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Research Article

Influence of Sleepers Shape and Configuration on Track-Train Dynamics

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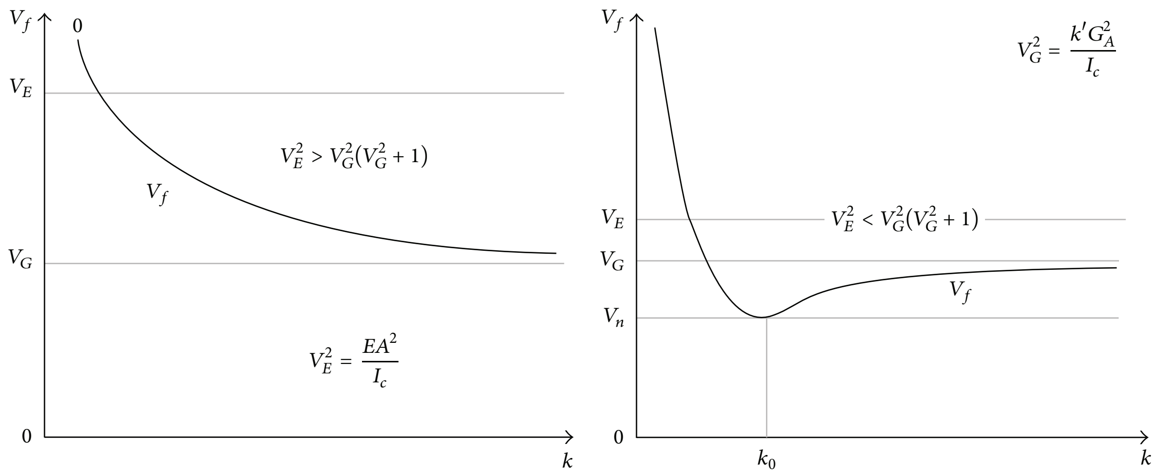
The paper is devoted to the study of dynamical behaviour of railway tracks as continuous systems (rails) supported by periodically spaced sleepers and subjected to moving concentrated loads. Several cases of dynamical problems, where elastically supported beams are excited by a moving concentrated force, are considered. In particular, the study is focused on interactions with structure periodic in the space. Results on one-dimensional structures are extended to the case of a two-dimensional system. The problems of stopping bands, passing bands, and mistuning are also mentioned.

1. Introduction

Nowadays, the load carrying capacity of trains, high-speed, and environment protection against the noise force rapid development of railway transportation. The classic and reinforced railway track is composed of two rails separated from the sleepers by viscoelastic pads. There are numerous simplifications in railway track modelling. The sleeper spacing and ballast stiffness are usually treated as uniform and repre- sented by constant parameters in the analyses. The rails are modelled as the infinite Euler-Bernoulli or Timoshenko beam models, sleepers by lumped masses or elastic bodies (beams), and ballast as viscoelastic foundation. The basic qualitative feature of the classic railway track is the periodicity of sleeper spacing. The sleeper spacing influences the periodicity of viscoelastic supports coefficient and additional mass of sleepers with rotational inertia. In the case of classic periodically supporting sleepers, one can observe passing bands in the frequency of moving and oscillating forces. The solution method which allows determining the stopping and passing bands in the case of tracks, proposed in [1], is based on direct application of Floquet's theorem. The motion of rails and sleepers in selected parts of excitation period T is shown in Figure 2. It is visible that, for the boundary value of frequency between passing and stopping bands, a qualitative change of solution describing rails and sleepers' vibrations occurs. The wheel/rail response, due to the parametric excitation by the varying dynamic stiffness of a periodically supported rail, has been studied using a spatially quasistatic method, based on the fact that the speed of wave propagation in the rail is much greater than the train speed, but as we can see in [1] or [2] this assumption is not adequate. From the study of the influence of random sleeper spacing [3] follows the fact that the phenomenon of the pinned-pinned resonance may be suppressed by the random sleeper spacing. Unfortunately, the random ballast stiffness distribution has no influence on the vibration behaviour. It seems to be obvious that some randomness usually occurs, but the deterministic spacing of supporting points can be the aim of engineering design. The difference between the mutual kinetic excitation of two wheelsets of the bogie in the stopping and passing band is significant. The passing band in track with classic sleepers is related to the rotation of rails in the classic fastening system. Some changes are possible using mistuning or replacing the single fastening system on the sleeper into the double-point



(a) (b)

Figure 1: Phase waves velocities V_f versus wave number dependent on the beam parameters (inequality (3)).

