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Synchronization of two asymmetric exciters in a vibrating system

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Abstract. We investigate synchronization of two asymmetric exciters in a vibrating system. Using the modified average method of small parameters, we deduce the non-dimensional coupling differential equations of the two exciters (NDDETE). By using the condition of existence for the zero solutions of the NDDETE, the condition of implementing synchronization is deduced: the torque of frequency capture is equal to or greater than the difference in the output electromagnetic torque between the two motors. Using the Routh-Hurwitz criterion, we deduce the condition of stability of synchronization that the inertia coupling matrix of the two exciters is positive definite. A numeric result shows that the structural parameters can meet the need of synchronization stability.

Keywords: Synchronization, asymmetric exciters, frequency capture, vibrating system

Nomenclature

f_{dj}	Damping constant of rotor of motor j, $j = 1, 2$;
f_{xi}	Damping constant of the ith spring in x-direction;
f_{yi}	Damping constant of the ith spring in y-direction;
g	Acceleration of gravity;
J_i	Moment of inertia of exciter i;
J_p	Moment of inertia of the machine body rotating about its centre-of-mass;
k_{e0i}	Stiffness coefficient of angular velocity for motor i when its angular velocity is a given value;
k_{xi}	Spring constant in x-direction;
k_{yi}	Spring constant in y-direction;
l_0	Distance between o''' and o';
l_i	Distance between the centre of exciter i and o;
m	Mass of the machine body;
m_{0i}	Mass of exciter i;
r_i	Eccentric radius of exciter i;
$T_{\mathrm{e}1}$	Electromagnetic torque of motor 1;
$T_{\mathrm{e}2}$	Electromagnetic torques of motor 2;
$\bar{v}_1$	Integrated average value of v_1 during the region of $\varphi = 0 \sim 2\pi$;
$\bar{v}_2$	Integrated average value of v_2 during the region of $\varphi = 0 \sim 2\pi$;
x	Displacement of the machine body in $x-$ and $y-$directions;
x_0	Initial displacements of the machine body in x-direction;
y	Displacement of the machine body in $y-$direction;
y_0	Initial displacement of the machine body in y-direction;

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$\bar{\alpha}$ Integrated average value of α;
β_0 Angle between oo''' and x-axis;
β_i Angle between the line from the rotating centre of exciter i to o and $x'-$axis;
φ_i Angular displacement of exciter i;
ρ_i Distance between o and the point where the ith spring is connected to the machine body;
θ_i Angle between the line from o' to the point where the ith spring is connected to the machine body and x-axis;
ε_i Disturbance coefficient of angular velocity around the a given value;
ψ Angular displacement rotating about the centre-of-mass of the machine body in ψ-direction;
ψ_0 Initial angle displacement of the machine body in ψ-direction;

1. Introduction

Synchronization was addressed as two or more systems possessing the same speed, phase, trajectory, or other physical states [1,2]. Starting with the work of Huygens, synchronization phenomena have attracted attention of many researchers, and much effort has been devoted to the study of synchronization problems. In early time, Van Der Pol studied synchronization of a certain electrical-mechanical system [3], and Rayleigh described in his famous treatise "The Theory of Sound" in 1877 that two organ tubes may produce a synchronized sound provided the outlets are closed to each other and called it "frequency capture" [4].

Synchronization of two self-excited oscillatory system is a classical problem in the theory of synchronization [5]. In the middle of the 20th century, Blekhman proposed the method of direct motion separation that has been proven very useful and descriptive [6–8]. In a vibrating system with the two identical unbalanced rotors, analytical investigation is greatly simplified by combining the differential equations of the two exciters into the differential equation of the phase difference between the two exciters and various types of vibrating machines have been developed in industrial engineering, such as self-synchronous vibrating feeders, self-synchronous vibrating conveyors, self-synchronous probability screens, self-synchronous vibrating coolers, etc. [9–14]. In addition, some synchronizations were achieved under the controllers, such as a neuron controller [15], an improved OPCL controller [16] and an adaptive controller [17]. Taking the disturbance parameters of the phase difference and average velocity of the two unbalanced rotors as the small parameters, the authors convert the problem of synchronizations of two unbalanced rotors into that of existence and stability of the zero solutions of the differential equations of the small parameters [18–20].

In this work, we investigate synchronization of two asymmetric exciters in a vibrating system. The rest of the paper is organized as follows: the mathematical model of the vibrating system is described and the non-dimensional coupling equation of the two exciters is derived in Section 2. The conditions of implementing synchronization and its stability are deduced in Section 3. Numeric results and discussions are shown in Section 4. Finally, conclusions are provided in Section 5.

2. Dynamical equations of the vibrating system

Figure 1 shows the dynamic model of a vibration system with two exciters, which are asymmetrically installed and driven by two induction motors respectively. The two motors rotate in opposite directions. o'' is the centre-of-mass of the machine body, o_1 and o_2 are the centre of spin axis of exciter 1 and 2 respectively, o''' is the centre of the line o_1o_2 and o' is the center-of-mass of the system. oxy is the fixed frame and $o'x'y'$ is the moving frame. The kinetic energy T of the vibrating system can be written as [12]

$$\begin{aligned} T &= \frac{1}{2}\left\{[\dot{x} - l_0\dot{\psi}\sin(\beta_0+\pi+\psi+\psi_0)]^2 + [\dot{y} + l_0\dot{\psi}\cos(\beta_0+\pi+\psi+\psi_0)]^2\right\} \\ &+\frac{1}{2}\sum_{i=1}^{2} m_{0i}\left\{[\dot{x}-l_i\dot{\psi}\sin(\beta_i+\psi+\psi_0)-r_i\dot{\phi}_i\sin\varphi_i]^2 + [\dot{y}+l_0\dot{\psi}\cos(\beta_0+\pi+\psi+\psi_0)+r_i\dot{\varphi}_i\cos\varphi_i]^2\right\} \quad (1) \\ &+\frac{1}{2}J_p\dot{\psi}^2 + \frac{1}{2}\sum_{i=1}^{2} J_i\dot{\varphi}_i^2 \end{aligned}$$

