Research Article

Analysis of Vertical-Horizontal Coupling Vibration Characteristics of Rolling Mill Rolls Based on Strip Dynamic Deformation Process

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Nonlinear dynamic rolling forces in the vertical and horizontal directions are, respectively, established, considering the impact of vertical and horizontal directions vibration of rolls. Then a vertical-horizontal coupling nonlinear vibration dynamic model of rolling mill rolls is proposed, based on the interactions between this dynamic rolling force and mill structure. The amplitude-frequency equations of the main resonance and inner resonance are carried out by using multiple-scale method. The characteristics of amplitude frequency under nonlinear stiffness, damping, and amplitude of the disturbance are obtained by adopting the actual parameters of 1780 rolling mills. Finally, the bifurcation behavior of the system is studied, and it is found that many dynamic behaviors such as period, period-3 motion, and chaos exist in rolling mill, and this behavior could be restrained effectively by choosing proper system parameters.

1. Introduction

The vibration of rolling mill often occurs in rolling process. The occurrence of vibration not only affects the quality of rolling products, but also leads to breakdown of the rolling equipment. In order to understand the vibration behaviors of mills, a number of models have been developed during the past few decades [1–5]. However, in most open literatures, the vibrations in the horizontal direction and in the vertical direction are studied separately. In recent years, the coupling relationship in rolling mill is proposed; Hu et al. studied the linear vibration characteristics in vertical and horizontal direction [6]. Yang et al. studied the stability of coupling dynamic vertical model of cold rolling mill, which consists of the rolling process model, the mill roll stand structure model, and the hydraulic servo system model [7]. In the process of studying rolling mill vibration, modeling of rolling force directly determines the accuracy of vibration model. In the early years, the rolling force in the rolling process is quasistatic, which assumes that only dynamic variations in roll spacing

produce variations in force, strip speed, and strip thickness to those occurring under steady-state conditions [8–10]. Yun et al. proposed a dynamic model of rolling mill, which considers the rate variation of change of the roll spacing. But in order to simplify calculation, he only selected the linear section of rolling force by using Taylor formula [11]. In fact, most of the literatures adopted the method of Yun, by taking the rolling force as a linear factor and neglecting nonlinear section.

In this paper, nonlinear dynamic rolling forces in the vertical and horizontal directions are proposed, respectively. Then a vertical-horizontal coupling vibration dynamic model of rolling mill rolls is constructed based on the interactions between this dynamic nonlinear rolling force and mill structure. Then the amplitude-frequency characteristics of the main resonance and inner resonance are analyzed under the nonlinear stiffness, damping, and the amplitude of the disturbance stiffness. Finally, the conditions of different dynamical motions are obtained by analyzing bifurcation behavior of the system, which could provide theoretical base for understanding of vibration mechanism of mill.

Then the speed v_x at any arbitrary position can be expressed as

$$v_x = \frac{v'_1 h_1 - \Delta \dot{V}}{h_x}. \quad (11)$$

When the horizontal component of surface speed of roll is equal to (11); namely,

$$v_x = \frac{v_R}{R} \sqrt{R^2 - (x - x_c)^2}, \quad (12)$$

at the moment, the position x is the position of neutral point x_n in (12). Due to $R \gg (x - x_c)$ and $|x - x_c| \ll 1$ and neglecting high-order item $(x - x_c)^2$, the neutral point can be derived as

$$x_n = x_1 - \frac{(h_1 - h_2) \sqrt{(4R^2 - 2)(C^2 - 1)}}{8R^2 - C^2}, \quad (13)$$

where

$$C = v_R h_2 - v'_1 h_1 + \frac{1}{2} \dot{x}_1 (h_1 + h_2) + x_1 \dot{y}_c. \quad (14)$$

2.2. Nonlinear Dynamic Rolling Force in Horizontal and Vertical Direction. According to slab method [10], take a slice of the strip in Figure 2.

Based on force balance theory in deformation zone by Von Karman [10], the force balance expression in horizontal direction can be expressed as

$$\frac{dh_x}{dx} (P + \tau_x) + h_x \frac{d\tau_x}{dx} \pm 2F_\mu = 0, \quad (15)$$

where $F_\mu = \mu\delta$; when $x < x_n$, take the negative and when $x > x_n$, take the positive.