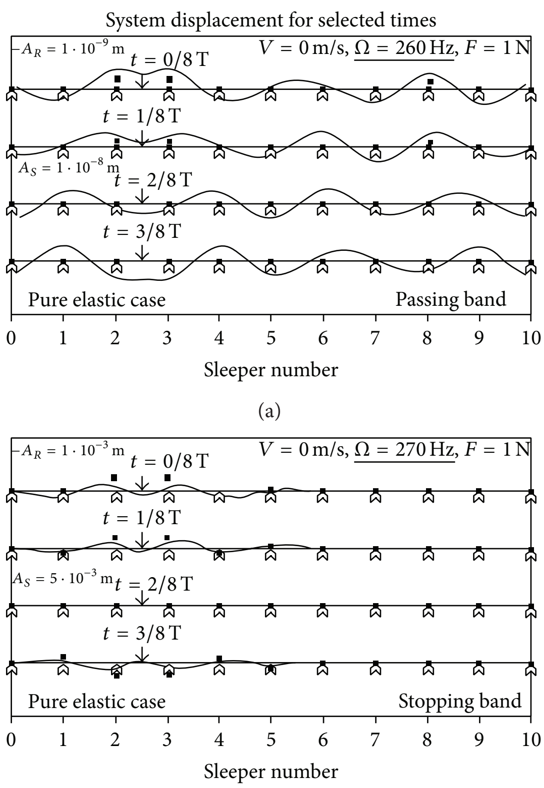


Figure 2: Waves in railway track as periodic structure-passing band and stopping band.

one, which transforms the features of railway tracks. The theory used in the investigation of the track dynamics is limited to linear analysis despite nonlinear characteristics of the pad. It is known that the wheelset dynamics in front of the train is different from the wheelset dynamics in the middle of the train. One of the reasons is connected with the above mentioned nonlinearity of the fastening system characteristics, in particular, the nonlinearity of the pad which changes the reference point of oscillation. This change is connected with a quasistatic preload under the train, which can be substituted by a distributed load [4].

2. Classic Design of Track: Response of Beam to Moving Load

The problem of a flexibly supported beam vibration, when the beam is subjected to the moving distributed load, can be composed of solution for the limiting case of load described by the following Heaviside function $F_0 H(x - Vt)$ and moving concentrated oscillating force described by the function $F_1(\cos \omega t)\delta(x - Vt)$:

$$\begin{aligned} EIw,_{xxxx} + Tw,_{xx,} + mw,_{tt} + hw,_t + cw \\ = F_0 H(x - Vt) + F_1(\cos \omega t)\,\delta(x - Vt), \end{aligned} \tag{1}$$

where w is the beam displacement, EI is the beam stiffness, T is the longitudinal compressive force in the beam, m is the mass density, h is the damping coefficient, c is the elasticity coefficient of the foundation, ω is the frequency of the force oscillation and V is velocity of load motion.

The first case and the case of the beam on a viscoelastic semispace were studied [4, 5]. The superposition of the obtained solution allows studying various kinds of moving loads distributed on a finite-length segment. The second term describing moving and oscillating load was discussed in [2]. The case of the Timoshenko beam on an elastic foundation subjected to uniformly distributed moving loads has been studied by several authors; see, for example, [4, 6]

$$\begin{aligned} EI\varphi,_{xx} + k' AG(w,_{,x} - \varphi) - mI\varphi,_{tt} = 0, \\ k' AG(w,_{xx} - \varphi,_x) - mw,_{tt} - hw,_t - cw \\ = -F_0 H(x - Vt) + F_1(\cos \omega t)\,\delta(x - Vt), \end{aligned} \tag{2}$$

where φ is the angle of rotation of beam due to pure shear, k' is the shear coefficient, G is the modulus of elasticity in shear, A is the cross-sectional area, and h is the damping coefficient.

