Research Article

Mathematical Modeling of an Active-Fiber Composite Energy Harvester with Interdigitated Electrodes

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The use of active-fiber composites (AFC) instead of traditional ceramic piezoelectric materials is motivated by flexibility and relatively high actuation capacity. Nevertheless, their energy harvesting capabilities remain low. As a first step toward the enhancement of AFC's performances, a mathematical model that accurately simulates the dynamic behavior of the AFC is proposed. In fact, most of the modeling approaches found in the literature for AFC are based on finite element methods. In this work, we use homogenization techniques to mathematically describe piezoelectric properties taking into consideration the composite structure of the AFC. We model the interdigitated electrodes as a series of capacitances and current sources linked in parallel; then we integrate these properties into the structural model of the AFC. The proposed model is incorporated into a vibration based energy harvesting system consisting of a cantilever beam on top of which an AFC patch is attached. Finally, analytical solutions of the dynamic behavior and the harvested voltage are proposed and validated with finite element simulations.

1. Introduction

Recently advances in smart microsensors and microelectronics contributed to the development of devices with reduced size and low energy consumption. This development encouraged researchers to innovate in the field of energy sources in order to overcome the use of batteries. Given that the technological evolutions of batteries were relatively slow during the last decade, use of piezoelectric devices as energy harvesters (EHs) has progressively attracted attention of researchers. Indeed, many studies indicate the feasibility of using piezoelectric materials as power sources at different scales [1–3].

Nevertheless, monolithic piezoelectric materials, and specifically piezoelectric ceramics such as PZT, present several integration difficulties to structures characterized by curved shapes and large displacements [4]. Furthermore, due to the brittle nature of PZT, monolithic energy harvesters cannot take advantage of relatively large stroke vibrations due

to their small allowable deformations. Thus, Bent et al. [5] developed a new piezoelectric composite known as active-fiber composite (AFC) which consists of PZT fibers with circular cross-section embedded into an epoxy matrix and sandwiched between two sets of interdigitated electrodes (IDE). A similar one, macrofiber composite (MFC), was also developed at NASA Langley Research Center, where PZT fiber has a square cross-section [6]. These particular designs, and thanks to the IDE, use the higher d_{33} mode for actuators [7] as well as energy harvesters [4].

When used as energy generators, piezocomposites with IDE are still far from fulfilling the power consumption requirements of most portable electronics, at present. In fact, several researches showed limitation for their use for energy harvesting applications [8, 9]. In order to improve the performance of piezocomposites used as EH or actuator, several studies were conducted to redesign AFC and MFC by studying their behavior through simulations. Unfortunately, most of the proposed models are based on FE analysis because

of the complexity of the piezocomposite structure and the presence of IDE which complicates the electric field solution [10, 11]. Few analytical models have been also reported on EHs using IDE. Two approaches are proposed in the literature. The first approach consists in studying a representative volume element (RVE) of the piezocomposite by benefiting from the periodicity of the material's microstructure. Den Otter [12] determined a polynomial analytic expression of the electric field and the capacitance in piezocomposite with IDEs. Later, Lloyd [13] determined an analytic solution of the electric field and potential distribution using complex analysis based on the Schwarz-Christoffel's method. Schaur and Jakoby [14] presented a more efficient and versatile semianalytical model, which yields approximate expressions for the capacitance of IDEs by integrating the electrode's thickness and the covering layer. The second approach consists in taking into account the whole microstructure of the piezocomposite. Erturk et al. [15] investigated the MFC configuration for EH by deriving a distributed-parameter electromechanical model. They assumed an empirical model of the electric field distribution in order to handle the nonuniform field. Using a quasistatic analysis, Mo et al. [16] developed an analytical model for an IDE energy harvester with monolithic piezoelectric material assuming constant and unidirectional electric field. Later, Knight et al. [17] developed an optimization of the IDE EH based on analytical modeling developed by Mo et al. [16].

In this paper, we seek a parameterized mathematical model, of an AFC microstructure, to be solved analytically in order to estimate the amount of electrical voltage that could

be generated by the system when excited under harmonic transversal displacement.

