Research Article

Nonlinear Regression Based Health Monitoring of Hysteretic Structures under Seismic Excitation

C. Xu,¹ J. Geoffrey Chase,² and Geoffrey W. Rodgers²

¹*School of Astronautics, Northwestern Polytechnical University, Xi'an 710072, China*

²*Department of Mechanical Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand*

Correspondence should be addressed to C. Xu; chao_xu@nwpu.edu.cn

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This paper presents a health monitoring method using measured hysteretic responses. Acceleration and infrequently measured displacement are integrated using a multirate Kalman filtering method to generate restoring force-displacement hysteresis loops. A linear/nonlinear regression analysis based two-step method is proposed to identify nonlinear system parameters. First, hysteresis loops are divided into loading/unloading half cycles. Multiple linear regression analysis is applied to separate linear and nonlinear half cycles. Preyielding stiffness and viscous damping coefficient are obtained in this step and used as known parameters in the second step. Then, nonlinear regression analysis is applied to identified nonlinear half cycles to yield nonlinear system parameters and two damage indicators: cumulative plastic deformation and residual deformation. These values are closely related to structural status and repair costs. The feasibility of the method is demonstrated using a simulated shear-type structure with different levels of added measurement noise and a suite of ground motions. The results show that the proposed SHM method effectively and accurately identifies physical system parameters with up to 10% RMS added noise. The resulting damage indicators can robustly and clearly indicate structural condition over different earthquake events.

1. Introduction

Whenever a strong motion earthquake occurs, buildings are expected to remain standing with various degrees of damage. Critical decisions must be made within a short period of time concerning whether the buildings are suitable for continued occupancy. Vibration-based structural health monitoring (SHM) has gained much interest and attention in the civil engineering community in recent years. It is recognised as a powerful tool to identify damage at its earliest stage and to determine the residual useful life of structures, especially for rapid evaluation after a major event [1].

Many vibration-based SHM methods for civil structures are based on identifying changes in modal characteristics [2–5]. However, only low frequency modes related to structural global deformation can be measured accurately, and these modal parameters are insensitive to localized damage in some cases and typically more applicable to structures where vibration response is highly linear [6]. Local diagnostic methods, such as impedance-based [7] and guided-wave

based [8] methods, have been developed to improve sensitivity to local failure modes. However, they rely on close proximity to damage location and typically require many sensors distributed throughout a structure, which is currently impractical.

Advanced signal processing tools, such as wavelet analysis [9], empirical mode decomposition, and Hilbert transform [10], are also being proposed. These techniques offer the advantage of determining both the location and time of the damage. However, they cannot directly identify physical system parameters and quantify the level of nonlinear damage due to the absence of a physical system model. Therefore, a number of model-based system identification methods have been presented, including a range of time-domain filters to track time-variant model parameters [11–18]. However, only a few address nonlinear hysteresis and hysteresis-based damage indicators [19].

Hysteretic behaviour plays a critical role not only in seismic performance-based analysis and design [20, 21], but also in capturing the nonlinear yielding and energy

absorption associated with damage [22]. A SHM method that captures hysteretic response would give more insight into structural nonlinearity and quantify the level of nonlinear damage.

Structural restoring force-displacement hysteresis loops can be constructed from measured responses [23–25]. Accelerometers are the most commonly used instruments in civil structures, and displacement and velocity have to be obtained from numerical integration. This procedure is fraught with major pitfalls due to the effects of noise, limiting accuracy of the hysteretic loops and damage detection methods based on hysteresis monitoring. However, recent advances in low-rate displacement sensors, such as GPS [26], enable sensor fusion methods that deliver accurate displacement, velocity, and acceleration. Several sensor fusion methods, such as the multirate Kalman filtering method [27], the cubic spline displacement correction method [28], the finite difference FIR filter method [29], and the finite element FIR filter method [30], have been proposed. These methods are expected to suppress measurement noise effectively and yield high quality hysteresis loops.

Structural damage indicators can be further extracted from constructed hysteresis loops. Secant stiffness was first calculated to determine the occurring of degradation and damage in [31]. System effective stiffness was extracted to describe the evolution of the structural stiffness in [32]. Evolution of hysteresis loop shape was considered as a rapid visual indicator of system degrading in [33]. Although these damage indicators can be used to indicate the occurrence of damage, they are largely qualitative. Damage indicators that can quantify structural damage and closely related to structural postevent safety and repair costs are urgently needed.

This research presents a simple and novel health monitoring method for hysteretic structures subjected to seismic excitation. A multirate Kalman filtering technique is applied to estimate high quality displacement and velocity from high-rate sampled acceleration and low-rate sampled displacement data. Hysteresis loops are constructed and a regression analysis based two-step method is proposed to identify preyielding, viscous damping coefficient, yielding displacement and postyielding stiffness, and resulting nonlinear damage indicators. The feasibility and robustness of the proposed method are illustrated for different noise levels over a suite of earthquake events.

