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Multilayered infinite medium subject to a moving load: Dynamic response and optimization using coiflet expansion

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Abstract. A wavelet based approach is proposed in this paper for analysis and optimization of the dynamical response of a multilayered medium subject to a moving load with respect to the material properties and thickness of supporting half-space. The investigated model consists of a load moving along a beam resting on a surface of a multilayered medium with infinite thickness and layers with different physical properties. The theoretical model is described by the Euler-Bernoulli equation for the beam and the Navier's elastodynamic equation of motion for a viscoelastic half-space. The moving load is modelled by a finite series of distributed harmonic loads. A special method based on a wavelet expansion of functions in the transform domain is adopted for calculation of displacements in the physical domain. The interaction between the beam and the multilayered medium is analyzed in order to obtain the vibration response at the surface and the critical velocities associated. The choice of the specific values of the design parameters for each layer, which minimize the vibration response of the multilayered medium, can be seen as a structural optimization problem. A first approach for using optimization techniques to explore the potential of the wavelet model is presented and briefly discussed. Results from the analysis of the vibration response are presented to illustrate the dynamic characterization obtained by using this method. Numerical examples reflecting the results of numerical optimizations with respect to a multilayered medium parameters are also presented.

Keywords: Multilayered half-space, wavelet-based approximation, vibrations, structural optimization, distributed moving load

1. Introduction

Due to the continuous development of train and road transportation the analysis of solids' dynamic behaviour became crucial for a construction of better and safer structures and vehicles [1−5] as well as for an improvement of environment protection methods. New modelling approaches and analytical methods of solution are sought for finding more efficient tools allowing the parametrical analysis of mathematical models.

This paper presents a wavelet based approach [6,7,8] for the analysis of beam-foundation interactions when the beam is subjected to a moving load. Similar methodology was previously used with success for the analysis of various dynamic systems [6,8,9,10]. In the present paper, a two-dimensional theoretical model is analyzed, where infinitely long beam rests on a surface of a viscoelastic half-space and it is subjected to a moving load. The model of a beam supported by a multilayered infinite medium [9], presented in this paper, try to reflect a more natural variability of the solid. At the same time, it leads to much more complex formulas. The type of load considered is represented by a series of distributed loads harmonically varying in time and it increases additionally the complexity of calculations. Therefore, numerical integration used for simpler models of moving loads becomes ineffective and an alternative method must be applied to obtain precise enough solutions.

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The previously developed wavelet-based method, with an application of coiflets filter, is used in the present paper as an efficient tool replacing numerical integration [11,12]. It allows to omit analytical singularities and alleviates numerical instabilities that are common in complex dynamic systems [6−9].

Having a mathematical model to analyse a beam-foundation structure dynamics, one can deal with the minimization problem of the vibration response, at least, within a frequency range of interest. Numerical optimization procedures could be applied and, of course in a theoretical way, one can look for solutions that minimize the system vibration. In the literature, one can find research to improve the performance of rail-soil and subsoil interactions [13] while this research is mostly concentrated on exploring wavelet model potentiality for being used with optimization. Although out of scope of this work, it would be interesting to have some propagation interruption or attenuation as it is found from badgap phenomena at certain periodic systems [14−18]. The wavelet method proposed in [7,9], here with an application of coiflet filter, is considered adequate to start studying this problem.

The aim of the present work is to develop an effective methodology for finding optimal parameters of medium layers in the sense of vibrations reduction. The most needed features of the developed method are the computational power and time efficiency allowing to execute the optimization procedure. This optimization process is very difficult with an application of the numerical integration instead of the wavelet-based approximation for the considered model due to numerical instabilities and increased time of calculations. The presented results are considered an important step for further analysis leading to modelling of composite materials that could be used for an improvement of beam-foundation systems. The performed investigations allowed to formulate basic guidelines for the wavelet-based optimization methodology in the analysis of dynamic systems related to structural dynamics problems.

2. Equations and conditions

The two-dimensional model investigated in this paper consists of an infinitely long beam resting on a surface of a viscoelastic multilayered infinite medium in the plane $x0z$ (Fig. 1).

