Research Article

Fundamental Frequencies of Vibration of Footbridges in Portugal: From *In Situ* Measurements to Numerical Modelling

C. S. Oliveira

ICIST/Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais, 1049-001 Lisbon, Portugal

Correspondence should be addressed to C. S. Oliveira; csoliv@civil.ist.utl.pt

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Since 1995, we have been measuring the *in situ* dynamic characteristics of different types of footbridges built in Portugal (essentially steel and precast reinforced concrete decks) with single spans running from 11 to 110 m long, using expedite exciting and measuring techniques. A database has been created, containing not only the fundamental dynamic characteristics of those structures (transversal, longitudinal, and vertical frequencies) but also their most important geometric and mechanical properties. This database, with 79 structures organized into 5 main typologies, allows the setting of correlations of fundamental frequencies as a negative power function of span lengths L ($L^{-0.6}$ to $L^{-1.4}$). For 63 footbridges of more simple geometry, it was possible to obtain these correlations by typology. A few illustrative cases representing the most common typologies show that linear numerical models can reproduce the *in situ* measurements with great accuracy, not only matching the frequencies of vibration but also the amplitudes of motion caused by several pedestrian load patterns.

1. Introduction

The great development of footbridges (also referred to as “pedestrian bridges”) in Portugal in the last 2 decades is related to the large amount of freeways or motorways constructed in recent years and the need for pedestrians to cross them. Also, footbridges were built over railways, in railway stations, near shopping centers and schools, and so forth. In a few cases, they were built as part of bicycle routes. Footbridges are in general quite different from viaducts over freeways, not only due to differences in the loading but also because the former can profit more easily from the advances in material developments and architectural creativity. In general, they are lighter, built with high strength materials, spanning quite large distances, and having a wide variety of structural designs. They tend to become slender structures with less mass, but they show more pronounced dynamic effects due to possible resonance in the passage of pedestrians. Large vibration amplitudes are in the range of discomfort, and so this phenomenon deserves much attention to understand the problems that may arise and the way they should be dealt with.

This paper is divided into two parts. In the first part we present and explore a database with *in situ* information on frequencies of vibrations. The second part is dedicated to numerical modelling of a few footbridges that were subjected to *in situ* testing.

For several years, we have been building a database on the main dynamic characteristics of different types of footbridges built in Portugal. In this paper, we report a group of 79 footbridges of different typologies, setting correlations between the fundamental frequencies in the three orthogonal directions and their larger span length, (Table 1 [1–5]). We also look at numerical modelling of a few structures representative of the most common footbridges in Portugal, and compare the results of these models with those of *in situ* testing, both for the fundamental frequencies and for the amplitudes produced by given pedestrian loading. The *in situ* experiments made consisted in single person walking, jogging or running and jumping.

We summarize the work by Silva [6], partially presented in Oliveira and Silva, [7], who has studied a steel and a reinforced concrete (RC) deck footbridge, subjected to

TABLE 1: Geometric characterization and first *in situ* transverse, longitudinal, and vertical frequencies.

Name	Place	Constr. year	Height (m)	Width (m)	Deck development	Deck cross section	Largest span (m)	Right stair position	Left stair position	Frequencies		
										f^- - transv. (Hz)	f^- -long (Hz)	f^- -Vert. (Hz)
Steel Box-Girders												
Relógio_Gago_Coutinho	Lisbon	1995	6.6	1.7	slight curve	trapezoidal	37	L	L	1.86	3.42	2.29
Entrepосто Olivais	Lisbon	2000	8.0	1.8	slight curve	trapezoidal	39	T	T	2.44	3.22	3.25
CP_Gago Coutinho	Lisbon	1998	8.3	2.2		trapezoidal	28	T	T	2.15	2.16	3.42
Relógio_Gomes Costa_West	Lisbon	2000	8.0	2.0	slight curve	trapezoidal	42	T	T	2.70	3.71	3.12
Av. Padre Cruz	Lisbon	2000	7.7	2.0		trapezoidal	37	TL	TL	2.25	5.08	3.42
Acesso 25 Abril Alvito	Lisbon	2000	8.0	2.0	large curve	trapezoidal	37	N	N	3.90	3.90	5.60
Belém (Café)	Lisbon	2001	6.7	1.7	large curve	trapezoidal	43	L	L	3.42	5.20	5.05
2ª Circular East Colegio Alemao*	Lisbon	2000	5.2	2.0	slight curve	trapezoidal	26	T	T	6.70	2.54	3.52
Termas S. Pedro Sul	S. Pedro Sul	1997	2.6	2.0		trapezoidal	53	L	RC	2.90	2.00	2.25
2ª Circular Colegio Alemao	Lisbon	2000	5.0	2.0	slight curve	rectangular	41	T	L	1.95	3.67	2.25
Belém FIL Café-Bar	Lisbon	2001	6.3	2.0		rectangular	14	TL	T	2.93	7.70	9.30
Churrasqueira Campo Grande	Lisbon	2000	5.0	2.0		rectangular	20	L	L	6.93	2.54	3.52
Av. Ceuta South	Lisbon	2002	5.3	2.1		rectangular	32	T	T	1.76	3.80	1.76
Faro.Calouste Gulbenkian	Faro	2003	3.5	2.0		rectangular	29	T	T	3.70	3.40	2.00
Lagos-Nova CML	Lagos	2004	4.6	2.0		rectangular	16	T	T	3.65	3.65	7.10

TABLE I: Continued.

Name	Place	Constr. year	Height (m)	Width (m)	Deck development	Deck cross section	Largest span (m)	Right stair position	Left stair position	Frequencies			
										f^- transv. (Hz)	f^- -long (Hz)	f^- -Vert. (Hz)	
Steel Truss I + I													
Teatro Camões Parque Nações	Lisbon	2010	7.0	2.5		I + I	21	L	L	6.80	4.30	4.30	
Coimbra 1A	Coimbra	2004	5.5	2.0		I + I	21	L	L	2.00	5.00	5.20	
Coimbra 1B	Coimbra	2004	5.5	2.0		I + I	28	L	L	2.00	5.00	4.08	
Coimbra 2A	Coimbra	2004	4.5	2.0		I + I	17	L	T	3.36	11.00	9.60	
Pupilos Exercito South	Lisbon	1999	10.0	1.7		I + slab	38	T	T	1.17	1.86	3.03	
Braga NE	Braga	2002	5.0	2.0		truss 2D	25	T	T	4.79	2.64	4.77	
Av Gomes Costa (East)	Lisbon	2002	5.6	2.8		truss 2D	30	T	T	2.73	2.73	3.42	
Rego	Lisbon	1999	6.3	2.4		truss 2D	46	Truss	Truss	1.95	1.95	3.81	
Conde Almoester over railway	Lisbon	2001	8.0	3.0	n/straight	truss 2D	50	T	T	2.25	2.93	2.93	
IP3-Lameira	Coimbra	2000	5.6	2.0		truss 2D	21	L	L	2.26	3.87	2.66	
Universidade Covilhã	Covilhã	2010	6.0	1.4		truss 2D	20	T	N	2.07		2.07	
IP3-CEPSA_Agueira	Coimbra	2000	5.8	2.0		truss 2D	22	T	T	5.03	3.22	7.72	
A1-Antuã	A1km 260	1995	4.3	2.5		truss 2D	42	RC	RC	2.73	2.91	4.98	
Bela Vista-Feira Nova	Lisbon	2008	12.0	2.5		truss 3D	48	N	N	2.20	3.00	5.60	
Massamá	Salg Maia		4.7	2.5		truss 3D	26	L	L	5.40	5.40	6.54	
EN-125_Chinicato	Lagos	2001	5.0	2.0		tubular	20	T	T	3.20	3.20	7.40	
Pinhão-Douro	Pinhão	2011	6	3.3	light curve	tubular	51	N	N	3.16	4.24	3.16	
Exit Camacha	Madeira	2002	4.0	2.0		triangle	28	N	N	2.44		1.86	
Aeroporto	Madeira	2002	4.0	2.0		triangle	19	T	T	3.71	4.32	2.50	
São Gonçalo	Madeira	2002	4.0	2.0		triangle	28	N	N	3.32	2.64	1.86	
Conde Almoester	Lisbon	2001	8.0	2.0		triangle	22	T	T	5.40	4.70	4.20	

