Research Article

Numerical Investigation of Rockburst Effect of Shock Wave on Underground Roadway

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Using UDEC discrete element numerical simulation software and a cosine wave as vibration source, the whole process of rockburst failure and the propagation and attenuation characteristics of shock wave in coal-rock medium were investigated in detail based on the geological and mining conditions of 1111(1) working face at Zhuji coal mine. Simultaneously, by changing the thickness and strength of immediate roof overlying the mining coal seam, the whole process of rockburst failure of roadway and the attenuation properties of shock wave were understood clearly. The presented conclusions can provide some important references to prevent and control rockburst hazards triggered by shock wave interferences in deep coal mines.

1. Introduction

Coal mining is closely related to blasting, transportation, mechanical operation, and other activities; therefore, the whole process of mining is accompanied with the generation and interference of shock wave. Due to diversity of vibration source and uncertainty of position, these dynamic loads will inevitably exert mutual interference in the process of coal-rock deformation and failure, and the disturbance effects on surrounding rock are very significant, especially for the surrounding rock with a free surface. Once the dynamic load is exerted, the surrounding rock may not generate macrodamage, but under the condition of stress wave disturbance, the cumulative damage of surrounding rock will rapidly increase and the local stress environment will be worsened on microcosmic or mesoscopic scale. Eventually, the instantaneous dynamic extension of crack is caused, and the transient strain energy rapidly releases in the form of kinetic energy by coal-rock violent ejection.

Once the mechanical system composed of coal and rock mass reaches its ultimate strength limit, the rockburst will

inevitably occur, and the vast majority of strain energy accumulated in coal and rock mass will, suddenly and sharply, be released, which can cause the instantaneous destruction of coal and rock in roadway, equipment damage, and miner casualty. Part of the energy will be released in the form of shock waves, which can generate dynamic impact on the surrounding coal and rock medium. In particular, shock waves might cause the deformation and damage of underground structures in far field. Once the residual shock wave intensity gets higher after attenuation, the energy input of coal and rock in higher stress concentration state can induce rockburst. In addition, rockburst may directly or indirectly trigger serious accidents such as coal/gas outburst, gas explosion, and water inrush. Increasingly, for gas-contained coal seam, the harm of rockburst is more remarkable [1, 2].

Because the rockburst occurrence relates closely to the dynamic instability of surrounding rock in higher static stress concentration state, the static stress combined with shock wave interference is extensively used to reveal the mechanism and influencing factors of rockburst failure of coal-rock, and many fruitful results have been obtained. In particular, a large

TABLE 1: Mechanical parameters of rock strata.

Stratum	Thickness/m	Density/(Kg·m ⁻³)	Bulk modulus/GPa	Shear modulus/GPa	Cohesion/MPa	Friction angle/°	Tensile strength/MPa
Tinea mudstone	11.5	2640	8	4.8	2.3	35	5
Finestone/siltstone	3.2	2700	12	8.1	4.0	37	7
Mudstone	10.0	2640	8	4.8	2.3	35	5
11 ⁻² coal seam	1.2	1300	8.5	4	0.5	28	2
Mudstone	4.5	2640	8	4.8	2.3	35	5
11 ⁻¹ coal seam	0.8	1300	8.5	4	0.5	28	2
Mudstone	9.0	2640	8	4.8	2.3	35	5