2. Construction of Hysteresis Loops

Toussi and Yao [23] first presented the idea of generating system hysteresis loops from recorded seismic response data. For this proof-of-concept study, it will be assumed that the structure in question can be adequately modelled as a single-degree-of-freedom (SDOF) system for simplicity and clarity. This situation is also true if the test structure responds primarily in a single mode and can be defined:

$$m\ddot{x} + f(x, \dot{x}) = -m\ddot{x}_g, \quad (1)$$

where x , $\dot{x}$, and $\ddot{x}$ are displacement, velocity, and acceleration related to the ground; f is the total restoring force; $\ddot{x}_g$ is ground acceleration; and m is the mass.

Rewriting (1) and including viscous damping restoring force yield

$$f(x, \dot{x}) = -m[\ddot{x}_g + \ddot{x}] = -mu = c\dot{x} + f_s(x, \dot{x}), \quad (2)$$

where u is absolute acceleration; c is viscous damping coefficient; and $f_s(x, \dot{x})$ is stiffness restoring force. Assuming m to be known *a priori* and u to be measured, f is consequently obtained. Dynamic displacement and velocity can be obtained from measured sensor data by integration and correction. Thus, hysteresis loops can be constructed by graphing the restoring force versus displacement with time as an implicit parameter.

Direct integration of measured acceleration to obtain velocity and displacement is sensitive to noise and can cause significant distortion of estimated displacement [34]. Data fusion of high-rate acceleration and low-rate displacement measurements can effectively suppress noise and yield good estimates of velocity and displacement. If high-rate acceleration and low-rate displacement measurements are available, estimation of displacement and velocity from the measurements can be modelled by a discrete dynamic system:

$$\mathbf{x}_{k+1} = \mathbf{A}^s \mathbf{x}_k + \mathbf{B}^s u_k + \boldsymbol{\alpha}_k, \quad (3)$$

$$z_k = \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\beta}_k, \quad (4)$$

where (3) is the system equation and (4) is the observation equation; u_k is the measured acceleration and z_k is the measured displacement. The state vector $\mathbf{x}_k$ comprises the displacement d_k and the velocity v_k ; that is,

$$\mathbf{x}_k = \begin{bmatrix} d_k \\ v_k \end{bmatrix}. \quad (5)$$

Note that the subindex k indicates a progression in time. $\mathbf{A}^s$ is a 2×2 matrix describing the system dynamics, $\mathbf{B}^s$ is 2×1 input matrix, and $\mathbf{H}_k$ is 1×2 design matrix, defined as

$$\begin{aligned} \mathbf{A}^s &= \begin{bmatrix} 1 & \tau_a \\ 0 & 1 \end{bmatrix}; \\ \mathbf{B}^s &= \begin{bmatrix} \frac{\tau_a^2}{2} \\ \tau_a \end{bmatrix}; \\ \mathbf{H}_k &= [1 \ 0], \end{aligned} \quad (6)$$

where τ_a is the acceleration sampling interval. In (3) and (4), $\boldsymbol{\alpha}$ is a vector of acceleration measurement noise with distribution $(\mathbf{0}, \mathbf{Q}^s)$ and $\boldsymbol{\beta}$ is the vector of displacement measurement noise with distribution $(\mathbf{0}, \mathbf{R}^s)$. Both are assumed

to be Gaussian white noise processes with covariance q and r . Thus, $\mathbf{Q}^s$ and $\mathbf{R}^s$ are given by

$$\mathbf{Q}^s = \begin{bmatrix} \frac{q\tau_a^3}{3} & \frac{q\tau_a^2}{2} \\ \frac{q\tau_a^2}{2} & q\tau_a \end{bmatrix}; \quad (7)$$

$$\mathbf{R}^s = \frac{r}{\tau_d},$$

where τ_d is the displacement sampling interval.

With (3) to (7), a discrete time multirate Kalman filter can be used to estimate the displacement and velocity at each acceleration sampling instant [27, 35].

3. SHM Based on Regression Analysis of Hysteresis Loops

Many civil structures exhibit hysteresis when subject to severe cyclic loading. Figure 1 shows general hysteretic loops without considering system stiffness or strength degradation. A hysteretic cycle consists of a loading and an unloading half cycle. Any loading/unloading half cycle can be further divided into two nearly linear regimes: elastic and plastic, governed by k_e , the preyielding stiffness, and k_p , the postyielding stiffness, respectively. The elastic-plastic transition is generally smooth and gradual, but small. Omitting the transition process, the original half cycle can be represented by two line segments with different slopes, as shown in Figure 1, to capture the essential system dynamics.

