Shock and Vibration 18 (2011) 3–12
DOI 10.3233/SAV-2010-0568
IOS Press

Nonlinear dynamics of a vibratory cone crusher with hysteretic force and clearances

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Received 12 February 2010
Revised 21 April 2010

Abstract. Based on the analysis on crushing process and hysteresis of material layers, a hysteretic model with symmetrical clearances is presented. The mechanical model of two-degree of freedom with bilinear hysteresis and its dynamical equations of system are proposed. In order to further investigate the dynamic characteristics of the novel vibratory cone crusher, the system is also simplified into a dynamical system of single degree of freedom with a bilinear hysteretic component together with clearances. According to some nonlinear dynamic analysis tools such as bifurcation diagram, Lyapunov exponents, Poincare section, *etc.*, different motion patterns of the system are discussed, including periodic, periodic doubling, chaos and other characteristics. These theoretical results will provide readers with deep understanding on the regular and complex dynamical behaviors of the vibratory cone crusher due to the hysteresis with clearances.

Keywords: Vibratory cone crusher, bilinear hysteretic model with clearances, nonlinear vibration, chaos

1. Introduction

The vibrating system with hysteretic nonlinear force has a wide range of applications in the industrial sectors, such as vibratory moulding machines to compact loose materials, vibratory road rollers to compact soil or gravel, and so on [1]. They all utilized vibrating to produce the plastic deformation of compacted materials or the slippage between material particles so as to complete the material compaction. For the vibratory rolling mills to roll steel or non-ferrous materials, in the process of rolling, accompanied by the occurrence of plastic deformation of materials, the relationship between restoring force and displacement takes on the hysteresis [2]. When the vibratory cone crusher is squeezing materials, the broken materials experience both energy storage and energy release. The materials play an important role in transferring energy and are regarded as an integral part of the vibratory cone crusher. The materials together with the crusher itself constitute a vibrating system. In the process of comminution, along with plastic deformation of materials, the relationship between restoring force and displacement shows the hysteresis. However, the previous dynamic analyses on the vibratory cone crusher system did not consider the impact of hysteretic nonlinear force of materials, which was not in line with the actual situation. The investigation on the dynamical characteristics of vibrating system with hysteretic nonlinear force is more realistic [3].

As well known, a lot of mechanisms and components with hysteretic force in engineering, such as viscoelastic materials for noise and vibration reduction, present nonlinear hysteresis between restoring force and displacement in the case of dynamic deformation because the damping inside the materials consume energy. The hysteresis between restoring force and displacement exists in the components such as reinforced concrete and steel, and also in the automobile shock absorbers which produce dry friction damping between the leaf springs consuming the energy of vibrating system [4].

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ISSN 1070-9622/11/$27.50

The first bilinear hysteretic model was proposed by Caughey in 1960 [5]. In this model system, which was the simplest mathematical model, the force-displacement curve was composed by several different segments [6]. Based on the bilinear hysteretic model, a number of scholars also proposed some improved hysteretic piecewise linear models [7–9], such as Neilsen degraded bilinear model, Clough degraded bilinear model, and many other linear hysteretic models. Different models have different characteristics and adapt to the needs of different situations, for example, multi-linear hysteretic model proposed by Takeda [10], two-parameter hysteretic model described with two smooth curves raised by Davidenkov [11], universal Bouc-Wen hysteretic model [12,13], polynomial hysteretic model, and so on.

In this paper, a novel vibratory cone crusher for processing metal ores is discussed for its dynamical behaviors. The crushing process on materials is studied and the hysteretic model of the material layers with clearances is proposed. Based on the above hysteretic model, the mechanical model of two-degree of freedom with bilinear hysteresis and its dynamical equations of system are established [14]. For the purpose of finding out the dynamic characteristics of the novel vibratory cone crusher, the system is also simplified into a dynamical system of single degree of freedom with a bilinear hysteretic component together with clearances. By means of nonlinear dynamic analysis tools such as bifurcation diagram, Lyapunov exponents, Poincare section, *etc.*, different motion patterns of the system are discussed, including periodic, periodic doubling, chaos and other characteristics. The results indicate that the hysteretic system with clearances contains very rich dynamical behaviors and that disorderly chaotic motions emerge in the system within the compass of particular parameters, which will greatly reduce the work efficiency and the processing capacity of the vibratory cone crusher system. Therefore, the existing machine set should be amended by a great deal of experiments and be determined a range of reasonable parameters through identification so that it can be applied in the field of engineering design and analysis.