number of researches have been conducted by the numerical simulation methods. For example, Qin and Mao [3] simulated rockburst induced by disturbing stress wave and analyzed the influence of depth and peak of stress wave on rockburst by using the software UDEC. Gao et al. [4] found that the energy attenuation index η was considerably small in the rock and soil media but apparently much larger in weak or soft media. According to the law of micromechanics of impact fracture on coal-rock, Grady and Kipp [5] studied the transmission characteristics of shock waves in sandstone and mudstone and so forth. As stated by Zhao et al. [6], the calibration work of UDEC modelling on P-wave propagation across single linearly and nonlinearly deformable fractures was conducted. Park and Jeon [7] proposed an air-deck method for attenuating blast-induced vibration waves in the direction of tunneling. Uysal et al. [8] aimed at investigating the effect of empty barrier holes alone on seismic vibration and detected a decrease in the peak particle velocity (PPV) of up to 18% just behind the barrier holes. Li et al. [9] proposed experimentally that the shock wave propagation will be reflected, refracted, and absorbed by the discontinuity in coal and rock mass. Brown et al. [10] presented methods for modeling rock fractures and their influence on rock masses has been a distinctive feature of rock mechanics and rock engineering. Schoenberg [11] and Pyrak-Nolte et al. [12] derived solutions of reflection and transmission coefficients for obliquely incident P-wave or S-wave on a dry or liquid-filled fracture between two dissimilar media. Nakagawa et al. [13] found that wave conversion occurs when P-wave or S-wave is normally incident on a fracture subjected to a shear movement. Meanwhile, Gu et al. [14] pointed out that an inhomogeneous P-interface wave appears when an SV-wave is incident upon a fracture at or beyond the critical angle, which is determined by Poisson's ratio of rock material. Pyrak-Nolte and Nolte [15] performed a wavelet analysis on experimental results of interface wave propagating along a single fracture. Cai and Zhao [16] and Zhao et al. [17] used the method of characteristics to study wave attenuation across the linearly deformable fractures, in which the multiple reflections have been considered. Later, the theoretical results obtained by Zhao et al. [18] were verified by a series of tests on ultrasonic wave attenuation across parallel artificial fractures. A detailed description of UDEC simulation can be found in the software manual [19]. Brady et al. [20] conducted a two-dimensional UDEC modelling on the slip at a single fracture under an explosive linear source. The numerical results were

accordant with theoretical solutions presented by Day [21]. Gao et al. [22] revealed that compressive shear failure, rather than tensile failure, is the dominant failure mechanism in the caved strata above the mined-out area by a UDEC model.

The purpose of our research is to provide the certain basis and reference for prevention of rockburst by using the UDEC numerical simulation modelling. Based on the geological and mining conditions of 1111(1) working face at Zhuji coal mine, the whole process of rockburst failure of surrounding rock in roadway subjected to the disturbance of shock wave was simulated and recurred, and the effect rules of the immediate roof thickness and strength overlying the mining coal seam and with or without bolt support on rockburst failure of roadway were analyzed in detail.

2. Numerical Modelling and Analysis

2.1. Modelling. The numerical model is established based on the actual geological conditions of 1111(1) working face of Zhuji coal mine. The size (length $\times$ height) of model is 60 m $\times$ 40.2 m, and the shape of inside roadway is the semicircular arch with arch radius of 2 m and the size (width $\times$ height) of roadway is 4 m $\times$ 4 m. Mechanical parameters of the rock strata are shown in Table 1, and the models with or without bolt reinforcement are shown in Figure 1. Total 11 anchor rods were added in roadway, 5 of which were installed in semicircular arch area, and the rest were installed at left and right sides of roadway, respectively. The length of bolt is 2.4 m, the sectional area is $314e^{-6}$ m², and its pretightening force is set to be 40 KN.

The boundary conditions of model are as follows: the horizontal boundaries at left and right sides are constrained, the bottom boundary is fixed, and the upper boundary is free with the uniform load ($q = r \times H$), in which the average density r is 25 KN/m³ and the buried depth H of the upper boundary is 900 m. For the model of shock wave, in order to reduce the reflection effects of boundaries under the condition of dynamic loading, the boundaries were set up to be viscous to simulate infinity. The exerted shock was a cosine wave ($y = 20e^6 \times \cos(2\pi \times 10)$) with the peak stress of 20 MPa and the frequency of 10 Hz, and the source was located in 20–40 m scope of the upper boundary, respectively. Ten symmetrical points for monitoring the deformation and displacement in the vertical and horizontal directions were located at both sides of roadway (Figure 1(c)) as follows: (1) 5 points labeled A, B, C, D, and E were arranged at left side, and the

FIGURE 1: The mechanical models of surrounding rock with or without bolts.

two-dimensional coordinates were (27.5, 14.7), (27.5, 15.1), (27.5, 15.5), (27.5, 15.9), and (27.5, 16.3), respectively. (2) The rest labeled 1, 2, 3, 4, and 5 were arranged at right side, and the two-dimensional coordinates were (32.5, 14.7), (32.5, 15.1), (32.5, 15.5), (32.5, 15.9), and (32.5, 16.3), respectively. The critical damping ratio ε of the artificial shock as Rayleigh wave was set to be 0.1.

