Improving performance of cantilevered momentum wheel assemblies by soft suspension support

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Abstract. This paper focuses on improving the performance of the rigid support cantilevered momentum wheel assemblies (CMWA) by soft suspension support. A CMWA, supported by two angular contact ball bearings, was modeled as a Jeffcott rotor. The support stiffness, before and after in series with a linear soft suspension support, were simplified as two Duffing's type springs respectively. The result shows that the rigid support CMWA produces large disturbance force at the resonance speed range. The soft suspension CMWA can effectively reduce the force on the bearing (also disturbance forces produced by the CMWA) at high rotational speed, and also reduce the nonlinear characteristic of the stiffness. However, the instability of the soft suspension CMWA will limit the maximum rotational speed of the CMWA. Thus, a “proper” stiffness of the soft suspension system is a trade-off strategy between reduction of the force and extension of the speed range simultaneously.

Keywords: Cantilevered momentum wheel assemblies, ball bearing, soft suspension support, in series, instability, imbalance

Nomenclature

CMWA:	Cantilevered momentum wheel assemblies
CoM:	Centre of mass
c_n :	Nonrotational damping
c_r :	Rotational damping
$F(r)$:	Force-displacement relationship expression
F_a :	Axial preload
F_r :	Radial force
F_{sim} :	Fundamental harmonic force obtained by simulation
F_{input} :	Force input by mass imbalance
F_{test} :	Fundamental harmonic force obtained by experimental result
f_r :	Radial translation natural frequency
k :	Stiffness coefficient of Duffing's spring
k_0 :	Stiffness coefficient of the bearings
k_b :	Fitted stiffness coefficient of the rigid support CMWA
k_{bs} :	Fitted stiffness coefficient of the soft suspension CMWA

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k_r :	Stiffness coefficient of the ball bearings when the rotor is still
k_s :	Stiffness coefficient of the linear support
i :	Imaginary unit
m :	Mass of the rotor
$oxyz$:	Inertial ordinate
$o\epsilon\eta z$:	Rotational ordinate
R_+ :	Radius obtained by nonlinear model
R_∇ :	Radius obtained by liner model
r :	Displacement of the axis centre
r_0 :	Radius of circular whirling
t :	Time variable
U_{st} :	Mass imbalance
x_0, y_0, x'_0, y'_0 :	Initial condition of simulation
α :	Contact angular
ζ :	Damping radio
δ_0 :	Virtual radial displacement caused by the preload force
μ :	Nonlinear stiffness term of Duffing's spring
μ_b :	Fitted nonlinear stiffness term of rigid support CMWA
μ_{bs} :	Fitted nonlinear stiffness term of soft suspension CMWA
Ω :	Rotational speed
Φ :	Phase angle

1. Introduction

The microvibrations produced by reaction/momentum wheel assemblies (R/MWA) onboard spacecraft, for example reaction/momentum wheel assemblies (R/MWA), cryo-coolers, thrusters, solar array drive mechanisms, and mobile mirrors, can degrade the performance of instruments with high pointing precision and stability [1–4]. These disturbances are attracting more and more attentions, because they are significant in the 1k Hz frequency range [5] and excite flexible modes of the spacecraft, which cannot be controlled or reduced by the attitude and orbit control systems.

Reaction Wheel Assemblies (RWA) and Momentum Wheel Assemblies (MWA) are widely used in high precision and stability spacecrafts to provide attitude control or maintain stability. Generally, RWA and MWA are considered to be the largest micro-vibration source onboard spacecrafts [6–10]. Masterson et al. have conducted substantial researches on RWA in the area of empirical and analytical disturbance models, and have pointed out that mass imbalances are the most significant source of the disturbances [8]. Furthermore, Elias and Miller have proposed a coupled RWA disturbance analysis method using dynamic mass measurement techniques [9], and Shigemune et al. have established a method to measure low frequency disturbances induced by R/MWA [10], which are traditionally difficult to detect, and revealed that the internal dynamics of the R/MWA closely relate to the disturbances.

One of the most common disturbances from the RWA and MWA is residual imbalance [8], which causes disturbance force and torque, with amplitude proportional to the square of the rotational speed. Hence, the MWA, which operates at relatively higher rotational speed and produces greater disturbances than RWA.