2. Description of the vibratory crushing process

Under the action of the exciting force, the mantle of vibratory cone crusher revolves around the axis of crusher. In the longitude section of the cone crusher, the mantle performs a pendulum movement. As the mantle swings between the closed and open sides, it periodically approaches and leaves the concave surface. When the materials get into the crushing chamber, they keep falling until they meet the mantle. Then the materials are pushed against the concave surface by the mantle and get crushed [15–17]. The broken materials are bulk materials and form the material layers with a certain loose degree in the crushing chamber. The external excitation forces the mantle to move towards the internal surface of concave, which bring about the dynamic impact. The particles of various sizes are squeezed and crushed against the steel surface [18]. According to the wave theory, the stress waves act on the loose material layers to make them produce the physical movement of the greatest degree, accompanied by elasticity, non-elasticity, and sharp angle breakage caused by the compression of particles, this phase is regarded as the compression of clearances. In the phase, both the energy transferred to the material layers and the doing work on the material layers are very little, so the area for the compression phase in $\sigma - \varepsilon$ curve is correspondingly small, as shown in Fig. 1. The more the stress waves do work on the material layers, the more the compression and the sharp angle breakage occur. When the material layers are compacted to a certain extent, that is across the clearances, the mantle exerts a strong shock wave vibration on the material layers, the particles are crushed on a large scale under the action of increasing stress amplitude, the $\sigma - \varepsilon$ curve enters the phase of crushing, and in this phase the influence of strain rate on the $\sigma - \varepsilon$ curve is more significant than that in the compression phase.

After the breakage of particles, the volume of material layers expands and the stress redistributes. Only when the new particles are further compressed, the stress increases to produce further damage on particles. Therefore, the crushing phase is that the particles are constantly broken in the overlapping process of crushing-compression-crushing, generating new particles and superficial areas. From the beginning of crushing phase, the stress waves react on the particles compacted in the compression phase. The particles are then further crushed and compacted, accompanied by the emergence of new particles, so the fluctuation of slope is not obvious. With the continuous compression and crushing on the particles, the stress grows rapidly, while the increasing of strain becomes slowly, so the slope heightens gradually in the crushing phase. With the increasing of strain rate, the stress waves do more work on the material layers to enhance the crushing effects.

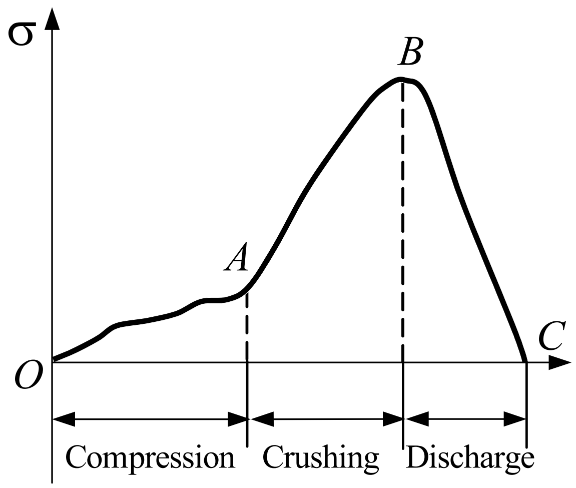


Fig. 1. Curve of whole process of dynamic crushing on materials.

The dynamic $\sigma - \varepsilon$ curve enters the discharge phase after the peak. The material layers store a large number of energy in the compression and crushing phase, the energy releases during the process of discharge for the further fragmentation of particles and for the formation and development of crackle [19].

3. A symmetric hysteretic model with clearances

In the process of crushing materials, there are many nonlinear factors and the most important one is the nonlinear characteristics of the material layers. It is essential to describe reasonably the nonlinear restoring force of material layers and analyze the dynamics of the whole vibrating system [20].