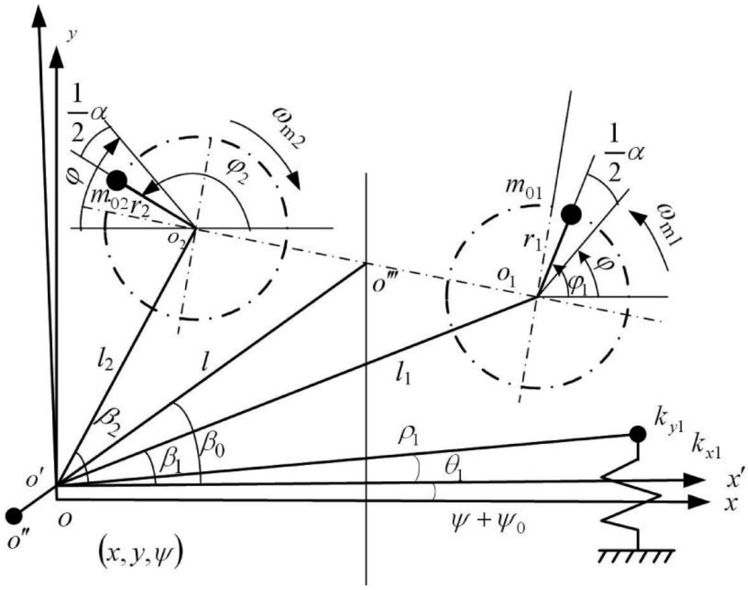

Fig. 1. Dynamic model of a vibrating system with two exciters asymmetrically installed.

The potential energy V of the system can be expressed as

$$V = \frac{1}{2}\left\{\sum_{i=0}^{2}[k_{xi}(x_0+x-\rho_i(\psi+\psi_0)\sin(\theta_i+\psi+\psi_0))^2 + k_{yi}[(y_0-y-\rho_i(\psi+\psi_0)\cos(\theta_i+\psi+\psi_0)]^2\right\}$$
$$+(m+\sum_{i=1}^{2}m_{0i})g[-y_0+y+\rho_i(\psi+\psi_0)\cos(\theta_i+\psi+\psi_0)] \quad (2)$$

The viscous dissipation function D of the system can be described as following:

$$D = \frac{1}{2}\sum_{j=1}^{2}f_{dj}\dot{\varphi}_j^2 + \frac{1}{2}\sum_{i=1}^{2}\left\{[f_{xi}(\dot{x}-\rho_i\dot{\psi}\sin(\dot{\theta}_i+\psi+\psi_0)]^2 + f_{yi}[-\dot{y}-\rho_i\dot{\psi}^2\cos(\dot{\theta}_i+\psi+\psi_0)]^2\right\} \quad (3)$$

Select x, y, ψ, φ_1 and φ_2 as the generalized coordinates, then the generalized forces are $\{\,Q_1\;Q_2\;Q_3\;Q_4\;Q_5\,\}^T = \{\,0\;0\;-T_{e1}-T_{e2}\;T_{e1}\;T_{e2}\,\}^T$. Compared with θ_i and β_i, ψ and ψ_0 in Eqs (1) through (3) are so small that they all can be neglected. If the origin of the fixed frame oxy is assumed to be the equilibrium position of centre-of-mass of the machine body, x_0, y_0 and ψ_0 are zero. Considering Eqs (1) through (3), the dynamical equations of the vibrating system are deduced by using Lagrange's equations as the following four differential equations:

$$(m+\sum_{i=1}^{2}m_i)\ddot{x} + f_x\dot{x} + f_{x\psi}\dot{\psi} + k_x x + k_{x\psi}\psi = \sum_{i=1}^{2}m_i r_i(\dot{\varphi}_i^2\cos\varphi_i + \ddot{\varphi}_i\sin\varphi_i)$$

$$(m+\sum_{i=1}^{2}m_i)\ddot{y} + f_y\dot{y} + f_{y\psi}\dot{\psi} + k_y y + k_{y\psi}\psi = \sum_{i=1}^{2}m_i r_i(\dot{\varphi}_i^2\cos\varphi_i + \ddot{\varphi}_i\sin\varphi_i)$$

$$J\ddot{\psi}+f_\psi\dot{\psi}+f_{\psi x}\dot{x}+f_{\psi y}\dot{y}+k_\psi\psi+k_{\psi x}x+k_{\psi y}y = -T_{e1}-T_{e2}+\sum_{i=1}^{2}m_i r_i l\left[\dot{\varphi}_i^2\sin(\varphi_i-\beta_i)-\ddot{\varphi}_i\cos(\varphi_i-\beta_i)\right] \quad (4)$$

$$J_{0j}\ddot{\varphi}_j + f_{dj}\dot{\varphi}_j = T_{ej} - m_{0j}r_j[(\ddot{x}-l_j\ddot{\psi}\sin\beta_j - l_j\dot{\psi}_2\cos\beta_j)\sin\varphi_j - (\ddot{y}+l_j\ddot{\psi}\cos\beta_j - l_j\dot{\psi}_j^2\sin\beta_j)\cos\varphi_j]$$
$$j = 1, 2$$

where,

$$
\begin{aligned}
&f_x = f_{x1} + f_{x2}\\
&f_{x\psi} = f_{\psi x} = f_{x1}\rho_1 \sin\theta_1 + f_{x2}\rho_2 \sin\theta_2\\
&f_{y\psi} = f_{\psi y} = f_{y1}\rho_1 \cos\theta_1 + f_{y2}\rho_2 \cos\theta_2\\
&f_\psi = f_{x1}\rho_1^2 \sin^2\theta_1 + f_{x2}\rho_2^2 \sin^2\theta_2 + f_{y1}\rho_1^2 \cos^2\theta_1 + f_{y2}\rho_2^2 \cos^2\theta_2
\end{aligned}
$$