2. Homogenization of Piezoelectric Fiber Composite

The first step toward the derivation of a mathematical model of the proposed AFC based energy harvester (EH) is to estimate the global effective physical properties of the unidirectional (1–3 periodic) piezoelectric fiber composite by homogenizing its internal microstructure in order to facilitate its integration into the equation of motion of the EH. The piezocomposite under study is formed by a transversely isotropic material (PZT-5A) embedded into an isotropic Epoxy matrix (see Table 1). Hence, the resulting composite is a transversely isotropic piezoelectric material too (Figure 1(a)).

Thus, the associated linear constitutive equations may be written as

$$\Sigma = E \Xi, \quad (1)$$

where in matrix form and using Voigt notation

$$\begin{aligned} \Sigma^t &= [\bar{\sigma}_1 \ \bar{\sigma}_2 \ \bar{\sigma}_3 \ \bar{\sigma}_4 \ \bar{\sigma}_5 \ \bar{\sigma}_6 \ \bar{D}_1 \ \bar{D}_2 \ \bar{D}_3], \\ \Xi^t &= [\bar{\epsilon}_1 \ \bar{\epsilon}_2 \ \bar{\epsilon}_3 \ \bar{\epsilon}_4 \ \bar{\epsilon}_5 \ \bar{\epsilon}_6 \ -\bar{E}_1 \ -\bar{E}_2 \ -\bar{E}_3] \end{aligned} \quad (2)$$

and, for the IDE configuration, the electromechanical constants matrix is given by

$$E = \begin{pmatrix} C_{11}^e & C_{12}^e & C_{31}^e & 0 & 0 & 0 & 0 & 0 & e_{31}^e \\ C_{12}^e & C_{11}^e & C_{31}^e & 0 & 0 & 0 & 0 & 0 & e_{31}^e \\ C_{31}^e & C_{31}^e & C_{33}^e & 0 & 0 & 0 & 0 & 0 & e_{33}^e \\ 0 & 0 & 0 & C_{44}^e & 0 & 0 & 0 & e_{15}^e & 0 \\ 0 & 0 & 0 & 0 & C_{44}^e & 0 & e_{15}^e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15}^e & 0 & -\epsilon_{11}^e & 0 & 0 \\ 0 & 0 & 0 & e_{15}^e & 0 & 0 & 0 & -\epsilon_{11}^e & 0 \\ e_{31}^e & e_{31}^e & e_{33}^e & 0 & 0 & 0 & 0 & 0 & -\epsilon_{33}^e \end{pmatrix}, \quad (3)$$

where $\bar{\sigma}_i$, $\bar{D}_i$, $\bar{\epsilon}_i$, and $\bar{E}_i$ denote average values of the homogenized material for stress, electric displacement, strain, and electric field components, respectively. C_{ij}^e , e_{ij}^e , and ϵ_{ij}^e denote the effective coefficients of elastic stiffness and piezoelectric and permittivity components at constant strain, respectively. The mean value of an effective physical property, bar notation, is defined as $\bar{f} = (1/V) \int_V f dV$.

Various approaches were proposed in the literature to calculate the homogenized parameters of a two-phase piezocomposite using analytical and/or numerical homogenization techniques [18, 19]. In this paper, we use both techniques to calculate the effective material coefficients of the piezocomposite. The obtained values are compared to each other for validation purposes. As an analytical approach, we propose the use of the Mori-Tanaka method (MTM). On the other hand, the finite element method (FEM),

with the commercial package ANSYS, will be used as a numerical approach. In fact, several studies showed that these approaches are suitable for piezocomposites with short and long fibers [19, 20].

The use of the FEM as periodic homogenization technique consists of taking advantage of the periodicity of the physical properties of the piezocomposite by reducing the studied domain to a representative volume element (RVE) that should faithfully emulate the behavior of the entire piezocomposite. Here, the criterion for a good choice of the RVE is the equality in strain energy for the piezocomposite and homogenized structures. Figure 1(b) shows the finite element meshed model used in ANSYS for the RVE using a fiber volume fraction (FVF) of 50%. The 3D 8-node coupled-field solid finite element SOLID5 was used to mesh all volumes. Here, 2580 elements and 3216 nodes were used.

FIGURE 1: Schematic of the active-fiber composite and its RVE.

TABLE 1: Material properties of the piezocomposite constituents: Fiber (PZT-5A) and Matrix (Epoxy) [23].

