Research Article

A Cable-Passive Damper System for Sway and Skew Motion Control of a Crane Spreader

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While the crane control problem is often approached by applying a certain active control command to some parts of the crane, this paper proposes a cable-passive damper system to reduce the vibration of a four-cable suspended crane spreader. The residual sway and skew motions of a crane spreader always produce the angle deflections between the crane cables and the crane spreader. The idea in this paper is to convert those deflections into energy dissipated by the viscous dampers, which connect the cables and the spreader. The proposed damper system is effective in reducing spreader sway and skew motions. Moreover, the optimal damping coefficient can be found analytically by minimizing the time integral of system energy. The numerical simulations show that the proposed passive system can assist the input shaping control of the trolley motion in reducing both sway and skew responses.

1. Introduction

Because the crane spreader suspended by cables is highly flexible in nature, the external disturbances, such as wind or motion of the support unit (e.g., the bridge or trolley or tower) often cause the residual oscillation. In general, the control strategies proposed in the literature are often approached by applying control command to some parts of crane. The control command can be applied to the cables [1, 2], to the trolley [3, 4], to the boom [5, 6], or to the active mass damper [7]. The control algorithm can be based on open-loop or closed-loop techniques [8]. The closed-loop (feedback) techniques use the crane measurements (e.g., deflection, and position) to generate the control command. The feedback control provides disturbance rejection but it is not easy to obtain accurate measurements of payload deflection and other system states. The sensors can be expensive and difficult to mount, calibrate, and maintain. Feedback control can also cause unexpected motions that make it difficult for the human operator to drive the crane. On the other hand, a typical open-loop technique, namely, input shaping, modifies the desired velocity command before it is issued to the crane motors [9–12]. The input

shaping techniques are easy to apply and do not require the sensors. However, they lack the ability to handle external disturbances or initial conditions. While many other studies on open- and closed-loop control of crane can be referred to in the references of the aforementioned papers, this paper proposes a rather more conventional damping system.

It is well-known that the passive viscous damper is the most popular and simple device used in vibration control. Because the crane payload is often modeled as a single pendulum, it is not easy to imagine how to install a passive damper into a single cable. Some recent approaches propose the Coriolis damper to the rigid pendulum [13–15]. In fact, many actual crane mechanisms consist typically of a set of four hoisting cables dropping from four points on a trolley and attached to four points on a spreader bar. Some studies have considered multicables mechanism [1, 2, 16, 17]. The idea of this paper is based on a reality that when the spreader oscillates, the angles between the cables and the spreader change. It is noted that if the spreader is treated as a point mass, the mentioned angles are meaningless. If we consider the actual multicables mechanism of the crane, the opportunity is opened for converting the relative movements between

FIGURE 1: Modified spreader to implement the cable damper system.

the cables and the spreader into the energy dissipation in the viscous dampers. The proposed damping system in this paper is purely passive and it can not replace the active control schemes. Instead of that, because the passive devices do not rely on sensors and external energy, it is expected that the proposed damping system can be used to improve any active control scheme acting on the crane. To demonstrate this fact, in the numerical simulation in this paper, the proposed system is used to improve an input shaping control of the trolley motion.

In the multicables mechanism, the crane spreader has six degrees of freedom: three translational motions and three rotational motions. Three rotational motions consisting of roll, pitch, and yaw (skew) can be caused by the uneven distribution of the materials inside the container or misaligned ropes or other external disturbances like the wind or interaction of the load with other rigid obstacles. Sometimes they make it difficult to position the spreader fast and accurately. Among these three rotational motions, the skew motion is the most critical in the operation of a crane.

The novelty of this paper is the proposal and design of a cable-damper system which can regulate effectively both sway and skew motions of a four-cable suspended crane spreader. The cable damper system consists of 4 short cables and 8 viscous dampers connecting the crane long cables and the crane spreader through spherical joints (Figure 1). The structure of paper is as follows. At first, the full linearized equations of motion containing 18 equations are written in nondimensional vector matrix form. Then the formula of the stored energy of sway motion and skew motion are derived based on Lyapunov matrix equation. The optimal damping coefficient is obtained analytically by minimizing the stored energy. At last, the damper effectiveness is verified by nonlinear numerical simulations.