$$
\begin{aligned}
&k_x = k_{x1} + k_{x2}\\
&k_{x\psi} = k_{\psi x} = k_{x1}\rho_1 \sin\theta_1 + k_{x2}\rho_2 \sin\theta_2\\
&f_{y\psi} = k_{\psi y} = k_{y1}\rho_1 \cos\theta_1 + k_{y2}\rho_2 \cos\theta_2\\
&k_\psi = k_{x1}\rho_1^2 \sin^2\theta_1 + k_{x2}\rho_2^2 \sin^2\theta_2 + k_{y1}\rho_1^2 \cos^2\theta_1 + k_{y2}\rho_2^2 \cos^2\theta_2
\end{aligned}
$$

$$J_{0j} = J_j + m_{0j} r_j^2$$

$$J = J_p + \sum_{i=1}^{2} m_{0i} l_i^2$$

In Eq. (4), the electromagnetic torque of the two motors in the vicinity of a given angular velocity can be approximately expressed as [6–11]

$$T_{ei} = T_{e0i} - k_{e0i}\varepsilon_i \ (i = 1, 2) \tag{5}$$

Herein, we investigate the case that the operation frequency of the vibrating system is much greater than its natural frequency. In this case, the response of the vibrating system can be approximately represented by the sum of the responses excited by the two exciters operating at their average angular velocity [18,19]. The differential equations of the two exciters in Eq. (4) are mathematically treated by the following steps [18–20]:

(1) Assume that the average angular velocity of the two exciters is ω_m and the phase difference between the two exciters is 2α, (where $2\alpha = \varphi_1 - \varphi_2$) when the vibrating system operates in the steady state;
(2) Solve the responses of the vibrating system, x, y and ψ when the angular velocity of the two exciters both are ω_m;
(3) Assume that the instantaneous change coefficients of average phase φ and phase difference α of the two exciters are ε_1 and ε_2 respectively, i.e., $\dot{\varphi} = (1 + \varepsilon_1)\,\omega_{m0}$ and $\dot{\alpha} = \varepsilon_2\omega_{m0}$;
(4) Differentiate the response of the vibrating system by the chain rule (applied to each component of α and φ) to obtain $\ddot{x}, \ddot{y}$ and $\ddot{\psi}$;
(5) Substitute $\ddot{x}, \ddot{y}$ and $\ddot{\psi}$ into the differential equations of the two exciters in Eq. (4) and integrate them between $\varphi = 0$ and $\varphi = \pi$;
(6) Assume $v_1 = \varepsilon_1 + \varepsilon_2$ and $v_2 = \varepsilon_1 - \varepsilon_2$;
(7) Compared with $m_i r^2$, J_{0i} is so small that it can be neglected;
(8) Neglect the high order terms of v_1 and v_2.

Then we obtain

$$
\begin{bmatrix} \mu_1 & W_c\cos(2\bar{\alpha}+\theta_c)/2 \\ W_c\cos(2\bar{\alpha}+\theta_c)/2 & \mu_2 \end{bmatrix}
\begin{Bmatrix} \bar{v}_1 \\ \bar{v}_2 \end{Bmatrix}
= -\omega_m
\begin{bmatrix} \kappa_1 & -W_c\sin(2\bar{\alpha}+\theta_c) \\ W_c\sin(2\bar{\alpha}+\theta_c) & \kappa_2 \end{bmatrix}
\begin{Bmatrix} \bar{v}_1 \\ \bar{v}_2 \end{Bmatrix} +
\tag{6}
$$

$$
\begin{Bmatrix}
\dfrac{T_{e01} - f_1\omega_m}{m_1 r_0^2 \omega_m} - \frac{1}{2}\omega_m(W_{s1} + W_s\cos(2\bar{\alpha}_0 + \theta_S) + W_c\sin(2\bar{\alpha}_0 + \theta_c)) \\
\dfrac{T_{e02} - f_2\omega_m}{m_1 r_0^2 \omega_m} - \frac{1}{2}\omega_m(W_{s2} + W_s\cos(2\bar{\alpha}_0 + \theta_S) - W_c\sin(2\bar{\alpha}_0 + \theta_c))
\end{Bmatrix}
$$

where

$$W_{s1} = r_m(\mu_x \sin\gamma_x + \mu_y \sin\gamma_y + r_{l1}^2 \mu_\psi \sin\gamma_\psi),$$

$$W_{s2} = \eta^2 r_m(\mu_x \sin\gamma_x + \mu_y \sin\gamma_y + r_{l2}^2 \mu_\psi \sin\gamma_\psi),$$

$$W_{c1} = r_m(\mu_x \cos\gamma_x + \mu_y \cos\gamma_y + r_{l1}^2 \mu_\psi \cos\gamma_\psi),$$

$$W_{c2} = r_m(\mu_x \cos\gamma_x + \mu_y \cos\gamma_y + r_{l2}^2 \mu_\psi \cos\gamma_\psi),$$

$$a_s = -\mu_x \sin\gamma_x + \mu_y \sin\gamma_y + r_{l1} r_{l2} \mu_\psi \sin\gamma_\psi \cos(\beta_1 + \beta_2),$$

$$b_s = r_{l1} r_{l2} \mu_\psi \sin\gamma_\psi \sin(\beta_1 + \beta_2), \quad W_s = \eta r_m \sqrt{a_s^2 + b_s^2},$$

$$a_c = \mu_x \cos\gamma_x - \mu_y \cos\gamma_y - r_{l1} r_{l2} \mu_\psi \cos\gamma_\psi \cos(\beta_1 + \beta_2),$$

$$b_c = r_{l1} r_{l2} \mu_\psi \cos\gamma_\psi \sin(\beta_1 + \beta_2), \quad W_c = \eta r_m \sqrt{a_c^2 + b_c^2},$$

$$\theta_s = \begin{cases} \arctan(-b_s/a_s) \ a_s \geqslant 0; \\ \pi + \arctan(-b_s/a_s) a_s < 0. \end{cases}, \quad \theta_c = \begin{cases} \arctan(b_c/a_c) \ a_c \geqslant 0; \\ \pi + \arctan(b_c/a_c) \ a_c < 0. \end{cases}.$$

$$\mu_1 = 1 - W_{c1}/2, \quad \mu_2 = \eta - W_{c1}/2, \quad \kappa_1 = k_{e01}/m_1 r_0^2 \omega_{m0}^2 + W_{s1}, \quad \kappa_2 = k_{e01}/m_1 r_0^2 \omega_{m0}^2 + W_{s2}.$$

