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Design of a random decrement method based structural health monitoring system

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Abstract. Structural Health Monitoring (SHM) has reached a high importance in numerous fields of civil and mechanical engineering. Promising damage detection approaches like the Damage Index Method, Gapped Smoothing Technique and Modal Strain Energy Method require the structure's mode shapes [1].

Long term modal data acquisition on real life structures requires a computational efficient system based on a measuring method that can easily be installed. Systems using the Random Decrement Method (RDM) are composed of a decentralized network of smart acceleration sensors applied for both, triggering and pure measuring. They allow the reduction of cabling effort and computational costs to a minimum.

In order to design a RDM measuring network efficiently, an approved procedure for defining hardware as well as measuring settings is required. In addition, optimal sensor positions have to be defined. However, today those decisions are mostly based on expert's knowledge. In this paper a systematic and analytical procedure for defining the hardware requirements and measuring settings as well as optimal sensor positions is presented. The proposed routine uses the outcome of an Experimental Modal Analysis (EMA).

Due to different requirements for triggering and non-triggering sensors in the RDM network a combination of two approaches for sensor placement has to be used in order to find the best distribution of measurement points over the structure.
A controllability based technique is used for placing triggering sensors, whereas the Effective Independence (EI) is utilized for the placement of non-triggering sensors.
The combination of these two techniques selects the best set of measuring points for a given number of sensors out of all possible sensor positions.

Damage detection itself is not considered within the scope of this paper.

Keywords: Structural health monitoring, test planning, effective independence

1. Introduction

In this paper a systematic procedure for generating all information that is needed for implementing a RDM system on an arbitrary structure is presented. Its focus is on the system design. Further a comparison of the results obtained with an EMA and the presented system is published in [2].

Today Structural Health Monitoring (SHM) is applied in several fields of civil and mechanical engineering. One promising method for long term data capture on all kinds of structures is the Random Decrement Method (RDM), a method based on a network of acceleration sensors divided in triggering and pure measuring sensor nodes. To test the capability of this method a complete sensor network shall be installed on the pedestrian steel bridge shown in Fig. 1. This bridge is a frequently used connection between two buildings about 24 m long and 6 m high. The investigation focuses on the main span between the 4 pillars. As depicted in the geometry model (Fig. 1) the roof is not taken into consideration.

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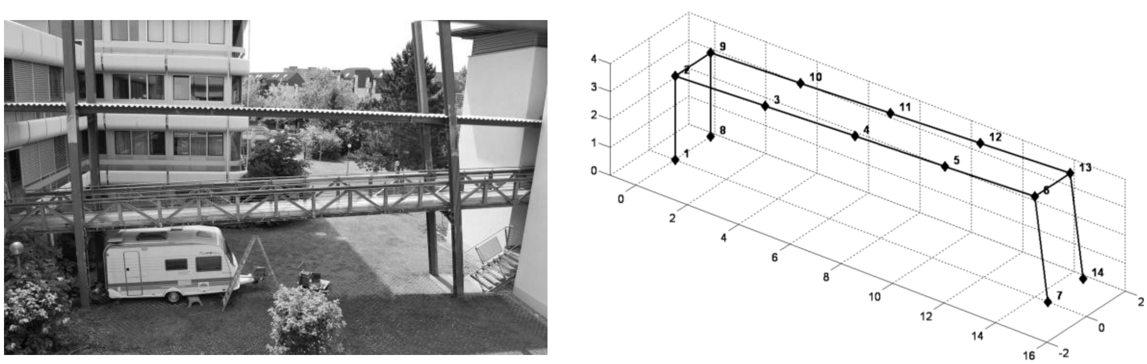


Fig. 1. Pedestrian Steel Bridge and corresponding geometry model.

Figure 5 gives an overview over the complete routine developed in this paper.

