

## Research Article

# Intelligent Vibration Control for High-Speed Spinning Beam Based on Fuzzy Self-Tuning PID Controller

**Lanwei Zhou and Guoping Chen**

*State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*

Correspondence should be addressed to Lanwei Zhou; [zhoulanwei@nuaa.edu.cn](mailto:zhoulanwei@nuaa.edu.cn)

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Spinning structures play an increasingly prominent role in modern engineering. In order to suppress the inevitable vibration of a high-speed spinning flexible beam along the longitudinal direction, intelligent vibration controller is investigated. To design an intelligent vibration controller, the original system is reduced first using the internal balance model reduction method. A fuzzy self-tuning PID controller is further designed based on the reduced-order model. The results show that the reduced-order model derived from the internal balance model reduction method is a good approximation of the original system. In addition, the proposed controller is effective in suppressing the vibration and improves by about 22% compared with the traditional PID controller.

## 1. Introduction

A spinning structure is widely used in modern engineering, for instance, spindles of gyroscopes for control of satellites, satellite booms, shafts of combustion engines, DC and AC electric motors, turbine blades, rotating shafts, and propellers. Accurate dynamic prediction and vibration control of such structures are critical. In recent years, many control methods have been proposed to suppress the vibration of spinning structures [1–5]. It has also been proved that piezoelectric actuator can be used to deal with the vibration control of spinning structures. Zhou and Shi [1] presented a review of the active vibration control and active balancing for rotating structures. Song et al. [2] studied the vibration and stability control of a spinning shaft using both structural tailoring and piezoelectric strain actuation with highlights on the strong and synergistic effects on vibration response and stability boundaries of the advanced composite materials and piezoelectric actuation. Kunze et al. [3] used piezoelectric devices and active vibration reduction instead of viscoelastic materials and proof mass absorbers to reduce the noise levels of rotating shaft. The carried out experiment showed that a remarkable reduction of interior sound pressure level had been achieved. Horst and Wölfel [4] added surface-bonded piezoceramic actuator patches on the shaft surface

to suppress lateral bending vibrations of the elastic shaft, and this approach was proved to be effective through simulation and experimental tests. Sloetjes and de Boer [5] studied a flexible shaft with surface-mounted piezoceramic sheets and strain sensors and analyzed active modal damping and active modal balancing methods. Intelligent controller for spinning beams is seldom carried out in the existing research.

Passive control and active control are the most popular control strategies in the vibration control field. However, passive control methods have poor adaptability and may even become invalid in complex environments. Active control is introduced to cope with this situation due to its good adaptability. Most present researchers focus on adding damping to original system using feedback control method through piezoelectric actuator. Proportional-integral-derivative (PID) controller is one of the most popular feedback control methods in industrial systems for its simplicity, convenience, good adaptive ability, and strong robustness. However, the use of PID controller is usually limited in linear and constant dynamic characteristic system. PID controller could hardly deal with complex system. Spinning system is usually with rigid-flexible coupling and nonlinear characteristic. Simply adding damping or traditional PID controller may not be the best choice. The development of intelligent control methods, such as fuzzy control, provides

a new way to study the vibration control of such complicated system. Fuzzy control is an intelligent control method by imitating fuzzy inferences and decision-making process of human. In addition, it is independent of system models and it can deal with nonlinear characteristic and parameter uncertainty. Fuzzy control combining both traditional controller and fuzzy logic principles has developed markedly and been widely used in the field of vibration control [6–10]. Zheng et al. [6] studied a fuzzy neural network controller and intelligent vibration control of truss structure. Li et al. [7] presented the vibration suppression of large space truss structure using decentralized adaptive fuzzy control methods. Chen [8] developed a simple, convenient fuzzy controller for interconnected systems and established criteria to ensure the stability using the fuzzy Lyapunov functions. Chen et al. [9] designed an intelligent controller for a pneumatic vibration isolator system with nonlinear and time-varying characteristics by combining a fuzzy adaptive rule and sliding-mode control, which showed online learning ability during the vibration control process. Kumar and Chhabra [10] developed an adaptive fuzzy sliding-mode multiple-input multiple-output controller to suppress the vibration of a rectangle plate using the MATLAB/Simulink platform. Shaharuddin and Darus [11] present the fuzzy PID control of flexibly mounted rigid pipe and demonstrated that the vibration of the system was attenuated by the developed controller effectively. Sheng et al. [12] studied the combined fuzzy PID and fuzzy adapt PID controllers based on flexible beam structure in order to suppress wings vibration. Shao et al. [13] designed a fuzzy PID controller a flexibly supported parallel manipulator considering the time-variability, nonlinearity, and dynamic coupling of the system. It is believed that fuzzy control could deal with the vibration suppression problem of spinning structures.