In Eq. (6), $\bar{v}_1$ and $\bar{v}_2$ are the disturbance parameters of angular velocities of the two exciters around ω_m. Equation (6) is the non-dimensional coupling differential equations of the two exciters.

3. Synchronization and its stability

When the two exciters rotate synchronously, $\bar{v}_1 = 0$ and $\bar{v}_2 = 0$. Substitute $\bar{v}_1 = 0$ and $\bar{v}_2 = 0$ into Eq. (6), we have

$$\begin{aligned} &\frac{T_{e01} - f_1\omega_{m0}}{m_1 r_0^2 \omega_{m0}} - \frac{1}{2}\omega_{m0}(W_{s1} + W_s \cos(2\bar{\alpha}_0 + \theta_S) + W_c \sin(2\bar{\alpha}_0 + \theta_c)) \\ &\frac{T_{e02} - f_2\omega_{m0}}{m_1 r_0^2 \omega_{m0}} - \frac{1}{2}\omega_{m0}(W_{s2} + W_s \cos(2\bar{\alpha}_0 + \theta_S) - W_c \sin(2\bar{\alpha}_0 + \theta_c)) \end{aligned} \tag{7}$$

Subtracting the second formula from the first one in Eq. (7) and rearranging it, we obtain

$$2\bar{\alpha} + \theta_c = \arcsin T_D/T_S \tag{8}$$

where $T_S = m_1 r_0^2 \omega_{m0}^2 W_c$, $T_D = T_{R1} - T_{R2}$, $T_{R1} = T_{e01} - f_1\omega_{m0} - m_1 r_0^2 \omega_{m0}^2 W_{s1}/2$, $T_{R2} = T_{e02} - f_2\omega_{m0} - m_1 r_0^2 \omega_{m0}^2 W_{s2}/2$.

Since $|\sin(2\bar{\alpha} + \theta_c)| \leqslant 1$, it can be found that when $T_S \geqslant |T_D|$, there is $\bar{\alpha}$ satisfying Eq. (8). Hence, $T_S \geqslant |T_D|$ is the condition of implementing the synchronization of the two coupled exciters. In this case, we can obtain the synchronous angular velocity ω_{m0} and the phase difference $2\bar{\alpha}_0$ by the numeric solution of Eqs (7) and (8).

Linearize Eq. (8) around $\bar{\alpha} = \alpha_0$, then add the two rows as the first new row and subtract the second row from the first row as the second new one, append the third row, $\Delta\dot{\alpha} = \omega_{m0}^* \bar{\varepsilon}_2 (\Delta\alpha = \bar{\alpha} - \alpha_0)$. Assume $\boldsymbol{z} = \{\varepsilon_1 \ \varepsilon_2 \ \Delta\alpha\}$, then we obtain

$$\dot{\boldsymbol{z}} = \boldsymbol{C}\boldsymbol{z} \tag{9}$$

where $\boldsymbol{C} = \boldsymbol{A}^{-1}\boldsymbol{B}$, and the elements of matrix $\boldsymbol{A}$ and $\boldsymbol{B}$ are presented as follows:

$$A = \begin{bmatrix} \mu_1 & W_c \cos(2\bar{\alpha} + \theta_s)/2 & 0 \\ W_c \cos(2\bar{\alpha} + \theta_s)/2 & \mu_2 & 0 \\ 1 & -1 & 0 \end{bmatrix},$$

$$B = -\omega_m \begin{bmatrix} \kappa_1 & -W_c \sin(2\bar{\alpha} + \theta_c) & -W_c \cos(2\bar{\alpha} + \theta_c) \\ W_c \cos(2\bar{\alpha} + \theta_c) & \kappa_2 & W \cos(2\bar{\alpha} + \theta_c) \\ 1 & -1 & 0 \end{bmatrix}.$$

Exponential time-dependence of the form $\dot{\boldsymbol{z}} = \boldsymbol{u}\exp(\lambda t)$ is now assumed, and can be substituted into Eq. (9). From the determinant equation $\det(\boldsymbol{C} - \lambda\boldsymbol{I}) = 0$, the characteristic equation for the eigenvalue λ is deduced as follows:

$$\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0 \tag{10}$$

where $c_1 = 4\omega_{m0}H_1/H_0$, $c_2 = 2\omega_{m0}^2 H_2/H_0$, $c_3 = 2\omega_{m0}^3 H_3/H_0$ and

$$\begin{aligned} &H_0 = 4\mu_1\mu_2 - W_c^2\cos^2(2\alpha_0 + \theta_c) \\ &H_1 = \mu_1\kappa_2 + \mu_2\kappa_2 H_2' = 2\kappa_1\kappa_2 + (\mu_1 + \mu_2)W_c\cos(2\alpha_0 + \theta_c) + W_c^2 + W_c^2\cos^2(2\alpha_0 + \theta_c) \\ &H_3 = (\kappa_1 + \kappa_2)W_c\cos(2\alpha_0 + \theta_c) \end{aligned} \tag{11}$$

