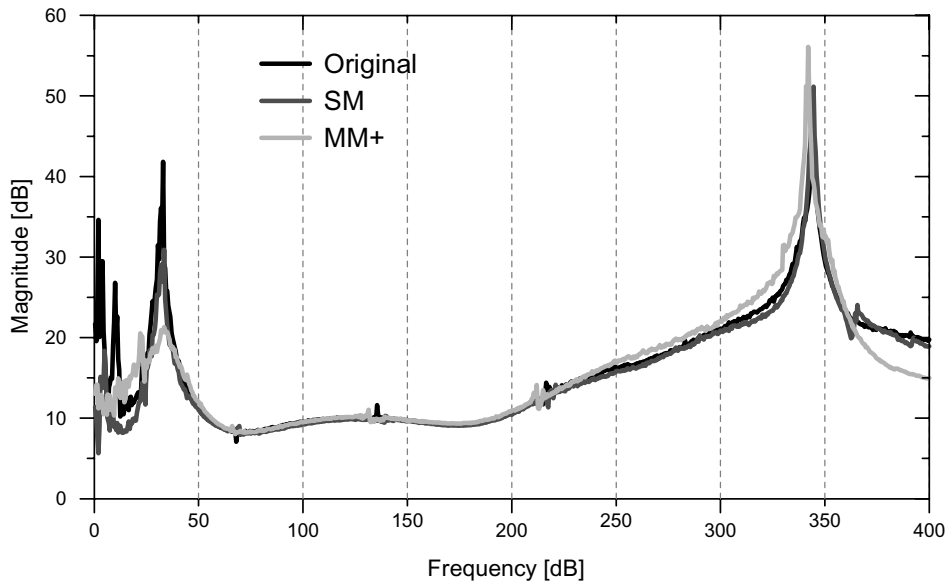


Fig. 7. Transmissibility Indicator (TI) for the three case studies, original structure and modified structures SM and MM+.

$$\begin{aligned} TI\left(\omega_i\right) &= 20\log_{10}\left(\left|\left|\boldsymbol{T}_{UK}^{(A)}\left(\omega_i\right)\right|\right|_F\right) = 20\log_{10}\left(\sqrt{\sum_{r=1}^{n_U}\sum_{s=1}^{n_K}\left|\boldsymbol{T}_{rs}\left(\omega_i\right)\right|^2}\right) \\ &= 20\log_{10}\left(\sqrt{\sum_{r=1}^{n_U}\sum_{s=1}^{n_K}\left(\left(\mathrm{Re}\left(\boldsymbol{T}_{rs}\left(\omega_i\right)\right)\right)^2 + \left(\mathrm{Im}\left(\boldsymbol{T}_{rs}\left(\omega_i\right)\right)\right)^2\right)}\right) \end{aligned} \tag{15}$$

When applying this indicator to the three situations presented in Fig. 6 the results are now clearer as it is shown in Fig. 7.

As it can be concluded, the transmissibility curves of original and modified structures are nearly coincident, thus the transmissibility matrix of the original structure will be used to calculate the receptance matrix $\boldsymbol{H}'_{UA}$, for both modified structures. In Fig. 8, three elements of that matrix such as H'_{16}, H'_{36} and H'_{56} are presented and compared with the measured ones.

By observing Fig. 8, two important facts can be highlighted:

The first one is that the false peaks observed in the estimated FRFs ($\boldsymbol{H}'_{UA} = T_{UK}^{(original)}\,\boldsymbol{H}'_{KA}$) correspond to the maxima of the transmissibilities of the original structure. These maxima correspond to the zeros of the characteristic polynomial generated by the determinant $|Z_{BU}|$ or $|\boldsymbol{H}_{KA}|$ according to Eq. (9):

$$\boldsymbol{T}_{UK}^{(A)} = \boldsymbol{H}_{UA}\,\boldsymbol{H}_{KA}^{+} = \boldsymbol{H}_{UA}\left(\frac{1}{|\boldsymbol{H}_{KA}|}\,adj\left(\boldsymbol{H}_{KA}\right)\right) = -\boldsymbol{Z}_{BU}^{+}\boldsymbol{Z}_{BK} = -\left(\frac{1}{|\boldsymbol{Z}_{BU}|}\,adj\left(\boldsymbol{Z}_{BU}\right)\right)\boldsymbol{Z}_{BK}. \tag{16}$$

The second fact is related to the FRFs of the mass modified structure (MM+). A comparison of these FRF data, Fig. 8(b), shows a deviation in the range 250–400 Hz. A possible reason for this may be due to the fact that, in the modified system MM+, the added mass has not only a translational effect but also a rotational one. In fact, from a theoretical point of view, one is intending that the modification only affects translational motion, neglecting any rotational inertia effect.

3.4.1. New mass modified structure (*MM-*)

In order to confirm the above assertion about the possible effect of rotational inertia in the estimated FRFs, it was decided to modify the original structure by removing the masses (MM-) at coordinate 6, as it can be seen in Fig. 9.