If the approximated two lines and their interaction point are found, the nonlinear plastic deformation during the half cycle can be easily calculated. Damage indicators related to postevent structural safety and repair costs, such as residual deformation and cumulative plastic deformation, can then be directly obtained by summing identified nonlinear deformation from all half cycles. Thus, the SHM problem is converted to a search for this approximation for each half cycle.

Hysteretic loops can be divided into many loading or unloading half cycles by identifying the points where the sign of the velocity changes. During a seismic event, structural behaviour is linear for most half cycles and nonlinear for fewer others. For linear half cycles, a single segment line approximation is enough. For nonlinear half cycles, a broken line approximation is needed. Hence, a two-step approximation method is developed to optimally approach the original half cycle. In the first step, linear and nonlinear half cycles are separated. In the second step, identified nonlinear half cycles are further estimated.

Regression analysis is a powerful tool for modelling the relationship between a dependent variable and one or more independent variables [36]. Recalling (2), displacement x and velocity $\dot{x}$ are defined as the independent variables and restoring force f is defined as the dependent variable. Thus, the optimal approximation of each half cycle formulates a regression problem.

FIGURE 1: Hysteretic loops for arbitrary response.

3.1. Step 1: Linear Regression to Each Half Cycle. Multiple linear regression is applied to each half cycle. It is equal to using an equivalent linear system assumption to each half cycle. Thus, (2) can be rewritten:

$$-m [\ddot{x}_g + \ddot{x}] = c_l \dot{x} + k_l x, \quad (8)$$

where k_l is the effective system stiffness and c_l is the effective system damping. All observation variables ($x, \dot{x}, \ddot{x}, \ddot{x}_g$) can be obtained directly or indirectly from measurements. Structural mass is assumed to be known a priori. Equation (8) holds at each sampling instant k , with variables defined:

$$y_k = -m [\ddot{x}_{gk} + \ddot{x}_k], \quad (9a)$$

$$x_{1,k} = \dot{x}_k, \quad (9b)$$

$$x_{2,k} = x_k. \quad (9c)$$

The optimal approximation problem can be formulated as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \quad (10a)$$

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \\ \vdots \\ y_n \end{bmatrix},$$

$$\mathbf{X} = \begin{bmatrix} 1 & x_{1,1} & x_{2,1} \\ 1 & x_{1,2} & x_{2,2} \\ \vdots & \vdots & \vdots \\ 1 & x_{1,k} & x_{2,k} \\ \vdots & \vdots & \vdots \\ 1 & x_{1,n} & x_{2,n} \end{bmatrix},$$

$$\boldsymbol{\beta} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{Bmatrix}, \quad \boldsymbol{\epsilon} = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_k \\ \vdots \\ \epsilon_n \end{Bmatrix}, \quad (10b)$$

where n is the number of all observed response variable pairs, $\mathbf{Y}$ is regressand, $\mathbf{X}$ is the regressor, $\boldsymbol{\beta}$ is the regression coefficients vector to be estimated, and $\boldsymbol{\epsilon}$ is the vector of estimation error due to measurement noise and model error and is random and normally distributed. The least squares method then finds the unbiased estimates of the regression coefficients:

$$(b_0, b_1, b_2) = \arg \min_{\boldsymbol{\beta}} [(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})], \quad (11)$$

where the vector (b_0, b_1, b_2) is the estimates of the regression coefficients of $(\beta_0, \beta_1, \beta_2)$.

Comparing (10a), (10b) and (8), it is clear that the least squares estimates, b_1 and b_2 , are the effective linear system damping and the effective stiffness coefficient, respectively. When there is no plastic deformation presented in the half cycle, k_l should approach the system true preyielding elastic stiffness, k_e , and when there is nonlinear plastic deformation, k_l should capture a secant average stiffness of k_e and k_p . Therefore, k_l is similar to the secant stiffness in [31] but derived in a least squares sense here.

The estimated equivalent stiffness k_l can vary over different half cycles. Varying k_l is a significant indicator of the nature of the dynamic system. It is reasonable that the hysteresis curve is linear when the half-cycle displacement increment Δd is small and nonlinear when Δd is larger than the structural yield displacement. A rapid drop in k_l at large displacement increment can be viewed as a good indicator of occurring inelastic behaviour during that half cycle. Thus, the plot of k_l versus Δd will be used to identify the potential nonlinear half cycle.

In addition, the estimated equivalent linear damping coefficient c_l is the measure of system energy dissipation. System energy dissipation capacity will increase due to the added hysteretic damping. The plot of c_l versus Δd may also be used as another indicator of the inelastic half cycles. Thus, linear and nonlinear half cycles can be separated by using a threshold determined from these indicators.

