Theoretical and experimental study on synchronization of the two homodromy exciters in a non-resonant vibrating system

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Received 14 April 2012
Revised 28 September 2012
Accepted 3 October 2012

Abstract. In this paper we give some theoretical analyses and experimental results on synchronization of the two non-identical exciters. Using the average method of modified small parameters, the dimensionless coupling equation of the two exciters is deduced. The synchronization criterion for the two exciters is derived as the torque of frequency capture being equal to or greater than the absolute value of difference between the residual electromagnetic torques of the two motors. The stability criterion of synchronous state is verified to satisfy the Routh-Hurwitz criterion. The regions of implementing synchronization and that of stability of phase difference for the two exciters are manifested by numeric method. Synchronization of the two exciters stems from the coupling dynamic characteristic of the vibrating system having selecting motion, especially, under the condition that the parameters of system are complete symmetry, the torque of frequency capture stemming from the circular motion of the rigid frame drives the phase difference to approach $\pi$ and carry out the swing of the rigid frame; that from the swing of the rigid frame forces the phase difference to near zero and achieve the circular motion of the rigid frame. In the steady state, the motion of rigid frame will be one of three types: pure swing, pure circular motion, swing and circular motion coexistence. The numeric and experiment results derived thereof show that the two exciters can operate synchronously as long as the structural parameters of system satisfy the criterion of stability in the regions of frequency capture. In engineering, the distance between the centroid of the rigid frame and the rotational centre of exciter should be as far as possible. Only in this way, can the elliptical motion of system required in engineering be realized.

Keywords: Synchronization, vibrating system, stability, vibratory synchronization transmission

1. Introduction

In the natural world, human society, or fields of engineering and technology, the synchronous phenomena or synchronous problems can be found everywhere. The earliest detailed accounts on synchronized motion was made by Huygens [1]. Since 1894, the synchronous phenomena was also found in nonlinear circuits by scientists, such as Rayleigh [2] found that two organ tubes could produce a synchronized sound when the outlets are close to each other, and Pol [3] observed the synchronization of certain electrical-mechanical system. They called this phenomena as “frequency capture”. “Frequency capture” or “synchronization”, therefore, is a unique phenomenon of nonlinear system. In the 1960s, Blekhman [4–10] in Soviet Union proposed the synchronization theory of mechanical exciters. Chinese scholar Prof. Wen, applied such synchronization theory to engineering and established a branch of vibration...
utilization engineering [11–15]. Balthazar [16] has also given some short comments on self-synchronization of two non-ideal sources by means of numerical simulations. Many theories of synchronization of more than two exciters are studied by scholars, in which the main methods used are the method of direct motion separation [4–10] and the averaging method of small parameters [17–20]. In Refs. [17–19] synchronization of the two non-identical exciters with rotating oppositely in a non-resonant vibrating system of plane motion was investigated, and that of the two exciters of spatial motion was also discussed in Ref. [20]. In this paper, synchronization of the two Homodromy exciters is studied by using the average method of modified small parameters. We present that the vibrating system driven by the two exciters has the coupling dynamic characteristic of selecting motion, and the theory method used is verified to be feasible and descriptive by experiment.

This paper contains the following elements: The dynamic model and motion equations of system are given firstly. The criterion of implementing frequency capture and that of stability of the synchronous system are deduced by followed. Next section, a quantitative numeric discussions are provided. Experiments are given in Section 5. Finally, concludes this paper.

2. The dynamic model and motion equations of system

The dynamic model of the considered vibrating system is illustrated in Fig. 1. Springs are connected to a rigid frame. The two induction motors, are supplied with the same electrical source at the same time, and installed symmetrically in the rigid frame, rotate in the same directions and drive two eccentric lumps (two exciters) to excite the vibrating system. The frame $oxy$ is a fixed frame, and its origin $o$ is the equilibrium point of centroid of the rigid frame. The motions of the rigid frame are vibrations in $x$- and $y$-directions, denoted by $x$ and $y$, and swing about its centroid, denoted by $\psi$. Each eccentric lump rotates about its spin axis, denoted by $\varphi_i$, $i = 1, 2$. Using Lagrange’s equations, and choosing the $x, y, \psi, \varphi_1$, and $\varphi_2$ as the generalized coordinates, we derive the differential equations of motion of the vibrating system in the form:

$$
M \ddot{x} + f_x \dot{x} + k_x x = m_1 r_1 (\dot{\varphi}_1^2 \cos \varphi_1 + \dot{\varphi}_1 \sin \varphi_1) + m_2 r_2 (\dot{\varphi}_2^2 \cos \varphi_2 + \dot{\varphi}_2 \sin \varphi_2)
$$

$$
M \ddot{y} + f_y \dot{y} + k_y y = m_1 r_1 (\dot{\varphi}_1^2 \sin \varphi_1 - \dot{\varphi}_1 \cos \varphi_1) + m_2 r_2 (\dot{\varphi}_2^2 \sin \varphi_2 - \dot{\varphi}_2 \cos \varphi_2)
$$

$$
J \ddot{\psi} + f_\psi \dot{\psi} + k_\psi \psi = m_1 r_1 l_0 [\dot{\varphi}_1^2 \sin (\varphi_1 - \beta) - \dot{\varphi}_1 \cos (\varphi_1 - \beta)] + m_2 r_2 l_0 [\dot{\varphi}_2^2 \sin (\varphi_2 + \beta) - \dot{\varphi}_2 \cos (\varphi_2 + \beta)] + m_1 r_1 l_0 \dot{\varphi}_1 \cos \varphi_1 + m_2 r_2 l_0 \dot{\varphi}_2 \cos \varphi_2
$$

$$
J_{01} \ddot{\varphi}_1 + f_{1 \varphi_1} = T_{c1} - m_1 l_0 \dot{\varphi}_1 \cos \varphi_1 - l_0 \dot{\psi} \cos (\varphi_1 - \beta) + l_0 \dot{\psi}^2 \sin (\varphi_1 - \beta)
$$

$$
J_{02} \ddot{\varphi}_2 + f_{2 \varphi_2} = T_{c2} - m_2 l_0 \dot{\varphi}_2 \cos \varphi_2 - l_0 \dot{\psi} \cos (\varphi_2 + \beta) - l_0 \dot{\psi}^2 \sin (\varphi_2 + \beta)
$$

where

$$
M = m + \sum_{i=1}^{2} m_i, J = (m_1 + m_2) \omega_i^2, f_\psi = \frac{1}{2} (f_x l_x^2 + f_y l_y^2), k_\psi = \frac{1}{2} (k_x l_x^2 + k_y l_y^2), J_{0j} = m_i r_j^2 + j_{0j}.
$$