Angular contact ball bearings are widely used in MWA. Although many researchers investigate the performance of the MWA by a linear model [7–9], all the bearings present a more or less pronounced nonlinear behaviour at high rotational speed [11,12]. The vibrations emitted by the CMWA are relative small at low rotational speed, because almost all MWAs are well balanced before they are in use. However, the disturbance forces increase abruptly when the rotational speed is closed to the critical speed, which cause damage of the ball bearing and failure of the CMWA. Thus, the nonlinear stiffness characteristic of the ball bearing should be considered at high rotational speed because the amplitude of vibration increases fast with the rotational speed.

A great many vibration isolation systems have been developed to isolate the disturbances produced by RWA and MWA [13–15]. However, the extra vibration isolation systems will add unfavourable mass and structure to the

Table 1
Parameters of the CMWA

Parameter	m	f_r	U_{st}	ζ	c_n	c_r
Value	0.76 kg	224.2 Hz	0.64 g cm	0.025	$2\zeta\sqrt{k_r m}$	$0.2c_n$

Fig. 1. The “rigid” support and “soft” suspension CMWAs.

spacecraft. An alternative design for RWA/MWA is to replace the “rigid” support by a “soft” suspension support in the wheel assemblies, which operates as a vibration isolator and filters the higher-frequency disturbances [16].

The soft suspension supports are widely used in high rotational speed rotor systems, such as squirrel cage rotor, domestic washing machine, and so on. However, there are few publications investigate the soft suspension support in the MWA, except Zhou et al. [16], who have built up the mathematical model of soft suspension support cantilevered momentum wheel assemblies (CMWA) and tested corresponding microvibrations. However, Zhou et al. [16] have built up the equations of motion by making a linear assumption, ignoring the stiffness of the ball bearing and its nonlinearity. In this paper, we investigated the stiffness of the angular contact ball bearing and its nonlinearity, and then analyzed the performance of the CMWA with a soft suspension support in series.

The paper proceeds as follows: Section 2 analyzes the stiffness of the angular ball bearing rigid support and the soft suspension support CMWA, and presents the motion of equations. Section 3 represents and discusses the displacement of the rotating axis centre, the force acted on the bearing, and the stability of the system before and after in series with the soft suspension support. Section 4 gives the conclusions.

2. Models and method

2.1. Description of the CMWA

Figure 1(a) depicts the basic configuration of the rigid support CMWA. The flywheel is driven by the motor through its shaft. The shaft is fixed in the middle of two angular contact ball bearings, and two ball bearings are fixed on the base by rigid supports. Figure 1(b) shows the configuration of the soft suspension CMWA. The rigid supports are replaced by soft suspension ones.

The data in [16] is employed to analyse the characteristics of rigid and soft suspension CMWA. Table 1 gives the parameters of the CMWA used in this paper. The radial translation frequencies of the rigid and soft suspension CMWA are 224.2 Hz and 52.1 Hz respectively, as reported in [16].

Note that the static imbalance ($U_{st} = 0.64$ g cm) is fitted by the method of least minimum square using the experimental data at rotational speed range from 300 rev/min to 5000 rev/min, thus it is different from the one in [16], which is calculated by the data at the rotational speed of 1500 rev/min. As the damping coefficient is

Fig. 2. Force analysis for the bearing.

Fig. 3. Sketch of equivalent support effect.

difficult to obtained, it is chosen as a random value, 0.025 in this paper. Although the CMWA is not symmetrically supported in the axial in [16], the CMWA is assumed to be symmetrically supported in this paper.

2.2. Ball bearing stiffness analysis

To simplify the problem, we assume the contact angle of the bearing is constant during rotation. Using Hertzian elastic contact deformation theory, force-displacement characteristic of the ball bearing can be obtained as follows:

$$F(r) = -k_0 |r|^{\frac{1}{2}} \cdot r \quad (1)$$

The stiffness coefficient of the bearings is calculated using the method in [17]. In this paper, it is determined to be $k_0 = 2.0 \times 10^8$ N/m.

Generally, the angular contact ball bearing in the CMWA is preloaded by axial force to increase the stiffness and remove the clearance, as shown in Fig. 2. The axial preload F_a introduces a radial force F_r simultaneously. Assuming the contact angle is constant ($\alpha = 45^\circ$), the relationship of the axial preload and the radial preload can be written as follows:

$$F_r = \frac{F_a}{\tan \alpha} \quad (2)$$

Considering a preload is acted in the axial, the force-displacement characteristic of the ball bearing can be obtained as follows:

$$F(r) = -k_0 \begin{cases} \left[(|r| + \delta_0)^{\frac{3}{2}} - (\delta_0 - |r|)^{\frac{3}{2}} \right] \cdot \frac{r}{|r|} & (|r| < \delta_0) \\ (|r| + \delta_0)^{\frac{3}{2}} \cdot \frac{r}{|r|} & (|r| \geq \delta_0) \end{cases} \quad (3)$$

where $\delta_0 = \left[\frac{F_a}{k_0} \right]^{\frac{2}{3}}$ is the virtual radial displacement caused by the preload force.

