Research Article

Numerical Study of the Active Tendon Control of a Cable-Stayed Bridge in a Construction Phase

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This paper investigates numerically the active tendon control of a cable-stayed bridge in a construction phase. A linear Finite Element model of small scale mock-up of the bridge is first presented. Active damping is added to the structure by using pairs of collocated force actuator-displacement sensors located on each active cable and decentralized first order positive position feedback (PPF) or direct velocity feedback (DVF). A comparison between these two compensators showed that each one has good performance for some modes and performs inadequately with the other modes. A decentralized parallel PPF-DVF is proposed to get the better of the two compensators. The proposed strategy is then compared to the one using decentralized integral force feedback (IFF) and showed better performance. The Finite Element model of the bridge is coupled with a nonlinear cable taking into account sag effect, general support movements, and quadratic and cubic nonlinear couplings between in-plane and out-of-plane motions. Finally, the proposed strategy is used to control both deck and cable vibrations induced by parametric excitation. Both cable and deck vibrations are attractively damped.

1. Introduction

In the past few decades, design and construction of civil structures showed a very deep evolution because of the technological progress in materials and devices. Cable-stayed bridges increased considerably their center span from 182.6 m (Stromsund Bridge in Sweden) to 1104 m (Russky Bridge in Russia). These structures are getting more slender, light, and flexible which makes them sensitive to vibrations induced by wind, traffic, waves, or even earthquakes. Consequently, vibration control has become a major issue in civil engineering.

Vibrations in cable-stayed bridges may be reduced using passive [1–4], semiactive [5–12], and active methods [13]. Active control uses a set of actuators and sensors connected by feedback or feed forward loops. Among the proposed devices to control vibrations of cable-stayed bridges are the active mass dampers [14], active aerodynamic appendages

[15], and active tendons. Several strategies have been proposed for the active tendon control of the global modes of bridges, as well as for the in-plane and out-of-plane cable vibrations. Yang and Giannopolous [16] were the first to propose active tendon control to reduce vibration induced by strong wind gusts. They studied the feedback control of a simple continuous beam model suspended by four stay cables using four active tendons equipped with servohydraulic actuators. With respect to the motion of the bridge deck detected by the sensors installed at the anchorage of each cable, the actuators actively change the cable tension and apply time-varying forces to the deck in order to reduce the vibrations. Fujino and Susumpow [17] carried out an experimental study on active control of planar cable vibration by axial support motion. Using a cable-supported cantilever beam model, Warnitchai et al. [18] performed an analytical and experimental study on active tendon control of cable-stayed bridges. They demonstrated that the vertical global

mode of the bridge can be damped with a linear feedback of the girder velocity on the active tendon displacement and that the in-plane local cable vibration can be controlled efficiently by sag induced forces. Kobayashi et al. [19] studied the tendon control of cable-stayed bridges by setting active cables parallel to stay cables. They conducted an experimental study on a 1/100 scale half-span model of a 410 m center span cable-stayed bridge to demonstrate the effectiveness of their control strategy using a tendon control force proportional to the velocity of the girder.

All the studies on active tendon control presented above used noncollocated pairs of actuator sensor which may destabilize the structure for certain gain values and may also cause spillover instability. Achkire and Preumont [20] solved this problem using a collocated displacement actuator-force sensor configuration. They considered the active vibration control of cable-stayed bridges with an active tendon controlling the axial displacement of the cable anchor point. By using a piezoelectric actuator collocated with a force sensor measuring the cable tension, integral force feedback (IFF) is applied to offer an active damping control. An experimental setup consisting of a cable in connection with a spring-mass system was tested to evaluate control efficiency. Bossens and Preumont [21] proposed a simplified linear theory to predict the closed-loop poles with a root locus technique and reported an experimental study of two cable-stayed bridge models using active tendon control. The first one is a small size mock-up (3 m-length) representative of a cable-stayed bridge in a construction phase. The second mock-up is a 30 m length cable-stayed cantilever structure, equipped with hydraulic actuators. The experimental results showed that the active tendon control brought a substantial reduction in the deck and cable vibration amplitudes. Using the same control strategy (decentralized collocated IFF), El Ouni et al. [22, 23] studied numerically and experimentally the effect of active tendon control on the principal parametric resonance of a stay cable using a small scale mock-up of a cable-stayed bridge [24]. They showed that the threshold excitation amplitude of the deck, needed to trigger the parametric excitation, increases by an increase of the active damping in the structure. Other active control laws can be also used in a similar way as the IFF, such as first order positive position feedback (PPF) proposed by Baz et al. [25] and direct velocity feedback (DVF) proposed by Balas [26].