$m$ is the mass of the rigid frame; $m_i$ the mass of the exciter $i, i = 1, 2$; $l_0$ the distance between the rotational centre $o_i$ of the exciter-i and the mass centre $o$ of the rigid frame; $r_1 = r_2 = r$ the eccentric radius of two eccentric lumps; $\beta$ the angle between $o_1 o$ and $x$-axis; $\beta_1$ the angle between $Ao$ and $x$-axis; $k_x, k_y$ and $k_\psi$ the constants of springs, and $f_x, f_y$ and $f_\psi$ the damping constants in $x$-, $y$- and $\psi$-directions, respectively; $f_j$ the damping constant of rotor of the motor $j$ and $J_{0j}$ its moment of inertia; $j_{0j}$ the moment of inertia of motor-oj axis which can be neglected, $j = 1, 2; l_e$ the equivalent rotating radius of the vibrating system about the centroid of the rigid frame; $T_{cj}$ the electromagnetic torque of the motor $j$. ($\bullet$) and ($\otimes$) signify $d \bullet /dt$ and $d^2 \bullet /dt^2$, respectively.
3. Frequency capture of the two exciters and stability of synchronous state

As illustrated in Fig. 1, assuming the average phase of the two exciters and their phase difference to be \( \varphi \) and \( 2\alpha \), respectively, then we obtain

\[
\varphi_1 = \varphi + \alpha, \varphi_2 = \varphi - \alpha
\]

The average mechanical angular velocity of the two exciters therefore, is \( \dot{\varphi} \). Due to the periodical motion of the vibrating system, the mechanical angular velocities of the two motors change periodically. If the least common multiple period of the two motors is assumed to be \( T_{\text{LCMP}} \), the average value of their average angular velocity over time \( T_{\text{LCMP}} \), then must be a constant, i.e.

\[
\omega_{m0} = \frac{1}{T_{\text{LCMP}}} \int_0^{T_{\text{LCMP}}} \dot{\varphi}(t) \, dt = \text{constant}
\]

Assuming the instantaneous change coefficients of \( \dot{\varphi} \) and \( \dot{\alpha} \) to be \( \varepsilon_1 \) and \( \varepsilon_2 \) (\( \varepsilon_1 \) and \( \varepsilon_2 \) are the functions of time \( t \)), i.e., \( \dot{\varphi} = (1 + \varepsilon_1) \omega_{m0}, \dot{\alpha} = \varepsilon_2 \omega_{m0} \), respectively, we have

\[
\begin{align*}
\dot{\varphi}_1 &= (1 + \varepsilon_1 + \varepsilon_2) \omega_{m0} = (1 + \nu_1) \omega_{m0}, \quad \nu_1 = \varepsilon_1 + \varepsilon_2 \\
\dot{\varphi}_2 &= (1 + \varepsilon_1 - \varepsilon_2) \omega_{m0} = (1 + \nu_2) \omega_{m0}, \quad \nu_2 = \varepsilon_1 - \varepsilon_2
\end{align*}
\]

The two exciters can operate synchronously, if the average values of \( \varepsilon_1 \) and \( \varepsilon_2 \) over the single period (\( T_0 = 2\pi/\omega_{m0} \)) are zero, i.e., \( \varepsilon_1 = 0 \) and \( \varepsilon_2 = 0 \). Generally in engineering, the masses of eccentric lumps are far smaller than that of the rigid frame [11–14], the coupling terms of \( \dot{\psi} \) in the first two formulae of Eq. (1) and that of \( \dot{x} \) and \( \dot{y} \) in the third formula, therefore, have been neglected. On the other hand, the slip of an induction motor usually ranges from 2% to 8% [21], i.e., \( \varepsilon_1 \ll 1 \) and \( \varepsilon_2 \ll 1 \), so \( \dot{\varphi}_1 \) and \( \dot{\varphi}_2 \) can be neglected in the first three formulae of Eq. (1) when the vibrating system operates in the steady-state. We assume \( m_1 \) is \( m_0 \) and \( m_2 \) is \( \eta m_0 \) (\( 0 < \eta \leq 1 \)), and Eq. (4) are inserted into the first three formulae of Eq. (1) to yield

\[
\begin{align*}
M\ddot{x} + f_x\dot{x} + k_x x &= m_0 r \omega_{m0}^2 [(1 + \varepsilon_1 + \varepsilon_2)^2 \cos(\varphi + \alpha) + \eta(1 + \varepsilon_1 - \varepsilon_2)^2 \cos(\varphi - \alpha)] \\
M\ddot{y} + f_y\dot{y} + k_y y &= m_0 r \omega_{m0}^2 [(1 + \varepsilon_1 + \varepsilon_2)^2 \sin(\varphi + \alpha) + \eta(1 + \varepsilon_1 - \varepsilon_2)^2 \sin(\varphi - \alpha)] \\
J\ddot{\psi} + f_\psi \dot{\psi} + k_\psi \psi &= m_0 r \omega_{m0}^2 l_0 [(1 + \varepsilon_1 + \varepsilon_2)^2 \sin(\varphi + \alpha - \beta) - \eta(1 + \varepsilon_1 - \varepsilon_2)^2 \sin(\varphi - \alpha - \beta)]
\end{align*}
\]

For a non-resonant machinery, the operating frequency of system is about (3~10) times of its natural frequency and the damping constants of springs are very small [11–14] according to Ref. [17], the responses of Eq. (5) can be expressed in the form:

\[
\begin{align*}
x &= -\frac{r_{m}}{\mu_y} \cos(\varphi + \alpha + \gamma_x) + \eta \cos(\varphi - \alpha + \gamma_x), \\
y &= -\frac{r_{m}}{\mu_y} \sin(\varphi + \alpha + \gamma_y) + \eta \sin(\varphi - \alpha + \gamma_y), \\
\psi &= -\frac{r_{m}}{l_y \mu_\psi} \sin(\varphi + \alpha - \beta + \gamma_\psi) - \eta \sin(\varphi - \alpha + \beta + \gamma_\psi),
\end{align*}
\]