According to the Routh-Hurwitz criterion, we can obtain

$$c_1 > 0, c_3 > 0 \text{ and } c_1c_2 > c_3 \tag{12}$$

The equilibrium solution $z_i = 0$ is stable. According to the sign of H_0, Eq. (12) can be rewritten as Eqs (13) and (14), respectively.

$$H_0 > 0, H_1 > 0, H_3 > 0 \text{ and } 4H_1H_2 - H_0H_3 > 0 \tag{13}$$

$$H_0 < 0, H_1 < 0, H_3 < 0 \text{ and } 4H_1H_2 - H_0H_3 > 0 \tag{14}$$

Because $\kappa_1 > 0$ and $\kappa_2 > 0$, from $H_0 > 0$ and $H_1 > 0$, it can be deduced that

$$\mu_1 > 0, \mu_2 > 0 \text{ and } 4\mu_1\mu_2 - W_c^2\cos^2(2\alpha_0 + \theta_c) > 0 \tag{15}$$

From $H_3 > 0$, it can be deduced that

$$\cos(2\alpha_0 + \theta_c) > 0 \text{ or } 2\alpha_0 + \theta_c \in (-\pi/2, \pi/2) \tag{16}$$

Substituting H_0, H_1, H_2 and H_3' into $4H_1H_2 - H_0H_3 > 0$ and rearranging it, we can obtain

$$\begin{aligned} &[4\mu_1^2\kappa_2 + 4\mu_2^2\kappa_2 + (\kappa_1 + \kappa_2)W_c^2\cos^2(2\alpha + \theta_c)]W_c\cos(2\alpha + \theta_c) > \\ &-4(\mu_1\kappa_2 + \mu_2\kappa_1)(2\kappa_1\kappa_2 + W_c^2 + W_c^2\sin^2(2\alpha + \theta_c)) \end{aligned} \tag{17}$$

Obviously, when $\cos(2\alpha + \theta_c) > 0$, the left hand side of Eq. (17) is greater than zero, and its right hand side is less than zero when $\mu_1 > 0$ and $\mu_2 > 0$. Therefore, Eqs (15) and (16) can guarantee the correctness of Eq. (17).

When $H_0 < 0$, $H_1 < 0$ requires $\mu_1\kappa_2 + \mu_2\kappa_1 < 0$ and $H'_3 < 0$ requires $\cos(2\alpha + \theta_c) < 0$. In such case, the left hand side of Eq. (17) is less than zero and its right hand side is greater than zero. Hence, $H'_0 < 0$, $H'_1 < 0$ and $H'_3 < 0$ can not meet the need of $4H_1H_2 - H_0H_3 > 0$.

There are two solutions of $2\alpha + \theta_c$ to meet the need of Eq. (17) when the torque of frequency capture, T_C, is greater than or equal to the difference between the residual electromagnetic torques of the two motors, T_D, while Eq. (16) decides the stable phase difference. Therefore, Eq. (15) is the condition of stability of synchronization.

4. Numeric results and discussions

In Section 3, the stability condition of synchronization in the simplified form is theoretically discussed. In this section, we will quantitatively compare the numeric results of stability domain of synchronization with its analytical results in its simplified form. An example is used in order to confirm the main results of the above theoretical analysis clearly. To verify the feature of frequency capture of the vibrating system, the parameters of the two motors are assumed to be different. The parameters of the vibrating system are: $m = 2400$ kg, $J_p = 1800$ kg $\cdot$ m$_2$, $k_x = 1247$ kN/m, $k_y = 1247$ kN/m, $k_\psi = 935$ kN/rad, $f_x = f_y = 7.66$ kN $\cdot$ s/m, $f_\psi = 5.745$ kN $\cdot$ s/rad. The motors are two three-phase squirrel-cage (380 V, 50Hz, 6-pole, and Δ-connected), The parameters of motor 1 (3.7 kW, and the rated speed 980 r/min) are: $R_{s1} = 0.56\Omega$, $R_{r1} = 0.54\Omega$, $L_{s1} = 141$ mH, $L_{r1} = 143$ mH, $L_{m1} = 138$ mH, $f_{d1} = 0.01$. The parameters of motor 2 (0.75 kW, rated speed 980 r/min): $R_{s2} = 3.35\Omega$, $R_{r2} = 3.40\Omega$, $L_{s2} = 170$ mH, $L_{r2} = 170$ mH, $L_{m2} = 164$ mH, $f_{d2} = 0.005$.