The first stationary solution obtained for the case of the Timoshenko beam on an elastic foundation was obtained by Achenbach and Sun [6]. The solution obtained in [6] is valid in full range of velocity but only for the set of parameters fulfilling the following inequality:

$$E > k'G\left(1 + k'GA^2(Ic)^{-1}\right). \tag{3}$$

The dependence of phase velocity versus wave number in this case is shown on left side of Figure 1, where wave velocities V_E and V_G are expressed as in Figure 1.

The generalisation of the results obtained by Achenbach and Sun and the discussion of qualitatively different travelling wave solution depending on the beam parameters are presented in [4]. The results of this study can be used for determination of quasistatic preload under the train.

2.1. Response of Periodic Beam Structure to Moving Concentrated Loads. The guideways for high-speed vehicles are composed of repetitive elements or cells which form a periodic structure. The steady-state system response is determined for a moving disturbances source in the form of constant and periodic force (1).

The equation of motion is completed by interface conditions at the supports which depend on the model assumed, for example, for the railway track condition of continuity (4) and equilibrium of vertical forces (5), which are required:

$$\begin{aligned} w(nl+,t) &= w(nl-,t);\\ w,_x(nl+,t) &= w,_x(nl-,t);\\ w,_{xx}(nl+,t) &= w,_{xx}(nl-,t); \end{aligned} \tag{4}$$

$$w,_{xxx}(nl-,t) - w,_{xxx}(nl+,t) = R(nl,t), \tag{5}$$

while for the supports of maglev model, one requires continuity of position, vanishing bending moment, and equilibrium of vertical forces.

The solution method proposed in such a case is based on the direct application of Floquet's theorem to the differential equations of motion with periodic parameters [1, 7] describing periodicity in space. Another approach (by the use of perturbation method) for periodic mass and stiffness distribution along the beam was applied by Popp and Mueller [8] in order to approximate the sleepers in the track. In this case, for the realistic system of parameters, the differences were very small. With some extended study of railway track and maglev track, we can state that the application of Floquet's theorem allows solving the problem of free and forced vibration of periodic structures subjected to moving load [1]. The motion of harmonic travelling load generates the set of stopping or passing bands, but from engineering point of view it is sufficient to take into consideration waves corresponding to the first and second passing bands.

As examples of qualitative difference of solution in the passing band and stopping band in Figure 2 are shown, the shapes of rail for two frequencies and time are $0, T/8, T/4$, and $3T/8$.

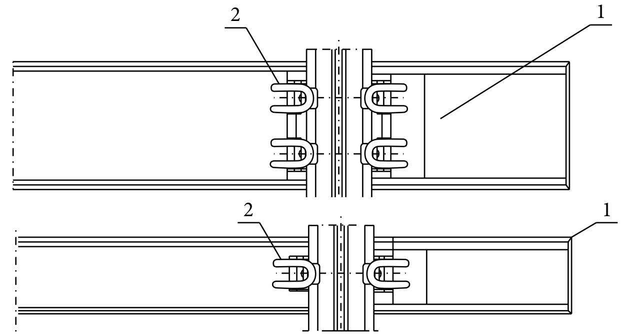


Figure 3: Two kinds of sleepers with different fastening systems.

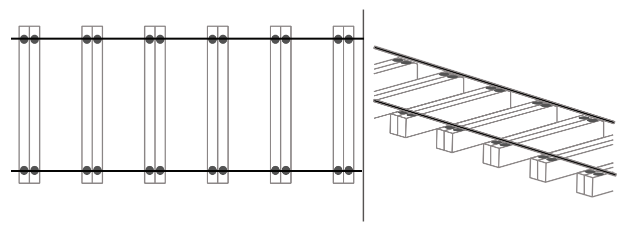


Figure 4: Railway track with stiff type of sleepers with double-point fastening systems.