The Euler-Bernoulli equation for the beam can be written as [6,19]:

$$EI\frac{\partial^4 W}{\partial x^4}+\rho_B\frac{\partial^2 W}{\partial t^2}=P(x,t)-\bar{a}\sigma_{zz}(x,0^+,t) \tag{1}$$

where $W(x,t)$, $\sigma_{zz}(x,z,t)$, $P(x,t)$, EI, ρ_B, and $\bar{a}$ are the vertical displacement of the beam, the vertical stress, the vertical point load, the bending stiffness, the mass per unit length of the beam and the thickness of the beam in y direction, respectively. The equation of motion for a viscoelastic medium can be described by the elastodynamic equation:

$$(\hat{\lambda}+\hat{\mu})\nabla_{xz}(\nabla_{xz}\mathbf{u})+\hat{\mu}\nabla_{xz}^2\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{2}$$

where operators $\hat{\lambda}=\lambda+\lambda^*\partial/\partial t$ and $\hat{\mu}=\mu+\mu^*\partial/\partial t$ represent the viscoelastic behaviour of the layer, λ, μ and ρ are the Lamé constants and the mass density of the medium, respectively and $\mathbf{u}(x,z,t)=[u(x,z,t),0,w(x,z,t)]$ is the displacement vector for the viscoelastic solid.

The boundary and continuity conditions are formulated as follows:

$$u(x,0,t)=0,\ w(x,0,t)=W(x,t), \tag{3a}$$

$$u\left(x,\left(\sum_{k=1}^{K}h_k\right)^-,t\right)=u\left(x,\left(\sum_{k=1}^{K}h_k\right)^+,t\right),\ w\left(x,\left(\sum_{k=1}^{K}h_k\right)^-,t\right)=w\left(x,\left(\sum_{k=1}^{K}h_k\right)^+,t\right), \tag{3b}$$

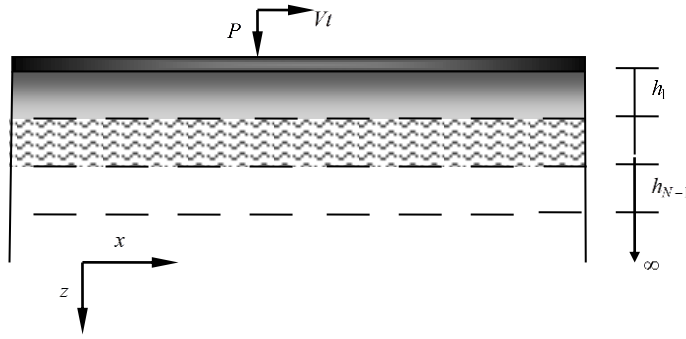

Fig. 1. The beam on surface of multilayered half-space with moving load.

$$\sigma_{xz}\left(x,\left(\sum_{k=1}^{K}h_k\right)^{-},t\right)=\sigma_{xz}\left(x,\left(\sum_{k=1}^{K}h_k\right)^{+},t\right),\ \sigma_{zz}\left(x,\left(\sum_{k=1}^{K}h_k\right)^{-},t\right)=\sigma_{zz}\left(x,\left(\sum_{k=1}^{K}h_k\right)^{+},t\right), \tag{3c}$$

$$\lim_{z\to\infty}u(x,z,t)=0\,,\ \lim_{z\to\infty}w(x,z,t)=0 \tag{3d}$$

where $K=1,2,...,N-1$ and N is the number of layers.

The system of Eqs (1−3) allows to obtain the solution for the steady state response and therefore the Fourier transform can be used for further calculations. The finite sum of the distributed harmonic loads is considered (Fig. 2), giving the possibility to represent the more complex and realistic excitation:

$$P(x,t)=\sum_{l=0}^{L-1}\frac{P_0}{2r}\cos^2\left(\frac{\pi(x-Vt-(2r+s)l)}{2r}\right)H\left(r^2-(x-Vt-(2r+s)l)^2\right)e^{i\Omega t} \tag{4}$$

where $H(.)$, $2r$, V and Ω are the Heaviside function, the span of the load, the velocity and the frequency of the moving load, respectively. L is a number of separated impulses and s is the distance between them.