2. Equations of Free Vibration

The proposed cable-damper system consists of 4 new cables and 8 viscous dampers as shown in Figure 1. The original

cables are proposed not to connect directly to the spreader but through the cable-damper system (Figure 1).

Let us consider the details in Figure 2.

Two coordinate reference frames are assigned. They are the global reference frame ($Oxyz$) (called T-frame), on the base of the rail, and the spreader attached reference frame ($Px_s y_s z_s$) (called S-frame), in the geometric center of the top of the spreader. Assume that the principal axes of the spreader are coincided with the coordinate axes of S-frame, meaning that the moment of inertia matrix $\mathbf{J}$ of the spreader has diagonal form and is denoted by $\mathbf{J} = \text{diag}(J_x, J_y, J_z)$ in the S-frame. The length and width between the trolley cabling points are denoted by $2a_t$ and $2b_t$, respectively. The length and width between the spreader cabling points are denoted by $2a$ and $2b$, respectively. The spreader mass is denoted by m . The distance between the trolley and the spreader is denoted by l . The elastic modulus and the cross-sectional area of the cable are denoted by E and A_c , respectively. Let A_i ($i = 1, \dots, 4$) and B_i ($i = 1, \dots, 4$) denote the locations of cables on the trolley and on the spreader, respectively.

The proposed damper system consists of 4 short cables and 8 identical viscous dampers connecting the main cables and the spreader. Each main cable connects with one damper in X -direction and another damper in Y -direction. Figure 2 shows a typical set of one short cable and two dampers connecting to the main cable number 4. Denote D_i ($i = 1, \dots, 4$) as the connection points between the main cables and the damper, while B_{ix} ($i = 1, \dots, 4$) and B_{iy} ($i = 1, \dots, 4$); denote the connection points between the spreader and the dampers. Two ends of each damper use spherical joints to allow damper to rotate freely. The damping coefficient of each damper is denoted by c . The system in Figures 1 and 2 has 18 degrees of freedom, including three spreader translations, three spreader rotations, and three translations of each in four connection points D_i ($i = 1, \dots, 4$). Let x, y, z denote three spreader translations measured from static position, ϕ, θ, ψ denote the angles representing the roll, pitch, and yaw (skew) motions of the spreader, and x_i, y_i, z_i ($i = 1, \dots, 4$) denote three translations of the connection points D_i ($i = 1, \dots, 4$) measured from static position. The derivation of equations of motion is explained in the appendix. The nonlinear equations are very complicated to write in the full forms. However, the linearized equations can be written in compact forms by using the following fundamental assumption: four main cables are parallel in static position. This assumption is valid if the main crane cables are long enough. Let us introduce the following parameters:

$$\omega_s = \sqrt{\frac{g}{l}};$$

$$\tau = \omega_s t;$$

$$\alpha_s = \sqrt{\frac{EA_c}{mg}};$$

$$\zeta = \frac{c}{2m\omega_s};$$

$$\begin{aligned}
\gamma &= \frac{l - l_d}{l}; \\
d &= \frac{u_d}{l}; \\
\beta &= \frac{u_d^2}{u_d^2 + l_d^2}; \\
r_a &= \frac{a}{l}; \\
r_b &= \frac{b}{l}; \\
J_1 &= \frac{J_x}{ml^2}; \\
J_2 &= \frac{J_y}{ml^2}; \\
J_3 &= \frac{J_z}{ml^2},
\end{aligned} \tag{1}$$

in which, ω_s is the spreader sway frequency, τ is the normalized time with time scale ω_s^{-1} and the dot operator from now denoting the differentiation with respect to τ , α_s is the ratio between the vertical vibration frequency and the sway frequency, ζ is the damping ratio of each viscous damper, γ

is the location parameter, d , r_a , r_b are normalized lengths, J_i ($i = 1, \dots, 3$) are normalized moment inertias, and β is the cosine of the angle between damper and spreader. The linearized equations can be expressed in vector matrix form as follows:

$$\begin{aligned}
&\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{s}}_1 \\ \ddot{\mathbf{s}}_2 \\ \ddot{\mathbf{s}}_3 \\ \ddot{\mathbf{s}}_4 \end{bmatrix} \\
&+ 2\zeta\beta \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_{12} & \mathbf{C}_{13} & \mathbf{C}_{14} \\ \mathbf{C}_{12}^T & \mathbf{C}_2 & \mathbf{C}_{23} & \mathbf{C}_{24} \\ \mathbf{C}_{13}^T & \mathbf{C}_{23}^T & \mathbf{C}_3 & \mathbf{C}_{34} \\ \mathbf{C}_{14}^T & \mathbf{C}_{24}^T & \mathbf{C}_{34}^T & \mathbf{C}_4 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \\ \dot{\mathbf{s}}_3 \\ \dot{\mathbf{s}}_4 \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_{12} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{12}^T & \mathbf{K}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_3 & \mathbf{K}_{34} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{34}^T & \mathbf{K}_4 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \\ \mathbf{s}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},
\end{aligned} \tag{2}$$

in which the superscript “ T ” denotes transpose matrix and

$$\begin{aligned}
\mathbf{s}_1 &= [x \ y \ l\psi]^T, \\
\mathbf{s}_2 &= [x_1 \ y_1 \ x_2 \ y_2 \ x_3 \ y_3 \ x_4 \ y_4]^T, \\
\mathbf{s}_3 &= [z \ l\phi \ l\theta]^T, \\
\mathbf{s}_4 &= [z_1 \ z_2 \ z_3 \ z_4]^T, \\
\mathbf{M}_1 &= \text{diag}(1, 1, J_3), \\
\mathbf{M}_3 &= \text{diag}(1, J_1, J_2), \\
\mathbf{K}_1 &= \frac{1}{\gamma} \text{diag}(1, 1, r_a^2 + r_b^2), \\
\mathbf{K}_{12} &= \frac{1}{4\gamma} \begin{bmatrix} -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ r_b & -r_a & r_b & r_a & -r_b & r_a & -r_b & -r_a \end{bmatrix}, \\
\mathbf{K}_2 &= \frac{1}{4\gamma(1-\gamma)} \mathbf{I}_8, \\
\mathbf{K}_3 &= 4\frac{\alpha_s^2}{\gamma} \text{diag}(1, r_b^2, r_a^2), \\
\mathbf{K}_{34} &= \frac{\alpha_s^2}{\gamma} \begin{bmatrix} -1 & -1 & -1 & -1 \\ -r_b & -r_b & r_b & r_b \\ r_a & -r_a & -r_a & r_a \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\mathbf{K}_4 &= \frac{\alpha_s^2}{\gamma(1-\gamma)} \mathbf{I}_4, \\
\mathbf{C}_1 &= 4 \times \text{diag}(1, 1, r_a^2 + r_b^2), \\
\mathbf{C}_{12} &= \begin{bmatrix} -1 & 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 & -1 \\ r_b & -r_a & r_b & r_a & -r_b & r_a & -r_b & -r_a \end{bmatrix}, \\
\mathbf{C}_2 &= \mathbf{I}_8, \\
\mathbf{C}_{13} &= \frac{\gamma}{d} \begin{bmatrix} 0 & 0 & 4(r_a - d) \\ 0 & -4(r_b - d) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathbf{C}_{14} &= \frac{\gamma}{d} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ r_a - r_b & r_b - r_a & r_a - r_b & r_b - r_a \end{bmatrix}, \\
\mathbf{C}_{23} &= \frac{\gamma}{d} \begin{bmatrix} 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ r_b & r_b - d & -r_b & r_b - d & r_b & r_b - d & -r_b & r_b - d \\ d - r_a & -r_a & d - r_a & r_a & d - r_a & -r_a & d - r_a & r_a \end{bmatrix}^T, \\
\mathbf{C}_{24} &= \frac{\gamma}{d} \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}^T, \\
\mathbf{C}_3 &= 4 \frac{\gamma^2}{d^2} \text{diag}(2, r_b^2 + (r_b - d)^2, r_a^2 + (r_a - d)^2), \\
\mathbf{C}_{34} &= \frac{\gamma^2}{d^2} \begin{bmatrix} -2 & -2 & -2 & -2 \\ -2r_b + d & -2r_b + d & 2r_b - d & 2r_b - d \\ 2r_a - d & -2r_a + d & -2r_a + d & 2r_a - d \end{bmatrix}, \\
\mathbf{C}_4 &= 2 \frac{\gamma^2}{d^2} \mathbf{I}_4,
\end{aligned} \tag{3}$$