By taking account of the assumption of homogeneous deformation [11], the distribution of stress can be written by von Mises yield criterion as follows:

$$\frac{d\tau_x}{dx} = \frac{2\delta}{h_x} \left(\mp \mu - \frac{2(x - x_c)}{R} \right). \quad (16)$$

Integrating (16), the tension τ_x can be express as

$$\tau_x = \tau_b + \int_{x_1}^x \frac{2\delta}{h_x} \left(\mp \mu - \frac{2(x - x_c)}{R} \right) dx. \quad (17)$$

The unit rolling force by (17) can be expressed as

$$P(x) = 2\delta - \left[\tau_b + \int_{x_1}^x \frac{2\delta}{h_x} \left(\mp \mu - \frac{2(x - x_c)}{R} \right) dx \right]. \quad (18)$$

The rolling force in horizontal and vertical direction can be obtained as follows:

$$\begin{aligned} F_x &= - \int_{x_2}^{x_1} P(x) \tan \theta dx + \int_{x_2}^{x_1} \mp \mu \delta dx, \\ F_y &= \int_{x_2}^{x_1} P(x) dx + \int_{x_2}^{x_1} \mp \mu \delta \tan \theta dx, \end{aligned} \quad (19)$$

where

$$\tan \theta = \frac{x - x_c}{\sqrt{R^2 - (x - x_c)^2}}. \quad (20)$$

In (19), integrating zone is composed of two sections, namely, $x_2 \sim x_n$ and $x_n \sim x_1$. Integrating (19), we obtained

$$\begin{aligned} F_x &= -\mu\delta h_2 \sqrt{\frac{R}{h_0 + 2y_c}} \left[2 \tan^{-1} \left(\frac{x_n - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right. \\ &\quad \left. - \tan^{-1} \left(\frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right] \\ &\quad + \frac{\tau_b}{2} (h_1 - h_2) - \delta h_2 \ln \frac{h_1}{h_2}, \\ F_y &= \left[2\delta \ln \left(\frac{h_1}{h_2} \right) - 2\delta - \tau_b \right] (x_1 - x_c) \\ &\quad + 4\delta \sqrt{Rh_0 + 2Ry_c} \tan^{-1} \left(\frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \\ &\quad + 2\mu\delta \sqrt{\frac{R}{h_0 + 2y_c}} (x_1 - x_c) \\ &\quad \times \left[2 \tan^{-1} \left(\frac{x_n - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) - \tan^{-1} \left(\frac{x_1 - x_c}{\sqrt{Rh_0 + 2Ry_c}} \right) \right] \\ &\quad + \mu\delta R \ln \left(\frac{h_1 h_2}{h_n^2} \right). \end{aligned} \quad (21)$$

3. The Vertical-Horizontal Coupling Dynamic Equation of Mill Rolls

Set x_{c0} and y_{c0} which are the balance points of rolls in the horizontal and vertical direction, respectively. Under steady conditions, $\dot{x}_{c0} = 0$ and $\dot{y}_{c0} = 0$; by using Taylor formula, (21) can be expressed as

$$\begin{aligned} F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) &= F_x(x_{c0}, 0, y_{c0}, 0) + \Delta F_x(x_c, \dot{x}_c, y_c, \dot{y}_c), \\ F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) &= F_y(x_{c0}, 0, y_{c0}, 0) + \Delta F_y(x_c, \dot{x}_c, y_c, \dot{y}_c), \end{aligned} \quad (22)$$

where $F(x_{c0}, 0, y_{c0}, 0)$ is rolling force when there is no vibration and $\Delta F(x_c, \dot{x}_c, y_c, \dot{y}_c)$ is dynamic parts of rolling force, because too many parameters, for simplify the calculation, take the parts of first and third order as follows:

$$\begin{aligned} \Delta F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) &= a_1 x_c + a_2 \dot{x}_c + a_3 y_c + a_4 \dot{y}_c + a_5 x_c^3 + a_6 y_c^3, \\ \Delta F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) &= b_1 x_c + b_2 \dot{x}_c + b_3 y_c + b_4 \dot{y}_c + b_5 x_c^3 + b_6 y_c^3, \end{aligned} \quad (23)$$

FIGURE 2: The pressure diagram of strip.