2.2. Simulation Scheme. According to the geological and mining conditions of 1111(1) working face, the original stress of surrounding rock of model was simulated and calculated firstly, and then the roadway was excavated; after that, the artificial shock wave was exerted. Eventually, by collecting the displacement and stress of monitoring points, the effect rules of dynamic stress disturbance on the stability of surrounding rock were analyzed in detail. The simulation scheme was as follows.

- (1) Under the conditions of the artificial shock wave, the effect rules of surrounding rock stability of roadway with and without bolt reinforcement were simulated and analyzed.
- (2) The effect rules of immediate roof thickness on the stability of roadway were simulated and analyzed.

- (3) The effect rules of immediate roof strength on the stability of roadway were simulated and analyzed.

3. Simulation Results and Analysis

3.1. Process Simulation of Rockburst Failure of Roadway Surrounding Rock. In the primitive condition of surrounding rock without bolt, the buried depth of roadway is about 900 m, and the peak stress of the exerted shock wave is 20 MPa. The simulation results are shown in Figure 2.

From Figure 2, the whole process of rockburst failure of surrounding rock in roadway induced by shock wave disturbance clearly recurred, and the impact time is 1.5 s. It is obviously observed that cracks firstly began nearby both sides of roadway as the high stress concentration areas, and the failure points located at two junctions between the arch and both sides. With further disturbance of shock wave, cracks began to expand rapidly from the starting positions to deep coal and rock mass of two sides and formed two macrocracks symmetrically distributed at both sides, respectively. Meanwhile, due to the effects of shock wave, coal and rock were firstly ejected outward from the top of roadway and gradually evolved to two sides and thrown into roadway at a certain velocity when the rockburst is triggered.

FIGURE 2: Process simulation of rockburst in the primitive condition.

Because the symmetrical dynamic stress is correspondingly generated, the deformation and failure distribution of roadway are also basically symmetrical. To reveal the positions of coal and rock failure once rockburst occurs, the displacements of 5 monitoring points at the right side were obtained and analyzed, as shown in Figure 3.

From Figure 3, the x - and y -directional displacements of monitoring points 1 and 5 are approximately the same, and their curves almost overlap. Comparatively, the displacement of point 2 is relatively bigger, and the displacements of points 3 and 4 are the biggest with the peak value of 0.8 m. Therefore, it is concluded that the coal and rock mass eject outward mainly from the interface between coal seam and its overlying stratum once rockburst occurs for the thin coal seam. Because the mechanical parameters of interface are smaller, the rockburst failure is easily induced by shock wave.

3.2. Simulation of Rockburst Failure of Roadway with Bolt Supporting. Bolts at the top and two sides of roadway were installed, and the pretightening force was exerted. Under the same conditions of the buried depth of 900 m, peak stress of 20 MPa, and the impact time of 1.5 s, the simulation results are shown in Figure 4.

At the different acting time of shock wave, the shear force and y -directional displacement of each bolt are shown in Figures 5 and 6, respectively.

In Figures 5 and 6, the numbers from 1# to 11# represent the serial numbers of bolts. For the roadway with bolt supporting, when it is disturbed by shock wave, the following conclusions can be drawn: (1) for the geological and mining conditions of 1111(1) working face, bolt supporting can reinforce the stability of surrounding rock mass in roadway and improve its ability to undergo shock wave, (2) for the semicircular arch roadway in shape, the shear stress of bolts in arch is smaller, and its distribution is basically symmetrical, and (3) y -directional displacements of each bolt along with shock time ($t = 0.5$ s and 1.0 s) are approximately symmetrical.