where

\[
\begin{align*}
r_m &= \frac{m_0}{M} l_c = \sqrt{\frac{J}{M}}, \\
r_1 &= \frac{l_0}{l_c}, \quad \mu_x = 1 - \frac{\omega_{nx}^2}{\omega_{m0}^2}, \quad \mu_y = 1 - \frac{\omega_{ny}^2}{\omega_{m0}^2}, \quad \mu_\psi = 1 - \frac{\omega_{n\psi}^2}{\omega_{m0}^2}, \quad \omega_{nx} = \sqrt{\frac{k_x}{M}}, \\
\omega_{ny} &= \sqrt{\frac{k_y}{M}}, \quad \omega_{n\psi} = \sqrt{\frac{k_\psi}{M}}, \quad \gamma_i = \frac{2\kappa_{ni}(\omega_{ni}/\omega_{m0})}{1 - (\omega_{ni}/\omega_{m0})^2}, i = x, y, \psi
\end{align*}
\]
ξ_x, ξ_y and ξ_ψ are the damping ratios of springs (ξ_x ≤ 0.07, ξ_y ≤ 0.07 and ξ_ψ ≤ 0.07) [11,12], π – γ_t denotes the phase angle in i-direction, i = x, y, ψ.

Differentiating x, y and ψ in Eq. (6) with respect to time t by the chain rule, respectively, we obtain ̇x, ̇y and ̇ψ, which are inserted into the last two formulae of Eq. (1) and integrating them over φ = 0 ∼ 2π, and neglecting the higherorder terms of ɛ_1 and ɛ_2, the balanced equations of the two exciters are expressed as

\[
\begin{align*}
J_{01}\omega_{m0}(\dot{\xi}_1 + \dot{\xi}_2) + J_{11}\omega_{m0}(1 + \dot{\xi}_1 + \dot{\xi}_2) &= T_{e1} - \ddot{T}_{L1} \\
J_{02}\omega_{m0}(\dot{\xi}_1 - \dot{\xi}_2) + J_{22}\omega_{m0}(1 + \dot{\xi}_1 - \dot{\xi}_2) &= T_{e2} - \ddot{T}_{L2}
\end{align*}
\]  

(7)

with

\[
\begin{align*}
\ddot{T}_{L1} &= \chi'_{11}\dot{\xi}_1 + \chi'_{12}\dot{\xi}_2 + \chi_{11}\ddot{\xi}_1 + \chi_{12}\ddot{\xi}_2 + \chi_a + \chi_f1 \\
\ddot{T}_{L2} &= \chi'_{21}\dot{\xi}_1 + \chi'_{22}\dot{\xi}_2 + \chi_{21}\ddot{\xi}_1 + \chi_{22}\ddot{\xi}_2 - \chi_a + \chi_f2
\end{align*}
\]  

(8)

where

\[
\begin{align*}
\chi'_{11} &= m_0\omega^2_{m0}[-W_{c0} - W_x\sin(2\alpha + \theta_c) + W_c\cos(2\alpha + \theta_c)]/2, \\
\chi'_{12} &= m_0\omega^2_{m0}[-W_{c0} - W_x\sin(2\alpha + \theta_c) - W_c\cos(2\alpha + \theta_c)]/2, \\
\chi'_{21} &= m_0\omega^2_{m0}[-\eta^2W_{c0} + W_x\sin(2\alpha + \theta_c) + W_c\cos(2\alpha + \theta_c)]/2, \\
\chi'_{22} &= m_0\omega^2_{m0}[-\eta^2W_{c0} + W_x\sin(2\alpha + \theta_c) - W_c\cos(2\alpha + \theta_c)]/2, \\
\chi_{11} &= m_0\omega^2_{m0}[W_{s0} + W_x\cos(2\alpha + \theta_c) + W_c\sin(2\alpha + \theta_c)], \\
\chi_{12} &= m_0\omega^2_{m0}[W_{s0} - W_x\cos(2\alpha + \theta_c) - W_c\sin(2\alpha + \theta_c)], \\
\chi_{21} &= m_0\omega^2_{m0}[\eta^2W_{s0} + W_x\cos(2\alpha + \theta_c) - W_c\sin(2\alpha + \theta_c)], \\
\chi_{22} &= m_0\omega^2_{m0}[\eta^2W_{s0} - W_x\cos(2\alpha + \theta_c) + W_c\sin(2\alpha + \theta_c)], \\
\chi_f1 &= m_0\omega^2_{m0}[W_{s0} + W_x\cos(2\alpha + \theta_c)]/2, \\
\chi_f2 &= m_0\omega^2_{m0}[\eta^2W_{s0} + W_x\cos(2\alpha + \theta_c)]/2, \\
\chi_a &= m_0\omega^2_{m0}W_c\sin(2\alpha + \theta_c)/2, \\
W_{c0} &= r_m\left(\frac{\cos \gamma_x}{\mu_x} + \frac{\cos \gamma_y}{\mu_y} + \frac{r^2_0 \cos \gamma_\psi}{\mu_\psi}\right), \\
W_x &= \eta r_m \sqrt{a^2_x + b^2_x}, \\
\theta_c &= \begin{cases} 
\arctan(-b_x/a_x), & a_x \geq 0 \\
\pi + \arctan(-b_x/a_x), & a_x < 0 
\end{cases}, \\
p_c &= \begin{cases} 
\arctan(b_c/a_c), & a_c \geq 0 \\
\pi + \arctan(b_c/a_c), & a_c < 0 
\end{cases}, \\
\alpha_s &= \frac{\sin \gamma_x}{\mu_x} - \frac{\sin \gamma_y}{\mu_y} - \frac{r^2_0 \sin \gamma_\psi}{\mu_\psi} \cos(2\beta), \\
b_x &= -\frac{r^2_0 \sin \gamma_\psi}{\mu_\psi} \sin(2\beta), \\
a_c &= -\frac{\cos \gamma_x}{\mu_x} - \frac{\cos \gamma_y}{\mu_y} + \frac{r^2_0 \cos \gamma_\psi}{\mu_\psi} \cos(2\beta), \\
b_c &= -\frac{r^2_0 \cos \gamma_\psi}{\mu_\psi} \sin(2\beta). 
\end{align*}
\]

Compared with the change of φ (φ = ωm0) with respect to time t, that of ɛ_1, ɛ_2, ̇ɛ_1 and ̇ɛ_2 are very small. ɛ_1 and ɛ_2, therefore, are considered to be slow-changing parameters, while the change of φ is considered as fast-changing parameter, in this study. According to the method of direct separation of motions [4–10], ɛ_1, ɛ_2, ̇ɛ_1, ̇ɛ_2 and α are
assumed to be the middle values of their integrations \( \dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_1, \dot{\xi}_2 \) and \( \dot{\alpha} \) respectively during the aforementioned integration.