where

$$\begin{aligned}
 a_1 &= \frac{\partial}{\partial x_c} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_1 &= \frac{\partial}{\partial x_c} F_y(x_{c0}, 0, y_{c0}, 0), \\
 a_2 &= \frac{\partial}{\partial \dot{x}_c} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_2 &= \frac{\partial}{\partial \dot{x}_c} F_y(x_{c0}, 0, y_{c0}, 0), \\
 a_3 &= \frac{\partial}{\partial y_c} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_3 &= \frac{\partial}{\partial y_c} F_y(x_{c0}, 0, y_{c0}, 0), \\
 a_4 &= \frac{\partial}{\partial \dot{y}_c} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_4 &= \frac{\partial}{\partial \dot{y}_c} F_y(x_{c0}, 0, y_{c0}, 0), \\
 a_5 &= \frac{1}{6} \frac{\partial^3}{\partial x_c^3} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_5 &= \frac{1}{6} \frac{\partial^3}{\partial x_c^3} F_y(x_{c0}, 0, y_{c0}, 0), \\
 a_6 &= \frac{1}{6} \frac{\partial^3}{\partial y_c^3} F_x(x_{c0}, 0, y_{c0}, 0), \\
 b_6 &= \frac{1}{6} \frac{\partial^3}{\partial y_c^3} F_y(x_{c0}, 0, y_{c0}, 0).
 \end{aligned} \tag{24}$$

Based on the assumption that the mass of working rolls are much smaller than that of backup rolls, the mass of the

working rolls may be neglected [6]. The vertical-horizontal coupling nonlinear vibration model of rolling mill with nonlinear dynamic rolling force is illustrated in Figure 3.

The dynamic equation in Figure 3 can be written as

$$\begin{aligned}
 m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 (x_c + x_{c0}) + F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) &= 0, \\
 m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 (y_c + y_{c0}) + F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) &= T_1, \\
 m_2 \ddot{x}_c + c_3 \dot{x}_c + k_4 (x_c + x_{c0}) + F_x(x_c, \dot{x}_c, y_c, \dot{y}_c) &= 0, \\
 -m_2 \ddot{y}_c - c_4 \dot{y}_c - k_4 (y_c + y_{c0}) + F_y(x_c, \dot{x}_c, y_c, \dot{y}_c) &= T_2.
 \end{aligned} \tag{25}$$

Assuming that the structure of rolling mill and vibration are symmetrical in relation to the rolled strip [6], then there exist $k_1 = k_3, k_2 = k_4, c_1 = c_3, c_2 = c_4, m_1 = m_2$, and $T_1 = -T_2$, and (25) can be simplified as

$$\begin{aligned}
 m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 (x_c + x_{c0}) + F_x(x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) &= 0, \\
 m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 (y_c + y_{c0}) + F_y(x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) &= T_1.
 \end{aligned} \tag{26}$$

Under steady conditions, the external disturbance force $T_1 = 0$, there exist $\ddot{x}_c = \ddot{y}_c = 0, \dot{x}_c = \dot{y}_c = 0$, and $x_c = y_c = 0$, and the balance equation can be obtained as follows:

$$\begin{aligned}
 k_1 x_{c0} + F_x(x_{c0}, 0, 2y_{c0}, 0) &= 0, \\
 k_2 y_{c0} + F_y(x_{c0}, 0, 2y_{c0}, 0) &= 0.
 \end{aligned} \tag{27}$$

Substituting (27) into (26), (26) can be expressed as

$$\begin{aligned}
 m_1 \ddot{x}_c + c_1 \dot{x}_c + k_1 x_c + \Delta F_x(x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) &= 0, \\
 m_1 \ddot{y}_c + c_2 \dot{y}_c + k_2 y_c + \Delta F_y(x_c, \dot{x}_c, 2y_c, 2\dot{y}_c) &= T_1.
 \end{aligned} \tag{28}$$

FIGURE 3: The vertical-horizontal coupling dynamic model of rolling mill rolls.

Substitute (23) and (27) into (28), and set

$$\begin{aligned}
 \omega_1^2 &= \frac{(k_1 + a_1)}{m_1}, & \alpha_1 &= \frac{(c_1 + a_2)}{m_1}, \\
 \beta_1 &= \frac{a_3}{m_1}, & \gamma_1 &= \frac{a_4}{m_1}, & \eta_1 &= \frac{8a_5}{m_1}, \\
 \varsigma_1 &= \frac{8a_6}{m_1}, & \omega_2^2 &= \frac{(k_2 + b_3)}{m_1}, \\
 \alpha_2 &= \frac{(c_2 + b_4)}{m_1}, & \beta_2 &= \frac{b_1}{m_1}, & \gamma_2 &= \frac{b_2}{m_1}, \\
 \eta_2 &= \frac{8b_5}{m_1}, & \varsigma_2 &= \frac{8b_6}{m_1}, & T &= \frac{T_1}{m_1}.
 \end{aligned} \quad (29)$$

Equation (28) can be rewritten as

$$\begin{aligned}
 \ddot{x}_c + \omega_1^2 x_c + \alpha_1 \dot{x}_c + \beta_1 y_c + \gamma_1 \dot{y}_c + \eta_1 x_c^3 + \varsigma_1 y_c^3 &= 0, \\
 \ddot{y}_c + \omega_2^2 y_c + \alpha_2 \dot{y}_c + \beta_2 x_c + \gamma_2 \dot{x}_c + \eta_2 x_c^3 + \varsigma_2 y_c^3 &= T.
 \end{aligned} \quad (30)$$

Equation (28) is vertical-horizontal coupling dynamic equation of mill rolls under vibration conditions, where β , γ , ς are coupling coefficients.