2.2. Mistuning and Change of the Track Periodicity. The passing bands occurring in the track with classic sleepers in the ballast or in slab track are related to the rotation of rails in the classic fastening system. Some changes are possible using mistuning or change of single fastening system described by the conditions:

$$\begin{aligned} \big(w,_x(nl+,t) &= w,_x(nl-,t);\\ w,_{xx}(nl+,t) &= w,_{xx}(nl-,t)\big). \end{aligned} \tag{6}$$

This is possible by the change of sleepers with single support to the system of sleepers with double-point fastening. It seems to be successful in the change of railway track features. Such sleepers were shown in Figure 3.

The sleeper with double-point fastening system is much stiffer and heavier compared to the classic concrete sleeper (Figure 4). The double-point fastening system influences the periodicity track which becomes also double-periodic. Additionally, the track with such sleepers is much more convenient for the ballast [9], due to lower pressure. The initial experimental investigation of such sleepers and fasteners confirms advantages in application for the high-speed trains' lines. The results will be described in next papers.

A very important problem in railway engineering is connected with the transition zones, when the foundation stiffness changes more rapidly, that is, before or behind of the bridge abutment, than the dynamical behaviour of sleepers and ballast during exploitation which lead to the plastic

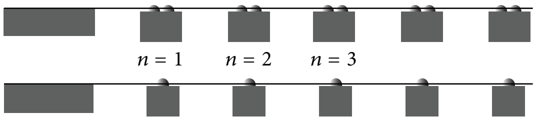


FIGURE 5: Scheme of track in transition zone with two types of sleepers and fastening systems.

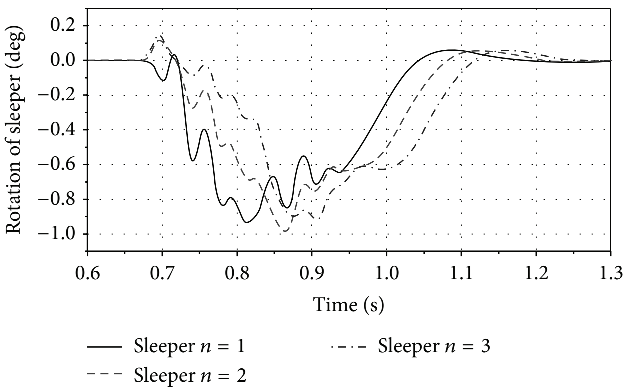


FIGURE 6: Rotation of three sleepers with double-point fastening during motion in the zone behind the bridge abutment.

deformations of the track. A simple model of such zones is shown in Figure 5.

As an example, the results of numerical study of the sleepers' motion in the transition zone behind the bridge and of the stiff sleepers with double-point fastening system are shown in Figures 6 and 7. The speed of the vehicle is assumed equal to 20 m/s. In Figure 6, are shown some results of three successive sleepers' rotation, which are located in the transition zone, just after the rigid foundation. We can see that the response of consecutive sleepers is dependent on the time and distance of the sleeper from the boundary of rigid foundation. The vertical displacement of sleepers is presented in Figure 7.

The above shown results indicate that use of sleepers with double fastening system induces decreasing vertical displacements and rotation of sleepers in comparison with classic track (Figures 6 and 7). In Figure 8, a comparison of the sleeper rotation of two different sleepers with different types of fastening systems and different weights is presented.

The differences of sleepers' rotation (Figure 8) and differences of the rail rotation around the horizontal axis perpendicular to the direction of vehicle motion (Figure 9) for different fastening systems and different sleepers are essential.