For all identified linear half cycles, the multiple linear regression process yields many estimates of viscous damping coefficient c and preyielding elastic stiffness k_e . The statistical mean of c and k_e over all these linear half cycles will be

considered “true” values and be used as known parameters for the next step.

3.2. Step 2: Nonlinear Regression Analysis to Identified Nonlinear Half Cycles. The postyielding stiffness is typically about a 5–10% of preyielding stiffness for many civil structures. Thus, the slope of the hysteresis curve for a nonlinear half cycle will undergo sudden change. To optimally approximate the nonlinear half cycles, data points in these nonlinear half cycles must be divided into multiple segments and regress a different linearly parameterized polynomial for each segment. k_e and k_p can be obtained directly from estimated regression coefficients. The difficulty is associated with the unknown interaction point of each segment and the joint point of the segmented regression lines has to be estimated. It is actually a special nonlinear regression problem, named multiphase linear regression.

This nonlinear regression problem has a long history in mathematics [37–39] and has been applied in some engineering fields [40]. However, it has not been used extensively in civil engineering. Let (x_k, y_k) , $k = 1, \dots, n$, be n pairs of observation values of displacement and the restoring force within a nonlinear half cycle. Because the viscous damping coefficient c is estimated from the first identification step, the stiffness restoring force can be calculated:

$$f_s(x, \dot{x}) = -m[\ddot{x}_g + \ddot{x}] - \tilde{c}\dot{x}, \quad (12)$$

where $\tilde{c}$ is the estimated viscous damping coefficient from the first step. To optimally approximate the nonlinear half cycles, a multiphase linear regression model can be defined:

$$\begin{aligned} f_1 &= a_1x + b_1, & x_1 \leq x \leq x_0, \\ f_2 &= a_2x + b_2, & x_0 < x \leq x_n, \end{aligned} \quad (13)$$

where $\boldsymbol{\alpha} = \{a_1, b_1, a_2, b_2\}^T$ is the set of the unknown regression coefficients of each segment and x_0 is the unknown interaction point. The interaction point satisfies the linear constraint to ensure the continuity of the solution at the interaction point:

$$a_1x_0 + b_1 = a_2x_0 + b_2. \quad (14)$$

Using a least squares method, it is possible to seek the best estimate of the vector $\boldsymbol{\alpha}$, which minimizes the residual sum

$$\begin{aligned} R(\boldsymbol{\alpha}) &= \sum_{x_1 \leq x_k \leq x_0} [y_k - (a_1x_k + b_1)]^2 \\ &+ \sum_{x_0 < x_k \leq x_n} [y_k - (a_2x_k + b_2)]^2 \end{aligned} \quad (15)$$

and is subject to the constraint (14).

To minimize the function $R(\boldsymbol{\alpha})$, a method similar to the one implemented in [40] is used here. Conceptually, if the transition point is known, the minimum of $R(\boldsymbol{\alpha})$ can be found by computing a standard linear regression for each segment. Thus, given a specific division between data points I and $I + 1$, the residual sum can be minimized over $\bar{\boldsymbol{\alpha}}_I =$

$\{a_1, b_1, a_2, b_2\}^T$, and this outcome yields a sequence of residual sum functions $R_I(\alpha)$ ($I = 2, \dots, n-2$). The goal is to pick I that gives the minimum value for $R_I(\alpha)$. Note that this is true only when $x_I \leq x_0 \leq x_{I+1}$. The estimator of x_0 has to be computed using the linear constraint (14) from the elements of $\tilde{\alpha}_I$ to check that x_0 is in fact between the two data points I and $I+1$ to ensure the solution is the final solution. Using the proposed nonlinear regression analysis method, each nonlinear half cycle is approached by a two-segment broken line. This process yields the estimates of postyielding stiffness and yielding turning point on each nonlinear half cycle.

3.3. Damage Indicators. Information obtained from the proposed two-step method can be used to derive important damage indicators related to damage severity and repair cost of the target structure. In particular, consider the following:

- (1) The preyielding and postyielding stiffness, k_e and k_p , give good approximation of the actual system mechanical behaviour. k_p clearly indicates the system residual load carrying capacity after yielding. The k_p to k_e ratio, like bilinear factor α , can be used as a damage indicator to represent the sacrificial or residual stiffness during seismic events. Finally, changes in k_p and k_e over time indicate system stiffness/strength degradation.
- (2) The yielding turning points identified in Step 2 are related to system yield deformation, d_y . It can be seen from Figure 1 that, for an unloading half cycle i ,

$$d_{yi} = \frac{x_{\max,i} - x_{0,i}}{2}, \quad (16)$$

where $x_{\max,i}$ is the displacement history maximum during the half cycle and $x_{0,i}$ is estimated interaction point of the half cycle i . For a loading nonlinear half cycle i ,

$$d_{yi} = \frac{x_{0,i} - x_{\min,i}}{2}, \quad (17)$$

where $x_{\min,i}$ is the displacement history minimum during the half cycle i .