This paper investigates numerically the active tendon control of a small scale mock-up of a cable-stayed bridge in a construction phase. Active damping is added to the structure by using pairs of collocated force actuator-displacement sensors located on each active cable. This configuration is first examined with decentralized PPF and DVF. Then, a parallel PPF-DVF is proposed to get the better of the two compensators and compared to the one using decentralized IFF. A Finite Element model of the bridge is coupled with a nonlinear cable which takes into account sag effect, general support movements, and quadratic and cubic nonlinear couplings between in-plane and out-of-plane motions. Finally, the proposed strategy is used to control both deck and cable vibrations induced by parametric excitations.

FIGURE 1: Description of the main components of the bridge.

FIGURE 2: FE model of the bridge created by Matlab/SDT.

2. The 3D Finite Element Model

A model of a smart cable-stayed bridge was developed in Active Structure Laboratory at ULB [24]. This mock-up represents a small scale model of the bridge in a construction phase. The bridge is made of a central steel pillar resting on a concrete block and a deck supported by 8 stainless steel cables. The deck is made of two U-shaped aluminum beams, steel rectangular stiffeners, and forty additional masses (see Figure 1). The height of the pillar is 1.6 m; the total length and width of the deck are, respectively, 3 and 0.32 m. The Matlab/SDTools software has been used to build a 3D Finite Element (FE) model of the bridge (see Figure 2). Shell elements are used to model the pillar, the U-shaped beams, and the stiffeners. The additional masses are modeled by 3D elements and the cables are represented by linear bars. A clamped support condition at the lower end of the pillar is adopted. Thus, the final bridge model is composed of 29172 nodes and 23743 elements and has 112980 degrees of freedom. The natural damping of all modes of the structure is equal to 1%. For more details about the bridge demonstrator, the FE

model, and the numerical and experimental modal analysis see [23, 24].

3. Active Tendon Control Using Decentralized PPF and DVF

The global equation of motion of the linear cable-stayed bridge equipped with pairs of a force actuator and a displacement sensor in the chosen active cables (n) can be written as follows:

$$M\ddot{x} + C\dot{x} + Kx = B^n F_{\text{cont}} + F_{\text{excit}}, \quad (1)$$

where M , C , and K are, respectively, the mass, damping, and the stiffness of the bridge. $\ddot{x}$, $\dot{x}$, and x are, respectively, the acceleration, velocity, and displacement vectors. B is the influence matrix relating the local coordinate systems of the active tendons to the global coordinates. F_{excit} is the excitation force vector. ${}^n F_{\text{cont}}$ are the control forces.

The control forces of the decentralized DVF [27] are

$${}^n F_{\text{cont}} = {}^n H_1(s) ({}^n x_i - {}^n x_j) = -{}^n g_1 s ({}^n x_i - {}^n x_j), \quad (2)$$

where ${}^n H_1(s)$ is the feedback control law of the DVF, s is the Laplace variable, ${}^n g_1$ are the controller gains, and $({}^n x_i - {}^n x_j)$ are the relative displacements of the extremities (i and j) of the cables projected on the chord lines.

The control forces of the decentralized first order PPF [27] are

$${}^n F_{\text{cont}} = {}^n H_2(s) ({}^n x_i - {}^n x_j) = \frac{{}^n g_2}{1 + \tau s} ({}^n x_i - {}^n x_j), \quad (3)$$

where ${}^n H_2(s)$ is the feedback controller law of the PPF, ${}^n g_2$ are the controller gains, and τ is a design parameter which decides the damping ratio, defines the position of the pole of the first order PPF on the real axis, and fixes the stability margin.

The main idea in developing a decentralized parallel PPF-DVF strategy is as follows: can we get the better of the two compensators in order to control the maximum of modes?

The control forces of the proposed decentralized parallel PPF-DVF strategy are

$$\begin{aligned} {}^n F_{\text{cont}} &= {}^n H_3(s) ({}^n x_i - {}^n x_j) \\ &= \left[-{}^n g_1 s + \frac{{}^n g_2}{1 + \tau s} \right] ({}^n x_i - {}^n x_j), \end{aligned} \quad (4)$$

where ${}^n H_3(s)$ is the feedback controller law of the proposed concept and the controller gains ${}^n g_1$ and ${}^n g_2$ can be tuned to get optimal damping on the target modes. The block diagram of the proposed control system is given in Figure 3.