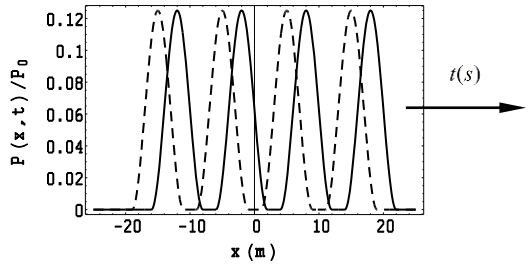

Fig. 2. The sample series of distributed harmonic loads for $L=4$.

The closed form solution obtained in this paper allows parametrical analysis and the assessment of the influence of various physical parameters on the displacements. This paper is mainly concerned with the response of the system for the moving load defined by Eq. (4) with relatively low frequency Ω .

3. Solution

By using the Fourier transforms

$$\tilde{\tilde{f}}(k,\omega)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,t)e^{i(\omega t-kx)}dxdt\,,\quad f(x,t)=\frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{\tilde{f}}(k,\omega)e^{-i(\omega t-kx)}d\omega dk \tag{5}$$

where k and ω are the variables in the transform domain, the problem (Eqs (1−3)) can be reformulated in terms of Lame' potentials [20] in the transform domain:

$$\frac{d^2\tilde{\tilde{\phi}}}{dz^2}-R_L{}^2\tilde{\tilde{\phi}}=0\,,\ \frac{d^2\tilde{\tilde{\psi}}}{dz^2}-R_T{}^2\tilde{\tilde{\psi}}=0\,, \tag{6a}$$

$$(EIk^4-\rho_B\omega^2)\tilde{\tilde{W}}(k,\omega)=\tilde{\tilde{P}}(k,\omega)-\bar{a}\tilde{\tilde{\sigma}}_{zz}(k,h_1^+,\omega)\,, \tag{6b}$$

where the coefficients $R_L=\sqrt{k^2-\omega^2/(c_L{}^2-i\omega(\lambda^*+2\mu^*)/\rho)}$ and $R_T=\sqrt{k^2-\omega^2/(c_T{}^2-i\omega\mu^*/\rho)}$ describe the physical properties of medium and are specific for each layer. $c_L=\sqrt{(\lambda+2\mu)/\rho}$ and $c_T=\sqrt{\mu/\rho}$ are velocities of the longitudinal and the shear waves in the medium, respectively.

By assumption of different material properties for each layer (Fig. 1), one can write solutions of Eq. (6a) as follows:

$$\tilde{\tilde{\phi}}_j(k,z,\omega)=A_{1+4(j-1)}(k,\omega)e^{R_{Lj}z}+A_{2+4(j-1)}(k,\omega)e^{-R_{Lj}z}\,, \tag{7a}$$

$$\tilde{\tilde{\psi}}_j(k,z,\omega)=A_{3+4(j-1)}(k,\omega)e^{R_{Tj}z}+A_{4+4(j-1)}(k,\omega)e^{-R_{Tj}z}\,, \tag{7b}$$

where $j=1,...,N$, and R_{Lj}, R_{Tj} are the roots of the characteristic Eq. (6a) for each layer separately. Then one can obtain the transformed solutions for displacements and stresses in the layers. The use of the boundary and continuity conditions leads to the system of algebraic equations with respect to $A_{n+4(j-1)}(k,\omega)$ that can be solved easily by using the Cramer's rule:

$$A_{n+4(j-1)}(k,\omega)=\tilde{\tilde{P}}(k,\omega)D_{n+4(j-1)}(k,\omega)/D(k,\omega) \tag{8}$$

with the determinants $D, D_{n+4(j-1)}$ ($n=1,...,4;\ j=1,...,N$) depending on the system parameters. Hence, one can obtain a solution in the transform domain for horizontal and vertical displacements at the surface:

$$\tilde{\tilde{u}}(k,0,\omega)=\tilde{\tilde{P}}(k,\omega)\tilde{\tilde{u}}_0(k,\omega)=\tilde{\tilde{P}}(k,\omega)(ik(D_1+D_2)+R_{T1}(D_3-D_4))/D \tag{9}$$

$$\tilde{\tilde{w}}(k,0,\omega)=\tilde{\tilde{P}}(k,\omega)\tilde{\tilde{w}}_0(k,\omega)=\tilde{\tilde{P}}(k,\omega)(R_{L1}(D_1-D_2)-ik(D_3+D_4))/D\,. \tag{10}$$