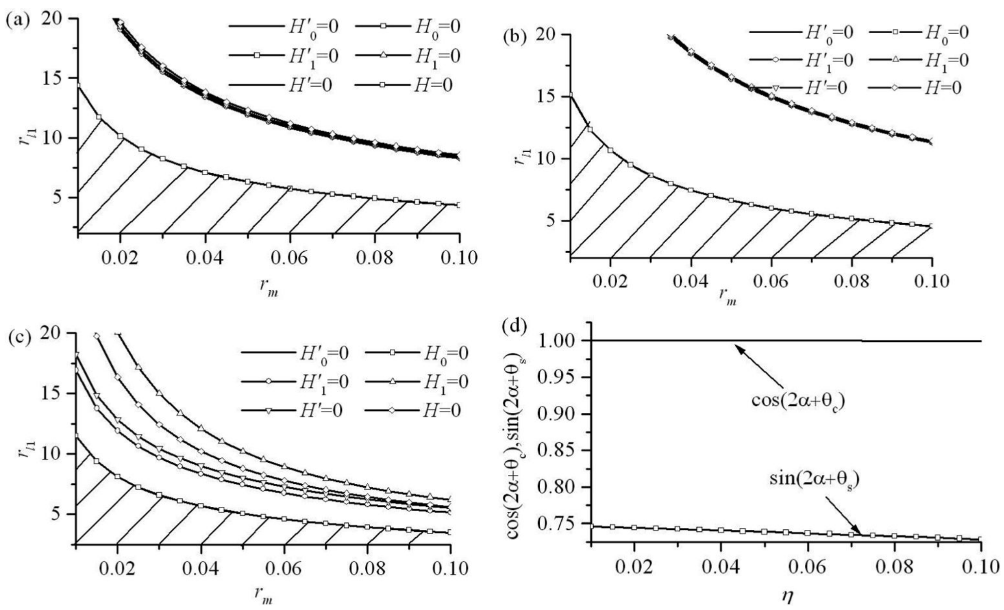

Fig. 2. Regions of stability of synchronization in $r_m r_{l1}$ – plane for different parameters: (a) $\eta = 0.5$, $r_{l1}/r_{l2} = 2$, $\beta_1 + \beta_2 = 2\,\pi/3$; (b) $\eta = 1.0$, $r_{l1}/r_{l2} = 5$, $\beta_1 + \beta_2 = 2\pi/3$; (c) $\eta = 0.8$, $r_{l1}/r_{l2} = 1$, $\beta_1 + \beta_2 = \pi/3$; (d) values of $\cos^2(2\alpha + \theta_c)$ and $\sin^2(2\alpha + \theta_s)$ along the curve of $H_0 = 0$ in Fig. (c).

4.1. Stability domain of synchronization

As shown in Eqs (13) and (14), the stability of synchronization is dependent on the signs of H_0, H_1, H_3 and $H = 4H_1H_2 - H_0H_3$, which are the functions of the parameters W_{s1}, W_{s2}, W_{c1}, W_{c2}, W_s and W_c. In a non-resonant vibrating system, $1 - \omega_{nx}^2/\omega_{m0}^2$, $1 - \omega_{ny}^2/\omega_{m0}^2$ and $1 - \omega_{n\psi}^2/\omega_{m0}^2$ usually range from 15/16 to 24/25 and the spring constants and damping constants in x- and y-directions are usually approximately equal to each other [18, 19], then W_s and W_c can be simplified as follows

$$W_s \approx \frac{\eta r_m r_{l1} r_{l2} \sin\gamma_\psi}{1 - \omega_{n\psi}^2/\omega_{m0}^2} \tag{18}$$

$$W_c \approx \frac{\eta r_m r_{l1} r_{l2} \cos\gamma_\psi}{1 - \omega_{n\psi}^2/\omega_{m0}^2} \tag{19}$$

Hence, the main parameters that affect stability of synchronization are r_m, η, r_{l1}, and r_{l2}. In order to determine the boundary of the stability domain that satisfies Eq. (16), we give fixed η-value and r_{l1}/r_{l2}-value to find the values of r_m and r_l satisfying $H_0 = 0$, $H_1 = 0$, $4H_1H_2 - H_0H_3 = 0$, and $H_3 = 0$. From the expression of H_0 in Eq. (11), it can be deduced that H_0 is the quadratic function of r_{l1}^2 and r_{l2}^2. Hence, there are two curves satisfying $H_0 = 0$ in $r_m r_{l1}-$ plane for the given η-value and r_{l1}/r_{l2}-value, denoted by $r_{l1a}(H_0 = 0)$ and $r_{l1b}(H_0 = 0)$ respectively. H_0 is less than zero in the region between $r_{l1a}(H_0 = 0)$ and $r_{l1b}(H_0 = 0)$. While r_{l1}-value in the curve $r_{l1b}(H_0 = 0)$ is far greater than that in the curve $r_{l1a}(H_0 = 0)$, it takes unimportant part in engineering. Therefore, the curve $r_{l1b}(H_0 = 0)$ is neglected in this paper, i.e., when r_{l1} is less than r_{l1}-value in the curve $r_{l1a}(H_0 = 0)$, $H_0 > 0$; otherwise, $H_0 \leqslant 0$. As mentioned above, $c_3 > 0$ decides the position of the phase difference, $2\alpha_0 + \theta_c$, viz., $2\alpha_0 + \theta_c$can be adjusted in the intervals of $(-\pi/2, \pi/2)$ and $(\pi/2, 3\pi/2)$ to guarantee that the sign of H_3 is the same as that of H_0.

Figure 2 (a) displays comparisons of the curves of $H_3H_0/|H_0|$ for $\eta = 0.5$, $r_{l1}/r_{rl2} = 2$, $\beta_1 + \beta_2 = 2\pi/3$ with different r_m-values. The phase difference is assumed to be $2\alpha + \theta_c \in (-\pi/2, \pi/2)$ and $2\alpha + \theta_c \in (\pi/2, 3\pi/2)$ below and above the dash line respectively to guarantee $c_3 > 0$. Therefore, the stability domain of synchronization of the two motors depends on the signs of H_0, H_1 and $H = 4H_1H_2 - H_0H_3$. H is a complicated function of the parameters η, $r_{l1}$and r_{l2}. Figure 2 (b) shows the comparisons of H with H' in ηr_{l1}-plane for the different r_m-values. As illustrated in Fig. 2 (b), there is only one zero-point in each curve, denoted by r_{l10} and r'_{l10} for $H = 0$ and $H' = 0$, respectively. Furthermore, r'_{l10} is very close to r_{l10}, especially, when r_m is greater.

Figure 2 (a), (b) and (c) shows the stability regions (diagonally hatched) in terms of the parameters r_m and r_{l1} for three groups of the parameters, η, r_{l1}/r_{l2} and $\beta_1 + \beta_2$, respectively. As illustrated in Fig. 2 (a), (b) and (c), H_0 and H'_0 first pass through zero at the same time and become negative, then followed by H, H', H_1 and H'_1 in turn. c_3 is always greater than zero in the global region, see Fig. 2 (a). H is greater than zero in the region between the curve $H_0 = 0$ and $H = 0$, but H_1 is greater than zero and H_0 is less than zero in this region, i.e., $c_1 < 0$. Hence, the synchronous operation in this region is unstable.