The stiffness coefficient of the ball bearings when the rotor is still can be assumed to be:

$$k_r = \left. \frac{dF(r)}{dr} \right|_{r=0} \quad (4)$$

Although it is unable to define the natural frequency in nonlinear system, the radial translation natural frequency of the CMWA can be approximately assumed to be:

$$f_r = \sqrt{\frac{k_r}{m}} \quad (5)$$

From Eqs (3)–(5), the radial stiffness of the CMWA can be changed by acting different preload; thus the radial natural frequency of the CMWA is determined by the preload.

Using method of least minimum square, the $F(r)$ can be approximated by a cubic polynomial as the same type of Duffing's equation as follows:

$$F(r) = -k_b(1 + \mu_b |r|^2)r \quad (6)$$

where k_b is the fitted stiffness coefficient of the bearing, μ_b is the fitted nonlinear stiffness term. $\mu_b > 0$ represents a “hardening” spring, $\mu_b < 0$ represents a “soft” one, and $\mu_b = 0$ represents a “linear” one.

Comparing Eqs (3)–(5) with Eq. (6), it can be inferred that when $r \rightarrow 0$, $k_b \rightarrow k_r$. Thus, the radial translation natural frequency (also it is assumed as the critical speed) can be approximated as follows:

$$f_r \approx \sqrt{\frac{k_b}{m}} \quad (7)$$

2.3. Soft suspension CMWA

The rotational speed range of the CWMA may increase by decreasing the stiffness of the support, to make the CMWA operate at the super critical speed. Figure 3 shows a linear spring suspension support in series with the ball bearings, with the assumption that the stiffness of the ball bearing in the CMWA can be fitted by a Duffing's type spring.

As shown in Fig. 3(b), the relationship between force and displacement after in series can be obtained as follows:

$$\begin{aligned} F &= -k_s \Delta r_s \\ F &= -k_b(1 + \mu_b \Delta r_b^2) \Delta r_b \end{aligned} \quad (8)$$

Hence, the relationship between displacement of the soft suspension support and the bearing can be written as follows:

$$\Delta r_s = \frac{k_s}{k_b} (1 + \mu_b |\Delta r_b|^2) \Delta r_b \quad (9)$$

The relationship between the force $F(r)$ and the displacement $r = (\Delta r_s + \Delta r_b)$ is also assumed as follows:

$$F(r) = -k_{bs}(1 + \mu_{bs} |r|^2)r \quad (10)$$

Note that the coefficients k_{bs} and μ_{bs} are also fitted by the method of least minimum square in this paper.

2.4. Equations of motion

The main parts of the CMWA can be divided into three parts: the rotor (including the flywheel, the shaft, and the motor), the support system, and the base. Assuming the CoM (centre of mass) of the rotor is designed to be at the centre of the motor, the CMWA is modelled as a Jeffcot rotor model in the inertial coordinate $oxyz$ and the rotating coordinate $o\epsilon\eta z$ respectively, which are as shown in Fig. 4.

In the complex coordinate, assuming that $r = x + yi$, the motion of the CMWA can be obtained as follows [11]:

$$m\ddot{r} + (c_n + c_r)\dot{r} + F(r) - i\Omega c_r r = U_{st}\Omega^2 e^{i\Omega t} \quad (11)$$

One of the closed-form solutions of Eq. (11) is the circular whirling, which can be written as:

$$r = r_0 e^{i(\Omega t + \Phi)} \quad (12)$$

where $r_0 = \sqrt{x_0^2 + y_0^2}$, is the radius of the circular whirling, $\Phi \in [-\pi, 0]$ is the corresponding phase.

Fig. 4. Jeffcott rotor model.