In this study, a fuzzy self-tuning PID controller aiming at vibration control of a high-speed spinning shaft in a cyclopter shown in Figure 1 is developed and analyzed. A fuzzy self-tuning PID controller can tune the parameters of PID controller online, realize optimum adjustment of governor parameters, and achieve the desired suppression effect of spinning structure. This control algorithm has merits of simple structure, easily adjusting parameters, and stronger ability of antijamming and stabilization. In Section 2, internal balance model reduction method and fuzzy self-tuning PID controller are briefly studied, followed by numerical simulation conducted in Section 3. Finally, a concluding remark is given in Section 4.

## 2. Methodology

**2.1. Finite Element Model.** As shown in Figure 2, finite element equations of a spinning flexible beam are derived using Hamilton principle, which can be expressed as

$$\begin{aligned} \mathbf{M}\dot{\boldsymbol{\eta}}(t) + \mathbf{D}\dot{\boldsymbol{\eta}}(t) + \mathbf{K}\boldsymbol{\eta}(t) &= \bar{\mathbf{B}}\mathbf{u}, \\ \mathbf{y} &= \bar{\mathbf{C}}_d\boldsymbol{\eta} + \bar{\mathbf{C}}_v\dot{\boldsymbol{\eta}}, \end{aligned} \quad (1)$$

where  $\boldsymbol{\eta} \in R^n$  is the vector of physical coordinates and  $\mathbf{M}, \mathbf{D}, \mathbf{K} \in R^{n \times n}$  are the mass matrix, damping matrix, and

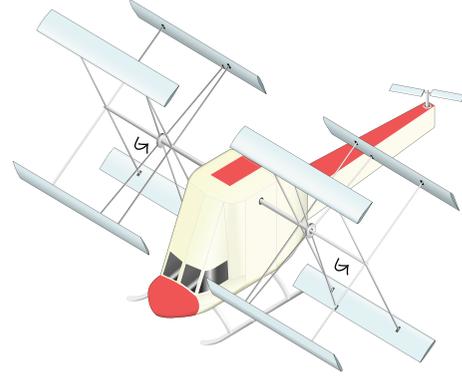


FIGURE 1: A spinning beam in a cyclopter [14].

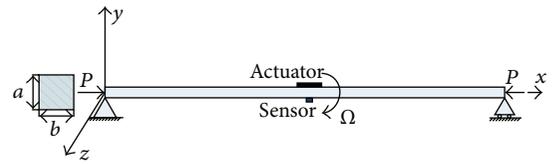


FIGURE 2: Spinning flexible beam model.

stiffness matrix, respectively.  $n$  is the degree of freedom of the system,  $\mathbf{u} \in R^{m_1}$  is control vector,  $\bar{\mathbf{B}} \in R^{n \times m_1}$  is a matrix that denotes the locations of the actuator, and  $m_1$  is the number of actuators.  $\mathbf{y} \in R^{m_2 \times 1}$  is an output vector,  $\bar{\mathbf{C}}_d, \bar{\mathbf{C}}_v \in R^{m_2 \times n}$  are observability matrices of displacement and velocity, respectively, and  $m_2$  is the number of sensors.

Translating (1) into state-space function

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \\ \mathbf{y} &= \mathbf{C}\mathbf{x}, \end{aligned} \quad (2)$$

where  $\mathbf{x} = [\dot{\boldsymbol{\eta}} \ \boldsymbol{\eta}]^T$ ,  $\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{D} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$ ,  $\mathbf{B} = \begin{bmatrix} -\mathbf{M}^{-1}\bar{\mathbf{B}} \\ \mathbf{0} \end{bmatrix}$ , and  $\mathbf{C} = \begin{bmatrix} \bar{\mathbf{C}}_v & \bar{\mathbf{C}}_d \end{bmatrix}$ .

**2.2. Internal Balance Model Reduction.** A majority of mathematical models in industrial applications are extremely high-order systems, which make implementation and computation of such systems complicated and time consuming. Thus, obtaining suitable low-order approximation of such systems is essential. Compared with original system structure, low-order models result in several advantages such as reduction in computational complexity and easy analysis [15]. Spinning flexible beam is one of the typical high-order systems. Given that the mass matrix, damping matrix, and stiffness matrix are not symmetrical and the system has low damping, the internal balance technique is introduced to reduce the original system.

Firstly proposed by Moore [16], the internal balance technique ensures that controllability and observability Gramian matrices of the system are equal diagonal matrices through internal balance transformation. Since the numerical size of the diagonal elements represents controllability and observability of the system, a reduced-order model for the original

system is composed by preserving the states corresponding to the larger diagonal elements.