Therefore, $\mu_1 > 0$, $\mu_2 > 0$ and $H_0 > 0$ are the sufficient conditions of stability of synchronization. In Fig. 2 (d) are shown $\cos^2(2\alpha + \theta_c)$-value and $\sin^2(2\alpha + \theta_s)$ along the curve of $H_0 = 0$ in Fig. 2 (c). As illustrated in Fig. 2 (d), $\cos^2(2\alpha + \theta_c)$ can be considered to be 1 along the curve of $H_0 = 0$. All these facts demonstrate that the structural parameters of the vibrating system at the boundary of synchronization stability make the torque of frequency capture so great that the phase difference between the two exciters almost reaches the orientation angle of dynamic symmetry axis of the vibrating system. While $\sin^2(2\alpha + \theta_s)$-value along the curve of $H_0 = 0$ ranges from 0.723 to 0.745 and H'_0-curves is almost superimposed on H_0-curve in $\eta r_{l1}-$plane, see Fig. 2 (a), (b) and (c). This demonstrates that the effect of W_s on the non-dimensional moment of coupled inertia of the two exciters can be neglected. Hence, $H_0 > 0$ can be simplified as the following:

$$4\mu_1\mu_2 > W_c^2 \tag{20}$$

Therefore, the condition of stability of synchronization is that the non-dimensional moments of inertia of the two exciters are all greater than zero and four times their product is greater than the square of the coefficient of coupled cosine effect. From Fig. 2 (a), (b) and (c), it can be seen that the less the non-dimensional parameters, r_m and r_l, the stronger the stability of the synchronization. To guarantee the reliable stability of synchronous operation and exciting force of the two exciters, it is recommended that the exciters should be designed to have great eccentric radius but small mass.

4.2. Computer simulations

Further analysis has been performed by computer simulations [21–23]. Here, the parameters of the two eccentric lumps are: $m_1 = 60$ kg ($r_m = 0.025$), $m_2 = 30$ kg ($\eta = 0.5$), $r = 0.2$ m, $J_1 = 2.4$ kgm^2, $J_2 = 1.2$ kgm^2, $\beta_1 = 0$, $\beta_2 = \pi/3$, $l_1 = 1.0(r_{l1} = 1.155)$ and $l_2 = 0.5(r_{l1}/r_{l2} = 2)$. The calculated value of θ_c is $-\pi/3(-60°)$. Figure 2 shows that the above parameters of the exciters are in the stability region of synchronization. Simulation results are shown in Fig. 3, in which the phase difference is 0 when the system begins to start. As illustrated in Fig. 3, when time is less than 2 seconds, the system operates at the starting process. During the starting process, the two exciters cause the resonant responses of the vibrating system in x-, y- and ψ-directions when their rotational velocities pass through the natural frequency, see Fig. 3 (c), (d) and (e). During this stage, the vibrations of the system have larger amplitude, lower frequency and lower acceleration. Hence, the load torques that the system acts on the two motors are smaller and the angular acceleration of motor 2 is greater than that of motor 1 due to that the moment of inertia of exciter 2 is less that of exciter 1, see Fig. 3 (a), and the phase difference between the two exciters changes from $-180°$ to $180°$. But when the starting process ends and the vibrations of higher frequency are excited, the torque of frequency capture plays the role of adjusting the load torques that the system acts on the two motors. From then on, the angular velocities of the two motors and the phase difference fluctuate periodically around their balance points, n_0^* and θ_c, respectively. When time is greater than 3 seconds, the system operates in the steady-state that the rotational velocity is 988.4 r/min and the phase difference is 64.1°, see the amplified diagrams in Fig. 3 (a) and (b). When time is greater 6 seconds, the power supply of motor 2 is cut off. The system adjusts the phase differences and the synchronous rotation of the two motors continues, i.e., the system implements the vibratory synchronization transmission of the steady-state that the rotational velocity is 985.5 r/min and the phase difference is 68.2°.