The research of railway track dynamics is carried out intensively in several railway centres. However, in engineering practice the attention and understanding of wave phenomena is very limited. There are also much simpler ways to change the features of the track. One of them is mistuning used by the change of geometry which can be supplemented by the change of stiffness parameters. An example of change of the spacing in order to obtain track with better dynamic behaviour is shown in Figure 10. The optimal difference of

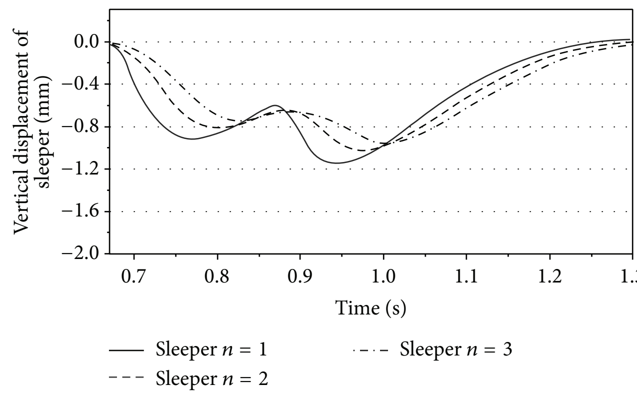


FIGURE 7: Vertical displacement of the sleepers with double-point fastening during motion in the zone behind the rigid base.

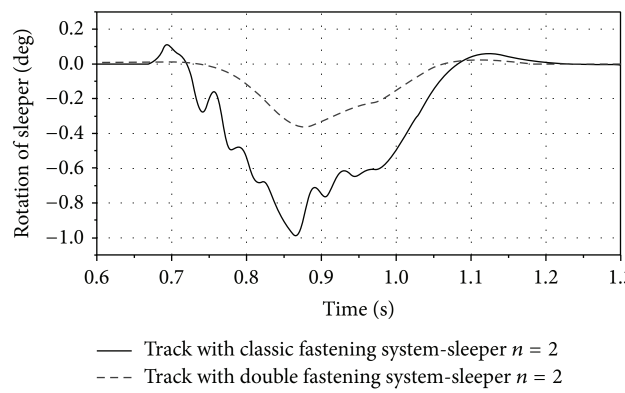


FIGURE 8: Comparison of rotation of two sleepers with different weight and fastening systems—vehicle speed 20 m/s.

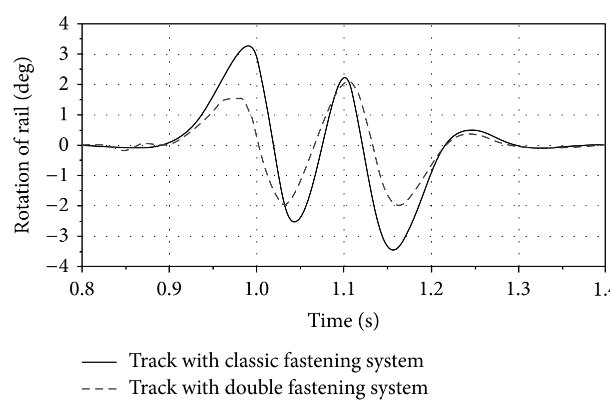


FIGURE 9: Rail rotation in the tracks with different sleepers and fastening systems at the vehicle speed 20 m/s.

distance between sleepers seems to be stochastic in definite range but in engineering practice the spacing can be taken as follows: $d_i = d_1 + \Delta_i$; $\Delta_1 = 0$, $\Delta_2 = 25$ mm, and $\Delta_3 = 50$ mm (Figure 10).

The change of stiffness parameters without change of geometry can influence only the selected modes of travelling waves. For example, the pin-pin mode is not sensitive to

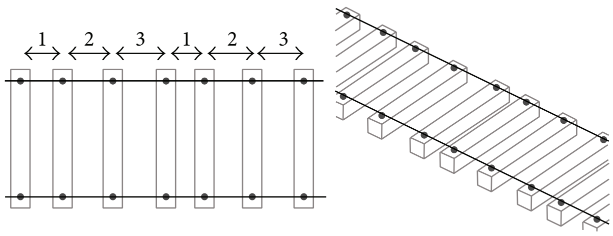


FIGURE 10: Scheme of rack with the change of sleeper spacing, distances $d_i = d_1 + \Delta_i$.