The steady state vibrations can be analysed considering an arbitrarily chosen point at the surface and the point $x=0$ is assumed in this paper. One should note that the method of solution applied in this paper allows to obtain the system response for each point z inside the medium and the surface behaviour ($z=0$) is analyzed by the authors due to its importance for the structure analysis. The inverse Fourier transform Eq. (5) leads to the solution for the displacements at the surface in the following integral form:

$$\mathbf{u}(0,0,t)=\frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{\tilde{P}}(k,\omega)[\tilde{\tilde{u}}_0(k,\omega),0,\tilde{\tilde{w}}_0(k,\omega)]e^{-i\omega t}dkd\omega\,. \tag{11}$$

The integrand in Eq. (11) has strong singularity for $\Omega=0$ [6−9] in case of half-space under the beam, whereas the higher load frequency $\Omega>0$ increases extremely the complexity of formulas leading to numerical instabilities. Therefore the integral cannot be effectively derived by using classical numerical integration. The analysis becomes ineffective due to the loss of important information about system's behaviour. One can propose an alternative

method of calculation allowing further parametrical analysis. This wavelet-based method is introduced in the next section.

4. Wavelet estimation of integrals

The method based on a wavelet expansion [10,11,21] of functions in the transform domain is adopted here for calculation of integrals (Eq. (11)), allowing the effective parametrical analysis of the system. This semi-analytical method was already successfully applied to solution of many dynamic problems, giving possibility of an efficient numerical analysis and the recognition of detailed features of systems that are lost during the process of the classical numerical integration [6−10].

The inverse Fourier transform $f(x)$ of function $\tilde{f}(\omega)$ can be represented by a series

$$f(x)=\left(\frac{2^{-n/2-1}}{\pi}\right)\tilde{\Phi}\left(\frac{-x}{2^n}\right)\sum_{k=-\infty}^{+\infty}c_{n,k}\exp\left(\frac{ixk}{2^n}\right)+\frac{1}{2\pi}\sum_{j=n}^{\infty}2^{-\frac{j}{2}}\tilde{\Psi}\left(-\frac{x}{2^j}\right)\sum_{k=-\infty}^{+\infty}d_{j,k}\exp\left(\frac{ixk}{2^j}\right) \tag{12}$$

for each function $f(x)\in L^2(\mathbf{R})$, where Φ and Ψ are the scaling function and the wavelet function, respectively. The wavelet coefficients $c_{n,k}=\int_{-\infty}^{+\infty}\tilde{f}(\omega)\Phi_{n,k}(\omega)d\omega$ and $d_{j,k}=\int_{-\infty}^{+\infty}\tilde{f}(\omega)\Psi_{j,k}(\omega)d\omega$ are usually difficult to obtain but they can be effectively estimated by using the properties of coiflet filters { p_0, p_1, ..., p_{17} } defining one of wavelet bases [7,12].

This sequence of numbers can be used for construction of the scaling function Φ and the wavelet function Ψ by using the refinement equations (two-scale relations) [21]:

$$\Phi(x)=\sum_{k=0}^{K}p_k\Phi(2x-k)\,,\quad \Psi(x)=\sum_{k=0}^{K}q_k\Phi(2x-k)\,, \tag{13}$$

where $K=3\tilde{N}-1$ and $\tilde{N}$ is an order of coiflet filter, in this paper equal to 6 [7]. The sequence (q_k) can be found from relation $q_k=(-1)^k p_{2l+1-k}$ for some integer l that is determined depending on specific properties of wavelets.