In the initial phase of crusher running, the mantle operates on the material layers with some loose degree to ascertain the periodic external load, which reduces the clearances among the particles of material layers and couples with elasticity, non-elasticity, sharp angle breakage, and particles breaking dissociation along the lattice defects from micro-perspective point of view; Macro-perspectively, the density of material layers increases, the material layers approach continuously the internal surface of concave and induce the deformation, and this phase is regarded as the clearances; When the material layer is compacted to a certain extent, that is across the clearances, the mantle exerts the forward load on the material layer, here the strain accumulation of the material layers increases constantly, and this phase is looked as the elastic deformation; When the stress level exceeds the elastic limit of materials, the material layers suffer the irreversible plastic deformation, which leads to the fracture of materials. At this time, the whole material layers take on the hysteresis related to their historical path. The hysteretic restoring force of materials is not the single-valued relationship with the relative displacement between the mantle and the concave, but the hysteretic relationship relevant to the loading history. Considering the material layers as elastic-plastic materials and their strengthened nature, the bilinear model can be used to replace approximately the actual hysteretic loops. During the process of discharge, the unloading stiffness can be equivalent to the loading stiffness. The material layers come into the plastic deformation and are then unloaded, the hysteresis force will eventually be reduced to zero.

Due to the symmetrical structure of the vibratory cone crusher and the crushing characteristics, the hysteretic force is symmetrical and is piecewise linear odd function which is symmetrical about the origin in one cyclical movement of crushing materials. Because of the symmetry of the crusher structure, when the mantle moves to the opposite direction, its movement characteristics are similar to the previous ones. If the movement of the entire system is observed from *xoy* plane, in the x, y positive direction or negative direction, the hysteretic force characteristics are consistent no matter loading or unloading. The only difference is that the phase angle of hysteresis force between positive direction and negative direction is just 180°. Therefore, for the vibrating system of vibratory cone crusher,

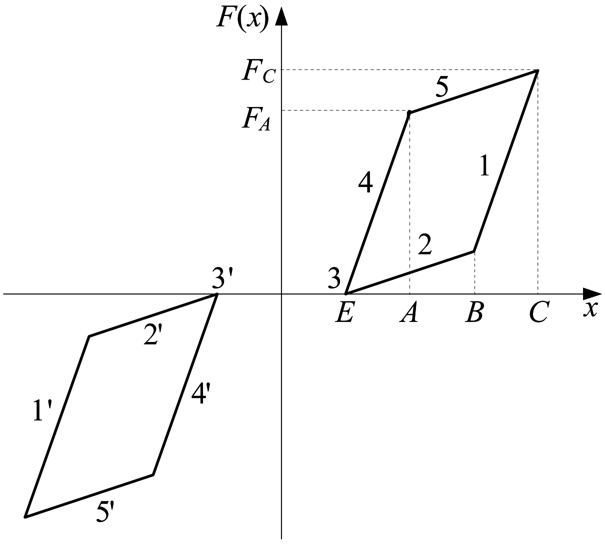


Fig. 2. Symmetric hysteretic model with clearances.

the resilience of materials is a special form of hysteretic restoring force due to the influence of some factors such as forward loading and unloading, reverse loading and unloading, and compression of the loose material layers, and so on. The symmetric hysteretic model with clearances can be simply demonstrated by Fig. 2 as above. The hysteretic force $F(x)$ can be expressed as Eq. (1):

$$F(x) = \begin{cases} k_1 x + a_1 sgn(x) & \text{interval } 1, 1' \quad x_B \leqslant |x| < x_C \quad x\dot{x} < 0 \\ k_2 x + a_2 sgn(x) & \text{interval } 2, 2' \quad e \leqslant |x| < x_B \quad x\dot{x} < 0 \\ 0 & \text{interval } 3, 3' \quad |x| < e \\ k_1 x + a_3 sgn(x) & \text{interval } 4, 4' \quad e \leqslant |x| < x_A \quad x\dot{x} > 0 \\ k_2 x + a_4 sgn(x) & \text{interval } 5, 5' \quad x_A \leqslant |x| < x_C \quad x\dot{x} > 0 \end{cases} \tag{1}$$

where, k_1 is the elastic load stiffness; k_2 is the plastic load stiffness; e is the clearance value; $a_1 = -k_1 x_C + F_C$; $a_2 = -k_2 e$; $a_3 = -k_1 e$; $a_4 = -k_2 x_A + F_A$; sgn is the sign function.