	C_{11} (GPa)	C_{12} (GPa)	C_{31} (GPa)	C_{33} (GPa)	C_{44} (GPa)	C_{66} (GPa)
PZT-5A	120.35	75.18	75.1	110.87	21.05	22.57
Epoxy	9.48	7.75	7.75	9.48	0.86	0.86
	e_{31} (C/m ²)	e_{33} (C/m ²)	e_{15} (C/m ²)	ϵ_{11} (10^{-9} F/m)	ϵ_{33} (10^{-9} F/m)	
PZT-5A	-5.35	15.78	12.3	9.16	8.30	
Epoxy	—	—	—	0.885	0.885	

Next, we calculate the effective electroelastic coefficients corresponding to tensile deformations of the chosen RVE, using ANSYS for different FVF [21]. To do so, the symmetry boundary conditions are firstly imposed on the planes $x_1 = 0$ and $x_2 = 0$. For a particular loading situation, only one value in the strain or electric field tensors is nonzero and all others become zero [19, 21].

In Figure 2, we plot the mechanical, electrical, and piezoelectric ratios between the effective values and the purely piezoelectric values, as a function of the FVF. We note that for high values of the FVF, the MTM is less relevant than FEM, because it assumes that the stress and strain fields inside the inclusions (fiber) are supposed to be constant, which is not the case of the FEM [19]. Thus, the FEM values corresponding to tensile deformations will be retained for the rest of this paper. However, since this FEM is not suitable to evaluate shear coefficients, where it results in a large overestimation [22], these coefficients (e_{15} , C_{44} and C_{66}) will be only determined by the MTM. In fact, while the high aspect ratio EH at hand is excited in the bending mode, structural shear deformations are not expected to be pronounced [15]. These latter will be used for 3D FEM of the EH.

3. Electric Field Considerations in the Homogenized AFC

In the case of homogenized AFC with IDEs, the corresponding electric field forms a set of curved lines, in the $(x_3 - x_2)$ plane, along the direction of the fiber. Previous studies have shown that the electric field direction within the RVE is inhomogeneous and anisotropic [11]. In our case, for simplification purposes, the homogenized material is considered to be uniformly poled in the x_3 -direction [10]. This simplification is adequate for large electrodes distances, as shown later.

An appropriate electrical representative volume element (eRVE) is chosen for the electric field analysis, in which we take advantage of the symmetry inside the AFC and the associated IDE. The eRVE is composed of a monolithic homogenized material with 50% of FVF. The corresponding material properties are shown in Table 2 where the material properties corresponding to shear deformation are computed using the MTM. The electrodes were not modeled explicitly, but voltages were applied directly on nodes located at the surface where electrodes are patterned. The symmetry boundary conditions are imposed on the eRVE at the planes $x_1 = 0$ and

FIGURE 2: Material mechanical, piezoelectric, and electrical properties of the piezocomposite as a function of the fiber volume fraction.

TABLE 2: Material properties of the homogenized material with 50% of fiber.

C_{11}^e (GPa)	C_{12}^e (GPa)	C_{31}^e (GPa)	C_{33}^e (GPa)	C_{44}^e (GPa)	C_{66}^e (GPa)
19.97	12.77	13.59	39	2.42	3.8
e_{31}^e (C/m ²)	e_{33}^e (C/m ²)	e_{15}^e (C/m ²)	ϵ_{11}^e (10 ⁻⁹ F/m)	ϵ_{33}^e (10 ⁻⁹ F/m)	
-0.465	9.485	0.172	2.33	4.69	

$x_2 = 0$ (Figure 6) and coupled displacements are applied on plane $x_3 = 0$ of the eRVE. The eRVE is meshed using 64256 nodes and 56250 elements with a 3D coupled-field element SOLID5 (Figure 3(a)).

For modeling purposes, we consider that the electric field is significant and homogeneous in the x_3 direction only. To justify this hypothesis, we study the effect of electrode separation d for a range of electrode width b_e on the field-elongation coefficient α that characterizes the electric field shape's curvature in the x_3 direction

$$\alpha = \frac{|\bar{E}_3|}{|\bar{E}_2| + |\bar{E}_3|}. \quad (4)$$

In Figure 3(b), we can note that as the electrode separation is increased, α increases significantly. For large values of electrode separation d , the electric field becomes globally unidirectional and uniform along the x_3 -axis. Thus, we can model the homogenized AFC as a piezoelectric material with transverse and parallel electrodes, in which the electrical field is uniform and aligned in the x_3 -direction (Figure 5(a)). This component is formed by a series of constant electric fields

connected through the electrodes separated by the distance d and electric potential V , that is,

$$E_3 = \frac{V}{d} h(x_3) (H(x_2) - H(x_2 - h_p)), \quad (5)$$

where $h(x_3)$ accounts for the spatial distribution of the electric field in the x_3 -direction, and

$$h(x_3) = \sum_{i=1}^{(N-1)/2} \{2H(x_3 - (2i-1)d) - H(x_3 - 2id) - H(x_3 - (2i-2)d)\}. \quad (6)$$

Here N is the number of electrodes (odd number), H is the Heaviside step function, and $V(t)$ is the resulting potential difference between negative and positive electrodes.