If the two motors are supplied with the same electric source and have identical pole pairs, their electromagnetic torques can be expressed as follows:

\[
T_{e1} = T_{e01} - k_{e01}(\dot{\xi}_1 + \dot{\xi}_2), \quad T_{e2} = T_{e02} - k_{e02}(\dot{\xi}_1 - \dot{\xi}_2)
\]

where \( T_{e01} \) and \( T_{e02} \) are the electromagnetic torques, \( k_{e01} \) and \( k_{e02} \) the stiffness of angular velocity when the two motors operate at the angular velocity of \( \omega_{m0} \).[20]

We choose exciter 1 as the standard exciter \((m_1 = m_0, m_2 = \eta m_0, 0 < \eta \leq 1)\) to normalize Eq. (7) in the following manner: firstly substituting Eqs (8) and (9) into Eq. (7), and then dividing Eq. (7) by the moment of the standard exciter, \( m_0 \omega^2 \omega_{m0} \), after that, adding two formulae as the first row, subtracting the second formula from the first one as the second row, next introducing the non-dimensional parameters \( \rho_1, \rho_2, \kappa_1, \kappa_2, \) and \( \dot{\nu}_1, \dot{\nu}_2 \)

\[
\rho_1 = 1 - W_{e0}/2, \quad \rho_2 = \eta - \eta^2 W_{e0}/2, \quad \dot{\nu}_1 = \dot{\xi}_1 + \dot{\xi}_2, \quad \dot{\nu}_2 = \dot{\xi}_1 - \dot{\xi}_2
\]

and finally writing them into the matrix form, frequency capture equation of the two exciters can be expressed in the form:

\[
A \dot{\nu} = B \nu + u
\]

(10)

where

\[
\nu = \{\dot{\nu}_1, \dot{\nu}_2\}^T, \quad u = \{u_1, u_2\}^T,
\]

\[
a_{11} = \rho_1 + \rho_2 + W_c \cos(2\dot{\alpha} + \theta_c), \quad a_{12} = \rho_1 - \rho_2 + W_c \cos(2\dot{\alpha} + \theta_c),
\]

\[
a_{21} = \rho_1 - \rho_2 - W_c \sin(2\dot{\alpha} + \theta_c), \quad a_{22} = \rho_1 + \rho_2 - W_c \cos(2\dot{\alpha} + \theta_c),
\]

\[
b_{11} = -\omega_{m0}(\kappa_1 + \kappa_2 - 2W_c \cos(2\dot{\alpha} + \theta_c)), \quad b_{12} = -\omega_{m0}(\kappa_1 - \kappa_2 - 2W_c \sin(2\dot{\alpha} + \theta_c)),
\]

\[
b_{21} = -\omega_{m0}(\kappa_1 - \kappa_2 + 2W_c \sin(2\dot{\alpha} + \theta_c)), \quad b_{22} = -\omega_{m0}(\kappa_1 + \kappa_2 + 2W_c \cos(2\dot{\alpha} + \theta_c)).
\]

\[
u_1 = [T_{e01}/(m_0 \omega^2 \omega_{m0}) - f_1/(m_0 r^2)] + [T_{e02}/(m_0 \omega^2 \omega_{m0}) - f_2/(m_0 r^2)] - \omega_{m0} W_{s0}(1 + \eta^2)/2 - \omega_{m0} W_{c0} \cos(2\dot{\alpha} + \theta_c),
\]

\[
u_2 = [T_{e01}/(m_0 \omega^2 \omega_{m0}) - f_1/(m_0 r^2)] - [T_{e02}/(m_0 \omega^2 \omega_{m0}) - f_2/(m_0 r^2)] - \omega_{m0} W_{s0}(1 - \eta^2)/2 - \omega_{m0} W_{c0} \sin(2\dot{\alpha} + \theta_c).
\]

Equation (10) describes the coupling relation of the two exciters and is referred to as the dimensionless coupling equation of the two exciters.

### 3.1. The criterion of implementing frequency capture

Substituting \( \dot{\nu}_1 = 0 \) and \( \dot{\nu}_2 = 0 \) into Eq. (10), we have \( u_1 = 0 \) and \( u_2 = 0 \). Calculating the sum and the difference of \( u_1 \) and \( u_2 \), and rearranging them as follows

\[
T_{e01} + T_{e02} - (f_1 + f_2) \omega_{m0} - [m_0 \omega^2 \omega_{m0} W_{s0}(1 + \eta^2)/2 + m_0 \omega^2 \omega_{m0} W_{c0} \cos(2\dot{\alpha} + \theta_c)] = 0
\]

(11)

\[
(T_{e01} - T_{e02}) - (f_1 - f_2) \omega_{m0} - m_0 \omega^2 \omega_{m0} W_{s0}(1 - \eta^2)/2 - m_0 \omega^2 \omega_{m0} W_{c0} \sin(2\dot{\alpha} + \theta_c) = 0
\]

(12)

In the above Eq. (11), \( T_{e01} + T_{e02} \) is the sum of electromagnetic torques of the two motors; \( (f_1 + f_2) \omega_{m0} \) is that of the rotors damping torques of the two motors; the last terms, \( \chi f_1 + \chi f_2 \), is the sum of the load torques that the
vibrating system acts on the two motors. Equation (11), therefore, is the equation of torque balance of the vibrating system operating in the steady-state.

Rewriting Eq. (12), we have

\[
\sin(2\tilde{\alpha} + \theta_c) = T_D / T_C, \quad 2\tilde{\alpha} = \arcsin(T_D / T_C) - \theta_c
\]  

(13)

where

\[
T_C = m_0 r^2 \omega_m^2 W_c, \quad T_D = T_{R1} - T_{R2}, \quad T_{R1} = T_{e01} - f_1 \omega_m - m_0 r^2 \omega_m^2 W_{s0}/2,
\]

\[
T_{R2} = T_{e02} - f_2 \omega_m - m_0 r^2 \omega_m^2 \eta^2 W_{s0}/2.
\]

\(T_C\) is the torque of frequency capture; \(T_D\) the difference between the residual electromagnetic torques of the two motors; \(T_{R1}\) and \(T_{R2}\) the residual electromagnetic torques of the motors 1 and 2, respectively.