4. Differential equations of the system with hysteretic force and clearances

During the squeezing motion, the intervening materials in the crushing chamber will be subjected to a compressive stress field and thereby be comminuted. During the releasing motion, the materials flowing through the chamber are driven by the influence of gravity together with the motion of the mantle [21]. As known from the whole process of dynamic crushing on materials, the materials suffer extrusion and vibration shock from the mantle and deform as a result, which transfer simultaneously energy to the concave and cause very small movement of the concave. It elucidates that for the self-synchronizing vibratory cone crusher, the material layers are not only the object for processing, but also an integrated part of the vibrating system [22,23]. It is of great significance to introduce the acting force of material layers into the vibrating system of crusher and to establish the mechanical model and the nonlinear equations of vibrating system considering the acting force of material layers.

Under the role of the exciting force, the mantle executes reciprocating motion along the x direction and the y direction in the *xoy* plane. The energy is transferred by the materials between the mantle and the concave. Considering the role of the hysteretic force of material layers, the mechanical model of vibrating system is shown in Fig. 3.

Due to the structural symmetry of the vibratory cone crusher, the crushing process and the dynamic characteristics no matter in x or y direction are identical. Therefore, the research on dynamic characteristics is carried out only in

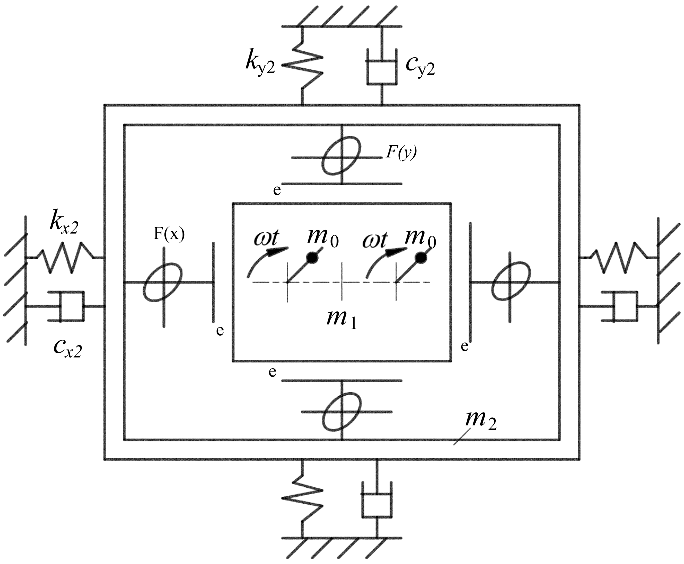


Fig. 3. Mechanical model of vibrating system.

one direction. The equation of motion of vibrating system in x direction can be described by Eq. (2):

$$\begin{aligned} &m_1\ddot{x}_1 + c_{x1}\dot{x}_1 - c_{x1}\dot{x}_2 + F(x) = 2m_0 r\omega^2 \sin\omega t \\ &m_2\ddot{x}_2 - c_{x1}\dot{x}_1 + (c_{x1} + c_{x2})\dot{x}_2 + k_{x2}x_2 - F(x) = 0 \end{aligned} \tag{2}$$

Where, x_1, x_2, and x are respectively the displacement of frame 1, the displacement of frame 2, and their relative displacement, $x = x_1 - x_2$; m_1 and m_2 are respectively the mass of frame 1 and frame 2 involved in vibration; c_{x1} and c_{x2} are respectively the equivalent linear damping coefficient of frame 1 and frame 2; k_{x2} is the spring stiffness between frame 2 and motor base in x direction, so is k_{x1}; m_0 is the mass of eccentric block of vibration exciter; $r$is the radius of gyration to the center of mass of eccentric block; ω is the angular velocity of gyration of eccentric block; $F(x)$ is the hysteretic force of material layers in x direction as shown in Eq. (1).