4. Energy Harvester Model and Analytical Solution

We consider a bilayered cantilever beam in which one layer is made of aluminum and the other layer is made of an AFC piezocomposite patch partially covering the length of

FIGURE 3: Electric field studies in the homogenized AFC.

the beam and placed near the clamped end. For modeling purposes, the piezocomposite is replaced by a homogenized material at 50% of FVF and crossed by N transversal IDEs perpendicular to the x_3 -direction, which corresponds to the x -axis of the beam, as shown in Figure 4(a). The clamped end of the beam is excited using a transverse harmonic displacement $Y(t) = Y_0 \sin(\Omega t)$, where Ω is the driving frequency and $Y_0 = 0.1$ mm. In Table 3, we give the material and geometrical properties of the bilayered cantilever beam, where the subscript a refers to the aluminum layer and the subscript p refers to the piezoelectric layer.

The constitutive equations of an isotropic material (aluminum layer) and transversely isotropic (piezoelectric layer) material are, respectively, represented by

$$\begin{aligned} \sigma_{11}^a &= E_a^* \varepsilon_{11} = \frac{E_a}{1 - \nu_a^2} \varepsilon_{11}, \\ \sigma_{11}^p &= E_p^* \varepsilon_{11} - \bar{e}_{33}^e E_1, \end{aligned} \quad (7)$$

where E_p^* is the effective Young's modulus and $\bar{e}_{33}^e$ is the piezoelectric constant at the plane-stress assumption.

To analytically model our system, we start by assuming a beam that undergoes out-of-plane bending motion only characterized by the following displacement vector:

$$\mathbf{R} = (\nu + Y) \mathbf{y} + y \mathbf{y}_1, \quad (8)$$

where $\nu(x, t)$ is the transverse displacement. $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ is the inertial base frame, while the $(\mathbf{x}_1, \mathbf{y}_1, \mathbf{z})$ is the local frame attached to the beam's cross-section obtained by a rotation with respect to the z -axis. Assuming small displacements,

neglecting the rotary inertia, and applying Hamilton's principle, we use the Euler-Bernoulli beam theory to write the equation of motion

$$\begin{aligned} & \left[H_1 E y_3 + H_2 E_a^* \frac{b}{3} (y_1^3 - y_0^3) \right] v^{(4)} \\ & + \left[H_1 \bar{e}_{33}^e \frac{V(t)}{2d} b h''(x) (y_2^2 - y_1^2) \right] \\ & + [H_1 m_1 + H_2 m_2] (\ddot{v} + \ddot{Y}) = 0, \end{aligned} \quad (9)$$

where $m_1 = \rho_a b(y_0 - y_1) + \rho_p b(y_2 - y_1)$, $m_2 = \rho_a b h_a$, $E y_3 = b E_a^* \int_{y_0}^{y_1} y^2 dy + b E_p^* \int_{y_1}^{y_2} y^2 dy$, $H_1 = H(x) - H(x - L_1)$, and $H_2 = H(x - L_1) - H(x - L)$. y_i denotes the different layers positions (see Figure 4(b)).

To develop a reduced-order model of the system, we use the Galerkin procedure and let the displacement $\nu(x, t)$ be expressed as follows:

$$\nu(x, t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t), \quad (10)$$

where $\phi_r(x)$ is the r th normalized mode shape of the free undamped eigenvalue problem associated with (9), when $L_1 = L$ and $\eta_r(t)$ is the modal displacement [24].

Equation (10) is substituted into the equation of motion (9) and multiplied by the mode shape function; we use the orthogonality conditions and integrate over the beam's length. The coupled electromechanical ordinary differential equation of the modal response of the EH is obtained as

$$M_r \ddot{\eta}_r(t) + C_r \dot{\eta}_r(t) + K_r \eta_r(t) + F_r = 0, \quad (11)$$

FIGURE 4: Schematic of the energy harvester with its neutral axis position.