The dynamic equations of the original system are represented in (2). Assuming that the system is asymptotically stable, the controllability and observability Gramian matrices can be written as

$$\begin{aligned} \mathbf{W}_c &= \int_0^{\infty} e^{\mathbf{A}t} \mathbf{B} \mathbf{B}^T e^{\mathbf{A}^T t} dt, \\ \mathbf{W}_o &= \int_0^{\infty} e^{\mathbf{A}^T t} \mathbf{C}^T \mathbf{C} e^{\mathbf{A}t} dt. \end{aligned} \quad (3)$$

$\mathbf{W}_c$  and  $\mathbf{W}_o$  satisfy the Lyapunov equation:

$$\begin{aligned} \mathbf{A} \mathbf{W}_c + \mathbf{W}_c \mathbf{A}^T &= -\mathbf{B} \mathbf{B}^T, \\ \mathbf{A}^T \mathbf{W}_o + \mathbf{W}_o \mathbf{A} &= -\mathbf{C}^T \mathbf{C}. \end{aligned} \quad (4)$$

Transforming (2) through a nonsingular linear transformation  $\mathbf{T}_b$

$$\mathbf{x} = \mathbf{T}_b \mathbf{x}_b \quad (5)$$

we have

$$\begin{aligned} \dot{\mathbf{x}}_b &= \mathbf{A}_b \mathbf{x}_b + \mathbf{B}_b \mathbf{u}, \\ \mathbf{y} &= \mathbf{C}_b \mathbf{x}_b, \end{aligned} \quad (6)$$

where  $\mathbf{A}_b = \mathbf{T}_b^{-1} \mathbf{A} \mathbf{T}_b$ ,  $\mathbf{B}_b = \mathbf{T}_b^{-1} \mathbf{B}$ ,  $\mathbf{W}_b = \mathbf{T}_b^{-1} \mathbf{W}$ , and  $\mathbf{C}_b = \mathbf{C} \mathbf{T}_b$ .  $\mathbf{T}_b$  is the balance transformation matrix from balanced coordinates to physical coordinates.

Laub A [17] and Bartels and Stewart [18] proposed an effective approach to solve  $\mathbf{T}_b$ :

$$\mathbf{T}_b = \mathbf{L} \mathbf{U} \mathbf{\Lambda} \quad (7)$$

in which  $\mathbf{L}$  is the Cholesky decomposition of  $\mathbf{W}_c$ , which means  $\mathbf{W}_c = \mathbf{L} \mathbf{L}^T$ .  $\mathbf{U}$  is an orthogonal matrix; that is,  $\mathbf{U}^{-1} = \mathbf{U}^T$ .  $\mathbf{\Lambda}$  is a diagonal matrix, satisfying

$$\mathbf{U}^T (\mathbf{L}^T \mathbf{W}_o \mathbf{L}) \mathbf{U} = \mathbf{\Lambda}^2, \quad \mathbf{\Lambda} = \text{diag}(\sigma_i), \quad (8)$$

where the diagonal element  $\sigma_i$  is the  $i$ th singular value.

The controllability and observability Gramian matrices of (6) satisfy that

$$\mathbf{W}_c^b = \mathbf{W}_o^b = \mathbf{\Sigma} = \text{diag}(\sigma_i), \quad (9)$$

where  $\mathbf{\Sigma}$  is a diagonal matrix. Equation (6) is the internal balance system, and the corresponding transform (5) is defined as internal balance transform. It is believed that all linear, time-invariant, asymptotically stable state-space systems can be transformed into an internal balance form through an internal balance transformation.

The controllability and observability Gramian matrices of (6) can be obtained from (4):

$$\begin{aligned} \mathbf{A}_b \mathbf{W}_c^b + \mathbf{W}_c^b \mathbf{A}_b^T &= -\mathbf{B}_b \mathbf{B}_b^T, \\ \mathbf{A}_b^T \mathbf{W}_o^b + \mathbf{W}_o^b \mathbf{A}_b &= -\mathbf{C}_b^T \mathbf{C}_b. \end{aligned} \quad (10)$$

Inserting  $\mathbf{A}_b = \mathbf{T}_b^{-1} \mathbf{A} \mathbf{T}_b$ ,  $\mathbf{B}_b = \mathbf{T}_b^{-1} \mathbf{B}$ , and  $\mathbf{C}_b = \mathbf{C} \mathbf{T}_b$  into (10) and using the matrix transform method

$$\begin{aligned} \mathbf{A} (\mathbf{T}_b \mathbf{W}_c^b \mathbf{T}_b^T) + (\mathbf{T}_b \mathbf{W}_c^b \mathbf{T}_b^T) \mathbf{A}^T &= -\mathbf{B} \mathbf{B}^T, \\ \mathbf{A}^T ((\mathbf{T}_b^{-1})^T \mathbf{W}_o^b \mathbf{T}_b^{-1}) + ((\mathbf{T}_b^{-1})^T \mathbf{W}_o^b \mathbf{T}_b^{-1}) \mathbf{A} &= -\mathbf{C}^T \mathbf{C}. \end{aligned} \quad (11)$$