In summary, for roadway without bolt supporting, the resonant effect commonly triggering rockburst failure of surrounding rock will form when the dominant frequency of residual shock wave reaches or approaches the natural vibration frequency of surrounding rock of roadway. After installing bolts as deformation constraint and antishear structure, the integrity and strength of surrounding rock can be significantly enhanced. Simultaneously, the natural vibration frequency of surrounding rock obviously reduces,

FIGURE 3: The displacement curves of 5 monitoring points at the right side.

FIGURE 4: The disappearance of rockburst failure of roadway with bolt supporting.

FIGURE 5: Shear stress curves with shock time of each bolt.

FIGURE 6: y-directional displacement curves with shock time of each bolt.

and thus the dominant frequency of the residual shock wave is higher than the natural vibration frequency. Therefore, the resonant effect does not occur, and rockburst failure of roadway is also not induced. In addition, if the resonant frequency of the surrounding rock increases by relief-shots, the dominant frequency of residual shock wave will be smaller than resonant frequency of roadway, and thus both resonant effect and rockburst do not occur. In conclusion, without bolt supporting, the resonant effect of roadway will induce rockburst failure of surrounding rock. When roadway is reinforced by bolt supporting, its integrity and strength are enhanced, and the natural vibration frequency obviously reduces to avoid resonant effect. Therefore, under the certain

conditions, bolt supporting can improve the stability of surrounding rock and prevent the occurrence of rockburst.

3.3. Effect of Immediate Roof Thickness on Rockburst Failure of Roadway. To discover the influences of immediate roof thickness on propagation and attenuation characteristics of shock wave, we keep the mechanical parameters of rock strata unchanged, only adjust the thickness of immediate roof to be 2 m, 6 m, and 10 m, respectively, and analyze the deformation and failure characteristics of surrounding rock under the conditions of different immediate roof thickness.

According to the simulation results, under the conditions of the buried depth of 900 m and the cosine wave with peak stress of 20 MPa, when the immediate roof thickness is set to be 2 m, 6 m, and 10 m, respectively, the rockburst failure form of roadway is approximately the same and is similar with the original condition. Figure 7 shows the rockburst failure of roadway at $t = 0.3$ s and 0.5 s of shock wave acting time, respectively.

From Figure 7, it is well known that the smaller the immediate roof thickness is, the stronger rockburst failure intensity of roadway is. Based on the histogram of 1111(1) working face, the overlying two strata from downward to upward of 11⁻² coal seam are mudstone with thickness of 10 m and finestone with thickness of 3.2 m, respectively. Based on the simulated fact that the total thickness of two strata is fixed, when the thickness of immediate roof mudstone decreases, the thickness of finestone will accordingly increase. Because the mechanical parameters of finestone are commonly larger than those of mudstone, the attenuation index of shock wave propagated in finestone is smaller compared with mudstone. So, under the condition of the same shock wave, the smaller the thickness of immediate roof mudstone is, the higher the residual shock wave intensity is and, accordingly, the stronger rockburst failure intensity of roadway is.

In summary, for the certain geological and mining conditions of 1111(1) working face, the smaller the immediate roof thickness is, the weaker the attenuation level of shock wave is, and the stronger rockburst failure of roadway is.

Figure 8 shows the changing curves of y-dimensional displacements of monitoring points 3 and 4.

From Figure 8, for the monitoring points 3 and 4 located at the right side of roadway, under the condition of the same calculation step, the displacement of immediate roof with thickness of 2 m is the largest, followed by immediate roof with thickness of 6 m, and the immediate roof with thickness of 10 m is the smallest. Based on the above-mentioned fact that the thickness of immediate roof mudstone is smaller and the corresponding thickness of overlying finestone is larger, due to the bigger mechanical parameters of finestone, the attenuation coefficient of shock wave propagated in overlying strata increases along with the increase of immediate roof thickness, and the intensity of residual shock wave decreases. Therefore, the displacement of surrounding rock of roadway will obviously reduce along with the increase of immediate roof thickness from 2 m to 10 m.