4. Comparison between Different Control Strategies

Figure 4(a) shows the root locus of the DVF added through the four small tendons. This control law is unconditionally

FIGURE 3: Block diagram of the proposed control system.

stable for all gain values, since all loops are contained in the left side of the imaginary axis. Figure 4(b) shows the root locus of the first order PPF added through the four small tendons. The PPF is conditionally stable and has the same poles and zeros as in the DVF case, because the two controllers use the same actuator and sensor configuration. When the pole travelling on the real axis reaches the origin (the stability limit), the controller becomes unstable. In fact, when the pole reaches the stability limit the negative stiffness of the controller should exceed the static stiffness of the system, which leads to the static collapse of the bridge. For some modes, only the initial part of the loop is available, because of the stability condition. Note also that in the PPF case, the loops do not leave the open loop poles orthogonal to the imaginary axis as in the DVF case (as a result of the negative stiffness which softens the system), which suggests that the control effort may be larger [27].

Figure 4(c) shows the root locus of the proposed strategy. It is also conditionally stable but the loops of the high frequencies are wider than those of the PPF and similarly for the low frequencies which seem wider than those of the DVF. The major advantage of the proposed strategy is that the size of the loops can be tuned not only through τ but also through ${}^n g_1$ and ${}^n g_2$.

The maximum damping ratio for decentralized DVF, PPF, and parallel PPF-DVF is determined for the first 17 modes using the root locus technique and is plotted in Figure 5 as a function of mode number. The PPF seems more efficient for modes 1, 3, 5, 6, and 9. The DVF is more efficient for modes 8, 10, 11, 13, 14, and 15. The parallel PPF-DVF has important damping for many modes. With all the control strategies, weak controllability is observed for modes 2, 4, 12, 16, and 17. Figure 6 shows the FRF between the white noise excitation (F_{excit}) through the cable number 2 and the vertical displacement of the girder (U_z) in point A with different control strategies. Using the parallel PPF-DVF, the negative stiffness effect of the PPF is reduced and an average FRF is obtained between the FRF with PPF and the one with DVF.

5. Comparison between Parallel PPF-DVF and IFF

The decentralized IFF [21] uses a collocated pairs of displacement actuator-force sensor in each active cable. The IFF

FIGURE 4: Root locus of the DVF (a), the first order PPF (b), and the parallel PPF-DVF (c) added through four small tendons.

is unconditionally stable but suffers from negative stiffness problem which may be solved by adding a 2nd order high pass filter (2 Hz) in series with the IFF [21]. The maximum damping ratio for decentralized IFF and parallel PPF-DVF are compared for the first 17 modes (see Figure 7). Both strategies successfully provide the cable-stayed bridge with

active damping but the parallel PPF-DVF shows better performance for all modes except mode number 7. The proposed strategy is conditionally stable and also has a problem of negative stiffness which must be treated carefully for real applications. The FRF between the force of excitation (F_{excit}) and the vertical displacement of the deck (U_z) in point A is

FIGURE 5: Maximum damping ratio as a function of mode number for different active control strategies.

FIGURE 6: FRF between the force of excitation (F_{excit}) and the vertical displacement of the deck (U_z) in point A, for different active control strategies.

plotted in Figure 8 for the two strategies with a maximum damping on mode number 1.

6. Active Tendon Control of a Nonlinear Cable-Stayed Bridge under Parametric Excitation

6.1. Nonlinear Modelling of an Inclined Small Sag Cable. The nonlinear model of the inclined cable takes into account general support movement, sag effect, and quadratic and cubic nonlinear couplings between in-plane and out-of-plane motions. The cable model is presented in Figure 9. The local coordinate system is chosen such that the x -axis is defined along the chord line and y -axis in the horizontal plane.

FIGURE 7: Maximum damping ratio as a function of mode number for IFF and parallel PPF-DVF concepts.

FIGURE 8: FRF between the force of excitation (F_{excit}) and the vertical displacement of the deck (U_z) in point A for IFF and parallel PPF-DVF concepts.

The z -axis is then taken perpendicular to the chord line, in the gravity plane. The cable displacements are separated into three parts: the static, the quasi-static, and the dynamic contributions (for more details see [28]).