TABLE 1: Continued.

Name	Place	Constr. year	Height (m)	Width (m)	Deck development	Deck cross section	Largest span (m)	Right stair position	Left stair position	Frequencies		
										f - transv. (Hz)	f -long (Hz)	f -Vert. (Hz)
Reinforced Concrete												
Alfredo Bensaude (West)	Lisbon	1997	5.0	2.1		I + slab + I	30	T	T	4.32	2.93	2.88
Alfredo Bensaude (East)	Lisbon	1997	5.0	2.1		I + slab + I	30	T	T	3.91	2.05	2.73
Av. EUA_Chelas East	Lisbon	2000	9.0	2.5		I + slab + I	25	TL	T	2.91	1.70	4.00
Vale Formoso (Chelas)	Lisbon	2000	7.2	2.5		I + slab + I	28	T	T	1.76	2.00	3.81
Figueira_Foz_Lidl	Figueira Foz	2002	6.0	2.5		I + slab + I	19	T	T	5.57	3.42	6.45
Lusófona (Campo Grande)*	Lisbon	2001	6.0	2.0		I + slab + I	26	T	T	3.73	2.77	3.79
EN-125 Faro_Mercedes	Faro	2003	6.3	1.5		I + slab + I	29	T	T	1.90	2.90	1.90
Gambelas Rotunda Aeroporto	Faro	2005	5.8	1.8		I + slab + I	38	T	T	2.06	2.51	2.30
EN-125 Faro-Metalo-Farense	Faro	2005	6.0	1.8		I + slab + I	32	T	T	1.80	2.20	3.10
EN-125 Faro-Patacao (center)*	Faro	2005	5.0	2.0		I + slab + I	22	TL	TL	2.60	2.60	3.58
EN-125 Faro-Patacao (N part)	Faro	2005	5.0	2.0		I + slab + I	13	TL	TL	2.60	2.35	10.70
EN-125 Faro-Patacao (S part)	Faro	2005	5.0	2.0		I + slab + I	11	TL	TL	2.60	2.50	13.50
EN-125 Faro-Paga Pouco	Faro	2005	5.0	2.0		I + slab + I	22	T	T	3.55	2.25	5.05
EN-125 Faro-Coord Seguros Eng	Faro	2005	5.0	1.5		I + slab + I	23	T	T	4.10	2.15	3.50
Pupilos Exército North	Lisbon	1999	10.0	1.7		I + slab	31	T	T	1.37	1.86	4.49
Conde Almoster	Lisbon	2001	7.0	2.0	n/straight	triangle	22	RC	RC	5.40	4.70	4.20

TABLE I: Continued.

Name	Place	Constr. year	Height (m)	Width (m)	Deck development	Deck cross section	Largest span (m)	Right stair position	Left stair position	f^- transv. (Hz)	f^- -long (Hz)	f^- -Vert. (Hz)
Reinforced Concrete												
IP25-Aveiro	Aveiro	1999	4.0	2.0	large curve	trapezoidal	36	L	L	1.24	1.44	2.57
EN-125 Exit Aeroporto (Ibis)	Faro	2003	5.0	2.0		U	37	T	T	2.10	1.65	1.90
Circular Praia da Rocha	Portimão	1970	5.0	2.0		U	21	L	L	6.90	5.10	6.90
Entrepósito	Lisbon	2001	5.7	2.3	T + T	T + T	20	T	T	1.60	3.12	3.50
Escola_Herc_Carvalho	Amadora	1995	6.0	4.0		rectangular	22	T	T	6.30	6.20	3.71
Amadora Cacém	N117	2001	6.0	2.0		U	31	T	T	2.52	2.40	2.76
IC-19 Queluz	Exit 7	2001	6.0	2.0	U/V	U/V	32	L	T	2.52	4.12	3.13
IC-19 Estação	Exit 9	2001	6.0	2.2		U/V	41	T	T	2.52	1.84	2.32
Massamá-Barcarena	Exit 9	2001	6.0	2.4		U/V	46	T	T	2.07	1.80	2.12
IC-19 Cacém West Cemitério)	Exit 8	2001	6.0	2.0	U/V	U/V	39	T	T	2.76	2.34	2.58
IC-19 Cacém Leste	S-Exit A7	2001	6.0	2.0		U/V	25	T	T	3.00	2.12	3.76
IC-19 Cacém Leste Gasolina BP												
Guimarães												
Steel variable cross-section												
SMPC_Av Calouste Gulbenkian	Lisbon	2010	8.6	4.0	trapezoidal I + I	trapezoidal	38	N	N	7.30	2.80	3.86
Olaias-Bela Vista	Lisbon	2011	15.0	4.0		I + I	56	N	N	2.16	3.04	1.90

TABLE I: Continued.

Name	Place	Constr. year	Height (m)	Width (m)	Deck development	Deck cross section	Largest span (m)	Right stair position	Left stair position	Frequencies		
										$f_{\text{transv.}}$ (Hz)	$f_{\text{-long}}$ (Hz)	$f_{\text{-Vert.}}$ (Hz)
Other Types												
Parque Nações near Vodafone	Lisbon	2010	7.0	2.5		Timber U	25	L	L	5.80		5.80
Green park_Benavente	Benavente	2007	3.0	2.5		Steel ribbon	48	N	N	1.12	1.76	2.36
FEUP Campus* [1]	Oporto	2001	5.0	2.8		RC ribbon	32	N	N	2.15	2.54	0.81
CC Vasco Gama North*	Lisbon	1998	8.0	2.4		Steel arch	75	N	N	1.83		0.94
CC Vasco Gama South	Lisbon	1998	8.0	2.4		Steel arch	75	N	N	1.70		0.88
Guarda near railway station* [2]	Guarda	2007	7.0	2.0		Steel bow-string	90	N	N	0.63		2.33
Microsoft Porto Salvo	Oeiras	2001	6.0	2.0		Steel	25	T	L	3.22	5.66	3.22
Infante D. Henrique	Lisbon	1978	5.6	2.5		bow-string	30	L	L	2.70	8.00	5.37
Av. Marech Gomes Costa (East)	Lisbon	1978	5.6	2.5		Fiber-glass	32	T	T	4.00	3.90	5.37
Ribeira Carpinteira	Covilhã	2009	39.0	4.4	S-shape	Fiber-glass	49	N	N	1.36	2.11	2.47
Pedro e Inês* [3]	Coimbra	2006	10.0	4.0	n/straight	Steel [+] 2D	110	N	N	0.91		1.95
Movable cable-stayed—Open* [4]	Viana Castelo	2007	8.0	2.5		S + RC arch-girder	36	N	N	0.98		1.03
Movable cable-stayed—Closed*	Viana Castelo	2007	8.0	2.5		Steel cantilever	36	N	N	4.09		3.17
Circular ribbon* [5]	Aveiro	2009	5.0	2.0	ring	Steel cantilever	ext diameter = 13	L	N			3.17