Comparing (11) with (10), we have

$$\begin{aligned} \mathbf{W}_c &= \mathbf{T}_b \mathbf{W}_c^b \mathbf{T}_b^T, \quad \mathbf{W}_c^b = \mathbf{T}_b^{-1} \mathbf{W}_c (\mathbf{T}_b^T)^{-1}, \\ \mathbf{W}_o &= (\mathbf{T}_b^{-1})^T \mathbf{W}_o^b \mathbf{T}_b^{-1}, \quad \mathbf{W}_o^b = \mathbf{T}_b^T \mathbf{W}_o \mathbf{T}_b. \end{aligned} \quad (12)$$

From (9) and (12), we have

$$\mathbf{\Sigma}^2 = \mathbf{W}_c^b \mathbf{W}_o^b = \mathbf{T}_b^{-1} (\mathbf{W}_c \mathbf{W}_o) \mathbf{T}_b. \quad (13)$$

It indicates that every column of the internal balance transformation  $\mathbf{T}_b$  is the eigenvector of the matrix  $\mathbf{W}_c \mathbf{W}_o$ , and  $\sigma_i^2$  is the corresponding eigenvalue from (13). With respect to system (6), the controllability and observability of the system mode could be represented by a singular value. The larger the singular value is, the more controllable the corresponding mode would be. Thus, the system order could be truncated by reserving the most ‘‘controllable’’ states.

The coordinate  $\mathbf{x}$  in (2) is called standard modal coordinates. Correspondingly, the coordinate  $\mathbf{x}_b$  in (6) is called the internal balance modal coordinates. Assuming retaining  $m$  modes, the state-space function of reduced-order system can be defined as

$$\begin{aligned} \dot{\mathbf{x}}_b^m &= \mathbf{A}_b^m \mathbf{x}_b^m + \mathbf{B}_b^m \mathbf{u}, \\ \mathbf{y} &= \mathbf{C}_b^m \mathbf{x}_b^m. \end{aligned} \quad (14)$$

The internal balance technique could largely reduce the order of the original system and preserve relevant dynamics in the meantime. The intelligent controller designed for the reduced-order system would also be valid in suppressing the original system.

**2.3. Design of a Fuzzy Self-Tuning PID Controller.** A fuzzy self-tuning PID controller is composed of a PID controller and a fuzzy logic inference. It has the dual advantages of both PID controllers and fuzzy control. The main idea of parameter fuzzy automatic adjustable PID control is to find the fuzzy relationship between the three parameters  $k_p$ ,  $k_i$ , and  $k_d$  and the input deviation  $e$  and deviation rate of change  $ec$ . Three correction parameters  $\Delta k_p$ ,  $\Delta k_i$ , and  $\Delta k_d$  obtained by fuzzy logic inference rules are entered into the PID controller to correct the parameters  $k_p$ ,  $k_i$ , and  $k_d$  of the PID controller online, respectively, and in real time.

This research studied a spinning beam structure, and the difference between the given elastic deformation and the actual output deformation of the middle point of the beam in  $y$  direction is used as a feedback. The controller corrects the three parameters of the PID controller online based on the control error  $e = r - y$  ( $r$  is often defined as 0) and the deviation rate of change  $ec$ . Then the output of

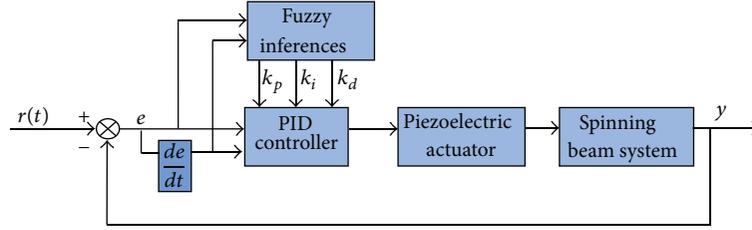


FIGURE 3: The fuzzy self-tuning PID controller for spinning beam structure.

TABLE 1: Fuzzy control rule table for  $\Delta k_p$ .

$e$	$ec$						
	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PM	PM	PS	ZO	ZO
NM	PB	PB	PM	PS	PS	ZO	NS
NS	PM	PM	PM	PS	ZO	NS	NS
ZO	PM	PM	PS	ZO	NS	NM	NM
PS	PS	PS	ZO	NS	NS	NM	NM
PM	PS	ZO	NS	NM	NM	NM	NB
PB	ZO	ZO	NM	NM	NM	NB	NB

TABLE 2: Fuzzy control rule table for  $\Delta k_i$ .

$e$	$ec$						
	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NB	NM	NM	NS	ZO	ZO
NM	NB	NB	NM	NS	NS	ZO	ZO
NS	NB	NM	NS	NS	ZO	PS	PS
ZO	NM	NM	NS	ZO	PS	PM	PM
PS	NM	NS	ZO	PS	PS	PM	PB
PM	ZO	ZO	PS	PS	PM	PB	PB
PB	ZO	ZO	PS	PM	PM	PB	PB

TABLE 3: Fuzzy control rule table for  $\Delta k_d$ .