2. Random decrement method

Most of the structures which are interesting for SHM cannot be excited for structural analysis. They are either too large (e.g., infrastructure objects) or it is impractical to apply a vibration exciter during operation. Thus only the output signals, i.e., the vibrations excited by operational loads, can be used in order to estimate the system's behavior. An effective way in terms of computational costs for extracting this information is the RDM. It is a simple method that averages time data series $x(t)$ measured on the system under random input loads when a given trigger condition is fulfilled (see Eq. (1) as an example for a level crossing trigger of trigger level a). The result of this averaging process is called a Random Decrement (RD) Signature $D_{XX}(\tau)$. A descriptive explanation of the method is given in [1].

$$D_{XX}(\tau) = \frac{1}{N}\sum_{n=1}^{N} x(t_n + \tau),\ x(t_n) \geqslant a \tag{1}$$

Here, N is the total number of averages, $x(t)$ the measured time data and τ the length of the recorded signal. Asmussen [3] proved that out of the RD signatures D_{XX} the correlation functions R_{XX} can be calculated using the trigger level a itself and the variance σ_x^2 of the triggered signal, (see Eq. (2)).

$$R_{XX}(\tau) = D_{XX}(\tau)\frac{\sigma_x^2}{a} \tag{2}$$

Therefore, instead of measuring and transferring whole time blocks in real time, it is sufficient to interchange trigger information between the sensors in order to acquire correlation functions. Having estimated the RD signatures, they are transferred to a central processing unit and the correlation functions are calculated. This allows to reduce the cabling effort and computational cost. Hence, RDM is ideally suited for long term measurements.

As described in [4], the concept may be extended from autocorrelation functions to the estimation of crosscorrelation functions using single trigger information.

The derived correlation functions are processed with an Operational Modal Analysis algorithm, namely the Frequency Domain Decomposition (FDD), to extract eigenfrequencies and modeshapes of the structure under natural excitation [4].

The following sections describe the design of a RDM based measuring system.

3. Experimental modal analysis

The first step is to identify the modes, which are planned to be monitored in the SHM project and their corresponding frequency band. In the application considered here, this data is derived by an EMA.

The EMA is performed with ten triaxial-accelerometers comprising a sensitivity of 100 mV/g. The excitation of the bridge can be done with a modal hammer of 1.5 kg. The measurement ranges from 0–128 Hz with a resolution of 0.25 Hz. The data acquisition and analysis is performed with the LMS Testlab system. Five eigenmodes below 20 Hz, listed in Table 1, are identified.

Both, visualization of all five modes and the model assurance criterion [6] show, that the three torsion modes comprise a very similar shape. It is assumed that neglecting the bridge's roof causes spatial aliasing which makes it impossible to see differences in these modes. The sensor positioning routines used in this paper are based on the linear independence of eigenvectors. Since it is known that the three torsion vectors are not independent it is not necessary to consider all three modes. Therefore two translation modes and only one torsion mode are chosen for the following procedure. The forms are shown in the geometric model in Fig. 2.

The described methods are not limited to EMA gained data. Any approved method providing eigenfrequencies and modeshapes can be used, i.e. operational modal analysis or numerical and analytical calculations. Suitable excitation and measuring methods, especially for civil engineering structures, that are often too large to be excited by means of an impact hammer, are listed in [5].

Table 1
List of identified eigenmodes

Frequency [Hz]	Damping [%]	Description	Picked for further use
8.0	0.63	Horizontal translation	X
9.8	0.43	Vertical translation	X
13.5	0.31	Torsion	−
15.2	1.23	Torsion	X
18.25	0.87	Torsion	−

4. Trigger sensor placement using the controllability criterion

As pointed out in Section 2 the triggering sensors play an important role for the RDM. After providing the data base, the next step of the presented procedure is to define the triggering sensors in the smart sensor network.

The quality of the determined modeshapes acquired with the cross-RD-signatures strongly depends on the accurateness of the estimated phase relationships. Therefore the acquired cross-RD-signatures have to contain information about all modes of interest in adequate quality. This is achieved by choosing the triggering sensors or reference degrees of freedom (refDOF) in such a way that each mode of interest is excitable at least at one refDOF. A similar demand is described by the controllability criterion of an arbitrary Linear Time Invariant (LTI)-system.