5. Numerical simulations

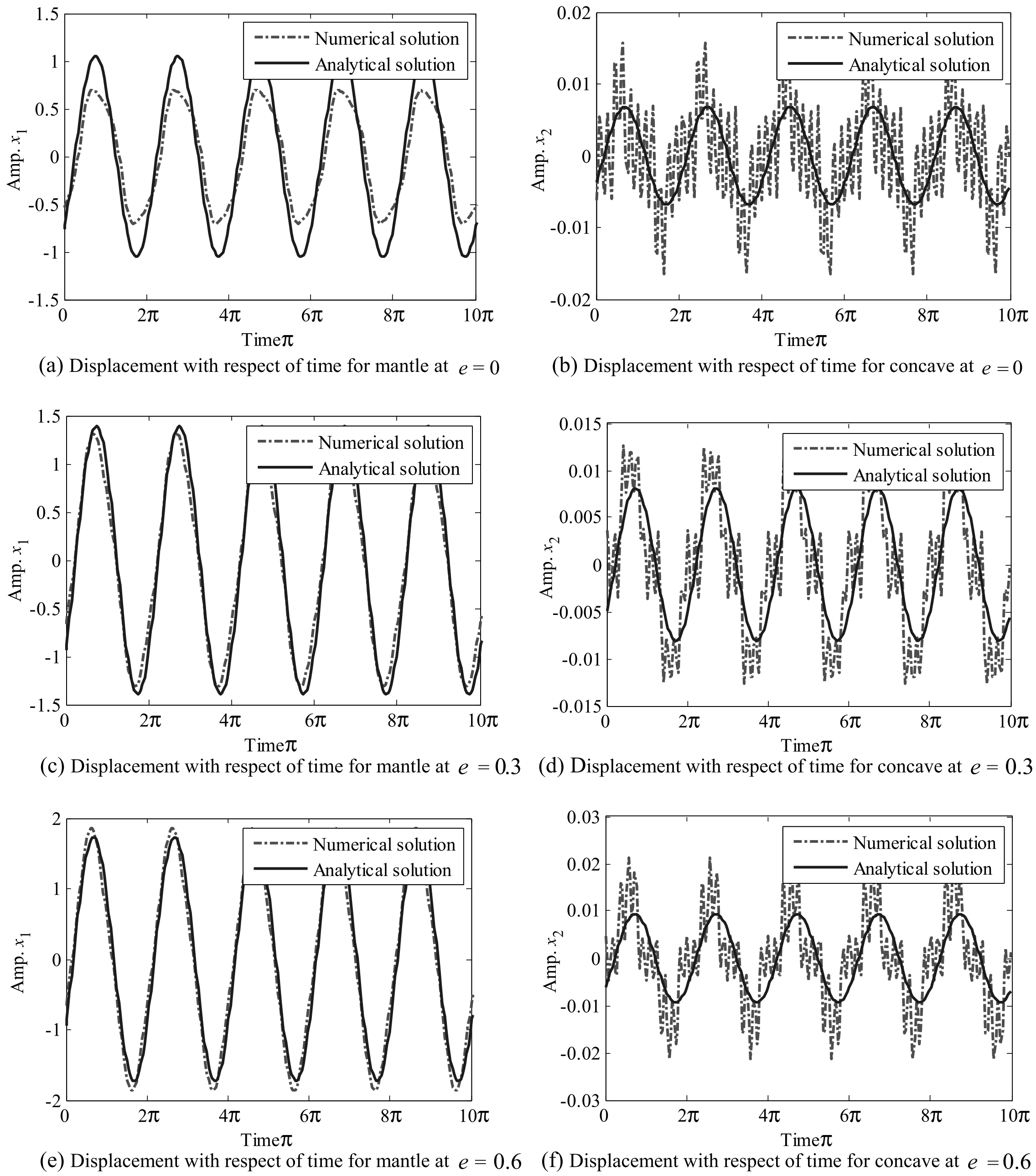
The following simulations are carried out according to direct integration method such as Runge-Kutta method [24]. In numerical process, the system parameter values are taken as follows: $m_1 = 1$, $m_2 = 2$, $c_{x1} = 0.01$, $c_{x2} = 0.01$, $k_{x1} = 0$, $k_{x2} = 100$, $m_0 = 1$, $r = 0.55$, $k_1 = 1$, $k_2 = 0.1$, $x_A = e+ 0.5$, $x_B = e+ 0.5$, $x_C = e+ 1.0$, $\omega = 0.2\pi$.

The steady-state displacement responses of the system at different clearance values are shown as dot and dash lines in Fig. 4. The numerical results from the original system Eq. (2) with the steady-state displacement responses of system are also shown as full lines in Fig. 4. The vertical coordinates represent for the displacement, the horizontal ordinates denote the dimensionless time, and $\tau = \omega t$.

6. Complex dynamic behaviors

The displacement of the frame 2 is much larger than that of the frame 1, as shown in Fig. 5, so the system can be simplified as a single-dof system.