Since \(|\sin(2\tilde{\alpha} + \theta_c)| \leq 1\), the criterion of implementing vibrating synchronization is

\[
T_C \geq |T_D|
\]  

(14)

Equation (14) indicates that the criterion of synchronization for the two exciters is that the torque of frequency capture is equal to or greater than the absolute value of difference between the residual electromagnetic torques of the two motors.

Equations (11) and (12) are nonlinear functions of \(\omega_m\) and \(\tilde{\alpha}\), their solutions, \(\omega_m^*\) and \(\tilde{\alpha}_0\), can be determined by numeric method.

3.2. Stability of synchronous state

Linearizing Eq. (10) around \(\tilde{\alpha} = \tilde{\alpha}_0\) and appending the third row, \(\Delta \dot{\tilde{\alpha}} = \omega_m^* \bar{\varepsilon}_2 \) \((\Delta \tilde{\alpha} = \tilde{\alpha} - \tilde{\alpha}_0)\), after that, writing them as a system of the three first-order differential equations, and using the notation \(z = \{\varepsilon_1 \ v_2 \ \Delta \alpha\}\) yields

\[
\dot{z} = A'^{-1} B' z
\]  

(15)

where

\[
z = \{\varepsilon_1 + \varepsilon_2, \varepsilon_1 - \varepsilon_2, \tilde{\alpha} - \tilde{\alpha}_0\}^T, \quad A' = \begin{bmatrix} a'_{11} & a'_{12} & 0 \\ a'_{21} & a'_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B' = \begin{bmatrix} b'_{11} & b'_{12} & -2\omega_m^* W_s \sin(2\tilde{\alpha}_0 + \theta_s) \\ b'_{21} & b'_{22} & 2\omega_m^* W_c \cos(2\tilde{\alpha}_0 + \theta_c) \\ 0 & 0 & -\omega_m^* \end{bmatrix}.
\]

It should be noted that, \(a'_{ij}\) and \(b'_{ij}\) denote the values of \(a_{ij}\) and \(b_{ij}\) in matrix \(A\) and \(B\) for \(\tilde{\alpha} = \tilde{\alpha}_0\) and \(\omega_m = \omega_m^*\). Exponential time-dependence of the form \(\dot{z} = u \exp(\lambda t)\) is now assumed, and inserted into Eq. (15). Solving the determinant equation \(\det(\lambda A^{-1} B^{-1} - \lambda I) = 0\), we deduce the characteristic equation for eigenvalue \(\lambda\)

\[
\lambda^3 + c_1 \lambda^2 + c_2 \lambda + c_3 = 0
\]  

(16)

where

\[
c_1 = 4\omega_m^* H_1 / H_0, \quad c_2 = 2\omega_m^* H_2 / H_0, \quad c_3 = 2\omega_m^* H_3 / H_0
\]

\[
H_0 = 4\rho_1 \rho_2 - W_c^2 \cos^2(2\tilde{\alpha}_0 + \theta_c) + W_s^2 \sin^2(2\tilde{\alpha}_0 + \theta_s), \quad H_1 = \rho_1 \kappa_2 + \rho_2 \kappa_2 - W_s W_c \cos(\theta_c - \theta_s),
\]

\[
H_2 = 2\kappa_1 \kappa_2 + (\rho_1 + \rho_2)W_c \cos(2\tilde{\alpha}_0 + \theta_c) + (\rho_1 - \rho_2)W_s \sin(2\tilde{\alpha}_0 + \theta_s) - W_s^2 - W_s^2 \sin^2(2\tilde{\alpha}_0 + \theta_s) + W_c^2 + W_c^2 \cos(2\tilde{\alpha}_0 + \theta_c),
\]

\[
H_3 = (\kappa_1 + \kappa_2)W_c \cos(2\tilde{\alpha}_0 + \theta_c) + (\kappa_1 - \kappa_2)W_s \sin(2\tilde{\alpha}_0 + \theta_s) + 2W_s W_c \cos(\theta_c - \theta_s).
\]  

(17)
In engineering, compared with $W_ε$ in the expression of $c_1, c_2$ and $c_3$, $W_ε$ is so small that it can be neglected [17–20]. $H_0, H_1, H_2$ and $H_3$, then, can be simplified as:

$$
H_0' = 4ρ_1ρ_2 - W_c^2 \cos^2(2\bar{α}_0 + \theta_c),
$$

$$
H_1' = ρ_1κ_2 + ρ_2κ_1,
$$

$$
H_2' = 2κ_1κ_2 + (ρ_1 + ρ_2)W_c \cos(2\bar{α}_0 + \theta_c) + W_c^2 + W_ε^2 \cos^2(2\bar{α}_0 + \theta_c),
$$

$$
H_3' = (κ_1 + κ_2)W_c \cos(2\bar{α}_0 + \theta_c).
$$

Based on the Routh-Hurwitz criterion [23], it can be seen that if and only if all roots of $λ$ in Eq. (16) have the negative real part, i.e., Eq. (19) is satisfied, the zero solution of Eq. (15), $z = 0$, is stable. It should be noted that, since the two motors having the identical rated speed are supplied with the same electric source, the difference between the two motors’ speed is the rather small. Furthermore, due to the periodical rotations of the two motors, and based on the method of direct separation of motions by averaging, when the time $t \to +\infty$, Eq. (19) can guarantee that the synchronous state of the vibrating system caused by the torque of frequency capture is stable, i.e.,

$$
limit_{t \to +\infty} z = 0
$$

means $\varepsilon_i |_{t \to +\infty} = 0$ and $ν_i |_{t \to +\infty} = 0$, $i = 1, 2$, we have $\textbf{A}ν = 0$.

According to the sign of $H_0'$, Eq. (19) can be rewritten as Eqs (20) and (21):

$$
H_0' > 0, H_1' > 0, H_2' > 0 \text{ and } 4H_1'H_2' - H_0'H_3' > 0
$$

$$
H_0' < 0, H_1' < 0, H_3' < 0 \text{ and } 4H_1'H_2' - H_0'H_3' > 0
$$

From $H_0' > 0$ and $H_1' > 0$ ($κ_1 > 0$ and $κ_2 > 0$), we can deduce $ρ_1 > 0, ρ_2 > 0$ and $4ρ_1ρ_2 - W_c^2 \cos^2(2\bar{α}_0 + \theta_c) > 0$.