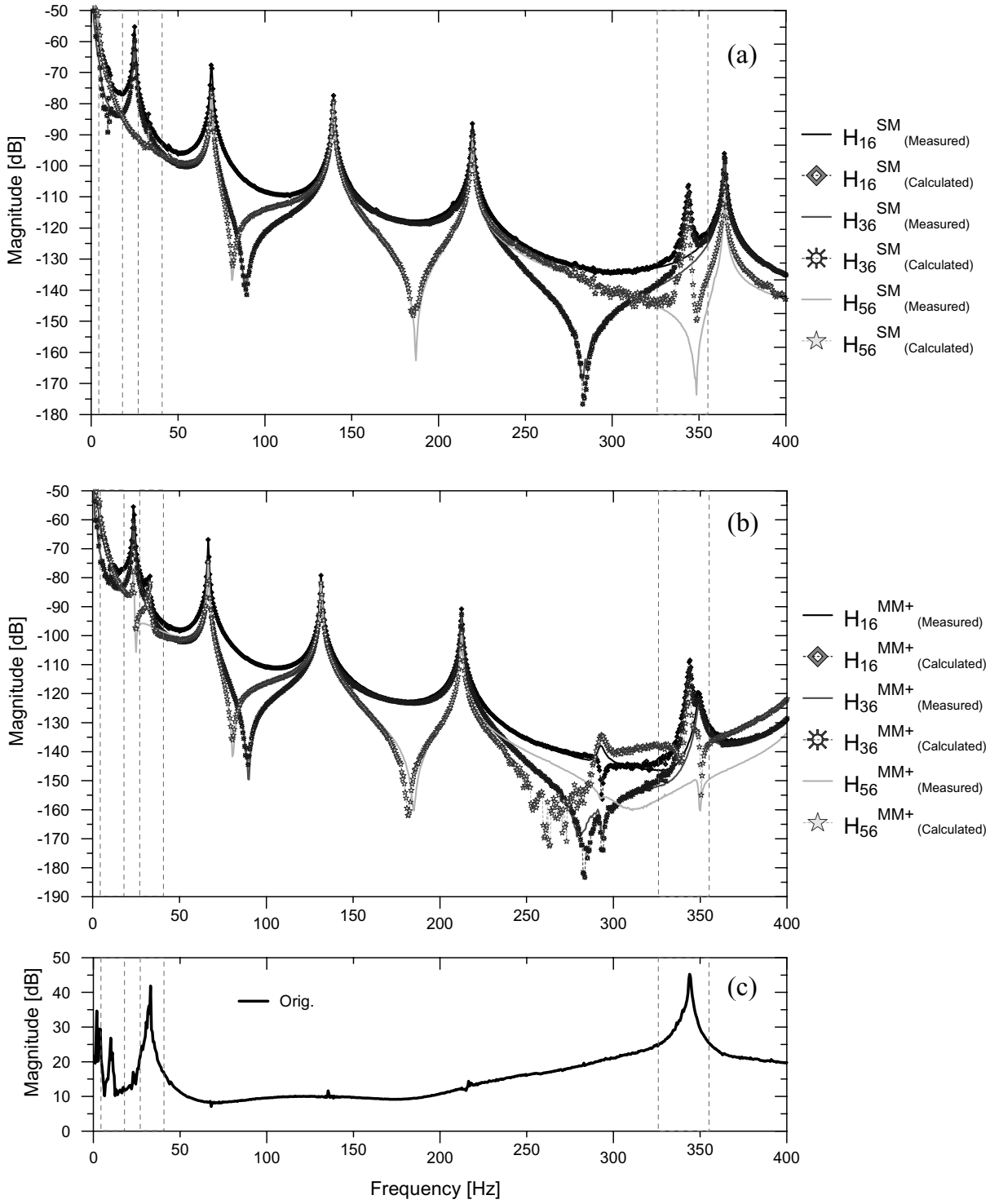


Fig. 8. (a) Measured and estimated Receptances H'_{14}, H'_{36} and H'_{56} for the (b) stiffness modified structure (SM) and (c) mass modified structure (MM+), (c) Transmissibility Indicator for the original structure.

Fig. 9. System with mass modification in coordinate 6 (mass removal).

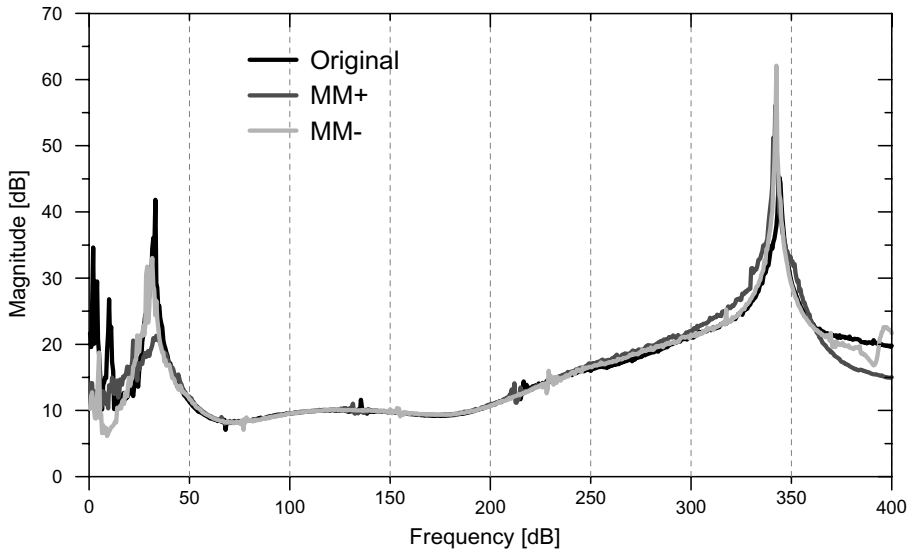


Fig. 10. Application of Transmissibility Indicator to the three case studies, original structure, MM+ and MM-.