4. The Resonance Characteristics of Equation

Assuming that the external disturbance $T = \varepsilon F \cos \omega t$ and the system is a weak nonlinear system, (30) can be expressed as

$$\begin{aligned}
 \ddot{x}_c + \omega_1^2 x_c + \alpha_1 \dot{x}_c + \beta_1 y_c + \gamma_1 \dot{y}_c &= -\varepsilon (\eta_1 x_c^3 + \varsigma_1 y_c^3), \\
 \ddot{y}_c + \omega_2^2 y_c + \alpha_2 \dot{y}_c + \beta_2 x_c + \gamma_2 \dot{x}_c &= -\varepsilon (\eta_2 x_c^3 + \varsigma_2 y_c^3 + F \cos \omega t).
 \end{aligned} \quad (31)$$

By using multiple scales method, one has

$$\begin{aligned}
 T_n &= \varepsilon^n t, \quad n = 0, 1, \dots, \\
 \frac{d}{dt} &= D_0 + \varepsilon D_1,
 \end{aligned} \quad (32)$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 (D_1^2 + 2D_0 D_1) + \dots,$$

where D_n is defined as $\partial/\partial T_n$ and $n = 0, 1$.

Set (31) which has solution as follows:

$$\begin{aligned}
 x_c &= x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + \dots, \\
 y_c &= y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1) + \dots.
 \end{aligned} \quad (33)$$

Substituting (32) and (33) into (31) and separating terms of each order of ε , one has

$$\begin{aligned}
 D_0^2 x_0 + \omega_1^2 x_0 &= 0, \\
 D_0^2 y_0 + \omega_2^2 y_0 &= 0,
 \end{aligned} \quad (34)$$

$$\begin{aligned}
 D_0^2 x_1 + \omega_1^2 x_1 &= -2D_0 D_1 x_0 - \alpha_1 D_0 x_0 \\
 &\quad - \beta_1 y_0 - \gamma_1 D_0 y_0 - \eta_1 x_0^3 - \varsigma_1 y_0^3, \\
 D_0^2 y_1 + \omega_2^2 y_1 &= -2D_0 D_1 y_0 - \alpha_2 D_0 y_0 \\
 &\quad - \beta_2 x_0 - \gamma_2 D_0 x_0 - \eta_2 x_0^3 - \varsigma_2 y_0^3 - F \cos \omega t.
 \end{aligned} \quad (35)$$

Set the solution of (34) as

$$x_0 = A(T_1) e^{i\omega_1 T_0} + cc, \quad y_0 = B(T_1) e^{i\omega_2 T_0} + cc, \quad (36)$$

where cc represents complex conjugate of former term and $A(T_1)$ and $B(T_1)$ are undetermined complex function.

Substituting (36) into (35), the following equation can be expressed as

$$\begin{aligned}
 D_0^2 x_1 + \omega_1^2 x_1 &= (-2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \bar{A}) e^{i\omega_1 T_0} \\
 &\quad - (3\varsigma_1 B^2 \bar{B} + \beta_1 B + i\omega_2 \gamma_1 B) e^{i\omega_2 T_0} \\
 &\quad - \eta_1 A^3 e^{3i\omega_1 T_0} - \varsigma_1 B^3 e^{3i\omega_2 T_0} + cc, \\
 D_0^2 y_1 + \omega_2^2 y_1 &= (-2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\varsigma_2 B^2 \bar{B}) e^{i\omega_2 T_0} \\
 &\quad - (\beta_2 A + i\omega_1 \gamma_2 A + 3\eta_2 A^2 \bar{A}) e^{i\omega_1 T_0} \\
 &\quad - \varsigma_2 B^3 e^{3i\omega_2 T_0} - \eta_2 A^3 e^{3i\omega_1 T_0} + \frac{F e^{i\omega T_0}}{2} + cc.
 \end{aligned} \quad (37)$$

