Research Article

Detection and Localization of Tooth Breakage Fault on Wind Turbine Planetary Gear System considering Gear Manufacturing Errors

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Sidebands of vibration spectrum are sensitive to the fault degree and have been proved to be useful for tooth fault detection and localization. However, the amplitude and frequency modulation due to manufacturing errors (which are inevitable in actual planetary gear system) lead to much more complex sidebands. Thus, in the paper, a lumped parameter model for a typical planetary gear system with various types of errors is established. In the model, the influences of tooth faults on time-varying mesh stiffness and tooth impact force are derived analytically. Numerical methods are then utilized to obtain the response spectra of the system with tooth faults with and without errors. Three system components (including sun, planet, and ring gears) with tooth faults are considered in the discussion, respectively. Through detailed comparisons of spectral sidebands, fault characteristic frequencies of the system are acquired. Dynamic experiments on a planetary gear-box test rig are carried out to verify the simulation results and these results are of great significances for the detection and localization of tooth faults in wind turbines.

1. Introduction

Planetary gear systems have been widely used in wind power systems because of the advantages of compact structure, large carrying capacity, and high transmission efficiency [1]. In recent years, the tooth faults that have occurred in planetary gear systems have brought numerous troubles to wind power plants [2–4]. Therefore, fault diagnosis of the planetary gear system is of great significant to the safe operation of wind turbine.

Tooth pitting, spalling, cracking, and breakage are some of the common fault modes that have occurred in wind turbine planetary gear systems. Detection and localization of those faults are of great significance. Vibration based diagnosis is one of the most effective and frequently used health monitoring technologies. In recent years, much attention has been paid on analyzing the vibration signal in time and frequency domains. To name a few, Lei et al. [5] extracted two diagnostic parameters based on the examination of

vibration characteristics of a planetary gearbox in both time and frequency domains. Experiments revealed that the proposed diagnostic parameters performed better than other parameters. Lin and Zuo [6] introduced a method based on the Morlet wavelet time-frequency analysis. The method was found more effective in detection of tooth cracks than two other types of discrete wavelet transform. Liu et al. [7] carried out fault diagnosis based on the local mean decomposition (LMD) method. The method was suitable for obtaining instantaneous frequencies in frequency domains.

The above-mentioned studies are helpful for the fault detection of planetary gear system. In addition, the sidebands of vibration signals also attract the attention of many researchers, because the sidebands contain abundant information which is useful for the detection and localization of faults in planetary gear systems. McFadden and Smith [8] found that the modulation sidebands of the planetary gear system are typically asymmetric and the frequency

FIGURE 2: A sun-planet mesh.

$\mathbf{M}$, $\mathbf{G}$ represent the mass and gyroscopic matrices, respectively, and are expressed by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_c & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{M}_r & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{M}_s & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \mathbf{M}_1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{M}_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \mathbf{M}_N \end{bmatrix},$$

$$\mathbf{M}_j = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & \frac{I_j}{r_j^2} \end{bmatrix}, \quad j = c, r, s, 1, \dots, N$$

$$(3)$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_c & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{G}_r & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{G}_s & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \mathbf{G}_1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{G}_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \mathbf{G}_N \end{bmatrix},$$

$$\mathbf{G}_j = \begin{bmatrix} 0 & -2m_j & 0 \\ 2m_j & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad j = c, r, s, 1, \dots, N,$$

where m_j and r_j are the mass and the base radius of part j , I_j is the inertial moment of part j with respect to its rotational axis.

$\mathbf{K}_b$ is the bearing stiffness matrices and are expressed by

$$\mathbf{K}_b = \begin{bmatrix} \mathbf{K}_{cb} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{K}_{rb} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{K}_{sb} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\mathbf{K}_{jb} = \begin{bmatrix} k_{jx} & 0 & 0 \\ 0 & k_{jy} & 0 \\ 0 & 0 & k_{ju} \end{bmatrix}, \quad j = c, r, s,$$

where k_{jx} , k_{jy} , k_{ju} represent the bearing supporting stiffness along the three degrees of freedom.

The centripetal stiffness matrix due to the carrier rotation is denoted by $\mathbf{K}_\Omega$. Consider

$$\mathbf{K}_\Omega = \begin{bmatrix} \mathbf{K}_{\Omega c} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{K}_{\Omega r} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{K}_{\Omega s} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \mathbf{K}_{\Omega 1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{K}_{\Omega 2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{K}_{\Omega N} \end{bmatrix},$$

$$\mathbf{K}_{\Omega j} = \begin{bmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad j = c, r, s, 1, \dots, N.$$