The transmissibility indicator shown in Fig. 10 for the MM- structure becomes much closer to the original one than the MM+ one, reinforcing our supposition. This is also clear when one displays the measured and estimated FRFs for this modified structure (see Fig. 11).

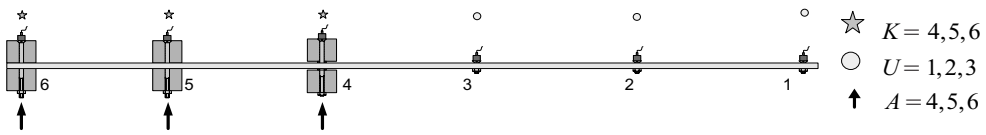
In order to check that it is always possible to calculate the FRFs $\boldsymbol{H}'_{UA}$, using the formulation presented in Eq. (10), regardless of the choice of co-ordinates – known and unknown – and the co-ordinates where the forces can be applied, a variety of studies were performed, involving SM and MM- structures, some of them are presented next.

3.5. Assessment for different location of applied forces/known responses

A large number of sets of applied forces as well as sets of known co-ordinates (☆) and unknown ones (○) have been assumed leading to different case studies, for the SM and MM- structures, and will be presented in the following sub-sections.

3.5.1. All of A co-ordinates coincident with all of K

Case I (SM) → $U = B = 1,2,3$



$$\begin{bmatrix} T_{14}^{(A)} & T_{15}^{(A)} & T_{16}^{(A)} \\ T_{24}^{(A)} & T_{25}^{(A)} & T_{26}^{(A)} \\ T_{34}^{(A)} & T_{35}^{(A)} & T_{36}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{24} & H_{25} & H_{26} \\ H_{34} & H_{35} & H_{36} \end{bmatrix} \begin{bmatrix} H_{44} & H_{45} & H_{46} \\ H_{54} & H_{55} & H_{56} \\ H_{64} & H_{65} & H_{66} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}^{+} \begin{bmatrix} Z_{14} & Z_{15} & Z_{16} \\ Z_{24} & Z_{25} & Z_{26} \\ Z_{34} & Z_{35} & Z_{36} \end{bmatrix} \quad (17)$$

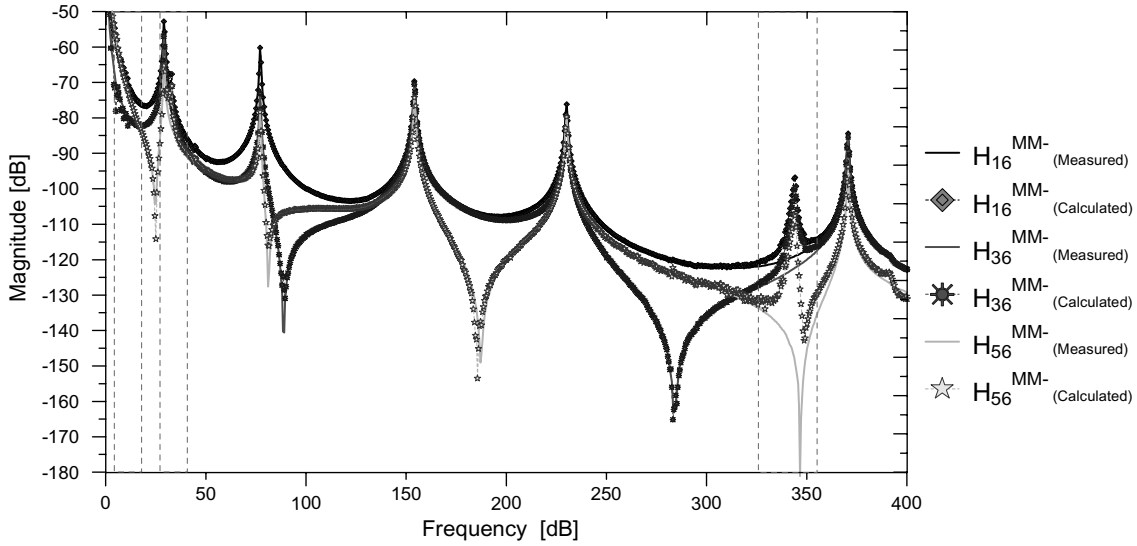


Fig. 11. Measured and estimated Resceptances H'_{16}, H'_{36} and H'_{56} for mass modified structure (MM-).

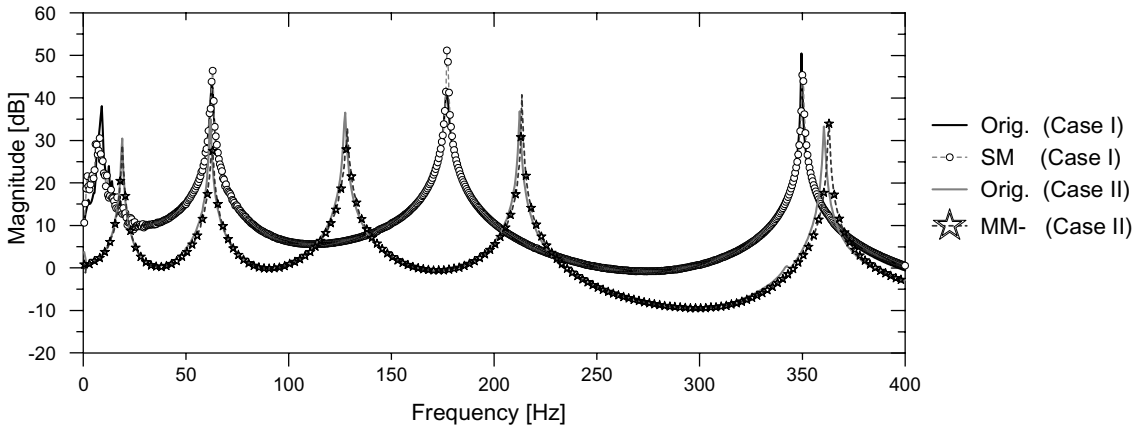
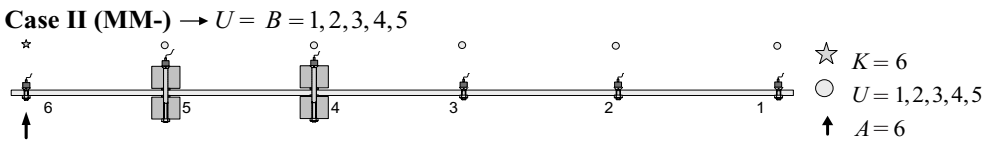