4.1. The Analysis of Main Resonance. In the case of main resonance, set $\omega = \omega_2 + \varepsilon\sigma$ and eliminating secular term of (37), one can obtain

$$\begin{aligned} -2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \bar{A} &= 0, \\ -2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\zeta_2 B^2 \bar{B} + \frac{Fe^{i\sigma T_1}}{2} &= 0. \end{aligned} \quad (38)$$

The polar coordinate form of A and B in (36) is introduced as follows:

$$A = \frac{1}{2}a(T_1)e^{i\varphi_1(T_1)}, \quad B = \frac{1}{2}b(T_1)e^{i\varphi_2(T_1)}. \quad (39)$$

By substituting (39) into (38) and separating the real and imaginary parts, the average equation of coupling system can be obtained as follows:

$$\begin{aligned} \dot{a} &= -\frac{1}{2}\alpha_1 a, \\ a\dot{\varphi}_1 &= (\sigma - \sigma_1)a + \frac{3}{8\omega_1}\eta_1 a^3, \\ \dot{b} &= -\frac{1}{2}\alpha_2 b + \frac{F}{2\omega_2} \sin \theta, \\ b\dot{\varphi}_2 &= \sigma b + \frac{3}{8\omega_2}\zeta_2 b^3 - \frac{F}{2\omega_2} \cos \theta, \end{aligned} \quad (40)$$

where $\theta = \sigma T_1 - \varphi_2$. In the steady state, existing $\dot{a} = \dot{b} = 0$ and $\dot{\theta}_2 = 0$ and eliminating θ_2 from (40), then the amplitude frequency response equation of the system can be written as

$$\frac{9}{16}\zeta_2^2 b^6 + 3\omega_2 \sigma \zeta_2 b^4 + \omega_2^2 (\alpha_2^2 + 4\sigma^2) b^2 - F^2 = 0. \quad (41)$$

4.2. The Analysis of Inner Resonance. Assuming that $\omega_1 = \omega_2 + \varepsilon\sigma_1$ and $\omega = \omega_2 + \varepsilon\sigma$, in order to solve the secular term of (37), A and B must meet conditions as follows:

$$\begin{aligned} -2i\omega_1 D_1 A - i\omega_1 \alpha_1 A - 3\eta_1 A^2 \bar{A} \\ - (3\zeta_1 B^2 \bar{B} + \beta_1 B + i\omega_2 \gamma_1 B) e^{-i\sigma_1 T_1} &= 0, \\ -2i\omega_2 D_1 B - i\omega_2 \alpha_2 B - 3\zeta_2 B^2 \bar{B} \\ - (\beta_2 A + i\omega_1 \gamma_2 A + 3\eta_2 A^2 \bar{A}) e^{i\sigma_1 T_1} &= \frac{-Fe^{i\sigma T_1}}{2}. \end{aligned} \quad (42)$$

Substituting (39) into (42), the average equation can be obtained under the polar coordinate; that is,

$$\begin{aligned} \dot{a} &= -\frac{1}{2}\alpha_1 a - \frac{2}{2\omega_1} \left[\left(\frac{3}{4}\zeta_1 b^2 + \beta_1 \right) \sin \theta_1 + \omega_2 \gamma_1 \cos \theta_1 \right], \\ a\dot{\varphi}_1 &= (\sigma - \sigma_1)a + \frac{3}{8\omega_1}\eta_1 a^3 \\ &+ \frac{b}{2\omega_1} \left[\left(\frac{3}{4}\zeta_1 b^2 + \beta_1 \right) \cos \theta_1 - \omega_2 \gamma_1 \sin \theta_1 \right], \end{aligned}$$

$$\begin{aligned} \dot{b} &= -\frac{1}{2}\alpha_2 b + \frac{F_0}{2\omega_2} \sin \theta_2 \\ &+ \frac{a}{2\omega_2} \left[\left(\frac{3}{4}\eta_2 a^2 + \beta_2 \right) \sin \theta_1 - \omega_1 \gamma_2 \cos \theta_1 \right], \\ b\dot{\varphi}_2 &= \sigma b + \frac{3}{8\omega_2}\zeta_2 b^3 - \frac{F}{2\omega_2} \cos \theta_2 \\ &+ \frac{a}{2\omega_2} \left[\left(\frac{3}{4}\eta_2 a^2 + \beta_2 \right) \cos \theta_1 + \omega_1 \gamma_2 \sin \theta_1 \right], \end{aligned} \quad (43)$$

where $\theta_1 = \varphi_2 - \varphi_1 - \sigma_1 T_1$ and $\theta_2 = \sigma T_1 - \varphi_2$.