Sensor positions, that assure an optimal controllability, are derived from the input matrix of a state-space representation [7]. Here, the state-space-model is derived from the result of an experimental modal analysis of the bridge structure as described in Herold [8] or Bartel et al. [9]. It is shown that the input matrix B is equal to the transposed eigenmatrix of the identified system. Thus, the columns of the input matrix represent the eigenvectors $\vec{b}_i$ and, the generalized deflection of the considered DOFs of the *i*th mode, accordingly. Once the input matrix is built up, the modal controllability index vector of each mode $\vec{s}_i$ is defined by

$$\vec{s}_i = \frac{\vec{b}_i}{\max(\vec{b}_i)} \tag{3}$$

with $\max(\vec{b}_i)$ being the DOF with the highest deflection of the *i*th mode. Hence, the modal controllability index gives the relation of the obtained controllability at each DOF of the modeshape in comparison to the DOF with the maximal controllability. Accordingly $\vec{s}_{ij}(DOF) = 1$ indicates for a DOF j optimal controllability of the *i*th mode.

In some applications the number of refDOFs may be restricted, so the overall controllability given by

$$S = \frac{\sum_{i}^{i_{\max}} \max(s_{ij})}{i_{\max}} \tag{4}$$

is used to estimate the quality of the chosen sensor placement. If the number of refDOFs is lower than the number of regarded modeshapes, *S* might be smaller than one. A permutation algorithm can thus be applied in order to find the optimal refDOFs.

In the case of the bridge considered here, the three modes mentioned above are used for calculating the controllability of a single DOF. The result is shown in Table 2. It turns out, that for the second and third mode DOF 11 (shown in Table 2) is the best refDOF.

Table 2
Calculated controllability index for the optimal sensor position

	Controllability index of sensor 1 @ point 11 in z direction	Controllability index of sensor 2 @ point 12 in y direction
Mode 1	0.30	1.00
Mode 2	1.00	0.05
Mode 3	1.00	0.40

The controllability index of 1 at each single mode also yields to an overall controllability index of 1, indicating that the optimal refDOF placement is found. The identified modeshapes and the calculated refDOFs are depicted in Fig. 2.

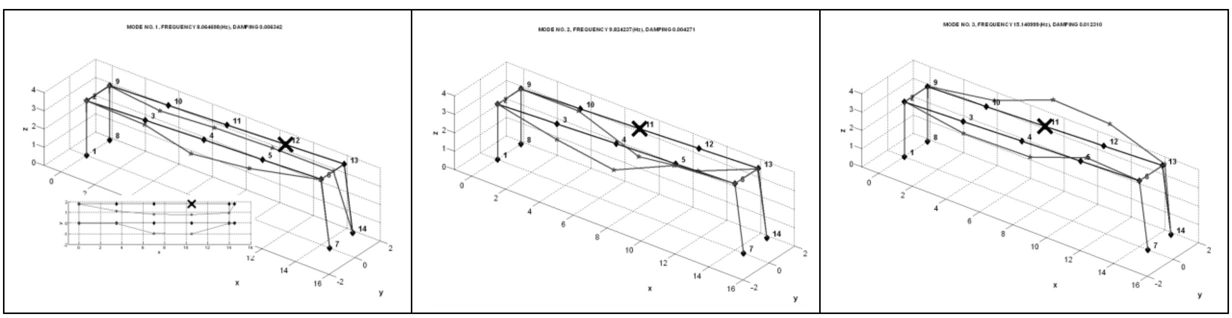


Fig. 2. Modeshapes with the calculated optimal reference degrees of freedom (marked with black crosses).

5. Measuring sensor placement using effective independence

In general the total number of sensors that are used in a SHM project is limited due to economic reasons such as the high price of sensors suitable for small excitation levels or the data acquisition hardware limiting the number of transducers. In contrast an EMA is often performed with a high number of DOFs, especially when numerical investigations by means of FE analysis are carried out. Taking the limited number of sensors into account, the best possible set of sensor positions for SHM has to be extracted.