6.1.1. Out-of-Plane Cable Motion. The transverse out-of-plane displacements of the cable are described by the following equation of motion governing the generalized coordinates y_n of the n th out-of-plane mode of vibration:

$$\frac{1}{2}ml \left\{ \ddot{y}_n + 2\xi_{yn}\omega_{y_n}\dot{y}_n + \frac{n^2\pi^2}{ml^2} (T_0 + T_q + T_d) y_n \right\} = F_{y_n} - \frac{ml}{n\pi} (\ddot{v}_a + (-1)^{n+1}\ddot{v}_b), \quad (5)$$

FIGURE 9: 3D model of an inclined cable with general support movements.

where m is the mass per unit length; l is the chord length of the cable; ξ_{y_n} , ω_{y_n} , and F_{y_n} are, respectively, the modal damping, the frequency, and the modal component of the external forces applied to the cable, associated with the generalized coordinates y_n of the cable mode n ; T_0 , T_q , and T_d are, respectively, the static tension in the cable at its equilibrium, the tension increment induced by the support movement, and the tension increment induced by the dynamic motion of the cable; T_d is responsible for the quadratic and cubic nonlinear couplings between in-plane and out-of-plane motions; $\ddot{v}_a$ and $\ddot{v}_b$ are, respectively, the transverse acceleration of the anchorage points a and b according to the y -axis.

The expressions of T_q and T_d are given in the Appendix.

6.1.2. In-Plane Cable Motion. The in-plane displacements of the cable (perpendicular to its chord line) is described by the following equation of motion governing the generalized coordinates z_n of the n th in-plane mode of vibration and accounting for the gravity effect ($\gamma = \rho g \cos \theta$):

$$\begin{aligned} \frac{1}{2}ml \left\{ \ddot{z}_n + 2\xi_{z_n} \omega_{z_n} \dot{z}_n + \frac{n^2 \pi^2}{ml^2} (T_0 + T_q + T_d) z_n \right\} \\ = F_{z_n} - \frac{ml}{n\pi} (\ddot{w}_a + (-1)^{n+1} \ddot{w}_b) \\ + \frac{ml^2 E_q \gamma (1 + (-1)^{n+1})}{(\sigma^s)^2 (n\pi)^3} (\ddot{u}_b - \ddot{u}_a) \\ - \frac{\gamma A l (1 + (-1)^{n+1})}{T_0 n\pi} T_d, \end{aligned} \quad (6)$$

where E_q is the effective modulus of elasticity (see Appendix), γ is the component of distributed weight along the cable, ρ is the cable density, g is the gravity, θ is the angle of the chord line with respect to the horizontal, A is the cross section of the cable and σ^s is the static stress, ξ_{z_n} , ω_{z_n} , and F_{z_n} are, respectively, the modal damping, the frequency, and the

FIGURE 10: Principle of coupling between the FE model of the bridge and the nonlinear cables.

modal component of the external forces applied to the cable, associated with the generalized coordinates z_n of the cable mode n . $\ddot{w}_a$ and $\ddot{w}_b$ are, respectively, the in-plane acceleration of the anchorage points a and b according to the z -axis. $\ddot{u}_a$ and $\ddot{u}_b$ are, respectively, the longitudinal acceleration of the anchorage points a and b according to the y -axis.

6.2. Coupling between the Nonlinear Cables and the FE Model of the Bridge. As an alternative to a general nonlinear Finite Element approach which would be extremely time consuming, we had developed, using SDTools [29] and Matlab/Simulink, software which combines a Finite Element model of the linear structure with a nonlinear analytical model of the cables accounting for general support movement and cubic and quadratic couplings between in-plane and out-of-plane motions of the cable. Figure 10 shows the principle of coupling between the FE model of the bridge and the nonlinear cables: the structure motion imposes displacements to the cables supports and the reactions of the cables supports act like external forces to the structure. Using SDTools, it can be achieved numerically by creating pairs of collocated force actuator-displacement sensors in the anchorage points and coupling the cables to the rest of the structure through Simulink (for more details about the coupling see [23]).

Taking into account the nonlinear dynamics of the n_c cables and active damping, the global equation of motion of the cable-stayed bridge can be expressed in modal coordinates as follows:

$$\begin{aligned} \mu_i \{ \ddot{e}_i + 2\xi_i \omega_i \dot{e}_i + \omega_i^2 e_i \} \\ = F_i + \phi_i^T B^n F_{\text{cont}} \\ - \phi_i^T \sum_k^{n_c} \left[(L_a^{kT}, L_b^{kT}) (F_{u_a}^k, F_{v_a}^k, F_{w_a}^k, F_{u_b}^k, F_{v_b}^k, F_{w_b}^k)^T \right], \end{aligned} \quad (7)$$

where μ_i , ξ_i , ω_i , and F_i are, respectively, the modal mass, the modal damping, the frequency, and the modal component of the external forces applied to the bridge without cables, associated with the generalized coordinates e_i of the bridge mode i . ϕ_i represents the mode shapes of the bridge without cables. L_a^{kT} and L_b^{kT} are the transformation matrices allowing the transformation from the global coordinates of the bridge to the local coordinates of the cable k . $F_{u_a}^k$, $F_{v_a}^k$, $F_{w_a}^k$, $F_{u_b}^k$, $F_{v_b}^k$,

FIGURE 11: Description of the numerical experience.

and $F_{w_b}^k$ are the reaction forces on the anchorage points (a and b) written in the local coordinates of the cable k (see Appendix). ${}^n F_{\text{cont}}$ are the control forces of the n active cables and are given in (4).