Fig. 12. Transmissibility Indicator for case studies I and II.



$$\begin{bmatrix} T_{16}^{(A)} \\ T_{26}^{(A)} \\ T_{36}^{(A)} \\ T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{16} \\ H_{26} \\ H_{36} \\ H_{46} \end{bmatrix} [H_{66}]^{+} = - \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{bmatrix}^{+} \begin{bmatrix} Z_{16} \\ Z_{26} \\ Z_{36} \\ Z_{46} \\ Z_{56} \end{bmatrix} \tag{18}$$

In Fig. 12, the Transmissibility Indicator (TI) is shown for the modified structures MM- and SM and for the original structure.

It can be observed that the transmissibility indicator of the original and modified structures, for both Cases I and II, is nearly coincident at all frequencies. The small discrepancies occur at the maxima of these curves, especially at

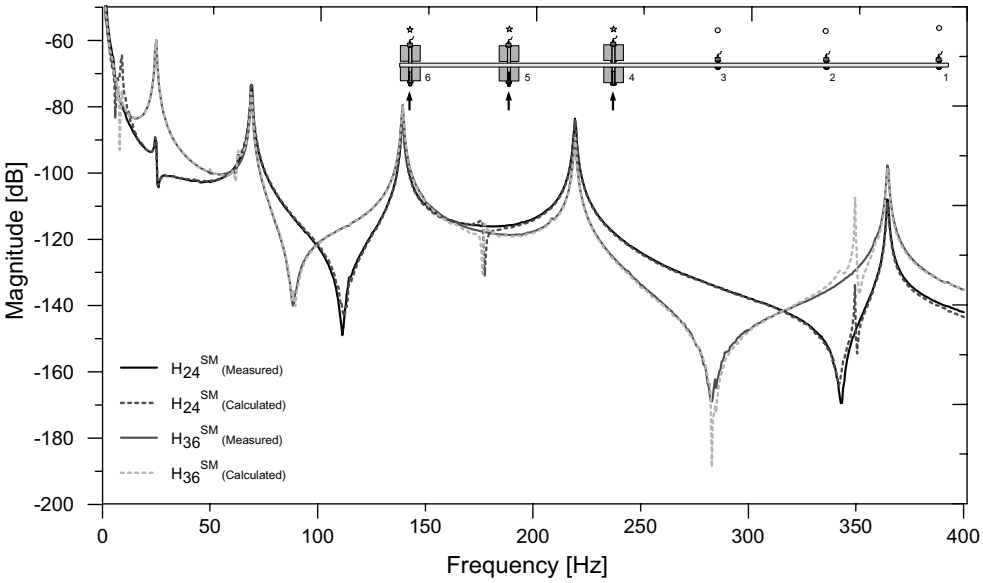


Fig. 13. Measured and estimated Receptances H'_{24} and H'_{36} for the SM structure (Case I).

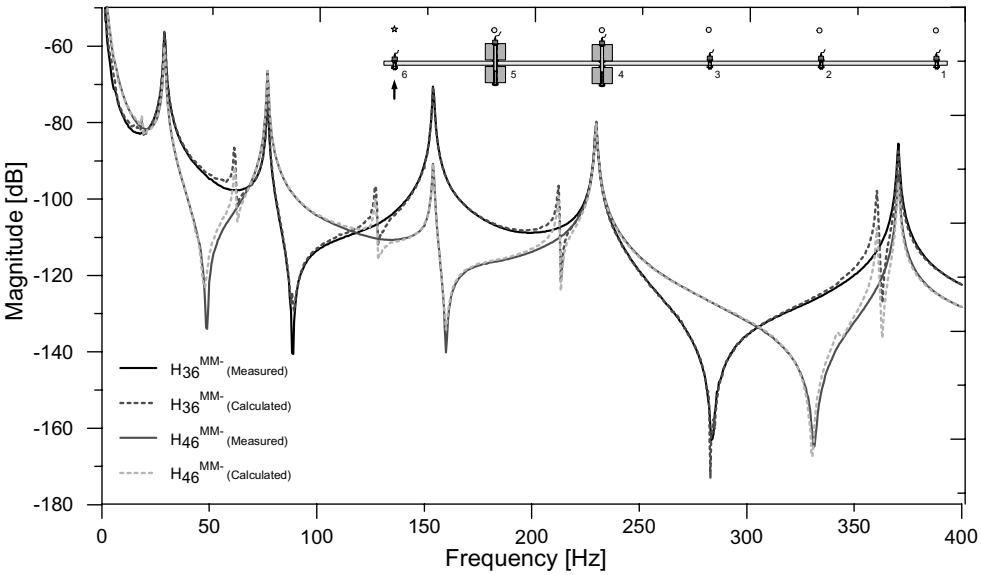


Fig. 14. Measured and estimated Receptances H'_{36} and H'_{46} for the MM- structure (Case II).

higher frequencies.