TABLE 3: Material and geometrical properties of the bi-layered cantilever beam.

L (mm)	L_1 (mm)	b (mm)	h_p (mm)	h_a (mm)	ρ_p (kg/m ³)	ρ_a (kg/m ³)	E_a (GPa)	ν_a
100	60	20	0.33	1	4475	2700	68	0.35

where the modal mass term is

$$M_r = \int_0^L [H_1 m_1 + H_2 m_2] \phi_r^2(x) dx, \quad (12)$$

the modal damping coefficient is

$$C_r = 2 \zeta_r \omega_r M_r, \quad (13)$$

where ω_r is the r th natural frequency and ζ_r is the modal damping ratio ($\zeta_1 = 0.017$), the modal mechanical stiffness term is

$$K_r = \int_0^L \left[H_1 E y_3 + H_2 E_a \frac{b}{3} (y_1^3 - y_0^3) \right] \beta_r^4 \phi_r^2(x) dx, \quad (14)$$

where β_r is the r th frequency number given in [24], and the modal electromechanical force is expressed as

$$F_r = \int_0^L H_1 \bar{e}_{33} \frac{b}{2d} (y_2^2 - y_1^2) h''(x) \phi_r(x) dx V(t) + \int_0^L [H_1 m_1 + H_2 m_2] \phi_r(x) dx \ddot{Y}(t). \quad (15)$$

For the electrical modeling of the homogenized AFC, we use the simplification adopted in this paper when the electrical field is parallel to the x -axis. Therefore, we can model the AFC as a series of capacitors and current sources connected in parallel (Figure 5(b)).

The output voltage $V(t)$, also shown in Figure 5(b), represents the harvested voltage from the proposed EH. This voltage is measured across the external resistor R representing the external load. Applying Gauss's law on the whole piezoelectric patch yields

$$\frac{d}{dt} \sum_{j=1}^{N-1} \int_{A_j} \mathbf{D}_j \cdot \mathbf{n}_j dA_j = \frac{V(t)}{R}, \quad (16)$$

where $\mathbf{D}_j$ is the electrical displacement vector between the electrodes, $\mathbf{n}_j$ is the electrodes normal vector, and the integration is performed over the electrode area A_j . Taking into account that the electrodes are parallel to the (y, z) plane and considering the homogenized piezoelectric constitutive equation, we obtain

$$\mathbf{D}_j \cdot \mathbf{n}_j = D_1 = \bar{e}_{33}^e \epsilon_{11} - \bar{e}_{33}^e \frac{V(t)}{d}, \quad (17)$$

where $\bar{e}_{33}^e$ is the permittivity component at constant strain with the plane-stress assumption. Therefore, (15) is rewritten as

$$(N-1) \bar{e}_{33}^e \frac{b}{d} (y_1 - y_2) \dot{V}(t) + \sum_{j=1}^{N-1} \int_{(j-1)d}^{jd} \bar{e}_{33}^e (y_1^2 - y_2^2) \frac{b}{2d} \dot{v}''(x, t) dx = \frac{V(t)}{R}. \quad (18)$$

Applying Kirchhoff laws to the equivalent electrical circuit and using Galerkin decomposition, we obtain the following equation:

$$C_P \dot{V}(t) + \frac{V(t)}{R} = i(t), \quad (19)$$

where the internal equivalent capacitance term is expressed as

$$C_P = (N-1) \bar{e}_{33}^e \frac{b}{d} (y_2 - y_1), \quad (20)$$

and the equivalent current source is expressed as

$$i(t) = \sum_{r=1}^{\infty} \left(\sum_{j=1}^{N-1} \bar{e}_{33}^e (y_1^2 - y_2^2) \frac{b}{2d} \frac{d\phi_r(x)}{dx} \Big|_{x=(j-1)d}^{x=jd} \right) \dot{\eta}_r(t). \quad (21)$$

FIGURE 5: Modeling of the AFC patch.