in which $\mathbf{0}$ defines the zeros matrix with appropriate dimension while $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. The motion equations can be simplified more by assumption that the cables are stiff enough to make the heave, the roll, and the pitch motions be small in comparison with the sway and the skew motions. Ignoring the vectors $\mathbf{s}_3$ and $\mathbf{s}_4$ gives the following simplified equations:

$$\begin{aligned}
&\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{s}}_1 \\ \ddot{\mathbf{s}}_2 \end{bmatrix} + 2\zeta\beta \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_{12} \\ \mathbf{C}_{12}^T & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{s}}_1 \\ \dot{\mathbf{s}}_2 \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_{12} \\ \mathbf{K}_{12}^T & \mathbf{K}_2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \tag{4}
\end{aligned}$$

which can be expressed in state space form as follows:

$$\dot{\mathbf{p}} = \mathbf{A}\mathbf{p}, \tag{5}$$

in which $\mathbf{p}$ is a 14×1 state vector and $\mathbf{A}$ is a 14×14 system matrix defined as

$$\begin{aligned}
\mathbf{p} &= \begin{bmatrix} \mathbf{s}_1 \\ \dot{\mathbf{s}}_1 \\ \mathbf{s}_2 \end{bmatrix}; \\
\mathbf{A} &= \begin{bmatrix} \mathbf{I}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_1 & 2\zeta\beta\mathbf{C}_{12} \\ \mathbf{0} & \mathbf{0} & 2\zeta\beta\mathbf{C}_2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} & \mathbf{I}_3 & \mathbf{0} \\ \mathbf{K}_1 & 2\zeta\beta\mathbf{C}_1 & \mathbf{K}_{12} \\ \mathbf{K}_{12}^T & 2\zeta\beta\mathbf{C}_{12}^T & \mathbf{K}_2 \end{bmatrix}. \tag{6}
\end{aligned}$$

The state space form (5) is used in damper design, but the original nonlinear full equation is used in numerical simulation to check the accuracies of the simplified equation (5).

3. Optimal Damping

Imagine what would happen with the increased damping in the dampers. Without damping, the response obviously is the same as that in the uncontrolled case. Conversely, when the damping becomes infinite, the two connection points of a damper are virtually fused to each other and the result is a crane system with the cable length shorter but without damping again. Therefore, somewhere between these extremes, there must be an optimal damping value. In this section, the optimal value is obtained by minimizing the time integration of the stored energy in sway and skew motions.

In linear region, the stored energy in sway and skew motions, which consists of kinetic and potential energies can be expressed by

$$E_s = \dot{\mathbf{s}}_1^T \mathbf{M}_1 \dot{\mathbf{s}}_1 + \begin{bmatrix} \dot{\mathbf{s}}_1^T & \dot{\mathbf{s}}_2^T \end{bmatrix} \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_{12} \\ \mathbf{K}_{12}^T & \mathbf{K}_2 \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} = \mathbf{p}^T \mathbf{Q} \mathbf{p}, \quad (7)$$

in which,

$$\mathbf{Q} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{K}_{12} \\ \mathbf{0} & \mathbf{M}_1 & \mathbf{0} \\ \mathbf{K}_{12}^T & \mathbf{0} & \mathbf{K}_2 \end{bmatrix}. \quad (8)$$