In the specific process of numerical solution, the whole calculated time can be divided into very small time step Δt_k. It is required that the restoring force $f(x)$ is a constant in each time step Δt_k, that is, to get approximate solution with piecewise linear method. After the end of calculation at each time step, $f(x)$ is redefined according



(a) Displacement with respect of time for mantle at $e = 0$ (b) Displacement with respect of time for concave at $e = 0$

(c) Displacement with respect of time for mantle at $e = 0.3$ (d) Displacement with respect of time for concave at $e = 0.3$

(e) Displacement with respect of time for mantle at $e = 0.6$ (f) Displacement with respect of time for concave at $e = 0.6$

Fig. 4. Displacement responses of system.

to the changes in displacement x and then enters the calculation of next stage. A variety of hysteretic broken line of piecewise linear of stiffness non-degradation can be dealt with the classical Runge-Kutta method. The modified fourth-order Runge-Kutta method is adopted to calculate the system.

In order to investigate whether such a nonlinear hysteretic force will produce chaos, the nonlinear hysteretic force is supposed to bear all the restoring forces of system. The Eq. (2) can be simplified as Eq. (3):

$$\ddot{x} + 2\zeta\omega_n\dot{x} + f(x) = p\sin\omega t \tag{3}$$

Here, $p = P/m$, $2\zeta\omega_n = c$, $f(x) = F(x)/m$, $\omega_1^2 = k_1/m$, $\omega_2^2 = k_2/m$, $a_1 = -\omega_1^2 x_C + F_C$, $a_2 = -\omega_2^2 e$,

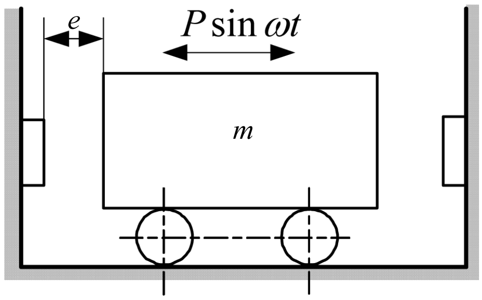


Fig. 5. Mechanical model of single degree of freedom of vibratory cone crusher.

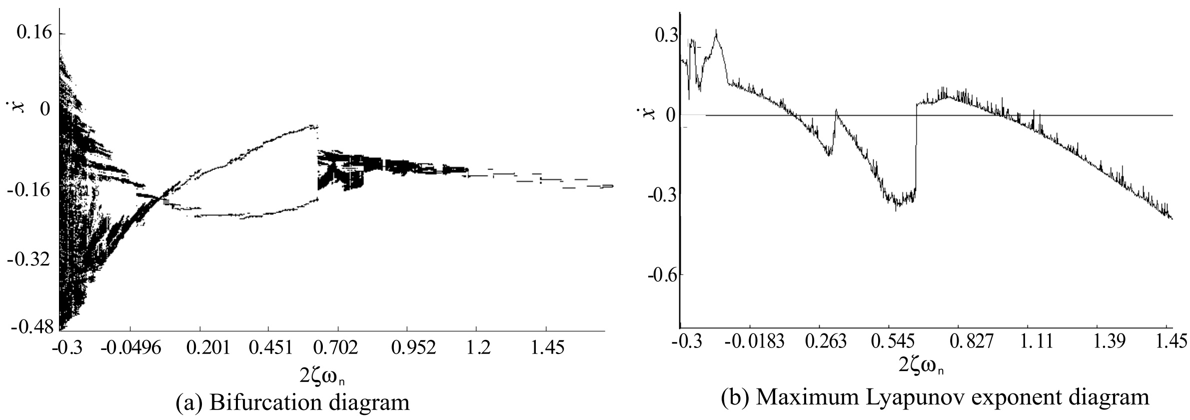


Fig. 6. Nonlinear dynamic characteristics of system at 2 $\zeta\omega \in (-0.3, 1.7)$.