$e$	$ec$						
	NB	NM	NS	ZO	PS	PM	PB
NB	PS	NS	NB	NB	NB	NM	PS
NM	PS	NS	NB	NM	NM	NS	ZO
NS	ZO	NS	NM	NM	NS	NS	ZO
ZO	ZO	NS	NS	NS	NS	NS	ZO
PS	ZO	ZO	ZO	ZO	ZO	ZO	ZO
PM	PB	NS	PS	PS	PS	PS	PB
PB	PB	PM	PM	PM	PS	PS	PB

the controller  $u$  is converted into the piezoelectric actuator. The piezoelectric actuator generates control force by the use of inverse piezoelectric effect in piezoceramics to suppress the vibration of the spinning flexible beam, as shown in Figure 3.

**2.3.1. Fuzzification of Input Signal.** The controller studied in this paper is a two-input ( $e$  and  $ec$ ) and three-output ( $\Delta k_p$ ,  $\Delta k_i$ , and  $\Delta k_d$ ) self-tuning PID controller, with input and output in exact values, while the fuzzy logic ruler is aimed at the fuzzy variable. Thus the exact input should be converted to the fuzzy variable, and this step is called fuzzification.

The domain of input and output is selected based on the dynamic characteristic of the spinning system. The input deviation  $e$  and deviation rate of change  $ec$  are converted to the input domain of the fuzzy controller by the corresponding quantification factor. The domain of the input variables of the fuzzy self-tuning PID controller is defined as  $[-0.3, 0.3]$ , and the corresponding domain of the output is  $[-0.3, 0.3]$ ,  $[-0.003, 0.003]$ , and  $[-1.5, 1.5]$ , respectively. The fuzzy sets of the variables are negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive middle (PM), and positive big (PB).

**2.3.2. Fuzzy Control Rule.** The fuzzy control rules of  $\Delta k_p$ ,  $\Delta k_i$ , and  $\Delta k_d$  based on engineering technology and experience are shown in Tables 1, 2, and 3, respectively. Gravity method is used for defuzzification of the fuzzy self-tuning PID controller with the Mamdani Fuzzy Inference Method. The fuzzy correction of the three parameters of the controller,  $\Delta k_p$ ,  $\Delta k_i$ , and  $\Delta k_d$ , is obtained through the fuzzy control

rules; then the exact output is obtained by gravity method. The three parameters of the controller can be written as

$$\begin{aligned} k_p &= k_{p0} + \Delta k_p, \\ k_i &= k_{i0} + \Delta k_i, \\ k_d &= k_{d0} + \Delta k_d \end{aligned} \quad (15)$$

in which  $k_{p0}$ ,  $k_{i0}$ , and  $k_{d0}$  are the initial value of the three parameters and  $\Delta k_p$ ,  $\Delta k_i$ , and  $\Delta k_d$  are the corresponding corrections.

### 3. Results and Discussion

To verify the validity of the model reduction method and the proposed controller, a high spinning flexible beam is analyzed. In this section, the data of the spinning beam is given as follows: the density is  $2600 \text{ Kg/m}^3$ , the length, width, and thickness of the beam are  $1200 \text{ mm} \times 8 \text{ mm} \times 8 \text{ mm}$ , and Young's modulus is  $70 \text{ GPa}$ . The moment of inertia is  $3.413 \times 10^{-10} \text{ m}^4$ . The beam is spinning about its own longitudinal