The time-varying mesh stiffness matrix is represented by $\mathbf{K}_e(t)$. The matrices are expressed as

$$\mathbf{K}_e(t) = \begin{bmatrix} \sum \mathbf{K}_{c1}^i & 0 & 0 & \mathbf{K}_{c2}^1 & \mathbf{K}_{c2}^2 & \cdots & \mathbf{K}_{c2}^N \\ 0 & \sum \mathbf{K}_{r1}^i & 0 & \mathbf{K}_{r2}^1 & \mathbf{K}_{r2}^2 & \cdots & \mathbf{K}_{r2}^N \\ 0 & 0 & \sum \mathbf{K}_{s1}^i & \mathbf{K}_{s2}^1 & \mathbf{K}_{s2}^2 & \cdots & \mathbf{K}_{s2}^N \\ \mathbf{K}_{c2}^1 & \mathbf{K}_{r2}^1 & \mathbf{K}_{s2}^1 & \mathbf{K}^1 & 0 & \cdots & 0 \\ \mathbf{K}_{c2}^2 & \mathbf{K}_{r2}^2 & \mathbf{K}_{s2}^2 & 0 & \mathbf{K}^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{c2}^N & \mathbf{K}_{r2}^N & \mathbf{K}_{s2}^N & 0 & 0 & \cdots & \mathbf{K}^N \end{bmatrix}, \quad (6)$$

where

$$\begin{aligned}
\mathbf{K}_{r1}^i &= k_{rp_i}(t) \begin{bmatrix} \sin^2 \varphi_{rp_i} & -\cos \varphi_{rp_i} \cos \alpha_r & \sin \varphi_{rp_i} \\ -\cos \varphi_{rp_i} \cos \varphi_r & \cos^2 \varphi_{rp_i} & \cos \varphi_{rp_i} \\ \sin \varphi_{rp_i} & \cos \varphi_{rp_i} & 1 \end{bmatrix}, \\
\mathbf{K}_{c1}^i &= k_{p_i} \begin{bmatrix} 1 & 0 & -\sin \varphi_{p_i} \\ 0 & 1 & \cos \varphi_{p_i} \\ -\sin \varphi_{p_i} & \cos \varphi_{p_i} & 1 \end{bmatrix}, \\
\mathbf{K}_{s1}^i &= k_{sp_i}(t) \begin{bmatrix} \sin^2 \varphi_{sp_i} & -\cos \varphi_{sp_i} \sin \varphi_{sp_i} & -\sin \varphi_{sp_i} \\ -\cos \varphi_{sp_i} \sin \varphi_{sp_i} & \cos^2 \varphi_{sp_i} & \cos \varphi_{sp_i} \\ -\sin \varphi_{sp_i} & \cos \varphi_{sp_i} & 1 \end{bmatrix}, \\
\mathbf{K}_{c2}^i &= k_{p_i} \begin{bmatrix} -\cos \varphi_{p_i} & \sin \varphi_{p_i} & 0 \\ -\sin \varphi_{p_i} & -\cos \varphi_{p_i} & 0 \\ 0 & -1 & 0 \end{bmatrix}, \\
\mathbf{K}_{s2}^i &= k_{sp_i}(t) \begin{bmatrix} \sin \varphi_{sp_i} \sin \alpha_s & \sin \varphi_{sp_i} \cos \alpha_s & -\sin \varphi_{sp_i} \\ -\cos \varphi_{sp_i} \sin \alpha_s & -\cos \varphi_{sp_i} \cos \alpha_s & -\cos \varphi_{sp_i} \\ -\sin \alpha_s & -\cos \alpha_s & 1 \end{bmatrix}, \\
\mathbf{K}_{r2}^i &= k_{rp_i}(t) \begin{bmatrix} -\sin \varphi_{rp_i} \sin \alpha_r & \sin \varphi_{rp_i} \cos \alpha_r & \sin \varphi_{rp_i} \\ \cos \varphi_{rp_i} \sin \alpha_r & -\cos \varphi_{rp_i} \cos \alpha_r & -\cos \varphi_{rp_i} \\ \sin \alpha_r & \cos \alpha_r & -1 \end{bmatrix}, \\
\mathbf{K}^i &= \mathbf{K}_{c3}^i + \mathbf{K}_{r3}^i + \mathbf{K}_{s3}^i, \quad \mathbf{K}_{c3}^i = \begin{bmatrix} k_{p_i} & 0 & 0 \\ 0 & k_{p_i} & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\mathbf{K}_{s3}^i &= k_{sp_i}(t) \begin{bmatrix} \sin^2 \alpha_s & \cos \alpha_s \sin \alpha_s & -\sin \alpha_s \\ \cos \alpha_s \sin \alpha_s & \cos^2 \alpha_s & -\cos \alpha_s \\ -\sin \alpha_s & -\cos \alpha_s & 1 \end{bmatrix}, \\
\mathbf{K}_{r3}^i &= k_{rp_i}(t) \begin{bmatrix} \sin^2 \alpha_r & -\cos \alpha_r \sin \alpha_r & -\sin \alpha_r \\ -\cos \alpha_r \sin \alpha_r & \cos^2 \alpha_r & \cos \alpha_r \\ -\sin \alpha_r & \cos \alpha_r & 1 \end{bmatrix}, \tag{7}
\end{aligned}$$

where $k_{jp_i}(t)$ ($j = s, r$) is the meshing stiffness between the part j and the i th planet. k_{p_i} is the bearing supporting stiffness of the i th planet in the transverse and longitudinal direction. α_j is the pressure angle of part j . $\varphi_{sp_i} = \varphi_{pi} - \alpha_s$ and $\varphi_{rp_i} = \varphi_{pi} + \alpha_r$, φ_{pi} is illustrated in a sun-planet mesh (Figure 2). $i = 1, 2, \dots, N$, N is the number of the planet gears.