- (3) Cumulative plastic deformation can be used to capture the accumulation of damage sustained during dynamic loading. It can be calculated by summing the absolute plastic deformation over all nonlinear half cycles. The nonlinear plastic deformation d_p for an unloading half cycle i can be calculated:

$$d_{pi} = (x_{\min,i} - x_{0,i}) \left(1 - \frac{k_p}{k_e} \right), \quad (18)$$

and for a loading half cycle i ,

$$d_{pi} = (x_{\max,i} - x_{0,i}) \left(1 - \frac{k_p}{k_e} \right). \quad (19)$$

Thus, the cumulative plastic deformation d_{net} is defined:

$$d_{\text{net}} = \sum_{i=1}^{nl} |d_{pi}|, \quad (20)$$

and the residual deformation d_{rel} is defined:

$$d_{\text{rel}} = \sum_{i=1}^{nl} d_{pi}, \quad (21)$$

where nl is the number of identified half cycles.

Other damage indices may also be easily obtained based on identified parameters. The more important point is that, with the estimated physical system parameters, model validation and response prediction for future seismic event are also possible, which will give a further critical reference for evaluation of structural safety and repair costs. Finally, quantified knowledge of these values could provide a better foundation for decision making by building owners, tenants, and insurers, reducing debate and speeding up recovery.

4. Simulated Proof-of-Concept Structure

The simulated proof-of-concept structure is a SDOF moment-resisting frame model of a five-story building shown in Figure 2. The seismic weight per floor is 1692 kN for the roof level and 2067 kN for all other levels. The frame system is designed using the displacement-based design approach to sustain a target drift of 2% under a 500-year return period earthquake. A push-over analysis shows bilinear behaviour between base-shear and roof displacement with yield deformation $d_y = 46.5$ mm, preyielding stiffness $k = 27300$ kN/m, and bilinear factor $\alpha = 0.065$. The estimated linear structural fundamental period is ~ 1.20 s. The detailed nonlinear push-over results can be found in [19]. A damping ratio of 5% is assumed which is common for civil structures and the corresponding viscous damping coefficient c is 521 kN-s/m.

Structural displacement and acceleration response is obtained through Newmark numerical integration. The sampling frequency is 200 Hz for the measurement of acceleration and is 20 Hz for the displacement. The objective of applying the proposed SHM method is to determine the structural properties of the preyielding stiffness, bilinear factor, and yielding deformation and estimate cumulative plastic deformation and residual deformation to indicate potential structural damage. The proposed method is implemented in MATLAB.

First, the targeted structure was subjected to the 1987 Superstition Hill earthquake with peak ground acceleration (PGA) of 0.358 g (EQ1 in Table 1). The SHM method was first demonstrated for proof of the concept using noise-free response signals. The effect of choices of the threshold to separate linear and nonlinear half cycles is investigated. Next, the effect of measurement noise was studied by adding a white noise process to acceleration and displacement response and ground acceleration, respectively. Four noise levels of 3%, 5%, 10%, and 20% RMS noise-to-signal ratio are considered. This

TABLE 1: Selected 20 ground motions.

EQ	Event	Year	M_W	Station	R-distance (km)	Soil type	Duration (s)	PGA (g)
EQ1	Superstition Hill	1987	6.7	EI Centro Imp. Co. Cent	13.9	D	40.0	0.358
EQ2				Brawley	18.2	D	22.0	0.156
EQ3				Plaster City	21.0	D	22.2	0.121
EQ4	Northridge	1994	6.7	Beverly Hills 14145 Mulhol	19.6	C	30.0	0.516
EQ5				Canoga Park-Topanga Can	15.8	D	25.0	0.356
EQ6				Glendale-Las Palmas	25.4	D	30.0	0.206
EQ7				LA-Hollywood Stor. FF	25.5	D	40.0	0.231
EQ8				N. Hollywood-Coldwater Can	14.6	C	21.9	0.273
EQ9				LA-N Faring Rd	23.9	D	30.0	0.298
EQ10				Sunland-Mt Gleason Ave	17.7	C	30.0	0.127
EQ11	Loma Prieta	1989	6.9	Capitola	14.5	D	40.0	0.529
EQ12				Gilroy Array #3	14.4	D	39.9	0.555
EQ13				Gilroy Array #4	16.1	D	40.0	0.417
EQ14				Gilroy Array #7	24.2	D	40.0	0.226
EQ15				Hollister Diff. Array	25.8	D	39.6	0.269
EQ16				Saratoga-W Valley Coll.	13.7	C	40.0	0.332
EQ17	Cape Mendocino	1992	7.1	Fortuna-Fortuna Blvd	23.6	C	44.0	0.116
EQ18				Rio Dell Overpass-FF	18.5	C	36.0	0.171
EQ19	Landers	1992	7.3	Desert Hot Springs	23.3	C	50.0	0.385
EQ20				Yermo Fire Station	24.9	D	44.0	0.245

FIGURE 2: The simulated five-storey shear-type building.

case was repeated for 100 Monte-Carlo runs to find the effect of noise and the range of possible variation at the given noise level.