Because the stored energy is reduced over time due to the damping, the integral of energy should be considered. The following nondimensional performance index is introduced:

$$J(\mathbf{p}_0, \zeta) = \frac{\int_0^\infty E_s d\tau}{E_s(0)} = \frac{\int_0^\infty \mathbf{p}^T \mathbf{Q} \mathbf{p} d\tau}{\mathbf{p}_0^T \mathbf{Q} \mathbf{p}_0}, \quad (9)$$

where $\mathbf{p}_0 = \mathbf{p}(0)$ is the vector containing initial conditions. The performance index J depends on both initial conditions $\mathbf{p}_0$ and damping coefficient ζ . Let us do some transformation to remove the integral. Consider matrix $\mathbf{P}$ being the solution of Lyapunov matrix equation as

$$\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P} + \mathbf{Q} = \mathbf{0}. \quad (10)$$

The matrix $\mathbf{P}$ can be obtained analytically with the aid of Symbolic Math Toolbox in Matlab. Substituting (10) into the integral in (9) and using state space equation (5) give

$$\begin{aligned} \int_0^\infty \mathbf{p}^T \mathbf{Q} \mathbf{p} d\tau &= - \int_0^\infty \mathbf{p}^T (\mathbf{P} \mathbf{A} + \mathbf{A}^T \mathbf{P}) \mathbf{p} d\tau \\ &= - \int_0^\infty (\mathbf{p}^T \mathbf{P} \dot{\mathbf{p}} + \dot{\mathbf{p}}^T \mathbf{P} \mathbf{p}) d\tau \\ &= (\mathbf{p}^T \mathbf{P} \mathbf{p}) \Big|_0^\infty \\ &= \mathbf{p}_0^T \mathbf{P} \mathbf{p}_0 - \mathbf{p}(\infty)^T \mathbf{P} \mathbf{p}(\infty). \end{aligned} \quad (11)$$

It is assumed that the system is asymptotically stable due to the presence of damping, we have $\mathbf{p}(\infty) = \mathbf{0}$ and the

performance index (9) can be written without integral as follows:

$$J(\mathbf{p}_0, \zeta) = \frac{\mathbf{p}_0^T \mathbf{P} \mathbf{p}_0}{\mathbf{p}_0^T \mathbf{Q} \mathbf{p}_0}. \quad (12)$$

Let us find the extrema of J over the change of initial condition $\mathbf{p}_0$. Equation (12) can be rewritten as

$$\mathbf{p}_0^T (\mathbf{P} - J(\mathbf{p}_0, \zeta) \mathbf{Q}) \mathbf{p}_0 = 0. \quad (13)$$

Use vector and matrix derivative [18], taking the derivative of (13) with respect to vector $\mathbf{p}_0$ gives

$$2(\mathbf{P} - J(\mathbf{p}_0, \zeta) \mathbf{Q}) \mathbf{p}_0 + \mathbf{Q} \mathbf{p}_0 \mathbf{p}_0^T \frac{\partial J(\mathbf{p}_0, \zeta)}{\partial \mathbf{p}_0} = \mathbf{0}. \quad (14)$$

By denoting J_m as the extremum value of J over the change of initial condition $\mathbf{p}_0$, at the extremum point, we have

$$\left. \frac{\partial J(\mathbf{p}_0, \zeta)}{\partial \mathbf{p}_0} \right|_{J=J_m} = \mathbf{0}. \quad (15)$$

Equations (14) and (15) show that J_m is exactly an eigenvalue of the general eigenvalue problem as

$$\det(\mathbf{P} - J_m \mathbf{Q}) = 0. \quad (16)$$

In brief, since the matrix $\mathbf{P}$ has 14 eigenvalues, there are 14 extrema of J , which are denoted by J_{mi} ($i = 1, \dots, 14$). The damping ratio ζ is found to minimize the average value of the extrema J_{mi} ($i = 1, \dots, 14$); that is, the optimization problem is

$$J_{\text{opt}} = \min_{\zeta} \left(\frac{1}{14} \sum_{i=1}^{14} J_{mi} \right). \quad (17)$$