For coiflets, Eq. (12) can be written in more practical form:

$$f(x)=\lim_{n\to\infty}f_n(x)=\lim_{n\to\infty}\frac{1}{\pi 2^{n+2}}\prod_{j=1}^{\tilde{J}}\left(\sum_{k=0}^{3\tilde{N}-1}p_k\exp\left(\frac{ikx}{2^{j+n}}\right)\right)\sum_{k=k_{\min}}^{k_{\max}}\tilde{f}\left(\frac{k+M}{2^n}\right)\exp\left(\frac{ixk}{2^n}\right) \tag{14}$$

where $M=\frac{1}{2}\sum_{k=0}^{3\tilde{N}-1}kp_k$ and $\tilde{J}$ is an integer associated with the class and properties of the used filter [7]. The range of summation $k_{\min}(n)=\omega_{\min}2^n-16$, $k_{\max}(n)=\omega_{\max}2^n-1$ is determined by the interval $[\omega_{\min},\omega_{\max}]$ which must cover the set of variable ω influencing the original function. This interval can be found by analysing the integrand of the Fourier integral, i.e. placing the main part of its support inside the interval [7]. The fifth term, f_4 of the approximating sequence Eq. (14) is usually precise enough for simpler problems and the primary criterion for the approximation order choice is a stabilization of the obtained solutions f_n (see Section 7) [7]. Parametric simulations show that, also for the model considered in this paper, at least the fifth term f_4 should be used for numerical estimation of displacements at the surface (Eq. (11)).

5. Optimization

Using the wavelet model analysis described in the previous sections, the system response under various parameters' choice can be simulated and, by this way, it is possible to give answers to the required questions regarding

dynamics of the considered system. For example: can the maximum amplitude be minimized for a given velocity by optimizing the set of physical parameters of the layers? It is expected that this technique allows also to analyse the problem of the increase of the critical value of the velocity by changing physical parameters of the system with constraints related to real constructions.

In this paper, a structural optimization problem is formulated to be applied to the simplified problem of two viscoelastic layers under the beam (Fig. 1). The first layer is characterized by its dynamic Young's modulus $\tilde{E}$, mass density $\tilde{\rho}$ and thickness h_1. The second layer is characterized by its dynamic Young's modulus E, mass density ρ, and an infinite thickness. It is considered the objective of minimizing the vertical displacement (vibration) amplitude for a given load velocity V and the following relation between the layers: $\tilde{E} = c^3 E$ and $\tilde{\rho} = c\,\rho$. These relations are considered to simulate a nonlinear relation of Young's modulus between two layers under the beam, with the admissible range of the parameter c, in order to recognize the solid stiffness optimised for the vibration reduction.

The determination of the optimal parameters (c and h_1) for the first layer can be formulated as the following optimization problem. For a load moving along the beam with a given velocity V,

$$Minimize[\mathrm{Re}(w)] \tag{15}$$

subject to the side constraint on the parameter c within $c_{\min} \leqslant c \leqslant c_{\max}$ and on the thickness h_1 of the first layer within $h_{\min} \leqslant h_1 \leqslant h_{\max}$. The problem is subject to the state Eqs (1−4) which allows to obtain the objective function, i.e. the vibration amplitude.

In order to solve this optimization problem, the Mathematica software [22] was used with its function "FindMinimum". This routine searches for a local minimum in the objective function, using the "QuasiNewton" method, for the automatic option, given with one starting point. In order to deal with the local minima problem, several starting points were tested.

6. Numerical examples

The following system of parameters previously used in other papers [3,5,6,8,9], are taken for numerical calculations: the mass density $\rho = 1700$ kg/m^3, Young's modulus $E = 3\cdot 10^7$ N/m^2, $\mu^* = \lambda^* = 3\cdot 10^4$ kg/ms and Poisson's ratio $\nu = 0.3$ for the foundation; the mass density $\rho_B = 120$ kg/m and the bending stiffness $EI = 6.4\cdot 10^6$ Nm^2 for the beam. The width of the track structure in y direction is $\bar{a} = 4$ m and the thickness of the first layer under the beam is $h_1 = 10$ m. The parameters regarding the moving load are: $P_0 = 4\cdot 10^4$ N, the load frequency $\Omega = 10^{-10}$ rad/s, the span of the load $2r = 0.15$ m, the number of separated impulses $L = 10$ and the distance between distributed loads $s = 2$ m.