* Analytical studies; N: no stair; L: longitudinal; T: transversal; TL: oblique; Right: pointing W-N-E-; []: reference.

FIGURE 1: Largest span of analyzed footbridges (circular ribbon footbridge not included).

a pedestrian-load pattern and to the seismic action corresponding to the Portuguese Code [8]. Other analytical studies made in more complex structures [9] or as part of a campaign for evaluating seismic vulnerability of typical footbridges [10] are also synthesized.

This work aims to contribute to the understanding of footbridge behavior under pedestrian loading, verifying the reliability of standard structural analysis programs to obtain a correct representation of these types of structures. This is in line of recent studies merging analytical modelling with *in situ* experiments (see [1]). The correlations obtained between frequencies in the vertical direction and span lengths are quite good for certain typologies and can therefore be used as a first estimation for design or amplitude assessment.

Although in many cases the footbridges do not show structural problems with pedestrian crossings, they may notice excessive vibration problems under specific loading, making the crossing uncomfortable and even scary. The case that called attention to this phenomenon was the Millennium Bridge, in London. In the inauguration's day, when thousands of pedestrians crossed the footbridge, excessive horizontal vibrations were observed caused by a synchronized transverse movement of the crowd. This effect was known as "lock-in effect" [11].

2. Main Footbridge Typologies Existing in Portugal

The present database refers to 79 footbridges of several geometric layouts, structural types, deck cross sections, materials, and largest span length varying from 11 to 110 m, Figure 1. The most common typologies are the steel box-girder deck and the precast RC deck with prestressed beams. Other typologies used different materials such as fibre-glass and timber or combinations of tubular supporting structures. A classification of decks of the most common footbridge typologies in Portugal (Figure 2), whose images are shown in Figure 3, is as follows: (a) steel box-girders (BG) with trapezoidal and rectangular cross sections; (b) steel truss into a 2D or 3D special geometry; (c) RC lateral precast beams with concrete slabs; (d) steel variable cross sections; and (e) other types for larger spans, such as bow strings, suspended,

and cable-stay. In this category we also consider footbridges of materials different from steel and RC.

Even though more than 50% of the reported footbridges refer to the region of Lisbon, the author has selected a number of cases in each typology in other regions of the country to gain some statistical significance. Also, there is a set of recently built footbridges, which represent landmarks in the modern Portuguese scenario for their outstanding design, such as the stress-ribbon in the FEUP Campus (Oporto) [1], the "Pedro e Inês" in Coimbra [3], the circular footbridge in Aveiro [5], the movable cable-stayed footbridge in Viana do Castelo [4], the bow-string in Guarda [2], and the S-shape Carpinteira footbridge in Covilhã [12], which were added to the database for completeness. The main properties of some of these structures were taken from data and results published in the literature (see Table 1).

The above most common (a) to (d) typologies built in Portugal are also seen in many other countries and, consequently, the results presented herein may be extrapolated outside Portugal. These footbridges, in a total of 63 cases, are essentially single span long over an entire free-way, supported in lateral columns or pillars, made of cylindrical hollow steel or of RC precast elements, with cylindrical or elongated cross sections. The access to the deck in an elevated level is made in different ways, commonly by RC stairways or ramps, running longitudinal with the axis of the footbridge (L), transversal to it (T), or at an angle (LT). For bicycle routes generally there is no ramp (N) and the deck sits directly in the abutments.

Foundations of columns vary from case to case but, in general, they are made of concrete blocks. Connections at the top of the columns and to the stairways or ramps also differ quite considerably. But as the majority of these structures are precast, these connections are weak points of the structure especially for seismic loadings.

The dynamic characterization of these structures is of most importance for a number of reasons. As they are slender structures, with continuous distribution of mass and stiffness, most of them spanning lengths of 20 to 60 m (Figure 1), the passage of pedestrians may cause large amplitude vibrations. In some cases, resonance effects in the vertical and/or transverse directions, beyond becoming uncomfortable, they may induce damage.

Frequencies and damping characteristics are probably the most important parameters controlling the dynamic behavior, together with the frequency of walking pedestrians (number of people, velocity, synchronization of stepping, etc.). The frequencies of the structure depend on the geometry and mechanical properties. For geometry they vary with the number of spans, the type of connections, the lateral pathway (ramp and stairways in the longitudinal or transversal directions), the height of columns, the width of deck, and the curvature in elevation and its development in plan. For mechanical properties, the main characteristics to be accounted are the weight and the modulus of elasticity.

The main information contained in the database for each one of the 5 above mentioned typologies refers to the following parameters, obtained either from design drawings (a few cases) or from direct measurements of structural elements: (1) identification—name; (2) location—place; (3) construction

FIGURE 2: Main structural deck types for footbridges in Portugal.

year; (4) height of deck; (5) width; (6) deck development; (7) deck cross section; (8) largest span; (9) and (10) lateral access; (11) to (13) frequencies of first mode in the transversal, longitudinal, and vertical directions. Table 1 presents this information for the 79 footbridges in the database, the majority of them subjected to *in situ* dynamic testing. The entire database contains more information such as the designer/contractor; state of conservation; total length; number of spans; deck cross section and columns dimensions; date of testing; other identified modal frequencies, especially the ones with more pronounced effect; and amplitudes of vibration for 1 pedestrian walking at “normal” speed. For most cases, GIS information is used to locate the footbridge and a photo or sketch is annexed.

3. In Situ Campaign and Analysis of Results

3.1. In Situ Experiments and Data Treatment. *In situ* expedite experiments were performed with a single 3-component accelerometric station equipped with a “force-balance” 3-component sensors with a resolution below 0.5×10^{-2} mg, acquiring data at 200 Hz. Samples of 60 sec duration were averaged out after band-pass filtered with Butterworth with 4-poles between 0.03 and 25 Hz. Samples were visually reviewed to avoid the ones with anomalous spikes.