$\mathbf{T}(t)$ represents the force matrix and can be expressed by

$$\mathbf{T}(t) = \left[0, 0, \frac{T_c}{r_c}, 0, 0, \frac{T_r}{r_r}, 0, 0, \frac{T_s}{r_s}, 0, \dots, 0 \right]^T, \tag{8}$$

where T_j ($j = c, s, r$) is the external torques applied on part j . r_j is the base radius of part j .

$\mathbf{F}_W(t)$ is the impact force caused by faults, the mathematical expression of which will be given in Section 3.

$\mathbf{F}_e(t)$ is the excitation force induced by manufacturing errors and can be written as

$$\mathbf{F}_e(t) = [0, \mathbf{F}_c, \mathbf{F}_s, \mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N], \tag{9}$$

where

$$\begin{aligned}
\mathbf{F}_c &= \left[\sum_{i=1}^N k_{rp_i}(t) e_{ri}(t) \sin(\varphi_{rp_i}), \right. \\
&\quad \left. - \sum_{i=1}^N k_{rp_i}(t) e_{ri}(t) \cos(\varphi_{rp_i}), - \sum_{i=1}^N k_{rp_i}(t) e_{ri}(t) \right]^T, \\
\mathbf{F}_s &= \left[\sum_{i=1}^N k_{sp_i}(t) e_{si}(t) \sin(\varphi_{sp_i}), \right. \\
&\quad \left. - \sum_{i=1}^N k_{sp_i}(t) e_{si}(t) \cos(\varphi_{sp_i}), - \sum_{i=1}^N k_{sp_i}(t) e_{si}(t) \right]^T, \\
\mathbf{F}_n &= \left\{ \sum_{i=1}^N -k_{rp_i}(t) e_{ri}(t) \sin(\alpha_r) + k_{sp_i}(t) e_{si}(t) \sin(\alpha_s), \right. \\
&\quad \sum_{i=1}^N k_{rp_i}(t) e_{ri}(t) \sin(\alpha_r) - k_{sp_i}(t) e_{si}(t) \cos(\alpha_s) \\
&\quad \left. \sum_{i=1}^N k_{rp_i}(t) e_{ri}(t) - k_{sp_i}(t) e_{si}(t) \right\} \\
&\quad n = 1, 2, \dots, N \tag{10}
\end{aligned}$$

$e_{ji}(t)$ ($j = s, r$) is the errors between the part j and the i th planet and can be written as

$$\begin{aligned}
e_{si}(t) &= E_{p_i s}(t) - E_{sp_i}(t) + e_{sp_i}(t) \\
e_{ri}(t) &= E_{rp_i}(t) - E_{p_i r}(t) + e_{rp_i}(t), \tag{11a}
\end{aligned}$$

where $E_{jp_i}(t)$ and $E_{p_i j}(t)$ ($j = s, r$) are errors associated with rotational frequencies of gears. A unified mathematical formula which is able to express those errors (gear eccentricities, pitch-line run-out errors, tooth spacing, and indexing errors) was extracted by Inalpolat and Kahraman [17],

$$\begin{aligned}
E_{jp_i}(t) &= E_j \sin \left[\frac{f_{\text{mesh}}}{z_j} t + \varepsilon_j - \psi_{ji} \right], \\
E_{p_i j}(t) &= E_{p_i} \sin \left[\frac{f_{\text{mesh}}}{z_p} t + \varepsilon_{p_i} \pm \alpha_j \right], \tag{11b}
\end{aligned}$$

where E_j and E_{p_i} are the amplitudes of these errors, f_{mesh} is mesh frequency, z_j and z_p are tooth numbers of the component j and planet, ε_j and ε_{p_i} are phase angles, ψ_{ji} is relative position angle, and the arithmetic sign of the pressure angle α_j is plus for $j = r$ and minus for $j = s$.

In (11a), e_{sp_i} and e_{rp_i} are gear transmission errors. Inalpolat and Kahraman [17] believed that those errors were

amplitude and frequency modulated by $E_{jp_i}(t)$ and $E_{pi,j}(t)$. Therefore, another equation is able to express the gear transmission errors as given by Inalpolat and Kahraman [17],

$$\begin{aligned} e_{sp_i}(t) &= A_{sp_i} e_{sp_i} \sin [B_{sp_i}(t) f_{\text{mesh}} t + z_s \varphi_{p_i} + \phi_{sp_i}] \\ e_{rp_i}(t) &= A_{rp_i} e_{rp_i} \sin [B_{rp_i}(t) f_{\text{mesh}} t + z_r \varphi_{p_i} + \gamma_{sr} + \phi_{rp_i}], \end{aligned} \quad (11c)$$