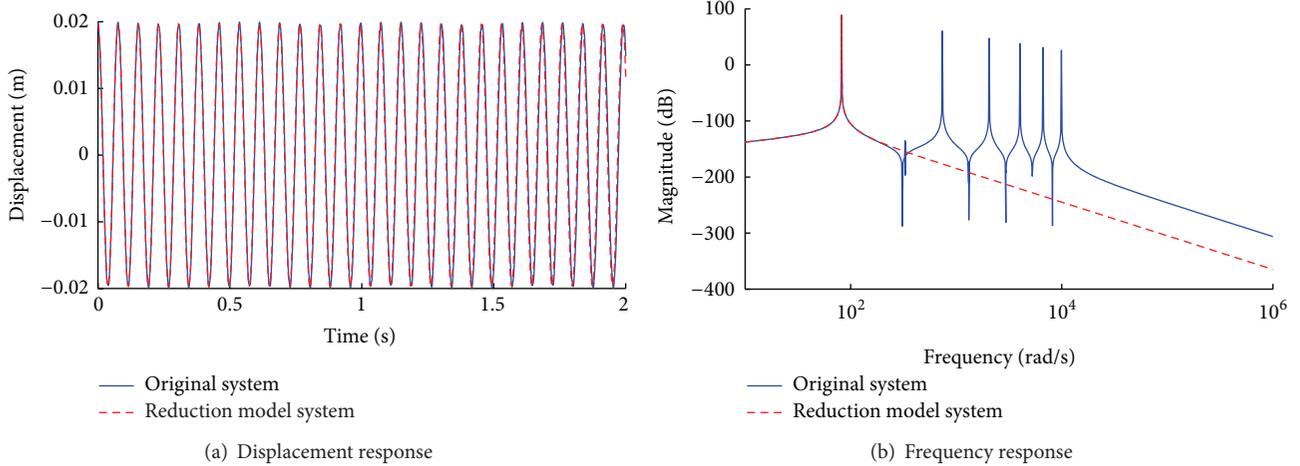


FIGURE 4: The midpoint responses of the beam using the original system model and reduced-order model.

axis at a speed of  $\Omega = 150$  rad/s. All simulations are implemented using MATLAB 2012a programming language.

**3.1. Simulation of Internal Balance Model Reduction.** This section concerns self-excited vibration without external excitation. It is assumed that the initial deformation field applied at the middle point of the beam is 20 mm. The internal balance model reduction is used for the order reduction of the spinning beam. Since the lower-order modal responses mainly dominate the response of the beam, the first six normal modes are chosen to represent the real response of the beam. Thus, the original system is denoted by the dynamic model consisting of the first six modes. The reduced-order system is the reduced-order model obtained using the internal balanced method. The modes corresponding to the largest singular values remained, while the modes corresponding to the smaller singular values are truncated based on internal balance model reduction method. Therefore, the first mode corresponding to the four biggest singular values remained and the dimension of the state vector  $x_b$  in (14) is 2. The state-space function of reduced-order system is also obtained.

Figure 4 illustrates (a) displacement responses and (b) frequency responses of the midpoint of the beam using the original system model and reduced-order model, where the solid line is the result obtained using the original system model and the dashed line is that obtained using the reduced-order model. It can be observed that the responses using two different models show a well agreement, which means that the reduced-order model can effectively predict the original system model.

### 3.2. Simulation of the Fuzzy Self-Tuning PID Controller

**3.2.1. Initial Displacement Field.** The displacement field is the same as that in Section 3.1. A piezoelectric actuator is deployed in the midpoint of the spinning beam, as well as a piezoelectric sensor. Then a fuzzy self-tuning PID controller based on the reduced-order model is proposed. The responses obtained by PID controller and fuzzy self-tuning PID controller are compared with the response of the beam without

TABLE 4: Comparison of control effect of traditional PID and fuzzy self-tuning PID controllers.

	Open loop	Traditional PID	Fuzzy self-tuning PID
Error norm	0.62	0.31	0.21
Control effort norm	0	5.20	7.60

control, respectively. The three parameters of PID controller are defined as  $k_p = 0.02$ ,  $k_i = 0$ , and  $k_d = 0.1$ . The three initial parameters of fuzzy self-tuning PID controller are set as  $k_{p0} = 0.02$ ,  $k_{i0} = 0$ , and  $k_{d0} = 0.1$ .

Figure 5 gives the responses obtained by PID controller and fuzzy self-tuning PID controller and the response of the beam without control, respectively. Figure 5 suggests that the vibration of the beam has been depressed remarkably, which demonstrates that PID controller and fuzzy self-tuning PID controller are effective for the vibration reduction of the spinning flexible beam. Particularly, the displacement using fuzzy self-tuning PID controller is reduced more obviously compared to the response corresponding to PID controller. The fuzzy self-tuning controller can deliver better performance than that with just a PID controller in vibration suppression, for further reducing the maximum amplitude in less time. Figure 6 shows the control forces applied to both PID controller and fuzzy self-tuning PID controller. The control force through the fuzzy self-tuning PID controller is bigger than that of the PID controller in the first 0.47s and gradually decreases to be smaller than that of the PID controller. Besides, Figure 7 illustrates the fuzzy self-tuning of  $k_p$  (a) and  $k_d$  (b).

The effort norm and control effort norm are shown in Table 4; error norm using traditional PID controller and fuzzy self-tuning PID controller is 0.31 and 0.21 while it is 0.61 in open loop. For control effort norm, fuzzy self-tuning PID controller is 7.60 compared with 5.20 of traditional PID controller.