Today this decision still is often based on an engineer's guess. However, several methods provide a systematic solution. Kammer for example shows two ways how to use model based data from numerical simulations to find the best possible sensor set of a given number of sensors [10,11]. Within the scope of this paper experimental based data from an EMA is used instead.

The EI is a sensor placement method based on the linear independence of candidate sensor sets. In an iterative process a large starting set is reduced to a given number of sensors [10]. With small adjustments in the algorithm the method can also be used to expand a small number of starting sensors to a given amount [11].

The basic theory of both methods is the same: The m chosen mode shapes with n starting candidate DOFs (to guarantee full rank: $n \rangle m$) form the initial mode shape matrix $[\Phi]_{nxm}$ ($[\Phi]_{nxm}$ is equal to the output matrix C of state space system) that is used to calculate the Fisher Information Matrix $[A]_{mxm}$ [10]:

$$[A]_{mxm} = \left[\Phi^T \right]_{mxn} [\Phi]_{nxm} \tag{5}$$

Out of this matrix the so called Prediction Matrix, $[E]_{nxn}$ is calculated:

$$[E]_{nxn} = [\Phi]_{nxm} [A]_{mxm} \left[\Phi^T \right]_{mxn} \tag{6}$$

Every diagonal element of the matrix $[E]_{nxn}$ can be seen as the contribution of its corresponding degree of freedom to the global rank of the mode shape matrix [2].

The classical EI method starts with an initial mode shape matrix $[\Phi]_{nxm}$ that contains all original candidate DOFs. Now the prediction matrix $[E]_{nxn}$ is calculated and the sensor ranked lowest is located. The corresponding candidate DOF is eliminated from the mode shape matrix and a new prediction matrix is calculated. This routine is executed until the set is reduced to the given number of DOFs.

In contrast the expanding EI (eEI) method starts with a small group of sensors that represent the initial mode shape matrix $[\Phi]_{nxm}$. The other DOFs that were used in the EMA represent the candidate DOFs. Now, the iteration starts with the calculation of the additional distribution to the prediction matrix of all candidate DOFs. The DOF with the highest contribution is added to the initial mode shape matrix and deleted from the candidate group. This is done until the given number of DOFs is reached.

In case of preliminary investigation for RD projects, the eEI sensor placement method offers an advantage compared to the classical method. When the classical EI method is applied, the refDOFs chosen by the controllability method are not a mandatory subset of the sensor set calculated. However if the eEI method is applied, the refDOFs present an ideal initial mode shape matrix $[\Phi]_{nxm}$. It is also possible that sensor positions have to be considered although they are not selected by one of the two routines. This may happen in case of high additional excitation levels on that spot. These DOFs can also form the initial mode shape matrix and the eEI determines the best additional measuring sensors.

The minimum number of selected sensors is equal to the number of observed modes; the maximum number is the amount of DOFs used in the EMA. Since the reliability of the eEI was already proven by Kammer [11], it is implemented as the standard routine.

In the case of the footbridge, two trigger sensors are sufficient to measure all three modes. They form the matrix $[\Phi]_{nxm}$, the remaining sensor positions are picked with the eEI method as described above.

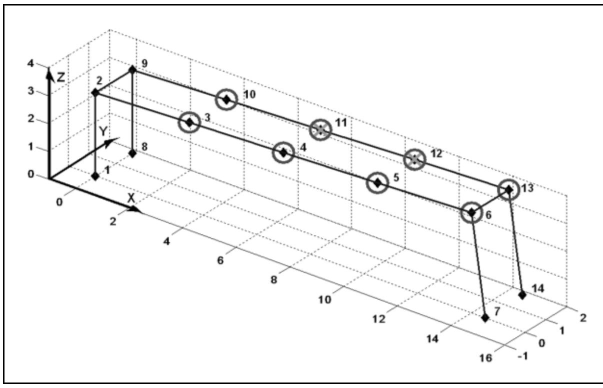
In Fig. 3 the result of the routine is illustrated for a total number of 14 sensors. Here, diamonds indicate all candidate sensor positions, measuring positions are marked by circles and triggering positions are denoted by crosses.