where

$$\begin{aligned} A_{sp_i}(t) &= 1 + \beta_s \sin \left[\frac{f_{\text{mesh}}}{z_s} t + \phi_{\beta si} \right] \\ &\quad + \beta_{pi} \sin \left[\frac{f_{\text{mesh}}}{z_p} t + \phi_{\beta pi} \right] \\ B_{sp_i}(t) &= 1 + \hat{\beta}_s \sin \left[\frac{f_{\text{mesh}}}{z_s} t + \hat{\phi}_{\beta si} \right] \\ &\quad + \hat{\beta}_{pi} \sin \left[\frac{f_{\text{mesh}}}{z_p} t + \hat{\phi}_{\beta pi} \right] \end{aligned} \quad (11d)$$

$$\begin{aligned} A_{rp_i}(t) &= 1 + \beta_r \sin \left(\frac{f_{\text{mesh}}}{z_c} t + \phi_{\beta ri} \right) \\ &\quad + \vartheta_{pi} \sin \left[\frac{f_{\text{mesh}}}{z_p} t + \phi_{\vartheta pi} + \pi \right] \\ B_{rp_i}(t) &= 1 + \hat{\beta}_r \sin \left[\frac{f_{\text{mesh}}}{z_c} t + \hat{\phi}_{\beta ri} \right] \\ &\quad + \hat{\vartheta}_{ri} \sin \left[\frac{f_{\text{mesh}}}{z_p} t + \hat{\phi}_{\vartheta pi} + \pi \right], \end{aligned} \quad (11e)$$

where ϕ_{jp_i} ($j = s, r$) is the phase angle of $e_{jp_i}(t)$, γ_{sr} is the phase angle between $e_{sp_i}(t)$ and $e_{rp_i}(t)$, β_s , β_{pi} , β_r , ϑ_{pi} are amplitude coefficients, $\phi_{\beta si}$, $\phi_{\beta pi}$, $\phi_{\beta ri}$, $\phi_{\vartheta pi}$ are the phase angles of amplitude modulation, $\hat{\beta}_s$, $\hat{\beta}_{pi}$, $\hat{\beta}_r$, $\hat{\vartheta}_{ri}$ are amplitude coefficients, and $\phi_{\beta ri}$, $\phi_{\vartheta pi}$, $\hat{\phi}_{\beta ri}$, $\hat{\phi}_{\vartheta pi}$ are the phase angles of frequency modulation. The research results by Conry and Seireg [21] showed that the peak-to-peak amplitude of $e_{sp_i}(t)$ might experience up to 4–6% variation due to the influence of $E_{sp_i}(t)$ and $E_{pi,s}(t)$ even with the modest error magnitudes.

3. Modeling of Tooth Faults

In this section, the typical tooth fault of tooth breakage is modeled to illustrate the influence of tooth faults on gear mesh. The tooth breakage can be modeled by the reduction of the time-varying mesh stiffness and the generation of additional impact forces ($F_w(t)$).

3.1. The Influence of Tooth Breakage on Time-Varying Stiffness. As shown in Figure 3, a tooth is often considered as a cantilever beam with varying section when calculating

FIGURE 3: The schematic diagram of a cantilever beam.

deformation. Deformation consists of five parts: bending, compressive, shear, fillet-foundation, and contact deformations.

The total deformation of the point B is expressed by

$$\delta_{Bi} = u_B \sin \gamma_i + v_B \cos \gamma_i + w_B \sin \gamma_i + \delta_f + \delta_c, \quad (12)$$

where u_B , v_B , w_B , δ_f , δ_c represent bending, compressive, shear, fillet-foundation, and contact deformations, respectively, which can be calculated by the method used in [12], γ_i (see Figure 3) defines the angle between the tooth normal force and the tooth centerline. The stiffness of one gear tooth at point B is calculated as $k_i = F/\delta_{Bi}$. The total stiffness of one pair of meshing tooth is obtained by $k_{\text{pair}} = (k_1 \times k_2)/(k_1 + k_2)$ according to the serial relation. The stiffness when double-teeth meshing is $k = k_{\text{pair1}} + k_{\text{pair2}}$ according to the parallel relation.

The parameters of the studied planetary gear system are listed in Table 1. Young's modulus $E = 2.06 \times 10^{11}$ Pa, Poisson's ratio $\nu = 0.3$. Figure 4 shows the mesh stiffness of the sun gear and ring gear with planet 1 (k_{sp_1} and k_{rp_1}). The planet 1 with and without tooth breakage is considered and the mesh stiffness is seriously affected by tooth breakage.

3.2. The Impact Force Generated by Tooth Breakage. The impact force is generated during the approaching and separation of the meshing teeth [22]. As one gear tooth is broken, the normal gear meshing process is interrupted. When the next gear tooth is going to mesh, additional impacts would occur between the driven and driving gears. The dynamic response of the system will be influenced by the additional impact force. Thus, only the change of mesh stiffness could not reflect the effect of tooth breakage upon the dynamic response of the planetary gear system. In this section, the additional tooth impact forces are modeled.

The additional impact force $F_{W_{pf}}(t)$ generated by the tooth breakage of the planet gear can be expressed as (planet 1 is assumed to be the fault gear)

$$\begin{aligned} F_{W_{s1}}(t) &= [0, 0, F_{w_s}(t), 0, F_{w_1}(t), 0, 0]^T \\ F_{W_{r1}}(t) &= [0, F_{w_r}(t), 0, 0, F_{w_1}(t), 0, 0]^T, \end{aligned} \quad (13)$$

TABLE 1: Parameters of the planetary gear system.