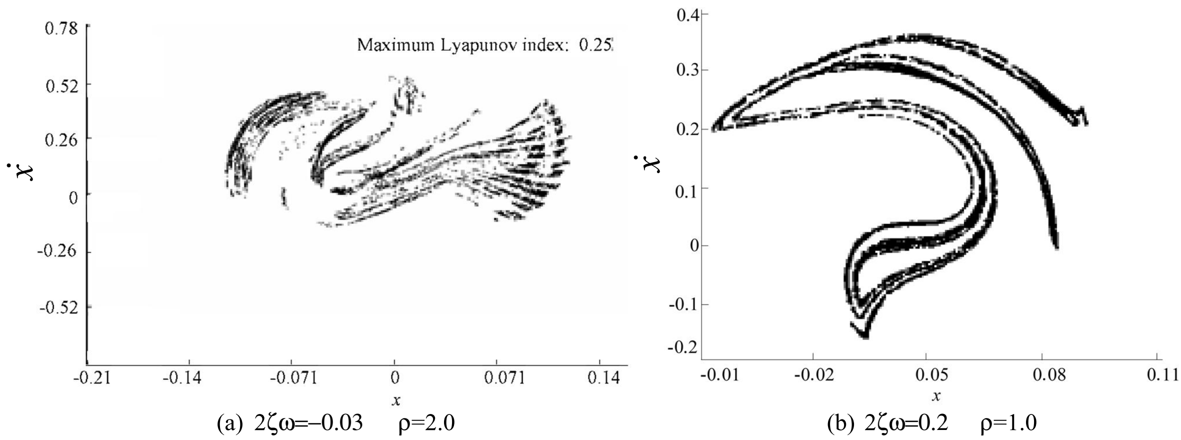


Fig. 7. Poincare section of system under the conditions of different parameters.

$a_3 = -\omega_1^2 e$, $a_4 = -\omega_2^2 x_A + F_A$. When the damping $2\zeta\omega_n$ changes between -0.3 and 1.7, $p = 2.0$, $\omega = 2.0$, $\omega_1^2 = 40.0$, $\omega_2^2 = 20.0$, $e = 0.03$. When $x_A = x_B = 0.1$, the bifurcation diagram and the maximum Lyapunov exponent diagram are shown in Fig. 6.

When the system is vibrating, the hysteretic restoring force consumes the energy of system. When the damping of system is negative, such as the self-excited vibrating system [25], chaotic behaviors can be easily generated [26]. The Poincare section, obtained with at $2\zeta\omega_n = -0.3$, $p = 2.0$, shown in Fig. 7 (a) indicates a typical strange chaotic

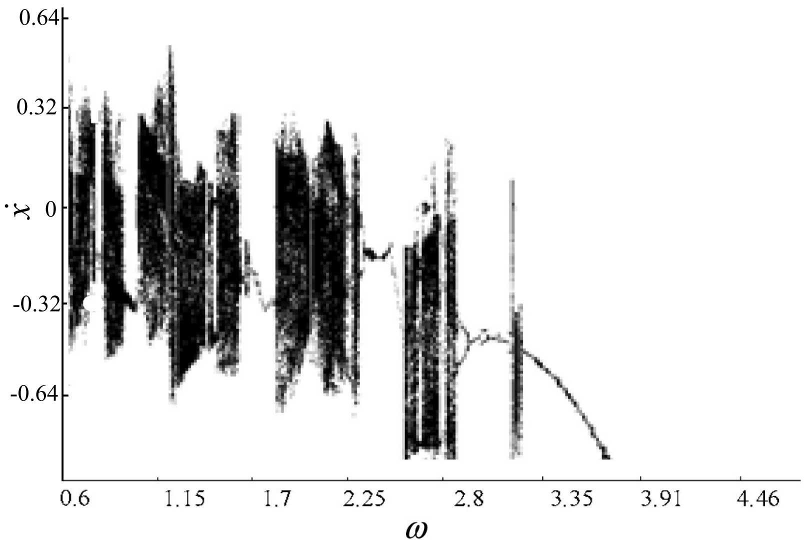


Fig. 8. Bifurcation diagram of system at ω ∈(0.6, 5.0).

