Research Article

Probabilistic Neural Network and Fuzzy Cluster Analysis Methods Applied to Impedance-Based SHM for Damage Classification

Lizeth Vargas Palomino, Valder Steffen Jr., and Roberto Mendes Finzi Neto

School of Mechanical Engineering, Federal University of Uberlândia, Campus Santa Mônica, 38400-902 Uberlândia, MG, Brazil

Correspondence should be addressed to Valder Steffen Jr.; vsteffen@mecanica.ufu.br

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Impedance-based structural health monitoring technique is performed by measuring the variation of the electromechanical impedance of the structure caused by the presence of damage. The impedance signals are collected from patches of piezoelectric material bonded on the surface of the structure (or embedded). Through these piezoceramic sensor-actuators, the electromechanical impedance, which is directly related to the mechanical impedance of the structure, is obtained. Based on the variation of the impedance signals, the presence of damage can be detected. A particular damage metric is used to quantify the damage. Distinguishing damage groups from a universe containing different types of damage is a major challenge in structural health monitoring. There are several types of failures that can occur in a given structure, such as cracks, fissures, loss of mechanical components (e.g., rivets), corrosion, and wear. It is important to characterize each type of damage from the impedance signals considered. In the present paper, probabilistic neural network and fuzzy cluster analysis methods are used for identification, localization, and classification of two types of damage, namely, cracks and rivet losses. The results show that probabilistic neural network and fuzzy cluster analysis methods are useful for identification, localization, and classification of these types of damage.

1. Introduction

Failures occurring in industrial equipment and structures in general are associated with friction, fatigue, impact, and crack growth or with other reasons. For an appropriate functioning of the system, the failure should be located and repaired timely. In general terms, the problem of damage monitoring consists in locating and measuring the fault and estimating the remaining life of the system (damage prognosis). One of the most important ambitions of modern engineering is to perform structural health monitoring in real time in structural components of high cost and considerable responsibility. Thus, the creation or improvement of techniques that enhance the accuracy and reliability of the tracking process is highly desirable and is the subject of several studies both in industry and academic environments [1].

There are several techniques for monitoring the occurrence and propagation of structural damage. One of these

techniques is the so-called impedance-based structural health monitoring [2]. The basic idea behind this technique is monitoring the changes in the mechanical impedance of the structure as caused by the presence of damage. As the direct measurement of the mechanical impedance of the structure is a difficult task, the method uses piezoelectric ceramics (PZT patches) bonded to or incorporated into the structure, allowing the measurement of the electromechanical impedance. As this measure is related to the structure variation of the impedance signals, the presence of damage can be detected. A particular damage metric is used to quantify the damage [3].

The impedance-based SHM technique was first proposed by Liang et al. [4] and subsequently the method was extended by Chaudhry et al. [5, 6], Sun et al. [7], Park et al. [8–11], Giurgiutiu and Zagari [12], Soh et al. [13], Bhalla et al. [14], Giurgiutiu et al. [15, 16], Moura Jr. and Steffen Jr. [17], Peairs [18], Moura Jr. [19], and Neto et al. [20].

Distinguishing damage groups from a universe containing different types of damage is a major challenge in structural health monitoring. There are various types of failures, which may occur in a given structure, such as cracks, fissures, loss of joining components (rivets), corrosion, and wear. In the case of composite structures delamination is a major concern. It is important to characterize each type of damage for defining appropriate correction efforts. In order to distinguish the different damage types, probabilistic neural network and fuzzy cluster analysis methods for classification can be used [21, 22].