It is well known that the trace of a square matrix is exactly equal to the sum of all its eigenvalues. Therefore we have

$$\sum_{i=1}^{14} J_{mi} = \text{trace}(\mathbf{Q}^{-1} \mathbf{P}). \quad (18)$$

With the aid of Symbolic Math Toolbox in Matlab, the optimal condition,

$$\frac{\partial}{\partial \zeta} \left(\sum_{i=1}^{14} J_{mi} \right) = 0, \quad (19)$$

leads to the following analytical optimal solution:

$$\zeta_{\text{opt}} = \frac{\sqrt{2 + J_3 / (r_a^2 + r_b^2)}}{4\beta\gamma\sqrt{(\gamma + 2)(6 - 5\gamma)}}, \quad (20)$$

$$\begin{aligned} J_{\text{opt}} &= \left(\frac{1}{14} \sum_{i=1}^{14} J_{mi} \right) \Big|_{\zeta=\zeta_{\text{opt}}} \\ &= \frac{\sqrt{2 + J_3 / (r_a^2 + r_b^2)} \sqrt{(\gamma + 2)(6 - 5\gamma)}}{7\gamma}. \end{aligned} \quad (21)$$

TABLE 1: Numerical values used in simulation.

Parameter	Notation	Value
Acceleration of gravity	g	9.80665 (m/s ²)
Distance between trolley and spreader	l	Minimum: 15 (m) Maximum: 25 (m) Used for design: 20 (m)
Cable diameter	d	30 (mm)
Modulus of elasticity	E	2×10^{11} (N/m ²)
Trolley cabling point: length $\times$ width	$2a_t \times 2b_t$	5×2.9 (m $\times$ m)
Spreader cabling point: length $\times$ width	$2a \times 2b$	5×2.5 (m $\times$ m)
Spreader dimension	s_x, s_y, s_z	$6 \times 2.5 \times 1.25$ (m $\times$ m)
Spreader mass	m	Used for design: 20 (tons), taken two values in simulation: 20 (tons), 60 (tons)
Spreader inertias in S-frame	J_x, J_y, J_z	$m(s_y^2 + 4s_z^2)/12, m(s_x^2 + 4s_z^2)/12, m(s_y^2 + s_x^2)/12$
Length of short cables	$l - l_d$	Taken two values in simulation: 2 (m), 4 (m)
Distance from the short cables to the connection points between the dampers and the spreader	u_d	1 (m)
Optimal damping coefficient	c_{opt}	Calculated from (1) and (20)

TABLE 2: Maximum value of the responses after 50 s (the percentage beside it is the vibration reduction).

Case	Response		
	X-offset (m)	Y-offset (m)	Skew angle (rad)
Without ZVD and damper	0.52 (0%)	0.16 (0%)	0.10 (0%)
With ZVD	0.16 (69%)	0.16 (0%)	0.10 (0%)
With ZVD and damper ($m = 20$ tons, $\gamma = 0.1$)	0.08 (85%)	0.06 (63%)	0.02 (80%)
With ZVD and damper ($m = 60$ tons, $\gamma = 0.1$)	0.07 (87%)	0.09 (44%)	0.03 (70%)
With ZVD and damper ($m = 20$ tons, $\gamma = 0.2$)	0.06 (88%)	0.01 (94%)	0.002 (98%)

FIGURE 3: (a) The predefined velocity, (b) the predefined trajectory, (c) and the predefined trolley motion.

FIGURE 4: Spreader X-offset in the case $m = 20$ tons, $\gamma = 0.1$.

FIGURE 8: Effect of mass variance on the spreader Y-offset.

FIGURE 5: Spreader Y-offset in the case $m = 20$ tons, $\gamma = 0.1$.

FIGURE 9: Effect of mass variance on the spreader skew angle.

FIGURE 6: Spreader skew angle in the case $m = 20$ tons, $\gamma = 0.1$.

FIGURE 10: Effect of location parameter on the spreader X-offset.

FIGURE 7: Effect of mass variance on the spreader X-offset.