When the system has a periodic motion, (43) will exist, $\dot{a} = \dot{b} = \dot{\theta}_1 = \dot{\theta}_2 = 0$; eliminate θ_1 and θ_2 from (43) and then the frequency response equation of the coupling system can be obtained as

$$\omega_1^2 \alpha_1^2 a^2 + 4\omega_1^2 G^2 = b^2 M, \quad (44)$$

$$\begin{aligned} \left(\frac{3}{4}\eta_2 a^3 + \beta_2 a \right)^2 + \left(2\omega_2 \sigma b + \frac{3}{4}\zeta_2 b^3 \right)^2 \\ + \omega_1^2 \gamma_2^2 a^2 + \omega_2^2 \alpha_2^2 b^2 + \frac{\omega_1 a}{M} \left(2\omega_2 \sigma + \frac{3}{4}\zeta_2 b^2 \right) \\ \times (2\alpha_1 a Z - GN) - \frac{\omega_1 \omega_2 \alpha_2 a}{M} (2\alpha_1 a N + GZ) = F^2, \end{aligned} \quad (45)$$

where

$$\begin{aligned} M &= -\omega_2^2 \gamma_1^2 + \left(\frac{3}{4}\zeta_1 b^2 + \beta_1 \right)^2, \\ N &= \left(\frac{3}{4}\zeta_1 b^2 + \beta_1 \right) \left(\frac{3}{4}\eta_2 a^2 + \beta_2 \right) - \omega_1 \omega_2 \gamma_1 \gamma_2, \\ Z &= -\omega_2 \gamma_1 \left(\frac{3}{4}\eta_2 a^2 + \beta_2 \right) - \omega_1 \gamma_2 \left(\frac{3}{4}\zeta_1 b^2 + \beta_1 \right), \\ G &= (\sigma - \sigma_1)a + \frac{3}{8\omega_1}\eta_1 a^3. \end{aligned} \quad (46)$$

5. Numerical Experiments

Taking the 1780 rolling mills of Chengde Steel Co as an example, the parameters of this mill are listed as follows:

$h_1 = 0.0141$ m, $h_0 = 0.0082$ m, $m = 14.4$ t, $v_R = 2.5$ m/s, $\mu = 0.26$, $\tau_b = 3.8$ Mpa, $\tau_f = 5.5$ Mpa, $R = 0.42$ m, $\delta = 30$ Mpa, $c_1 = 2 \times 10^5$ N·s/m, $k_1 = 2.01 \times 10^{11}$ N/m, $k_2 = 2.08 \times 10^{11}$ N/m, and $c_2 = 8.65 \times 10^5$ N·s/m.

Parameters of rolling force can be listed as follows: $\varepsilon = 0.01$, $a_1 = 5.8 \times 10^6$ N/m, $a_2 = 1.46 \times 10^8$ N·s/m, $a_3 = 213.6$ N/m, $a_4 = -1131$ N·s/m, $\varepsilon a_5 = 4 \times 10^6$ N/m³, $\varepsilon a_6 = -7.47 \times 10^{11}$ N/m³, $b_1 = 6.978 \times 10^7$ N/m, $b_2 = -2593$ N·s/m, $b_3 = -2.16 \times 10^9$ N/m, $b_4 = 1.4 \times 10^4$ N·s/m, $\varepsilon b_5 = 2.65 \times 10^9$ N/m³, and $\varepsilon b_6 = -3.7 \times 10^{15}$ N/m³.

Figures 4~6 show the curve of main resonance amplitude frequency of rolling mill under different parameters.

In Figure 4, the main resonance amplitude-frequency response curves for several values of nonlinear stiffness b_3 are illustrated by (40). It can be seen that the nonlinear stiffness

FIGURE 4: Main resonance amplitude-frequency curve of different nonlinear stiffness b_3 .

b_3 directly affects the vertical natural frequency of the rolling mill. The natural frequency decreases with the decrease of the stiffness b_3 , and the main resonance amplitude becomes larger and the backbone of rolling mill curve deviates toward right, simultaneously.

In Figure 5, the main resonance amplitude-frequency response curves for different values of parameter c_2 are illustrated by (44). With the increasing of the parameter c_2 , the resonance amplitude of vibration decreases.

Figure 6 shows the curve of the main resonance amplitude frequency in coupling rolling mill system for different nonlinear parameter b_6 . It can be seen that with the decrease of b_6 , the frequency response curve deviates toward right and the jump phenomenon becomes more obvious. When the disturbance frequency σ changes negative to positive, the amplitude changes according to 1-2-3-5 route and jumps in 3-5; when the disturbance frequency σ is changed from positive to negative, the amplitude changes according to 5-4-2 route and jumps in 4-2, which will lead to the jump phenomenon.