Data treatment consisted in analyzing peak acceleration values, predominant frequencies of vibrations in the three orthogonal directions through FFT techniques, and damping from amplitude decay in a few cases. A cross examination of results, together with the interpretation of modal shapes for the most simple geometrical layouts, led to the identification of first modal frequencies in the three directions of space (T: transversal; L: longitudinal; V: vertical). For all analyzed structures, damping is quite small with values varying in the interval $1\% > \zeta > 0.5\%$, even for large amplitude motions. All these values were transported to the database. Whenever an analytical model was available, identification of various modal shapes was achieved with this single 3-component instrument.

In several situations we repeat the *in situ* testing not only to check the robustness of the readings, but also to use different resolution instrumentation. In the first measurements we used 12-bit instruments with 100 mg of full scale and more

recently we used 18-bit at a 1000 mg full scale, which allows a much better signal/noise ratio. Frequencies were obtained in general with an error of ± 0.05 Hz.

The experiments were of two types, with measurements performed with the accelerometric station located at mid-span and at quarter-span as follows:

- (1) measurements for noise vibration produced by car traffic passing underneath, for mode identification;
- (2) measurements for a set of typified tests with the passage of pedestrians at different velocities: (a) one person at slow walking; (b) one person at normal walking; (c) one person at fast walking; (d) one person jogging (slow running); (e) forced vibration caused by the movement of one person in resonance conditions; and (f) impulsive action derived from “jumping.” For details, see [6].

This technique, using a single instrument, can only be used with confidence for footbridges with simple geometric layouts, in which modes are easily separated into the three directions and no interaction is taking place. For more complex geometries, either in plan or in the vertical direction, multiple instrumentation with common time is required for identification of frequencies and modal shapes. The case of the Ribeira da Carpinteira (Covilhã) footbridge, a S-shape plan view with two lateral steel beams ([+] cross section) is the situation where modes are coupled in the 3 directions of space with an important participation of torsion of deck [12]. Only the frequencies were identified, but it was impossible with a single 3-component instrument to assign them to any particular mode shape with the exception of the first ones. Most of the “other types” footbridges (Figure 2) fall into this category.

3.2. Analysis of Results. The main results are presented in Figure 4, where the frequencies of first mode in the T, L, and V directions are plotted against the largest span length. Various representations are made to understand the correlation of measured frequencies with the larger span. Most of the footbridges can be considered as simple supported beams with constant mass and stiffness. A curve fitting of the first frequency of vibration (f) versus the span length (L) was

FIGURE 3: Main typologies of footbridges in Portugal.

assayed for the frequency in each direction and for each typology class (Figure 2). A power function ($f = a \times L^{-\alpha}$) was used. As expected, the correlation for the vertical frequency of steel box-girders (Figure 4(e)) (especially if the two larger curvature footbridges are eliminated from the plot, $R^2 = 0.667$, not presented in Figure 4) and RC footbridges (Figure 4(g)) is much higher than for the other directions. However, for steel truss structures, the existing variety, with 2D, 3D, and so forth cases, is so large that no correlation can be observed even in the V direction. A more detailed analysis entering with other parameters, namely, the material and the location of stairways, would certainly increase

the correlation, especially in the T and L directions. But to do that and keep some statistical insight we need to perform more tests, increasing our database.

Even though the steel structures are much lighter than RC, a comparison of their vertical frequencies shows similar results (Figure 5). As the ratio (pedestrian)/(full dead-load) is much higher for steel structures than for RC structures, this causes higher amplifications in steel structures (see Section 4.2).

Figure 6 shows that the V and L frequencies do not correlate with each other, probably because the frequency in the L direction depends very much on the stiffness of columns

FIGURE 4: Frequencies of 1st mode in the three directions (T, L, and V) as a function of largest span: (a) to (c) all typologies for V, T, L, and (d) V + L; (e) to (h) vertical for steel box-girders, truss structures, RC structures, and other structures, respectively. Red curves are explained in Section 3.3.

FIGURE 5: Comparison of frequencies for steel box-girders and RC footbridges.

FIGURE 6: Correlation of V and L frequencies for all footbridges.

and on the position of stairs or ramps. If we took these aspects into consideration, the correlations would probably be much better. Numerical modelling, presented in Section 5 for various typologies and stairways locations, shows that the analytical frequencies are very similar to the *in situ* measurements in all three directions of space, precluding the better above-mentioned correlation.

3.3. Estimating the Natural Vibration Frequency of Footbridges. An empirical formula to compute the fundamental vertical frequency of a footbridge would be very useful for a designer to quickly assess its response to pedestrian crossing.

Let us consider a footbridge as a simple supported beam (length L) with constant mass $\bar{m}$ per unit length and constant inertia I , made of material with modulus of elasticity E .

The frequency (Hz) of the first mode is given by Clough and Penzien [13]:

$$f = \frac{1}{2\pi} \times \pi^2 \sqrt{\frac{EI}{\bar{m}L^4}} \quad (1)$$

which corresponds to a sinusoidal shape. Higher modes have frequencies 4, 9, and so forth times the fundamental frequency. Assuming that the maximum deflection of the beam under dead load should not surpass $L/1000$; then the frequency is only function of L and given by

$$f = 17.9 \times L^{-0.5} \quad (f \text{ in Hz; } L \text{ in m}). \quad (2)$$

For built-in supports (clamp-clamp), the first frequency is about 2.27 times larger than for simple supported. And the effect of changing the assumption of maximum deflection permitted, or of the stiffening of the supports, would only affect the initial constant, not the exponent of L .

Expression (2) is in the lower bound of decrease of frequency with length ($L^{-0.5}$). Usually the exponent, observed from the fitting ([14], after *in situ* testing of 67 footbridges, arrived to a fitting $f = 33.6 \times L^{-0.73}$, exhibiting large dispersion and not making any distinction among typologies), is higher denoting that frequency attenuates faster with L . Steel box-girders attenuate with $L^{-0.9}$, RC with $L^{-1.4}$, and other types with $L^{-0.8}$. It means that the assumption of maximum deflection which implies (3) given by

$$\frac{I}{\bar{m}} \cong f(L^3) \quad (3)$$

is not suitable and the ratio $I/\bar{m}$ is more likely proportional to L^2 . In many cases the cross section varies along the length, and the maximum deflection is not given by a simple expression as used for the simple supported beam with constant mechanical properties. Also the principle of displacement-controlled design may not apply, prevailing a constant extension (ϵ) assumption.

We plotted the values produced by (2) in Figures 4(e) and 4(h) to check the differences to the empirical fitting.

4. Characterization of Pedestrian Loading

The dynamic loading in footbridges is essentially due to the passage of persons alone, in groups, randomly walking, jogging, running, or a combination of all previous cases. Also, sudden loads provoked by jumps, fall of objects, or rhythm action may arise. Sometimes the passage of bicycles or motorbikes may be observed. The “lock in effect” is another resonant effect, induced by the bridge itself, which influences the walking pattern. We will concentrate only on the passage of a single person walking, jogging, or running. Also excitation near to resonance by one single person was also performed. However, the response of footbridges for groups of pedestrians walking in rhythm or randomly walking was not analyzed.