In Figs 13 and 14, two receptances estimated for the modified structures SM (H'_{24} and H'_{36}) and MM- (H'_{36} and H'_{46}) are presented and compared with the measured receptances of the actual modified structures.

Remarks

It can be observed that the estimated FRFs for these case studies present a good agreement with the measured ones, except at certain frequencies where false peaks appear, corresponding to the maxima of the transmissibilities of the original structure.

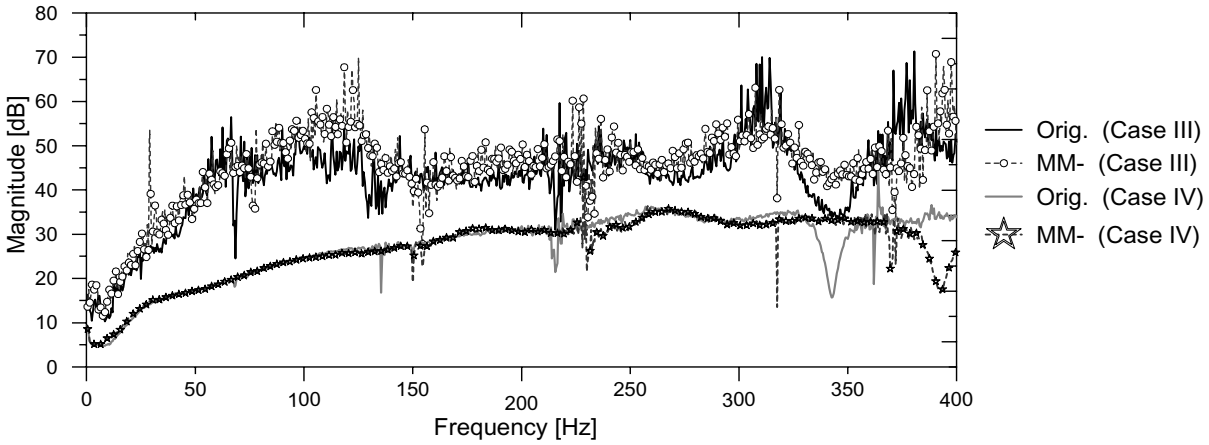
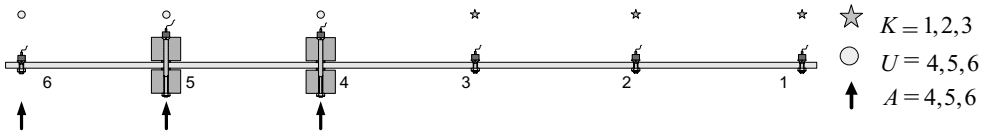


Fig. 15. Transmissibility Indicator for case studies III and IV.

It is important to highlight that, if theoretical data had been used, these discrepancies would not occur (see Maia et al. [4]). The possible cause for the appearance of false peaks, by using experimental data, is due to the fact that the transmissibility functions do not really coincide, mainly at their maxima, leading to the false peaks in the estimated FRFs.

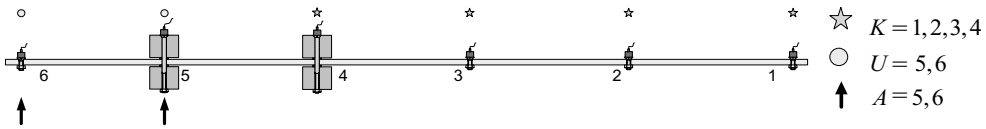
3.5.2. All of A co-ordinates not coincident with all of K

Case III (MM-) → (Z_{BU} square matrix to be inverted)



$$\begin{bmatrix} T_{41}^{(A)} & T_{42}^{(A)} & T_{43}^{(A)} \\ T_{51}^{(A)} & T_{52}^{(A)} & T_{53}^{(A)} \\ T_{61}^{(A)} & T_{62}^{(A)} & T_{63}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{44} & H_{45} & H_{46} \\ H_{54} & H_{55} & H_{56} \\ H_{64} & H_{65} & H_{66} \end{bmatrix} \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{24} & H_{25} & H_{26} \\ H_{34} & H_{35} & H_{36} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{14} & Z_{15} & Z_{16} \\ Z_{24} & Z_{25} & Z_{26} \\ Z_{34} & Z_{35} & Z_{36} \end{bmatrix}^{+} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \quad (19)$$

Case IV (MM-) → (Z_{BU} *rectangular matrix* to be inverted)



$$\begin{bmatrix} T_{51}^{(A)} & T_{52}^{(A)} & T_{53}^{(A)} & T_{54}^{(A)} \\ T_{61}^{(A)} & T_{62}^{(A)} & T_{63}^{(A)} & T_{64}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{55} & H_{56} \\ H_{65} & H_{66} \end{bmatrix} \begin{bmatrix} H_{15} & H_{16} \\ H_{25} & H_{26} \\ H_{35} & H_{36} \\ H_{45} & H_{46} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{15} & Z_{16} \\ Z_{25} & Z_{26} \\ Z_{35} & Z_{36} \\ Z_{45} & Z_{46} \end{bmatrix}^{+} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \quad (20)$$

The TI has been calculated for case studies III and IV and the results are presented in Fig. 15, showing a irregular behavior.