Since the system is assumed to be linear, we assume that $\eta_r(t) = \text{Im}(\eta_r^0 e^{j\Omega t})$ and $V(t) = \text{Im}(V_0 e^{j\Omega t})$, where Im stands for the imaginary part, $j = \sqrt{-1}$ is the unit imaginary number, η_r^0 is the complex modal amplitude of the transverse displacement, and V_0 is the complex amplitude of the harmonic voltage across the resistive load. Solving the previous system analytically, we obtain the voltage V_0 amplitude as

$$V_0 = \frac{\sum_{r=1}^{\infty} (k_r m_r \Omega^2 Y_0 / (j\Omega C_r - M_r \Omega^2 + K_r))}{C_p - (j/\Omega R) + \sum_{r=1}^{\infty} (k_r f_r / (j\Omega C_r - M_r \Omega^2 + K_r))}. \quad (22)$$

Assuming that excitation frequencies from the environment are usually low, the fundamental mode response will be dominant and constitute our main concern in this study ($r = 1$).

5. Typical Results and FE Validation

5.1. FE Model. The FEM is used to validate the harvested voltage for different excitations near the fundamental frequency. To do so, a 3D FE model with 17 electrodes ($N = 17$ and $\alpha = 0.92$) is developed using ANSYS. Solid coupled field elements (SOLID5) are used for the piezoelectric behavior and circuit element (CIRCU94) for the piezoelectric-circuit analysis. A nil potential is applied at the nodes belonging to the negative electrodes. Also, we use equipotentiality condition for the positive electrodes' nodes. We then relate the resistive element R to the electrodes through two active nodes located on the surfaces of negative and positive electrodes of the EH. A base displacement Y_0 is applied to the clamped side of

the beam at frequency Ω . The model, shown in Figure 6, has 22800 elements and 26313 nodes.

We start by calculating the mode shape and the natural frequencies of the system according to the FE model. The corresponding natural frequencies are shown in Table 4; they are compared to the ones obtained by the analytical model. As observed a mismatch of 10% is obtained. It is mainly due to the fact that the length of the AFC patch was taken to be equal to the aluminum beam in the analytical model.

5.2. Typical Results. Now, we study the response of the system to a harmonic excitation using the proposed analytical model, with one mode approximation in the Galerkin procedure, and compare the obtained results with those obtained using FE model at the same excitation conditions.

The results are given as a function of the normalized excitation frequency, obtained by dividing Ω by a factor so that the corresponding natural frequency is normalized to the FE short circuit natural frequency ω_{SC} , given in Table 4.

We plot in Figure 7(a) the frequency response curves of the maximum output amplitude voltage $V_{\max}$ for different values of load resistance R , when the number of electrodes is $N = 17$. In Figure 7(b), we vary the load resistance and look at the variation of $V_{\max}$ at the peak of the first resonant frequency; we denote this value by $V_{\max}^{\omega_1}$. As shown in Figure 7, a good agreement is obtained between analytical and FE simulations demonstrating the effectiveness of the proposed model. However, the proposed model is limited to an IDE EH with relatively large electrode's distance and very small electrode's width. The IDE EH performances could be improved by selecting the correct design parameters. Hence,

FIGURE 6: FE electromechanical model of the EH for $N = 17$ electrodes (P: poling direction).

FIGURE 7: Analytical and FE harvested voltage at different excitation frequencies and load resistances: analytical prediction (continuous and dashed lines) and FE model (circles).

TABLE 4: Natural frequencies for open-circuit and short-circuit configurations when $N = 17$ electrodes.

	$\omega_1/2\pi$ (FE)	$\omega_1/2\pi$
Short-circuit frequency (Hz)	100.95	89.21
Open-circuit frequency (Hz)	109.62	101.41

the theoretically developed model can be used as basis for optimal design analysis.

6. Conclusion

In this paper, an analytical model of a vibrating structure, composed of an aluminum beam on top of which

a piezoelectric composite patch, namely, AFC, has been attached, is derived and a closed-form solution is proposed. We use homogenization techniques to determine the effective properties of the piezoelectric composite, leading to a homogeneous model based on properties of the fiber and matrix constituents. We then used numerical simulations with ANSYS to extract the homogenized mechanical and electrical properties of the AFC. A simplified electrical field model is proposed in order to overcome the difficulty of resolving Gauss's equation for interdigitated-electrode configuration. We show that the proposed procedure could be applied to a complex piezoelectric microstructure in order to analytically derive and solve the equations of motion of such systems. We proposed closed-form solutions of the harvested electrical voltage. The proposed solutions were validated

using finite element analysis for the same homogenized material. We demonstrated that the proposed solution is in good agreement with the finite element one even for critical load resistances.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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