An artificial neural network is a mathematical model, computational model, or metamodel that mimics the structure by using functional aspects of biological neural networks. It consists of an interconnected group of artificial neurons and processes information by using a connectionist approach for computation [23]. In most cases an artificial neural network is an adaptive system that changes its structure based on external or internal information that flows through the network during the learning phase. Modern neural networks can be understood as nonlinear statistical data modeling tools. They are usually used to model complex relationships between input and output or to find patterns in data [23]. There are several types of artificial neural network; one of them is the probabilistic neural network. This artificial neural network can be used for classification tasks. The network is an implementation of the statistical algorithm called kernel discriminate analysis [24] in which the operations are organized into a multilayered feedforward network with four layers, namely, the input layer, pattern layer, summation layer, and output layer [25].

Fuzzy clustering is an unsupervised learning operation that aims at decomposing a given set of objects into subgroups or clusters based on similarity. The goal is to divide the dataset in such a way that objects or cases belonging to the same cluster are as similar as possible, whereas objects belonging to different clusters are dissimilar [26]. The main potential of clustering is to detect the underlying structure in data, not only for classification and pattern recognition, but also for model reduction and optimization.

In the present paper, the probabilistic neural networks and the fuzzy cluster analysis methods are used for identification, localization, and classification of damage in metallic aeronautic structures. The impedance signal measurement set is used as the input of the probabilistic neural network and the output is the type of damage (crack, rivet loss, or pristine condition). The Gustafson-Kessel fuzzy clustering algorithm was also implemented. The impedance signal measurement set is used as the object to be classified by the fuzzy cluster analysis algorithm and the results represent the type of damage. The results show that the methods are useful for identification, localization, and classification of damage.

1.1. Probabilistic Neural Network. Artificial neural networks are parallel distributed systems composed of simple processing elements (neurons) that calculate given mathematical functions (usually nonlinear). Such units are arranged in one or more layers and interconnected by a large number of connections, usually unidirectional. In most models, these

connections are associated with weights, which store the knowledge represented in the model and consider the input received by each neuron in the network. The operation of these networks is inspired by a physical structure designed by nature, the human brain [27]. There are different types of neural networks; the probabilistic neural network is one of them [25].

The probabilistic neural network is predominantly a classifier. It is based on the probability distribution function, and is an implementation of a statistical algorithm known as kernel discriminating analysis [24], in which the operations are organized into a multilayered feedforward network with four layers, namely, the input layer, pattern layer, summation layer, and output layer. The architecture for this system is shown in Figure 1.

When a sample X is presented, the input layer distributes this sample to the pattern layer neurons (second layer). The function described in the following equation is calculated for each j -neurons of the i -class in the pattern layer:

$$ft_{i,j}(X) = \frac{1}{(2\pi)^{d/2} \sigma_i^d} \exp \left[\frac{-(X - W_{i,j})^T (X - W_{i,j})}{2\sigma_i^2} \right], \quad (1)$$

where $ft_{i,j}(X)$ is the contribution of the j -neuron in the i -class; σ is the transfer function and $W_{i,j}$ is the weight of the j -neuron of the i -class. In each i -neuron of the summation layer the contribution of each neuron of the pattern layer that belongs to the i -class is added. In the output layer, the sample X is associated with the class with the highest probability [25].

The training process consists in a unique step, that is, the weight of each pattern layer neuron is formed by the characteristic vector of each training sample [25].

1.2. Fuzzy Cluster Analysis Method. In clustering analysis the sampled points (or the population) are divided into a quantity of defined groups by using the similarities between these members. In many fields of knowledge, these clustering techniques have been used to distinguish groups by their features [28]. The clustering analyses can be divided into two subclassifications, the hierarchical and nonhierarchical clustering techniques [28]. Both methods considered in the present contribution are nonhierarchical techniques.

The nonhierarchical techniques find directly the N elements of the k clusters or groups in such a way that these partitions follow two criteria, namely, the similarity (or internal cohesion) and separation of the formed groups [28]. The Gustafson-Kessel algorithm is based on the behavior of the objective function. The basic idea considers an objective or evaluation function that assigns to each possible cluster partition a quality or error value that has to be optimized. The optimal solution is the cluster partition that obtains the best evaluation. In this sense, an optimization problem is to be solved when cluster analysis is performed [29]. The corresponding objective function is given by

$$J = \sum_{i=1}^k \sum_{j=1}^N (u_{ij})^m d^2(x_j, v_i), \quad (2)$$

FIGURE 1: Probabilistic neural network architecture.