In Fig. 16, two receptances estimated for the modified structure MM-, using cases III and IV, of are presented and compared with the measured ones.

Remarks

It can be observed that there is a better agreement of the estimated FRFs for case IV rather for case study III. This is in accordance with the behavior of the TI shown in Fig. 15. A possible explanation for this resides on

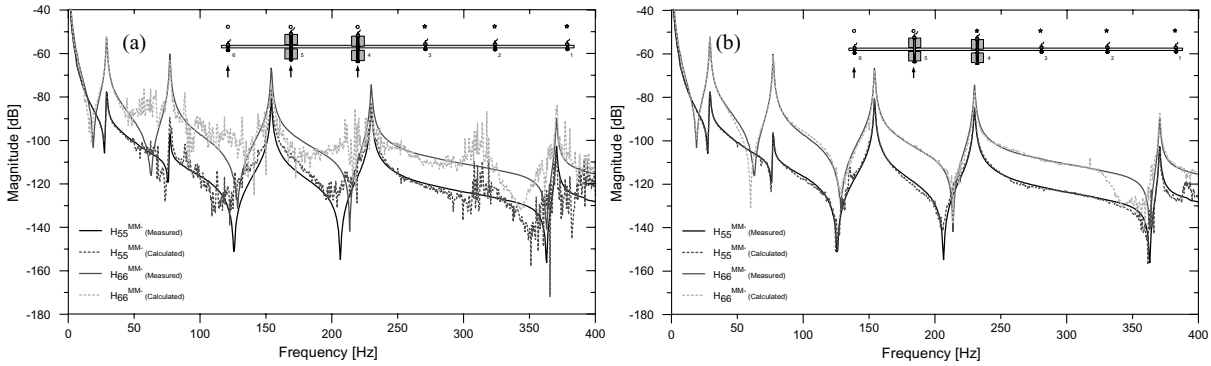


Fig. 16. Measured and estimated Receptances H'_{55} and H'_{66} for the MM- structure (a) Case III and (b) Case IV.

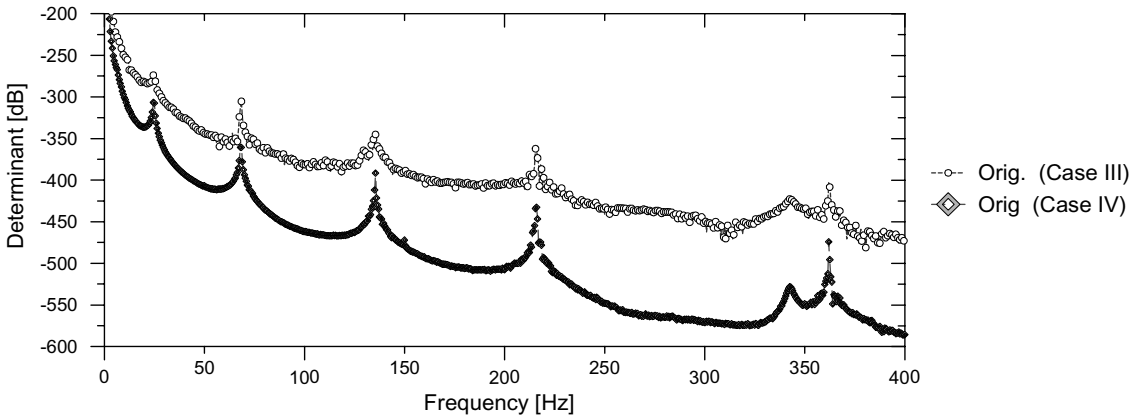


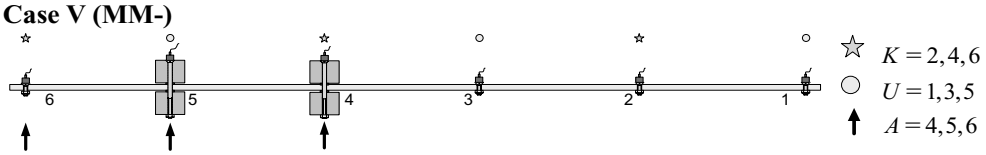
Fig. 17. Determinant curves of matrix H_{KA} for case study III and $H^T_{KA} \cdot H_{KA}$ for case study IV.

the inversion process which is intimately related to the calculation of the determinant of matrices $\boldsymbol{H}_{KA}$ for the traditional inversion and $\left|\boldsymbol{H}^T_{KA} \cdot \boldsymbol{H}_{KA}\right|$ for the pseudo-inverse, respectively for cases III and IV, which behavior is shown in Fig. 17. The small values demonstrate that one is dealing with nearly singular matrices, although in case IV a smooth behavior may explain the better estimation of the corresponding FRFs.

In an attempt to prove what was previously mentioned that the pseudo-inverse applied on rectangular matrices $\boldsymbol{H}_{KA}$ or $\boldsymbol{Z}_{BU}$ improves the results, other cases [with $\#B > \#U$ and $\#K > \#A$] were studied and the results are presented next.