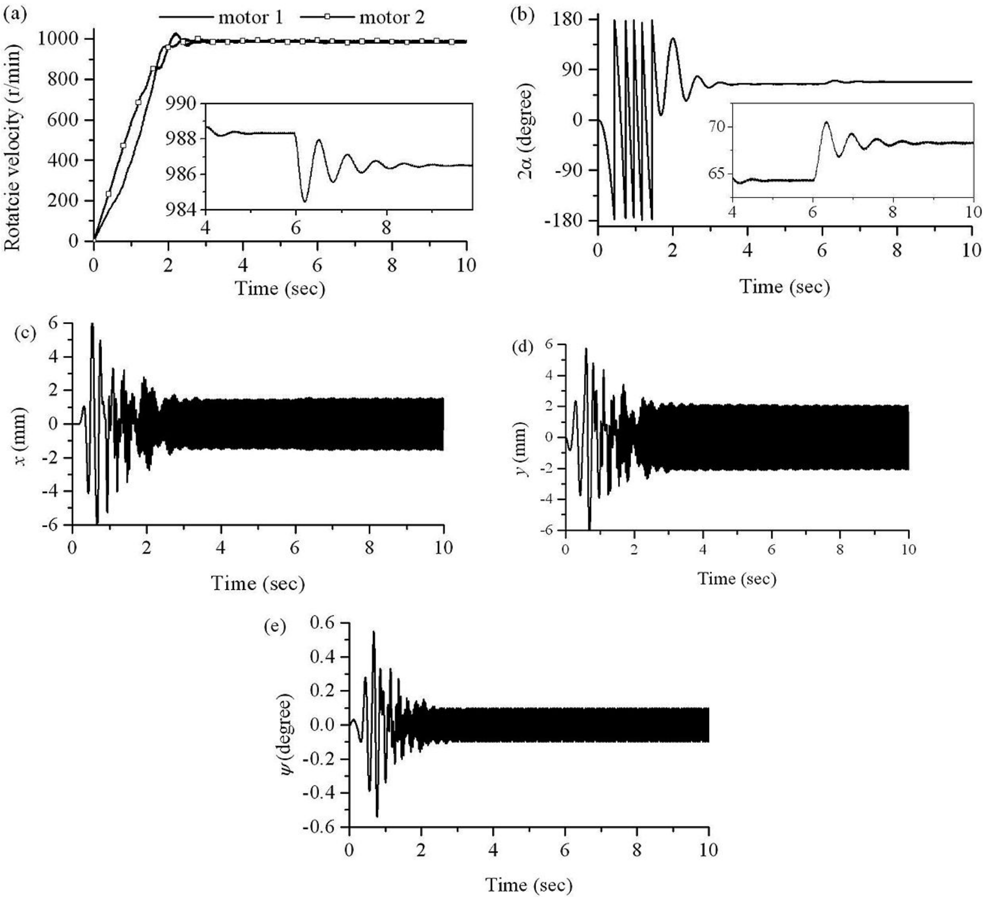

Fig. 3. Results of computer simulation: (a) rotation angular velocities of the two motors; (b) phase difference between the two exciters; (c) displacement in x-direction; (d) displacement in y-direction; (e) angular displacement in ψ-direction.

5. Conclusions

From the results of the theoretical and numeral investigation given in the above sections, the following remarks can be stressed:

(1) In the vibration system driven by two motors installed asymmetrically and rotating in the inverse directions, the condition of realizing frequency capture to reach the synchronous operation of the motors is that the torque of frequency capture of the system is greater than the difference between the electromagnetic torques and that of damping torques of the two motors, namely, the synchronization index D_a is greater than 1.

(2) During the operation of the vibrating system, the system realizes the frequency capture by means of the torque of frequency capture to reach the synchronous operation of the two motors. One half of the torque of frequency capture and the sine of two exciters phase are acted on the motor of the phase leading as the load torque, and another is acted on one of the phase lagging as the driving torque. Under the steady-state of the vibrating system, the torque of frequency capture does not do work.

(3) When the system realizes the frequency capture to reach the synchronization of two coupled exciters and operates in the steady-state, the balance equation of torque of the system is that the sum of electromagnetic torque of the two motors is equal to that of the damping loads of the motor axes and load torques of the two motors resulting from the damping of the vibration system.

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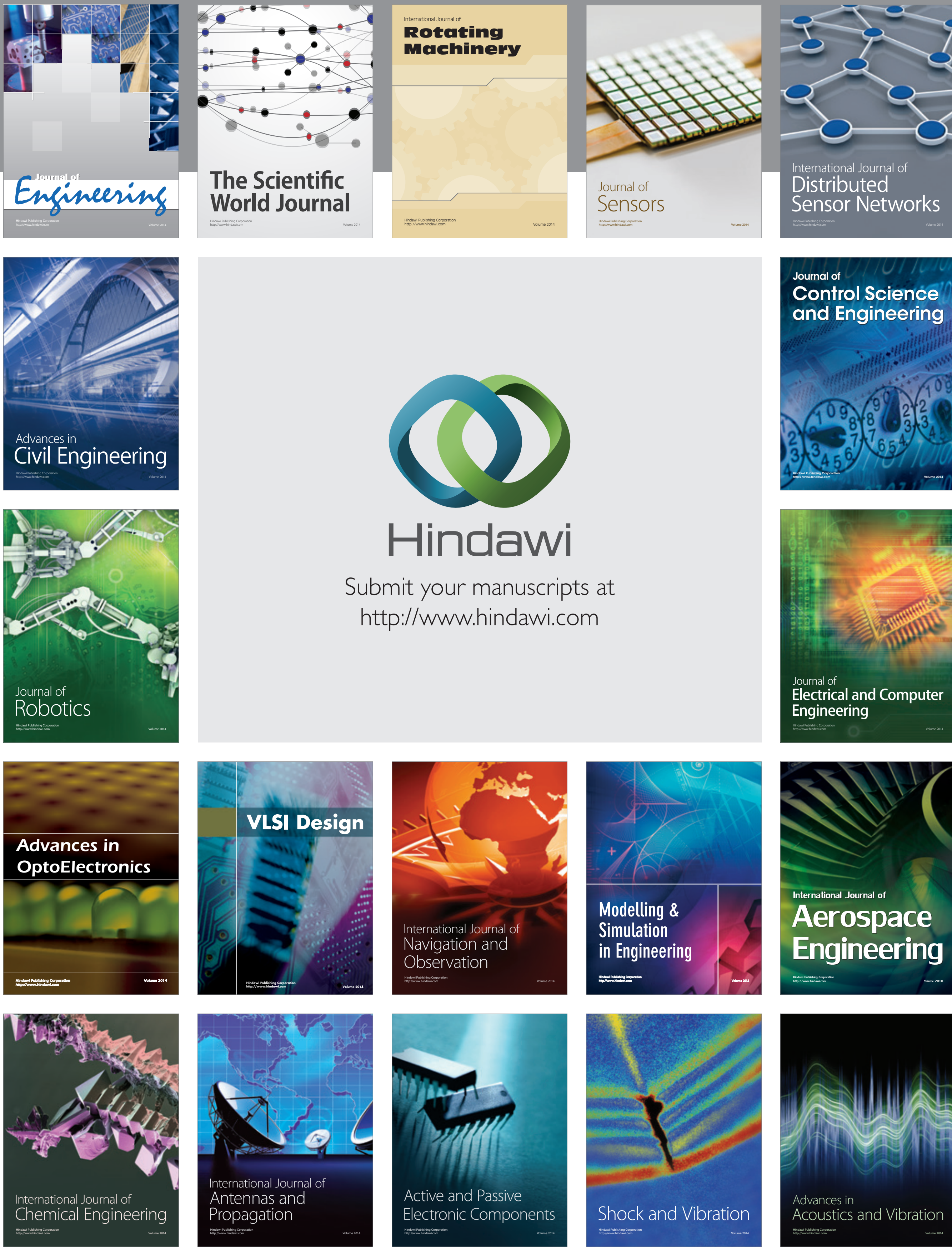
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