FIGURE 2: Aluminum aircraft panel equipped with eight PZT patches.

TABLE 1: States of the aircraft panel.

Number	State	Description	Measurements number
1	Baseline	The panel with all rivets	1–200
2	Damage 1	The panel without one of the rivets (Figure 2(c))	201–400
1	Baseline	The panel with all rivets	401–600
3	Damage 2	The panel with all rivets and localized corrosion	601–800

FIGURE 3: Initial degree of pertinence for aircraft panel.

TABLE 2: Probabilistic neural network for damage classification.

Layer	Number of neurons		
Input	190		
Pattern	570		
Summation	3		
Output	1		
	Baseline	Damage 1	Damage 2
Training set	360	180	180
Test set	40	20	20

FIGURE 4: Final degree of pertinence for the aircraft panel.

where v_i is the center of the cluster i , x_j is the data j , $d(x_j, v_i)$ is the distance between x_j and cluster center v_i , m is the fuzzy parameter, and u_{ij} is the probability of the element x_j to pertain to the cluster i . The objective function constraints are presented in the following:

$$0 \leq u_{ij} \leq 1, \quad (3)$$

$$\sum_{i=1}^k u_{ij} = 1, \quad \forall j = 1, \dots, N. \quad (4)$$

Mahalanobis distance used in the Gustafson-Kessel (GK) algorithm and the corresponding formulation is presented in the equation below. This technique provides greater flexibility to adapt to the shape and dimensions of each cluster but has higher computational complexity [28]. Consider the following:

$$d^2(x_j, v_i) = (x_j - v_i)^T P_i (x_j - v_i), \quad (5)$$

FIGURE 5: Aluminum aircraft window containing ten PZT patches.

where P_i are the fuzzy covariance matrices that are obtained from

$$P_i = \frac{\sum_{j=1}^N u_{ij}^m (x_j - v_i)(x_j - v_i)^T}{\sum_{j=1}^N u_{ij}^m}. \quad (6)$$

The process consists in minimizing the objective function, (4), and the results obtained are the cluster centers v and the pertinence matrix u .

2. Case Study Number 1: Aluminum Aircraft Panel

The first test presented in this work corresponds to an 80×80 cm aircraft panel, as shown in Figure 2(a). The structure was tested by using eight PZT patches to capture the impedance signals. A first type of damage was simulated by removing a rivet located close to PZT3, as shown in Figure 2(c). After the measurements have been made for this state, the rivet was reattached at its former position. Then, to simulate a corrosion type of damage, hydrochloric acid was spread in the vicinity of the rivet. A localized corrosion area was obtained (see Figure 2(c)).

A description of each state of the structure is presented in Table 1. Two hundred (200) measurements were taken for each state. Every measured signal contains 200 points.

To classify damage, the impedance signals measured in the panel were used as inputs of the neural networks. Eight probabilistic networks (one for each PZT) were implemented to analyze the structure. All these networks were built with the same architecture, since they were all intended to the same purpose (classifying the damage in the panel). The descriptions of the networks together with their training sets are presented in Table 2. The results obtained with the test set for each of the eight probabilistic neural networks are presented in Table 3. The error percentages found in damage classification for PZT1, PZT2, and PZT6 were greater than 48%, which means that they did not detect the damage. The PZT3 and PZT4 showed error percentages greater than 10%, although it should be noted that the damage 1 (loss of the rivet) was perfectly detected by both of these patches (PZT3 and PZT4). PZT3 and PZT4 were dedicated to this specific damage. The PZT5, PZT7, and PZT8 had error percentages smaller than 4%, similar to the results obtained for simpler structures (beam and plate) [30]. One can thus conclude that these three PZT patches succeeded to properly detect the types of damage that were inserted into the structure.