The equations of motions of the cables and the bridge are solved simultaneously and interactively using the fourth and fifth order Dormand-Prince Runge-Kutta method.

6.3. Parametric Excitation. In cable-stayed bridges, the presence of many low frequencies in the deck or tower and in the stay cables may give rise to parametric excitation. The coupling between a local cable and a global structure makes the bridge sensitive to very small motion of the deck or tower which may cause dynamic instabilities and very large oscillations of the stay cables (see Figure 11). This may occur when the frequency of the anchorage motion is close to the fundamental frequency or twice the first natural frequency of the cable.

In order to produce a principal (first order) parametric excitation corresponding to a fundamental natural frequency of the in-plane mode (6.77 Hz) equal to the half of the frequency of the first symmetric flexural mode shape of the bridge (13.55 Hz), the tension of cable number 2 is tuned. Active damping is added through the four short active cables using decentralized parallel PPF-DVF strategy. Then, the global flexural mode had been harmonically excited by a frequency equal to 13.55 Hz and force amplitude of 2 N through the actuator of cable number 1. Finally, the in-plane midspan motion of cable number 2 and the deck vibration in the anchorage point A in the vertical direction had been recorded. Figure 11 describes the principle of the numerical experience. In order to produce a fundamental (second order) parametric excitation, the same numerical experience described above is repeated but the cable tension is tuned to obtain a fundamental natural frequency of the in-plane mode equal to the frequency of the first symmetric flexural mode shape of the bridge. The evolution in time of the in-plane motion of cable number 2 at midspan ($L/2$) and the deck vibration in the anchorage point A in the vertical direction, under principal and fundamental parametric excitations, are plotted in Figures 12(a) and 13(a) for both cases, with and without active control. The amplitude of the deck is well

damped and the parametric resonance is cancelled. Figures 12(b) and 13(b) show the trajectory of the cable at midspan before control triggering and then during the first 10 seconds after switching on the control and finally during the last 10 seconds. The cable is attractively damped for both in-plane and out-of-plane motions.

7. Conclusions

The active tendon control of a cable-stayed bridge in a construction phase had been investigated numerically. Active damping is added to the structure by using pairs of collocated force actuator-displacement sensor located on each active cable and decentralized first order positive position feedback (PPF) or direct velocity feedback (DVF). A comparison between these two compensators showed that each one has good performance for some modes and performs inadequately with the other modes. A parallel PPF-DVF is proposed to get the better of the two compensators. The proposed strategy is then compared to the one using decentralized integral force feedback and showed better performance. Finally, the proposed strategy is applied to a nonlinear model of a cable-stayed bridge in order to control both deck and cable vibrations induced by parametric excitation. Both cable and deck vibrations are attractively damped. As a future work, a modal analysis of the cable-stayed bridge will be carried out during all the construction phases. The proposed control strategy will be improved to be adaptive to different phases of construction and semiactive tendon control of the cable-stayed bridge using MR dampers will also be investigated.

Appendix

The Irvine parameter is

$$\lambda^2 = \left(\frac{\rho l g \cos \theta}{\sigma^s} \right)^2 \frac{E}{\sigma^s}, \quad (\text{A.1})$$

where ρ is the cable density, g is the gravity, θ is the angle of the chord line with respect to the horizontal, E is the modulus of elasticity, and σ^s is the static stress.

The effective modulus of elasticity is

$$E_q = \frac{1}{1 + (\lambda^2/12)} E. \quad (\text{A.2})$$

The tension increment induced by the support movement is

$$T_q = T_q^{(1)} + T_q^{(2)}, \quad (\text{A.3})$$

where

$$\begin{aligned} T_q^{(1)} &= E_q A \frac{u_b - u_a}{l}, \\ T_q^{(2)} &= EA \left(1 + \frac{E_q}{\sigma^s} \frac{\lambda^2}{12 + \lambda^2} \right) \frac{(u_b - u_a)^2}{2l^2} + EA \frac{(v_b - v_a)^2}{2l^2} \\ &\quad + EA \left(1 + \frac{\lambda^2 \sigma^s}{12E} \right) \frac{(w_b - w_a)^2}{2l^2}, \end{aligned} \quad (\text{A.4})$$

FIGURE 12: (a) Evolution in time of the vertical deck vibration and the in-plane cable vibration at $L/2$ (control on at 20 s); (b) cable trajectory at $L/2$ before and after control (case of principal parametric excitation).