To assess the robustness of the proposed method over different ground motions, the simulated structures were subjected to a suite of 20 ground motions with different spectral characteristics and PGA, as shown in Table 1. These earthquake records are widely used in earthquake engineering [19]. In each case, the noise level of 10% RMS is considered.

It is noted that in this study the 10% RMS Gaussian white noise was selected as a base and typical noise level and added to measurements of ground acceleration, structural acceleration, and displacement because it is considered to be a normal working range for civil engineering applications [27] and big enough for typical acceleration and displacement sensors accuracy [26, 28–30]. The reason of studying different

noise levels is to prove the robustness of the proposed method and clarify the sensitivity of the method to noise.

5. Results and Discussions

5.1. Validation of the Proposed Method Using Noise-Free Response. Simulated noise-free high-rate acceleration and low-rate displacement responses were first used as inputs to reconstruct high-rate displacement and velocity using the multirate Kalman filtering method. These reconstructed responses, together with ground and response acceleration, are input to SHM procedure.

Figure 3 plots the identified equivalent linear system stiffness k_l and equivalent viscous damping coefficient c_l for each half cycle versus half-cycle displacement Δd , respectively. The points in Figure 3 can be divided into two regimes

TABLE 2: Estimations of preyielding stiffness [KN/m] and viscous damping coefficient [KN·s/m].

	Mean	Mean error	St. d.	95% confidence interval	True value
Preyielding stiffness	27335	0.1%	271	[27263, 27407]	27300
Viscous damping coefficient	482	7.5%	108	[453, 510]	521

TABLE 3: Estimated structural performance parameters from multiphase linear regression analysis.

Half cycle #	Bilinear factor	Yield displacement (mm)	Plastic displacement (mm)
31	0.062	46.7	-64.4
32	0.061	47.3	+104.7
Mean	0.062	47.0	—
Mean error	4.6%	1.1%	—
True values	0.065	46.5	—

FIGURE 3: Linear regression results: (a) effective stiffness with half-cycle displacement; (b) effective viscous damping with half-cycle displacement.

according to the trend of variation. When the half-cycle displacement is small, k_l and c_l are nearly constant because the structure behaves linearly. Both drop rapidly as the amplitude of displacement exceeds a critical value. Therefore, Figure 3 can be used as a qualitative indicator of system linear or nonlinear behaviour during an earthquake. If all points are around a horizontal line, the structure can be viewed as linear or nearly linear. Otherwise, nonlinear deformation should be considered. Based on Figure 3, a threshold can be assigned to separate linear and nonlinear regimes. In this case, the threshold of 0.11 m is used and the effect of the choices will be investigated in next section.

All points in linear regions directly give estimates of preyielding stiffness and viscous damping coefficient. Table 2 gives the statistical results of these two parameters. It can be seen that the identified mean of k_e is very close to the true model values with relative error of 0.1%. Since viscous damping restoring force forms a very small part of the total restoring force, the mean estimate error of the viscous damping coefficient is a little larger but still satisfying. Overall, the results demonstrate that the proposed method can give good estimates of system preyielding stiffness and viscous damping coefficient.

Half cycles in nonlinear regime are identified using Step 2. Figure 4 shows the simulated true hysteresis loops. Figure 5 shows the identified nonlinear half cycles and multiphase linear regression approximation results. It can be seen that, using the threshold, the main nonlinear hysteretic half cycles

FIGURE 4: Simulated hysteresis loops.

are captured (#31, 32) that dominate the nonlinear structural behaviour.

Multiphase linear regression results approach the identified hysteresis half cycles very well. Table 3 shows the detailed

FIGURE 5: Identified nonlinear half cycles and multiphase linear regression results for 2 half cycles. The line is the identified model and the circles are the simulated data.

multiphase regression results for two half cycles, #31~32. It can be seen that the estimated bilinear factor and yield displacement are very close to the true parameters. The derived plastic displacement in each nonlinear half cycle can be summed to obtain the cumulative plastic deformation and residual displacement. In this case, the cumulative plastic deformation is 169.1 mm and the residual displacement is +40.3 mm.