In Figure 7, it can be seen that when disturbance amplitude εF is small value, the frequency response curve is single value of system. With the increase of εF , the main resonance amplitude increases gradually, and the resonance point of coupling system shifts right, and a clear jump phenomenon arises.

Figures 8 and 9 show the curves of inner resonance amplitude-frequency under different parameters in horizontal and vertical directions, respectively. It can be seen that the system will raise resonance when disturb frequency is near to or equal to natural frequency ω_1 or ω_2 , and it has two resonance zones in coupling system.

In Figure 8, when $\varepsilon F = 0.3$ MN, the curve in horizontal direction has two resonance points, and the result is unique and stable. With the increase of disturb amplitude εF , when $\varepsilon F = 0.5$ MN, the right resonance zone becomes bend to right, and the jump phenomenon arises in Figure 8(b), and then the system is unstable.

In Figure 9, the change of tendency of curve in vertical direction is similar to the curve in horizontal direction.

FIGURE 5: Main resonance amplitude frequency curve of different structural damping c_2 .

FIGURE 6: Main resonance amplitude frequency curve of different nonlinear stiffness b_6 .

FIGURE 7: Curve of main resonance amplitude frequency with different disturb amplitude εF .

FIGURE 8: Curve of inner resonance amplitude frequency in horizontal direction with disturb amplitude εF .

FIGURE 9: Curve of inner resonance amplitude frequency in vertical direction with disturb amplitude εF .

FIGURE 10: Bifurcation characteristics of coupling system with disturb amplitude εF when $\sigma = 50$ Hz.

FIGURE 11: Periodic motion when $\sigma = 50$ Hz and $\epsilon F = 0.529$ MN. FIGURE 12: Period-3 motion when $\sigma = 50$ Hz and $\epsilon F = 1.06$ MN.

According to (31), Figure 10 shows the bifurcation diagram with the change of disturb parameter ϵF in the condition of $\sigma = 50$ Hz. It can be seen that the rolling mill may have different motions when it adopts different disturb parameter ϵF . When ϵF adopts value from range 0.47 to 0.62, the system becomes periodic motion, and then it becomes chaos motion. When ϵF adopts value from range 0.9 to 0.92, the system becomes period-2 motion. When ϵF adopts value from range 1.07 to 1.24, the system becomes period-3 motion.

The phase diagrams and Poincaré maps are shown in Figures 11~13 when the system adopts different values of ϵF in Figure 10.

Figure 11 is periodic motion when $\epsilon F = 0.529$ MN, and it can be seen that the phase diagram has one closed curve in Figure 11(a) and the Poincaré maps have one single point in

Figure 11(b). Figure 12 shows a period-3 motion when $\epsilon F = 1.06$ MN, and it has three single points in Poincaré map. Figure 13 illustrates chaos motion when $\epsilon F = 0.45$ MN.

6. Conclusions

- (1) The nonlinear rolling force model of rolling mill in the vertical and horizontal directions is built. On this basis, the dynamic model of nonlinear vertical-horizontal coupling vibration model of rolling mill is proposed, considering the influence of mill structure.
- (2) By means of multiple-scale method, the amplitude-frequency equations of main resonance and inner resonance of coupling system of rolling mill rolls are carried out. The simulation adopting the actual

FIGURE 13: Chaotic motion when $\sigma = 50$ Hz and $\epsilon F = 0.45$ MN.

parameters of rolling mill is analyzed. It is found that the amplitude of vibration increases with an increase of stiffness and external disturb; but the maximum value of the main resonance will decrease as the increase of structure damp; when changing nonlinear stiffness, jump phenomenon will arise both in main resonance and in inner resonance, so choosing proper parameter will restrain resonance vibration of rolling mill.

- (3) The bifurcation characteristics of vertical-horizontal coupling system of rolling mill roll are studied, and it is found that the system has different motions such as period motion, period-3 motion, and chaos, and choosing proper parameters may change the motion state of rolling mill.