Two parameters, $a = \tilde{\rho}/\rho$ and $c = \tilde{E}/E$, are introduced for taking into account an admissible range of physical properties of the medium layers. $\tilde{\rho}$ and $\tilde{E}$ denote the mass density and the Young's modulus of the arbitrarily chosen layer (h_i; $i = 1,2,...,N$). It is assumed, for parametrical analysis, that the first layer (with the thickness h_1) differs from others. Some preliminary results for the dynamic analysis of the considered model were presented in [9]. In the present paper, some plots are shown to highlight the influence of changes in physical properties of the foundation on the system behaviour. The numerical simulations were performed by using the adopted coiflet-based approximation described in Section 4. As expected, it achieved considerable efficiency compared with the numerical integration (see also Section 7) and showed a strong potential to analyse the investigated system. To illustrate it, Fig. 3 shows that the vibration amplitudes are very sensitive to parametric changes of the system and this feature justifies the application of the proposed optimization.

Figure 3 shows the maximum amplitude of the vertical displacement at the surface ($z = 0$) depending on the load velocity V in case of identical layers ($a = c = 1$). One can observe that the critical velocity, defined here as the velocity for which the level of vibrations reaches the maximum value, remains very close to the velocity c_T of shear waves in the supporting medium [8]. The value of the displacements amplitude strongly depends on the parameters describing the foundation and its maximum can be considerably high. This situation is recognized as extremely dangerous and cannot be accepted in reality.

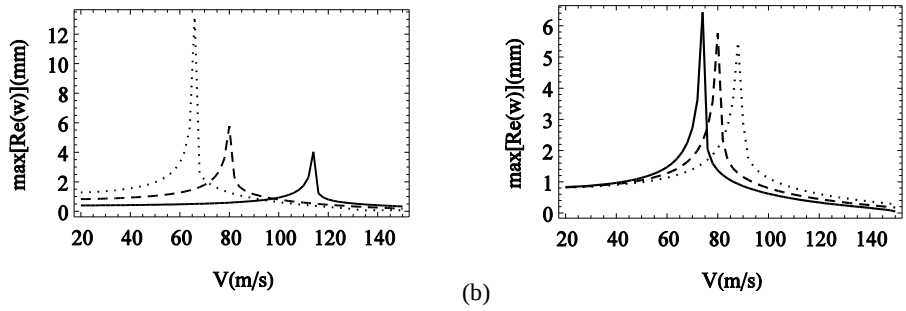

Fig. 3. The vibrations amplitude versus velocity: (a) all with $\rho = 1700$ kg/m^3 and $E = 10^7$ N/m^2 (dotted), $E = 3 \cdot 10^7$ N/m^2 (dashed), $E = 15 \cdot 10^7$ N/m^2 (solid); (b) all with $E = 3 \cdot 10^7$ N/m^2 and $\rho = 1400$ kg/m^3 (dotted), $\rho = 1700$ kg/m^3 (dashed), $\rho = 2500$ kg/m^3 (solid).

For the numerical optimization the design variables c and h_1 of the first layer beneath the beam are used, as described in Section 5.

The vertical displacements (vibration) amplitude for a given load velocity V is minimized running the wavelet model (with an order of approximation $n = 4$). An appropriate number of the approximating sequences are considered for the recognition of main features of the optimization procedure. The choice of the velocity for testing is very important due to the fact that strong variations of the system behaviour are expected in the area of the critical velocities [2,5,7,8], however they can be observed also for relatively low velocities for the used system of parameters. The sub-critical velocities: 20 m/s, 50 m/s, 60 m/s and 70 m/s, and the super-critical velocity 100 m/s, were chosen for this application.

Results of the optimization were obtained considering the following design domain given by the size constraints of $0.75 < c < 1.3$ and $9.85 \text{ m} < h_1 < 10.15 \text{ m}$. Note that a value of c smaller than 1 means that the first layer has a lesser mass density as well as a less stiff material. For values of c bigger than one is the contrary. The given properties of the reference layer with c equal to one are associated with the infinite layer and kept fixed.