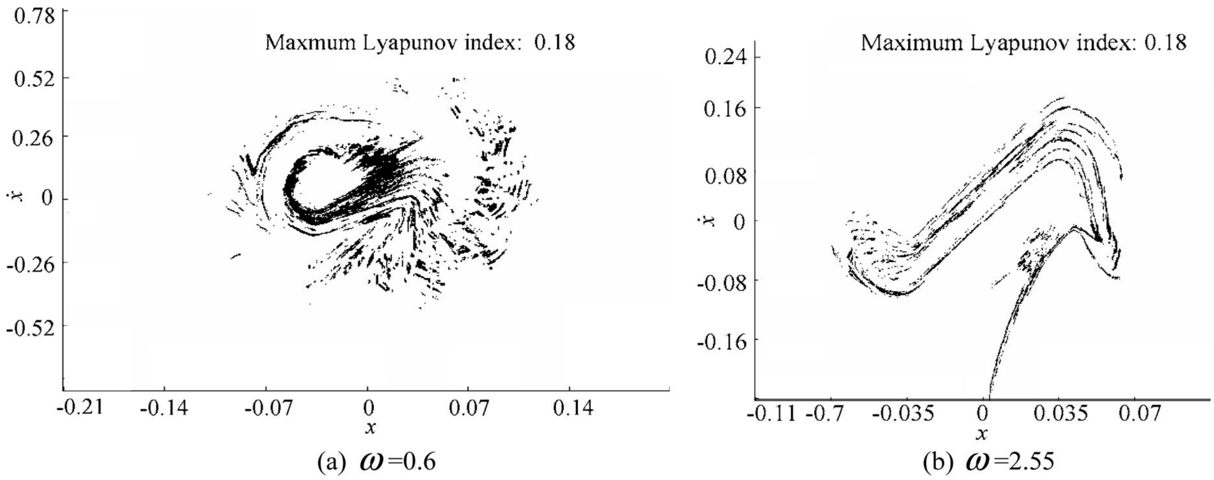


Fig. 9. Poincare section of system under the conditions of different parameters.

attractor. Then the periodic motion emerges in the system; However, at about $2\zeta\omega_n = -0.2$, that is when the damping of system is positive, chaotic behaviors can still generated; The shape of corresponding chaotic attractor can be seen in Fig. 7 (b). Chaotic motion is a kind behavior of overall stability and local instability, while damping generally plays a role in the maintenance of system stability [27,28]. In nonlinear vibrating systems of large energy-consuming with hysteretic restoring force, the chaotic behaviors still occur when the damping is positive, which is caused by the non-smooth characteristics of the system at this time.

When the damping is $2\zeta\omega_n = -0.3$, $p = 2.0$, $\omega_1^2 = 40.0$, $\omega_2^2 = 20.0$, $e = 0.03$, $x_A = x_B = 0.1$, the bifurcation diagram is shown in Fig. 8 with ω changing from 0.6 to 5.0. As can be seen in Fig. 8, when the system frequency changes from 0.6 to 5.0 under the external stimulation, several chaotic areas appear continuously in the system at the beginning, and then the system transits constantly from chaos to cycles, then cycles to chaos. When the frequency ω is near 2.8, the system emerges the reverse period-doubling bifurcation. The Poincare section diagram of system and the corresponding maximum Lyapunov exponent diagram are shown in Fig. 9, at $\omega = 0.6$ and $\omega = 2.55$ respectively. Compared with the shape of chaotic attractor of other nonlinear systems, this chaotic attractor has infinite hierarchy in the geometric structure because the hysteretic restoring force is piecewise. This is a unique phenomenon only belong to the piecewise hysteretic system [29].

7. Conclusions

(1) Through the analysis on the whole process of crushing materials, the stress effects are elucidated. The fact can be obtained that the materials are involved in vibration and constitute a vibrating system of two-degree of freedom together with the vibratory cone crusher.
(2) The role of material layers is simplified into the form of symmetrical hysteretic force with clearances, which possesses theoretical advancement and conforms to reality. Using bilinear model with clearances to describe the hysteretic force, the dynamic characteristics of the vibrating system can be reflected in engineering.
(3) The mechanical model of two-degree of freedom with bilinear hysteresis and its dynamical equations of system are proposed. The results of numerical simulations of the original system are obtained and shown.
(4) The hysteretic system with clearances contains very rich dynamical behaviors. In order to further investigate the dynamic characteristics of the novel vibratory cone crusher, the system is also simplified into a dynamical system of single degree of freedom with a bilinear hysteresis together with clearances. According to some nonlinear dynamic analysis tools such as bifurcation diagram, Lyapunov exponents, Poincare section, *etc.*, different motion patterns of the system are discussed, including periodic, periodic doubling, chaos and other characteristics. The results show that disorderly chaotic motions emerge in the system within the compass of particular parameters, which will greatly reduce the work efficiency and the processing capacity of the vibratory cone crusher system. Therefore, the existing machine set should be amended by a great deal of experiments and be determined a range of reasonable parameters through identification so that it can be applied in the field of engineering design and analysis.

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