FIGURE 13: (a) Evolution in time of the vertical deck vibration and the in-plane cable vibration at $L/2$ (control on at 20 s); (b) cable trajectory at $L/2$ before and after control (case of fundamental parametric excitation).

where u_a, u_b, v_a, v_b, w_a , and w_b are the movements imposed to anchorage points.

The tension increment induced by the dynamic motion of the cable is

$$T_d = T_d^{(1)} + T_d^{(2)}, \quad (\text{A.5})$$

where

$$\begin{aligned} T_d^{(1)} &= \frac{EA^2\gamma}{T_0} \sum_n \left[\frac{z_n}{n\pi} (1 + (-1)^{n+1}) \right], \\ T_d^{(2)} &= \frac{EA}{2} \sum_n \left(\gamma_n^2 \frac{n^2 \pi^2}{2l^2} \right) + \frac{EA}{2} \sum_n \left(z_n^2 \frac{n^2 \pi^2}{2l^2} \right) \\ &\quad - \frac{E_q EA\gamma}{(\sigma^s)^2} \frac{u_b - u_a}{l} \sum_n \left[\frac{z_n}{n\pi} (1 + (-1)^{n+1}) \right]. \end{aligned} \quad (\text{A.6})$$

The reaction forces on the cable anchorage points a and b are expressed as follow:

$$\begin{aligned} F_{u_a} &= \frac{1}{2} ml \left\{ c_1 (\ddot{u}_b - \ddot{u}_a) + \ddot{u}_b + c_2 (\ddot{w}_b - \ddot{w}_a) \right. \\ &\quad \left. + c_3 \ddot{w}_a + c_4 \sum_n \frac{1 + (-1)^{n+1}}{(n\pi)^3} \ddot{z}_n \right\} \\ &\quad + (T_0 + T_q + T_d) \\ &\quad \times \left\{ c_5 + c_6 \sum_n \frac{1 + (-1)^{n+1}}{(n\pi)} z_n + c_7 (u_b - u_a) \right\}, \\ F_{u_b} &= \frac{1}{2} ml \left\{ -c_1 (\ddot{u}_b - \ddot{u}_a) + \ddot{u}_b + c_8 (\ddot{w}_b - \ddot{w}_a) \right. \\ &\quad \left. - c_3 \ddot{w}_a - c_4 \sum_n \frac{1 + (-1)^{n+1}}{(n\pi)^3} \ddot{z}_n \right\} \\ &\quad - (T_0 + T_q + T_d) \\ &\quad \times \left\{ c_5 + c_6 \sum_n \frac{1 + (-1)^{n+1}}{(n\pi)} z_n + c_7 (u_b - u_a) \right\}, \\ F_{v_a} &= \frac{1}{2} ml \left\{ -\frac{2}{3} (\ddot{v}_b - \ddot{v}_a) + \ddot{v}_b + 2 \sum_n \frac{1}{n\pi} \ddot{y}_n \right\} \\ &\quad - (T_0 + T_q + T_d) \frac{v_b - v_a}{l}, \\ F_{v_b} &= \frac{1}{2} ml \left\{ \frac{2}{3} (\ddot{v}_b - \ddot{v}_a) + \ddot{v}_a + 2 \sum_n \frac{(-1)^{n+1}}{n\pi} \ddot{y}_n \right\} \\ &\quad + (T_0 + T_q + T_d) \frac{v_b - v_a}{l}, \end{aligned}$$

$$\begin{aligned} F_{w_a} &= \frac{1}{2} ml \left\{ c_9 (\ddot{w}_b - \ddot{w}_a) + \ddot{w}_b - c_{10} (\ddot{u}_b - \ddot{u}_a) \right. \\ &\quad \left. + c_{11} \ddot{u}_a + 2 \sum_n \frac{1}{n\pi} \ddot{z}_n \right\} \\ &\quad + c_{12} (T_0 + T_q + T_d) (w_b - w_a), \\ F_{w_b} &= \frac{1}{2} ml \left\{ -c_9 (\ddot{w}_b - \ddot{w}_a) + \ddot{w}_a + c_8 (\ddot{u}_b - \ddot{u}_a) \right. \\ &\quad \left. - c_{11} \ddot{u}_a + 2 \sum_n \frac{(-1)^{n+1}}{n\pi} \ddot{z}_n \right\} \\ &\quad - c_{12} (T_0 + T_q + T_d) (w_b - w_a), \end{aligned} \quad (\text{A.7})$$