4.1. Pedestrian Loading Pattern. There are three levels to be considered in the definition of pedestrian loading. The first

FIGURE 7: (a) Force-time typical diagram for different movements: left-step frequency < 2.2 Hz, right-step frequency > 2.2 Hz; (b) variation in time-space imposed by the walking movement [15].

TABLE 2: Frequency range (Hz) for different patterns of movement (adapted from [14]).

	Slow	Normal	Fast	Total
Walk	1.4–1.7 Hz	1.7–2.2 Hz	2.2–2.4 Hz	1.4–2.4 Hz
Jogging	1.9–2.2 Hz	—	—	1.9–2.2
Running	—	—	3.0–3.4 Hz	3.0–3.4 Hz

one attends to the frequency of movement, resulting from a speed of 0.5 m/s to 0.8 m/s (for slow walk) to 3.5 m/s (for jogging-slow running) with a step size from 0.65 m for slow walk and not exceeding 1.7 m for fast running (Table 2).

The second and third levels are related to the contact form of the foot with the deck, with one vertical and two

horizontal components. This contact form depends on the pattern of movement [2, 15]: the contact time between the foot and the floor; the time interval among two consecutive steps; the relation between the applied force and the pedestrian's weight; and the step length. Figure 7 shows the form of how we applied the foot loading along the time-space.

We generated a load curve of the form shown in Figure 7(a) and applied it at successive nodal points, at different time intervals, Figure 7(b).

Whereas the vertical component of the load always applies in each step due to the gravitational force in the same direction, the horizontal action introduces a force alternating to the right and left, according to the stepping foot.

These functions were programmed to be used with standard linear dynamic analysis software using the time history

FIGURE 8: Amplitude values in mg of the measured maximum motion at mid-span. Series 1: noise; Series 2: 1 person walking at “normal” pace (2 Hz); Series 3: 1 person jumping from 30 cm; Series 4: excitation close to resonant conditions.

FIGURE 9: Amplitude values in mg of the measured maximum motion at mid-span (T; L; V): 1 person walking at “normal” pace (2 Hz) in different typologies (Series 2).

integration. For the L direction the intensity of loading is 50% of V, whereas in the T direction it is between 3 and 10% of V. The frequency in L direction is equal to the V direction, whereas in T direction the frequency is half of the V direction [16, 17]. These patterns are also observed in the database of the present work.

4.2. In Situ Amplitude Values. Table 3 and Figure 8 present the values of the measured maximum amplitudes of motion at mid-span for a group of 10 footbridges in the T, L, and V directions, for the following loading conditions: (1) noise; (2) 1 person walking at “normal” pace (2 Hz); (3) 1 person jumping from 30 cm; and (4) excitation close to resonant conditions. It is observed that amplitudes in the vertical direction are almost 10 times the ones in the transversal direction, and walking produces lower amplitudes than jumping or exciting in resonant conditions.

Figure 9 shows the amplitudes by typology for 1 person walking at “normal” pace. It is clear that RC structures have

much lower amplitudes than all other typologies. We can look at these differences as a phenomenon of amplification, in which the amplification of response for the same load type of steel box-girders and steel truss is much larger than for RC footbridges.

As far as amplitudes are concerned, if we take the excitation amplitude given by the equivalent single degree of freedom under resonant conditions, the values are also similar to those indicated by measurements, for the low damping observed ($1\% > \xi > 0.5\%$).

5. Code Provisions and Acceptable Comfort Levels

Currently, there are two different design procedures for footbridges under dynamic loading, which are contemplated by the international standards.

They are essentially based on discomfort and resonance: (i) peak acceleration values in the vertical and horizontal directions should not surpass certain limits for given load patterns and (ii) fundamental frequencies should be outside the so-called *interval of critical frequencies*. There are several international recommendations practiced in various countries such as BSI [18], AASHTO [19], EN 1990, 2003 [20], DIN-ENV [21], ISO 10317 [22], OHBDC, 1983 [23], or the Japanese Footbridge Design Code, 1979 [24]. If the criterion is based on the amplitude value, these recommendations fix values quite different from one another [25], such as in the case of a peak acceleration that should be below, say, 70 mg in vertical and 10–20 mg in horizontal, or a function of the footbridge frequency, Figure 10(a) and Table 4. If the criterion is separation of resonant conditions [26], the recommendations are as shown in Figure 10(b). In a different approach, Kazakevitch and Zakora [27] establish a maximum load (k), dependent on natural frequency (f), to reduce the resonance effect, given by $k = 4/(1 + 1.5f^2)$ kN/m².

TABLE 3: Amplitude values in mg of the measured maximum motion at mid-span for a group of 10 footbridges in the transverse (T), longitudinal (L), and vertical (V) directions.

Name	Length (m)	Noise			1 person walk			Jump			Excitation			Typology
		T	L	V	T	L	V	T	L	V	T	L	V	
CP_Gago Coutinho	28	0.5	0.5	0.9	2.5	2.5	12				8	5	109	Steel BG
Relógio_Gomes Costa_W	42				6.5	2.5	7	12	6	45	7	3	27	Steel BG
Braga NE	25	1.5	1	7	4	1.4	17	24	9.6	138	19	18	40	Steel truss
Av Gomes Costa (Leste)	30	1.8	2	7	5.2	5.4	32	36	7.1	79	6.7	7.6	97	Steel truss
Rego	46	3.3	1.1	5.5	4.8	1.9	7.5	10.6	10	77	4	8.1	76	Steel truss
Av. EUA_Chelas Leste	25	0.2	0.2	0.2	0.8	1.3	2.8	2.7	1	24				RC
Entrepasto Escola_Herc_Carvalho	20										2.6	4	31	RC
Vale Formoso (Chelas)	28	0.5	0.2	0.5	0.6	1	3.5	0.8	0.9	7.5	1.6	1.4	22	RC
Microsoft Porto Salvo	25	2.9	0.9	10.8	11	4.8	3.8	35	9.6	155	24	7.6	90	Other types
CC Vasco Gama Norte	75	1.6		1.9	4		5				5.2		12.5	Other types

(a)

(b)

FIGURE 10: (a) Allowable vertical accelerations as a function of frequency of vertical mode [26], (b) frequencies to be avoided [14, 17].

TABLE 4: Acceleration limits recommended in some codes (Figure 10(a)).

Regulation	Vertical acceleration (m/s ²)	Horizontal acceleration (m/s ²)
BS5400 [18]	$a_{\max} \leq 0.5 \cdot \sqrt{f}$	—
OHBDC (1983) [23]	$a_{\max} \leq 0.5 \cdot (f_0)^{0.28}$	—
Eurocode [20]	$a_{\max} \leq 0.7$	$a_{\max} \leq 0.2$
Hong Kong (2002) [28]	$a_{\max} \leq 0.5 \cdot \sqrt{f}$	$a_{\max} \leq 0.15$

Živanović et al. [11], Heinemayer et al. [29], SÉTRA [30], and HIVOSS Project [2] are three important references summarizing topics and restrictions for comfort and safety of footbridges. More recent advancements were produced and are presented in publications such as Footbridge-2008 [31] and Footbridge-2011 [32].