Fig. 5. Force-displacement relationship under different preloads, (a) $F_a = 0$ N, $\mu_b = 3.08 \times 10^8$, $k_b = 8.3 \times 10^5$ N/m, (b) $F_a = 5$ N, $\mu_b = 8.03 \times 10^7$, $k_b = 1.5 \times 10^6$ N/m, (c) $F_a = 15$ N, $\mu_b = -1.82 \times 10^7$, $k_b = 2.3 \times 10^6$ N/m, (d) $F_a = 25$ N, $\mu_b = -4.85 \times 10^7$, $k_b = 2.9 \times 10^6$ N/m.

Assume that $F(r)$ is approximated by a cubic polynomial as the same type of well-known Duffing's equation [11]:

$$F(r) = -k(1 + \mu|r_0|^2)r_0 \quad (13)$$

where k is the stiffness coefficient, μ is the nonlinear term which represents the characteristic of the nonlinearity of the stiffness.

Fig. 6. Force comparison obtained by simulation and experiment.

The equation of the radius can be obtained as follows:

$$\begin{aligned} [-m\Omega^2 + k(1 + \mu r_0^2)] r_0 &= U_{st}\Omega^2 \cos \Phi \\ c_n\Omega r_0 &= -U_{st}\Omega^2 \sin \Phi \end{aligned} \quad (14)$$

Solving Eq. (14), we can obtain the radius of the circular whirling r_0 . Furthermore, the bearing force, which is equal to the disturbance force, can be calculated by Eq. (13).

The equations of the imbalance response, backbone, limit envelope, and stability can be found in [11], thus they are omitted in this paper.

3. Result and discussion

3.1. Preload

The preload not only affects the stiffness coefficient of the CMWA, but also changes the nonlinear characteristic. Figures 5(a)–(d) shows the relationships between force and displacement under different preload values. The stiffness coefficient of the ball bearings k_b increases with the increasing preloaded force. While the nonlinear term μ_b decreases as the preloaded force increases at first; and when $\mu_b = 0$, the “hard” spring changes to a “soft” one; subsequently, the absolute value of μ_b increases with the increase of the preload, and the nonlinear characteristic of the spring becomes “softening”.

From Eqs (3)–(5), the stiffness coefficient of the CMWA is determined by the preload, and so is the natural frequency of the CMWA. When the natural frequency of the CMWA is 224.2 Hz, the preload can be calculated approximately to be 5 N.

3.2. Imbalance response

To verify the nonlinear characteristic of the rigid support CMWA, the simulation results of the imbalance response is compared with the experimental data in [16]. Further more, it is extended to a broad speed range, to investigate the dynamic characteristic of the CMWA at virtual rotational speed, at which may cause damage to the bearing.

Figure 6 shows the force comparison obtained by simulation and experiment. Where $F_{input} = U_{st}\Omega^2$, F_{test} is the fundamental harmonic force obtained by experimental test in [16], and F_{sim} is the force calculated by simulation. It is found that F_{test} is consistent with the parabola at low rotational speed; however, it increases fast when the rotational speed is above 6500 rev/min. The maximum rotational speed of the rigid support CMWA is set to be 7800 rev/min, because it may cause damage to the bearing if the CMWA is speeded up further. Figure 6 indicates the force obtained

Fig. 7. Imbalance response before and after in series.

by simulation F_{sim} has the same trend with the tested force F_{test} ; thus, it is used to analyze the characteristic at higher rotational speed.

Extending the rotational speed to 30000 rev/min, Figs 7(a) and (b) show the radius and force response of the rigid support CMWA calculated by simulation. Figure 7(a) shows that the amplitude of the circular radius increases with the rotational speed slowly at first and then rises rapidly when the rotational speed approaches to $\sqrt{\frac{k_r}{m}}$. If the rotational speed is even larger than point B, the amplitude of radius jumps down and reaches self-centre, and then decreases with the rotational speed. Note that three solutions are generated when the rotational speed is in the range of point A and point B due to the nonlinearity of the stiffness. Two of them are stable solutions, the other one is unstable, which means the rotational speed cannot maintain at that radius.

Figure 7(c) shows the results of the radius for the CMWA with a soft suspension support in series. R_+ is obtained by the nonlinear model, while R_{∇} is obtained by the linear model (setting $\mu = 0$). The amplitude of the circular radius increases with the rotational speed and reaches the maximum point at first, and then, decreases and maintains at a very low value. However, it reaches the unstable point when the rotational speed reaches 18100 rev/min. The locus of the axis centre, when the rotational speed is in the unstable range, will be analysed in Section 3.3. R_+ and R_{∇} are nearly coincided with each other, which verifies that the nonlinear characteristic almost can be ignored for the soft suspension CMWA.