Parameters	Sun gear	Ring	Carrier	Planets
Number of tooth	17	79	—	31
Module (mm)	3	3	—	3
Face width (mm)	35	35	—	35
Mass (kg)	1.75	3.552	7	0.979
Mass moment of inertia (kg/m ²)	1	3	4.56	0.544
Bearing support stiffness (N/m)	$K_p = K_{sx} = K_{sy} = K_{rx} = K_{ry} = K_{cx} = K_{cy} = 2 \times 10^8$			
Torsional stiffness (N/m)	$K_{ru} = 10^9; K_{su} = K_{cu} = 2 \times 10^7$			
Pressure angle (°)	20°	20°	—	20°
Helix angle (°)	0°	0°	—	0°

— Healthy
--- Breakage

(a)

— Healthy
--- Breakage

(b)

FIGURE 4: The mesh stiffness of planet 1 with and without tooth breakage: (a) planet-sun gear pair, (b) planet-ring gear pair.

where

$$\begin{aligned}
 \mathbf{F}_{w_s}(t) &= [F_{s1}(t) \sin(\varphi_{sp_1}), -F_{s1}(t) \cos(\varphi_{sp_1}), -F_{s1}(t)]^T \\
 \mathbf{F}_{w_r}(t) &= [F_{r1}(t) \sin(\varphi_{rp_1}), -F_{r1}(t) \cos(\varphi_{rp_1}), F_{r1}(t)]^T \\
 \mathbf{F}_{w_i}(t) &= [-F_{r1}(t) \sin(\alpha_r) + F_{s1}(t) \sin(\alpha_s), F_{r1}(t) \sin(\alpha_r) \\
 &\quad + F_{s1}(t) \cos(\alpha_s), F_{r1}(t) - F_{s1}(t)]^T,
 \end{aligned} \tag{14}$$

where $\mathbf{F}_{W_{j1}}(t)$ represents the impact force vector of gear j ($j = s, r$), $F_{j1}(t)$ ($j = s, r$) denotes the impact force and can be expressed as

$$F_{j1}(t) = \begin{cases} C_{j1}(t) & t_{Ib_{j1}} < t < t_{If_{j1}} \\ 0 & \text{else,} \end{cases} \tag{15}$$

where $t_{Ib_{j1}}$ and $t_{If_{j1}}$ represent the beginning and finishing time of the impact force of gear j . The relationship between $C_r(t)$ and $C_s(t)$ is expressed by $C_r(t) = H(t)C_s(t)$. Because the impact acceleration will be generated, $H(t) > 1$ when the fault tooth meshes with ring gear and $H(t) < 1$ when the fault tooth meshes with sun gear.

The additional impact force $\mathbf{F}_{W_{rf}}(t)$ generated by the tooth breakage of the ring gear could be expressed by

$$\begin{aligned}
 \mathbf{F}_{W_{r1}}(t) &= [0, \mathbf{F}_{w_{r1}}(t), \mathbf{F}_{w_{s1}}(t), 0, \mathbf{F}_{w_1}(t), 0, 0]^T \\
 \mathbf{F}_{W_{r2}}(t) &= [0, \mathbf{F}_{w_{r2}}(t), \mathbf{F}_{w_{s2}}(t), 0, 0, \mathbf{F}_{w_2}(t), 0]^T \\
 \mathbf{F}_{W_{r3}}(t) &= [0, \mathbf{F}_{w_{r3}}(t), \mathbf{F}_{w_{s3}}(t), 0, 0, 0, \mathbf{F}_{w_3}(t)]^T,
 \end{aligned} \tag{16}$$

where

$$\begin{aligned}
 \mathbf{F}_{w_{ri}}(t) &= [F_{ri}(t) \sin(\varphi_{rp_i}), -F_{ri}(t) \cos(\varphi_{rp_i}), -F_{ri}(t)]^T \\
 \mathbf{F}_{w_{si}}(t) &= [F_{si}(t) \sin(\varphi_{sp_i}), -F_{si}(t) \cos(\varphi_{sp_i}), -F_{si}(t)]^T \\
 \mathbf{F}_{w_i}(t) &= [-F_{ri}(t) \sin(\alpha_r) + F_{si}(t) \sin(\alpha_s), \\
 &\quad F_{ri}(t) \sin(\alpha_r) + F_{si}(t) \cos(\alpha_s), F_{ri}(t) - F_{si}(t)]^T,
 \end{aligned} \tag{17}$$

where $\mathbf{F}_{W_{ri}}(t)$ represents the impact force matrix when planet i meshes with ring gear, $F_{si}(t)$ and $F_{ri}(t)$ are the pulsating impact forces, the mathematical expressions of which are similar to (15) and could be expressed by

$$F_{ji}(t) = \begin{cases} C_{ji}(t) & t_{Ib_{ji}} < t < t_{If_{ji}} \\ 0 & \text{else.} \end{cases} \tag{18}$$

Similarly, there are several coefficients $H_{ji}(t)$ used to describe the relation among $C_{ji}(t)$. The starting time of $F_{si}(t)$ and $F_{ri}(t)$ varies with i .