5.2. Effect of Threshold Chosen. The effect of the choice of threshold is investigated by varying its values between 0.09 m and 0.13 m. The results are listed in Table 4. It can be seen that there is little effect on the identification accuracy of the linear parameters, preyielding stiffness, and viscous damping coefficient when the threshold chosen varies from 0.09 m to 0.13 m. However, identified bilinear factor and yield displacement show a larger error when the threshold is lower because some linear or nearly linear half cycles are identified as nonlinear. In this situation, multiphase linear regression will give poor results due to wrong regression model used. Thus, identification accuracy improves when only large displacement nonlinear half cycles are considered (thresholds larger than 0.11 m). However, it is noted that a very large threshold will mean some large displacement half cycles lost and underestimate the cumulative plastic deformation.

5.3. Effect of Noise on Parameter Identification. Figure 6 shows the multiphase linear regression analysis results for 100 runs at different noise levels. It can be seen that as the noise level increases, the nonlinear regression accuracy and

consistency both decrease. A threshold of 0.11 m was used in all cases.

The statistical summary of identified system parameters compared to the true model parameters is listed in Table 5. It can be seen from Table 5 and Figure 6 that the numerical accuracy of the identified parameters is generally very good and the two-step identification method proposed can give robust system performance parameters even at 10% RMS added noise level. In particular, the identified preyielding stiffness and yield displacement are less sensitive to noise than the viscous damping coefficient and bilinear factor. Even with added 20% RMS noise, the mean relative error of preyielding stiffness and yield displacement is within 2%. Thus, the identification of preyielding stiffness and yield deformation using the proposed method is highly robust to measurement noise. The identified bilinear factor is also excellent to 10% noise and good at 20%. It is more sensitive to noise due to the fact that there are far less data points in the nonlinear regime than in elastic regime, and regression analysis is sensitive to the number of data points. Identification accuracy would be improved if there were more large plastic displacements to provide a larger number of data points.

5.4. Identification Results over 20 Seismic Events. Table 6 lists the identified system parameters and damage indicators over 20 seismic events with 10% RMS added noise. A “—” is presented where the structure is identified as remaining linear during the event. The structure was identified as remaining linear for all of EQ2, 3, 5, 6, 9, 10, 12, 14, 17, and 19, and as nonlinear for the other events. Therefore, the proposed method can directly detect whether the structure undergoes nonlinear deformation. The identified system model parameters match very well with true model parameters and demonstrate the proposed method is robust to ground motions. The method can derive two damage indicators: cumulative plastic deformation and residual deformation, used to assess structural damage severity and repair costs. For example, estimated maximum cumulative plastic deformation is 500.4 mm for EQ11, which indicates the structure is significantly damaged, while it is much lower for EQ7.

It is noted that there is no direct comparative assessment of the proposed method against any existing SHM techniques. The primary reason is that no prior methods split the linear half cycles from the nonlinear half cycles of response and pull out nonlinear half-cycle displacement and postyielding stiffness, except Nayyerloo et al. [19], which is a much more complex, but real-time, algorithm. Equally importantly, the work of Nayyerloo et al. [19] is restricted to fitting a Bouc-Wen model, whereas this approach is more general to any nonlinear, elastoplastic method. Finally, it is important to note that we found no prior works that directly identified nonlinear stiffness in this fashion making direct comparison very difficult for those that do address nonlinear behaviour.

Although the efficiency of the method is demonstrated using a simple closed-formed problem, the value of the proposed method can be evaluated from three perspectives. First, the key of the method is to capture half cycles and get elastoplastic properties from them. It is not dependent on a specific

TABLE 4: Effect of threshold chosen on identification results.

	Preyielding stiffness (KN/m)	Viscous damping (KN-s/m)	Yield deformation (mm)	Bilinear factor
True	27300	521	46.5	0.065
Threshold = 0.09 m				
Mean	27380	464	39.2	0.284
Coefficient of variation	0.007	0.174	0.356	1.581
Mean error	0.3%	10.9%	15.7%	336.9%
Threshold = 0.10 m				
Mean	27371	468	46.7	0.079
Coefficient of variation	0.007	0.176	0.010	0.702
Mean error	0.3%	10.2%	0.4%	21.5%
Threshold = 0.11 m				
Mean	27335	482	47.0	0.062
Coefficient of variation	0.010	0.224	0.008	0.008
Mean error	0.1%	7.5%	1.1%	4.6%
Threshold = 0.12 m				
Mean	27335	482	47.0	0.061
Coefficient of variation	0.010	0.224	0.008	0.008
Mean error	0.1%	7.5%	1.1%	6.2%
Threshold = 0.13 m				
Mean	27335	482	47.0	0.061
Coefficient of variation	0.010	0.224	0.008	0.008
Mean error	0.1%	7.5%	1.1%	6.2%

TABLE 5: Statistical summary of estimated system parameters for 100 Monte-Carlo runs (threshold = 0.11 m).