By $H_3' > 0$, we obtain

$$
\cos(2\bar{α}_0 + \theta_c) > 0.
$$

Inserting $H_0', H_1', H_2'$ and $H_3'$ into $4H_1'H_2' - H_0'H_3' > 0$ and rearranging it, we have

$$
[4ρ_1^2κ_2 + 4ρ_2^2κ_1 + (κ_1 + κ_2)W_c^2 \cos^2(2\bar{α}_0 + \theta_c)]W_c \cos(2\bar{α}_0 + \theta_c) > -4(ρ_1κ_2 + ρ_2κ_1)(2κ_1κ_2 + W_c^2 + W_ε^2 \sin^2(2\bar{α}_0 + \theta_c)).
$$

As shown in Eq. (24), if $\cos(2\bar{α}_0 + \theta_c) > 0$, the left hand-side of Eq. (24) is greater than zero, and its right hand-side is less than zero when $ρ_1 > 0$ and $ρ_2 > 0$. Hence, Eqs (22) and (23) satisfy Eq. (24).

When $H_0' < 0$, from $H_1' < 0$, we have $ρ_1κ_2 + ρ_2κ_1 < 0$, and $H_3' < 0$ requires $\cos(2\bar{α}_0 + \theta_c) < 0$. In this case, the left hand-side of Eq. (24) is less than zero and its right hand-side is greater than zero. $H_0' < 0, H_1' < 0$ and $H_3' < 0$ hence, can not meet the need of $4H_1'H_2' - H_0'H_3' > 0$.

Besides, one can see that, $2\bar{α}_0 + \theta_c \in (-\pi/2, \pi/2)$ satisfies Eq. (23), from which the interval of $2\bar{α}_0$ is determined by $\theta_c$. Equations (22) and (23), therefore, are the stability criterions of synchronous states for the two exciters.
4. Numeric discussions: The regions of frequency capture and characteristic of selecting motion

Section 3 has given some theoretical analyses in the simplified form on synchronization problem. This section will compare quantitatively the numeric results of the regions of frequency capture and the motion types of the vibrating system, with its above theoretically analytical results in the simplified form.

Based on the expression of \( r_{l_{\text{max}}} \), its maximum can be simplified in the form

\[
 r_{l_{\text{max}}}^2 = \lim_{l_0 \to \infty} r_l^2 = \frac{1 + r_m(1 + \eta)}{r_m(1 + \eta)}
\]

If \( r_{l_{\text{max}}}^2 \) satisfies Eqs (22) and (23), the synchronous state of system is always stable. As shown in Fig. 2, \( r_{l_{\text{max}}} \approx 7 \) for \( \eta = 1 \), the value of \( r_l \) can be arranged from 0 to 7 in the following discussions.

From Eq. (14), the main parameters that influence frequency capture of system are \( W_{s_0} \) and \( W_c \), which are functions of the dimensionless structural parameters \( r_m \), \( r_l \), \( \eta \), \( u_x \), \( u_y \) and \( u_\psi \). In a non-resonant vibrating system, however, \( u_x \), \( u_y \) and \( u_\psi \) change little (24/25–99/100) [11,12], we focus on investigating the effect of dimensionless parameters \( r_m \), \( r_l \), \( \eta \) on the frequency capture. To guarantee the torque of frequency capture, \( T_C \) in Eq. (13) should sufficiently overcome \( T_D \), i.e., \( T_C > |T_D| \). When the two identical motors are taken to drive the two non-identical unbalanced rotors, we have

\[
 T_D = T_{R1} - T_{R2} = -m_0 r^2 \omega_{m_0}^2 W_{s_0}(1 - \eta^2)/2
\]

Here, we assume that \( T_{s_01} - f_1 \omega_{m_0} - (T_{s_02} - f_2 \omega_{m_0}) \approx 0 \) just for convenient discussion (in engineering, actually, the difference between the electromagnetic torques of the two identical motors is not complete zero).

Equation (14), therefore, can be simplified in the form

\[
 W_c \geq |W_{s_0}(1 - \eta^2)/2|
\]

Figure 3(a) shows the regions of implementing frequency capture in \( \eta r_l \)-plane for \( \beta = 0 \). From Eq. (27), \( r_m \) has no effect on the frequency capture. \( \eta r_l \)-plane is divided into three Regions: I, II and III. The regions of implementing frequency capture are in Regions I and II, and the phase difference in the steady state is \( 2\alpha_0 \in (-\pi/2, \pi/2) \) in Region I, \( 2\alpha_0 \in (\pi/2, 3\pi/2) \) in Region II. The vibrating system can not implement frequency capture in Region III. Regions I, II and III converge into a point \( (\eta = 1.0, r_l = 1.414 \approx \sqrt{2}) \), at this point, \( T_D = 0 \), in other words, when the two motors are identical, this point is the best-matching parameters of the two exciters that can enhance the ability of the frequency capture.

Figure 3(b) shows the regions of the frequency capture with \( \beta \neq 0 (\beta \in (0, \pi/4]) \). Here, it is noteworthy that, \( \eta \) has little effect on the frequency capture by the fact that \( W_s \) is far smaller than \( W_c \), i.e., Eq. (27) is always satisfied in Regions I and II.
Table 1  
The motion types of the vibrating system

<table>
<thead>
<tr>
<th>Parameters of vibrating system</th>
<th>2α₀</th>
<th>Motion types of vibrating system</th>
</tr>
</thead>
<tbody>
<tr>
<td>η = 1.0</td>
<td>β = 0°, r₁ &lt; √2 (i.e., a_c &lt; 0)</td>
<td>π Pure swing</td>
</tr>
<tr>
<td></td>
<td>r₁ &gt; √2 (i.e., a_c &gt; 0)</td>
<td>0 Circular motion</td>
</tr>
<tr>
<td>0° &lt; β &lt; 90°</td>
<td>(0, π)</td>
<td>Swing and circular motion</td>
</tr>
<tr>
<td>β = 90°</td>
<td>π Pure swing</td>
<td></td>
</tr>
<tr>
<td>0° &lt; β &lt; 90°</td>
<td>(0, π)</td>
<td>Swing and circular motion</td>
</tr>
<tr>
<td>β = 90°</td>
<td>π Pure swing</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. The values of a_c for β and r₁.

Fig. 5. Approximate values of 2α₀ with the two identical motors: (a) η = 0.2; (b) η = 0.5; (c) η = 0.8; (d) η = 1.0.

We assume that \( \rho_C = T_C \sin(2\alpha + \theta_c)/2 \), if the structural parameters of system are Region I and II in Figs 3(a) and (b), from Eqs (11) and (12), we can see that, \( \rho_C \) acting on the phase-leading exciter is the load torque that limits the increase of its angular velocity. Meanwhile, it also acts on the other phase-lagging exciter, in which \( \rho_C \) is the driving torque that limits its the decrease of the angular velocity. Finally, the synchronous and stable operation of the two exciters is reached. When the two exciters operate in the steady state, the torque of frequency capture does no work.