The allocation of the chosen DOFs is reasonablel. The picked DOFs are the ones with the highest amplitude, DOFs, which are close to the nodes of the modeshape, e.g., DOF2 or DOF9 are not part of the sensor set.

6. Dimension of measuring hardware

The main parameters of the operational monitoring system are derived from the results of the Experimental Modal Analysis (EMA). The process of calculating these parameters are described in the following section.

In a first step the range of modes to be monitored is selected. This choice should be made on the experience of a monitoring engineer as well as on information about the critical parts of the observed system.



Measuring sensors Z-direction	Measuring sensors Y-direction	Trigger sensors
3 Z	3 Y	11 Z
4 Z	4 Y	12 Y
5 Z	5 Y	
6 Z	10 Y	
10 Z	11 Y	
11 Z	12 Y	
12 Z	13 Y	

Fig. 3. Candidate (diamonds) −, Triggering (crosses) − and Measuring sensors (circles).

Afterwards, parameters such as cut-off frequency of the anti-aliasing filters, sampling frequency, and block length of RD signatures are calculated from the result of the EMA. For this purpose, the decay behavior of a one mass oscillator with the lowest measured damping ratio is observed. The time t_x after which the amplitude is reduced to a reasonable value (an engineering guess would be 10% of the original value) multiplied with the sampling frequency f_s yields the block length lRD (Table 3).

Table 3
Setup of the monitoring system derived from the Experimental Modal Analysis

Parameter	Symbol	Value
Frequency of lowest mode of interest	f_l	8.0 Hz
Frequency of highest mode of interest	f_h	15.2 Hz
Anti-aliasing cut-off frequency	f_{co}	32 Hz
Sampling frequency	f_s	128 Hz
Block length of RD signatures	lRD	2048

For monitoring the pedestrian bridge under operational conditions, the RD signal processing algorithm is implemented on an embedded PC by automatic code generation from Matlab/Simulink. In general the resources of the used embedded PCs are limited. For this reason, the maximum number of sensors that can be handled by the hardware is also calculated from EMA results. To assure real-time capability the worst-case sensor task execution time must be less than the sensor signal sampling time. In case of the considered bridge the designated hardware can handle a maximum number of 14 sensors, two reference and 12 response sensors.

In order to define sensor parameters like required sensitivity and resolution, the characteristic of the natural excitation need to be known. Furthermore, the optimum trigger levels of both reference nodes are calculated from measured acceleration, too. In case of the considered bridge there are two characteristic types of natural excitation, pedestrians crossing the bridge and wind (see Fig. 4).

Asmussen [3] proposed the optimum trigger level as $a = \sqrt{2}\,\sigma_x$ with σ_x being the standard variation of the measured acceleration signal at the trigger position. In real life applications it is inappropriate to calculate the standard derivation σ_x from the total sensor signal period, because of the insufficient signal-to-noise ratio (SNR) due to the measuring equipment over a wide range. For this reason an experimental trigger level a_e is calculated only from sections with a sufficient signal-to-noise ratio. In the present context this is achieved when a pedestrian passes the bridge.

Consequently, the experimental trigger level is calculated as follows, with N being the number of averages, i the index of the triggering sensor and k the index of standard variations calculated out of signals with good SNR:

$$a_{e,i} = \frac{\sqrt{2}}{N}\sum_{k=1}^{N}\sigma_k. \tag{7}$$

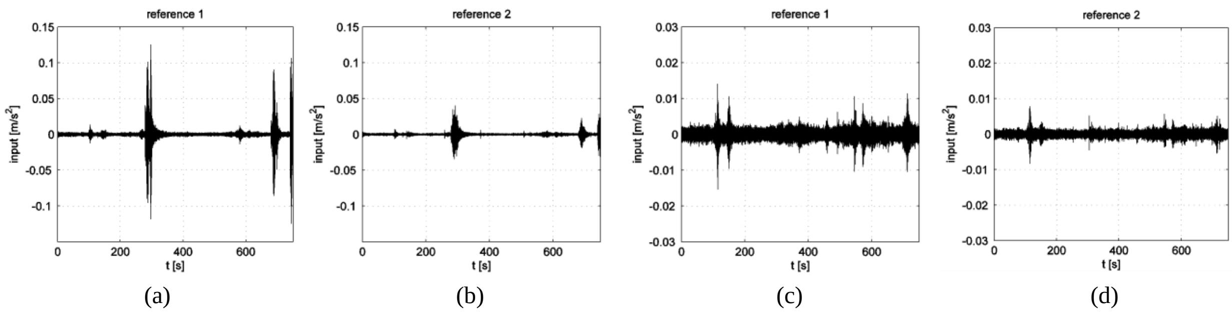


Fig. 4. On the reference nodes there are two characteristic types of natural excitation on the bridge. (a) pedestrian excitation reference 1; (b) pedestrian excitation reference 2; (c) wind excitation reference 1; (d) wind excitation reference 2.

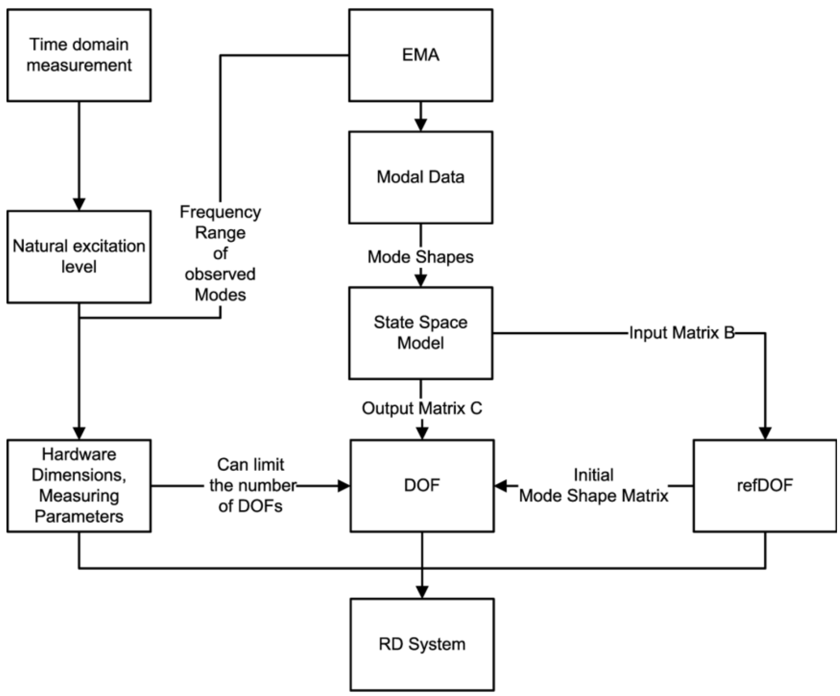


Fig. 5. Overview of the developed routine for the design of a RDM based SHM system.

Table 4 contains the calculated experimental trigger levels for both reference nodes.

Table 4
Setup of the monitoring system derived from the initialization

Parameter	Symbol	Value
Experimental trigger level (reference 1)	$a_{e,1}$	0.0346 m/s^2
Experimental trigger level (reference 2)	$a_{e,2}$	0.0107 m/s^2

7. Summary

In this paper a systematic approach for the design of RDM based SHM systems is proposed. A solution that uses the outcome of an EMA to calculate all the required input for the measuring system, like the hardware dimension of the target PC, the degrees of freedom and reference degrees of freedom, is presented. Missing system parameters like trigger levels are generated in the initialization process of the system. The complete routine can be implemented analytically, leading to a reduction of decisions that have to be done by expert's knowledge to a minimum.

The approach is tested and presented on a pedestrian steel bridge and achieved satisfactory and reasonable results [2]. Its general character makes it suitable for any structure to be observed by RDM based SHM.

Figure 5 gives an overview over the complete routine developed in this paper.

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