6. Dynamic Modelling

Four types of structures were the object of a detailed analytical study: two of them correspond to the most common types built recently in Portugal, (1) a steel box-girder and (2) a prefabricated RC I + I beam; the third (3) is a steel inclined arch with a large span; and the fourth (4) is a RC I + I beam, similar to (2) but with 3 spans and inclined stairways. Essentially, we were interested in checking how analytical models could be validated by *in situ* measurements of several modal frequencies. Also, for three cases, the response of the footbridge for pedestrian crossing was compared with the measured values.

In model definition several parameter values are uncertain, such as the elastic material properties (E , modulus of elasticity) and the detail of boundary conditions including the foundation stiffness. Another important issue is the level of discretization of the model, represented in its finite element (FE) description, which has to be sufficiently detailed to account, for instance, with torsion. In this study we just adjusted E and checked the boundary conditions. Model calibration was done taking into account the measured frequency for one specific mode shape, usually the fundamental one. Once the analytical frequency of this mode is tuned to the *in situ* measurement (sources of errors were essentially due to the supporting connections), the other analytical frequencies will adjust to the *in situ* identified frequencies, without need for further corrections to the model.

6.1. Steel Box-Girder [6]. This structure, with 2.0 m wide and spanning 25.5 m at a height of 5.2 m, has a steel box-girder deck supported in two cylindrical, partially hollow columns and connected to 2 adjacent stairways, one at each side of the deck (Figure 11). The main geometric characteristics of deck cross section are the trapezoidal shape with 65 cm height and the columns with diameter $\phi = 29.5$ cm and thickness of 1.4 cm, filled up almost to the top with concrete.

The analytical model of this footbridge, including the stairways, was made with SAP2000 [33], using both “beam” and “plate” elements (Figure 11). Table 5 compares analytical and measured frequency values. With the exception of torsion modes (which require the use of more than

one instrument), the agreement is very good, also for high frequency modes.

This structure was also subjected to a set of *in situ* experimental testings to determine the peak amplitude of motion ($a_{\max}$) for different situations as defined in Table 2. Table 6 presents the results for the three orthogonal directions and marks the values exceeding the recommended limits according to several codes. It is clear that in many instances the limits of discomfort are widely exceeded, especially for the cases of stepping at a frequency close to resonance.

Tests of the model feasibility were made to reproduce the walking of a person at different speeds, by comparing the peak acceleration amplitudes obtained in the model with the measurement *in situ* [6]. Even though difficulties arise in controlling the experimental testing (correct speed, frequency of walking), the results (Figure 12) show a good agreement with the analytical model for the vertical direction. The results are not so good for the longitudinal and transversal directions, probably due to the difficulty in reproducing the pedestrian loading in those directions. Also, in the *in situ* case, the resonance situation was never attained with walking experiments.

6.2. RC Structure [6]. The analyzed structure spans 25.6 m, has a height of approximately 5-6 m, and is 2.0 m wide, Figure 13. The deck, formed by two precast and prestressed I beams, $h = 1.23$ m, is supported by two RC columns. Between the columns and beams there are neoprene pads. The pavement is made of RC plates, with thickness of 13 cm, supported on the lower flanges (Figure 13(b)). The columns have rectangular cross section with variable dimensions 1.40×0.45 m² at the base and 0.7×0.45 m² at the top, just below the opening up for the support of the deck (Figure 13(c)).

Concrete is a B45.1 for the beams and B30.1 for the precast elements (plates, columns, and stairways). Stairways run perpendicular to the bridge axis and are supported in square columns at 1/3 height.

Similarly to what was done for the steel structure, we compare in Table 7 results (SAP2000 [33]) of measurements *in situ* for mode frequencies and present in Table 8, with analytical values for peak acceleration amplitudes. Figure 14 shows the comparison for walking situations.

From the analysis of Tables 7 and 8 and Figure 14, we can say that the analytical model reproduces quite well the measurements *in situ* for frequencies, but not so accurately for pedestrian loading. The difficulties in controlling with accuracy the experimental conditions, in what concerns the frequency of the excitation and the intensity of the pedestrian steps, are the main reasons for the deviations observed.

FIGURE 11: Steel box-girder structure under study: (a) analytical model; (b) view of column, deck, and stairway.

TABLE 5: Comparison of frequencies (Hz) between measurements *in situ* and the analytical model.

Mode direction	Measurements <i>in situ</i>	Analytical model
Longitudinal	2.54	2.55
Vertical	3.42 to 3.52	3.52
Torsion about x (longitudinal)	?	5.27
Torsion about z (vertical)	?	5.74
Transversal	6.74 to 6.84	6.38

TABLE 6: Peak acceleration amplitudes for different tests.

Walking load	$a_{\max}$ (mg)		
	Vertical	Longitudinal	Transversal
1 Person*—walk at $f = 1.8$ Hz	18.9	2.7	3.8
1 Person—walk at $f = 2.2$ Hz	29.1	5.2	6.1
1 Person—jogging at $f = 2.5$ Hz	38.2	20.2 [#]	12.1
1 Person—jogging at $f = 3.3$ Hz	251.3 [†]	16.5	21.4 [#]
1 Person—jogging at $f = 3.7$ Hz	252.6 [†]	12.5	24.5 [#]
2 Person—walk at $f = 1.7$ Hz	26.3	6.4	5.8
2 Person—walk at $f = 2.1$ Hz	38.3	25.3 [#]	7.0
2 Person—jogging at $f = 2.25$ Hz	74.4 [‡]	28.9 [#]	19.0
3 Person—walk at $f = 2.1$ Hz	71.5 [‡]	29.1 [#]	14.1
7 or 8 kids (30 to 40 kg)—slow walk	16.3	6.2	10.3
Noise	6.8	0.9	2.6
Jump	198.1 [†]	21.1 [#]	83.4 [#]
Excitation in transversal direction (resonance)	27.9	3.2	15.5
Impulse in transversal direction	39.8	3.8	17.0
Excitation in vertical direction (resonance)	239.2 [†]	11.7	16.1

*74 kg.

[†] Values of vertical accelerations exceeding the less stringent limit (93.8 mg); [‡] the most stringent limit (66.7 mg); [#] values exceeding the code ones for horizontal vibrations (20 mg, EN 1990 [20]).

TABLE 7: Comparison of frequencies (Hz) between measurements *in situ* and the analytical model.

Mode direction	Measurements <i>in situ</i>	Analytical model
Longitudinal	2.77	2.77
Vertical	3.79	3.34
Transversal	3.73	3.73
Torsion about z (vertical)	?	5.71
Torsion about x (longitudinal)	7.91	8.80

TABLE 8: Peak acceleration amplitudes for different tests.

Walking load	$a_{\max}$ (mg)		
	Vertical	Longitudinal	Transversal
1 Person*—walk at $f = 2.0$ Hz	7.3	1.7	2.9
1 Person*—walk at $f = 2.2$ Hz	10.1	1.8	3.1
1 Person*—walk at $f = 2.5$ Hz	8.3	4.9	8.0
1 Person*—walk at $f = 2.6$ Hz	11.6	12.2	13.5
1 Person*—walk at $f = 3.0$ Hz	15.5	7.8	15.8
1 Person*—walk at $f = 3.7$ Hz	32.7	11.8	25.4 [†]
2 Jumps at mid-span	61.1	34.2 [†]	90.2 [†]
Impulse in transversal direction	8.5	5.2	15.6

*74 kg.