When the two motors are identical, according to Eq. (13), the approximate values of 2α₀ for r₁, η and β are shown in Fig. 5.

Based on the sign of a_c and the value of 2α₀ in Figs 4 and 5, respectively, the motion types of the vibrating system are expressed in Table 1.

In Table 1, when the structural parameters of system are complete symmetry, i.e., η = 1.0, β = 0° (Wc = ηr_m |a_c|), according to the expression of the torque of frequency capture, which is determined by a_c. In this case,
Table 2
The parameters of experimental system

<table>
<thead>
<tr>
<th>Contents</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>The mass of vibrating system: ( M ) (kg)</td>
<td>152</td>
</tr>
<tr>
<td>The moment of vibrating system: ( J ) (kg·m²)</td>
<td>17</td>
</tr>
<tr>
<td>The mass of standard eccentric lamp ((r = 0.05 \text{ m})): ( m_0 ) (kg)</td>
<td>4</td>
</tr>
<tr>
<td>The constant of springs in ( x )-direction: ( k_x ) (N/m)</td>
<td>77000</td>
</tr>
<tr>
<td>The constant of springs in ( y )-direction: ( k_y ) (N/m)</td>
<td>77650</td>
</tr>
<tr>
<td>The constant of springs in ( \psi )-direction: ( k_\psi ) (Nm/rad)</td>
<td>3000</td>
</tr>
<tr>
<td>The damping constant in ( x )-directions: ( f_x ) (N/(m/s))</td>
<td>270</td>
</tr>
<tr>
<td>The damping constant in ( y )-directions: ( f_y ) (N/(m/s))</td>
<td>270</td>
</tr>
<tr>
<td>The damping constant in ( \psi )-directions: ( f_\psi ) (N/(m/s))</td>
<td>220</td>
</tr>
</tbody>
</table>

Fig. 6. The angle of the general dynamic symmetry.

Fig. 7. A laboratory model of a vibrating system: (a) Schematic of experimental setup; (b) Vibrating synchronization bedstand.

the motions of the rigid frame excited by the two exciters are the circular motion \((x \text{- and } y \text{-directions})\) and the swing \((\psi \text{-direction})\) about its centroid. The torque of frequency capture resulting from the circular motion of the rigid frame drives the phase difference to approach \( \pi \) and implement the swing of the rigid frame; that from the swing of the rigid frame forces the phase difference to approach 0 and carry out the circular motion of the rigid frame. In the expression of \( a_c \), the terms related to the circular motion of the rigid frame are negative, and that referred to the swing are positive. \( a_c < 0 \) (i.e., \( r_l < \sqrt{2} \)) means that the contribution of the circular motion on the \( \rho_C \) is greater than that of the swing, the vibrating system, thus, selects the swing of the rigid frame, vice versa.

When \( \eta = 1.0, 0^\circ < \beta < 90^\circ \), the torque of frequency capture is determined by \( a_c \) and \( b_\psi \) \((b_\psi \text{ is related to the swing of rigid frame})\), in this case, the system shares the circular motion and the swing of the rigid frame. The phase difference of the two exciters and the motion types of rigid frame are shown in the relevant figures of Fig. 5 and Table 1.

If \( \eta = 1.0, \beta = 90^\circ \), we have \( a_c < 0 \) (as shown in Fig. 4), the vibrating system still selects the swing of the rigid frame.

Under the condition of \( \eta \neq 1.0 \), the torque of frequency capture decreases to \( \eta \) \((0 < \eta < 1)\) times of its original value. Based on the above principle discussed, the torque of frequency capture drives the phase difference to approach the corresponding value shown in Fig. 5. The motion types of the rigid frame are shown in Table 1, here, we do not discuss in more detail.

The above facts demonstrate that the vibrating system has the coupling dynamic characteristic of selecting motion. Under the case that the criterions of the frequency capture and that of the stability of the two exciters are all
Fig. 8. The experimental results of the two non-identical exciters rotating in the same directions ($\eta = 0.5$): (a) Rotational velocities of the two exciters; (b) The phase difference between the two exciters ($2\alpha = \phi_1 - \phi_2$); (c) Acceleration in $x$-direction; (d) Acceleration in $y$-direction; (e) Acceleration in $\psi$-direction.

satisfied, the vibrating system will select one of the following three types of motion: pure circular motion, pure swing, the swing and the circular motion coexistence. The synchronization of the two exciters stems from such coupling dynamic characteristic of selecting motion. It is of great significance to design in engineering. In order to realize the feasible circular motion of the rigid frame, $\beta$ should be zero and the longer $l_0$.

Figure 6 shows the angle of general dynamic symmetry [19], the better the structural symmetry of system, the better the tendency that the phase difference of the two exciters approaches $-\theta_c$. By the comparison of Figs 4 and 5, we can see that $2\alpha_0$ is greatly close to $-\theta_c$ when $\beta = 0$, in other words, the structure of system is complete symmetry. Otherwise, the case is reverse.

5. Experiments

In this section, we address the validity of the simplified model and the theoretical and numerical results of the above sections, by comparing to experimental results for a laboratory model of a vibrating system.

5.1. Experiment illustration

Figure 7(a) shows the setup schematically, three acceleration sensors and two Hall-sensors are used to measure the acceleration of experimental system in $x$-, $y$- and $\psi$-directions and the phases of the two exciting motors, respectively. At the same time, the instantaneous phases of the two exciting motors with synchronous operation are continuously recorded by high-speed camera. Vibrating synchronization bedstand is shown in Fig. 7(b). The two exciting motors (exciters) are installed symmetrically in the main rigid body and rotating in the same directions, as shown in Fig. 7(b).

At first, it is necessary to introduce the measuring method of swinging acceleration, $\ddot{\psi}$: Two acceleration sensors are installed on the centroid position of the vibrating system (one is responsible for measuring acceleration in $x$-direction, the other in $y$-direction); the third acceleration sensor for $y$-direction is installed on the point $A$ in Fig. 1, denoted by $\ddot{y}_A$. According to the vector mechanic analysis method, $\ddot{\psi}$ can be expressed as

$$\ddot{y}_A = \ddot{y} - l\dot{\psi}\cos\beta_1, \quad \ddot{\psi} = (\ddot{y} - \ddot{y}_A)/(l\cos\beta_1) \quad (28)$$
Each exciting motor has two pairs eccentric lumps which are symmetrical distribution on both ends of axis. We can adjust the included angle between two eccentric lumps to accommodate certain excited force.