FIGURE 5: The acceleration spectrum of the ring gear in longitudinal direction in the system with and without tooth breakages (without errors): (a) without breakage, (b) planet gear with and without tooth breakages, (c) sun gear with and without tooth breakages, (d) ring gear with and without tooth breakages.

The impact force generated by the tooth breakage of the sun gear is $F_{W_{sf}}(t)$, the mathematical expression of which is the same as that of $F_{W_{rf}}(t)$. However, the values for beginning and finishing times of $F_{ji}(t)$ are different.

The research results by Yao and Wei [23] showed that the action time of impact force is usually about 5–10% of one mesh cycle (T_m). Wang et al. [24] pointed out that the maximal impact force induced by tooth breakage is about twice the average mesh force. Thus, we can assume $t_{If_{ji}} - t_{Ib_{ji}} = T_m/10$ and $C_{ji}(t) = 2\bar{F}_{jp_i} = \text{const.}$

4. The Dynamic Response of Planetary Gear System without Errors

Physical parameters of the studied planetary gear system are listed in Table 1. The ring gear is fixed and there are three planets rotating around the sun gear. The speed of the input part (carrier) is 2.77 rad/s (Ω_c) and the load of the output part (sun gear) is 200 N·m. The rotating frequency of the carrier

(f_c), sun gear (f_s), planets (f_p), and the mesh frequency (f_{mesh}) can be calculated by

$$\begin{aligned}
 f_c &= \frac{\Omega_c}{2\pi}, \\
 f_s &= \frac{z_s + z_r}{z_s} f_c, \\
 f_p &= \frac{f_s z_s - f_c (z_s + z_p)}{z_p}, \\
 f_{\text{mesh}} &= f_c z_r.
 \end{aligned} \tag{19}$$

Obviously, these frequency values are $f_c = 0.441$ Hz, $f_s = 2.49$ Hz, and $f_p = 0.68$ Hz, respectively, and the mesh frequency is $f_{\text{mesh}} = 34.9$ Hz.

The errors e_{sp_i} , e_{rp_i} , E_{p_i} , E_s , E_r are set to be zero. The dynamic equation of the model is solved by the NEWMARK method. Figure 5(a) shows the acceleration spectrum of the ring gear in longitudinal direction (without breakage). The spectrum is occupied by the mesh frequency and no

FIGURE 6: The spectrum of the system with and without tooth breakages (with errors): (a) without breakage, (b) planet gear with and without tooth breakages (the right side of f_{mesh}), (c) sun gear with and without tooth breakages (the right side of f_{mesh}), (d) ring gear with and without tooth breakages.

TABLE 2: Parameters of the parallel-stage gear.

Level	Stage1		Stage2	
Gear	Input	Output	Input	Output
Number of tooth	73	21	66	23

sidebands are found around the mesh frequency. Then tooth breakage is added to the planet gear, sun gear, and ring gear successively, and the acceleration spectrum (the ring gear in longitudinal direction) is showed in Figures 5(b)–5(d). Compared with the spectrum of the normal system (without breakage), modulation sidebands appeared in the spectrum of the breakage system and the peaks of the sidebands are at the frequencies of $f_{\text{mesh}} \pm n(f_p + f_c)$, $f_{\text{mesh}} \pm n(f_s - f_c)$ and $f_{\text{mesh}} \pm nf_c$ ($n = 1, 2, \dots$) successively. Therefore, the mesh frequency f_{mesh} is modulated by those frequencies, which are caused by the influence of tooth breakage on mesh stiffness and additional impact force. So, the sidebands of the planetary gear system are comparatively simple if the errors are not included in the system.

5. The Dynamic Response of Planetary Gear System with Errors

In order to display the modulation effect of errors more reasonably, a formula was given by Inalpolat and Kahraman [17] to express the acceleration response of the system

$$a(t) = \sum_{i=1}^N w_i(t) \left(\bar{C}_i k_{sp_i}(t) \delta_{sp_i}(t) + \bar{D}_i k_{rp_i}(t) \delta_{rp_i}(t) \right), \quad (20)$$

where $\bar{C}_i$ and $\bar{D}_i$ are constants, $\delta_{sp_i}(t)$ and $\delta_{rp_i}(t)$ are defined as

$$\begin{aligned} \delta_{sp_i}(t) &= y_s \cos \varphi_{sp_i} - x_s \sin \varphi_{sp_i} - x_i \sin \alpha_s + u_s + u_i - e_{si}(t), \\ \delta_{rp_i}(t) &= y_r \cos \varphi_{rp_i} - x_r \sin \varphi_{rp_i} - x_i \sin \alpha_r + u_r - u_i + e_{ri}(t) \end{aligned} \quad (21)$$

x_j, y_j, u_j ($j = s, r, i$) can be obtained by the dynamic model.

FIGURE 7: The test bench of the planetary gear system.

(a)

(b)

(c)

FIGURE 8: The acceleration sensor and breakage tooth: (a) acceleration sensor position, (b) planet gear with tooth breakage, and (c) ring gear with tooth breakage.