3.4. Effect of Immediate Roof Strength on Rockburst Failure of Roadway. To reveal the effect of immediate roof strength on

(a) The acting time of shock wave is $t = 0.3$ s (b) The acting time of shock wave is $t = 0.5$ s

FIGURE 7: Rockburst failure of roadway at $t = 0.3$ s and 0.5 s of shock wave acting time. Note: 2 m, 6 m, and 10 m represent the immediate roof thickness, respectively.

(a) The y -dimensional displacement curves recorded by point 3 (b) The y -dimensional displacement curves recorded by point 4

FIGURE 8: The changing curves of y -dimensional displacements of monitoring points 3 and 4. 2 m, 6 m, and 10 m in legend represent the immediate roof thickness, respectively.

FIGURE 9: Rockburst failure of roadway at $t = 0.3$ s and 0.5 s of shock wave acting time. Note: the numbers of 1.0, 5.0, and 10.0 m represent the multiple of the original uniaxial tensile strength of immediate roof, respectively.

attenuation characteristics of shock wave, we maintain the thickness of overlying roof strata unchanged, only adjust the uniaxial tensile strength of immediate roof mudstone to be 5 times and 10 times of the original value, respectively, and analyze the deformation and failure of roadway under the conditions of different immediate roof strength.

According to the simulation results, under the conditions of the buried depth of 900 m and the cosine wave with peak stress of 20 MPa, when the uniaxial tensile strength of immediate roof is set to be the original value, 5 times and 10 times of the original value, respectively, the rockburst failure form of roadway is basically the same and is similar to the original condition. Figure 9 shows the rockburst failure of roadway at $t = 0.3$ s and 0.5 s of shock wave acting time, respectively.

From Figure 9, it is well known that the smaller the uniaxial tensile strength of immediate roof is, the weaker the rockburst failure intensity of roadway is. The evident reason is that the attenuation index of shock wave gradually reduces

with the increase of immediate roof strength. Therefore, the dynamic stress of surrounding rock of roadway will rise along with the increase of immediate roof strength, and thus rockburst is easily triggered. Meanwhile, when the strength of immediate roof increases, it will be easier to accumulate a large amount of elastic energy in surrounding rock. Once the stored energy reaches or exceeds its limit, rockburst failure of roadway will inevitably occur.

In summary, for the certain geological and mining conditions of 1111(1) working face, the smaller the immediate roof strength is, the larger the attenuation index of shock wave is and thus the weaker rockburst failure intensity of roadway is.

Figure 10 shows the changing curves of y -dimensional displacements of monitoring points 3 and 4.

From Figure 10, it is obviously shown that the displacement of surrounding rock significantly rises along with the increase of uniaxial tensile strength of immediate roof. Due to the decreasing attenuation index of shock wave with the increase of immediate roof strength, the residual

(a) The y -dimensional displacement curves recorded by point 3 (b) The y -dimensional displacement curves recorded by point 4FIGURE 10: The changing curves of y -dimensional displacements of monitoring points 3 and 4.

shock wave intensity will significantly improve, and thus the displacement of surrounding rock rapidly rises. Ultimately, the rockburst failure is easily induced.

4. Conclusions

- (1) For roadway without bolt support, due to the small mechanical parameters of interface between coal seam and its overlying stratum, the coal and rock mass will eject outward mainly from the interface once rockburst induced by shock wave occurs.
- (2) For roadway with bolt support, its integrity and strength are obviously enhanced, and the natural vibration frequency of surrounding rock significantly reduces to avoid the resonant effect triggering rockburst.
- (3) The smaller the immediate roof thickness is, the higher the intensity of the residual shock wave is and, accordingly, the stronger rockburst failure intensity of roadway is. Moreover, the displacement of roadway obviously reduces with the increase of immediate roof thickness.
- (4) The attenuation index of shock wave gradually reduces with the increase of immediate roof strength, and the dynamic stress accretion of surrounding rock correspondingly rises, and thus rockburst failure is easily triggered. In other words, the higher the immediate roof strength is, the stronger rockburst failure of roadway is.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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