where

$$\begin{aligned} c_1 &= -2 \left(\frac{1}{1 + (\lambda^2/12)} \right)^2 \left(\frac{1}{3} + \frac{\lambda^2}{18} + \frac{\lambda^4}{432} + \frac{\lambda^2}{120} \frac{E}{\sigma^s} \right), \\ c_2 &= \frac{\lambda}{12 + \lambda^2} \sqrt{\frac{\sigma^s}{E}} \left(-\frac{\lambda^2}{12} - 1 + \frac{E}{\sigma^s} \right), \\ c_3 &= \frac{2\lambda}{12 + \lambda^2} \sqrt{\frac{E}{\sigma^s}}, \\ c_4 &= \frac{2\gamma l E_q}{(\sigma^s)^2}, \\ c_5 &= -\frac{1}{1 + (\lambda^2/12)}, \\ c_6 &= \frac{\gamma A^2 E_q}{T_0^2}, \\ c_7 &= -\frac{1}{l} \left(\frac{\lambda^2}{12 + \lambda^2} \frac{E}{\sigma^s} + 1 \right), \\ c_8 &= -\frac{\lambda}{12 + \lambda^2} \sqrt{\frac{\sigma^s}{E}} \left(\frac{E}{\sigma^s} + \frac{\lambda^2}{12} + 1 \right), \\ c_9 &= -\frac{2}{3} - \frac{\lambda^2 \sigma^s}{60E}, \\ c_{10} &= \frac{\lambda}{12 + \lambda^2} \sqrt{\frac{\sigma^s}{E}} \left(-\frac{E}{\sigma^s} + \frac{\lambda^2}{12} + 1 \right), \\ c_{11} &= -\frac{\lambda}{6} \sqrt{\frac{\sigma^s}{E}}, \\ c_{12} &= -\frac{1}{l} \left(1 + \frac{\lambda^2 \sigma^s}{12E} \right). \end{aligned} \quad (\text{A.8})$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