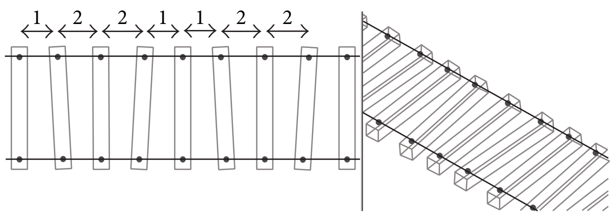


FIGURE 11: Track mistuning by the change of sleeper spacing (rotation of every second sleeper).

the stiffness of supports. That is why the change of spacing ought to be used. The next possibility of change of periodicity is loss of symmetry as the factor that eliminates the regularity and periodicity of the motion. The change of periodic property of the track due to change of geometry of sleeper spacing can be obtained by rotation of every second or third sleeper in the positive or negative direction; such an example is shown in Figure 11.

It is also important that the length of the wheel circumferences is about 3 m (in classic track, it is equal to distance of five sleepers). Also the dynamic coupling between both axles of the bogie is important issue.

It can be done by application of "Y-shaped" sleepers made of steel. This kind of sleepers is made of steel. Such spacing is, similarly to the case shown in Figure 11, asymmetric to the left and right wheels of the wheelset. The dynamic response in this case is strongly dependent on vehicle speed [10].

The main advantages of the track with "Y-shaped" sleepers are

(i) increased resistance to horizontal forces;

(ii) increased inertia by incorporation of the ballast into the vertical and horizontal vibrations;

(iii) higher stiffness of the track becoming a plate-like structure;

(iv) stabilization of the vehicle motion due to the alternate periodic vertical stiffness of both rails—the vehicle geometrical centre exhibits considerably lower oscillations.

The principal idea of using the "Y-shaped" sleepers is to increase the transversal stiffness and to enlarge the inertia of the track by incorporating the ballast into the C-shaped parts of sleepers. Experimental parts of the track exhibit lower

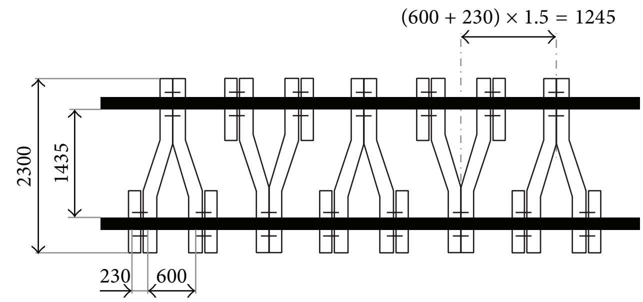


FIGURE 12: Track "Y-shaped" sleepers spacing as visible above.

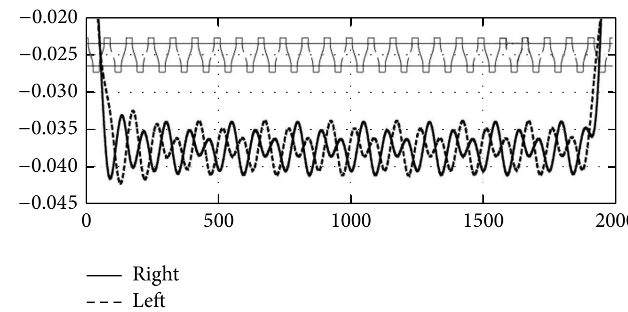


FIGURE 13: Vertical displacements of the vehicle/track contact points at the speed 30 m/s [11, 12].

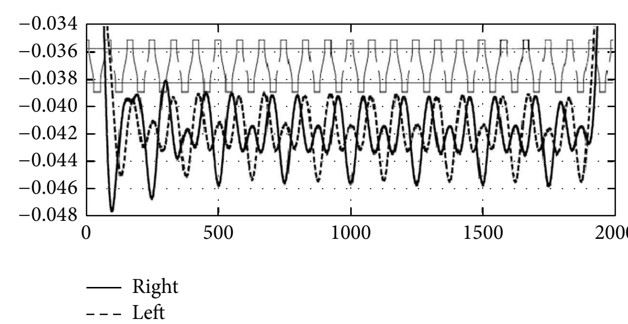


FIGURE 14: Vertical displacements of the vehicle/track contact points at the speed 50 m/s [11, 12].

noise level. Numerical simulations show reduced vertical amplitudes. The Y-type track is designed for moderate speed.