From the results, the fuzzy self-tuning PID controller gets an about 22.3% improvement compared with traditional

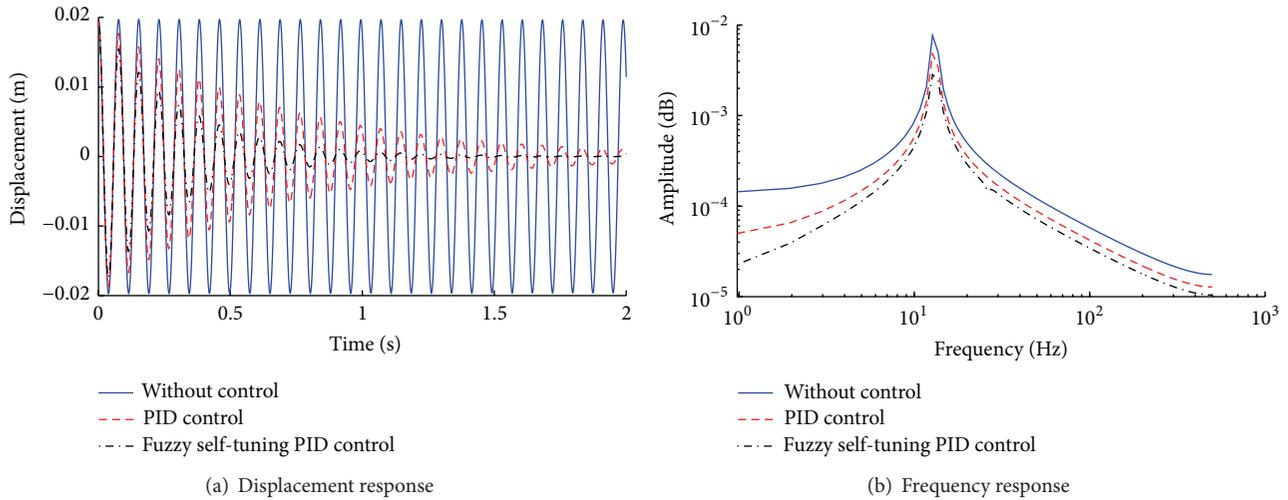


FIGURE 5: The midpoint response with and without control in initial displacement field.

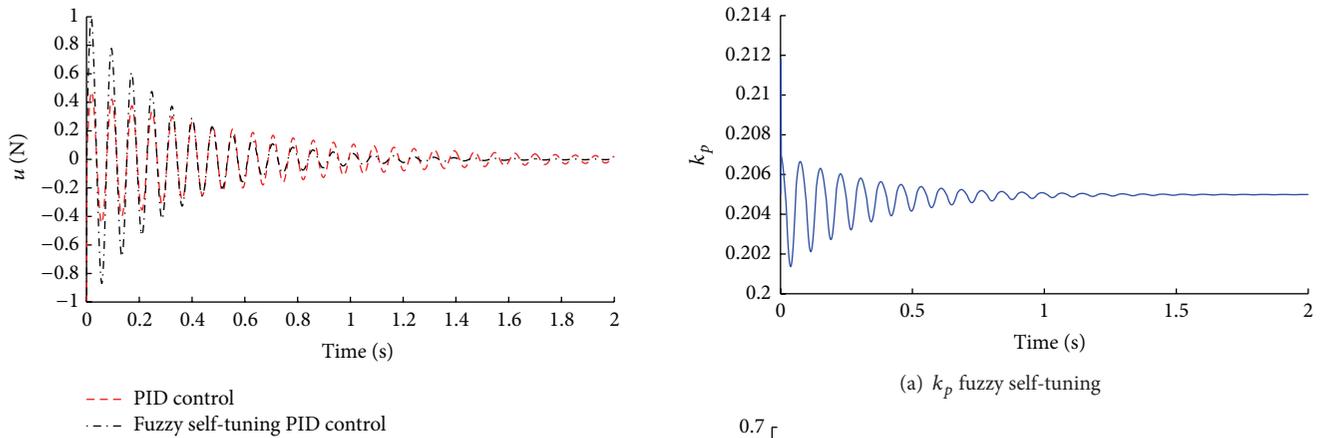


FIGURE 6: The control forces in initial displacement field.

PID controller. The error norm is reduced markedly than traditional PID controller in the same time. Besides, the fuzzy self-tuning PID controller also has more adaptivity to different models in complex environments.

**3.2.2. White Noise Disturbance.** In order to verify the robustness to noise of the proposed controllers, a white noise point force disturbance with a variance of  $2 \times 10^{-2} \text{ N}^2$  is applied in the middle of the beam in this section. The parameters of both PID controller and fuzzy self-tuning PID controller are the same as those in previous section.