The parameters of the two exciting motors are identical, model VB-326-W (380 V, 50 Hz, 6-pole, Y-connected, 0.82 A, the rated speed 950 r/min, exciting force 0~3 kN, mass 29 kG, protection grade IP54), \( f_1 = f_2 = 0.002 \), \( \beta = 15^\circ \), \( l_0 = 0.36 \) m, \( \xi_{nx} = 0.07 \), \( \xi_{ny} = 0.07 \), and \( \xi_{n\psi} = 0.07 \), \( r_1 = 1.08 \), \( u_x = 0.97 \), \( u_y = 0.93 \) and \( u_\psi = 0.98 \). The other experimental system parameters are shown in Table 2. The two exciting motors are regulated into operating with 40 Hz by converter.

5.2. Experiment results

Figure 8 shows some experimental results of the two non-exciter rotating in the same directions. Here, the masses of the two exciters are \( m_1 = m_0 = 4 \) kg, \( m_2 = 2 \) kg (\( \eta = 0.5 \)), and their equivalent eccentric radius are same (\( r = 0.053 \) m). During the starting few seconds, the two exciters are supplied with the electric source at the same time, when their angular velocities pass through the resonant region of system, the two exciters excite the resonant acceleration responses in \( x \)-, \( y \)- and \( \psi \)-directions, and angular acceleration of exciter 2 is greater than that of exciter 1 since the moment of the exciter 2 (0.0056 kg·m²) is less than that of the exciter 1 (0.0112 kg·m²), the corresponding phase difference changed periodically, as shown in Fig. 8. Because of the damping effect, the resonant responses resulting from the starting process gradually disappear. With the time being, the two exciters reach the synchronous operation by self-adjusting of \( T_C \), the responses of system in \( x \)-, \( y \)- and \( \psi \)-directions rapidly stabilize, the rotational velocity of synchronization nears 791 r/min and the phase difference in the steady-state nears 145°. In this case, the motion type of the rigid frame is swing and circular motion coexistence, and swing has priority because that \( 2\alpha \) nears \( \pi \). The above facts coincide with the contents in Fig. 3(b) and Table 1. When time reaches 5 s, the power supply of
the motor 2 is cut off, the phase difference decreases from 145° to 132.9° and the synchronous rotational velocity increases to 795.5 r/min, as shown in Figs 8(a) and (b). But the synchronization of the two exciters continues, this is so-called vibratory synchronization transmission [11], during which, the torque of capture, $T_C$ transmits the driving torque from motor 1 to motor 2 to overcome the load torque of motor 2. The accelerations of $x(t)$, $y(t)$ and $\psi(t)$ are shown in the relevant figures in Fig. 8, respectively. As shown in Figs 8(a) and (b) during vibratory synchronization transmission, the stability of its synchronous state is better than that for the case before cutting off, as for the reason, is the absence of a motor’s disturbance, in my opinion.

In the above experiment for $\eta = 0.5$, before exciter 2 is cut off, we recorded continuously phases of the two exciters within two cycles by high-speed camera in the steady-state, which are shown in Fig. 9. Here, high-speed camera shooting frequency is 50/s, by Fig. 9, the phase difference of the two exciters is $2\alpha = \varphi_1 - \varphi_2 \approx 143° \sim 146°$, which is roughly the same as that of in Figs 8(b) and 5(b) by comparing. The two exciters, hence, can operate stably and synchronously.

Figure 10 is the experimental results for $\eta = 1.0$ and $\eta = 0.8$. In Fig. 10(a), $\eta = 1.0$, the synchronous phase difference of the two exciters is $2\alpha = 152°$, at 50 s, the exciter 2 is cut off, $2\alpha$ increases from $152°$ to 164.3°; In Fig. 10(b), $\eta = 0.8$ the synchronous phase difference is $2\alpha = 160°$, at 50 s, the exciter 2 is cut off, $2\alpha$ increases from 160° to 173.7°. Compared the results in Fig. 10 with that in Figs 5(c) and (d), $2\alpha$ has the error of 2°~5°, why? Personally I think, although the models of the two motors are completely identical, electromagnetic torques of the two motors are not complete equality in practice, i.e., $T_{e01} - f_1 \omega_{n0} - (T_{e02} - f_2 \omega_{n0}) \neq 0$ in Eq. (13). Here, the motion type of the vibrating system in Figs 10(a) and (b) is still swing and elliptical motion coexistence.

6. Conclusions

By the theoretical investigation and experiment, the following remarks should be stressed:

With the introduction of the average method of modified small parameters, the frequency capture equation of the vibrating system is deduced. The criterion of implementing synchronization for the two exciters is derived, and that of stability of synchronous state satisfies Routh-Hurwitz criterion. These criterions can be used to evaluate and discriminate whether a self-synchronous machine used in industries is able to achieve vibratory synchronization or not, as well as to supervise the design of a self-synchronous vibrating machine has the capacity of achieving vibratory synchronization.

The regions of implementing frequency capture are presented by numeric method, as well as corresponding stabilized regions of phase difference of the two exciters.

The coupling dynamic characteristic that the vibrating system has selecting motion is discussed. Especially, in light of the case that the parameters of system are complete symmetry, the torque of frequency capture resulting from the circular motion of the rigid frame drives the phase difference to approach $\pi$ and implement the swing of the rigid frame; that from the swing of the rigid frame forces the phase difference to near 0 and achieve the
circular motion of the rigid frame. When the parameters of system satisfy the criterion of stability in the regions of frequency capture, and the torque of frequency capture resulting from the circular motion is prior to that from the swing motion, the rigid frame embodies mainly the swing motion, otherwise, the circular motion. The motion of the rigid frame will be one of the following three types: pure swing, pure circular motion, swing and elliptical one coexistence. The synchronization of the two exciters stems from such dynamic characteristic of selecting motion. The corresponding motion type of the vibrating system can be achieved to meet the requirements in engineering by adjusting its structural parameters $r_l$, $\eta$ and $\beta$. In engineering, $\beta$ should near zero and $l_0$ as far as possible.

By the comparison of theory, numeric and experiment results, the feasibility of theory method is proved.

Acknowledgment

This work is supported by the National Natural Science Foundations of China (No. 51075063 and No. 50975045).

References

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