The obtained results are presented in Tables 1 to 3. Several starting points have been examined showing that the result strongly depends on starting point (Table 1). The procedure must be repeated then in order to reach the sought value (Table 2).

One should note that this algorithm cannot be effectively applied with the numerical integration used instead of the coiflet-based approximation due to strong variations of the integrands to be evaluated (Eq. (11)). One can note that the value of the parameter c did not converge to 1, but resulted in a stiffer layer with the higher thickness at sub-critical velocities (Table 3). The presented example illustrates the potential of the proposed methodology. A number of simulations carried out by using the developed method show that the presented optimization problem is nontrivial and many areas with local minima of the vibrations amplitude can be found for the considered model. Therefore an optimization method for finding optimal foundation design parameters is considered important.

The obtained results allow to describe the advantages of the developed approach and problems that might appear when applying the coiflet-based optimization procedure. The presented semi-analytical coiflet-based method of solution is considered efficient and may be successfully applied to the analysis of complex dynamic systems. It is computationally faster and more effective than many numerical methods, as for example, when compared with the direct application of the Finite Element Method. The simulations carried out allow to observe a strong efficiency of the developed method, as it is expected when compared with the numerical integration applied instead of the coiflet-based expansion. The results obtained with an application of the wavelet expansion give the possibility of the detailed parametrical analysis by using various systems of parameters. Nevertheless, one must remember that the features of the system solution must be well analyzed in the transform domain before the wavelet approximation is applied, in order to determine the range of summation in the approximating formulas and the appropriate order of approximation n that must secure a stabilisation of the solution (Eq. (14)).

Table 1

Optimization results for the velocity $V = 60$ m/s and a set of design variables $0.75 < c < 1.3$ and $9.85 \text{ m} < h_1 < 10.15$ m

Starting point (c, h_1)	(0.75,9.85)	(0.75,10.15)	(1.3,9.85)	(1.3,10.15)	(1,10)
Amplitude $Abs[\mathrm{Re}[w]]$[mm]	0.184962	0.181735	0.181735	0.210262	2.05122
Design variable c	1.3	1.29746	1.29746	1.28296	0.93009
Design variable h_1[m]	9.88622	10.15	10.15	10.15	9.85

Table 2

Global extremes within the constraints domain $0.75 < c < 1.3$ and $9.85 \text{ m} < h_1 < 10.15$ m for the velocity $V = 60$ m/s

Numerically determined value	Global minimum	Global maximum
Amplitude $Abs[\mathrm{Re}[w]]$[mm]	0.180615	2.7801
Design variable c	1.29011	0.75
Design variable h_1[m]	10.15	10.15

Table 3

Global minima within the constraints domain $0.75 < c < 1.3$ and $9.85 \text{ m} < h_1 < 10.15$ m obtained via wavelet-based optimization procedure

Velocity V m/s	20 m/s	50 m/s	70 m/s	100 m/s
Amplitude Abs $[\mathrm{Re}[w]]$[mm] (global minimum)	3.78865×10^{-13}	0.0139345	0.385787	0.967631
Design variable c	1.26531	1.26303	1.3	0.75
Design variable h_1 [m]	10.0699	10.15	10.1259	9.86917

7. Validation

The coiflet-based approximation proposed in this paper has been already examined for different systems described by differential equations [6−10].

In order to illustrate the influence of the order n of the approximation on the accuracy of the wavelet solution, one can show a comparison of the proposed coiflet approach to the numerical one presented in [19]. In the numerical simulations the half-space is modelled by a considerable thickness of the layer under the beam ($h = 10^5$ m). A moving distributed load harmonically varying in time is considered in both cases. The assumption of considerable thickness of the layer under the beam is used for elimination of computational issues, when the numerical integration is applied [6,19]. The results presented in Fig. 4 show a good agreement of the wavelet-based results with those obtained by using the numerical approach.

The system of parameters used for the presented numerical example is similar to the described in previous section with $\Omega = 4\pi$ rad/s, $L = 1$ and only one homogeneous layer with finite thickness under the beam [23]. One can observe that the fifth term w_4 of the approximating sequence (14) applied to the vertical component of the integral (11), can be successfully used instead of the numerical integration proposed by other authors [5,6,19]. For the points near $t = 0$ the lower order of approximation could be also used [7].