$w_i(t)$ is a weighting function for a planet gear positioned at the angle of φ_{p_i} and $w_i(t)$ can be defined as

$$w_i(t) = W\left(t - \frac{\varphi_{p_i}}{f_c}\right) U_i(t), \quad (22)$$

where $W(t)$ is a Hanning function used to represent the influence of rolling planets on fixed accelerometer (scrolling

FIGURE 9: The acceleration spectrum of the normal system.

effect) and $U_i(t)$ is the unit step function. Both of them are given as follows:

$$W(t) = \frac{1}{2} - \frac{1}{2} [\cos(2\pi N f_c t)]$$

$$U_i(t) = \sum_{n=1}^{\infty} \left\{ u\left[t - \left(\frac{(n-1)N + i - 1}{N}\right) T_c\right] - u\left[t - \left(\frac{(n-1)N + i}{N}\right) T_c\right] \right\}, \quad (23)$$

where $u(t-a) = 1$ for $t > a$ and $u(t-a) = 0$ for $t < a$, N is the number of planets. In this study, N is equal to 3 and i is 1, 2, and 3.

Run-out errors of the planets, sun gear, and ring gear are taken into account and $e_{sp_i} = 0.6 \mu\text{m}$, $e_{rp_i} = 0.3 \mu\text{m}$, $E_{p_i} = 10 \mu\text{m}$, and $E_s = E_r = 15 \mu\text{m}$. According to Conry's study [21], modulation coefficients are set as $\beta_s = 0.033$, $\phi_{\beta si} = 2(i-1)\pi/3$, $\beta_{pi} = 0.034$, $\phi_{\beta pi} = 2(i-2)\pi/3$, $\beta_{ri} = 0.033$, $\phi_{\beta ri} = 2(i-2)\pi/3$, $\beta_r = 0.035$, $\phi_{\beta ri} = 2(i-1)\pi/3$ and all other modulation coefficients are set to be zero. The dynamic equation of the model is solved to get the $\delta_{sp_i}(t)$ and $\delta_{rp_i}(t)$, then acceleration is calculated by (20). Figure 6(a) shows the system acceleration spectrum of a normal system (without breakage). Compared with Figure 5(a), complex sidebands appeared around the mesh frequency due to the existence of errors. They are symmetrically arranged on both sides of the mesh frequency. The sidebands of the system have undergone tremendous changes when errors are taken into consideration. The phenomenon is bound to influence the fault characteristic frequencies of the system components.

Figure 6(b) shows the acceleration spectrum of the system with and without tooth breakage on planet 1. Since the mesh frequency f_{mesh} is modulated by the frequencies of weighting function (which characteristic frequencies are mf_c) and the frequencies of $n(f_p + f_c)$ (caused by the influence of tooth breakage on mesh stiffness and additional impact force), the amplitudes of the tooth breakage system are much larger than those of the normal (without breakage) system at the frequencies of $f_{\text{mesh}} \pm mf_c \pm n(f_p + f_c)$ ($m, n = 1, 2, \dots$). Therefore, the fault characteristic frequencies of the planet gear are $f_{pf} = f_{\text{mesh}} \pm mf_c \pm n(f_p + f_c)$ and can be simplified as $f_{\text{mesh}} \pm mf_c \pm nf_p$ ($m, n = 1, 2, \dots$).

FIGURE 10: The spectrum of the system with and without tooth breakage on planet at different speeds and loads: (a) load = 200 N·m, input speed = 2.74 rad/s, (b) zoom the right side of (a), (c) load = 229 N·m, input speed = 2.74 rad/s, and (d) load = 229 N·m, input speed = 2.22 rad/s.

Likewise, the amplitudes of the sun gear tooth breakage system are found to be larger than those of the normal system (without breakage) at the frequencies of $f_{\text{mesh}} \pm nf_c \pm mf_s$ ($m, n = 1, 2, \dots$) in Figure 6(c). The main reason is that the mesh frequency f_{mesh} is modulated by the frequencies of weighting function (mf_c) and the frequencies of $n(f_s - f_c)$ (caused by the influence of tooth breakage on mesh stiffness and additional impact force). Therefore, the fault characteristic frequencies of the sun gear are $f_{\text{mesh}} \pm mf_c \pm n(f_s - f_c)$ and can be as $f_{sf} = f_{\text{mesh}} \pm nf_c \pm mf_s$ ($m, n = 1, 2, \dots$).

Similarly, because the mesh frequency f_{mesh} is modulated by the frequencies of the weighting function mf_c and the frequencies of nf_c (caused by the influence of tooth breakage on mesh stiffness and additional impact force), the amplitudes of the ring gear tooth breakage system are larger than those of the normal system (without breakage) at frequencies of $f_{\text{mesh}} \pm nf_c \pm mf_c$ ($m, n = 1, 2, \dots$), as shown in Figure 6(d). Therefore, the fault characteristic frequencies of the ring gear are $f_{rf} = f_{\text{mesh}} \pm nf_c \pm mf_c$ and can be simplified as $f_{rf} = f_{\text{mesh}} \pm nf_c$ ($n = 1, 2, \dots$).