Fig. 8. Locus of axis centre of the rigid support CMWA.

Fig. 9. Locus of axis centre of the soft suspension CMWA.

Table 2
Performance of CMWA with different soft suspension support in series

Coefficient of elasticity k_s (N/m)	8×10^4	4×10^4	1.0×10^4
Stiffness coefficient of system k_{bs} (N/m)	7.59×10^4	3.90×10^4	9.93×10^3
Nonlinear term μ_{bs}	2.07×10^5	5.46×10^4	3.55×10^3
Resonance rotational speed (rev/min)	3100	2300	1200
Maximum rotational speed (rev/min)	18100	13000	6700
Force output at supercritical speed (N)	0.667	0.334	0.084

Figures 7(b) and (d) show the input force caused by mass imbalance, and output forces (also the acted force at the bearing) before and after in series respectively. Figure 7(b) shows that the output force of the rigid support CMWA increases at an exponential rate when the rotational speed is approaching to 13400 rev/min, which is assumed to be the critical speed in this paper. Although the input force is relatively low, the bearing force is multiple greater and may damage the ball bearing. Hence, in general, the rigid support CMWA should operate at the speed range that much lower than the critical speed. Figure 7(d) shows that the output force of the soft suspension CMWA performs similar as a one degree of freedom system, it reaches the maximum force at 3100 rev/min, and then decreases and maintains at a relatively low value, which is much lower than the increasing input force.

3.3. Locus of the axis centre

Considering $x_0 = y_0 = x'_0 = y'_0$ initially, Figures 8 and 9 show the locus of the axis centre of the CMWA before and after in series with the soft suspension support respectively. Figure 8 shows that the stable rotating radius increases with the rotational speed firstly; then, there are two stable radius when the rotational speed is in the range of A and B corresponding to Fig. 7(a), for example 18000 rev/min, which is as shown in Figs 8(g)–(j).

Figure 9 shows the locus of the rotating axis centre of the soft suspension CMWA. The radius of locus increases with the rotational speed at first, and reaches the maximum at 3100 rev/min; then, the rotor keeps self-centre and the radius decreases. However, when the rotational speed is beyond the stable speed, the radius increases fast, and keeps stable in a large range, which is as shown in Figs 8(i) and (j). Actually, the unstable rotational speed can be considered to be the maximum speed because the motor can not provide adequate torque to keep stable in the unstable range, which has been verified by [18]. The maximum rotational speed of soft suspension CMWA can be increased by increasing the nonrotating damping or the stiffness of the soft suspension system, however, they may also increase the disturbance force output at supercritical speed range simultaneously.

3.4. Different stiffness of the soft suspension support

Changing the stiffness coefficients of soft suspension support, Table 2 gives the performance of the different stiffness of the soft suspension CMWA. Table 2 shows that if the stiffness of the soft suspension support is lower, then the nonlinear characteristic is smaller, and the stiffness of the system is closer to that of the soft suspension support. The lower stiffness suspension CMWA produces the smaller disturbance force, but has lower maximum rotational speed. Increasing the stiffness of the soft suspension support improves the maximum rotational speed, but increases the fundamental harmonic disturbance force at supercritical speed range. Therefore, in a practical CMWA, the stiffness of the soft suspension support is determined by the trade-off of reduction of the disturbance force and the extension of the rotational speed simultaneously.

4. Conclusion

In this paper, we analysed the rigid support and soft suspension CMWA respectively. The following can be concluded.

- (1) The nonlinear stiffness characteristic of the angular ball bearings changes from “hardening” to “softening” with the increase of the preload.

- (2) The nonlinear characteristic of the ball bearing stiffness is reduced by in series with a soft suspension system, and the nonlinear character can be further reduced by decreasing the stiffness of the soft suspension system.
- (3) The bearing force at the rigid support CMWA increases fast when the rotational speed approaches to $\sqrt{\frac{k_b}{m}}$, which produces large disturbance force, even may cause damage to the bearing and ruin the CMWA, which limits the maximum rotational speed. The soft suspension CMWA (in series with the ball bearing system) can effectively reduce the force on the bearing at high rotational speed. However, the instability of the soft suspension CMWA system, which is partly determined by the stiffness of the support, limits the maximum rotational speed. Thus, a proper stiffness of the soft suspension support is a trade-off between the reduction of force on the bearing (also disturbance forces produced by the CMWA) and extension of the rotational speed range simultaneously.

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