The open- and closed-loop displacements of the middle point of the beam and control forces for the white noise force disturbance evaluated by PID controller and fuzzy self-tuning PID controller are presented in Figures 8 and 9, respectively. In Figure 8, the displacements are derived by two different controllers. Obviously, the proposed controllers manage to reduce the deformation significantly. The maximum displacements derived by PID controller and fuzzy self-tuning PID

controller are 0.7738 mm and 0.5741 mm, while the open-loop displacement is 1.507 mm. The fuzzy self-tuning PID controller could obtain a better performance for its capability of further suppressing the vibration of the spinning beam than conventional PID controller. However, if we focus on the utilized control forces, the relatively more inefficient

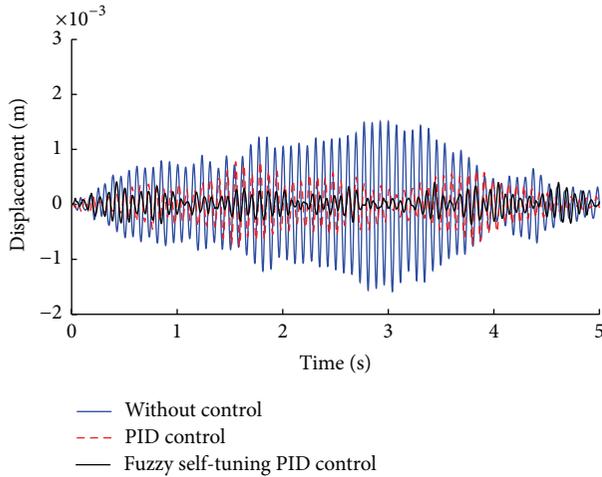


FIGURE 8: The midpoint response with and without control with white noise disturbance.

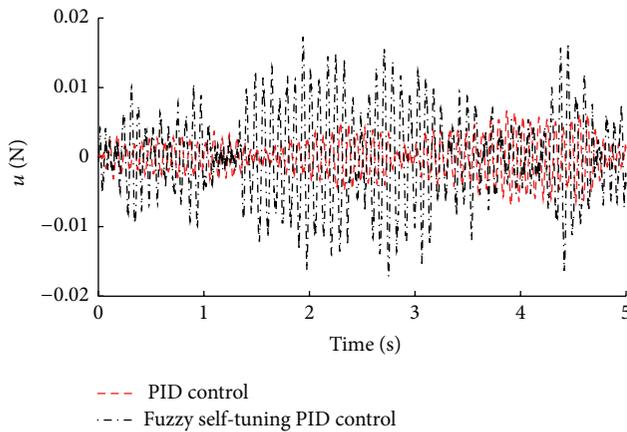
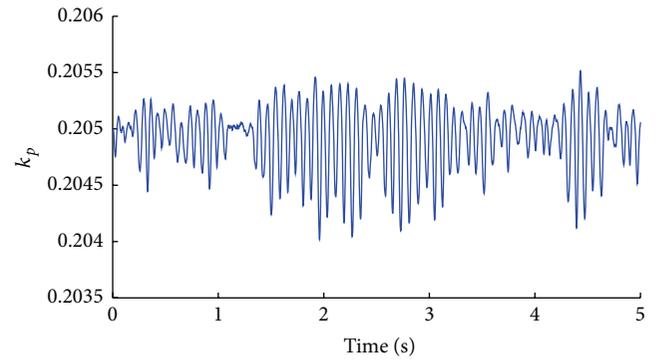


FIGURE 9: The control forces with white noise disturbance.

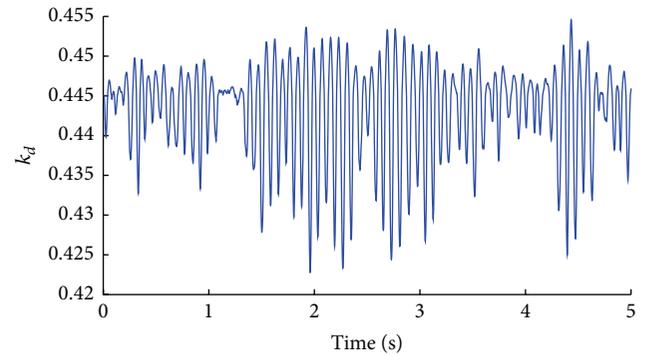
controller needs only about 30% of the energy used by the fuzzy self-tuning PID controller. Figure 10 shows the fuzzy self-tuning of  $k_p$  (a) and  $k_d$  (b) with white noise disturbance.

#### 4. Summary and Conclusions

The investigation of vibration control for a spinning flexible beam is carried out in this paper. Internal balanced model reduction is studied to realize model reduction considering of rigid-flexible coupling and nonlinear characteristic of spinning system firstly. Further, a fuzzy self-tuning PID controller is proposed to control the vibration based on the reduced-order model. The numerical results prove that the reduced-order model obtained by internal balanced model reduction method could well represent the original spinning system model. It is also demonstrated that the fuzzy self-tuning PID controller is effective in suppressing the vibration of the spinning beam and significantly outperforms the traditional PID controller.



(a)  $k_p$  fuzzy self-tuning



(b)  $k_d$  fuzzy self-tuning

FIGURE 10: Fuzzy self-tuning of controller parameters with white noise disturbance.

#### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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