3.5.3. *How to improve the estimation of* H'_{UA}

Case V (MM-)



$$\begin{bmatrix} T^{(A)}_{12} & T^{(A)}_{14} & T^{(A)}_{16} \\ T^{(A)}_{32} & T^{(A)}_{34} & T^{(A)}_{36} \\ T^{(A)}_{52} & T^{(A)}_{54} & T^{(A)}_{56} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{15} & H_{16} \\ H_{34} & H_{35} & H_{36} \\ H_{54} & H_{55} & H_{56} \end{bmatrix} \begin{bmatrix} H_{24} & H_{25} & H_{26} \\ H_{44} & H_{45} & H_{46} \\ H_{64} & H_{65} & H_{66} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{11} & Z_{13} & Z_{15} \\ Z_{21} & Z_{23} & Z_{25} \\ Z_{31} & Z_{33} & Z_{35} \end{bmatrix}^{+} \begin{bmatrix} Z_{12} & Z_{14} & Z_{16} \\ Z_{22} & Z_{24} & Z_{26} \\ Z_{32} & Z_{34} & Z_{36} \end{bmatrix} \quad (21)$$

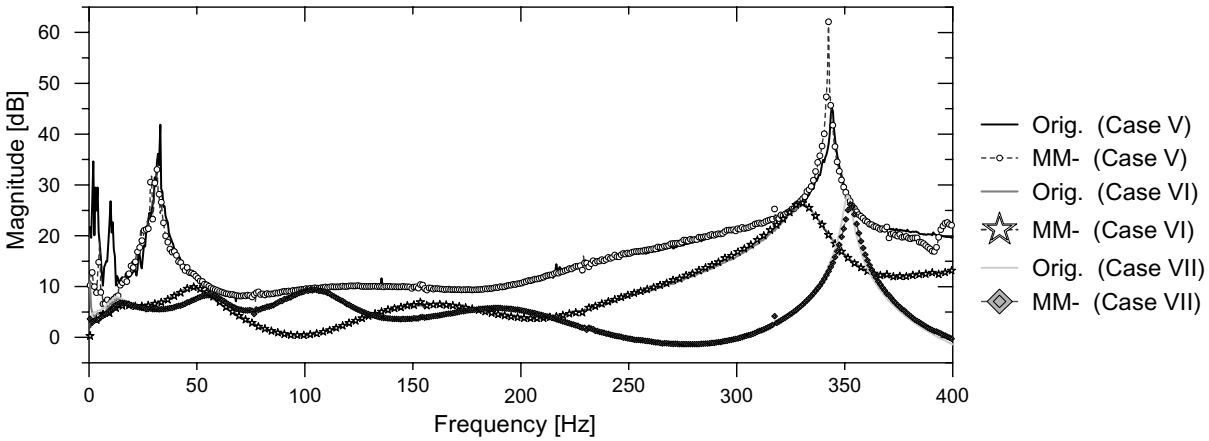
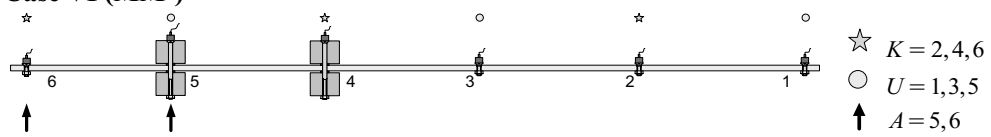


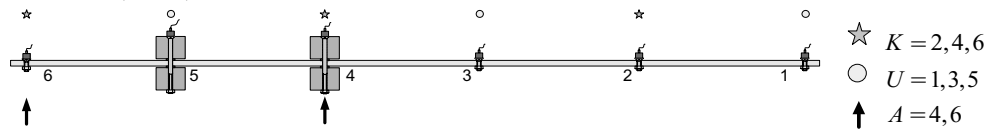
Fig. 18. Application of Transmissibility Indicator to case studies V, VI and VII.

Case VI (MM-)



$$\begin{bmatrix} T_{12}^{(A)} & T_{14}^{(A)} & T_{16}^{(A)} \\ T_{32}^{(A)} & T_{34}^{(A)} & T_{36}^{(A)} \\ T_{52}^{(A)} & T_{54}^{(A)} & T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{15} & H_{16} \\ H_{35} & H_{36} \\ H_{55} & H_{56} \end{bmatrix} \begin{bmatrix} H_{25} & H_{26} \\ H_{45} & H_{46} \\ H_{65} & H_{66} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{11} & Z_{13} & Z_{15} \\ Z_{21} & Z_{23} & Z_{25} \\ Z_{31} & Z_{33} & Z_{35} \\ Z_{41} & Z_{43} & Z_{45} \end{bmatrix}^{+} \begin{bmatrix} Z_{12} & Z_{14} & Z_{16} \\ Z_{22} & Z_{24} & Z_{26} \\ Z_{32} & Z_{34} & Z_{36} \\ Z_{42} & Z_{44} & Z_{46} \end{bmatrix} \tag{22}$$

Case VII (MM-)



$$\begin{bmatrix} T_{12}^{(A)} & T_{14}^{(A)} & T_{16}^{(A)} \\ T_{32}^{(A)} & T_{34}^{(A)} & T_{36}^{(A)} \\ T_{52}^{(A)} & T_{54}^{(A)} & T_{56}^{(A)} \end{bmatrix} = \begin{bmatrix} H_{14} & H_{16} \\ H_{34} & H_{36} \\ H_{54} & H_{56} \end{bmatrix} \begin{bmatrix} H_{24} & H_{26} \\ H_{44} & H_{46} \\ H_{64} & H_{66} \end{bmatrix}^{+} = - \begin{bmatrix} Z_{11} & Z_{13} & Z_{15} \\ Z_{21} & Z_{23} & Z_{25} \\ Z_{31} & Z_{33} & Z_{35} \\ Z_{51} & Z_{53} & Z_{55} \end{bmatrix}^{+} \begin{bmatrix} Z_{12} & Z_{14} & Z_{16} \\ Z_{22} & Z_{24} & Z_{26} \\ Z_{32} & Z_{34} & Z_{36} \\ Z_{52} & Z_{54} & Z_{56} \end{bmatrix} \tag{23}$$

The TI is applied to case studies V, VI and VII and the results are presented in Fig. 18.