8. Conclusions

A closed form solution for a multilayered viscoelastic solid's displacements is found for the case of the moving load represented by a finite series of separated distributed loads harmonically varying in time. The obtained wavelet approximation allows an efficient analysis and optimization of the system response by changing physical properties of the multilayered foundation. It allows to omit the numerical integration and therefore the analysis is possible

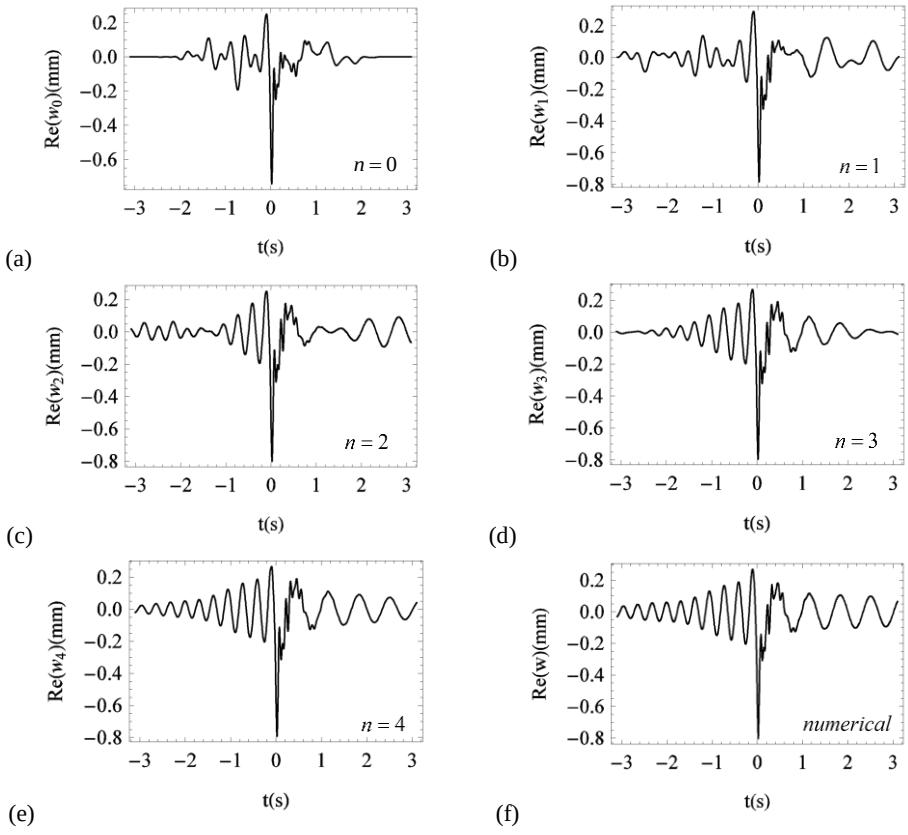

Fig. 4. Validation of the proposed method: (a)−(e) the wavelet-based solution (Eqs. (11) and (14)); (f) the numerical solution [19,23].

without a loss of important details. The developed methodology may be applied to relatively complex systems, e.g. when the layers of supporting medium have different characteristics. In such cases, the integrands reflect strong variations and the numerical evaluation of integrals becomes ineffective, leading to numerical instabilities. Such numerical instabilities are not a problem when the wavelet approximation is applied.

Using the proposed wavelet model analysis, a structural optimization method is applied to the simplified system of two viscoelastic layers under the beam. An optimization problem of minimizing the vertical displacements (vibrations) amplitude for a given load velocity V is considered, by assuming the first layer to be modelled by the design parameters describing its physical properties while the infinite layer is kept fixed. A few examples are presented taking into account various velocities of the moving load. These examples illustrate the potential of the developed optimization methodology. The performed investigations allowed to formulate basic guidelines for the wavelet-based optimization methodology in the analysis of dynamic systems related to structural dynamics problems. The obtained results point to the minimization of the surface vibration amplitude when an adequate (optimized) layer is between the beam and the lower semi-infinite layer.

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