Some results of simulations for selected values of speed are shown in Figures 12, 13, and 14. It is visible that, with increasing speed and time of motion, the effect of synchronization occurs. The next important dynamical feature is associated with decreasing of vibration with the distance between excitation points. Simulations show (Figures 15 and 16) that vertical displacements of rails in the front of the contact points of the buggy are much larger in classic track than in the case of track with "Y" shaped sleepers. The great advantage of the "Y-shaped" track is connected with application on curved track because of its higher resistance to horizontal forces.

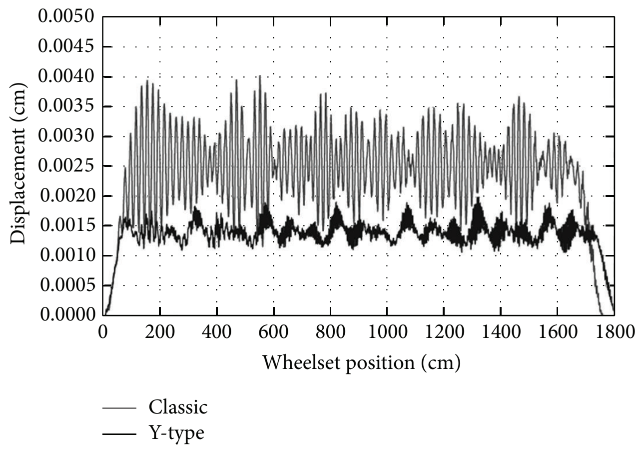


Figure 15: Vertical displacements of rails registered 120 cm in front of the contact points of the buggy for classic and "Y"-type track at the speed 40 m/s [12].

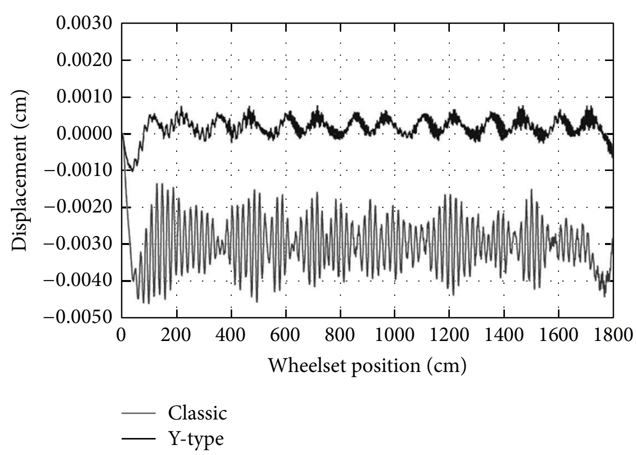


Figure 16: Vertical displacements of rails registered 180 cm in front of the contact points of the buggy for classic and "Y"-type track at the speed 40 m/s [12].