In Figs 19 and 20, various receptances estimated for the modified structure MM- in cases V, VI and VII are presented and compared with the measured receptances of the actual modified structure.

Remarks

Observing Figs 19 and 20 it can be verified that the estimated FRFs for case studies VI e VII are much more coincident with the measured ones then the FRFs obtained for case study V. In these two case studies (VI and VII) the matrices to be inverted are rectangular, so the pseudo-inverse was used; for case study V the standard inverse is used because matrix is square. It can be seen that, when the pseudo-inverse is applied on the rectangular matrix H_{KA}, sharp peaks do not appear in these curves; this can be seen more clearly in the TI curves (see Fig. 18). The presence of smoother transmissibility curves may lead to more reliable estimation of the FRFs.

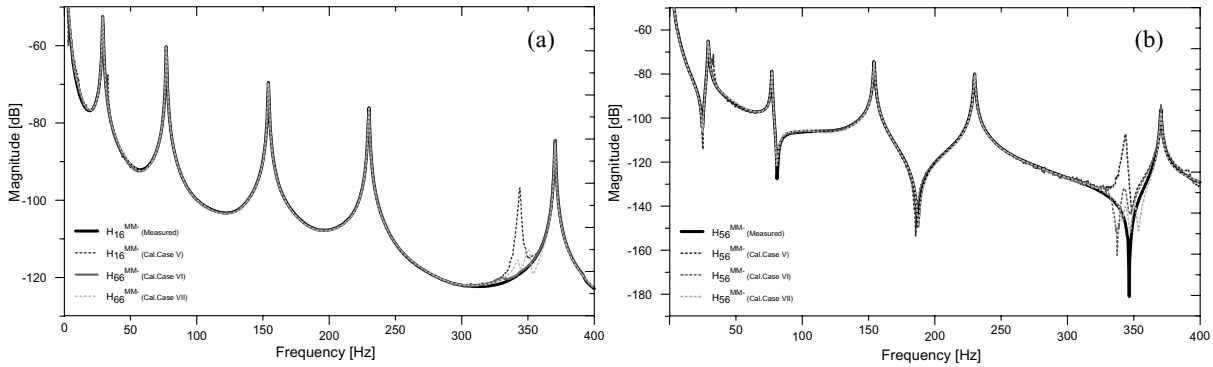


Fig. 19. Measured and estimated receptances (a) H'_{16} and (b) H'_{56}, for cases studies V, VI and VII, for the MM- structure.

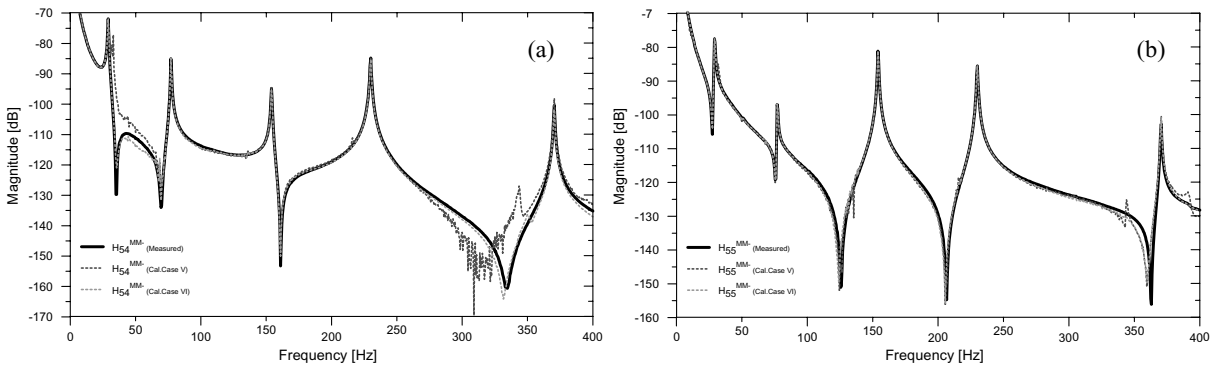


Fig. 20. Measured and estimated Receptance (a) H'_{54} for cases V and VII and (b) H'_{55}. for cases V and VI, for the MM- structure.

4. Final conclusions

In this work it has been shown that it is possible to predict response data (FRFs) in certain inaccessible locations of modified systems, provided that the transmissibility matrix of the original system is known, by invoking properties 1 and 2. However, it was observed that *the predicted FRF data present a high sensitivity to any small discrepancy between the transmissibilities of the structure before and after modification, particularly in the region of their maxima.* The various case studies allowed us to identify that the prediction of the FRFs improved whenever the pseudo inverse was used, i.e., whenever the matrices to be inverted were rectangular. This happens when the set of co-ordinates where the forces can be applied (set A) is a subset of the known co-ordinates (set K).

The Transmissibility indicator is an easy and important tool to be used during the comparison stage of the transmissibility properties of the original and modified structures.

Acknowledgement

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