The initial degrees of pertinence for the Gustafson-Kessel algorithm are shown in Figure 3. After several iterations (Table 4) the algorithm was interrupted by the convergence of the process for every PZT patch that was used. One may then observe the final degree of pertinence as shown in Figure 4. The results of PZT1, PZT2, and PZT6 confirm again that these

TABLE 3: Classification of test set of probabilistic neural networks for each PZT patch in the aircraft panel.

		Baseline	Damage 1	Damage 2	Error%
PZT1	✓	23	10	5	52,5%
	✗	17	10	15	
Total		40	20	20	
PZT2	✓	20	9	8	55%
	✗	20	11	12	
Total		40	20	20	
PZT3	✓	35	20	16	11,25%
	✗	5	0	4	
Total		40	20	20	
PZT4	✓	32	20	14	17,5%
	✗	8	0	6	
Total		40	20	20	
PZT5	✓	40	19	18	3,75%
	✗	0	1	2	
Total		40	20	20	
PZT6	✓	40	20	18	48,75%
	✗	19	8	12	
Total		40	20	20	
PZT7	✓	40	20	19	1,25%
	✗	0	0	1	
Total		40	20	20	
PZT8	✓	40	20	18	2,5%
	✗	0	0	2	
Total		40	20	20	

TABLE 4: Optimization results of the Gustafson-Kessel algorithm for the aircraft panel.

	Iteration	Initial objective function value	Final objective function value
PZT1	139	3173,262	446,395
PZT2	102	831,342	118,823
PZT3	67	532244,526	5803,566
PZT4	47	602945,975	2788,759
PZT5	18	186,055	95,991
PZT6	131	3644,445	1594,554
PZT7	13	1041994,032	23013,747
PZT8	25	1452335,476	10885,113

PZTs failed to detect damage and thus made the classification impossible. The PZT3 and PZT4 correctly identified the damage 1; nevertheless the damage 2 was impossible to be distinguished from the Baseline. Finally, the PZT5, PZT7, and PZT8 managed to correctly classify the two types of damage with a degree of pertinence greater than 80%.

3. Case Study Number 2: Aluminum Aircraft Window

A second aircraft structure was used to test the artificial intelligence techniques in structural health monitoring for damage classification. For this aim a window located in an aluminum aircraft structure, as illustrated in Figure 5(a),

was used. Due to the size and complexity of the structure, ten PZT patches were considered in the experiment. This number of PZT patches was arbitrary since no preliminary study was performed to optimize the test configuration. Since the beginning of the tests, the PZT10 showed poor stability and repeatability and has therefore been ignored in the test. To simulate two different types of damage in the structure, two experiments were performed as follows. First, a weight was added to the structure as shown in Figure 5(b). Second, after the mentioned weight was removed, one of the clamps (located close to the PZT2) was removed (Figure 5(c)). For every state of the structure 200 measurements were made as shown in Table 5. For every measurement 200 points were taken.

TABLE 5: States of the aircraft window.

Number	State	Description	Measurements number
1	Baseline	Window with all the clamps	1–200
2	Damage 1	Window with all the clamps and the weight	201–400
1	Baseline	Window with all the clamps	401–600
3	Damage 2	Window with one clamp missing near PZT2 (Figure 5(c))	601–800

TABLE 6: Classification of test set of probabilistic neural networks for each PZT patch of the aircraft window.