FIGURE 11: Effect of location parameter on the spreader Y-offset.

FIGURE 12: Effect of location parameter on the spreader skew angle.

- (iii) The damper is designed for the spreader mass of 20 tons. In the case of three-time larger mass ($m = 60$ tons), the designed damper can have not enough damping and the vibration reduction is poorer than one of the case $m = 20$ tons. In the future, the robust design of the damper should be considered.
- (iv) The effect of the location parameter γ (denoted in (1)) is predicted in the end of Section 3 and is verified in the results in Table 2. The larger value of γ , the better vibration reduction.

6. Conclusions

This paper approaches the antisway and antiskew control of a four-cable suspended crane spreader by introducing a passive damping system. By considering the actual mechanism of a multicable crane, a set of four short cables and eight viscous dampers is introduced to connect the main crane cables and the spreader. The deformation angles between the cables and the spreader in spreader oscillation provide the energy dissipation in the dampers. The optimal damping coefficient of the dampers can be found analytically by minimizing the stored energy of sway and skew motions. Numerical simulations on a crane moving in a predefined trajectory show that the proposed cable-damper system can improve the active control using input shaping technique and is effective in reducing both spreader sway and skew motions.

Appendix

Derivation of Equations of Motion

Because there are totally 8 dampers and 8 cable parts, it is not convenient if the forces in each part are written in detail. Instead of that, the forces in the general links as shown in Figure 13 are calculated. Two general links consist of the link AD representing the main cable and the link DB representing the short cable or the viscous damper. Two coordinate reference frames are assigned. They are the global reference frame ($Oxyz$) (called T-frame), on the base of the rail, and the spreader attached reference frame ($Px_s y_s z_s$) (called S-frame), in the geometric center of the top of the spreader. In the T-frame, the coordinates of points A, D, P are denoted as

FIGURE 13: Two general links connect the trolley and the spreader and three Euler angles.

$A(x_A, y_A, 0), D(x_D, y_D, z_D), P(x_P, y_P, z_P)$. In the S-frame, the coordinates of point B are denoted as $B(x_B, y_B, 0)$.

The 3-2-1 choice of Euler angles is used to derive the motion equations. Three successive elementary rotations defined by the yaw angle ψ , the pitch angle θ , and the roll angle ϕ are shown in Figure 13. The coordinate transformation matrix from the S-frame to the T-frame is defined as follows:

$\mathbf{R}$

$$= \mathbf{R}(z, \psi) \mathbf{R}(y, \theta) \mathbf{R}(x, \phi)$$

$$= \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}. \quad (\text{A.1})$$

A vector $\mathbf{V}$ will be denoted by $\mathbf{V}^{(T)}$ or $\mathbf{V}^{(S)}$ if it is defined in the global reference frame (T-frame) or in the spreader reference (S-frame), respectively. A vector $\mathbf{V}$ connecting any two points A and B will be denoted by $\mathbf{V}_{AB}$. Some vectors are calculated as follows:

$$\mathbf{V}_{OA}^{(T)} = [x_A \ y_A \ 0]^T,$$

$$\mathbf{V}_{OP}^{(T)} = [x_P \ y_P \ z_P]^T,$$

$$\mathbf{V}_{PB}^{(T)} = \mathbf{R} \cdot \mathbf{V}_{PB}^{(S)} = \mathbf{R} [x_B \ y_B \ 0]^T,$$

$$\mathbf{V}_{AD}^{(T)} = [x_D - x_A \ y_D - y_A \ z_D]^T, \quad (\text{A.2})$$

$$\begin{aligned} \mathbf{V}_{DB}^{(T)} &= \mathbf{V}_{OP}^{(T)} + \mathbf{V}_{PB}^{(T)} - \mathbf{V}_{OA}^{(T)} - \mathbf{V}_{AD}^{(T)} \\ &= [x_P \ y_P \ z_P]^T + \mathbf{R} [x_B \ y_B \ 0]^T \\ &\quad - [x_D \ y_D \ z_D]^T. \end{aligned}$$