[†] Values exceeding the code limits for horizontal vibration (20 mg). EN-1990 [20]).

FIGURE 12: Comparison of amplitudes at mid-span (vertical acceleration) between *in situ* measurements and the analytical model: 1 person walking at various speeds (steel box-girder structure). Red circles: analytical model; blue squares; *in situ* measurements; vertical line at 2.2 Hz separates “walk” from “running.”

Comparing the RC structure with the steel structure, we see that the former is much more rigid, with peak values almost 1/2 to 1/3 below the latter, depending on the direction considered. This means that RC footbridges amplify much more the response for the same loading characteristics than the steel box-girders, as already mentioned in Section 4.2.

6.3. Passage between the Orient Station and the Shopping Centre “CC Vasco da Gama”. This footbridge is located in Avenue D. João II in Lisbon and makes the connection between the Orient Station and the Shopping Centre Vasco da Gama. There are two identical footbridges, with length of 86.6 m and usable width of 2.4 m. Each one is constituted essentially by three tubular steel sections and thirty-seven pieces with variable “I” section that make the connection with the previous ones. Two of the tubular sections are straight and support the bridge deck, while the third one forms an arch that rises through the footbridge making an angle of 31.9° to the ground. The deck and lateral guards are made by thick glasses supported by secondary metallic profiles (Figure 15).

The footbridge is simply supported in the two extremes, still having an intermediate support at 1/4 span, counted

from the West side (Orient Station). The material used for the construction is the structural steel S355 (for details, see [9]).

The analytical model of this footbridge was made with SAP2000 [33]. After defining all the structure with 3D “beam” elements, the remaining permanent load of 2.48 kN/m has been applied, corresponding to the weight of the floor glasses, the guards, and the small metallic profiles that support the pavement glass plates. The final analytic model of the footbridge is presented in Figure 15(b).

The frequencies of the first 15 modes obtained by the analytical model and by *in situ* measurements are compared in Table 9.

The first vibration mode corresponds essentially to a vertical oscillation (Figure 16) and there is no significant torsion. In the higher modes the significant displacements are due essentially to lateral oscillations and rotation around the several axes, sometimes occurring the coupling of translational and torsional modes.

It should be noted that, even though the two footbridges connecting the Orient Station to Vasco da Gama Shopping Centre are essentially identical, the *in situ* measured first vibration frequency of the one located to the north is about 6% higher than the other to the south.

To simulate a pedestrian’s passage on the structure, the load functions as mentioned in Section 4.1 were used to represent the several movement types. For each analytical run the acceleration envelope was obtained for the three directions. The computed accelerations for the several movement types of a single pedestrian are always below 0.15 m/s² in the vertical and lateral directions. The value is much lower than that required by the regulations for the vertical direction. However, for the lateral direction, this value corresponds exactly to the maximum limit indicated by the Hong Kong Structures Design Manual for Highways and Railways, [28].

In general, it is verified that the results from the analytic model are conservative, because the acceleration is, in most cases, larger than the values measured experimentally. Table 10 shows the values obtained for the “normal” walking with two different acceleration transducers (number 133 and number LN874) to check the robustness of the measurements. The measurements were

FIGURE 13: RC structure under study: (a) analytical model; (b) deck cross section; (c) view of column, deck, and stairway.

TABLE 9: Comparison of frequencies (Hz) between the analytical model and measurements *in situ*.

MODES	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Analytical	0.84	1.23	1.48	2.23	2.59	3.30	3.56	4.22	4.89	5.27	5.49	6.04	6.53	6.98	7.52
<i>In situ</i>	0.88		1.7		3.12	3.55	4.52	5.06	5.66			6.59

FIGURE 14: Comparison of amplitudes at mid-span (vertical acceleration) between *in situ* measurements and the analytical model: one person walking at various speeds (RC structure). Red circles: analytical model; blue squares: *in situ* measurements; vertical line at 2.2 Hz separates “walk” from “running.”

TABLE 10: Acceleration measured at 1/2 span for the normal walking.

Direction	Acceleration at 1/2 span [mg]		
	Measuring device	Analytic model	
	133	LN874	
Longitudinal	0.25	0.20	0.24
Lateral	2.00	2.00	2.00
Vertical	3.50	3.20	4.50

carried out at the same location but in a different epoch of the year.

These acceleration obtained for one or two pedestrians is below the maximum values indicated by the design guidelines, so for one or two persons the footbridge accomplishes the regulations. However, it is foreseen that for people's groups these limits are no longer respected. According to research related to the Millennium Bridge, there is a critical number N of pedestrians needed to bring about the situation

of lateral *lock-in* where feedback of lateral forces cancels out the positive action of damping of the structure, resulting in unbounded growth of response [34]:

$$N = \frac{8\pi \cdot f_i \cdot \xi \cdot m_i}{k}, \quad (4)$$

where f_i , ξ , and m_i are lateral mode i frequencies, damping, and unit-normalized modal masses, and k is an empirical constant estimated as 300 Ns/m.

Considering that the vibration mode which has significant lateral oscillation is mode 4, $f = 2.23$ Hz, with a participation mass about 35% in this direction, the estimated N value for this footbridge is 6 pedestrians. Assuming a structure total span of 87 m (74 m largest span), it is easy to reach that number in normal conditions of use, because the total time needed to cross the bridge, is very large and many pedestrians can be present simultaneously.

The solution to this type of problem can be found by introducing mass-tuning dampers, which will reduce significantly the vibration levels compatible to the estimated pedestrian flux [11, 35]).

6.4. Overpass RC Footbridge with Precast ($I + \text{slab} + I$). The work developed by Oliveira [10] focuses on the vulnerability of a footbridge located near Faro, south of Portugal, serving as an overpass on EN125. This footbridge was chosen based on its location, in a region of high seismic load. In a circle of 8 km there are 9 very identical footbridges, which vary only in the span length and disposition of the lateral access that can be in form of stairs or ramps. These ways of communication are very important for the region in a scenario of catastrophe caused by a seismic event, especially because EN125 is the main route for accessing the hospital and for evacuating the population in case of tsunami.

In here we concentrate only on the comparison of frequency values from *in situ* and from a simplified analytical model through a dynamic linear analysis with SAP2000 [33].

FIGURE 15: (a) Vision of the CC Vasco da Gama (Calatrava) structure; and (b) analytical model.

FIGURE 16: First vibration mode with $f = 0.84$ Hz with predominant vertical movement: (a) front view; (b) from above; (c) lateral view.

FIGURE 17: RC footbridge in Faro: (a) front view; (b) cross-section with precast (I + slab + I); (c) central column (longitudinal view); (d) column-beam connection (units in m).

FIGURE 18: Modes of vibration: (a) 1st longitudinal; (b) 2nd transversal; (c) 3rd vertical, central span.