6. Experimental Validations

Figure 7 shows the test bench of a planetary gear system. The drive motor is controlled by frequency converter and provides power for the system. The speed of the system is slowed down by the speed reducer and speeded up by the speed multiplier. A loading motor continuously pumps high-pressure oil into the loader to induce a workload. The structure and the size of the speed multiplier are proportional to the gearbox of a real wind turbine and it consists of a planetary and a parallel gear boxes. The parallel gear box is composed of two pairs of parallel-stage gearing. The parameters of the planetary gear system are listed in Table 1 and those of the parallel-stage are listed in Table 2. The structure style and parameter setting of the speed reducer are the same as those of the speed multiplier.

As shown in Figure 8(a), an acceleration sensor is mounted on the planetary gear box of the speed multiplier. The tooth breakage of planet gear and ring gear are added to the planetary gear system separately, as shown in Figures 8(b) and 8(c). Vibration signals are collected for a normal mode

FIGURE 11: The spectrum of the system with and without tooth breakage on ring gear at different loads and speeds: (a) load = 200 N·m, input speed = 2.77 rad/s, (b) load = 229 N·m, input speed = 2.77 rad/s, and (c) load = 229 N·m, input speed = 2.24 rad/s.

(without breakage) and the two fault modes. The sampling frequency is 16384 Hz.

Figure 9 shows the acceleration spectrum of the normal system (without breakage). The drive motor speed is 156.24 rad/s and the input speed of the speed multiplier is 2.77 rad/s. Therefore, the mesh frequency is 34.9 Hz. Figure 9 shows that sidebands appeared around the mesh frequency. The results indicate the existence of errors in the system.

6.1. Planet Gear with Tooth Breakage. Figure 10 shows the acceleration spectrum of the system with and without tooth breakage on planet.

In Figure 10(a), the load and the speed of the system are 200 N·m and 2.74 rad/s, respectively. Therefore, the mesh frequency is 34.87 Hz. Compared with the amplitudes of the normal system (without breakage), those of the tooth breakage system change significantly. In order to make it more clear, the right side of Figure 10(a) is zoomed in, as shown in Figure 10(b). All amplitudes increase at the frequencies of $f_{\text{mesh}} \pm mf_c \pm nf_p$ ($m, n = 1, 2, \dots$), except for the frequencies of $f_{\text{mesh}} + 2f_c$ and $f_{\text{mesh}} + f_c + 3f_p$.

Then, the load is increased to 229 N·m and the speed remains the same. The differences of amplitudes between the normal system and the tooth breakage system are increased at the frequencies of $f_{\text{mesh}} \pm mf_c \pm nf_p$ ($m, n = 1, 2, \dots$), as shown in Figure 10(c).

Finally, the load is kept unchanged at a value of 229 N·m and the input speed is decreased to 2.22 rad/s. Therefore, the mesh frequency is changed to 27.86 Hz. The amplitudes of the tooth breakage system are also larger than those of the normal system at the frequencies of $f_{\text{mesh}} \pm mf_c \pm nf_p$ ($m, n = 1, 2, \dots$) significantly, as shown in Figure 10(d).

Figure 10 verified the correctness of the simulation results in Section 5. The fault characteristic frequencies of the planet gear with tooth breakage are $f_{pf} = f_{\text{mesh}} \pm mf_c \pm nf_p$ ($m, n = 1, 2, \dots$).

6.2. Ring Gear with Breakage. Figure 11 shows the acceleration spectrum of the system with and without tooth breakage on the ring gear.

In Figure 11, the load and the speed of the system are 200 N·m and 2.77 rad/s, respectively; therefore, the mesh

frequency is 34.9 Hz, similar with the method used in Section 6.1. The amplitudes of the acceleration spectrum are compared between the normal system and the tooth breakage system. The fault characteristic frequencies of $f_{\text{mesh}} \pm nf_c$ ($n = 1, 2, \dots$) are further confirmed by the experiment results.

The change law of amplitudes with loads and speeds of the normal and the tooth breakage systems is same as the law described in Section 6.1. The conclusion can be obtained through Figures 11(a)–11(c).

7. Conclusions

Detection and localization of tooth fault on wind turbine planetary gear system are investigated numerically and experimentally in this paper. Through detailed comparisons on both numerical and experimental results, the fault characteristic frequencies can be obtained after considering the gear manufacturing errors.

- (1) The sidebands of the planetary gear system become much more complex when manufacturing errors are taken into consideration.
- (2) The fault characteristic frequencies of planet gears, sun gear and ring gear with tooth breakage are $f_{pf} = f_{\text{mesh}} \pm mf_c \pm nf_p$, $f_{sf} = f_{\text{mesh}} \pm nf_c \pm mf_s$ and $f_{rf} = f_{\text{mesh}} \pm nf_c$, respectively.
- (3) As the fault characteristic frequencies for the sun, planet, and ring gears are different, so the tooth breakage fault could also be located by monitoring the values of these frequencies.

The above results are useful for the detection and localization of tooth faults in actual planetary gear system of wind turbines.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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