	Preyielding stiffness (KN/m)	Viscous damping (KN-s/m)	Bilinear factor	Yield deformation (mm)
Actual model	27300	521	0.065	46.5
Noise-free				
Mean	27335	482	0.062	47.0
Coefficient of variation	0.0000	0.0000	0.0000	0.0000
Mean error	0.1%	7.5%	4.6%	1.1%
3% RMS white noise				
Mean	27335	481	0.062	47.1
Coefficient of variation	0.0000	0.0124	0.0212	0.002
Mean error	0.1%	7.7%	4.6%	1.3%
5% RMS white noise				
Mean	27336	482	0.064	47.2
Coefficient of variation	0.0018	0.0210	0.0345	0.0035
Mean error	0.1%	7.5%	1.5%	1.5%
10% RMS white noise				
Mean	27237	474	0.071	47.3
Coefficient of variation	0.0082	0.0625	0.0706	0.0080
Mean error	0.2%	9.0%	9.2%	1.7%
20% RMS white noise				
Mean	26825	440	0.092	46.9
Coefficient of variation	0.0142	0.1293	0.0855	0.0130
Mean error	1.7%	15.5%	41.5%	0.9%

FIGURE 6: Identified nonlinear half cycles and multiphase linear regression results.

mechanics model but instead relies on direct measurements and identified half cycles. Thus, the proposed method can be generalized to identify similar hysteretic systems, where nonlinear half-cycle shape can be approximated by a bilinear shape. Second, the identification procedure is carried out from half cycle to half cycle. It thus can capture time-variant physical parameters to characterize a degrading hysteretic system. Third, the identification procedure is essentially performed storey by storey. Therefore, the proposed method is completely generalizable to overall nonlinear multistorey structures and a wide range of mechanics.

However, the robustness of the method to real data is still unproven since the significant plastic real data is available in a limited way. The proposed identification procedure remains to be experimentally validated and further tested before implementation in the field for final performance evaluation.

6. Conclusions

This paper develops a novel SHM method for civil structures using hysteresis loops reconstructed from seismic response

data. Low-rate sampled displacement and high-rate sample acceleration are fused by the multirate Kalman filtering method to construct high quality hysteresis loops. A two-step regression analysis based method is developed to identify nonlinear system parameters and extract damage indicators related to structural health status and repair costs.

To apply linear and nonlinear regression analysis, system hysteretic loops are split into many half cycles where restoring force is a monotone function of displacement. A special nonlinear regression method, named multiphase linear regression, is used to directly estimate turning points and postyielding stiffness. This approach significantly simplifies the nonlinear system identification procedure for obtaining preyielding stiffness, viscous damping coefficient, postyielding stiffness, and the yield displacement simultaneously.

From the results obtained in this study, it is clear that the proposed method is feasible and effective for nonlinear system identification and damage indicator extraction. When no measurement noise is added, the proposed SHM procedure can identify system physical parameters with very high

TABLE 6: Identification results for 20 seismic events with 10% RMS added noise.

Event #	Preyielding stiffness (KN/m)	Yield displacement (mm)	Bilinear factor	Viscous damping coefficient (KN-s/m)	Estimated cumulative plastic deformation (mm)	Residual deformation (mm)
EQ1	27237	47.3	0.071	474	168.1	42.0
EQ2	27421	—	—	448	—	—
EQ3	27218	—	—	500	—	—
EQ4	26870	51.0	0.108	544	391.5	54.7
EQ5	26971	—	—	557	—	—
EQ6	28119	—	—	480	—	—
EQ7	27411	45.8	0.141	475	21.1	20.0
EQ8	26967	46.0	0.200	577	214.7	33.4
EQ9	27622	—	—	461	—	—
EQ10	27531	—	—	453	—	—
EQ11	26723	46.7	0.154	512	500.4	9.3
EQ12	27571	—	—	453	—	—
EQ13	27346	44.6	0.131	468	198.1	30.0
EQ14	27819	—	—	478	—	—
EQ15	26976	47.1	0.112	506	98.3	60.8
EQ16	27016	47.4	0.123	521	368.0	91.9
EQ17	27470	—	—	463	—	—
EQ18	27289	46.6	0.126	480	234.6	64.0
EQ19	27540	—	—	459	—	—
EQ20	26974	46.9	0.128	503	307.7	97.9
True	27300	46.5	0.065	521		

precision. Even with 10% or 20% RMS noise, the method identifies some system parameters with good precision and has good repeatability and robustness. The proposed method is also robust over different ground motions and can directly detect whether nonlinear response occurs.

Overall, the proposed SHM method is simple, direct, and robust. The identification procedure is performed time segment by time segment, which provides the possibility for it to be implemented in real-time or near real-time. Although the concept is proven focusing on structural systems that display nondegraded hysteresis behaviour, it can be easily extended to degrading structures and arbitrary changes in system parameters, including degradation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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