Nomenclature

x : Arbitrary distance from the centerline of the rolls
 x_1 : Distance of the exit plane from the centerline of the rolls
 x_2 : Distance of the entry plane from the centerline of the rolls
 x_n : Distance of the neutral plane from the centerline of the rolls
 x_c : Variation of the horizontal displacement of rolls
 $\dot{x}_1$: Rate of change of the horizontal position of roll bite
 $\dot{x}_c$: Rate of change of roll horizontal displacement
 y : Arbitrary distance from the asymmetry line of the rolls
 y_c : Roll vertical displacement

v_0 : Strip velocity at exit
 v_1 : Strip velocity at entry
 v_R : Roll velocity
 v_x : Strip horizontal velocity at any arbitrary position from the centerline of the rolls
 v'_1 : The equivalent horizontal velocity at entry
 h_0 : Variation of the strip thickness at exit
 h_1 : Strip thickness at entry
 h_x : Strip thickness at any arbitrary distance from the centerline of the rolls
 τ_f : Forward tensile stress at exit
 τ_b : Backward tensile stress at entry
 τ_x : Horizontal tensile stress at any arbitrary distance from the centerline of the rolls
 F_μ : Shear stress
 P : Interface pressure
 μ : Friction factor
 δ : Shear yield strength
 F_x : The rolling force in horizontal direction
 F_y : Rolling force in vertical direction
 R : Roll radius
 ΔV : Volume flow in deformation zone range from x_1 to x
 $\Delta \dot{V}$: The rate of volume flow change in deformation zone range from x_1 to x
 k_1 : Equivalent stiffness between upper rolls and upper supporting posts
 k_2 : Equivalent stiffness between upper rolls and upper beam
 k_3 : Equivalent stiffness between lower rolls and lower supporting posts
 k_4 : Equivalent stiffness between lower rolls and lower supporting posts
 c_1 : Equivalent damping between upper rolls and upper supporting posts
 c_2 : Equivalent damping between upper rolls and upper beam

- c_3 : Equivalent damping between lower rolls and lower supporting posts
- c_4 : Equivalent damping between lower rolls and lower supporting posts
- m_1 : Equivalent mass of upper rolls
- m_2 : Equivalent mass of lower rolls
- T_1 : External disturbance of upper rolls
- T_2 : External disturbance of lower rolls.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

- [1] P. M. Shi, J. Z. Li, J. S. Jiang, B. Liu, and D. Y. Han, "Nonlinear dynamics of torsional vibration for rolling mill's main drive system under parametric excitation," *Journal of Iron and Steel Research International*, vol. 20, no. 1, pp. 7–12, 2013.
- [2] Z. Drzymala, A. Swiatoniowski, and A. Bar, "Nonlinear vibration in cold rolling mills," *Mechanique & Industries*, vol. 4, no. 2, pp. 151–158, 2003.
- [3] J. L. Sun, P. Y. Peng, and H. M. Liu, "Coupled dynamic modeling of rolls model and metal model for four high mill based on strip crown control," *Chinese Journal of Mechanical Engineering*, vol. 26, no. 1, pp. 144–150, 2013.
- [4] H. Li, B. Wen, and J. Zhang, "Asymptotic method and numerical analysis for self-excited vibration in rolling mill with clearance," *Shock and Vibration*, vol. 8, no. 1, pp. 9–14, 2001.
- [5] Q. Y. Wang, Z. Y. Jiang, J. W. Zhao, and M. Fang, "Multi-factor coupling system characteristic of the dynamic roll gap in the high-speed rolling mill during the unsteady lubrication process," *Tribology International*, vol. 67, pp. 174–181, 2013.
- [6] P.-A. Hu, H. Zhao, and K. F. Ehmann, "Third-octave-mode chatter in rolling—part 1: chatter model," *Proceedings of the Institution of Mechanical Engineers B*, vol. 220, no. 8, pp. 1267–1277, 2006.
- [7] X. Yang, C.-N. Tong, G.-F. Yue, and J.-J. Meng, "Coupling dynamic model of chatter for cold rolling," *Journal of Iron and Steel Research International*, vol. 17, no. 12, pp. 30–34, 2010.
- [8] E. Orowan, "The calculation of roll pressure in hot and cold flat rolling," *Proceedings of the Institution of Mechanical Engineers*, vol. 150, no. 4, pp. 140–167, 1943.
- [9] R. B. Sims, "Calculation of roll force and torque in cold rolling by graphical and experimental methods," *Journal of Iron and Steel Institute*, vol. 178, pp. 19–34, 1954.
- [10] T. Von Karman, "Beitrag zur theorie des walzorganges," *Zeitschrift fur Angewandte, Mathematik und Mechanik*, vol. 5, pp. 1939–1141, 1925.
- [11] I.-S. Yun, W. R. D. Wilson, and K. F. Ehmann, "Chatter in the strip rolling process—part 1: dynamic model of rolling," *Journal of Manufacturing Science and Engineering, Transactions of the ASME*, vol. 120, no. 2, pp. 330–336, 1998.