Let us denote $\mathbf{u}_{DB}$ as the unit vector and ε_{DB} as the stretch of the link DB , we have

$$\mathbf{u}_{DB} = \frac{\mathbf{V}_{DB}^{(T)}}{|\mathbf{V}_{DB}^{(T)}|}, \quad (\text{A.3})$$

$$\varepsilon_{AD} = |\mathbf{V}_{DB}^{(T)}| - |\mathbf{V}_{DB}^{(T)}|_0,$$

in which $|\mathbf{V}_{DB}^{(T)}|_0$ is the length of the link DB in static position. In the T-frame, the tension force vector and the torque vector of the link DB are given by

$$\begin{aligned} \mathbf{F}_{DB}^{(T)} &= -(c_{DB}\dot{\varepsilon}_{DB} + k_{DB}\varepsilon_{DB}) \mathbf{u}_{DB}, \\ \mathbf{M}_{DB}^{(T)} &= \mathbf{V}_{DB}^{(T)} \times \mathbf{F}_{DB}^{(T)}, \end{aligned} \quad (\text{A.4})$$

where $\times$ denotes the cross-product operation, c_{DB} and k_{DB} are the damping coefficient and the stiffness coefficient of the link DB , respectively. The same procedure can be applied to find the tension force vector and the torque vector of any link.

After calculating all the force and torque, let us calculate the kinetic and potential energies of the spreader mass. The spreader angular velocity vector $\boldsymbol{\omega}$ can be expressed in the inverse kinematics form as [20]

$$\boldsymbol{\omega} = [\dot{\phi} - \dot{\psi} \sin \theta \quad \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi \quad -\dot{\theta} \sin \phi + \dot{\psi} \cos \theta \cos \phi]^T. \quad (\text{A.5})$$

The kinetic energy T is given by

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2} \boldsymbol{\omega}^T \text{diag}(J_x, J_y, J_z) \boldsymbol{\omega}, \quad (\text{A.6})$$

in which “diag” denotes the diagonal matrix. Assume that the roll angle and pitch angle are small enough to approximate the spreader height by $l - z$, the potential energy U is expressed as

$$U = mg(l - z), \quad (\text{A.7})$$

in which g is the acceleration of gravity. The Lagrange motion equations for 6 degrees of freedom of spreader motion have forms:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{x}} \right) - \frac{\partial (T - U)}{\partial x} &= F_x; \\ \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{y}} \right) - \frac{\partial (T - U)}{\partial y} &= F_y; \\ \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{z}} \right) - \frac{\partial (T - U)}{\partial z} &= F_z; \\ \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{\phi}} \right) - \frac{\partial (T - U)}{\partial \phi} &= M_x; \\ \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{\theta}} \right) - \frac{\partial (T - U)}{\partial \theta} &= M_y; \\ \frac{d}{dt} \left(\frac{\partial (T - U)}{\partial \dot{\psi}} \right) - \frac{\partial (T - U)}{\partial \psi} &= M_z, \end{aligned} \quad (\text{A.8})$$

in which F_x , F_y , and F_z are three components of the total force of every link connecting to the spreader, M_x , M_y , and M_z are three components of the total torque of every link connecting to the spreader. Besides, by assuming that the cable mass and the damper mass are ignored, there are three equations of force equilibrium in each connection point between the dampers and the cables. For example, three force equilibrium equations for point D in Figure 13 can be expressed as

$$F_{Dx} = F_{Dy} = F_{Dz} = 0, \quad (\text{A.9})$$

in which F_{Dx} , F_{Dy} , and F_{Dz} are three components of the total force of every link connecting to the point D . Since there are four connection points between the dampers and the cables, a total of 12 equations can be obtained. Including 6 Lagrange equations (A.8), the mechanical system has 18 equations for 18 degrees of freedom. The linearized equations can be derived by assuming that the vibration states have small perturbations from their values at the equilibrium point. With the aid of Symbolic Math Toolbox in Matlab, after some manipulations to the first order, the linearized equations can be written in matrix form (2).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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