- [1] H. M. Ali and A. M. Abdel-Ghaffar, "Seismic energy dissipation for cable-stayed bridges using passive devices," *Earthquake Engineering and Structural Dynamics*, vol. 23, no. 8, pp. 877–893, 1994.
- [2] M. Abdel-Rohman and H. Askar, "Control by passive TMD of wind-induced nonlinear vibrations in cable stayed bridges," *Journal of Vibration and Control*, vol. 2, no. 2, pp. 251–267, 1996.
- [3] F. Casciati, G. P. Cimellaro, and M. Domaneschi, "Seismic reliability of a cable-stayed bridge retrofitted with hysteretic devices," *Computers and Structures*, vol. 86, no. 17-18, pp. 1769–1781, 2008.
- [4] A. M. Sharabash and B. O. Andrawes, "Application of shape memory alloy dampers in the seismic control of cable-stayed bridges," *Engineering Structures*, vol. 31, no. 2, pp. 607–616, 2009.
- [5] W. L. He, A. K. Agrawal, and K. Mahmoud, "Control of seismically excited cable-stayed bridge using resetting semiactive stiffness dampers," *Journal of Bridge Engineering*, vol. 6, no. 6, pp. 376–384, 2001.
- [6] H. Iemura and M. H. Pradono, "Passive and semi-active response control of a cable-stayed bridge," *Earthquake Engineering and Structural Dynamics*, vol. 9, pp. 189–204, 2002.
- [7] J. M. Caicedo, S. J. Dyke, S. J. Moon, L. A. Bergman, G. Turan, and S. Hague, "Phase II benchmark control problem for seismic response of cable-stayed bridges," *Journal of Structural Control*, vol. 10, no. 3-4, pp. 137–168, 2003.
- [8] S. J. Moon, L. A. Bergman, and P. G. Voulgaris, "Sliding mode control of cable-stayed bridge subjected to seismic excitation," *Journal of Engineering Mechanics*, vol. 129, no. 1, pp. 71–78, 2003.
- [9] A. K. Agrawal, J. N. Yang, and W. L. He, "Applications of some semiactive control systems to benchmark cable-stayed bridge," *Journal of Structural Engineering*, vol. 129, no. 7, pp. 884–894, 2003.
- [10] H. Kim and H. Adeli, "Wavelet-hybrid feedback linear mean squared algorithm for robust control of cable-stayed bridges," *Journal of Bridge Engineering*, vol. 10, no. 2, pp. 116–123, 2005.
- [11] S. Y. Ok, D. S. Kim, K. S. Park, and H. M. Koh, "Semi-active fuzzy control of cable-stayed bridges using magnetorheological dampers," *Engineering Structures*, vol. 29, no. 5, pp. 776–788, 2007.
- [12] M. El-Katt, R. Shaker, and Y. Kassem, "Optimum control system for seismic and aerodynamic flutter response of cable-stayed bridge using magnetorheological dampers," *Advanced Materials Research*, vol. 163–167, pp. 4269–4279, 2011.
- [13] V. Gattulli and A. Paolone, "Planar motion of a cable-supported beam with feedback controlled actions," *Journal of Intelligent Material Systems and Structures*, vol. 8, no. 9, pp. 767–774, 1997.
- [14] S. Qi, W. Lu, P. Chen, L. Ming, D. Ding, and J. He, "Study on cable-stayed bridge flutter active control by a single group of ADM," in *Proceedings of the 1st International Conference on Computing Control and Industrial Engineering (CCIE '10)*, pp. 426–430, Wuhan, China, June 2010.
- [15] K. Wilde and Y. Fujino, "Aerodynamic control of bridge deck flutter by active surfaces," *Journal of Engineering Mechanics*, vol. 124, no. 7, pp. 718–727, 1998.
- [16] J. N. Yang and F. Giannopolous, "Active control and stability of cable-stayed bridge," *ASCE Journal of Engineering Mechanics and Division*, vol. 105, no. 4, pp. 677–694, 1979.
- [17] Y. Fujino and T. Susumpow, "Experimental study on active control of in-plane cable vibration by axial support motion," *Earthquake Engineering and Structural Dynamics*, vol. 23, no. 12, pp. 1283–1297, 1994.
- [18] P. Warnitchai, Y. Fujino, B. M. Pacheco, and R. Agret, "An experimental study on active tendon control of cable-stayed bridges," *Earthquake Engineering and Structural Dynamics*, vol. 22, no. 2, pp. 93–111, 1993.
- [19] H. Kobayashi, M. Hosomi, and K. Koba, "Tendon control for cable-stayed bridge vibration," in *Proceedings of the 1st World Conference on Structural Control*, pp. 23–29, Los Angeles, Calif, USA, 1994.
- [20] Y. Achkire and A. Preumont, "Active tendon control of cable-stayed bridges," *Earthquake Engineering and Structural Dynamics*, vol. 25, no. 6, pp. 585–597, 1996.
- [21] F. Bossens and A. Preumont, "Active tendon control of cable-stayed bridges: a large scale demonstrator," *Earthquake Engineering and Structural Dynamics*, vol. 30, no. 7, pp. 961–979, 2001.
- [22] M. H. El Ouni, A. Deraemaeker, A. Preumont, N. Ben Kahla, and M. Abdelghani, "An experimental and numerical study of the effect of active damping on the parametric excitation," in *Proceedings of the 3rd International Congress Design and Modeling of Mechanical Systems*, Hammamet, Tunisia, 2009.
- [23] M. H. El Ouni, N. Ben Kahla, and A. Preumont, "Numerical and experimental dynamic analysis and control of a cable stayed bridge under parametric excitation," *Engineering Structures*, vol. 45, pp. 244–256, 2012.
- [24] A. Deraemaeker, M. H. El Ouni, I. Romanescu, and A. Preumont, "The Smart Bridge demonstrator: description of the experimental setup," in *Proceedings of the International Conference on Noise and Vibration Engineering*, Leuven, Belgium, 2008.
- [25] A. Baz, S. Poh, and J. Fedor, "Independent modal space control with positive position feedback," *Journal of Dynamic Systems, Measurement and Control*, vol. 114, no. 1, pp. 96–112, 1992.
- [26] M. Balas, "Direct velocity feedback control of large space structures," *Journal of Guidance, Control, and Dynamics*, vol. 2, no. 3, pp. 252–253, 1979.
- [27] A. Preumont and K. Seto Wiley, Chichester, UK.
- [28] F. Bossens, *Amortissement actif des structures câblées: de la théorie à l'implémentation [Ph.D. thesis]*, Université Libre de Bruxelles, 2001.
- [29] <http://www.sdtools.com>.