The whole structure consists of prefabricated elements of reinforced concrete. The deck is 1.63 m wide and 45.3 m long, developed in a straight line, divided in 3 spans, Figure 17. The cross section of deck is made of precast RC (I + slab + I) and columns have an elongated shape almost constant in height, with $0.82 \times 0.5 \text{ m}^2$ at the base, but forming an enlargement when approaching the deck support.

The connection between the beams and columns is provided by two vertical bars of 20 mm on top of these columns, crossing the ends of each beam through holes made during the molding of the pieces. The holes were filled during the assembly phase, by a grout. A neoprene plaque was interposed between the column and the beam to distribute the contact stresses.

The access to the deck is made through four flights of stairs in each side. The stairs are almost perpendicular to the deck. Each set of stairs is formed by an independent prefabricated part, supported only on the ends. The steel used in reinforcing concrete was A500NR. Three types of concrete were used in this footbridge: C20/25 for the foundations, C35/45 in the prestressed beams, and C25/30 in the other parts. The concrete cover is 2.5 cm in the columns and beams and 5 cm in the foundations elements. Each span of the deck is formed by two prestressed reinforced concrete I beams.

The deck was modulated as a single bar with cross section as given in Figure 17(a). This option has been taken to ensure the simplicity of the model, considering that the links are used to guarantee that the deck behaves as only one piece. To simplify the model, the section of the columns was considered constant, with $0.82 \times 0.5 \text{ m}^2$. The height of the columns in the model is equal to reality. The stairs were modulated as a beam element with $1.6 \times 0.2 \text{ m}^2$ and the stairs columns with a square section of $0.3 \times 0.6 \text{ m}^2$ throughout its height.

The connection between the columns and other elements was made through steel bars. To simulate the contribution of these bars in the absorption of flexural moments, without giving excessive rigidity to allow for some flexibility, the modelling was done considering that on top of the columns there is an elastic support given by springs, which absorb some of the existing moments. These springs also help to simulate the effect of the slab, because the beams have no continuity above the columns, but the slab does.

The analysis of the *in situ* records led to the following values (Figure 18 and Table 11): the first mode is longitudinal, with a frequency of 2.2 Hz; the second mode is transversal, with a frequency of 2.6 Hz; the third mode is vertical in the central span (22 m), with a frequency of 3.6 Hz; the right lateral span presents a vertical vibration mode with a frequency of 10.7 Hz; the left lateral span has a vertical vibration mode with a frequency of 13.9 Hz. The damping ratio is 1.0%.

From Table 11 we can see an excellent agreement between the *in situ* and the analytical model for different spans. It is worth noting that the presence of stairways influences the first two mode shapes (L and T directions).

7. Conclusions and Final Remarks

We have been creating a database with geometrical and mechanical properties of many footbridges built in Portugal in the last two decades, representing the most common structural types. These structures, with larger spans varying from 11 to 110 m long, tend to become light and slender structures made of diverse materials and cross sections. They may show a pronounced dynamic amplification with the passage of pedestrians due to possible resonant effects.

TABLE II: Comparison of frequencies (Hz) between measurements *in situ* and the analytical model.

Mode direction	Span 22 m		Span 12.5 m		Span 10.7 m	
	<i>In situ</i>	Analytical	<i>In situ</i>	Analytical	<i>In situ</i>	Analytical
Longitudinal	2.2	2.25	—	—	—	—
Transversal	2.6	2.69	—	—	—	—
Vertical	3.58	3.51	10.7	10.64	13.5	14.5

These large vibration amplitudes may be excited to a point of discomfort or even structural failure, and so the phenomenon deserves special attention [7].

We present a simple method to derive the main dynamic properties of these structures based on experimental *in situ* testing for loading caused by pedestrian crossing, allowing the estimation of most important frequencies and damping and amplification factors. This database, with 79 structures organized in 5 typologies, allow the setting of correlations of fundamental frequencies as a negative power function of span lengths L ($L^{-0.5}$ to $L^{-1.5}$), for different typologies and access situations. An analytical expression for the vertical direction, the sole direction exhibiting a reasonable correlation with span length, (2), based on simplified assumptions, is in the lower bound of attenuation with length ($L^{-0.5}$). Usually, frequency decreases faster with L : steel box-girders attenuate with $L^{-0.8}$, RC with $L^{-1.4}$, and “other types” with $L^{-0.8}$.

The experimental results are essential to calibrate analytical studies (presented for four cases) and detect sources of errors, essentially due to the supporting connections and a few simplifications in the model. The localization and type of stairways influences drastically the frequencies and modal shapes, especially in the transversal and the longitudinal directions of the structures and, consequently, the vibration levels.

Once the analytical model has been calibrated for the first frequency, agreement of other higher frequencies between measured and modelling was very good.

In relation to amplitudes, the results are not always so good, especially for RC structures, due to difficulty in setting proper pedestrian loading, so that the loading model well represents the *in situ* testing. Only with an accurate measurement of the input loading during the pedestrian passage, as Caetano et al. 2011 [1] tried to do, it would be possible to get better matching results between *in situ* and analytical modelling. Due to the difficulty in measuring accurately the input of a single pedestrian loading, modern guidelines are increasingly relying on the use of stochastic models instead of single pedestrian loading [2].

Despite these small details which need improvements in both the experimental techniques (for measuring the dynamic applied forces) and in the analytical modelling, we can say that both the used *in situ* expedite technique and linear modelling are quite robust for footbridges of simple geometry. For complex geometries *in situ* multiple instrumental setup is required for modal identification, together with a force-measurement setup provoked by the pedestrian passage.

We believe that the information contained in this database, involving a number of different structural types,

allows the extrapolation of results to similar footbridges in other countries.

As a final result of practical importance, the measured peak acceleration values under pedestrian loading for steel structures are well above the limits defined in the international recommendations, especially for the case of jogging. This aspect confirms the idea, already stated in previous studies [7], that the majority of the steel footbridges in Portugal and in particular in the region of Lisbon suffer of excessive amplitude of vibration.

For the RC structures the situation is reversed. The measured peak acceleration values under pedestrian loading are well contained within the limits of the international recommendations, causing a better sensation of comfort and structural safety.

Several topics should be developed in future work:

- (i) develop standard experimental techniques for routine testing of footbridges based on simple methods as the one presented to obtain the transfer function of these types of structures, making use of *in situ* load/deflection input and frequency/damping output estimations;
- (ii) use more parameters of the collected data to reduce dispersion on the correlations;
- (iii) from the convolution of the results obtained with the passage of a single person, obtain a simple analytical formula to estimate peak amplitude motion as function of number of people and velocity of crossing. Stochastic representation of pedestrian crossing can be analyzed in future work if we use the response of a footbridge produced by a single person at different speeds as a “green function” to obtain the stochastic formulation;
- (iv) develop dissipation systems to damp out the large amplitudes of vibration observed in steel and long spanned structures, as suggested by Caetano et al. [35];
- (v) estimate seismic vulnerability functions for these structures, as their collapse over main road access lines may be critical in case of earthquake emergency.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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