3. Conclusions

The periodicity of sleeper spacing is the basic qualitative feature of the classic railway track [13, 14]. It influences the periodicity of viscoelastic supports, coefficients, and additional mass of sleepers with rotational inertia. In the case of classic periodically supporting sleepers, we can observe the passing bands in the frequency of moving and oscillating forces. The solution method which allows determining the stopping and passing bands in the case of track with periodically spaced sleepers and stationary motion is based on direct application of Floquet's theorem. The difference between the mutual kinetic excitation of two wheelsets of the bogie in the stopping and passing bands is significant. Loss of the periodicity of spacing is connected to a mistuning of wave propagation and irregular contact forces, which can be positive, from the dynamical point of view, process for vehicle-track interaction. In the case of transient motion, as in the case of a transition zone, an advantage of double fasteners is obtained and visibly different sleeper motion dependent on the distance is shown. The vertical displacements of rails registered in front and behind of the contact points of the buggy for classic and "Y"-type track show one of the advantages of the track with "Y"-type sleepers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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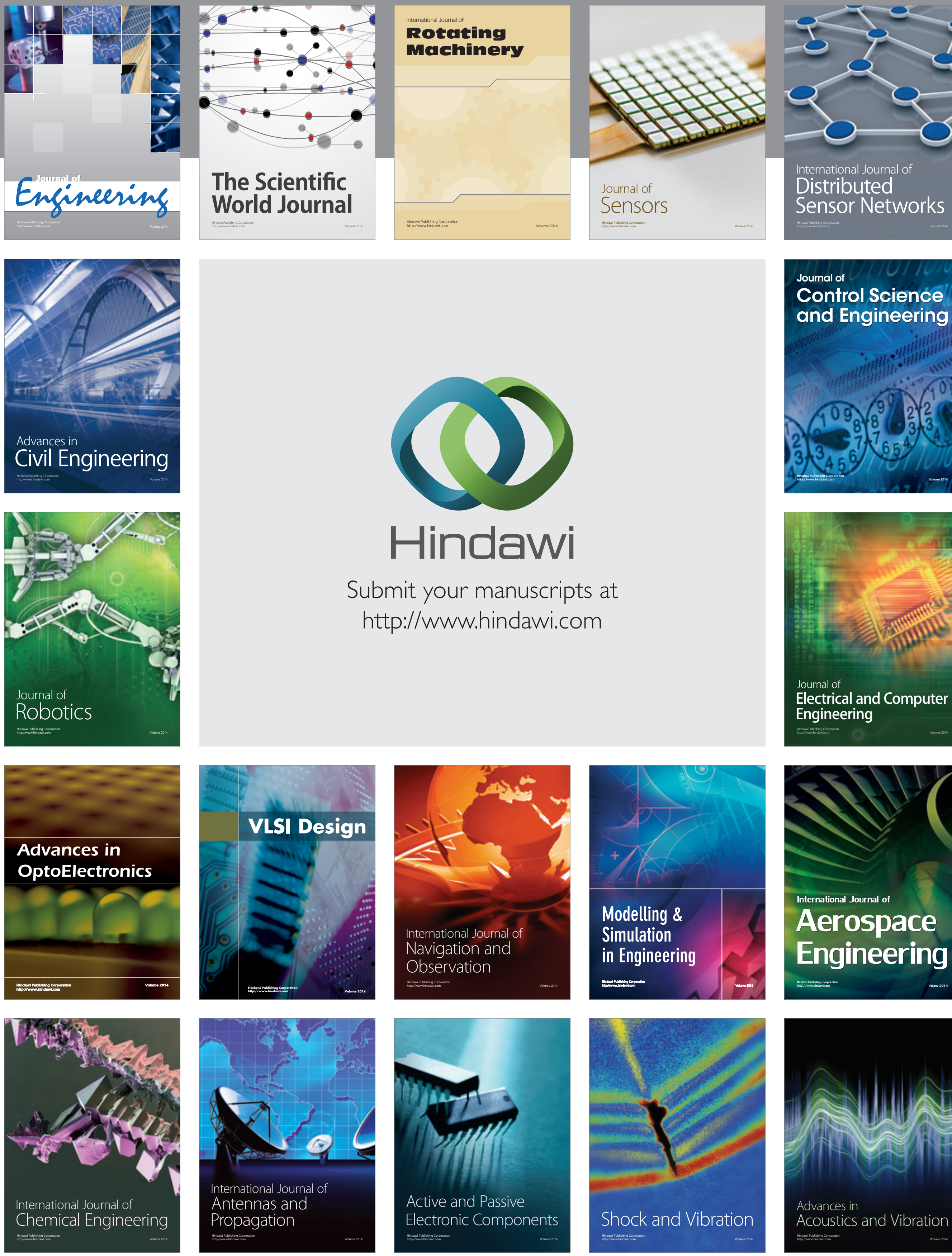
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