		Baseline	Damage 1	Damage 2	Error%
PZT1	✓	21	8	12	48,75%
	✗	19	12	8	
Total		40	20	20	
PZT2	✓	36	16	20	10%
	✗	4	4	0	
Total		40	20	20	
PZT3	✓	20	3	7	62,5%
	✗	20	17	13	
Total		40	20	20	
PZT4	✓	19	7	13	51,25%
	✗	21	13	7	
Total		40	20	20	
PZT5	✓	21	8	13	47,5%
	✗	19	12	7	
Total		40	20	20	
PZT6	✓	17	9	9	56,25%
	✗	23	11	11	
Total		40	20	20	
PZT7	✓	40	19	20	1,25%
	✗	0	1	0	
Total		40	20	20	
PZT8	✓	33	15	20	18,75%
	✗	7	8	0	
Total		40	20	20	
PZT9	✓	33	15	20	11,25%
	✗	7	5	0	
Total		40	20	20	

TABLE 7: Optimization results of the Gustafson-Kessel algorithm for the aircraft window.

	Iteration	Initial objective function value	Final objective function value
PZT1	130	107992,545	13922,783
PZT2	44	1951882,844	3723,708
PZT3	112	11418,915	1011,319
PZT4	131	14063,77	1106,733
PZT5	105	73130,147	3914,543
PZT6	83	187017,9	8330,349
PZT7	9	209498,097	1474,566
PZT8	101	71861,057	7189,752
PZT9	40	149831,047	652,42

FIGURE 6: Initial degrees of pertinence for the aircraft window.

In this case, nine probabilistic networks were implemented (one for each PZT) to analyze this structure as shown in Table 2. The results obtained with the test set for each one of the nine probabilistic neural networks are presented in Table 6.

The damage misclassification percentages of PZT1, PZT3, PZT4, PZT5, and PZT6 were greater than 48%. These PZT patches are not meant to detect damage. The types of damage (structural modifications in this case) were inserted in the back panel and the PZT patches were bonded to the reinforcing beams. The PZT2, while installed in a reinforcement

beam, was able to detect the damage 2 without errors; this success is due to the fact that this sensor is close to the clamp position. The PZT8 and PZT9 detected only the clamp removal without errors, with an overall error percentage of less than 20%. Finally, the PZT7, which was bonded directly onto the panel and close to the removed clamp, was able to identify all states with an error percentage of 1.25%.

The initial degrees of pertinence for the Gustafson-Kessel algorithm are shown in Figure 6. After several iterations (Table 7), the algorithm was interrupted by the convergence of the process for each of the PZT patches. The final degrees

FIGURE 7: Final degrees of pertinence for the aircraft window.

of pertinence are shown in Figure 7. The results of the PZT1, PZT3, PZT4, PZT5, and PZT6 confirm once again that these PZT patches failed to detect damage, which made the classification impossible due to the position of the patches on the structure. The PZT2, PZT8, and PZT9 correctly classified damage 2, with a degree of pertinence of 99%. However, the damage 1 was impossible to discriminate from the baseline. Finally, for the PZT7, the Gustafson-Kessel algorithm was able to correctly classify all measurements with a degree of pertinence greater than 95%.

4. Conclusion

The probabilistic neural network and fuzzy cluster analysis methods were applied to real-world structures in the context of impedance-based structural health monitoring for damage detection, localization, and classification purposes in metallic aeronautic structures. Impedance signal responses were used as the input of the probabilistic neural network. The output was the type of damage (crack, rivet loss, or pristine condition). The Gustafson-Kessel fuzzy clustering algorithm was

also implemented. The results demonstrated the efficiency of these techniques in accomplishing these tasks. It should be pointed out that the tests were performed at constant room temperature (approximately 20°C). The PZT patches that presented the largest error percentages for both techniques used were the ones that did not succeed to detect damage due to their inappropriate location along the structure [30]. This means that the location of the PZT patches is a major concern in impedance-based structural health monitoring. Further studies will focus on temperature compensation regarding its influence on the damage classification approach. Also, the authors have designed a compact network signal analyzer for electromechanical impedance measurements, which includes post-processing computation for damage metrics calculation and temperature compensation, aiming at on board/online structural health monitoring.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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